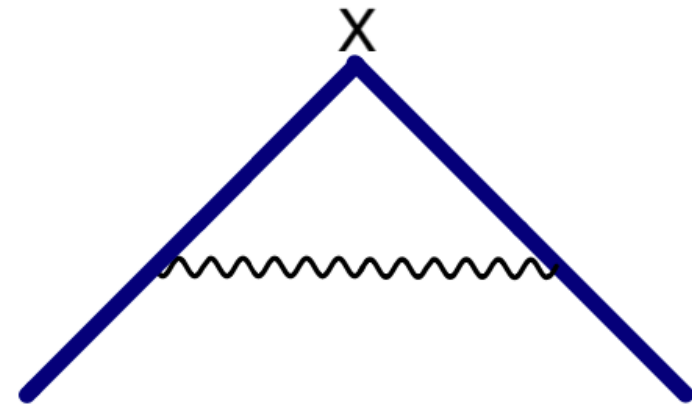


Roma, 15/2/24

ENP: attività' Giancarlo D'Ambrosio e Luigi Cappiello

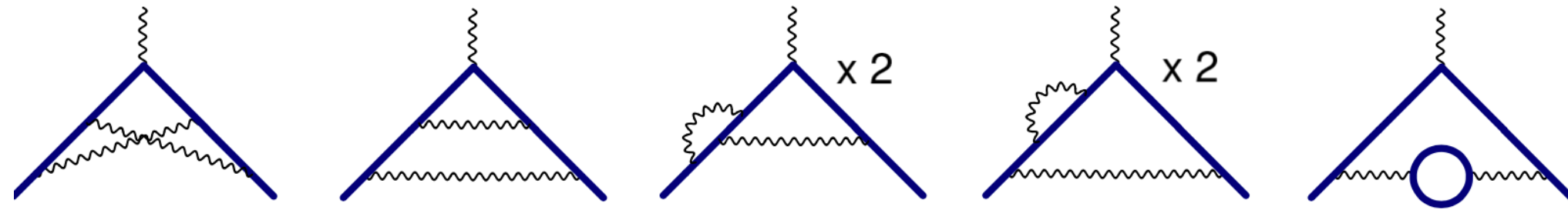
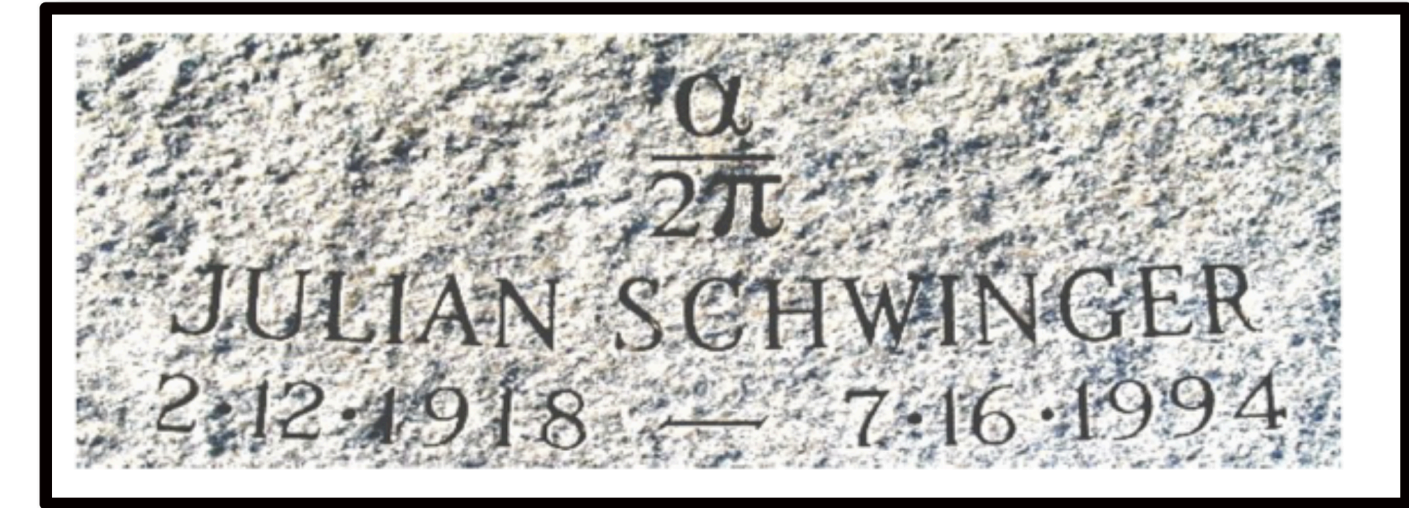
g-2: contributo hlbl adronico con l olografia e LFUV nei K

Muon g-2: the QED contribution



Schwinger 1948

$$\frac{1}{2} \frac{\alpha}{\pi}$$



$$\left\{ \frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2} \ln 2 + \frac{3}{4} \zeta(3) \right\} \left(\frac{\alpha}{\pi} \right)^2$$

0.328 478 444 002 89(60)

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; Passera 04

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek '99; Passera '04

Friot, Greynat & de Rafael '05, Ananthanarayan, Friot, Ghosh 2020

$$+24.05050988(28) \left(\frac{\alpha}{\pi} \right)^3$$

..

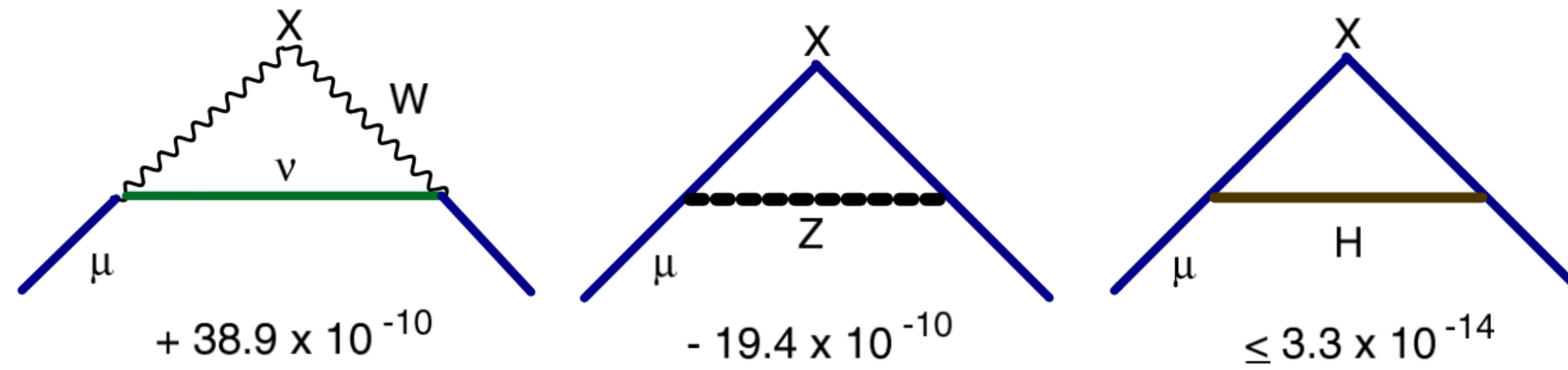
$$+750.86(88) \left(\frac{\alpha}{\pi} \right)^5$$

Kinoshita et al. '90, Remiddi, Laporta

$$116584718.9(1) \times 10^{-11} \quad 0.001 \text{ ppm}$$

EW contribution to g-2 muon

from Eduardo de Rafael



$$a_{\mu}^{EW} \sim \frac{G_F}{\sqrt{2}} \frac{m_{\mu}^2}{8\pi^2} \left[\frac{5}{3} + \frac{1}{3} (1 - 4 \sin^2 \theta_W)^2 \dots \right] = (153.6 \pm 1.0) \times 10^{-11}$$

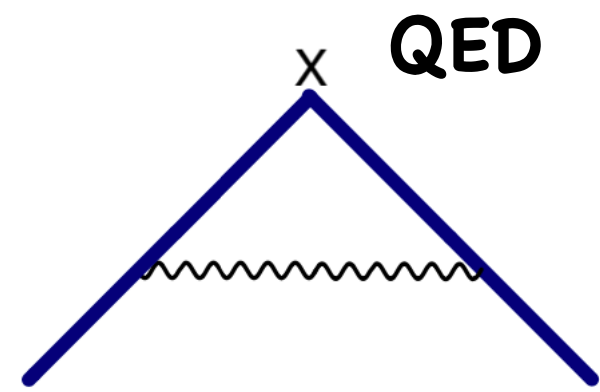
New Physics : naive scaling

$$\frac{\delta a_l^{NP}}{a_l} \sim \frac{m_l^2}{M_{NP}^2} \quad l = e, \mu, \tau$$

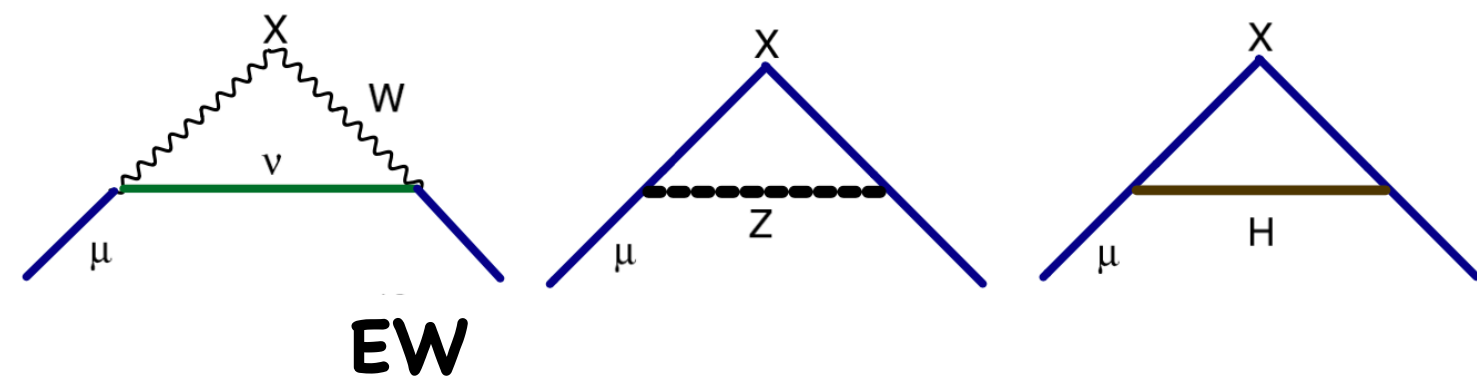
$$a_{\mu}^{SUSY} \approx \frac{g^2 m_{\mu}^2 \tan \beta}{32\pi^2 \tilde{m}^2} \approx 2 \times 10^{-9}$$

$\tilde{m} = 500 \text{ GeV} \ \& \ \tan \beta = 40$

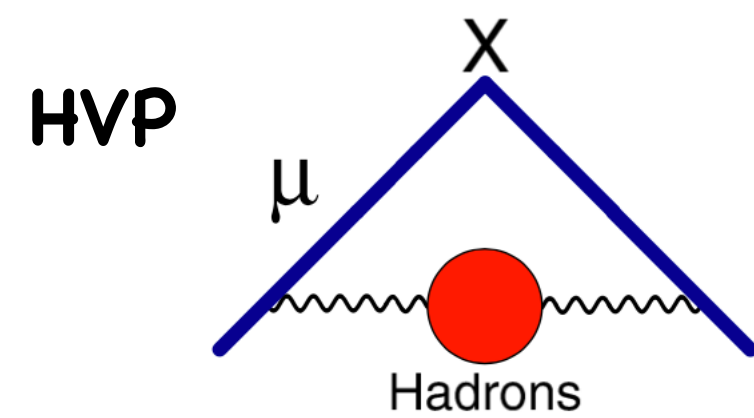
Contributions from known particles: The Standard Model



$$116584718.9(1) \times 10^{-11} \quad 0.001 \text{ ppm}$$

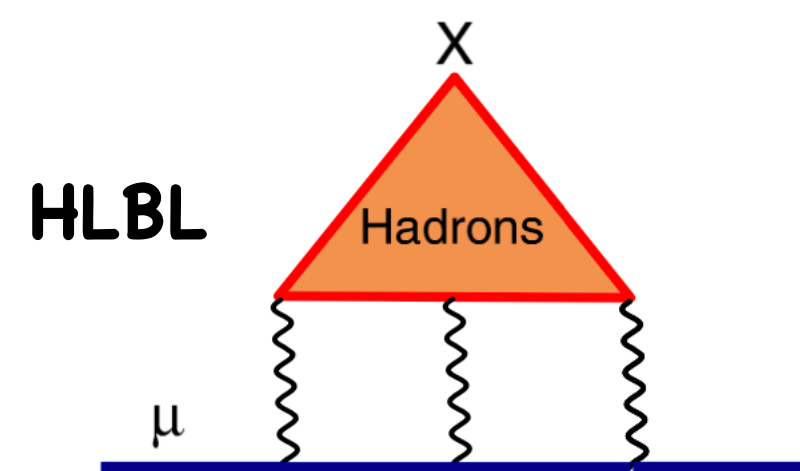


$$153.6(1.0) \times 10^{-11} \quad 0.01 \text{ ppm}$$



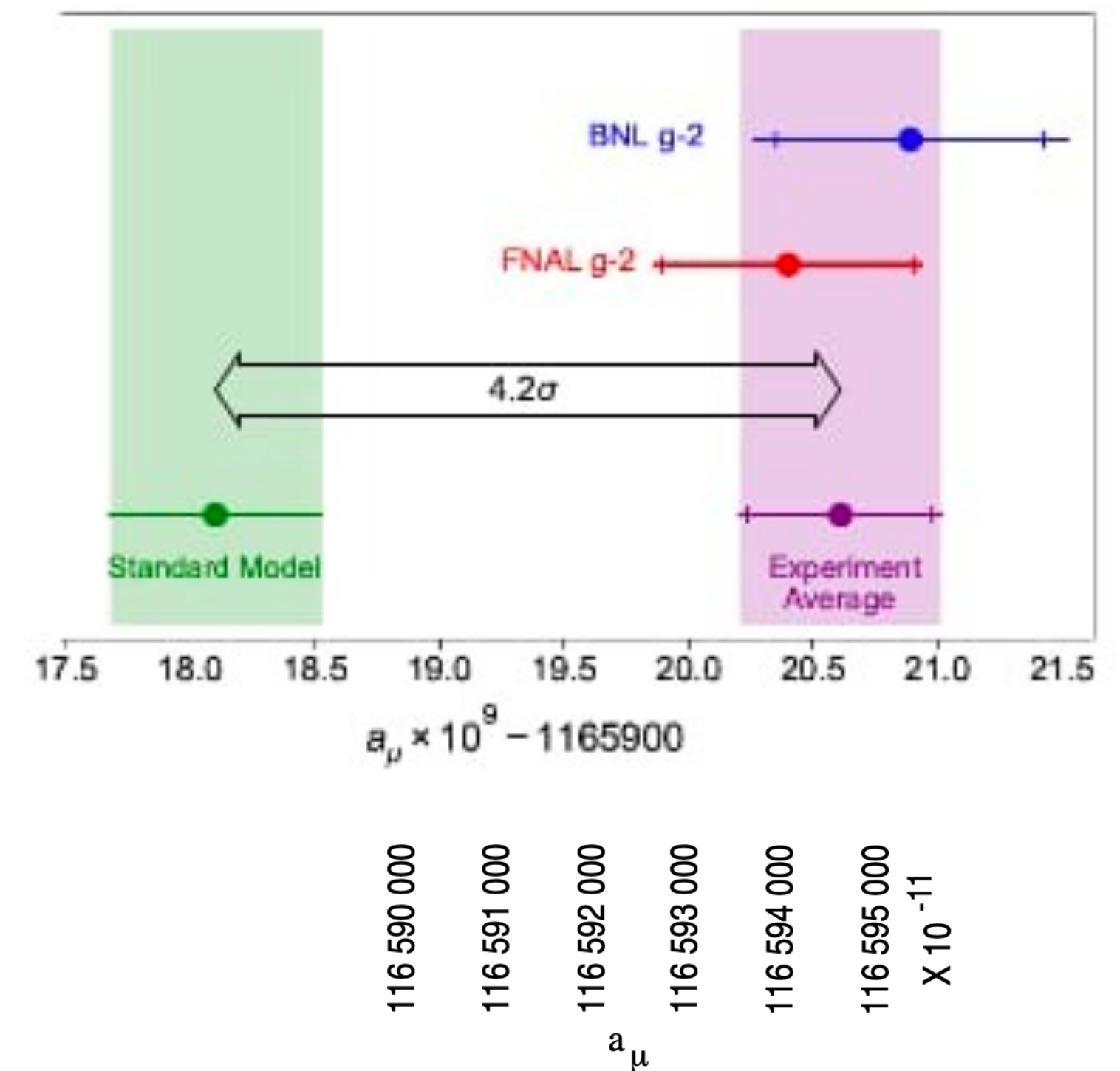
$$6845(40) \times 10^{-11} \quad 0.01 \text{ ppm}$$

[0.6%]



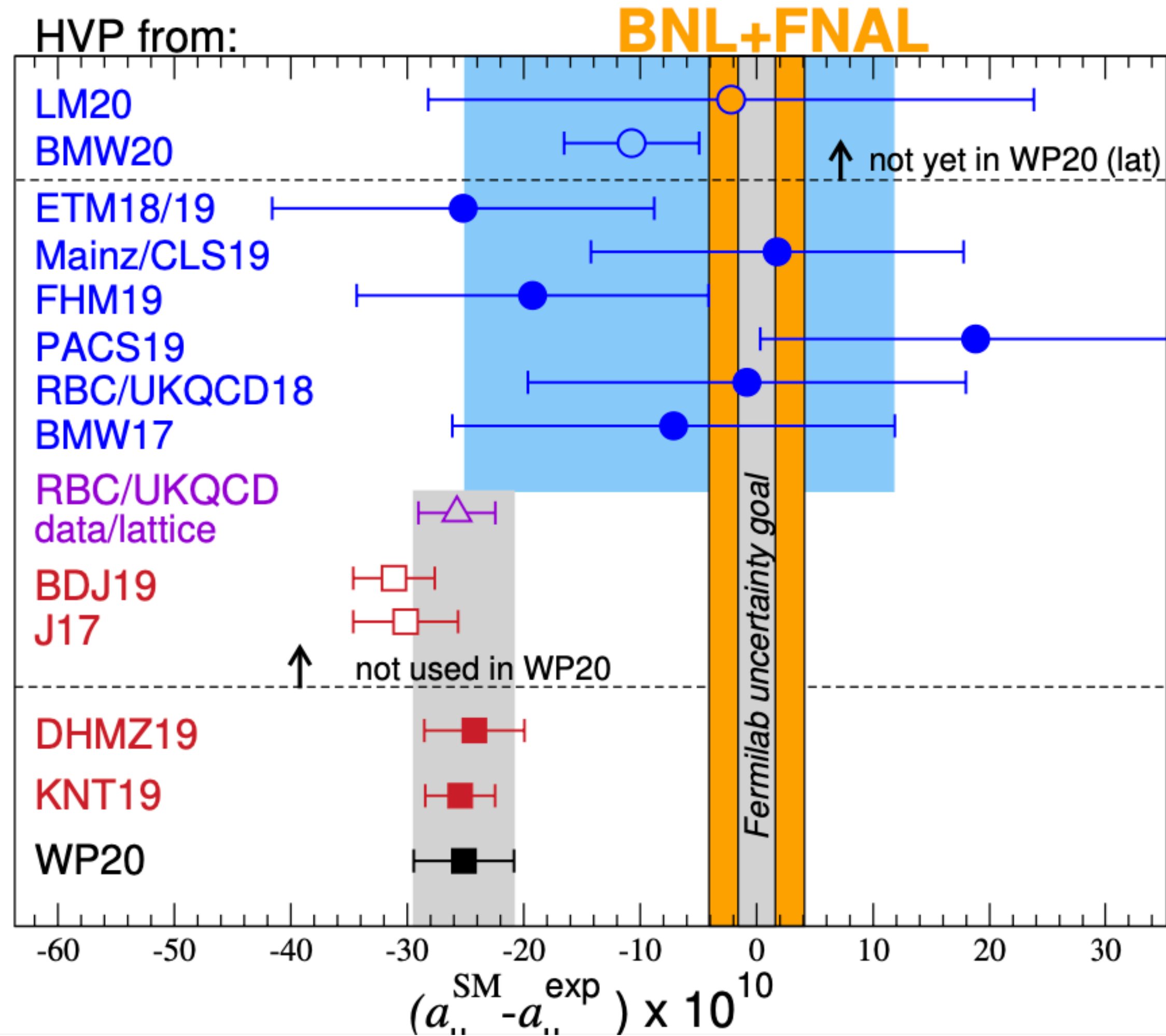
$$92(18) \times 10^{-11} \quad 0.15 \text{ ppm}$$

[20%]



Present status of $(g - 2)_\mu$: experiment vs SM

After the Fermilab result





Laboratori Nazionali di Frascati

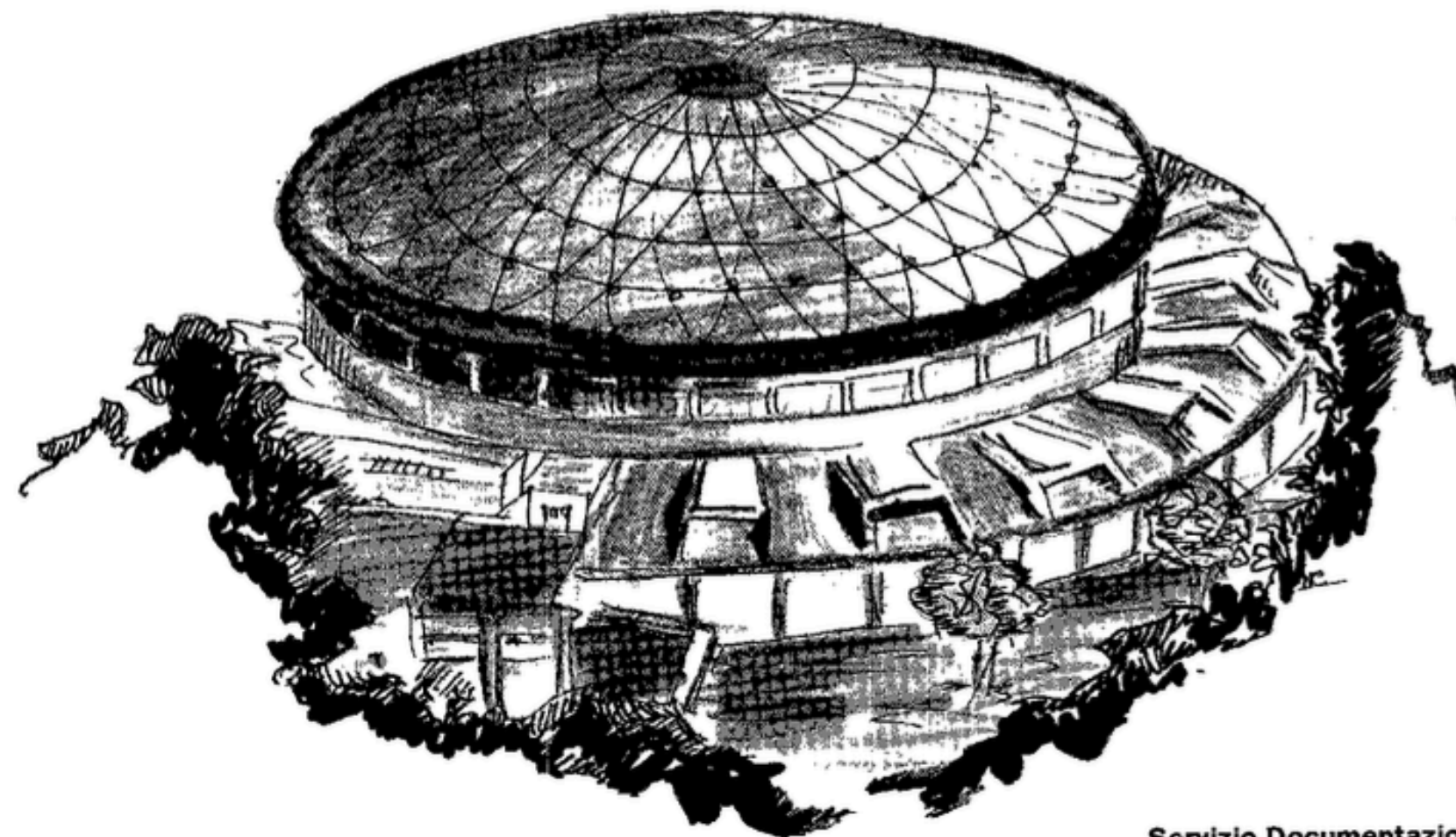


LNF-91/020 (R)
3 Maggio 1991

LNF-90/041 (R)
29 Maggio 1990

R. Barbieri, L. Maiani, G. Martinelli, L. Paoluzi, N. Paver, R. Petronzio, E. Remiddi:

REPORT FROM THE Φ FACTORY WORKING GROUP

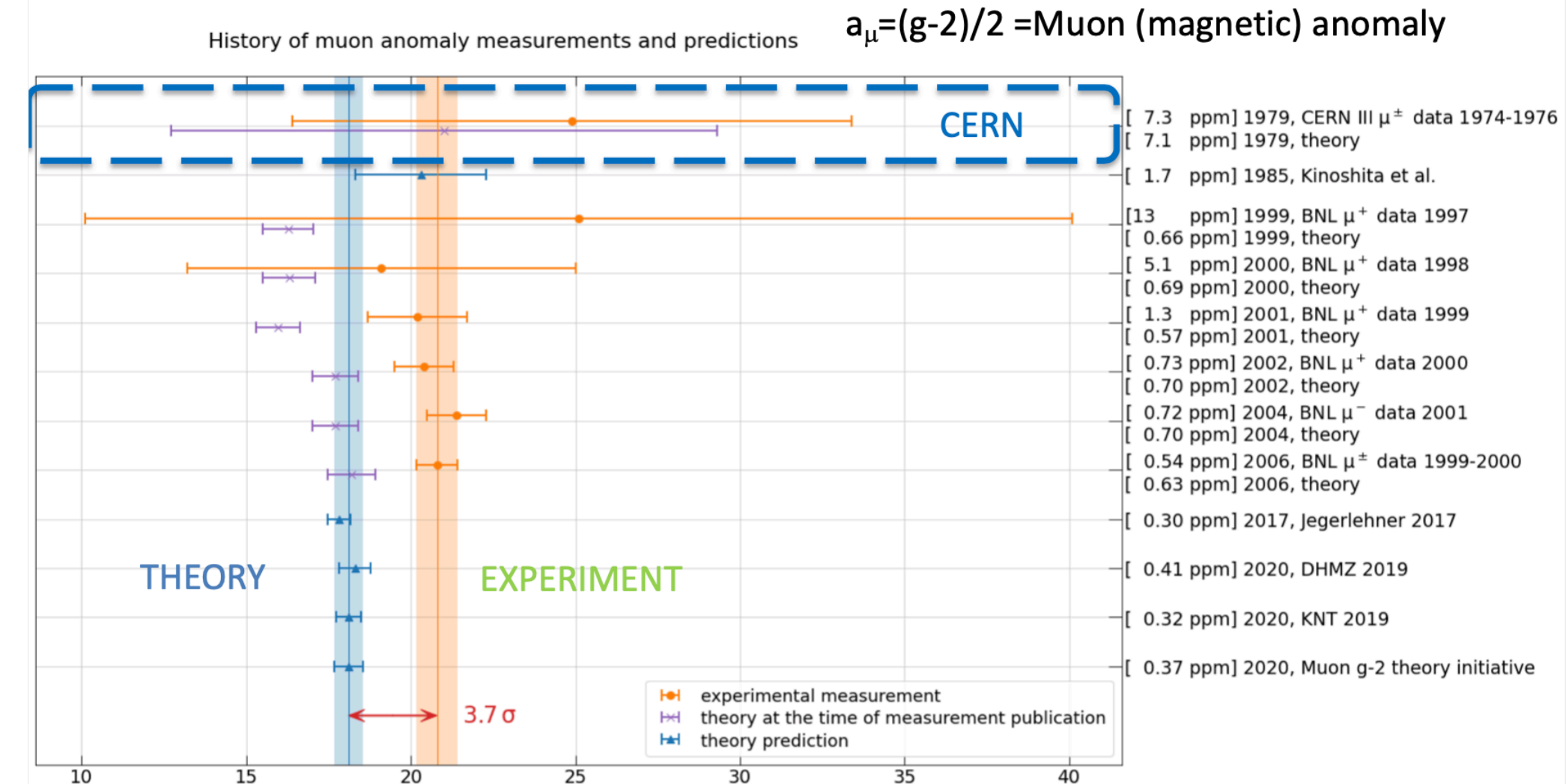


Servizio Documentazione
dei Laboratori Nazionali di Frascati
P.O. Box, 13 - 00044 Frascati (Italy)

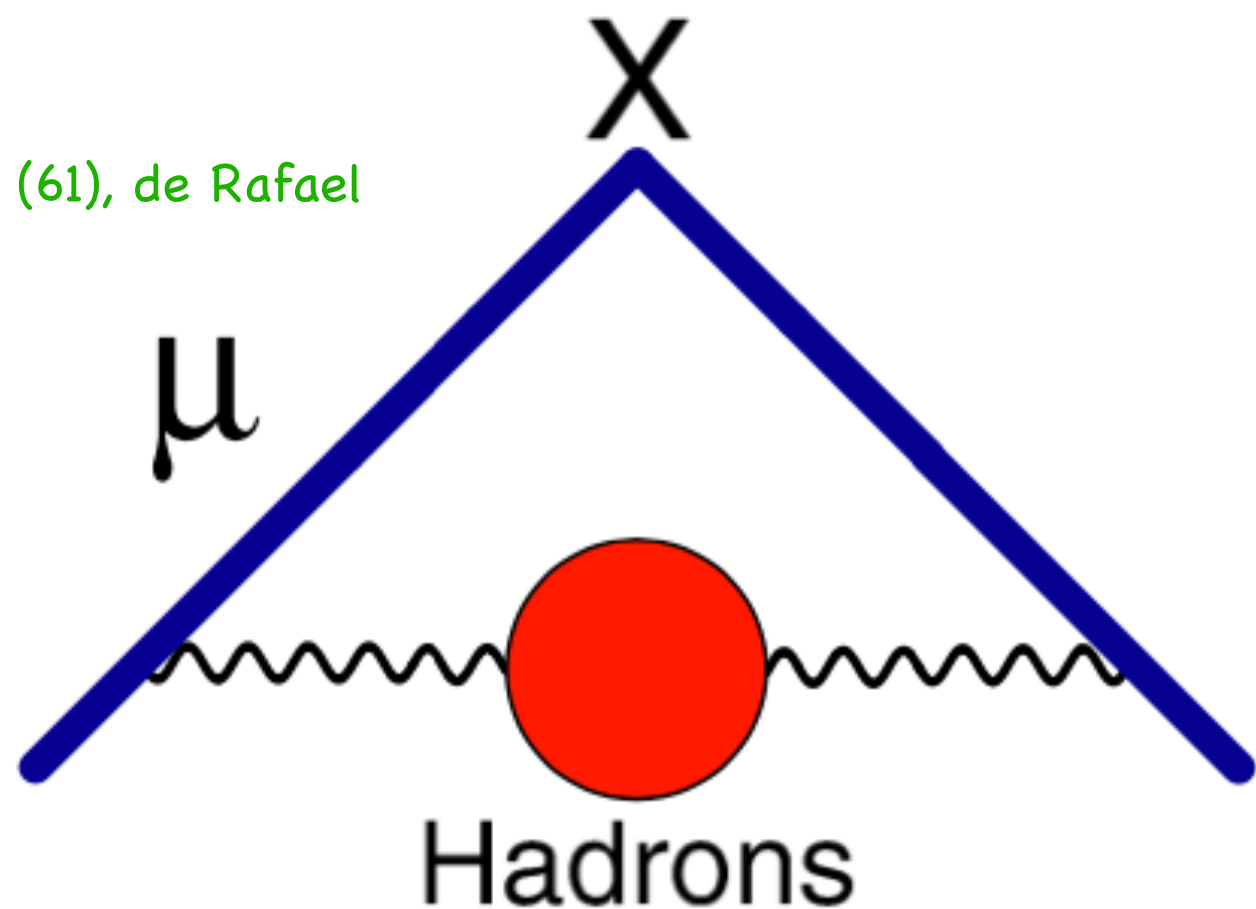
ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE - ISTITUTO NAZIONALE DI FISICA NUCLEARE

After CERN but before Brookhaven

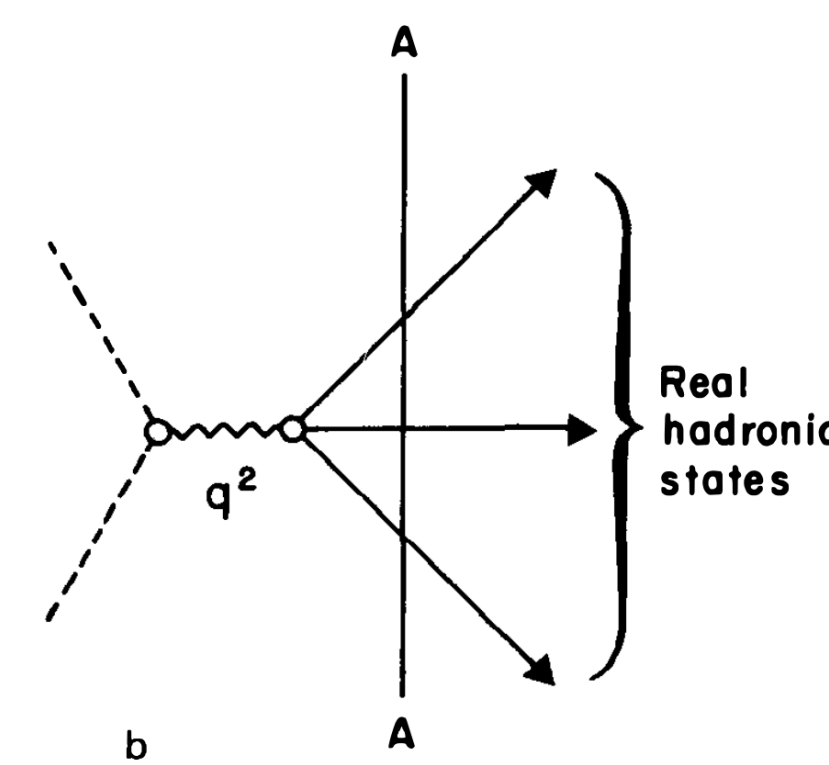
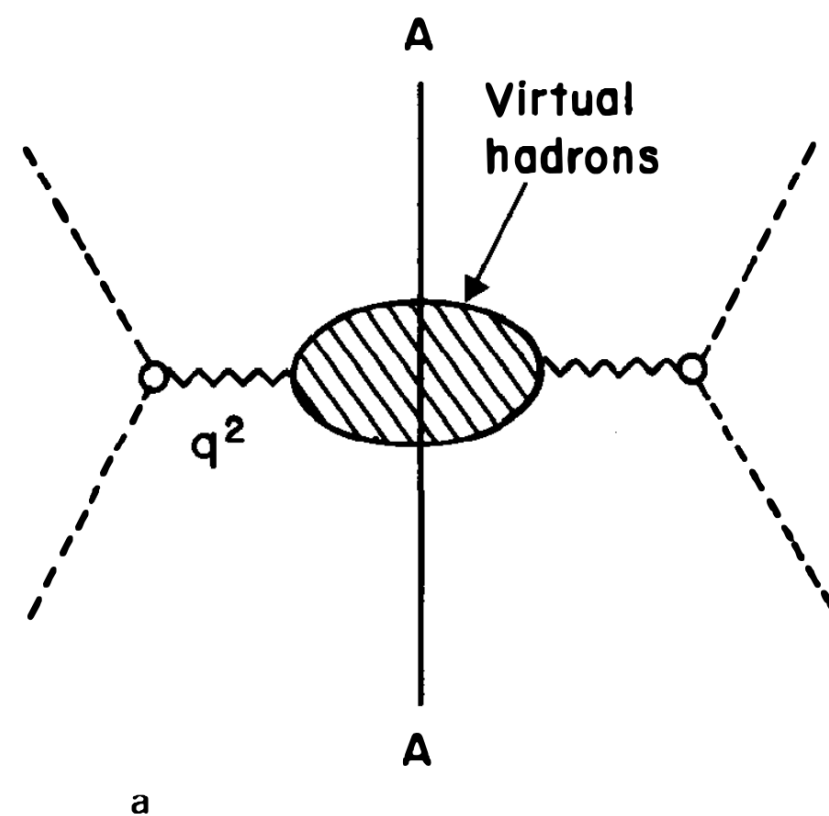
How good we know hadronic contributions?



Bouchiat, Michel (61), de Rafael



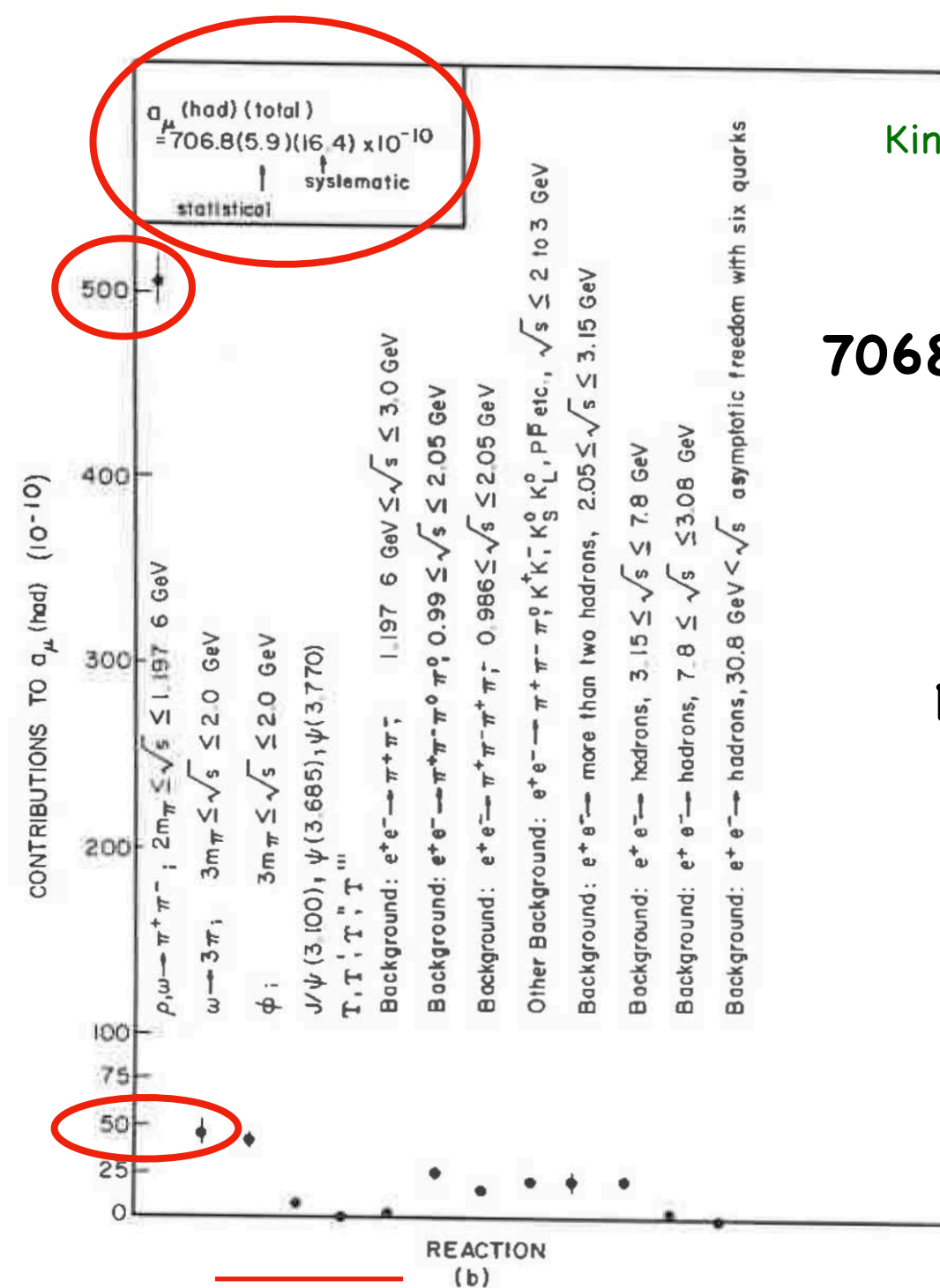
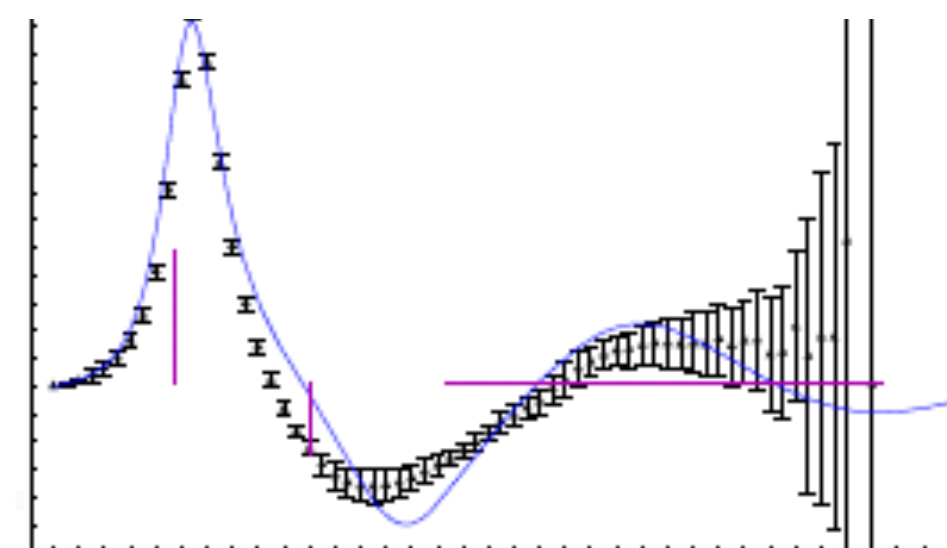
Strong contribution to vacuum polarization



Hadron production (measurable) dispersion theory relates $\sigma(q^2)$ to vacuum polarization

$$a_{\mu}^{(h. v.p.)} = \frac{\alpha}{\pi} \int_0^{\infty} \frac{dt}{t} \frac{1}{\pi} \text{Im}\Pi(t) \int_0^1 dx \frac{x^2(1-x)}{x^2 + \frac{t}{m_{\mu}^2}(1-x)},$$

$$\sigma(t)_{e^+e^- \rightarrow \text{hadrons}} = \frac{4\pi^2\alpha}{t} \frac{1}{\pi} \text{Im}\Pi(t)$$

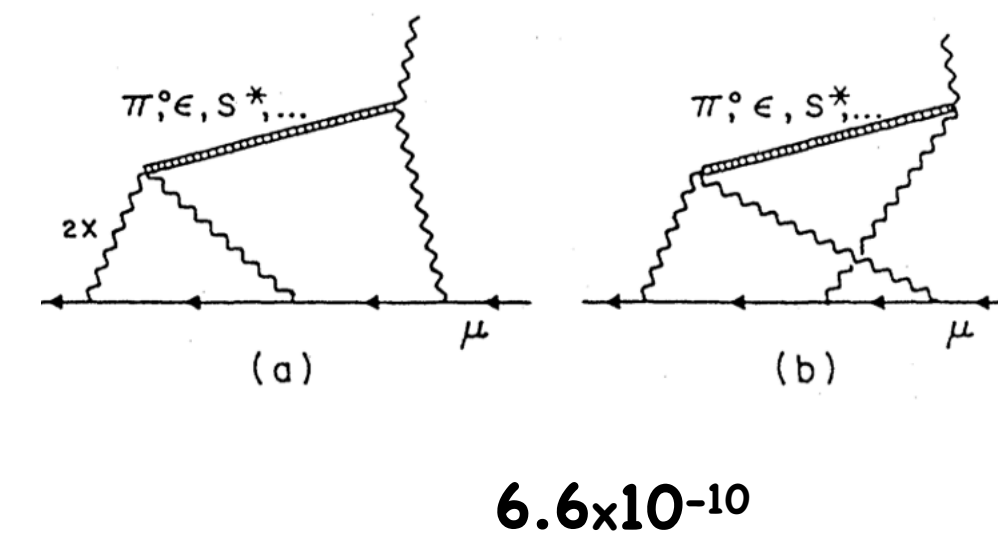
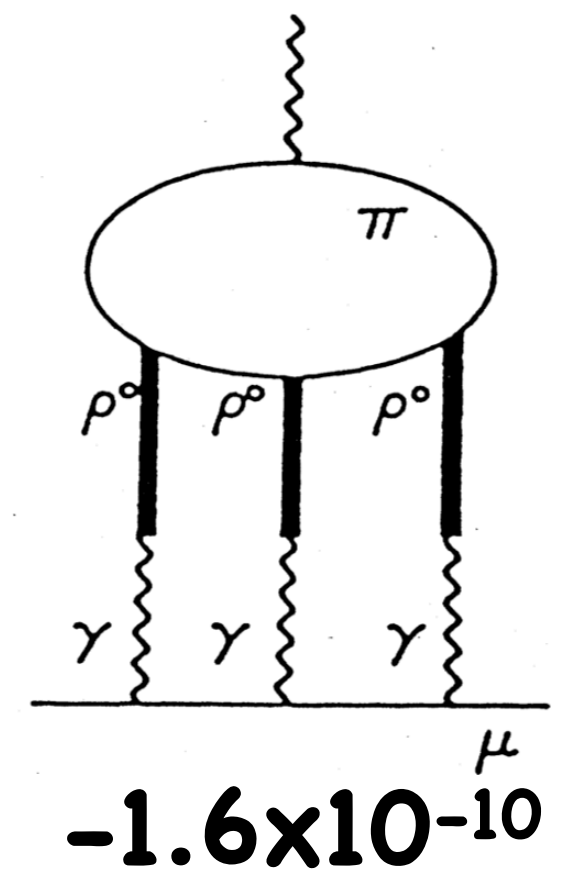
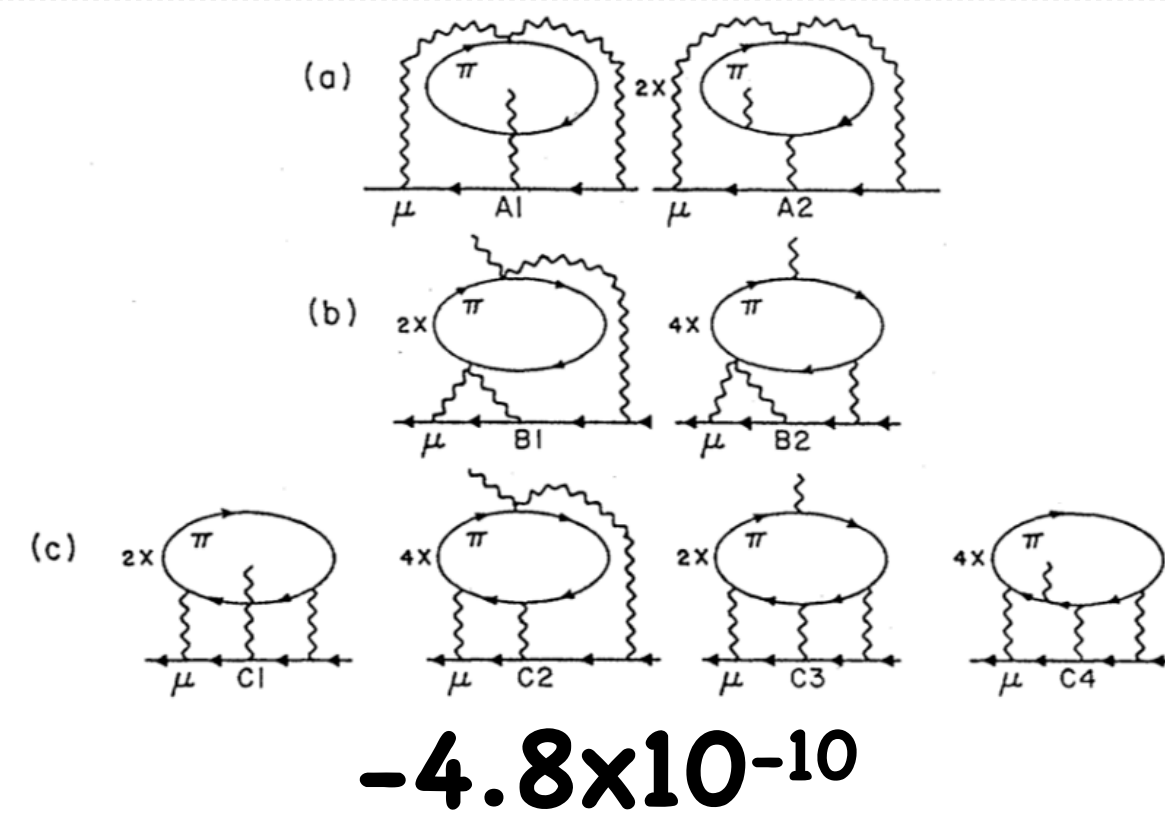
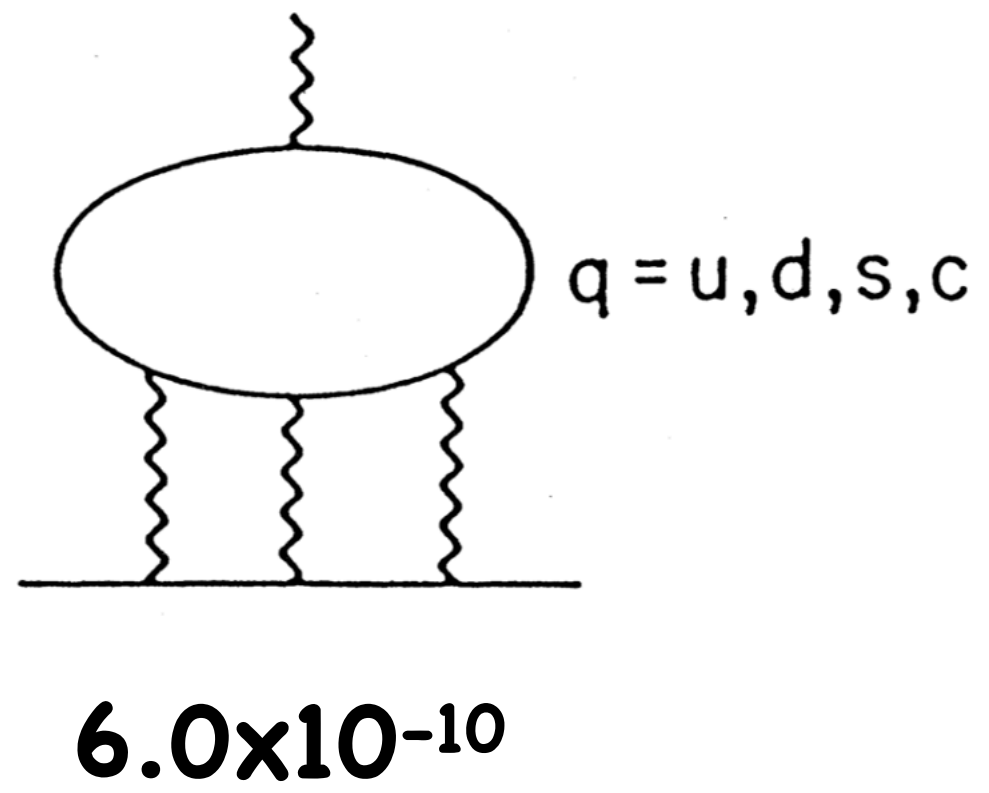
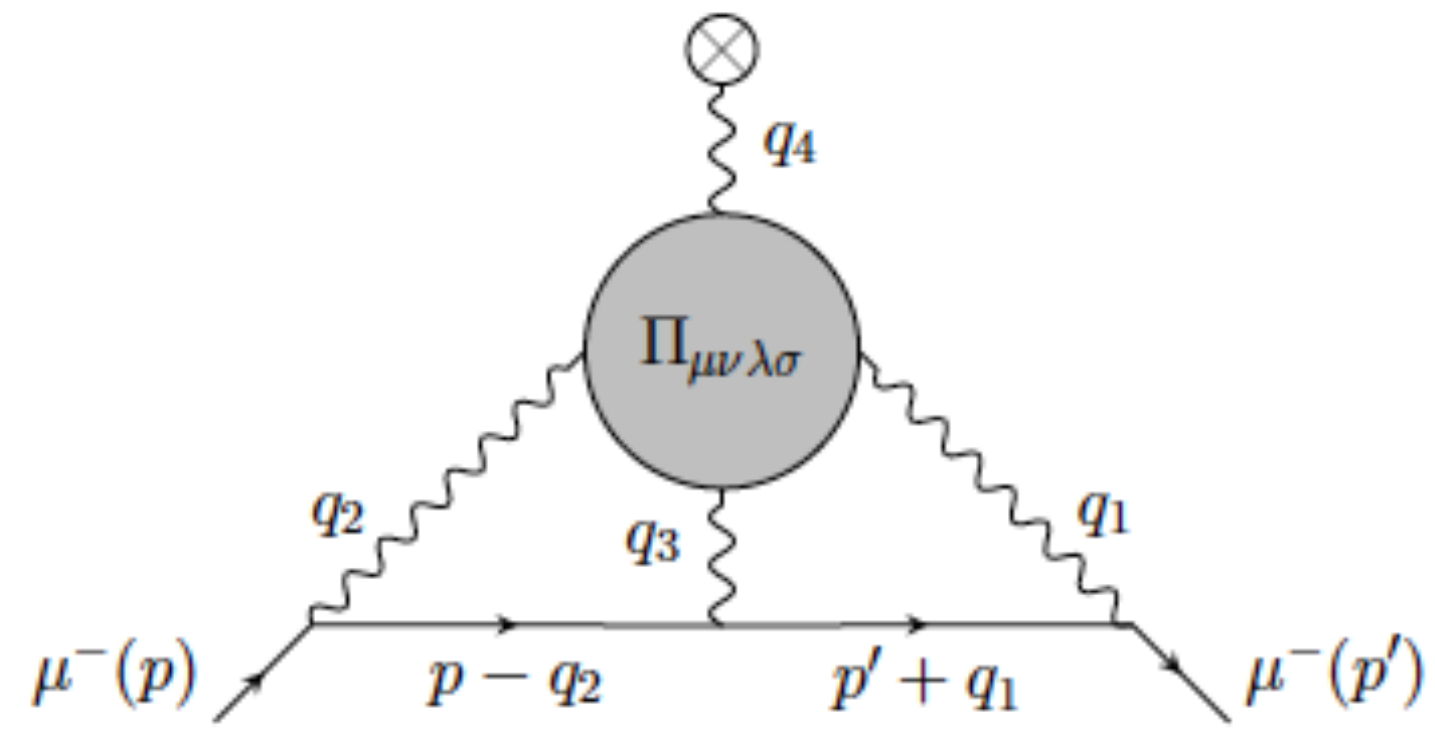


Kinoshita '85

7068.0(59)(164)x10^-11

EW 195x10^-11

Kinoshita '85



Quark-hadron duality

- ChPT '80
- VMD '89
- g-2

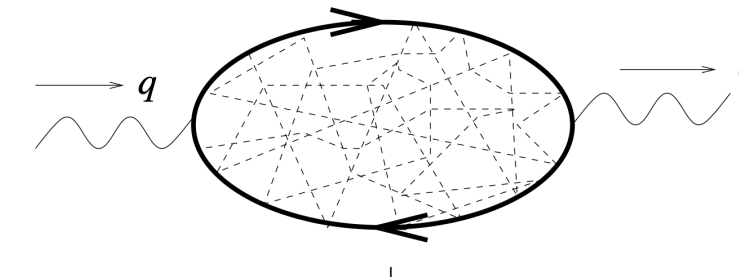
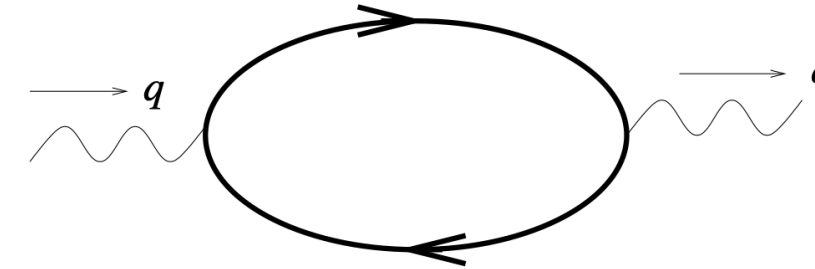
Short distance

QCD

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma_\mu D^\mu - m)q - \frac{G^{\mu\nu} G_{\mu\nu}}{4}$$

UV

↓



Vector Meson Dominance

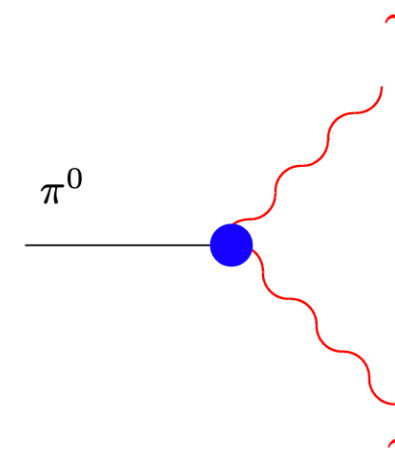
- g_4 Gasser Leutwyler coeff.
- VMD => predictions
- Short distance matching VMD

$$\mathcal{L}_{\text{ChPT}}(\pi) = \mathcal{D}^\mu \pi \mathcal{D}_\mu \pi + g_4 (\mathcal{D}^\mu \pi \mathcal{D}_\mu \pi)^2$$

$$\mathcal{D}^\mu \pi = \frac{\partial^\mu \pi}{1 + \pi^2}$$

Long distance

IR



Quark-hadron duality

- ChPT '80
- VMD '89
- g-2

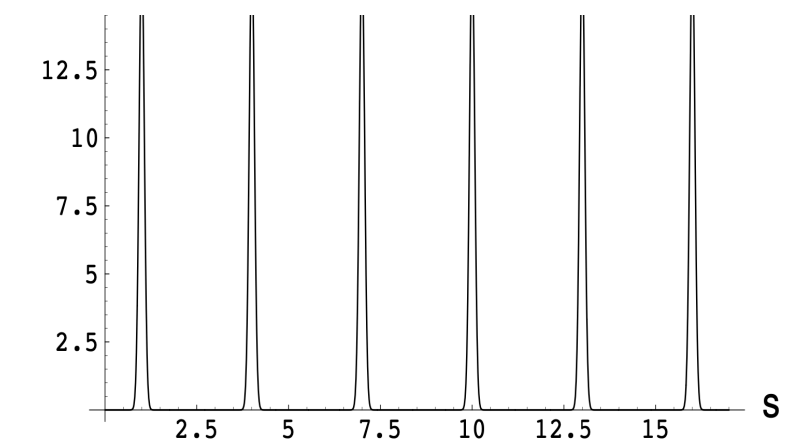
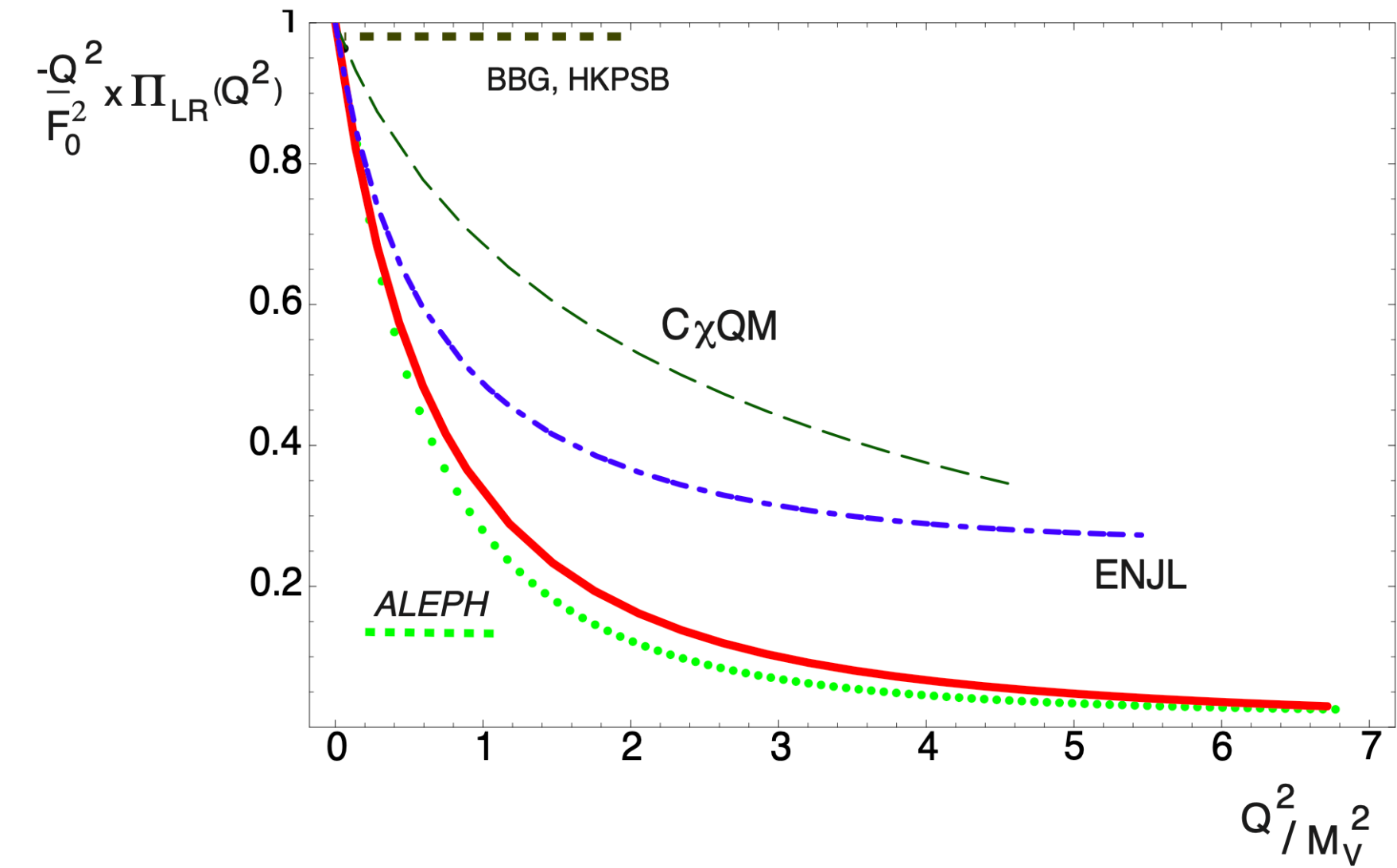
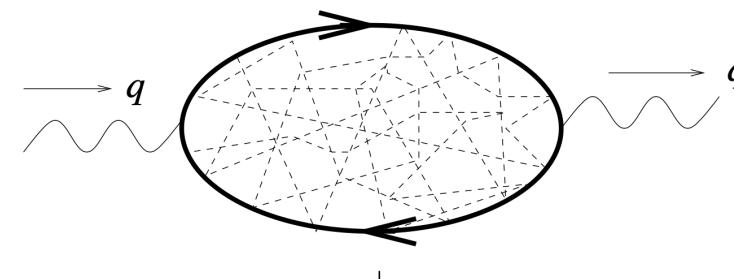
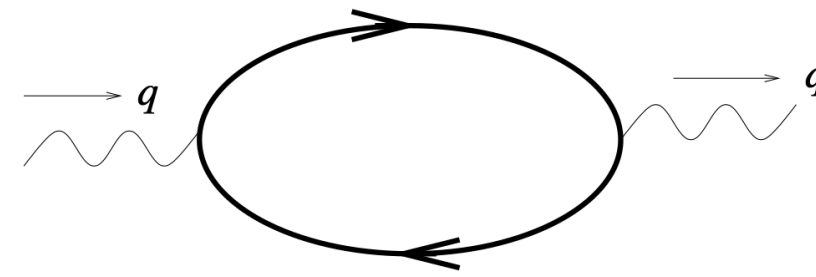
Short distance

QCD

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma_\mu D^\mu - m)q - \frac{G^{\mu\nu} G_{\mu\nu}}{4}$$

UV

↓



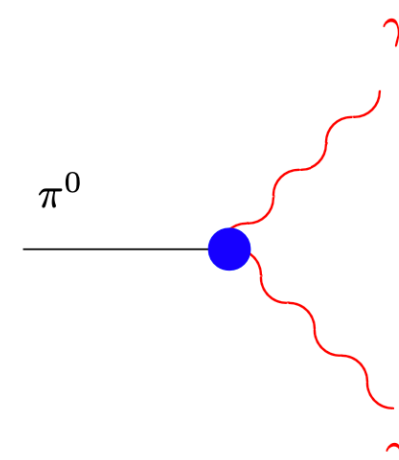
Vector Meson Dominance

$$\mathcal{L}_{ChPT}(\pi) = \mathcal{D}^\mu \pi \mathcal{D}_\mu \pi + g_4 (\mathcal{D}^\mu \pi \mathcal{D}_\mu \pi)^2$$

$$\mathcal{D}^\mu \pi = \frac{\partial^\mu \pi}{1 + \pi^2}$$

Long distance

IR



Light by light contribution

- ChPT '80
- VMD '89
- g-2

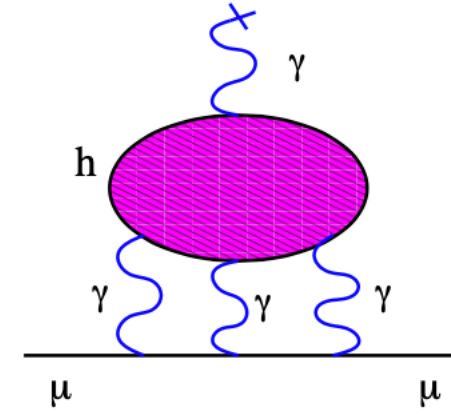
Short distance

QCD

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma_\mu D^\mu - m)q - \frac{G^{\mu\nu} G_{\mu\nu}}{4}$$

UV

↓



$$\mathcal{L}_{\text{ChPT}}(\pi) = \mathcal{D}^\mu \pi \mathcal{D}_\mu \pi + g_4 (\mathcal{D}^\mu \pi \mathcal{D}_\mu \pi)^2$$

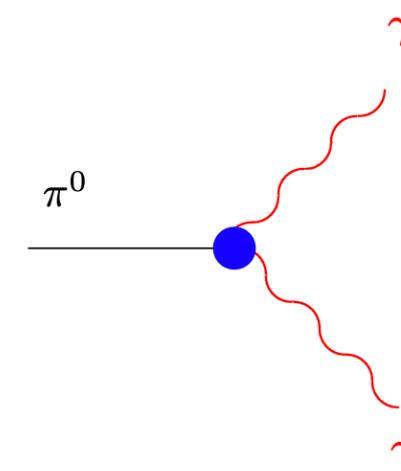
$$\mathcal{D}^\mu \pi = \frac{\partial^\mu \pi}{1 + \pi^2}$$

Long distance

IR

↓

$$a_\mu^{\text{LbyL}} = \left(\frac{\alpha}{\pi}\right)^3 \left(\frac{N_c}{4\pi}\right)^2 \frac{1}{3} \left(\frac{m}{F_\pi}\right)^2 \left[\ln^2\left(\frac{M}{m}\right) + c_\chi \ln\left(\frac{M}{m}\right) + K \right] + \mathcal{O}(N_c^0)$$



Light by light contribution

- ChPT '80
- VMD '89
- g-2

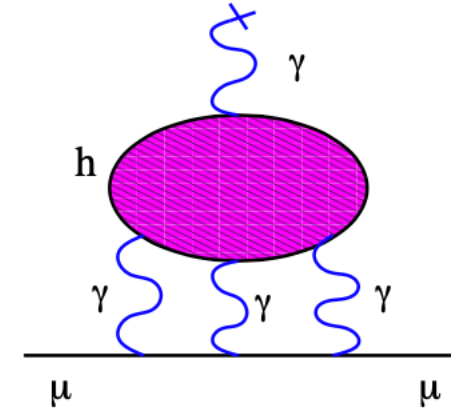
Short distance

QCD

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma_\mu D^\mu - m)q - \frac{G^{\mu\nu} G_{\mu\nu}}{4}$$

UV

↓



Long distance

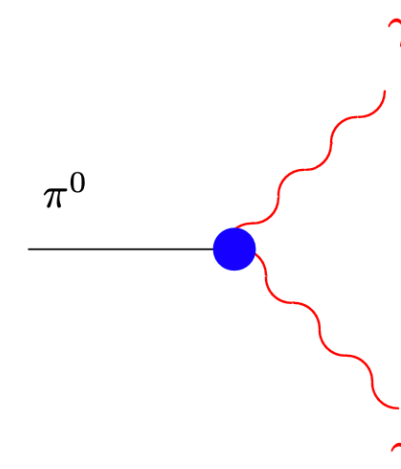
$$\mathcal{L}_{\text{ChPT}}(\pi) = \mathcal{D}^\mu \pi \mathcal{D}_\mu \pi + g_4 (\mathcal{D}^\mu \pi \mathcal{D}_\mu \pi)^2$$

$$\mathcal{D}^\mu \pi = \frac{\partial^\mu \pi}{1 + \pi^2}$$

↓

IR

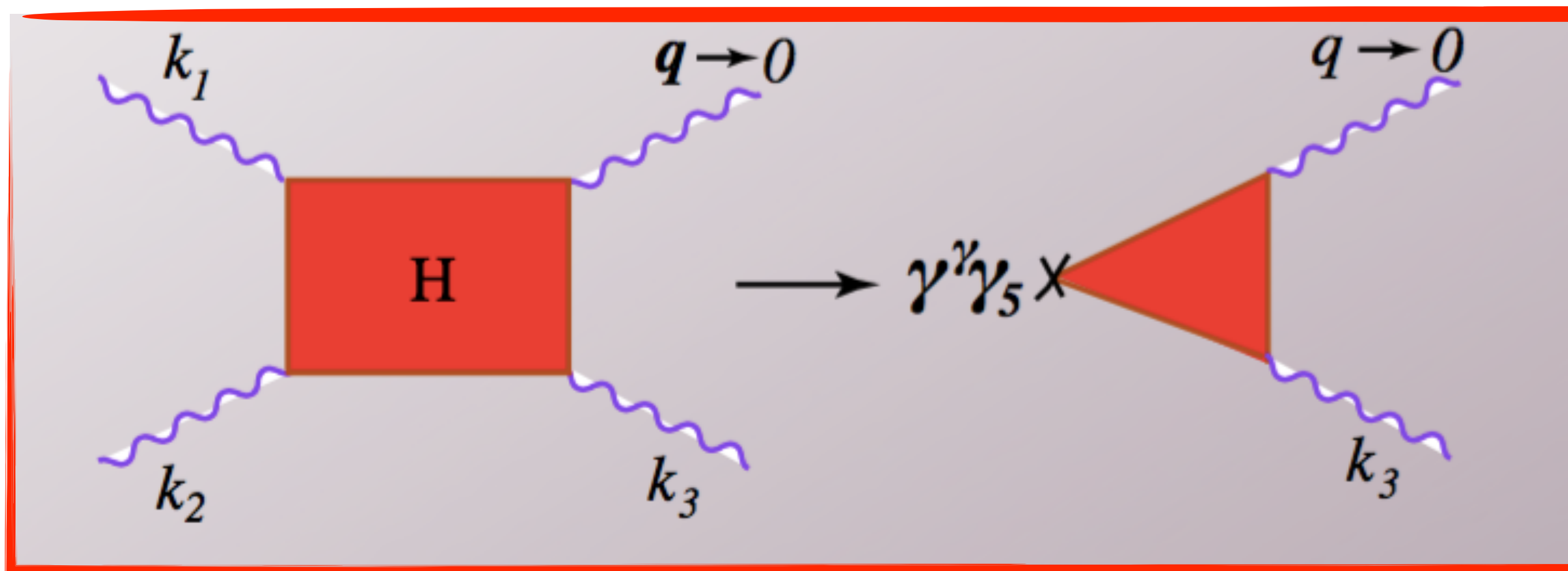
$$a_\mu^{\text{LbyL}} = \left(\frac{\alpha}{\pi}\right)^3 \left(\frac{N_c}{4\pi}\right)^2 \frac{1}{3} \left(\frac{m}{F_\pi}\right)^2 \left[\ln^2\left(\frac{M}{m}\right) + c_\chi \ln\left(\frac{M}{m}\right) + K \right] + \mathcal{O}(N_c^0)$$



Short distance constraints
Melnikov Vainshtein

Melnikov-Vainshtein Limit $q_1^2 \approx q_2^2 \gg q_3^2$

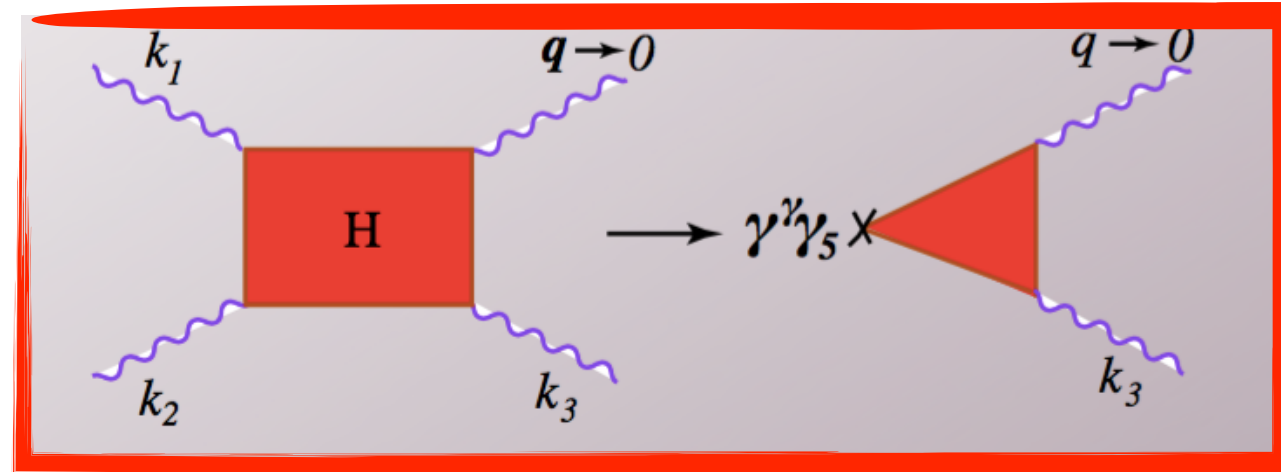
$$a_\mu^{\text{LbyL}} = \left(\frac{\alpha}{\pi}\right)^3 \left(\frac{N_c}{4\pi}\right)^2 \frac{1}{3} \left(\frac{m}{F_\pi}\right)^2 \left[\ln^2\left(\frac{M}{m}\right) + \dots \right]$$



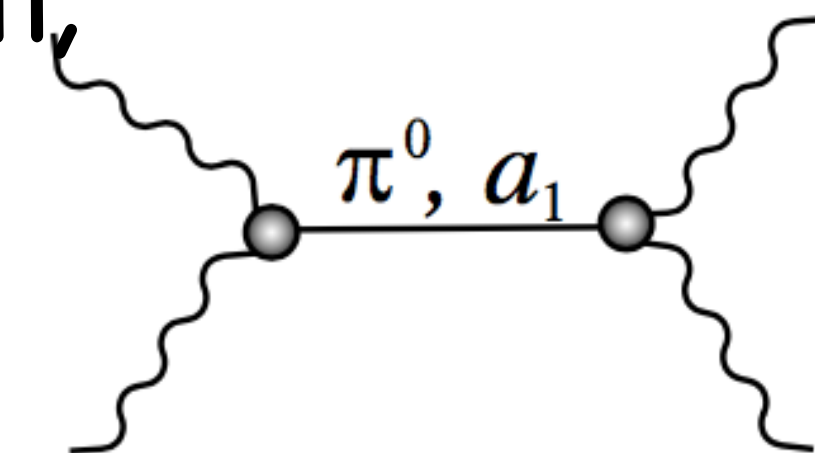
$$q_1^2 \approx q_2^2 \gg q_3^2$$

in the 4-point

Melnikov-Vainshtein Limit



- Contribution to **two** helicity amplitudes: π^0 and a_1 exchange
- Model to correctly reproduce this s.d. limit, numerically important



Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	—	114 ± 13	99 ± 16
π, K loops	-19 ± 13	-4.5 ± 8.1	—	—	—	-19 ± 19	-19 ± 13
π, K l. + subl. in Nc	—	—	—	0 ± 10	—	—	—
axial vectors	2.5 ± 1.0	1.7 ± 1.7	—	22 ± 5	—	15 ± 10	22 ± 5

Hlbl: status 2020

Contribution	PdRV(09) <i>Glasgow consensus</i>	N/JN(09)	J(17)	WP(20)
π^0, η, η' -poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K -loops/boxes	-19(19)	-19(13)	-20(5)	-16.4(2)
S -wave $\pi\pi$ rescattering	-7(7)	-7(2)	-5.98(1.20)	-8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	-	-	-	} - 1(3)
tensors	-	-	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s -loops / short-distance	-	21(3)	20(4)	15(10)
c -loop	2.3	-	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

Collaboration with L. Cappiello O. Cata, D.Greynat, A. Iyer

arXiv:1912.02779 PRD

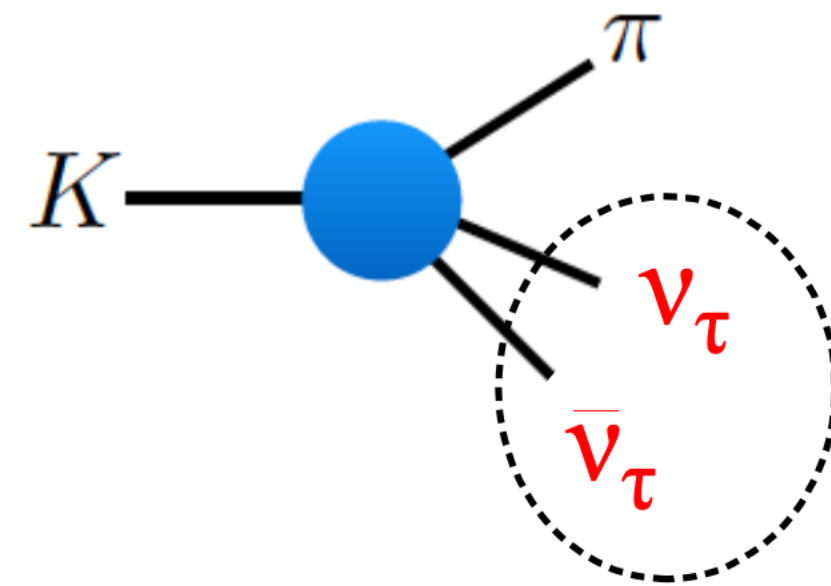
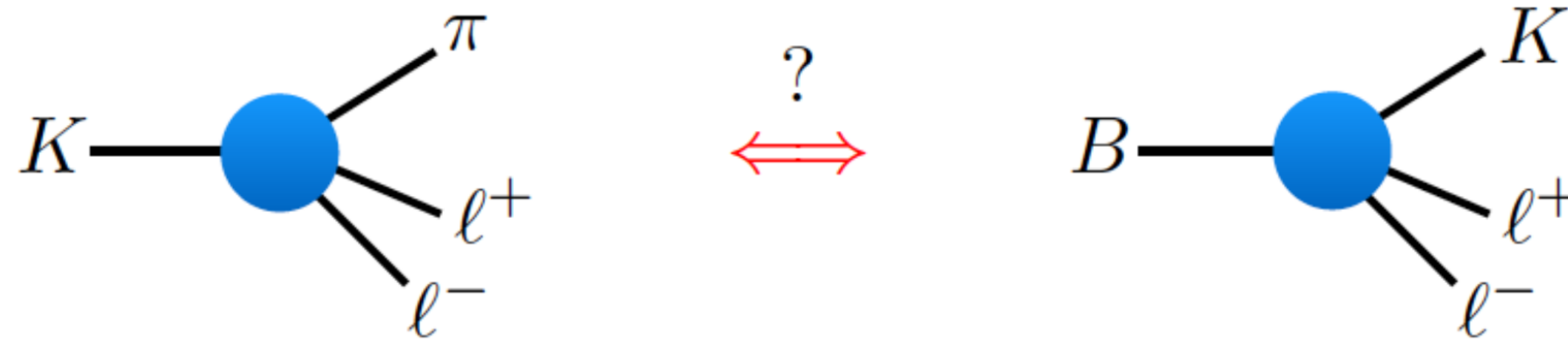
Hlbl: status 2020

Contribution	BPP(96)	HKS(96)	KnN(02)	MV(04)	BP(07)	PdRV(09)	N/JN(09)
π^0, η, η'	85(13)	82.7(6.4)	83(12)	114(10)		114(13)	99(16)
π, K loops	19(13)	1.5(8.1)				-19(19)	-19(13)
π, K loops + subleading in N_c				0(10)			
axial vectors	2.5(1.0)	1.7(1.7)		22(5)		15(10)	22(5)
scalars	-6.8(2.0)					-7(7)	-7(2)
quark loops	21(3)	9.7(11.1)				2.3	21(3)
total	83(32)	89.6(15.4)	80(40)	136(25)	110(40)	105(26)	116(39)

Contribution	PdRV(09)	N/JN(09)	J(17)	WP(20)
	<i>Glasgow consensus</i>			
π^0, η, η' -poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K -loops/boxes	-19(19)	-19(13)	-20(5)	-16.4(2)
S-wave $\pi\pi$ rescattering	-7(7)	-7(2)	-5.98(1.20)	-8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	-	-	-	} - 1(3)
tensors	-	-	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	
u, d, s -loops / short-distance	-	21(3)	20(4)	15(10)
c -loop	2.3	-	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

► Lepton Flavor Universality

Example-II: neutral currents, $\mu^+\mu^-$ vs. e^+e^-



Access to 3rd gen. leptons
as in R(D) & R(D*)

*...but a potential more promising effect could appear in
our beloved $K \rightarrow \pi\nu\nu$ decays....*

► SM, BSM, & “non-standard BSM” in $K \rightarrow \pi\nu\nu$

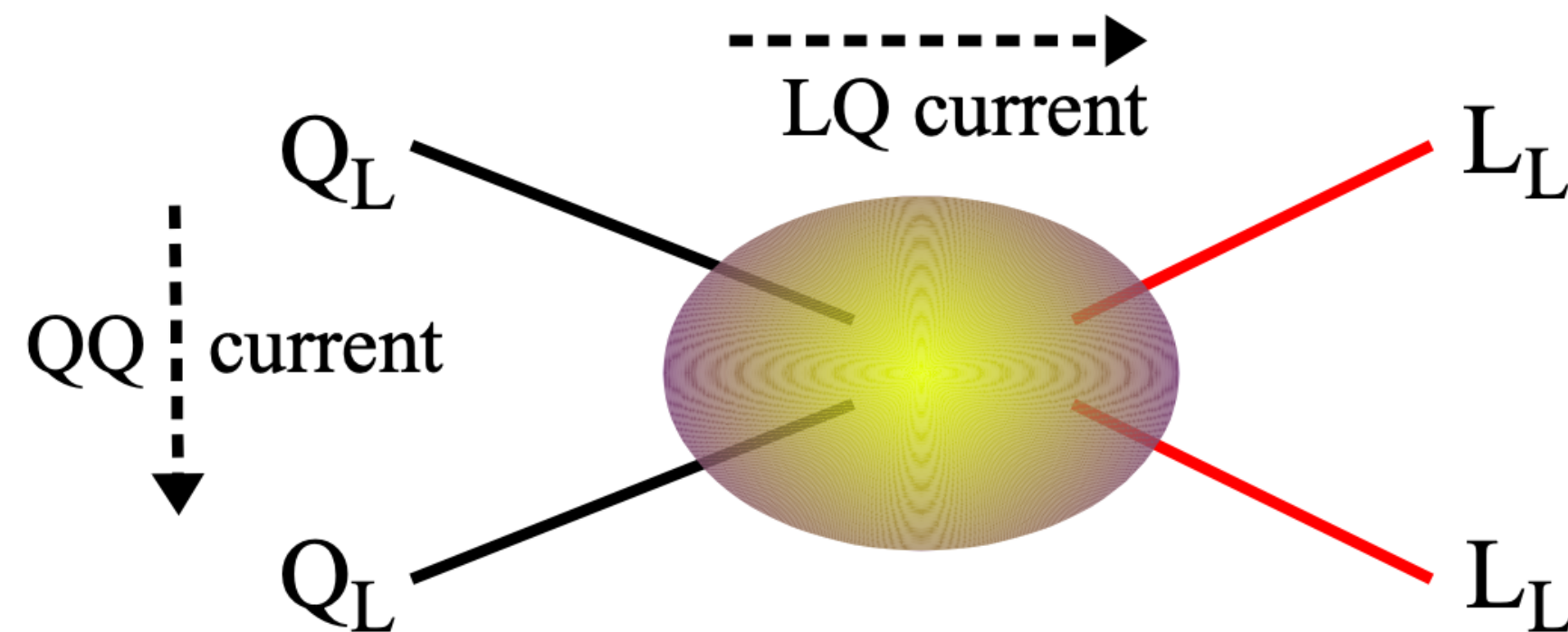
But what I find even more interesting, is the natural link with LFU effects in B-physics, thanks to the presence of 3rd generation leptons in the final state

$$\Gamma(K \rightarrow \pi\nu\nu) = \Gamma(K \rightarrow \pi\nu_e\bar{\nu}_e) + \Gamma(K \rightarrow \pi\nu_\mu\bar{\nu}_\mu) + \Gamma(K \rightarrow \pi\nu_\tau\bar{\nu}_\tau)$$

SM like

few %
deviation
as in $b \rightarrow s\mu\mu$

possible **O(1) deviation**
from SM
expected also in $b \rightarrow s\tau\tau$



Explicit (UV) models:

- LQ (composite) mediators
Barbieri, GI, Pattori, Senia '16
- Z', W' (composite) mediators
GI *et al.* - work in prog.

Anatomy of kaon decays and prospects for lepton flavour universality violation

GD, A.M. Iyer, F. Mahmoudi, S. Neshatpour

arxiv 2206.14748

- Motivated by B-anomalies we study LFUV Kaon decays

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \lambda_t^{sd} \frac{\alpha_e}{4\pi} \sum_k C_k^\ell O_k^\ell,$$

- $O_9^\ell = (\bar{s}\gamma_\mu P_L d) (\bar{\ell}\gamma^\mu \ell), \quad O_{10}^\ell = (\bar{s}\gamma_\mu P_L d) (\bar{\ell}\gamma^\mu \gamma_5 \ell),$

$$O_L^\ell = (\bar{s}\gamma_\mu P_L d) (\bar{\nu}_\ell \gamma^\mu (1 - \gamma_5) \nu_\ell),$$

$$\delta C_L^{\ell-} \equiv \delta C_9^\ell = -\delta C_{10}^\ell.$$

$$\delta C_L^{\tau-} = \delta C_L^{\mu-}.$$

$$K \rightarrow \pi \nu \bar{\nu}$$

arxiv 2206.14748

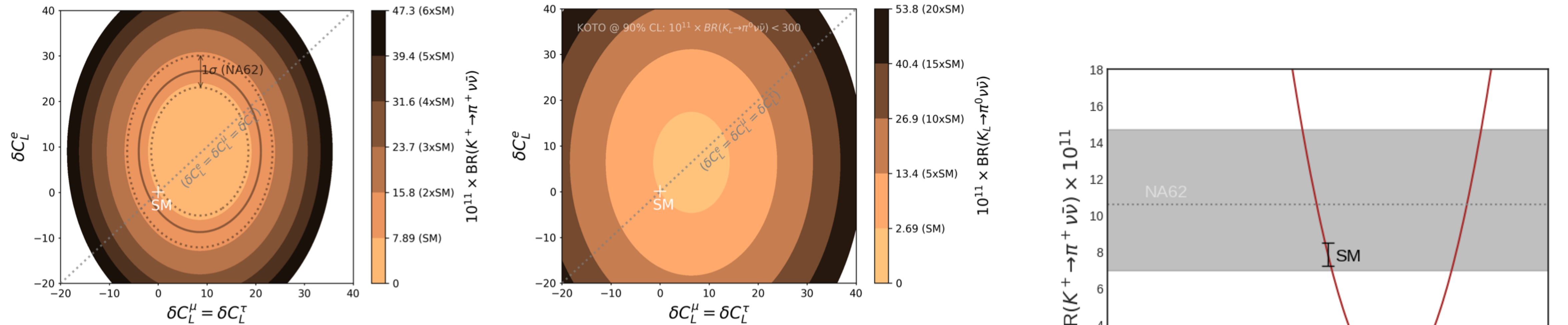
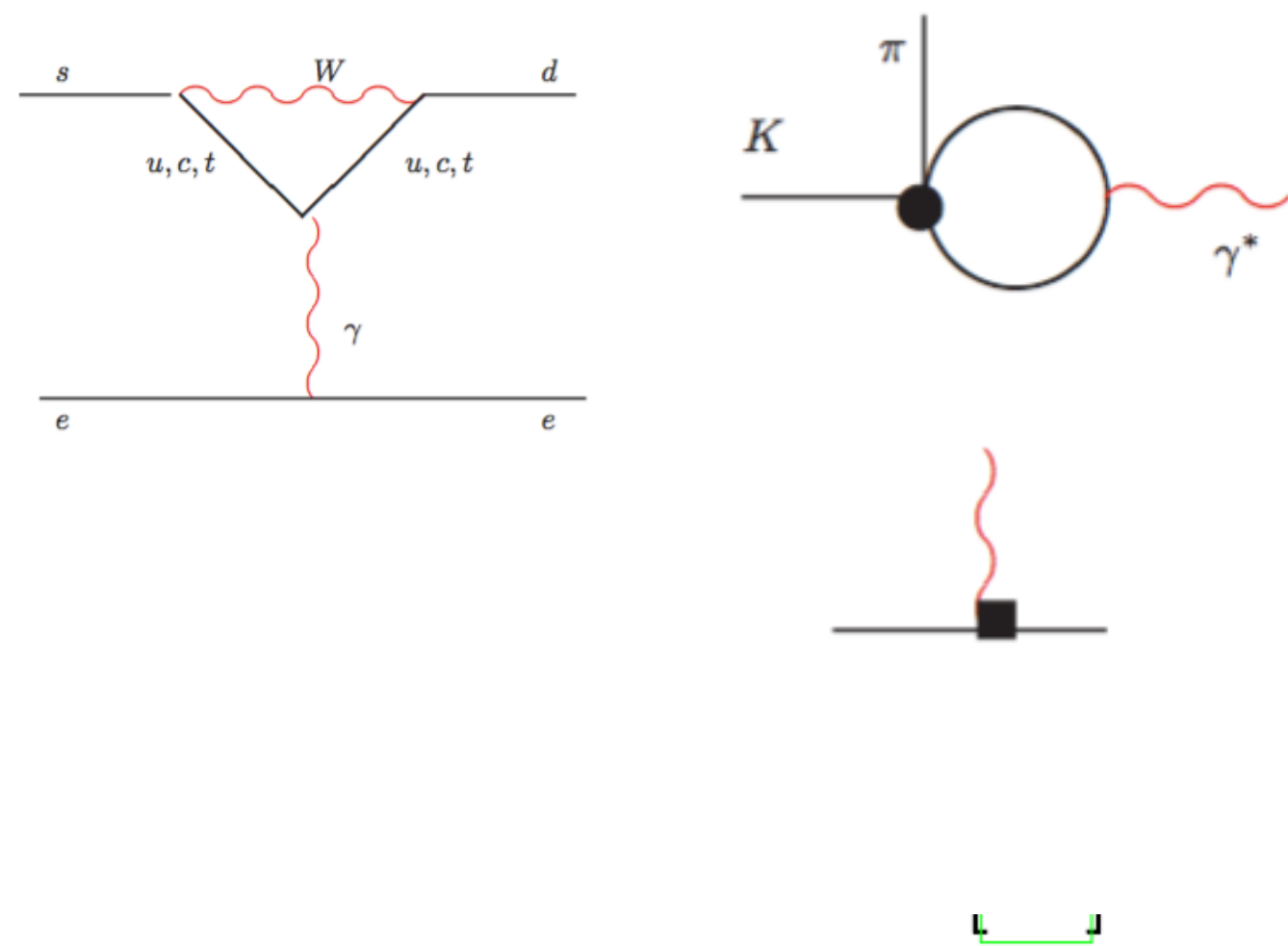


Figure 1: $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ (left) and $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ (right) as a function of δC_L^e and $\delta C_L^\mu = \delta C_L^\tau$. The dotted grey line represents the lepton flavour universality scenario. In the left plot, the brown solid (dotted) line corresponds to the measured central value (1σ experimental uncertainty) by NA62 [15]. In the right plot, the upper bound on $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is not visible for the scanned values.

LFUV in Kaons

$$\frac{\Gamma(K^+ \rightarrow \pi^+ \mu^+ \mu^-)}{\Gamma(K^+ \rightarrow \pi^+ e^+ e^-)}$$

SD \ll LD



$$A_V^{K^+ \rightarrow \pi^+ \gamma^*} = -\frac{G_f \alpha}{4\pi} V_+(z) \bar{u}_l(p_-) (\gamma_\mu k^\mu + \gamma_\mu p^\mu) v_l(p_+)$$

GD, Ecker, Isidori, Portoles

$$V_+(z) = a_+ + b_+ z + V_+^{\pi\pi}(z)$$

a_+ and b_+ Short distance

$$a_+^{\mu\mu} - a_+^{ee} = -\sqrt{2} \text{Re} [V_{td} V_{ts}^* (C_9^\mu - C_9^e)]$$

Crivellin, GD, Hofrichter, Tunstall

<i>Historical progression</i>				<i>Current situation</i>			
Channel	a_+	b_+	Reference	Channel	a_+	b_+	Reference
ee	-0.587 ± 0.010	-0.655 ± 0.044	E865 [32]	ee	-0.561 ± 0.009	-0.694 ± 0.040	comb. [42]
ee	-0.578 ± 0.016	-0.779 ± 0.066	NA48/2 [33]	$\mu\mu$	-0.592 ± 0.015	-0.699 ± 0.058	NA62 [16]
$\mu\mu$	-0.575 ± 0.039	-0.813 ± 0.145	NA48/2 [34]				

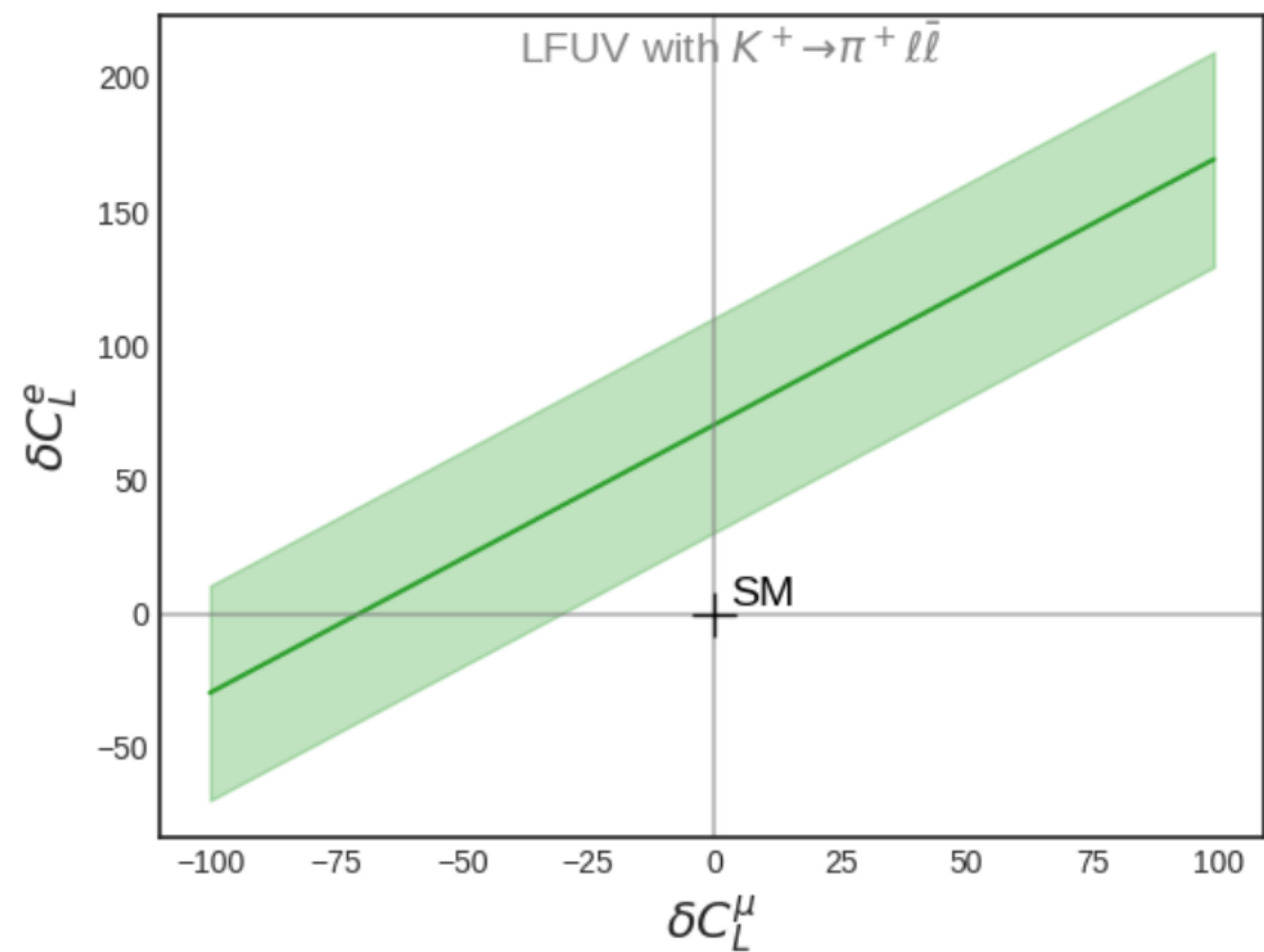
Table 1: Summary of the estimation of vector form factors for $K^+ \rightarrow \pi^+ \ell \bar{\ell}$. The left panel gives the historical progression and the right panel gives the current status.

$$a_+ = -0.575 \pm 0.013, b_+ = -0.722 \pm 0.043$$

NA62
 Measurement of the rare decay $K^+ \rightarrow \pi^+ \mu^+ \mu^-$
 at the NA62 experiment
 Newly published: arXiv:2209.05076

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KAON2022

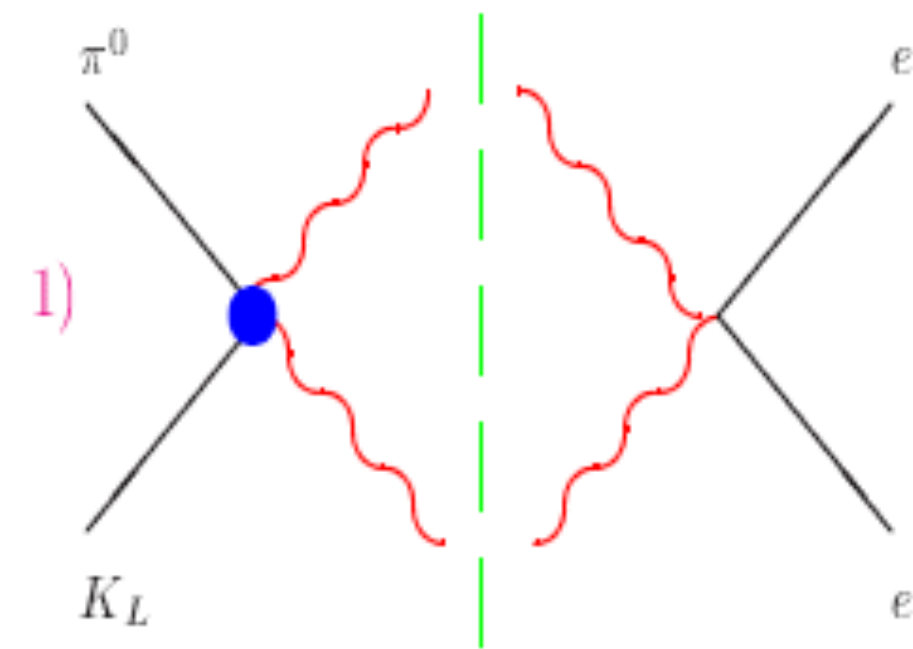


arxiv 2206.14748

Figure 2: Region consistent with the estimation of the LFUV variable in $K^+ \rightarrow \pi^+ \ell \bar{\ell}$ decays.

$K_L \rightarrow \pi^0 e^+ e^-$: summary

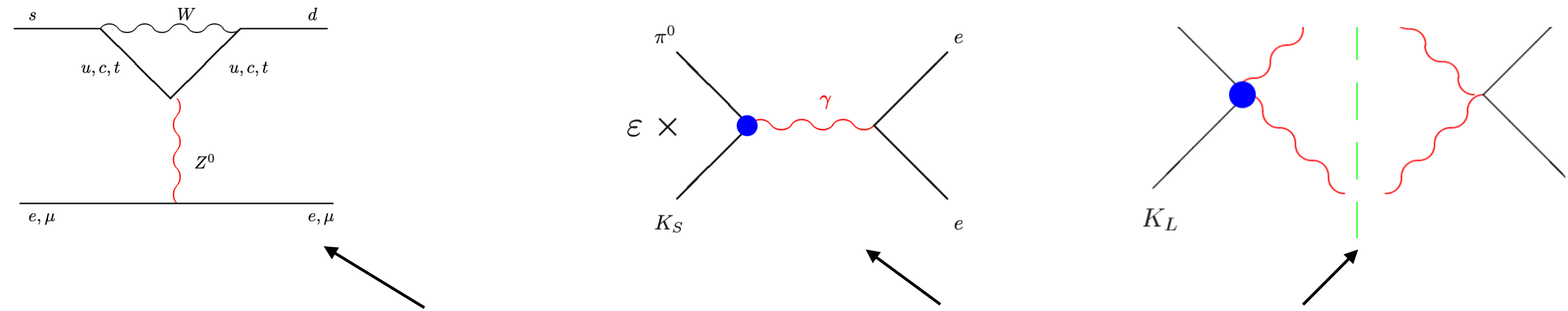
$$\text{Br}(K_L \rightarrow \pi^0 e^+ e^-) \leq 2.8 \cdot 10^{-10} \text{ at 90\% CL} \quad \text{KTeV}$$



CP conserving NA48

$$\text{Br}(K_L \rightarrow \pi^0 e^+ e^-) < 3 \cdot 10^{-12}$$

$$V-A \otimes V-A \Rightarrow \langle \pi^0 e^+ e^- | (\bar{s}d)_{V-A} (\bar{e}e)_{V-A} | K_L \rangle \text{ violates CP}$$



$$\text{BR}(K_L \rightarrow \pi^0 l \bar{l}) = (C_{\text{dir}}^\ell \pm C_{\text{int}}^\ell |a_S| + C_{\text{mix}}^\ell |a_S|^2 + C_{\gamma\gamma}^\ell) \cdot 10^{-12},$$

$$|a_S| = 1.20 \pm 0.20,$$

	C_{dir}^ℓ	C_{int}^ℓ	C_{mix}^ℓ	$C_{\gamma\gamma}^\ell$
$\ell = e$	$(4.62 \pm 0.24)(w_{7V}^2 + w_{7A}^2)$	$(11.3 \pm 0.3)w_{7V}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05)(w_{7V}^2 + 2.32w_{7A}^2)$	$(2.63 \pm 0.06)w_{7V}$	3.36 ± 0.20	5.2 ± 1.6

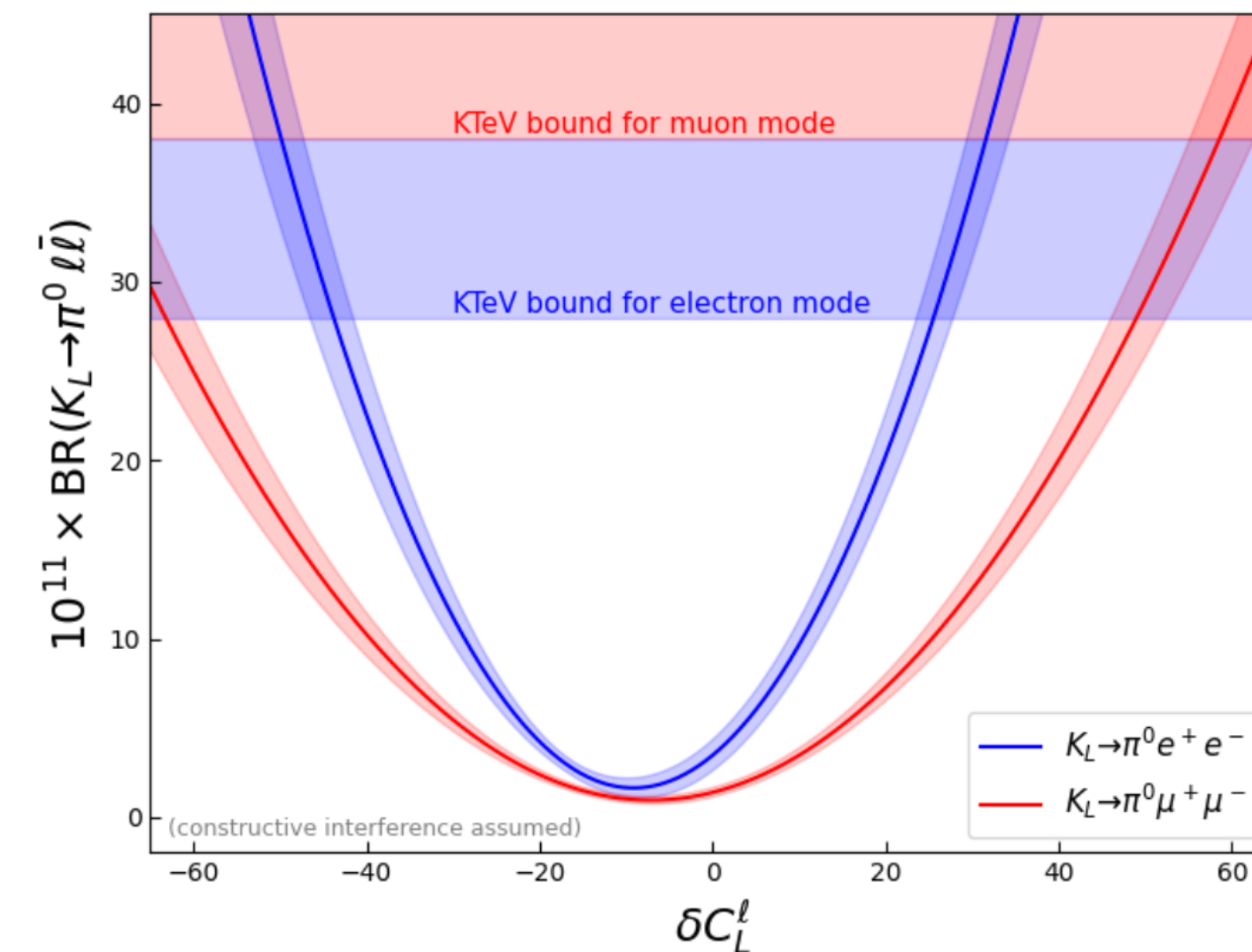
arxiv 2206.14748

$$\text{BR}^{\text{SM}}(K_L \rightarrow \pi^0 e \bar{e}) = 3.46_{-0.80}^{+0.92} (1.55_{-0.48}^{+0.60}) \times 10^{-11}$$

$$\text{BR}^{\text{SM}}(K_L \rightarrow \pi^0 \mu \bar{\mu}) = 1.38_{-0.25}^{+0.27} (0.94_{-0.20}^{+0.21}) \times 10^{-11}$$

$$\text{BR}^{\text{exp}}(K_L \rightarrow \pi^0 e \bar{e}) < 28 \times 10^{-11} \quad \text{at 90\% CL.}$$

$$\text{BR}^{\text{exp}}(K_L \rightarrow \pi^0 \mu \bar{\mu}) < 38 \times 10^{-11} \quad \text{at 90\% CL.}$$



Observable	SM prediction	Exp results	Ref.	Experimental Err. Projections
$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6_{-3.5}^{+4.0} \pm 0.9) \times 10^{-11}$	[15]	10%(@2025) 5%(CERN; long-term) [58]
$\text{BR}(K_L^0 \rightarrow \pi^0 \nu \bar{\nu})$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 3.0 \times 10^{-9}$ @90% CL	[17]	20%(CERN; long-term [58]) 15% (KOTO [61])
$\text{LFUV}(a_+^{\mu\mu} - a_+^{e\bar{e}})$	0	-0.031 ± 0.017	[16, 42]	± 0.007 (assuming ± 0.005 for each mode)
$\text{BR}(K_L \rightarrow \mu\mu) (+)$	$(6.82_{-0.29}^{+0.77}) \times 10^{-9}$	$(6.84 \pm 0.11) \times 10^{-9}$	[43]	experimental uncertainty kept to current value
$\text{BR}(K_L \rightarrow \mu\mu) (-)$	$(8.04_{-0.98}^{+1.47}) \times 10^{-9}$			
$\text{BR}(K_S \rightarrow \mu\mu)$	$(5.15 \pm 1.50) \times 10^{-12}$	$< 2.1(2.4) \times 10^{-10}$ @90(95)% CL	[44]	$< 8 \times 10^{-12}$ @95% CL (CERN; long-term [51])
$\text{BR}(K_L \rightarrow \pi^0 e\bar{e})(+)$	$(3.46_{-0.80}^{+0.92}) \times 10^{-11}$	$< 28 \times 10^{-11}$ @90% CL	[56]	
$\text{BR}(K_L \rightarrow \pi^0 e\bar{e})(-)$	$(1.55_{-0.48}^{+0.60}) \times 10^{-11}$			observation (CERN; long-term [58]) (we assume 100% error)
$\text{BR}(K_L \rightarrow \pi^0 \mu\bar{\mu})(+)$	$(1.38_{-0.25}^{+0.27}) \times 10^{-11}$	$< 38 \times 10^{-11}$ @90% CL	[57]	
$\text{BR}(K_L \rightarrow \pi^0 \mu\bar{\mu})(-)$	$(0.94_{-0.20}^{+0.21}) \times 10^{-11}$			

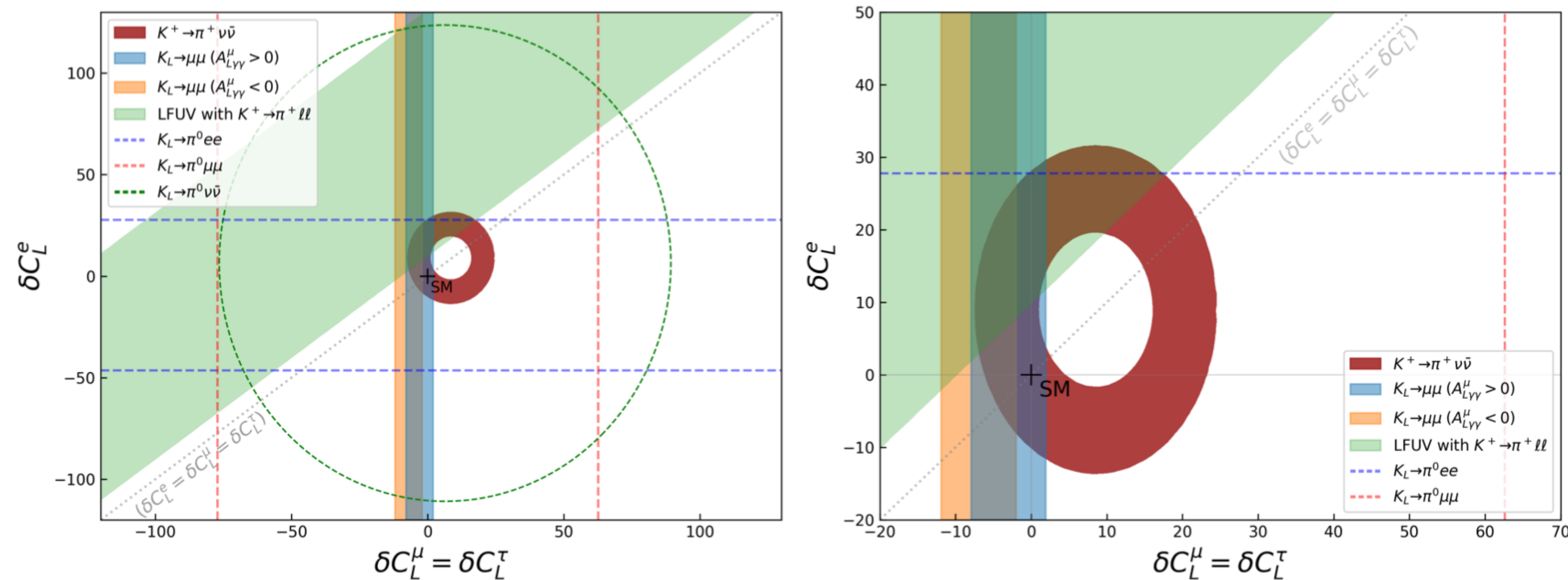


Figure 7: The bounds from individual observables. The right panel is the zoomed version of the left panel. The coloured regions correspond to 68% CL when there is a measurement and the dashed ones to upper limits at 90% CL. $K_L \rightarrow \mu\bar{\mu}$ has been shown for both signs of the long-distance contribution. For $K_L \rightarrow \pi^0 e\bar{e}$ and $K_L \rightarrow \pi^0 \mu\bar{\mu}$, constructive interference between direct and indirect CP-violating contributions has been assumed.

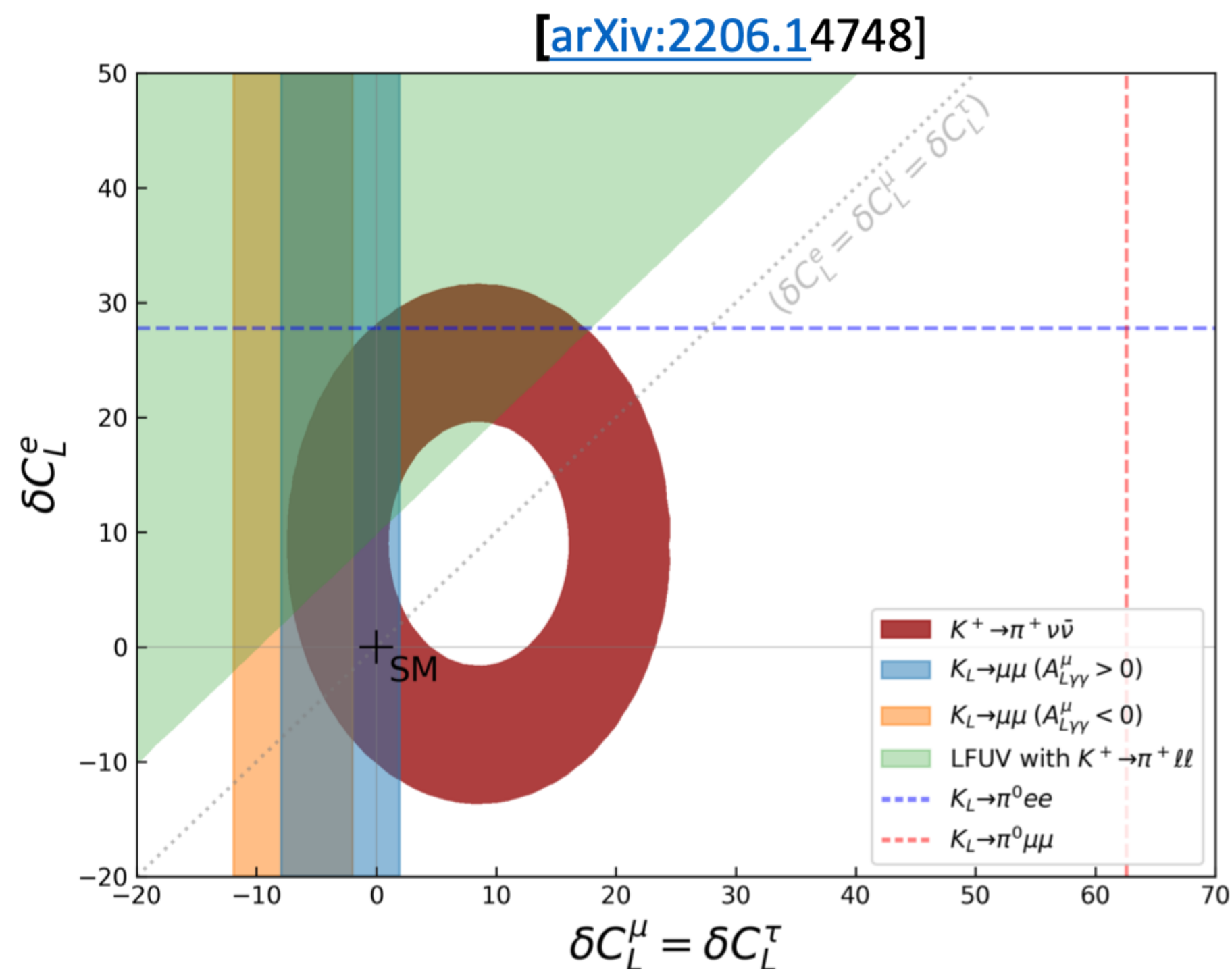
Kaon Global Fit

For example, recent paper with global fits to set of kaon measurements
 Deviation of Wilson coefficients from SM, for NP scenarios with only left-handed quark currents.

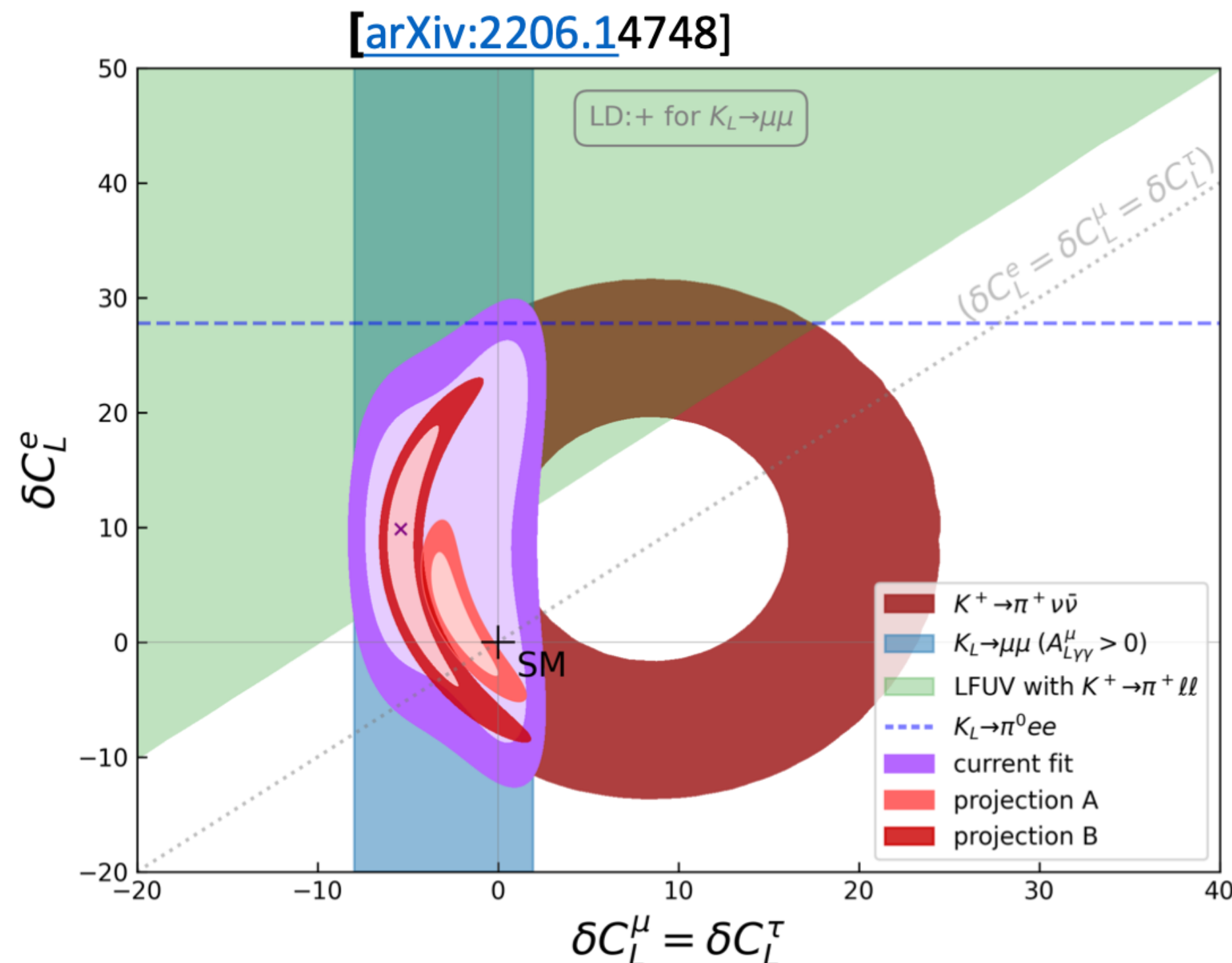
$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \lambda_t^{sd} \frac{\alpha_e}{4\pi} \sum_k C_k^\ell O_k^\ell$$

$$O_L^\ell = (\bar{s} \gamma_\mu P_L d) (\bar{\nu}_\ell \gamma^\mu (1 - \gamma_5) \nu_\ell)$$

$$C_k^\ell = C_{k,\text{SM}}^\ell + \delta C_k^\ell$$



Bounds from individual observables.
 Coloured regions are 68%CL measurements
 Dashed lines are 90%CL upper limits



With projections: central value for existing measurements kept the same, A upper bounds extrapolated to central value consistent with SM, B central value of all observables is projected to the best-fit points obtained from fits to existing data