QUANTUM COMPUTING WITH CAVITIES

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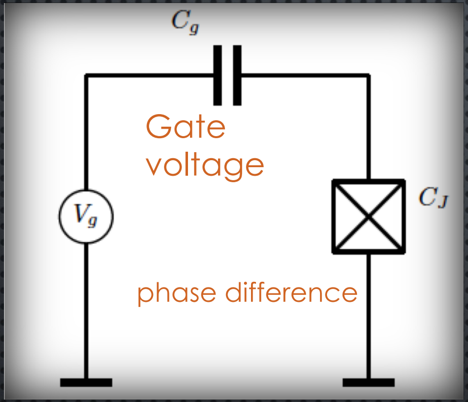
OUTLINE

- Cavities coupled to transmons for dispersive readout
- Optimal control (driving pulses engineering)
- Scattering of two neutrons (a case study)
- Improving readout from cavities via neural networks (two cents on error correction)

COOPER PAIR BOX



In the superconductive regime charge is carried by Cooper pairs. It is possible to write the Hamiltonian of the circuit:

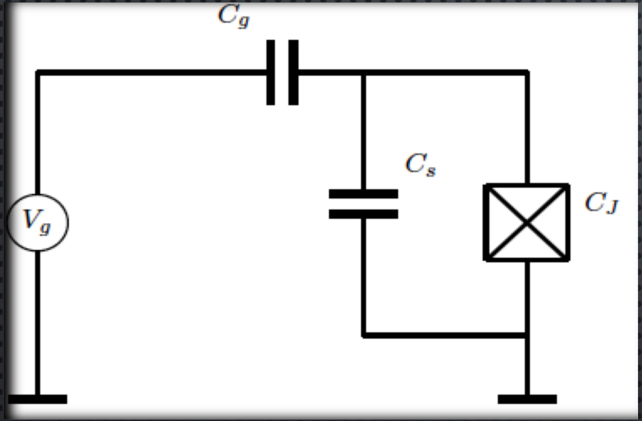
$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos(\delta\phi)$$

$E_C = \frac{e^2}{2C_{\text{tot}}}$

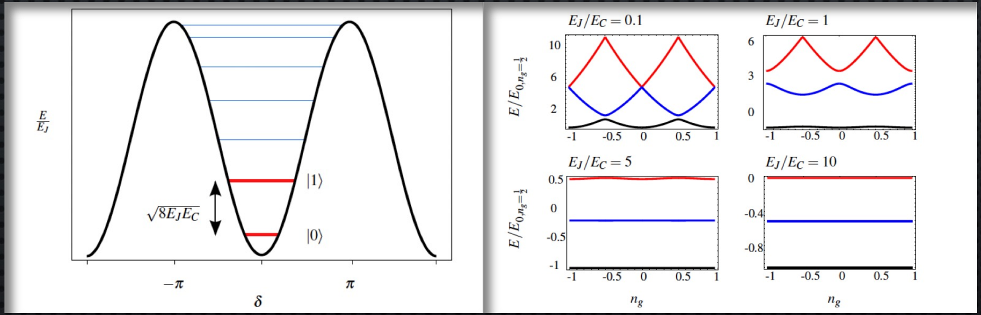
n_g: gate voltage induced charge

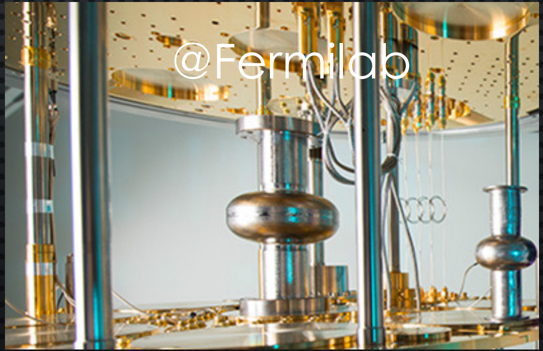
CPB regime: EC>>EJ: energy determined by the number of transferred pairs

It is possible to increase the ratio between EJ and EC by shunting the circuit with a capacitor.



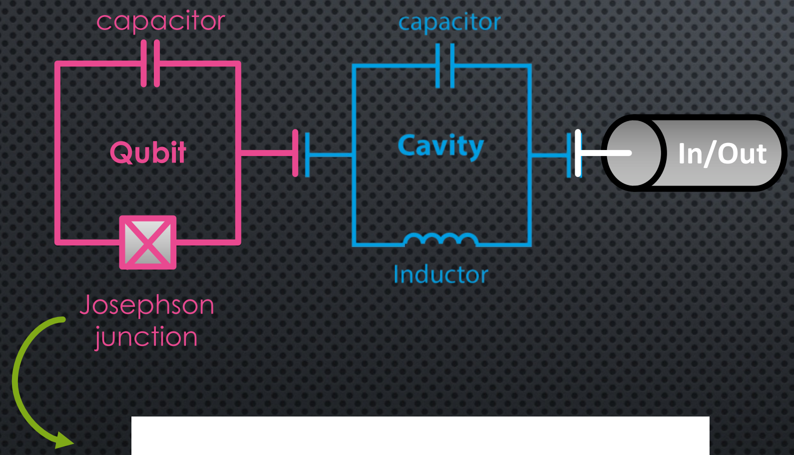
In this way it is possible to make the cosine term in the potential become prevalent. This means that the energy will be essentially determined by the phase jump through the JJ. This behaves like an artificial atom with many levels!!!

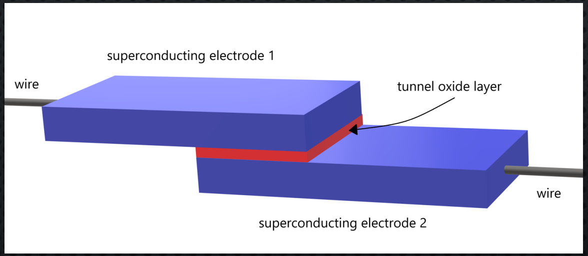


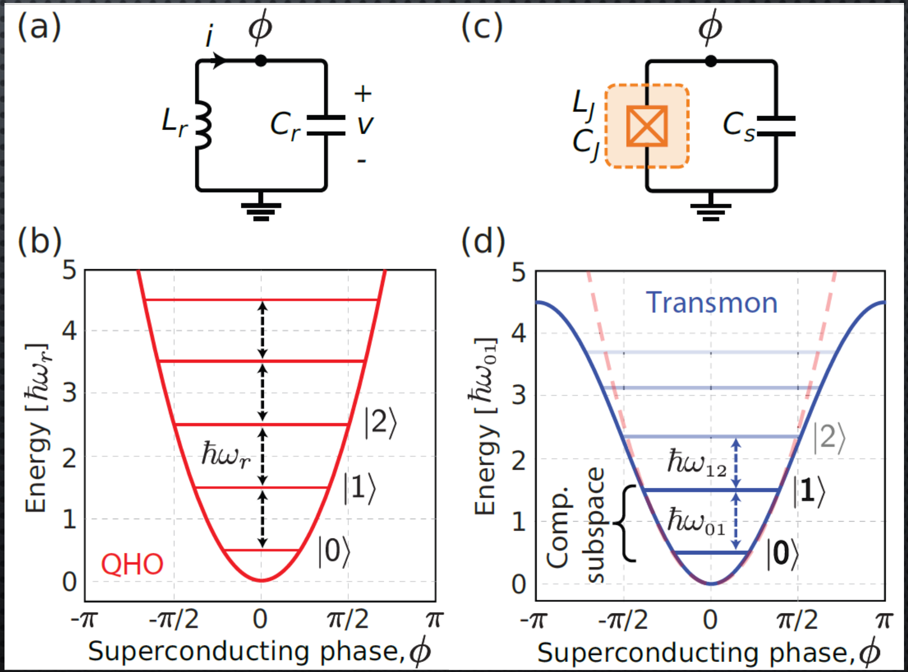


TRANSMON QUBIT-CAVITY SYSTEM

The transmon can be coupled to a resonant cavity ➡ (dispersive readout)







HAMILTONIAN

$$H_{QPU} = H_d + H_c(t)$$

- Hamiltonian for a 3D transmon coupled to a readout cavity
- Time-dependent drive for a single-mode transmon (in the frame of the trasmon)

$$H_d = \hbar\omega_T \hat{a}_T^{\dagger}\hat{a}_T + \hbar\omega_R \hat{a}_R^{\dagger}\hat{a}_R - E_J\left[\cos\hat{\varphi}_J + \frac{\hat{\varphi}_J^{\,2}}{2}\right]$$

$$H_c(t) = \hbar\varepsilon_I(t)(\hat{a}_T^{\dagger} + \hat{a}_T) + i\hbar\varepsilon_Q(t)(\hat{a}_T^{\dagger} - \hat{a}_T)$$

Hamiltonian and Connectivity

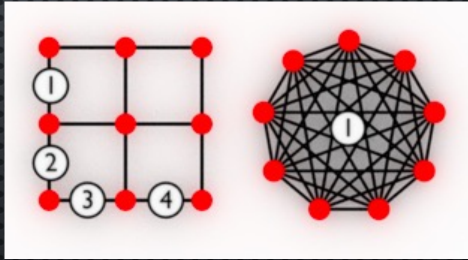
Typical undriven Hamiltonian for superconducting qudits / qubits:

$$H_0 \approx \sum_i^N \omega_i a_i^\dagger a_i - \chi_{ii} a_i^\dagger a_i^\dagger a_i a_i - \sum_{j \neq i}^N \chi_{ij} a_i^\dagger a_i a_j^\dagger a_j$$

Each (bosonic) mode is a qubit. The number of photons (0 or 1, but could be more!) determines the qubit state.

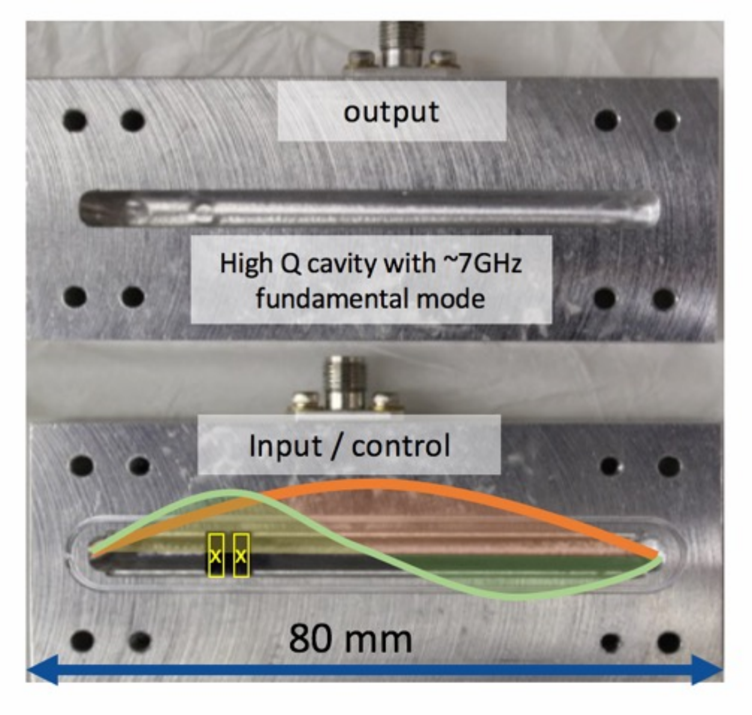
$$\omega_i \gg \chi_{ii} \gg \chi_{ij}$$

- Single qubit gates are fast
- Entangling gates are slower



H_0 *and* choice of gate set strongly influence connectivity / unit time

FIRST GENERATION QUANTUM TESTBED AT LLNL (OLD)



Two multilevel transmons coupled to a qudit register and a readout mode.

Demonstrated effective $SU(4)^3$ computational Hilbert space equivalent to six qubits

99% multi-qudit gate fidelity

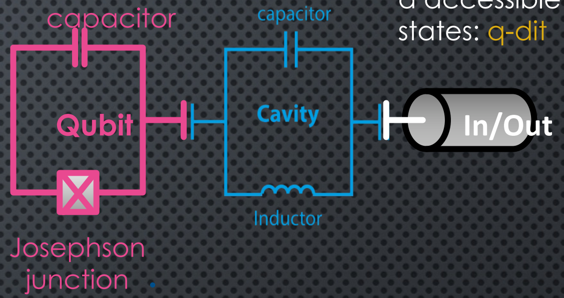
"CONTROL-CENTRIC" APPROACH TO QUANTUM COMPUTATION

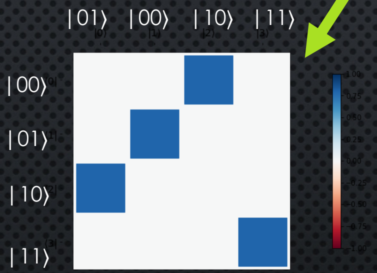
Given some control parameterization $f(t,\alpha) = \sum_k \alpha_k \phi_k(t)$

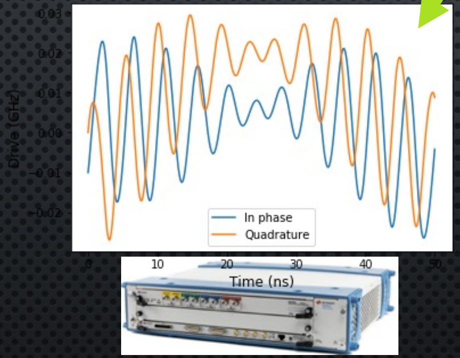
all unitary transformations in time T can be expressed as

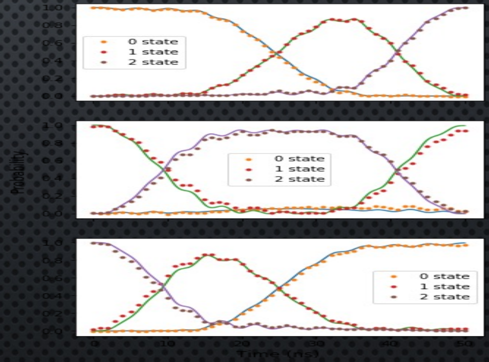
$$U(t,\alpha) = \mathcal{T} e^{-i\int_0^t dt' [\hat{H}_0 + f(t',\alpha)\hat{H}_c]}$$

Device with d accessible states: q-dit









See e.g. S. Schirmer, "Hamiltonian Engineering for Quantum Systems." , Proceedings of the 7th International Conference on Cooperative Control and Optimization (2007)

Optimization methods, machine learning (Q@Tn ML-QFORGE project (Jonathan Dubois, Kyle Wendt (LLNL), Simone Taioli, Paolo Trevisanutto (ECT*-LISC), FP, Piero Luchi, (UNITN)) – PRA 101, 062307 (2020)

OPTIMAL CONTROL STUDIES

- Computation of control pulses is quite expensive. In general, the cost scales exponentially with the number of qubits involved
- In many cases Hamiltonians depend on some external parameters that vary in time, and controls need to be constantly recomputed.
- In this case the time required to compute pulses if some wort of efficient fitting procedure can be found

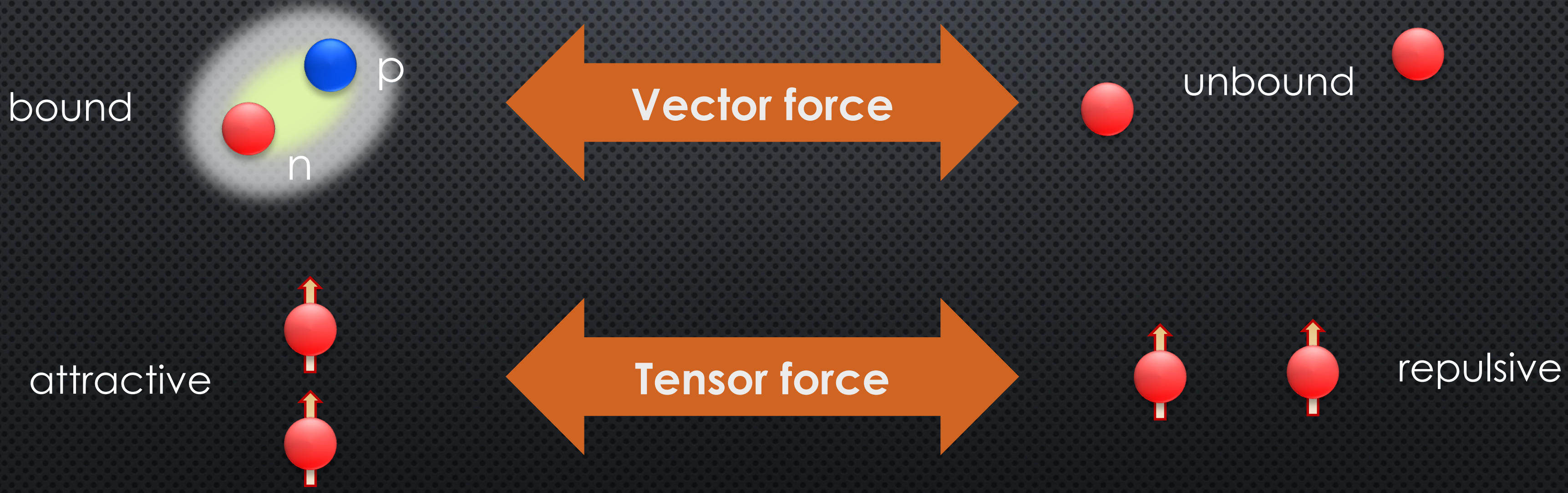
A SIMPLE, YET NON-TRIVIAL, NUCLEAR PHYSICS PROBLEM…

Describe how two neutrons evolve in time under the effect of their mutual interaction

- Interaction at leading-order (LO) of chiral effective field theory (spin tensor!)
- Implementation of real-time evolution of the system of 2 neutrons
- Realistic device-level simulations gauged on LLNL's QPU without/with measured noise
- Measurements/calculations on the LLNL testbed
- A simple extension on a digital machine (AQT testbed @ LBL) including some kind of evolution of the coordinates

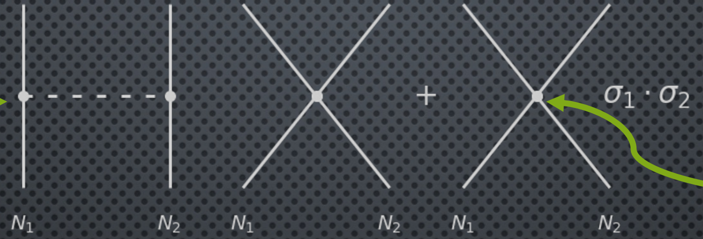
THE STRONG NUCLEAR INTERACTION HAS A NON-TRIVIAL DEPENDENCE ON THE QUANTUM STATE OF THE NUCLEONS

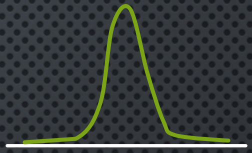
- ONE OF THE MAIN FEATURES OF THE NUCLEON-NUCLEON INTERACTION IS ITS SPIN/ISOSPIN DEPENDENCE. THIS DEPENDENCE ACCOUNTS FOR SOME VERY BASIC PHENOMENOLOGY.



NUCLEON NUCLEON INTERACTION (LO)

One-pion exchange (OPE)





Regularized contact (cutoff in momentum)

regulator function

$$H_{\rm int}^{\rm LO} = V_{\rm OPE}\,[1 - \delta_{R_0}(\vec{r})] + [C_0 + C_1\,\vec{\sigma}^1 \cdot \vec{\sigma}^2]\,\delta_{R_0}(\vec{r})$$

$$V_{\rm OPE} = \frac{f_\pi^2 m_\pi}{12\pi}\left[T_\pi(r)\,S_{12} - \left(Y_\pi(r) - \frac{4\pi}{m_\pi^3}\,\delta(\vec{r})\right)\,\vec{\sigma}^1 \cdot \vec{\sigma}^2\right]\vec{\tau}^1 \cdot \vec{\tau}^2$$

TIME PROPAGATION

Formal solution of the time-dependent Schroedinger equation:

$$\exp\left(-\frac{i}{\hbar}\hat{H}t\right) = \exp\left[-\frac{i}{\hbar}\left(\hat{T}+\hat{V}_{SI}+\hat{V}_{SD}\right)t\right]$$

- V_{SI}: **SPIN-INDEPENDENT** part of the interaction
- V_{SD}: **SPIN-DEPENDENT** part of the interaction

In the short-time limit, we can separate the terms depending on V_{SI} and V_{SD}:

$$\exp\left[-\frac{i}{\hbar}\left(\hat{T}+\hat{V}_{SI}\right)\delta t\right]\exp\left[-\frac{i}{\hbar}\hat{V}_{SD}\delta t\right]+o(\delta t^2)$$

TIME PROPAGATION

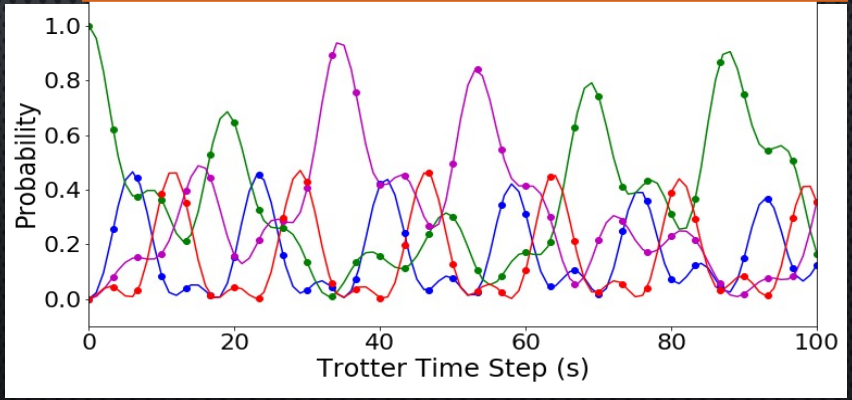
First application: "frozen" nucleons → Spin/isospin Hamiltonian only

$$\exp\left[-\frac{i}{\hbar}\hat{V}_{SD}\delta t\right] = \exp\left[-\frac{i}{\hbar}\left(\sum_{i,j=1}^{A}\sum_{\alpha,\beta=x,y,z}\sigma_{i\alpha}A(r_{ij})_{ij;\alpha\beta}\sigma_{j\beta}\right)\delta t\right]$$

coordinates appear as "parameters"

Computer simulation of the actual device

Occupation probability



Lines: analytic results
Circles: synthetic data: probability of measuring state |α⟩ after a time t (obtained by repeating the process over and over)

The time evolution with a tensor Hamiltonian introduces spin-parallel components

$|0\rangle = |\uparrow\uparrow\rangle$ $|1\rangle = |\uparrow\downarrow\rangle$ $|2\rangle = |\downarrow\uparrow\rangle$ $|3\rangle = |\downarrow\downarrow\rangle$

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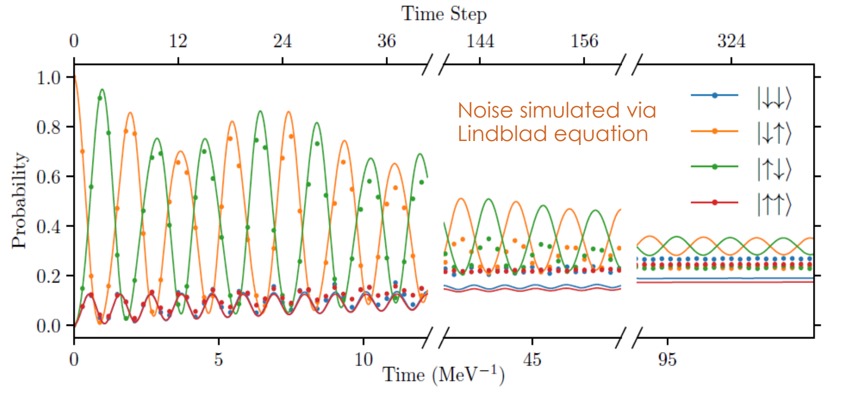

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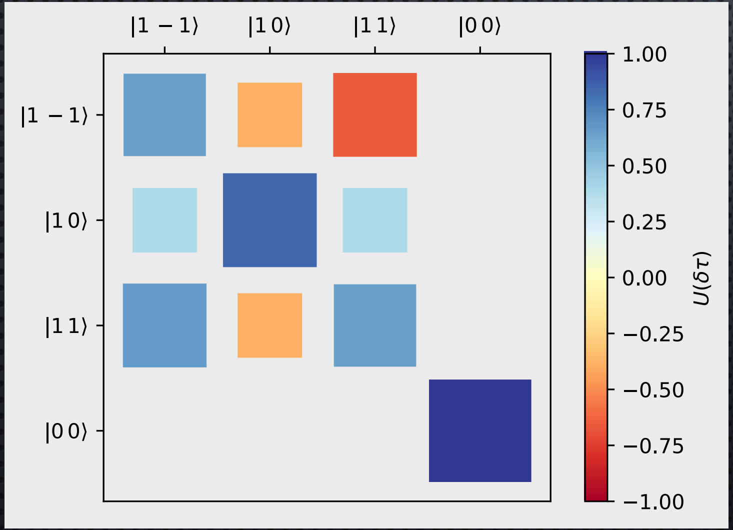
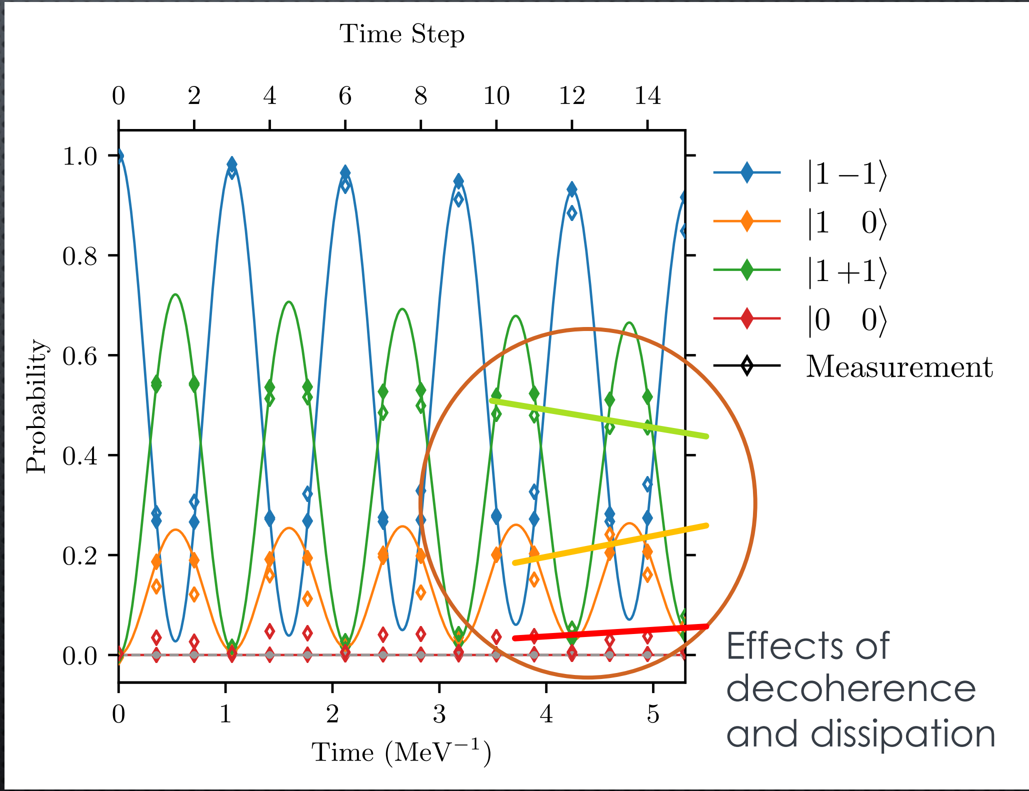
RESULTS ON LLNL TEST BED SIMULATION

- Minor Snag with test bed: 4th state was temporarily lost (now working)
- Transform Hamiltonian to coupled spin basis





"COPROCESSING" SCHEME FOR FULL TIME EVOLUTION

$$\exp\left(-\frac{i}{\hbar}\hat{H}t\right) = \exp\left[-\frac{i}{\hbar}\left(\hat{T}+\hat{V}_{SI}+\hat{V}_{SD}\right)t\right]$$

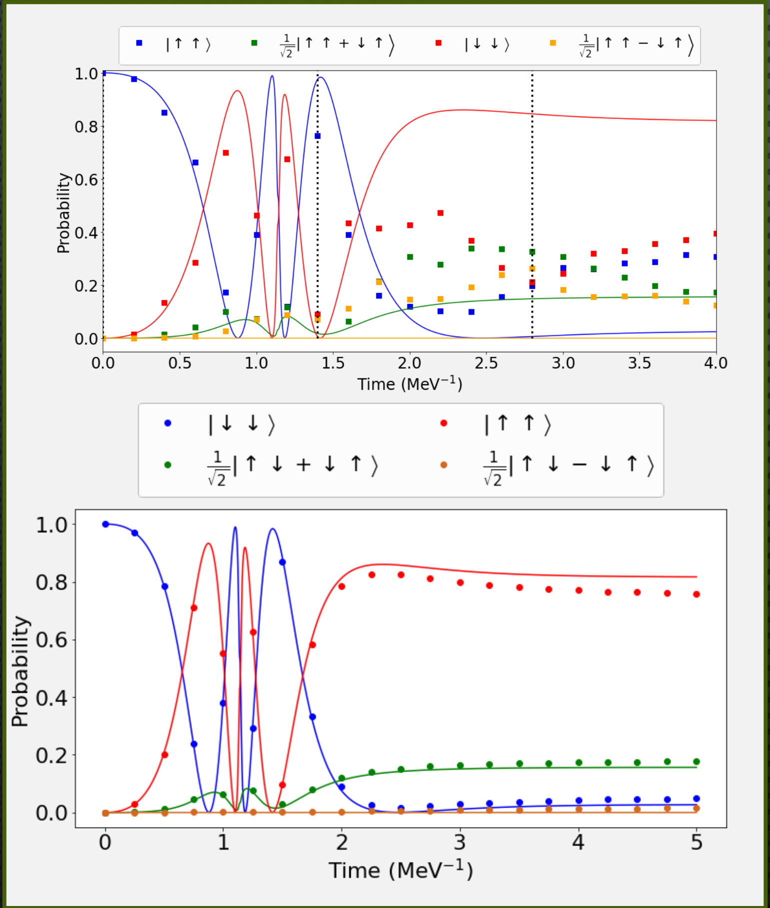
As a first step in the direction of simulating the full dynamical evolution of a scattering process, we employed a mixed scheme in which the coordinates are evolved classically.

Remind that:

$$\exp\left[-\frac{i}{\hbar}\left(\hat{T}+\hat{V}_{SI}\right)\delta t\right]\exp\left[-\frac{i}{\hbar}\hat{V}_{SD}\delta t\right] + o(\delta t^2)$$

We can make the (very crude) approximation of evolving the coordinates of the nucleons classically (using V_{SI}), and evolving the spin of the nucleons with the second factor in the propagator. Since the coordinates are evolved on a classical computer, we call this "coprocessing" scheme.

TIME EVOLUTION OF NN SPIN IN A COPROCESSING SCHEME



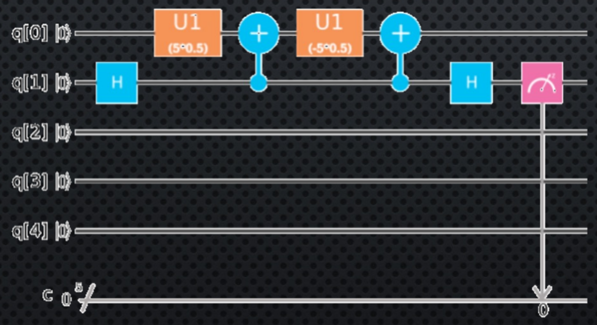
Simulation on a real quantum computer (AQT testbed at LBL). Results can be improved a bit by applying several error correction techniques

Simulated results using optimal control (noise included)

Different models in quantum computing (recap)

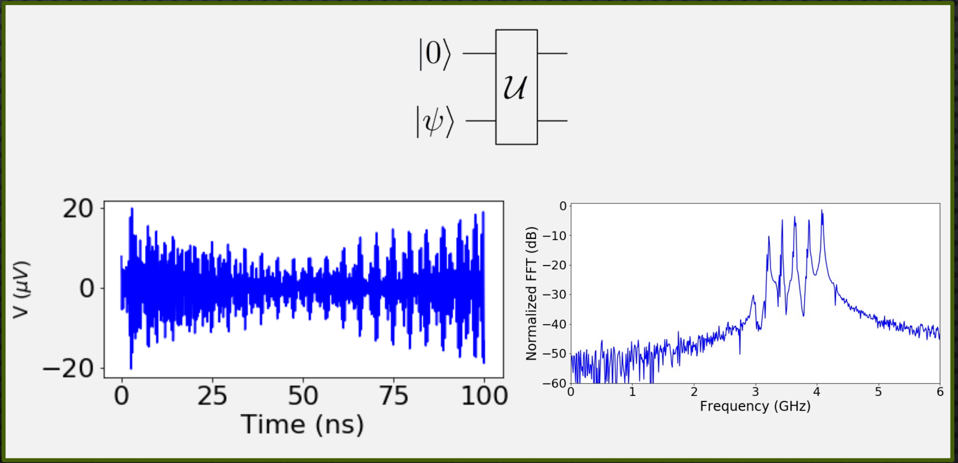
Traditional (circuit model of QC)

- Discrete, finite predetermined set of quantum logical operations (gates)
- Many-body dynamics to be simulated implemented through a circuit involving multiple gates



Alternative (control-centric model of QC)

- Software reconfigurable, continuous unitary transformation (gate)
- Many-body dynamics to be simulated implemented with a single gate on a qudit

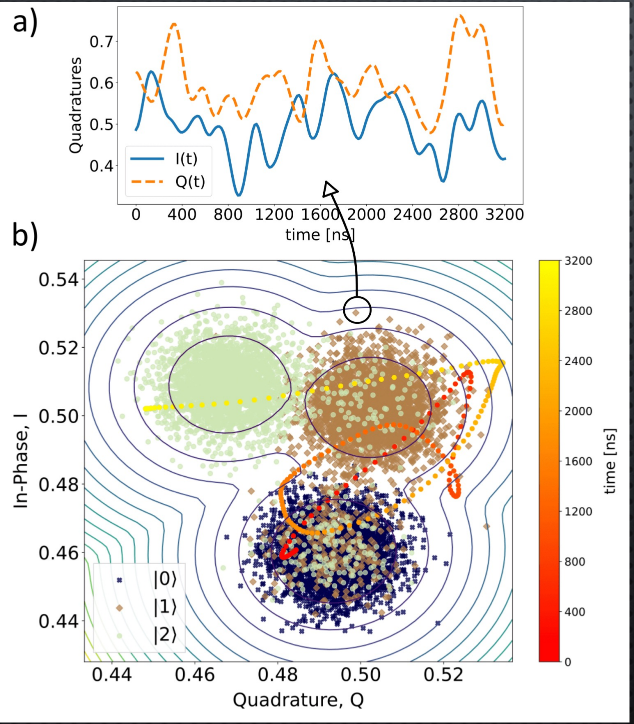


CONCLUSIONS

- Quantum testbeds based on high-Q cavities are a simple and effective way to experiment quantum computing techniques that are not easily accessible using commercial machines.
- Optimal control and the creation of customized gates are an interesting tool to shortcut some of the limitations due to the need of creating deep circuits to implement complex transformations and to study test cases of physical interest in view of better quantum computers.
- Theorists can contribute at technical level by developing software to be applied as online or post-processing of experimental output.

IMPROVING READOUT FORM CAVITIES WITH AUTOENCODERS

Autoencoders can be used as a way to reduce misclassifications when reading out the state of a cavity.



Demodulated signal [I(t): in-phase Q(t): quadrature]. The average of the signals identifies a point in the panel below.

Readout points from many shots of a given experiment.

- Notice the evident effects of the decay of the state along the measurement and the consequent possible misreadings

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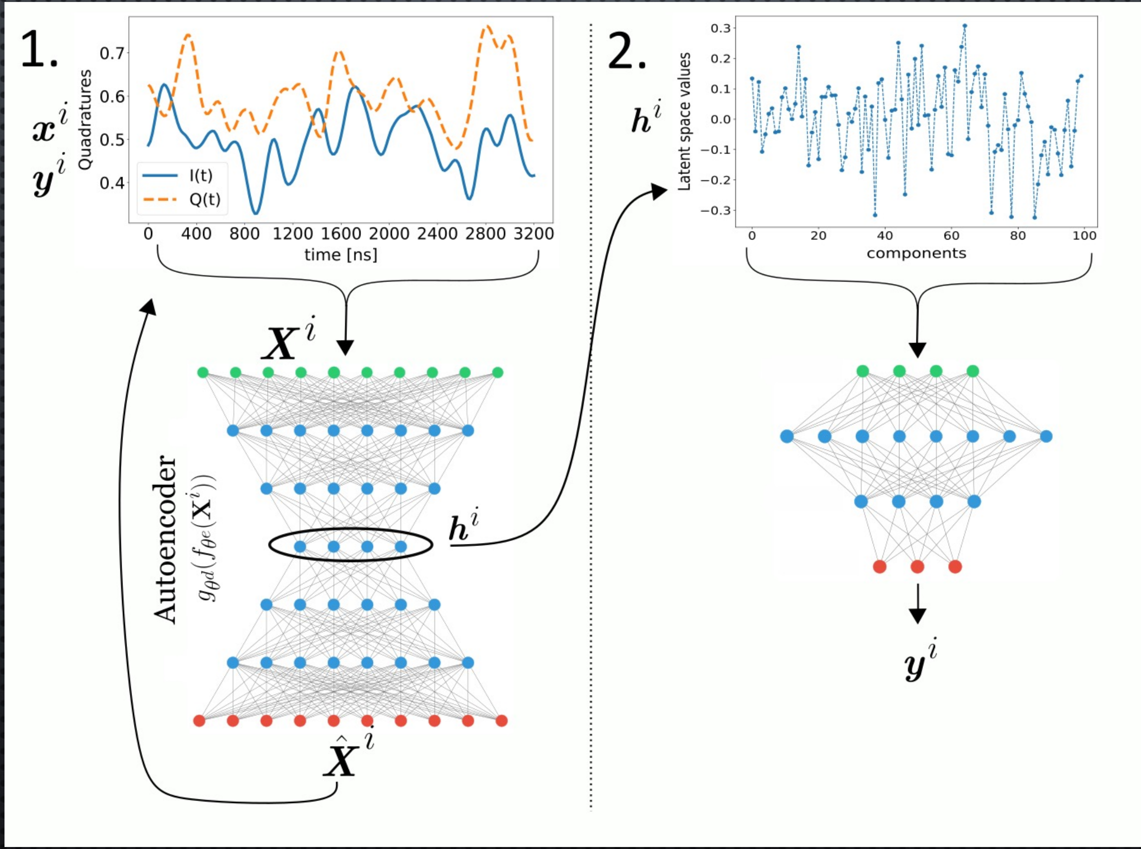

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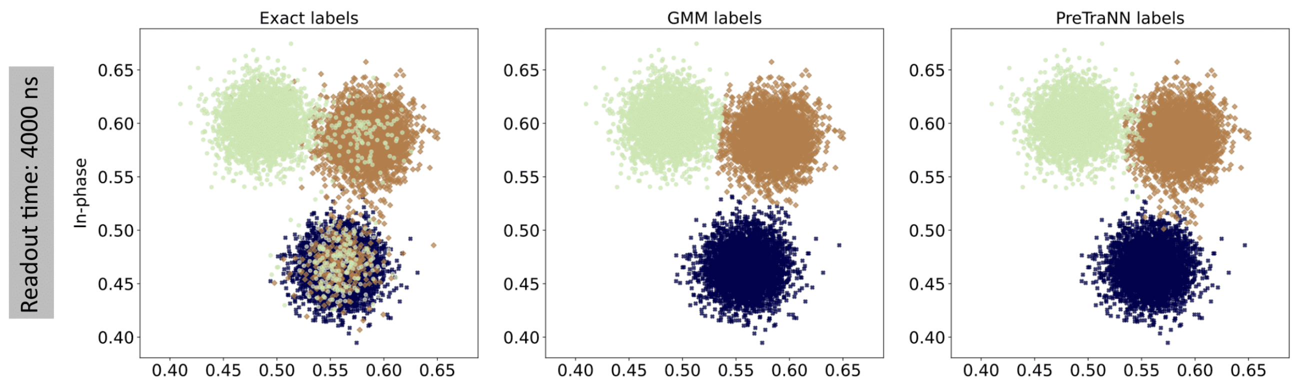
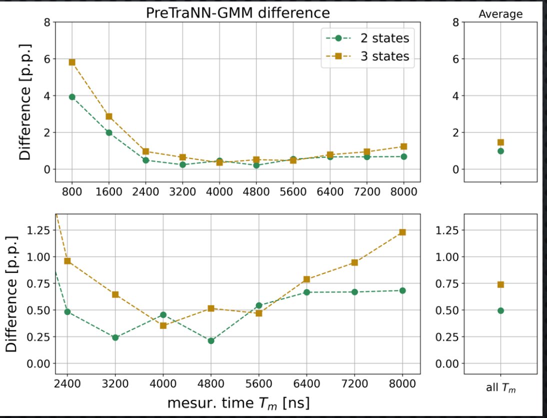


	Average Class. Accuracy
GMM	93.2 %
PreTraNN	93.3 %



Different schemes have been tested and compared:

- **PreTraNN:** Autoencoder pre-training
- **GMM:** Gaussian Mixture Model