Perturbation of the Vaidya metric in the frequency domain: QNMs and Tidal Response

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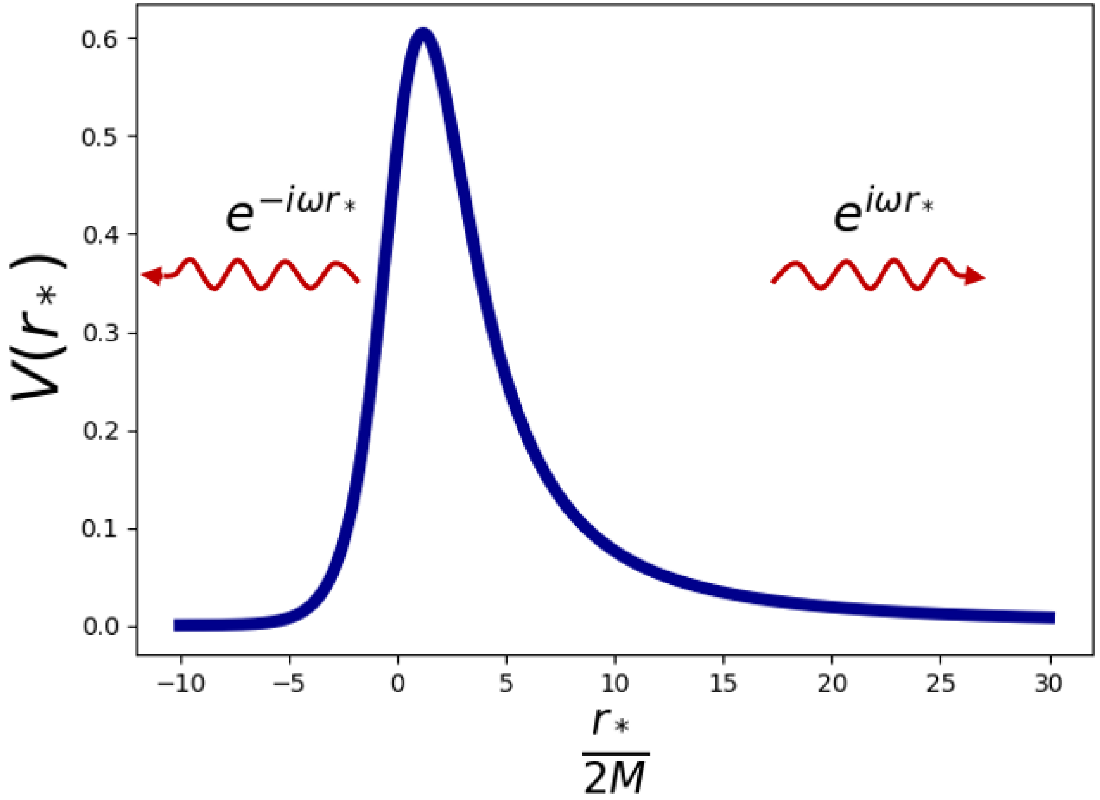
From arXiv:2407.06009
LC, Luca Santoni, Enrico Barausse

Quasi-Normal Modes

Schrödinger-like equation:

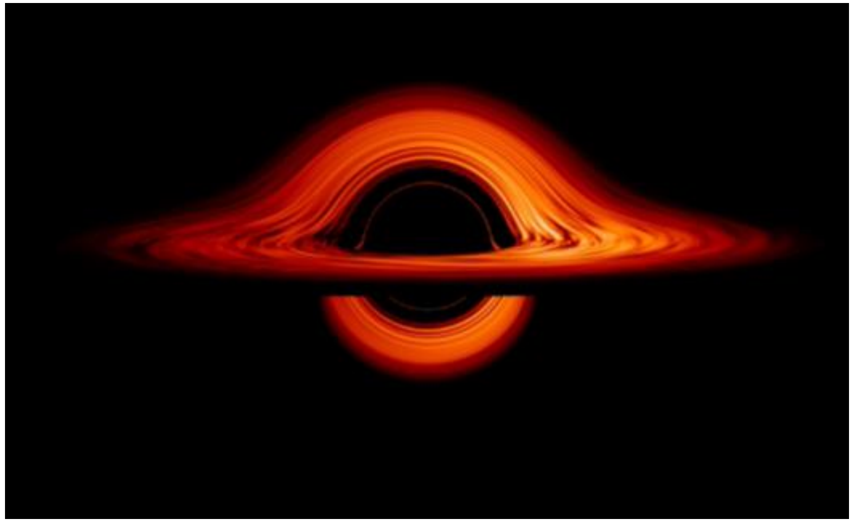
$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r_*^2} - V(r_*)\right)\psi(r_*) = -\omega^2\psi(r_*)$$



Dynamical BHs

Motivation

- Accretion
- Hawking evaporation
- Superradiance
- GW absorption



Credits: NASA

The Vaidya solution

- **Metric in Eddington-Finkelstein coordinates:**

$$\mathrm{d}s_V^2 = -\left(1 - \frac{2M(u)}{r}\right)\mathrm{d}u^2 \pm 2\mathrm{d}u\,\mathrm{d}r + r^2\mathrm{d}\Omega_{S^2}^2$$

- **Stress-Energy Tensor:**

$$T_{\mu\nu}^V = \pm\frac{M'(u)}{4\pi r^2}\,k_\mu k_\nu$$

Time domain approach

- **Solve perturbation equation numerically as IVP**
  Abdalla, Chirenti and Saa, 2006
  Lin, Sun and Zhang, 2021
  Redondo-Yuste, Pereñiguez and Cardoso, 2023

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- **Any time-dependent mass function can be chosen**

- **Doesn’t allow high accuracy frequency extraction**

Constant rate approximation

- **Rescaled coordinates:**

$$\begin{cases} T = \int \frac{du}{2M(u)} + \int \frac{dx}{f(x)} \\ x = \frac{r}{2M(u)} \end{cases}$$

Constant rate approximation

- **Rescaled coordinates:**

$$\begin{cases} T = \int \frac{du}{2M(u)} + \int \frac{dx}{f(x)} \\ x = \frac{r}{2M(u)} \end{cases}$$

- **Conformal transformation of the metric:**

$$\mathrm{d}s_V^2 = M(T,x)^2 \left[-f(x)\mathrm{d}T^2 + f(x)^{-1}\mathrm{d}x^2 + x^2 \mathrm{d}\Omega_{S^2}^2 \right]$$

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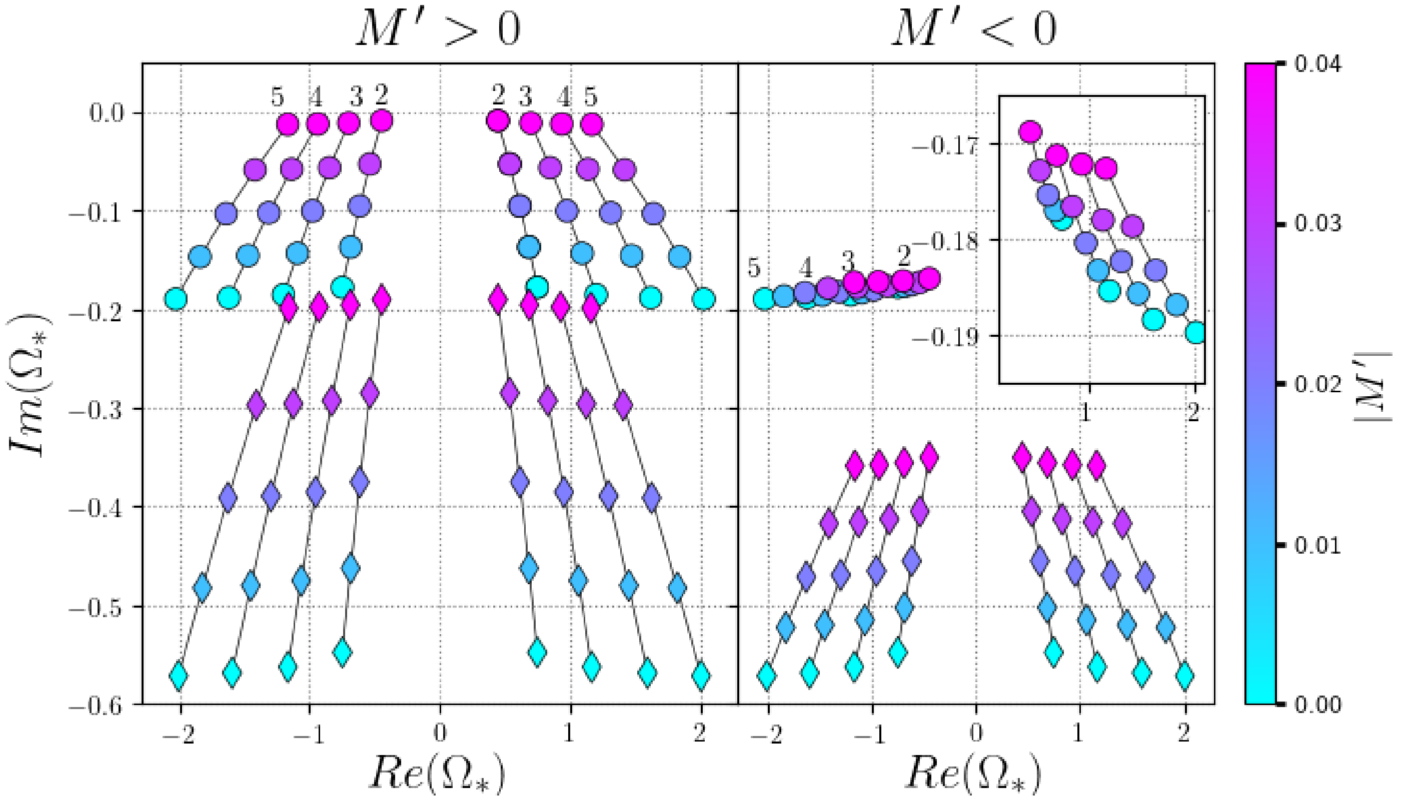
- **Perturbation master equation:**

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x_*^2} + \Omega^2 - f(x)V(x) \right] R(x) = 0$$

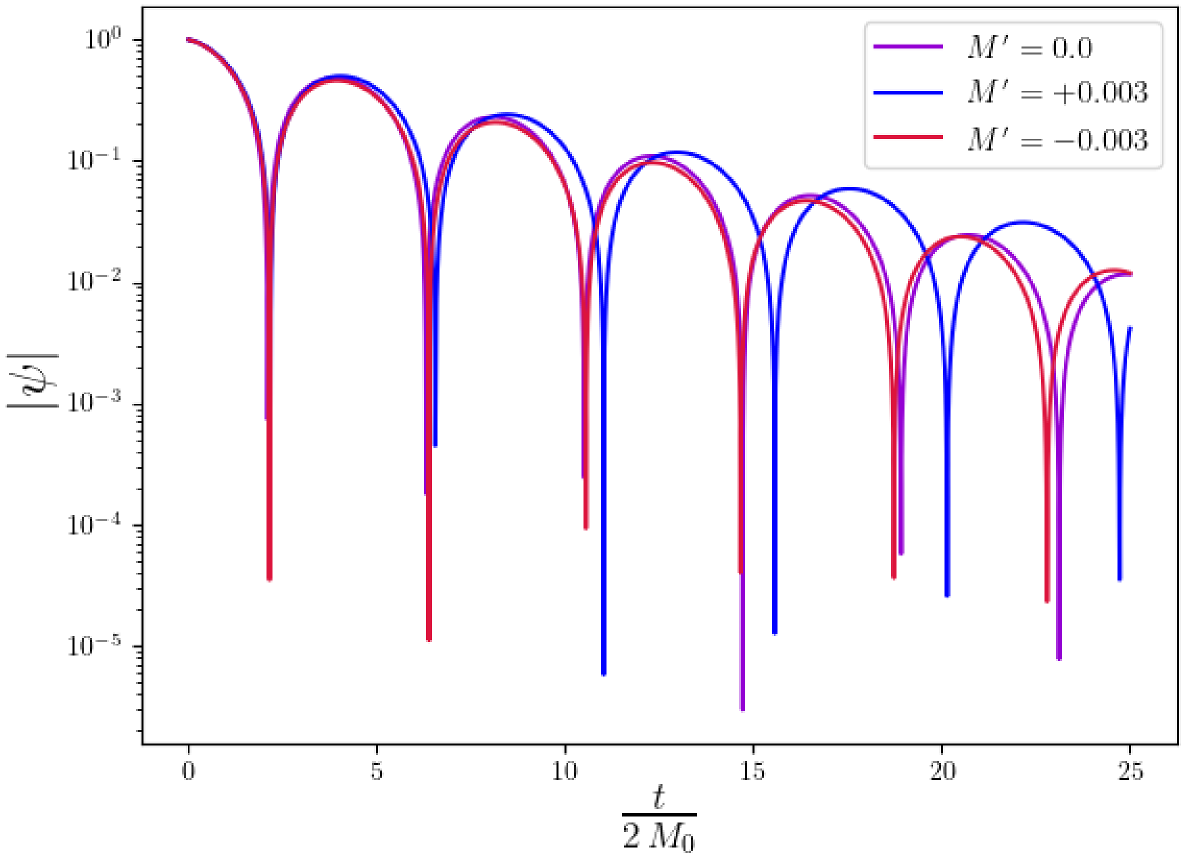
RESULTS: QNMs



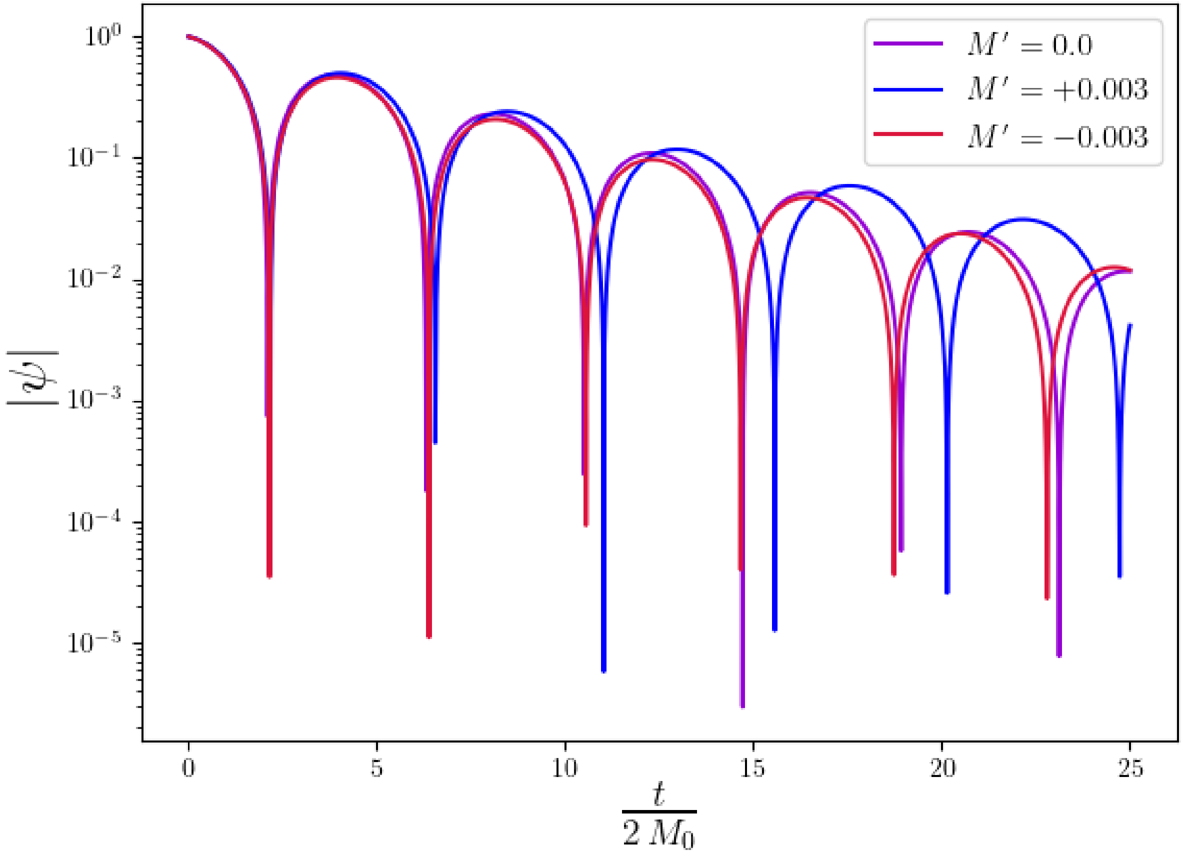
INSTABILITIES

- Scalar: possible for decreasing mass
- Electromagnetic: no instability
- Gravitational: possible for increasing mass

RESULTS: Time Domain



RESULTS: Time Domain



$$\omega(u) \propto M(u)^{-1}$$

TIDAL RESPONSE

General treatment

Static solution:

$$\psi_{\mathrm{static}} \sim x^{\ell+1} + k\, x^{-\ell}$$

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Tidal field

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Tidal field

Response

TIDAL RESPONSE

General treatment

Static solution:

$$\psi_{\text{static}} \sim x^{\ell+1} +$$

In GR the LNs vanish!

TIDAL RESPONSE

The Vaidya case

$$\psi(u,r) \sim \left(\frac{r}{2M_0}\right)^{\ell+1}\left[1 + w(u,r) + \mathcal{O}\left(\frac{2M_0}{r}\right) + \right.$$
$$\left. + k\left(\frac{r}{2M_0}\right)^{-(2\ell+1)}\left(1 + \mathcal{O}\left(\frac{2M_0}{r}\right)\right)\right].$$

TIDAL RESPONSE

The Vaidya case

Correction to tidal field

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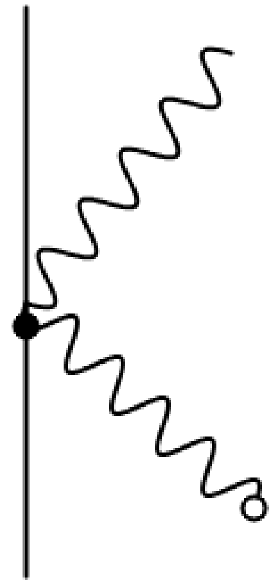
Constant correction to Schwarzschild
Love numbers

Point-particle EFT

- **Full action:**

$$S_{\text{total}} = S_{\text{scalar}} + S_{\text{gravity}} + S_{\text{SET}} - \int \mathrm{d}T\, e(x) \left[M_0 + \sum_{\ell=1}^{\infty} \frac{\mu_\ell}{\ell!} \left(\partial_{(a_1} \ldots \partial_{a_\ell)} \varphi \right)^2 \right]$$



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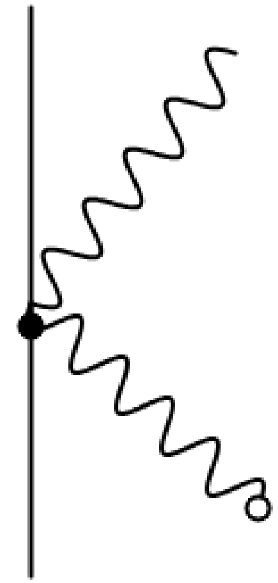
- **Solve KG equation perturbatively in the rate:**

$$\frac{1}{\sqrt{-g}} \partial_i \left(\sqrt{-g}\, g^{ij} \partial_j \varphi \right) = -\mu_\ell (-1)^\ell\, \partial^{(\ell)} \delta_D^{(3)} (x - \tilde{x})$$



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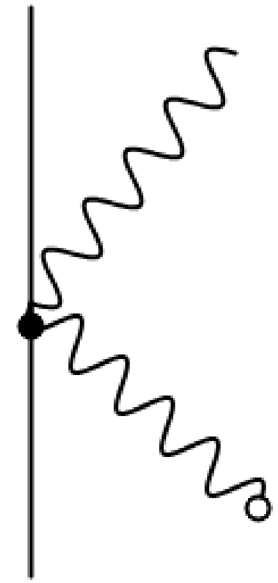
$$g = \text{Minkowsky} + M'\, \delta g + \ldots$$



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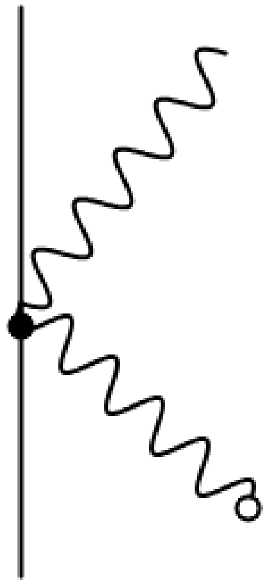

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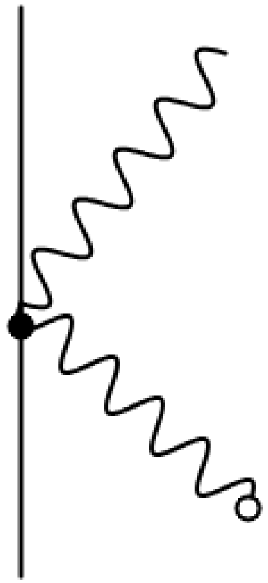

- **Match couplings with other computation**

$$\varphi_{\text{resp}}^{(1)} \sim \mu_\ell^{(1)} x^{-\ell}$$

CONCLUSIONS

- **Revisited perturbation of Vaidya metric with constant rate approximation**
- **Computed QNMs in the frequency domain**
- **Computed the linear Tidal Response**


That's all Folks!