# Einstein Maxwell Scalar Black Hole Collisions and Gravitational Echoes

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# Intro

## 1 History and Motivation

- Einstein-Maxwell-Dilaton (EMD) theory originated as a low energy limit of string theory.
- EMD theory describes a spacetime with elctromagnetism and a scalar field called a dilaton; the dilaton has a specific coupling to the electro-magnetic field tensor in the lagrangean.
- In a similar fashion to Kaluza-Klein (KK) theory the dilaton represents the size of a *curled up* higher dimension.
- It provides black hole (BH) solutions with scalar hair aswell as the usual mass, electromagnetic charge and spin.
- BH solutions found by G. W. Gibbons and C. M. Hull in 1982.

# Motivation

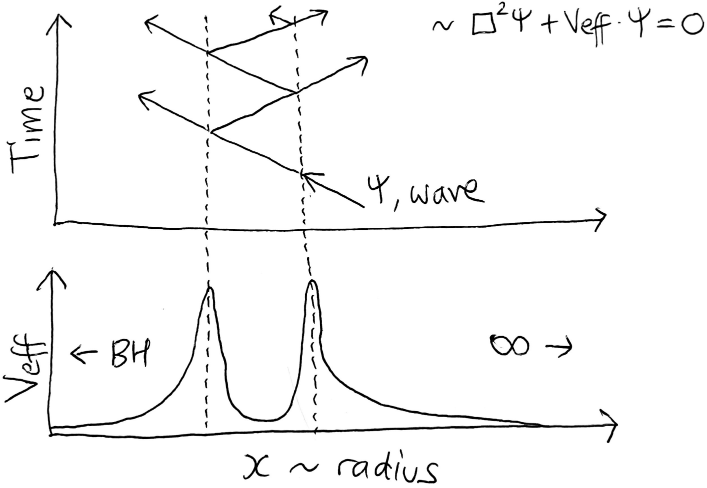
- Why do we care about EMD's (apart from string theory)?
- From a theoretical point of view they are an extension of General Relativity (GR) and can be used to gauge the nature of event horizons, singularity formation, black hole thermodynamics and more.
- They can be used to test strong gravity by simulating their gravitational wave (GW) signals and collision phenomonology.
- Current GW detectors are suited to binary inspirals of compact objects (i.e. EMD BH's).
- The number of theories deviating from GR that are well posed is limited; even if this exact theory is not viewed as correct it serves as a simple proxy for a large class of theories.

# EMS Theory and Echoes

- Einstein-Maxwell-Scalar (EMS) theory is a generalisation of EMD theory in which the dilaton coupling to electromagnetism is generic.
- This allows for a wider range of phenomenon to be investigated such as spontanious scalarisation and echoes.
- Echoes provide a *smoking gun* for a detector; for example they can imprint a periodic modulation on gravitational ringdown signals.
- Careful tuning of the dilaton coupling is required to support echoes. This often leads to near-critical solutions where the horizon shrinks to near zero and the mass approaches the theoretical limit.

# Pictorial Demonstration of Echoes





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2 Mathematical formulation

# Lagrangean

- The EMS lagrangean consist of a real scalar field and the electromagnetic field,

$$\mathcal{L} = \left( \frac{R}{8\pi} - 2g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - e^{-2\alpha\lambda(\phi)} F_{\mu\nu} F^{\mu\nu} \right) \sqrt{-g}. \tag{1}$$

- $\phi$ - real scalar field.
- $A_\mu$ - electromagnetic 4-vector.
- $F_{\mu\nu}$ - electromagnetic field tensor, $:= \nabla_\mu A_\nu - \nabla_\nu A_\mu$.
- $\alpha$ - dilaton/scalar coupling constant.
- Both $\phi$ and $F_{\mu\nu}$ are minimally coupled to gravity through $\sqrt{-g}$.
- Coupling function $\exp(-2\alpha\lambda(\phi)) \sim 1 - 2\alpha\lambda(\phi) + \ldots$ is between $\phi$ and $F_{\mu\nu}$.
- In the limit $\alpha = 0$ we have pure electromagnetism in curved space.

- The Euler-Lagrange equation for $g^{\mu\nu}$ gives the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{2}$$

$$T_{\mu\nu} = 2\nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \nabla\phi \cdot \nabla\phi + 2e^{-2\alpha\lambda(\phi)} F_{\mu\rho} F_\nu{}^\rho - \frac{1}{2} g_{\mu\nu} e^{-2\alpha\lambda(\phi)} F^2. \tag{3}$$

- The Euler-Lagrange equation for $\phi$ gives the Klein-Gordon equation,

$$g^{\mu\nu} \nabla_\mu \nabla_\nu \phi = -\frac{1}{2} \alpha \lambda'(\phi) e^{-2\alpha\lambda(\phi)} F^2. \tag{4}$$

- The Euler-Lagrange equation for $A_\mu$ gives the Maxwell equations,

$$\nabla_\mu \left( e^{-2\alpha\lambda(\phi)} F^{\mu\nu} \right) = 0. \tag{5}$$

## The Einstein Maxwell Dilaton Solution



- The EMD BH using the choice $\lambda(\phi) = \phi$ and has the ansatz,

$$\mathrm{d}s^2 = -f^2\mathrm{d}t^2 + \frac{1}{f^2}\mathrm{d}r^2 + \rho^2\mathrm{d}\Omega^2,$$

$$f^2 = \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{\frac{1-\alpha^2}{1+\alpha^2}} \quad \text{and} \quad \rho^2 = r^2\left(1 - \frac{r_-}{r}\right)^{\frac{2\alpha^2}{1+\alpha^2}},$$

$$A_\mu = \left(\frac{Q}{r}, 0, 0, 0\right) \quad \text{and} \quad F_{tr} = -\frac{Q}{r^2},$$

$$e^{2\alpha\phi} = \left(1 - \frac{r_-}{r}\right)^{\frac{2\alpha^2}{1+\alpha^2}}.$$

- More general choices of $\lambda(\phi)$ require numerical techniques to solve for black hole solutions.

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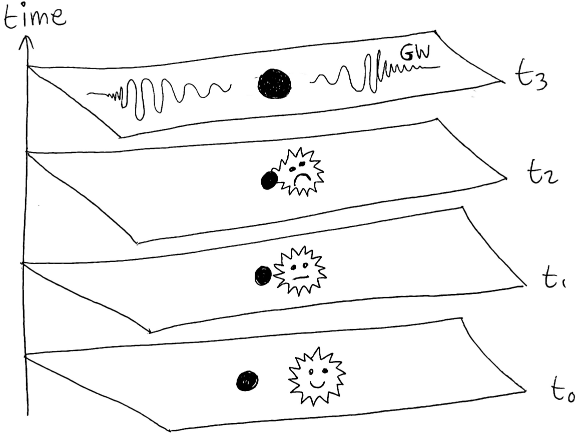
# Spacetime Foliation / Initial Value Problem






- Want initial data that covers an instance of time, $t_0$.
- Credit to F. Corelli (Sapienza) for creating initial data.
- Need 3+1 adapted evolution equations to evolve data foreward in time.
- System of PDE's must be strongly hyperbolic / well posed.

- To evolve the EMS system on a timelike hypersurface $\Sigma$ we need to decompose our tensor fields into temporal and spatial components.
- The metric is decomposed in the Arnowitt-Deser-Misner (ADM) form,

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -N^2\mathrm{d}t^2 + \gamma_{ij}\left(\mathrm{d}x^i + \beta^i \mathrm{d}t\right)\left(\mathrm{d}x^j + \beta^j \mathrm{d}t\right). \tag{6}$$

- Define the future directed, timelike unit vector $n_\mu = -N\mathrm{d}t$ which is normal to $\Sigma$. Alternatively $n^\mu = N^{-1}\{1, -\beta^i\}$.
- The electromagnetic field is decomposed as,

$$F_{\mu\nu} = n_\mu E_\nu - n_\nu E_\mu + {}^{(3)}\epsilon_{\mu\nu\rho}B^\rho. \tag{7}$$

- The scalar field $\phi$ does not need decomposing, but it is helpful to define the momemntum $\Pi = -n^\mu \partial_\mu \phi$.

# Numerical Evolution Scheme



- The code used is GRChombo, a modern fully adaptive mesh refinement (AMR) for numerical relativity (NR).
- GRChombo uses the CCZ4 formulation (BSSN + Z4 constraint damping) and the moving puncture gauge (MPG).
- Matter evolution equations for $\phi$ and $F_{\mu\nu}$ are given next.
- Comparing to the traditional Maxwell equations, we recover the evolution for $E_i$ and $B_i$ with two constraint equations.
- The constraints are promoted to evolution variables $\Lambda$ and $\Xi$ that are driven to $\Lambda \to 0$ and $\Xi \to 0$. Analogous to the traditional constraints $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$ and $\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho$.

# EMS Evolution Equations Summary



$$\partial_t \phi = \mathcal{L}_\beta \phi - N\Pi,$$

$$\partial_t \Pi = \mathcal{L}_\beta \Pi - \gamma^{\mu\nu}(D_\mu \phi)\partial_\nu N + N\left[\mathcal{K}\Pi - \gamma^{\mu\nu} D_\mu D_\nu \phi - \alpha\lambda'(\phi)e^{-2\alpha\lambda(\phi)}(B^2 - E^2)\right],$$

$$\partial_t E_i = \mathcal{L}_\beta E_i + NKE_i - 2NE^i K_{ij} + \epsilon_i^{\ jk}\left[B_k D_j N + N D_j B_k\right] + N D_i \Xi$$
$$- 2\alpha\lambda'(\phi) N\left[\epsilon_i^{\ jk} B_k D_j \phi + \Pi E_i\right],$$

$$\partial_t B_j = \mathcal{L}_\beta B_j + NKB_j - 2NB^i K_{ij} - \epsilon_j^{\ ik}\left[E_k D_i N + N D_i E_k\right] + N D_j \Lambda,$$

$$\partial_t \Xi = \mathcal{L}_\beta \Xi + N\left[D_i\left(E^i e^{-2\alpha\lambda(\phi)}\right) - \kappa \Xi\right],$$

$$\partial_t \Lambda = \mathcal{L}_\beta \Lambda + N\left[D_i B^i - \kappa_B \Lambda\right],$$

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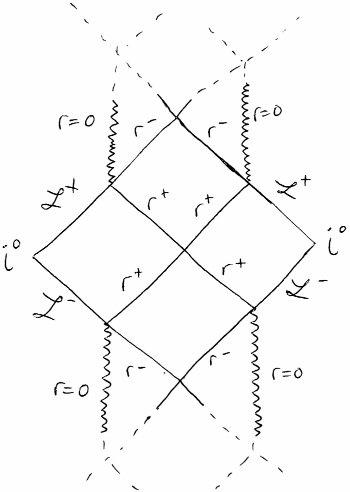
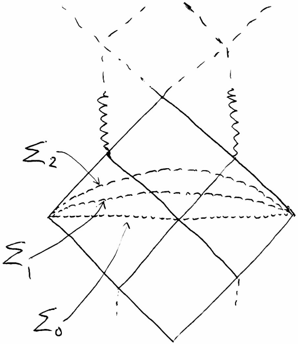
# Reissner Nordstrom Penrose Diagram

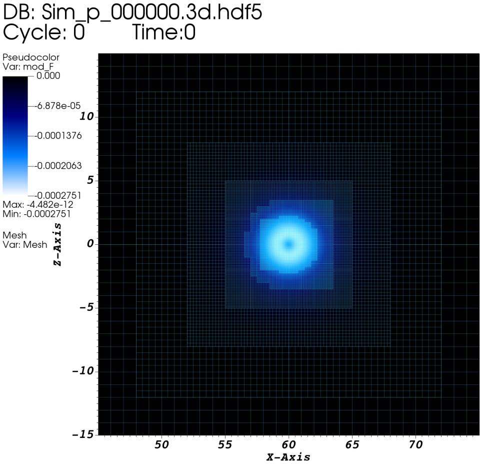
4 Numerical Relativity Example Simulation



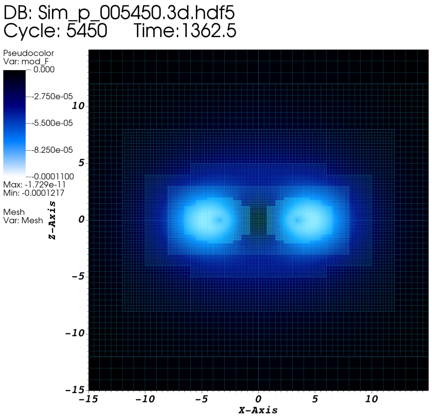
# Initial Data - Electromagnetic Field ($F_{\mu\nu}F^{\mu\nu}$)





# Pre and Post Merger Plot - $F_{\mu\nu}F^{\mu\nu}$

## 4 Numerical Relativity Example Simulation



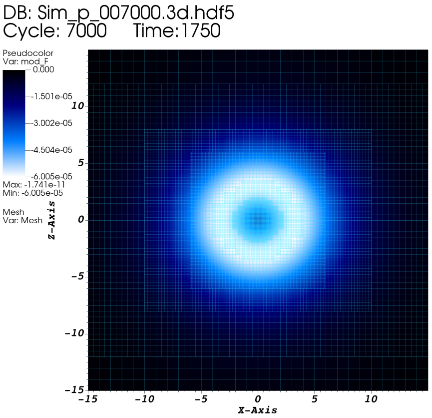
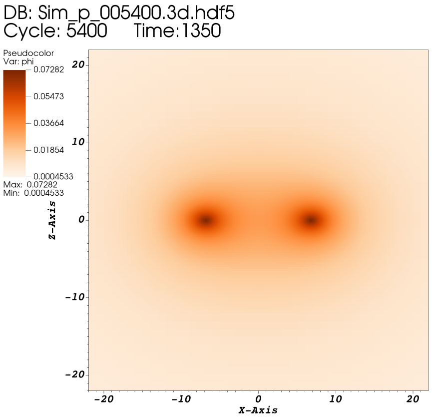




**Figure:** Left: Pre-Merger, Right: Post-Ringdown

# Pre and Post Merger Plot - $\phi$



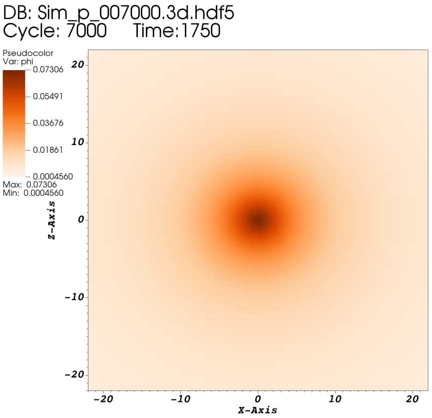




**Figure:** Left: Pre-Merger, Right: Post-Ringdown

# Gravitational Wave Signal

## 4 Numerical Relativity Example Simulation

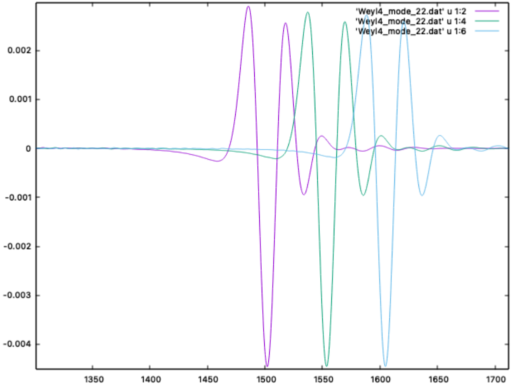



Figure: Gravitational radiation: x-axis time, y-axis $\psi_4$ Weyl scalar.

# Outlook

- Currently we can evolve EMS BH's and their collisions.
- Sadly the solutions with double peaked potentials we have found posses very small horizons and require extreme resolution to run if we want to see echoes. A potential workaround is to try find less extreme EMS BH's that also have a double peaked potential.
- Additionally I'm using a spherically symmetric evolution code for use with a perturbative approach - credit to M. Melis (Sapienza) for supplying the perturbative equations of motion and numerical effective potentials.
- The perturbative code is working well as a useful probe to see scalar/axial (soon polar?) echoes in the linear (soon nonlinear?) regime to give an idea of what to expect in 3D collisions or perturbation simulations.

# Einstein Maxwell Scalar Black Hole Collisions and Gravitational Echoes

*Thanks for listening!*

*Any questions?*

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