

Investigating core excitation in halo nuclei using halo-EFT

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Overview

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 - Halo Nuclei
 - Halo-EFT description of ^{11}Be
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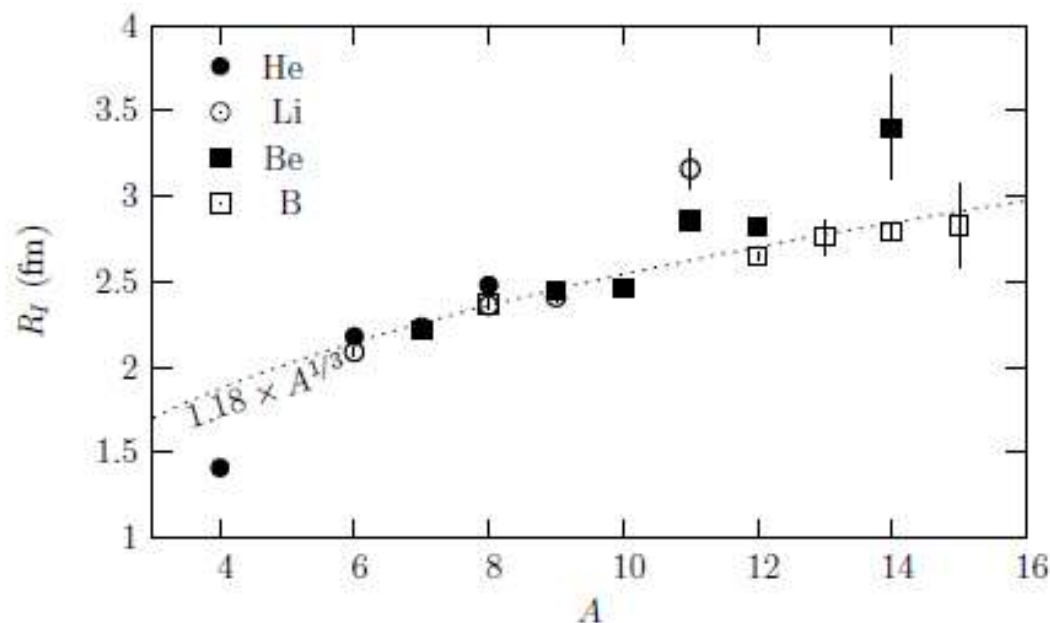
Unstable nuclei

1985: Measurements of interaction cross sections σ_I of light exotic nuclei

[I. Tanihata PRL 55, 2676 (1985)]

In a simple geometrical model:

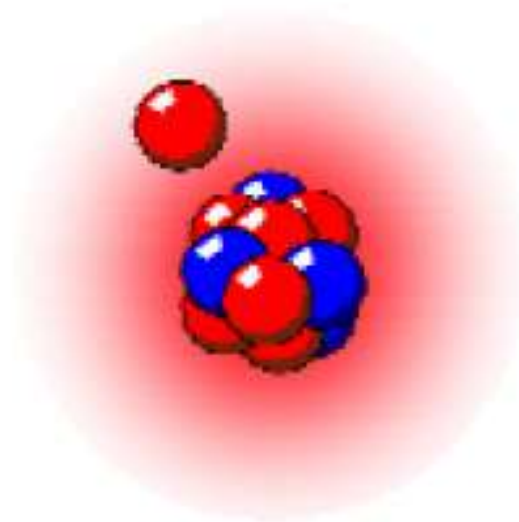
$$\sigma_I = \pi (R_P + R_T)^2$$



Some nuclei deviate from the $A^{1/3}$ trend: ${}^6\text{He}$, ${}^{11}\text{Be}$, ${}^{11}\text{Li}$, ${}^{14}\text{Be}$...
→ these larger nuclei are: **halo nuclei**

Halo nuclei

- Light, neutron-rich nuclei with large matter radius
- Low S_n or S_{2n} : one or two loosely-bound neutrons
- Clusterised structure: neutrons can tunnel far from the core
→ halo-nucleus \equiv compact core + valence neutron(s)



- Our case study : $^{11}\text{Be} \equiv ^{10}\text{Be} + n$
- Short-lived → studied via reactions (e.g. **breakup**)
→ need of an **effective few-body** model for reaction calculations
→ **Halo-EFT**

Halo-EFT description of ^{11}Be

- Halo-structure \rightarrow separation of scales
 - \rightarrow small parameter $\eta = \frac{R_{\text{core}}}{R_{\text{halo}}} \simeq 0.4 < 1$
 - \rightarrow expansion of the core-neutron Hamiltonian along η

[Bertulani, Hammer, van Kolck, NPA 712, 37 (2002)]
Review: [Hammer, Ji, Phillips, JPG 44, 103002 (2017)]
- $^{11}\text{Be} = ^{10}\text{Be}(0^+) + n$ [core has no internal structure]
 - \rightarrow **single-particle description**: $H(\mathbf{r}) = T_{\mathbf{r}} + V_{\text{cn}}(\mathbf{r})$
- **Effective** Gaussian potentials in each partial wave ℓ_j @NLO ($\ell \leq 1$):

$$V_{\text{cn}}(\mathbf{r}) = V_{\ell_j}^{(0)} e^{-\frac{r^2}{2\sigma^2}} + V_{\ell_j}^{(2)} r^2 e^{-\frac{r^2}{2\sigma^2}}$$

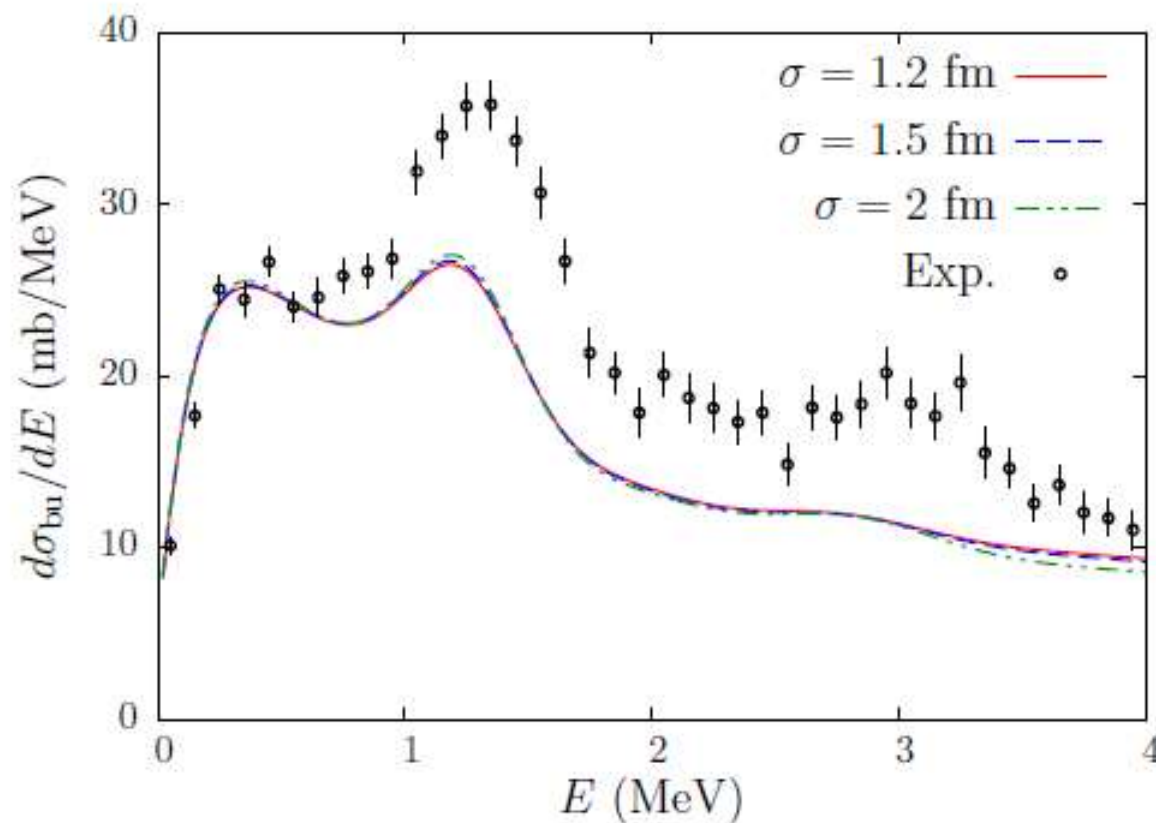
$V_{\ell_j}^{(0)}$ and $V_{\ell_j}^{(2)}$ fitted to reproduce:

- \rightarrow \mathbf{S}_n & asymptotic normalization coefficient (**ANC**) for bound states
- \rightarrow effective range parameters for continuum states

$\sigma :=$ **cut-off** \rightarrow evaluates sensitivity to short-range physics

What is the problem ?

- Assumption: ^{10}Be remains in its 0^+ ground state still valid ?
→ Nuclear breakup: $^{11}\text{Be} + \text{C} \rightarrow ^{10}\text{Be} + n + \text{C}$



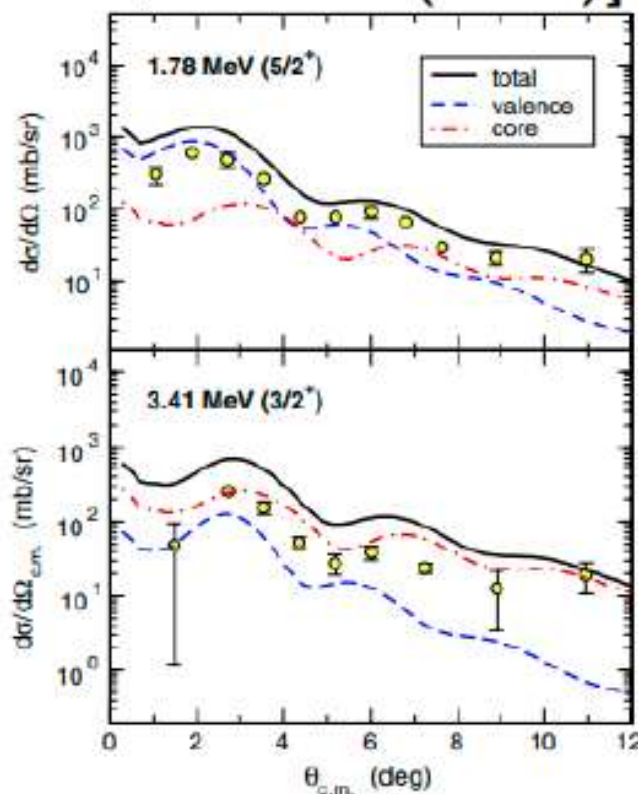
Exp: [Fukuda *et al.* PRC 70, 054606 (2004)]

Th.: [L.-P. Kubushishi and P. Capel, arXiv:2406.10168 (2024)]

⇒ Missing peaks at $\frac{5}{2}^+$ and $\frac{3}{2}^+$ resonances → is **s.p.** not enough ?

Nuclear breakup & core excitation

- Origin of these missing strengths ?
 - ⇒ missing degree of freedom [$^{10}\text{Be}(2^+)$]
 - ⇒ ^{10}Be core can be excited to its first 2^+ state
- [Moro & Lay, PRL 109, 232502 (2012)]



- To better understand **structure effects on reaction calculations** I develop a **Halo-EFT** few-body model including **core excitation**

Core excitation within Halo-EFT

- **Extension of Halo-EFT to include core excitation:**

$$H(\mathbf{r}, \xi) = T_{\mathbf{r}} + V_{\text{cn}}(\mathbf{r}, \xi) + h_{\text{core}}(\xi)$$

$h_{\text{core}}(\xi)$:= intrinsic Hamiltonian of the core with eigenstates $\chi_I^c(\xi)$

- **Halo-EFT particle-rotor model** [Bohr and Mottelson (1975)]:

$$V_{\text{cn}}(\mathbf{r}, \xi) = V_{\text{cn}}(r) + \beta\sigma Y_2^0(\hat{r}) \frac{d}{d\sigma} V_{\text{cn}}(r)$$

- Set of radial **coupled-channel** Schrödinger equations:

$$\left[T_{\mathbf{r}}^{\ell} + V_{\alpha\alpha}(r) + \epsilon_{\alpha} - E \right] \psi_{\alpha}(r) = - \sum_{\alpha' \neq \alpha} V_{\alpha\alpha'}(r) \psi_{\alpha'}(r)$$

with $V_{\alpha\alpha'}(r) = \langle \mathcal{Y}_{\alpha}(\hat{r}) \chi_{\alpha}(\xi) | V_{\text{cn}}(\mathbf{r}, \xi) | \mathcal{Y}_{\alpha'}(\hat{r}) \chi_{\alpha'}(\xi) \rangle$, $\alpha = \{\ell, s, j, I\}$

→ solved within the **R-Matrix method** on a Lagrange mesh

[D. Baye, Physics Reports 565 (2015) 1]

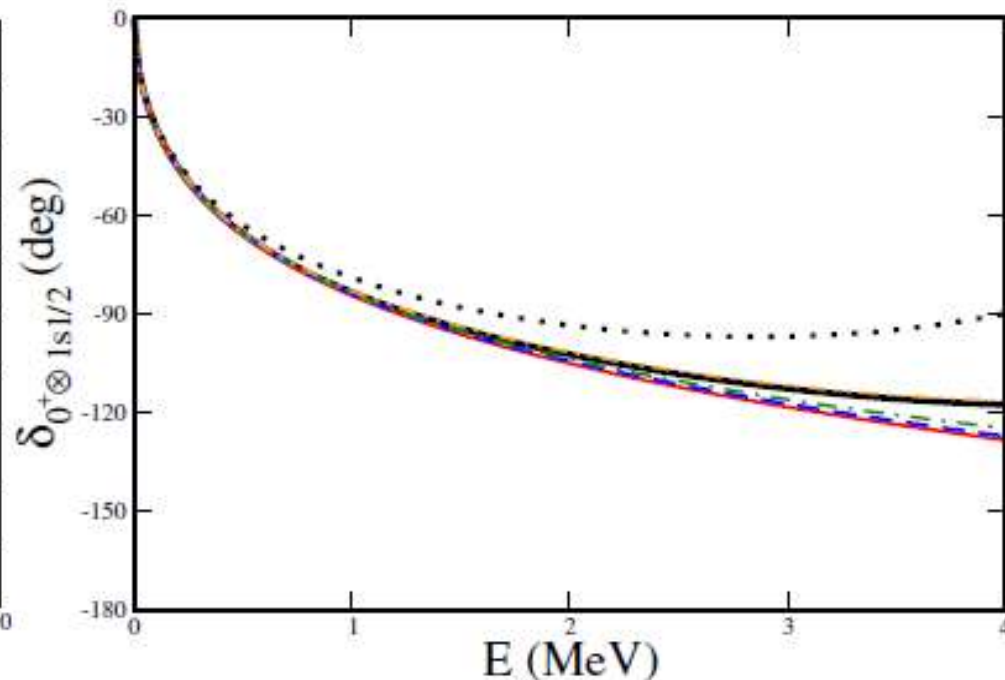
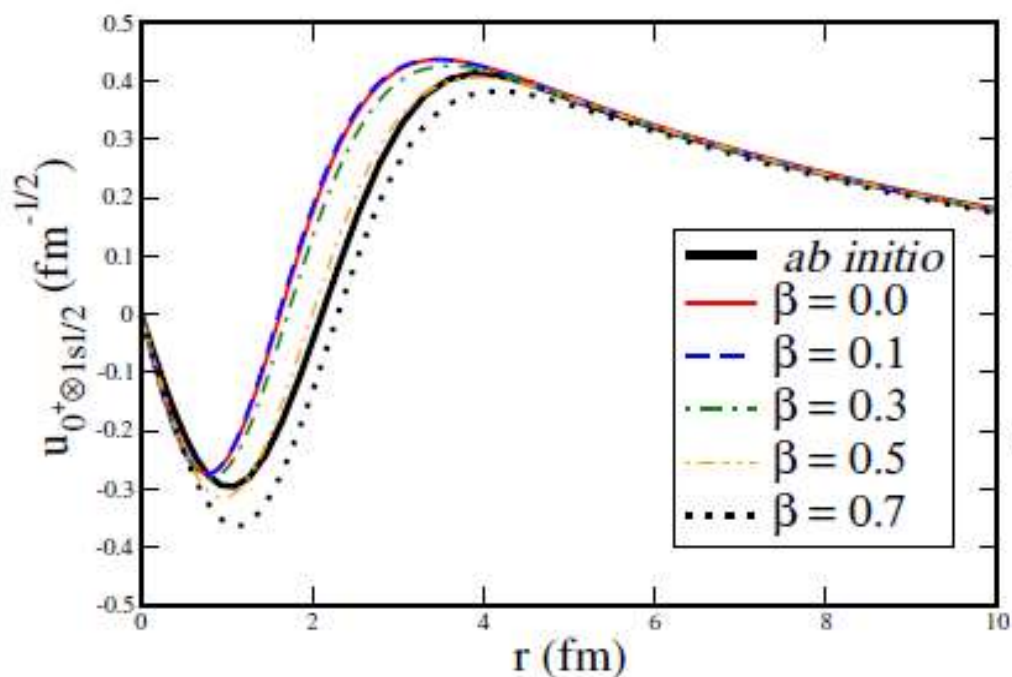
Study impact of core excitation on: ψ_{α} , δ_{α}



Ground state: $\frac{1}{2}^+$

Compare to *ab initio* predictions [Calci et al., PRL 117, 242501 (2016)]

- $\Psi_{1/2^+} = \psi_{1s1/2}(\mathbf{r}) \otimes \chi_{0^+}^{10\text{Be}} + \psi_{0d5/2}(\mathbf{r}) \otimes \chi_{2^+}^{10\text{Be}} + \psi_{0d3/2}(\mathbf{r}) \otimes \chi_{2^+}^{10\text{Be}}$
- NLO potentials **fitted to** reproduce S_n and *ab initio* **ANC** for $\neq \beta$



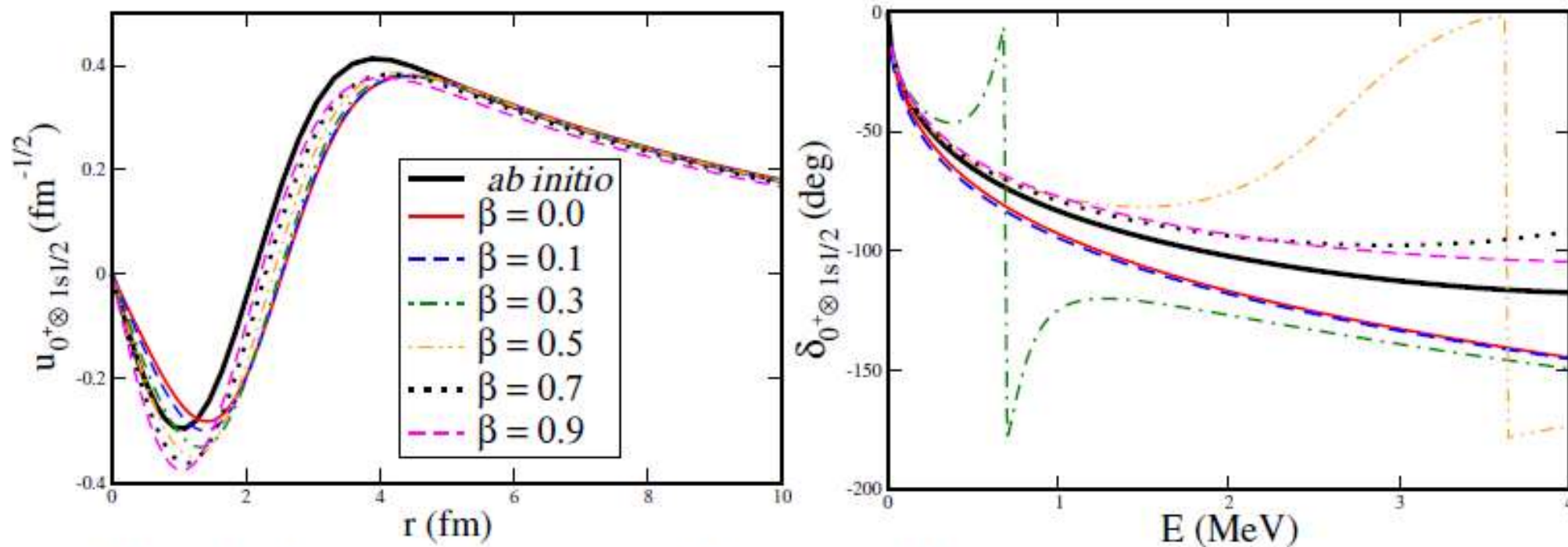
$\beta=0.5$ in excellent agreement with *ab initio* for both ψ_α , δ_α

\Rightarrow Including core **dof** improves both ψ_α , δ_α with 1 added parameter: β

\Rightarrow Results are σ -independent

Ground state: $\frac{1}{2}^+$ - Type 2 solution

- $\Psi_{1/2^+} = \psi_{1s1/2}(\mathbf{r}) \otimes \chi_{0^+}^{10\text{Be}} + \psi_{0d5/2}(\mathbf{r}) \otimes \chi_{2^+}^{10\text{Be}} + \psi_{0d3/2}(\mathbf{r}) \otimes \chi_{2^+}^{10\text{Be}}$
- Another type of solutions can be found:
→ when potential hosts a 0d **bound state** (expected in shell model)



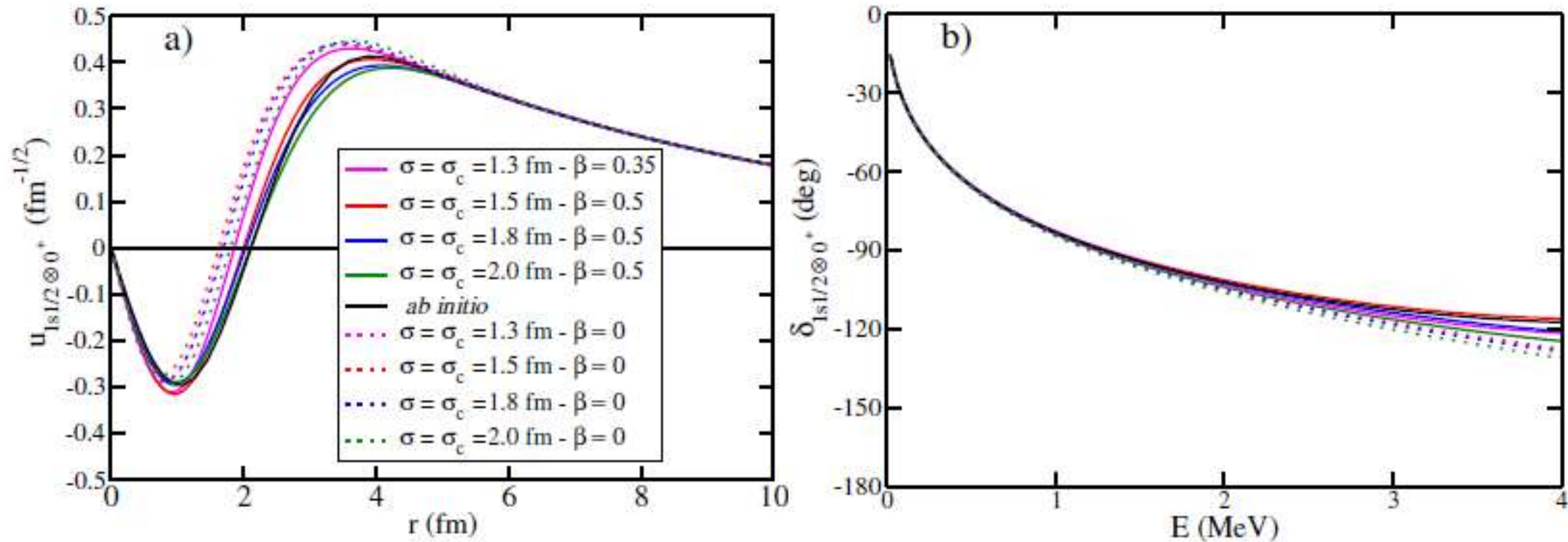
- Results less good than calculations without core excitation
→ this solution is rejected

Ground state and σ -dependency

Q1: In the spirit of the Halo-EFT, are our calculations σ -independent ?

Q2: Does adding a new degree of freedom lessen σ -dependency ?

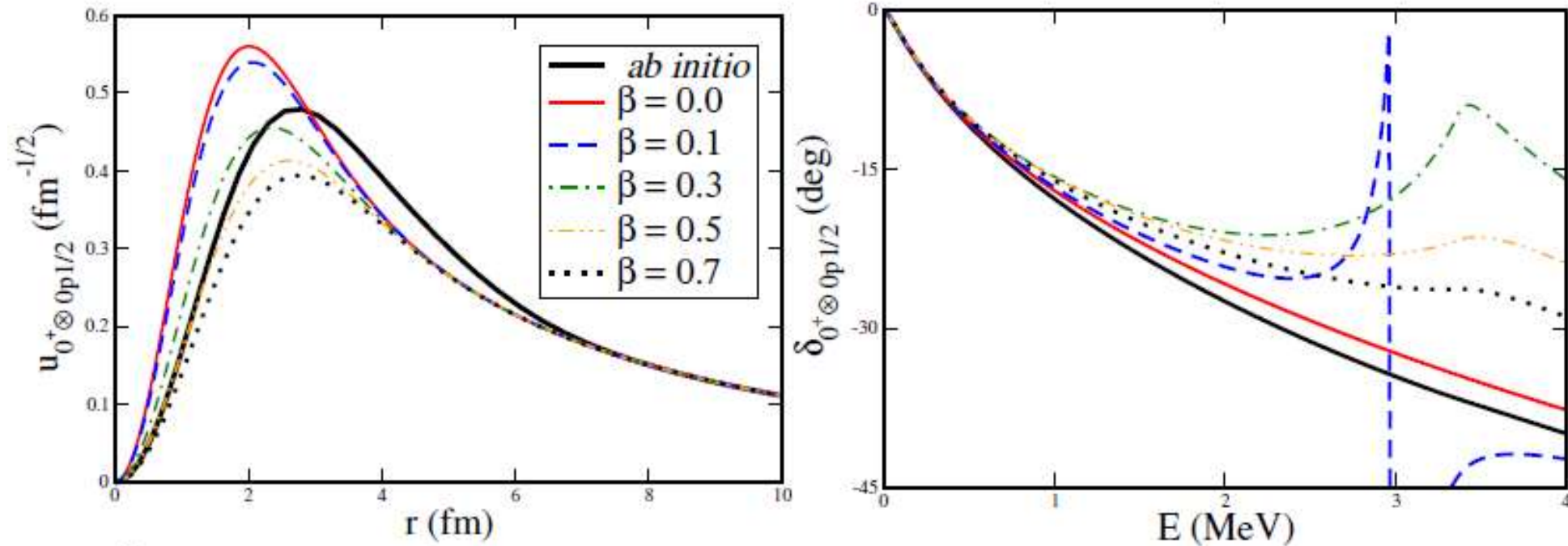
Idea: compare coupled-channel [**Type 1 solution**] to s.p. NLO results



Coupled-channel calculations reduce σ -dependency for both ψ_α and δ_α
 \rightarrow new degree of freedom decreases σ -dependency

First bound excited state: $\frac{1}{2}^-$

- $\Psi_{1/2^-} = \psi_{0p1/2}(\mathbf{r}) \otimes \chi_{0^+}^{10\text{Be}} + \psi_{0p3/2}(\mathbf{r}) \otimes \chi_{2^+}^{10\text{Be}} + \psi_{0f5/2}(\mathbf{r}) \otimes \chi_{2^+}^{10\text{Be}}$
- NLO potentials **fitted to** reproduce S_n and *ab initio* **ANC** for $\neq \beta$



- $\frac{1}{2}^- :=$ **core excitation does not improve the model:**
 - wfs: no improvement in the “pre-asymptotic” region (4-6 fm)
 - phaseshifts: less good than without core excitation
- **No “type 2” solution** because $E_{0p3/2}$ not at the right energy

Electric dipole transition probability: B(E1)

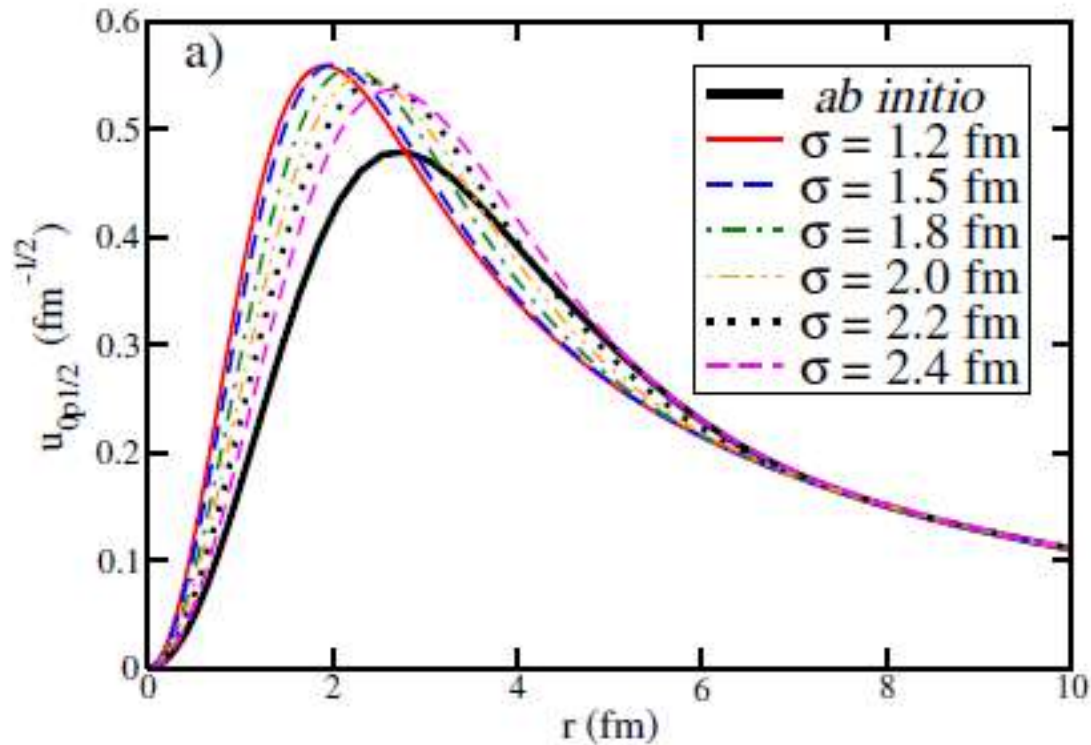
$$B(E\lambda; J \rightarrow J') = \frac{2J' + 1}{2J + 1} |\langle J' \| \mathcal{M}(E\lambda) \| J \rangle|^2$$

$$B(E1; \frac{1}{2}^+ \rightarrow \frac{1}{2}^-)$$

Different models/experiments	B(E1)[e ² fm ²]
Exp. (1983) from Ref. [114]	0.116(12)
Exp. (2007) from Ref. [113]	0.105(12)
Exp. (2014) from Ref. [112]	0.102(2)
Th. - F.M. Nunes (1996) - CC mean-field - from Ref. [70]	0.150
Th. - N.C. Summers (2014) - XCDCC - from Ref. [112]	0.098(4)
Th.- Calci <i>et al.</i> (2016) - "NCSMC" <i>ab initio</i> - from Ref. [22]	0.117
This work - CC ($\sigma=\sigma_c=1.3 - \beta_2=0.35$)	0.104
This work - CC ($\sigma=\sigma_c=1.5 - \beta_2=0.50$)	0.106
This work - CC ($\sigma=\sigma_c=1.8 - \beta_2=0.50$)	0.109
This work - CC ($\sigma=\sigma_c=2.0 - \beta_2=0.50$)	0.110

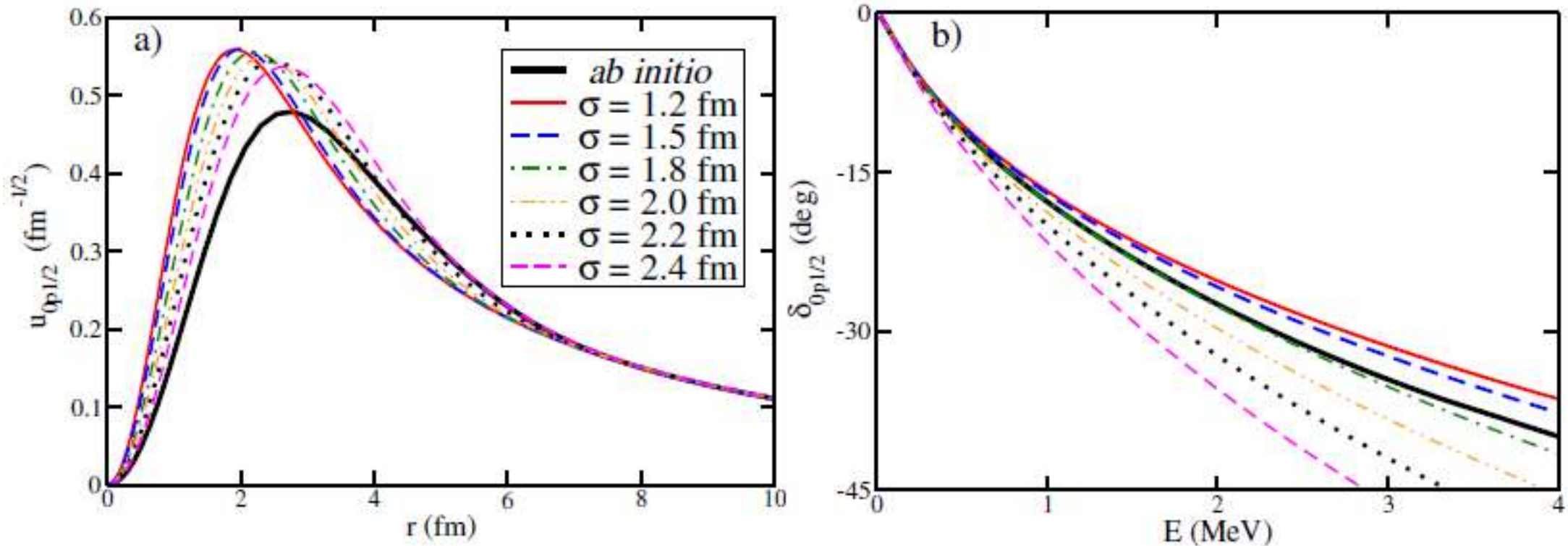
- Excellent agreement with experimental data BUT small discrepancies with *ab initio* value
- *Ab initio* B(E1) overestimates the data → due to their 1/2⁻ state (see A. Moro's talk)

σ -dependency already visible in s.p. NLO results:



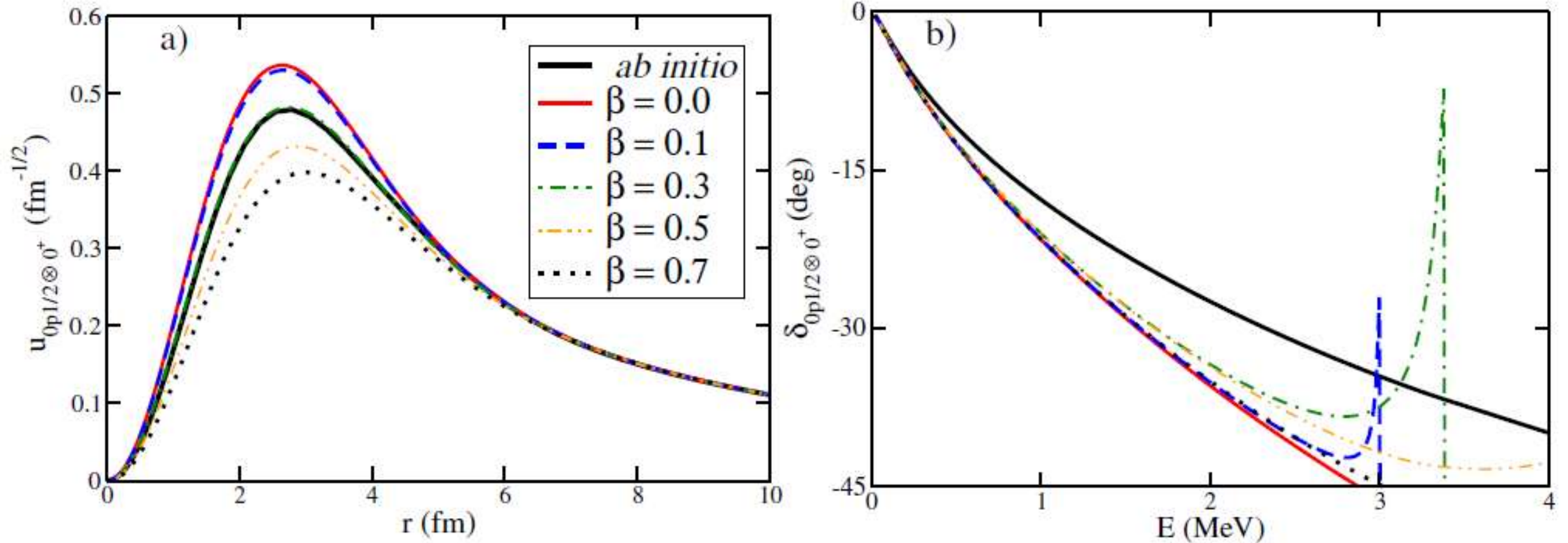
- Pre-asymptotic region only reproduced with large cut-off [$\sigma = 2.4$ fm]
→ $\frac{1}{2}^-$ state := mean-field state

σ -dependency already visible in s.p. NLO results:



- Pre-asymptotic region only reproduced with large cut-off [$\sigma = 2.4$ fm]
 $\rightarrow \frac{1}{2}^-$ state := mean-field state
- Phaseshifts: clear σ -dependency [best δ_α for $\sigma = 1.8$ fm]
 \rightarrow need of N²LO structure and **breakup** calculations

First bound excited state: $\frac{1}{2}^+$ - large $\sigma=2.4\text{fm}$



$\beta=0.3$:

- reproduces the *ab initio* wf very well [SF=0.86 vs 0.85 for *ab initio*]
- yields B(E1) in very good agreement with the *ab initio* prediction
- (and all β s) reproduces poorly the *ab initio* phaseshift

$\frac{1}{2}^+$ - large $\sigma=2.4\text{fm}$ \rightarrow my B(E1) vs *ab initio* B(E1)



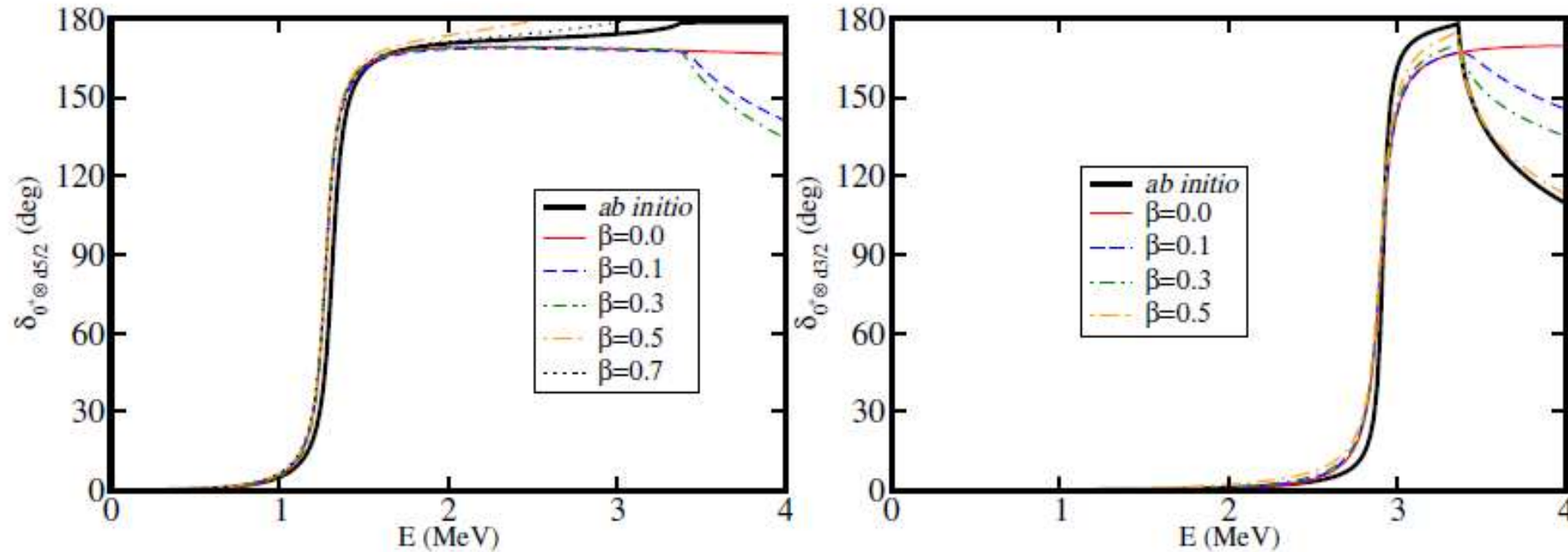
g.s	e.s	
$\sigma=\sigma_c$ [fm]	$\sigma=\sigma_c$ [fm]	$B(E1)$ [$e^2\text{fm}^2$]
1.3 - $\beta_2=0.35$	1.3 - $\beta_2=0.35$	0.104
1.5 - $\beta_2=0.50$	1.5 - $\beta_2=0.50$	0.106
1.8 - $\beta_2=0.50$	1.8 - $\beta_2=0.50$	0.109
2.0 - $\beta_2=0.50$	2.0 - $\beta_2=0.50$	0.110
1.3 - $\beta_2=0.35$	2.4 - $\beta_2=0.30$	0.115
1.5 - $\beta_2=0.50$	2.4 - $\beta_2=0.30$	0.114
1.8 - $\beta_2=0.50$	2.4 - $\beta_2=0.30$	0.114
2.0 - $\beta_2=0.50$	2.4 - $\beta_2=0.30$	0.113
<i>ab initio</i> from Ref. [22]	/	0.117
Exp. from Ref. [113]	/	0.105(12)
Exp. from Ref. [114]	/	0.116(12)
Exp. from Ref. [112]	/	0.102(2)

\rightarrow e.s. with large $\sigma=2.4\text{fm}$ \rightarrow ab initio B(E1) reproduced with max. 4% error !

Resonances @NLO: $\frac{5}{2}^+$, $\frac{3}{2}^-$, $\frac{3}{2}^+$

Compare to *ab initio* predictions [Calci et al., PRL 117, 242501 (2016)]

- NLO potentials **fitted to** reproduce exp. E_{res} and Γ_{res} for $\neq \beta$

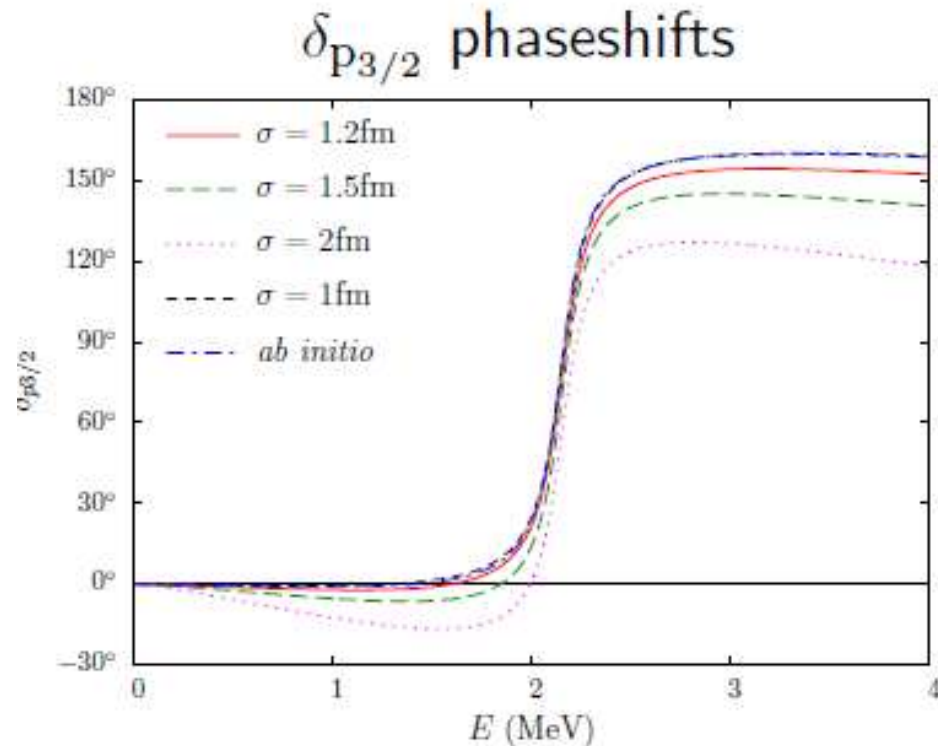


- Very good agreement for resonances too!
→ allows probing the **nature of resonances**
- Direct access to scattering wf, phaseshifts
→ $\frac{dB(E1)}{dE}$, cross sections for breakup, Coulomb excitation, transfer,...

s.p. @NLO: σ -dependency in p-waves

Sensitivity already seen in NLO calculations:

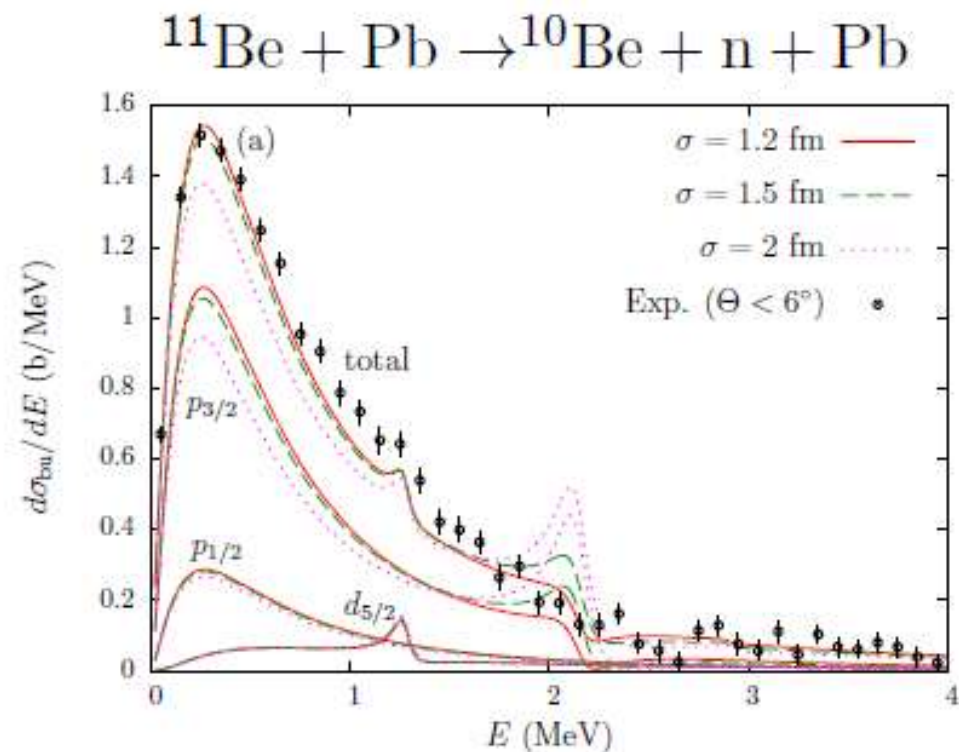
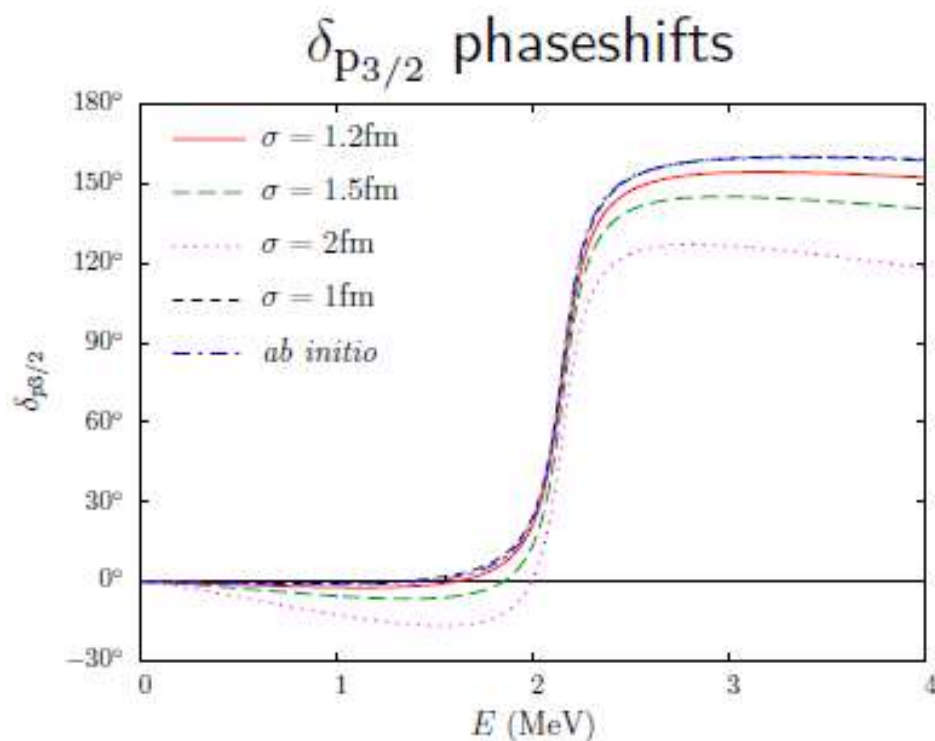
[Capel, Phillips, Hammer, PRC 98, 034610 (2018)]



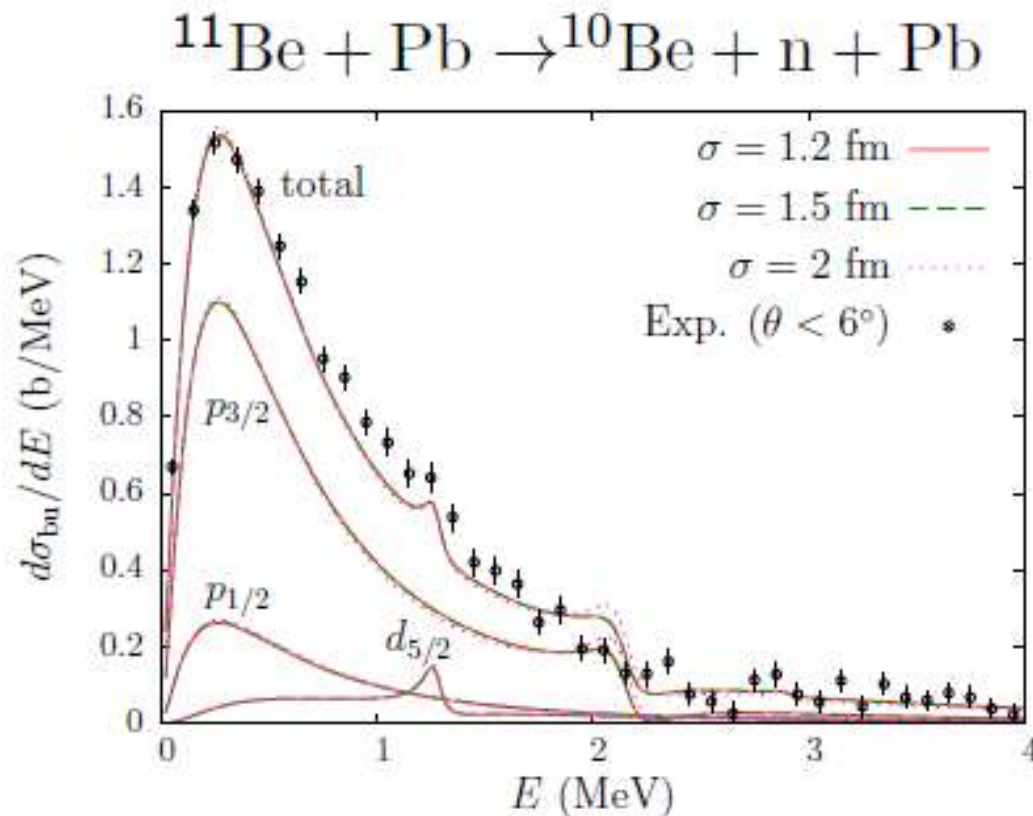
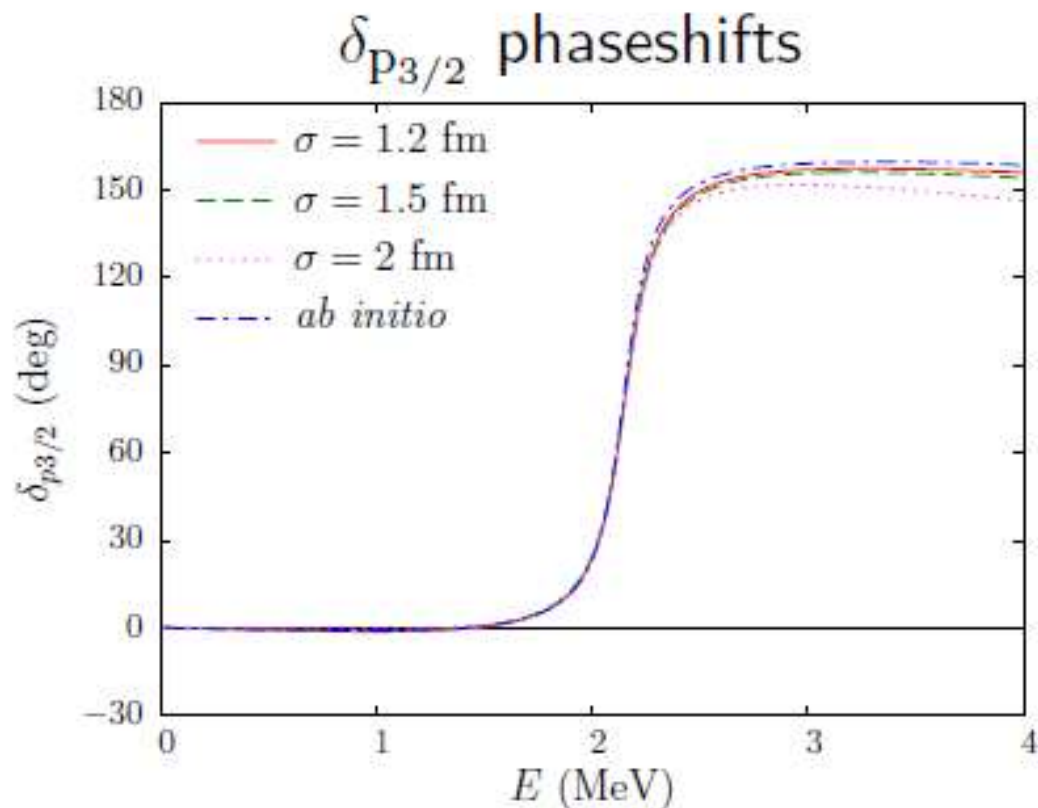
- @NLO: σ -dependency in δ

s.p. @NLO: σ -dependency in p-waves

Sensitivity already seen in NLO calculations:

[Capel, Phillips, Hammer, PRC 98, 034610 (2018)]

- @NLO: σ -dependency in δ leads to σ -dependency on cross sections
- @N²LO: strong reduction of the σ -dependency in $\delta_{p_{1/2}}$ and $\delta_{p_{3/2}}$?
→ Impact of N²LO structure model on reaction observables ?

s.p. @N²LO: σ -dependency in p-wavesDescription of ¹¹Be @N²LO:

- Suppression σ -dependency in p-waves phaseshifts
 \rightarrow same cross sections for all σ

[L.-P. Kubushishi and P. Capel, arXiv:2406.10168 (2024)]

Conclusion/outlook

I want to study reactions involving **one-neutron halo nuclei** :

- need of a **realistic few-body** model for reaction calculations
→ Halo-EFT

My model of one-neutron halo nuclei provides:

- perturbative inclusion of **core excitation within Halo-EFT**
- $\frac{1}{2}^+$ **state**: core excitation improves its few-body description
→ both wavefunction and phaseshift
- $\frac{1}{2}^-$ **state**: core excitation does not improve its few-body description
[L.-P. Kubushishi and P. Capel, (2024), (in preparation)]

Going @N²LO in **s.p.** description:

- removes σ -dependency in $\delta_{p_{1/2}}$ and $\delta_{p_{3/2}}$ phaseshifts/cross sections
[L.-P. Kubushishi and P. Capel, arXiv:2406.10168 (2024)]

Outlook:

- same formalism to study heavier haloes: ^{19}C , ^{31}Ne , ^{37}Mg , ^{34}Na
- include our model in reaction codes (**breakup**, Coulomb excitation,...)



Thank you!