Scattering amplitudes for black holes

Matteo Sergola
matteo.sergola@ipht.fr

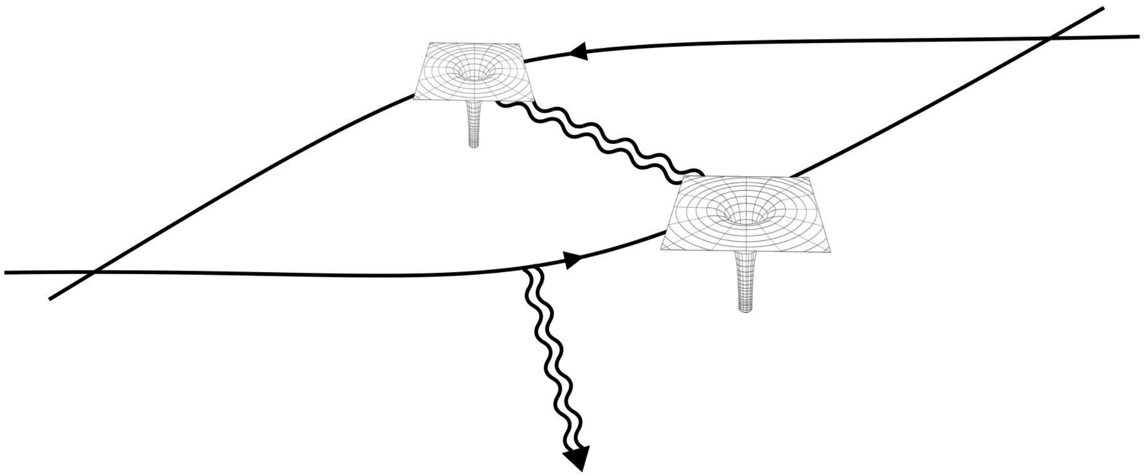
Institut de Physique Théorique, CEA, CNRS, Université Paris-Saclay

Based (mostly) on [2112.07556], [2303.06211]

Work with *Donal O'Connell, Ricardo Monteiro, Chris White, David Peinador Veiga, Ingrid A. Vazquez-Holm, Asaad Elkhidir, Nathan Moynihan, Alasdair Ross, Andrea Cristofoli, Riccardo Gonzo, Rafael Aoude*.

TEONGRAV, Roma 20/09/2024

Background and motivation



$$\Rightarrow \langle R_{\mu\nu\rho\sigma}(x) \rangle = \int_k \mathcal{A}(k)\, e^{-ik\cdot x}$$

Kosower, O'Connell, Gonzo & Cristofoli, 2019
Sergola, Peinador-Veiga, Monteiro & O'Connell, 2021
Sergola, Vazquez-Holm, Elkhidir & O'Connell, 2023

Classical observables from amplitudes

- Classical gravity/EM from amplitudes: $\Delta p^\mu, \ \Delta s^\mu, \ F^{\mu\nu}_{\text{rad}}, \ \Psi^{\alpha\beta\gamma\delta}_{\text{rad}}$

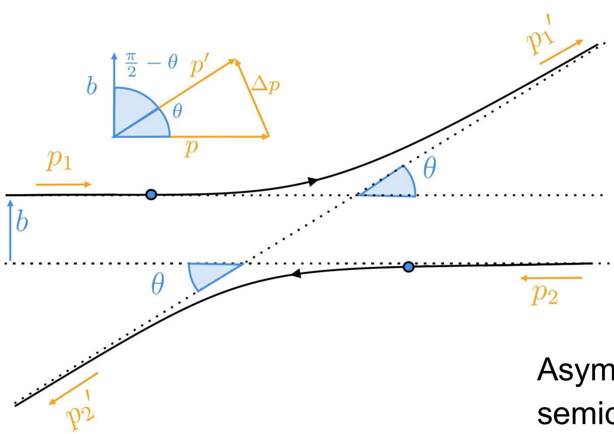
Kosower, Maybee & O'Connell, 2018.

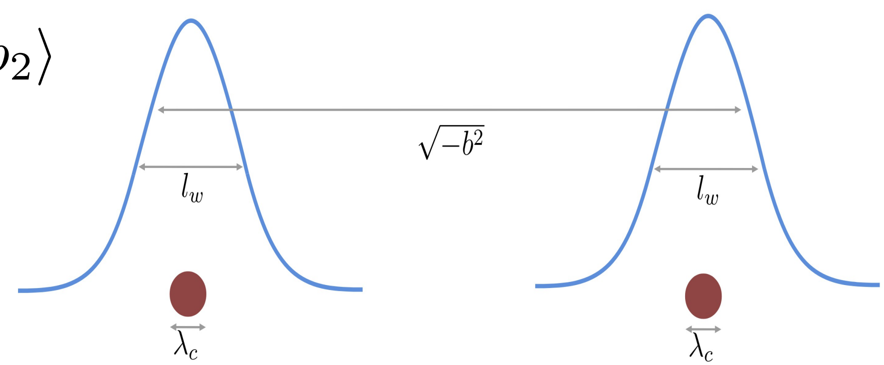
$$\Delta p_1^\mu = \langle\psi|S^\dagger \hat{P}^\mu S|\psi\rangle - p^\mu$$

Recall: $S = 1 + iT$

$$= \langle\psi|[\hat{P}^\mu, iT]|\psi\rangle + \langle\psi|T^\dagger \hat{P}^\mu T|\psi\rangle$$

$$|\psi\rangle \sim \varphi(p_1, p_2)|p_1\rangle \otimes |p_2\rangle$$





$$\lambda_c \ll l_w \ll \sqrt{-b^2}$$

Asymptotic states with semiclassical wavepackets

Kosower, Maybee, Vines, O'Connell, Cristofoli, Gonzo, Sergola, Ross, Moynihan, White, Vanhove, Planté, Porto, Damgaard, Bjerrum-Bohr, Zeng, Parra-Martinez, Hermann, Bern..

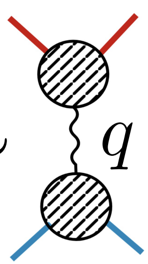
Classical observables from amplitudes

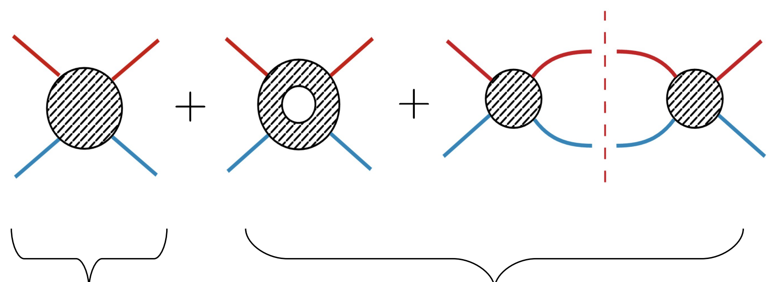
- Classical gravity/EM from amplitudes: $\Delta p^{\mu},\ \Delta s^{\mu},\ F^{\mu\nu}_{\rm rad},\ \Psi^{\alpha\beta\gamma\delta}_{\rm rad}$

Kosower, Maybee & O'Connell, 2018.

$$\Delta p_1^{\mu} = \int \hat{d}^4 q\, \hat{\delta}(p_1\cdot q)\hat{\delta}(p_2\cdot q)e^{-ib\cdot q}\Big[q^{\mu} i\mathcal{A}(p_1\to p_1+q)$$
$$+ \int \hat{d}^4 l\, \hat{\delta}(p_1\cdot l)\hat{\delta}(p_2\cdot l) l^{\mu}\mathcal{A}^*(p_1+q_1\to p_1+l)\mathcal{A}(p_1\to p_1+l)\Big]$$

Classical change in momentum: "$\hbar \to 0$"

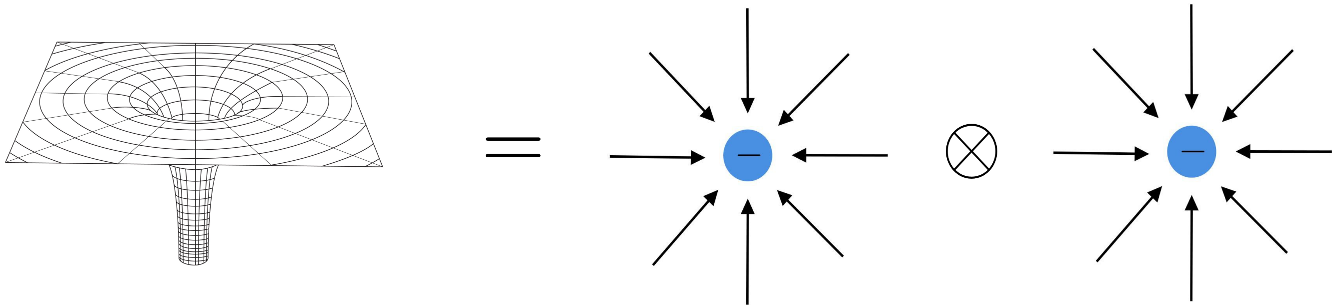
$\mathcal{A}_{tree} \sim$  $\sim \dfrac{1}{q^2}$

$\Delta p_1^{\mu} =$  $+ \cdots$

tree $\sim \mathcal{O}(G)$ 1-loop $\sim \mathcal{O}(G^2)$

Kosower, Maybee, Vines, O'Connell, Cristofoli, Gonzo, Sergola, Ross, Moynihan, White, Vanhove, Planté, Porto, Damgaard, Bjerrum-Bohr, Zeng, Parra-Martinez, Hermann, Bern..

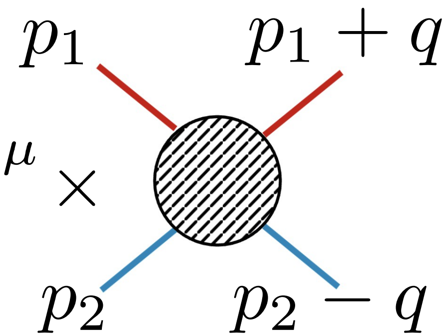
Double Copy duality: classical and quantum



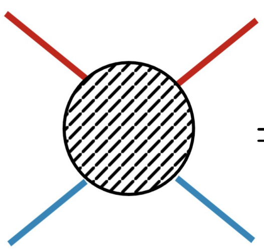
Kawai, Lewellen & Tye, 1986
Bern, Carrasco & Johansson, 2008
Monteiro, O'Connell & White, 2014

Momentum deflection from the double copy

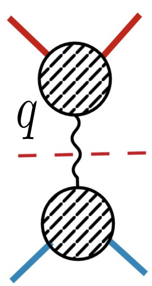
$$\Delta p_1^\mu = \int \hat{d}^4 q \hat{\delta}(2p_1 \cdot q)\hat{\delta}(2p_2 \cdot q) e^{-ib\cdot q} q^\mu \times$$



$\Rightarrow$



$= \frac{1}{q^2} \sum_h$



$2p_1 \cdot \epsilon_h(q)$

$2p_2 \cdot \epsilon_{-h}(q)$

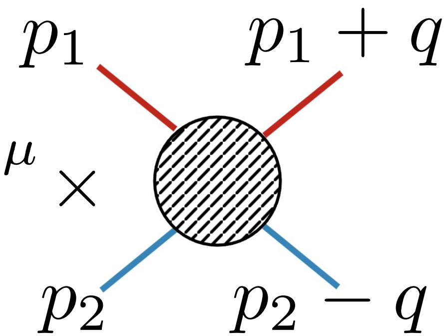
Impact parameter

$$\Rightarrow \Delta p_1^\mu \sim Q_1 Q_2 \frac{b^\mu}{b^2} \cosh \eta$$

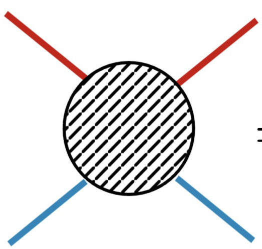
where $\frac{p_1 \cdot p_2}{m_1 m_2} = u_1 \cdot u_2 = \gamma = \cosh \eta$

Momentum deflection from the double copy

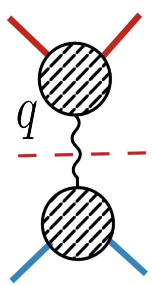
$$\Delta p_1^\mu = \int \hat{d}^4 q \, \hat{\delta}(2p_1 \cdot q)\, \hat{\delta}(2p_2 \cdot q)\, e^{-ib\cdot q} q^\mu \times$$



$\Rightarrow$



$$= \frac{1}{q^2} \sum_h$$



$$(2p_1 \cdot \epsilon_h(q))^2$$

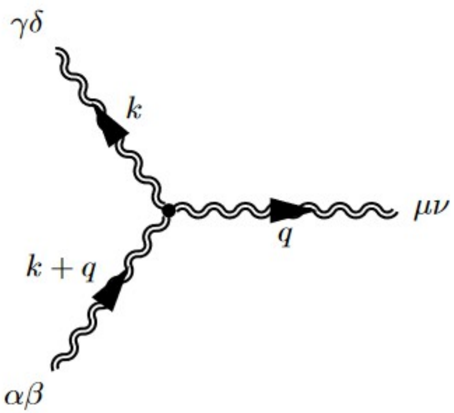
$$(2p_2 \cdot \epsilon_{-h}(q))^2$$

$$\Rightarrow \Delta p_1^\mu \sim G m_1 m_2 \frac{b^\mu}{b^2} \cosh 2\eta$$

Bern, Dixon, Dunbar & Kosower, 1994

Graviton diagrams = hard

Graviton Feynman rules, at the level of individual diagrams, are a nightmare:

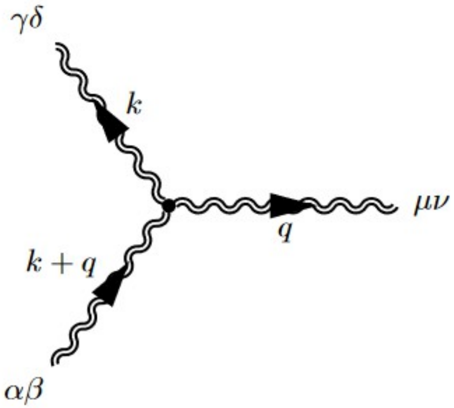


$$
\begin{aligned}
= -\frac{i\kappa}{2}\Big\{ & P_{\alpha\beta,\gamma\delta}\Big[k^\mu k^\nu + (k+q)^\mu (k+q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu} q^2\Big] \\
& + 2q_\lambda q_\sigma \Big[I^{\lambda\sigma,}{}_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta} + I^{\lambda\sigma,}{}_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta} \\
& \qquad - I^{\lambda\mu,}{}_{\alpha\beta} I^{\sigma\nu,}{}_{\gamma\delta} - I^{\sigma\nu,}{}_{\alpha\beta} I^{\lambda\mu,}{}_{\gamma\delta} \Big] \\
& + \Big[q_\lambda q^\mu (\eta_{\alpha\beta} I^{\lambda\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\nu,}{}_{\alpha\beta}) + q_\lambda q^\nu (\eta_{\alpha\beta} I^{\lambda\mu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\mu,}{}_{\alpha\beta}) \\
& \quad - q^2 (\eta_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta}) - \eta^{\mu\nu} q^\lambda q^\sigma (\eta_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta} I_{\alpha\beta,\lambda\sigma}) \Big] \\
& + \Big[2q^\lambda \Big(I^{\sigma\nu,}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\mu + I^{\sigma\mu,}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\nu \\
& \qquad - I^{\sigma\nu,}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k+q)^\mu - I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k+q)^\nu \Big) \\
& \quad + q^2 (I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I_{\alpha\beta,\sigma}{}^\nu I^{\sigma\mu,}{}_{\gamma\delta}) \\
& \quad + \eta^{\mu\nu} q^\lambda q_\sigma (I_{\alpha\beta,\lambda\rho} I^{\rho\sigma,}{}_{\gamma\delta} + I_{\gamma\delta,\lambda\rho} I^{\rho\sigma,}{}_{\alpha\beta}) \Big] \\
& + \Big[(k^2 + (k+q)^2) \Big(I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I^{\sigma\nu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\mu - \frac{1}{2}\eta^{\mu\nu} P_{\alpha\beta,\gamma\delta} \Big) \\
& \quad - ((k+q)^2 \eta_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta} + k^2 \eta_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta}) \Big] \Big\}
\end{aligned}
$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x), \quad \kappa = \sqrt{32\pi G}$$

Graviton diagrams = hard

Graviton Feynman rules, at the level of individual diagrams, are a nightmare:



$$= \kappa\left(\epsilon_q \cdot \epsilon_k \, \epsilon_{q+k} \cdot q + \epsilon_k \cdot \epsilon_{q+k} \, \epsilon_q \cdot k + \epsilon_q \cdot \epsilon_{q+k} \, \epsilon_k \cdot (q+k)\right)^2$$

$$e_k^{\mu\nu} = \epsilon_k^{\mu} \epsilon_k^{\nu}$$

$$\sim \left(\mathcal{A}_3^{YM} / f_{abc}\right)^2$$

The BCJ Double Copy of amplitudes

Any n-point, L-loop, Yang-Mills amplitude can be written as:

Kinematic numerators

$$\mathcal{A}_{n,L} = g^{n-2+2L} \sum_{i \in \Gamma} \int \prod_{j=1}^{L} \frac{d^D \ell}{(2\pi)^D} \frac{n_i c_i}{d_i}$$

Color factors

Propagators

$$c_\alpha \pm c_\beta \pm c_\gamma = 0$$

$$n_\alpha \pm n_\beta \pm n_\gamma = 0$$

The BCJ Double Copy of amplitudes

Kawai, Lewellen & Tye, 1986
Bern, Carrasco & Johansonn 2008

$$\mathcal{M}_{n,L} = \left(\frac{\kappa}{2}\right)^{n-2+2L} \sum_{i\in\Gamma} \int \prod_{j=1}^{L} \frac{d^D \ell}{(2\pi)^D} \frac{n_i \tilde{n}_i}{d_i}$$

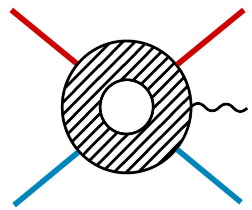
...Is a graviton scattering amplitude!

Waveforms

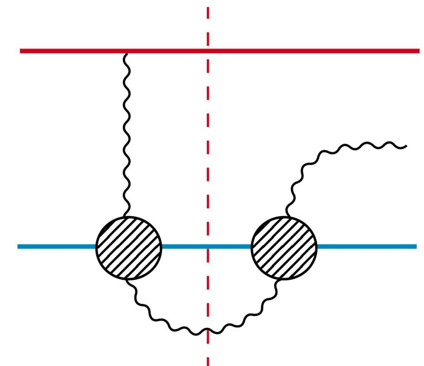
$$\equiv \langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x) S|\psi\rangle$$

$$R_{\mu\nu\rho\sigma}(x) = -\sum_\eta \int \mathrm{d}\Phi(k) \left(k_{[\mu}\varepsilon^\eta_{\nu]} k_{[\rho}\varepsilon^\eta_{\sigma]} \alpha_\eta(k) e^{-ik\cdot x} + \text{h.c.} \right)$$

$$\alpha_\eta(k) = \langle\psi|S^\dagger a_\eta(k) S|\psi\rangle =$$



$+i$



Radiation Kernel
Generated by
geodesic forces

Radiation Kernel
Generated by the black hole's
own field: radiation reaction

Vazquez-Holm, Elkhidir, O'Connell, Sergola, 2023
De Angelis, Travaglini, Brandhuber, Brown, Gowdy, Chen, 2023
Herdeschee, Roiban, Teng, 2023
but also Heissenberg, Russo, Georgoudis, Damour, Geralico,
Bini, Mizera, Hannesdottir, Geralico, Ita, Bohnenblust..

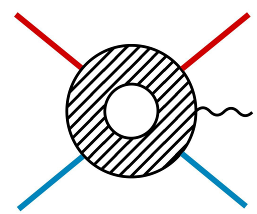
We compute via cuts:
Bern, Dixon, Dunbar, Kosower, 1994

Waveforms

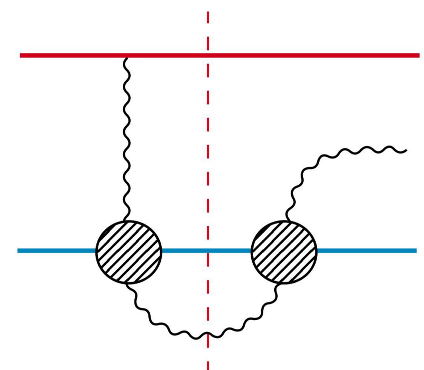
$$\equiv \langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x) S|\psi\rangle$$

$$R_{\mu\nu\rho\sigma}(x) = -\sum_\eta \int \mathrm{d}\Phi(k) \left(k_{[\mu}\varepsilon^\eta_{\nu]} k_{[\rho}\varepsilon^\eta_{\sigma]}\alpha_\eta(k) e^{-ik\cdot x} + \mathrm{h.c.}\right)$$

$$\alpha_\eta(k) = \langle\psi|S^\dagger a_\eta(k) S|\psi\rangle =$$



$+i$



Can't we just find the final state?

Radiation Kernel Generated by geodesic forces

Radiation Kernel Generated by the black hole's own field: radiation reaction

Vazquez-Holm, Elkhidir, O'Connell, Sergola, 2023
De Angelis, Travaglini, Brandhuber, Brown, Gowdy, Chen, 2023
Herdeschee, Roiban, Teng, 2023
but also Heissenberg, Russo, Georgoudis, Damour, Geralico, Bini, Mizera, Hannesdottir, Geralico, Ita, Bohnenblust..

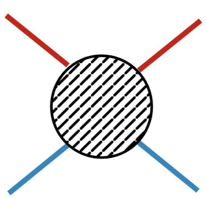
We compute via cuts:
Bern, Dixon, Dunbar, Kosower, 1994

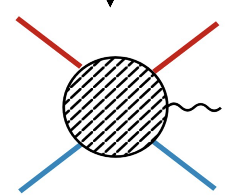
The final state ansatz

$$\langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x)\mathbb{R}_{\alpha\beta\gamma\delta}(y)S|\psi\rangle$$

$$\sim \langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle\langle\psi|S^\dagger \mathbb{R}_{\alpha\beta\gamma\delta}(y)S|\psi\rangle \; = R_{\mu\nu\rho\sigma}(x)R_{\alpha\beta\gamma\delta}(y)$$

$$\Rightarrow S|\psi\rangle = \int d\Phi(p)e^{i\chi(b)}\exp\left[\int d\Phi(k)\alpha_\eta(k)a^\dagger_\eta(k)\right]|p\rangle$$

$i\chi(b) \sim$  $+\cdots$

$\alpha_\eta(k) \sim$  $+\cdots$

Cristofoli, Gonzo, Moynihan, O'Connell, Ross, White, Sergola, 2021
Di Vecchia, Heissenberg, Russo, Veneziano, 2022

Confirmed by Daamgard, Vanhove, Hansen & Planté (3-loop impulse) and Georgoudis, Heissenberg, Holm (1-loop radiation)

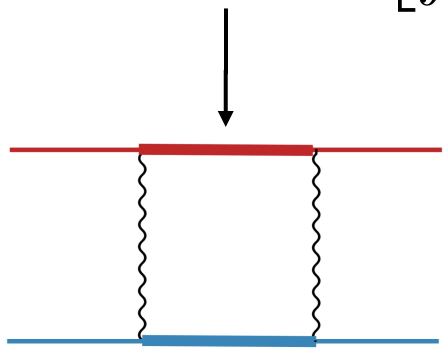
The final state ansatz

$$\langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x)\mathbb{R}_{\alpha\beta\gamma\delta}(y)S|\psi\rangle$$

$$\sim \langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle\langle\psi|S^\dagger \mathbb{R}_{\alpha\beta\gamma\delta}(y)S|\psi\rangle \; = R_{\mu\nu\rho\sigma}(x)R_{\alpha\beta\gamma\delta}(y)$$

$$S|\psi\rangle = \int d\Phi(p)e^{i\chi(b)}\delta(b)\exp\left[\int d\Phi(k)\alpha_\eta(k)a^\dagger_\eta(k)\right]|p\rangle$$

$\delta(b) \sim$



$+\cdots$

Aoude, Cristofoli, Elkhidir, Sergola,
coming soon!
Also see: Aoude & Ochirov, 2023
Jones & Ruf, 2023

First effects appear at $\mathcal{O}(G^7)$
we match to Goldberger & Rothstein, 2006

Conclusions

- The double copy is a very efficient tool useful to investigate fundamental questions about gravity, but its origin is still obscure
- On-shell methods and QFT technology can be powerful tools for computations of classical GW observables
- Next to do: *resummation of the probe limit, analytic continuation, spin effects...*

Thank you!