Recent advances in the modeling of reactions with weakly-bound nuclei

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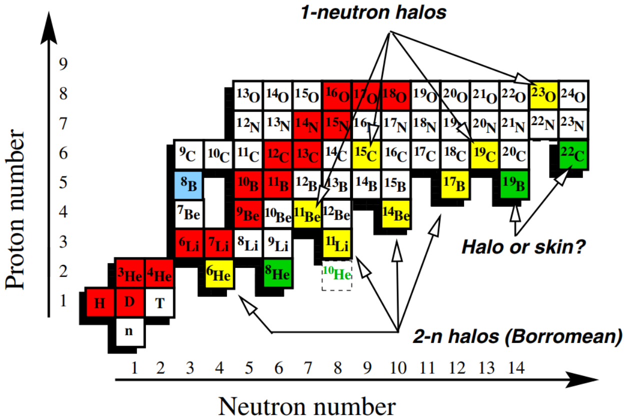
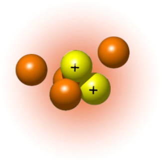
- Many physical processes occurring spontaneously in nature (e.g. stars) or artificially (e.g. nuclear reactor) involve nuclear reactions. We need theoretical tools to evaluate their rates and cross sections.
- Reaction theory provides the necessary framework to extract meaningful structure information from measured cross sections and also permits the understanding of the dynamics of nuclear collisions.
- The many-body scattering problem is not solvable in general, so specific models tailored to specific types of reactions are used (elastic, breakup, transfer, knockout...) each of them emphasizing some particular degrees of freedom.
- In particular, reactions with weakly-bound nuclei exhibit distinctive features which require a special treatment (usually accounting for the coupling the breakup channels).



☞*In the light region of the nuclear chart, weakly-bound nuclei display exotic features such as haloes.*

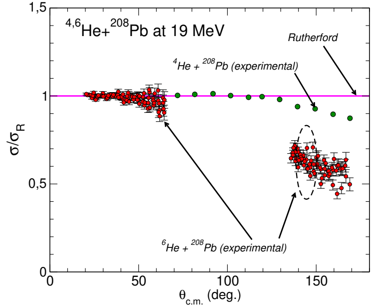
☞*Recall however that not all weakly-bound nuclei are unstable (e.g. deuterons) and many unstable nuclei are not weakly bound!*

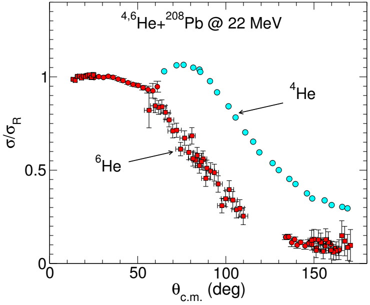
- Radioactive nuclei: they typically decay by β emission.
  E.g.: $^{6}\text{He} \xrightarrow{\beta^-} {}^{6}\text{Li}$ $(\tau_{1/2} \simeq 807 \text{ ms})$
- Weakly bound: typical separation energies are around 1 MeV or less.
- Spatially extended
- Halo structure: one or two weakly bound nucleons (typically neutrons) with a large probability of presence beyond the range of the potential.
- Borromean nuclei: Three-body systems with no bound binary sub-systems.



Signatures of weakly-bound nuclei in reaction observables

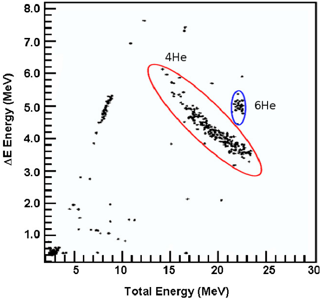
For ^{4,6}He+^{208}Pb, the Coulomb barrier is $V_b \approx 21$ MeV

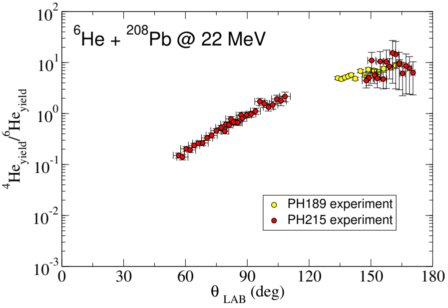




- ^{4}He follows Rutherford formula at 19 MeV and follows the "usual" Fresnel-type pattern at 22 MeV.
- ^{6}He drastically departs from Rutherford formula below the barrier and departs from Fresnel above barrier.

$$ {}^{6}\mathrm{He} + {}^{208}\,\mathrm{Pb} \rightarrow \alpha + \mathrm{X} $$





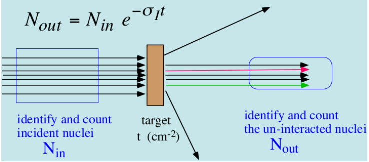
⇨At large angles, there are more α's than ^{6}He (elastic) !

⇨What are the mechanisms behind the α producion and how can we compute it?

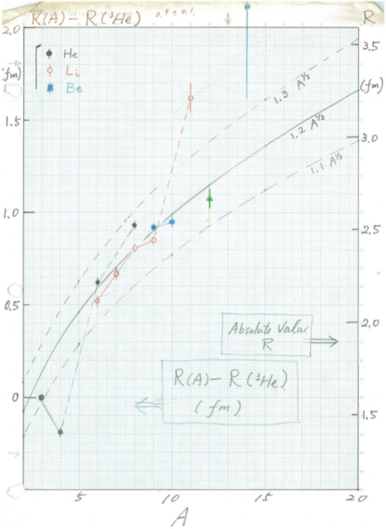
High-energy interaction cross sections with light targets

Interaction cross sections of nuclei on light targets and high energies are proportional to the size of the colliding nuclei.

$$\sigma_I \simeq \pi(R_p + R_t)^2$$



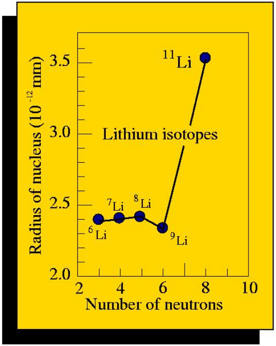
From I. Tanihata

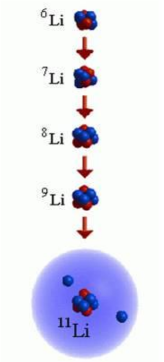


High-energy interaction cross sections with light targets

Interaction cross sections of nuclei on light targets and high energies (hundreds MeV/nucleon) are proportional to the size of the colliding nuclei.

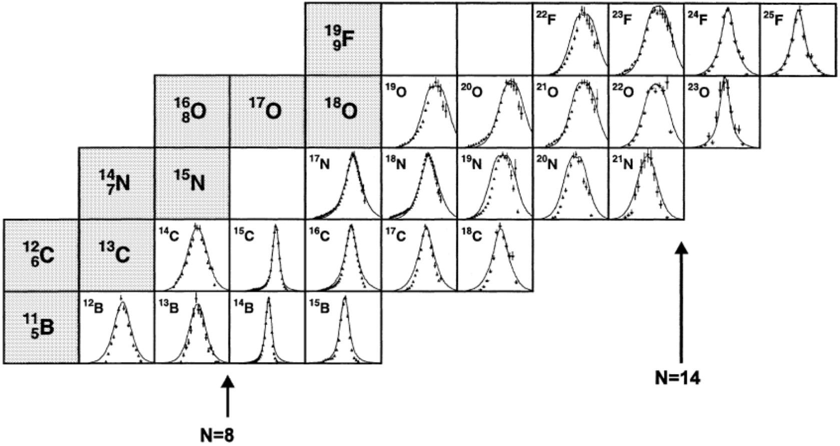
$$\sigma_I \simeq \pi(R_p + R_t)^2$$





Tanihata *et al*, PRL55, 2676 (1985)

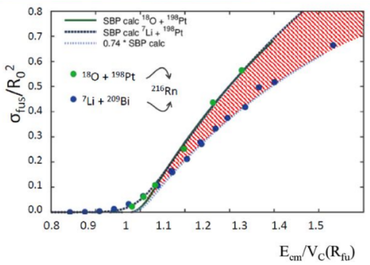
What do momentum distributions tell us about the size of the nucleus?



E. Sauvan et al., PLB 491, 1 (2000)

☞ *A narrow momentum distribution is a signature of an extended spatial distribution*

CF of weakly bound nuclei suppressed at energies above the Coulomb barrier-



- Observed for weakly-bound projectiles (^{6,7,8}Li,^{9}Be)
- CF reduced by ∼30% with respect to BPM or CC calculations.

M. Dasgupta et al., PRC 70, 024606 (2004)

Common interpretation:

⇨ CF is mostly reduced by breakup and incomplete fusion (ICF)

⇨ ICF is modeled as two-step process: breakup followed by capture of one charged fragment (breakup-fusion, BF).

Modelling nuclear reactions

Nuclear reaction theory:

- G.R. Satchler, *Introduction to nuclear reactions*, Macmillan (1990)
- G.R. Satchler, *Direct Nuclear Reactions*, Oxford University Press (1983)
- N. Glendenning, *Direct Nuclear Reactions*, World Scientific (2004)
- I.J. Thompson and F.M. Nunes, *Nuclear Reactions for Astrophysics*, Cambridge University Press (2009)

Reactions with weakly-bound nuclei

- A.M.M., *Models for nuclear reactions with weakly bound systems*, Proceedings of the International School of Physics Enrico Fermi Course 201 “Nuclear Physics with Stable and Radioactive Ion Beams” (arxiv:1807.04349).
- A.M.M., J. Casal, M. Gomez-Ramos, *The art of modeling nuclear reactions with weakly bound nuclei: status and perspectives*, Submitted to EPJA. (arXiv:2408.00175)

Direct versus compound reactions

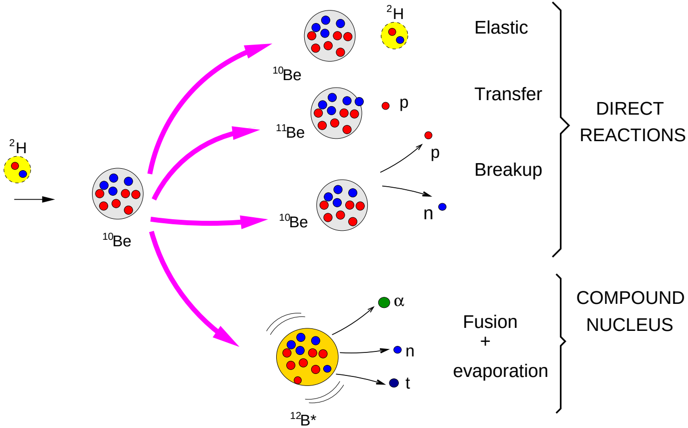
DIRECT: elastic, inelastic, transfer,...

- "fast" collisions (10^{-21} s).
- only a few modes (degrees of freedom) involved
- small momentum transfer
- angular distribution asymmetric about $\pi/2$ (forward peaked)

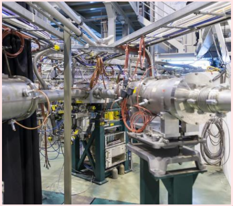
COMPOUND: complete, incomplete fusion.

- "slow" collisions ($10^{-18} - 10^{-16}$ s).
- many degrees of freedom involved
- large amount of momentum transfer
- "loss of memory" $\Rightarrow$ dominated by statistical decay of different of emitted particles; almost symmetric distributions forward/backward (in CM)

Example: the d+^{10}Be reaction

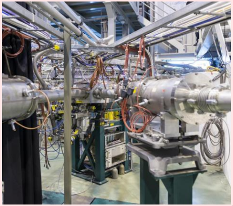


EXPERIMENT



THEORY
$(H\Psi = E\Psi)$

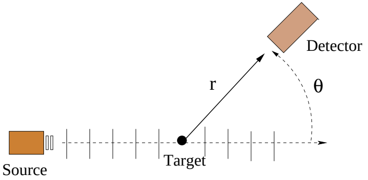
EXPERIMENT



THEORY
($H\Psi = E\Psi$)

CROSS SECTIONS

$$\frac{d\sigma}{d\Omega}, \frac{d\sigma}{dE}, etc$$



$$\Delta I = I_0 \, n_t \, \frac{d\sigma}{d\Omega} \Delta\Omega$$

- ΔI: detected particles per unit time in $\Delta\Omega$ (s^{-1})
- I_0: incident particles per unit time and unit area ($s^{-1}L^{-2}$)
- n_t: number of target nuclei within the beam
- $\Delta\Omega$: solid angle of detector ($=\Delta A/r^2$)
- $d\sigma/d\Omega$: differential cross section (L^2)

$$\frac{d\sigma}{d\Omega} = \frac{\text{flux of scattered particles through } dA = r^2 d\Omega}{\text{incident flux}}$$

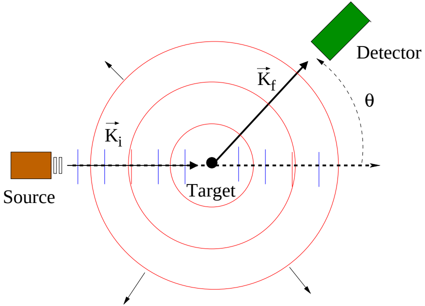
Full Hamiltonian

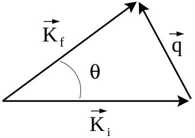
$$H = \underbrace{H_p(\xi_p) + H_t(\xi_t)}_{\text{internal dyn.}} + \underbrace{\hat{T}_{\mathbf{R}} + V(\mathbf{R}, \xi_p, \xi_t)}_{\text{relative motion}}$$

- $\hat{T}_{\mathbf{R}}$: proj.–target kinetic energy
- $H_p(\xi_p)$: projectile internal Hamiltonian
- $H_t(\xi_t)$: target internal Hamiltonian
- $V(\mathbf{R}, \xi_p, \xi_t)$: projectile–target interaction

Time-independent Schrödinger equation:

$$[H - E]\Psi(\mathbf{R}, \xi_p, \xi_t) = 0$$





Among the many mathematical solutions of $[H - E]\Psi = 0$ we are interested in those behaving asymptotically as:

$$\Psi^{(+)}_{\mathbf{K}_\alpha} \rightarrow \Phi_\alpha(\xi_\alpha) e^{i\mathbf{K}_\alpha \cdot \mathbf{R}_\alpha} + (\text{outgoing spherical waves in } \alpha, \beta, \ldots)$$

α=incident (elastic) channel, β,γ,... =nonelastic channels

$$\Psi^{(+)}_{\mathbf{K}_\alpha} \xrightarrow{R_\alpha \gg} \Phi_\alpha(\xi_\alpha) e^{i\mathbf{K}_\alpha \cdot \mathbf{R}_\alpha} + \Phi_\alpha(\xi_\alpha) f_{\alpha,\alpha}(\theta) \frac{e^{iK_\alpha R_\alpha}}{R_\alpha} \qquad \text{(elastic)}$$

$$+ \sum_{\alpha' \neq \alpha} \Phi_{\alpha'}(\xi_\alpha) f_{\alpha',\alpha}(\theta) \frac{e^{iK_{\alpha'} R_\alpha}}{R_\alpha} \qquad \text{(inelastic)}$$

$$\Psi^{(+)}_{\mathbf{K}_\alpha} \xrightarrow{R_\beta \gg} \sum_\beta \Phi_\beta(\xi_\beta) f_{\beta,\alpha}(\theta) \frac{e^{iK_\beta R_\beta}}{R_\beta} \qquad \text{(transfer)}$$

Cross sections:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\alpha\to\beta} = \frac{\mu_\alpha}{\mu_\beta} \frac{K_\beta}{K_\alpha} \left| f_{\beta,\alpha}(\theta) \right|^2 \qquad E = \frac{\hbar^2 K_\alpha^2}{2\mu_\alpha} + \varepsilon_\alpha = \frac{\hbar^2 K_\beta^2}{2\mu_\beta} + \varepsilon_\beta$$

$f_{\beta,\alpha}$ is called scattering amplitude

Ideally, the strategy would be:

1. Choose structure model for $H_\alpha(\xi)$
2. Compute $\Psi^{(+)}$ by solving $[H - E]\Psi^{(+)} = 0$
3. Consider the limit $R \gg$ of $\Psi^{(+)}$
4. Project it onto the desired final state to extract the scattering amplitude:

$$(\Phi_{\alpha'}|\Psi^{(+)}\rangle \equiv \int d\xi_\alpha \Phi^*_{\alpha'}(\xi_\alpha)\Psi^{(+)} = f_{\alpha',\alpha}(\theta)\frac{e^{iK_{\alpha'}R_\alpha}}{R_\alpha}$$

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$$\langle \Phi_{\alpha'} | \Psi^{(+)} \rangle \equiv \int d\xi_\alpha \Phi^*_{\alpha'}(\xi_\alpha) \Psi^{(+)} = f_{\alpha',\alpha}(\theta) \frac{e^{iK_{\alpha'} R_\alpha}}{R_\alpha}$$

But...

- Ψ is a solution of a complicated many-body problem, not solvable in most cases.
- The number of accessible channels and states can be enormous.

⇨*So, in practice, we will be content with an approximation of Ψ (or $f(\theta)$) in a restricted modelspace*

Defining our model space: Feshbach formalism

- Divide the full space into two groups: P and Q
  - ➩ P: channels of interest
  - ➩ Q: remaining channels
- Write $\Psi = \Psi_P + \Psi_Q$

$$\begin{aligned}(E - H_{PP})\Psi_P &= H_{PQ}\Psi_Q \\ (E - H_{QQ})\Psi_Q &= H_{QP}\Psi_P\end{aligned}$$

($H_{PP} = PHP$, $H_{PQ} = PHQ$, etc)

- Eliminate (formally) Ψ_Q:

$$\underbrace{\left[H_{PP} + H_{PQ}\frac{1}{E - H_{QQ} + i\epsilon}H_{QP}\right]}_{H_{\rm eff}}\Psi_P = E\Psi_P$$

- $H_{\rm eff}$ too complicated (complex, energy dependent, non-local) ⇒ needs to be replaced by a simpler Hamiltonian:

$$H_{\rm eff} \longrightarrow H_{\rm model} \quad \text{(complex, energy dependent)}$$

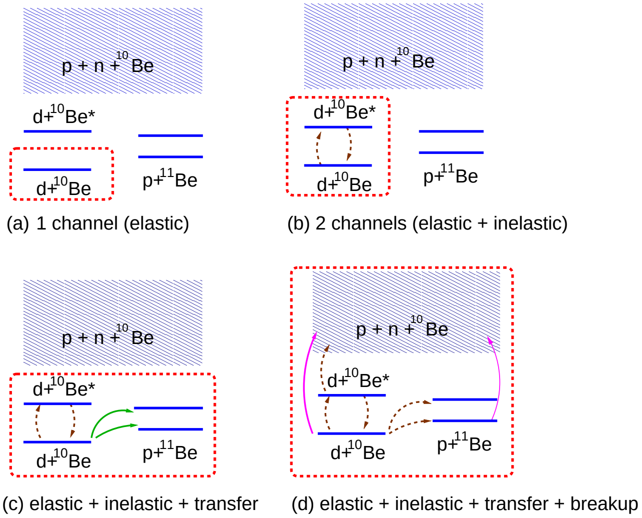
We need to make a choice for:

1. Modelspace: what channels are to be included?
2. Structure model: for projectile and target

   (Microscopic, collective, cluster...)
3. Reaction formalism

   (will depend on the process to be studied)

Choice of the modespace: the d+^{10}Be example



Modeling elastic scattering: the optical model

- Problem: How do we describe certain channels P, while leaving others apart Q
- P space represents just the ground state of projectile and target
- Wavefunction:

$$\Psi = \underbrace{\Psi_P}_{\text{elastic}} + \underbrace{\Psi_Q}_{\text{non-elastic}}$$

- Schrodinger equation in modelspace:

$$\left[T + H_\alpha(\xi_\alpha) + \mathcal{V}\right]\Psi_P = E\Psi_P$$

$$\mathcal{V} = \underbrace{V_{PP}}_{\text{Bare interaction}} + \underbrace{V_{PQ}\frac{1}{E - H_{QQ} + i\epsilon}V_{QP}}_{\text{“Polarization” potential}} \equiv V_{\text{bare}} + V_{\text{pol}}$$

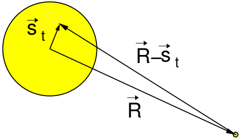
- Solution: Write the interaction as a bare potential plus a polarization potential.
- $\mathcal{V}$ too complicated ⇒ usually replaced by some phenomenological (complex) potential $U(\mathbf{R})$

Start from some (effective) nucleon-nucleon potential v_{NN} (JLM, M3Y, etc):

1. Single-folding potential:

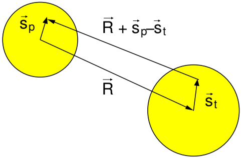
$$V(\mathbf{R}) = \int \rho_t(\mathbf{s}_t) v_{NN}(|\mathbf{R} - \mathbf{s}_t|) d\mathbf{s}_t$$

☞ $\rho_t(\mathbf{s}_t)$=target g.s. density.



2. Double-folding potential:

$$V(\mathbf{R}) = \int \rho_p(\mathbf{s}_p) \rho_t(\mathbf{s}_t) v_{NN}(|\mathbf{R} + \mathbf{s}_p - \mathbf{s}_t|) d\mathbf{s}_p d\mathbf{s}_t$$



- ⇨ If ρ_p and ρ_t are g.s. densities, $V(\mathbf{R})$ accounts only for the bare potential (V_{PP}) (P-space part) and ignores the effect of non-elastic channels.
- ⇨ A model for V_{pol} must be supplied.
- ⇨ If v_{NN} is real, $V(\mathbf{R})$ is also real.

Effective potential: $\mathcal{V} \approx U(R) = U_{\rm nuc}(R) + U_{\rm coul}(R)$

- Coulomb potential: charge sphere distribution

$$U_{\rm coul}(R) = \begin{cases} \frac{Z_1 Z_2 e^2}{2R_c}\left(3 - \frac{R^2}{R_c^2}\right) & \text{if } R \leq R_c \\ \frac{Z_1 Z_2 e^2}{R} & \text{if } R \geq R_c \end{cases}$$

- Nuclear potential (complex): Eg. Woods-Saxon parametrization

$$U_{\rm nuc}(R) = V(r) + iW(r) = -\frac{V_0}{1+\exp\left(\frac{R-R_r}{a_r}\right)} - i\,\frac{W_0}{1+\exp\left(\frac{R-R_i}{a_i}\right)}$$

- 6 parameters, fitted to data. Real and imaginary widths, radii and difuseness. $R_{c,r,i} = r_{c,r,i}(A_p^{1/3} + A_t^{1/3})$ ($r_{c,r,i}$=reduced radii) $r_c, r_r, r_i \sim 1.1 - 1.4$ fm. $a_r, a_i \sim 0.5 - 0.7$ fm.

- Real nuclear potential describes nuclear attraction. Imaginary nuclear potential describes the loss of particles in the elastic channel due to other reaction channels.

- Effective Hamiltonian:

$$H = T_{\mathbf{R}} + +U(\mathbf{R}) \qquad (U(\mathbf{R}) \text{ complex!})$$

- Schrödinger equation:

$$[T_{\mathbf{R}} + U(\mathbf{R}) - E_\alpha]\chi_0^{(+)}(\mathbf{K}, \mathbf{R}) = 0 \qquad (E_\alpha = \text{incident energy in CM})$$

- Boundary condition: plane wave plus spherical wave, multiplied by the **scattering amplitude** $f(\theta, \phi)$.

$$\chi_0^{(+)}(\mathbf{K}, \mathbf{R}) \rightarrow e^{i\mathbf{K}_i \cdot \mathbf{R}} + f(\theta)\frac{e^{iK_f R}}{R} \qquad K_i = K_f = \frac{\sqrt{2\mu E_\alpha}}{\hbar}$$

- For a central potential $[U(\mathbf{R}) = U(R)]$ the scattering wavefunction can be expanded in spherical harmonics (eigenfunctions of $\hat{L}^2$ and $\hat{L}_z$):

$$\chi_0^{(+)}(\mathbf{K},\mathbf{R}) = \frac{4\pi}{KR}\sum_\ell i^\ell \chi_\ell(K,R) \sum_m Y^*_{\ell m}(\hat{K}) Y_{\ell m}(\hat{R}) = \frac{1}{KR}\sum_\ell i^\ell \chi_\ell(K,R)(2\ell+1)P_\ell(\cos\theta)$$

- The radial wavefuntions $\chi_\ell(K,R)$ satisfy the equation:

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{R^2} + U(R) - E_0\right]\chi_\ell(K,R) = 0.$$

- For $U(R) = 0$, $\chi_0^{(+)}(\mathbf{K},\mathbf{R})$ must reduce to the plane wave:

$$e^{i\mathbf{K}\cdot\mathbf{R}} = \frac{1}{KR}\sum_\ell i^\ell (2\ell+1) F_\ell(KR) P_\ell(\cos\theta)$$

⇨ So, for $U = 0 \Rightarrow \chi_\ell(K,R) = F_\ell(KR) = (KR)j_\ell(KR) \rightarrow \sin(KR - \ell\pi/2)$

- For $R \gg \Rightarrow U(R) = 0 \Rightarrow \chi_\ell(K,R)$ will be a combination of F_ℓ and G_ℓ

$$F_\ell(KR) \to \sin(KR - \ell\pi/2) \qquad G_\ell(KR) \to \cos(KR - \ell\pi/2)$$

or their *outgoing/ingoing* combinations:

$$H^{(\pm)}(KR) \equiv G_\ell(KR) \pm iF_\ell(KR) \to e^{\pm i(KR-\ell\pi/2)}$$

- The physical solution is determined by the known boundary conditions:

$$\begin{array}{lllll} & \chi_0^{(+)}(\mathbf{KR}) & \to & e^{i\mathbf{K}\cdot\mathbf{R}} & + \; f(\theta)\dfrac{e^{iKR}}{R} \\ & \Downarrow & & \Downarrow & \quad \Downarrow \\ U = 0 & \chi_\ell(KR) & \to & F_\ell(KR) & + \; 0 \\ U \neq 0 & \chi_\ell(KR) & \to & F_\ell(KR) & + \; T_\ell H^{(+)}(KR) \end{array}$$

☞ The coefficients T_ℓ are to be determined by numerical integration.

Numerical procedure

1. Fix a *matching radius*, R_m, such that $U(R_m) \approx 0$
2. Integrate $\chi_\ell(R)$ from $R = 0$ up to R_m, starting with the condition:

$$\lim_{R\to 0} \chi_\ell(K,R) = 0$$

3. At $R = R_m$ impose the boundary condition:

$$\chi_\ell(K,R) \to F_\ell(KR) + T_\ell H_\ell^{(+)}(KR)$$
$$= \frac{i}{2}[H_\ell^{(-)}(KR) - S_\ell H_\ell^{(+)}(KR)]$$

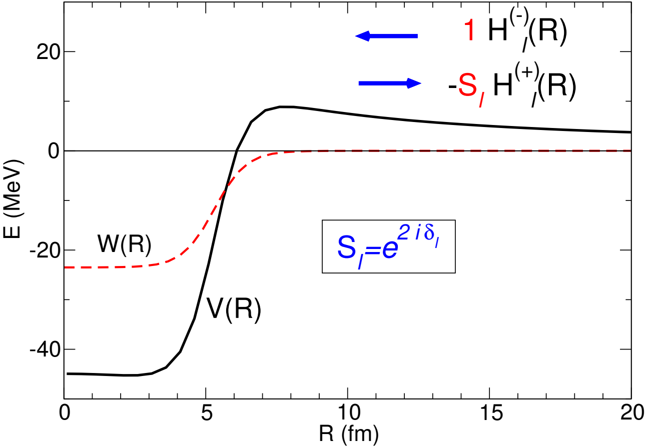
☞ $S_\ell = 1 + 2iT_\ell$ = **S-matrix**

4. Phase-shifts:

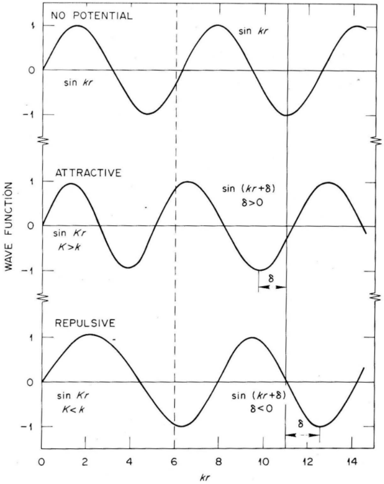
$$S_\ell \equiv e^{i2\delta_\ell} \qquad T_\ell = e^{i\delta_\ell}\sin(\delta_\ell)$$

$$\chi_\ell(K,R) \to e^{i\delta_\ell}\sin(KR + \delta_\ell - \ell\pi/2)$$

$$\chi_l(R) \rightarrow 1\, H^{(-)}_l(R) - S_l\, H^{(+)}_l(R)$$



- S_ℓ =coefficient of the outgoing wave for partial wave ℓ.
- $|S_\ell|^2$ is the *survival* probability for the partial wave ℓ:
  - U real $\Rightarrow |S_\ell| = 1 \Rightarrow \delta_\ell$ real
  - U complex $\Rightarrow |S_\ell| < 1 \Rightarrow \delta_\ell$ complex
- For $\ell \gg \Rightarrow S_\ell \to 1$
- Sign of $Re[\delta]$:
  - $Re[\delta] > 0 \Rightarrow$ attractive potential
  - $Re[\delta] < 0 \Rightarrow$ repulsive potential
  - $Re[\delta] = 0$ $(S_\ell = 1) \Rightarrow$ no potential $(U(R) = 0)$



- Replace the asymptotic $\chi_\ell(K,R)$ in the general expansion:

$$\chi_0^{(+)}(\mathbf{K},\mathbf{R}) \to \frac{1}{KR}\sum_\ell i^\ell(2\ell+1)\Big\{F_\ell(KR)+T_\ell H_\ell^{(+)}(KR)\Big\}P_\ell(\cos\theta)$$
$$= e^{i\mathbf{K}\cdot\mathbf{R}} + \frac{1}{K}\sum_\ell(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)\frac{e^{iKR}}{R}$$

(θ is the angle between **R** and **K**, which asymptotically corresponds to the scattering angle)

- The scattering amplitude is the coefficient of e^{iKR}/R.

$$f(\theta) = \frac{1}{K}\sum_\ell(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)$$
$$= \frac{1}{2iK}\sum_\ell(2\ell+1)(S_\ell-1)P_\ell(\cos\theta).$$

- Elastic cross section:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2.$$

Radial equation:

$$\left[\frac{d^2}{dR^2} + K^2 - \frac{2\eta K}{R} + \frac{2\mu}{\hbar^2}U(R) + \frac{\ell(\ell+1)}{R^2}\right]\chi_\ell(K,R) = 0$$

$$\eta = \frac{Z_p Z_t e^2}{\hbar v} = \frac{Z_p Z_t e^2 \mu}{\hbar^2 K}$$

(Sommerfeld parameter)

Asymptotic condition:

$$\chi^{(+)}(\mathbf{K},\mathbf{R}) \to e^{i[\mathbf{K}\cdot\mathbf{R}+\eta\log(kR-\mathbf{K}\cdot\mathbf{R})]} + f(\theta)\frac{e^{i(KR-\eta\log 2KR)}}{R}$$

$$\chi_\ell(K,R) \to e^{i\sigma_\ell}\left[F_\ell(\eta,KR) + T_\ell H_\ell^{(+)}(\eta,KR)\right] = \frac{i}{2}e^{i\sigma_\ell}\left[H_\ell^{(-)}(\eta,KR) - S_\ell H_\ell^{(+)}(\eta,KR)\right]$$

- ☞ $\sigma_\ell(\eta)$=Coulomb phase shift
- ☞ $F_\ell(\eta,KR)$=regular Coulomb wave
- ☞ $H_\ell^{(\pm)}(\eta,KR)$=outgoing/ingoing Coulomb wave

Total scattering amplitude:

$$f(\theta) = f_C(\theta) + \frac{1}{2iK} \sum_{\ell} (2\ell + 1) e^{2i\sigma_\ell} (S_\ell - 1) P_\ell(\cos\theta)$$

where $f_C(\theta)$ is the amplitude for pure Coulomb:

$$\frac{d\sigma_R}{d\Omega} = |f_C(\theta)|^2 = \frac{\eta^2}{4K^2 \sin^4(\frac{1}{2}\theta)} = \left(\frac{Z_p Z_t e^2}{4E}\right)^2 \frac{1}{\sin^4(\frac{1}{2}\theta)}$$

- Total elastic cross section (only uncharged particles!). Is a measurement of the particles that abandon the incident beam, to be scattered elastically to different angles.

$$\sigma_{el} = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{\pi}{K^2} \sum_{\ell} (2\ell + 1)|1 - S_\ell|^2 = \frac{4\pi}{K^2} \sum_{\ell} (2\ell + 1)|T_\ell|^2$$

  N.b.: when no potentials (real or imaginary) are present, $S_\ell = 1$, and the total elastic cross section is zero (no scattering).

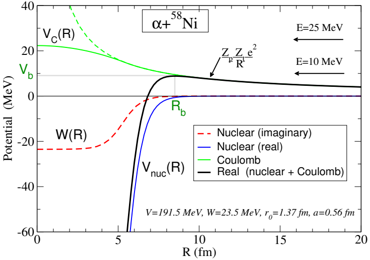
- Total reaction cross section. Is a measurement of the particles that abandon the incident beam, to give rise to different non-elastic reaction processes.

$$\sigma_{reac} = \frac{\pi}{K^2} \sum_{\ell} (2\ell + 1)(1 - |S_\ell|^2)$$

  N.b.: when there are no imaginary potentials, there is no loss of flux from the elastic channel, $|S_\ell| = 1$, and hence $\sigma_{reac} = 0$.

- The semi-classical vs quantum character of the scattering can be given in terms of the Sommerfeld parameter: $\eta = \frac{Z_p Z_t e^2}{4\pi\epsilon_0 \hbar v}$
- The Coulomb vs nuclear relevance, in terms of the energy of the Coulomb barrier: $V_b \simeq \frac{Z_p Z_t}{A_p^{1/3}+A_t^{1/3}}$ [MeV]
- Three distinct patterns appear for the elastic cross sections
  - Coulomb-dominated $E < V_b \Rightarrow$ Rutherford scattering
  - Nuclear relevant $E > V_b$, semiclassical $\eta \gg 1 \Rightarrow$ Fresnel scattering
  - Nuclear relevant $E > V_b$, quantum $\eta \lesssim 1 \Rightarrow$ Fraunhofer scattering

Effective potential: $U(R) = U_{nuc}(R) + U_{coul}(R)$



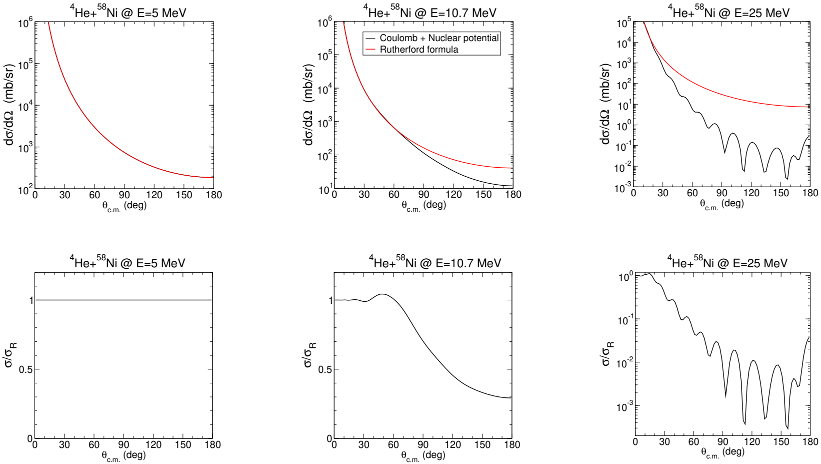
☞ The maximum of $V_{\rm nuc}(R) + V_C(R)$ defines the Coulomb barrier: V_b. From the plot:

$$R_b \approx 8.5 \text{ fm}; \qquad V_b \approx 10 \text{ MeV}$$

Approximately:

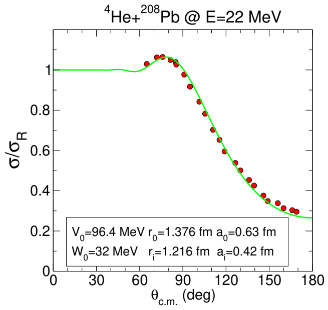
$$R_b \simeq 1.44(A_p^{1/3} + A_t^{1/3}) \text{ fm}$$

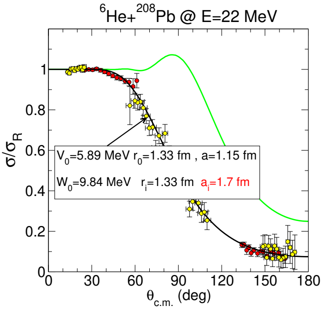
$$E_b \simeq \frac{Z_p Z_t e^2}{R_b} \approx \frac{Z_p Z_t}{(A_p^{1/3} + A_t^{1/3})} \text{ MeV}$$



Rutherford scattering **Fresnel Scattering** **Fraunhöfer Scattering**

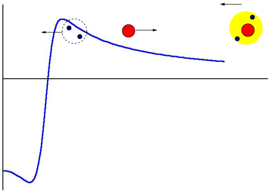
How does the halo structure affect the elastic scattering?





- ^{4}He+^{208}Pb shows typical Fresnel pattern and "standard" optical model parameters
- ^{6}He+^{208}Pb shows a prominent reduction in the elastic cross section, suggesting that part of the incident flux goes to non-elastic channels (eg. breakup)

  Understanding and disentangling these non-elastic channels requires going beyond the optical model (eg. coupled-channels method ⇒ next lectures)



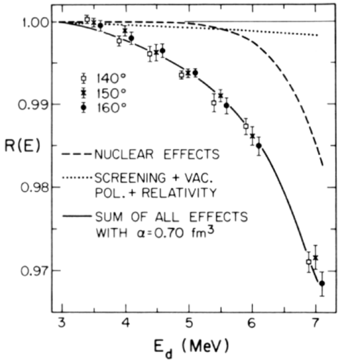
1. The strong Coulomb field will produce a polarization ("stretching") of the projectile, giving rise to a dipole contribution on the real potential:

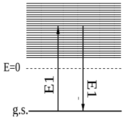
$$V(R) \approx \frac{Z_1 Z_2 e^2}{R} - \alpha \frac{Z_1 Z_2 e^2}{2R^4}$$

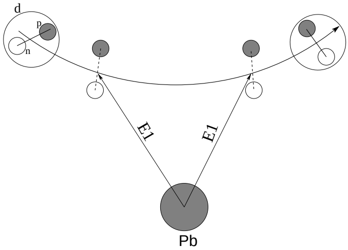
2. The weakly bound nucleus can eventually break up, leading to a loss of flux of the elastic channel ⇒ imaginary polarization potential.

Eg: deuteron polarizability from d+^{208}Pb:

☞ Adiabatic limit ($E_x \gg$): $V_{\rm pol}^{\rm dip} = -\alpha \frac{Z_1 Z_2 e^2}{2R^4}$



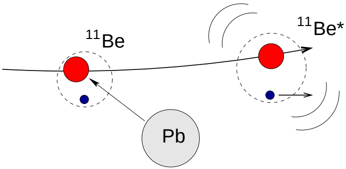


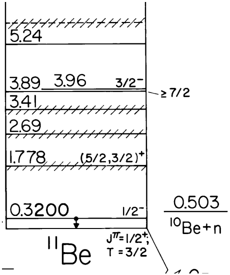


Rodning et al, PRL49, 909 (1982) ⇒ $\alpha = 0.70 \pm 0.05$ fm^3

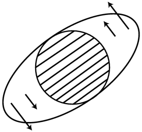
Inelastic scattering

- Nuclei are not inert or *frozen* objects; they do have an internal structure of protons and neutrons that can be modified (excited) during the collision.
- Quantum systems exhibit, in general, an energy spectrum with bound and unbound levels.

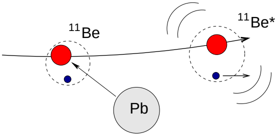




1. COLLECTIVE: Involve a collective motion of several nucleons which can be interpreted macroscopically as rotations or surface vibrations of the nucleus.



2. FEW-BODY/SINGLE-PARTICLE: Involve the excitation of a nucleon or cluster.



- By doing inelastic scattering experiments we *measure* the *response* of the nucleus to an external field (Coulomb, nuclear). This response is related to some structure property of the nucleus.

  Example: for a Coulomb field:

$$B(E\lambda; i \to f) = \frac{1}{2I_i + 1} |\langle \Psi_f \| \mathcal{M}(E\lambda) \| \Psi_i \rangle|^2_{BM}$$

  where $\mathcal{M}(E\lambda, \mu)$ is the electric multipole operator:

$$\mathcal{M}(E\lambda, \mu) \equiv e \sum_{i}^{Z_p} r_i^{\lambda} Y^*_{\lambda\mu}(\hat{r}_i)$$

- The structure $\Psi_{i,f}$ can be described in a collective, few-body or microscopic model.

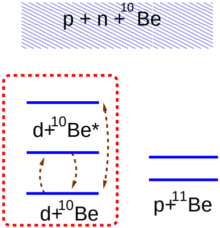
We need to incorporate explicitly in the Hamiltonian the internal structure of the nucleus being excited (e.g. target).

$$H = T_R + h(\xi) + V(\mathbf{R}, \xi)$$

- T_R: Kinetic energy for projectile-target relative motion.
- $\{\xi\}$: Internal degrees of freedom of the target (depend on the model).
- $h(\xi)$: Internal Hamiltonian of the target.

$$h(\xi)\phi_n(\xi) = \varepsilon_n \phi_n(\xi)$$

- $V(\mathbf{R}, \xi)$: Projectile-target interaction.



☞ P space composed by ground states (elastic channel) and some excited states (inelastic scattering)

Boundary conditions:

$$\Psi^{(+)}_{\mathbf{K}_0}(\mathbf{R},\xi) \xrightarrow{R\gg} \underbrace{e^{i\mathbf{K}_0\cdot\mathbf{R}}\phi_0(\xi)}_{\text{incident}} + \underbrace{f_{0,0}(\theta)\frac{e^{iK_0R}}{R}\phi_0(\xi)}_{\text{elastic}} + \underbrace{\sum_{n>0} f_{n,0}(\theta)\frac{e^{iK_nR}}{R}\phi_n(\xi)}_{\text{inelastic}}$$

Cross sections:

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{0\to n} = \frac{K_n}{K_0}\left|f_{n,0}(\theta)\right|^2$$

We expand the total wave function in a subset of internal states (the P space):

$$\Psi_{\text{model}}(\mathbf{R},\xi) = \phi_0(\xi)\chi_0(\mathbf{K}_0,\mathbf{R}) + \sum_{n>0} \phi_n(\xi)\chi_n(\mathbf{K}_n,\mathbf{R})$$

and impose the boundary conditions for the (unknown) $\chi_n(\mathbf{R})$:

$$\chi_0^{(+)}(\mathbf{K}_0,\mathbf{R}) \to e^{i\mathbf{K}_0\cdot\mathbf{R}} + f_{0,0}(\theta)\frac{e^{iK_0R}}{R} \qquad \text{for n=0 (elastic)}$$

$$\chi_n^{(+)}(\mathbf{K}_n,\mathbf{R}) \to f_{n,0}(\theta)\frac{e^{iK_nR}}{R} \qquad \text{for n>0 (non-elastic)}$$

- The model wavefunction must satisfy the Schrödinger equation:

$$[H - E]\Psi^{(+)}_{\rm model}(\mathbf{R},\xi) = 0$$

- Multiply on the left by each $\phi_n^*(\xi)$, and integrate over $\xi \Rightarrow$ coupled channels equations for $\{\chi_n(\mathbf{R})\}$:

$$[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})$$

- The structure information is embedded in the coupling potentials:

$$V_{n,n'}(\mathbf{R}) = \int d\xi \phi^*_{n'}(\xi) V(\mathbf{R},\xi)\phi_n(\xi)$$

☞ *$\phi_n(\xi)$ will depend on the assumed structure model (collective, few-body, etc).*

Optical Model

- The Hamiltonian:
  $H = T_R + V(\mathbf{R})$

- Internal states: Just $\phi_0(\xi)$

- Model wavefunction:
  $\Psi_{\rm mod}(\mathbf{R}, \xi) \equiv \chi_0(\mathbf{K}, \mathbf{R})\phi_0(\xi)$

- Schrödinger equation:
  $[H - E]\chi_0(\mathbf{K}, \mathbf{R}) = 0$

Optical Model

- The Hamiltonian:
  $H = T_R + V(\mathbf{R})$
- Internal states: Just $\phi_0(\xi)$
- Model wavefunction:
  $\Psi_{\rm mod}(\mathbf{R},\xi) \equiv \chi_0(\mathbf{K},\mathbf{R})\phi_0(\xi)$
- Schrödinger equation:
  $[H - E]\chi_0(\mathbf{K},\mathbf{R}) = 0$

Coupled-channels method

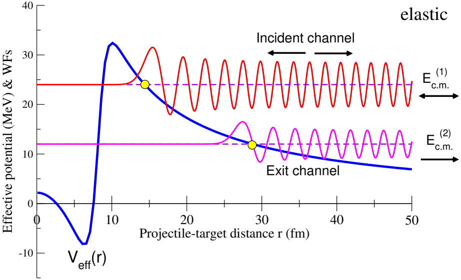
- The Hamiltonian:
  $H = T_R + h(\xi) + V(\mathbf{R},\xi)$
- Internal states:
  $h(\xi)\phi_n(\xi) = \varepsilon_n\phi_n(\xi)$
- Model wavefunction:
  $\Psi_{\rm model}(\mathbf{R},\xi) = \phi_0(\xi)\chi_0(\mathbf{K},\mathbf{R}) + \sum_{n>0}\phi_n(\xi)\chi_n(\mathbf{K},\mathbf{R})$
- Schrödinger equation:

$$[H - E]\Psi_{\rm model}(\mathbf{R},\xi) = 0$$

$$\Downarrow$$

$$[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{K},\mathbf{R}) = \sum_{n'\neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{K},\mathbf{R})$$

- Two-channel case with different internal energies: ε_1 and ε_2, with $\varepsilon_2 > \varepsilon_1$
- Energy conservation: $E^{(1)}_{\text{c.m}} + \varepsilon_1 = E^{(2)}_{\text{c.m}} + \varepsilon_2 \quad (E^{(2)}_{\text{c.m}} < E^{(1)}_{\text{c.m}})$
- Scattering wavefunction: $\Psi(r) = \underbrace{\chi_1(r)}_{\text{elastic}} |1\rangle + \underbrace{\chi_2(r)}_{\text{inelastic}} |2\rangle$



- Boundary conditions (incident chan. $|1\rangle$):

$$\begin{bmatrix}\chi_1(r)\\ \chi_2(r)\end{bmatrix} \rightarrow \begin{bmatrix} F_{\ell_1}(k_1 r) & + & T_{1,1} H^{(+)}_{\ell_1}(k_1 r) \\ & & T_{2,1} H^{(+)}_{\ell_2}(k_2 r)\end{bmatrix}$$

Alternatively, we can use the S-matrix

$$\begin{bmatrix}\chi_1(r)\\ \chi_2(r)\end{bmatrix} \rightarrow \begin{bmatrix} H^{(-)}_{\ell_1}(k_1 r) & - & S_{1,1} H^{(+)}_{\ell_1}(k_1 r) \\ & & S_{2,1} H^{(+)}_{\ell_2}(k_2 r)\end{bmatrix}$$

- Assume that we can write the p-t interaction as: $V(\mathbf{R},\xi) = V_0(R) + \Delta V(\mathbf{R},\xi)$
- Use central $V_0(R)$ part to calculate the (distorted) waves for p-t relative motion:

$$\left[\hat{T}_{\mathbf{R}} + V_0(R) - E_i\right]\chi_i^{(+)}(\mathbf{K}_i,\mathbf{R}) = 0 \qquad (E_i = \text{CM energy})$$
$$\left[\hat{T}_{\mathbf{R}} + V_0(R) - E_f\right]\chi_f^{(+)}(\mathbf{K}_f,\mathbf{R}) = 0 \qquad (E_f = E_i - E_x)$$

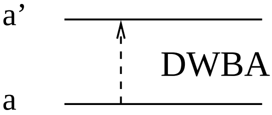
- In first order of $\Delta V(\mathbf{R},\xi)$ (DWBA) :

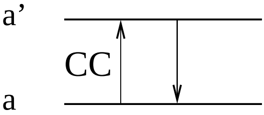
$$f_{i\to f}^{\text{DWBA}}(\theta) = -\frac{\mu}{2\pi\hbar^2}\int \chi_f^{(-)*}(\mathbf{K}_f,\mathbf{R})\,\Delta V_{if}(\mathbf{R})\,\chi_i^{(+)}(\mathbf{K}_i,\mathbf{R})\,d\mathbf{R}$$

with the coupling (or transition) potential:

$$\Delta V_{if}(\mathbf{R}) \equiv \int \phi_f^*(\xi)\,\Delta V(\mathbf{R},\xi)\,\phi_i(\xi)\,d\xi$$

- DWBA can be interpreted as a first-order approximation of a full coupled-channels calculation:





- The auxiliary potential U_β generating the entrance and exit distorted waves is usually chosen so as to reproduce the elastic scattering at the corresponding c.m. energy.

- In actual calculations, the internal states will have definite spin/parity:

$$\phi_i(\xi) = |I_i M_i\rangle \qquad \text{and} \qquad \phi_f(\xi) = |I_f M_f\rangle$$

- In many practical (and important) situations:

$$\Delta V(\mathbf{R}, \xi) = \sum_{\lambda>0} \underbrace{\mathcal{F}_\lambda(R)}_{\text{formfactor}} \sum_{\mu} \mathcal{T}_{\lambda,\mu}(\xi) Y_{\lambda\mu}(\hat{R})$$

$$\langle I_f M_f | \Delta V(\mathbf{R}, \xi) | I_i M_i \rangle = \sum_{\lambda>0} \mathcal{F}_\lambda(R) \langle I_f M_f | \mathcal{T}_{\lambda\mu}(\xi) | I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

- Wigner-Eckart theorem → reduced matrix elements (r.m.e.)*:

$$\langle I_f M_f | \mathcal{T}_{\lambda\mu}(\xi) | I_i M_i \rangle = (2I_f + 1)^{-1/2} \langle I_f M_f | I_i M_i \lambda\mu \rangle \underbrace{\langle I_f \| \mathcal{T}_\lambda(\xi) \| I_i \rangle_{\text{BM}}}_{\text{r.m.e}}$$

(*)*Bohr-Mottelson (BM) convention of r.m.e. assumed here.*

In general, we have both Coulomb and nuclear couplings

$$V_{if}(\mathbf{R}) = V_{if}^{C}(\mathbf{R}) + V_{if}^{N}(\mathbf{R})$$

1. Coulomb excitation → electric reduced matrix elements

$$V_{if}^{C}(\mathbf{R}) = \sum_{\lambda>0} \frac{4\pi\kappa}{2\lambda+1} \frac{Z_t e}{R^{\lambda+1}} \langle f; I_f M_f | \mathcal{M}(E\lambda,\mu) | i; I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

$$\langle f; I_f \| M(E\lambda) \| i; I_i \rangle = \sqrt{(2I_i+1) B(E\lambda; I_i \to I_f)}$$

2. Nuclear excitation (small deformations) → reduced deformation lengths

$$V_{if}^{N}(\mathbf{R}) \simeq -\frac{dV_0}{dR} \sum_{\lambda} \langle f; I_f M_f | \hat{\delta}_{\lambda\mu} | i; I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

DWBA scattering amplitude for a transition of multipolarity λ:

$$f_{iM_i \to fM_f}(\theta) = f^{C}_{iM_i \to fM_f}(\theta) + f^{N}_{iM_i \to fM_f}(\theta)$$

- **Coulomb:**

$$f^{C}_{iM_i \to fM_f}(\theta) = -\frac{\mu}{2\pi\hbar^2}\frac{4\pi Z_t e}{2\lambda+1}\langle f; I_f M_f|\mathcal{M}(E\lambda,\mu)|i; I_i M_i\rangle \int d\mathbf{R}\chi_f^{(-)*}(\mathbf{K}_f,\mathbf{R})\frac{Y_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}\chi_i^{(+)}(\mathbf{K}_i,\mathbf{R})$$

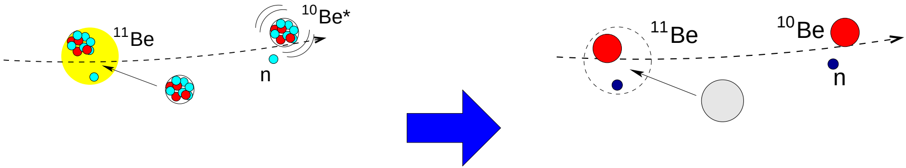
- **Nuclear:**

$$f^{N}_{iM_i \to fM_f}(\theta) = -\frac{\mu}{2\pi\hbar^2}\langle f; I_f M_f|\hat{\delta}_{\lambda\mu}|i; I_i M_i\rangle \int d\mathbf{R}\chi_f^{(-)*}(\mathbf{K}',\mathbf{R})\frac{dV}{dR}Y_{\lambda\mu}(\hat{\mathbf{R}})\chi_i^{(+)}(\mathbf{K},\mathbf{R})$$

Unpolarized cross section:,

$$\left(\frac{d\sigma}{d\Omega}\right)_{I_i \to I_f} = \frac{1}{(2I_i+1)}\frac{K_f}{K_i}\sum_{M_i,M_f}\left|f_{iM_i \to fM_f}(\theta)\right|^2$$

Single-particle and cluster excitations



$$\mathcal{V}_{pt} = \sum_{ij} V_{ij}(\mathbf{r}_{ij})$$

$$\mathcal{V}_{pt} = U_{ct}(\mathbf{r}_{ct}) + U_{nt}(\mathbf{r}_{nt})$$

- Effective three-body Hamiltonian:

$$H = T_{\mathbf{R}} + h_r(\mathbf{r}) + U_{ct}(\mathbf{r}_{ct}) + U_{nt}(\mathbf{r}_{nt})$$

- $U_{ct}(\mathbf{r}_{ct})$, $U_{nt}(\mathbf{r}_{nt})$ are optical potentials describing fragment-target elastic scattering (eg. target excitation is treated effectively, through absorption)

- Some nuclei allow a description in terms of two or more clusters: d=p+n, ^{6}Li=α+d, ^{7}Li=α+^{3}H.
- Projectile-target interaction:

$$V(\mathbf{R},\xi) \equiv V(\mathbf{R},\mathbf{r}) = U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2)$$

- Transition potentials:

$$V_{n,n'}(\mathbf{R}) = \int d\mathbf{r}\phi_n^*(\mathbf{r})\left[U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2)\right]\phi_{n'}(\mathbf{r})$$

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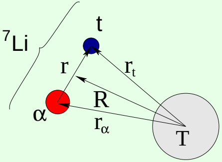
$$V_{n,n'}(\mathbf{R}) = \int d\mathbf{r}\phi_n^*(\mathbf{r})\left[U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2)\right]\phi_{n'}(\mathbf{r})$$

Example: ^{7}Li=α+t

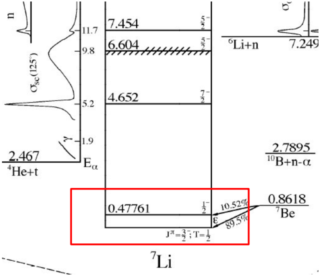
$$\mathbf{r}_\alpha = \mathbf{R} - \frac{m_t}{m_\alpha + m_t}\mathbf{r}; \quad \mathbf{r}_t = \mathbf{R} + \frac{m_\alpha}{m_\alpha + m_t}\mathbf{r}$$

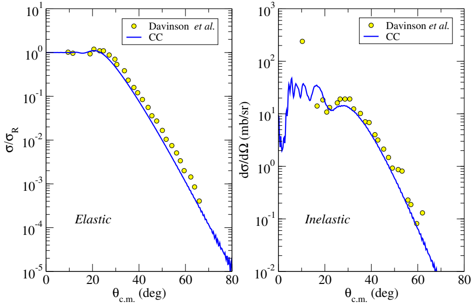


Internal states: (two-body cluster model)

$$[T_{\mathbf{r}} + V_{\alpha-t}(\mathbf{r}) - \varepsilon_n]\phi_n(\mathbf{r}) = 0$$

➩ CC calculation with 2 channels ($3/2^-$, $1/2^-$):

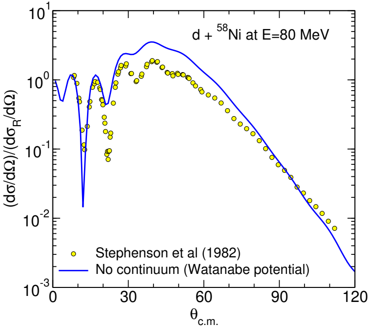


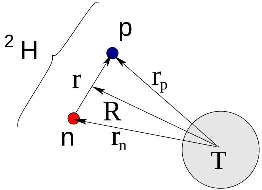


Data from Davinson et al, Phys. Lett. 139B (1984) 150)

Example: Three-body calculation (p+n+^{58}Ni) with Watanabe potential:

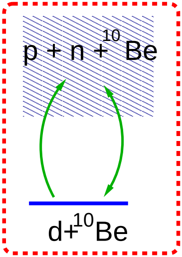
$$V_{dt}(\mathbf{R}) = \int d\mathbf{r}\phi^*_{\rm gs}(\mathbf{r}) \left\{ V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt}) \right\} \phi_{\rm gs}(\mathbf{r})$$



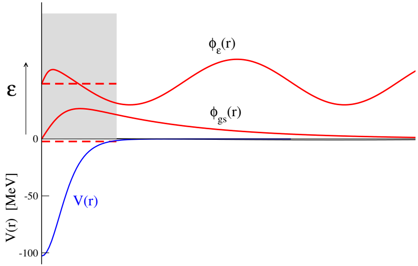


☞*Three-body calculations omitting breakup channels fail to describe the experimental data.*

Inclusion of breakup channels: the CDCC method



☞*We want to include explicitly in the modelspace the breakup channels of the projectile or target.*



Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm}$$

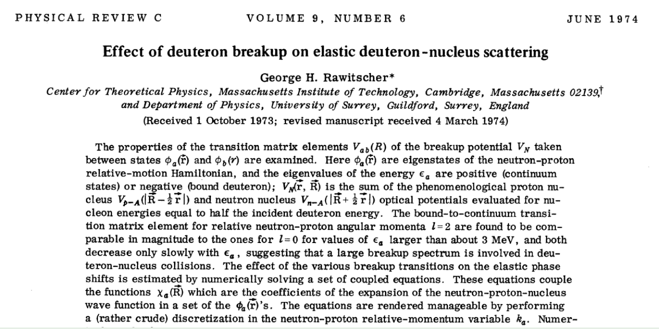
$$\varepsilon = \frac{\hbar^2 k^2}{2\mu}$$

Unbound states are not suitable for CC calculations:

- They have a continuous (infinite) distribution in energy.
- Non-normalizable: $\langle u_{k,\ell sj}(r) | u_{k',\ell sj}(r) \rangle \propto \delta(k - k')$

SOLUTION ⇒ continuum discretization

- Continuum discretization method proposed by G.H. Rawitscher [PRC9, 2210 (1974)] and Farrell, Vincent and Austern [Ann.Phys.(New York) 96, 333 (1976)] to describe deuteron scattering as an effective three-body problem $p + n + A$.



PHYSICAL REVIEW C VOLUME 9, NUMBER 6 JUNE 1974

Effect of deuteron breakup on elastic deuteron-nucleus scattering

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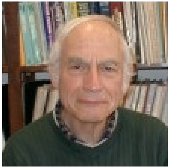
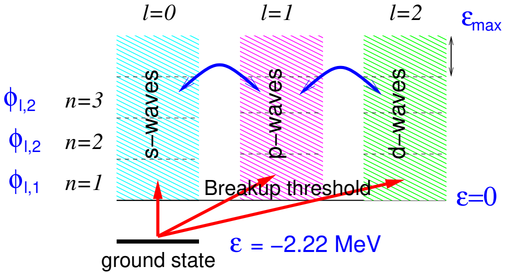
(Received 1 October 1973; revised manuscript received 4 March 1974)

The properties of the transition matrix elements $V_{ab}(R)$ of the breakup potential V_N taken between states $\phi_a(\vec{r})$ and $\phi_b(r)$ are examined. Here $\phi_a(\vec{r})$ are eigenstates of the neutron-proton relative-motion Hamiltonian, and the eigenvalues of the energy ϵ_a are positive (continuum states) or negative (bound deuteron); $V_N(\vec{r}, \vec{R})$ is the sum of the phenomenological proton nucleus $V_{p-A}(|\vec{R} - \frac{1}{2}\vec{r}|)$ and neutron nucleus $V_{n-A}(|\vec{R} + \frac{1}{2}\vec{r}|)$ optical potentials evaluated for nucleon energies equal to half the incident deuteron energy. The bound-to-continuum transition matrix element for relative neutron-proton angular momenta $l = 2$ are found to be comparable in magnitude to the ones for $l = 0$ for values of ϵ_a larger than about 3 MeV, and both decrease only slowly with ϵ_a, suggesting that a large breakup spectrum is involved in deuteron-nucleus collisions. The effect of the various breakup transitions on the elastic phase shifts is estimated by numerically solving a set of coupled equations. These equations couple the functions $\chi_a(\vec{R})$ which are the coefficients of the expansion of the neutron-proton-nucleus wave function in a set of the $\phi_a(\vec{r})$'s. The equations are rendered manageable by performing a (rather crude) discretization in the neutron-proton relative-momentum variable k_a. Numer-



George Rawitscher
(1928-2018)

- Full numerical implementation by Kyushu group (Sakuragi, Yahiro, Kamimura, and co.): Prog. Theor. Phys.(Kyoto) 68, 322 (1982)



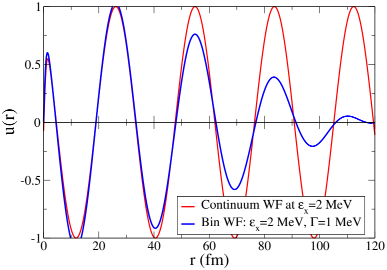
- Select a number of angular momenta ($\ell = 0, \ldots, \ell_{\rm max}$).
- For each ℓ, set a maximum excitation energy $\varepsilon_{\rm max}$.
- Divide the interval $\varepsilon = 0 - \varepsilon_{\rm max}$ in a set of sub-intervals (*bins*).
- For each bin, calculate a representative wavefunction.

Bin wavefunction:

$$\phi_{\ell jm}^{[k_1,k_2]}(\mathbf{r}) = \frac{u_{\ell j}^{[k_1,k_2]}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm} \qquad [k_1, k_2] = \text{bin interval}$$

$$u_{\ell sjm}^{[k_1,k_2]}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{k,\ell sj}(r) dk$$

- k: linear momentum
- $u_{k,\ell sj}(r)$: scattering states (radial part)
- $w(k)$: weight function



1. Choose a complete basis for the degree of freedom under consideration

   Eg: HO basis: $\{\phi^{HO}_{\ell,n}(r)\}$; $\quad n = 0, \ldots, \infty$

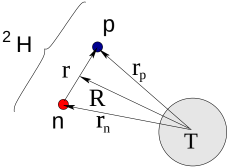
2. Truncate the basis: $n = 0, \ldots, N$

3. Diagonalize the Hamiltonian in the truncated basis

$$\{\phi^{HO}_{\ell,n}(r)\}_{n=0}^{N} \xrightarrow{\text{Diagonalize H}} \{\varphi_{\ell,n}(r)\}_{n=0}^{N} \begin{cases} \epsilon_0 \simeq \epsilon_{gs} < 0 & \Rightarrow \text{g.s.} \\ \epsilon_n < 0 \quad (n \neq 0) & \Rightarrow \text{Bound excited states} \\ \epsilon_n > 0 & \Rightarrow \text{Continuum states} \end{cases}$$

CDCC formalism for deuteron scattering

- Hamiltonian: $H = T_{\mathbf{R}} + h_r(\mathbf{r}) + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$
- Model wavefunction:



$$\Psi^{(+)}(\mathbf{R},\mathbf{r}) = \underbrace{\phi_{gs}(\mathbf{r})\chi_0(\mathbf{R})}_{\text{Ground state}} + \underbrace{\sum_{n>0}^{N} \phi_n(\mathbf{r})\chi_n(\mathbf{R})}_{\text{Discretized continuum}}$$

- Coupled equations: $[H - E]\Psi(\mathbf{R},\mathbf{r}) = 0$

$$\boxed{[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\,\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})}$$

- Coupling potentials:

$$V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\phi^*_n(\mathbf{r})\left[V_{pt}(\mathbf{R} + \frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R} - \frac{\mathbf{r}}{2})\right]\phi_{n'}(\mathbf{r})$$

- In practical calculations, the CDCC wf is expanded in the so-called channel basis $\langle \hat{R}, \mathbf{r}, |\beta; J_T\rangle = \left[Y_L(\hat{R}) \otimes \phi_{n,J_p}(\mathbf{r})\right]_{J_T}$:

$$\Psi_{\beta_0,J_T,M_T}(\vec{R},\vec{r},\xi) = \sum_{\beta} \frac{\chi^{J_T}_{\beta,\beta_0}(R)}{R} |\beta; J_T\rangle \quad \beta \equiv \{L, J_p, n\}$$

- The radial coefficients verify

$$\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + \frac{\hbar^2 L(L+1)}{2\mu R^2} + \varepsilon_n - E\right)\chi^{J_T}_{\beta,\beta_0}(R) + \sum_{\beta} V^{J_T}_{\beta,\beta'}(R)\chi^{J_T}_{\beta'}(R) = 0$$

with the coupling potentials:

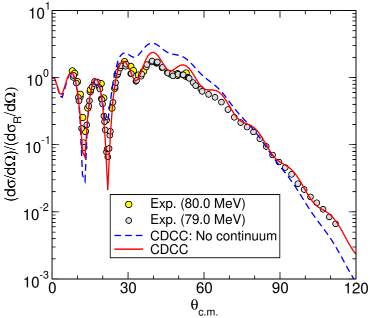
$$V^{J_T}_{\beta,\beta'}(R) = \langle \beta; J_T | V_1(\vec{R},\vec{r}) + V_2(\vec{R},\vec{r}) | \beta'; J_T\rangle$$

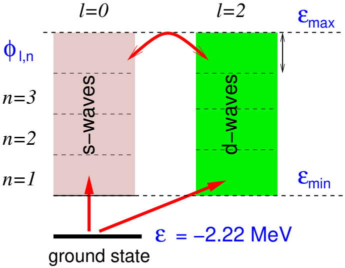
- Boundary conditions:

$$\chi^{J_T}_{\beta,\beta_0}(R) \rightarrow e^{i\sigma_L}\frac{i}{2}\left[H^{(-)}_L(K_\beta R)\delta_{\beta_0,\beta} - S^{J_T}_{\beta,\beta_0} H^{(+)}_L(K_\beta R)\right]$$

Coupling to continuum states produce:

- Polarization of the projectile (modification of real part)
- Flux removal (absorption) from the elastic channel (imaginary part)

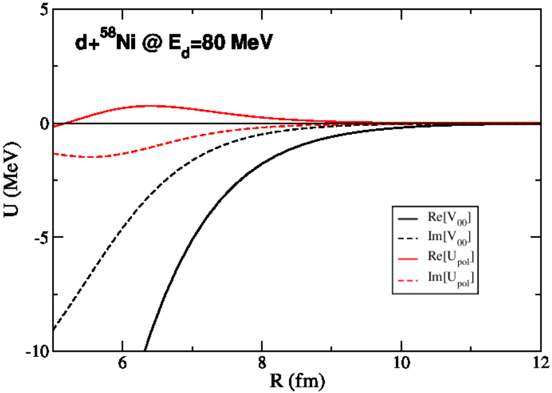




- From the elastic channel equation, a TELP can be defined as follows:

$$\Big[E - \varepsilon_0 - \hat{T}_{\mathbf{R}} - V_{0,0}(\mathbf{R})\Big]\chi_0(\mathbf{R}) = \sum_{i\neq 0} V_{i,0}(\mathbf{R})\chi_i(\mathbf{R}) \equiv U_{\mathrm{TELP}}(\mathbf{R})\chi_0(\mathbf{R}).$$

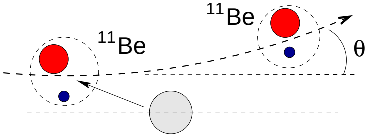
- In actual calculations, $U_{\mathrm{TELP}}(R)$ will depend on the total angular momentum, but a weighted average can be performed to obtain an approximate angular-momentum independent polarization potential
- A single channel calculation with the potential $U(\mathbf{R}) = V_{0,0}(\mathbf{R}) + U_{\mathrm{TELP}}(\mathbf{R})$ should reproduce approximately the elastic scattering cross section.

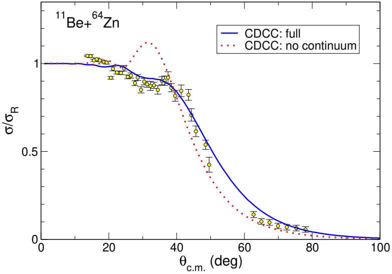


For this reaction, the TELP is complex:

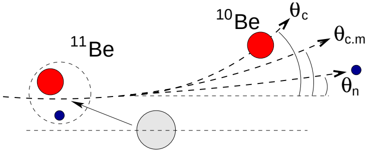
- The real part is repulsive (reduces projectile-target attraction)
- The imaginary part is absorptive (flux removal)

➩ Elastic scattering

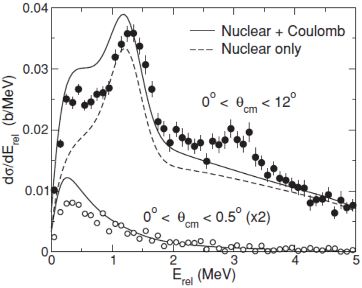




➩ Elastic breakup with respect with respect to $\theta_{\mathrm{c.m.}}$ or E_{rel}

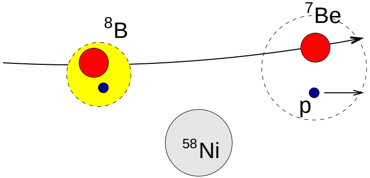


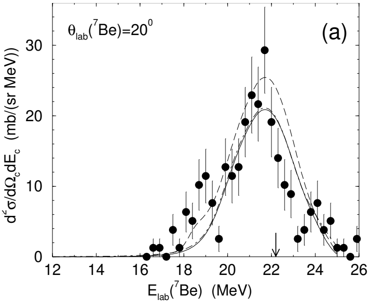
(requires coincidence measurements of n and ^{10}Be)



Howell et al, JPGG31, S1881 (2005)

➩ From the two-body scattering amplitudes, more complicated breakup observables can be obtained, such as angular/energy distribution of one of the fragments

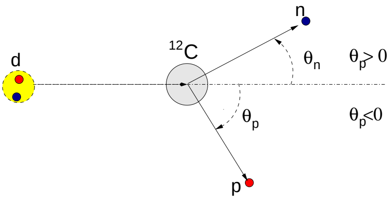


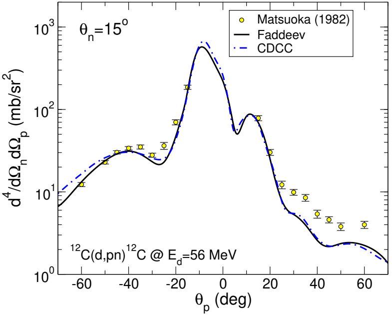


Tostevin et al,PRC 63, 024617

E.g.: CDCC calculations for **d+ ^{12}C** at 56 MeV:

Data: Matsuoka *et al.*, NPA391, 357 (1982).

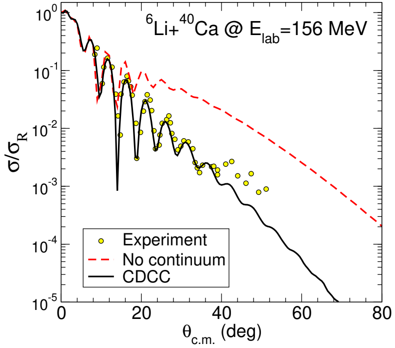


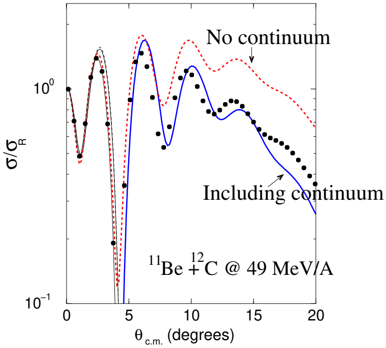


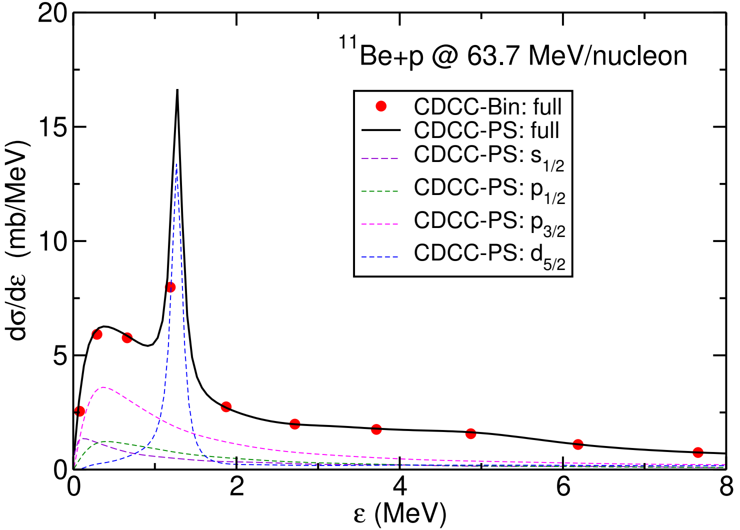
A. Deltuva et al, PRC 76, 064602 (2007)

☞ The CDCC has been also applied to nuclei with a cluster structure:

- ^{6}Li=α + d ($S_{\alpha,d}$=1.47 MeV)
- ^{11}Be=^{10}Be + n (S_n=0.504 MeV)







^{11}Be+p @ 63.7 MeV/nucleon
CDCC-Bin: full
CDCC-PS: full
CDCC-PS: $s_{1/2}$
CDCC-PS: $p_{1/2}$
CDCC-PS: $p_{3/2}$
CDCC-PS: $d_{5/2}$
dσ/dε (mb/MeV)
ε (MeV)
0
5
10
15
20
0
2
4
6
8

Extensions of standard CDCC

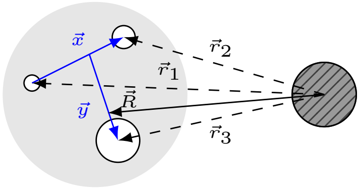
Recent (and no so recent) extensions of CDCC

1. Inclusion of target excitations, eg. $d + A \to p + n + A^*$
   - ☞ Kyushu: Yahiro *et al*, Prog. Theor. Phys. Suppl. 89, 32 (1986)
   - ☞ Seville: M. Gómez-Ramos and AMM, PRC95, 034609 (2017)

2. Inclusion of core excitations in the projectile (eg. ^{11}Be=^{10}Be* +n).
   - ☞ Michigan/Surrey (bins) : Summers *et al*, PRC74, 014606 (2006)
   - ☞ Seville (pseudo-states): R. de Diego *et al*, PRC 89, 064609 (2014)

3. Extension to 4-body reactions (3+1) (^{6}He, ^{11}Li, ^{9}Be), eg. ^{11}Li= ^{9}Li+n+n
   - ☞ Kyushu: Matsumoto i*et al*, NPA738, 471 (2004), PRC70, 061601(R) (2004).
   - ☞ Seville: Rodriguez-Gallardo *et al*, PRC72 (2005) 024007, PRC77, 064609 (2008).
   - ☞ Brussels: Descouvemont *et al*, PRC91, 024606 (2015)

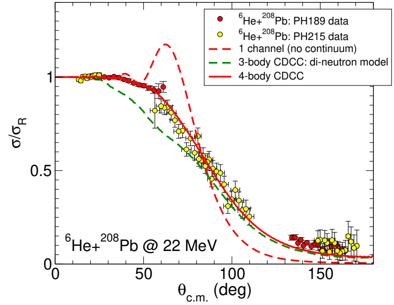
4. Use of microscopic projectile WFs (microscopic CDCC):
   - ☞ Kyushu: Sakuragi et al, Prog. Theor. Phys. Suppl. 89, 136 (1986).
   - ☞ Brussels: Descouvemont and Hussein, PRL 111, 082701 (2013)

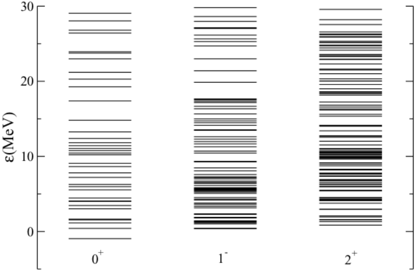


To extend the CDCC formalism, one needs to evaluate the new coupling potentials:

$$V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\, \phi_n^*(\mathbf{x},\mathbf{y})\, \{V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{\alpha t}(\mathbf{r}_3)\}\, \phi_{n'}(\mathbf{x},\mathbf{y})$$

- ☞ $\phi_n(\mathbf{x},\mathbf{y})$ three-body WFs for bound and continuum states: hyperspherical coordinates, Faddeev, etc (difficult to calculate!)
- ☞ 4b-CDCC calculations not included in FRESCO; require separate codes to compute the $\phi_n(\mathbf{x},\mathbf{y})$ wfs (e.g. FACE) and $V_{n;n'}(\mathbf{R})$ potentials

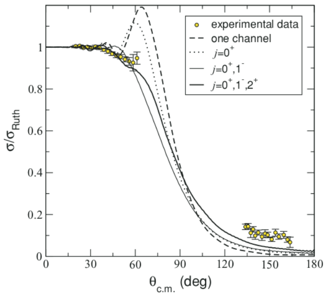


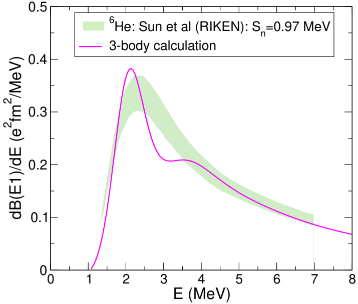


Data (LLN): NPA803, 30 (2008);PRC 84, 044604 (2011)
Calculations: PRC 80, 051601 (2009), PRC77, 064609 (2008)

N.b.: 1-channel potential considers only g.s. → g.s. coupling potential:

$$V_{00}(\mathbf{R}) = \int d\mathbf{r}\, \phi^*_{\rm g.s.}(\mathbf{x},\mathbf{y})\,\{V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{ct}(\mathbf{r}_3)\}\,\phi_{\rm g.s.}(\mathbf{x},\mathbf{y})$$

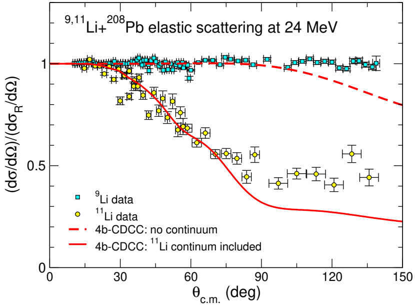


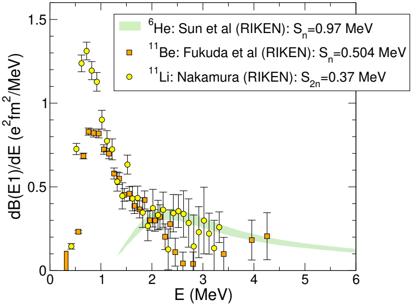


M. Rodriguez-Gallardo et al,PRC 77, 064609 (2008)

☞E1 couplings play a key role at explaining the elastic scattering behaviour of ^{6}He and other halo nuclei

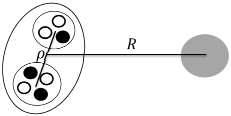
Four-body CDCC calculations for ^{11}Li (cont.)

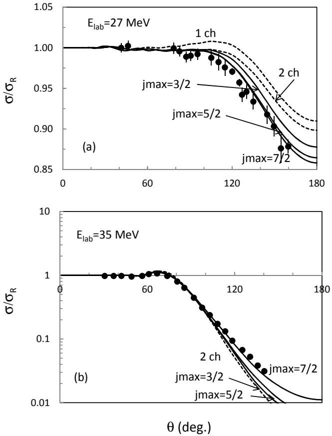




M Cubero et al, PRL109, 262701 (2012)

Eg.: Microscopic descrition of ^{7}Li:





$$\phi_i(\xi_p) = \mathcal{A}[[\phi_\alpha \otimes \phi_t]^{1/2} \otimes Y_\ell(\Omega_\rho)]^{jm} g_i^{\ell j}(\rho)$$

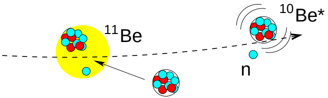
- ϕ_α, ϕ_α shell model wave functions for α and t
- $g_i^{\ell j}(\rho)$ relative inter-cluster wf.

P. Descouvemont, M.S. Hussein, PRL111, 082701 (2013)

Core excitations

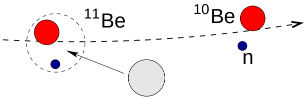
Deviations from the inert-cluster model are expected to show up when cluster d.o.f. are strongly excited during the reaction.

Microscopic models



- ✔ Fragments described microscopically
- ✔ Realistic NN interactions (Pauli properly accounted for)
- ✖ Numerically demanding / not simple interpretation.

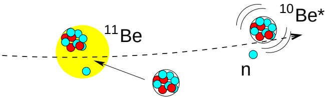
Inert cluster models



- ✖ Ignores cluster excitations (only few-body d.o.f).
- ✖ Phenomenological inter-cluster interactions (aprox. Pauli).
- ✔ Exactly solvable (in some cases).
- ✔ Achieved for $A \leq 5$ (eg. coupled-channels, Faddeev).

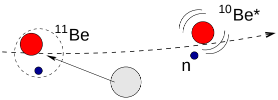
Deviations from the inert-cluster model are expected to show up when cluster d.o.f. are strongly excited during the reaction.

Microscopic models



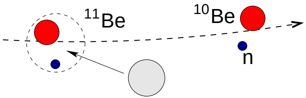
- ✔ Fragments described microscopically
- ✔ Realistic NN interactions (Pauli properly accounted for)
- ✖ Numerically demanding / not simple interpretation.

Non-inert-core few-body models



- ✔ Few-body + some relevant collective d.o.f.
- ✔ Implemented in extensions of Faddeev and CDCC (XCDCC)

Inert cluster models

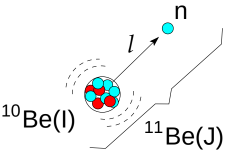


- ✖ Ignores cluster excitations (only few-body d.o.f).
- ✖ Phenomenological inter-cluster interactions (aprox. Pauli).
- ✔ Exactly solvable (in some cases).
- ✔ Achieved for $A \leq 5$ (eg. coupled-channels, Faddeev).

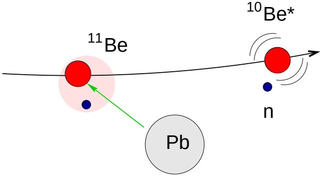
Core excitations will affect:

1. the structure of the projectile ➩ core-excited admixtures

$$\Psi_{JM}(\vec{r},\xi) = \sum_{\ell,j,I} \left[\varphi^{J}_{\ell,j,I}(\vec{r}) \otimes \Phi_I(\xi)\right]_{JM}$$



2. the dynamics ➩ collective excitations of the ^{10}Be during the collision compete with halo (single-particle) excitations.



➩ Both effects have been recently implemented in an extended version of of the CDCC formalism (CDCC): Summers *et al*, PRC74 (2006) 014606, R. de Diego *et al*, PRC 89, 064609 (2014)

Extending CDCC to include core excitations

- Standard CDCC ⇒ use coupling potentials:

$$V_{\alpha;\alpha'}(\mathbf{R}) = \langle \Psi^{\alpha'}_{J'M'}(\vec{r}) | V_{vt}(r_{vt}) + V_{ct}(r_{ct}) | \Psi^{\alpha}_{JM}(\vec{r}) \rangle$$

- Extended CDCC (XCDCC) ⇒ use generalized coupling potentials

$$V_{\alpha;\alpha'}(\mathbf{R}) = \langle \Psi^{\alpha'}_{J'M'}(\vec{r},\xi) | V_{vt}(r_{vt}) + V_{ct}(r_{ct},\xi) | \Psi^{\alpha}_{JM}(\vec{r},\xi) \rangle$$

  - $\Psi^{\alpha}_{JM}(\vec{r},\xi)$: projectile WFs involving core-excited admixtures (structure).

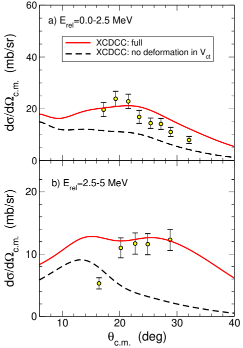
    (actual applications have employed particle-rotor models, or folded potentials with microscopic transition densities)

  - $V_{ct}(r_{ct},\xi)$: non-central potential allowing for core excitations/de-excitations (dynamic core excitation).

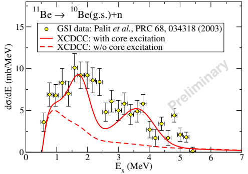
  - Summers *et al*, PRC74 (2006) 014606 (bins)
  - R. de Diego *et al*, PRC 89, 064609 (2014) (THO pseudo-states)

Effect of core excitations on breakup

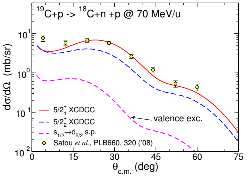
^{11}Be+ p @ 64 MeV/u



^{11}Be+ ^{12}C @ 520 MeV/u

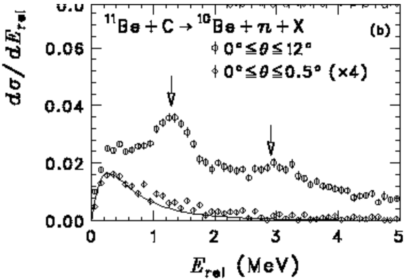


^{19}C+ p @ 69 MeV/u

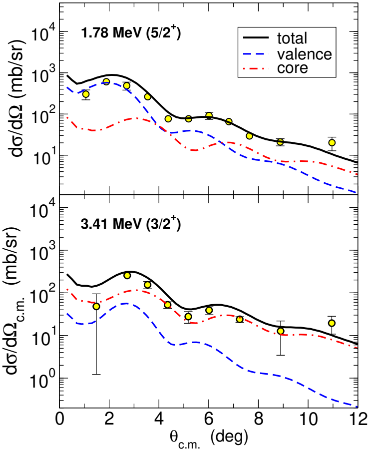


- ☞ De Diego et al, PRC85, 054613 (2014).
- ☞ Data from Shrivastava et al, PLB596 (2004) 54.
- ☞ JA Lay *et al*, PRC94,021602(R)(2016).

⇨ Core excitations may enhance dramatically the breakup cross sections in reactions of deformed halo nuclei with light targets

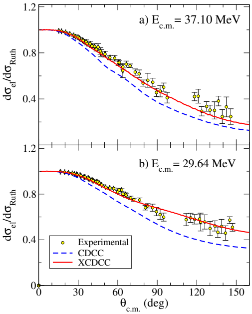


Fukuda et al, PRC70, 054606 (2004))

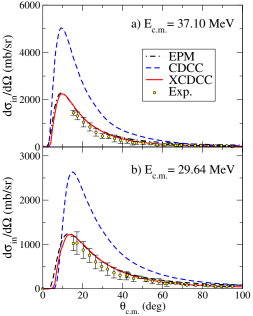


A.M.M. and J.A. Lay, PRL 109, 232502 (2012)

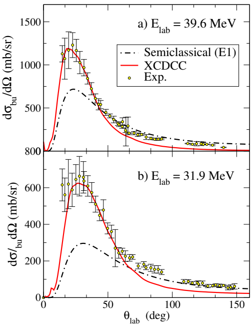
Elastic



Inelastic



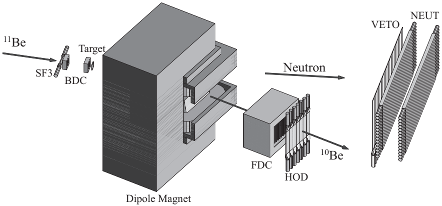
Breakup

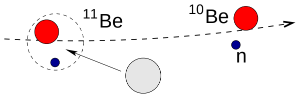


Pesudo et al, Phys. Rev. Lett. 118, 152502 (2017)

The problem of inclusive breakup

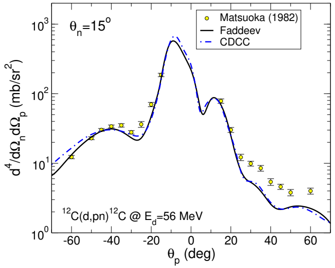
- Weakly-bound nuclei are known to break up easily in collisions with other nuclei due to the nuclear and Coulomb fields.
- When all fragments following dissociation are observed (exclusive breakup) different models have been developed and successfully applied: DWBA, CDCC, Faddeev, etc
- Full-kinematical measurements require dedicated experimental setups which are particularly involved when neutron detection is required:





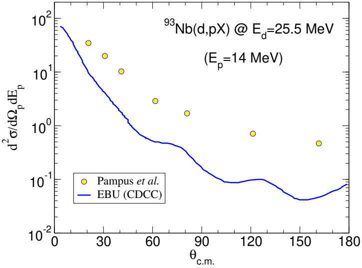
Fukuda et al,PRC 70, 054606 (2004)

Exclusive breakup:
$d + ^{12}C \rightarrow p + n + ^{12}C(g.s.)$



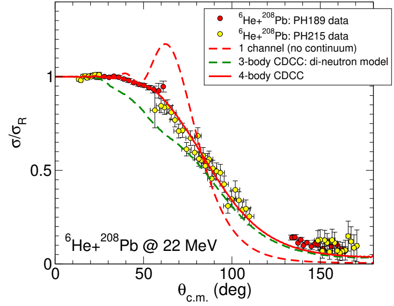
Data: Matsuoka et al, NPA391, 357 (1982)
Calc.: Deltuva et al,PRC76, 064602 (2007)

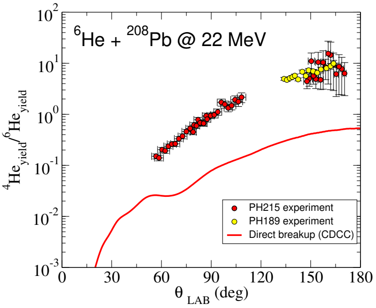
Inclusive breakup:
$d + ^{93}Nb \rightarrow p + X$



Data: Pampus et al, NPA311, 141 (1978)

$$^6\text{He} + ^{208}\text{Pb} \rightarrow \alpha + \text{X}$$

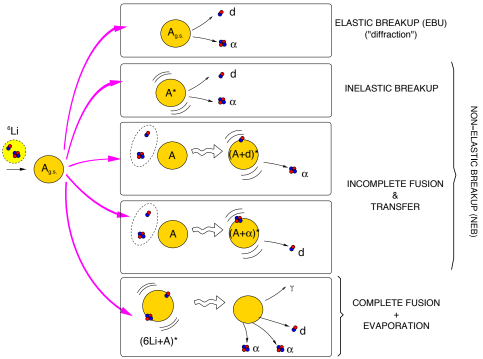




Data (LLN): Sánchez-Benítez et al, NPA 803, 30 (2008) L. Acosta et al, PRC 84, 044604 (2011), D. Escrig et al., NPA 792 (2007) 2

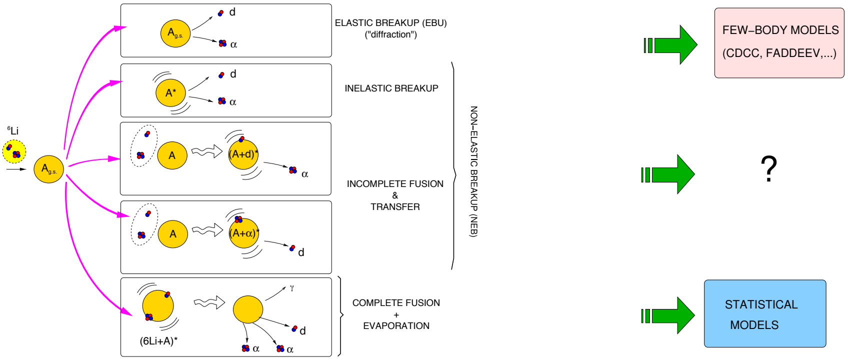
Calculations: Rodríguez-Gallardo et al, PRC 80, 051601 (2009)

☞*CDCC reproduces elastic scattering, but not inclusive α's.*



⇨ For a reaction of the form $a(= b + x) + A \to b +$ anything

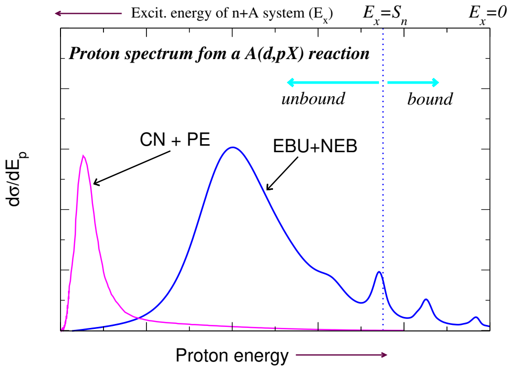
$$\sigma_b = \sigma_{EBU} + \sigma_{NEB} + \sigma_{CN}$$



➪ For a reaction of the form $a(= b + x) + A \to b +$ anything

$$\sigma_b = \sigma_{EBU} + \sigma_{NEB} + \sigma_{CN}$$

Expected proton energy spectrum in a $A(d,p)X$ reaction



- Highest energy protons from neutron transfer to bound states: $A(d,p)B$
- Lowest energy protons from compound nucleus (CN) and preequilibrium (PE)
- Intermediate energy protons from population of $x + A$ unbound states (including elastic breakup)

- Inclusive breakup could in principle be evaluated computing all contributing processes using standard reaction methods, such as DWBA or CRC.

- ...however, this procedure has serious shortcomings:
  - Final x+A states span a wide range of excitation energies and spins, so the number of populated states will in general be huge.
  - A significant part of the inclusive spectrum corresponds to x-A continuum.
  - An explicit calculation would require a detailed knowledge of the populated states: spin/parity, spectroscopic factors . . . , which are poorly known above a few MeV of excitation energy
  - Final states will include, in addition to direct processes, partial fusion ("incomplete fusion"), which are not easily accounted for by standard direct reaction theories.

Inclusive breakup models based on closed-form formulas provide an efficient (and elegant) alternative which exploit the fact that nonelastic $x-A$ processes are encoded in the imaginary part of the U_{xA} optical potential.

- Baur & co: DWBA sum rule with surface approximation.
    - Baur et al, PRC21, 2668 (1980).

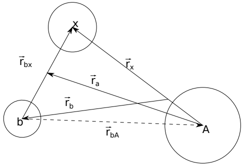
- Hussein & McVoy: extraction of singles cross section combining the spectator model with sum-rule over final states.
    - Nucl. Phys. A445, 124 (1985).

- Ichimura, Austern, Vincent (IAV): Post-form DWBA.
    - Ichimura, Austern, Vincent, PRC32, 431 (1985).
    - Austern *al*, Phys. Rep.154, 125 (1987).

- Udagawa, Tamura (UT): prior-form DWBA.
    - Udagawa and Tamura, PRC24, 1348 (1981).
    - Udagawa, Lee, Tamura, PLB135, 333 (1984).

⇨*Most of these theories have fallen into disuse and are now being revisited by several groups* (alternative derivation in Lei, AMM, PRC92, 044616 (2015)

- Inclusive reaction $\underbrace{(b+x)}_{a} + A \to b + (x+A)^*$
- b singles cross section: $\sigma_b^{inc} = \sigma_b^{\rm EBU} + \sigma_b^{\rm NBU}$



- EBU: $a + A \to b + x + A_{\rm g.s.}$ can be computed with CDCC, DWBA, etc
- $\sigma_b^{\rm NEB}$ can be interpreted as the absorption occurring in the $x + A$ channel:

$$\frac{d\sigma^{\rm NEB}}{d\Omega_b dE_b} = -\frac{2}{\hbar v_a}\rho_b(E_b)\langle\varphi_x(\vec{k}_b)|W_{xA}|\varphi_x(\vec{k}_b)\rangle \qquad W_{xA} = {\rm Im}[U_{xA}]$$

where $\varphi_x(\vec{k}_b, \mathbf{r}_x)$ describes x-A relative motion when b emerges with momentum $\vec{k}_b \equiv \{\Omega_b, E_b\}$.

$$(E_x^+ - K_x - U_{xA})\varphi_x^{\rm IAV}(\mathbf{r}_x) = \langle \mathbf{r}_x \chi_b^{(-)}(\vec{k}_b)|V_{\rm post}|\chi_a^{(+)}\phi_a\rangle \qquad V_{\rm post} \equiv V_{bx} + U_{bA} - U_{bB}$$

- For $E_x > 0$, U_{xA} is the usual optical model potential describing x+A elastics
  ☞ $\mathrm{Im}[U_{xA}]$ accounts for $x - A$ **absorption**.
- For $E_x < 0$, U_{xA} represents the distribution of single-particle (s.p.) states
  ☞ $\mathrm{Im}[U_{xA}]$ accounts for **s.p. fragmentation** (spreading widths).

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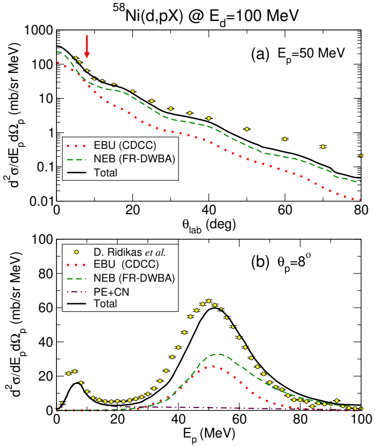
- For $E_x < 0$, U_{xA} represents the distribution of single-particle (s.p.) states
  ☞ Im[U_{xA}] accounts for **s.p. fragmentation** (spreading widths).

⇨ *These properties are naturally accommodated in dispersive optical model (DOM) potentials*

Mahaux, Bortignon, Broglia, Dasso, Phys. Rep. 120, (1985) 1-274

⇨ *Both "transfer" to unbound states and bound states described on an equal footing*

Application of IAV model to deuteron inclusive breakup



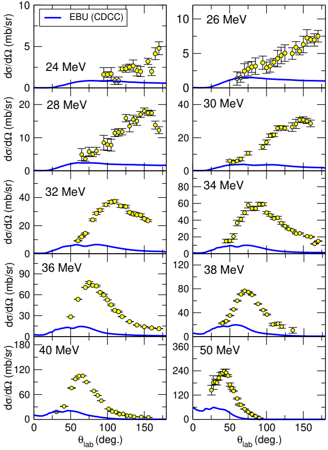
- EBU calculated with CDCC.
- NBU calculated with DWBA IAV model.
- Low-energy protons from preequilibrium (PE) and compound nucleus (CN) evaporation not accounted for by IAV model, but can be evaluated with HF theory.

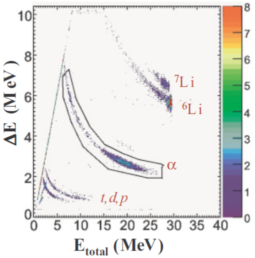
Data: Ridikas et al, PRC63, 014610 (2000)

Calc.: J. Lei, A.M.M., PRC 92, 044616 (2015); Jin Lei, Ph.D. thesis, https://idus.us.es/handle/11441/44344

Application to ^{209}Bi (^{6}Li,α)X

- Large α yield ($\sigma_\alpha \gg \sigma_d$) $\Rightarrow$ evidence of NEB channels
- EBU alone cannot explain the data.



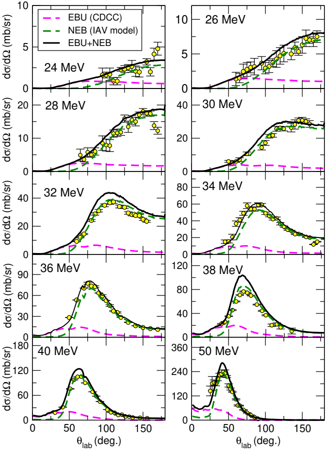


Santra et al. PRC85, 014612 (2012)

Application to ^{209}Bi (^{6}Li,α)X

Assume: $\sigma_\alpha \simeq \sigma^{EBU} + \sigma^{NEB}$:

- EBU ⇨ CDCC
- NEB ⇨ IAV

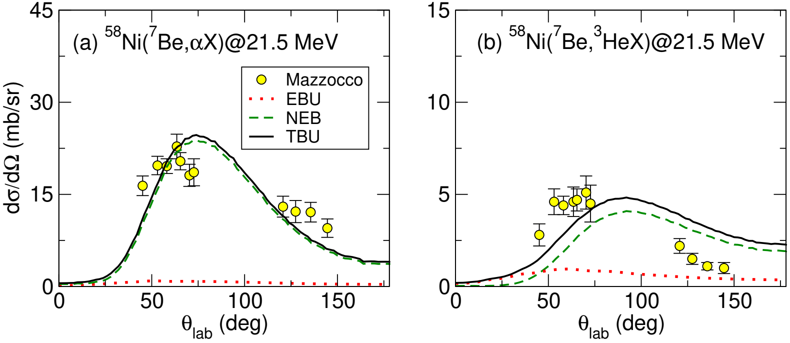


- Inclusive data are well accounted for by EBU+NEB
- Inclusive α's dominated by NEB
- EBU (^{6}Li $\rightarrow \alpha + d$) only relevant for small scattering angles.

J. Lei and AMM, PRC92, 044616 (2015)

^{58}Ni(^{7}Be,α)X @ LNL/INFN

$\sigma_\alpha \approx 5\sigma_{3\mathrm{He}}$ ☞ Evidence for non-elastic breakup mechanisms



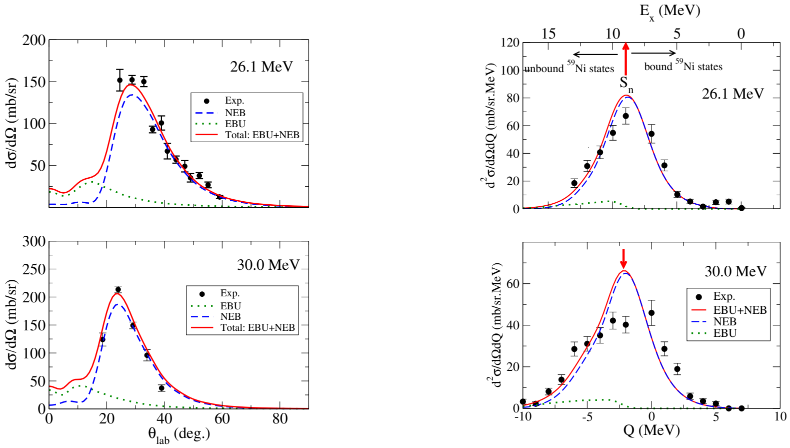
Data: M. Mazzocco et al, C 92, 024615 (2015)

Calculations: J. Lei, Ph.D. thesis, U. Sevilla.

Application to ^{58}Ni (^{8}Li, ^{7}Li)X

^{8}Li $\rightarrow$ ^{7}Li :

^{58}Ni(^{8}Li,^{7}Li)X @ RIBRAS

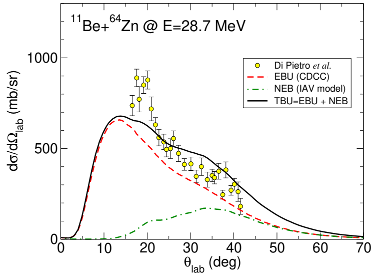


[O.C.B. Santos, PRC 103, 064601 (2021) and Ph.D. thesis]

- Inclusive breakup dominated by NEB.

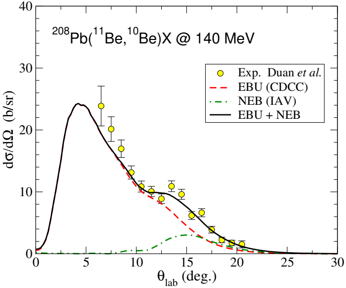
^{11}Be → ^{10}Be :

^{64}Zn(^{11}Be,^{10}Be)X @ ISOLDE-CERN



Di Pietro *et al.*, PLB 798 (2019) 134954

^{208}Pb(^{11}Be,^{10}Be)X @ HIRF-Lanzhou

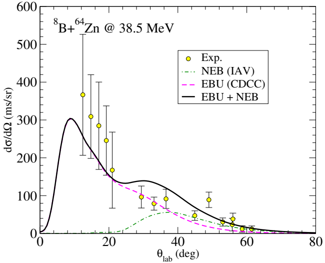


Duan *et al.*, PLB 811 (2020) 135942

- Inclusive breakup dominated by EBU.
- NEB only important around the grazing angle.

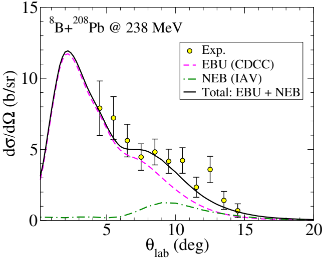
^{8}B → ^{7}Be :

^{64}Zn(^{8}B,^{7}Be)X @ ISOLDE-CERN



Sparta et al, PLB820, 136477 (2021)

^{208}Pb(^{8}B,^{7}Be)X @ HIRF-Lanzhou



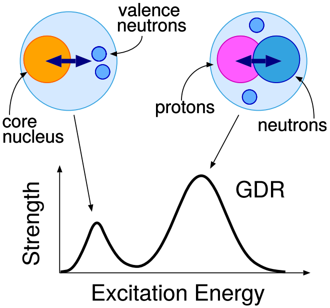
Wang et al, PRC 103, 024606 (2021)

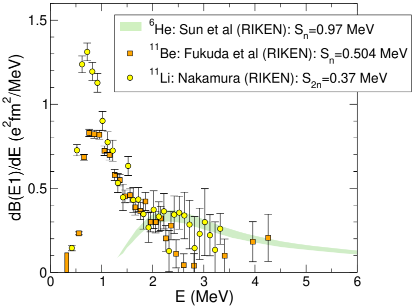
- Inclusive breakup dominated by EBU.
- NEB important around the grazing angle.

- For weakly-bound, non-halo nuclei (deuteron, ^{6,7,8}Li,..) the inclusive breakup is dominated by the NEB component.
- For very weakly-bound nuclei, such as halo nuclei (^{8}B,^{11}Be), the inclusive breakup is dominated by the EBU component.
- A significant part of the inclusive cross section corresponds to the population of continuum states of the residual (x+A) system.

Exploring the continuum with breakup reactions

1. Coulomb dissociation experiments
   - Semiclassical description: Alder and Winther
   - Quantum-mechanical description
   - Inferring radiative capture reaction rates from Coulomb dissociation
2. Exploring continuum structures: resonances and virtual states





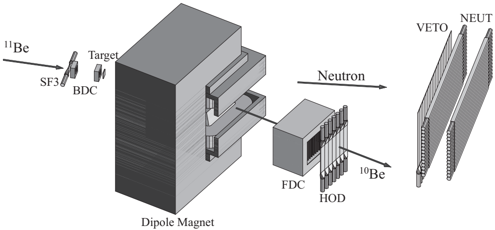
- The $E\lambda$ response can be quantified through the $B(E\lambda)$ probability:

$$B(E\lambda; i \to f) = \frac{1}{2I_i + 1} |\langle \Psi_f \| \mathcal{M}(E\lambda) \| \Psi_i \rangle|^2$$

- Neutron-halo nuclei have large $B(E1)$ strengths near threshold

How to probe/extract the $B(E1)$ of halo nuclei?

Example: ^{11}Be+^{208}Pb → ^{10}Be+ n+ ^{208}Pb measured at RIKEN (69 MeV/u).
Fukuda et al, PRC70, 054606 (2004))

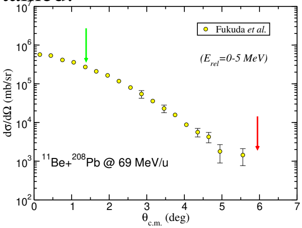


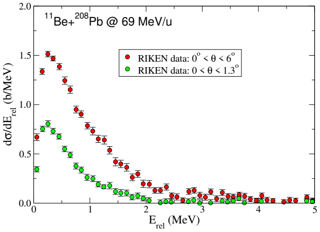
☞ ^{11}Be excitation energy can be reconstructed from core-neutron coincidences (*invariant mass method*)

- Experimentally, one measures angular and relative energy distribution of the ^{11}Be* system:

$$\frac{d^2\sigma}{d\Omega\, dE}$$

- Integrating over the angle or energy, single differential cross sections are obtained:





- In the Coulomb dominated region (i.e. small angles), the breakup cross section is expected to be dominated by the $dB(E\lambda)/dE$ distribution, but we need a theory that relates both observables.

- For Eλ excitation to bound states ($0 \to n$):

$$\left(\frac{d\sigma}{d\Omega}\right)_{0\to n} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0 \to n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta, \xi) \qquad \xi_{0\to n} = \frac{(E_n - E_0)}{\hbar}\frac{a_0}{v}$$

- For continuum states (breakup):

$$\frac{d\sigma(E\lambda)}{d\Omega dE} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{1}{e^2 a_0^{2\lambda-2}} \frac{dB(E\lambda)}{dE} \frac{df_\lambda(\theta, \xi)}{d\Omega}$$

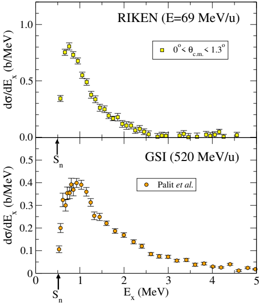
☞ $dB(E\lambda)/dE$ can be extracted from small-angle Coulomb dissociation data.

$$\frac{d\sigma}{dE}(\theta < \theta_{\rm max}) = \int_0^{\theta_{\rm max}} \frac{d\sigma(E\lambda)}{d\Omega dE} d\Omega \propto \frac{dB(E\lambda)}{dE}$$

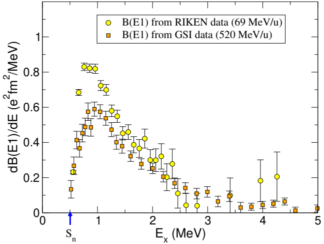
Assumptions:

- Breakup dominated by Coulomb excitation
- Nuclear excitation, if present, can be estimated and added incoherently

☞ If the assumptions above are fulfilled, the extracted $dB(E\lambda)dE$ should be independent of the incident energy and target employed, since it reflects a structure property of the weakly-bound projectile



$$\frac{dB(E\lambda)}{dE} \propto \frac{d\sigma}{dE}$$



RIKEN: Fukuda et al, PRC70 (2004) 054606

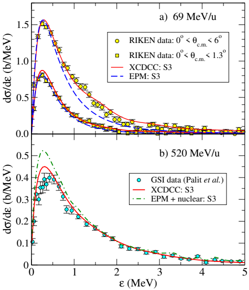
GSI: Palit et al, PRC68 (2003) 034318

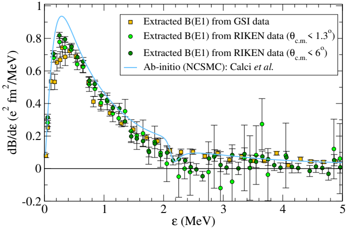
☞ *The extracted $dB(E\lambda)/dE$ distributions are reasonably compatible, but with apparent differences at the peak*

- Nuclear excitation not negligible, even for small θ
- Nuclear contribution interferes with Coulomb
- Higher-order couplings can affect the cross sections

☞ Alternatively, one may use fully QM approaches that incorporate Coulomb and nuclear couplings to all others (eg. CDCC).

E.g.: CDCC analysis based on two-body ^{10}Be+n model:



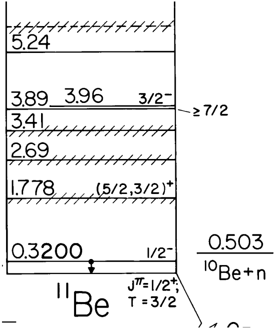


CDCC: AMM et al,PLB 811 (2020) 135959
Ab-initio: Calci et al, PRL117 (2016)

☞ Extracted $dB(E\lambda)dE$ from different cross section data compatible within errors.

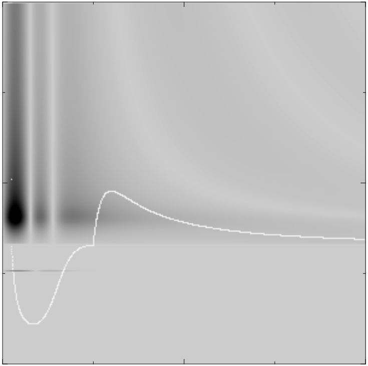
☞ Differences with ab-initio NCSM calls for further work.

The continuum spectrum is not "homogeneous"; it contains in general energy regions with special structures, such as resonances and virtual states

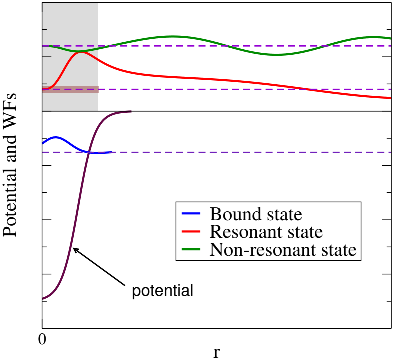


- It is a pole of the S-matrix in the complex energy plane.
- It is a structure on the continuum which may, or may not, produce a maximum in the cross section, depending on the reaction mechanism and the phase space available.
- The resonance occurs in the range of energies for which the phase shift is close to $\pi/2$.
- In this range of energies, continuum wavefunctions have a large probability of being in the radial range of the potential.
- The continuum wavefunctions are not square normalizable. For practical reasons, a normalized wave-packet (or "bin") can be constructed to represent the resonance.

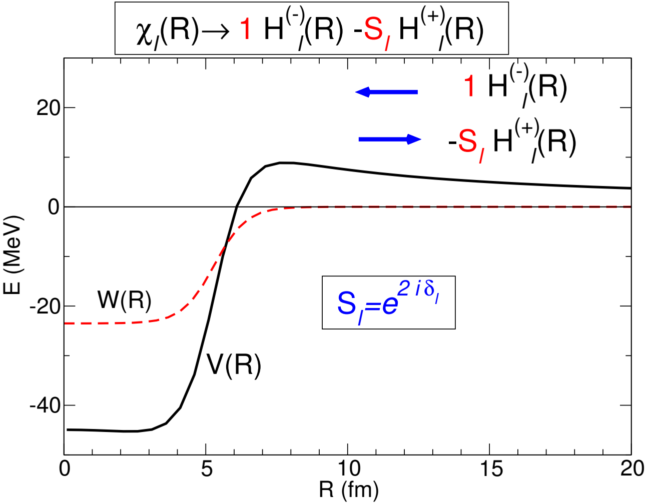
In the energy range of the resonance, the continuum wavefunctions have a large probability of being within the range of the potential.

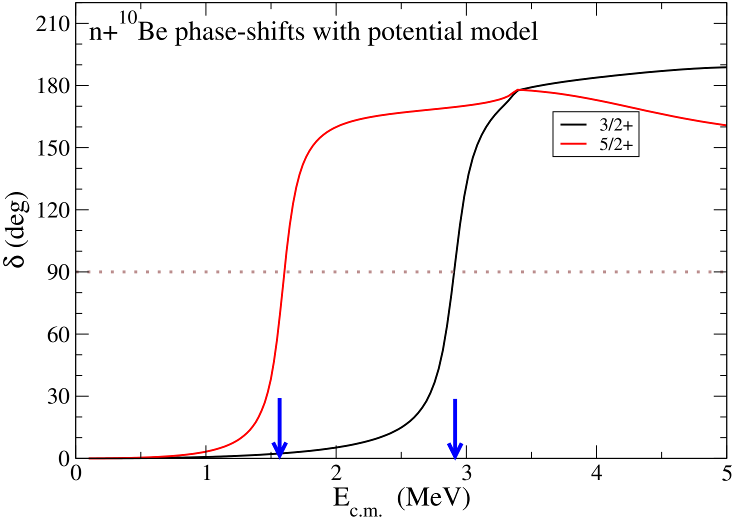


(Courtesy of C. Dasso)

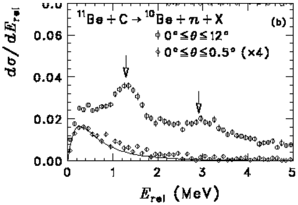


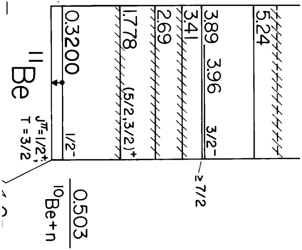
Resonances and phase-shifts




n+10Be phase-shifts with potential model
δ (deg)
210
180
150
120
90
60
30
0
0
1
2
3
4
5
E c.m. (MeV)
3/2+
5/2+

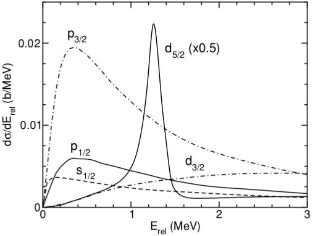
RIKEN data

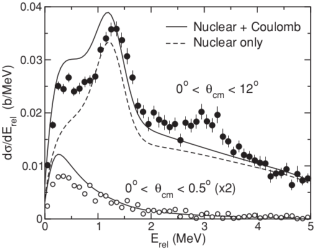




Fukuda *et al*, PRC70 (2004) 054606)

CDCC analysis



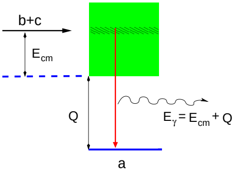


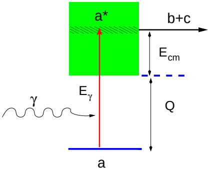
Howell *et al.*, JPG31 (2005) S1881

Application to radiative capture reactions

Radiative capture: $b + c \rightarrow a + \gamma$

Photodissociation: $a + \gamma \rightarrow b + c$





➩ Related by detailed balance:

$$\sigma^{(rc)}_{E\lambda} = \frac{2(2J_a+1)}{(2J_b+1)(2J_c+1)}\frac{k_\gamma^2}{k^2}\sigma^{(phot)}_{E\lambda} \qquad (\hbar k_\gamma = E_\gamma/c)$$

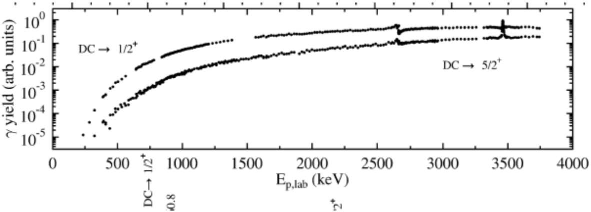
➩ Astrophysical S-factor:

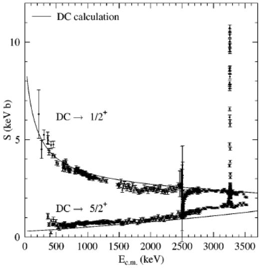
$$S(E_{\text{c.m.}}) = E_{\text{c.m.}}\sigma^{(rc)}_{E\lambda}\exp[2\pi\eta(E_{\text{c.m.}})]$$

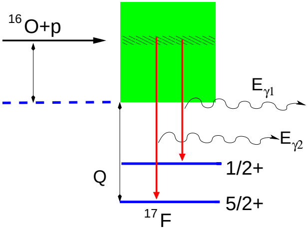
➩ Capture cross sections are difficult to measure because they are very small at relevant astrophysical energies.

Example: $p+{}^{16}O \rightarrow {}^{17}F + \gamma$

Morlock, PRL79, 3837 (1997)







➩ The photodissociation ($\gamma + a \to b + c$) cross section is related to the $B(E\lambda)$

$$\sigma^{\rm photo}_{E\lambda} = \frac{(2\pi)^3(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{dB(E\lambda)}{dE}$$

➩ Then, in 1st order semiclassical limit, the Coulomb breakup x-section is proportional to photodissociation x-section:

$$\boxed{\frac{d\sigma(E\lambda)}{d\Omega dE_\gamma} = \frac{1}{E_\gamma}\frac{dn_{E\lambda}}{d\Omega}\sigma^{\rm photo}_{E\lambda}}$$

(Equivalent Photon Method)

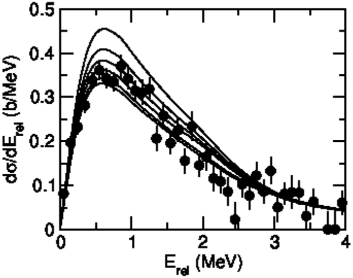
with the virtual photon number

$$\frac{dn_{E\lambda}}{d\Omega} = Z_t^2\alpha\frac{\lambda[(2\lambda+1)!!]^2}{(2\pi)^3(\lambda+1)}\xi^{2(1-\lambda)}\left(\frac{c}{v}\right)^{2\lambda}\frac{df_{E\lambda}}{d\Omega}$$

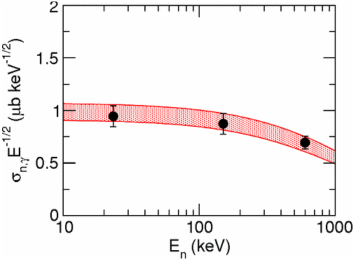
- ☞ Capture reactions have typically small cross sections
- ☞ Use breakup (Coulomb dissociation) reactions:

$$\frac{d\sigma}{d\Omega dE_{c.m.}} \rightarrow \sigma^{(\mathrm{phot})}_{E\lambda} \rightarrow \sigma^{(rc)}_{E\lambda} \rightarrow S(E_{\mathrm{c.m.}})$$

^{15}C+^{208}Pb → ^{14}C+n+^{208}Pb



^{14}C(n,γ)^{15}C



(dots: direct; shaded region: from Coulomb breakup)
Summers and Nunes, PRC 78, 011601(R), 2008