Spectroscopy of Magnetized Black Holes and Topological Stars

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In collaboration with:
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Based on:
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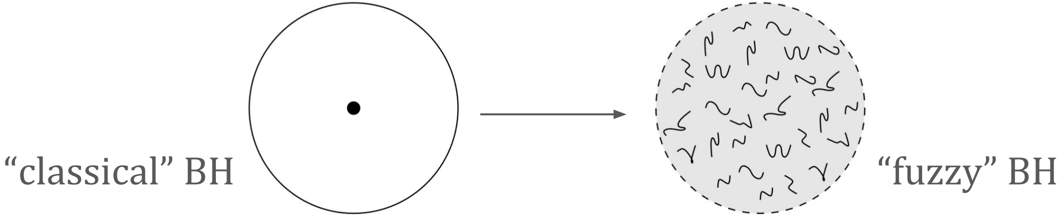
Outline

- The Big Picture: BH microstructure
- Topological Stars as toy models
- Linear stability analysis:
  - Regge-Wheeler-Zerilli perturbation scheme
  - Linear response in t-domain: stability, echoes & QNM spectrum
- Summary & Follow-up

The big picture: BH microstructure



Mathur (2025)

- **The Fuzzball program**: describing the BH microstructure
- BH as ensemble of **many**, **regular** and **horizonless** microstates
- Catch **B-H Entropy**, **Singularity** and **Information Paradox** with one stone!

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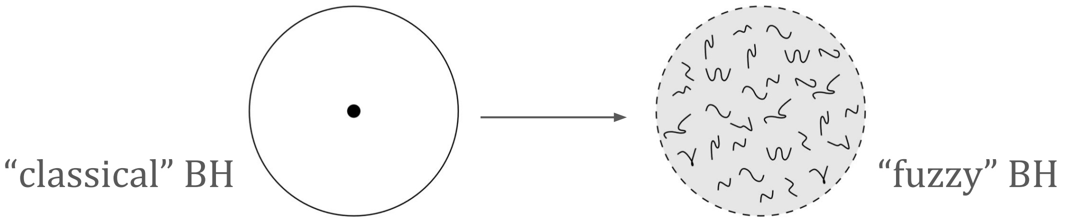

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Bah & Heidmann (2021)

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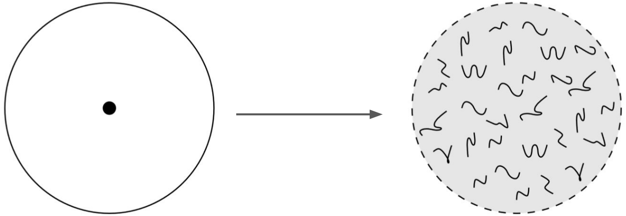

"classical" BH → "fuzzy" BH

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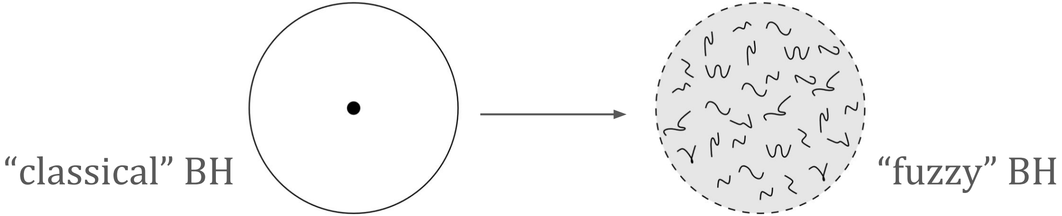

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CHALLENGE ACCEPTED!

Topological Stars in 5D

5D Einstein+Maxwell:

$$S_5 = \int d^5x\sqrt{-\mathbf{g}}\left(\frac{1}{2\kappa_5^2}\mathbf{R} - \frac{1}{4}\mathbf{F}_{AB}\mathbf{F}^{AB}\right)$$

$$ds^2 = -f_S(r)dt^2 + f_B(r)dy^2 + \frac{1}{h(r)}dr^2 + r^2 d\Omega_2^2 \qquad F = P\sin\theta\, d\theta \wedge d\phi$$

$$f_B(r) = 1 - \frac{r_B}{r}, \quad f_S(r) = 1 - \frac{r_S}{r}, \quad h(r) = f_B(r) f_S(r), \quad P = \pm\kappa_5^{-1}\sqrt{\frac{3}{2}r_B r_S}$$

Bah & Heidmann (2021)

- **horizon** r_S
- **end-of-spacetime** r_B

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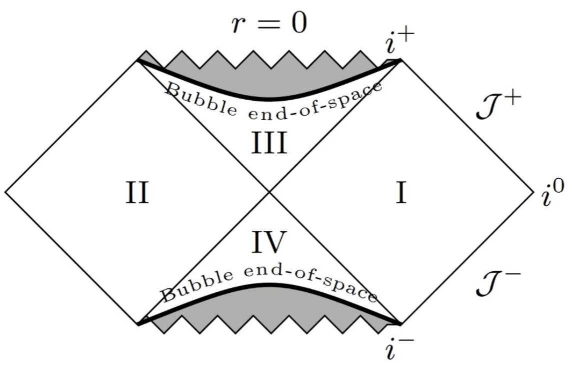
- **Magnetized Black String**: $r_B < r_S$



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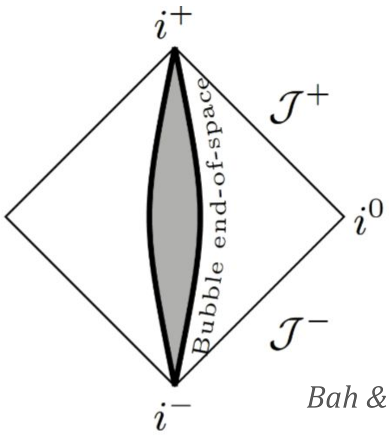
- **Magnetized Black String**: $r_B < r_S$
- **Topological Star**: $r_B > r_S$



Bah & Heidmann (2021)

Topological Stars in 5D

- **1st kind TS**: $\frac{3}{2}r_S \leq r_B \leq 2r_S$

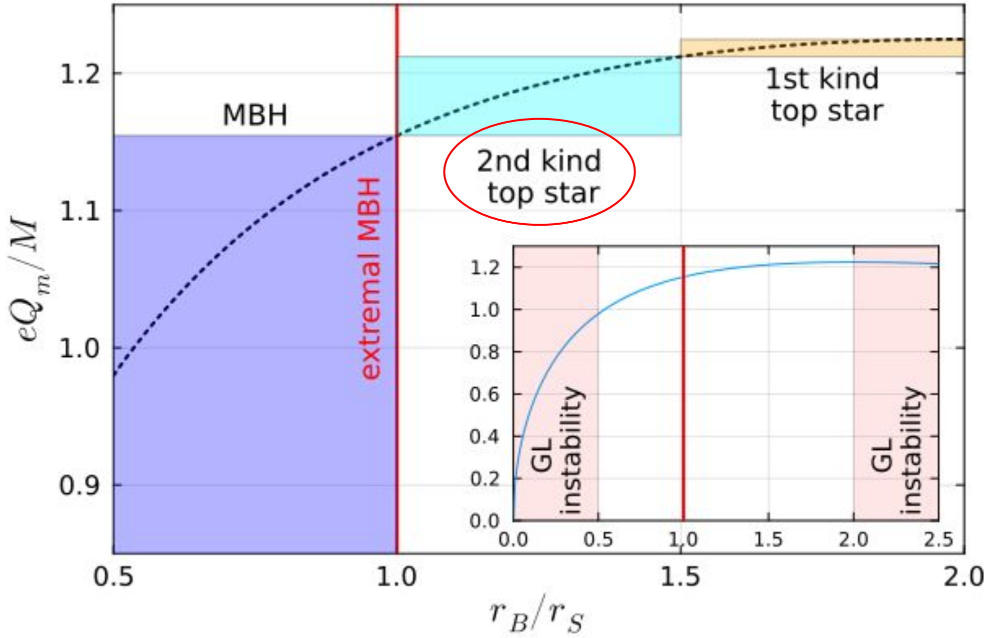
  $r_{(ph)} = r_B$

  (unstable)

- **2nd kind TS** : $r_S < r_B < \frac{3}{2}r_S$

  $r_{(ph)} = \frac{3}{2}r_S\,, \quad r_{(ph)} = r_B$

  (stable) (unstable)



Gregory-Laflamme instability:

$$r_B < \frac{1}{2}r_S\,, \quad r_B > 2r_S$$

Kaluza-Klein reduction to 4D

KK reduction (assume no y-dependence):

$$ds_5^2 = e^{-\frac{\sqrt{3}}{3}\Phi} ds_4^2 + e^{2\frac{\sqrt{3}}{3}\Phi}(dy + \cancel{A_\mu} dx^\mu)^2$$

$$\mathbf{F}_{AB} dx^A dx^B = F_{\mu\nu} dx^\mu dx^\nu + (\cancel{\partial_\mu \Xi} dx^\mu) \wedge (dy + A_\mu dx^\mu)$$

decouple!

Einstein-Maxwell-scalar:

$$\mathcal{S} = \int dx^4 \sqrt{-g} \left[\frac{1}{2\kappa_4^2} \left(R - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi \right) - \frac{1}{4e^2} e^{\frac{\sqrt{3}}{3}\Phi} F_{\mu\nu} F^{\mu\nu} \right]$$

Field equations:

$$G_{\mu\nu} - \frac{1}{2}\left(\partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} \partial_\rho \Phi \partial^\rho \Phi \right) + \frac{\kappa_4^2}{e^2} e^{\frac{\Phi}{\sqrt{3}}} \left(F_{\mu\rho} F^\rho{}_\nu + \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right) = 0$$

$$\Box \Phi - \frac{\sqrt{3}\kappa_4^2}{6e^2} e^{\frac{\Phi}{\sqrt{3}}} F_{\mu\nu} F^{\mu\nu} = 0 \qquad \nabla^\rho \left(e^{\frac{\sqrt{3}}{3}\Phi} F_{\mu\rho} \right) = 0$$

Topological Star in 4D

Compactified TS(/MBH):

$$ds_4^2 = -f_S f_B^{1/2} dt^2 + \frac{1}{f_S f_B^{1/2}} dr^2 + r^2 f_B^{1/2} d\Omega_2^2 \qquad F = \pm e Q_m \sin\theta \, d\theta \wedge d\phi \qquad \Phi = \frac{\sqrt{3}}{2} \log f_B$$

Mass and charge:

$$M = \frac{2\pi}{\kappa_4^2}(2r_S + r_B), \quad Q_m = \frac{1}{\kappa_4}\sqrt{\frac{3}{2} r_S r_B}$$

- Background for perturbation scheme
- Warning! **Compactification introduces singularities**!
- 5D solution implies regularity boundary conditions

Regge-Wheeler-Zerilli perturbation scheme

magnetic charge background → parity-mixing of perturbations $F = \pm e Q_m \sin\theta \, d\theta \wedge d\phi$

See **Pereñiguez (2021)** for alternative approach

Type-I

axial gravitational + polar EM (l≥1)

$$h_{\mu\nu}^{\rm odd} = \sum_{l,m} \begin{pmatrix} 0 & 0 & -h_0(t,r)/\sin\theta\partial_\phi & h_0(t,r)\sin\theta\partial_\theta \\ 0 & 0 & -h_1(t,r)/\sin\theta\partial_\phi & h_1(t,r)\sin\theta\partial_\theta \\ -h_0(t,r)/\sin\theta\partial_\phi & -h_1(t,r)/\sin\theta\partial_\phi & 0 & 0 \\ h_0(t,r)\sin\theta\partial_\theta & h_1(t,r)\sin\theta\partial_\theta & 0 & 0 \end{pmatrix} Y_{lm}(\theta,\phi)$$

$$f_{\mu\nu}^{\rm even} = \sum_{l,m} \begin{pmatrix} 0 & f_{01}^+(t,r) & f_{02}^+(t,r)\partial_\theta & f_{02}^+(t,r)\partial_\phi \\ -f_{01}^+(t,r) & 0 & f_{12}^+(t,r)\partial_\theta & f_{12}^+(t,r)\partial_\phi \\ -f_{02}^+(t,r)\partial_\theta & -f_{12}^+(t,r)\partial_\theta & 0 & 0 \\ -f_{02}^+(t,r)\partial_\phi & -f_{12}^+(t,r)\partial_\phi & 0 & 0 \end{pmatrix} Y_{lm}(\theta,\phi)$$

Type-II

polar gravitational + polar scalar + axial EM (l≥0)

$$h_{\mu\nu}^{\rm even} = \sum_{l,m} \begin{pmatrix} f_S f_B^{1/2} H_0(t,r) & H_1(t,r) & 0 & 0 \\ H_1(t,r) & f_S^{-1} f_B^{-1/2} H_2(t,r) & 0 & 0 \\ 0 & 0 & r^2 f_B^{1/2} K(t,r) & 0 \\ 0 & 0 & 0 & r^2 f_B^{1/2} \sin\theta^2 K(t,r) \end{pmatrix} Y_{lm}(\theta,\phi)$$

$$f_{\mu\nu}^{\rm odd} = \sum_{l,m} \begin{pmatrix} 0 & 0 & f_{02}^-(t,r)/\sin\theta\partial_\phi & -f_{02}^-(t,r)\sin\theta\partial_\theta \\ 0 & 0 & f_{12}^-(t,r)/\sin\theta\partial_\phi & -f_{12}^-(t,r)\sin\theta\partial_\theta \\ -f_{02}^-(t,r)/\sin\theta\partial_\phi & -f_{12}^-(t,r)/\sin\theta\partial_\phi & 0 & f_{23}^-(t,r)\sin\theta \\ f_{02}^-(t,r)\sin\theta\partial_\theta & f_{12}^-(t,r)\sin\theta\partial_\theta & -f_{23}^-(t,r)\sin\theta & 0 \end{pmatrix} Y_{lm}(\theta,\phi)$$

$$\delta\Phi = \sum_{l,m} \frac{\varphi(t,r)}{r} Y_{lm}(\theta,\phi)$$

Linear response in t-domain

Master equation for **Type-I (l>=1)** and **Type-II (l=0)**: $\Psi = \{\varphi_{l=0}\,, \mathcal{E}_{l=1}\,, Z_{1;l>1}\,, Z_{2;l>1}\}$

$$\left[\frac{d^2}{dt^2} - \frac{d^2}{d\rho^2} + V_{\text{eff}}\right] \Psi(t,\rho) = 0$$

Generalized tortoise coordinate: $$d\rho = \sqrt{\frac{g_{rr}}{g_{tt}}}dr = \frac{dr}{f_B^{1/2} f_S}$$

Effective potentials:

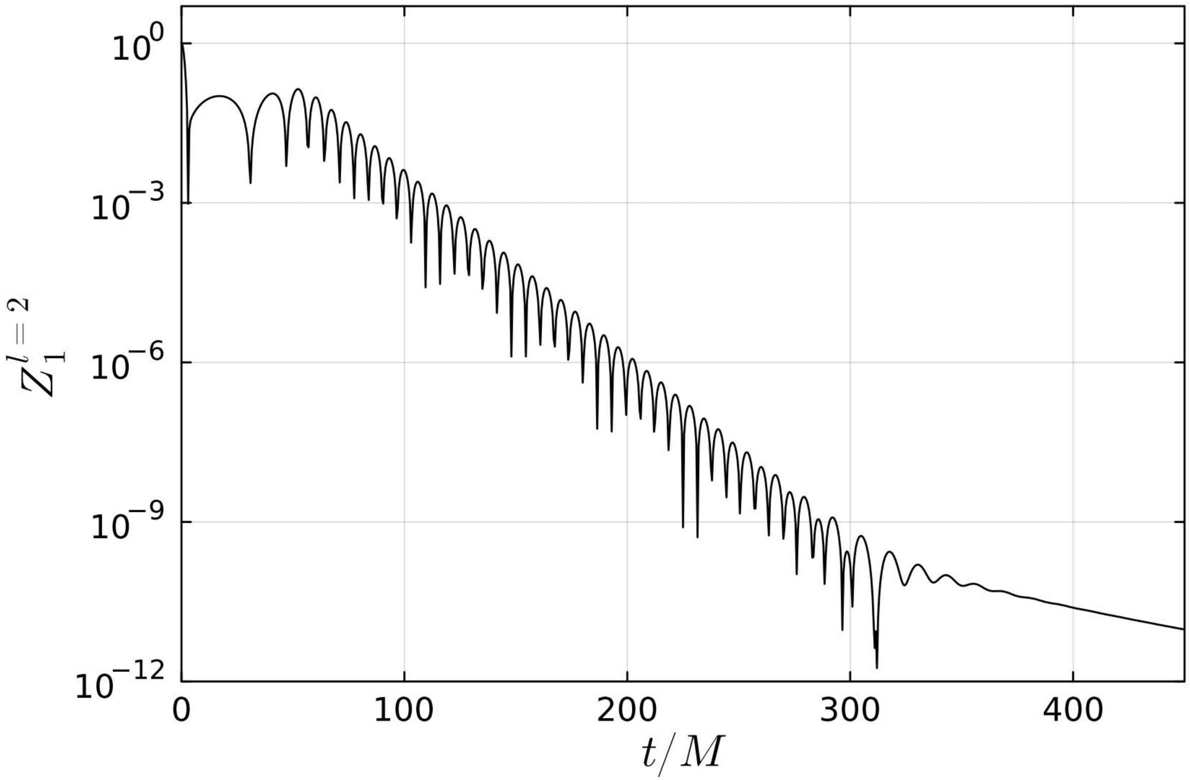
$l=0$	$V_{\text{eff}}^{l=0} = \frac{(r-r_S)\left(32r^3-24r^2r_B-39r_B^2r_S+rr_B(36r_S-5r_B)\right)}{16r^5(r-r_B)}$
$l=1$, Type-I	$V_{\text{eff}}^{l=1} = \frac{(r-r_S)\left(189r_B^4r_S+128r^4(r_B+2r_S)+64r^2r_B^2(6r_B+19r_S)-16r^3r_B(27r_B+52r_S)-9rr_B^3(9r_B+92r_S)\right)}{16r^5(4r-3r_B)^2(r-r_B)}$
$l\geq 2$, Type-I	$V_{\text{eff}}^{(1,2)} = \frac{(r-r_S)\left(16r^3\Lambda-r^2(8r_B+24r_S+16\Lambda r_B)+r(11r_B^2+60r_Br_S)-39r_B^2r_S\mp 8r(r-r_B)\sqrt{(2r_B-3r_S)^2+12r_Br_S\Lambda}\right)}{16r^5(r-r_B)}$

Type-II (l>0)? Coming soon!

Linear response in t-domain

Near-extremal MBH:

- (type-I) gravitational-induced perturbation
- fixed mass (M=1)
- ringdown + tail



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2nd kind Top Star:

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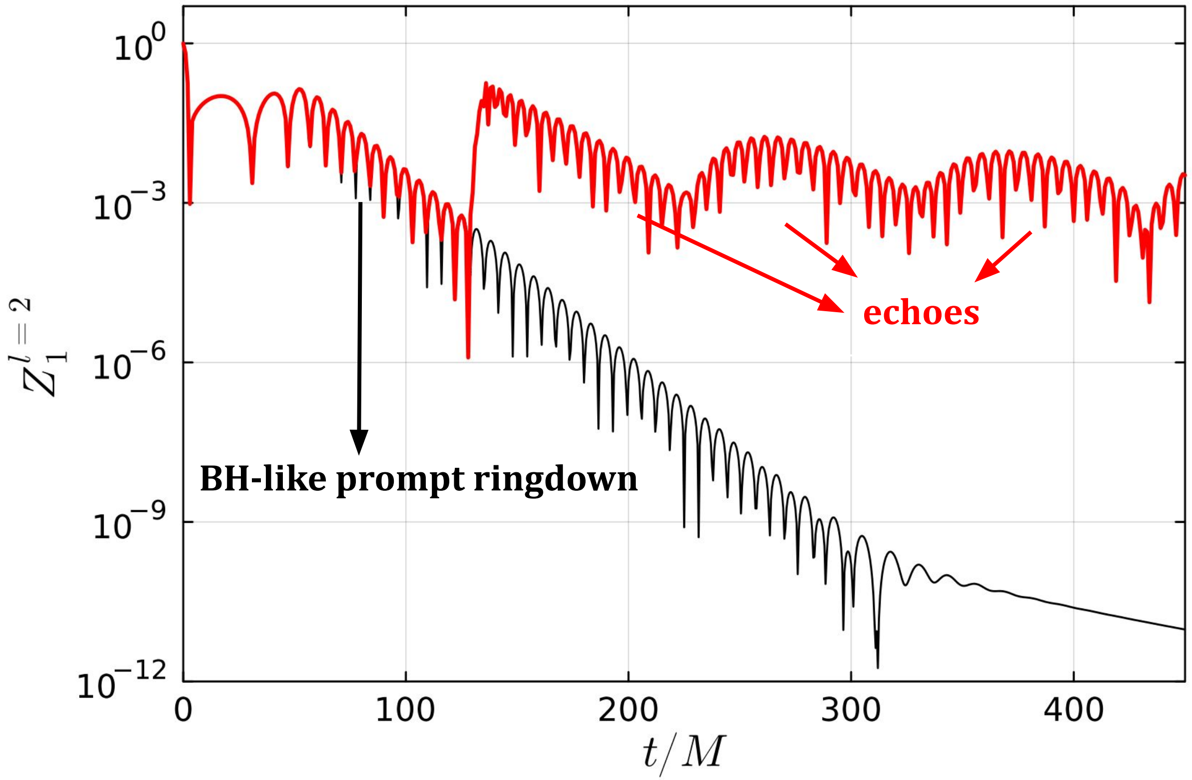
- very compact
- eΔQ/M≈0.3%
- initial **BH-like ringdown**
- long-lived modes: **echoes**!



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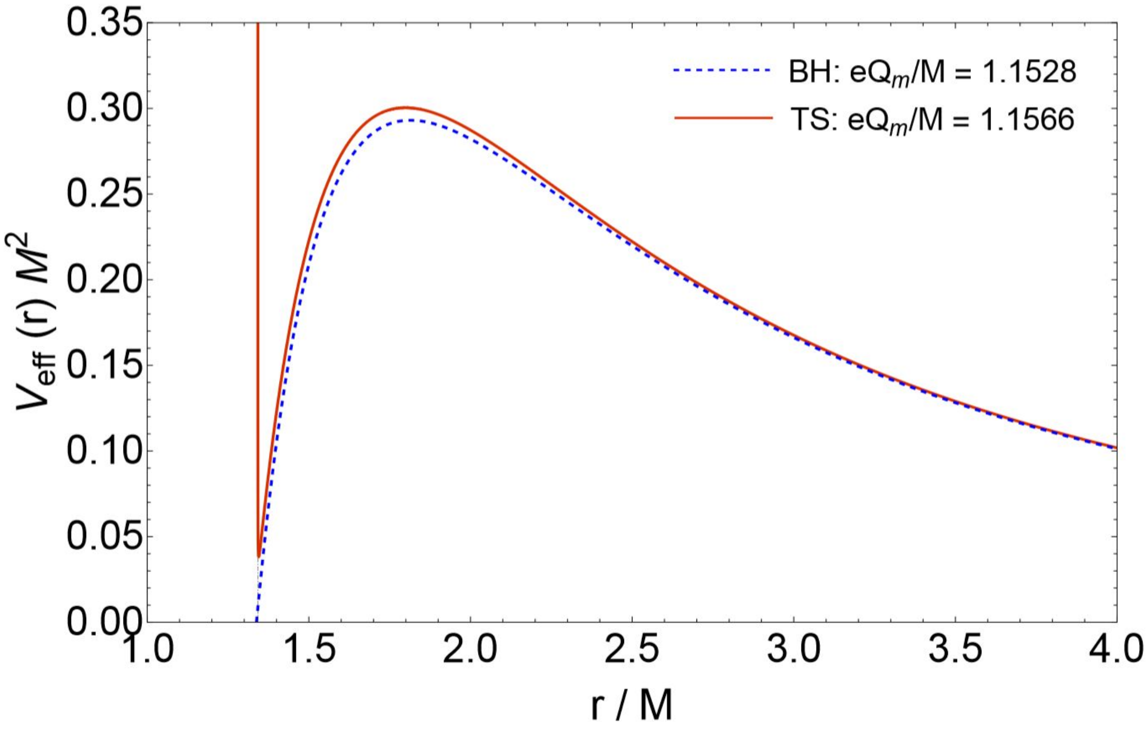

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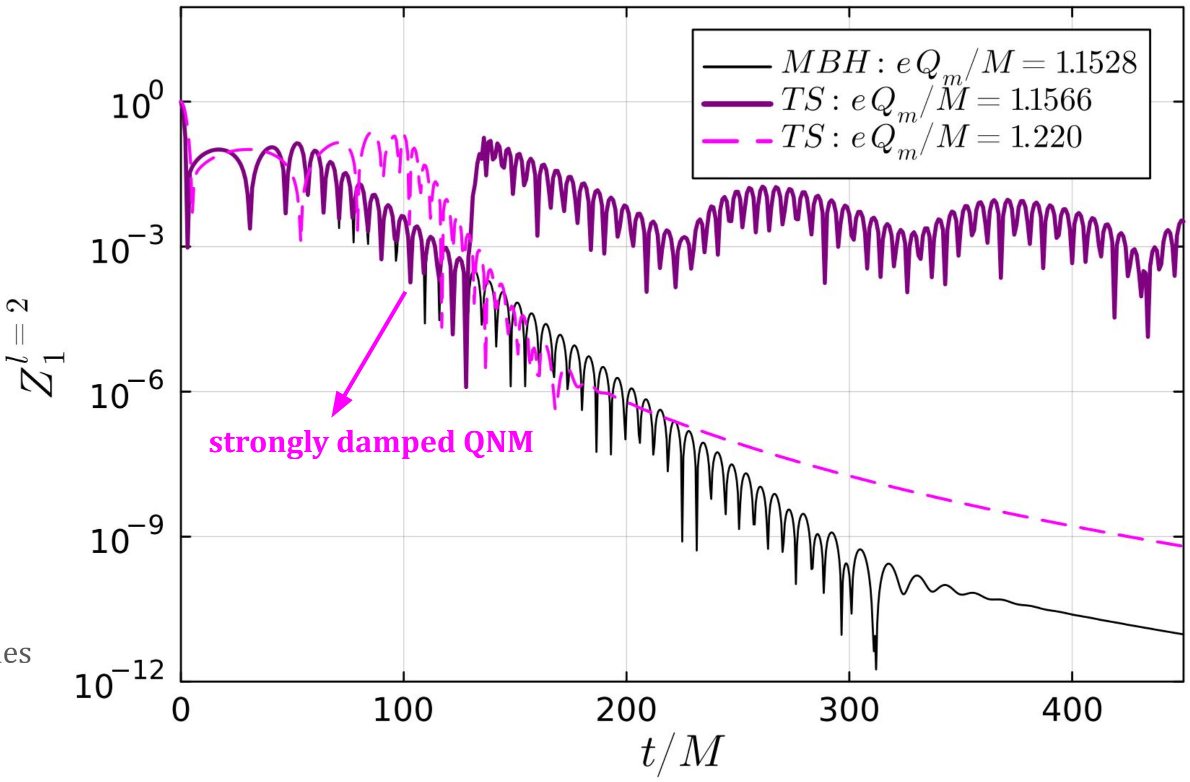
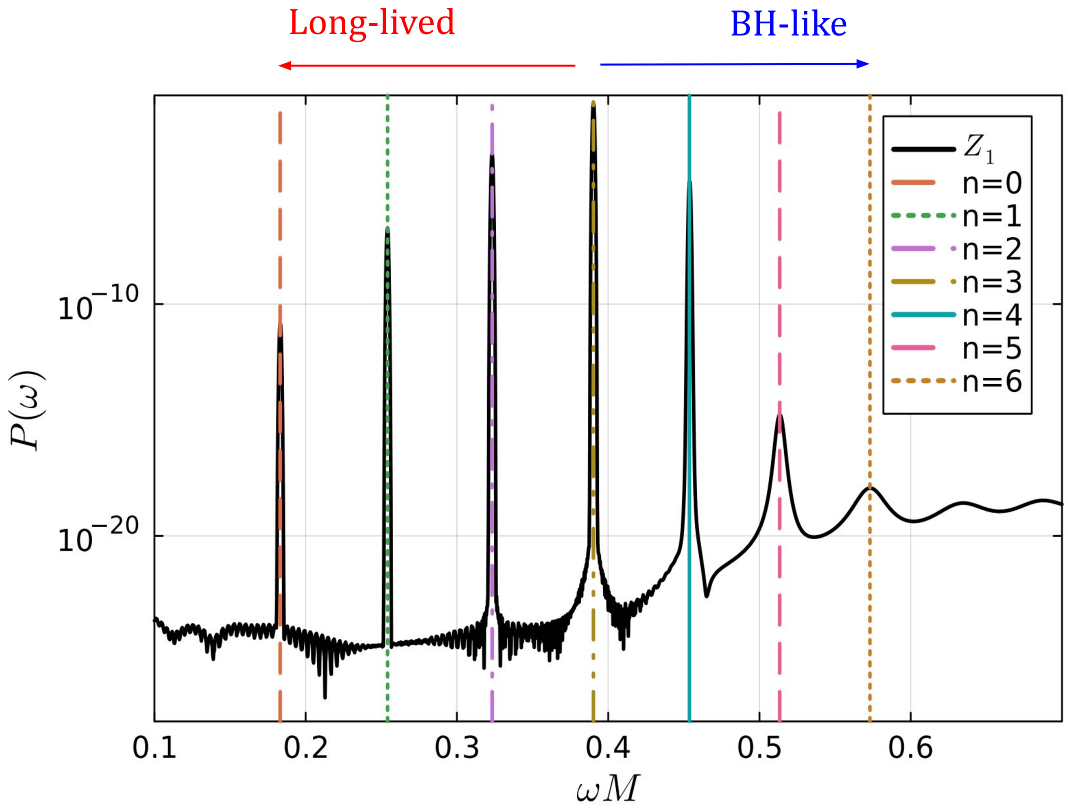
1st kind Top Star:

- fixed mass (M=1)
- star-like compactness
- highly-damped, higher frequencies (**w-modes**)



QNM Spectrum



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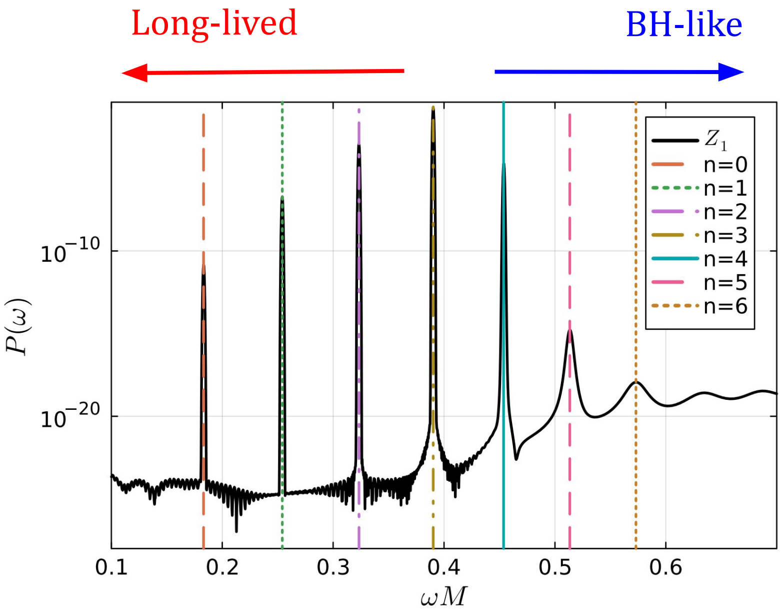

		Magnetized BH	TS, second kind	TS, first kind
$n=0$	f-domain	$0.489568 - i\,7.972 \times 10^{-2}$	$0.183217 - i\,4.674 \times 10^{-10}$	$0.644348 - i\,0.1551$
	t-domain	$0.489600 - i\,7.978 \times 10^{-2}$	$0.183219 - i\,3.349 \times 10^{-10}$	$0.643938 - i\,0.1665$
$n=1$	f-domain	-	$0.254071 - i\,6.001 \times 10^{-8}$	-
	t-domain	-	$0.254084 - i\,6.008 \times 10^{-8}$	-
$n=2$	f-domain	-	$0.323219 - i\,2.615 \times 10^{-6}$	-
	t-domain	-	$0.323263 - i\,2.622 \times 10^{-6}$	-
$n=3$	f-domain	-	$0.390169 - i\,6.116 \times 10^{-5}$	-
	t-domain	-	$0.390256 - i\,6.142 \times 10^{-5}$	-
$n=4$	f-domain	-	$0.453786 - i\,8.348 \times 10^{-4}$	-
	t-domain	-	$0.453832 - i\,8.340 \times 10^{-4}$	-
$n=5$	f-domain	-	$0.513765 - i\,5.463 \times 10^{-3}$	-
	t-domain	-	$0.513375 - i\,2.754 \times 10^{-3}$	-
$n=6$	f-domain	-	$0.574947 - i\,1.658 \times 10^{-2}$	-
	t-domain	-	$0.572869 - i\,1.140 \times 10^{-2}$	-

Summary & Follow-up

- **Topological Stars** as **toy models** of BH microstate geometries
- Two-parameter solution that **interpolates** between **regular UCOs** and **BHs**
- **Linear response in time** (this talk) and **frequency domain** (attend Marco's talk!)
- Verified **linear stability** under **Type-I** perturbations and **(radial) Type-II**
- **QNM spectrum** of MBH and Top Stars
  - **BH-like QNMs**
  - **Long-lived modes (echoes)**
  - **Strongly-damped (w-modes)**
- Next steps:
  - **Type-II, l>0**
  - **Nonlinear stability and 3+1 dynamics**
  - **More complex microstate geometries**

AD, M. Melis, P. Pani, 2406.19327
and
I. Bena, G. Di Russo, J. F. Morales, and
A. Ruiperez Vicente, 2406.19330