Hadronic contribution to a_μ^{HVP} and the R-ratio

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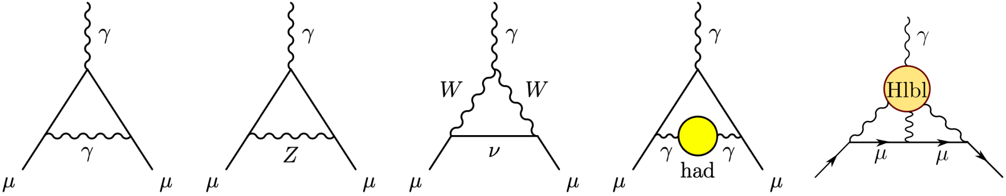
the response of a charged lepton to a static and uniform e.m. field is encoded in

$$\langle \ell(p_2)|J^{\nu}_{\text{em}}(0)|\ell(p_1)\rangle = -ie\bar{u}(p_1)\Gamma^{\nu}(p_1,p_2)u(p_2)$$

where the structure of the vertex Γ^{ν} is constrained by symmetries

$$\Gamma^{\nu}(p_1,p_2) = F_1(k^2)\gamma^{\nu} + \frac{i}{2m_\mu}F_2(k^2)\sigma^{\nu\rho}k_\rho + \text{P-violating terms}$$

$$a_\mu = \frac{g_\mu - 2}{2} = F_2(0)$$



largest contribution
computed up to 5–loops
[T. Aoyama et al. – PRL 109, 111807 – 2012]

small contribution
2–loops suppressed by m_W^{-2}
[Czarnecki et al. PRD (2006), Gnendiger et al. PRD (2013)]

α_{em}^2 α_{em}^3
non–perturbative contribution

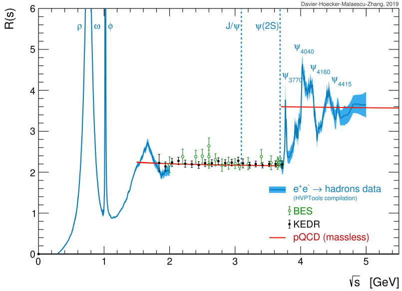
dispersive analysis

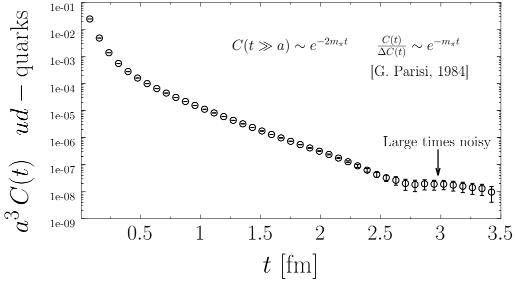
$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha_{\mathrm{em}}}{\pi}\right)^2 \int_{M_\pi^2}^{\infty} \mathrm{d}E\, \tilde{K}(E) R(E) \qquad R(E) = \frac{\sigma\left(e^+e^- \to \text{hadrons}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}$$

lattice analysis

$$a_\mu^{\mathrm{HVP}} = 2\alpha_{\mathrm{em}}^2 \int_0^{\infty} \mathrm{d}t\, t^2 K(m_\mu t) C(t) \qquad K(m_\mu t) \sim \begin{cases} t^2 & \text{if} \quad t \ll m_\mu^{-1} \\ 1 & \text{if} \quad t \gg m_\mu^{-1} \end{cases}$$

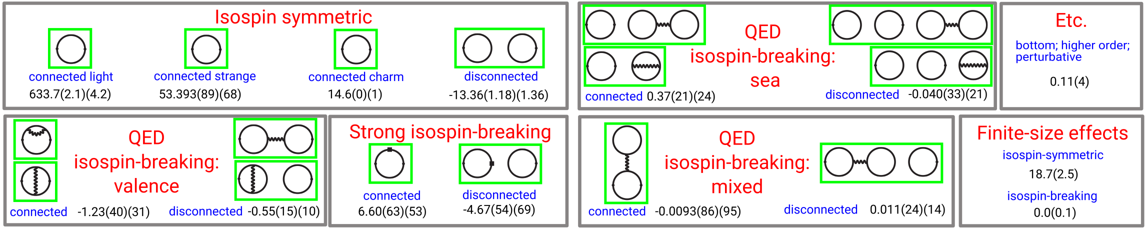
$$C(t) = \frac{1}{3} \int \mathrm{d}^3\boldsymbol{x} \sum_{i=1}^{3} \left\langle J_{\mathrm{em}}^i(\boldsymbol{x}, t) J^{i\,\dagger}_{\mathrm{em}}(\boldsymbol{0}, 0) \right\rangle \qquad J_{\mathrm{em}}^i = \frac{2}{3}\bar{u}\gamma^i u - \frac{1}{3}\bar{d}\gamma^i d - \frac{1}{3}\bar{s}\gamma^i s + \frac{2}{3}\bar{c}\gamma^i c$$





what about the full $a_\mu^{\mathrm{HVP-LO}}$?

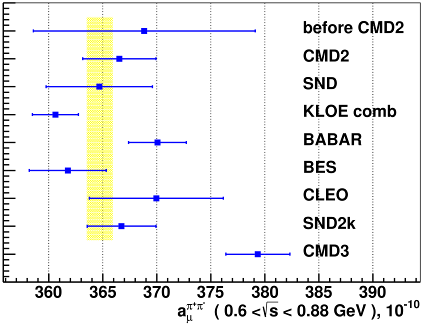
- signal to noise problem at large times.
- Large lattice volumes $V = L^3$ required to fit the light $\pi\pi$ states $\rightarrow$ avoid finite size effects (FSE).
- Leading Isospin-Breaking (LIB) effects $\mathcal{O}(\alpha_{\mathrm{em}}), \mathcal{O}\big((m_d - m_u)/\Lambda_{\mathrm{QCD}}\big) \rightarrow$ target accuracy 0.5%.

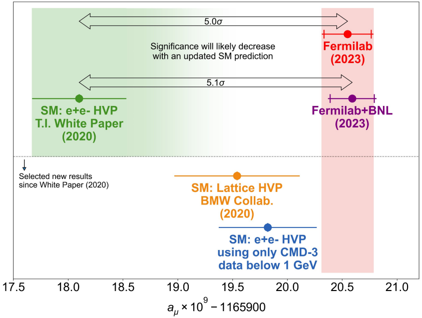


$10^{10} \times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$

[Sz. Borsanyi et al. – Nature 593, 51-55 – 2021]

- low–modes (dominating at large times) deflation to solve the signal to noise problem
- 2 different volumes to control and fit finite volume effects ($L \simeq 6$ fm and $L \simeq 11$ fm)
- LIB effects computed a la RM123 [G.M. de Divitiis et al. – PRD 87, 114505 – 2013]



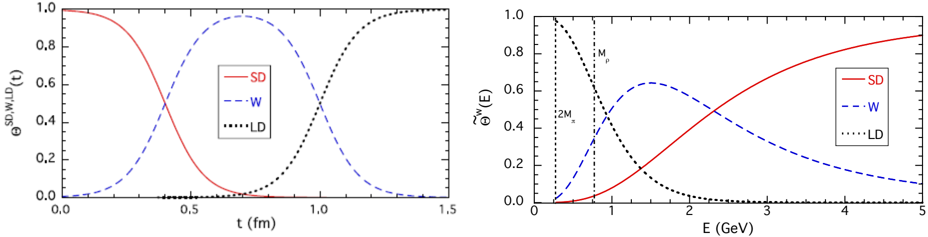


- the new CMD3 result increase the already present tension with previous measurements [F. Ignatov et al, arXiv:2302.08834]
- If confirmed it will drammatically reduce the strength of the a_μ anomaly, see e.g. conclusion of Muon g-2 Coll. – PRL 131, 161802 – 2023

$$\downarrow$$

At the moment the situation of exp. $e^+e^- \to$ hadrons needs to be clarified.

- short distance (SD) $\longrightarrow$ high energies: large lattice artifacts but excellent signal
- intermediate distance (W) $\longrightarrow$ $E \sim M_\rho$: controlled lattice artifacts and still good signal
- long distance (LD) $\longrightarrow$ low energies: small lattice artifacts but bad signal

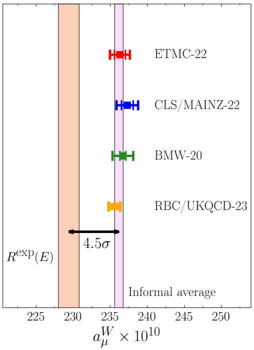


$$a_\mu^{\mathrm{w}} = 2\alpha_{\mathrm{em}}^2 \int_0^\infty \mathrm{d}t\; t^2 K(m_\mu t)\Theta^{\mathrm{w}}(t)\, C(t)\,, \qquad \mathrm{w} = \{\mathrm{SD, W, LD}\}$$

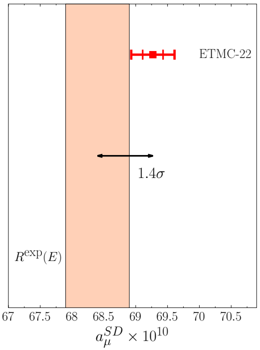
where the time-modulating function $\Theta^{\mathrm{w}}(t)$ is given by

$$\Theta^{\mathrm{SD}}(t) = 1 - \frac{1}{1+e^{-2(t-t_0)/\Delta}} \quad \Theta^{\mathrm{W}}(t) = \frac{1}{1+e^{-2(t-t_0)/\Delta}} - \frac{1}{1+e^{-2(t-t_1)/\Delta}} \quad \Theta^{\mathrm{LD}}(t) = \frac{1}{1+e^{-2(t-t_1)/\Delta}}$$

intermediate-distance $\Longrightarrow$ $E \lesssim 1$ GeV $(\pi\pi, \pi\pi\pi)$



short-distance $\Longrightarrow$ Large E $\gtrsim$ 1GeV



[C. Alexandrou et al. – PRD 107, 074506 – 2023]

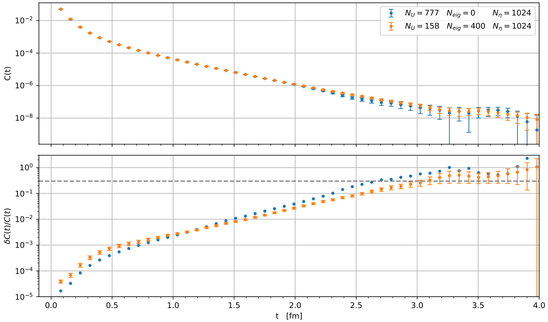
- Several lattice results (for individual flavor contributions more results available) all in agreement
  - Lattice results extremely solid: various groups use very different UV regularizations and simulation setups.
  - Striking $\sim 4.5\sigma$ tension with the data–driven $a_\mu^{\rm W}$ suggesting strong deviation of $R^{\rm exp}(E)$ from SM$^{\rm (LQCD)}$ at $E \sim m_\rho$.

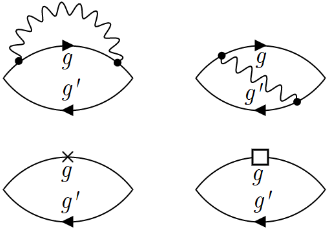
- $R^{\rm exp}(E)$ at high-energy in line with SM$^{\rm (LQCD)}$ $\longrightarrow$ EW precision tests not affected by the low-energy tension.

ETMC working on

- new generation of light–light flavour correlators with state of the art noise reduction techniques $\rightarrow$ reduce statistical errors by a factor $\sim 4 - 5$
- Leading Isospin–Breaking effects:
  - QED effects $\rightarrow$ numerical derivatives w.r.t. e^2
  - strong isospin effects $\rightarrow$ scalar operator insertions

  are evaluated via RM123 method after tuning action counterterms in order to match QCD+QED to the physical world





- R–ratio is the spectral desity of the vector-vector correlator

$$C(t) = \frac{1}{12\pi^2} \int_0^\infty \mathrm{d}E\; E^2 e^{-Et} R(E)$$

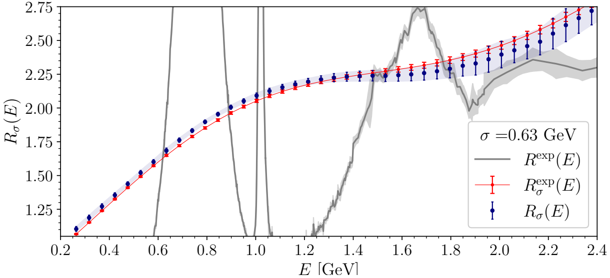
- inverting the previous relation to obtain $R(E)$ is a numerically ill-posed problem
  - lattice correlators are affected by statistical uncertainty and are evaluated only on a finite set of points
  - on a finite lattice, spectral densities are distributions

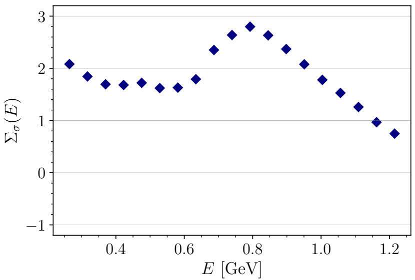
- energy smearing $R(E) \to R_\sigma(E)$ makes well-posed the inverse problem

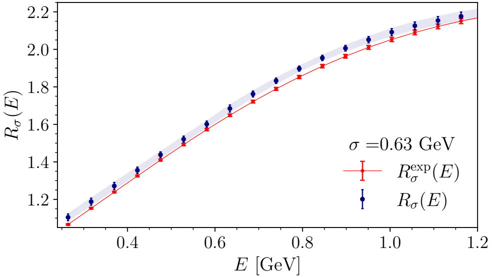
Hansen–Lupo–Tantalo method allows us to compute energy–smeared R–ratio

[M.Hansen PRD 99, 094508 – 2019, C. Alexandrou et al. – PRL 130, 241901 – 2023]

$$R_\sigma(E) = \int_0^\infty \mathrm{d}\omega\, G_\sigma(E,\omega)\; R(\omega) \qquad G_\sigma(E,\omega) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\omega - E)^2}{2\sigma^2}\right)$$







R-ratio results (with no LIB effects for now):

- $\sim 3\sigma$ tension at low–energy with $R^{\rm exp}(E)$
- result emphasized by CMD3 [arXiv:2302.08834]
- possibly NP? Same message as from $a_\mu^{\rm W}$

Outlook and conclusions:

- our numerical results for the window observables are competitive and in agreement with those from other lattice groups
- new CMD3 results increase the tension among experimental measures and question their overall credibility

on our side:

- improvement of statistical precision of lattice correlators
- computation of LIB effects a la RM123

$\downarrow$

compute both the full $a_\mu^{\rm LO-HVP}$ and R-ratio with a target accuracy of few per mill including LIB effects

while waiting for

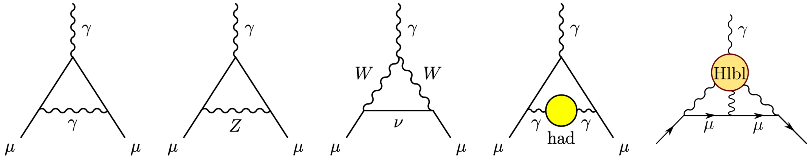
clarification of experimental R-ratio puzzle & confirmation of L-QCD+QED results for a_μ and R-ratio

Are the two puzzles of $R_{\rm had}(E)$ and of $g_\mu - 2$ the two sides of a unique coin?

- Very likely yes!
  if the problem stems from experimental studies (e.g. full inclusion of FSR only) of R_{had} at low E

- Unclear or likely no!
  if the experimental values of $R_{had}(E)$ in WP 2020 & 2205.12963 will be confirmed
  different options: e.g. [Di Luzio et al. – PLB 829 137037 – 2022 L. Darmé et al. – JHEP 06 122 – 2022]

note:
if a NP theory yields an effect ($\leftrightarrow$ a $d > 4$ term in the SM–EFT description) giving for R_{had} at $E < 1.5$ GeV a lower value than the SM, then dispersion relations imply a lower value than the SM for a_μ^{HVP-LO} too; unless the NP theory produces other effects ($\leftrightarrow$ another $d > 4$ term) that increase a_μ^{HVP-LO}: **an intriguing scenario!**



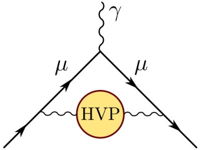
+ NP diagrams

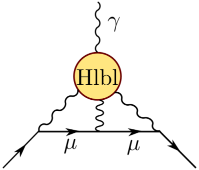
Thanks for the attention!

Backup

$$a_\mu^{\rm had} = \underbrace{a_\mu^{\rm HVP-LO}}_{\mathcal{O}(7\times10^{-8})} + \underbrace{a_\mu^{\rm Hlbl}}_{\mathcal{O}(10^{-9})} + \underbrace{a_\mu^{\rm HVP-NLO}}_{\mathcal{O}(10-9)} + \underbrace{a_\mu^{\rm HVP-NNLO}}_{\mathcal{O}(10-10)}$$

HVP-LO $\Longrightarrow$





$\Longleftarrow$ Hlbl

HVP–NLO, HVP–NNLO and Hlbl subdominat as source of uncertainty.

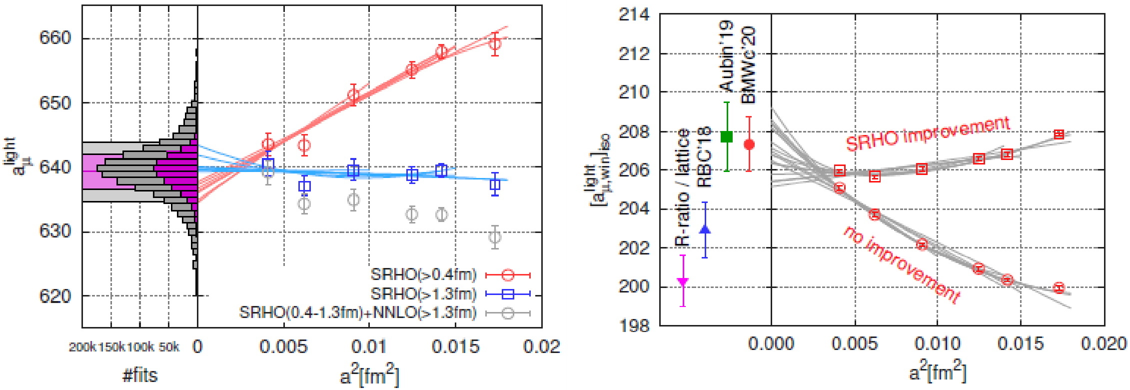
Dispersive approach:

- Relates full $a_\mu^{\rm HVP-LO}$ to $e^+e^- \to$ hadrons cross-section via optical theorem.
- For Hlbl (only) low-lying intermediate-states contributions can expressed in terms of transition form-factors TFFs.

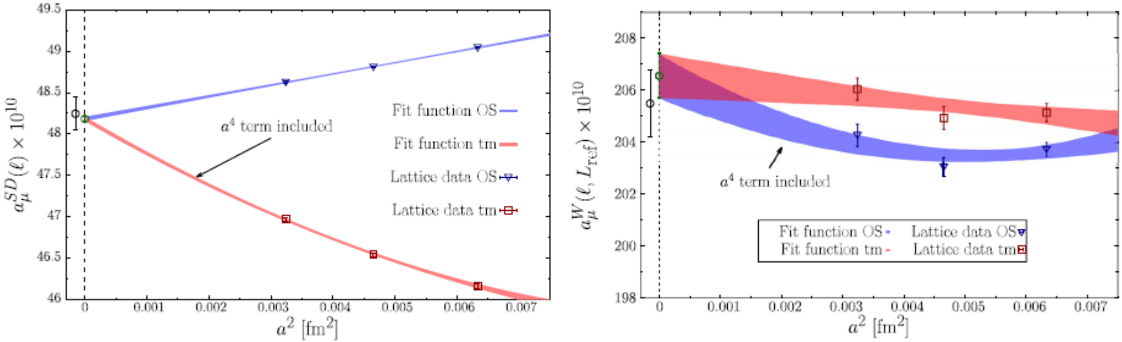
Lattice QCD:

- Only known first-principles SM method to evaluate both $a_\mu^{\rm HVP}$ and $a_\mu^{\rm Hlbl}$.
- In the past the accuracy of the predictions were not good enough. The situation has recently changed.

BMW continuum limits for the full a_μ^{light} and for the window quantity



ETMC continuum limits for a_μ^{light} in the SD and the window quantities



$$a_\mu^{\rm HVP} = \left(\frac{\alpha_{\rm em}}{\pi}\right)^2 \int_0^\infty {\rm d}Q^2\, \frac{1}{m_\mu^2}\, f\left(\frac{Q^2}{m_\mu^2}\right) \hat{\Pi}\left(Q^2\right)$$

$$\hat{\Pi}\left(Q^2\right) = 4\pi \left(\Pi(Q^2) - \Pi(0)\right) \qquad \Pi_{\mu\nu}\left(Q\right) = \left(Q_\mu Q_\nu - \eta_{\mu\nu} Q^2\right) \Pi(Q^2)$$

is vacuum polaritazion taht can be related to the R-ration through

$$\hat{\Pi}\left(Q^2\right) = \frac{Q^2}{3} \int_0^\infty {\rm d}s\, \frac{R(s)}{s\left(s+Q^2\right)} \qquad \sigma(e^+e^- \to \text{hadrons}) = \frac{4\pi\alpha_{\rm em}^2}{3s} R(s)$$

$$K(z) = 2\int_0^1 {\rm d}y\,(1-y)\left[1 - j_0^2\left(\frac{z}{2}\frac{y}{\sqrt{1-y}}\right)\right] \qquad j_0(y) = \frac{\sin(y)}{y}$$

$$\frac{G_\sigma\left(E,\omega\right)}{\omega^2} \sim \sum_{n=1}^{T} g_n(\sigma,E) e^{-\omega t_n}$$

$$R_\sigma(E) \sim \sum_{n=1}^{T} g_n(\sigma,E) \int_0^\infty {\rm d}\omega\, e^{-\omega t_n} \omega^2\, R(\omega) = \sum_{n=1}^{T} g_n(\sigma,E) C(t_n)$$