Probing time dependent scalar fields with extreme mass ratio inspirals

IN COLLABORATION WITH G.ANTONIOU, L.GUALTIERI AND A.MASELLI

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Istituto Nazionale di Fisica Nucleare

IN SUPREMÆ DIGNITATIS · 1343 ·

TEONGRAV, ROMA 2024

Contents

- **Scalar wigs solutions**: time dependent scalar clouds around black holes

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- Probing scalar wigs with **extreme mass ratio inspirals** (EMRIs): a time dependent scalar charge

Theoretical Setup

$$S[\mathbf{g}, \Phi] = S_{EH}[\mathbf{g}] + S_{\Phi}[\mathbf{g}, \Phi]$$

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$$\frac{1}{2}\int \sqrt{-g}\ \mathrm{d}^4x \left(\frac{1}{2} g^{\mu\nu}\partial_\nu \Phi \partial_\mu \Phi^* + g^{\mu\nu}\partial_\nu \Phi^* \partial_\mu \Phi - \frac{1}{2}\mu^2 |\Phi|^2 \right)$$

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minimally coupled **massive** scalar field

Theoretical Setup

Spherical symmetry

Asymptotically flat spacetime

Stationarity

Herdeiro +: Int.J.Mod.Phys.D 24 (2015) 09

No hair theorems

Spherical symmetry

Asymptotically flat spacetime

Stationarity

NO HAIR THEOREMS →

Schwarzschild metric

Constant scalar field

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No hair theorems

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No new physics !!!!!

Herdeiro +: Int.J.Mod.Phys.D 24 (2015) 09

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Spherical symmetry

Asymptotically flat spacetime

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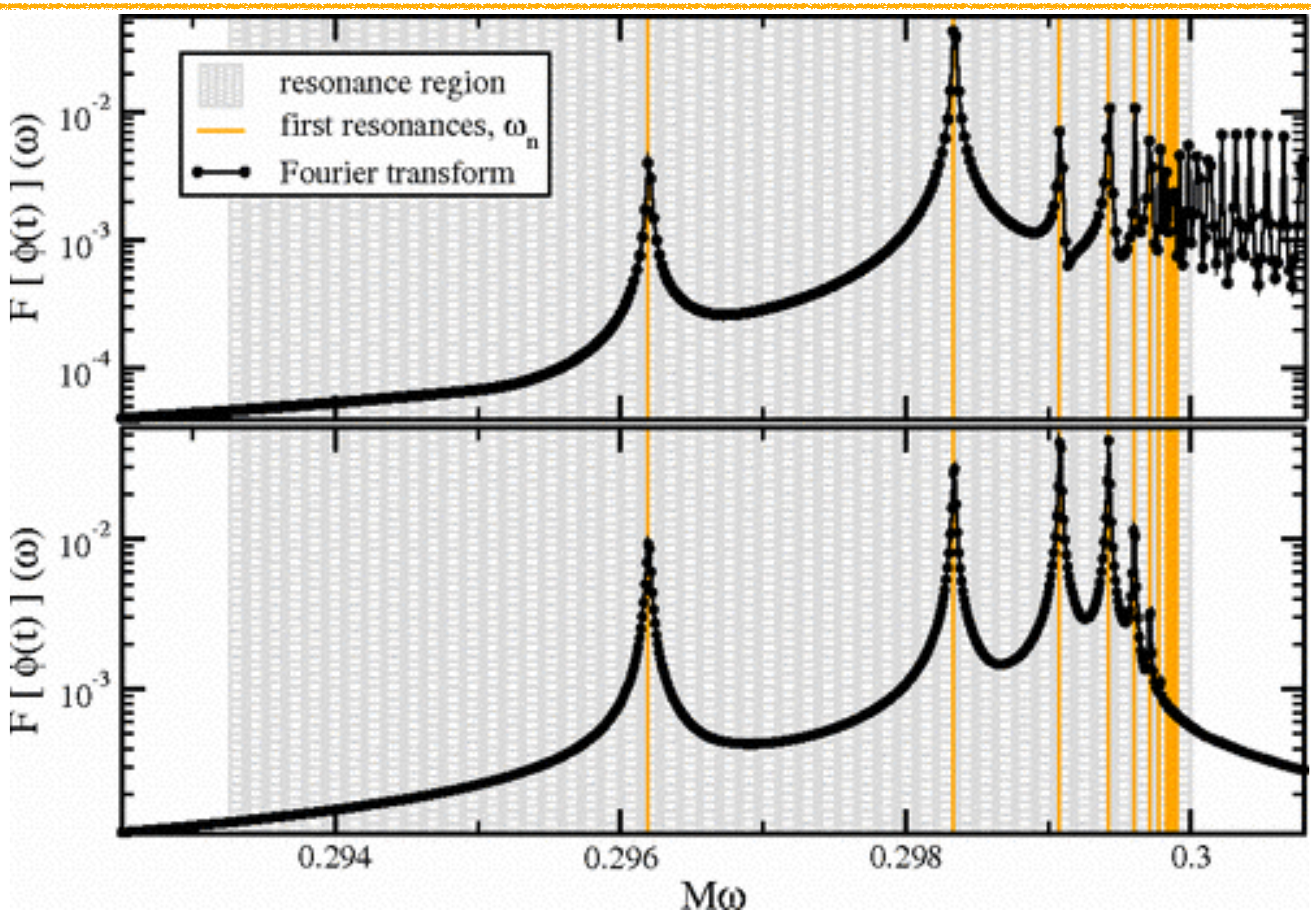
We can avoid the Birkhoff theorem: the metric can be time-dependent as well as the scalar field

Scalar wigs solution: quasi bound states

Given generic initial conditions at the horizon, the solution can be written as a some of quasi bound states at late time



Barranco +: Phys.Rev.Lett.109.081102 (2012)

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$$\psi \sim e^{-i\omega(t+r_*)} \qquad r_* \to r_H$$

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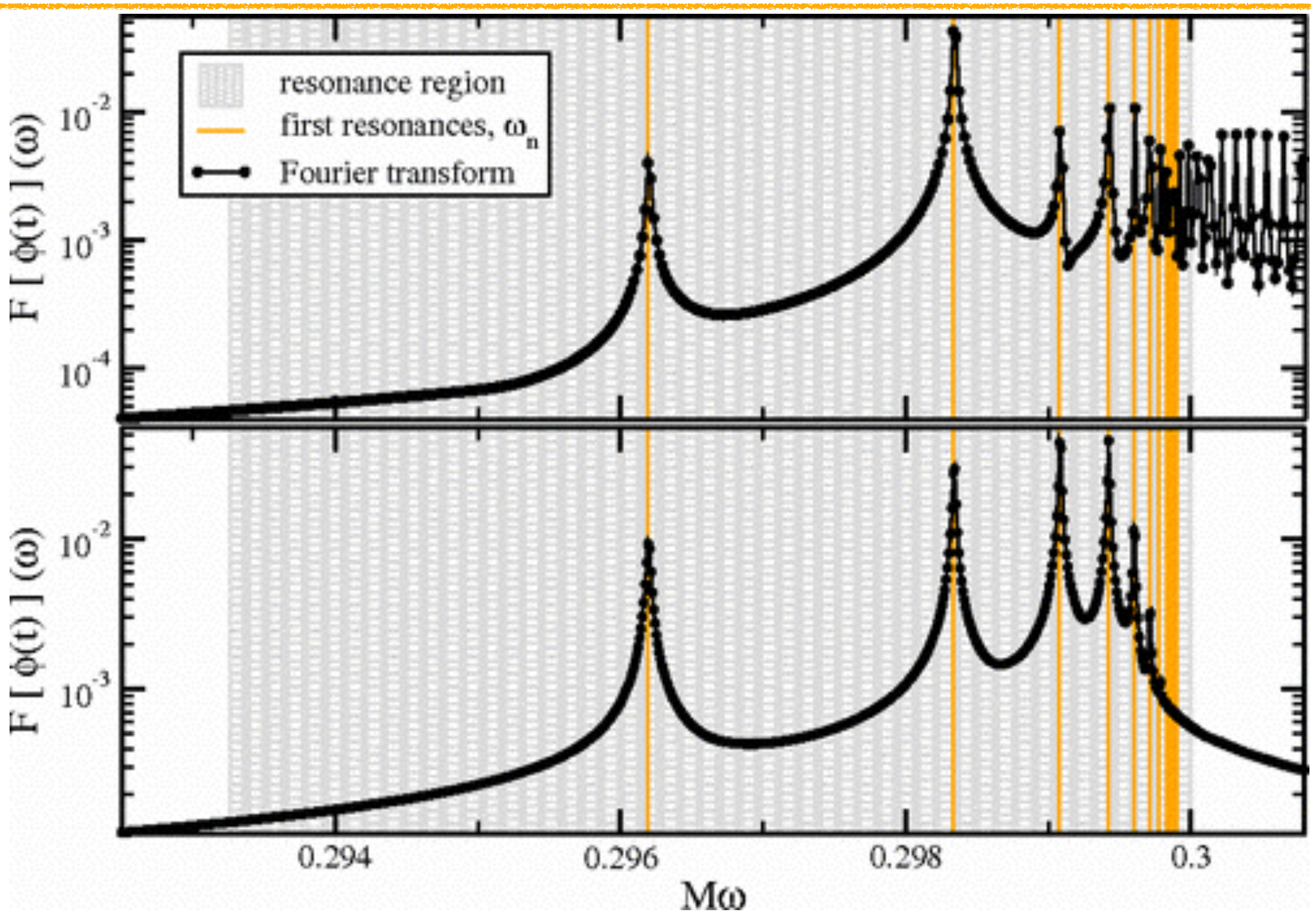

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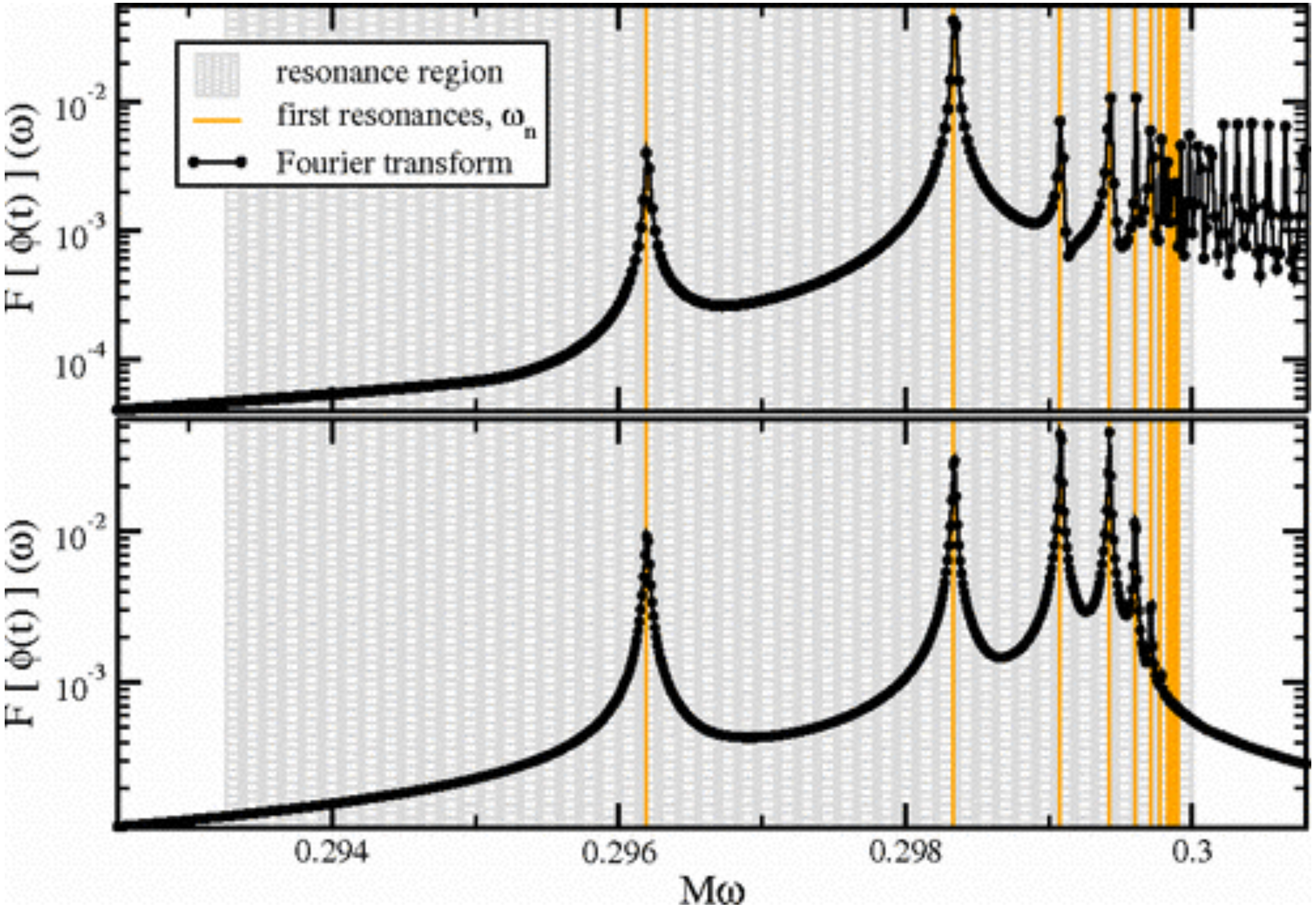
$\psi \propto$ Heun function



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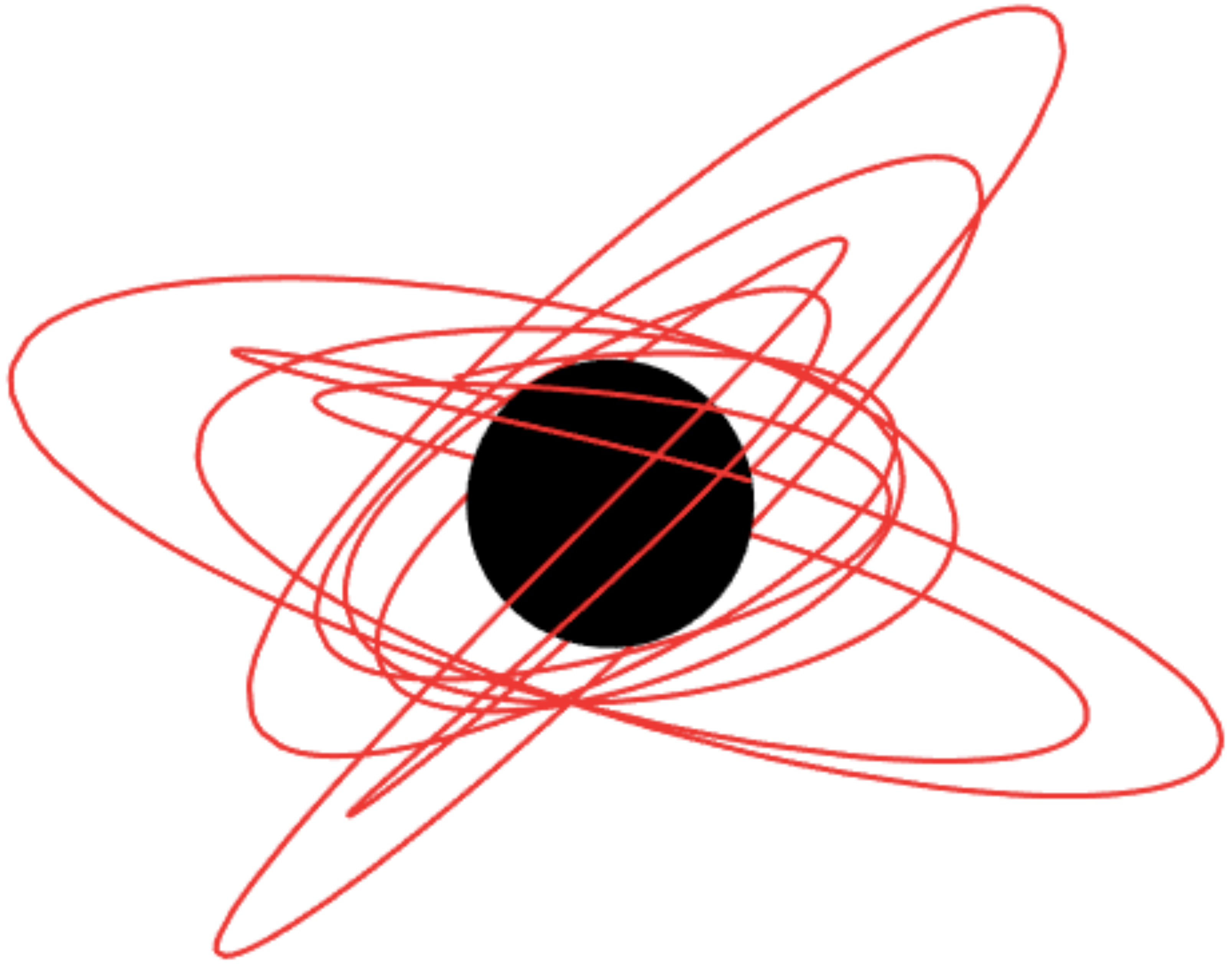
Extreme mass ratio inspirals

- Super Massive Black Holes

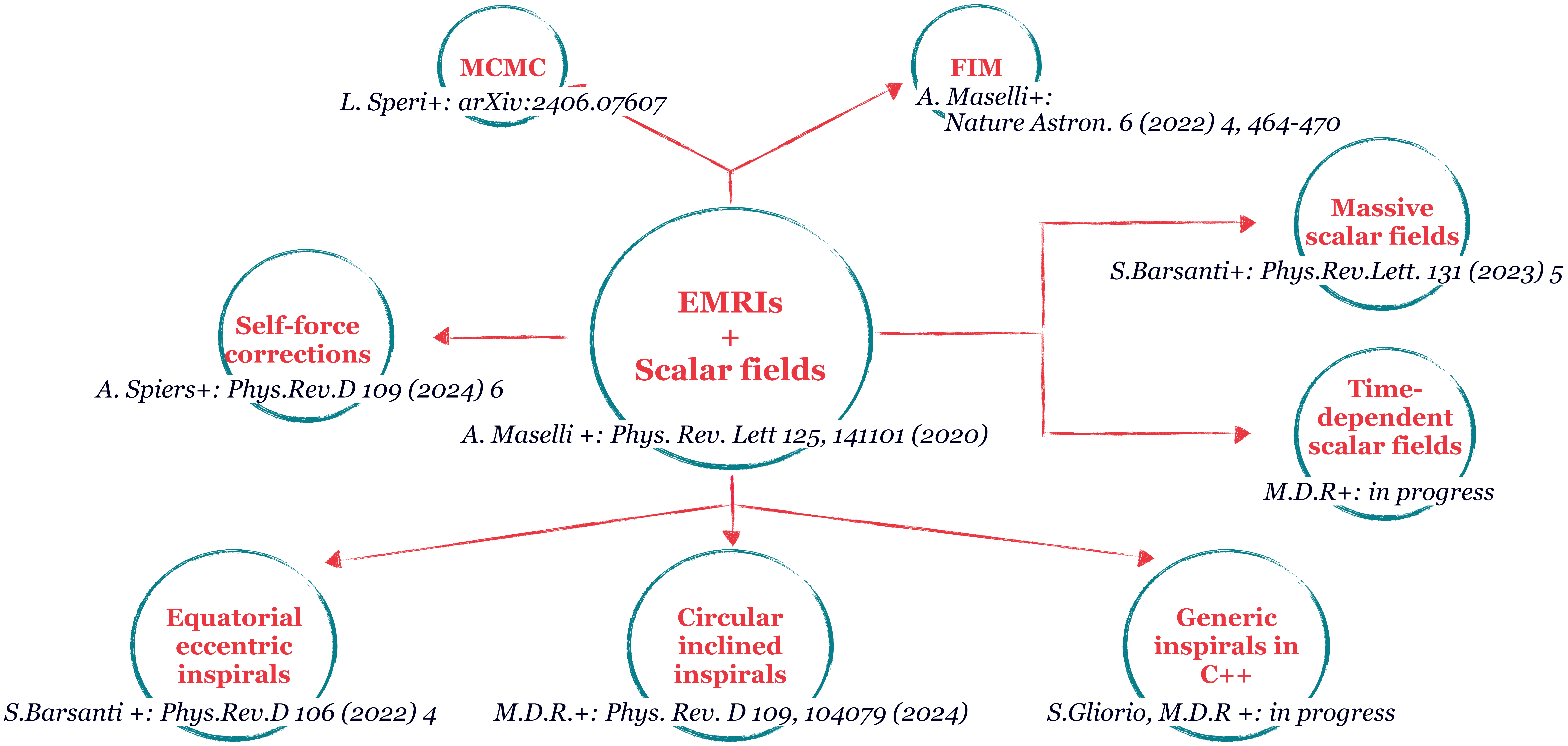
$$M \in (10^4, 10^9)\ M_\odot$$



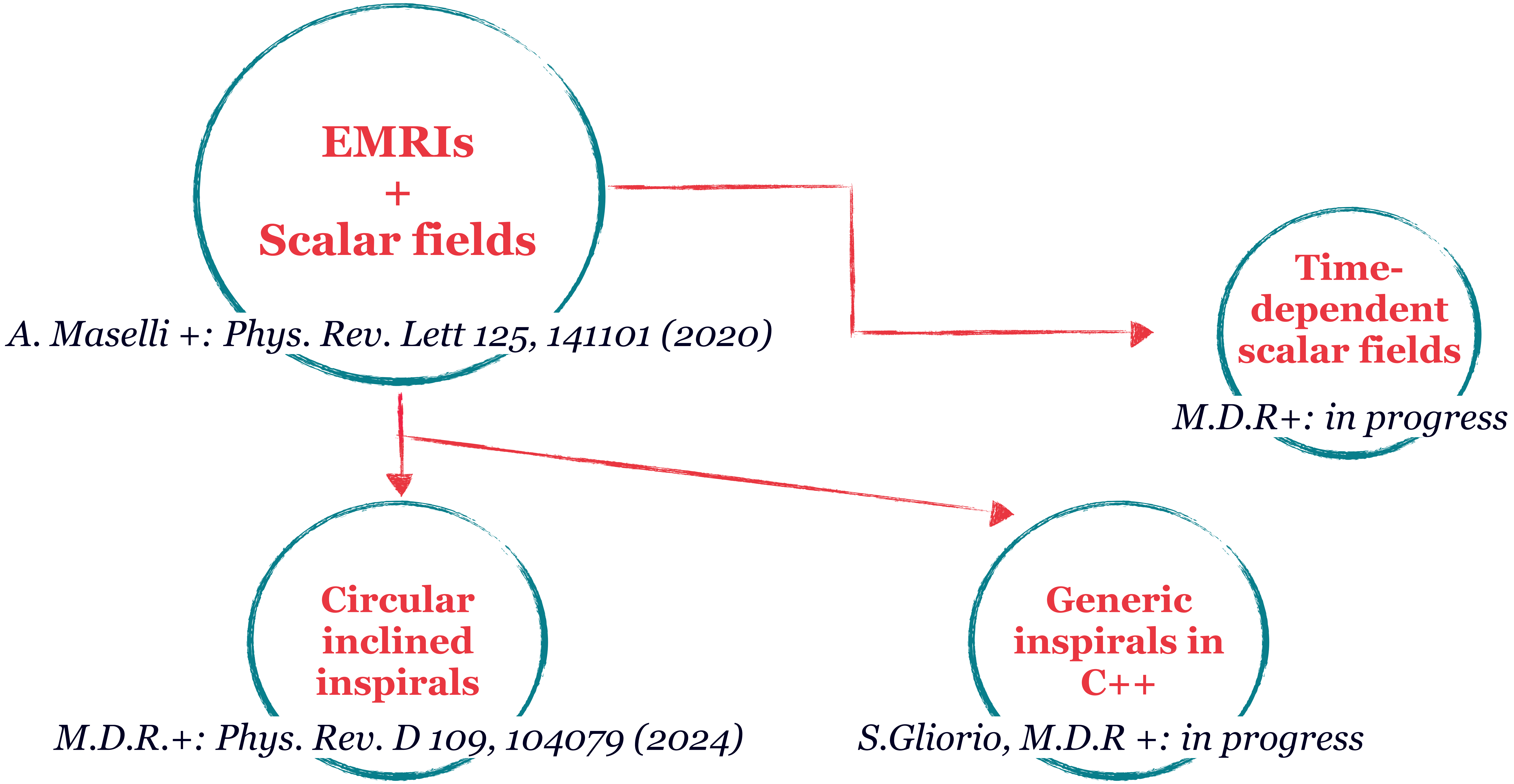
- Stellar Mass Compact Object

$$m_p \in (1, 10)\ M_\odot$$

Mindset


MCMC
L. Speri+: arXiv:2406.07607
FIM
A. Maselli+:
Nature Astron. 6 (2022) 4, 464-470
Massive scalar fields
S.Barsanti+: Phys.Rev.Lett. 131 (2023) 5
Self-force corrections
A. Spiers+: Phys.Rev.D 109 (2024) 6
EMRIs
+
Scalar fields
A. Maselli +: Phys. Rev. Lett 125, 141101 (2020)
Time-dependent scalar fields
M.D.R+: in progress
Equatorial eccentric inspirals
S.Barsanti +: Phys.Rev.D 106 (2022) 4
Circular inclined inspirals
M.D.R.+: Phys. Rev. D 109, 104079 (2024)
Generic inspirals in C++
S.Gliorio, M.D.R +: in progress

Mindset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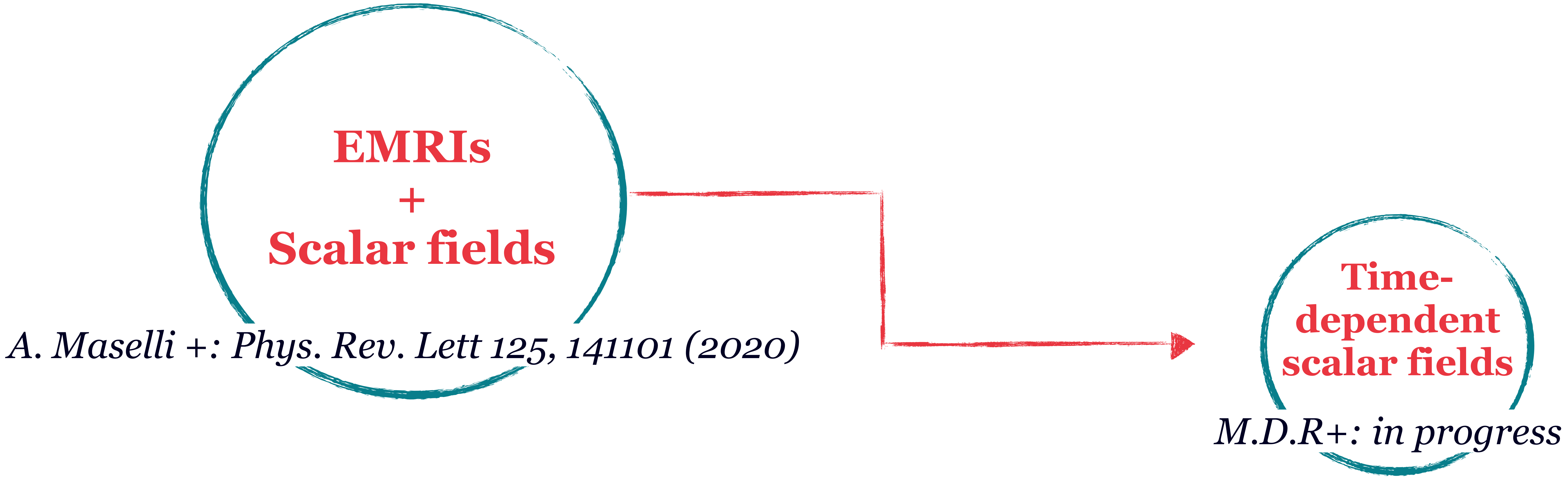
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A. Maselli +: Phys. Rev. Lett 125, 141101 (2020)
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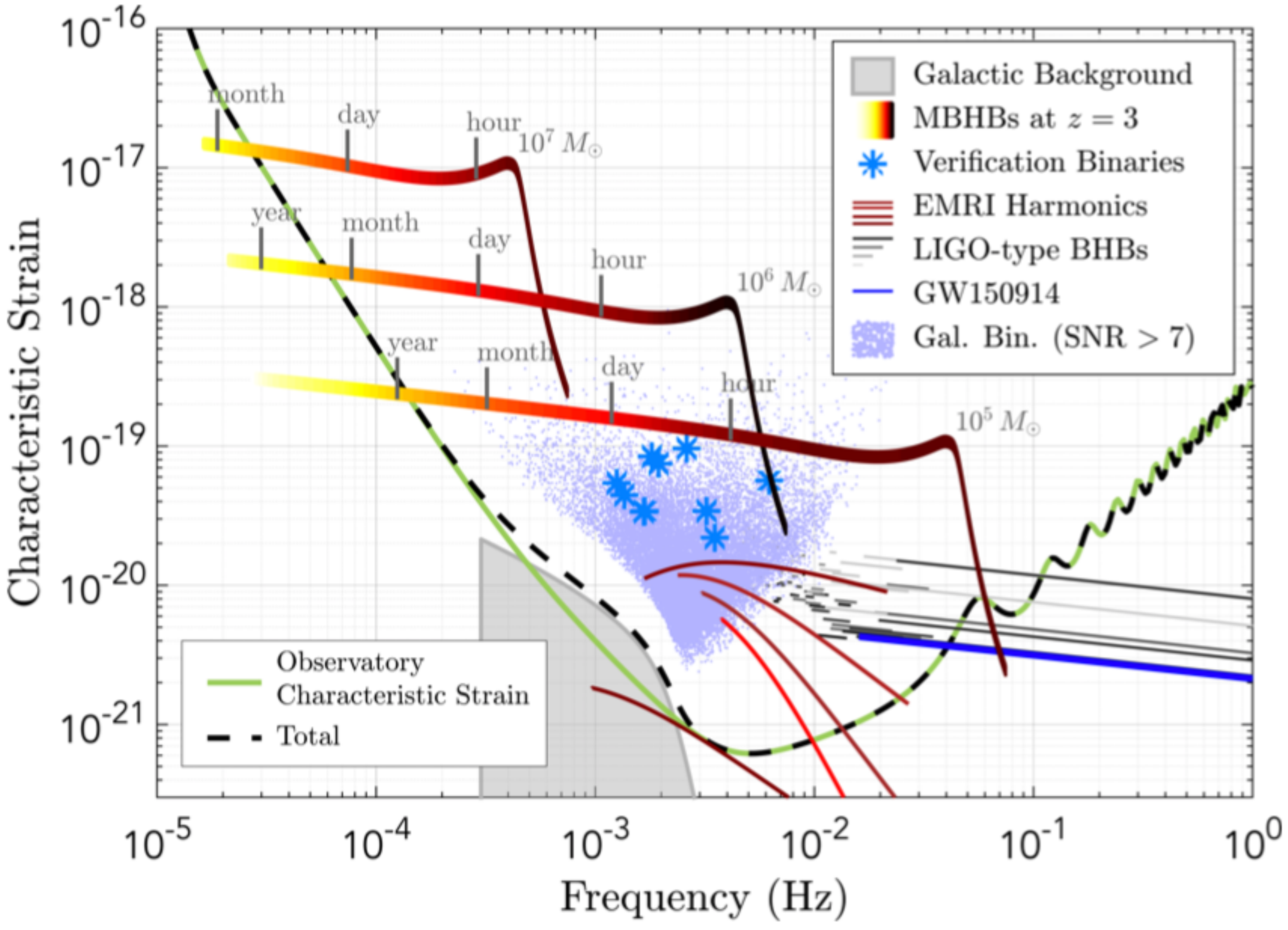
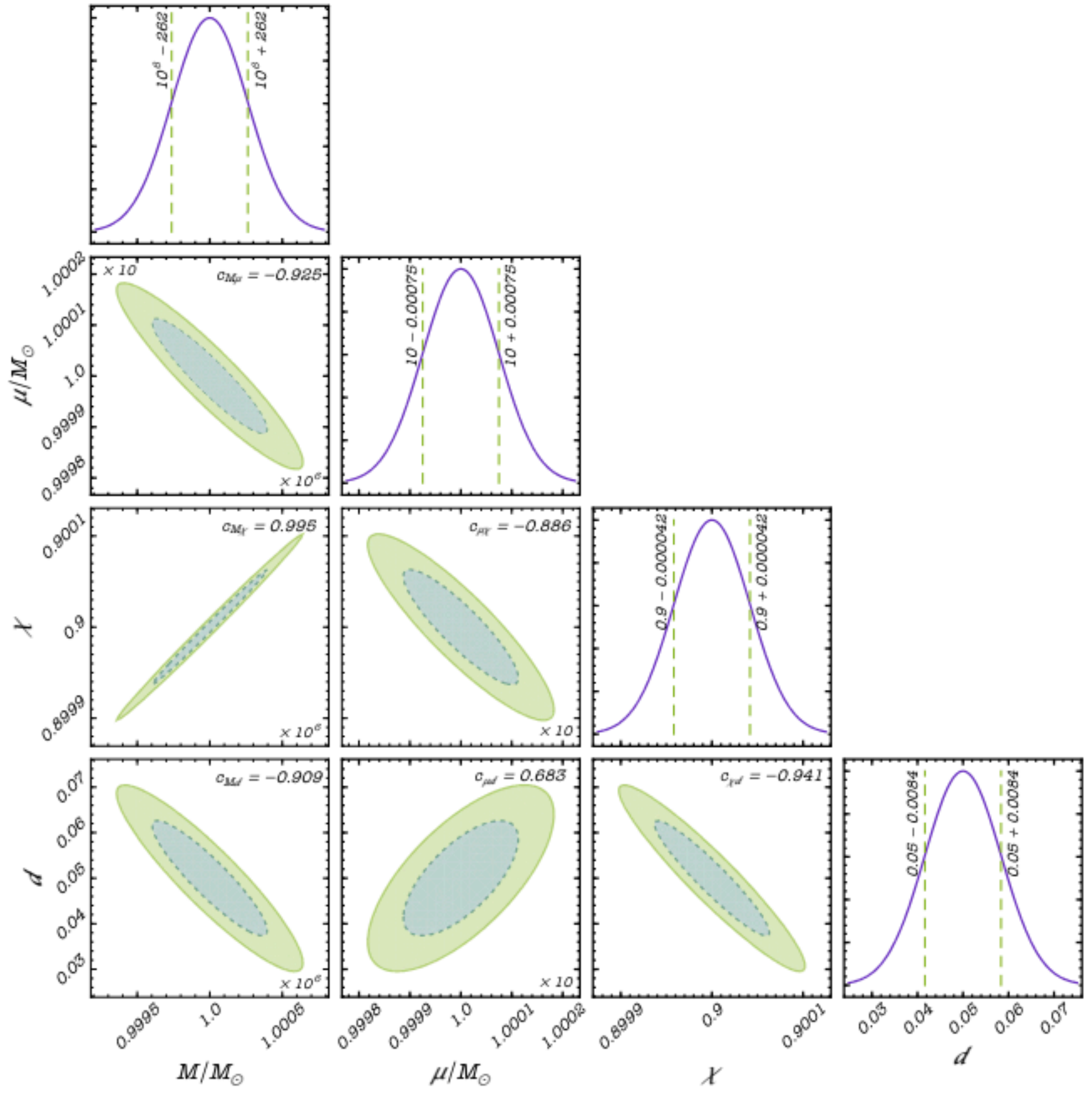
Testing black holes with EMRIs



10^5 Cycles in the Lisa sensitivity band

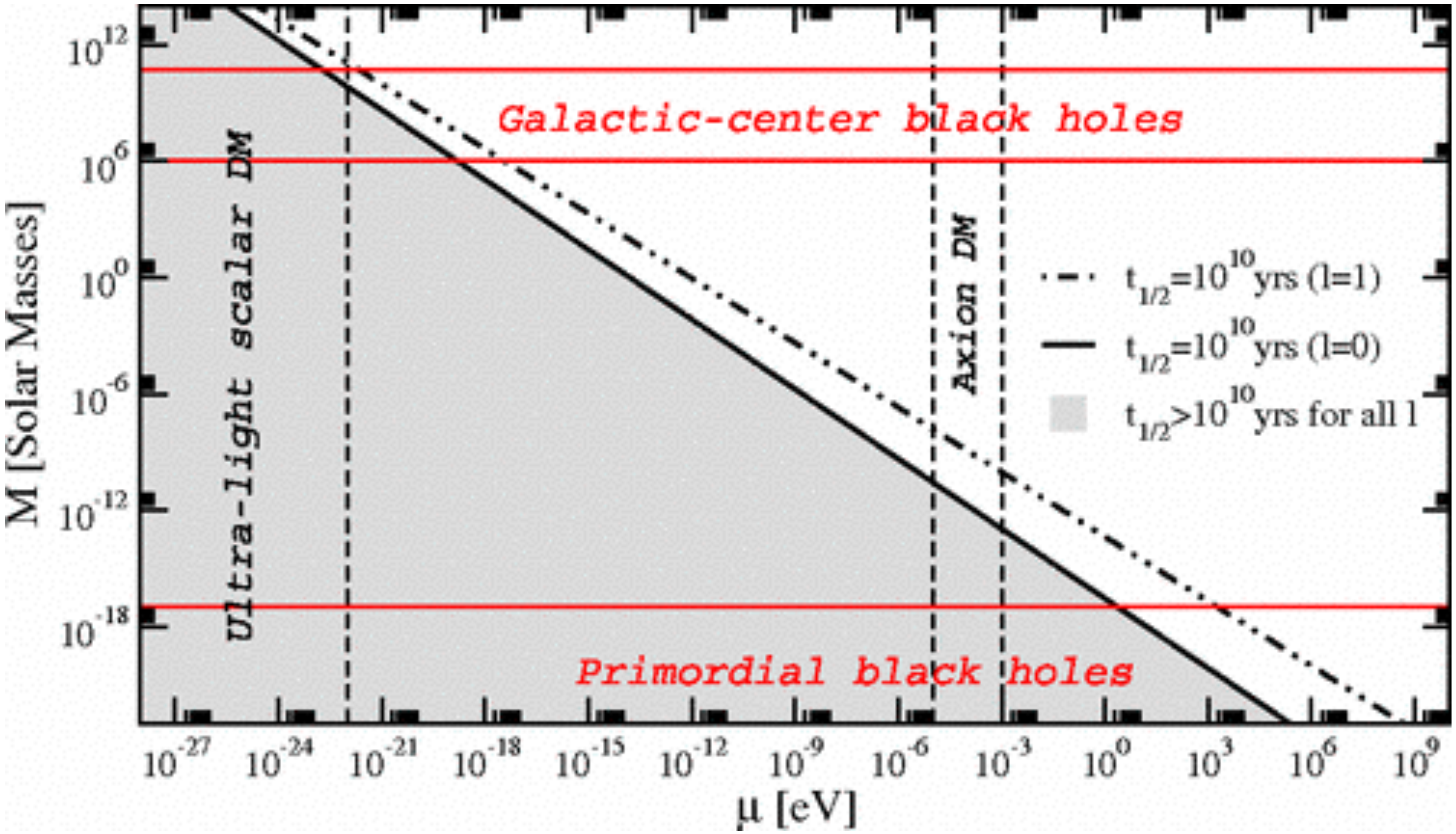
Robson+: Class.Quant.Grav. 36 (2019)



Undetectable gets detectable !

A.Maselli+: Nature Astron. 6 (2022) 4, 464-470

Theoretical framework



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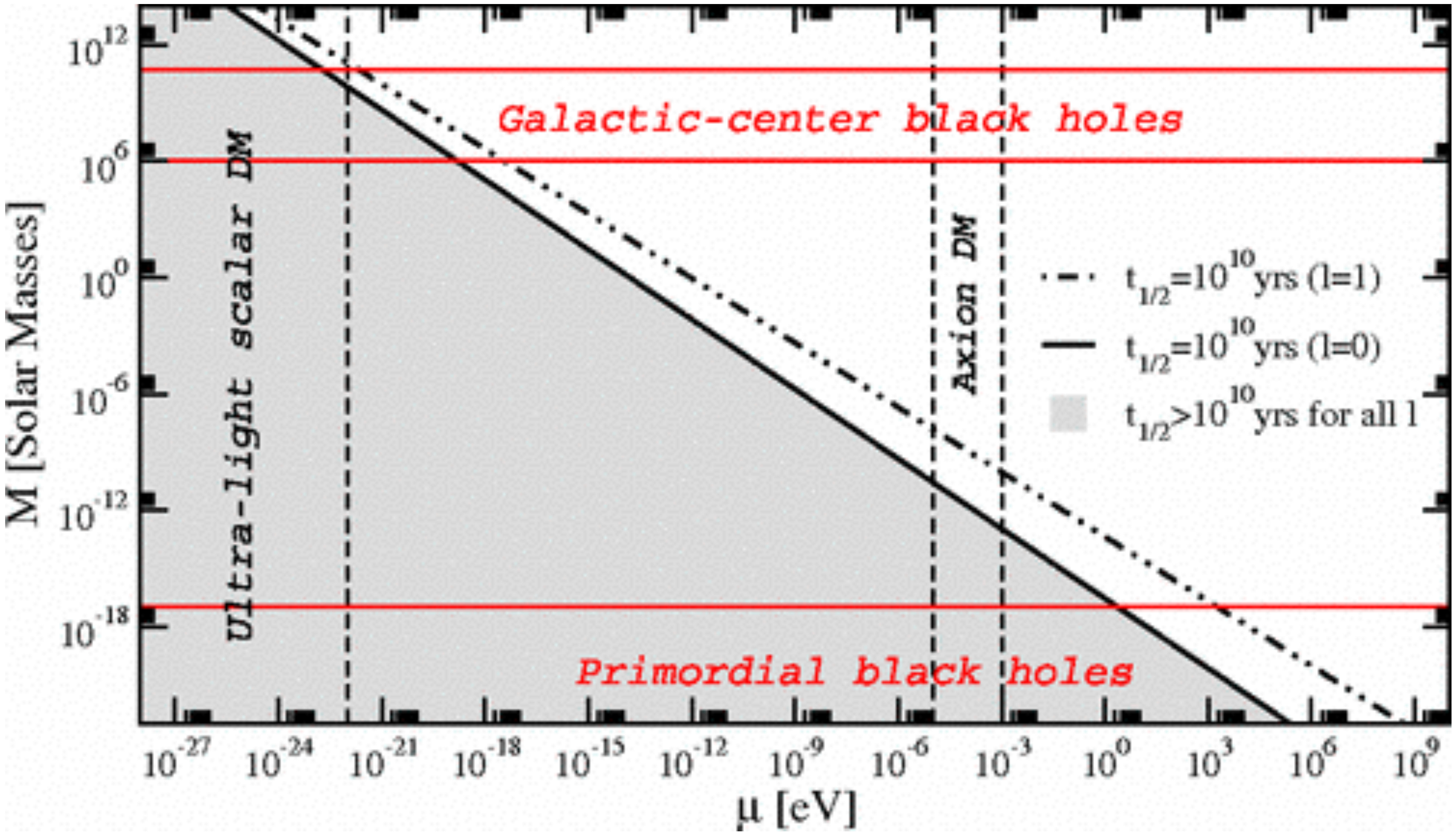

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$$g^0_{\mu\nu} = \text{Kerr Metric}$$

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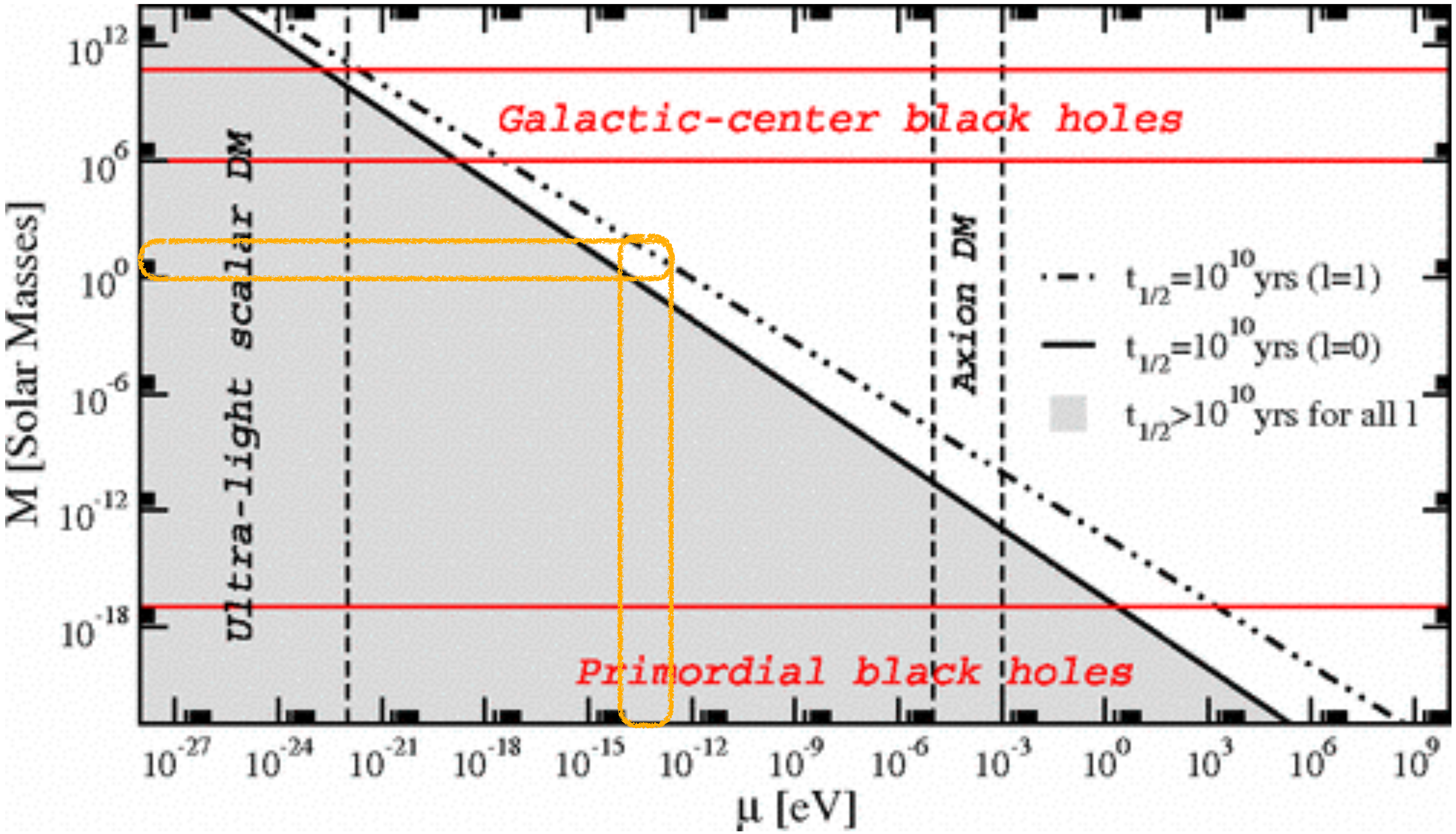

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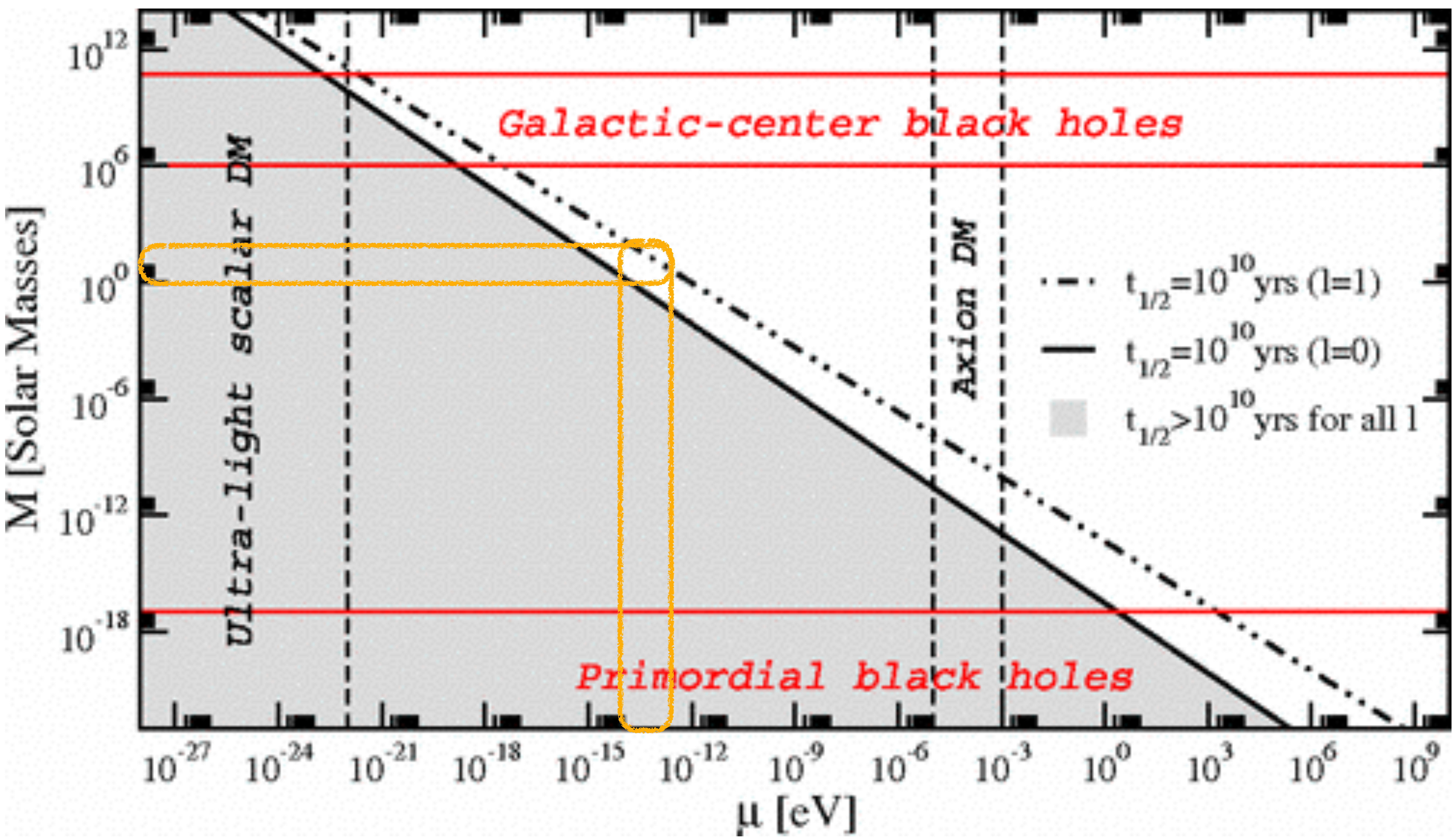
The scalar cloud forms around the stellar mass compact object

Theoretical framework



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Superrandiance is efficient when

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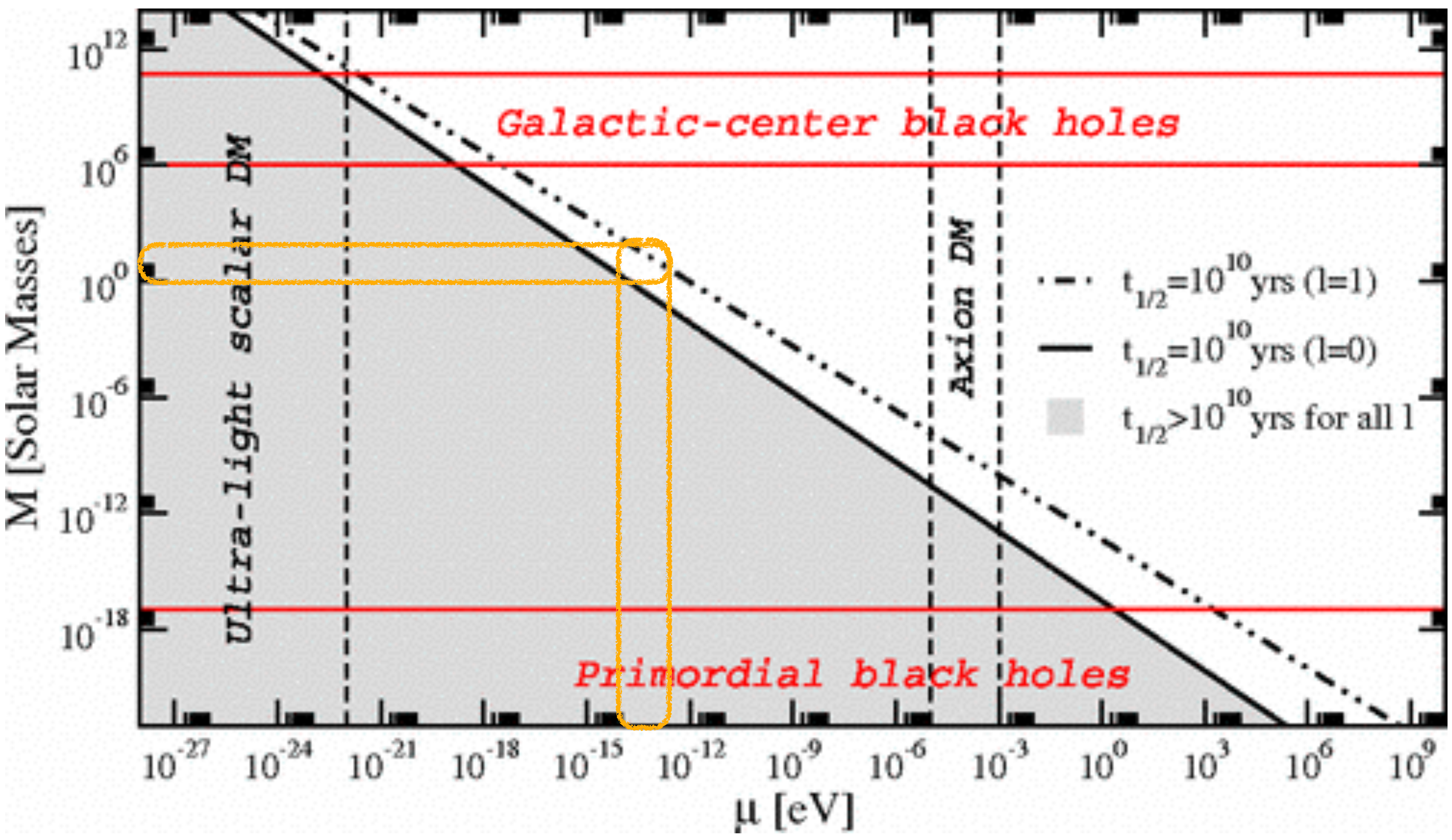

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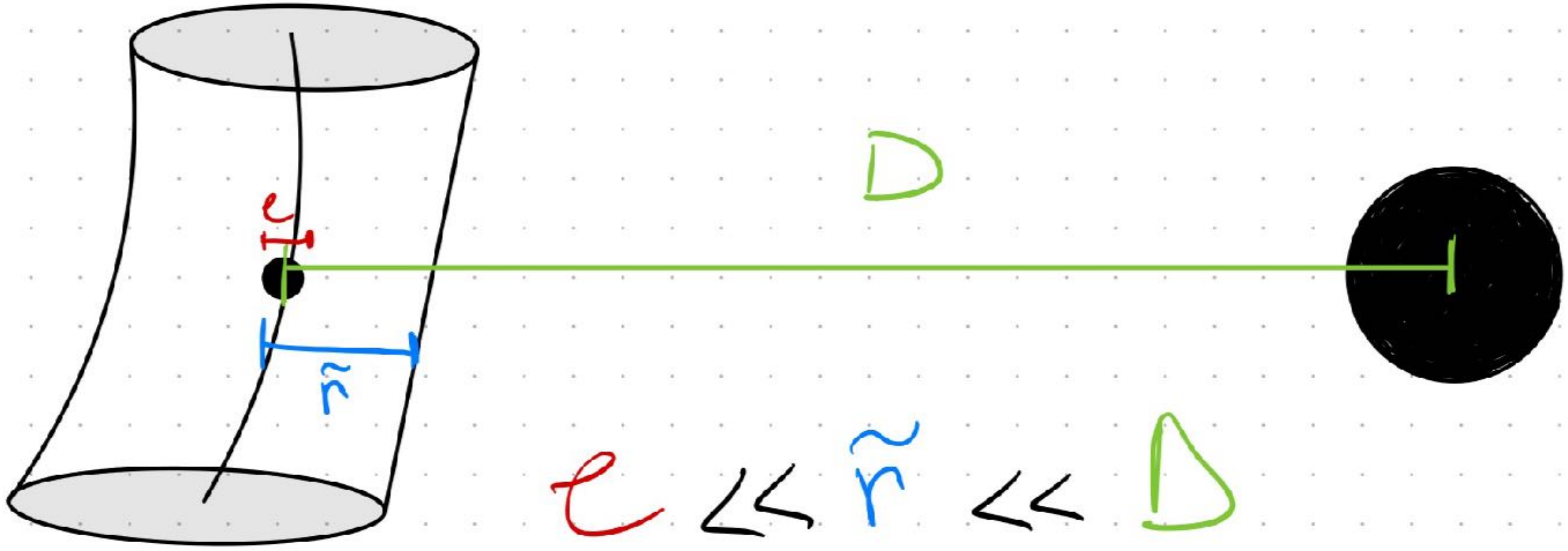
$$M\mu \sim 1$$

$$\mu \sim 10^{-14} \div 10^{-12}\ \mathrm{eV}$$

Scalar charge definition

T Damour and G Esposito-Farese
1992 Class. Quantum Grav. **9** 2093



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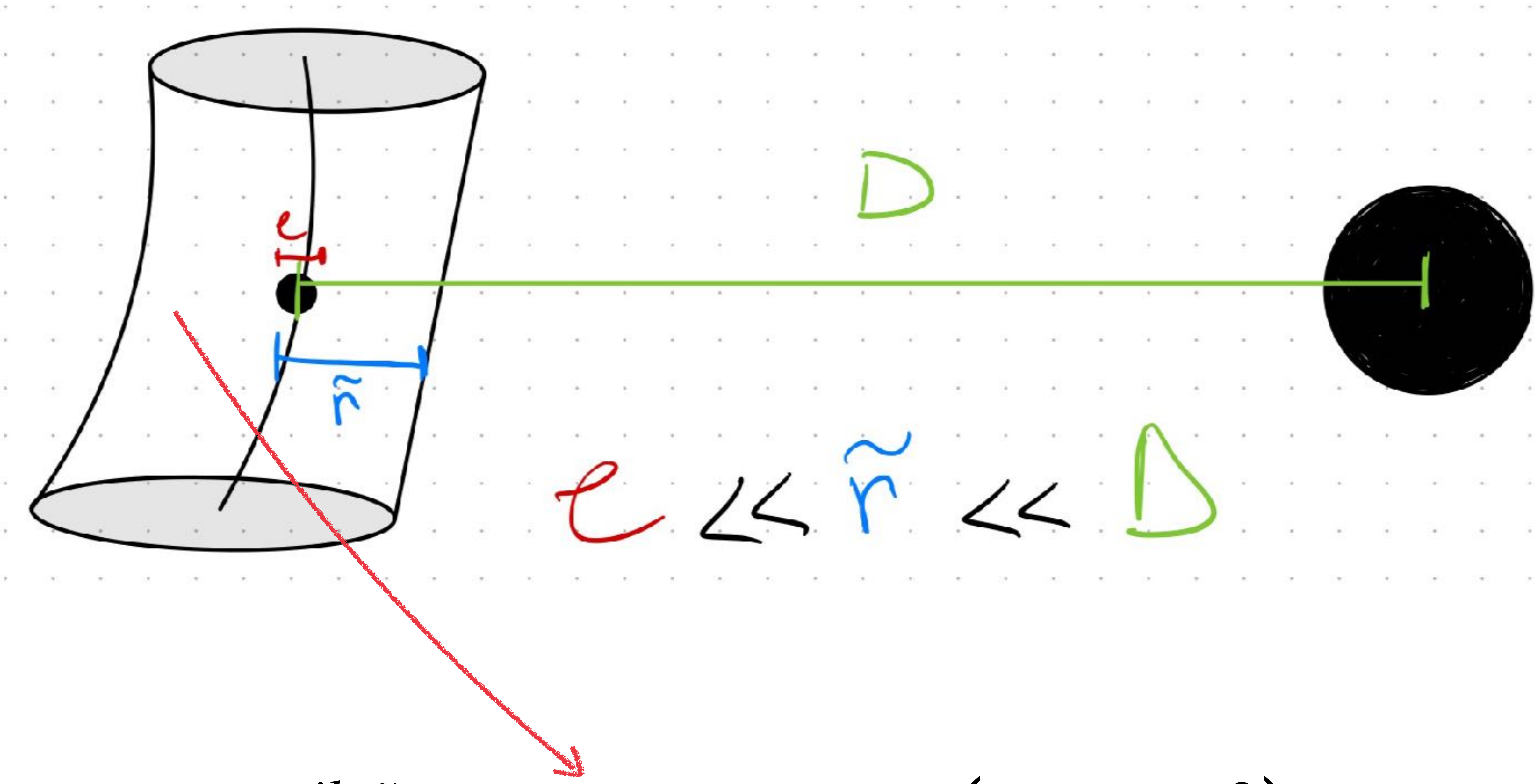

$$\varphi(\tilde{r}) = \varphi_0 + \frac{e^{ik_0\tilde{r}} m_p}{\tilde{r}} d_0 e^{-i\omega_0 \tilde{t}} + \mathcal{O}\left(\left(\frac{m_p}{\tilde{r}}\right)^2\right) + \mathcal{O}\left(\frac{\tilde{r}}{D}\right)$$

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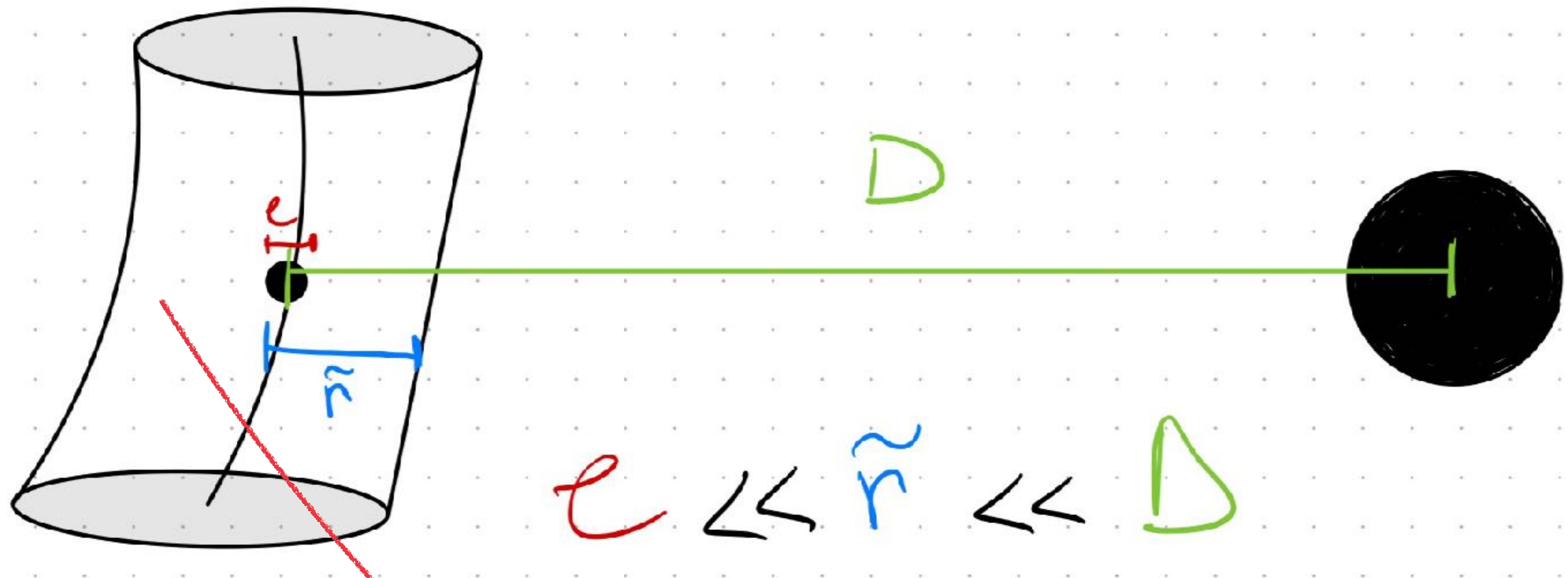

$$k_0 = \sqrt{\omega_0^2 - \mu^2}$$

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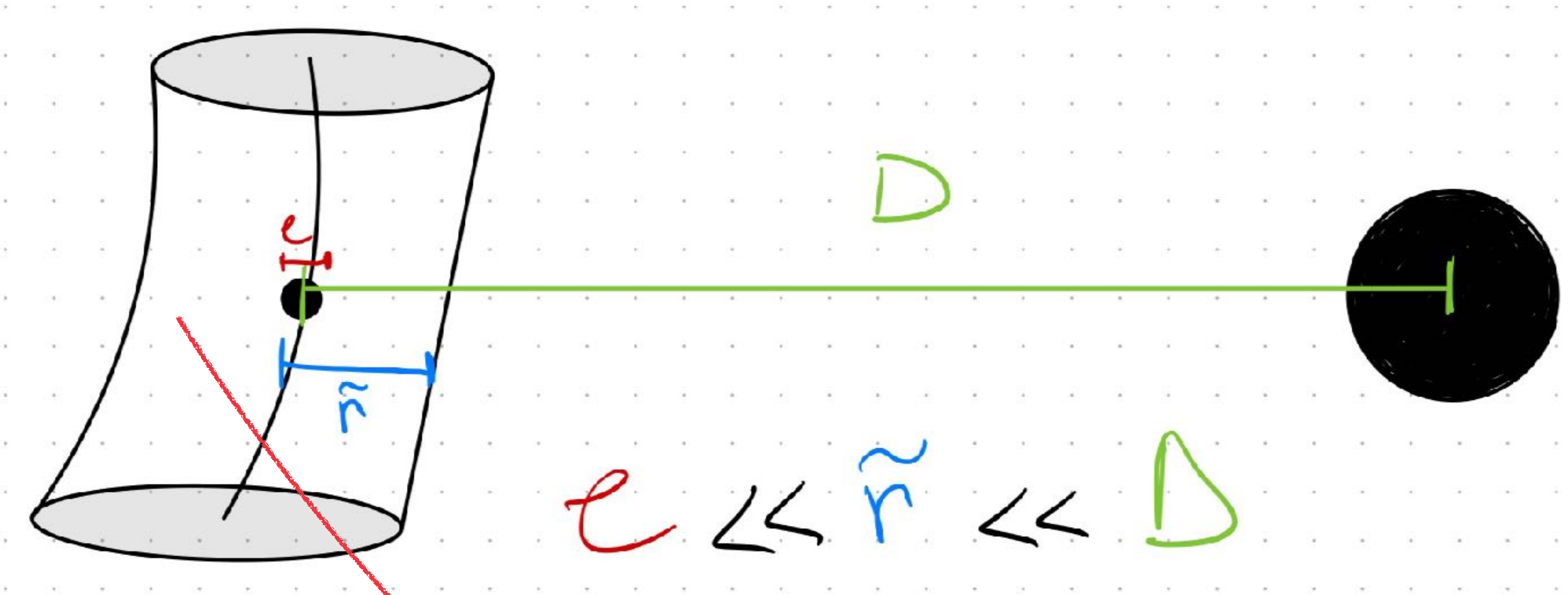

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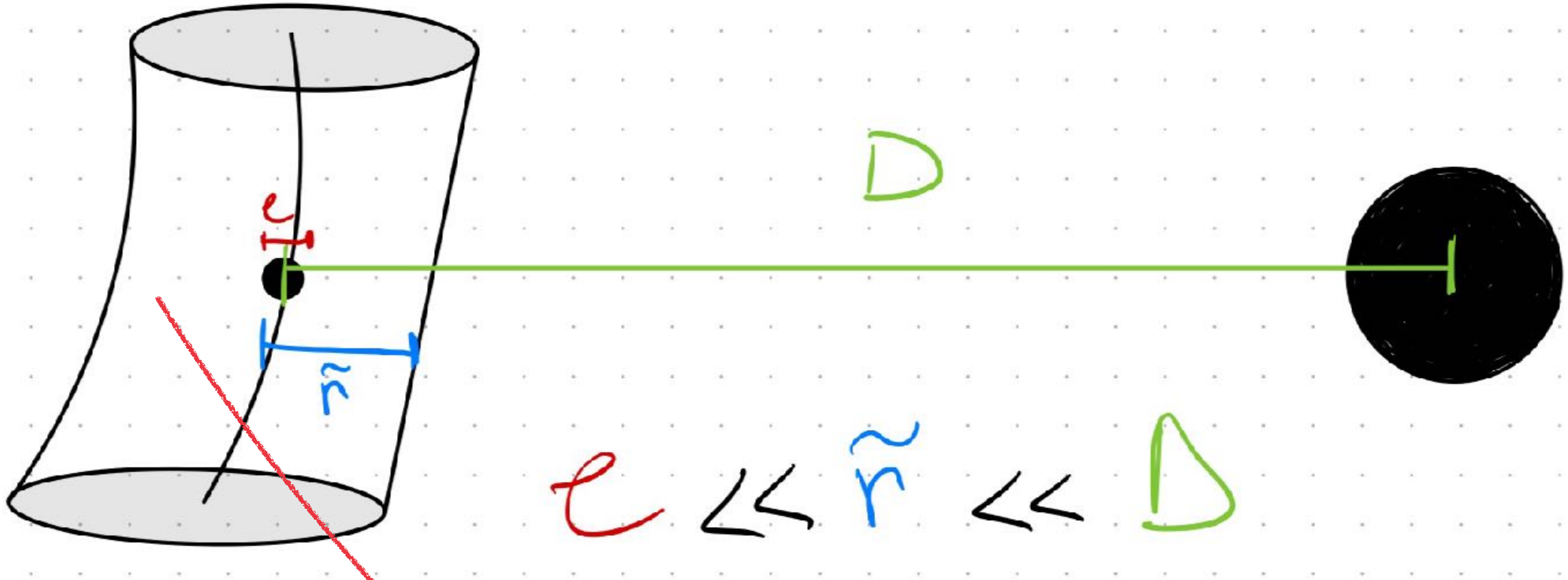

Stationary SC

S.Barsanti +: Phys.Rev.Lett. 131 (2023) 5

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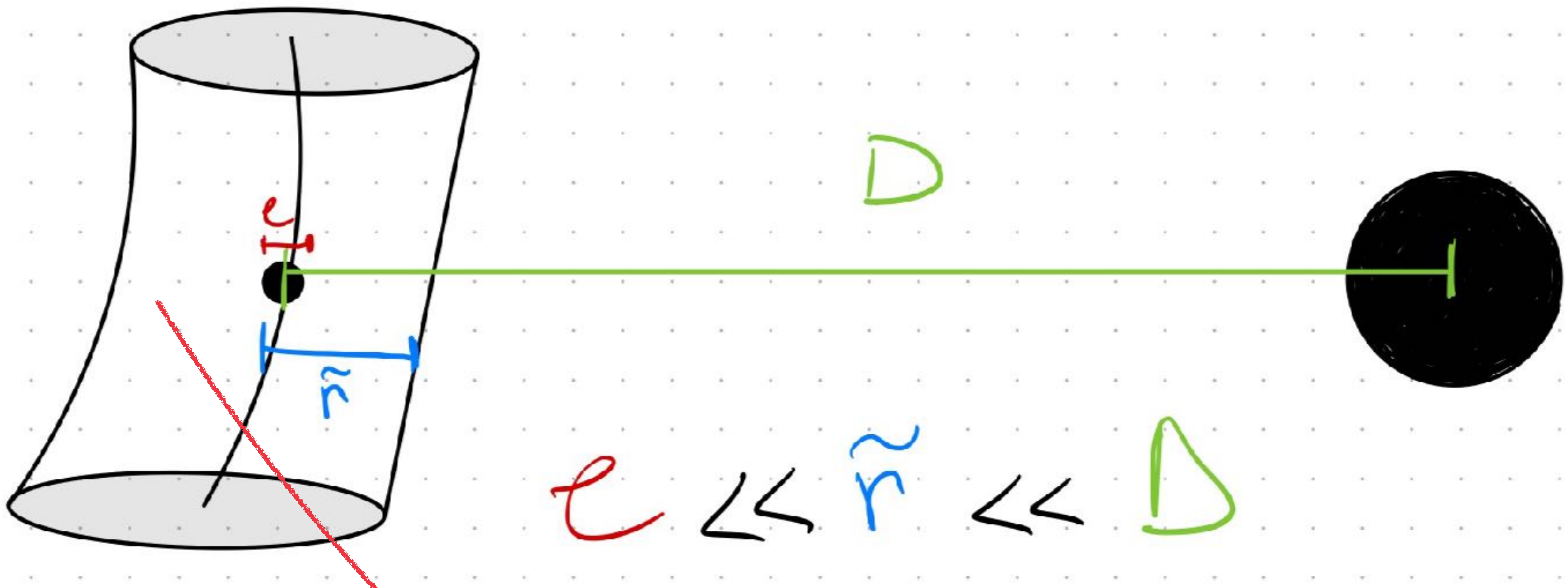

Massless SC

L.Speri+: arXiv: 2406.07607   M.D.R. +: Phys.Rev.D 109 (2024)

$$\varphi(\tilde r) = \varphi_0 + \frac{e^{-\mu_0\tilde r} m_p}{\tilde r} d_0 e^{-i\omega_0 \tilde t} + \mathcal{O}\left(\left(\frac{m_p}{\tilde r}\right)^2\right) + \mathcal{O}\left(\frac{\tilde r}{D}\right)$$

Time scale separation

J. Barranco +: Phys. Rev. D 84, 083008 (2011)

S. Detweiler: Phys. Rev. D 22, 2323 (1980)

Four typical time-scales

- **Oscillation period** of the scalar field

$$\Upsilon = 2\pi\gamma/\omega_0^R \simeq 2\pi/\mu_s$$

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- **Orbital period** $$P \sim 10M$$
- **Plunge time** $$t_{\rm plunge} \sim M^2/m_p$$

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$$\Upsilon \ll \tau$$

Scalar radiation is dominated by the **oscillation frequency** of the scalar field

$$\Upsilon \ll P$$

Phenomenology of time - dependent scalar charge

$\tau \gg P$

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- The scalar cloud **lasts** more than an orbital period
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$$\tau \ll P$$

- The scalar cloud **fades away** in an orbital period
- Scalar radiation may appear as a **glitch** in data stream

Phenomenology of time - dependent scalar charge

$$\tau \gg P \qquad\qquad \tau \ll P$$

Adiabatic regime:

$$\dot{C} = \dot{C}_{tens} + d^2\dot{C}_{scal}$$

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Adiabatic regime:

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Adiabatic regime:

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d can be assumed constant if $\tau \gg t_{\rm plunge}$

Phenomenology of time - dependent scalar charge

$\tau \gg P$

$\tau \ll P$

$$d(\omega) = \frac{1}{2\pi} \frac{d_0 \gamma \tau}{1 - i(\omega - \omega_0^R - m\Omega_\phi)\gamma\tau}$$

The scalar charge has a non trivial Fourier transform

A new source of dissipation

NON-ROTATING BH AT THE MOMENT

$$\dot{C}_{GW} = \sum_{i=+,-} [\dot{C}^{(i)}_{\rm grav} + \dot{C}^{(i)}_{\rm scal}] = \dot{C}_{grav} + \dot{C}_{scal} = -\dot{C}_{orb}$$

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$\omega \sim \Re[\omega_0]\ ,\ \omega < \mu$

Vanishing at infinity

S.Barsanti+: Phys.Rev.Lett. 131 (2023) 5

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S.Barsanti+: Phys.Rev.Lett. 131 (2023) 5

$$\frac{\mathrm{d}E_-}{\mathrm{d}t} = \frac{\hat{d}_0^2 m_p^2 \pi}{2} \frac{r_0 - 2M}{r_0 - 3M} \sum_{lm} \bar{\omega}_m^2 \frac{|R^+_{lm\bar{\omega}_m}(r_0)|^2}{|W_Y|^2} Y^2_{lm}(\pi/2)$$

$$\bar{\omega}_m = \omega_0^R + m\Omega_\phi$$

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Results and open problems

- ☑ We generalised the matching procedure to time dependent scalar fields
- ☑ We exploited time-scale separation to get formulae for the scalar flux

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- ☑ We generalised the matching procedure to time dependent scalar fields
- ☑ We exploited time-scale separation to get close formulae for the scalar flux
- ☐ We need to evaluate radial Regge-Wheeler function for values of $\omega \gg 1/M$
- ☐ How much is scalar flux different from those in the literature for stationary-massive scale?
- ☐ Generalise this formalism to rotating central black holes

Thank you for the attention

Theoretical Setup

$$S[\mathbf{g}, \Phi] = S_{EH}[\mathbf{g}] + S_{\Phi}[\mathbf{g}, \Phi]$$

Barranco +: PhysRevD.96.024049 (2017)

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spherically symmetric solution

$$\mathrm{d}s^2 = \alpha(t,r)^2\mathrm{d}t^2 + \gamma(t,r)^2\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \qquad \Phi = \Phi(t,r)$$

$$m(t,r) = \frac{r}{2}\left(1 - \frac{1}{\gamma^2}\right)$$

Barranco +: PhysRevD.96.024049 (2017)

Is there a variation of the mass?

Bondi: Nature 186, 535 (1960)
Mädler+:Scholarpedia 11 (2016) 33528

$$\mathrm{d}s^2 = -\frac{V}{r}e^{2\beta}\mathrm{d}u^2 - 2e^{2\beta}\mathrm{d}u\mathrm{d}r + r^2 h_{AB}\left(\mathrm{d}x^A - U^A\mathrm{d}u\right)\left(\mathrm{d}x^B - U^B\mathrm{d}u\right)$$

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Bondi News function

Is there a variation of the mass?

Bondi: Nature 186, 535 (1960)
Mädler+:Scholarpedia 11 (2016) 33528

$$\mathrm{d}s^2 = -\frac{V}{r}e^{2\beta}\mathrm{d}u^2 - 2e^{2\beta}\mathrm{d}u\mathrm{d}r + r^2 h_{AB}\left(\mathrm{d}x^A - U^A\mathrm{d}u\right)\left(\mathrm{d}x^B - U^B\mathrm{d}u\right)$$

$$\frac{\mathrm{d}m(u)}{\mathrm{d}u} = -\frac{1}{4\pi}\int \mathrm{d}\theta\mathrm{d}\phi\ \sin\theta\,|N|^2$$

Bondi News function

$$N \propto q^A q^B \partial_u c_{AB}$$

dyad element

$$c_{AB} = \lim_{r\to\infty} r(h_{AB} - q_{AB})$$

Flat space time

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Flat space time

No energy flux

scalar wigs go to zero exponentially at infinity ⇒ No variation of the mass