Young Researcher's Workshop
in XXI LNF SPRING SCHOOL
"BRUNO TOUSCHEK"

Reissner Nordström Blackhole in Lyra's Geometry

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Overview

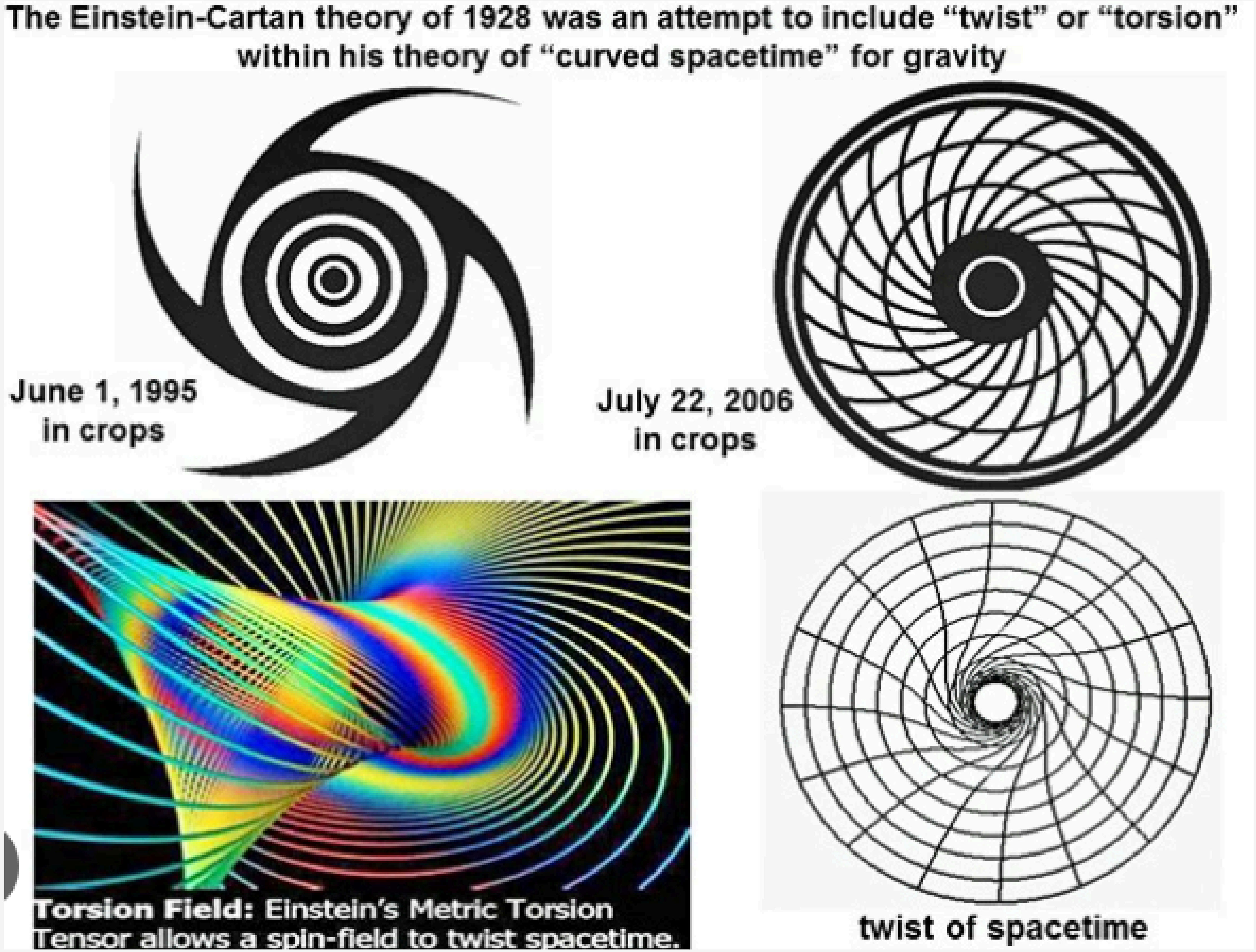
Why Lyra's Geometry?

- Lyra's geometry refers to a modification of standard general relativity proposed by the Norwegian physicist Gunnar Nordström in 1914.
- This modification introduces an additional tensor field, called the "Lyra vector," which interacts with the gravitational field.
- Tensor field is a more generalised form of both vector and scalar. Tensor calculus create a multilinear mappings between vector fields.
- Unlike in general relativity, where gravity is described solely by the curvature of spacetime, in Lyra's geometry, both curvature and torsion (associated with the Lyra vector) contribute to the gravitational field equations.
- In Lyra's geometry, the metric tensor $g\_\mu v$ is modified to include an extra term involving the Lyra vector field B_μ with a scalar field λ:

$$g_{\mu\nu} = e^{\lambda} \tilde{g}_{\mu\nu}$$

What is Torsion Tensor?

- Measures the asymmetry of the connection. It tells us if the connection has any "twisting" unlike “curvature” in the spacetime by Ricci/curvature tensor.
- Torsion is odd under parity, it must be represented by an odd-rank tensor
- F(t) gravity and F(t,B) gravity is a connected theory including both curvature and torsion



OUTLINE OF RN BLACKHOLES WITH LYRA’S GEOMETRY

- The Reissner-Nordström black hole is a solution to the equations of Einstein's general theory of relativity with an electromagnetic field.
- Mathematically, it is described by the Reissner-Nordström metric, which characterizes the spacetime geometry around a charged, spherically symmetric black hole.
- In this modified theory, the gravitational field equations are altered by introducing an extra scalar field ϕ into the theory involving lyra’ geometry.
- **The motivation behind Lyra's idea was to unify gravity with electromagnetism within a geometric framework.**

Sen and Dunn and W.D.Halford proposed a modification to the Einstein field equations by incorporating a scalar tensor theory of gravity which was named as Lyra’s scalar field,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \frac{3}{2} \phi_\mu \phi_\nu - \frac{3}{4} g_{\mu\nu} \phi_k \phi^k = -x T_{\mu\nu}$$

Introduction

The generalised Reissner metric can be written in the format of lyra's metric for a spherically symmetric system,

$$ds^2 = B(r)c^2dt^2 - A(r)dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2$$

Comparing both the equations, we will replace B(r) as e^λ and A(r) as e^ν later in our calculations. By considering the Lyra's field equations and assuming exponential terms for the **electromagnetic and gravitational potentials (e^λ and e^ν, respectively)**, expressions for these terms are derived.

$$ds^2 = e^{\lambda}dt^2 - e^{\nu}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2$$

This leads to the derivation of the modified metric for the Reissner-Nordström solution.

Mathematical aspects of RN blackhole

For Christoffel's symbols we will use,

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}\left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right)$$

After calculating the values of christoffel's symbols, we place them in Ricci tensor formula to get the ricci matrix which will be later used in modifying the einstein field equations ,

$$R_{ij} = \partial_k \Gamma^k_{ij} - \partial_j \Gamma^k_{ik} + \Gamma^k_{ij}\Gamma^m_{km} - \Gamma^k_{im}\Gamma^m_{jk}$$

Putting B as e^λ and A as e^ν to get the following derivatives, A'=$\nu' e$^ν, B'=$\lambda' e$^λ , B''=(λ'^2)e^λ+(e^λ)λ'' to use in the Ricci matrix as shown in the next slide.

We will consider the conditions for the Reissner metric:

$$\Lambda = 0$$

$$g_{\mu\nu} = \begin{bmatrix} B(r)c^2 & 0 & 0 & 0 \\ 0 & -A(r) & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin 2\theta \end{bmatrix}$$

$$T_{\mu\nu} = \frac{1}{\mu_0}\left[F_{\alpha\mu}F^{\alpha\beta}g_{\nu\beta} - \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right]$$

$$A_0 = \frac{Q}{4\pi\varepsilon_0 r}, A_i = 0$$

And, $$g^{\mu\nu} = \begin{bmatrix} \frac{1}{B(r)c^2} & 0 & 0 & 0 \\ 0 & \frac{-1}{A(r)c^2} & 0 & 0 \\ 0 & 0 & \frac{-1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{-1}{r^2 \sin 2\theta} \end{bmatrix}$$

$$
\begin{bmatrix}
\frac{-\left({\lambda'}^2 e^{\lambda}+e^{\lambda}\lambda''\right)}{2e^{\nu}}+\frac{\nu'\lambda' e^{\lambda}}{4e^{\nu}}+\frac{{\lambda'}^2 e^{\lambda}}{4e^{\nu}}-\frac{\lambda' e^{\lambda}}{re^{\nu}} & 0 & 0 & 0 \\
0 & \frac{{\lambda'}^2+\lambda''}{2}-\frac{\lambda'}{4}(\lambda'+\nu')-\frac{\nu'}{r} & 0 & 0 \\
0 & 0 & -1-\frac{r}{2e^{\nu}}(\nu'-\lambda')+\frac{1}{e^{\nu}} & 0 \\
0 & 0 & 0 & \sin^2\theta\left(-1-\frac{r}{2e^{\nu}}(\nu'-\lambda')+\frac{1}{e^{\nu}}\right)
\end{bmatrix}
$$

The **Ricci tensor** *R_μν* in terms of λ and ν.

SIMPLIFYING LYRA'S FIELD FROM LEAST ACTION

Considering the least action for lyra's field as given in D.K.Sen's paper,

$$s = s_m + s_\phi$$

$$s = \int \left[\sqrt{-g}(R - 2\wedge + \mathcal{L}_m) + \lambda(\phi_i\phi_i - \alpha^2)\right] d^4x,$$

where α is the constant parameter and λ is the lagrange multiplier, $\phi i \phi i$ is the sum of squares of components of scalar field ϕ.

By considering $\partial s/\partial gij$=0, we get the **modified Einstein tensor**,

$$G_{ij} = -8\pi G T_{ij} + \lambda T_{ij} - 2\lambda\phi_i\phi_j + 2\lambda\delta_{ij}\phi_i\phi_j$$

Now by putting Gij as (R−2∧+$\mathscr{L}m$) and Tij as λ($\phi i\phi i$−α^2) and a bit more simplification,

$$R_{\mu v} - \frac{1}{2}Rg_{\mu v} + \frac{3}{2}\phi_\mu\phi_v - \frac{3}{4}\phi_k\phi^k g_{\mu v} = -kT_{\mu v}$$

On the initial scalar conditions given R=0 when Λ=0,,

$$R_{\mu v} = kT_{\mu v} + \frac{3}{2}\phi_\mu\phi_v - \frac{3}{4}\phi_k\phi_k$$

The Faraday's tensor using $F\mu\nu = \partial\mu A\nu - \partial\nu A\mu$ and initial conditions $A0 = Q/4\pi\varepsilon\_0 r$, $Ai = 0$,

$$F^{\mu\nu} = \begin{bmatrix} 0 & \frac{-Q}{4\pi\varepsilon_0 r^2 AB} & 0 & 0 \\ \frac{Q}{4\pi\varepsilon_0 r^2 AB} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Using in $T\mu\nu = (1/\mu 0)[F\alpha\mu F\alpha\beta g\nu\beta - 1/4(g\mu\nu F\alpha\beta F\alpha\beta)]$ we get,

$$T^{\mu\nu} = \begin{bmatrix} \frac{-Q^2}{32\mu_0\pi^2\varepsilon_0^2 r^4 A} & 0 & 0 & 0 \\ 0 & \frac{Q^2}{32\mu_0\pi^2\varepsilon_0^2 r^4 B} & 0 & 0 \\ 0 & 0 & \frac{-Q^2}{32\mu_0\pi^2\varepsilon_0^2 r^2 AB} & 0 \\ 0 & 0 & 0 & \frac{-\sin^2\theta Q^2}{32\mu_0\pi^2\varepsilon_0^2 r^2 AB} \end{bmatrix}$$

Modified Field Equations

Let, $\phi = \sum_{n=0}^{\infty} a_n r^{-n}$,

where r is the radius of metric and a,n depends on coupling between gravity and electromagnetism.

As per our assumptions, **i) No vacuum energy ii) Charged point singularity iii) Static, spherically symmetric, the components of each of the scalar potentials are;**

$$\phi_0(n=0) = a_0, \qquad \phi_1(n=1) = \frac{a_1}{r},$$

$$\phi_2(n=2) = 0, \ [a_2 \ll r^2] \qquad \phi_3(n=3) = 0, \ [a_3 \ll r^3]$$

Therefore the modified field equations using the above conditions are,

$$\frac{-\left({\lambda'}^2 e^{\lambda} + e^{\lambda}\lambda''\right)}{2e^{\nu}} + \frac{\nu'\lambda' e^{\lambda}}{4e^{\nu}} + \frac{{\lambda'}^2 e^{\lambda}}{4e^{\nu}} - \frac{\lambda' e^{\lambda}}{re^{\nu}} = k\left(\frac{-Q^2}{32\mu_0\pi^2\varepsilon_0^2 r^4 e^{\nu}}\right) + \frac{3}{4}a_0^2 - \frac{3}{2}\frac{a_0 a_1}{r}$$

$$\frac{{\lambda'}^2 + \lambda''}{2} - \frac{\lambda'}{4}(\lambda' + \nu') - \frac{\nu'}{r} = k\left(\frac{-Q^2}{32\mu_0\pi^2\varepsilon_0^2 r^4 e^{\lambda}}\right) - \frac{3}{4}a_0^2 - \frac{3}{2}\frac{a_0 a_1}{r}$$

$$-1 - \frac{r}{2e^{\nu}}(\nu' - \lambda') + \frac{1}{e^{\nu}} = k\left(\frac{-Q^2}{32\mu_0\pi^2\varepsilon_0^2 r^2 e^{\nu} e^{\lambda}}\right) - \frac{3}{4}a_0^2 - \frac{3}{2}\frac{a_0 a_1}{r}$$

$$\sin^2\theta\left(-1 - \frac{r}{2e^{\nu}}(\nu' - \lambda') + \frac{1}{e^{\nu}}\right) = k\left(\frac{-\sin^2\theta\, Q^2}{32\mu_0\pi^2\varepsilon_0^2 r^2 e^{\nu} e^{\lambda}}\right) - \frac{3}{4}a_0^2 - \frac{3}{2}\frac{a_0 a_1}{r}$$

Computing the Modified Reissner Nordström metric

Dividing the first two equations of the modified field by e^λ and e^ν and doing some simple calculations we can get,

$$e^{\lambda} = \frac{kQ^2}{32\mu_0\pi^2\varepsilon_0^2 r^4\left(\frac{\lambda'^2+\lambda''}{2} - \frac{(\lambda'^2+\lambda'\nu')}{4} - \frac{\nu'}{r} + \frac{3}{4}a_0^2 + \frac{3}{2}\frac{a_0 a_1}{r}\right)}$$

$$e^{\nu} = \frac{\left(-\frac{\lambda\prime+\nu\prime}{r} + \frac{3}{4}a_0^2 + \frac{3}{2}\frac{a_0 a_1}{r}\right)e^{\lambda}}{\left(\frac{3}{4}a_0^2 - \frac{3}{2}\frac{a_0 a_1}{r}\right)}$$

Let, the constant a0 and a1 related to the gravitational potential and electromagnetic force respectively are considered as;

a0=k0M, where k0 represents a dimensionless constant and this expression contributes to the mass term.

And, **a1=k1(Q/R)^2**, where k1 is a dimensionless constant relating to the electromagnetic term.

Let, $\lambda(r)=A\mu= A0=Q/4\pi\varepsilon 0r$, $\partial\lambda/\partial t=0, \partial\lambda/\partial\phi=0, \partial\lambda/\partial\theta=0$, $\partial\lambda/\partial$r=0

and, $v(r)=V\mu=\alpha\phi$, where, α is the coupling constant, $\partial v/\partial t=\alpha$, $\partial v/\partial\theta=0, \partial v/\partial\phi=0$, $\partial v/\partial$r=0

By replacing the values of e^λ and e^v the modified Reissner Nordström metric in lyra's geometry is as follows;

$$ds^2 = \frac{kQ^2}{32\mu_0\pi^2\varepsilon_0^2 r^4\left(\frac{3}{4}a_0^2 + \frac{3}{2}\frac{a_0 a_1}{r}\right)}dt^2 - \frac{kQ^2}{\left(\frac{3}{4}a_0^2 - \frac{3}{2}\frac{a_0 a_1}{r}\right)(32\mu_0\pi^2\varepsilon_0^2 r^4)}dr^2 - r^2\, d\theta^2 - r\sin^2\theta\, d\phi^2$$

Comparing the Modified RN metric to the original RN metric

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{GQ^2}{4\pi\epsilon_0 c^4 r^2}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{r} + \frac{GQ^2}{4\pi\epsilon_0 c^4 r^2}\right)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

We will take e^ν as 1/e^ν' to compare it with the original format of the RN blackhole.

$$ds^2 = k\frac{Q^2}{\frac{3}{2}\mu_0\pi^2\epsilon_0^2 r^4\left(\frac{3}{4}a_0^2 + \frac{3}{2}a_0 a_1 r\right)}dt^2 - \frac{1}{k\frac{Q^2}{\left(\frac{3}{4}a_0^2 - \frac{3}{2}a_0 a_1 r\right)\frac{3}{2}\mu_0\pi^2\epsilon_0^2 r^4}}dr^2 - r^2 d\theta^2 - r^2\sin^2\theta d\phi^2$$

- **Both metrics contain terms related to the charge Q, radial coordinate r, and angular coordinates θ and ϕ but the gravitational terms get hidden.**

- **The modification in the provided metric includes terms of scalar fields a0 and a1, which do not appear in the standard Reissner-Nordström metric. But on reducing the metric on particular conditions we can get back the outline of the original metric .**

- **The coefficients multiplying the dt^2 and dr^2 terms in the provided metric are inversely proportional to the terms in the standard Reissner-Nordström metric. This suggests that there are differences in how time and radial distance are scaled in the modified metric compared to the standard metric.**

Geodesic Equations for modified RN metric

Using the general geodesic equation,

$$\ddot{x}^i = \sum_{j=0}^{3} \sum_{k=0}^{3} \Gamma^i_{jk} \dot{x}^j \dot{x}^k + qF^{ik} \dot{x}^k$$

therefore the geodesics of the modified RN are,

$$\dot{x} = \frac{Q}{8\pi\varepsilon_0 r^2} yx - \frac{qQ\left(32\mu_0\pi^2\varepsilon_0^2 r^4\right)^2}{4\pi\varepsilon_0 r^2 (KQ^2)^2}\left[\left(\frac{3}{4}k_0^2M^2\right)^2 - \left(\frac{3}{2}\frac{k_0k_1M\left(\frac{Q}{R}\right)^2}{r}\right)^2\right] y$$

$$\dot{y} = \frac{Q}{8\pi\varepsilon_0 r^2}\left(\frac{\frac{3}{4}k_0^2M^2 - \frac{3}{2}\frac{k_0k_1M\left(\frac{Q}{R}\right)^2}{r}}{\frac{3}{4}k_0^2M^2 + \frac{3}{2}\frac{k_0k_1M\left(\frac{Q}{R}\right)^2}{r}}\right)x^2 + \frac{qQ\left(32\mu_0\pi^2\varepsilon_0^2 r^4\right)^2}{4\pi\varepsilon_0 r^2 (KQ^2)^2}\left[\left(\frac{3}{4}k_0^2M^2\right)^2 - \left(\frac{3}{2}\frac{k_0k_1M\left(\frac{Q}{R}\right)^2}{r}\right)^2\right] x + \frac{r}{(kQ^2)}\left(32\mu_0\pi^2\varepsilon_0^2 r^4\right)\left[\frac{3}{4}k_0^2M^2 - \frac{3}{2}\frac{k_0k_1M\left(\frac{Q}{R}\right)^2}{r}\right] z^2$$

$$\dot{z} = \frac{-yz}{r}$$

Phase space analysis of the geodesic

- *The stability analysis was performed by evaluating the eigenvalues of the Jacobian matrix at each critical points.*

- *The presence of eigenvalues with negative real parts indicated stable equilibrium points, while eigenvalues with positive real parts indicated unstable equilibrium points.*

- *Complex eigenvalues suggested oscillatory behaviour.*

- *For computing the radius of the charged RN blackhole;*

$$R = \frac{GM}{c^2} + \sqrt{\left(\frac{GM}{c^2}\right)^2 - \left(\frac{Q^2}{G\,c^4}\right)^2}$$

Case	r (m)	k_0	k_1	Critical points	Eigenvalues	Stability analysis
Far from Horizon	10^{20}	10^{8}	10^{5}	[1, 1, 1]	[-9.100168550108147e+284, -5.477800218385041e+249, -1.6672496190120494e+245]	Very stable
At Horizon	10^{12}	100	90	[-1.19201108e-69 , 8.95269362e-91 , 1.00000000e+00]	[0.0, 2.4707558688213548e+265, 1.1344949965674093e+225]	Critically stable
Near Horizon	10^{5}	10	5	[-1.19201108e-61 , -2.05588159e-82 , 1.00000000e+00]	[4.455507965717163e+260, 3.599130618579141e+233, 1.6089368130980707e+226]	Very unstable
Near Singularity	10^{-30}	10^{-40}	10^{-60}	[2.51369975e-132, 2.66211189e-115, 3.05122660e-115]	[(-9.100168550108147e+284+0j), (1.4826166954317395e+248 +1.778524442542522e+254j), (1.4826166954317395e+248 -1.778524442542522e+254j)]	Highly complex system with more instability

Charge Analysis

Now we will take three cases of different test charge and keep r, k0 and kI as constant.

Case	q (C)	Critical points	Eigenvalues	Stability Analysis
q<Q	0	[-1.36531465e-120, -1.93225462e-121, -2.38129412e-147]	[4.9415117392496734e+263, 8.910855234581177e+258, 4.1811629740746546e+225]	Very Unstable
q>Q	10^{30}	[-1.19201107e-088, -2.16953014e-109, 9.99999998e-001]	[0.0, 0.0, 4.455507965730499e+260]	Unstable at beginning and then shifts to critically unstable state
q=Q	4×10^{8}	[-2.98002767e-67 , 5.75284390e-93 , 9.99999996e-01]	[0.0, 2.0260278323478486e+265, 6.1975694962367765e+239]	Critically unstable

Coupling constant with changing parameters

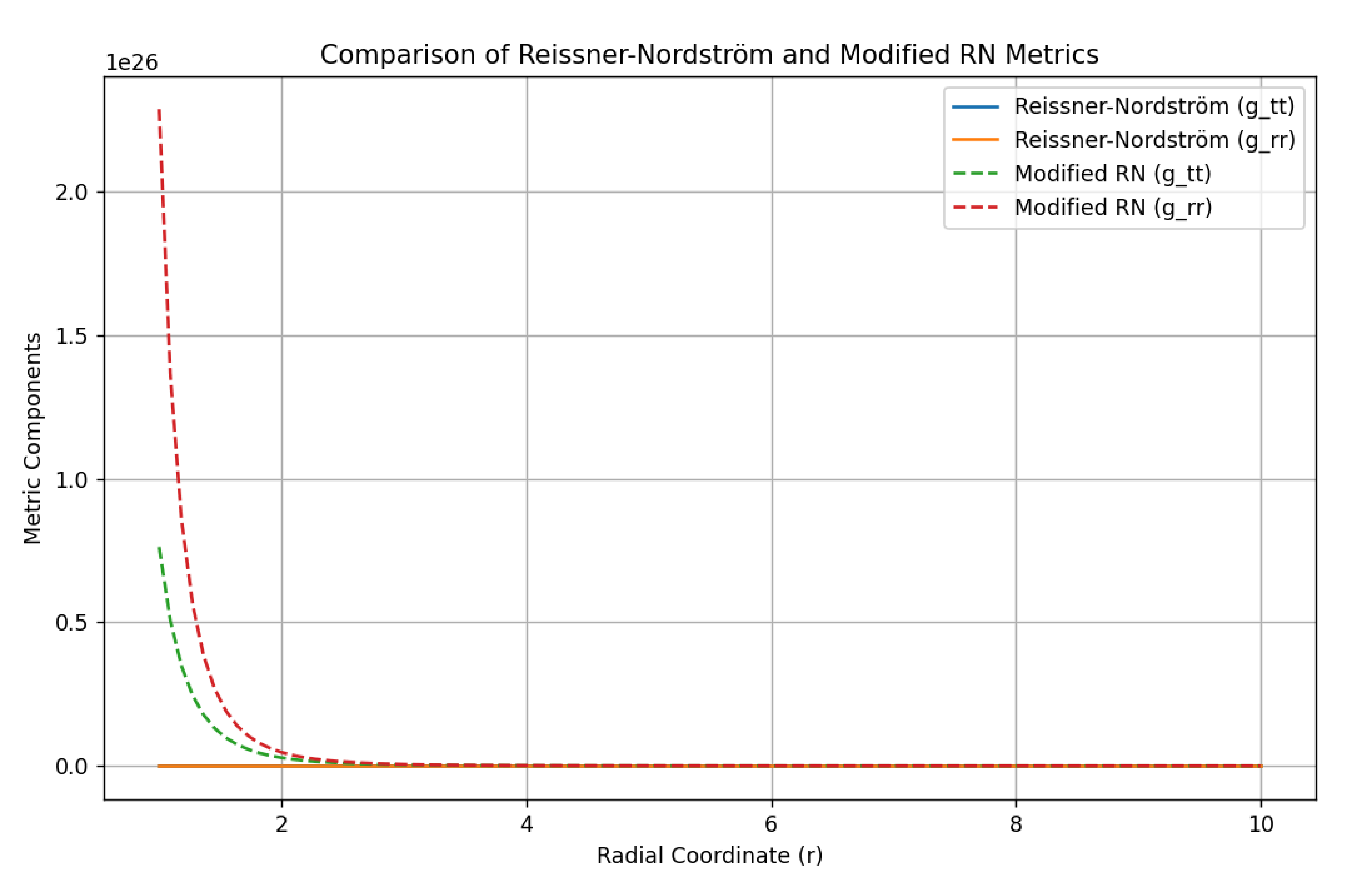
In the previous slides we have looked at the assumptions of λ and ν to be gravitational potential and electromagnetic potential respectively. The vector potential was assumed as $\lambda(r)=A\mu=Q/4\pi\varepsilon 0r$ and scalar gauge field as $\nu(r)=V\mu=\alpha\phi$, where α is the coupling constant and both λ and ν are functions of r and relations between λ and ν determines the value of α and to analyse the conditions.

Case	r (m)	k_0	k_1	Coupling constant (α)
Far from horizon	10^{20}	10^{8}	10^{5}	8.987551787368178e − 49
At horizon	10^{12}	100	90	$8.987551787368178e - 34$
Near horizon	10^{5}	10	5	$8.987551787368178e - 26$
Near the singularity	10^{-30}	10^{-40}	10^{-60}	$8.987551787368179e + 49$

From these four conditions, it's observed that when gravitational potential is more than electromagnetic potential then coupling constant is very less and vice-verse, and secondly when the distance r was large means when it's far from horizon α was very less and gradually increased we move towards the singularity and in the last case the coupling constant α suddenly jumped to a very higher value with a large exponential suggesting the coupling between gravitational potential and electromagnetic potential is very high near the singularity.

Rising values of the metric components for the modified RN blackhole

- Geometrical modifications
- Parameters and constants
- Additional Scalar Field



Quantum Corrections in RN metric

By EFT the Einstein-Hilbert action is modified with taking μ as energy scale,

$$\Gamma = \int d^4x \sqrt{-g}\left(\frac{R}{16\pi G_N} + c_1(\mu)R^2 + c_2(\mu)R_{\mu\nu}R^{\mu\nu} + c_3(\mu)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right) - \int d^4x\sqrt{-g}\left[\alpha R\ln\left(\frac{\Box}{\mu^2}\right)R + \beta R_{\mu\nu}\ln\left(\frac{\Box}{\mu^2}\right)R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma}\ln\left(\frac{\Box}{\mu^2}\right)R^{\mu\nu\rho\sigma}\right],$$

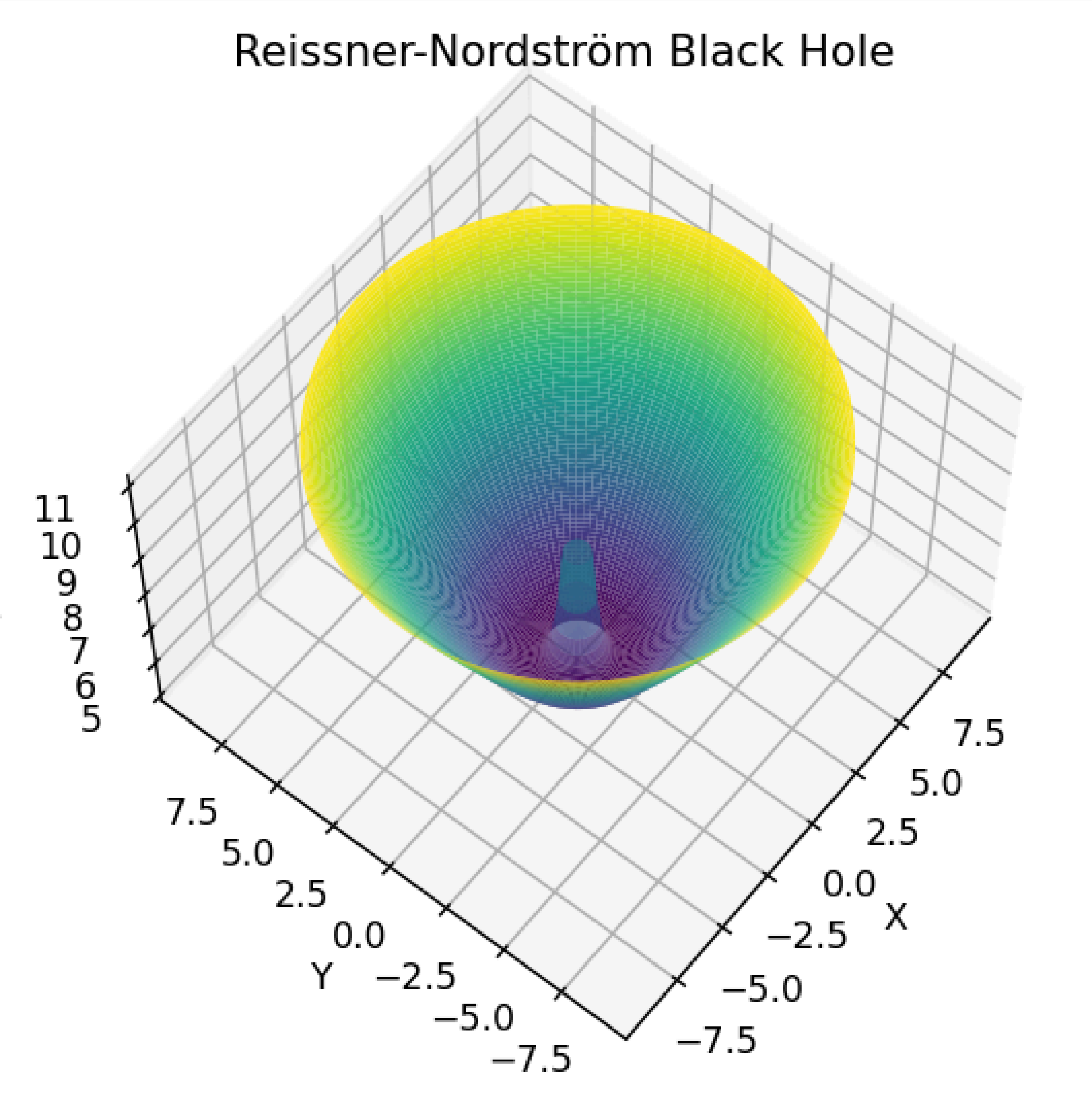
Correspondingly, the modified metric is

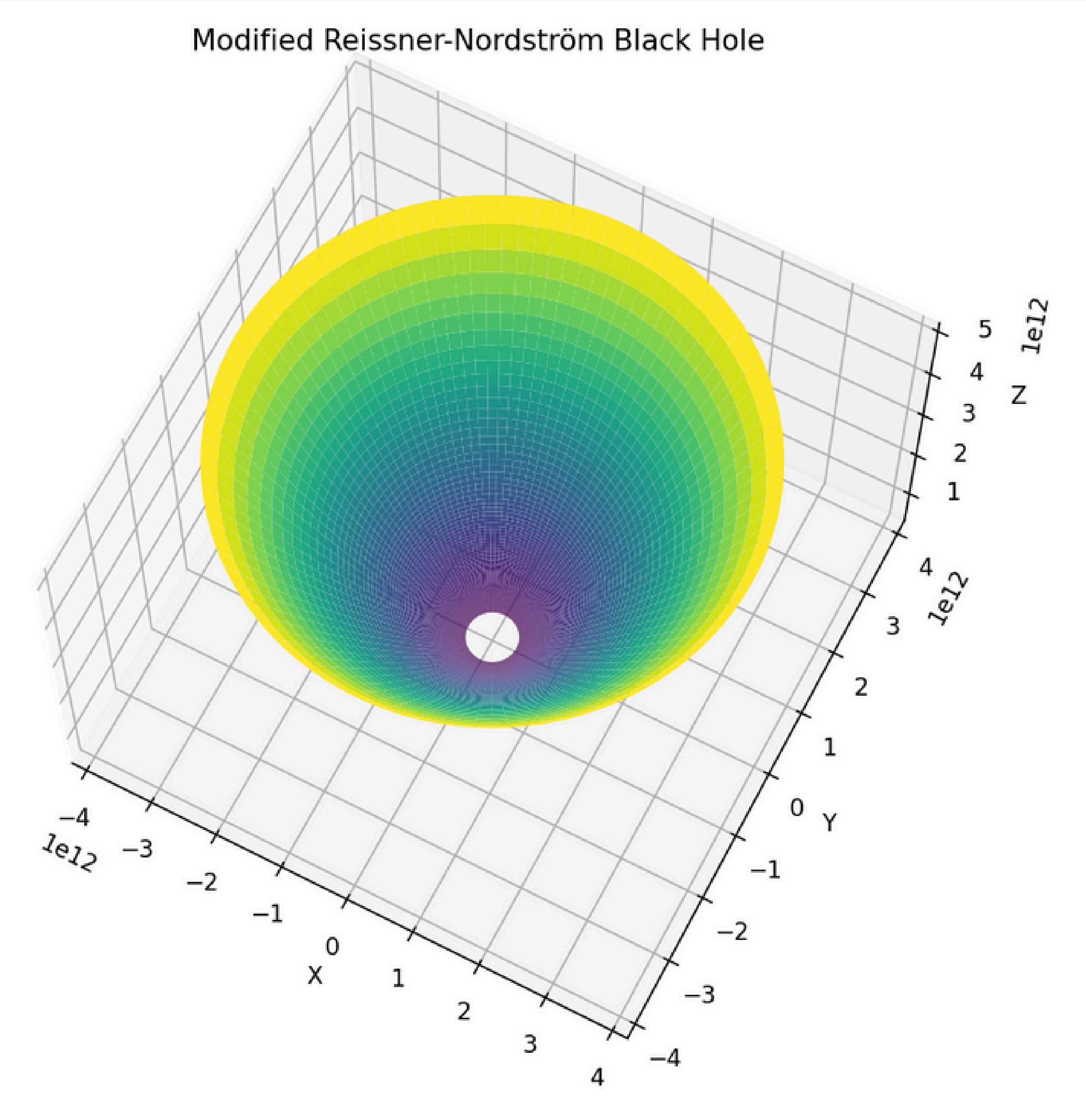
$$ds^2 = -f(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2,$$

with the following functions

$$f(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2} - \frac{32\pi G^2Q^2}{r^4}\left[c_2 + 4c_3 + 2(\beta+4\gamma)\left(\ln(\mu r) + \gamma_E - \frac{3}{2}\right)\right],$$

$$g(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2} - \frac{64\pi G^2Q^2}{r^4}\left[c_2 + 4c_3 + 2(\beta+4\gamma)(\ln(\mu r) + \gamma_E - 2)\right].$$


Reissner-Nordström Black Hole
11
10
9
8
7
6
5
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
X
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
Y


Modified Reissner-Nordström Black Hole
1e12
5
4
3
2
1
Z
1e12
4
3
2
1
0
−1
−2
−3
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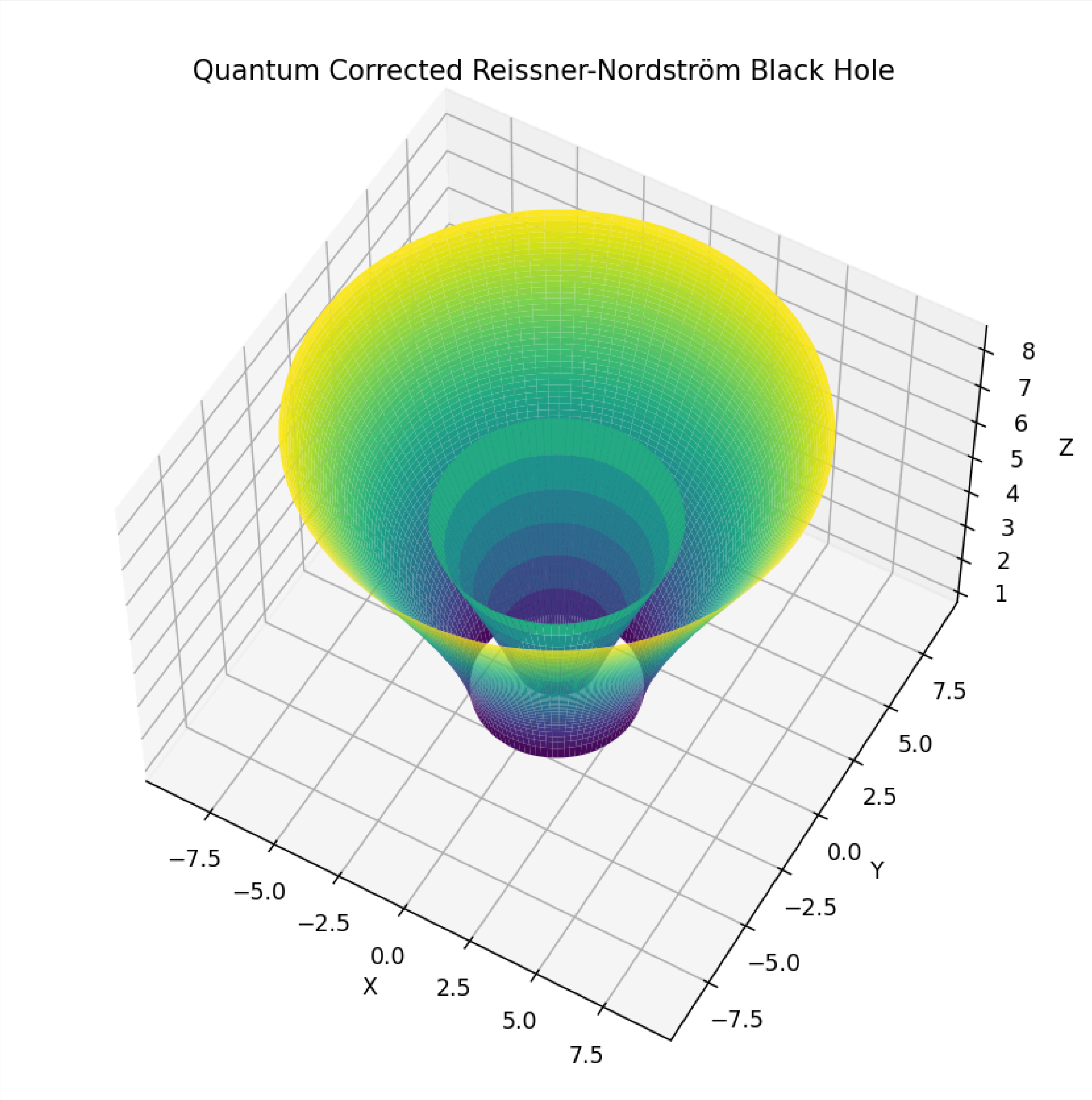
Y
1e12
−4
−3
−2
−1
0
1
2
3
4
X


Quantum Corrected Reissner-Nordström Black Hole
8
7
6
5
4
3
2
1
Z
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
Y
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
X

SUPERSYMMETRIC RN BLACK HOLES?

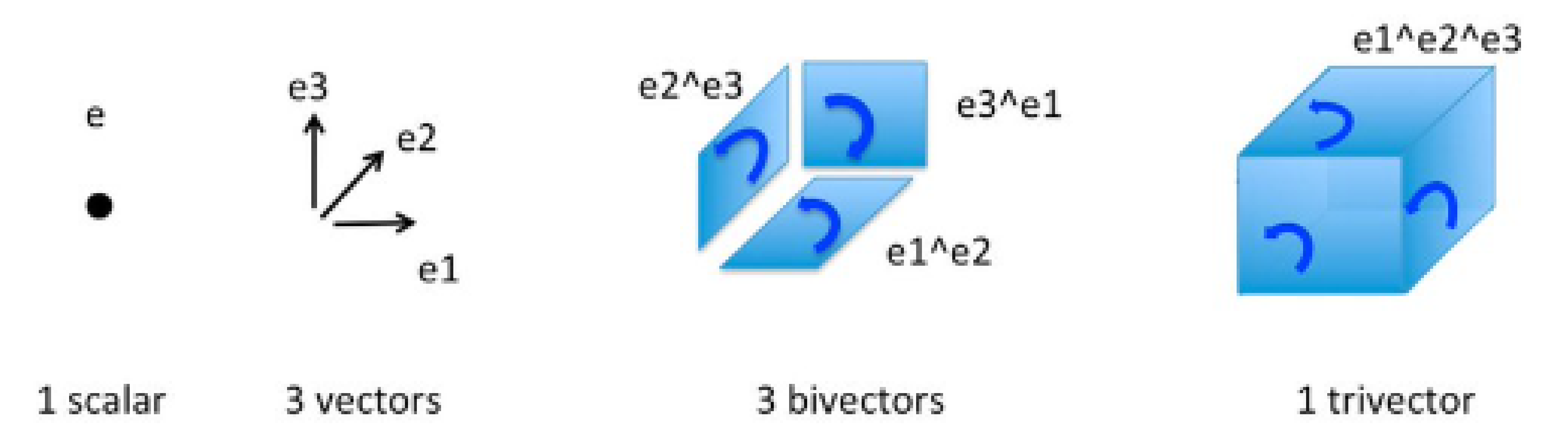
Grassmann Algebra

Superspace

Supersymmetry

SUSY RN Blackhole

v ∧ w=−w ∧ v , (u∧v)∧w=u∧(v∧w), u∧(v+w)=u∧v+u∧w , {1,ei ,ei∧ej, ei∧ej∧ek,…,ei∧ej∧...∧en} where the exterior product are vector, bivector, trivector,.....



Bosonic Coordinates(normal spacetime coordinates)
Fermionic (Grassmann) Coordinates (Grassmann Numbers)

$$(x^{\mu}, \theta_{\alpha}, \bar{\theta}_{\dot{\alpha}})$$

Supersymmetry is the symmetry group of grassmann coordinates.
SUSY generators mix bosonic and fermionic coordinates.

Maybe String theory and alternative gravity theories can help us formulate SUSY Blackhole

But there’s still no evidence for SUSY blackhole !!

Summary and Conclusion

- In this study, we have investigated the modified Reissner-Nordström metric in Lyra's geometry by computing the modified field equations and obtaining expressions for e^λ and e^ν.

- Furthermore, we conducted stability analysis to examine the stability of the system and phase space analysis to gain insights into the dynamical behaviour of the geodesics. Additionally, we studied the quantum corrections of RN metric.

- Expanding the previous knowledge of RN blackhole for a SUSY blackhole.

References

- W.D. Halford, "Cosmological Theory Based on Lyra's Geometry." April 20 ,1970 https://www.semanticscholar. org/paper/COSMOLOGICAL-THEORY-BASED-ON-LYRA'S-GEOMETRY.-Halford/acee750c8aa82aa784917b7 eefcbccd8682d158e
- F. Rahaman, A. Ghosh, and M. Kalam, "Lyra Black Holes." "[gr-qc/0612042] Lyra black holes - arXiv.org." 07 Dec. 2006, https://arxiv.org/abs/gr-qc/0612042
- R.R. Cuzinatto, E.M. de Morais, and B.M. Pimentel, PHYSICAL REVIEW D103,124002 (2021) "Lyra Scalar-Tensor Theory: A Scalar-Tensor Theory of Gravity on Lyra Manifold." 13 Apr. 2021, DOI: 10.1103/PhysRevD.103.124002
- D.K. Sen, “On geodesics of a modified Riemannian manifold”, Can. Math. Bull, 3, (1960) 255
- R.R. Cuzinatto, E.M. de Morais, and B.M. Pimentel, "LyST: A Scalar-Tensor Theory of Gravity on Lyra Manifold." 13 Apr. 2021, https://arxiv.org/abs/2104.06295v1
- Jonatan Nordebo, "The Reissner-Nordström Metric” Semantic Scholar. https://www.semanticscholar.org/paper/The-Reissner-Nordstr%C3%B6m-metric-Nordebo/9311976210064b18e4f59223665cdb14cffb6c26/figure/0.
- Ramesh Tikekar, "A Source for the Reissner-Nordstrøm Metric” Journal of Astrophysics and Astronomy· September 1984, DOI: 10.1007/BF0271454 https://link.springer.com/article/10.1007/BF02714543.
- D.K. Sen “A static cosmological model”, Zeit. f. Physik 149, (1957), 311
- “Charged black holes in GR and beyond” by N. K. Johnson-McDaniel Benasque meeting 04.06.2018 https://www. benasque.org/2018relativity/talks_contr/049_McDaniel.pdf.
- “Electric charge of black holes: Is it really always negligible?” By Michal Zajacek1 and Arman Tursunov2 09 Apr. 2019, https://arxiv.org/abs/1904.04654.
- PHYSICAL REVIEW D95,124060 (2017) “Dynamical analysis of an integrable cubic Galileon cosmological model” Alex Giacomini, Sameerah Jamal, Genly Leon, Andronikos Paliathanasis, and Joel Saavedra , DOI:10.1103/ PhysRev D.95.124060

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