Binary mergers in strong gravity background of Kerr black hole

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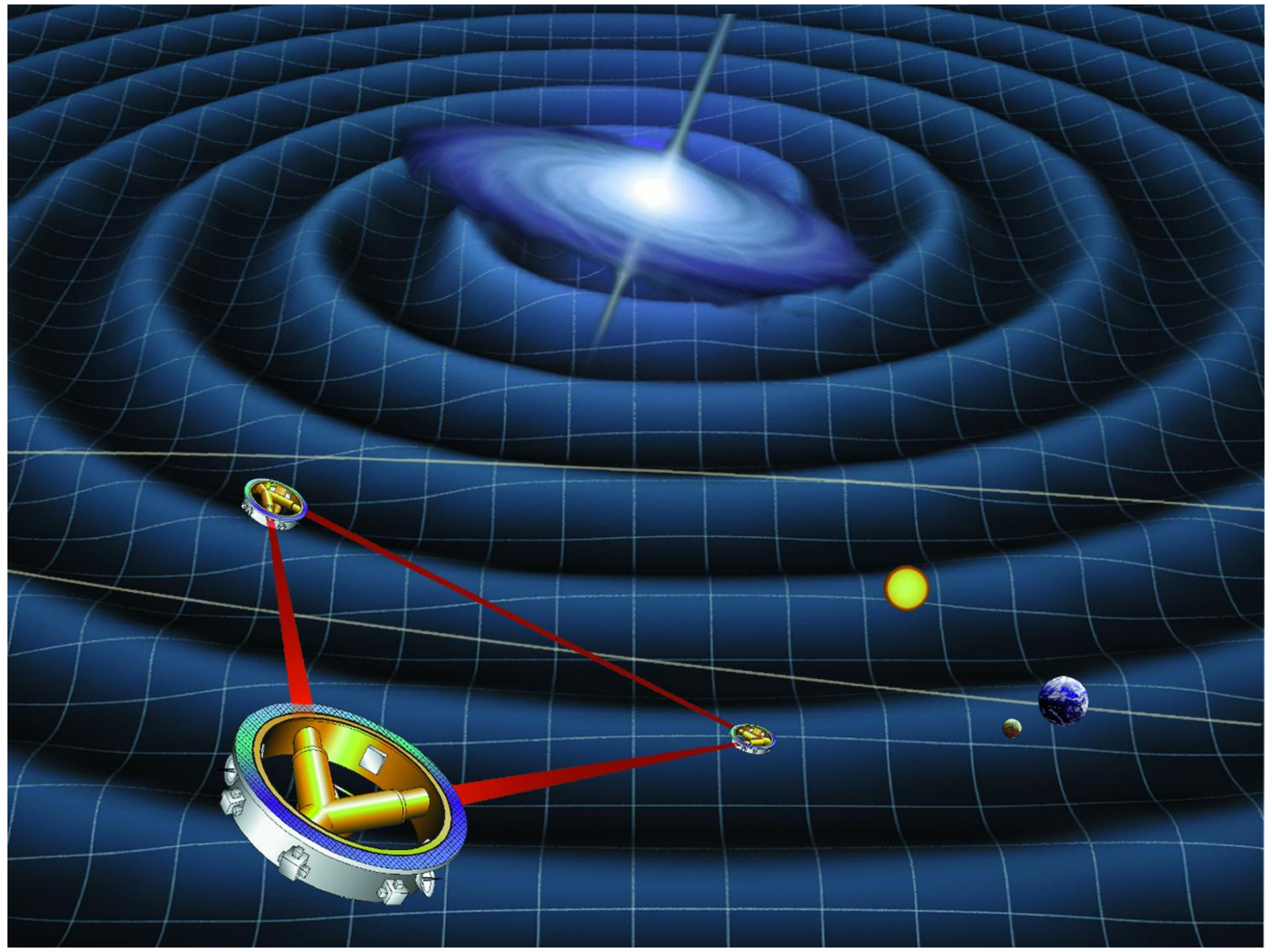
Why Three Body Systems?

- **Abundance of sources**: the density of stars in galactic nuclei can be 10^6 times the one in our solar system neighborhood (0.1stars/pc^3)



- Primary targets for **future GWs observations**
  - $\omega \sim 10^{-2} - 10$ **kHz** → **LIGO/VIRGO**
  - $\omega \sim 0.1 - 100$ **mHz** → **LISA**
- **Kozai-Lidov Mechanism** → very short merger timescale!
  - A circular binary of two $10M_\odot$ black holes orbiting a SMBH, with a separation of 1AU it would take 10^{14}years to merge.
  - A binary of two $10M_\odot$ black holes with eccentricity $e = 0.9995$ orbiting a SMBH, with a separation of 1AU it would take 4×10^3years to merge.

Strong Gravity Regime

- **Strong Gravity:** when GR effects become relevant

- **Small-tide approximation:** when the characteristic scale of the binary is much smaller than the radius of the curvature generated by the third body evaluated in the position of the binary system

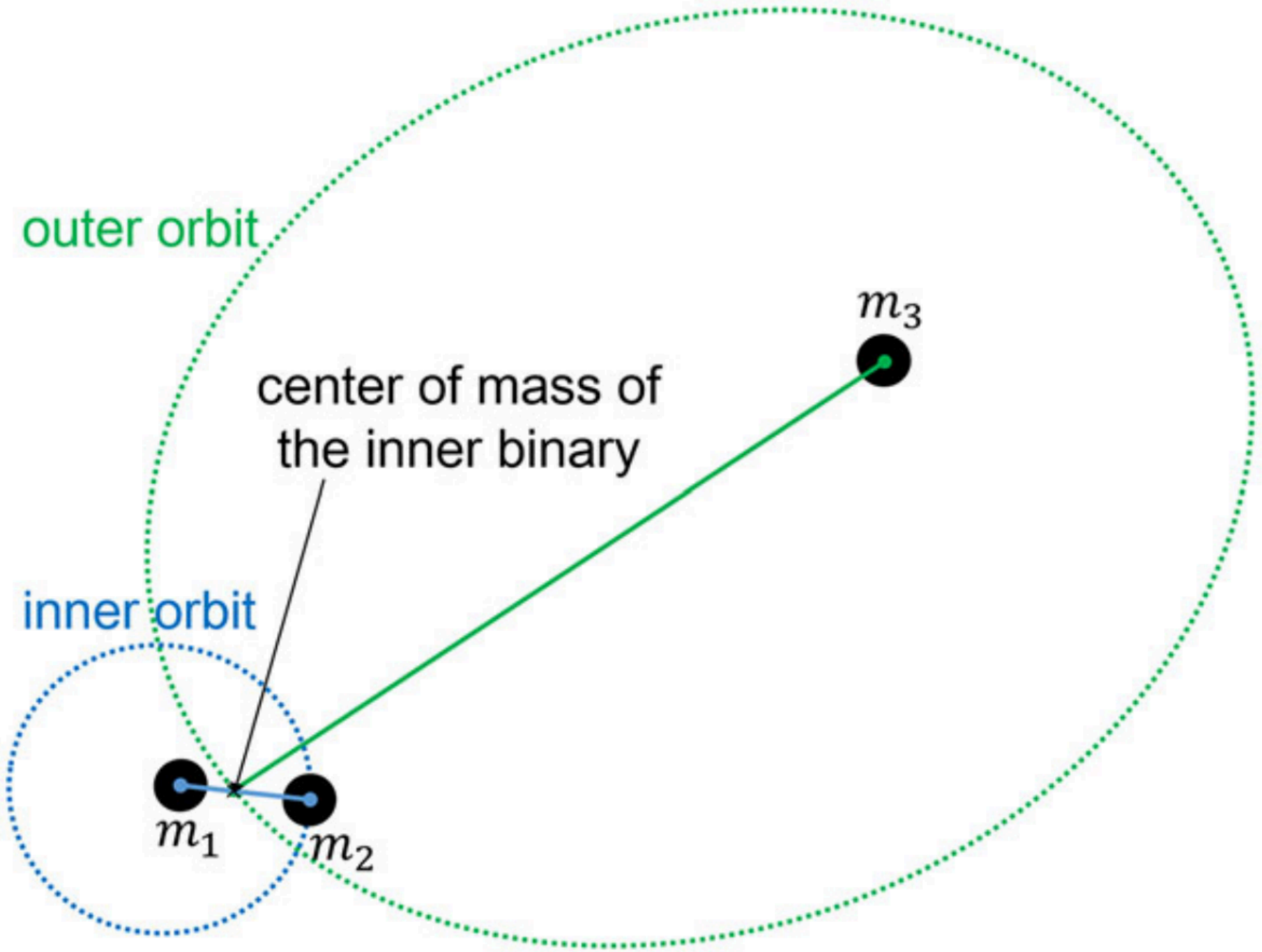
  - *Weak-field approximation* → the orbital velocity of the binary system must be small→ binary system far away from the perturber.

  - *Small-hole approximation* → dimension of the binary system is assumed to be much smaller than the external mass → the binary system can be close to the source of the tidal fields.

- Three body systems are usually studied as point particles in the weak-field approximation

Setup of the Triple System

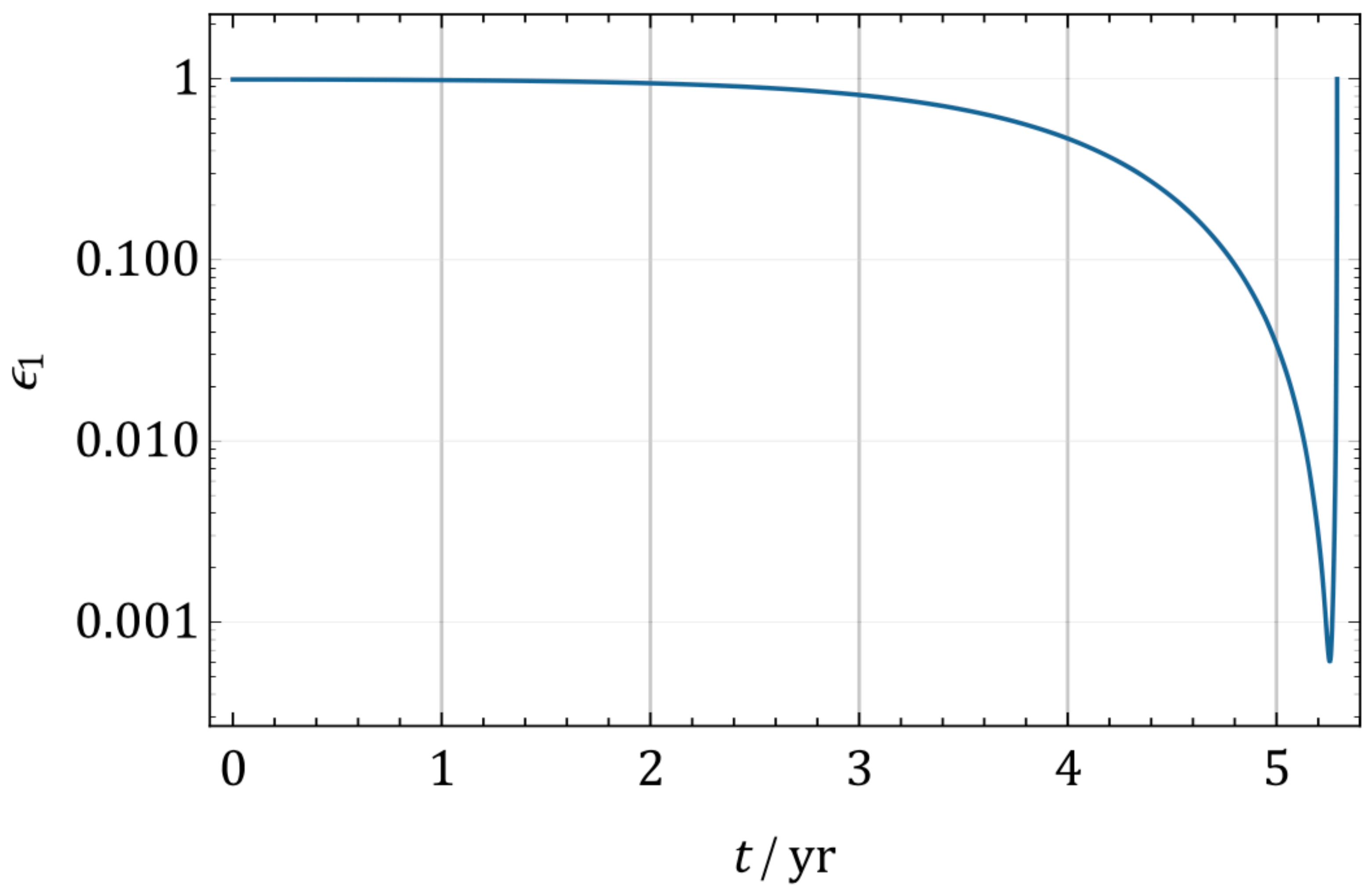
- Binary system of two black holes m_1 and m_2 moving along a geodesic around a SM Kerr BH m_3
    - *Inner orbit*
    - *Outer orbit*
- a semi-major axis of the binary, $\hat{r}$ distance between the binary and the SMBH
- **Two approximations**:
    - *Tidal Limit* $\rightarrow a \ll \frac{Gm_3}{c^2}$
    - *Point Particle Limit* $\rightarrow \frac{Gm_{1,2}}{c^2} \ll a$

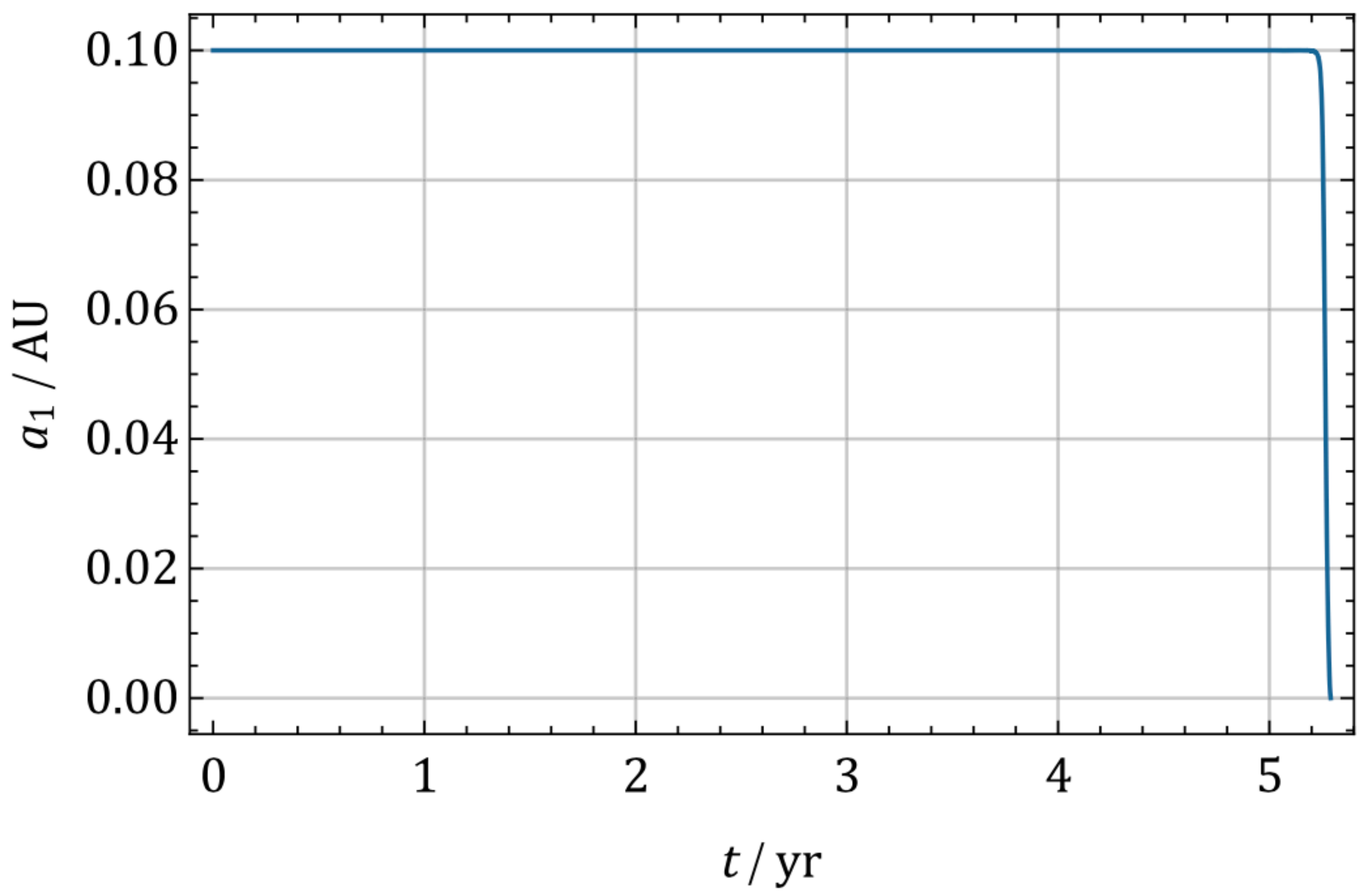


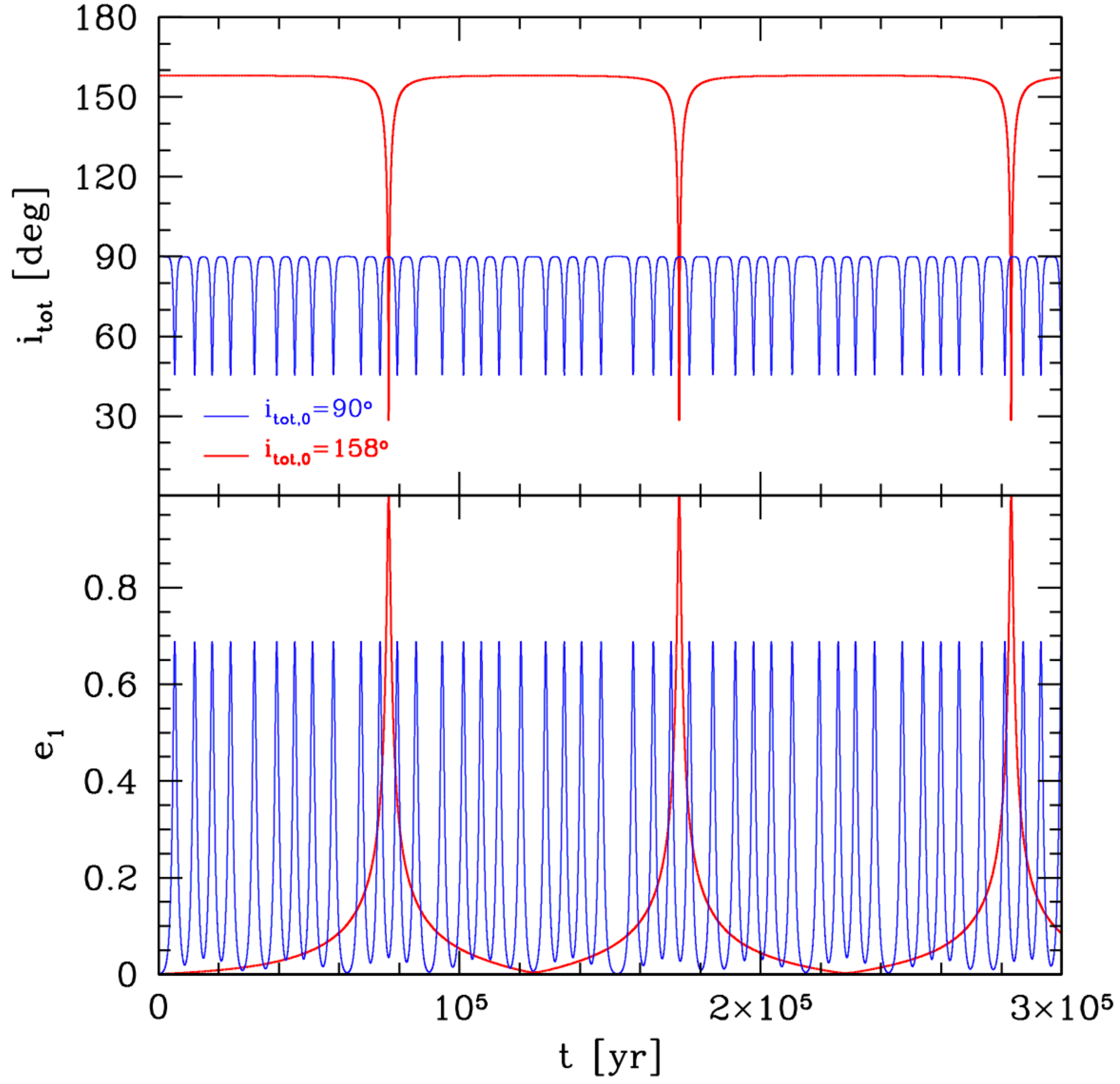
Kozai-Lidov Mechanism - Point Particle Approximation

- When the binary system has a high relative inclination to its orbit around the SMBH it will evolve over many orbits trading eccentricity for inclination in a periodic fashion

- Secular average over both the inner and outer orbit → Long timescale
- The BHB can reach eccentricity close to 1 → $e \sim 1$
- When including the emission of GW from the BHB the KL mechanism can speed up the merger!







Naoz (2016)

Kozai-Lidov Mechanism - Strong Gravity Regime

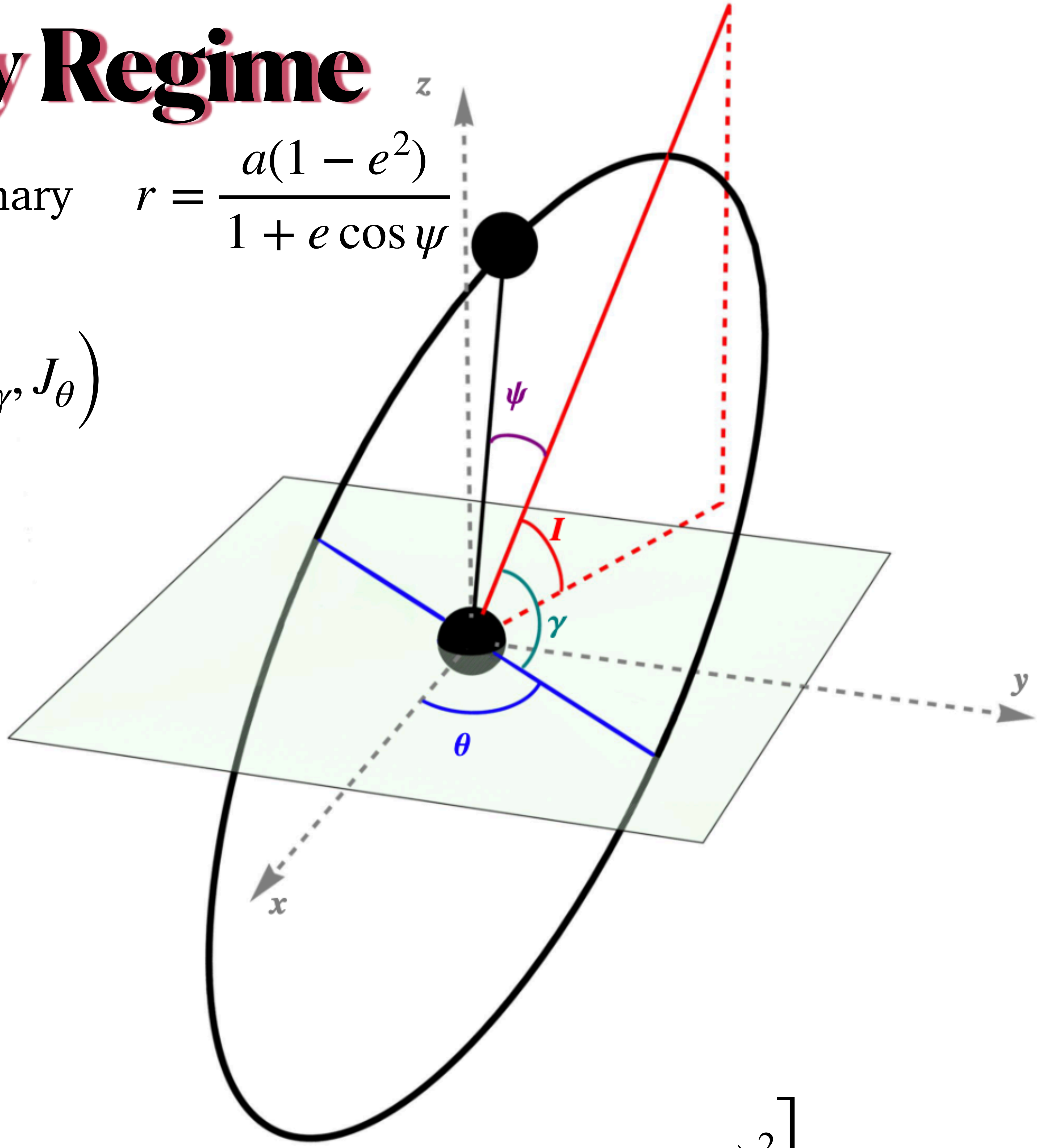
- The black holes in the binary are treated as point particles, the perturber as a rotating black hole
- The center of mass motion of the binary is decoupled from its relative motion
- The binary system is Newtonian, its Hamiltonian in a local inertial frame

$$H = \frac{p^2}{2\mu} - \frac{GM\mu}{r} + \frac{c^2}{2}\mu r^2 \mathcal{E}^q$$

- Where $p = \mu v$, (M, μ) the total and reduced mass of the binary, r the relative distance between m_1 and m_2, and $\mathcal{E}^q$ the quadrupole tidal potential which takes into account the interaction between the binary and the SMBH

Kozai-Lidov Mechanism - Strong Gravity Regime

- We introduce eccentricity e, semi-major axis a and true anomaly ψ for the inner binary $r = \dfrac{a(1-e^2)}{1+e\cos\psi}$



- We introduce the action-angle variables (*Delauney variables*): $(\beta, \gamma, \theta) \rightarrow (J_\beta, J_\gamma, J_\theta)$

- The Hamiltonian becomes $H = -\left(\dfrac{GM}{J_\beta}\right)^2 + H_q$

- Writing explicitly the tidal potential $\mathscr{E}^q$ in terms of the parameters of the SMBH

$$H_q = \frac{\mu}{2}\frac{Gm_3 r^2}{\hat{r}^3}\left[1 + 3\frac{K}{\hat{r}^2}\sin^2(\gamma+\psi)\sin^2 I - 3\left(1+\frac{K}{\hat{r}^2}\right)\left(\cos(\Psi-\theta)\cos(\gamma+\psi) + \sin(\Psi-\theta)\sin(\gamma+\psi)\cos I\right)^2\right]$$

Kozai-Lidov Mechanism - Strong Gravity Regime

- KL mechanism takes place on a longer time scale than both the inner and outer orbit period → Secular dynamics →average over both inner and outer orbits!
- The inner orbit is described by a Newtonian elliptic motion → average over the true anomaly ψ
- The outer orbit is described in full GR regime → average over the Marck's angle Ψ, which for a **circular orbit** grows uniformly with the proper time of the geodesic

$$W = (1 - e^2)(\cos^2 I - 2) - 5e^2 \sin^2 I \sin^2 \gamma$$

$$\langle H_q \rangle = -\Omega^{(GR)}_{ZLK} J_\gamma \left(W + \frac{5}{3} \right)$$

$$\Omega^{(GR)}_{ZLK} = \frac{3}{8J_\gamma} \left(\frac{Gm_3\mu}{\hat{r}} \right) \left(\frac{a}{\hat{r}} \right)^2 \left(1 + 3\frac{K}{\hat{r}^2} \right)$$

Kozai-Lidov Mechanism - Strong Gravity Regime

- In the weak field regime ($\hat{r} \to \infty$) one recovers the Newtonian frequency for the ZLK, this can be seen explicitly writing $\Omega^{(GR)}_{ZLK} = \Omega^{(N)}_{ZLK}\left(1 + 3\frac{K}{\hat{r}^2}\right)$

- Using the secular Hamiltonian we can compute the evolution equations for the inner orbit's parameters

$$\left\langle \frac{da}{d\tau} \right\rangle = -\frac{64}{5}\frac{G^3\mu M^2}{a^3c^5}\frac{1}{(1-e^2)^{7/2}}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) ,$$

$$\left\langle \frac{de}{d\tau} \right\rangle = 5\Omega^{(\mathrm{GR})}_{\mathrm{ZLK}}\, e(1-e^2)\sin^2 I \sin 2\gamma - \frac{304}{15}\frac{G^3\mu M^2}{a^4c^5}\frac{e}{(1-e^2)^{5/2}}\left(1 + \frac{121}{304}e^2\right) ,$$

$$\left\langle \frac{d\gamma}{d\tau} \right\rangle = 2\Omega^{(\mathrm{GR})}_{\mathrm{ZLK}}\left[2(1-e^2) - 5(1-e^2-\cos^2 I)\sin^2\gamma\right] + \frac{3}{ac^2(1-e^2)}\left(\frac{GM}{a}\right)^{3/2} ,$$

$$\left\langle \frac{dI}{d\tau} \right\rangle = -\frac{5}{2}\Omega^{(\mathrm{GR})}_{\mathrm{ZLK}}\, e^2 \sin 2I \sin 2\gamma ,$$

Kozai-Lidov Mechanism - Strong Gravity Regime

Where we included the gravitational back-reaction (Peter's equations) and the periastron precession of the inner binary

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Kozai-Lidov Mechanism - Strong Gravity Regime

New effects in the strong gravity regime!

Binary system close to the SMBH

- Binary on the ISCO
- Enhanced ZLK mechanism
- Faster mergers

Spin of the SMBH

- ISCO closer to the SMBH
- Higher frequency in e oscillations
- PN expansion to recover known results

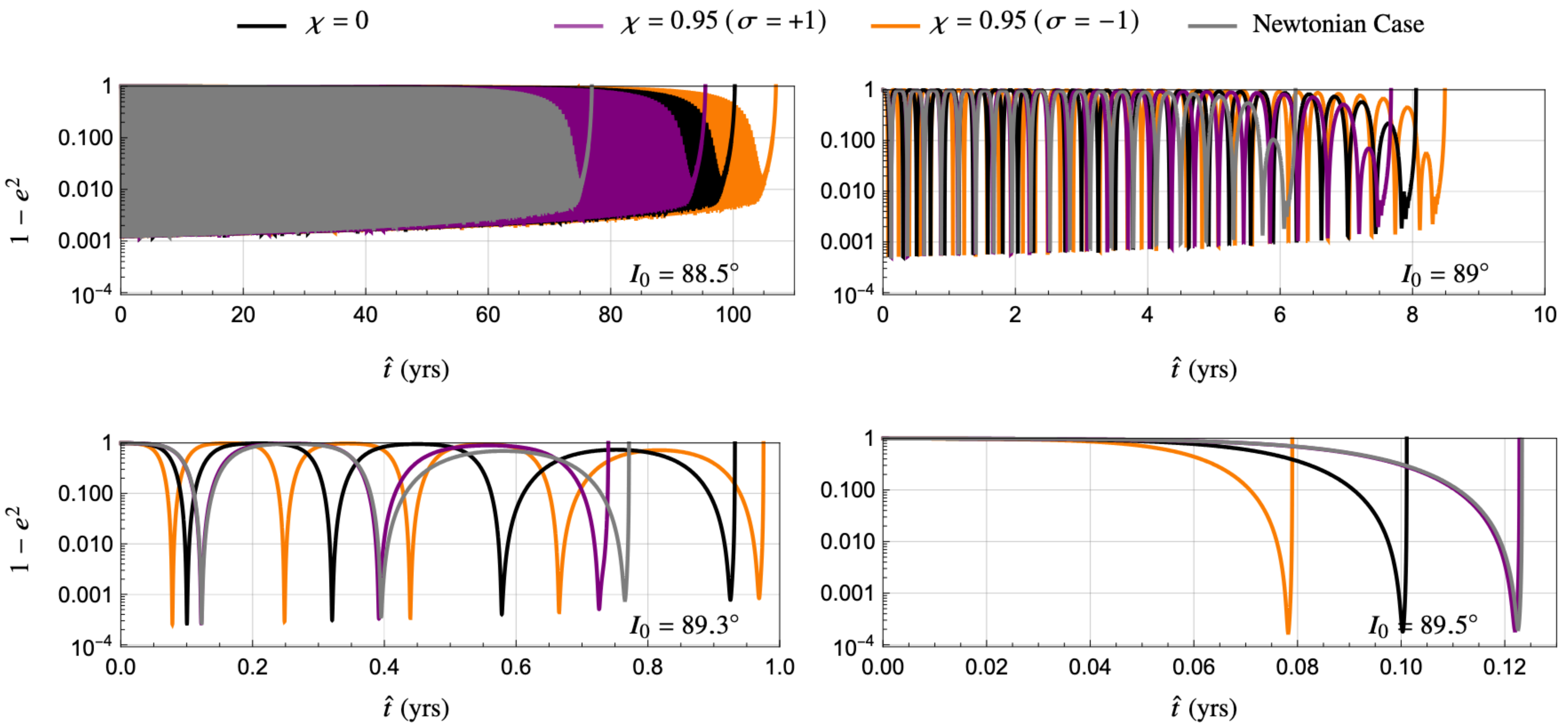
Gyroscope precession

- Precession of L_{in} due to the curvature of the background
- Fokker de Sitter precession
- Schiff precession

Redshift factor

- Gravitational redshift
- Longer merger times
- Suppressed GW frequency

Kozai-Lidov Mechanism - Strong Gravity Regime

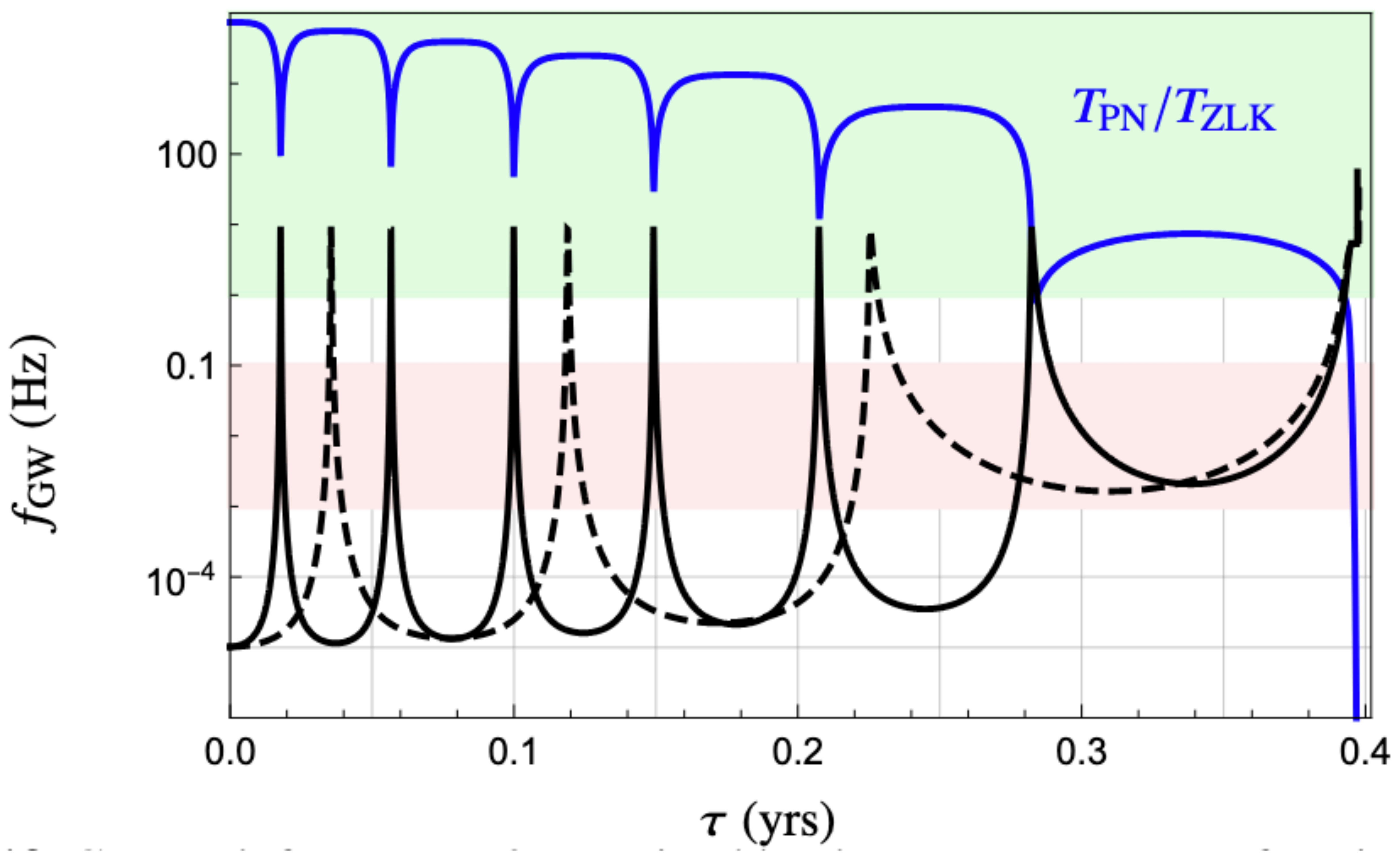


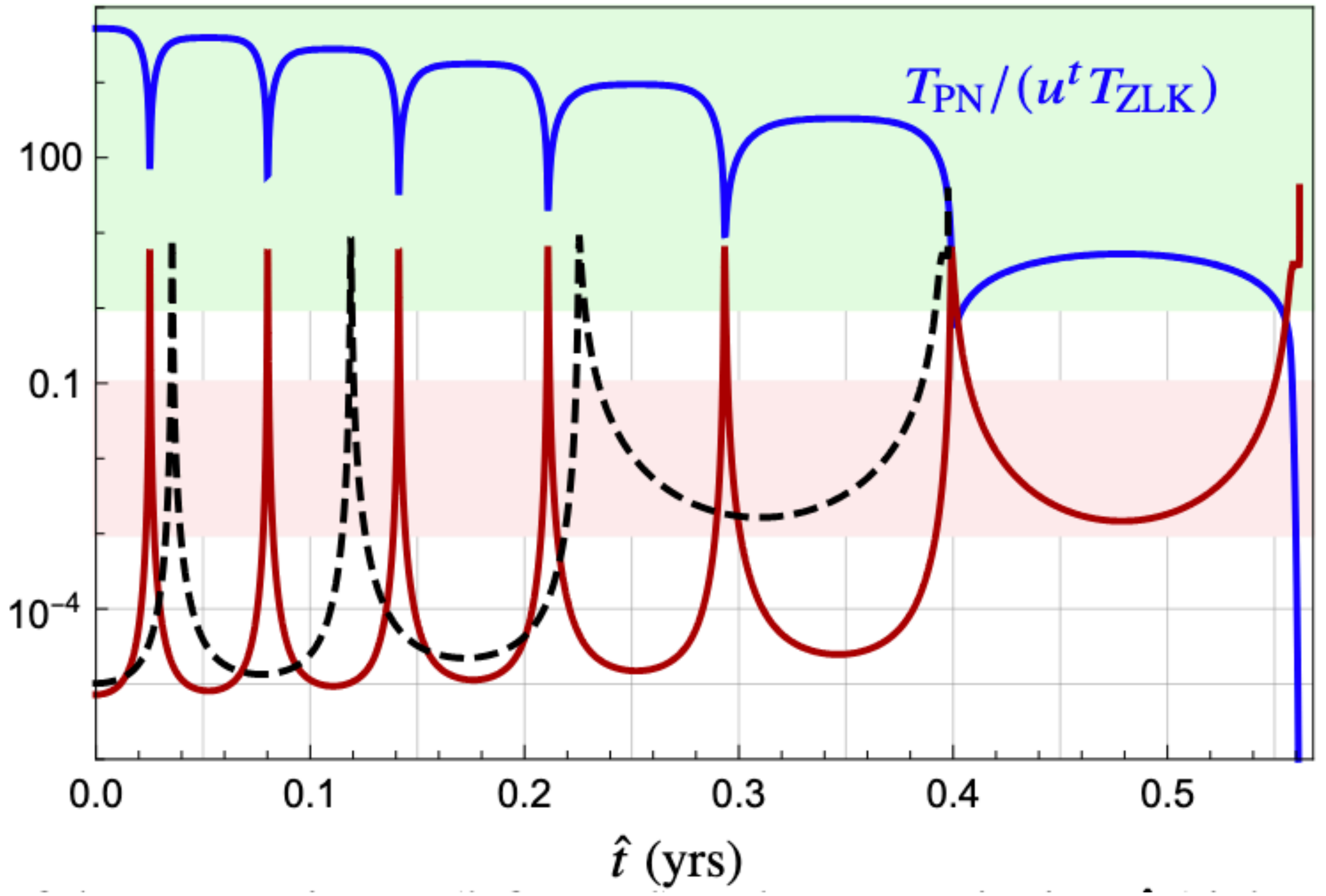
Kozai-Lidov Mechanism - Strong Gravity Regime

GW peak frequency at the ISCO

$$f_{GW} \simeq \frac{\sqrt{GM}}{\pi[a(1-e^2)]^{3/2}}(1+e)^{1.1954}$$

Wen (2003)





Future directions

Eccentric outer orbit! → Magnetic tidal potential

Resonances! → Non-secular effects, high eccentricities when ZKL not triggered

Octupole tidal potential! → New effects, orbital flip

Thanks for your attention!