Ringdowns for black holes with scalar hair: the large mass case

GIOVANNI D'ADDARIO
In collaboration with:
Antonio Padilla, Paul Saffin, Thomas Sotiriou & Andrew Spiers
University of Nottingham

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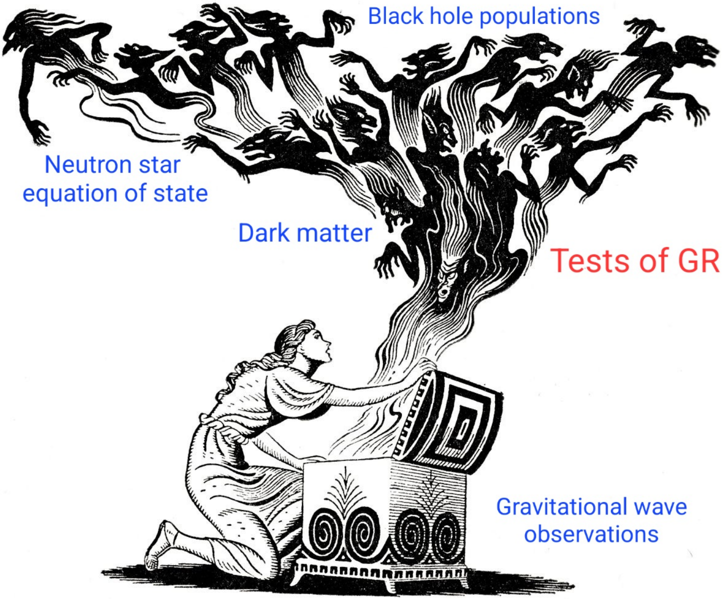

Black hole populations
Neutron star
equation of state
Dark matter
Tests of GR
Gravitational wave
observations

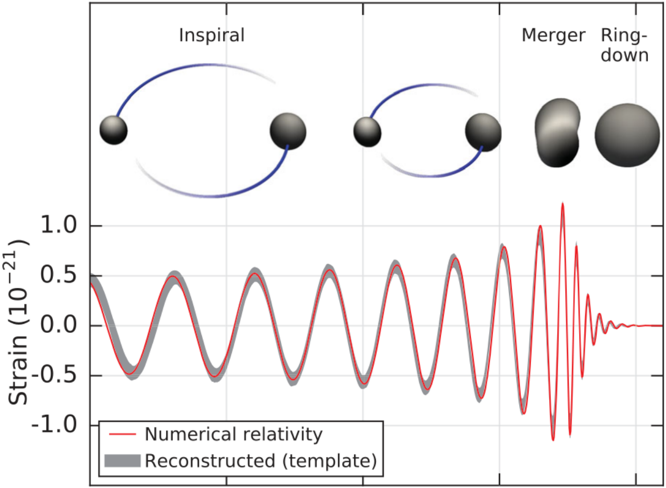


Figure: Estimated strain of GW150914 (Abbott et al., 2016a).

Outline

Perturbations in modified gravity can be extra complicated

Horndeski admits black holes with scalar hair set by the mass

Simplified formalism for perturbations in Horndeski

How feasible are future tests of GR with ringdown?

Black hole ringdown

Figure: Boundary conditions on solutions to linearised Einstein equations.

$$\delta G_{\mu\nu}[h_{\mu\nu}] = 0 \tag{1}$$

$$h_{\mu\nu} \sim e^{-i\omega t} \tag{2}$$

Frequencies ω are complex: quasi-normal modes (QNMs)

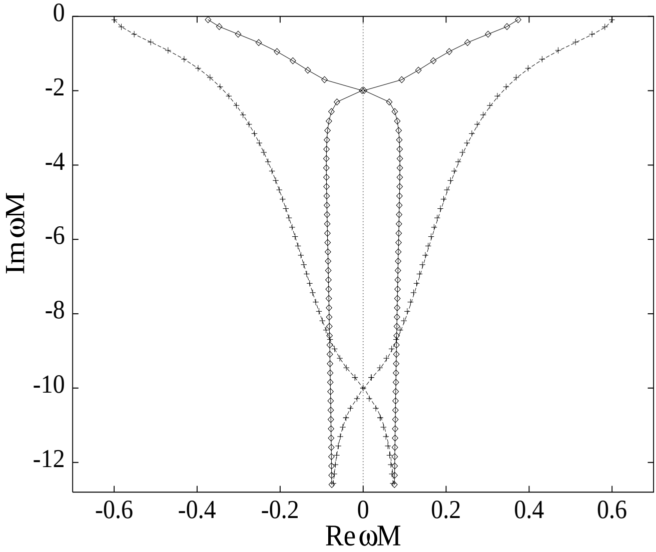


Figure: Spectrum of QNM frequencies for Schwarzschild, for $\ell = 2$ (diamonds) and $\ell = 3$ (crosses) (Kokkotas and Schmidt, 1999).

Black Hole Perturbation Theory in Modified Gravity

What is the background $\bar{g}_{\mu\nu}$?

Generically, the "nice" features of GR are not present

Theory	Coupling	Non-rotating n	ℓ	method	reference	Rotating a_{max}	n	ℓ	method	reference
GR + EM	Q	10^5	any	CF	[210, 46]	$\sqrt{1-Q^2}$	1	2	FDI	[116, 115]
Higher-derivative		0	4	DI, SC	[75]	0.4	0	3	DI, SR/SC	[69]
sGB	$f'(\phi) \neq 0$	0	3	DI	[53, 52]	0.7	0	3	DI, SR	[265]
	$f'(\phi) = 0$	0	2	DI	[51]					
dCS		2	4	CF, SC	[300]	0.2	0	4	DI, SR	[305, 284]
Horndeski					[286, 204]					

Figure: Status of QNM analyses in modified theories (Franchini and Völkel, 2023).

Are there any possible simplifications?

Shift-symmetric Horndeski

The action, with $c = G = 1$, is invariant under $\phi \to \phi +$ constant

$$S_H = \frac{1}{16\pi} \sum_{i=2}^{5} \int d^4x \sqrt{-g} \mathcal{L}_i + S_M \tag{3}$$

$$\begin{aligned}
X &= -\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi \\
\mathcal{L}_2 &= K(X) \\
\mathcal{L}_3 &= -G_3(X) \Box \phi \\
\mathcal{L}_4 &= G_4(X) R + G_{4X}(X) \left[(\Box \phi)^2 - \left(\nabla_\mu \nabla_\nu \phi \right)^2 \right] \\
\mathcal{L}_5 &= G_5(X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi \\
&\quad - \frac{G_{5X}}{6} \left[(\Box \phi)^3 - 3 \Box \phi \left(\nabla_\mu \nabla_\nu \phi \right)^2 + 2 \left(\nabla_\mu \nabla_\nu \phi \right)^3 \right]
\end{aligned} \tag{4}$$

Classification of shift-symmetric Horndeski theories

GR black holes obey no hair theorems...

(Saravani and Sotiriou, 2019) established the relation

$$\mathcal{L} = \tilde{\mathcal{L}} + \alpha\phi\mathcal{G} \,, \tag{5}$$

- $\tilde{\mathcal{L}}$: theories admitting $\phi = 0$ and hence all GR solutions
- $\mathcal{L}$: theories admitting $\phi = 0$ only for Minkowski
- $\mathcal{G} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2$

$\phi\mathcal{G}$ term gives solutions with scalar hair (Sotiriou and Zhou, 2014)

Just to drive it home...

$$\mathcal{L} = \tilde{\mathcal{L}} + \alpha\phi\mathcal{G} \tag{6}$$

$=$

$$+ \alpha\phi\mathcal{G} \tag{7}$$

Equations of motion & scalar charge

$$G_{\mu\nu} = T^{\phi}_{\mu\nu} \tag{8}$$

$$\nabla_\mu(\tilde{J}^\mu - \alpha\mathcal{G}^\mu) = 0 \qquad \mathcal{G} = \nabla_\mu\mathcal{G}^\mu \tag{9}$$

Dimensionless scalar charge:

$$4\pi q \propto \frac{\alpha}{M^2} \tag{10}$$

Perturbative formalism

$$4\pi q \propto \frac{\alpha}{M^2} \tag{11}$$

$$\phi = q\phi^{(0,1)} + \varepsilon\phi^{(1,0)} + q^2\phi^{(0,2)} + \varepsilon q\phi^{(1,1)} \tag{12}$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \varepsilon h^{(1,0)} + q^2 h^{(0,2)}_{\mu\nu} + \varepsilon q h^{(1,1)}_{\mu\nu} \tag{13}$$

For large black holes, q is small: $M \sim 10^6 M_\odot \implies q \lesssim 10^{-13}$

Only linear order perturbations in ε

Perturbed equations of motion

$$\Box\phi^{(1,0)} = 0 \tag{14}$$

$$\Box\phi^{(0,1)} = -\alpha^{(0,1)}\bar{R}_{\mu\nu\rho\sigma}\bar{R}^{\mu\nu\rho\sigma} \tag{15}$$

$$\delta G_{\mu\nu}[h^{(1,0)}_{\mu\nu}] = 0 \tag{16}$$

$$\delta G_{\mu\nu}[h^{(1,1)}_{\mu\nu}] = \alpha^{(0,1)}\kappa\big(\phi^{(1,0)}\big) \tag{17}$$

Implications:

$$\delta G_{\mu\nu}[h_{\mu\nu}^{(1,1)}] = \alpha^{(0,1)}\kappa\big(\phi^{(1,0)}\big) \tag{18}$$

The leading order corrections are suppressed by a factor of q

δG: GR operator! We are solving Kerr with a source

Source terms: contribution of $\alpha\phi\mathcal{G}$ only

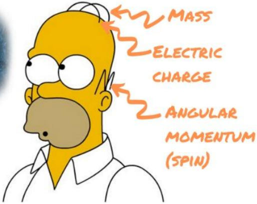
Implications for LISA

LISA is unlikely to detect scalar deviations from GR via

black hole ringdown analyses

The calculation of the QNMs is in progress!

RINGDOWNS FOR BLACK HOLES WITH SCALAR HAIR: THE LARGE MASS CASE



01 Black hole perturbation theory in modified gravity is hard!

02 Obtained a drastic simplification for a general class of scalar-tensor theories

03 LISA is unlikely to detect scalar deviations from GR with ringdown analyses

The University of Nottingham

Giovanni D’Addario

Linearised tensors

$$\delta G_{\mu\nu} = \delta\left[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right] = \left(g_\mu{}^\alpha g_\nu{}^\beta - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\right)\delta R_{\alpha\beta}\ , \quad (19)$$

with the Lichnerowicz operator

$$\delta R_{\mu\nu}[h] = -\frac{1}{2}\left(\bar{\Box}h_{\mu\nu} - 2h^\alpha{}_{(\mu;\nu)\alpha} + h^\alpha{}_{\alpha;\mu\nu}\right) \quad (20)$$

$$V^{-}_{s=2} = f(r)\left[\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right] \tag{14}$$

$$V^{+}_{s=2} = \frac{2f(r)}{r^3}\frac{9M^3 + 3\lambda^2 M r^2 + \lambda^2(1+\lambda)r^3 + 9M^2\left(\lambda r + \frac{r^3}{L^2}\right)}{(3M+\lambda r)^2} . \tag{15}$$

Figure: Potential V found by Regge-Wheeler (odd modes) (Regge and Wheeler, 1957) and Zerilli (even) (Zerilli, 1970)

$$(M\omega_n)^2 = V_\ell(r_0) - i\left(n + \frac{1}{2}\right)\left[-2\frac{d^2 V_\ell(r_0)}{dr_*^2}\right]^{1/2}, \tag{30}$$

Figure: WKB approximation for QNM frequencies.

$$
\begin{aligned}
&\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\varphi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\psi}{\partial\varphi^2} \\
&\quad - \Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{\partial\psi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) - 2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\psi}{\partial\varphi} \\
&\quad - 2s\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\psi}{\partial t} + (s^2\cot^2\theta - s)\psi = 4\pi\Sigma T\,. \qquad (4.7)
\end{aligned}
$$

Figure: Teukolsky equation for Kerr perturbations (Teukolsky, 1973).

The equations for R and S are

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{dR}{dr}\right) + \left(\frac{K^2 - 2is(r-M)K}{\Delta} + 4is\omega r - \lambda\right)R = 0\,, \qquad (4.9)$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dS}{d\theta}\right) + \left(a^2\omega^2\cos^2\theta - \frac{m^2}{\sin^2\theta} - 2a\omega s\cos\theta - \frac{2ms\cos\theta}{\sin^2\theta} - s^2\cot^2\theta + s + A\right)S = 0\,, \qquad (4.10)$$

Figure: Separate ODEs from the Teukolsky equation.

$$\begin{aligned} \tilde{K}(X) &= X + \mathcal{O}(X^2), \\ \tilde{G}_3(X) &= \tau_3 X + \mathcal{O}(X^2), \\ \tilde{G}_4(X) &= 1 + \tau_4 X + \mathcal{O}(X^2), \\ \tilde{G}_5(X) &= \tau_5 X + \mathcal{O}(X^2) \end{aligned} \tag{21}$$

$\mathcal{L} = \sum_k \frac{f^4}{\mu^k}\mathcal{O}_k[\frac{\varphi}{\nu}]$ where f, μ, ν are the corresponding mass scales of the theory, φ describes the canonically normalised bosonic degrees of freedom, and the operators $\mathcal{O}_k$ are assumed to contain k derivatives and coefficients of order one.

$f \sim \sqrt{\mu\nu}$

$\mathcal{L} = \mu^2 M_P^2 \mathcal{F}\left(g_{\alpha\beta}, \frac{R_{\alpha\beta\gamma\delta}}{\mu^2}, \phi, \frac{\nabla_\alpha\phi}{\mu}, \frac{\nabla_\alpha\nabla_\beta\phi}{\mu^2}\right)$ where the scalar ϕ is taken to be dimensionless and $\mathcal{F}$ admitting an expansion in $\frac{R_{\alpha\beta\gamma\delta}}{\mu^2}, \phi, \frac{\nabla_\alpha\phi}{\mu}$ and $\frac{\nabla_\alpha\nabla_\beta\phi}{\mu^2}$ with order one coefficients.

Naturalness considerations now imply that $\alpha \sim \tau_3 \sim \tau_4 \sim 1/\mu^2$ and $\tau_5 \sim 1/\mu^4$.

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We have constructed our expansion under the (small charge) assumption $\alpha \sim qM^2$. Assuming that the scale associated with α is roughly the same as those associated with the τ_i, would imply $\tau_3 \sim \tau_4 \sim qM^2$ and $\tau_5 \sim q^2M^4$. The absence of the scale hierarchy is indeed what one expects from the naturalness arguments we presented earlier, and it introduces a further suppression of the τ_i contributions.

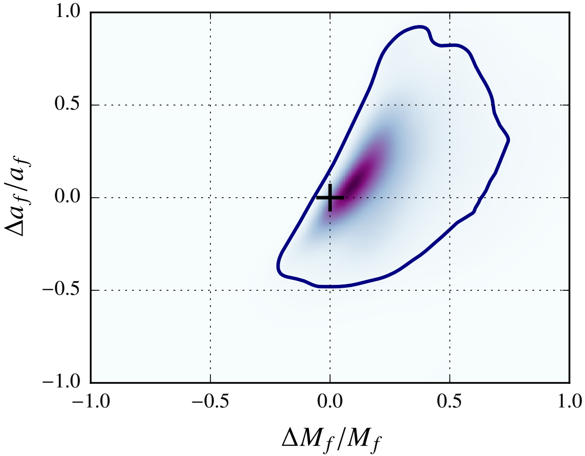

Figure: Posterior distribution for deviations from the GR values (plus sign) of the mass and spin of the remnant black hole from the analysis of GW150914 (Abbott et al., 2016b). The solid line is the 90% contour.

$$\delta G_{\mu\nu}[h^{(1,0)}_{\mu\nu}] = 0 \tag{22}$$

$$\begin{aligned}\delta G_{\mu\nu}[h^{(0,2)}_{\mu\nu}] = &-\frac{1}{2}\alpha^{(0,1)}\left[\bar{g}_{\rho\mu}\bar{g}_{\delta\nu} + \bar{g}_{\rho\nu}\bar{g}_{\delta\mu}\right] \\ &\cdot \bar{\nabla}_\sigma\left(\bar{\nabla}_\gamma\phi^{(0,1)}\epsilon^{\lambda\eta\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\bar{R}_{\lambda\eta\alpha\beta}\right) \\ &+ \frac{1}{2}\bar{\nabla}_\mu\phi^{(0,1)}\bar{\nabla}_\nu\phi^{(0,1)} - \frac{1}{4}\left(\bar{\nabla}_\alpha\phi^{(0,1)}\right)^2\bar{g}_{\mu\nu}\end{aligned} \tag{23}$$

$$\begin{aligned}\delta G_{\mu\nu}[h^{(1,1)}_{\mu\nu}] = &-\frac{1}{2}\alpha^{(0,1)}\left[\bar{g}_{\rho\mu}\bar{g}_{\delta\nu} + \bar{g}_{\rho\nu}\bar{g}_{\delta\mu}\right] \\ &\cdot \bar{\nabla}_\sigma\left(\bar{\nabla}_\gamma\phi^{(1,0)}\epsilon^{\lambda\eta\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\bar{R}_{\lambda\eta\alpha\beta}\right) \\ &+ \bar{\nabla}_{(\mu}\phi^{(0,1)}\bar{\nabla}_{\nu)}\phi^{(1,0)} - \frac{1}{2}\bar{\nabla}_\alpha\phi^{(0,1)}\bar{\nabla}^\alpha\phi^{(1,0)}\bar{g}_{\mu\nu}\end{aligned} \tag{24}$$

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