Neutron stars and the cosmological constant problem

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In collaboration with P.G.S. Fernandes, A. Maselli, A. Padilla and T.P. Sotiriou: arXiv:2404.19012

Phase transitions in the core of NS

In core of neutron stars, local pressure high enough to trigger the QCD phase transition ⟶ jump in vacuum energy!

Expected shift of cosmological constant of $\Lambda_{\rm QCD} \sim (200{\rm MeV})^4$.

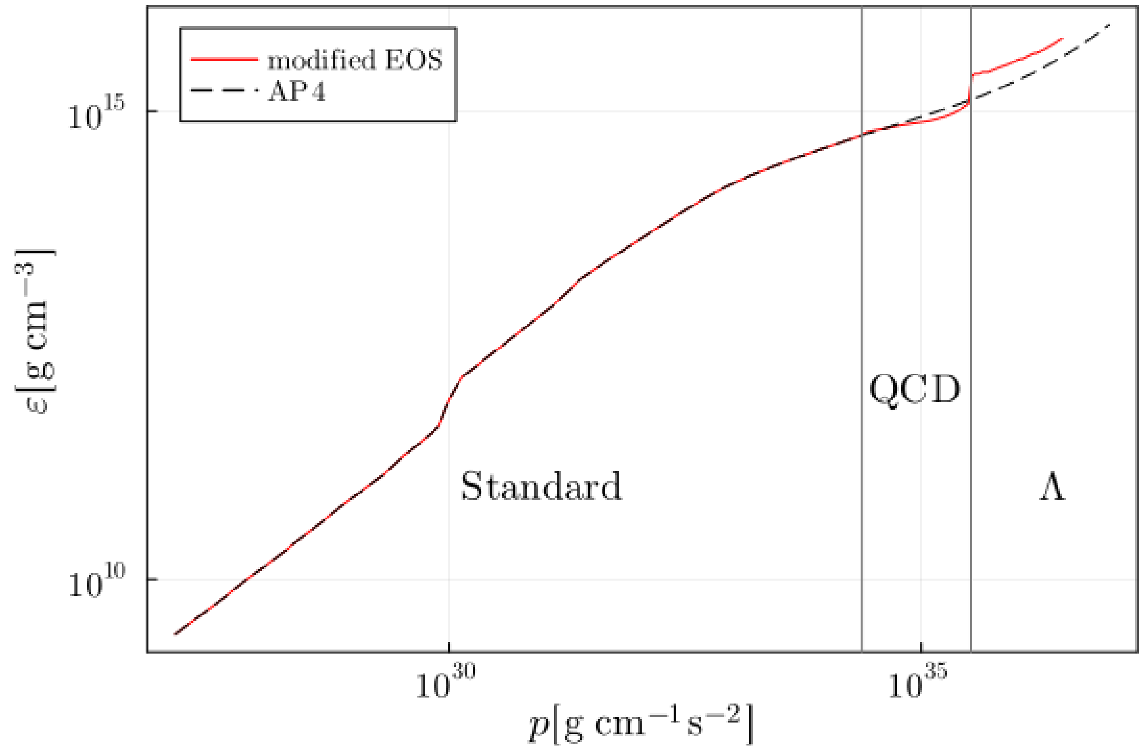
Imprints on properties of the star such as M-R relations, tidal deformability, GW observables.

⟶ NSs can be novel probes for cosmological constant problem!

Bellazzini et al, arXiv:1502.04702
Csaki et al, arXiv:1802.04813

Modified equation of state

To describe phase transition in the core, we modify the 'standard' EOSs by allowing first a QCD phase and on top of that a VE transtion.



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Low-density equation of state

Outer layers of the star described by either SLy or AP4:

- SLy, non-relativistic mean field approach.
- AP4, variational chain summation methods.

Note: difference with previous work of Csaki et al. – we use tables of EOS and not piecewise polytropes.

QCD phase transition

Tews et al., arXiv:1801.01923
Tews et al., arXiv:1804.02783

- For $\rho > \rho_{\rm tr} = 2\rho_0$ we extend EOS to higher density with speed-of-sound parametrization $c_s^2 = \partial p(\epsilon)/\partial\epsilon$.

- Randomly pick six reference point (ρ, c_s) in the range $\rho \in (\rho_{\rm tr}, 12\rho_0)$ and $c_s \in (0, 1)$ connect them with linear segments.

- It does not allow us to infer information on dense matter composition, but pragmatic technique and it allows us to create a large number of EOS.

Vacuum energy phase transition

VE phase transition triggered when $p > p_c = (200\mathrm{MeV})^4$. Only dense enough NS will develop a jump in VE:

$$p_\mathrm{t} = \begin{cases} p_\mathrm{fl} & p_\mathrm{t} < p_c \\ p_\mathrm{fl} - \Lambda & p_\mathrm{t} \geq p_c \end{cases} \qquad \epsilon_\mathrm{t} = \begin{cases} \epsilon_\mathrm{fl} & p_\mathrm{t} < p_c \\ \epsilon_\mathrm{fl} + \Lambda & p_\mathrm{t} \geq p_c \end{cases} \qquad \rho_\mathrm{t} = \begin{cases} \rho_\mathrm{fl} & p_\mathrm{t} < p_c \\ \rho_\mathrm{in} \exp\left(\int_{\epsilon *}^{\epsilon_\mathrm{t}} \frac{d\tilde{\epsilon}_\mathrm{t}}{\tilde{\epsilon}_\mathrm{t} + p_\mathrm{t}}\right) & p_\mathrm{t} \geq p_c \end{cases}$$

Difference with Csaki et al.

We require continuity in TOTAL pressure.

Reasonable to expect an increase of $(\mathcal{O}(100)\mathrm{MeV})^4$ but we explore also negative values.

Numerical implementation

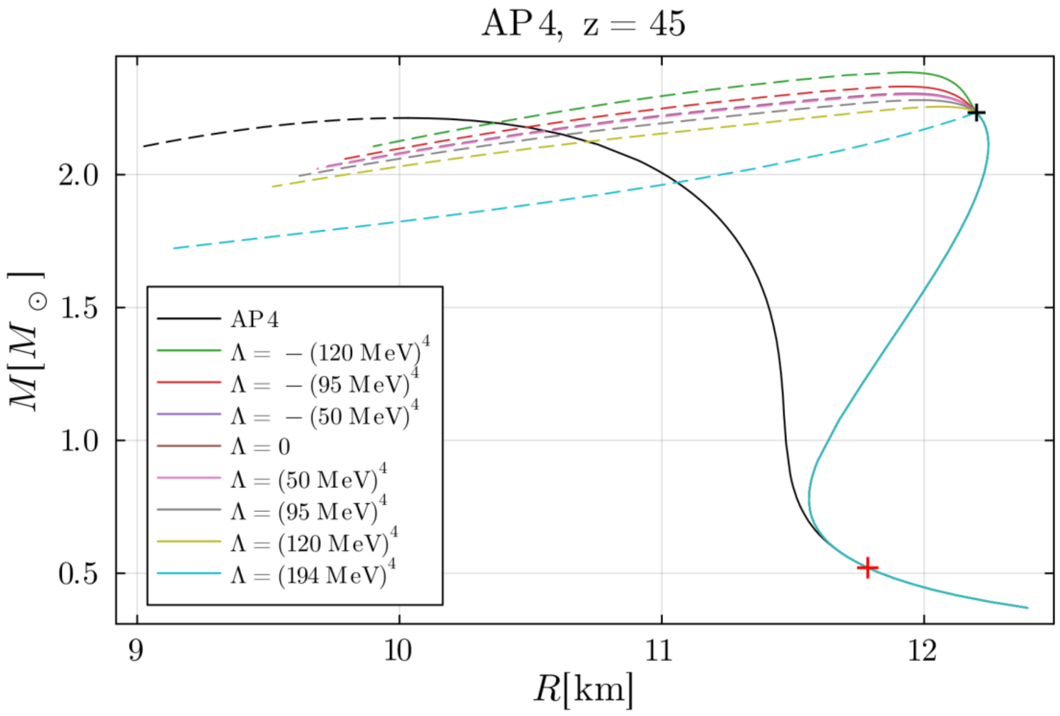
- For static stars, for each EOS we solve the TOV with RK4 in Julia.

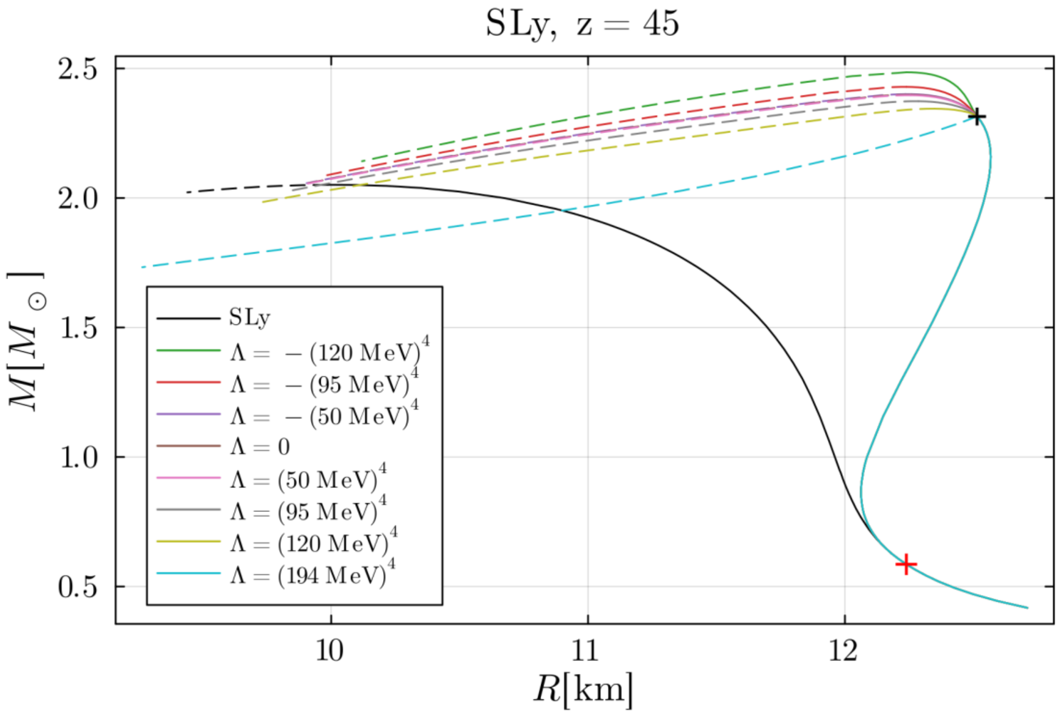
- Admissibility criterion (in agreement with PSR J0952-0607):
  $M_{\max} \in (2.18\, M_{\odot}, 2.52\, M_{\odot})$.

- For rotating stars we used the RNS code (Stergioulas, available at github.com/cgca/rns).

M-R curve for static stars

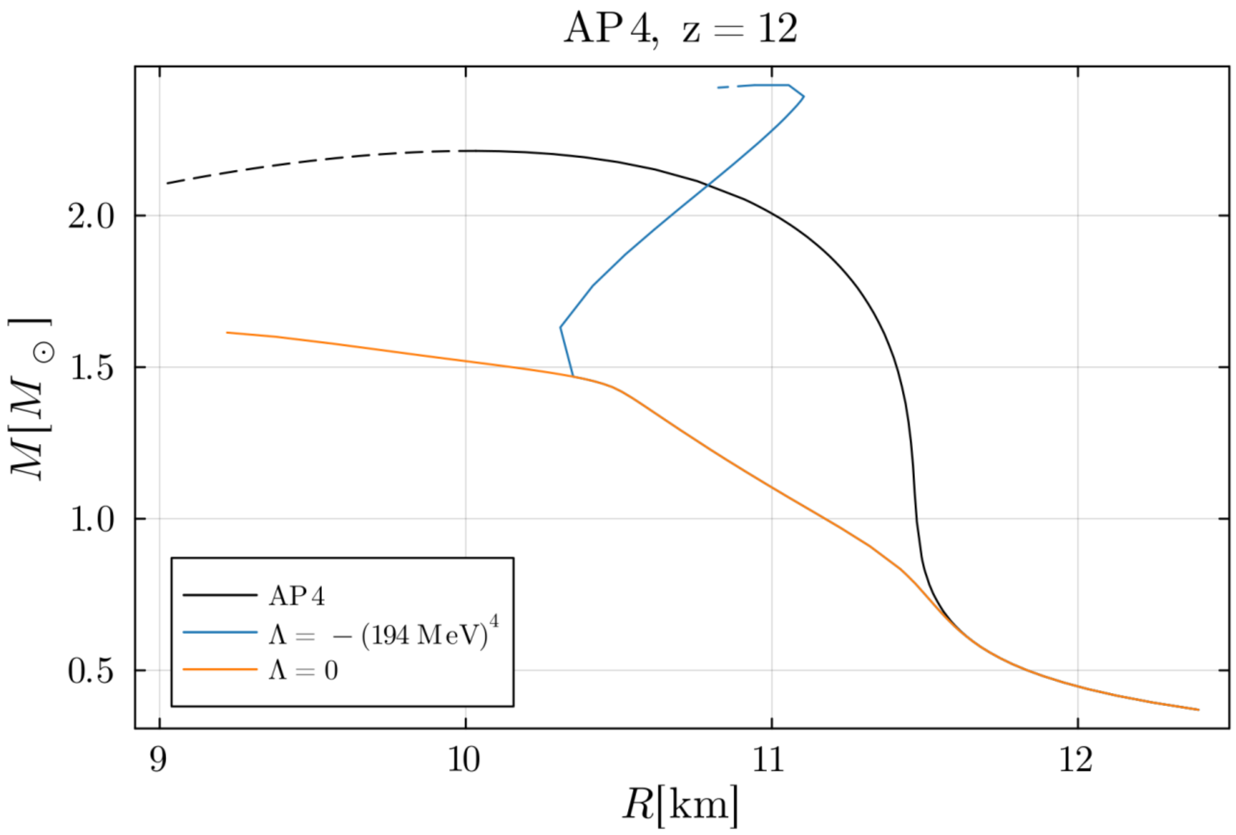
- First ‘family’ of EOS:





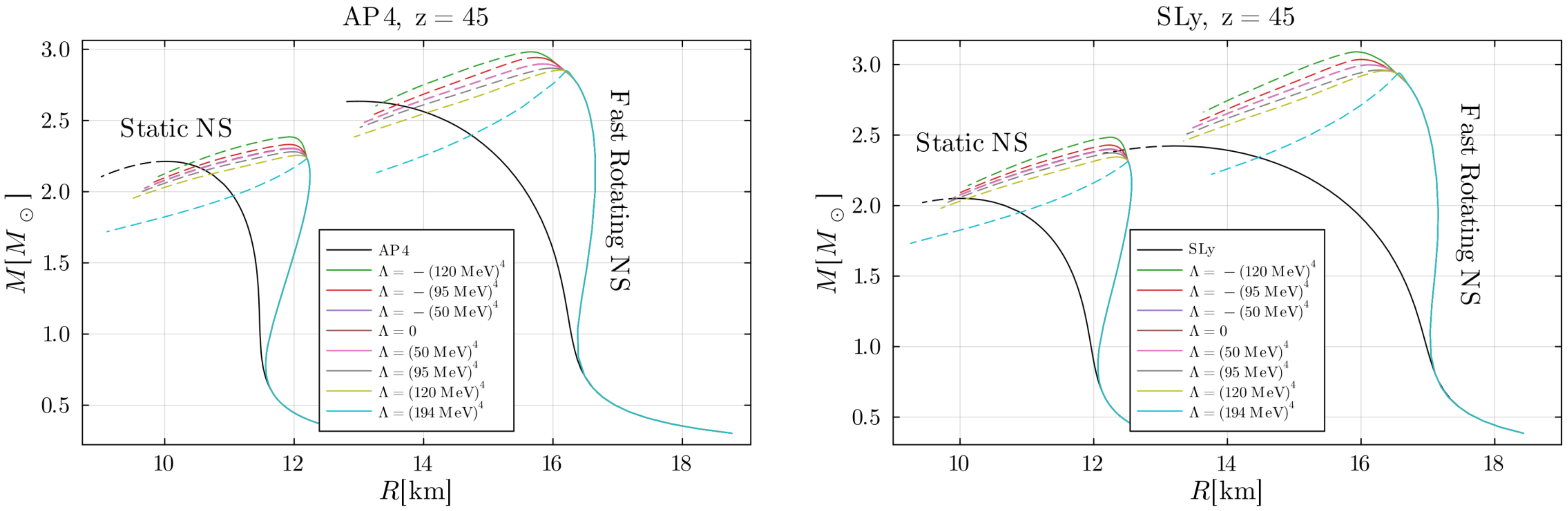
M-R curve for static stars

- Second ‘family’ of EOS:



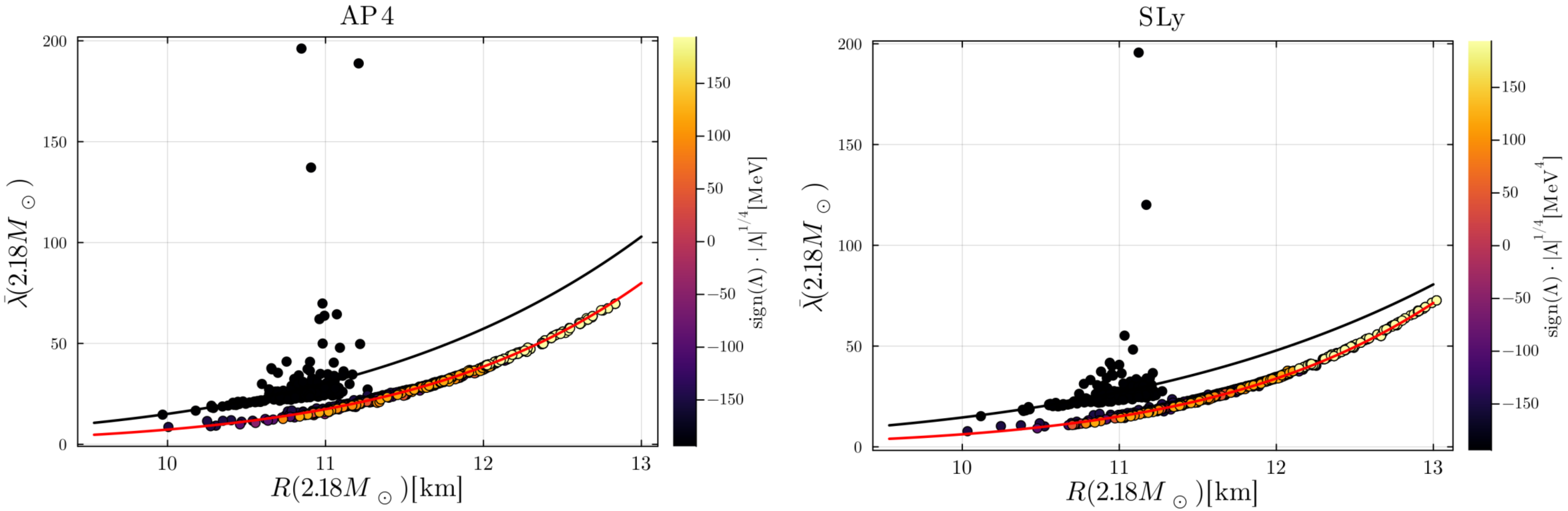
M-R curve for rotating stars

- Mass-radius relations for rotating stars for the first ‘family’ of EOS:



Tidal deformability

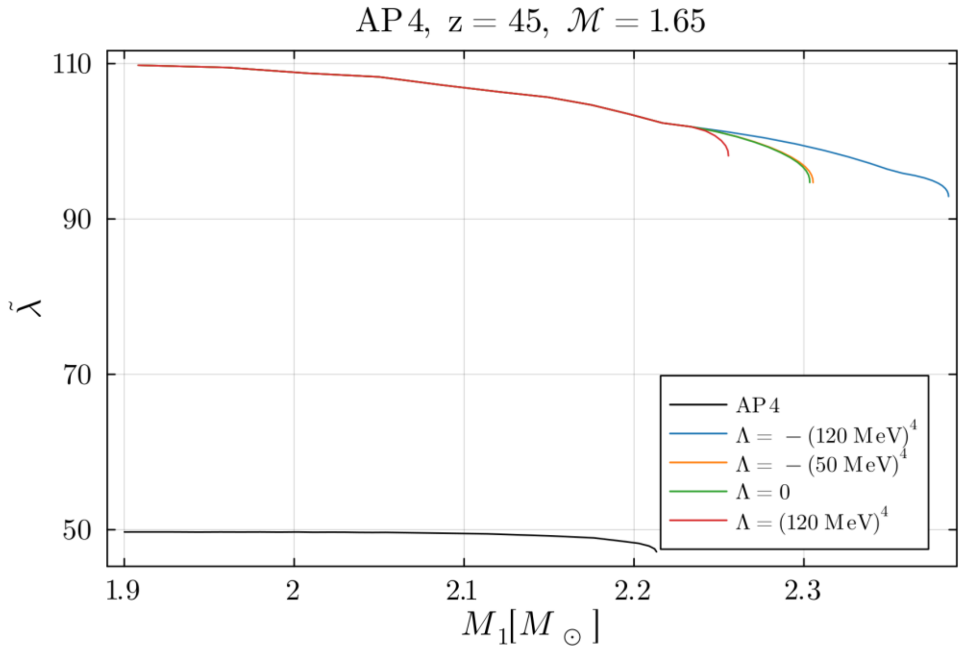
- We plot $\lambda(R)$ for a high fixed mass $M = 2.18\,M_\odot$:

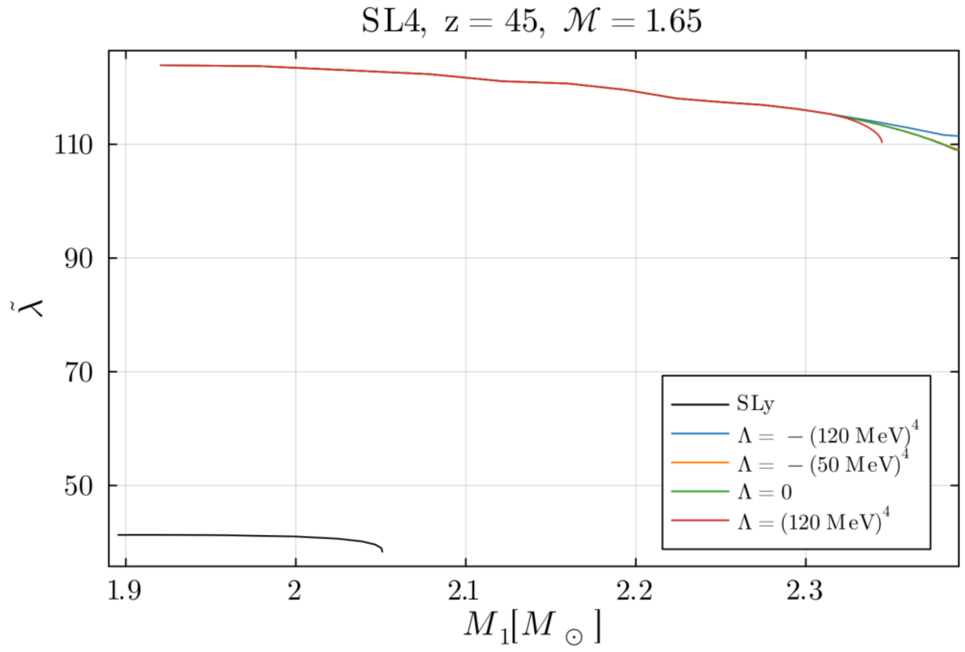


Combined tidal deformability

$$\tilde{\lambda} = \frac{16}{13}\left[(M_1 + 12\,M_2)M_1^4\bar{\lambda}_1 + (M_2 + 12\,M_1)M_2^4\bar{\lambda}_2\right]/(M_1 + M_2)^5$$

All results in agreement with GW170817.

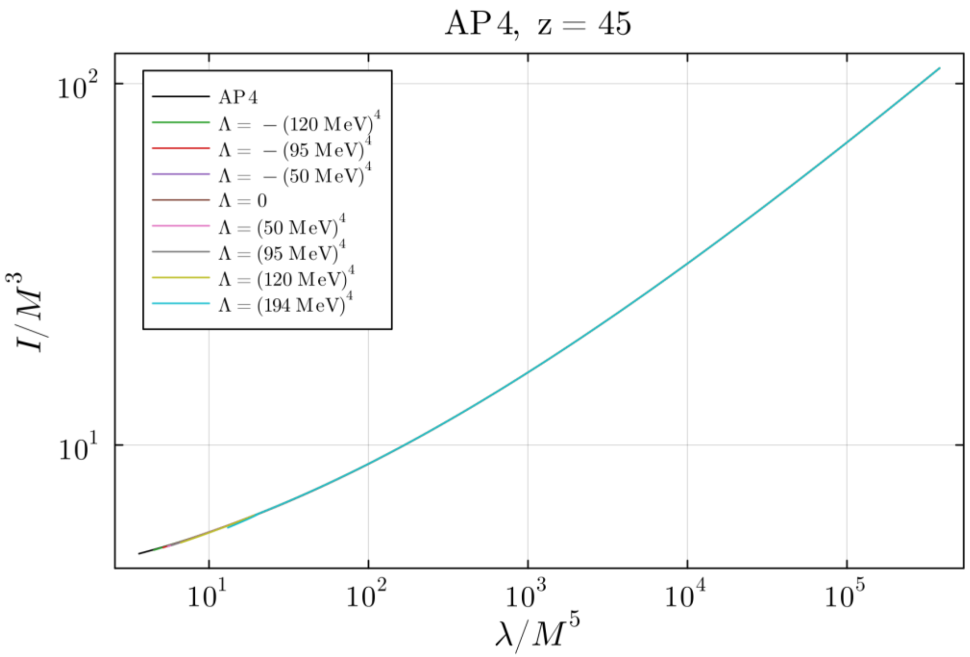


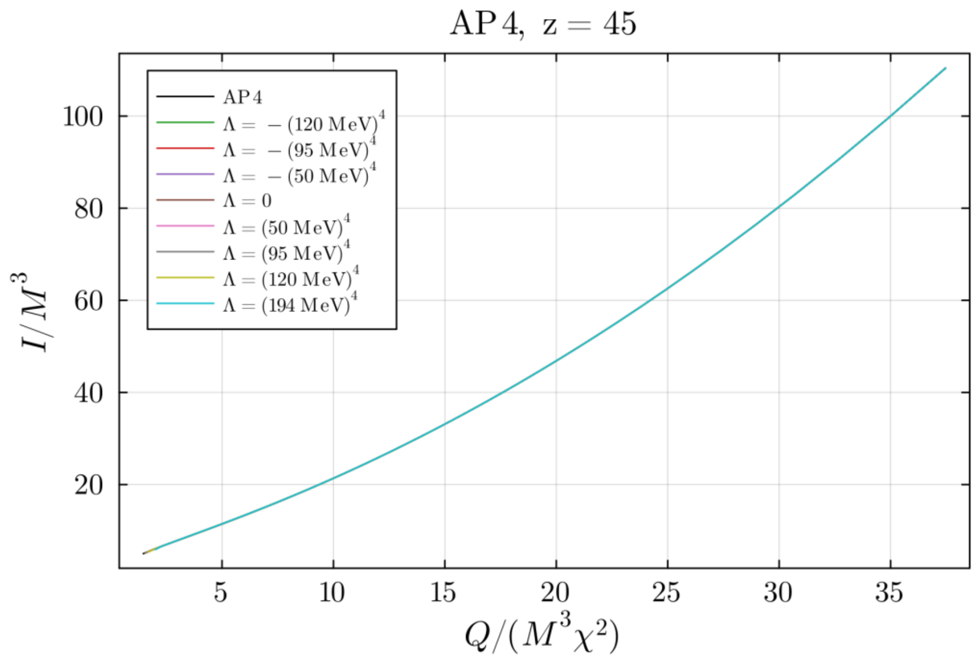


I-Love-Q relations

Yagi and Yunes, arXiv:1302.4499
Yagi and Yunes, arXiv:1303.1528

- Relations first discovered by Yagi and Yunes.





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Conclusion and perspectives

- We used NSs as probes of vacuum energy transitions at their core
- Two different ‘families’ of EOSs
- Shifts in M-R curve and combined tidal deformability
- I-Love-Q relations are not affected

For the future:

- Modified gravity sector
- Numerical simulations

Thank you!

Speed-of-sound formalism

Tews et al., arXiv:1801.01923
Tews et al., arXiv:1804.02783

- Once we randomly create the segments, we construct a new high-density table of EOS using the following algorithm:

$$\begin{cases} \rho_{i+1} = \rho_i + \Delta\rho \\ \epsilon_{i+1} = \epsilon_i + \Delta\epsilon = \epsilon_i + \Delta\rho\left(\frac{\epsilon_i + p_i}{\rho_i}\right) \\ p_{i+1} = p_i + c_s^2\Delta\epsilon \end{cases}$$

Setup: static NS

- Ansatz for the metric: $ds^2 = g^{(0)}_{\mu\nu} dx^\mu dx^\nu = -e^{\nu(r)} dt^2 + e^{\mu(r)} dr^2 + r^2 d\Omega^2$

- Matter described by perfect fluid: $T_{\mu\nu} = (\epsilon + p) u_\mu u_\nu + p\, g^{(0)}_{\mu\nu}$

- TOV equations: $$\begin{cases} m'(r) = 4\pi r^2 \epsilon(r), \\ p'(r) = -\frac{p(r)+\epsilon(r)}{r[r-2Gm(r)]} G[m(r) + 4\pi r^3 p(r)], \\ \nu'(r) = -\frac{2p'(r)}{p(r)+\epsilon(r)}. \end{cases}$$

Setup: perturbing static NS

Hinderer, arXiv:0711.2420
Hinderer et al, arXiv:0911.3535
Thorne and Campolattaro, ApJ 149, 591 (1967)

- Let us assume star is primary in NS binary. Each component is tidally deformed by the other.
- Primary is immersed in static external quadrupolar field E, in response it develops quadrupole moment.

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$k_2 = \frac{3}{2} G \lambda R^{-5}$$

$$-\frac{1+g_{tt}}{2} = -\frac{M}{r} - \frac{3}{2}\frac{Q_{ij}}{r^3} n^i n^j + \ldots + \frac{\mathcal{E}_{ij}}{2} r^2 n^i n^j + \ldots$$

Hinderer, arXiv:0711.2420
Hinderer et al, arXiv:0911.3535
Thorne and Campolattaro, ApJ 149, 591 (1967)

Setup: perturbing static NS

- We can introduce linear perturbation: $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$
- Perturbed metric is:

$$h_{\mu\nu} = e^{\nu(r)} H(r) Y_{20}(\theta,\phi) dt^2 + e^{\mu(r)} H(r) Y_{20}(\theta,\phi) dr^2 + r^2 K(r) Y_{20}(\theta,\phi) d\Omega^2$$

- Substituting in Einstein's equations one can solve for K and H.
- Comparing outside solution of H with $g_{tt}|_{r\to\infty}$ we can solve for k_2.

Setup: rotating NS

Komatsu et al, Mon. Not. Roy. Astron. Soc. 237, 355 (1989)

- Metric ansatz: $ds^2 = -e^{\gamma+\rho}dt^2 + e^{\gamma-\rho}r^2\sin^2\theta\,(d\phi-\omega dt)^2 + e^{2\alpha}\left(dr^2 + r^2 d\theta^2\right)$

- The field equations are:

$$\begin{cases} \nabla^2\left[\rho e^{\gamma/2}\right] = S_\rho(r,\theta) \\ \nabla^2\left[r\sin\theta\cos\phi\,\omega e^{(\gamma-2\rho)/2}\right] = r\sin\theta\cos\phi\, S_\omega(r,\theta) \\ \left(\partial_\varpi^2 + \partial_z^2\right)\left[\varpi\gamma e^{\gamma/2}\right] = \varpi S_\gamma(r,\theta) \\ \partial_\theta\alpha = S_\alpha\,(r,\theta) \end{cases}$$

with $\varpi = r\sin\theta$ and $z = r\cos\theta$.

Rotating stars

Komatsu et al, Mon. Not. Roy. Astron. Soc. 237, 355 (1989)

Using elliptic form of field equations, we can recast them as integral from using Green's functions:

$$\begin{cases} \rho = -\frac{e^{\gamma}/2}{4\pi} \int_0^\infty dr' \int_0^\pi d\theta' \sin\theta \int_0^{2\pi} d\phi' r'^2 \frac{S_\rho(r',\theta')}{|\mathbf{r}-\mathbf{r}'|} \\ \omega = -\frac{e^{(2\rho-\gamma)/2}}{4\pi r \sin\theta \cos\phi} \int_0^\infty dr' \int_0^\pi d\theta' \sin\theta \int_0^{2\pi} d\phi' r'^3 \cos\phi' \frac{S_\omega(r',\theta')}{|\mathbf{r}-\mathbf{r}'|} \\ \gamma = \frac{e^{-\gamma/2}}{2\pi r \sin\theta} \int_0^\infty dr' \int_0^{2\pi} d\theta' r'^2 \sin\theta' S_\gamma(r',\theta') \log|\mathbf{r}-\mathbf{r}'|. \end{cases}$$

Slowly rotating NS

$$ds^2 = -\, e^{\bar\nu(\bar r)} \left[1 + 2\bar h_2(\bar r) P_2(\cos\theta)\right] dt^2 + e^{\bar\lambda(\bar r)} \left[1 + \frac{2\bar m_2(\bar r) P_2(\cos\theta)}{r - 2\bar m(\bar r)}\right] d\bar r^2 +$$

$$\bar r^2 \left[1 + 2\bar K_2(\bar r) P_2(\cos\theta)\right] \times \left\{ d\theta^2 + \sin^2\theta \left\{ d\phi - \left[\Omega_* - \bar\omega_1(\bar r) P_1'(\cos\theta)\right] dt \right\}^2 \right\}$$

$$\begin{cases} m_2 = -r e^{-\lambda} h_2 + \frac{1}{6} r^4 e^{-(\nu+\lambda)} \left\{ r e^{-\lambda} \left(\frac{d\omega_1}{dr}\right)^2 + 16\pi r \omega_1^2 (\epsilon + p) \right\} \ , \\ \frac{dK_2}{dr} = -\frac{dh_2}{dr} + \frac{r - 3m - 4\pi p r^3}{r^2} e^{\lambda} h_2 + \frac{r - m + 4\pi p r^3}{r^3} e^{2\lambda} m_2 \ , \\ \frac{dh_2}{dr} = -\frac{r - m + 4\pi p r^3}{r} e^{\lambda} \frac{dK_2}{dr} + \frac{3 - 4\pi(\epsilon + p) r^2}{r} e^{\lambda} h_2 + \frac{2}{r} e^{\lambda} K_2 + \frac{1 + 8\pi p r^2}{r^2} e^{2\lambda} m_2 + \frac{r^3}{12} e^{-\nu} \left(\frac{d\omega_1}{dr}\right)^2 \end{cases}$$

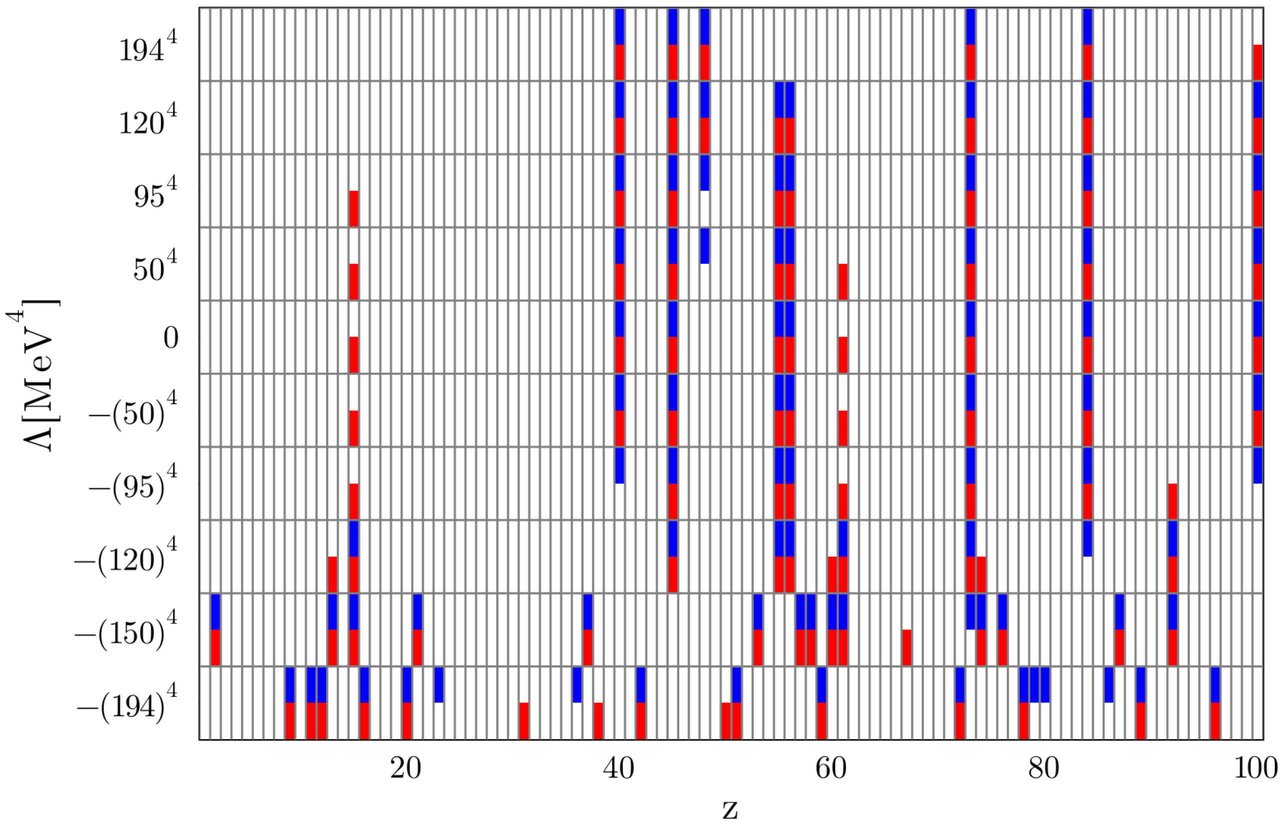
Modified EOS: the dataset

We can build modified tables of EOS:

Small dataset: $2\,\mathrm{std} \times 10^2\,\mathrm{QCD\ mod} \times 10\,\Lambda = 2 \times 10^3\,\mathrm{EOS}$

Large dataset: $2\,\mathrm{std} \times 10^3\,\mathrm{QCD\ mod} \times 10\,\Lambda = 2 \times 10^4\,\mathrm{EOS}$

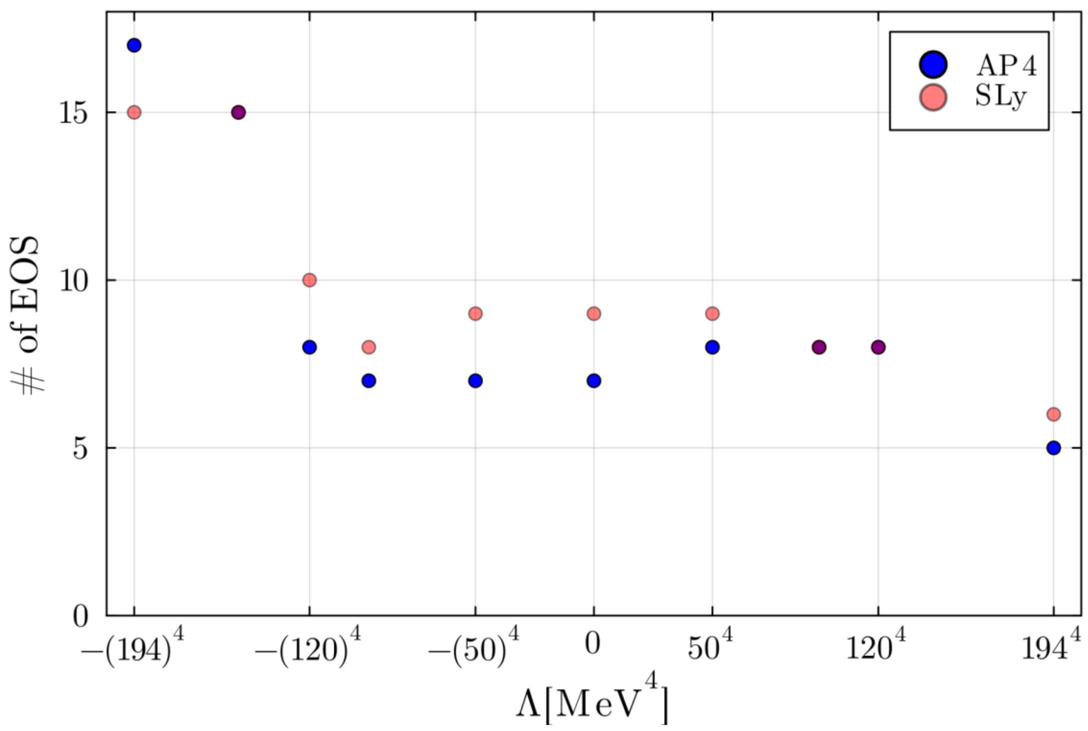
Analysis of dataset



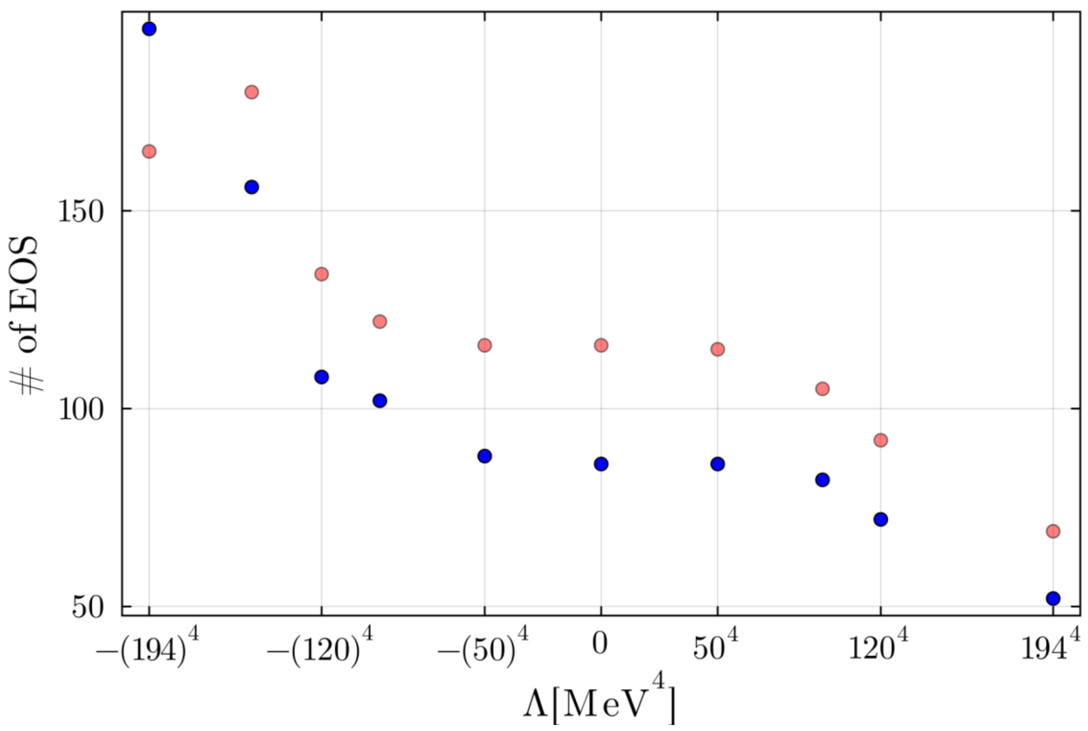
First 'family': wide range of vacuum energy

Second 'family': only very negative vacuum energy

Analysis of dataset



Smaller dataset

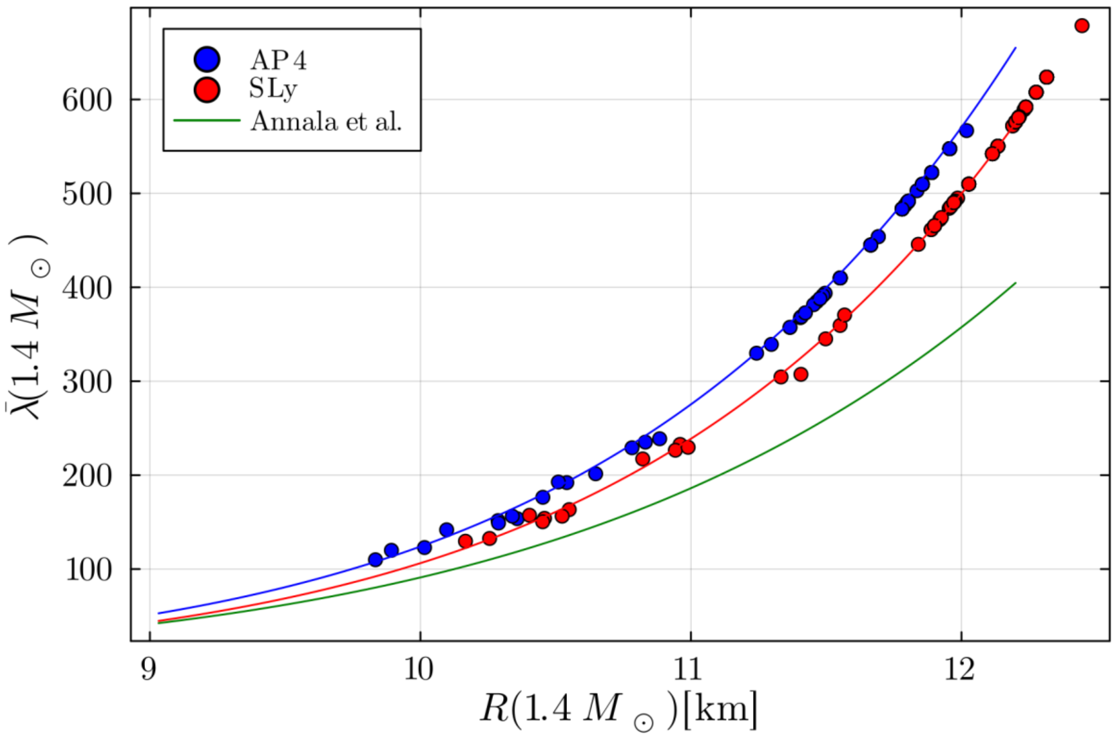


Larger dataset

Tidal deformability

Annala et al., arXiv:1711.02644

- We plot $\lambda(R)$ for a fixed mass $M = 1.14\, M_{\odot}$:



From GW170817, we have: $\bar{\lambda}(1.4 M_{\odot}) \lesssim 800$