Neutrino physics I

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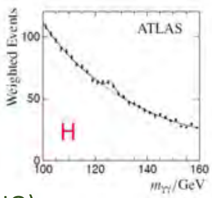
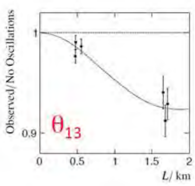
HIDDeN
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

VNIVERSITAT DE VALÈNCIA GENERALITAT VALENCIANA IFIC CSIC

2012 ~~One~~ two major discover~~y~~ies in particle physics

- A SM-like Higgs boson (ATLAS, CMS)
  The key to EWSB and a possible window to





- θ_{13} ~ 10^o (T2K, MINOS, Daya Bay, RENO)
  about as large as it could have been !
  The door to CP Violation in the leptonic sector

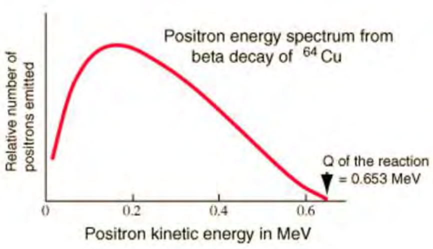
Spring Schools (if existed) were VERY short …..

β decay was supposed to be a two body decay

$$n \rightarrow p^+ + e^-$$

$$E_e = \frac{m_n^2 + m_e^2 - m_p^2}{2\, m_n}$$

Studies of β decay revealed a continuous energy spectrum.



Another anomaly was the fact that the nuclear recoil was not in the direction opposite to the momentum of the electron.

The emission of another particle was a probable explanation of this behaviour, but searches found no evidence of either mass or charge.

...desperate remedy to save the law of conservation of energy...

Neutron Decay:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Fermi postulated a theory for β decay in terms of spinors

$$\boxed{H_{ew} = \frac{G_F}{\sqrt{2}}\,\overline{\Psi}_p\ \gamma_\mu\,\Psi_n\quad \overline{\Psi}_e\gamma^\mu\ \psi_\nu}$$

A Dirac field is described by a four component spinor

$$\begin{pmatrix} e_L \\ e_R \\ \widehat{e}_L \\ \widehat{e}_R \end{pmatrix}$$

Standard Model of Particle Physics

Gauge Theory based on the group:

$$SU(3) \times SU(2) \times U(1)$$

$SU(3) \Rightarrow$ Quantum Chromodynamics

Strong Force (Quarks and Gluons)

$SU_L(2) \times U(1) \Rightarrow$ ElectroWeak Interactions broken to $U_{EM}(1)$

by HIGGS

$SU_L(2) \times U_Y(1) \Rightarrow U_{EM}(1)$

Force Carriers: $W^{\pm}$, Z^0 and γ masses: 80, 91 and 0 GeV

quark, SU(2) doublets: $\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$

up-quark, SU(2) singlets: u_R, c_R, t_R

down-quark, SU(2) singlets: d_R, s_R, b_R

lepton, SU(2) doublets: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$

neutrino, SU(2) singlets: $- - -$

charge lepton, SU(2) singlets: e_R, μ_R, τ_R

Electron mass

comes from a term of the form

$\bar{L}\phi e_R$

Absence of ν_R

forbids such a mass term (dim 4)

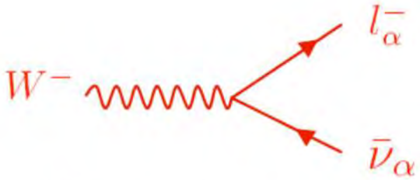
for the Neutrino

Therefore in the SM neutrinos are massless

and hence travel at speed of light.

Interactions:

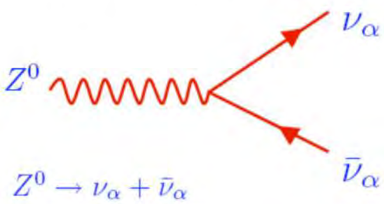
Charge Current (CC)



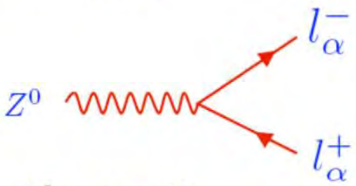
$W^- \to l_\alpha^- + \bar{\nu}_\alpha$

$$\Gamma(Z^0 \to f + \bar{f}) = K\frac{g_Z^2 M_Z}{48\pi}[|c_V^f|^2 + |c_A^f|^2]$$

Neutral Current (NC)



$Z^0 \to \nu_\alpha + \bar{\nu}_\alpha$



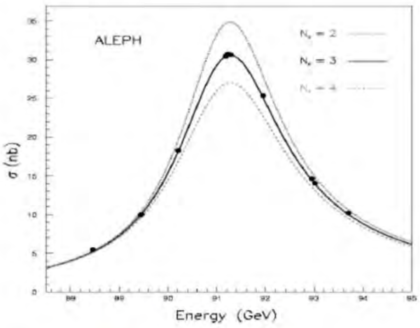
$Z^0 \to l_\alpha^- + l_\alpha^+$

$\alpha = e, \mu, \;or\; \tau$

Invisible width of Z plus other data from LEP:

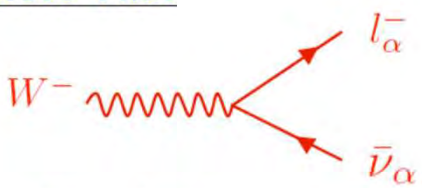
$Z^0 \to \nu\bar{\nu}$

Implies $N_\nu = 2.99 \pm 0.01$



<u>Three Active Neutrinos!!!</u> Sterile Neutrinos don't couple to Z^0

<u>Note That</u>

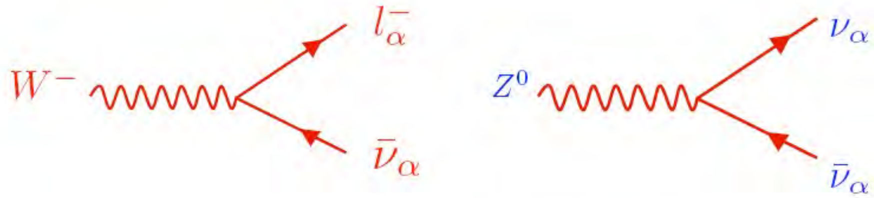


$W^- \to l_\alpha^- + \bar{\nu}_\alpha$

<u>Implies</u>

$$\begin{array}{ccccc} \nu_e & , & \nu_\mu & , & \nu_\tau \\ \updownarrow & & \updownarrow & & \updownarrow \\ e & , & \mu & , & \tau \end{array}$$

Standard Model

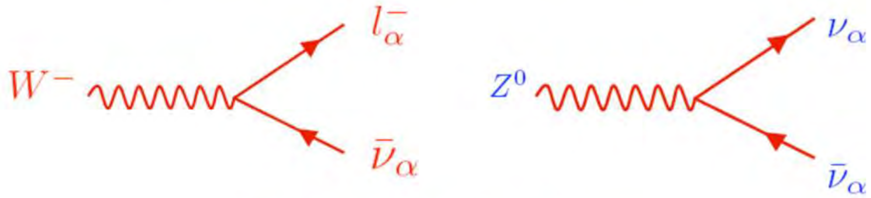


couplings conserve the Lepton Number L
defined by—

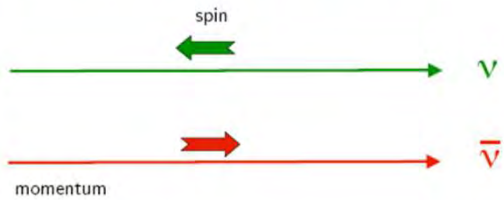
$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1.$$

Actually L_e, L_μ, and L_τ
separately

Left Handed Nature of The Neutrino



Produce Left-Handed Neutrinos
and Right-Handed Anti-Neutrinos



What about the RH neutrinos and LH anti-neutrino ????

There exist three fundamental and discrete transformations in nature:

- **Parity** $\mathcal{P}$ $\vec{x} \to -\vec{x}$
- **Time reversal** $\mathcal{T}$ $t \to -t$
- **Charge conjugation** $\mathcal{C}$ $q \to -q$

$\mathcal{P}$, $\mathcal{T}$ and $\mathcal{C}$ are conserved in the classical theories of mechanics and electrodynamics!

$\mathcal{CPT} \leftrightarrow$ Lorentz invariance $\oplus$ unitarity: is an essential building block of field theory

$\mathcal{CPT}$: L particle $\leftrightarrow$ R antiparticle

Neutrinos in the MSM are massless and exist only in two states: particle with negative helicity and antiparticle with positive one: Weyl fermion

Summary of ν's in SM:

Three flavors of massless neutrinos

$$W^- \to l_\alpha^- + \bar{\nu}_\alpha$$

$$W^+ \to l_\alpha^+ + \nu_\alpha$$

$$\alpha = e, \mu, \ or \ \tau$$

Anti-neutrino, $\bar{\nu}_\alpha$, has +ve helicity, Right Handed

Neutrino, ν_α, has -ve helicity, Left Handed

ν_L and $\bar{\nu}_R$ are CPT conjugates

massless implies helicity = chirality

Beyond the SM

What if Neutrino have a MASS?

speed is less than c therefore time can pass

and

Neutrinos can change character!!!

What are the stationary states?

How are they related to the interaction states?

NEUTRINO OSCILLATIONS:

Two Flavors

flavor eigenstates $\neq$ mass eigenestates

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

W's produce ν_μ and/or ν_τ's

but ν_1 and ν_2 are the states

that change by a phase over time, mass eigenstates.

$$|\nu_j\rangle \rightarrow e^{-ip_j \cdot x}|\nu_j\rangle \qquad p_j^2 = m_j^2$$

$\alpha, \beta \ldots$ flavor index $\qquad$ $i, j \ldots$ mass index

Production:

$$|\nu_\mu\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

Propogation:

$$\cos\theta e^{-ip_1\cdot x}|\nu_1\rangle + \sin\theta e^{-ip_2\cdot x}|\nu_2\rangle$$

Detection:

$$|\nu_1\rangle = \cos\theta|\nu_\mu\rangle - \sin\theta|\nu_\tau\rangle$$

$$|\nu_2\rangle = \sin\theta|\nu_\mu\rangle + \cos\theta|\nu_\tau\rangle$$

$$\begin{pmatrix}\nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

$$P(\nu_\mu \to \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2$$

$$P(\nu_\mu \to \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2$$

Same E, therefore $p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$

$$e^{-ip_j\cdot x} = e^{-iEt}e^{-ip_jL} \approx e^{-i(Et-EL)} \quad e^{-im_j^2L/2E}$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2\theta\cos^2\theta|e^{-im_2^2L/2E} - e^{-im_1^2L/2E}|^2$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2\frac{\delta m^2 L}{4E}$$

$\delta m^2 = m_2^2 - m_1^2$ and $\frac{\delta m^2 L}{4E} \equiv \Delta$ kinematic phase:

$$P(\nu_\mu \to \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2$$

Same E, therefore $p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$

$$e^{-ip_j\cdot x} = e^{-iEt}e^{-ip_jL} \approx e^{-i(Et-EL)} \quad e^{-im_j^2L/2E}$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2\theta\cos^2\theta|e^{-im_2^2L/2E} - e^{-im_1^2L/2E}|^2$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2\left(\frac{\delta m^2 L}{4E}\frac{c^4}{\hbar c}\right)$$

Appearance:

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

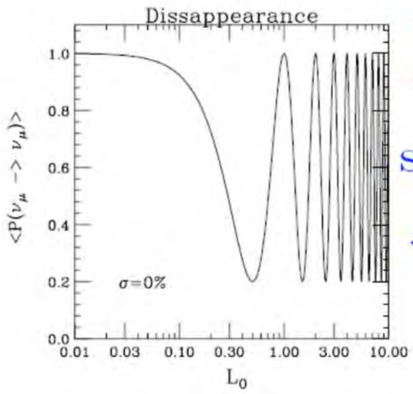
Disappearance:

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

Oscillation Length $L_0 = 4\pi E/\delta m^2$

Fixed E_ν



Amplitude of Oscillation

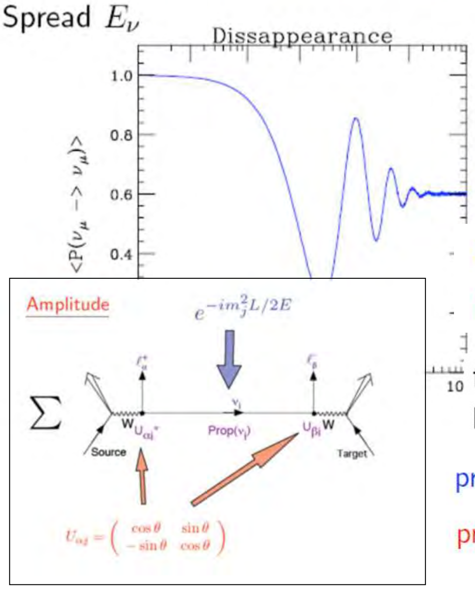
$\Uparrow$

$\sin^2 2\theta$

$\Downarrow$

$$\langle P(\nu_\mu \to \nu_\mu)\rangle = 1 - \sin^2 2\theta \left\langle \sin^2 \frac{\delta m^2 L}{4E} \right\rangle$$

Spread E_ν



effectively incoherent
mass eigenstates

$1 - \sin^2 2\theta(\frac{1}{2}) = \cos^4\theta + \sin^4\theta$

- $W^+ \to \mu^+ + \nu_1$ probability $\cos^2\theta$
- $W^+ \to \mu^+ + \nu_2$ probability $\sin^2\theta$

flavour fractions $|\nu_1\rangle$ and $|\nu_2\rangle$ during propagation remain unchanged

probability ν_1 contains ν_μ is $\cos^2\theta$

probability ν_2 contains ν_μ is $\sin^2\theta$

Using the unitarity of the mixing matrix: ($W^{jk}_{\alpha\beta} \equiv [V_{\alpha j} V^*_{\beta j} V^*_{\alpha k} V_{\beta k}]$)

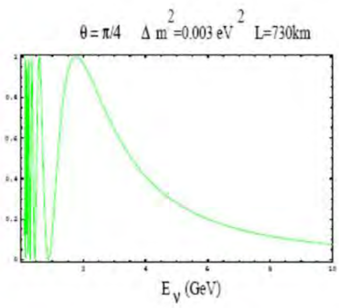
$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{k>j} \mathrm{Re}[\mathrm{W}^{\mathrm{jk}}_{\alpha\beta}] \sin^2\left(\frac{\Delta \mathrm{m}^2_{\mathrm{jk}} \mathrm{L}}{4\mathrm{E}_\nu}\right)$$

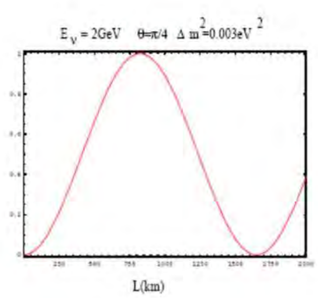
$$\pm \quad 2 \sum_{k>j} \mathrm{Im}[\mathrm{W}^{\mathrm{jk}}_{\alpha\beta}] \sin\left(\frac{\Delta \mathrm{m}^2_{\mathrm{jk}} \mathrm{L}}{2\mathrm{E}_\nu}\right)$$

For 2 families: $V_{MNS} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$$P_{\alpha\beta} = \sin^2 2\theta \;\; \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right) \rightarrow \text{appearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \rightarrow \text{disappearance}$$





Oscillation probabilities show the expected GIM suppression of any flavour changing process: they vanish if the neutrinos are degenerate

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \to \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \to \nu_\beta)|^2 =$$

$$P_{\alpha\beta} = \sin^2 2\theta \ \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right) \to \text{appearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \to \text{disappearance}$$

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \to \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \to \nu_\beta)|^2 =$$

$$P_{\alpha\beta} = \sin^2 2\theta \ \text{s} \quad \text{pearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} \quad \text{nce}$$

$$\left(\frac{\Delta m^2 \, L}{4\,E}\right)$$

$$\left(1.27\ \frac{\Delta m^2\ (eV^2)\ L(km)}{E(GeV)}\right)$$

L/E becomes crucial !!!

Evidence for Flavor Change:

⋆⋆⋆ Atmospheric and Accelerator Neutrinos with $L/E = 500$ km/GeV

⋆⋆⋆ Solar and Reactor Neutrinos with $L/E = 15$ km/MeV

Neutrinos from Stopped muons L/E= 2m/MeV (Unconfirmed)

Atmospheric neutrinos

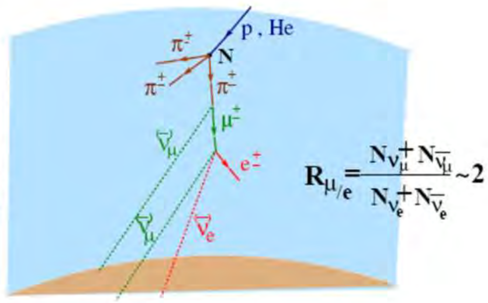
- Atmospheric neutrinos are produced by the interaction of *cosmic rays* (p, He, ...) with the Earth's atmosphere:

$\boxed{1}$ $A_{cr} + A_{air} \to \pi^{\pm}, K^{\pm}, K^0, \ldots$

$\boxed{2}$ $\pi^{\pm} \to \mu^{\pm} + \nu_{\mu},$

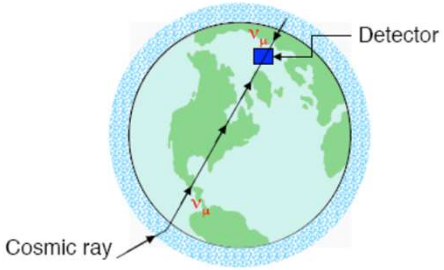
$\boxed{3}$ $\mu^{\pm} \to e^{\pm} + \nu_e + \nu_{\mu};$

- at the detector, some ν interacts and produces a **charged lepton**, which is observed.



A deficit was observed in the ratio μ/e events: Soudan2, IMB, Kamiokande

Atmospheric Neutrinos



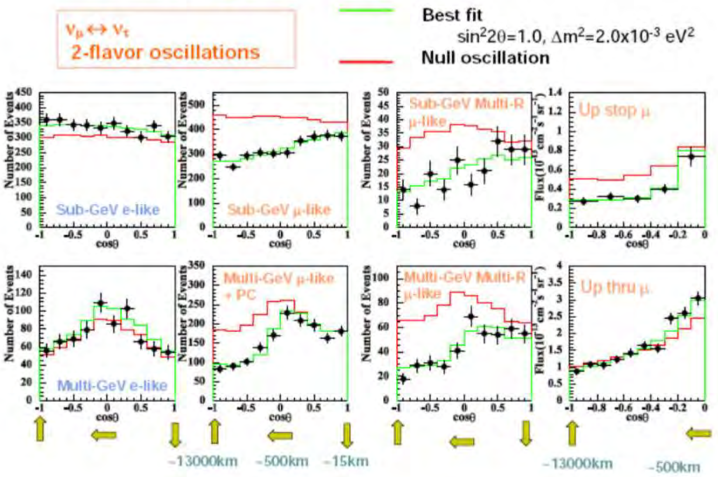
Isotropy of the $\gtrsim 2$ GeV cosmic rays + Gauss' Law + No ν_μ disappearance

$$\Rightarrow \frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 1 \, .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV

$$\frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 0.54 \pm 0.04 \, .$$

Zenith angle distributions



Half of the upward-going, long-distance-traveling ν_μ are disappearing.

Voluminous atmospheric neutrino data are well described by —

$$\nu_\mu \longrightarrow \nu_\tau$$

with —

$$\Delta m_{atm}^2 \cong 2.4\ 10^{-3}\ eV^2$$

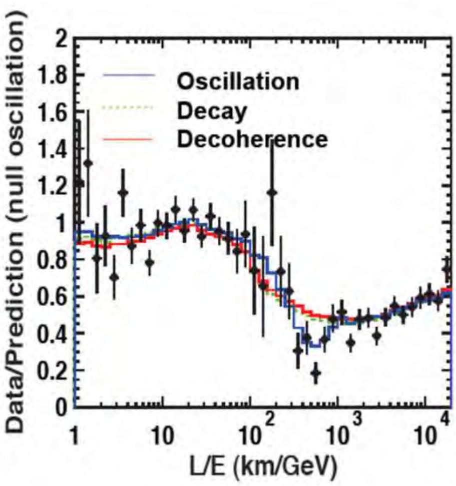
and —

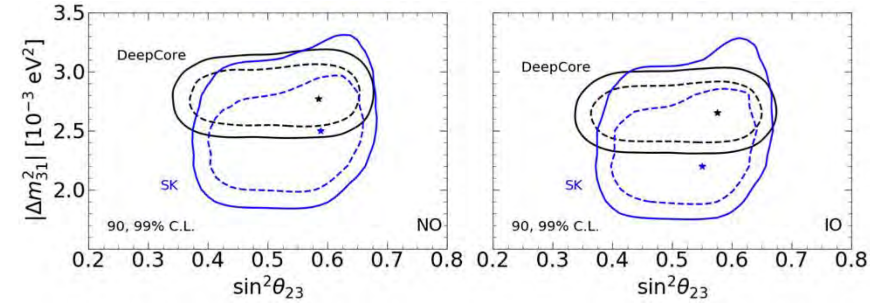
$$\sin^2 2\theta_{atm} \cong 1$$

L/E Analysis

- Oscillation, decay and decoherence models tested

χ^2_{osc} = 83.9/83
χ^2_{dcy} = 107.1/83, $\Delta\chi^2$ = 23.2(**4.8σ**)
χ^2_{dec} = 112.5/83, $\Delta\chi^2$ = 27.6(**5.3σ**)





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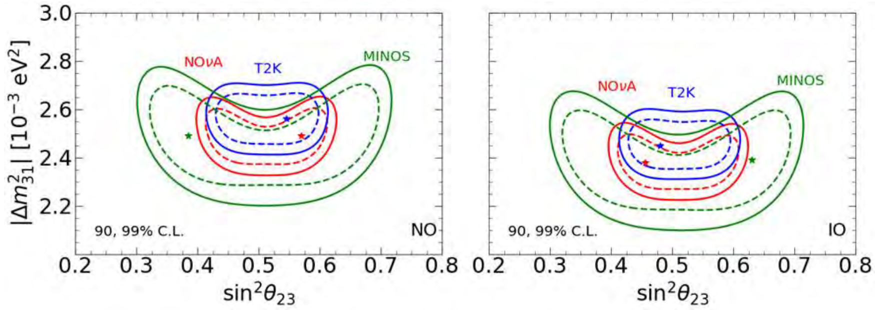


Solar Engine:

$$4p + 2e^- \rightarrow {}^4He + 2\nu_e + 26.7MeV$$

$$E = mc^2$$

1 ν_e for every 13.4 MeV (=2.1 $\times 10^{-12}$ J)

$\mathcal{L}_\odot$ at earth's surface 0.13 watts/cm^2

$$\phi_\nu = \frac{0.13}{2.1\times10^{-12}} = 6 \times 10^{10}/cm^2/sec$$

This corresponds to an average of 2 ν's per cm^3
since they are going at speed c.

$p+p \mapsto {}^2H + e^+ + \nu_e$

$p + e^- + p \mapsto {}^2H + \nu_e$

${}^2H + p \mapsto {}^3He + \gamma$

85 %

${}^3He + {}^3He \mapsto {}^4He + 2p$

15 %

${}^3He + {}^4He \mapsto {}^7Be + \gamma$

0.02 %

${}^7Be + p \mapsto {}^8B + \gamma$

${}^8B \mapsto {}^8Be^* + e^+ + \nu_e$

${}^8Be^* \mapsto {}^4He + {}^4He$

${}^7Be + e^- \mapsto {}^7Li + \nu_e$

${}^7Li + p \mapsto {}^4He + {}^4He$

${}^3He + p \mapsto {}^4He + e^+ + \nu_e$

Solar Spectrum:

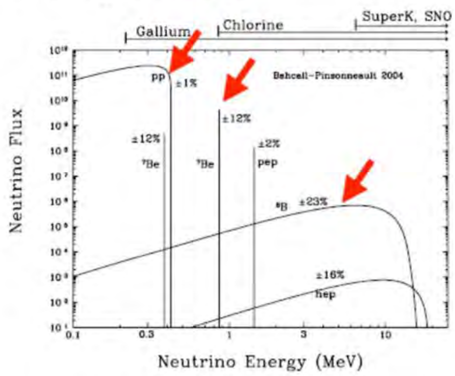


Figure 1. The predicted solar neutrino energy spectrum. The figure shows the energy spectrum of solar neutrinos predicted by the BP04 solar model [22]. For continuum sources, the neutrino fluxes are given in number of neutrinos $cm^{-2} s^{-1} MeV^{-1}$ at the Earth's surface. For line sources, the units are number of neutrinos $cm^{-2} s^{-1}$. Total theoretical uncertainties taken from column 2 of table 1 are shown for each source. To avoid complication in the figure, we have omitted the difficult-to-detect CNO neutrino fluxes (see table 1).

$$p + p \to {}^2H + e^+ + \nu_e$$

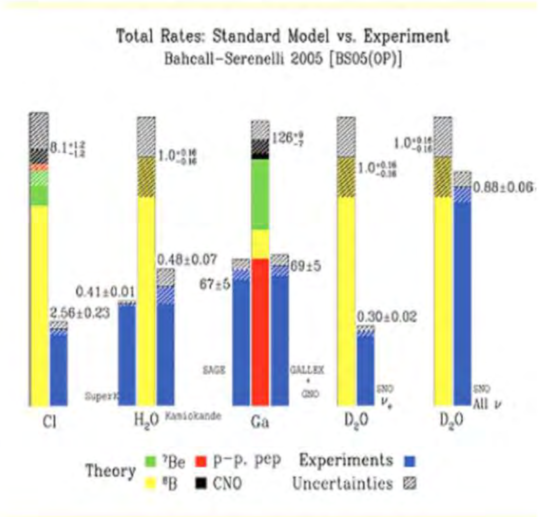
$$\phi_{pp} = 5.94(1 \pm 0.01) \times 10^{10} cm^{-2} sec^{-1}$$

$$ {}^7Be + e^- \to {}^7Li + \nu_e$$

$$\phi_{^7Be} = 4.86(1 \pm 0.12) \times 10^{9} cm^{-2} sec^{-1}$$

$$ {}^7Be + p \to {}^8B \to {}^8Be^* + e^- + \nu_e$$

$$\phi_{^8B} = 5.82(1 \pm 0.23) \times 10^{6} cm^{-2} sec^{-1}$$



Ray Davis & John Bahcall

Theory v Exp.

Neutrino Flavor Transistions!!!

Kinematical Phase:

$\delta m^2_\odot = 8.0 \times 10^{-5} eV^2$

$\sin^2 \theta_\odot = 0.31$

$$\Delta_\odot = \frac{\delta m^2_\odot L}{4E} = 1.27 \; \frac{8\times 10^{-5} \; eV^2 \; \cdot \; 1.5\times 10^{11} \; m}{0.1-10 \; MeV}$$

$$\Delta_\odot \approx 10^{7\pm 1}$$

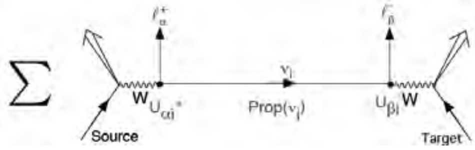
Effectively Incoherent !!!

Vacuum ν_e Survival Probability:

$$\langle P_{ee} \rangle = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot$$

where f_1 and f_2 are the fraction of ν_1 and ν_2 at production.

In vacuum $f_1 = $ co

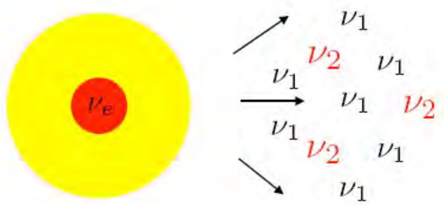


$$\langle P_{ee} \rangle = \cos^4\theta_\odot + \sin^4\theta_\odot = 1 - \tfrac{1}{2}\sin^2 2\theta_\odot$$

for pp and ^{7}Be this is approximately THE ANSWER.

$f_1 \sim 69\%$ and $f_2 \sim 31\%$ and $\langle P_{ee} \rangle \approx 0.6$

pp and ^{7}Be



$f_1 \sim 69\%$

$f_2 \sim 31\%$

$$\langle P_{ee} \rangle \approx 0.6$$

$f_3 = \sin^2\theta_{13} < 4\%$

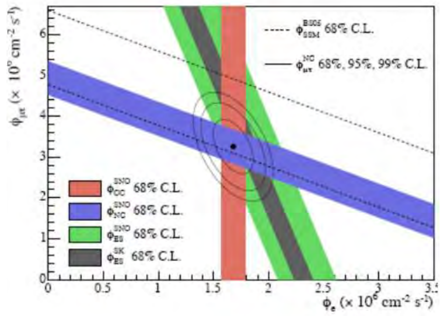
What about ^8B ?

SNO's CC/NC

CC: $\nu_e + d \rightarrow e^- + p + p$

NC : $\nu_x + d \rightarrow \nu_x + p + n$

ES: $\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$

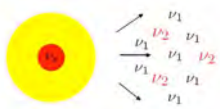


$\frac{CC}{NC} = \langle P_{ee} \rangle = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot$

$f_1 = \left(\frac{CC}{NC} - \sin^2\theta_\odot\right) / \cos 2\theta_\odot$

$= (0.35 - 0.31)/0.4 \approx 10$

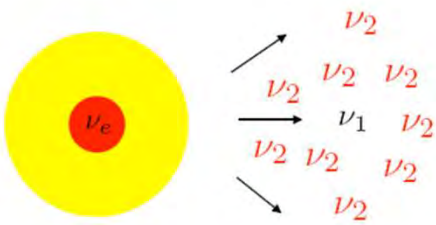
pp and ^{7}Be



$f_1 \sim 69\%$

$f_2 \sim 31\%$

^8B



$f_2 \sim 90\%$

$f_1 \sim 10\%$

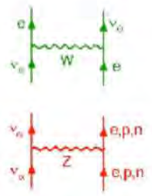
$\langle P_{ee} \rangle = \sin^2\theta + f_1 \cos 2\theta_\odot \approx \sin^2\theta_\odot = 0.31$

Wow!!! How did that happen???

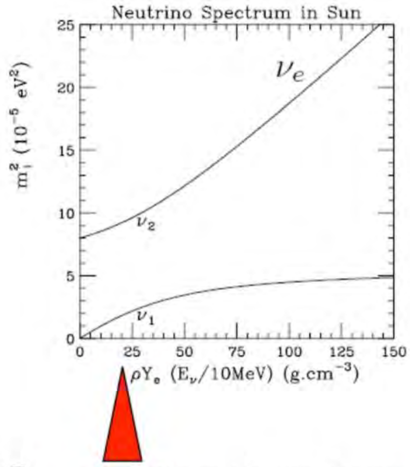
energy dependence!!!

MSW

Coherent Forward Scattering:



Wolfenstein '78



MATTER EFFECTS CHANGE THE NEUTRINO MASSES AND MIXINGS

Mikheyev + Smirnov Resonance WIN '85

Neutrino Evolution:

$$-i\frac{\partial}{\partial t}\nu = H\nu$$

in the mass eigenstate basis

$$\nu = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \text{ and } H = \begin{pmatrix} \sqrt{p^2+m_1^2} & 0 \\ 0 & \sqrt{p^2+m_2^2} \end{pmatrix}$$

$$E = \sqrt{p^2+m^2}$$

$$H = (p + \frac{m_1^2+m_2^2}{4p})I + \frac{1}{4E}\begin{pmatrix} -\delta m^2 & 0 \\ 0 & \delta m^2 \end{pmatrix}$$

$$\delta m^2 = m_2^2 - m_1^2 > 0$$

in the flavor basis

$$\nu \to U\nu \text{ and } H \to UHU^\dagger$$

where $\nu = \begin{pmatrix} \nu_e \\ \nu_\sigma \end{pmatrix}$ and $U = \begin{pmatrix} \cos\theta_\odot & \sin\theta_\odot \\ -\sin\theta_\odot & \cos\theta_\odot \end{pmatrix}$

$0 < \theta_\odot < \frac{\pi}{2}$

and therefore in flavor basis

$$H = \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}$$

i.e. $\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}_{mass} \Rightarrow \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}_{flavor}$

Coherent Forward Scattering:

dimensions $[G_F N_e] = M^{-2} L^{-3} = M$

Wolfenstein '78

$$\pm\sqrt{2} G_F N_e \ \delta_{ee}$$

N_e is number density of electrons

$+(-)$ for neutrinos (anti-neutrinos)

Same for all active flavors, therefore overall phases

$$\begin{pmatrix} +\sqrt{2}G_F N_e & 0 \\ 0 & 0 \end{pmatrix} \to \cancel{\frac{G_F N_e}{\sqrt{2}}} I_2 + \frac{1}{2} \begin{pmatrix} +\sqrt{2}G_F N_e & 0 \\ 0 & -\sqrt{2}G_F N_e \end{pmatrix}$$

Including Matter Effects in the Flavor Basis:

$$H_{flavor} = \frac{1}{4E_\nu}\begin{pmatrix} -\delta m^2 \cos 2\theta_\odot + 2\sqrt{2}G_F N_e E_\nu & \delta m^2 \sin 2\theta_\odot \\ \delta m^2 \sin 2\theta_\odot & \delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu \end{pmatrix}$$

Diagonalize by identifying with

$$H_{flavor} = \frac{1}{4E_\nu}\begin{pmatrix} -\delta m_N^2 \cos 2\theta_\odot^N & \delta m_N^2 \sin 2\theta_\odot^N \\ \delta m_N^2 \sin 2\theta_\odot^N & \delta m_N^2 \cos 2\theta_\odot^N \end{pmatrix}$$

Masses and Mixings in MATTER: δm_N^2 and $\theta_\odot^N$

$$\boxed{\begin{aligned} \delta m_N^2 \cos 2\theta_\odot^N &= \delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu \\ \delta m_N^2 \sin 2\theta_\odot^N &= \delta m^2 \sin 2\theta_\odot \end{aligned}}$$

Notice:
(1) Possible zero when $\delta m^2 \cos 2\theta_\odot = 2\sqrt{2}G_F N_e E_\nu$
(2) the invariance of the product $\delta m^2 \sin 2\theta_\odot$

ν_e disappearance in Loooong Block of Lead:

$$1 - P(\nu_e \to \nu_e) = \sin^2 2\theta_\odot^N \sin^2 \Delta_N$$

$$\Delta_N = \frac{\delta m_N^2 L}{4E}$$

same form as vacuum

The Solution:

$$\delta m_N^2 = \sqrt{(\delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu)^2 + (\delta m^2 \sin 2\theta_\odot)^2}$$

$$\sin^2 \theta_\odot^N = \frac{1}{2}\left(1 - \frac{(\delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu)}{\delta m_N^2}\right) \qquad \theta_\odot^N > \theta_\odot$$

Quasi-Vacuum: $2\sqrt{2}G_F N_e E_\nu \ll \delta m^2 \cos 2\theta_\odot$ — pp and ^{7}Be

$$\delta m_N^2 = \delta m^2 \text{ and } \theta_\odot^N = \theta_\odot$$

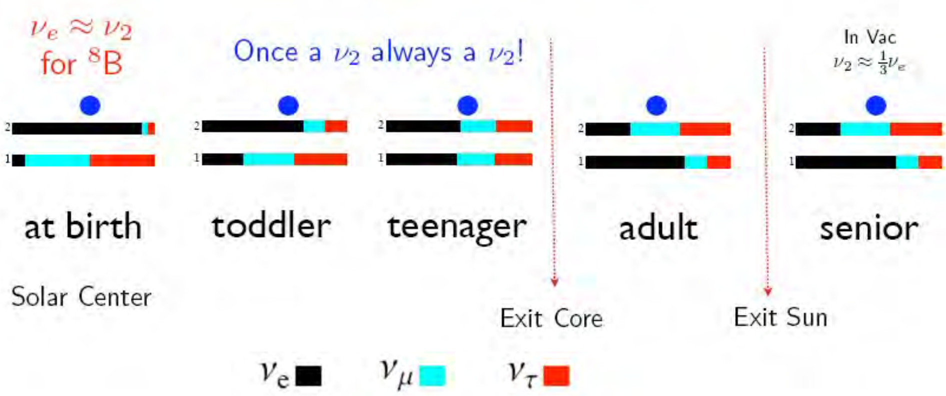
Resonance (Mikheyev + Smirnov '85): $2\sqrt{2}G_F N_e E_\nu = \delta m^2 \cos 2\theta_\odot$

$$\delta m_N^2 = \delta m^2 \sin 2\theta_\odot \text{ and } \theta_\odot^N = \pi/4$$

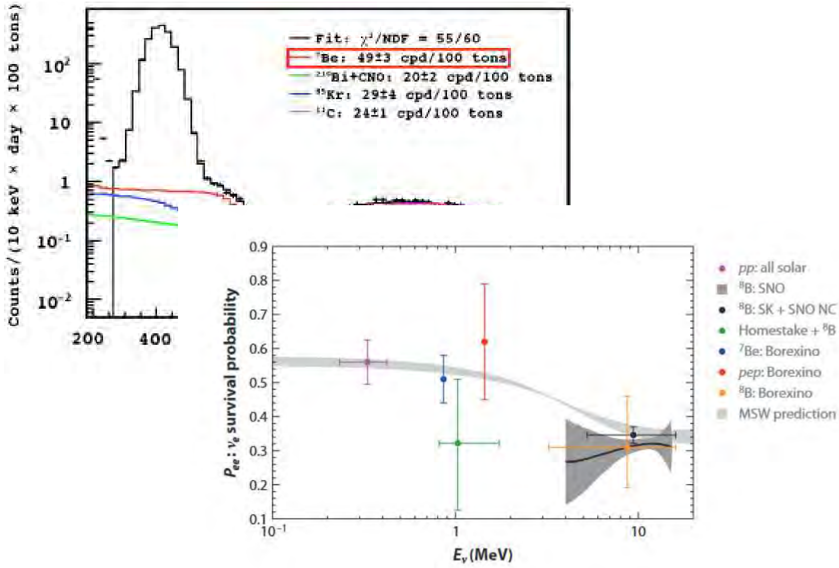
Matter Dominated: $2\sqrt{2}G_F N_e E_\nu \gg \delta m^2 \cos 2\theta_\odot$ — ^8B

$$\delta m_N^2 \to 2\sqrt{2}G_F N_e E_\nu \text{ and } \theta_\odot^N \to \pi/2$$

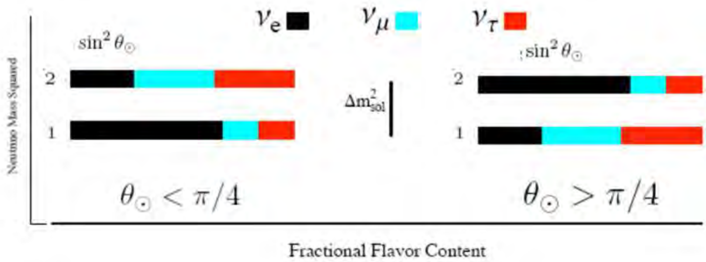
Life of a Boron-8 Solar Neutrino:



Borexino results



Solar Pair Mass Hierarchy:



Who cares ?

SNO does !!!

for neutrino in matter
$\theta_\odot^N > \theta_\odot$

$$\langle P_{ee}\rangle = \cos^2\theta_\odot^N \cos^2\theta_\odot + \sin^2\theta_\odot^N \sin^2\theta_\odot = \tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta_\odot^N \cos 2\theta_\odot$$

if $\theta_\odot < \pi/4$
$\langle P_{ee}\rangle \geq \sin^2\theta_\odot$

if $\theta_\odot > \pi/4$
$\langle P_{ee}\rangle \geq \frac{1}{2}(1+\cos^2 2\theta_\odot) \geq \frac{1}{2}$

SNO: $\langle P_{ee}\rangle_{day} = 0.347 \pm 0.038$

Solar Hierarchy Determined !!!

Day/Night Asymmetry:

$\sin^2\theta_\odot \to \sin^2\theta_\oplus = \sin^2\theta_\odot + \frac{1}{2}\sin^2 2\theta_\odot \left(\frac{A_\oplus}{\delta m^2_\odot}\right)$ in the earth.

A=2(D-N)/(D+N) expected to be few %

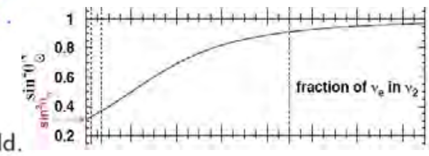
	Amplitude fit		separate D, N:
	Δm	Δm	(D-N)/((D+N)/2)
SK-I	-2.0±1.8±1.0%	-1.9±1.7±1.0%	-2.1±2.0±1.3%
SK-II	-4.4±3.8±1.0%	-4.4±3.6±1.0%	-5.5±4.2±3.7%
SK-III	-4.2±2.7±0.7%	-3.8±2.6±0.7%	-5.9±3.2±1.3%
SK-IV	-3.6±1.6±0.6%	-3.3±1.5±0.6%	-4.9±1.8±1.4%
comb	-3.3±1.0±0.5%	-3.1±1.0±0.5%	-4.1±1.2±0.8%
non-zero signif.	3.0σ	2.8σ	2.8σ

Spectral Distortion:

A characteristic of matter effects is that the Fraction of ν_2 is energy dependent .

Smaller at smaller E.

Implies an increase in P_{ee} near threshold.



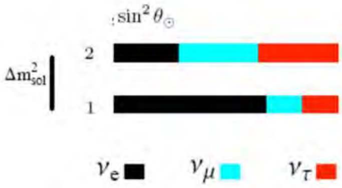
Summary:

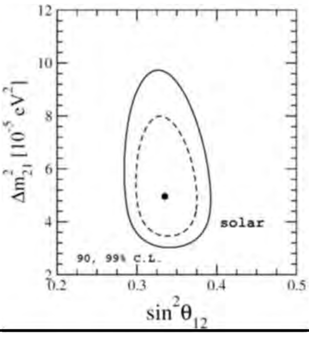
The low energy pp and ^{7}Be Solar Neutrinos exit the sun as two thirds ν_1 and one third ν_2 due to (quasi-) vacuum oscillations.

$f_1 = 65 \pm 2\%,\ f_2 = 35 \mp 2\%$ with $P_{ee} \approx 0.56$

The high energy ^{8}B Solar Neutrinos exit the sun as "PURE" ν_2 mass eigenstates due to matter effects.

$f_2 = 91 \pm 2\%$ and $f_1 = 9 \mp 2\%$ with $P_{ee} \approx 0.35$.





Testing solar neutrino oscillations with reactors

$$1 - P(\nu_e \to \nu_e) = \sin^2 2\theta_\odot \sin^2 \Delta$$

10^{-5} eV^2

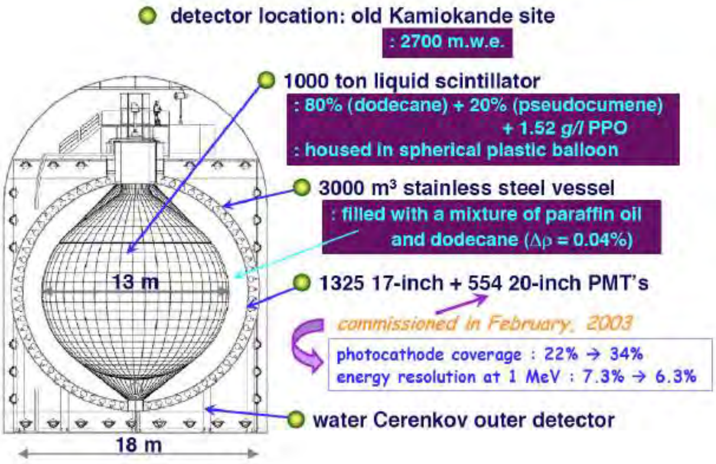
10^5m = 100 km

$$\Delta = \frac{\delta m^2 L}{4E}$$

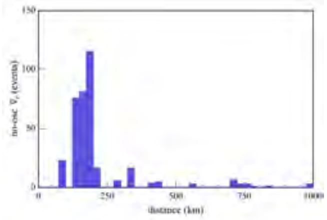
1 MeV

Reactor Neutrinos

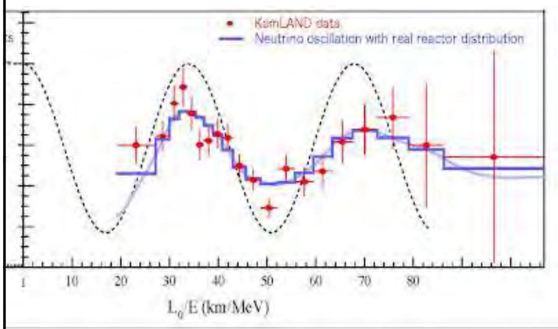
KamLAND Detector

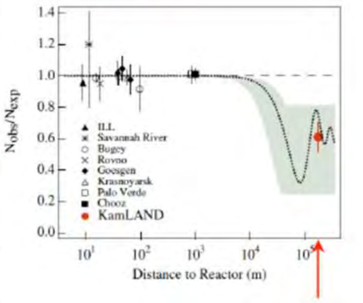


expected no-oscillation neutrino event rate at KamLAND



180 km





180 km

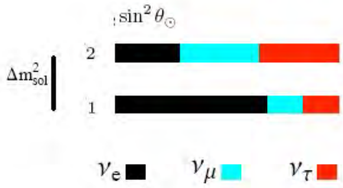
Summary:

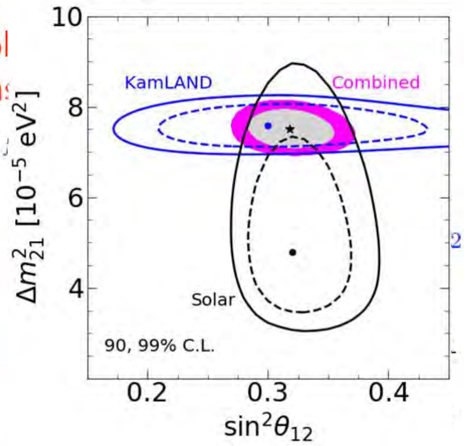
The low energy pp and ^{7}Be Solar Neutrinos exit the sun as two thirds ν_1 and one third ν_2 due to (quasi-) vacuum oscillations.

$f_1 = 65 \pm 2\%$, $f_2 = 35 \mp 2\%$ with $P_{ee} \approx 0.56$

The high energy ^{8}B Sol
"PURE" ν_2 mass eigen

$f_2 = 91 \pm 2\%$ and

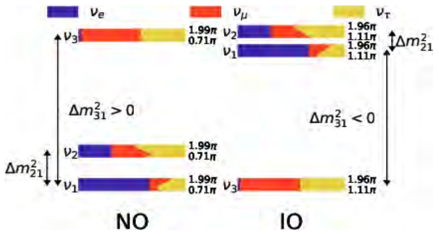




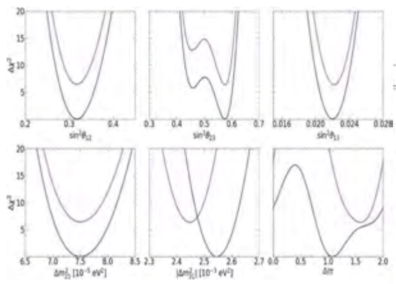
Three-neutrino oscillations

Neutrino mixing matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



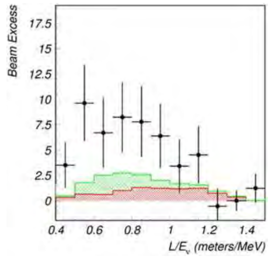
https://globalfit.astroparticles.es/

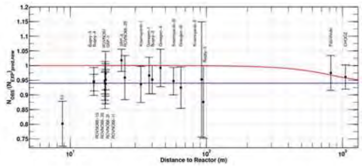


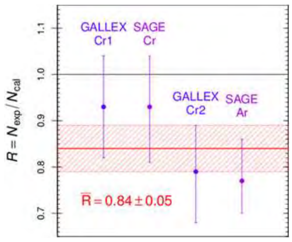
de Salas et al, JHEP 02 (2021) 071[arXiv:2006.11237]

parameter	best fit $\pm$ 1σ	2σ range	3σ range	Relative 1σ uncertainty
$\Delta m^2_{21}[10^{-5}\text{eV}^2]$	$7.50^{+0.22}_{-0.20}$	7.12–7.93	6.94–8.14	2.7% PRECISION
$\lvert\Delta m^2_{31}\rvert[10^{-3}\text{eV}^2]$ (NO)	$2.55^{+0.02}_{-0.03}$	2.49–2.60	2.47–2.63	1.1% ORDERING?
$\lvert\Delta m^2_{31}\rvert[10^{-3}\text{eV}^2]$ (IO)	$2.45^{+0.02}_{-0.03}$	2.39–2.50	2.37–2.53	
$\sin^2\theta_{12}/10^{-1}$	3.18 ± 0.16	2.86–3.52	2.71–3.69	5.2% PRECISION
$\sin^2\theta_{23}/10^{-1}$ (NO)	5.74 ± 0.14	5.41–5.99	4.34–6.10	5.1% OCTANT?
$\sin^2\theta_{23}/10^{-1}$ (IO)	$5.78^{+0.10}_{-0.17}$	5.41–5.98	4.33–6.08	
$\sin^2\theta_{13}/10^{-2}$ (NO)	$2.200^{+0.069}_{-0.062}$	2.069–2.337	2.000–2.405	3.0% PRECISION
$\sin^2\theta_{13}/10^{-2}$ (IO)	$2.225^{+0.064}_{-0.070}$	2.086–2.356	2.018–2.424	
δ/π (NO)	$1.08^{+0.13}_{-0.12}$	0.84–1.42	0.71–1.99	20% CPV?
δ/π (IO)	$1.58^{+0.15}_{-0.16}$	1.26–1.85	1.11–1.96	9.0%

Anomalies





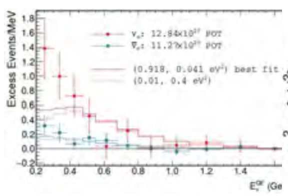


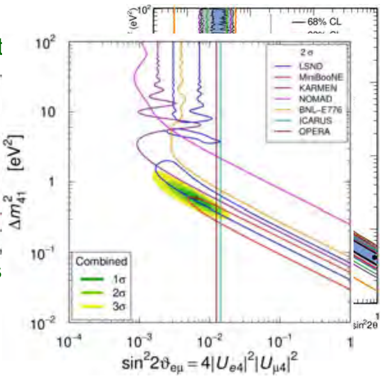
Need extra states !!!

MiniBooNE

MiniBooNE was built to check the LSND results with a different baseline, but similar L/E

MiniBooNE has no near det





MiniBooNE sees an excess at ~5σ at low energies

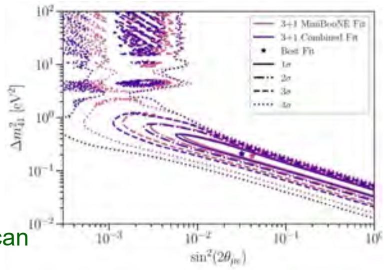
MicroBooNE

MicroBooNE was built to check the MiniBooNE results!

Looking for signals using several final state channels

The collaboration did not perform an oscillation analysis

A combined analysis shows that MicroBooNE can not exclude the region of parameter space preferred by MiniBooNE



2201.01724

The Gallium Anomaly

Tests of the solar neutrino detectors GALLEX (Cr1, Cr2) and SAGE (Cr, Ar)

ν_e Sources: $e^- + {}^{51}\mathrm{Cr} \to {}^{51}\mathrm{V} + \nu_e$ $\quad e^- + {}^{37}\mathrm{Ar} \to {}^{37}\mathrm{Cl} + \nu_e$

Detection Process: $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$

Deficit could be due to overestimate of $\sigma(\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-)$

Bahcall: [Bahcall, PRC 56 (1997) 3391, hep-ph/9710491]

$$\sigma({}^{51}\mathrm{Cr}) = 58.1 \times 10^{-46}\ \mathrm{cm}^2 \left(1^{+0.036}_{-0.028}\right)_{1\sigma} \implies \boxed{R_{\mathrm{Ga}} = 0.86 \pm 0.05}$$

[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]

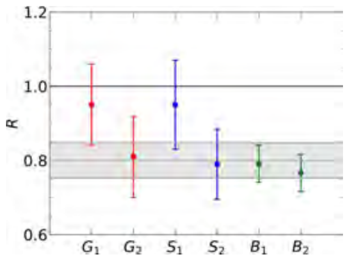
Haxton: [Hata, Haxton, PLB 353 (1995) 422, nucl-th/9503017; Haxton, PLB 431 (1998) 110, nucl-th/9804011]

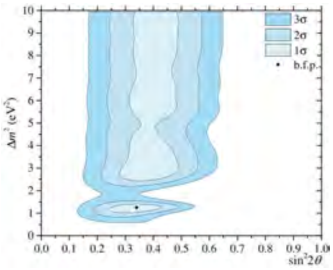
$$\sigma({}^{51}\mathrm{Cr}) = 63.9 \times 10^{-46}\ \mathrm{cm}^2\ (1 \pm 0.106)_{1\sigma} \implies \boxed{R_{\mathrm{Ga}} = 0.76^{+0.09}_{-0.08}}$$

[SAGE, PRC 59 (1999) 2246, hep-ph/9803418]

The Gallium anomaly

BEST coll., 2109.11482





The Gallium anomaly is now at more than 5σ significance