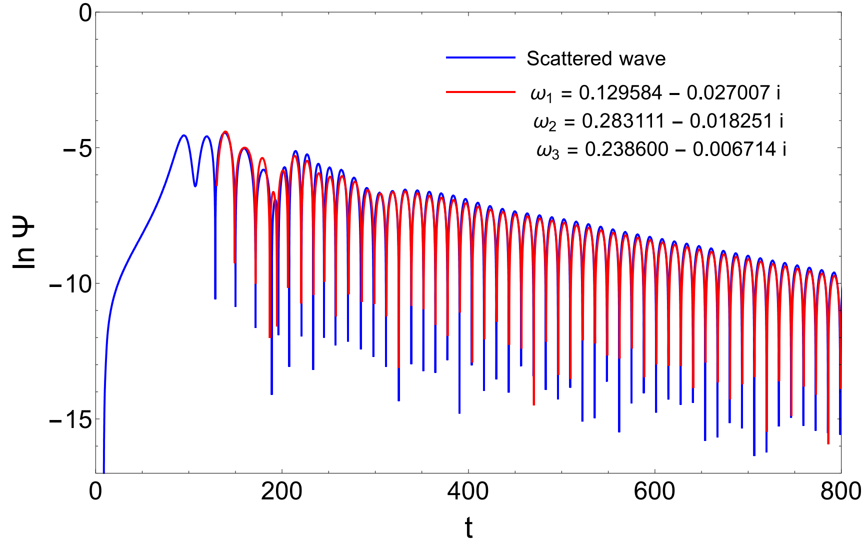
1^{st} Teongrav international workshop on theory of gravitational waves – September 17^{th}, 2024 – Rome

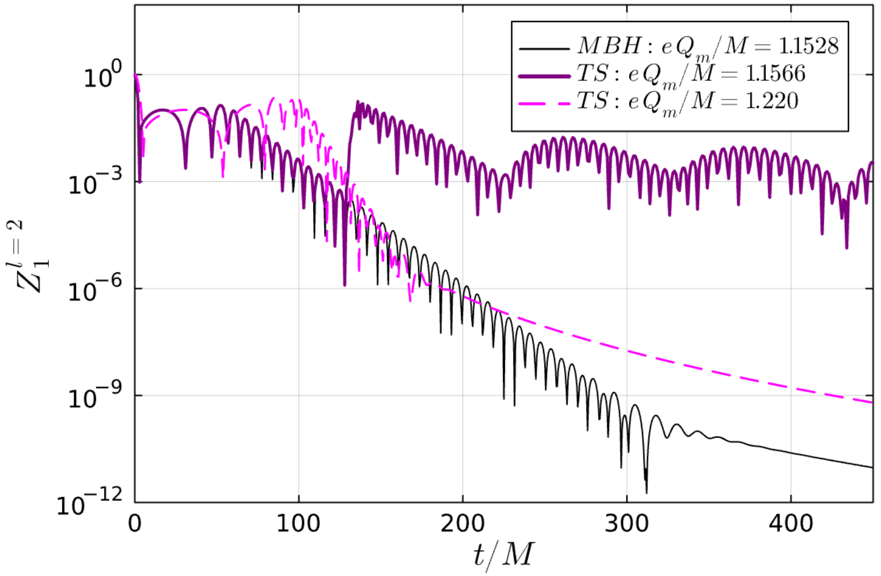
Spectroscopy in Einstein-Maxwell-scalar theories

based on:



Work in progress with
F. Corelli, **R. Croft** and **P. Pani**
arXiv:24XXxxxx

and:



arXiv:2406.19327
with **A. Dima** and **P. Pani**

Marco Melis
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SAPIENZA UNIVERSITÀ DI ROMA

INFN Istituto Nazionale di Fisica Nucleare

Motivations

- ***Fuzzball program of string theory***: classical BH horizon as a coarse-grained description of a superposition of regular quantum states:
  - Some microstates involve **smooth horizonless geometries**:
    gravity coupled with extra degrees of freedom (**gauge** and **scalar fields**).
  - ***Higher dimensions*** and ***non-trivial topologies***
  - These solutions are quite involved, and the **study of their dynamics is a hard task**!
- Study **toy theories** and **toy models** (gravity + extra fields) whose solutions share key aspects with the fuzzball paradigm:
  - **Einstein – Maxwell – scalar theory**: at least some models admits **scalarized BH solutions** (BHs with ***scalar hairs***)
  - **Einstein – Maxwell in 5D** → (4D reduction) **Einstein – Maxwell – scalar in 4D** that admits **magnetically charged BH** and regular solitons (**topological stars TS**)

Task: study *stability* and *spectroscopy* of these solutions.

Einstein – Maxwell – scalar theory

Consider the following action for the EMS theory

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\,[R - 2\partial_\mu\phi\partial^\mu\phi - F[\phi]F_{\mu\nu}F^{\mu\nu}]$$

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$$\Box\phi - \frac{1}{4}\frac{\delta F[\phi]}{\delta\phi}F_{\mu\nu}F^{\mu\nu} = 0$$

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The **linear scalar field equation** for a small $\delta\phi$ perturbation is

$$(\Box - \mu^2_{\mathit{eff}})\delta\phi = 0\ , \qquad \mu^2_{\mathit{eff}} = \frac{F_{\mu\nu}F^{\mu\nu}}{4}\frac{\delta^2 F[\phi]}{\delta\phi^2}\bigg|_{\phi=0}$$

If $\mu^2_{\mathit{eff}} < 0$ we encounter a ***tachyonic instability*** and the scalar perturbation exponentially grows:

- *non-linear contributions* become important

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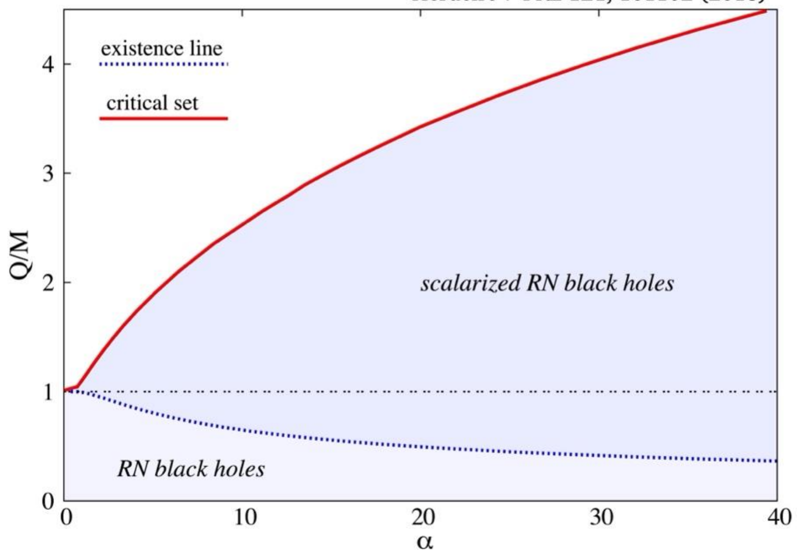
- *non-linear contributions* become important ⟶ **scalarization**

A specific EMS model for ***scalarization***

The model with $F[\phi] = e^{\alpha\phi^2}$ exhibits ***spontaneous scalarization.***

Herdeiro+ PRL **121**, 101102 (2018)



A specific EMS model for ***scalarization***

The model with $F[\phi] = e^{\alpha\phi^2}$ exhibits ***spontaneous scalarization.***

Consider a spherically symmetric ansatz for the scalarized BH solution

$$ds^2 = -N(r)e^{-2\delta(r)}dt^2 + \frac{1}{N(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$A(r) = V(r)dt \qquad \phi = \phi(r)$$

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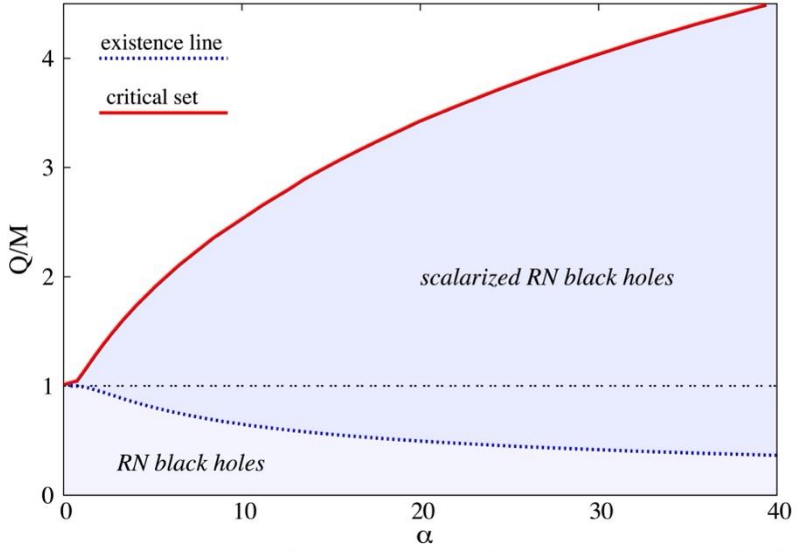


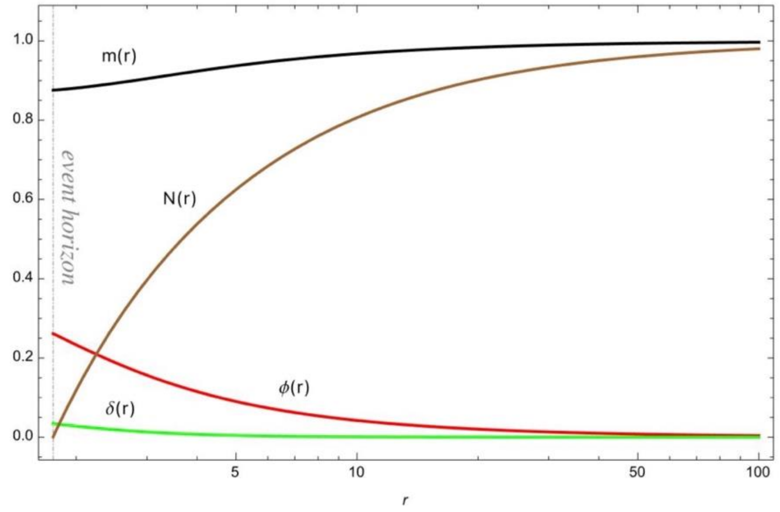
The field equations are given by ($N(r) = 1 - 2m(r)/r$)

$$\delta' + r\phi'^2 = 0$$

$$(e^{\delta}F[\phi]\,r^2V')' = 0$$

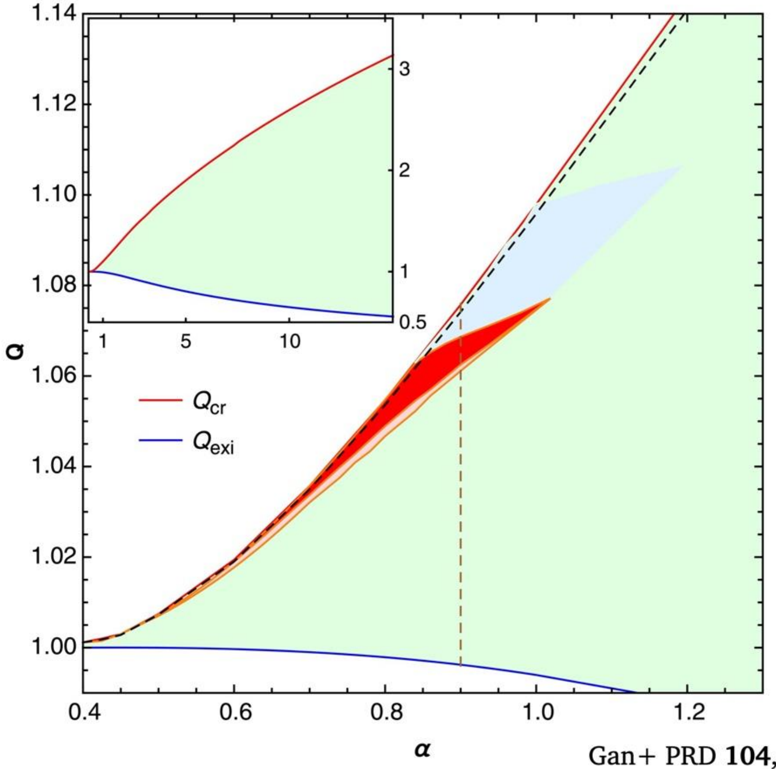
$$r(r-2m)\phi'^2 + r^2V'^2e^{2\delta}F[\phi] - 2m' = 0$$

$$r(r-2m)\phi'' - [2(m + rm' - r) + (r^2 - 2mr)\delta']\phi' + \frac{r^2V'^2e^{2\delta}}{2}\frac{\delta F[\phi]}{\delta\phi} = 0$$



Linear and spherical perturbations

- For scalarized BHs in the EMS model $F[\phi] = e^{\alpha\phi^2}$ there exist ***two unstable photon spheres*** in a small region of the parameter space.



Gan+ PRD **104**, 044049 (2021)

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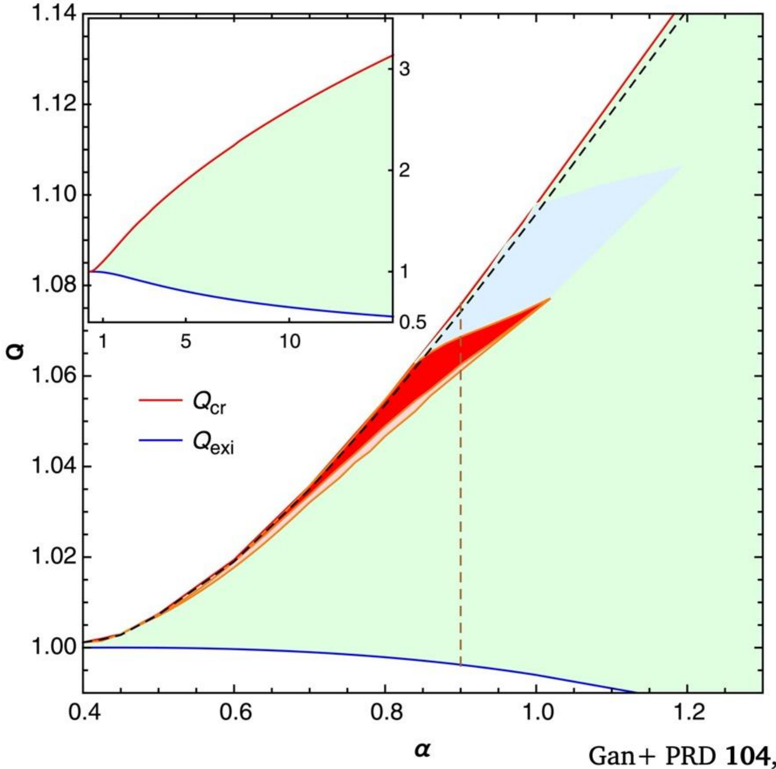
Two unstable photon spheres may trigger ***long-lived modes***!



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Two unstable photon spheres may trigger ***long-lived modes***!

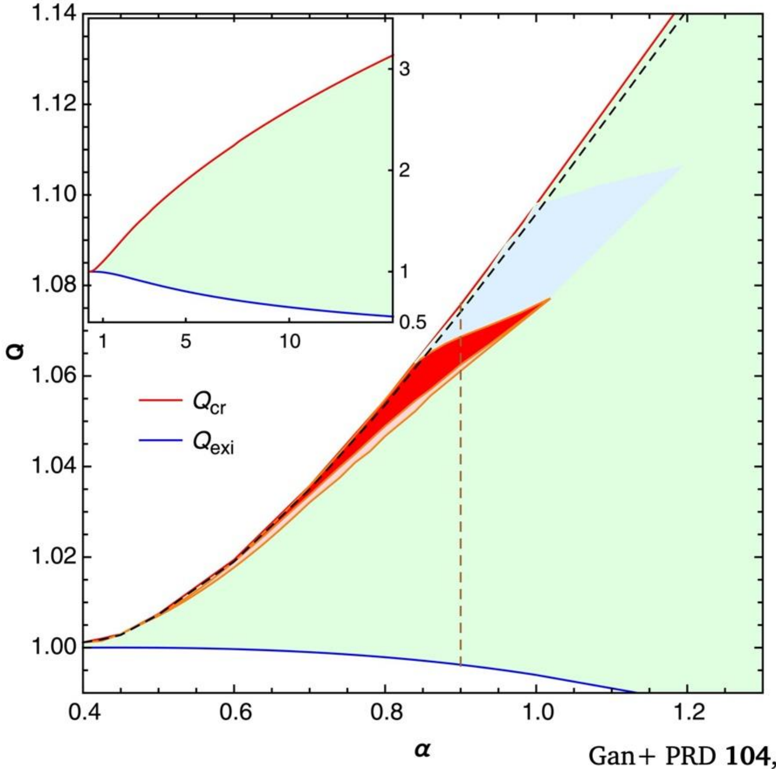
- Consider **spherical** and **linear** perturbations of the fields

$$ds^2 = -\tilde{N}(t,r)\, e^{-2\tilde{\delta}(t,r)} dt^2 + \frac{dr^2}{\tilde{N}(t,r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$A = \tilde{V}(t,r)dt\,, \qquad \phi = \tilde{\phi}(t,r)$$

$$\tilde{N}(t,r) = N(r) + \epsilon N_1(r) e^{-i\Omega t}\,, \qquad \tilde{\delta}(t,r) = \delta(r) + \epsilon\delta_1(r) e^{-i\Omega t}$$

$$\tilde{\phi}(t,r) = \phi(r) + \epsilon\phi_1(r) e^{-i\Omega t}\,, \qquad \tilde{V}(t,r) = V(r) + \epsilon V_1(r) e^{-i\Omega t}$$



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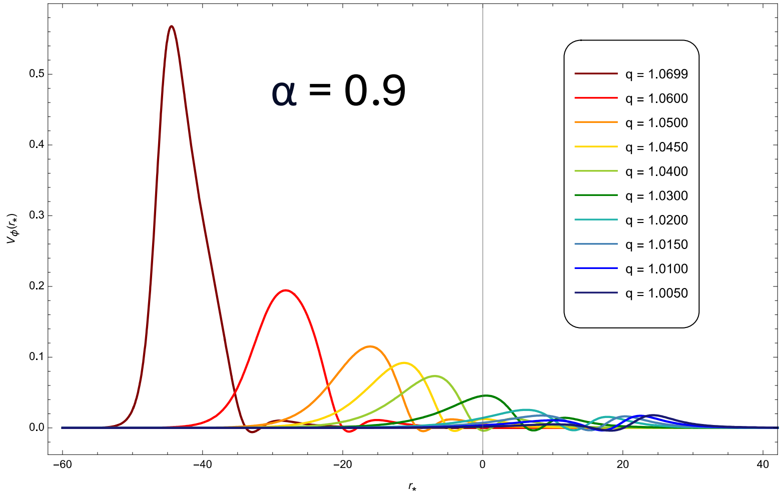
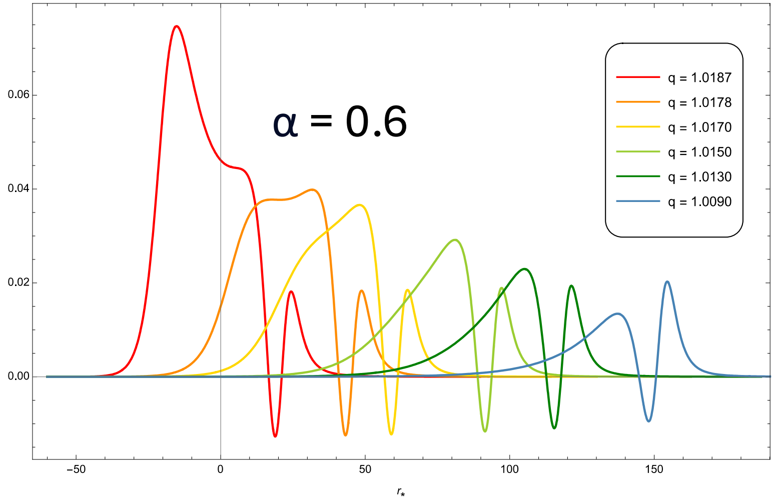
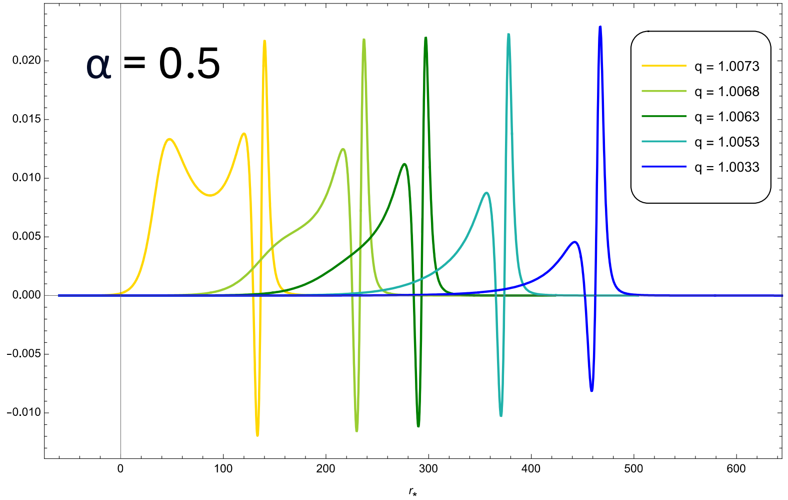
Master equation and effective potential

We get a single master equation for the *scalar field perturbation* of the **Schrödinger-like form**

$$\left(\frac{d^2}{dr_*^2}+\Omega^2\right)\Psi = V_\phi \Psi$$



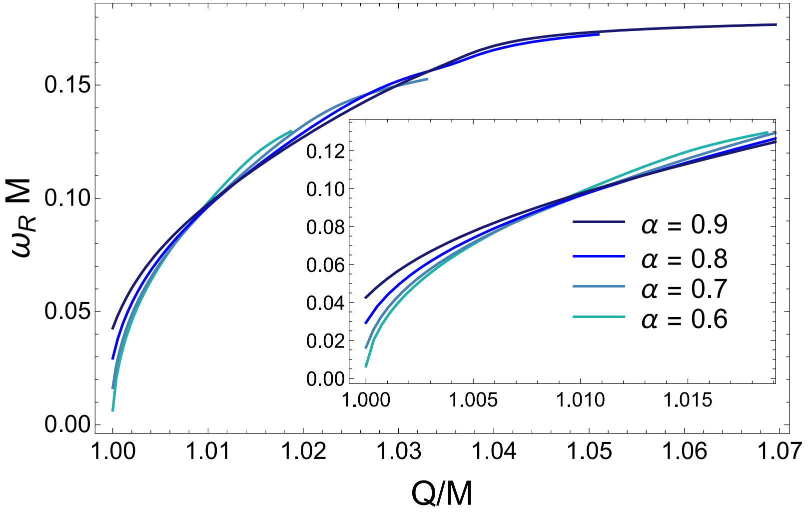


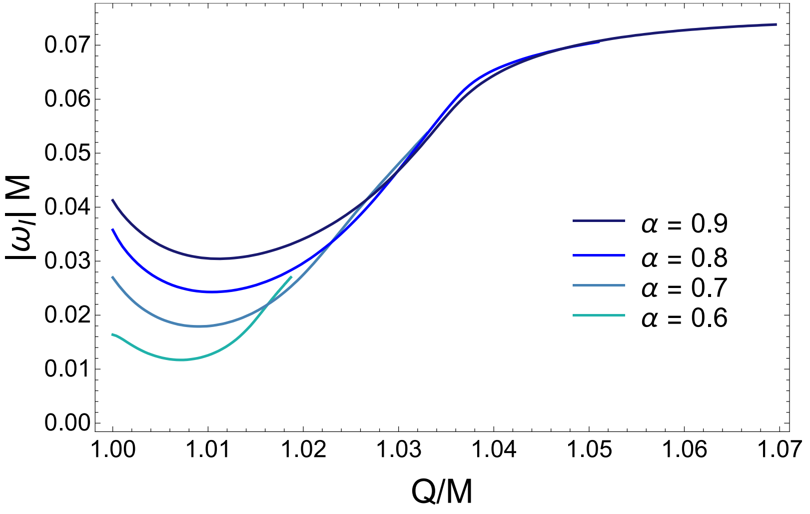


QNMs of radial perturbations

For the QNMs computation we use **direct integration** with ***proper boundary conditions***:

- *outgoing wave at infinity* $r_* \rightarrow +\infty$
- *ingoing wave at the horizon* $r_* \rightarrow -\infty$





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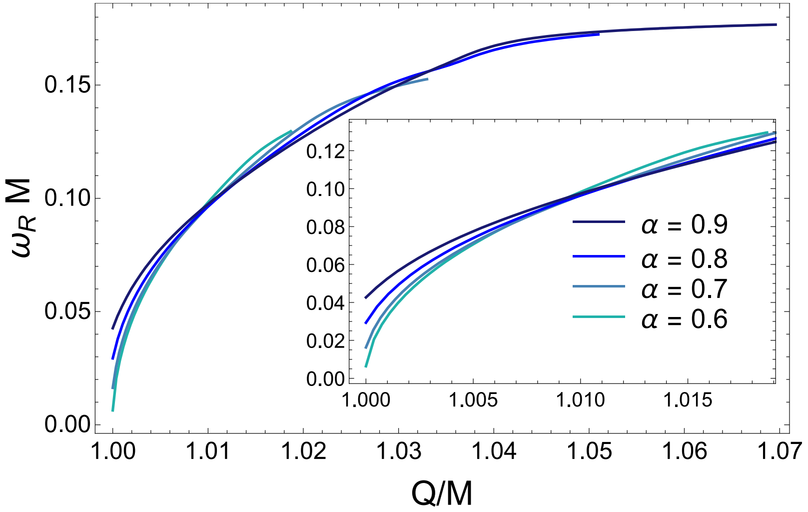
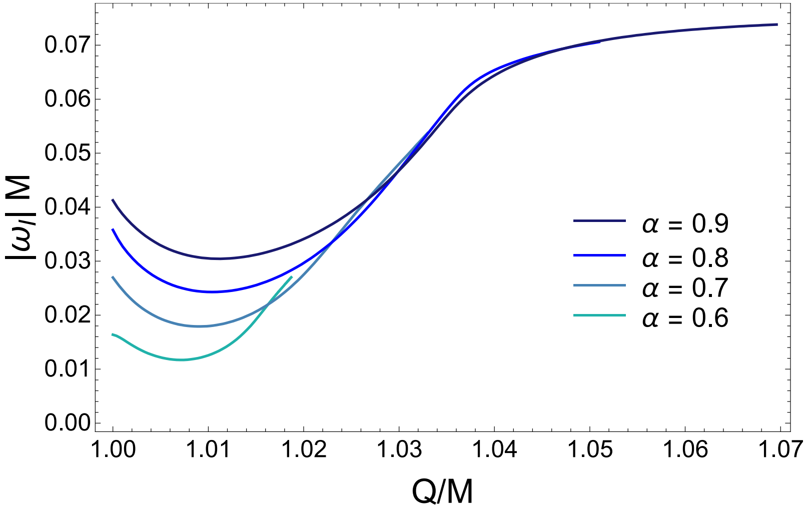
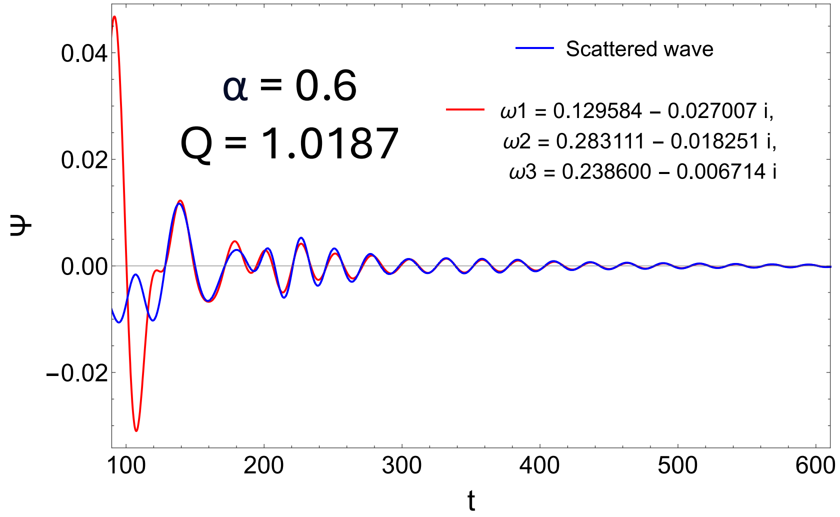
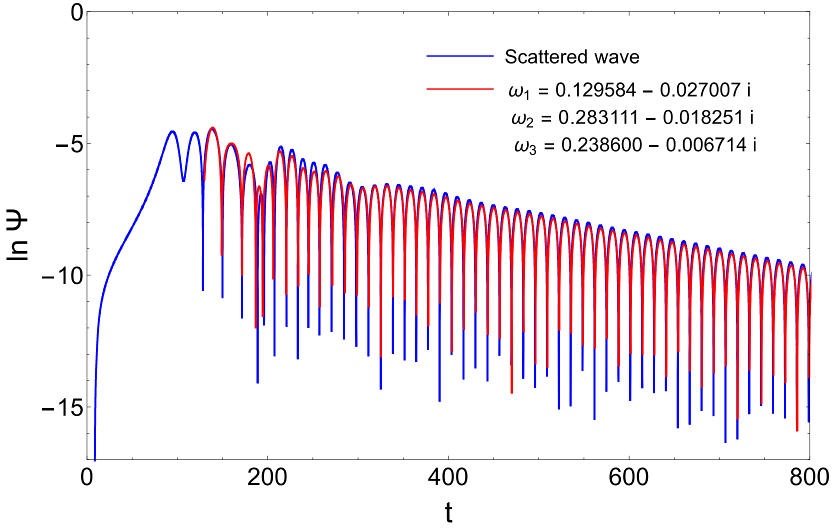




Solving the perturbation equation in the time-domain:





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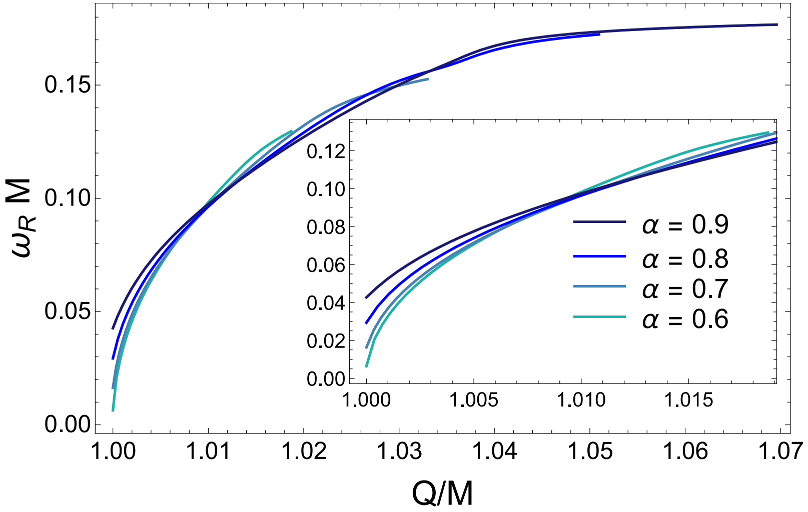

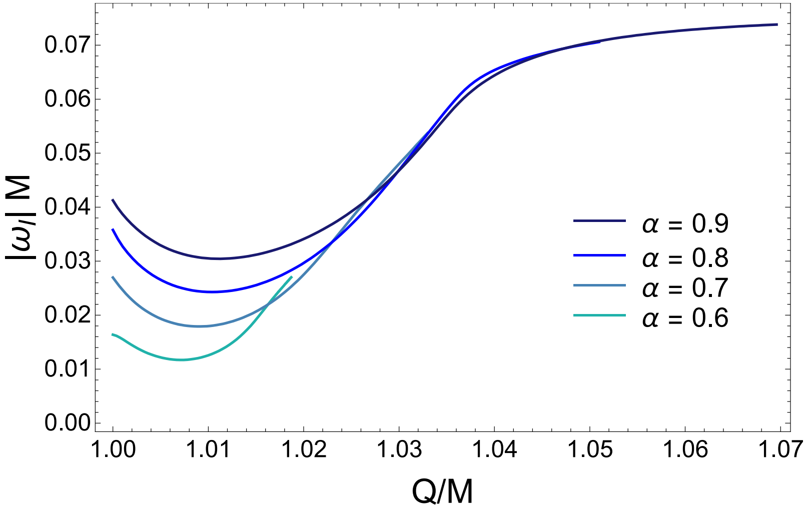

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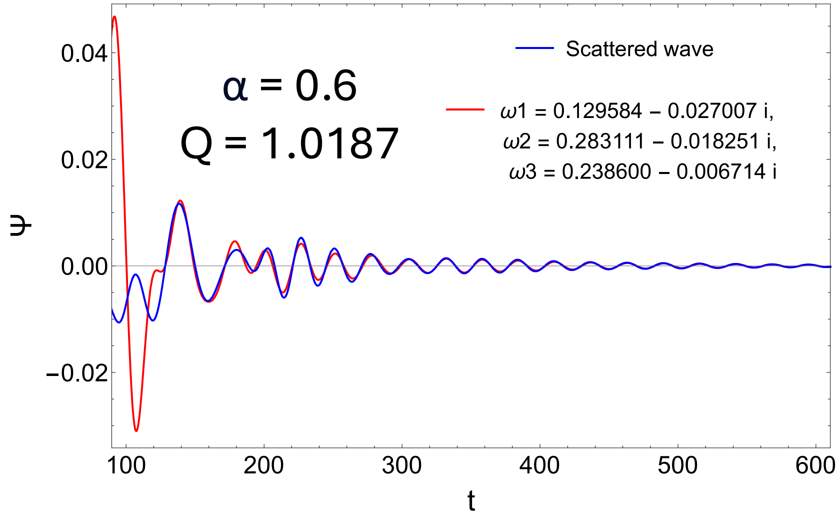

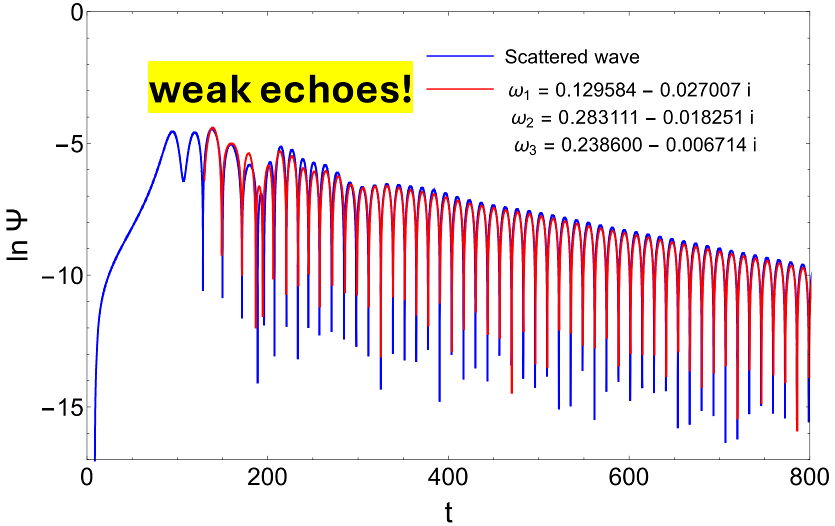

Linear and non-spherical perturbations

- Consider the following fields perturbations:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad A_\mu = \bar{A}_\mu + \delta A_\mu \qquad \phi = \bar{\phi} + \delta\phi$$

- Decompose the perturbations in terms of **scalar**, **vector** and **tensor spherical harmonics**:

  perturbations split into ”**axial**” $(-1)^{l+1}$ and “**polar**” $(-1)^{l}$

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- The linearized perturbation equations split into two sectors:

  - **Axial sector** : *system of two coupled ODEs of the second order:*

$$\left(\frac{d^2}{dr_*^2} + \omega^2\right)U(r) = V_{UU}U(r) + V_{UH}H(r)$$

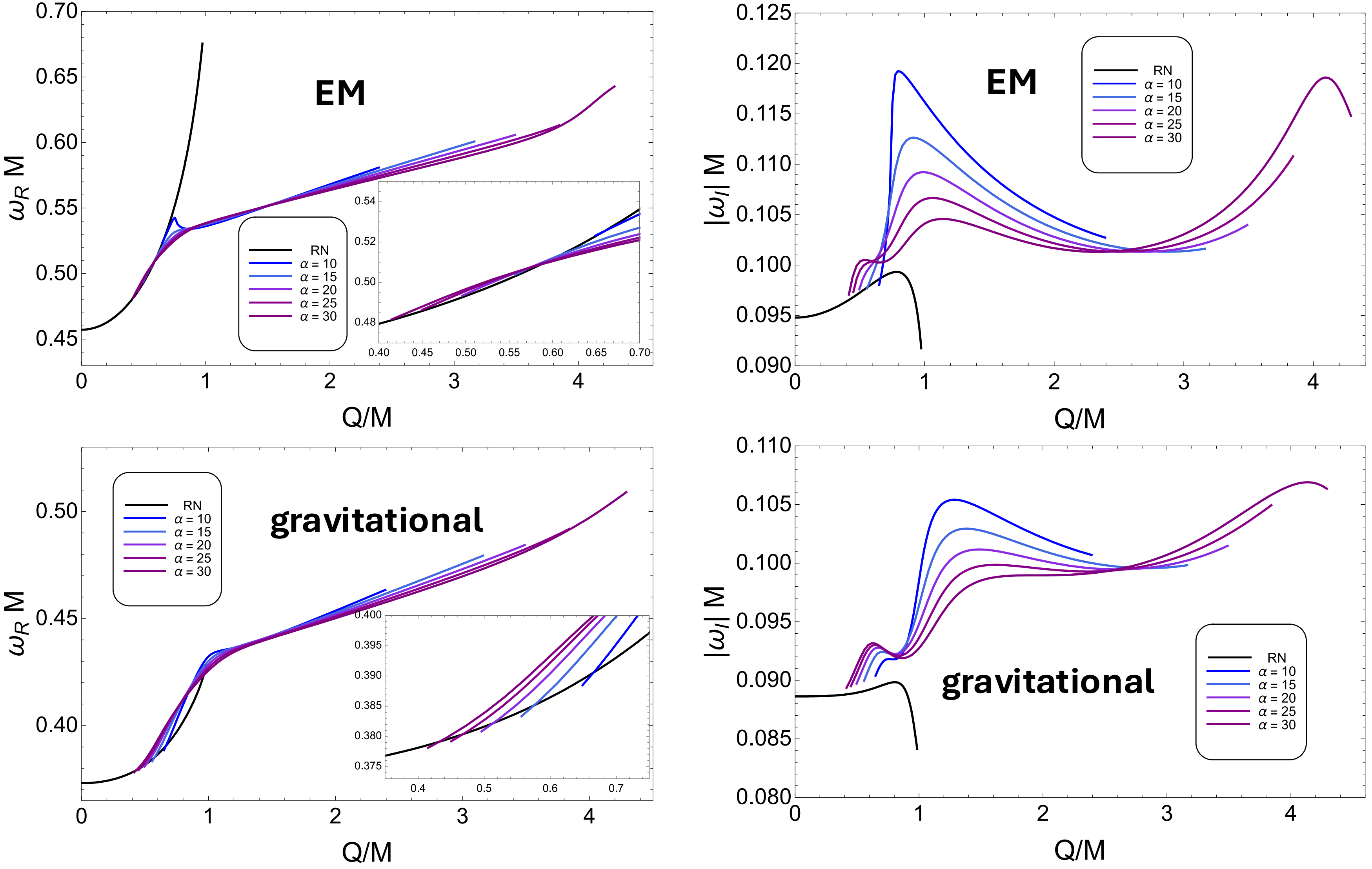
$$\left(\frac{d^2}{dr_*^2} + \omega^2\right)H(r) = V_{UH}U(r) + V_{HH}H(r)$$

$U(r)$ gravitational perturbation

$H(r)$ EM perturbation

  - **Polar sector** : *system of six coupled ODEs.*

Axial sector QNMs (EM and gravitational perturbations)



Topological stars (**TS**) and magnetized black string

Consider **Einstein – Maxwell theory in 5D**

$$S_5 = \int d^5x \sqrt{-\mathbf{g}} \left(\frac{1}{2\kappa_5^2} \mathbf{R} - \frac{1}{4} \mathbf{F}_{AB} \mathbf{F}^{AB} \right)$$

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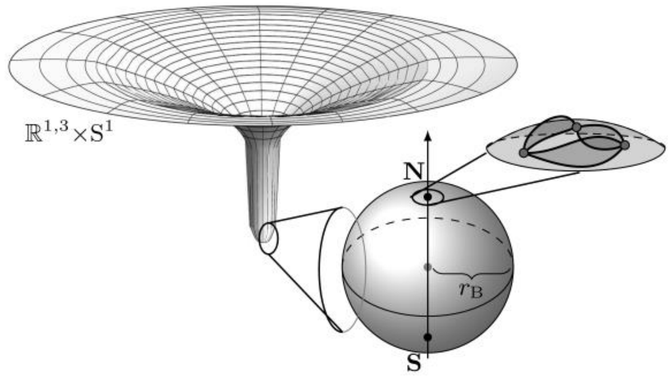

Bah, Heidmann PRL 2021

This theory admits black string with magnetic charge and **topological star**:

$$ds^2 = -f_S dt^2 + f_B dy^2 + \frac{1}{h}dr^2 + r^2 d\Omega_2^2$$

$$F = P\sin\theta\, d\theta\wedge d\phi$$

$$f_S = 1 - \frac{r_S}{r}\,, \quad f_B = 1 - \frac{r_B}{r}\,,$$

$$h = f_B f_S\,, \quad P = \pm\frac{1}{\kappa_5}\sqrt{\frac{3r_S r_B}{2}}\,.$$

- $r_B > r_S$ topological star
- $r_B \leq r_S$ magnetized black string

2 parameters family of solutions

Kaluza – Klein dimensional reduction to 4D

Let us assume *no y – dependence* in the involved fields:

$$ds_5^2 = e^{-\frac{\sqrt{3}}{3}\Phi} ds_4^2 + e^{2\frac{\sqrt{3}}{3}\Phi}(dy + \mathcal{A}_\mu dx^\mu)^2$$

$$\mathbf{F}_{AB} dx^A dx^B = F_{\mu\nu} dx^\mu dx^\nu + (\partial_\mu \Xi dx^\mu) \wedge (dy + A_\mu dx^\mu)$$

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4D metric (ds_4^2); *dilatonic field* (Φ); *extra gauge field* ($\mathcal{A}_\mu$); *4D Maxwell field* ($F_{\mu\nu}$)

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4D Maxwell field

extra scalar field

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4D Maxwell field — *extra scalar field*

The action in 4D reduces to

$$\mathcal{S} = \int dx^4 \sqrt{-g} \left[\frac{1}{2\kappa_4^2} \left(R - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{4} e^{\sqrt{3}\Phi} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} \right) + \frac{1}{e^2} \left(-\frac{1}{4} e^{\frac{\sqrt{3}}{3}\Phi} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} e^{-\frac{2\sqrt{3}}{3}\Phi} (\partial_\mu \Xi)^2 \right) \right]$$

4D background solution

The background solution in 4D that describes both the **magnetized BH** and the **topological star** is

$$ds_4^2 = -f_S f_B^{1/2} dt^2 + \frac{1}{f_S f_B^{1/2}} dr^2 + r^2 f_B^{1/2} d\Omega_2^2$$

$$\Phi = \frac{\sqrt{3}}{2} \log f_B$$

$$F = \pm e Q_m \sin\theta \, d\theta \wedge d\phi = \pm \frac{e}{\kappa_4} \sqrt{\frac{3}{2} r_B r_S} \sin\theta \, d\theta \wedge d\phi$$

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*odd parity (**axial**)*

decoupled

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ADM mass

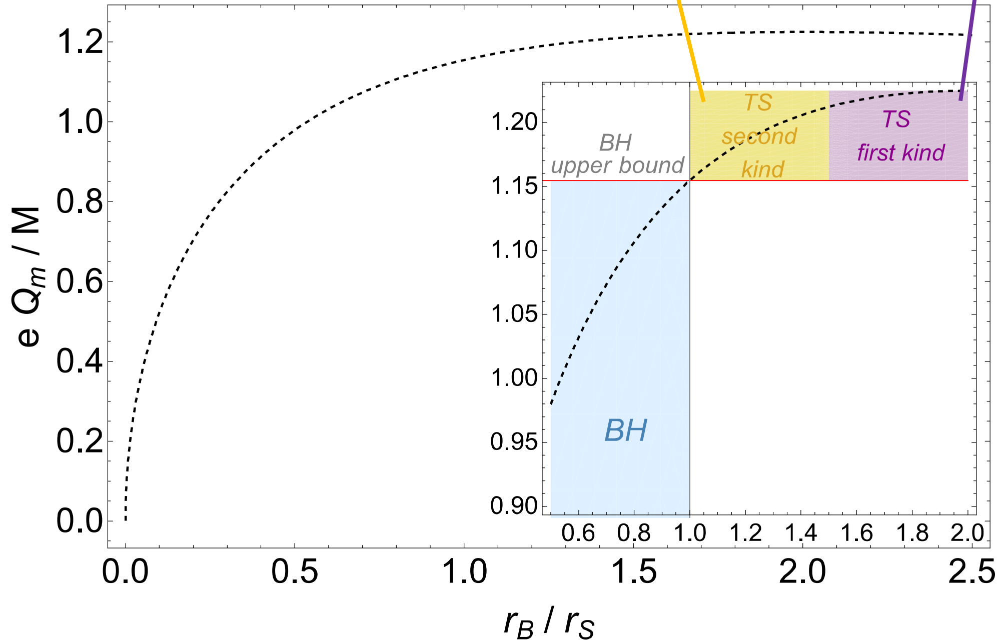
$$M = \frac{2\pi}{\kappa_4^2}(2r_S + r_B)$$

magnetic charge

$$Q_m = \frac{1}{\kappa_4}\sqrt{\frac{3}{2} r_S r_B}$$

2nd kind
unstable + stable photon spheres

1st kind
stable photon sphere



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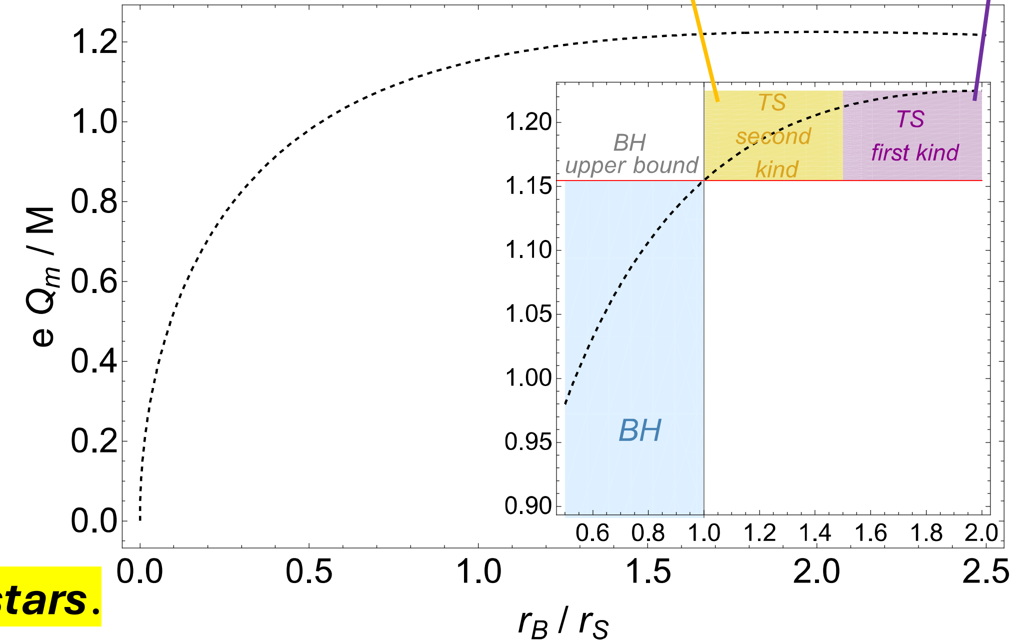
1st kind
stable photon sphere



*The solution interpolates between **BHs, ultracompact objects** and **stars**.*

Linear perturbations and master equations

Because of the presence of the *magnetic flux* in the background solutions **axial** (*odd parity*) and **polar** (even parity) perturbations are mixed:

- **Type I sector**: odd-parity metric + even-parity EM ($l \geq 1$)
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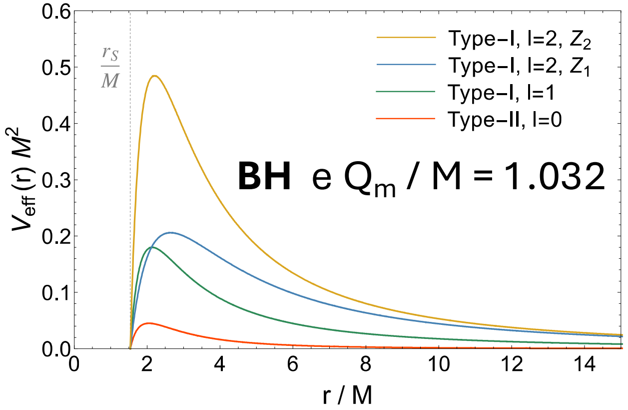
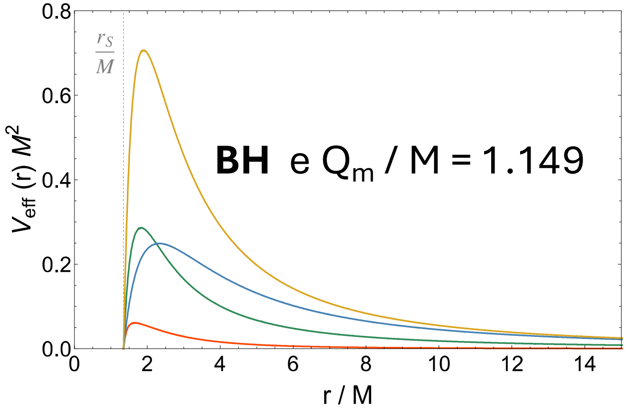
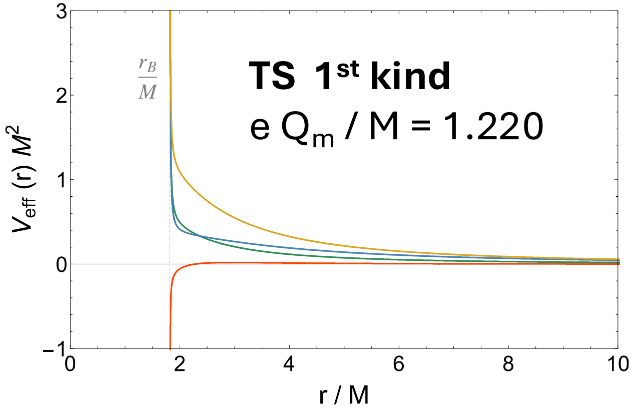
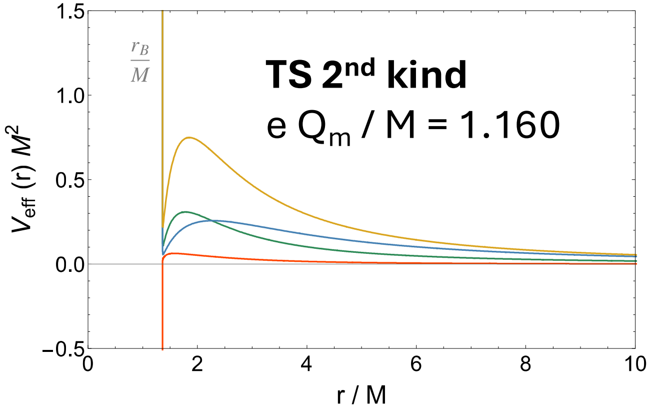




→ typical effective potential for a BH





→ **cavity for TS!**

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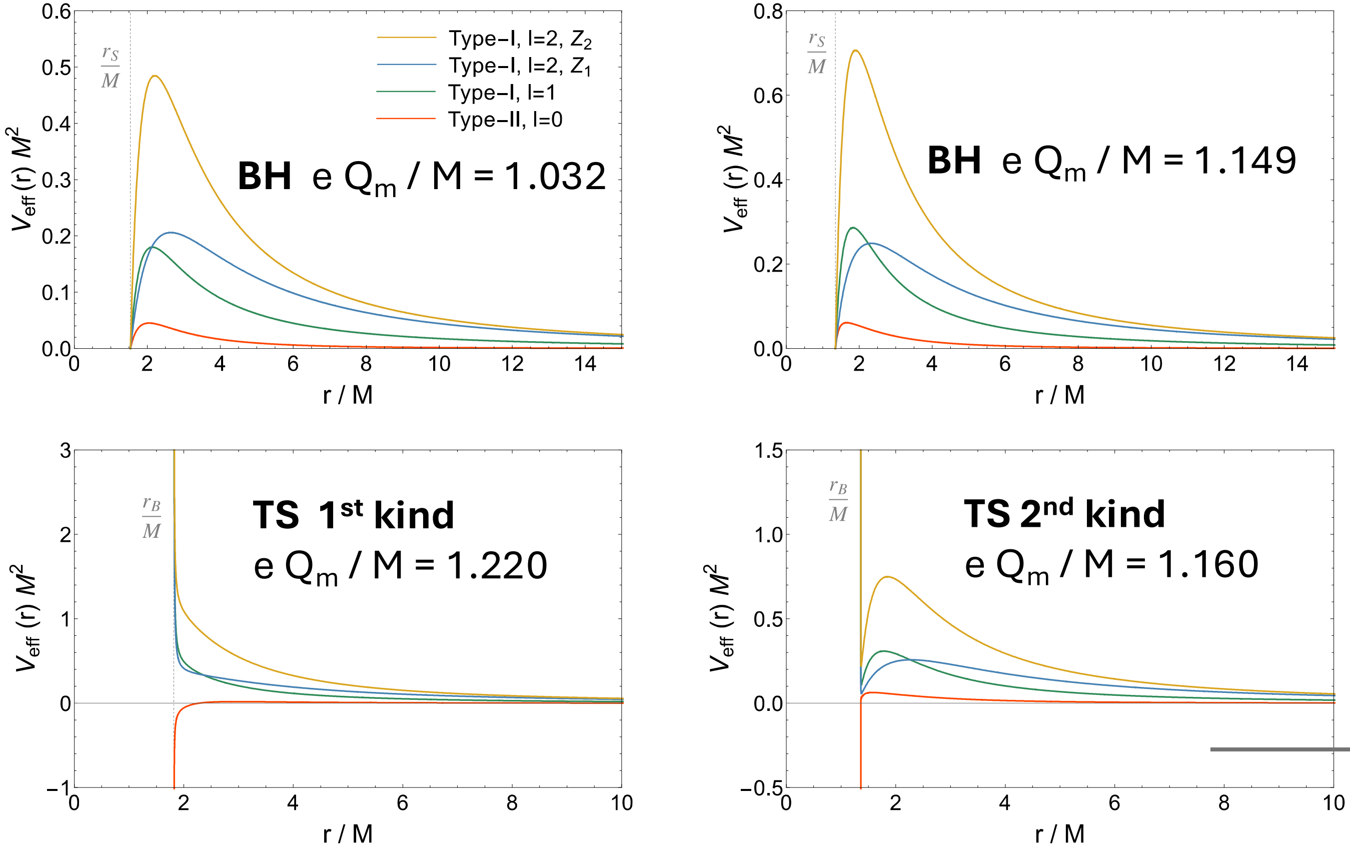
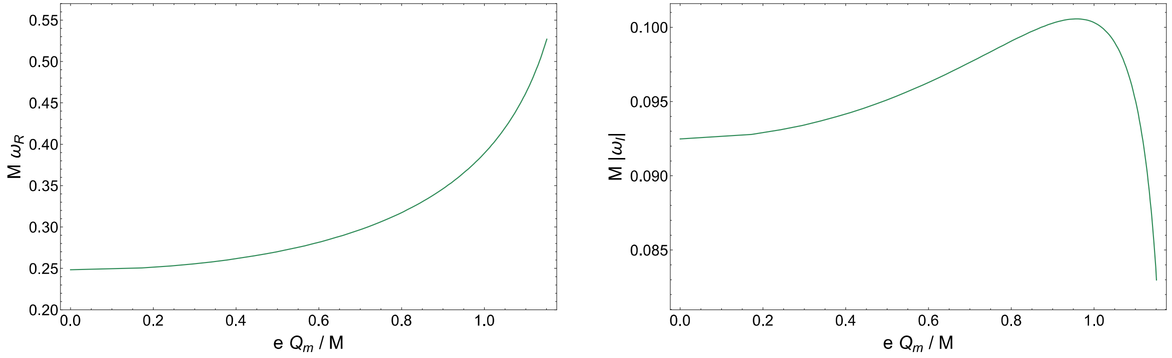


→ typical effective potential for a BH

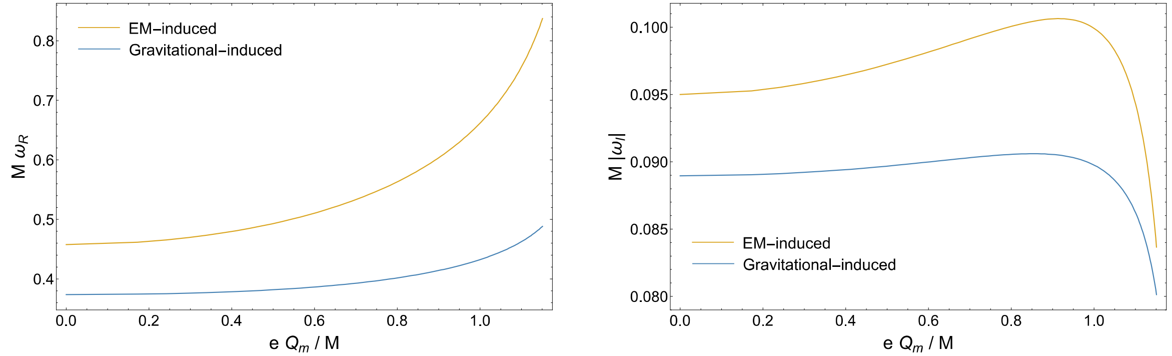
→ **cavity for TS!**

→ *peculiar negative divergent potential for radial perturbation*

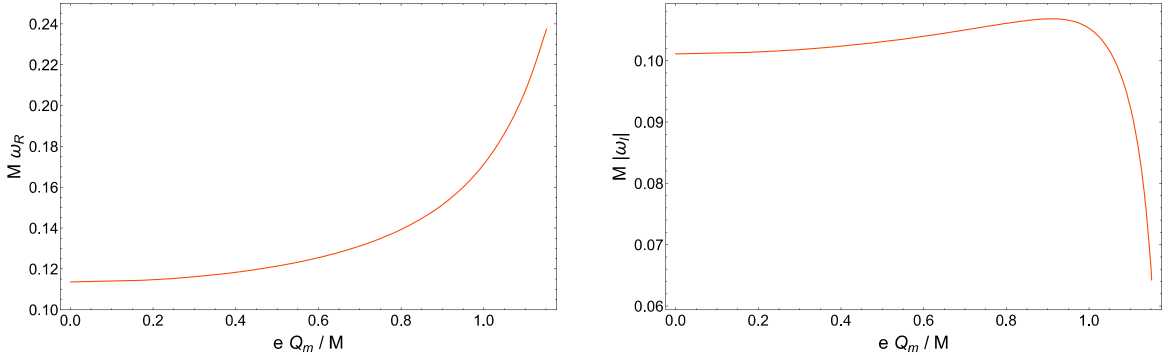
QNMs of magnetized BHs ($r_B < r_S$)



Type I l=1
(EM perturbation)

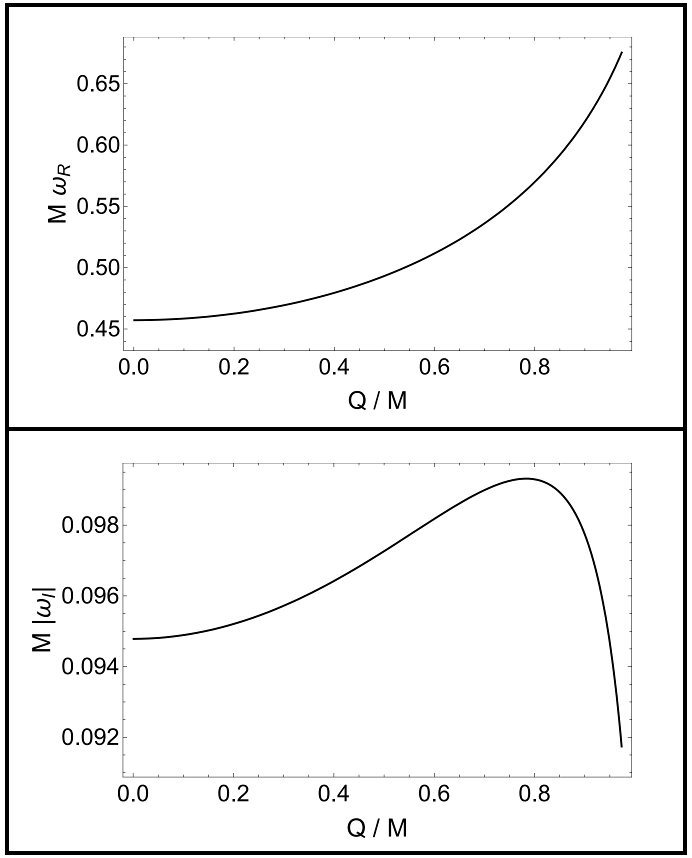


Type I l=2
(gravitational + EM perturbations)

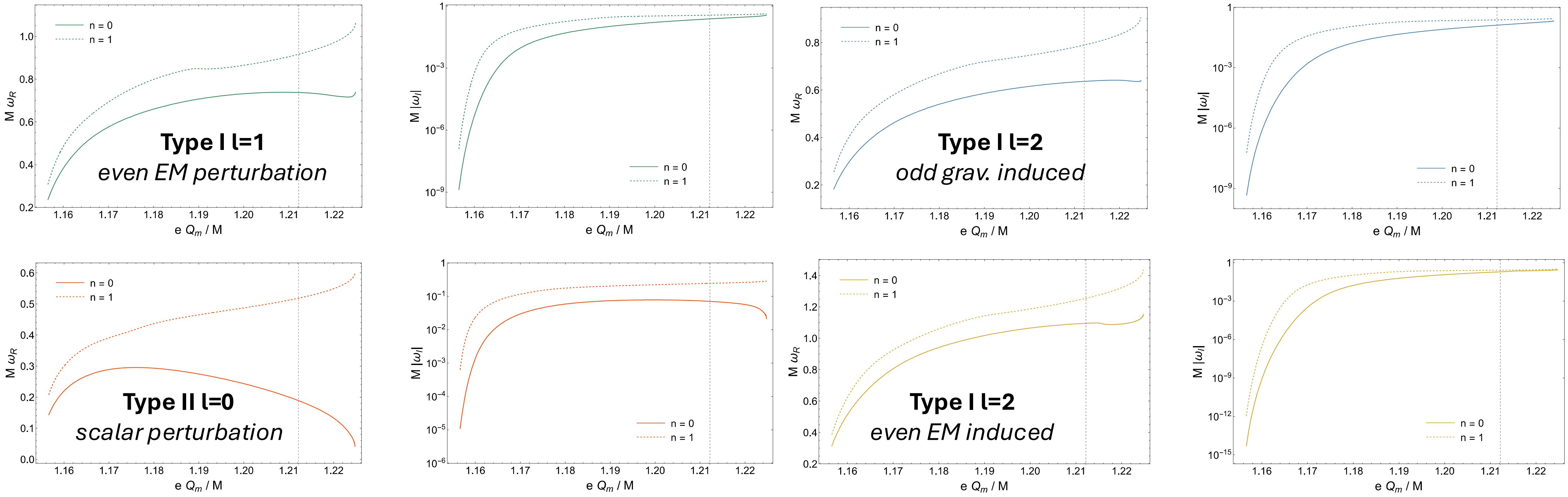


Type II l=0
(scalar perturbation)

RN BH QNMs spectrum



QNMs of TSs ($r_B \geq r_S$)



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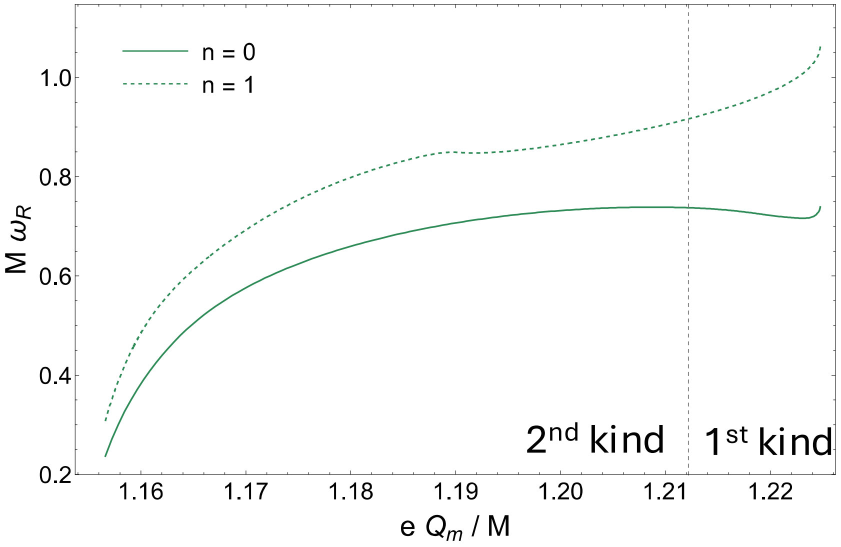

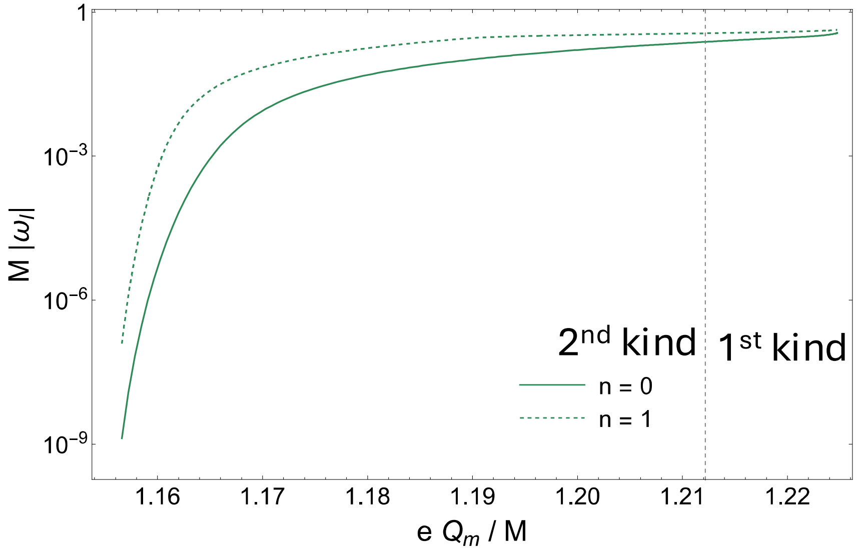

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even EM perturbation

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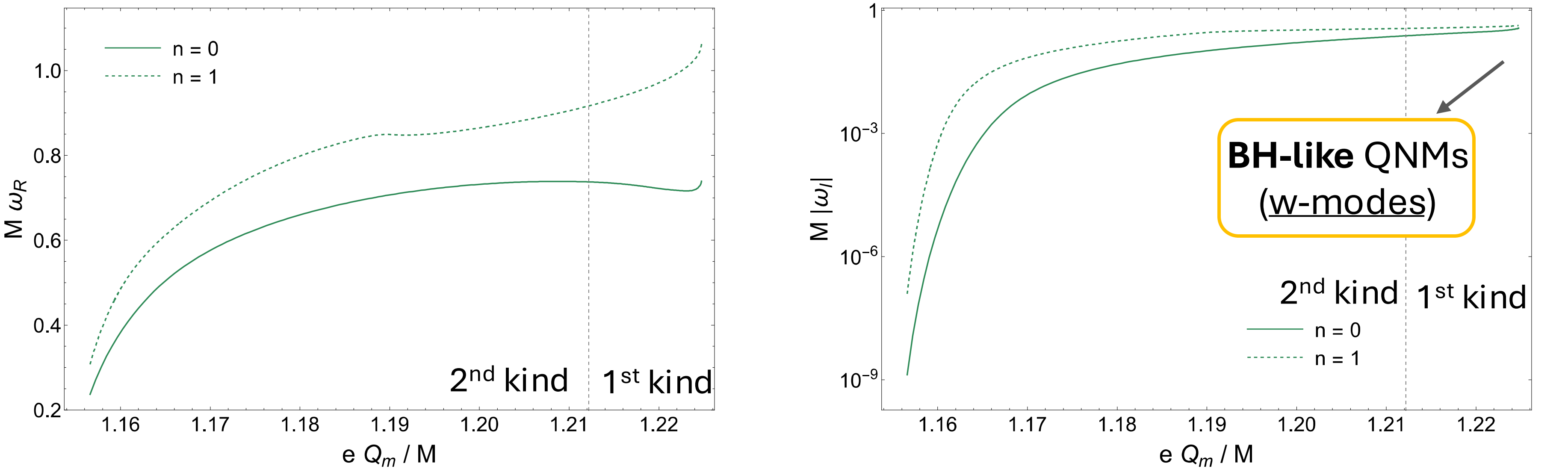

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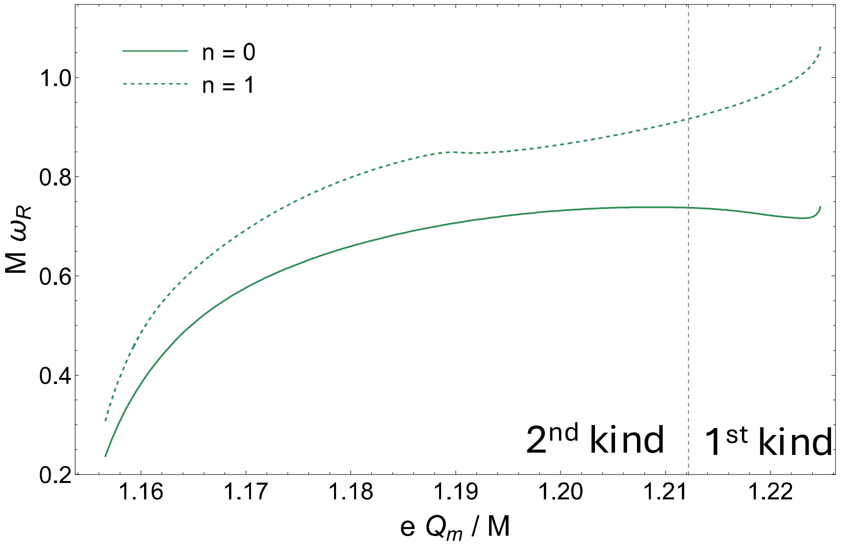

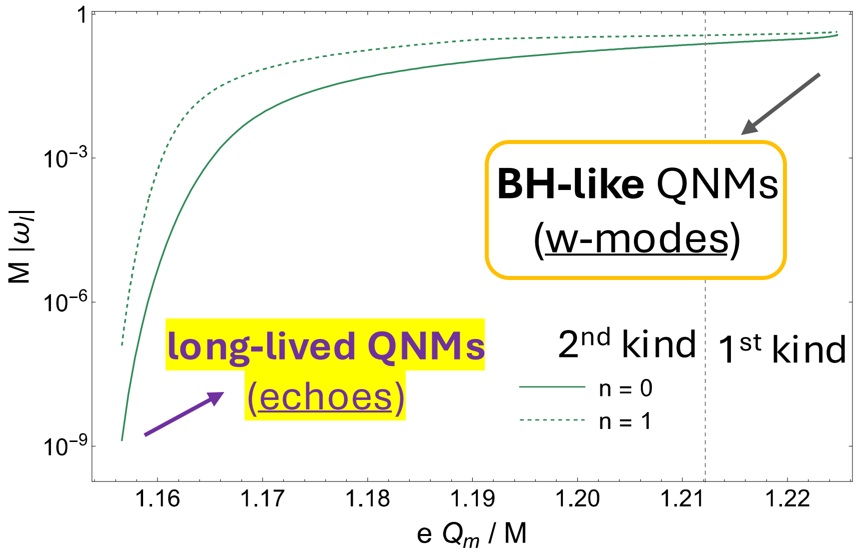

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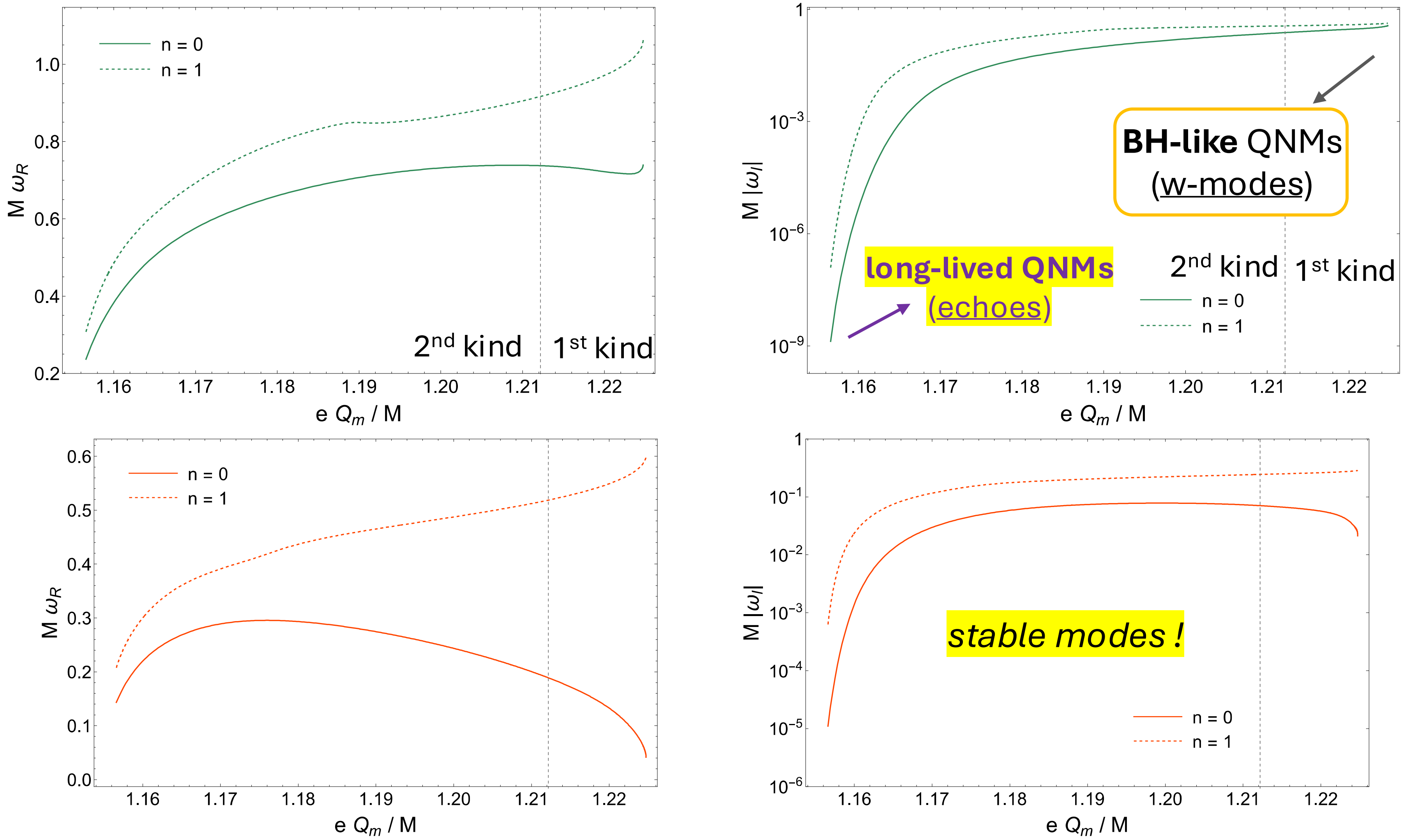
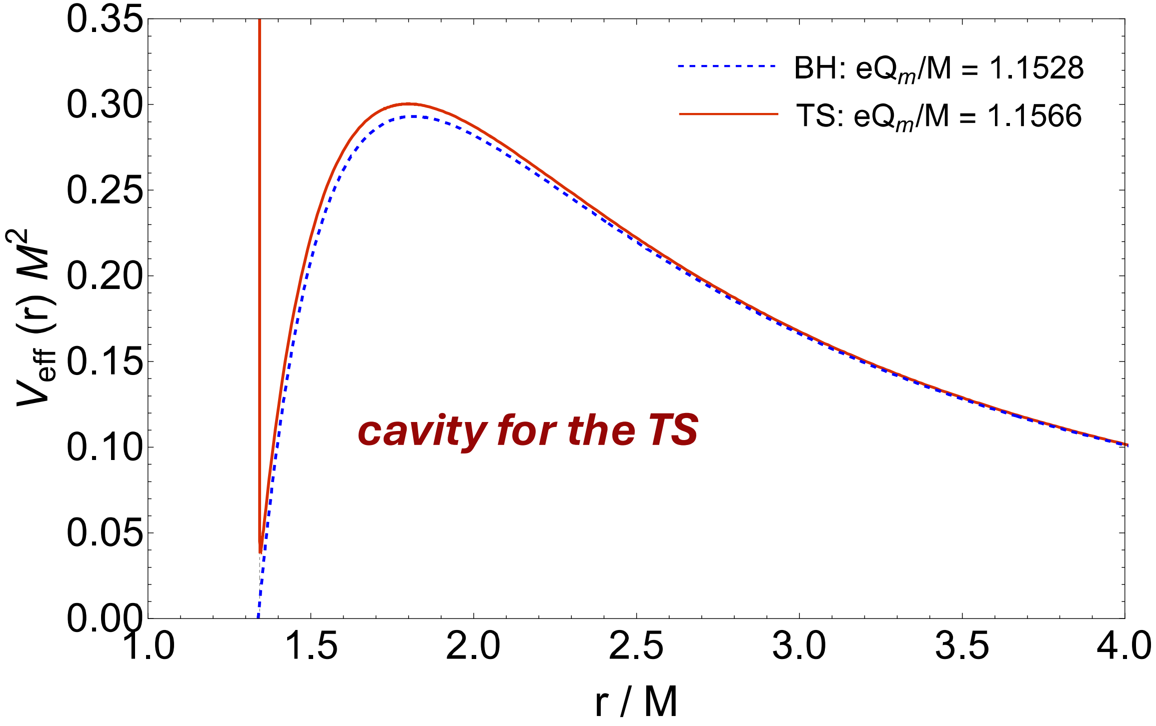


Type I l=1
even EM perturbation

Type II l=0
scalar perturbation

Echoes in the ringdown

A **time-domain analysis** (*see A. Dima talk*) confirms the QNMs spectrum and finds **echoes**!



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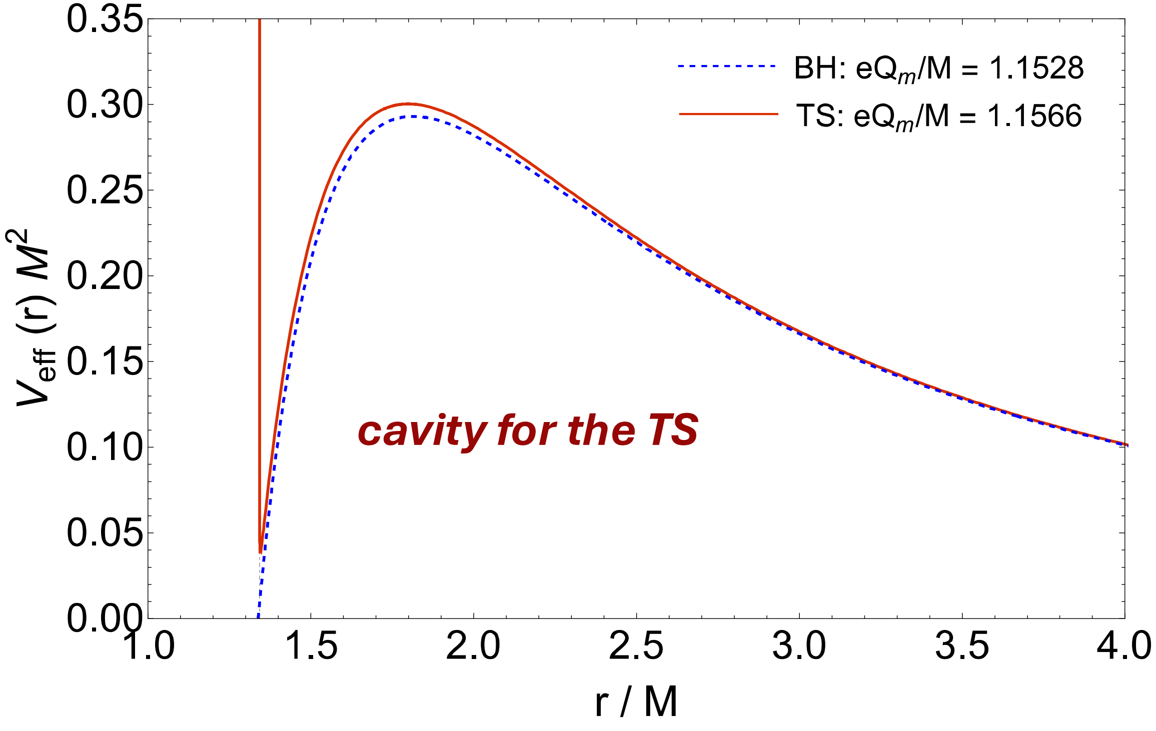

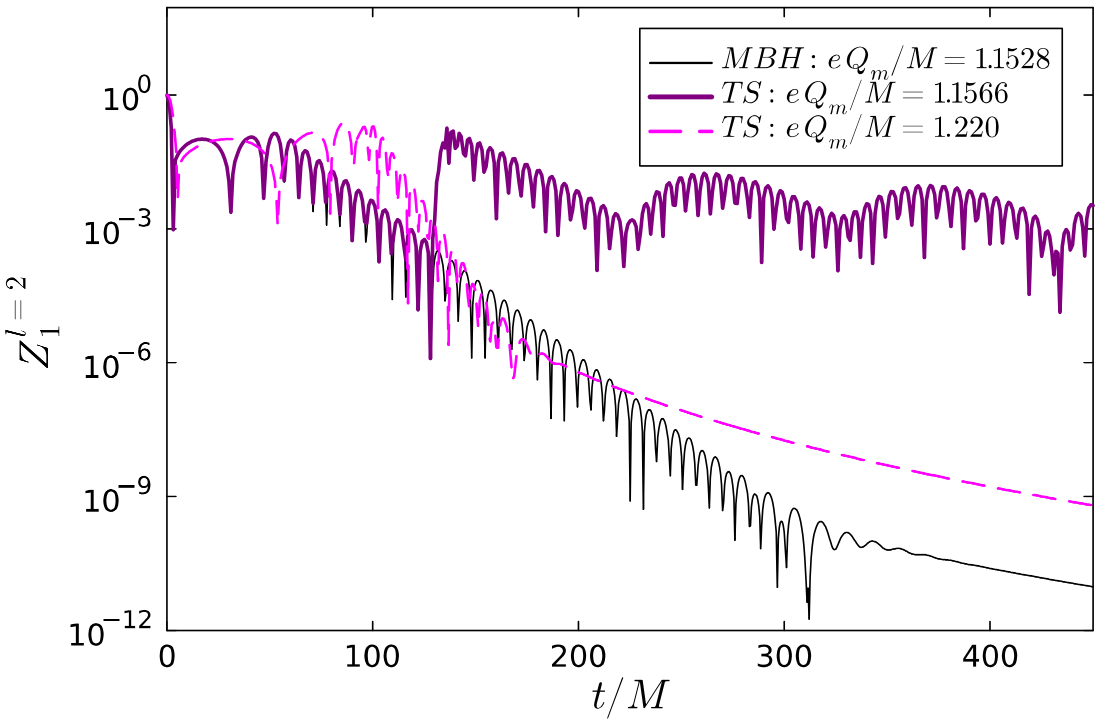

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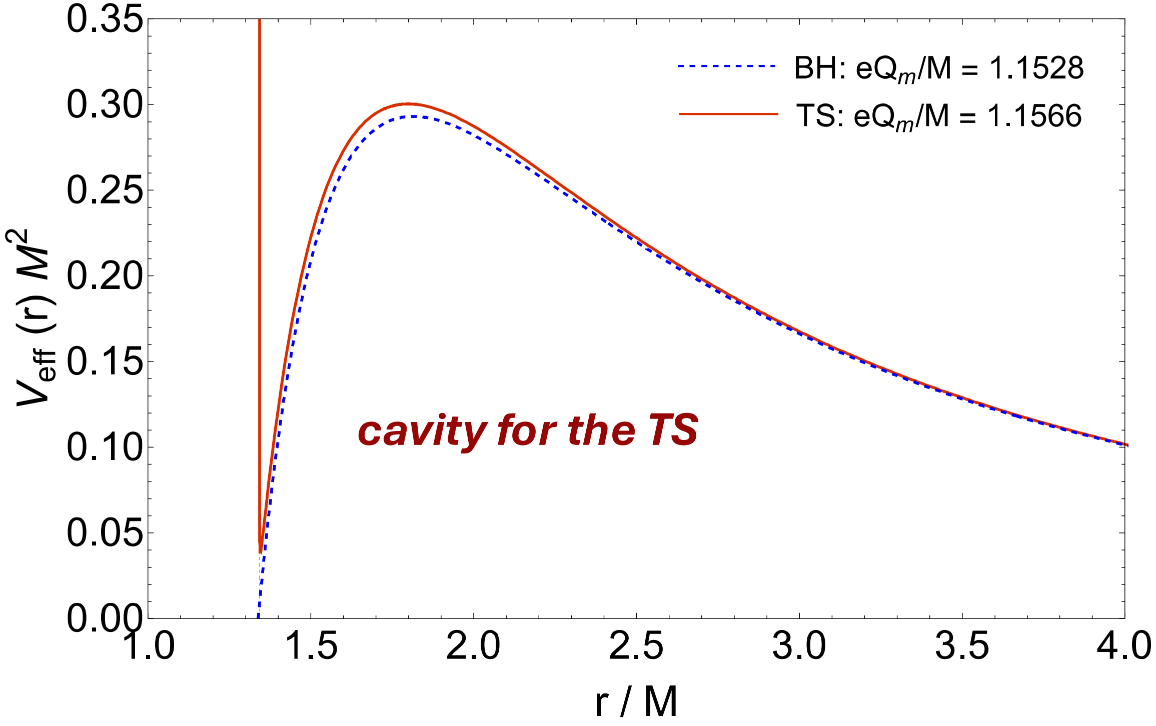

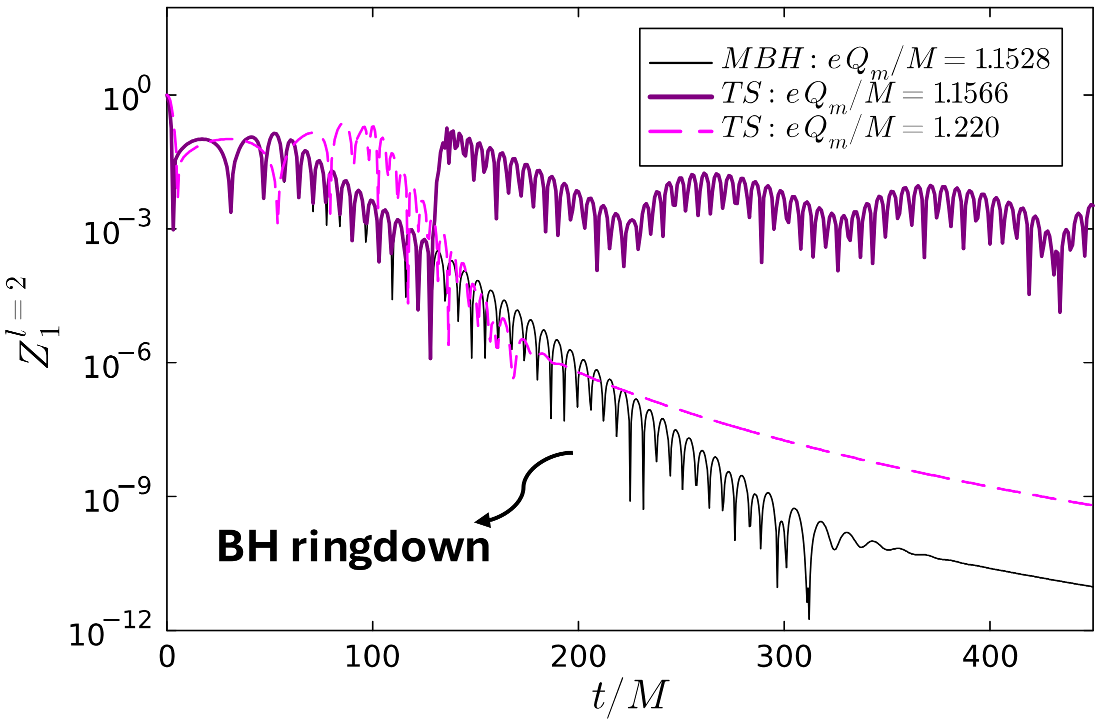

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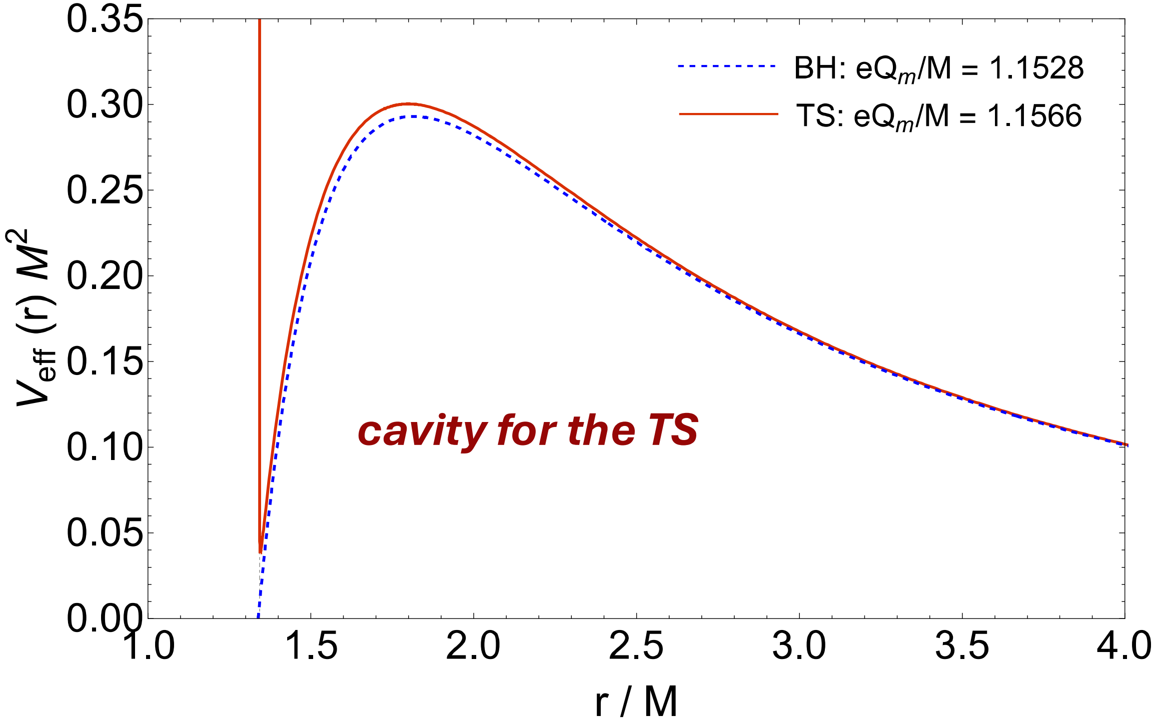

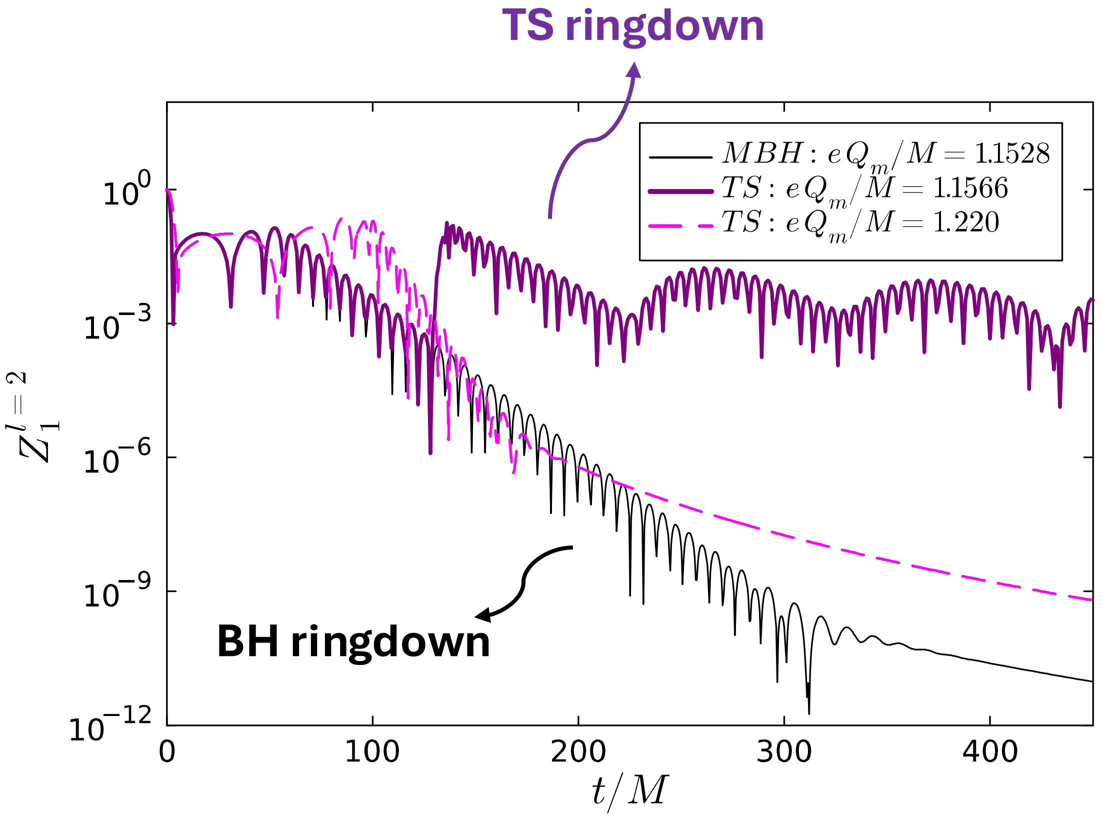

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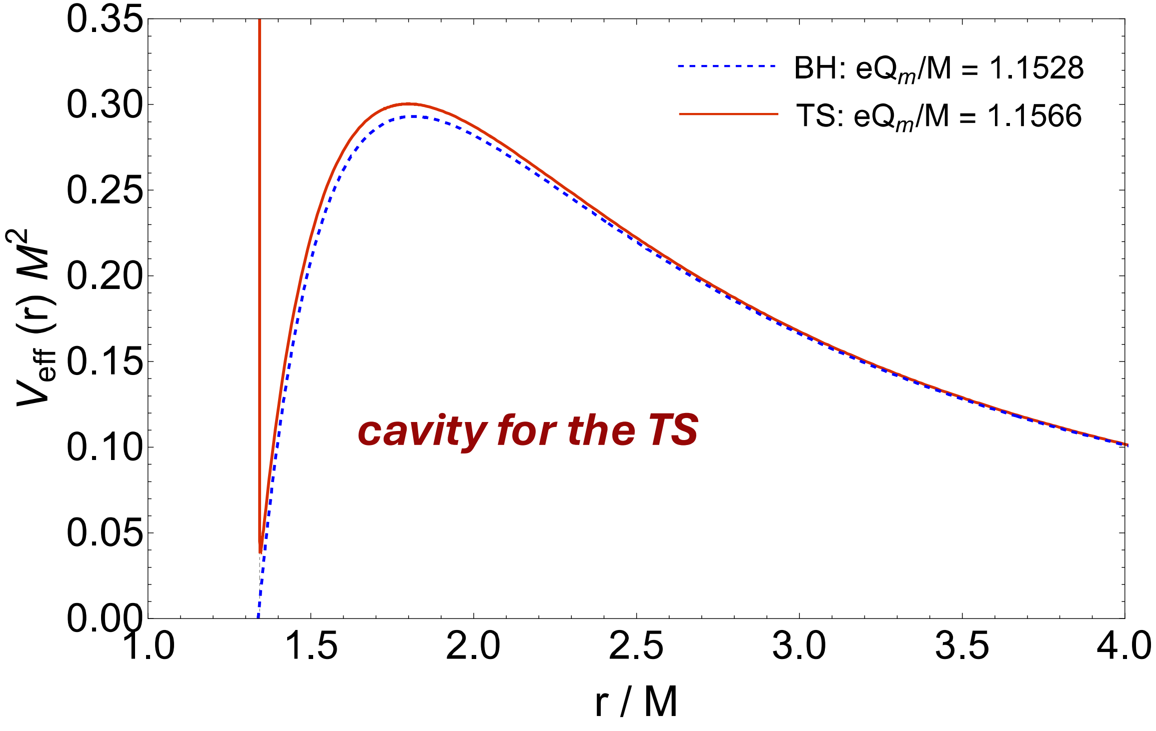

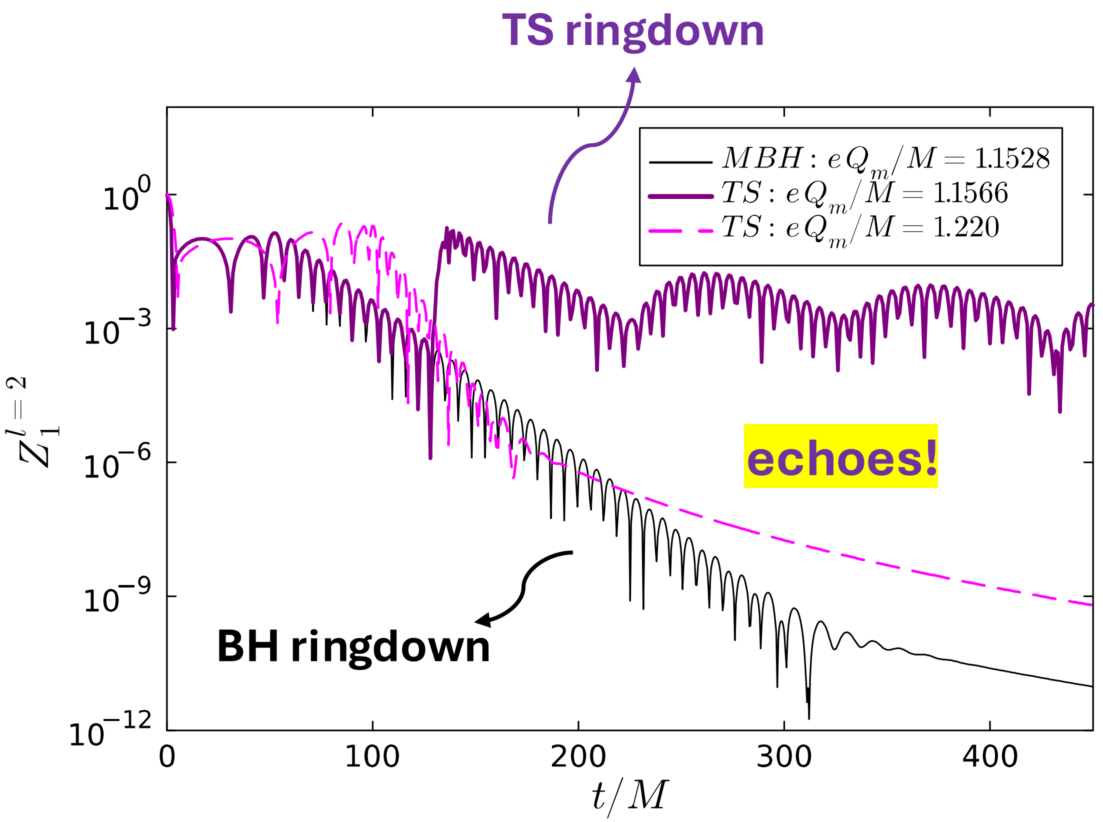

Conclusions

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  - Much *simpler* to study.
  - The solutions may come from a ***higher dimensional theory***.
  - They show attractive *beyond GR features* (e.g. ***echoes!***).

- *Next steps (ongoing)* :

- ***EMS* in 4D**: compute QNMs in the parameter space region where scalarized BHs may show echoes.

- ***Einstein-Maxwell* in 5D** (**topological star**) : complete the QNMs spectrum and t-domain analysis for **Type II sector**.

Thank you for the attention

Appendix A.1: Linear perturbations in Regge-Wheeler gauge (*EMS*)

$$h_{\mu\nu} = h^A_{\mu\nu} + h^P_{\mu\nu}$$

$$h^A_{\mu\nu} = \sum_{l,m} \int d\omega\, e^{-i\omega t} \begin{bmatrix} 0 & 0 & -\frac{h_0(r)\partial_\varphi Y_l^m}{\sin\theta} & h_0(r)\sin\theta\partial_\theta Y_l^m \\ * & 0 & -\frac{h_1(r)\partial_\varphi Y_l^m}{\sin\theta} & h_1(r)\sin\theta\partial_\theta Y_l^m \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}$$

$$h^P_{\mu\nu} = \sum_{l,m} \int d\omega\, e^{-i\omega t} Y_l^m \begin{bmatrix} e^{-2\delta(r)}N(r)H_0(r) & H_1(r) & 0 & 0 \\ * & \frac{H_2(r)}{N(r)} & 0 & 0 \\ * & * & r^2K(r) & 0 \\ * & * & * & r^2\sin^2\theta K(r) \end{bmatrix}$$

Appendix A.2: Linear perturbations in Regge-Wheeler gauge (*EMS*)

$$\delta F_{\mu\nu} = \delta F^{A}_{\mu\nu} + \delta F^{B}_{\mu\nu}$$

$$\delta F^{A}_{\mu\nu} = \sum_{l,m} \int d\omega\, e^{-i\omega t} \begin{bmatrix} 0 & 0 & -\frac{i\omega u_4(r)\partial_\varphi Y_l^m}{\sin\theta} & i\omega u_4(r)\sin\theta\partial_\theta Y_l^m \\ * & 0 & \frac{u_4'(r)\partial_\varphi Y_l^m}{\sin\theta} & -u_4'(r)\sin\theta\partial_\theta Y_l^m \\ * & * & 0 & l(l+1)u_4(r)\sin\theta Y_l^m \\ * & * & * & 0 \end{bmatrix}$$

$$\delta F^{P}_{\mu\nu} = \sum_{l,m} \int d\omega\, e^{-i\omega t} \begin{bmatrix} 0 & f_{01}(r)Y_l^m & f_{02}(r)\partial_\theta Y_l^m & f_{02}(r)\partial_\varphi Y_l^m \\ * & 0 & f_{12}(r)\partial_\theta Y_l^m & f_{12}(r)\partial_\theta Y_l^m \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}$$

$$\delta\phi = \sum_{l,m} \int d\omega\, e^{-i\omega t} z(r) Y_l^m$$

Appendix B: Linear perturbations in Regge-Wheeler gauge (*TS & MBH*)

Type I sector

$$h^{\text{odd}}_{\mu\nu} = \sum_{l,m} \begin{pmatrix} 0 & 0 & -h_0(t,r)/\sin\theta\partial_\phi & h_0(t,r)\sin\theta\partial_\theta \\ 0 & 0 & -h_1(t,r)/\sin\theta\partial_\phi & h_1(t,r)\sin\theta\partial_\theta \\ -h_0(t,r)/\sin\theta\partial_\phi & -h_1(t,r)/\sin\theta\partial_\phi & 0 & 0 \\ h_0(t,r)\sin\theta\partial_\theta & h_1(t,r)\sin\theta\partial_\theta & 0 & 0 \end{pmatrix} Y_{lm}(\theta,\phi)$$

$$f^{\text{even}}_{\mu\nu} = \sum_{l,m} \begin{pmatrix} 0 & f^+_{01}(t,r) & f^+_{02}(t,r)\partial_\theta & f^+_{02}(t,r)\partial_\phi \\ -f^+_{01}(t,r) & 0 & f^+_{12}(t,r)\partial_\theta & f^+_{12}(t,r)\partial_\phi \\ -f^+_{02}(t,r)\partial_\theta & -f^+_{12}(t,r)\partial_\theta & 0 & 0 \\ -f^+_{02}(t,r)\partial_\phi & -f^+_{12}(t,r)\partial_\phi & 0 & 0 \end{pmatrix} Y_{lm}(\theta,\phi)$$

Type II sector

$$h^{\text{even}}_{\mu\nu} = \sum_{l,m} \begin{pmatrix} f_S f_B^{1/2} H_0(t,r) & H_1(t,r) & 0 & 0 \\ H_1(t,r) & f_S^{-1} f_B^{-1/2} H_2(t,r) & 0 & 0 \\ 0 & 0 & r^2 f_B^{1/2} K(t,r) & 0 \\ 0 & 0 & 0 & r^2 f_B^{1/2} \sin\theta^2 K(t,r) \end{pmatrix} Y_{lm}(\theta,\phi)$$

$$f^{\text{odd}}_{\mu\nu} = \sum_{l,m} \begin{pmatrix} 0 & 0 & f^-_{02}(t,r)/\sin\theta\partial_\phi & -f^-_{02}(t,r)\sin\theta\partial_\theta \\ 0 & 0 & f^-_{12}(t,r)/\sin\theta\partial_\phi & -f^-_{12}(t,r)\sin\theta\partial_\theta \\ -f^-_{02}(t,r)/\sin\theta\partial_\phi & -f^-_{12}(t,r)/\sin\theta\partial_\phi & 0 & f^-_{23}(t,r)\sin\theta \\ f^-_{02}(t,r)\sin\theta\partial_\theta & f^-_{12}(t,r)\sin\theta\partial_\theta & -f^-_{23}(t,r)\sin\theta & 0 \end{pmatrix} Y_{lm}(\theta,\phi)$$

$$\delta\Phi = \sum_{l,m} \frac{\varphi(t,r)}{r} Y_{lm}(\theta,\phi)$$

Appendix C: Gregory-Laflamme instability

- **Black strings** suffer of the **Gregory-Laflamme instability** for $r_B \leq r_S/2$.

↓

magnetized BH (in 4D): instability against *spherical perturbations* with **e^{iky}**

- **Double Wick rotation**: BHs ⟷ TSs by $(t, y, r_S, r_B) \rightarrow (iy, it, r_B, r_S)$

If there is a **zero mode for BHs**

↓

purely imaginary mode for TSs for

$$r_B > 2r_S$$