Tidal deformability of black holes surrounded by thin accretion disks

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grit
gravitation in técnico

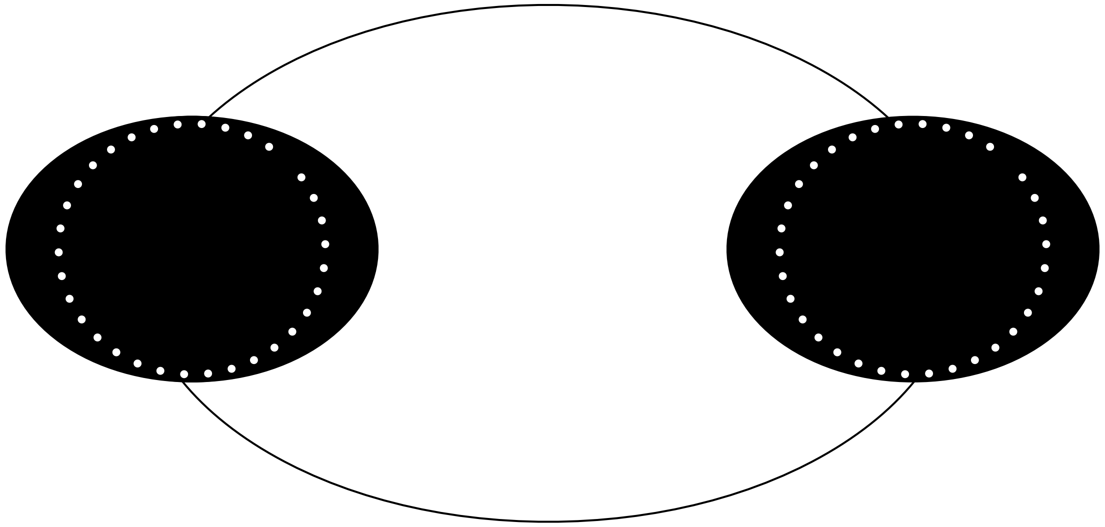
centra

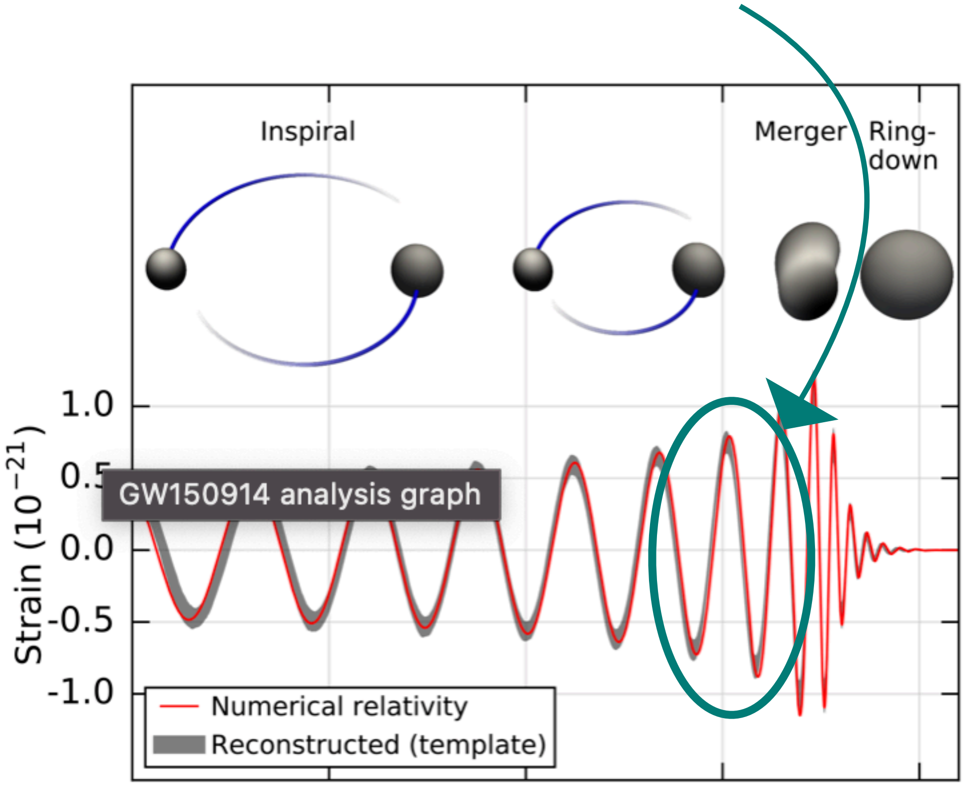
Tidal Deformability and Love Numbers

- Self-gravitating objects may be deformed in external tidal fields (e.g. companions in a binary system)

- Deformation is quantified using Tidal Love Numbers (TLNs), which describe the linear response of the body, akin to a "gravitational susceptibility"

- TLNs depend on the theory of gravity and the internal structure of the object, and they affect the orbital dynamics by leaving a footprint at 5PN order





Neutron Stars

Neutron stars can be tidally deformed, with a TLN dependent on their equation of state

$$\tilde{\Lambda} \propto O(EOS)\left(\frac{R_{NS}}{M_{NS}}\right)^5$$

Hinderer (2008)

Gravitational potential of deformed body (Newtonian):

$$U_{tot} = -\frac{GM}{r} - \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} Y_{\ell m}\left[\frac{(\ell-2)!}{\ell!}\mathcal{E}_{\ell m}r^{\ell} - \frac{(2\ell-1)!!}{\ell!}\frac{I_{\ell m}}{r^{\ell+1}}\right]$$

Tidal field Body response

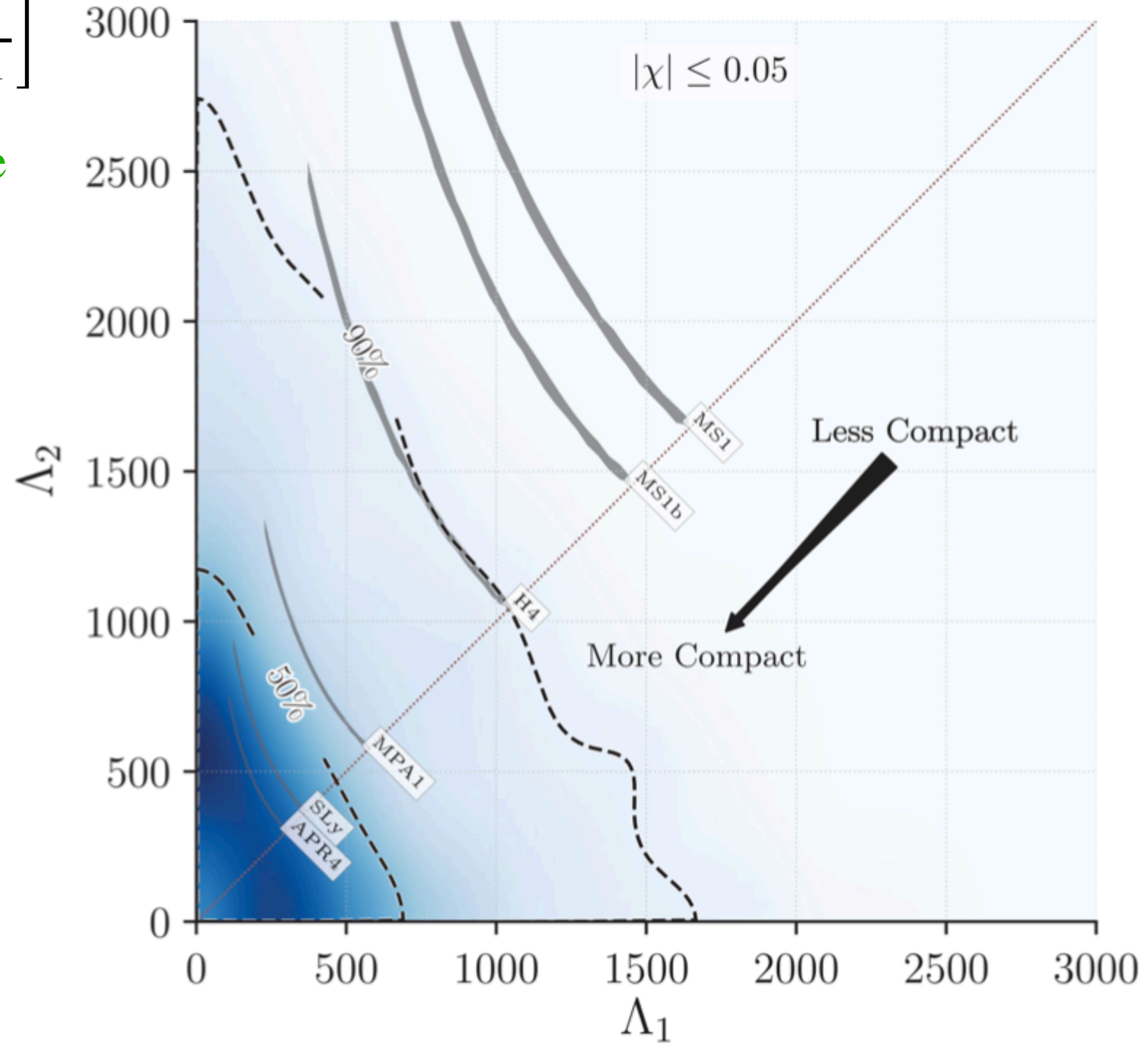
Linear response theory:

$$\boxed{I_{\ell m}(\omega) = -\frac{(\ell-2)!}{(2\ell-1)!}\lambda_{\ell m}(\omega)r_h^{2\ell+1}\mathcal{E}_{\ell m}(\omega)}$$

$$\boxed{k_{\ell m} = \mathrm{Re}(\lambda_{\ell \mathrm{m}})}$$

GW170817

Abbott et al. (2017)



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GR case: perturbation theory - massless fields in curved spacetime

Equation of motion: $\mathcal{O}_s \psi_s = 0$

Matching at the star surface: ψ_s regular at $\mathrm{r} = \mathrm{r_h}$

$$\psi_s \propto r^{\ell+1}\left[1 + k_s^{(\ell)}\left(\frac{r}{r_h}\right)^{-2\ell+1}\right]$$

Tidal deformability of black holes

In D=4, the static TLNs of asymptotically flat BHs in General Relativity vanish.

Fragile condition. $k_2 \neq 0$ in a plethora of different scenarios:

→	**Beyond GR theories**	*Cardoso+ (2017,2018), De Luca+(2023), Barura+(2024)*
→	**BH mimickers and exotic compact objects**	*Pani+(2015), Cardoso+ (2017, 2019), Herdeiro+(2020), Berti+(2024)*
→	**Higher dimensions**	*Hui+(2021), Kol+(2012), Cardoso+(2019), Rodriguez+(2023), Charalambous+(2023,2024), Ma+(2024)*

Implications in fundamental physics: Tidal tests of general relativity!

Why so vacuum?

Accretion disks are ubiquitous in astrophysics: Matter (dust, plasma) spirals around the BH due to gravity

$\dot{M}$

$\lesssim \dot{M}_{Edd}$	cold, geometrically thin disk
$\gtrsim \dot{M}_{Edd}$	hot, inflated thick disk
$\ll \dot{M}_{Edd}$	Advection Dominated accretion flow (quasi-spherical)

- Can feature very high densities ($n_e \approx 10^{19} M_\odot / M \text{ cm}^{-3}$)
- Less compact than neutron stars (more deformable?)

What is the deformability of a BH+disk system?

BH+thin disk geometry

“Non-linear superposition” of black hole and thin disk:

$$\mathrm{d}s^2 = -f(r)\mathrm{e}^{2\nu_{\mathrm{disk}}}\mathrm{dt}^2 + \mathrm{e}^{2\lambda_{\mathrm{ext}}-2\nu_{\mathrm{disk}}}\frac{\mathrm{dr}^2}{\mathrm{f(r)}} + \mathrm{r}^2\mathrm{e}^{-2\nu_{\mathrm{disk}}}\left(\mathrm{e}^{2\lambda_{\mathrm{ext}}}\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$$

Disk potential

Non-linear term (GR)

- Fully relativistic axisymmetric and static metric
- The disk stretches from the horizon to infinity, and vanish in the extremities
- Analytic expressions for the disk functions
- Density profile is astrophysically realistic (equatorial plane)

Model parameters of the disk:

$$\begin{cases} \epsilon = M_{\mathrm{disk}}/M_{BH} & \text{mass of the disk compared to BH mass} \\ \tilde{b} & \text{location of the disk peak} \end{cases}$$

Kotlarik+(2018,2022), Chen+(2023)

Tidal deformability of BHs with thin accretion disks

At leading order in ϵ:

Spin-0

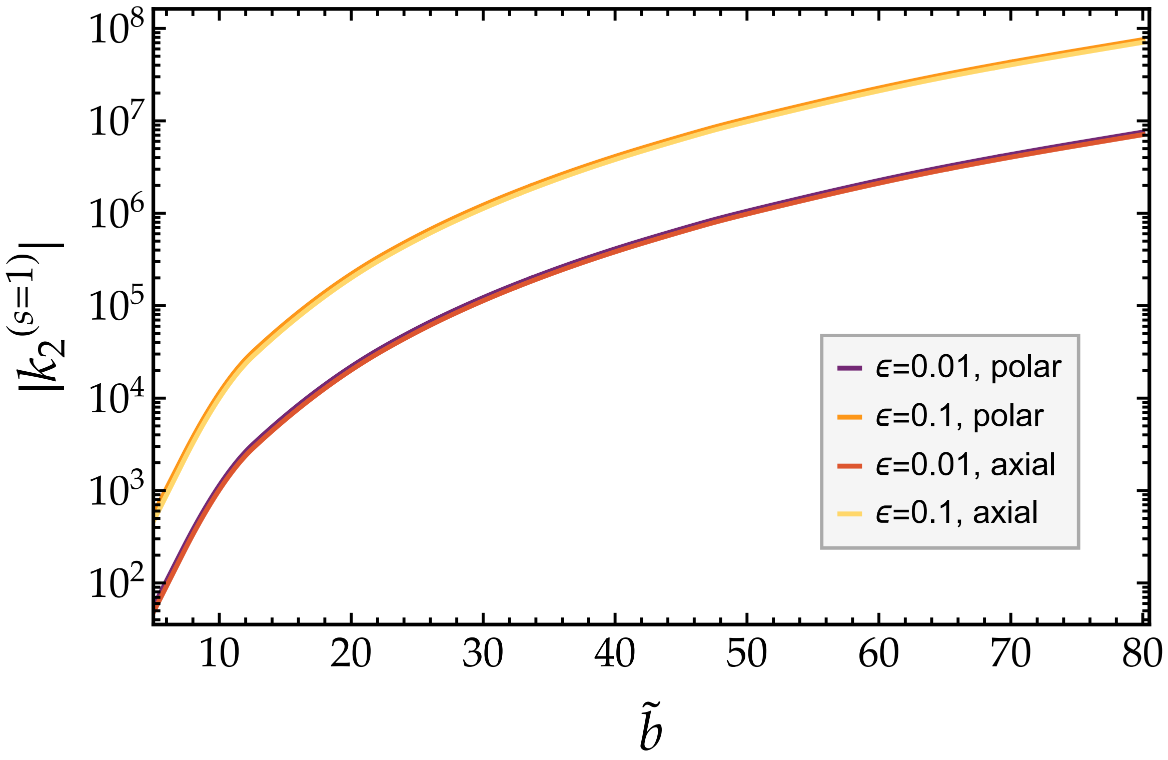
$$\boxed{k_2 = \frac{\epsilon}{768}(432\tilde{b}^4 - 208\tilde{b}^2 + 9)}$$

Typical values:

$\epsilon \rightarrow [10^{-4}, 10^{-1}]$

$b \rightarrow [r_{\rm ISCO},\ 30M]$

Spin-1



Scaling $k_2 \propto \epsilon\tilde{b}^4$ is in agreement with the scaling of TLNs of dressed BHs in other environments (boson clouds, thin shells, Einstein clusters) *De Luca+(2021), Duque+(2019,2021)*

Cannizzaro+(2024), arXiv: 2408.14208

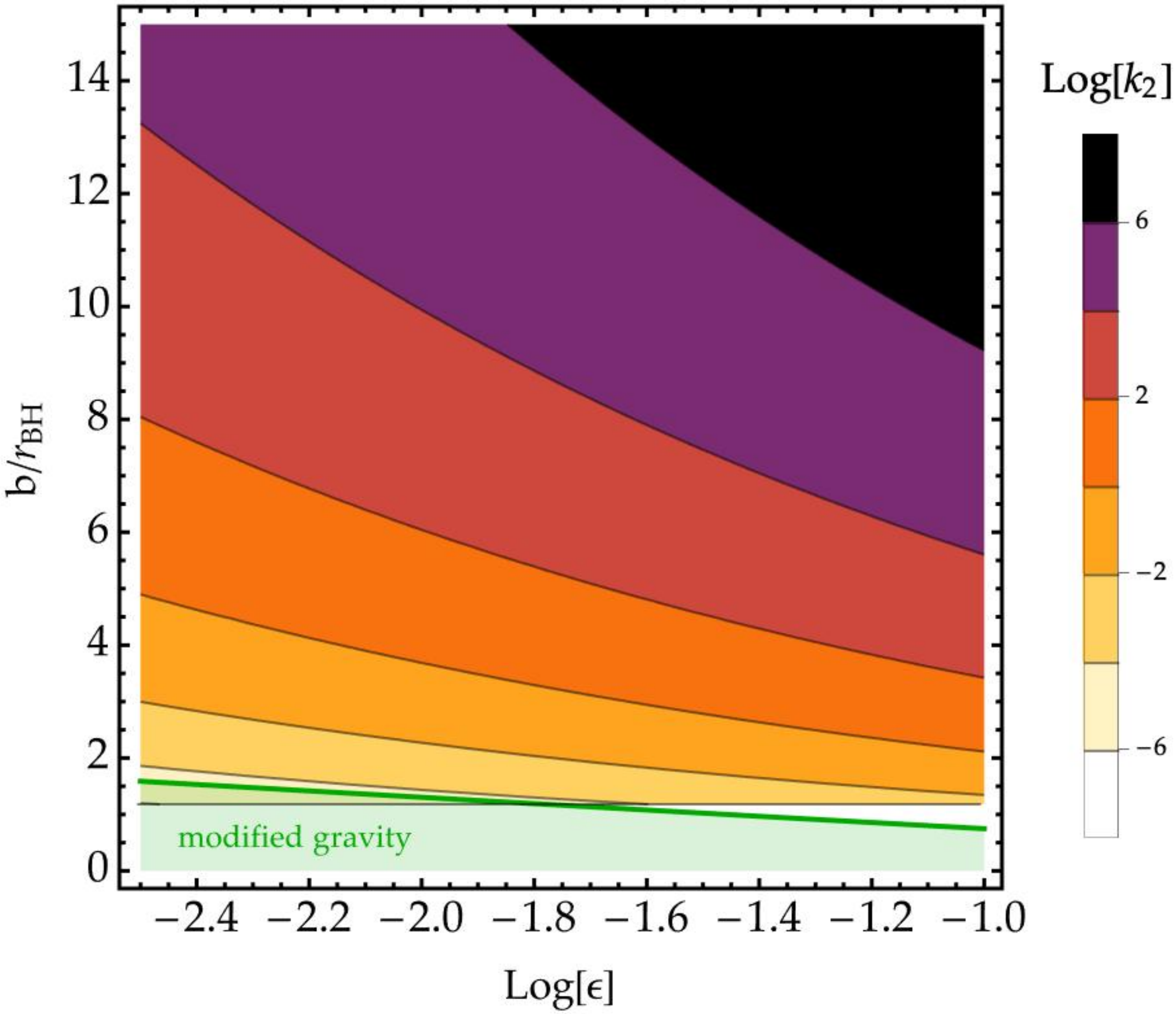
Environmental Effects vs Modified GR

Lots of beyond GR theories feature Non-vanishing Love Numbers

$$k_2^{\mathrm{MG}} \propto \frac{\alpha_{\mathrm{MG}}}{G\Lambda^2_{\mathrm{MG}}}\left(\frac{1}{Gm_{BH}}\right)$$

Can the TLN of the BH+disk system jeopardize tidal tests of GR?

Yes, whenever $\tilde{b} \gtrsim 5,\ \epsilon \gtrsim 10^{-2.5}$



Cannizzaro+(2024), arXiv: 2408.14208

Tidal disruption and frequency dependent TLNs

The accretion disks will get tidally disrupted at the Roche radius:

$$r_{Roche} = 2\gamma m_1 (1+\tilde{b}) \left[\frac{m_2}{m_1(1+\epsilon)} \right]^{1/3}$$

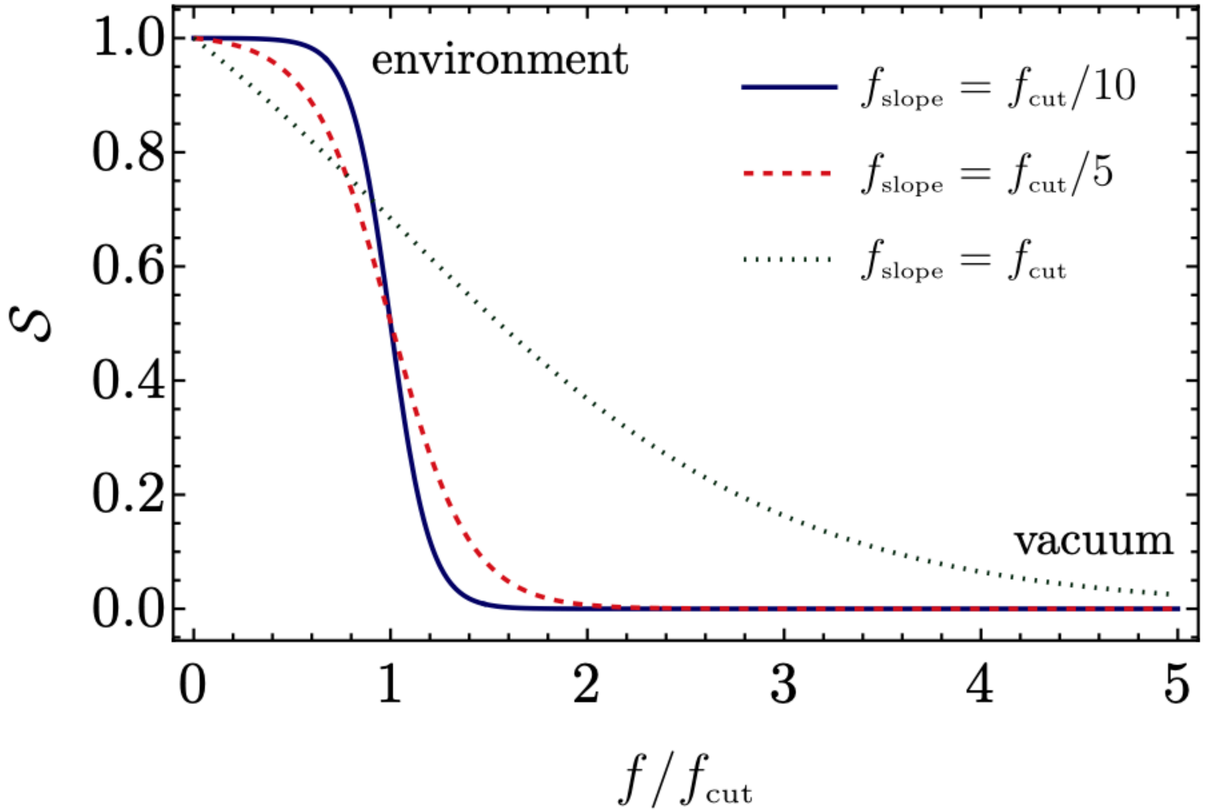
at a GW frequency

$$f_{\rm cut} = \frac{1}{2\sqrt{2}\pi\gamma^{3/2} m_1} \sqrt{\frac{(1+\epsilon)(m_1+m_2)}{(1+\tilde{b})^3 m_2}}$$

↓

Large $\tilde{b}$: Large TLN but small Roche frequency

Proxy for frequency dependent tidal deformability

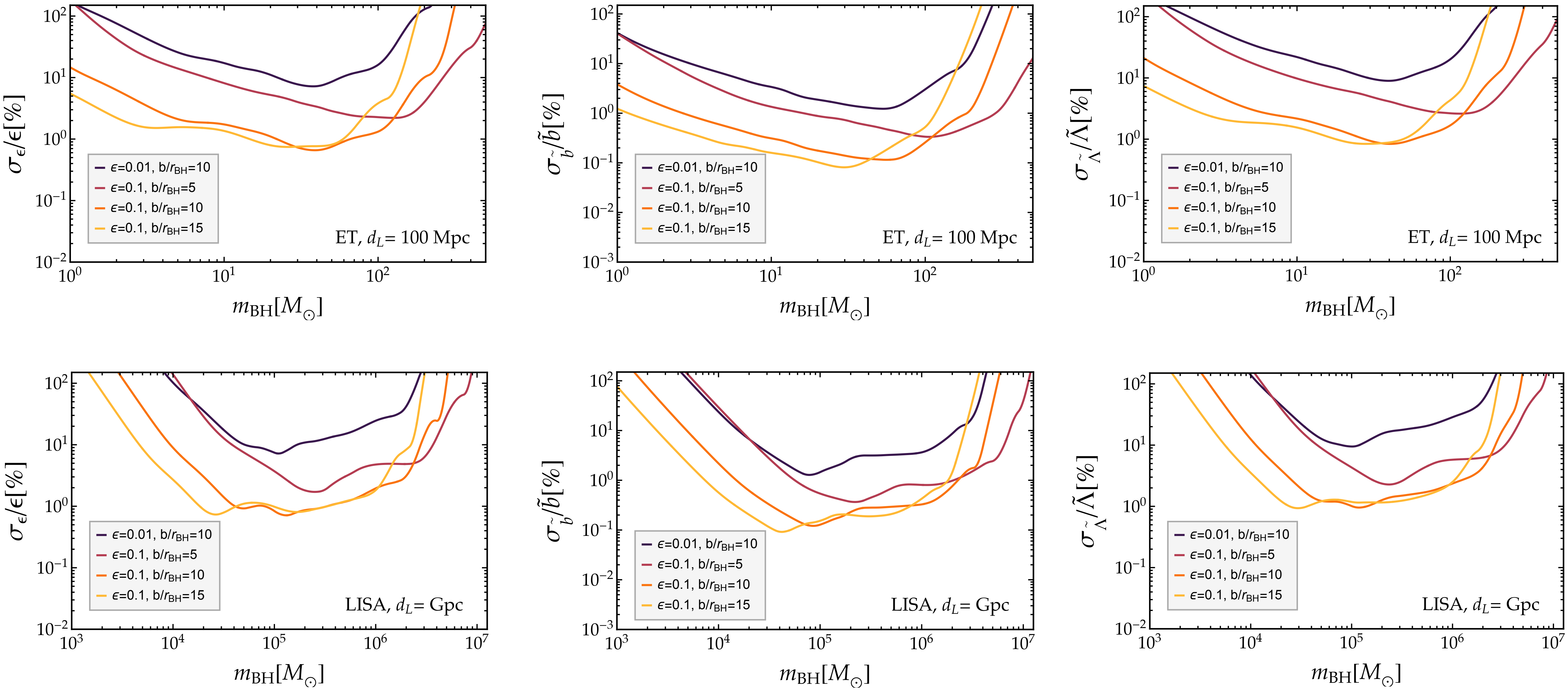


$$\tilde{\Lambda} \to \mathcal{S}(f) \cdot \tilde{\Lambda} = \frac{1 + e^{-f_{\rm cut}/f_{\rm slope}}}{1 + e^{(f - f_{\rm cut})/f_{\rm slope}}} \cdot \tilde{\Lambda}$$

De Luca+(2022)

Detectability at ET and LISA: Fisher matrix analysis

$$\tilde{h}(f) = C_{\Omega}\mathcal{A}_{\rm PN}e^{i\psi_{\rm PP}(f)+i\psi_{\rm Tidal}(f)}$$



All tidal parameters can be measured with high accuracy with next generation detectors!

Cannizzaro+(2024), arXiv: 2408.14208

Conclusions

- Presence of a thin disk around a BH induce a non-vanishing tidal deformability

Fundamental
Physics implications:
Such effect can easily
jeopardize tidal tests of theories beyond GR
and BH mimickers

Astrophysics
implications:
Disk parameters could be measured
with high accuracy with LISA and
third generation detectors

Thank you!