Development of Phenomenological Gravitational Wave Signals from Rotating Core-Collapse Supernovae in a Multi-Messenger Environment

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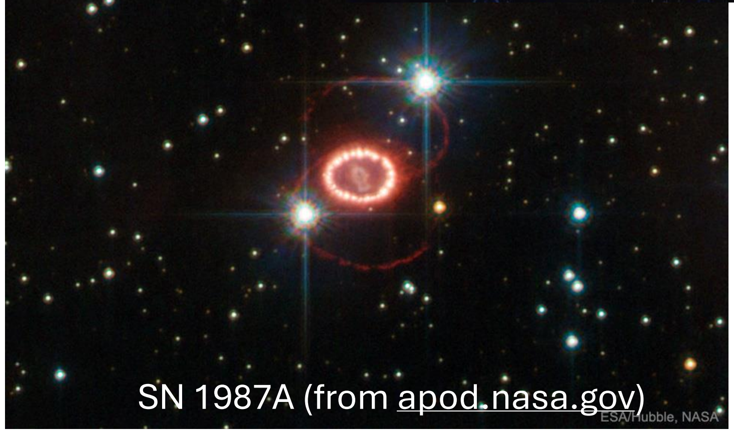
17/09/2024

Scientific case

- Core-Collapse Supernova (CCSN) features:
  - Type II SNe exceeding the Chandrasekhar limit ($1.4\ \mathrm{M}_\odot$) of their iron core
  - Massive stars ($M \sim 8 - 100\ \mathrm{M}_\odot$) with a rate of $\sim 1 - 3$ per century
  - Very weak GW signal of $\sim 10^{-21} - 10^{-23}$
- Multi-messenger sources producing both neutrinos and gravitational waves (along with an electromagnetic counterpart, as in the SN 1987A)
- CCSN waveforms difficult to construct:
  - Stochastic nature of the collapse due to non-radial instabilities
  - Lack of templates implying a detection efficiency of 50% at 10 kpc
  - Exclusion of modes different from the dominant and limited set of simulations used

Cassiopeia A (from apod.nasa.gov)



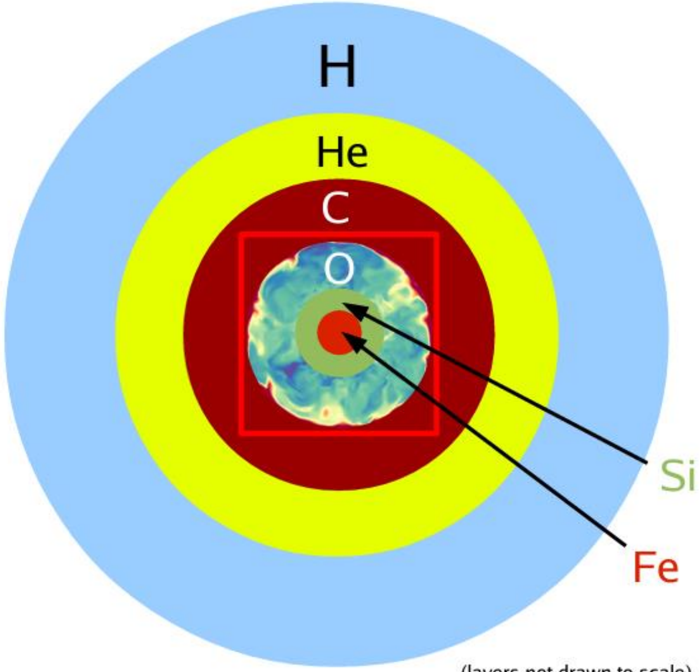
SN 1987A (from apod.nasa.gov)

ESA/Hubble, NASA

Neutrino-driven explosions

- Amount of energy released in GWs ($E_{\mathrm{GW}} \sim 10^{47}$ erg) not sufficient to explain these explosions
- Neutrinos are good candidates to drive the supernova explosions:
  - Cover the energies involved ($E_\nu \sim 10^{53}$ erg)
  - Hundreds of milliseconds after shock stagnation core contraction and heating causes the release of more powerful neutrinos
  - Low-mass progenitors ($M \sim 8 - 10\ \mathrm{M}_\odot$ with O-Ne-Mg or iron cores) well explained by the neutrino-driven mechanism
  - Massive iron-core progenitors ($M > 10\ \mathrm{M}_\odot$) might not be suited to this mechanism

Onion-shell structure of pre-collapse star



(layers not drawn to scale)

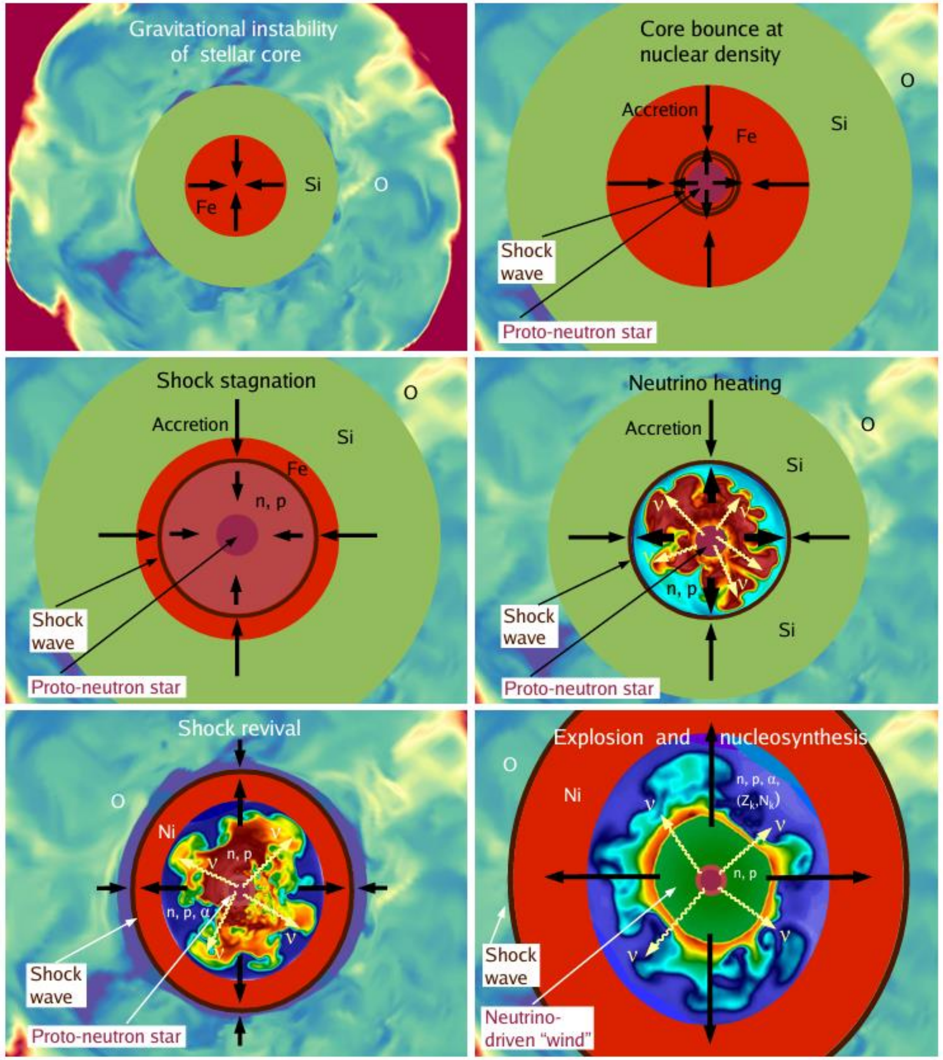
Evolution phases of collapsing stars

1. Collapse of stellar core:

$$e^- + p \rightarrow \nu_e + n$$

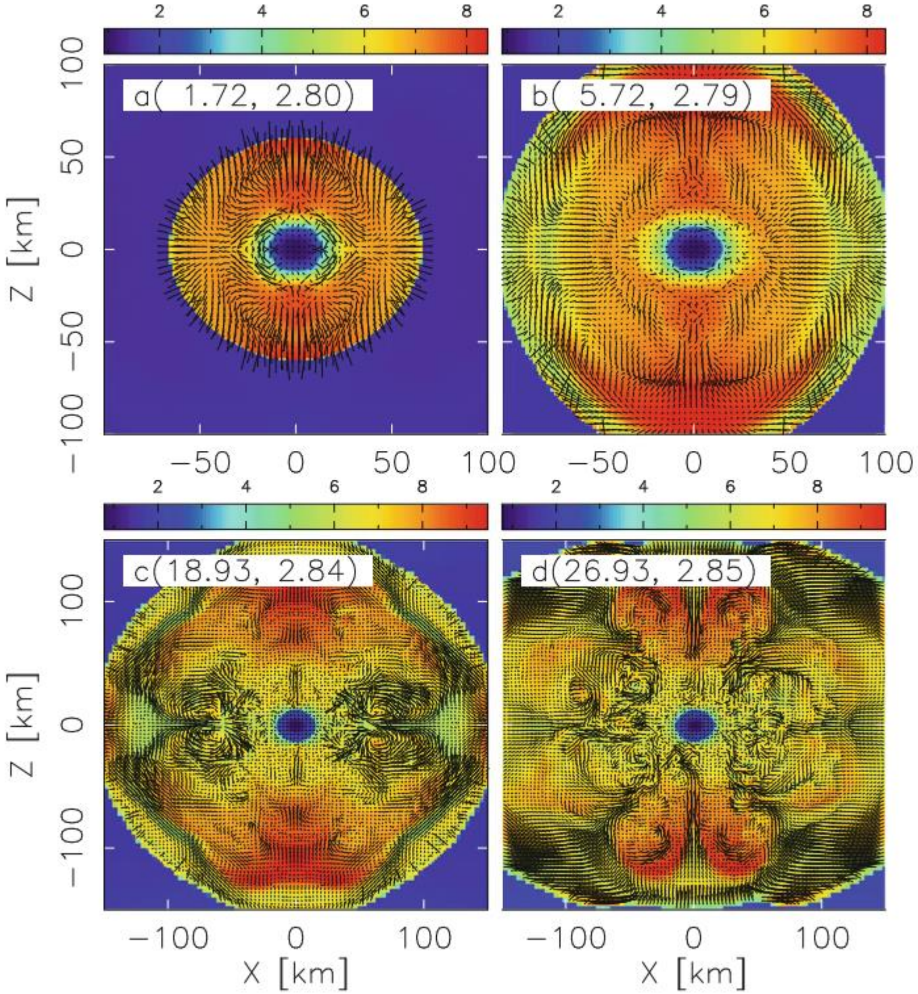
$$e^- + (A, Z) \rightarrow \nu_e + (A, Z-1)$$

2. Core bounce and shock formation:
   - Stop of the implosion at nuclear density $\rho_0 \approx 2.7 \times 10^{14}\ \mathrm{g/cm^3}$. Inner-core incompressibility ($M \sim 0.5\ \mathrm{M_\odot}$) creates a shock front
3. Shock stagnation and electron neutrino emission
4. Neutrino heating and accretion:
   - Reabsorption of ν_e and $\bar{\nu}_e$ by free neutrons and protons leads to convectively unstable layers (SASI, Rayleigh-Taylor plumes, ...)
5. Shock front revival and nucleosynthesis:
   - Non-radial instabilities favor neutrino heating with radioactive nuclei production (e.g. $^{56}\mathrm{Ni}$)
6. Explosion and compact remnant with neutrino wind

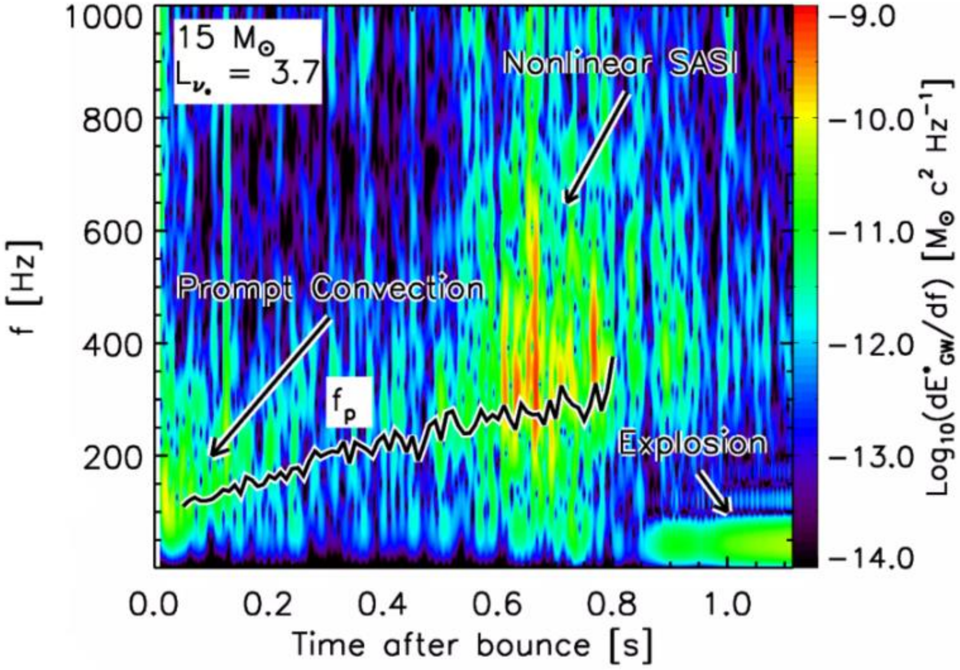


Rotating and non-rotating CCSNe

Rotating model



Non-rotating model

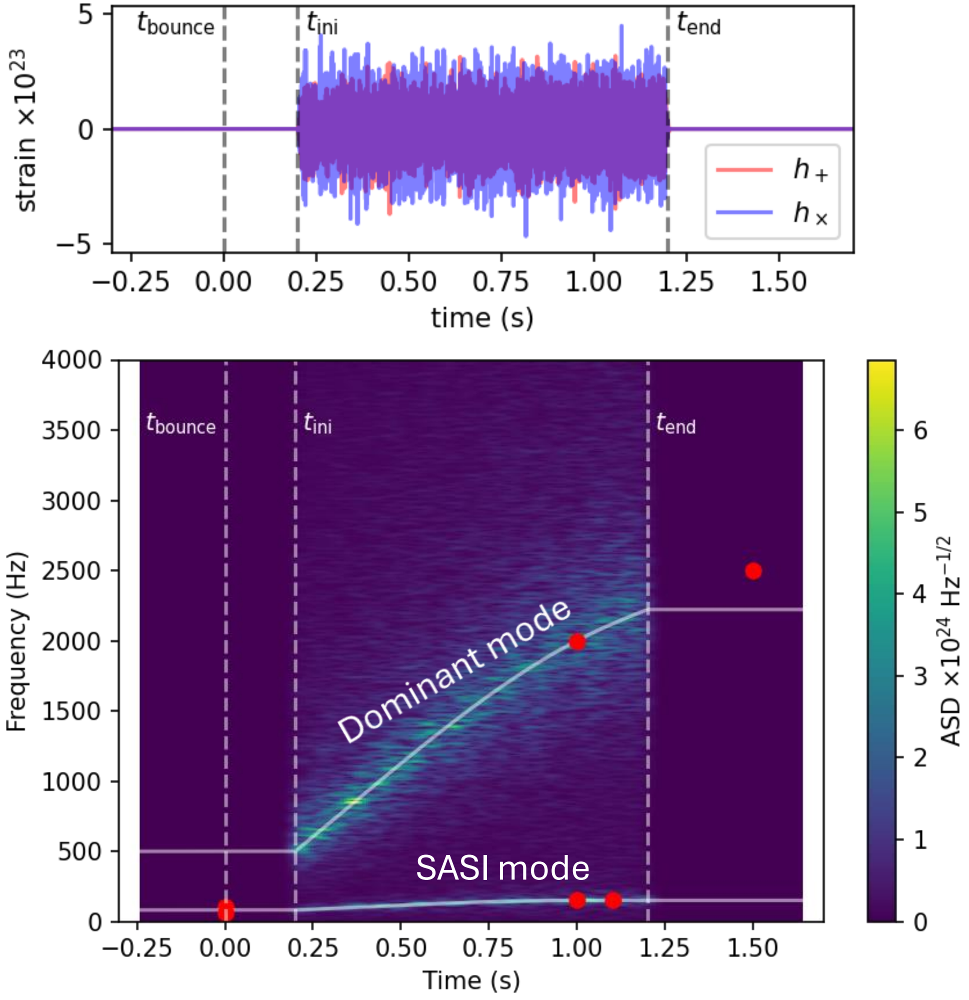


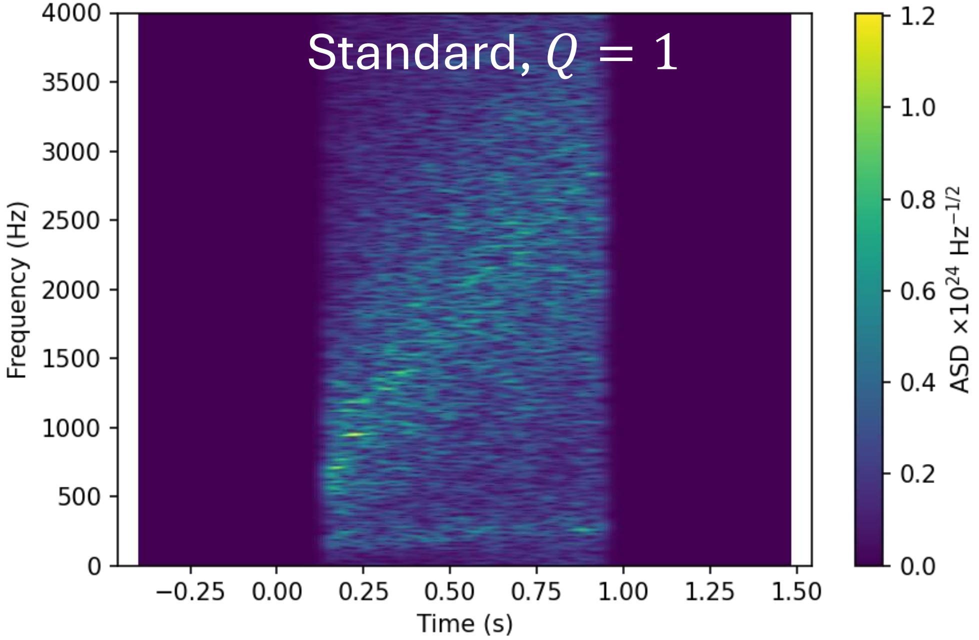
Waveforms from non-rotating progenitors

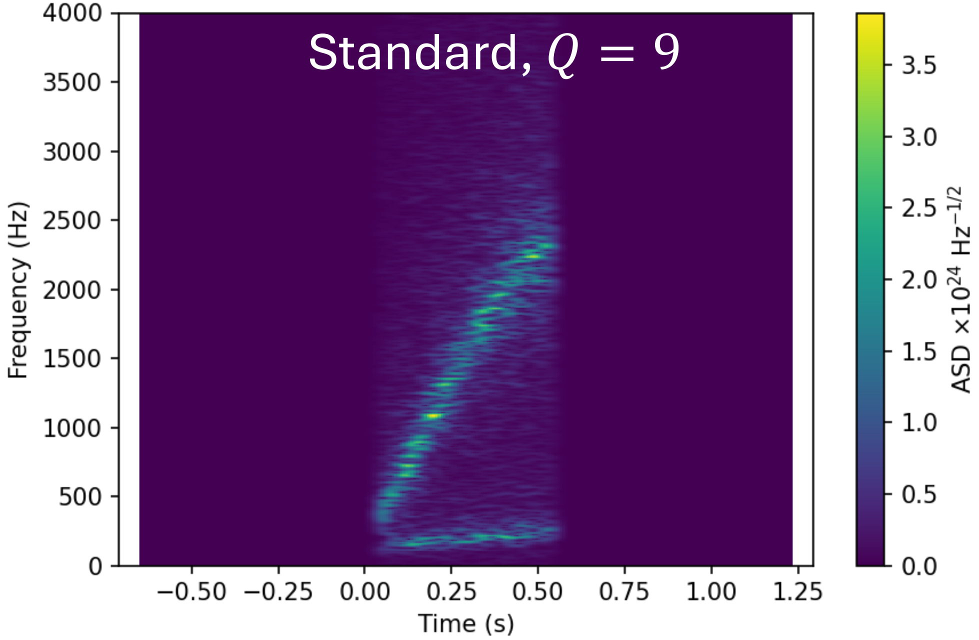
- The waveforms are generated with the *ccphen* code written by Prof. Pablo Cerdá-Durán
- Fundamental equation for time derivatives of the quadrupole moment I'_{lm} ($l = 2$) integrated during numerical analysis:

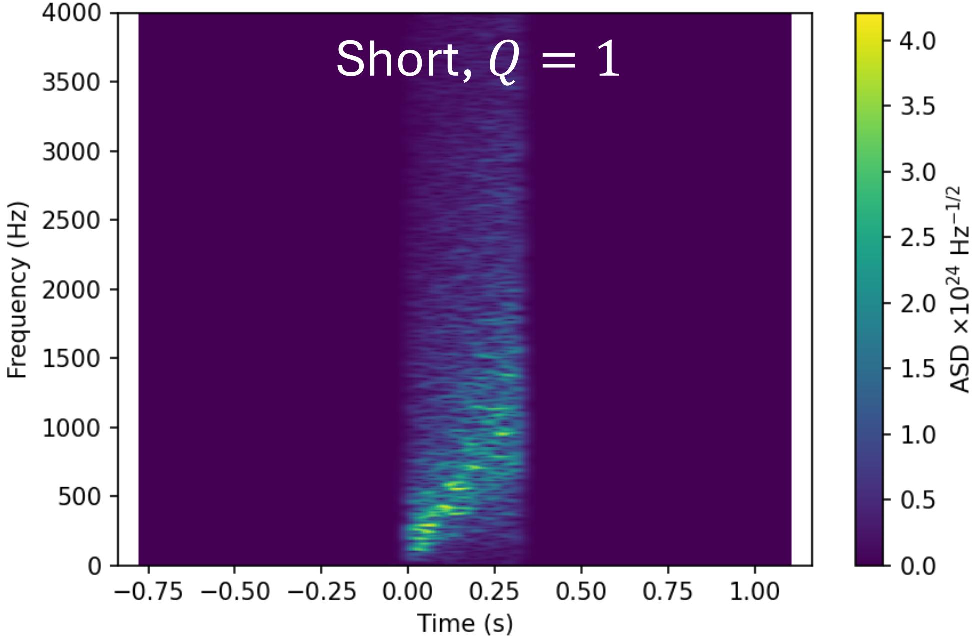
$$\boxed{\ddot{I}'_{lm} + \frac{\omega_{0,l}(t)}{Q_l}\dot{I}'_{lm} + \omega^2_{0,l}(t) I'_{lm} = W_{lm}(t)}$$

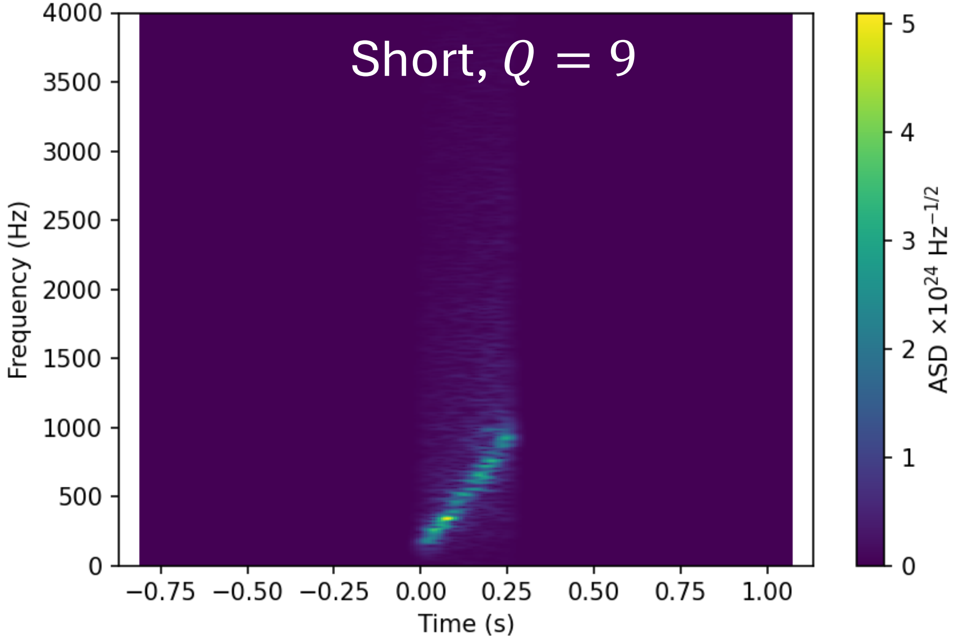
- Considered cases:
  - ➢ *Standard neutrino-driven supernovae with SASI*: suited to the most common cases ($M > 10\ \mathrm{M}_\odot$); minimum GW emission set to $0.4\ s$
  - ➢ *Short neutrino-driven supernovae*: adapted to low-mass progenitors ($M < 10\ \mathrm{M}_\odot$); duration limited to $0.1 - 0.4\ s$ without SASI (takes longer time to develop)




Standard, $Q = 1$
Frequency (Hz)
0
500
1000
1500
2000
2500
3000
3500
4000
Time (s)
−0.25
0.00
0.25
0.50
0.75
1.00
1.25
1.50
ASD ×10^24 Hz^−1/2
0.0
0.2
0.4
0.6
0.8
1.0
1.2


Standard, $Q = 9$
Frequency (Hz)
0
500
1000
1500
2000
2500
3000
3500
4000
Time (s)
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
1.25
ASD ×10^24 Hz^−1/2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5


Short, $Q = 1$
Frequency (Hz)
0
500
1000
1500
2000
2500
3000
3500
4000
Time (s)
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
ASD ×10^24 Hz^−1/2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0


Short, $Q = 9$
Frequency (Hz)
0
500
1000
1500
2000
2500
3000
3500
4000
Time (s)
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
ASD ×10^24 Hz^−1/2
0
1
2
3
4
5

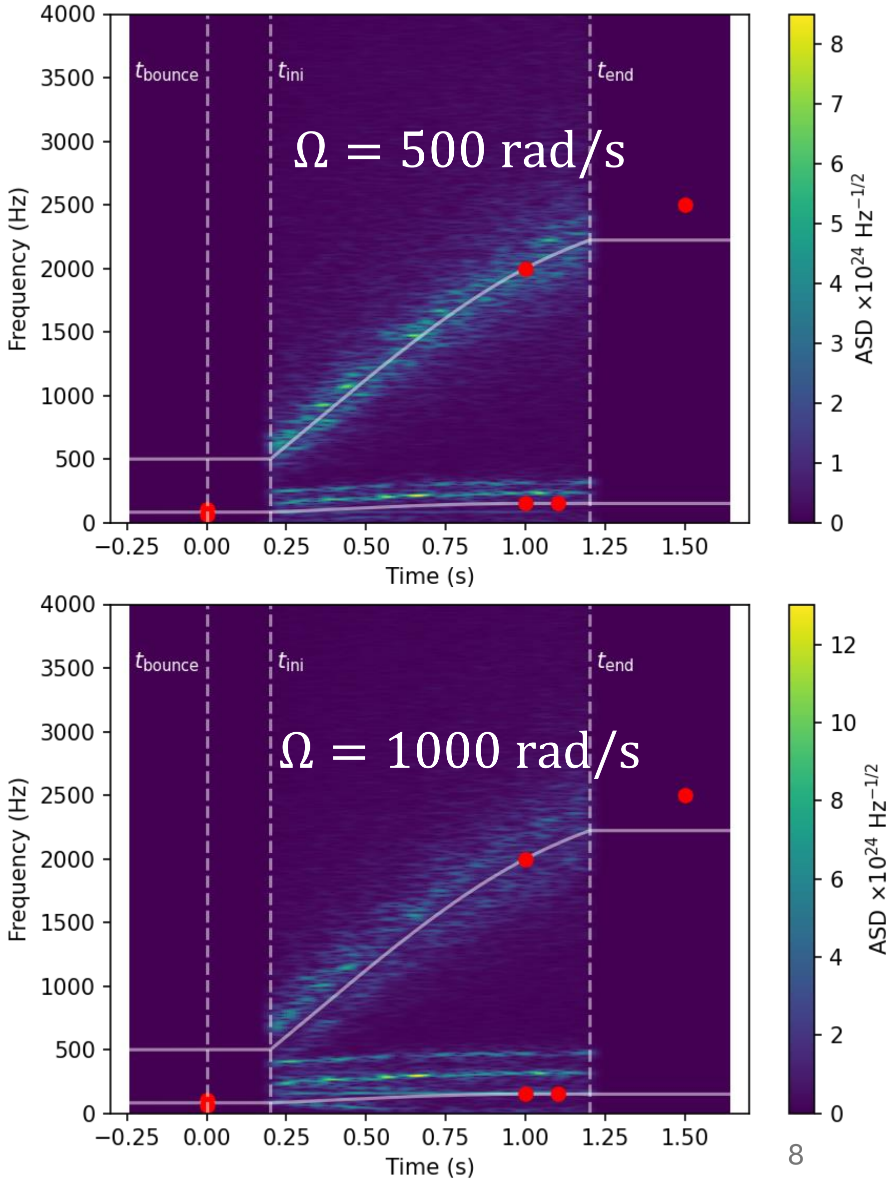
Waveforms from rotating models

- Two reference systems: co-rotating and laboratory frames
- Assuming constant rotation rate Ω
- Quadrupole moment in the laboratory frame:

$$\boxed{\ddot{I}_{lm} = (\ddot{I}^{\mathrm{co}}_{lm} - 2\mathrm{i}m\Omega\dot{I}^{\mathrm{co}}_{lm} - m^2\Omega^2 I^{\mathrm{co}}_{lm})e^{-\mathrm{i}m\Omega t}}$$

- If $I^{\mathrm{co}}_{lm} = Ae^{\mathrm{i}\sigma t}$, with A being a constant and σ the GW angular frequency, we have the so-called rotational splitting:

$$\boxed{\ddot{I}_{lm} = (-\sigma^2 + 2m\Omega\sigma - m^2\Omega^2)Ae^{\mathrm{i}(\sigma - m\Omega)t}}$$



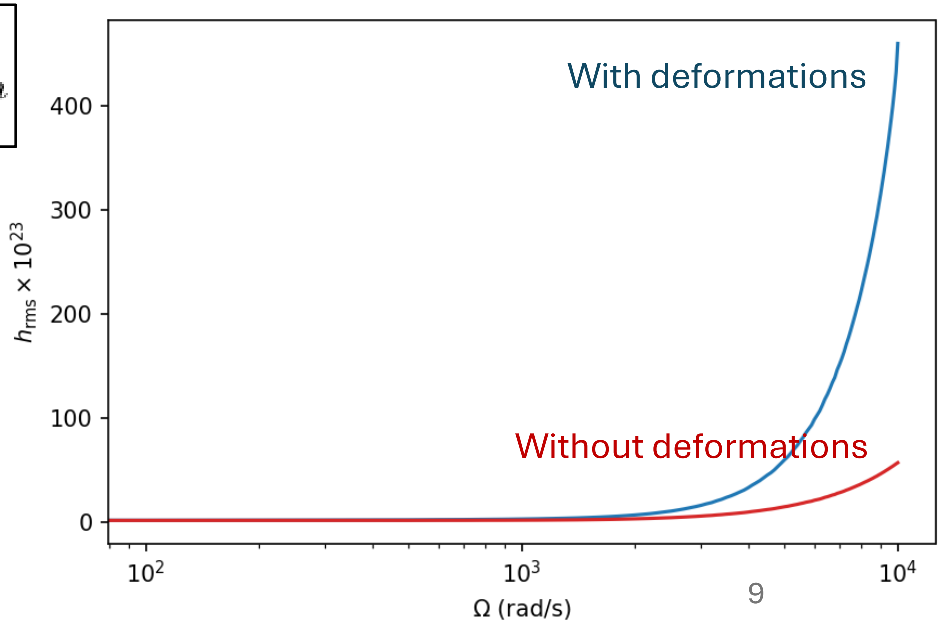
Waveforms from rotating models with deformations

- GW signal $(l = 2, m)$ axisymmetric perturbation from rotation-induced deformations of oscillation modes (l', m')
- Spherical PNS deformed to an oblate ellipsoid: displacement vector $\vec{X} = X^r(r)\hat{\mathbf{r}}$ decomposed in spherical harmonics, $X^r = \sum_{l=0}^{\infty} X_l^r(r) Y_{l0}(\theta, \phi)$
- Reference frame adapted to the star (r', θ', ϕ'), so to have isodensity surfaces at constant r'
- Variation of the quadrupole moment from dominant mode contribution $(l' = 2)$:

$$\boxed{\delta I_{2m} = \delta I_{2m,2m} = \left(1 + 5\mathcal{S}_{0m}\frac{X_0^r}{r'} + 5\mathcal{S}_{2m}\frac{X_2^r}{r'}\right) \cdot \int_0^{R_s} dr'\, r'^4\, \delta\rho_{2m}}$$

- Assuming constant c and ellipticity e (between 0 and 1):

$$\boxed{X_0^r = \sqrt{4\pi}\, c\, r'} \qquad \boxed{X_2^r = -\sqrt{\frac{16\pi}{5}}\frac{(3 - e^2) - 3\sqrt{1 - e^2}}{3 + e^2}(1 + c)\, r'}$$



Conclusions and future perspectives

- Core-Collapse Supernovae (CCSNe) are type II supernovae with masses $M \sim 8 - 100\ \mathrm{M}_{\odot}$ that have reached the Chandrasekhar limit of $1.4\ \mathrm{M}_{\odot}$ of their iron core
- The most prominent scenario is the revival of the stalling shock by neutrino heating, the so-called *neutrino-driven mechanism*: low-mass stars back up this mechanism, but massive progenitors couldn't be suited to this description
- Rotating CCSNe have a different gravitational wave (GW) emission as compared to non-rotating models: rotational splitting according to the value of the angular frequency Ω
- Proto-neutron star spherical shape alteration to an ellipsoidal one induced by rotation: the GW signal is amplified in terms of the ellipticity

- As a future outlook, the parameters controlling the GW emission in presence of rotation-induced perturbations could be related to the angular velocity
- Improvements of 3D simulations could sustain the neutrino-driven mechanism also for massive progenitors

References

[1] P. Cerdá-Durán et al. “Phenomenological gravitational waveforms for core collapse supernovae”. Draft to be presumably published in 2024.

[2] K. Kotake and T. Kuroda. “Gravitational Waves from Core-Collapse Supernovae”. In: *Handbook of Supernovae*. Springer International Publishing, 2017.

[3] H.-Th. Janka. “Neutrino-driven Explosions”. In: *Handbook of Supernovae*. Springer International Publishing, 2017.

[4] A. Torres-Forné et al. “Towards asteroseismology of core-collapse supernovae with gravitational wave observations – II. Inclusion of space–time perturbations”. In: *Monthly Notices of the Royal Astronomical Society* 482 (2018).