Quasi-local masses and cosmological coupling of astrophysical compact objects

based on:

- Cadoni, Sanna, Pitzalis, Banerjee, **Murgia**, Hazra, Branchesi
  JCAP 11 (2023) 007; **2306.11588**
- Cadoni, **Murgia**, Pitzalis, Sanna
  JCAP 03 (2024) 026; **2309.16444**

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Do local gravitational systems couple to large-scale cosmological dynamics?

- Cosmological expansion does not affect small-scale Newtonian systems. What about relativistic bodies such as black holes?

- A first attempt to address the question is done by McVittie by embedding the Schwarzschild solution in a FLRW background:

$$\mathrm{d}s^2 = -\frac{\left(1 - \frac{Gm(t)}{2r}\right)^2}{\left(1 + \frac{Gm(t)}{2r}\right)^2}\mathrm{d}t^2 + a^2\left(1 + \frac{Gm(t)}{2r}\right)^4\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right) ,$$

McVittie (1933)

Local and non-local: MS vs ADM mass

Arnowitt Deser Misner (ADM) mass

- Vacuum solution;
- Non-local quantity, as it is defined at the boundary of spacetime;
- Cannot be defined in non-asymptocally flat spacetimes as those corresponding to cosmologically embedded objects.

Misner Sharp (MS) mass

- Suitable for objects not in vacuum, as non-singular black holes;
- Quasi-local quantity, related to the astrophysical properties of virialized systems;
- Covariantly defined also in non-asymptocally flat spacetimes.

Common arguments against cosmological coupling

- There are exact solutions to Einstein's equations embedding black holes in expanding universes (McVittie, Kerr-deSitter): cosmological coupling does not occur for any of them (by looking at the ADM mass) Gaur&Visser 2308.07374

- Huge separation of scales between black hole physics and and cosmological dynamics
  - Heaviest known BH ⟶ ~ 10^{-3} parsec
  - Cosmological homogeneity scale L ⟶~ 10^{8} parsec

Our proposal: compact objects sourced by anisotropic fluids

- We describe the source of gravitational field with an anisotropic fluid

$$T_{\mu\nu} = (\rho + p_\perp)\, u_\mu u_\nu + p_\perp\, g_{\mu\nu} - (p_\perp - p_\parallel)\, w_\mu w_\nu \,,$$

- The spacetime is parametrized by

$$\mathrm{d}s^2 = a^2(\eta)\left[-e^{\alpha(\eta,r)}\mathrm{d}t^2 + e^{\beta(\eta,r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right] \,.$$

- Appropriate combinations of the Einstein's equation allows to reveal a coupling between the (large-scale) cosmological dynamics and the (small-scale) compact object properties

Our proposal: the Misner-Sharp mass

- Curvature term responsible for the coupling

Density profile of the object:

$$8\pi G\rho = \frac{8\pi G}{3}\rho_1 \frac{1}{r^2}\partial_r\left(r^3 e^{-\alpha}\right) + \frac{1}{a^2 r^2}\partial_r(r - re^{-\beta})\,.$$

- Purely cosmological contribution.
- Not relevant at scale L.

- Model-dependent correction (subleading term)

$$M(\eta) = 4\pi a^3(\eta)\int_0^L \mathrm{d}r\, r^2\,\rho(r,\eta) = \frac{4\pi}{3}\rho_1 a^3 L^3\, e^{-\alpha(L)} + M(a_i)\frac{a}{a_i}\left[1 - e^{-\beta_0(L)}a^{k_L}\right],$$

- Universal cosmological Schwarzschild mass: $\frac{a_i L}{2\,G}$.

Our prediction: linear mass growth of the MS mass

- **The equation is valid for any compact *regular* object sourced by anisotropic fluid allowed by General Relativity.**

$$M(a) = M(a_i)\left(\frac{a}{a_i}\right)^k , \qquad \text{with} \quad a \geq a_i ,$$

$$M(\eta) = 4\pi a^3(\eta)\int_0^L \mathrm{d}r\, r^2 \rho(r,\eta) = \frac{4\pi}{3}\rho_1 a^3 L^3 e^{-\alpha(L)} + M(a_i)\frac{a}{a_i}\left[1 - e^{-\beta_0(L)}a^{k_L}\right] ,$$

We predict a cosmological coupling constant k = 1

- **Length scale L:**
  - For black holes it is the event horizon
  - For horizonless compact object it represents the radius enclosing 99% of the mass

Non-local mass of black holes and cosmological coupling

- The ADM mass is a non-local quantity, therefore unable to quantify local properties
- The huge separation between the scales cannot justify the use of the ADM mass because it cannot be properly defined in non-asymptotically flat spacetimes (as those corresponding to cosmologically embedded objects), nor for compact objects sourced by anisotropic fluids

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Black hole event horizons are cosmologically coupled

2407.14549

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It is shown that an exactly static and spherically symmetric black hole event horizon cannot be embedded in a time-dependent geometry. Forcing it to do so results in a naked null singularity at the would-be horizon. Therefore, since the universe is expanding, black holes must couple to the cosmological expansion, which was suggested as the growth mechanism for supermassive black holes in galaxies, with implications for the dark energy puzzle.

MS mass for isotropic sources → no mass growth

- McVittie solution: perfect, isotropic and spherically-symmetric fluid as a source embedded in a FLRW background

$$\mathrm{d}s^2 = -\frac{\left(1-\frac{Gm(t)}{2r}\right)^2}{\left(1+\frac{Gm(t)}{2r}\right)^2}\mathrm{d}t^2 + a^2\left(1+\frac{Gm(t)}{2r}\right)^4\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right) ,$$

$$M_{\mathrm{MS}} = \frac{R}{2G}\left(\frac{2Gm_0}{R} + H^2R^2\right) = m_0 + \frac{H^2}{2G}R^3 .$$

McVittie (1933)

MS mass for anisotropic sources → linear mass growth

- Compact objects sourced by anisotropic fluids:

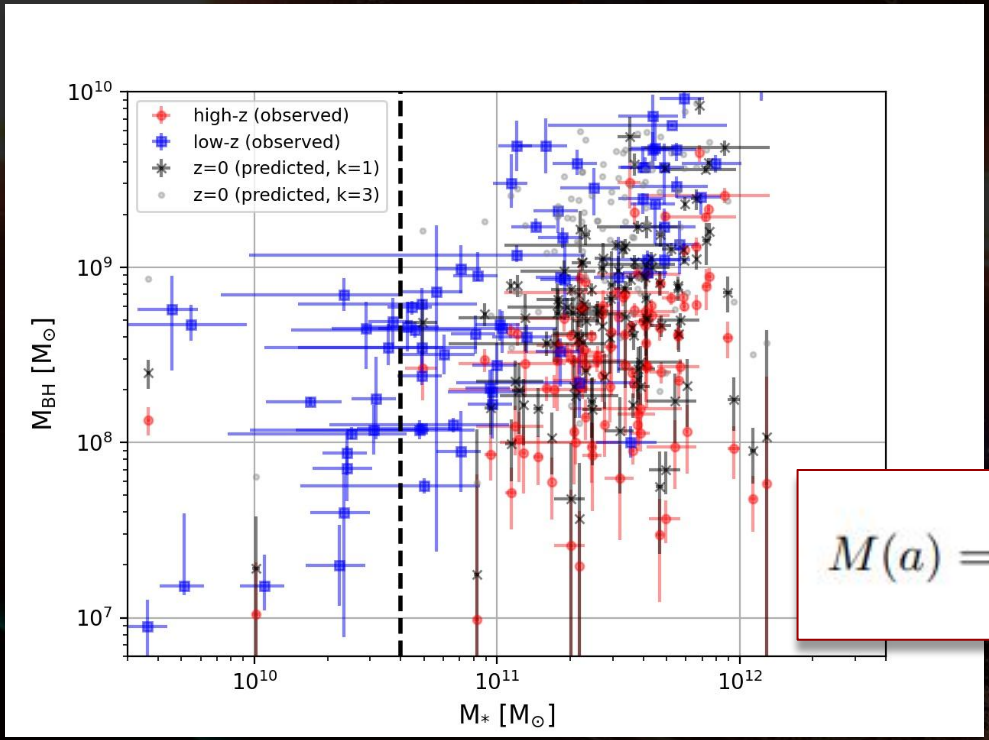
$$M_{\rm MS} = \frac{ar}{2G}\left[1 + \frac{\dot{a}^2}{a^2}r^2 e^{-\alpha} - \frac{a^2}{a^2}e^{-\beta}\right]$$
$$= \frac{4\pi}{3}\rho_1 a^3 r^3 e^{-\alpha} + \frac{ar}{2G}\left[1 - e^{-\beta_0(r)}a^{k(r)}\right],$$

- Cosmological coupling is present independently from the specific equation of state of the cosmological fluid
- The coupling is not due to some accretion flow, since we imposed the absence of radial fluxes

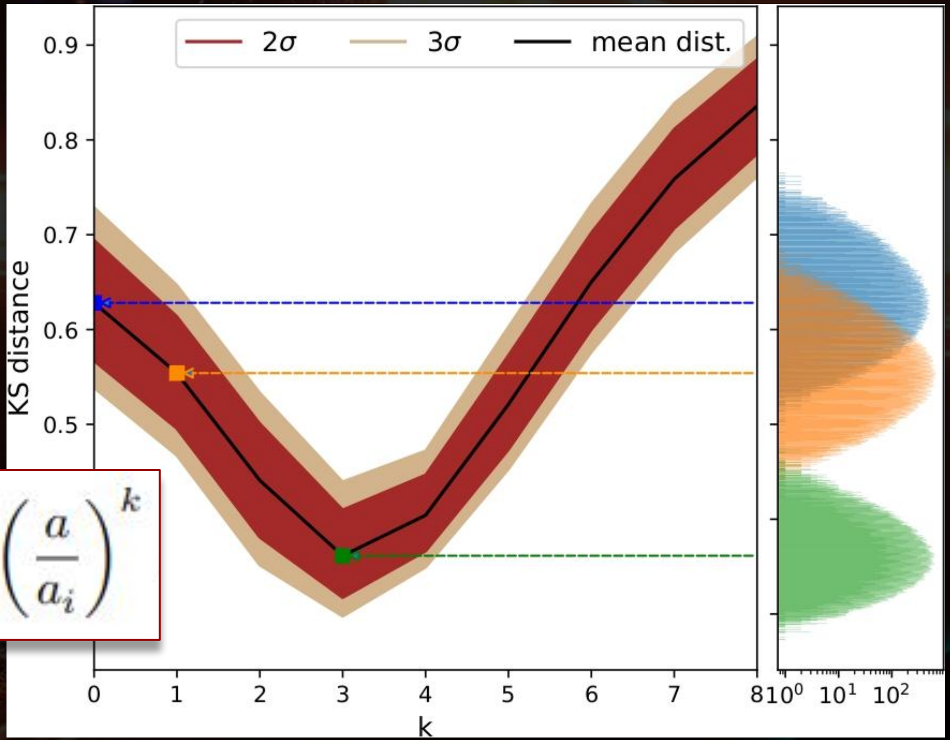
Constraints from SMBHs in quiescent elliptical galaxies

Comparison between theoretical predictions and mass measurements of SMBHs in elliptical galaxies.

- Red sequence elliptical galaxies (SMBHs growth via accretion is negligible):
  - $\mathbf{0.0016 \leq z \leq 0.19}$ (low redshift dataset, D. Farrah *et al.* The Astrophysical Journal 943, 133 (2023))
  - $\mathbf{0.8 \leq z \leq 0.9}$ (high redshift dataset, cross matching WISE (R.S. Barrows *et al.* (2021)) + SDSS (S. Rakshit *et al.* (2020)) survey.)



$$M(a) = M(a_i)\left(\frac{a}{a_i}\right)^k$$



Current limits on *k*

Black Holes as the source of the dark energy: a stringent test with the high-redshift JWST AGNs

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⟶ $k = -0.03 \pm 1.33$

OBSERVATIONAL IMPLICATIONS OF COSMOLOGICALLY COUPLED BLACK HOLES

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⟶ $k \leq 1$ (qualitative result)

Cosmologically coupled compact objects: a single parameter model for LIGO–Virgo mass and redshift distributions

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⟶ $k \sim 0.5$ (LVC)

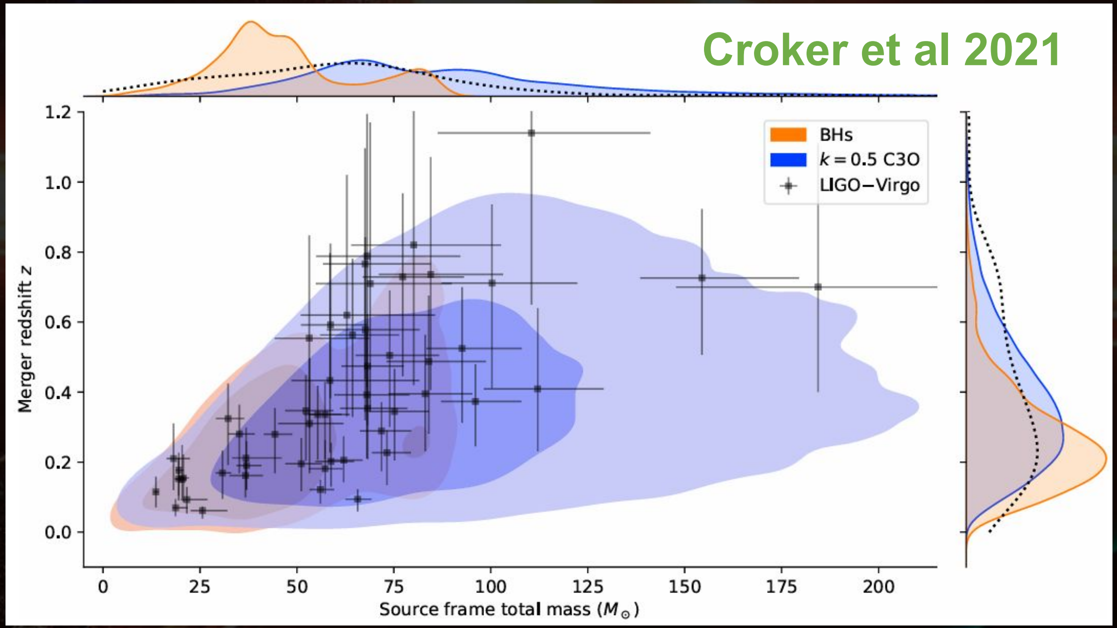
What now

- **Implement within BBH population synthesis codes an extra-term describing the adiabatic decay due to cosmological coupling**
  (cosmoRATE, F. Santoliquido et al (2021))

ONGOING WORK led by M. Pitzalis, U. Dupletsa, F. Santoliquido

$$\frac{\mathrm{d}R}{\mathrm{d}a} = \underbrace{\frac{\partial R}{\partial L}\frac{\mathrm{d}L}{\mathrm{d}a}\bigg|_{e,M} + \frac{\partial R}{\partial e}\frac{\mathrm{d}e}{\mathrm{d}a}\bigg|_{L,M}}_{\text{Radiative decay}} + \underbrace{\frac{\partial R}{\partial M}\frac{\mathrm{d}M}{\mathrm{d}a}\bigg|_{e,L}}_{\text{Adiabatic decay}}.$$

- **Perform full hierarchical Bayesian analyses on LVK data, as well as produce forecasts for next-gen GW observatories**



Conclusions

- We demonstrate that the Misner-Sharp mass is the correct quantity at play to fit astrophysical observations;
- We find that cosmological coupling is quite natural for compact objects sourced by local anisotropies like nonsingular black holes;
- We predict a universal linear growth of the MS mass as a function of the scale factor (with an additional subleading term for horizonless objects)

  i.e. $$M(a) = M(a_i)\left(\frac{a}{a_i}\right)^k$$ with k = 1;
- We are now looking into GW data to figure out whether:
  - k = 0 ⟶ nonsingular GR BHs incompatible with observations
  - k = 1 ⟶ smoking gun that GR BHs are nonsingular
  - k = other value ⟶ astrophysical BHs cannot be described with GR

BACKUP

MS mass for point-like objects

- Schwarzschild-de Sitter solution: mass particle embedded in a dS background (positive cosmological constant).

$$ds^2 = -\left(1 - \frac{2Gm}{r} - H^2 r^2\right) dt^2 + \frac{dr^2}{1 - \frac{2Gm}{r} - H^2 r^2} + r^2 d\Omega^2 .$$

$$M_{\rm MS} = m + \frac{H^2}{2G} r^3 .$$

MS mass for anisotropic sources

- **Sultana-Dyer solution: black hole embedded in a spatially flat FLRW background (J. Sultana and C. C. Dyer (2005)).**

$$ds^2 = -\frac{\left(1 - \frac{Gm_0}{2r}\right)^2}{\left(1 + \frac{Gm_0}{2r}\right)^2}dt^2 + a^2\left(1 + \frac{Gm_0}{2r}\right)^4\left(dr^2 + r^2 d\Omega^2\right).$$

$$M_{\rm MS} = a\, m_0 + \frac{H^2 R^3}{2G\left(1 - \frac{2Gam_0}{R}\right)}.$$

- **A physical consequence of the accretion flow of cosmic fluid onto the central object.**