Inspiral-inherited ringdown tails

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Simone Albanesi, Gregorio Carullo

[2406.17018]

1st TEONGRAV international workshop on theory of gravitational waves

strong. niels bohr institute

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Settings


m_1
m_2
Inspiral
Merger
Ringdown
(~0.2 ms if M=50 solar masses)
Time
Final state:
• Equilibrium configuration
Strain (10^{-21})
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−700
−600
−500
−400
−300
−200
−100
0
100
time [M]

Question...

Final state:

- **Equilibrium configuration**

What is the late-time signal ≡ tail?

Expectations

First order
perturbation theory

$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}^{(e/o)}(r_*)\right]\Psi_{\ell m}^{(e/o)}(t, r_*) = 0$$

$$\Psi_{\ell m}^{(e/o)}(t=0,r) = \psi_0 \qquad \partial_t \Psi_{\ell m}^{(e/o)}(t=0,r) = \zeta_0$$

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Prediction at $\mathscr{I}^+$:

- $\Psi_{\ell m} = \dfrac{A_{\text{tail}}}{\tau^{\ell+2}}$

$$\tau \equiv t - r_*$$

$$r_* \equiv r + 2M\ln(r-2M)$$

[Price, Phys. Rev. D 5, 2419]
[Leaver, Phys. Rev. D 34, 384]

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Price's law

[Price, Phys. Rev. D 5, 2419]
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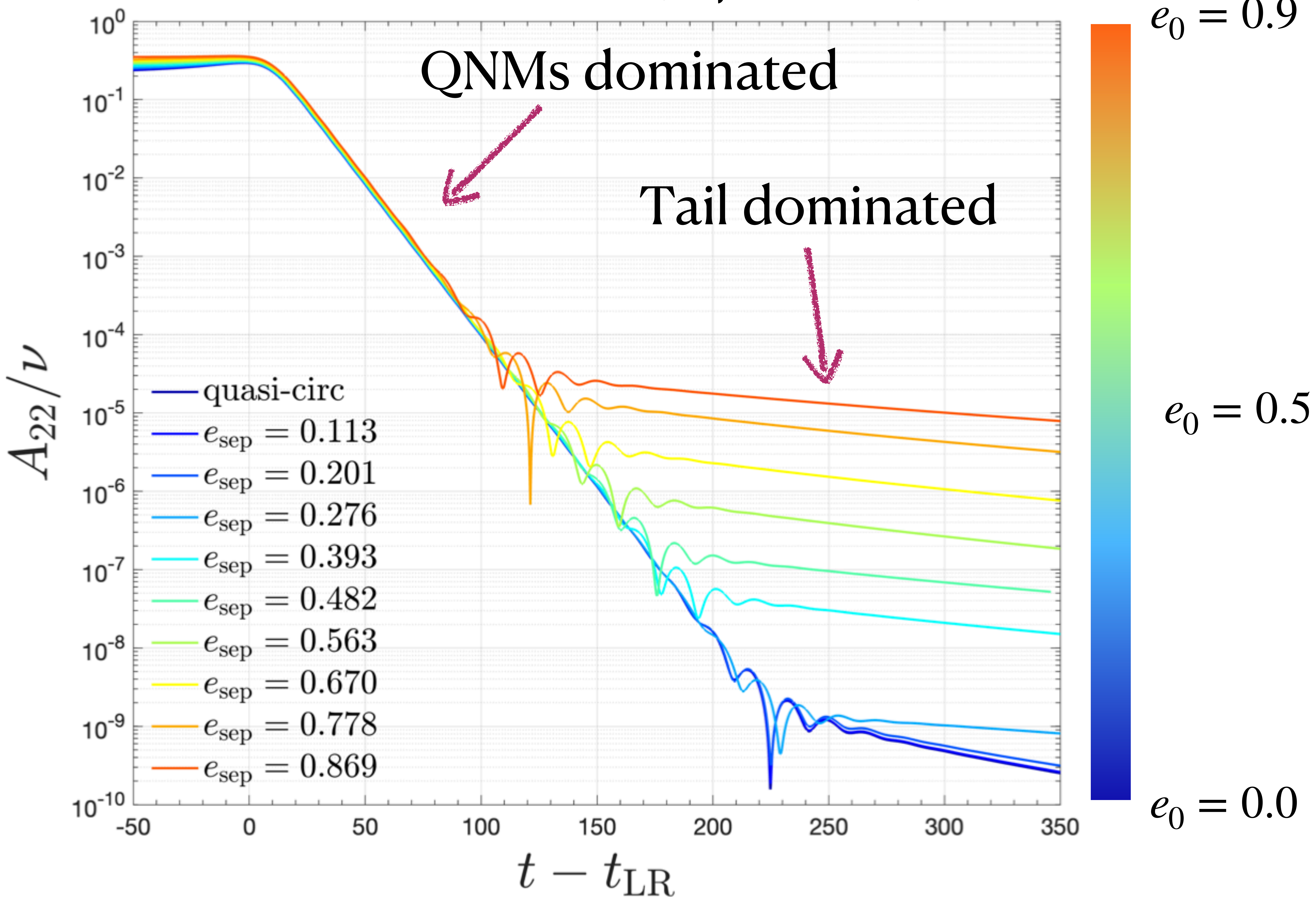
An exciting journey: previous results for EMR

[Albanesi et al, Phys. Rev. D 108, 084037]



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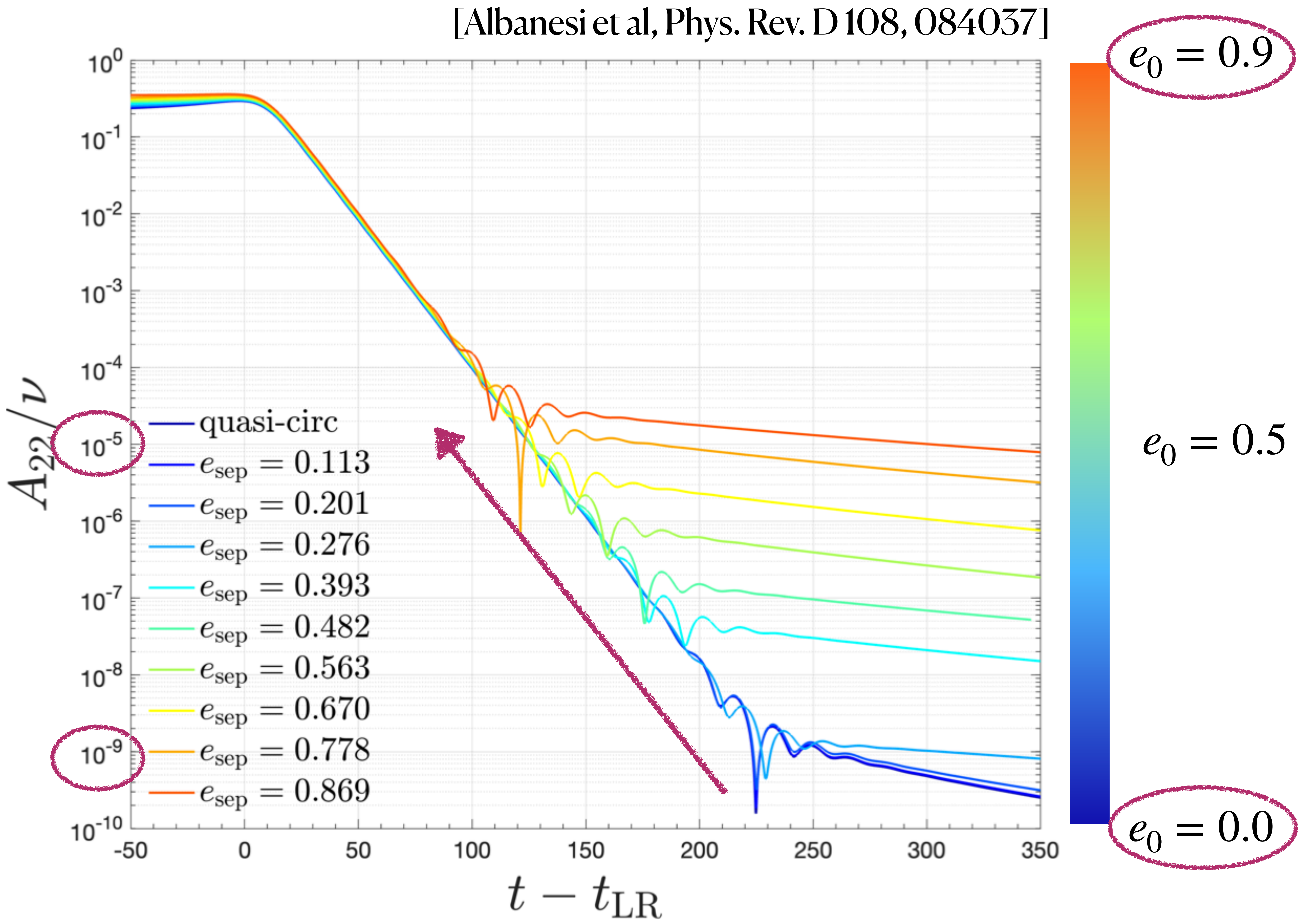

Amplitude

enhanced of several orders of magnitude by eccentricity

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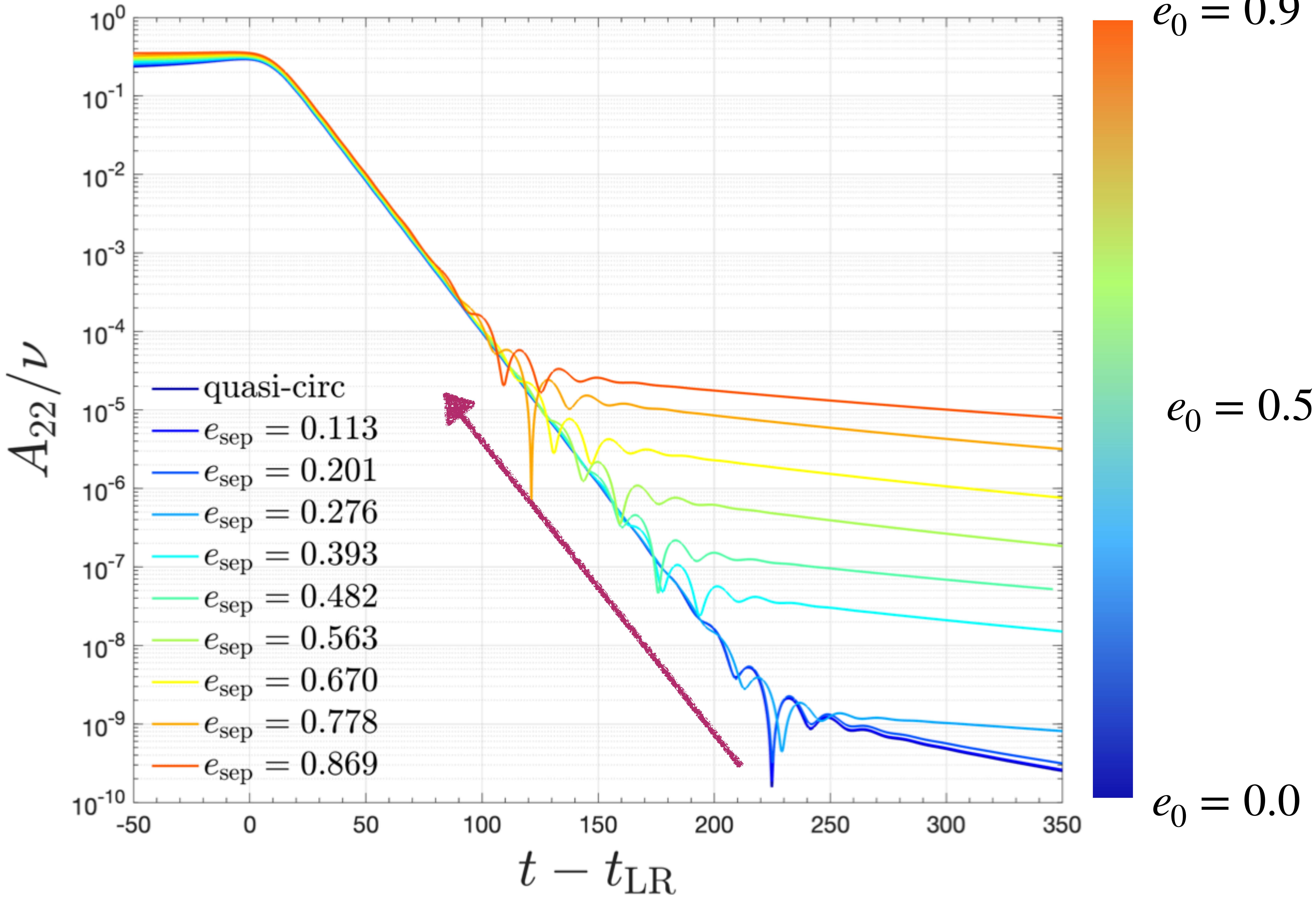

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What happens for comparable masses

An exciting journey: EMR vs comparable masses

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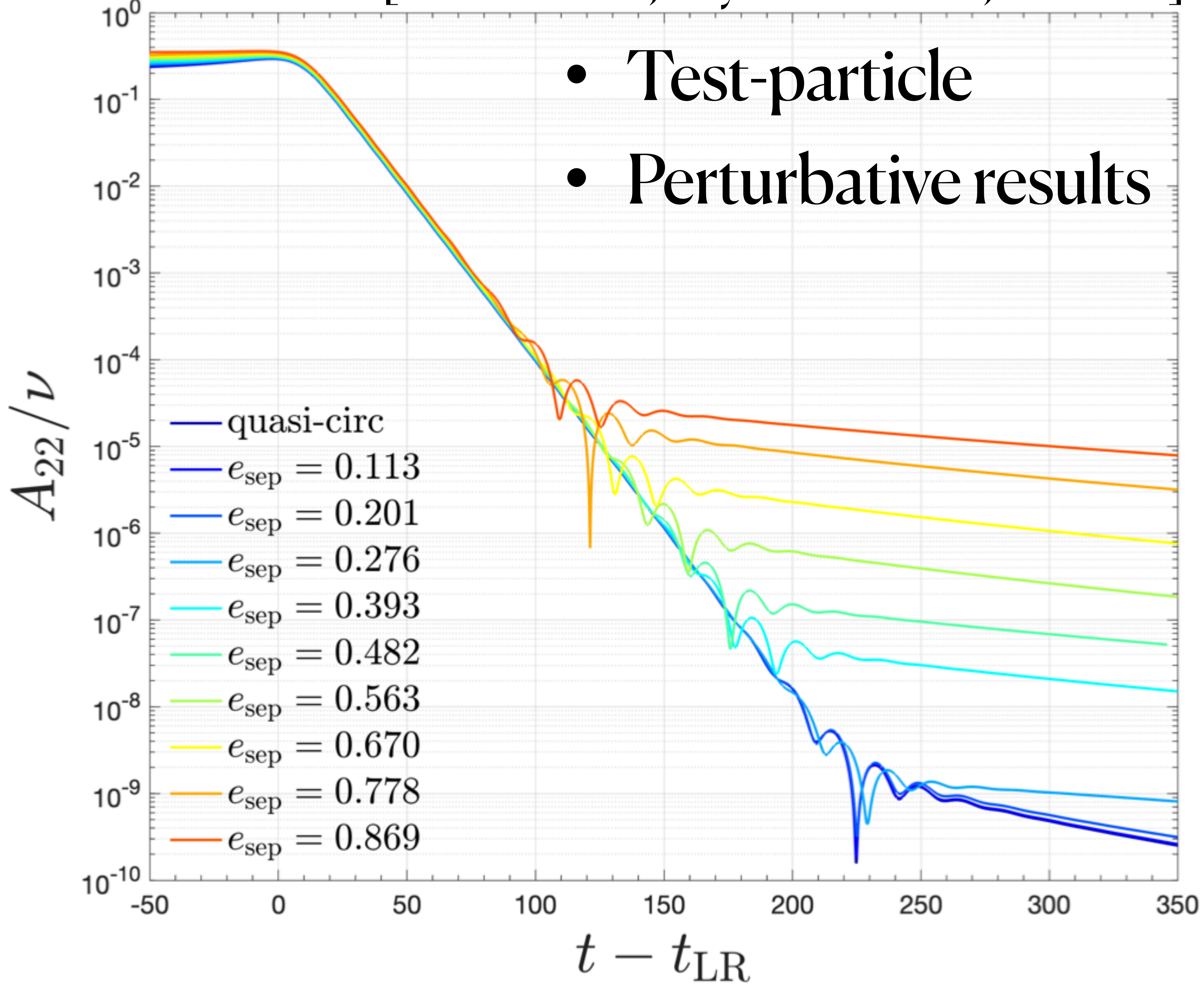
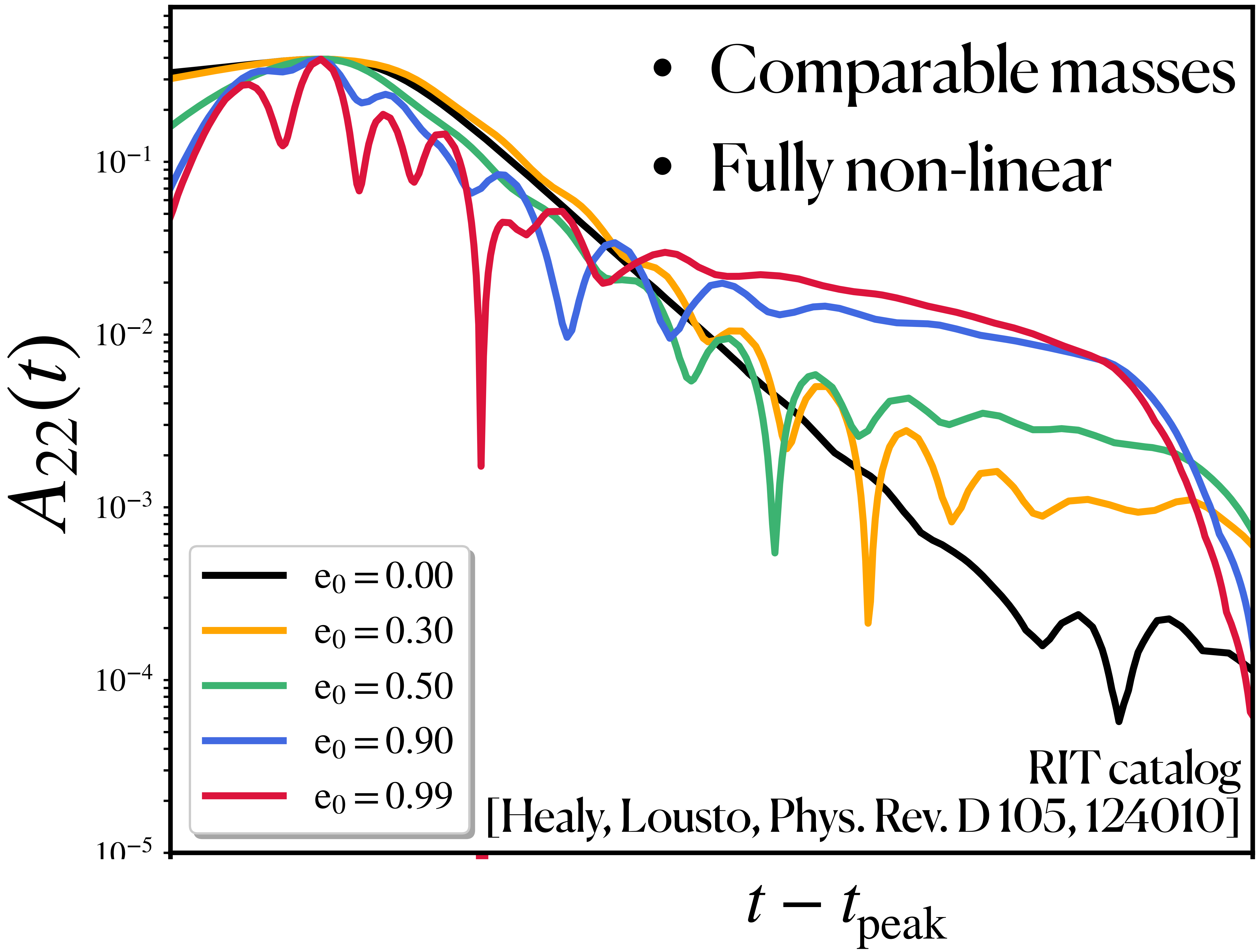


[Carullo and **De Amicis**, 2310.12968]



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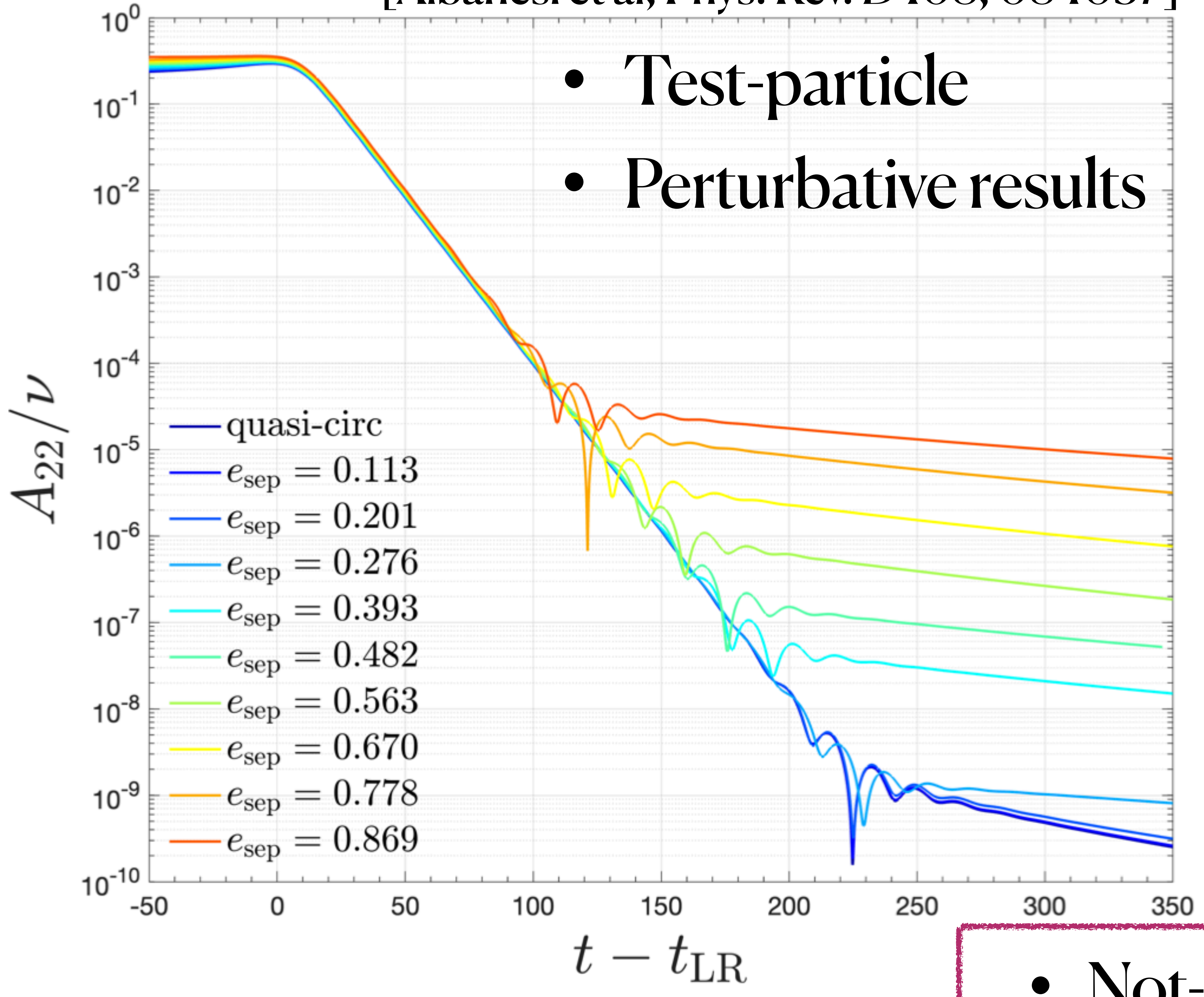
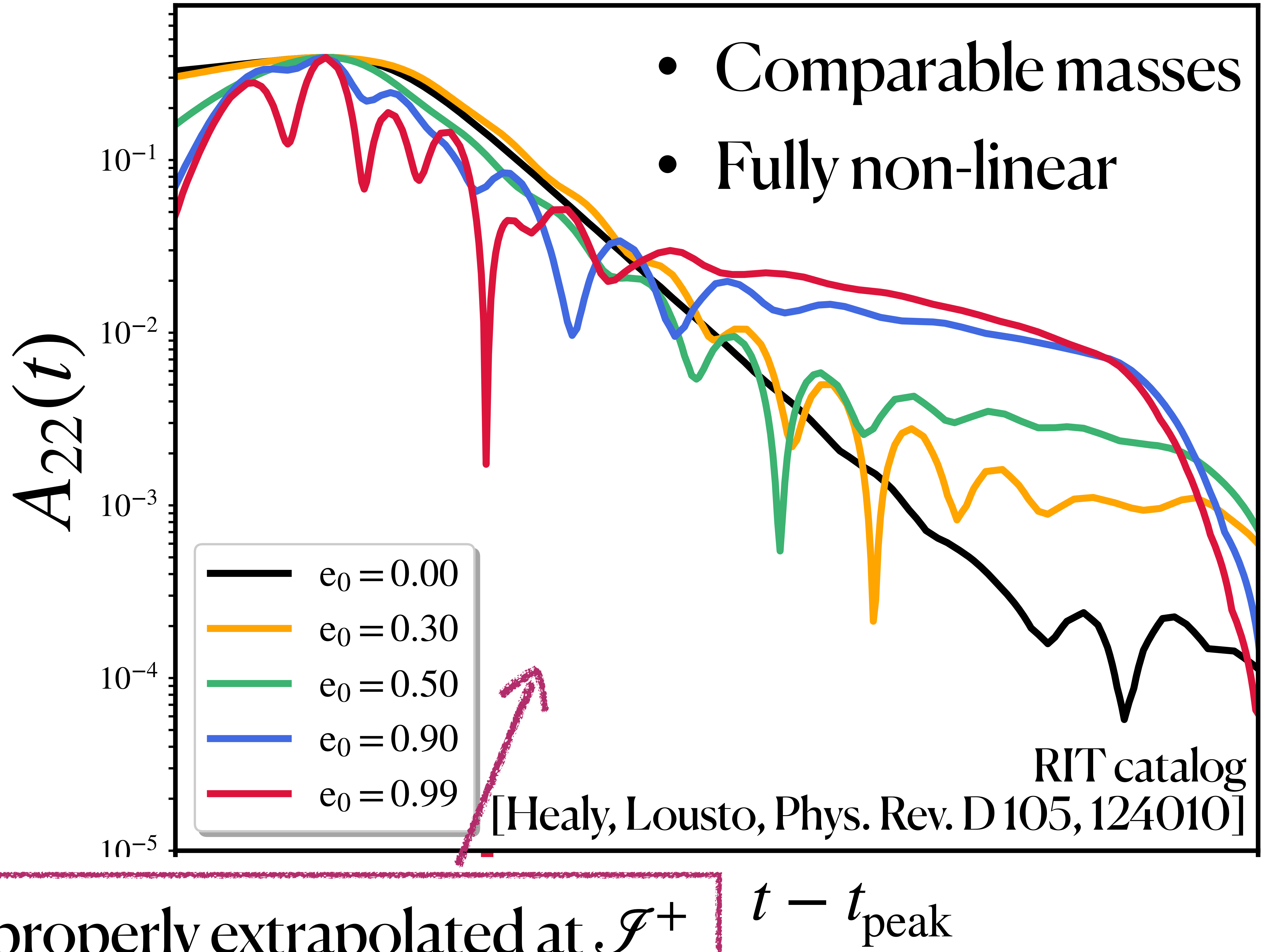


- Test-particle
- Perturbative results

[Carullo and **De Amicis**, 2310.12968]



[Healy, Lousto, Phys. Rev. D 105, 124010]

- Comparable masses
- Fully non-linear

- Not-properly extrapolated at $\mathscr{I}^+$

An exciting journey: EMR vs comparable masses

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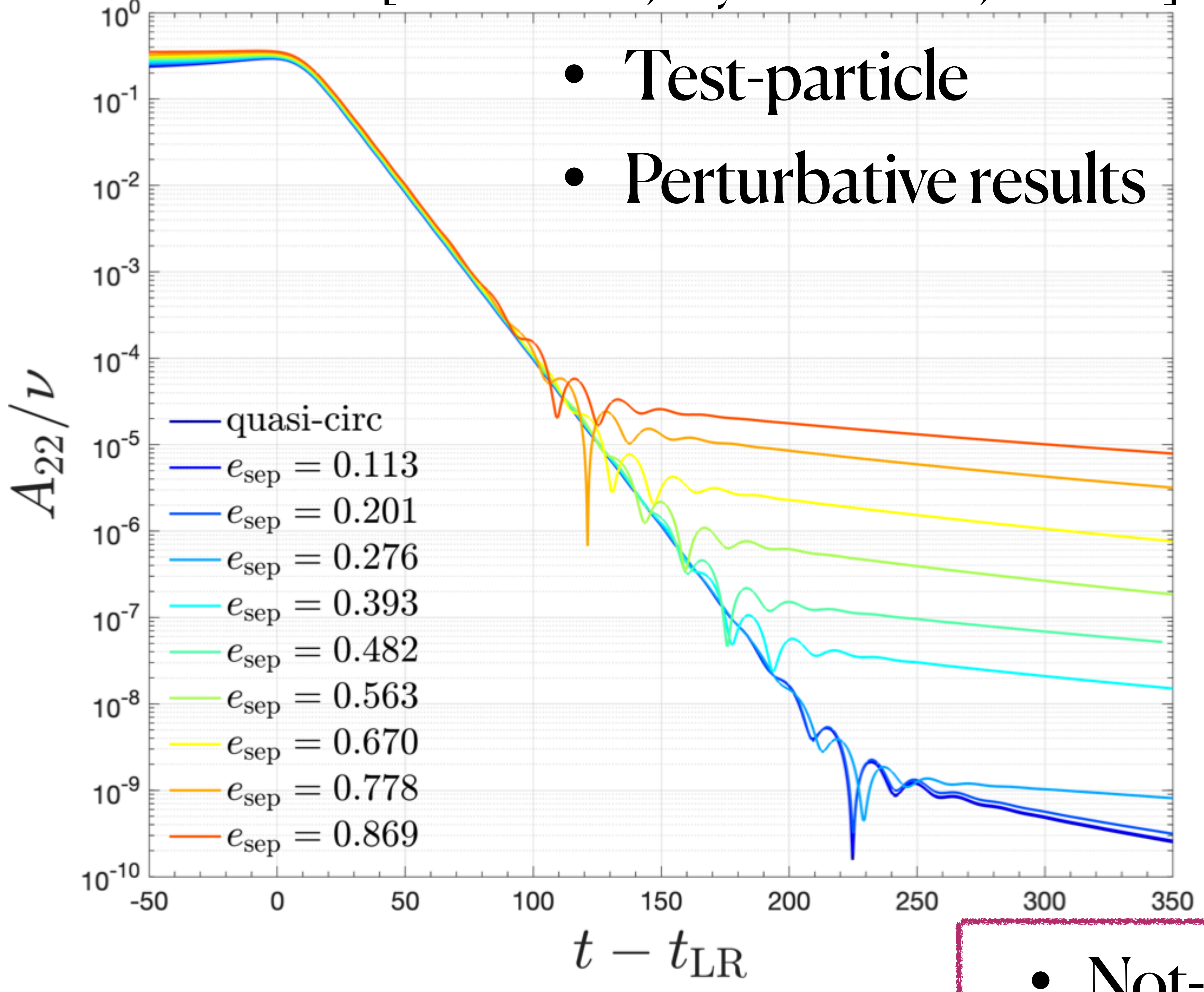

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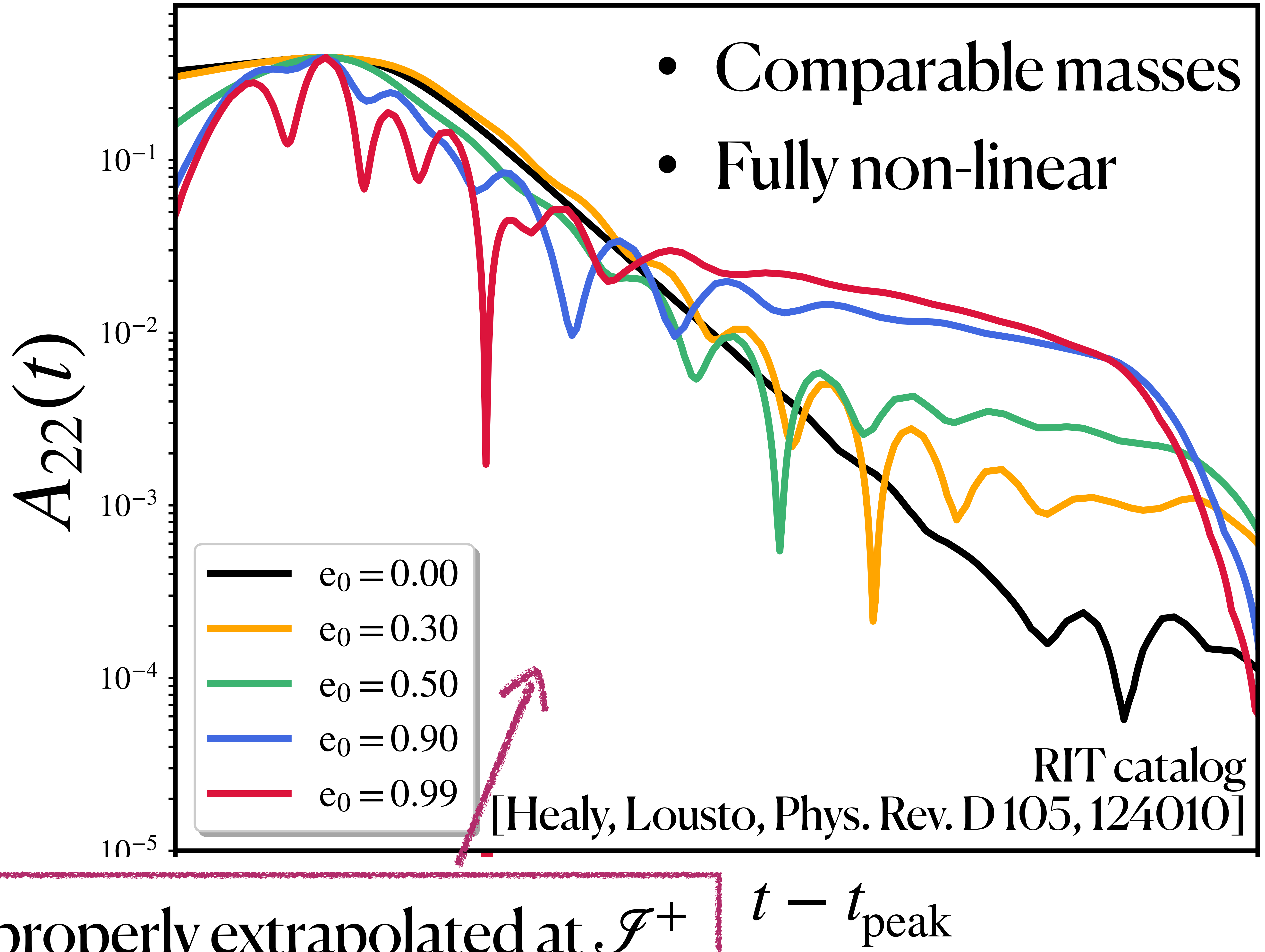

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- Is enhancement physical?

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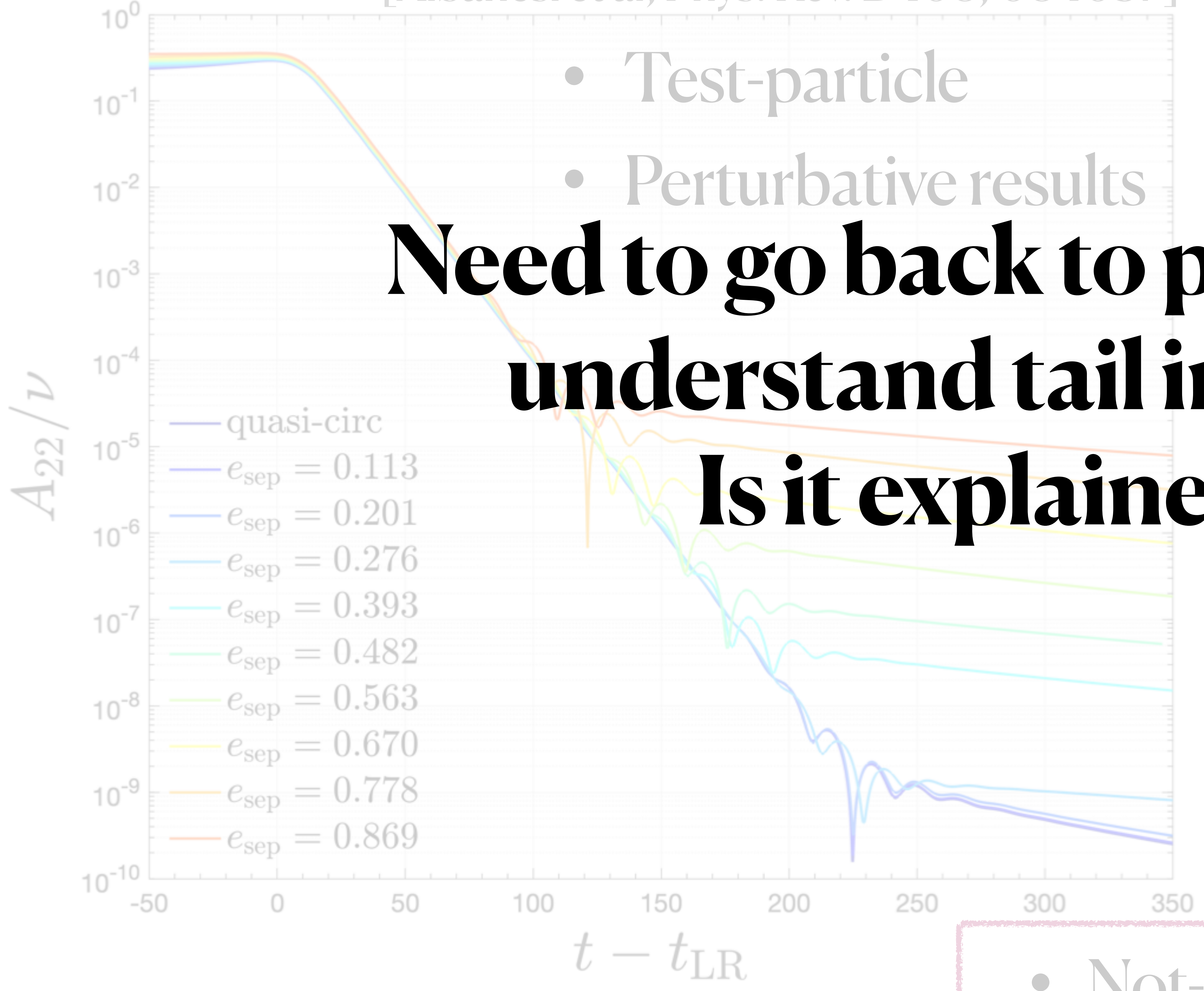

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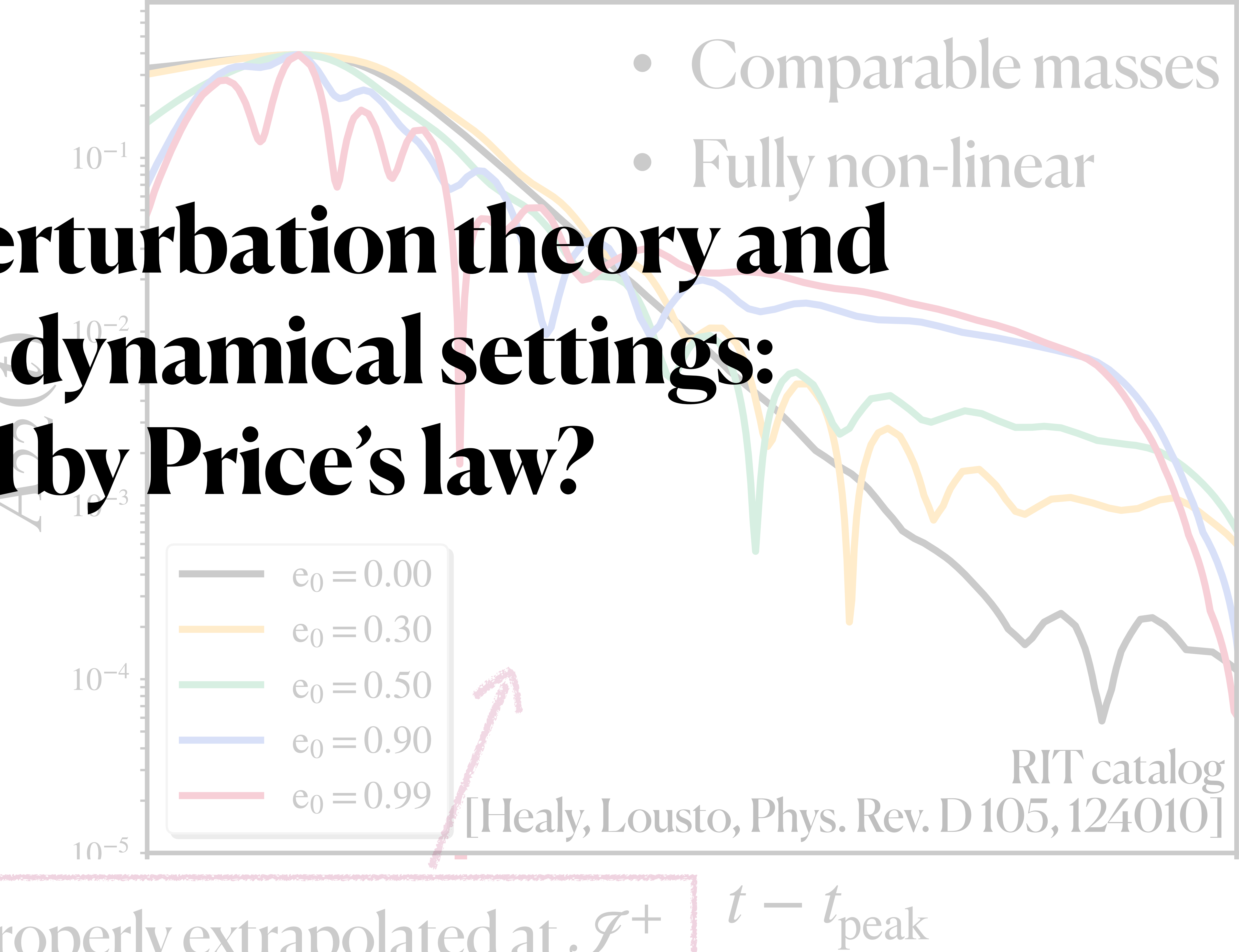

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Need to go back to perturbation theory and understand tail in dynamical settings: Is it explained by Price's law?

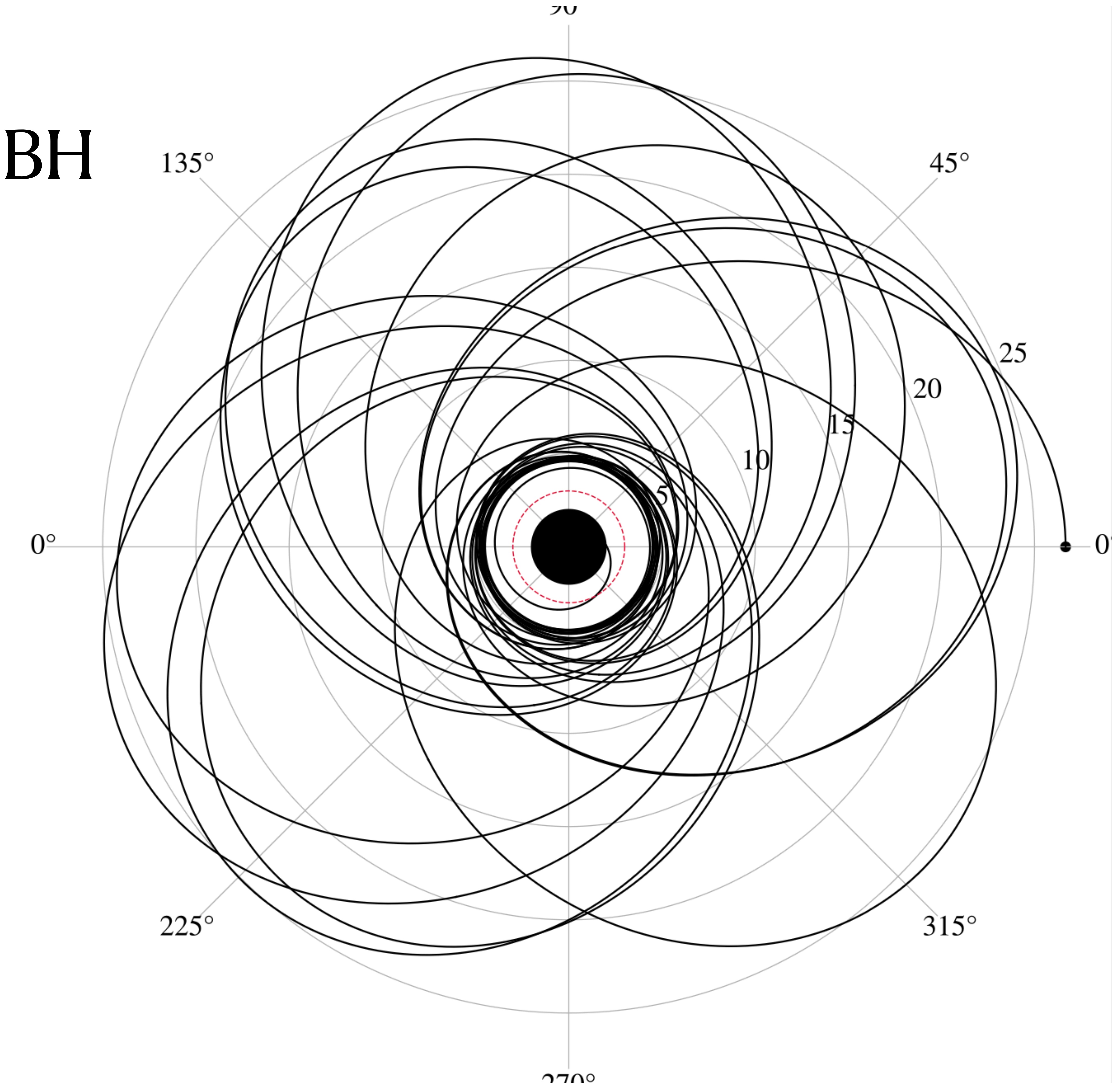
Why it matters?

- **Foundational problem** in General Relativity:
    - Two-body problem not fully understood

- Enhancement with eccentricity makes the tail **potentially observable**
    - Plenty possible channels of highly eccentric mergers
    - Could give constraints on inspiral parameters

Framework

- Test particle μ infalling in a Schwarzschild BH
- Perturbation theory



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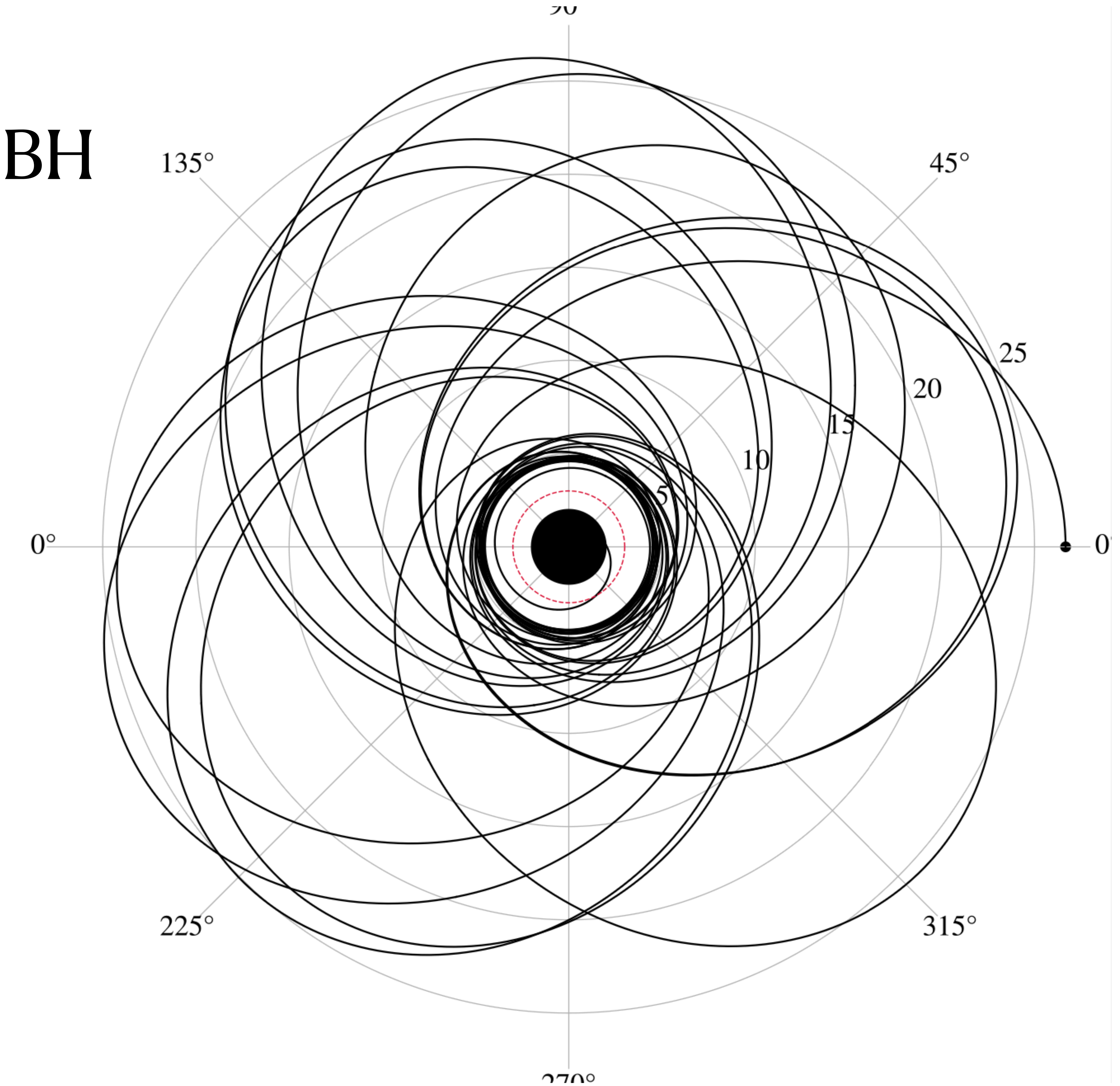
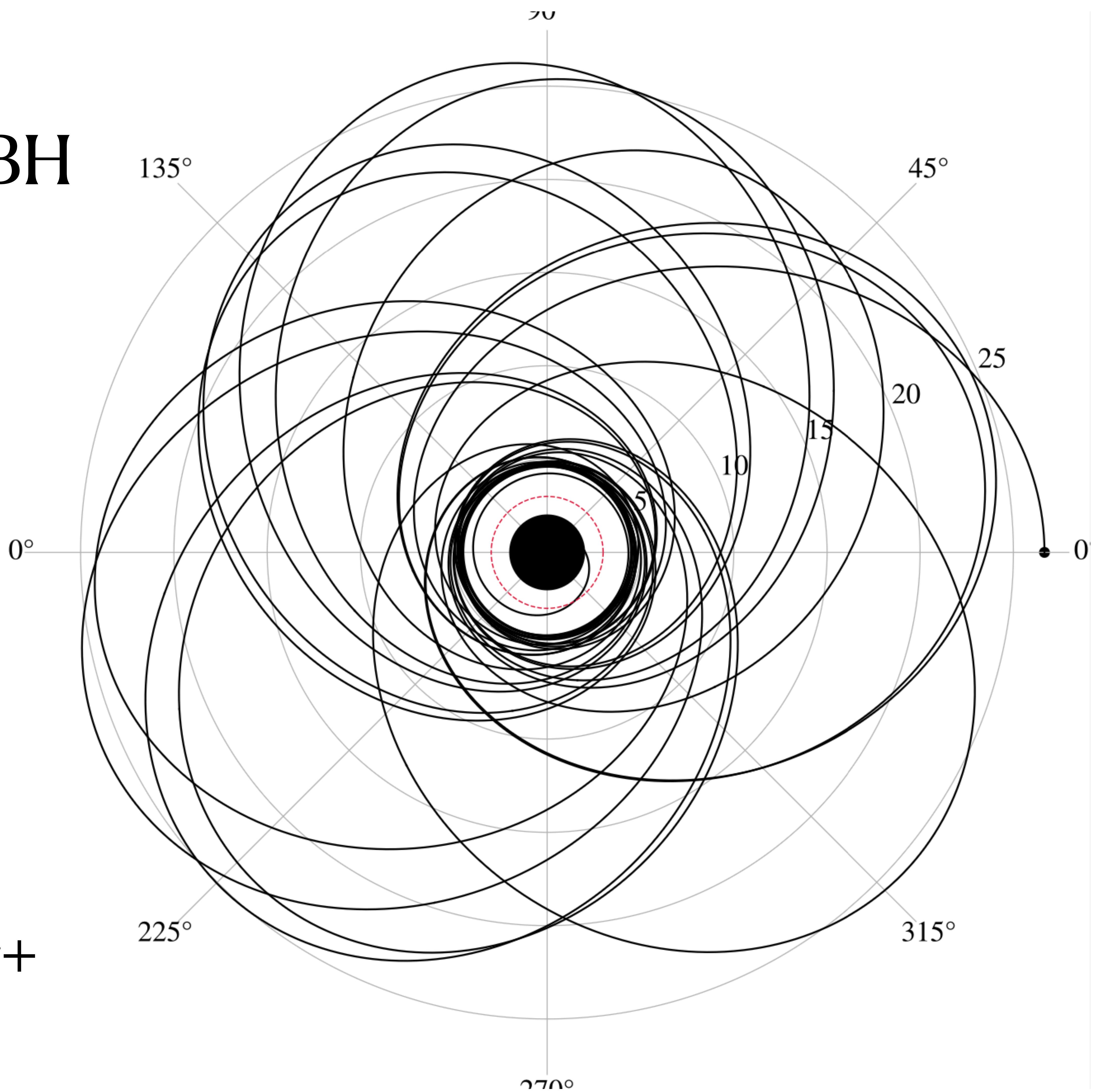


Framework

- Test particle μ infalling in a Schwarzschild BH
- Perturbation theory
- Signal extracted at scri+ (null infinity)
  - As observed by real detectors
  - Price's law:
    - $\Psi_{\ell m} \propto \frac{1}{\tau^{\ell+2}}, \quad \tau \equiv t - r_* \;$ at $\mathscr{I}^+$
    - $\Psi_{\ell m} \propto \frac{1}{t^{2\ell+3}}$ at finite distance → Suppressed!



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Numerical evolutions

$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}^{(e/o)}(r_*)\right]\Psi_{\ell m}^{(e/o)}(t, r_*) = S_{\ell m}^{(e/o)}(t, r)$$

$$\Psi_{\ell m}^{(e/o)}(t = 0, r) = \partial_t \Psi_{\ell m}^{(e/o)}(t = 0, r) = 0$$

\+ **Hamiltonian equations of motion** for the trajectory, driven **radiation-reaction**

[Chiaramello and Nagar, Phys. Rev. D 101, 101501 (2020)]
[Albanesi, Nagar, Bernuzzi, Phys. Rev. D 104, 024067 (2021)]

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- **Analytically** computed through Post-Newtonian expansions of fluxes extracted at infinity
- Allow to evolve a **generic orbit** up to merger

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- **Hyperboloidal layer** over which r_* is **compatified**
- Extract the radiative signal at $\mathscr{I}^+$

RWZhyp code:
[Bernuzzi and Nagar, Phys. Rev. D 81, 084056(2010)]
[Bernuzzi, Nagar and Zenginoglu, Phys. Rev. D 84, 084026(2011)]

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Analytical model

Regge-Wheeler/
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$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}(r_*)\right] \Psi_{\ell m}(t, r_*) = S_{\ell m}(t, r)$$

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Most general solution:

$$\Psi_{\ell m}(\tau, \rho_+) = \int_{T_{in}}^{\tau - \rho_+} dt' \int dr' S_{\ell m}(t', r')\, G_\ell(\tau, t'; r', \rho_+)$$

$\rho_+ \equiv$ location of $\mathscr{I}^+$ in the compactified coordinate

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Price's law propagator

$\rho_+ \equiv$ location of $\mathscr{I}^+$ in the compactified coordinate

Analytical model

$$S_{\ell m}(t,r) = f_{\ell m}(t,r)\delta(r-r(t)) + g_{\ell m}(t,r)\partial_r\delta(r-r(t))$$

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$$\Psi_{\ell m}(\tau,\rho_+) = \frac{(-1)^\ell 2^{\ell+1}\ell!(\ell+1)!}{(2\ell+1)!}\int_{T_{\rm in}}^{\tau-\rho_+} dt' \frac{r^\ell(t')\left[r\left(f_{\ell m}(t') - \partial_r g_{\ell m}(t')\right) - (\ell+1)\, g_{\ell m}(t')\right]_{r=r(t')}}{\left(\tau-\rho_+-t'\right)^{\ell+2}}$$

[De Amicis, Albanesi and Carullo, 2406.17018]

Analytical model

Analytical integral form of the tail:

$$\Psi_{\ell m}(\tau,\rho_+) = \frac{(-1)^\ell 2^{\ell+1} \ell!(\ell+1)!}{(2\ell+1)!} \int_{T_{\rm in}}^{\tau-\rho_+} dt' \frac{r^\ell(t')\Big[r\left(f_{\ell m}(t') - \partial_r g_{\ell m}(t')\right) - \left(\ell+1\right) g_{\ell m}(t')\Big]_{r=r(t')}}{\left(\tau-\rho_+-t'\right)^{\ell+2}}$$

- Tail as a **memory effect**

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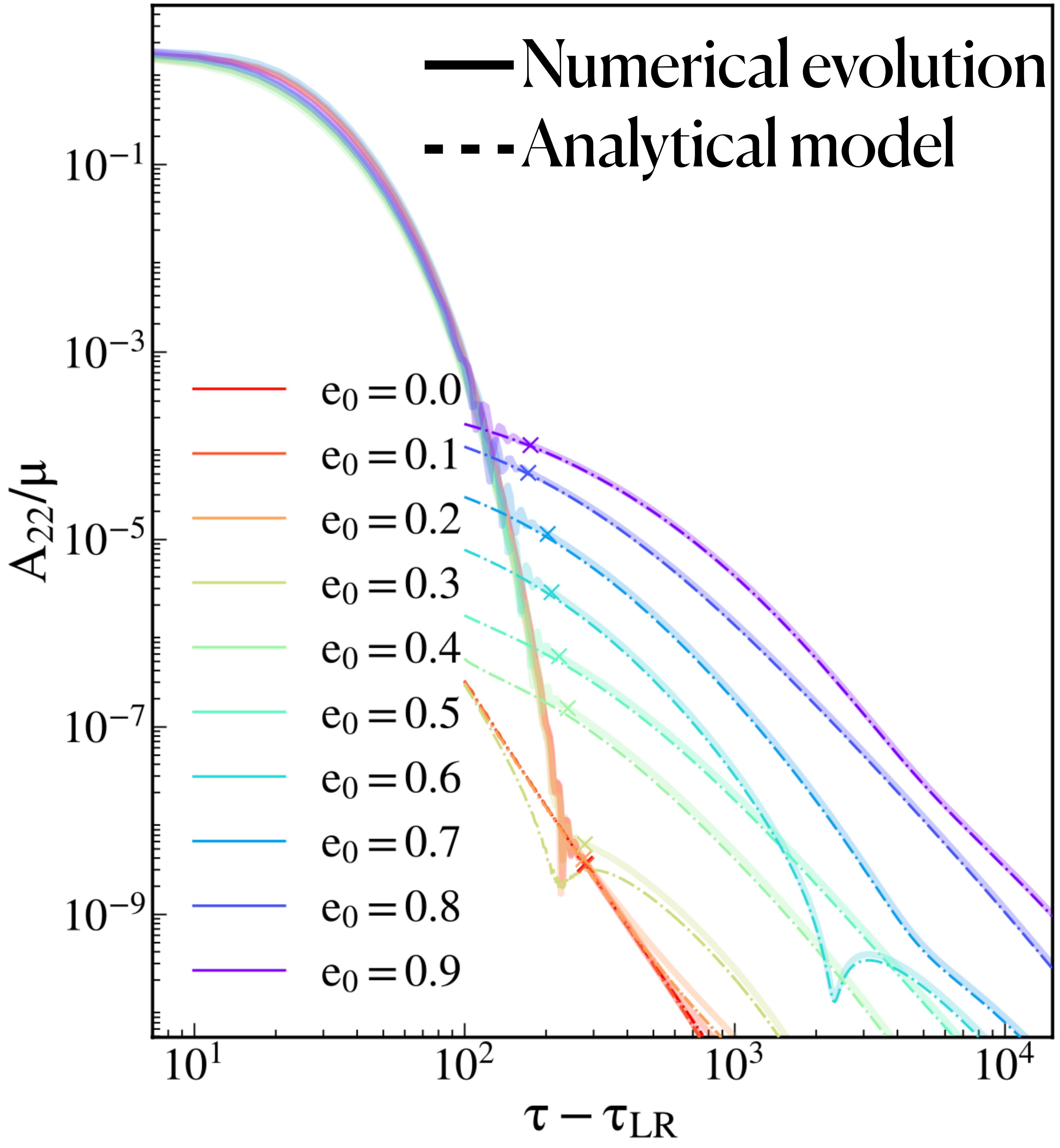
- Tail as a **memory effect**
- **Not** an **exact power-law** behavior

[De Amicis, Albanesi and Carullo, 2406.17018]

Model vs numerical evolutions: eccentric orbits



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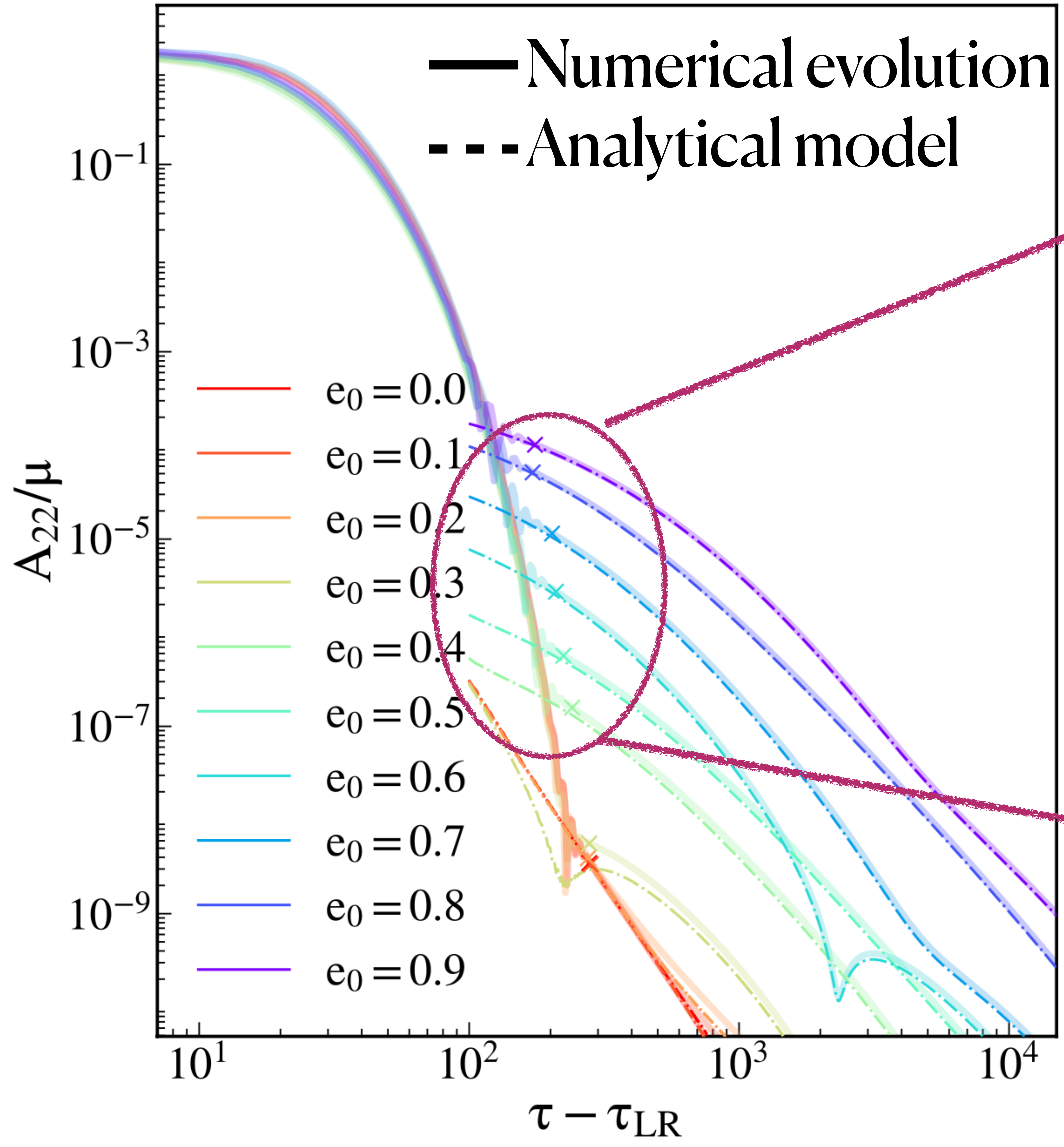

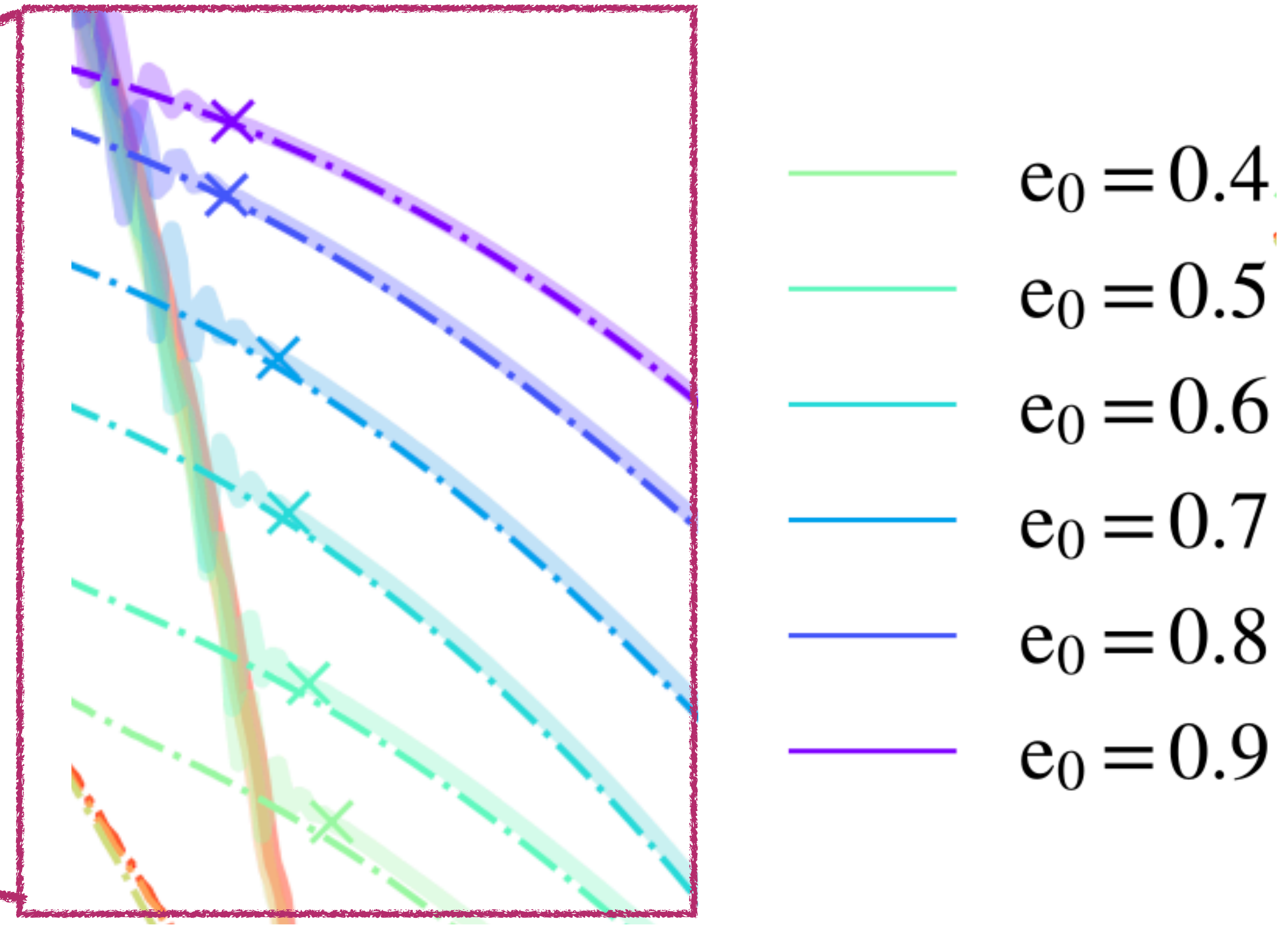

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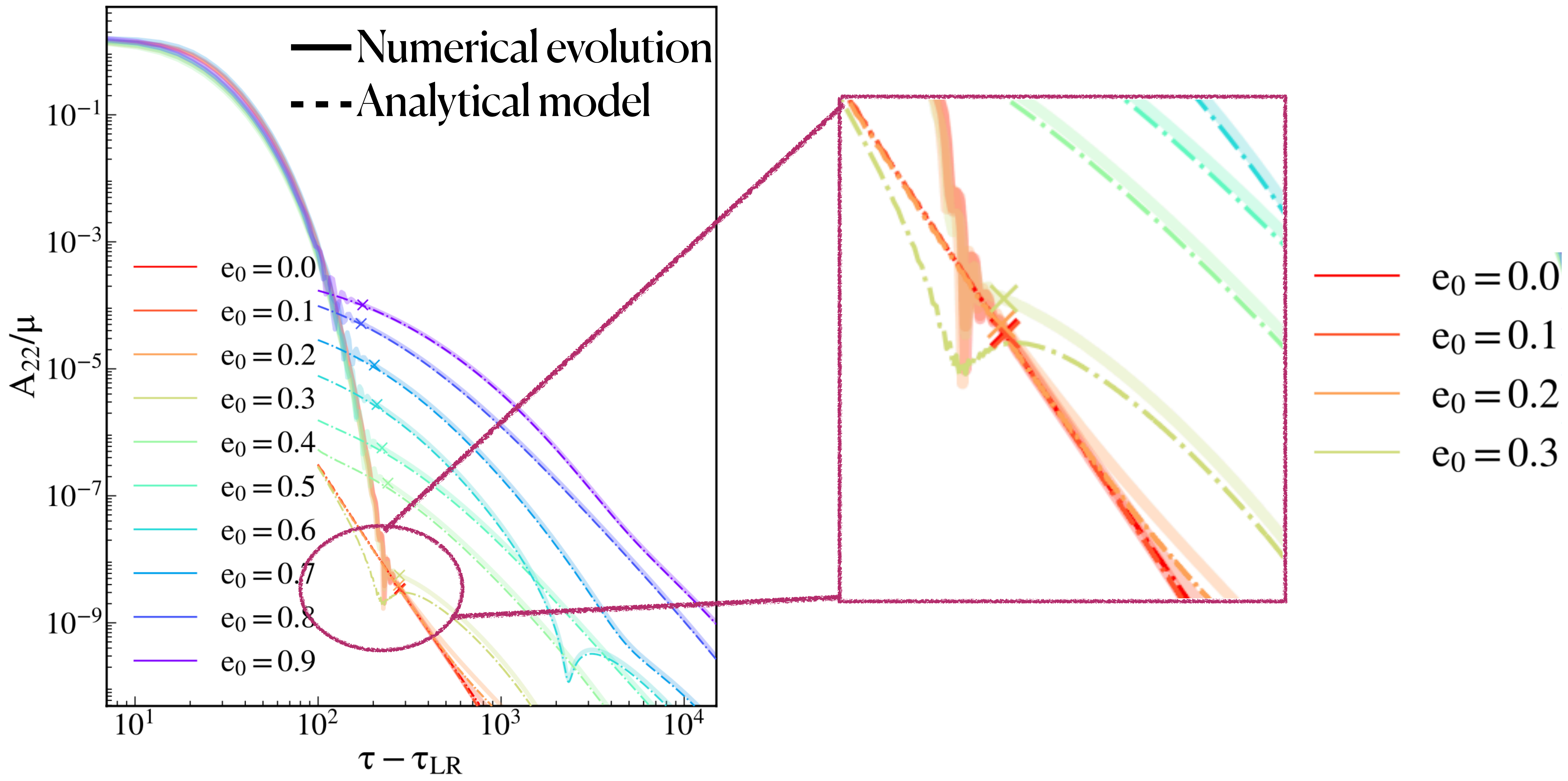

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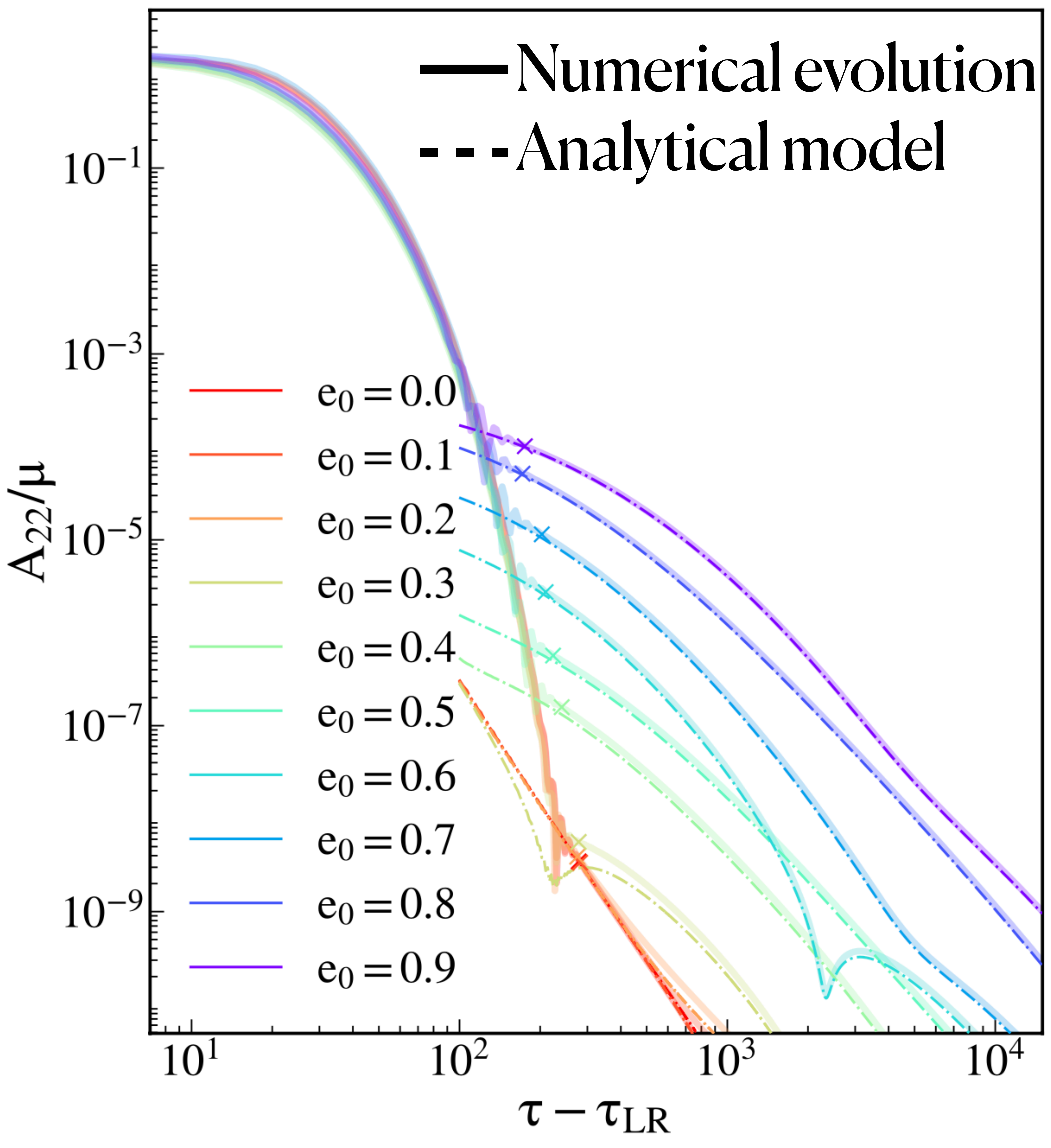

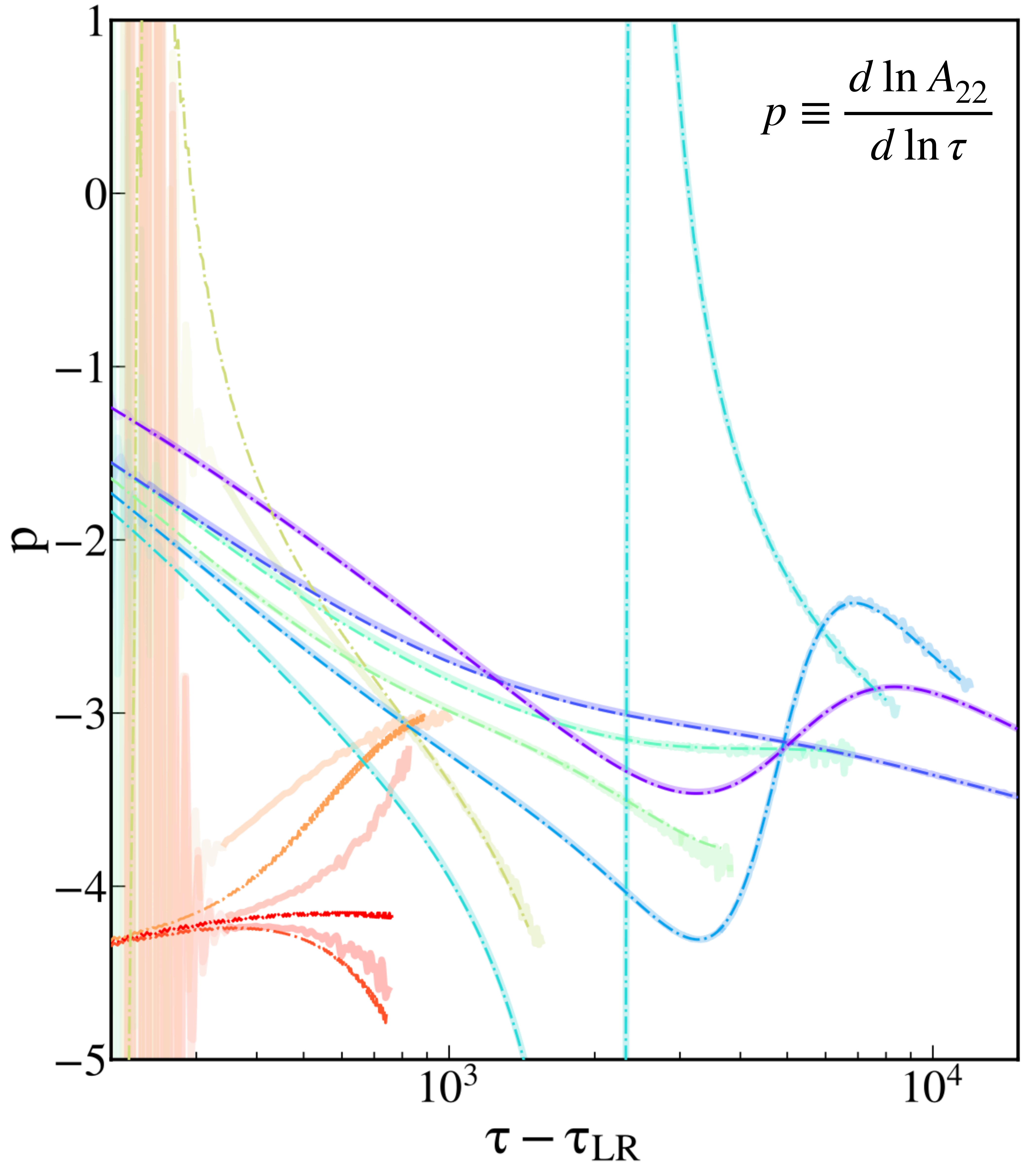

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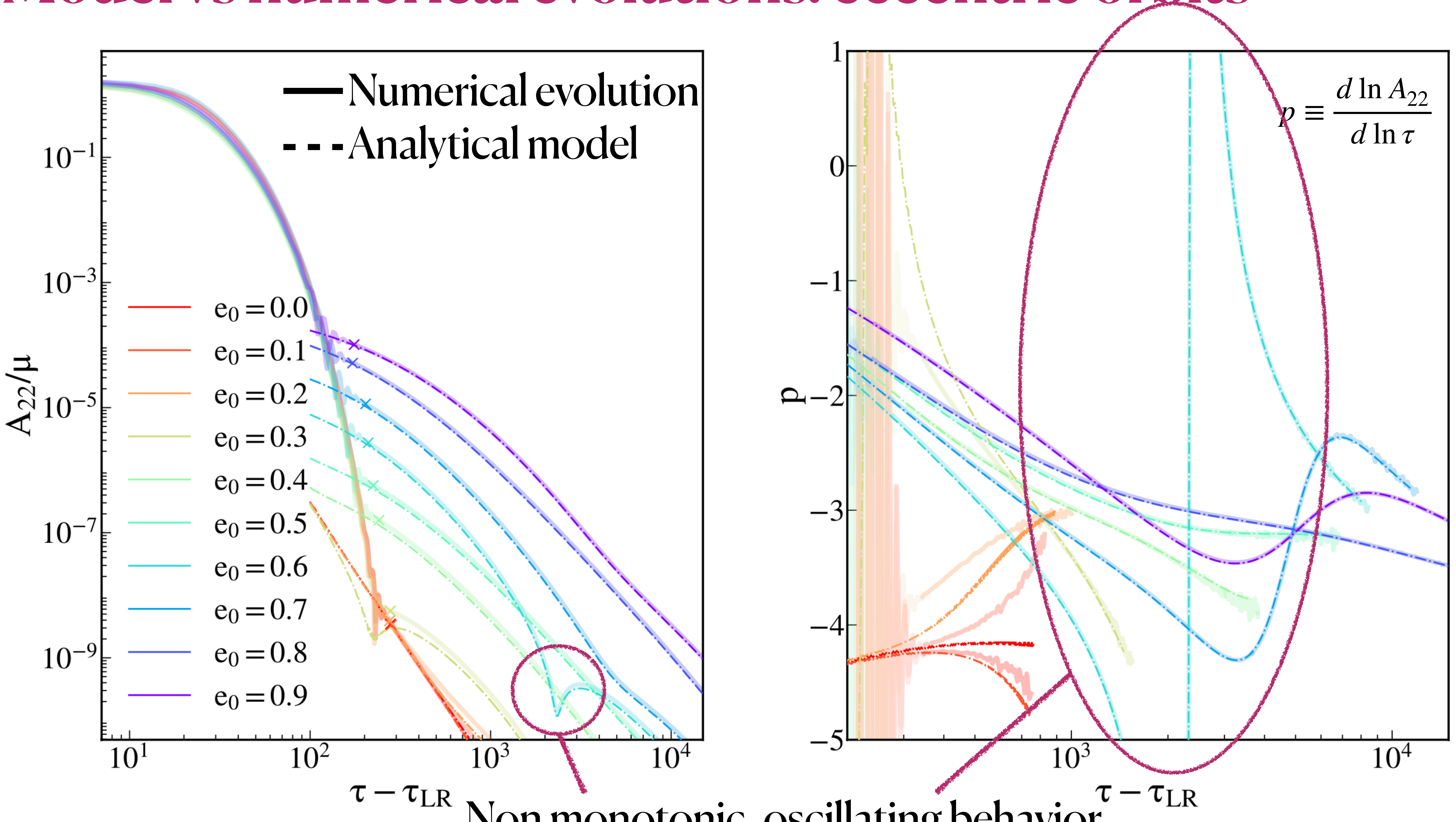

Non monotonic, oscillating behavior

Take a breather

- Integral model for tail in EMR, as a memory effect ✓
- Tail exponent is in general non monotonic
- Tail amplitude is enhanced by eccentricity

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Tail as superposition of power-laws

$\tau - \rho_+ \gg t_{\rm in}, t_f$

$t_{\rm in}$ = initial time

t_f = common horizon

Tail as superposition of power-laws

$$\Psi_{\ell m}(\tau,\rho_+) = \frac{c_\ell}{\tau^{\ell+2}} \int_{t_{\rm in}}^{t_f} dt'\ S_\ell(t') \left[1 + \sum_{n=1}^{\infty} \frac{(\ell+1+n)!}{n!(\ell+1)!} \left(\frac{t'+\rho_+}{\tau}\right)^n\right]$$

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- Superposition of power-laws
- Slowest decay is Price's law
- **Excitation coefficient** of each power-law depends on:
    - **amount of history**

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- Superposition of power-laws
- Slowest decay is Price's law
- **Excitation coefficient** of each power-law depends on:
  - **amount of history**
  - **specific orbit**

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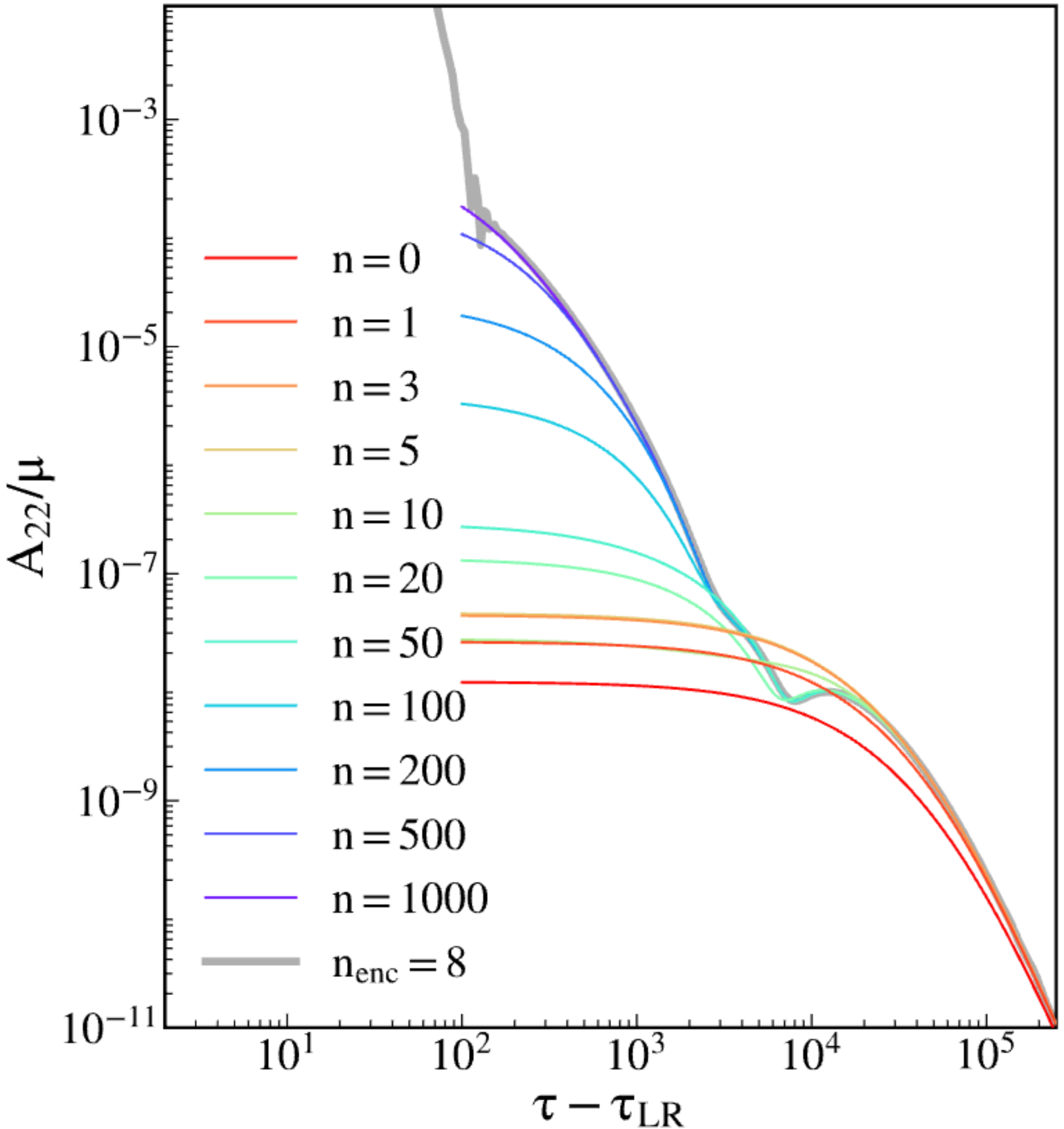

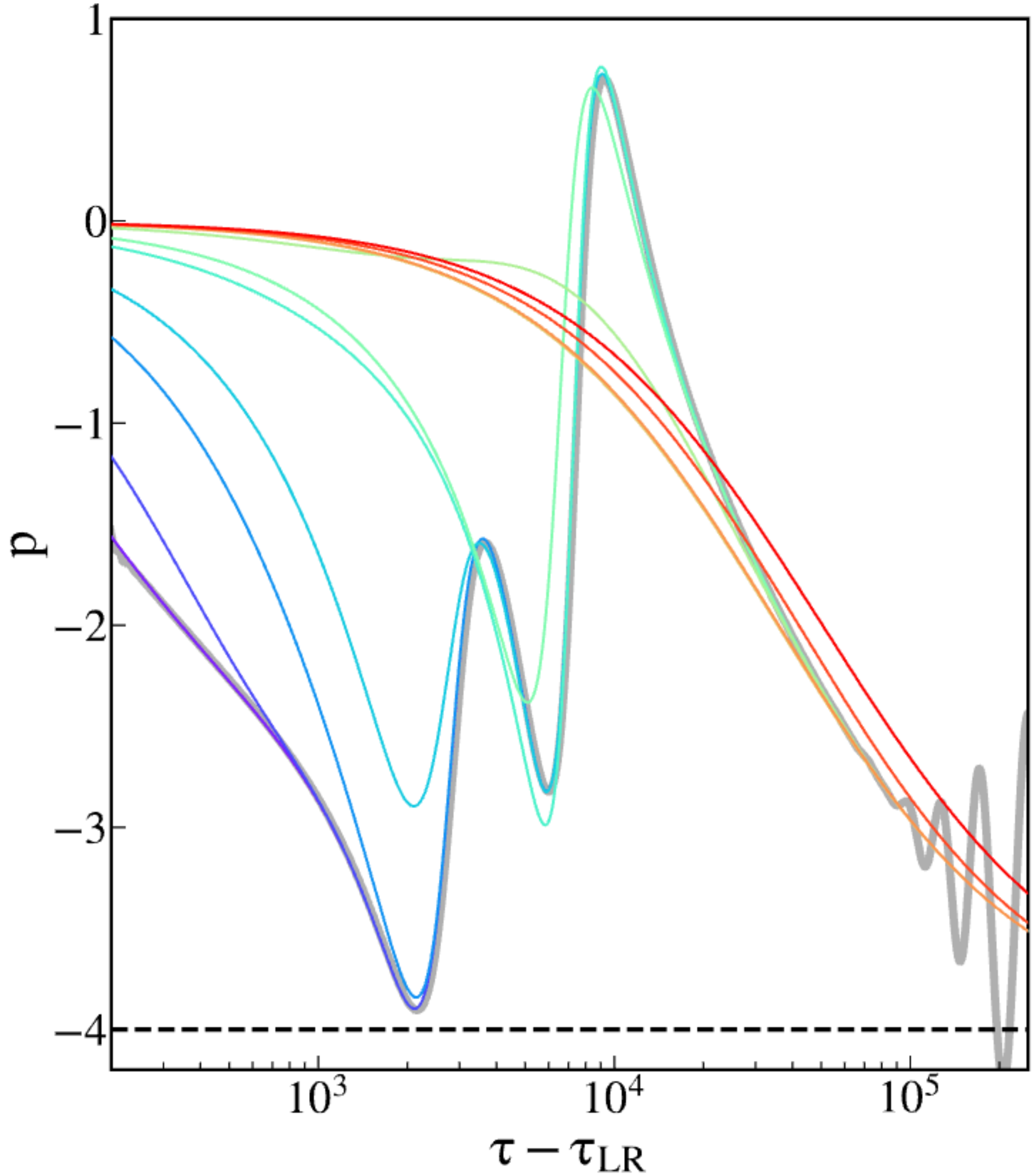

Take a breather

- Integral model for tail in EMR, as a memory effect
- Tail as superposition of power laws $\tau^{-\ell-2-n}$, with $n \geq 0$
- Tail amplitude is enhanced by eccentricity

Enhancement with eccentricity

Isolate the part of the trajectory which determines the amplitude at the transition from QNMs to tail

Enhancement with eccentricity

$$\Psi_{\ell m}(\tau, \rho_+) = \frac{(-1)^\ell 2^{\ell+1} \ell!(\ell+1)!}{(2\ell+1)!} \int_{t_{\rm in}}^{\tau-\rho_+} dt' \frac{r^\ell(t')\left[r\left(f_{\ell m}(t') - \partial_r g_{\ell m}(t')\right) - (\ell+1)\, g_{\ell m}(t')\right]_{r=r(t')}}{\left(\tau - \rho_+ - t'\right)^{\ell+2}}$$

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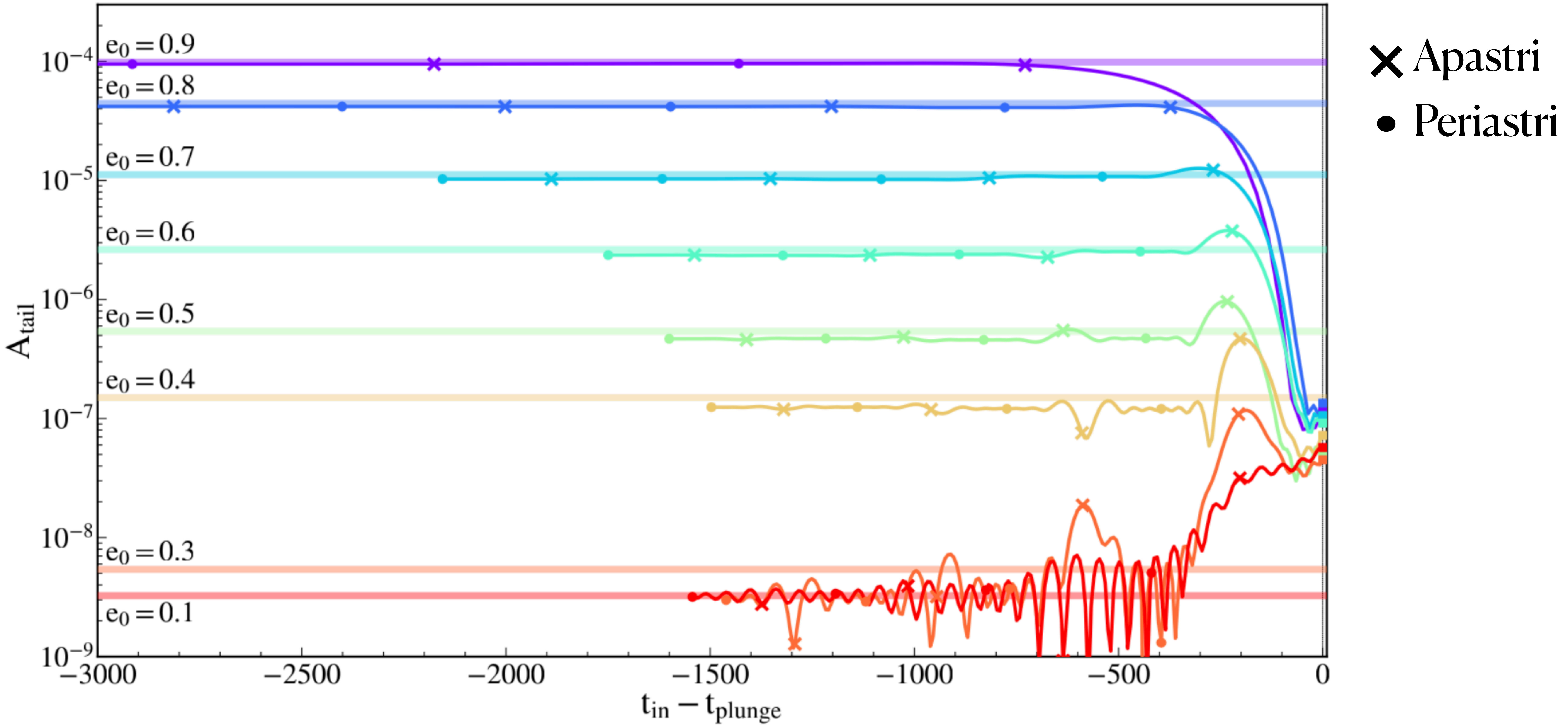

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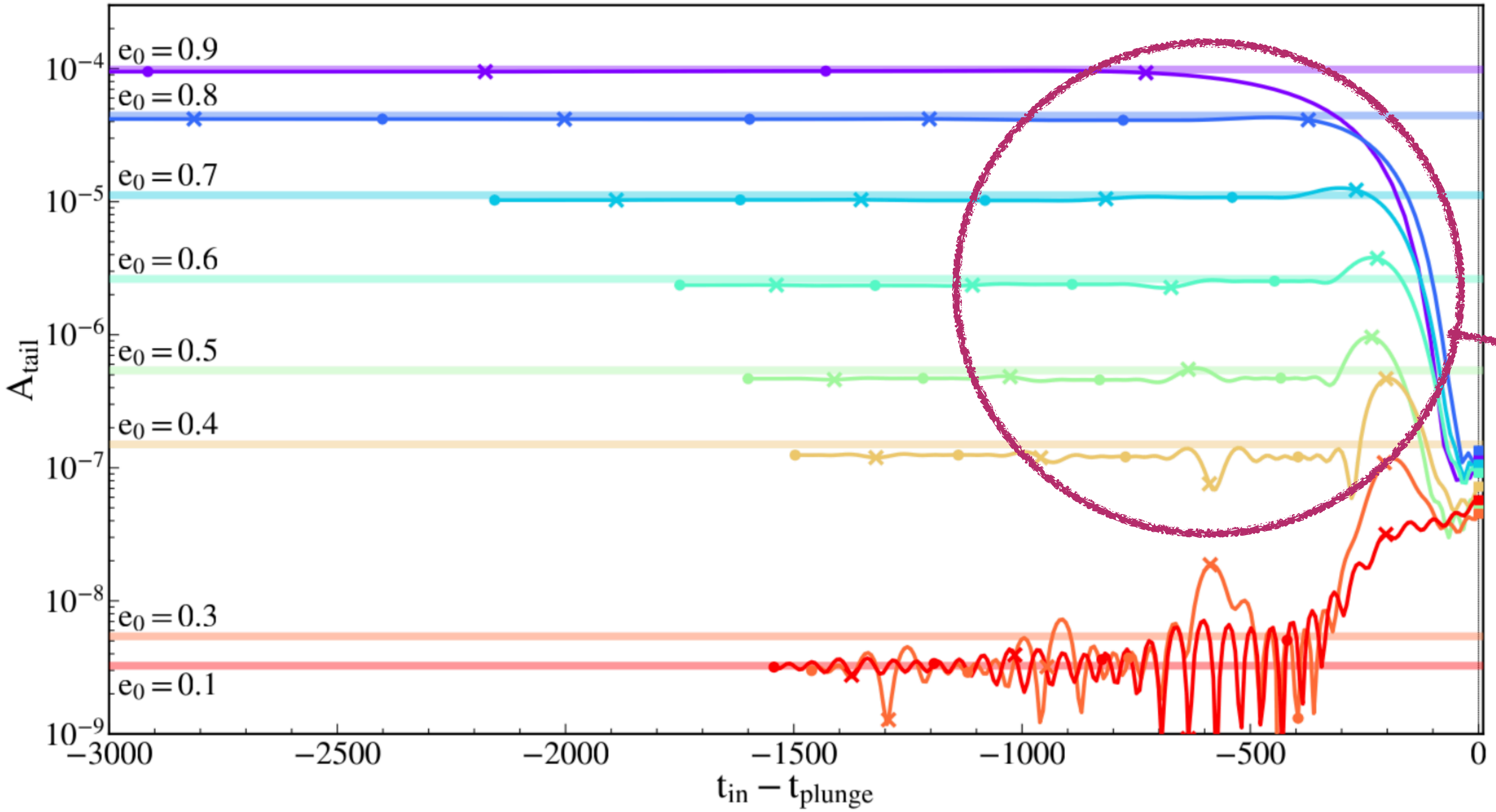
Isolate the part of the trajectory which determines the amplitude at the transition from QNMs to tail



× Apastri
• Periastri

Tail amplitude:

- Determined by motion near last apastron

Marina De Amicis

Enhancement with eccentricity

$$\Psi_{\ell m}(\tau,\rho_+) = \frac{(-1)^\ell 2^{\ell+1}\ell!(\ell+1)!}{(2\ell+1)!}\int_{t_{\rm in}}^{\tau-\rho_+} dt' \frac{r^\ell(t')\left[r\left(f_{\ell m}(t') - \partial_r g_{\ell m}(t')\right) - (\ell+1)\, g_{\ell m}(t')\right]_{r=r(t')}}{(\tau-\rho_+-t')^{\ell+2}}$$

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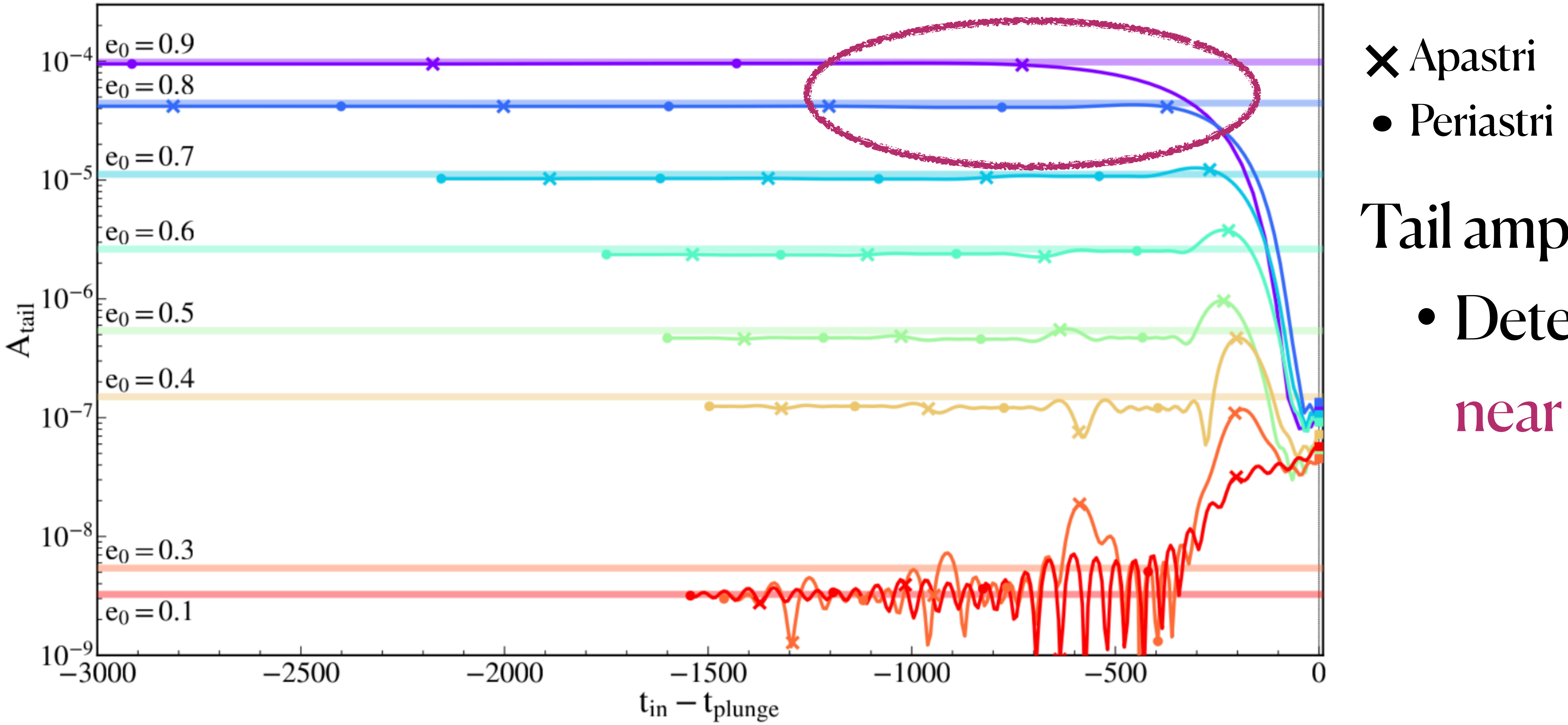


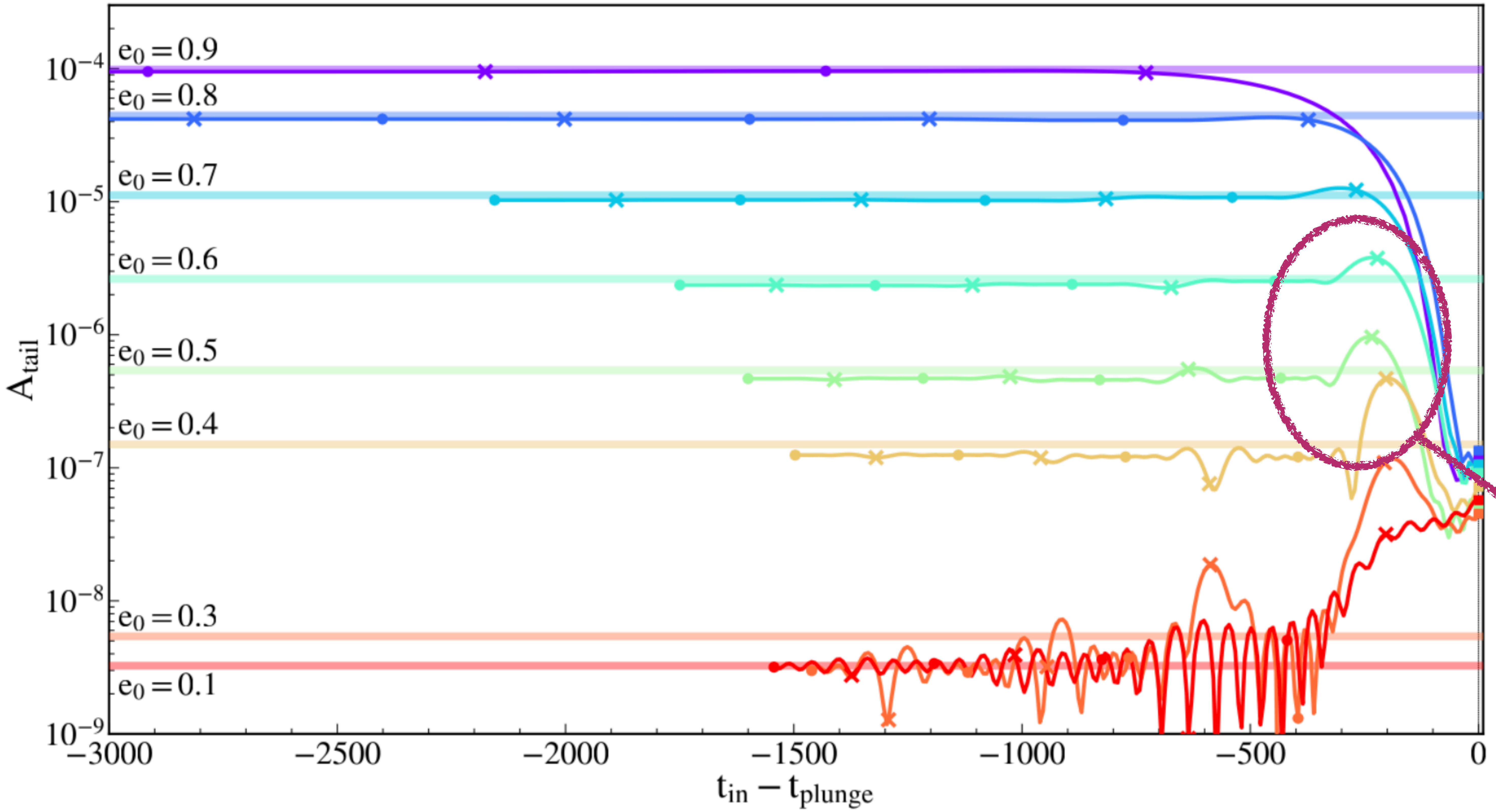
Tail amplitude:

- Determined by motion near last apastron

Enhancement with eccentricity

$$\Psi_{\ell m}(\tau,\rho_+) = \frac{(-1)^\ell 2^{\ell+1}\ell!(\ell+1)!}{(2\ell+1)!}\int_{t_{\rm in}}^{\tau-\rho_+} dt' \frac{r^\ell(t')\Big[r\left(f_{\ell m}(t') - \partial_r g_{\ell m}(t')\right) - (\ell+1)\, g_{\ell m}(t')\Big]_{r=r(t')}}{(\tau-\rho_+-t')^{\ell+2}}$$

Isolate the part of the trajectory which determines the amplitude at the transition from QNMs to tail



× Apastri

• Periastri

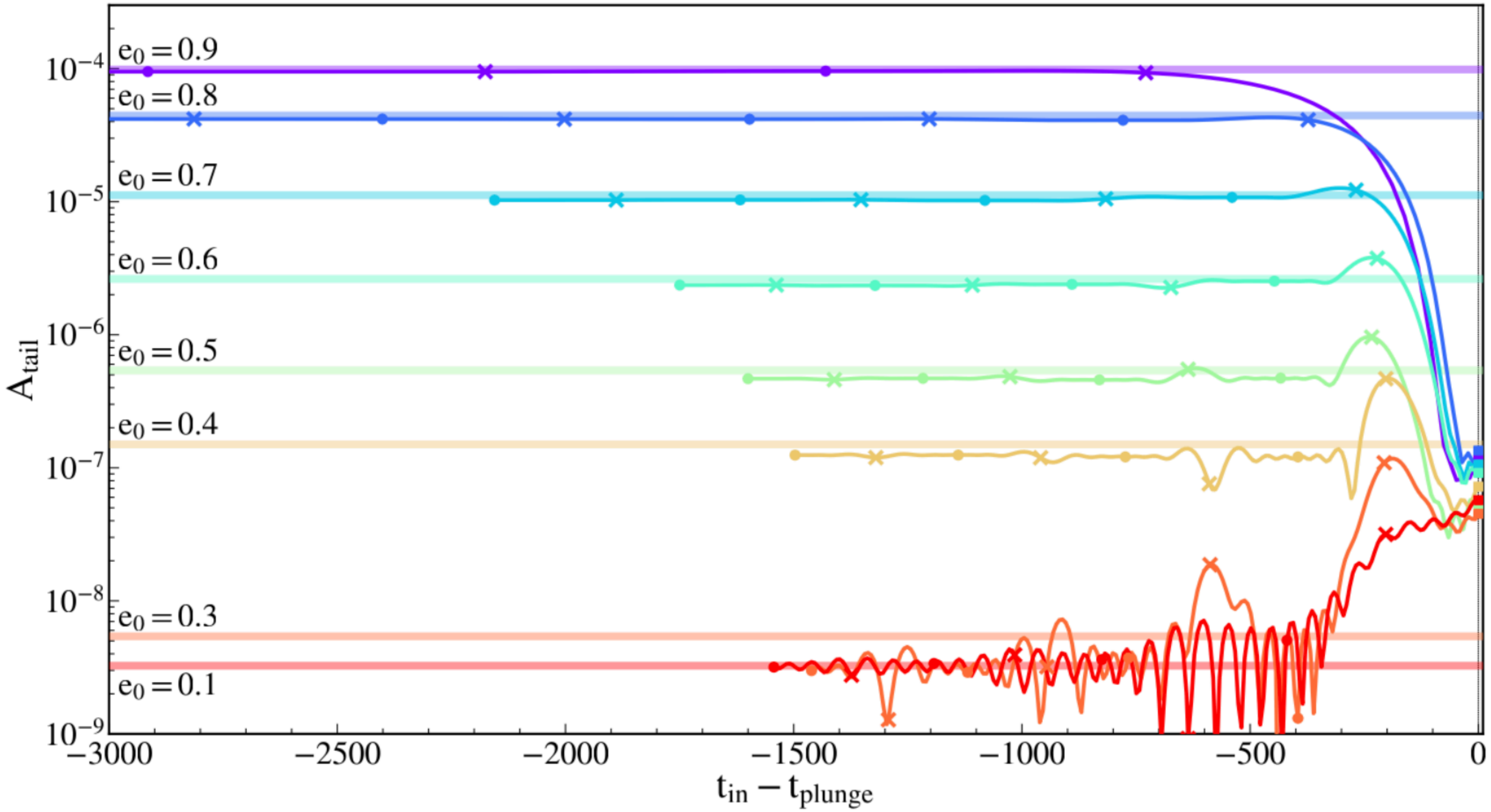
Tail amplitude:

- Determined by motion near last apastron
- Cancellation among in/outgoing motion

Enhancement with eccentricity

$$\Psi_{\ell m}(\tau,\rho_+) = \frac{(-1)^\ell 2^{\ell+1}\ell!(\ell+1)!}{(2\ell+1)!}\int_{t_{in}}^{\tau-\rho_+} dt' \frac{r^\ell(t')\left[r\left(f_{\ell m}(t') - \partial_r g_{\ell m}(t')\right) - (\ell+1)\, g_{\ell m}(t')\right]_{r=r(t')}}{\left(\tau-\rho_+-t'\right)^{\ell+2}}$$

Isolate the part of the trajectory which determines the amplitude at the transition from QNMs to tail



× Apastri

• Periastri

Tail amplitude:

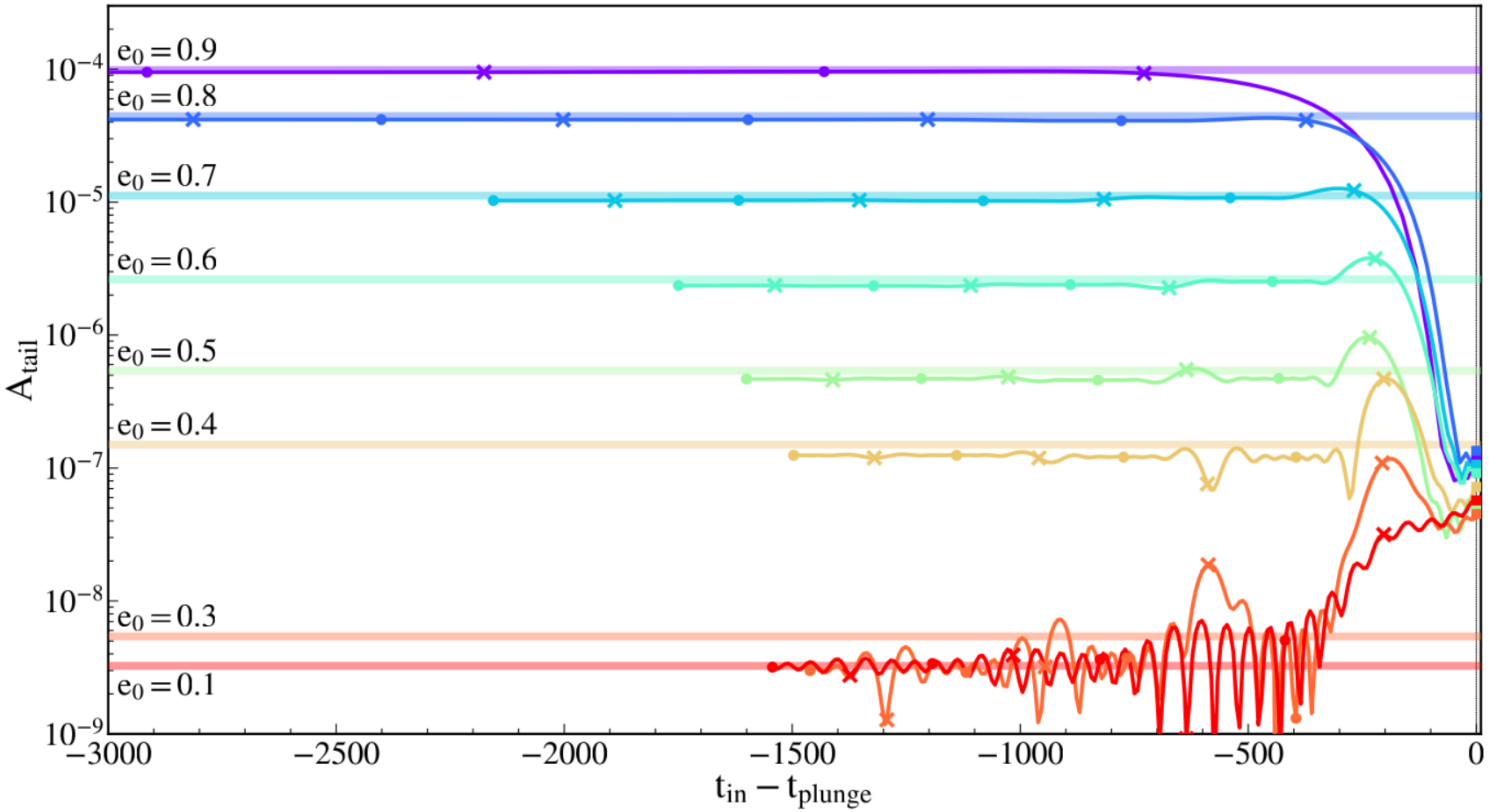
- Determined by motion near last apastron → $r \gg M$, $p_\varphi / r \ll 1$
- Cancellation among in/outgoing motion

Enhancement with eccentricity

Expand in large r and small p_φ/r: $\Psi_{\ell m}(\tau,\rho_+) = \int_{t_{\rm in}}^{t_f} dt' \frac{r^\ell(t') e^{-im\varphi(t')} P_{\ell m}(\cos\theta_0)}{(\tau - t' - \rho_+)^{\ell+2}} \cdot \left[a_1 - \frac{a_1}{2}\dot{r}^2 + a_2 \dot{r}\frac{p_\varphi}{r} + \left(a_3 + \frac{a_1}{2}\right)\frac{p_\varphi^2}{r^2} \right]$

× Apastri

• Periastri

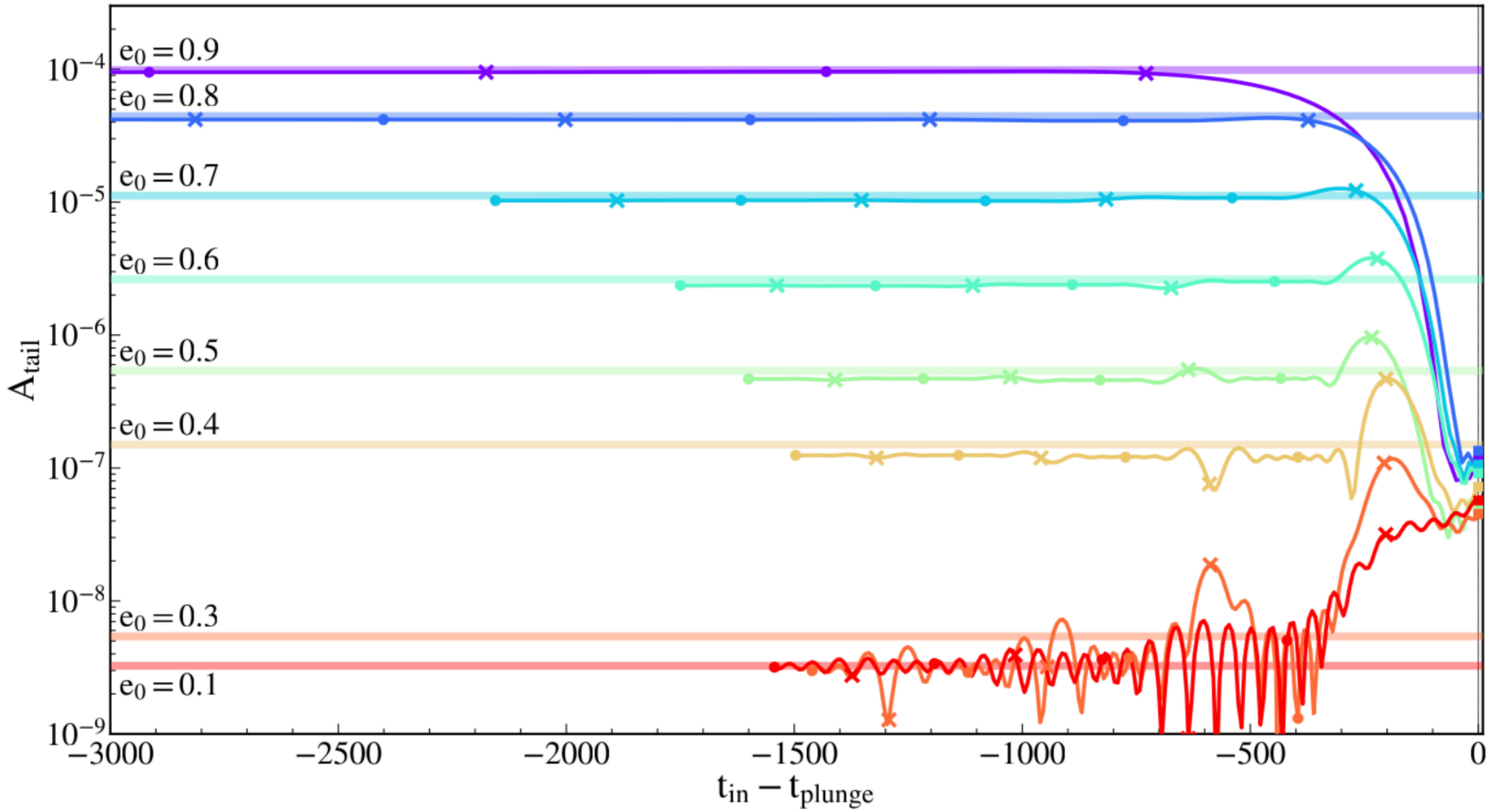


Enhancement with eccentricity

Expand in large r and small p_φ/r: $\Psi_{\ell m}(\tau,\rho_+) = \int_{t_{\rm in}}^{t_f} dt' \frac{r^\ell(t')e^{-im\varphi(t')}P_{\ell m}(\cos\theta_0)}{(\tau - t' - \rho_+)^{\ell+2}} \cdot \left[a_1 - \frac{a_1}{2}\dot{r}^2 + a_2\dot{r}\frac{p_\varphi}{r} + \left(a_3 + \frac{a_1}{2}\right)\frac{p_\varphi^2}{r^2}\right]$

× Apastri

• Periastri



Oscillanting contribution can induce destructive interference

↓

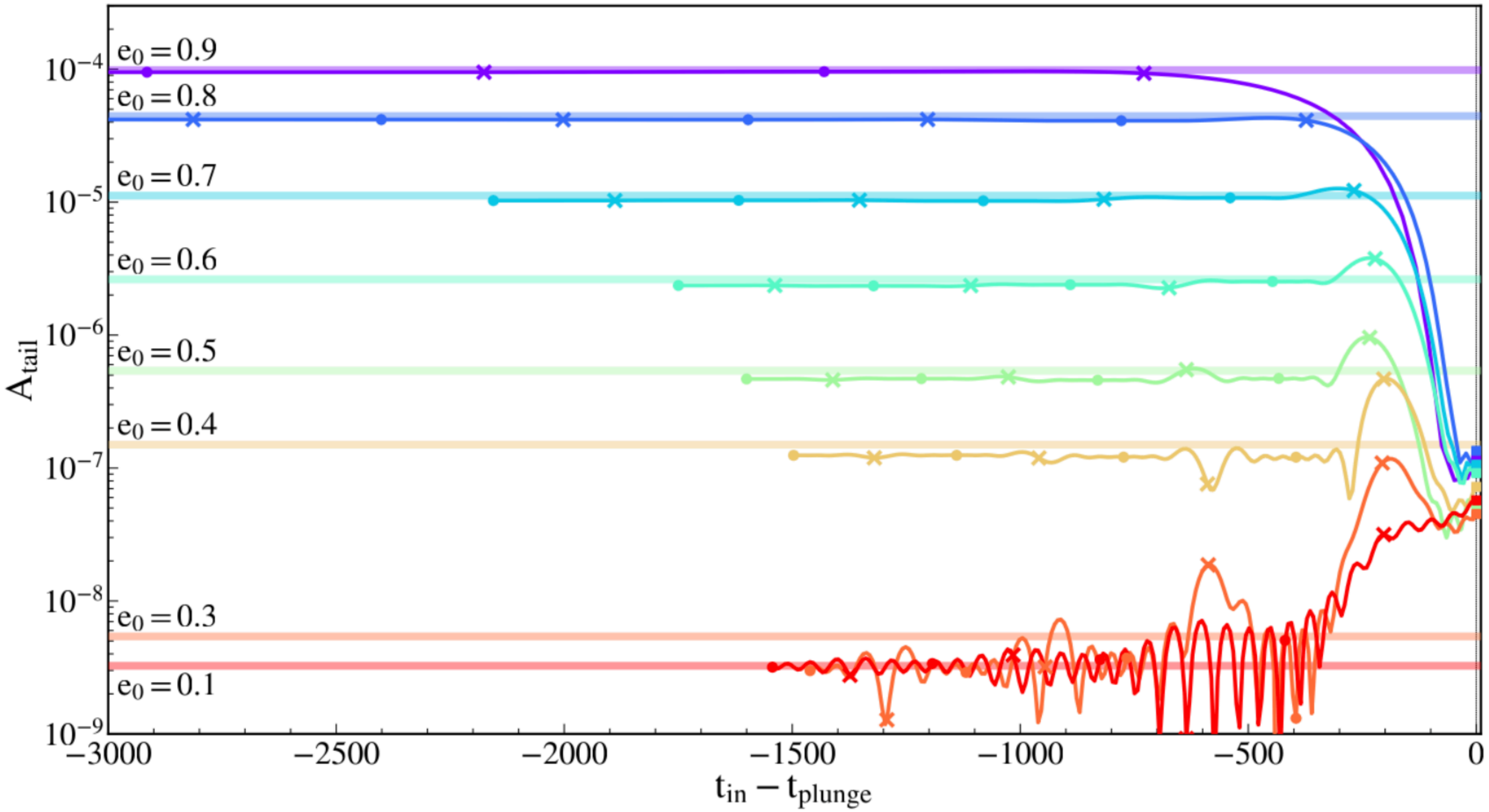
Tail maximied for radial infall!

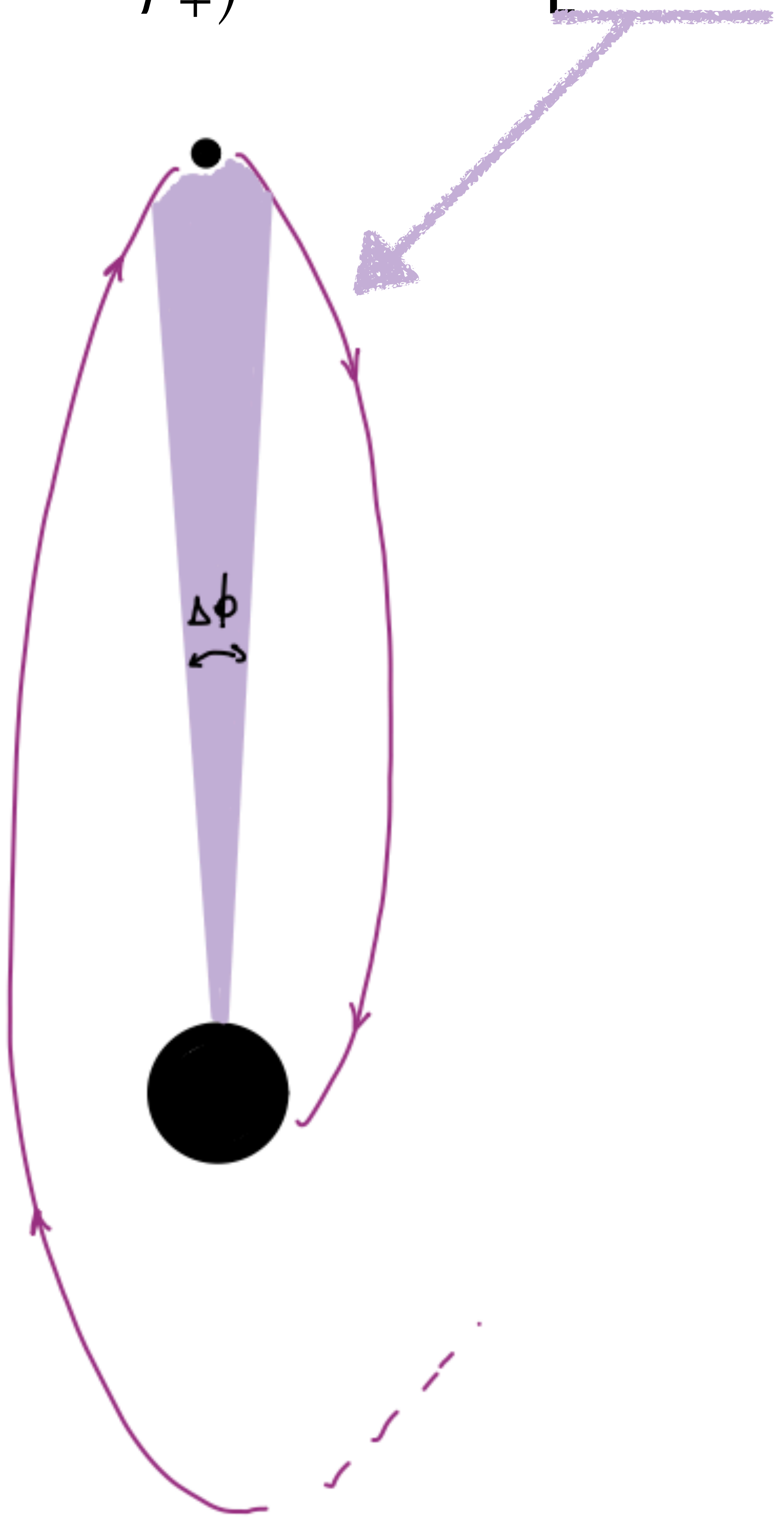
Enhancement with eccentricity

Expand in large r and small p_φ/r: $\Psi_{\ell m}(\tau,\rho_+) = \int_{t_{in}}^{t_f} dt' \frac{r^\ell(t') e^{-im\varphi(t')} P_{\ell m}(\cos\theta_0)}{(\tau - t' - \rho_+)^{\ell+2}} \cdot \left[a_1 - \frac{a_1}{2}\dot{r}^2 + a_2 \dot{r}\frac{p_\varphi}{r} + \left(a_3 + \frac{a_1}{2}\right)\frac{p_\varphi^2}{r^2}\right]$

× Apastri

• Periastri



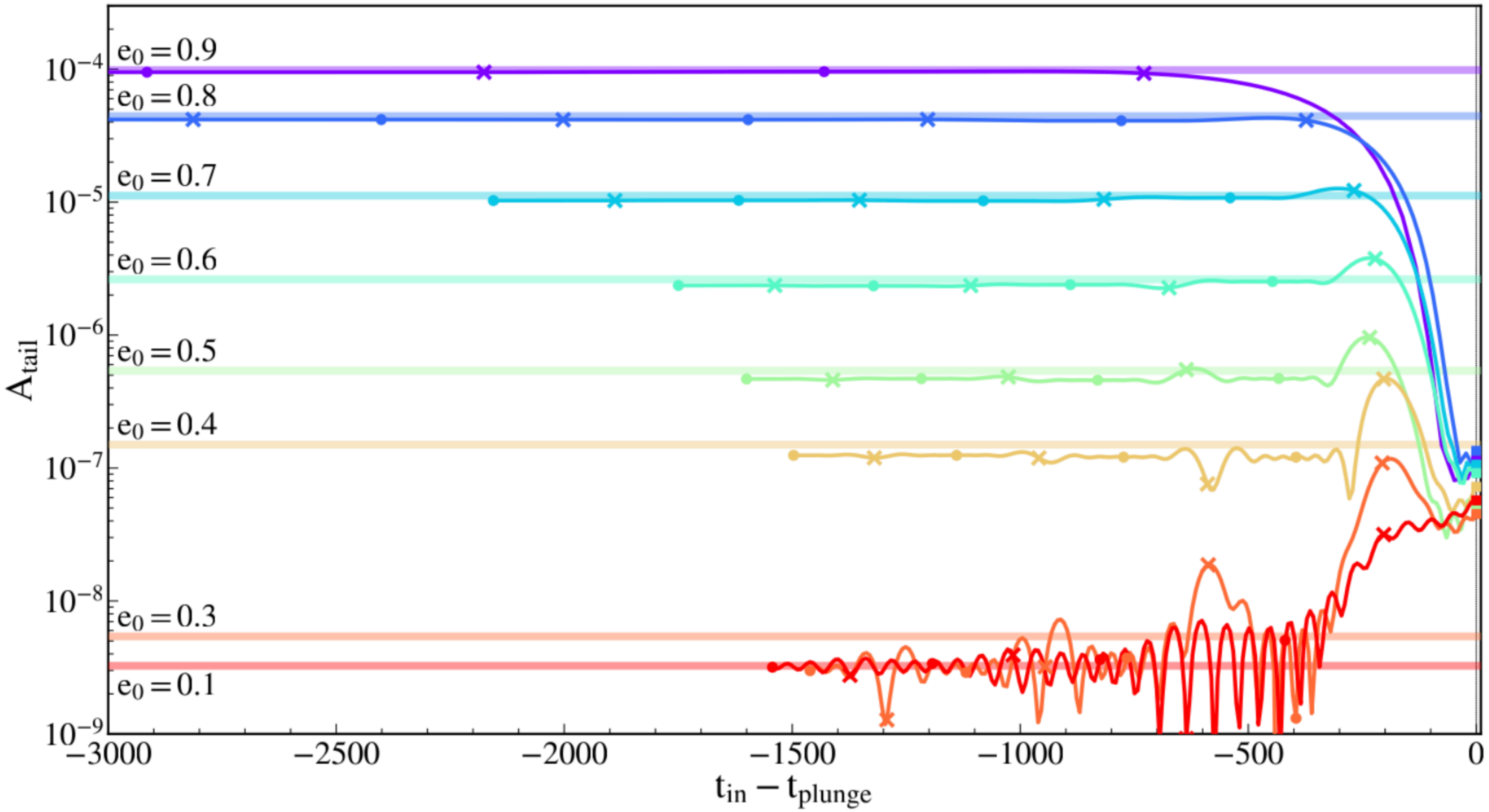


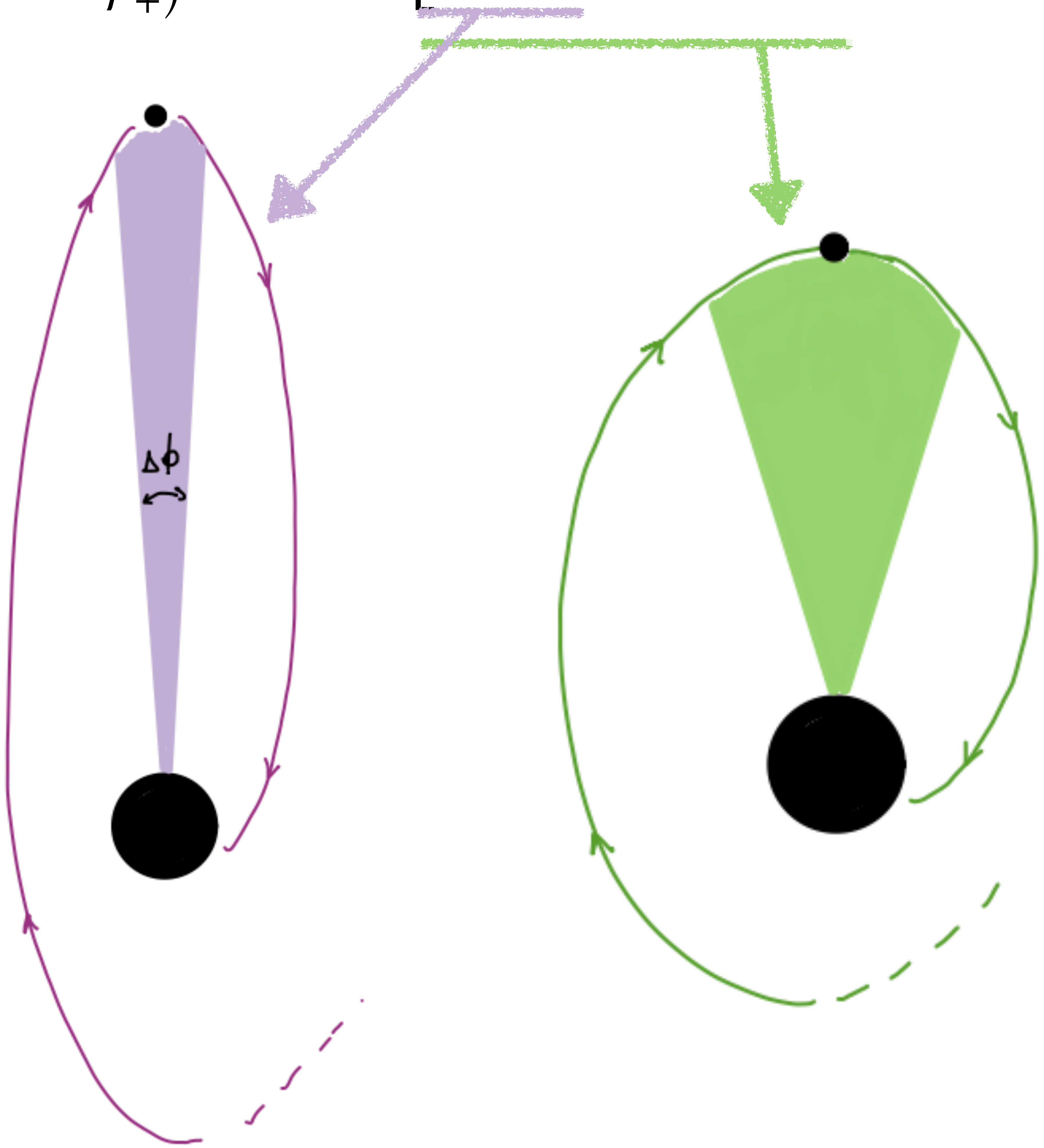
Enhancement with eccentricity

Expand in large r and small p_φ/r: $$\Psi_{\ell m}(\tau,\rho_+) = \int_{t_{in}}^{t_f} dt' \frac{r^\ell(t') e^{-im\varphi(t')} P_{\ell m}(\cos\theta_0)}{(\tau - t' - \rho_+)^{\ell+2}} \cdot \left[a_1 - \frac{a_1}{2}\dot{r}^2 + a_2 \dot{r}\frac{p_\varphi}{r} + \left(a_3 + \frac{a_1}{2}\right)\frac{p_\varphi^2}{r^2} \right]$$

× Apastri

• Periastri





Conclusions

- Integral model for tail in EMR, as a memory effect ✓
- Tail as superposition of power laws $\tau^{-\ell-2-n}$, with $n \geq 0$ ✓
- Tail emission enhanced for motion at large distances $r \gg \mathrm{M}$, with small tangential velocity. Hence, emission is maximized at apastra ✓

Future directions III

- Go back to comparable masses
  - More accurate simulations SXS (with Keefe Mitman, Hannes Rüter, Leo Stein, Melize Ferrus…)
  - Proper extraction at $\mathscr{I}^+$ using Cauchy Characteristic Extraction method (CCE) [Bishop et al,Phys. Rev. D 54, 6153(1996)]
  - Compare different eccentricities:
    - What happens for an head-on?
    - Can evolve only from last orbit
- Extend the model to Kerr
  - Long-range propagator in Kerr
  - Test for EMR against Teukode [Harms et al,CQG 31, 245004(2014)]
- Estimate the observability