Exploring the DETECTABILITY of ASYMMETRIC BINARIES surrounded by DARK MATTER HALOS

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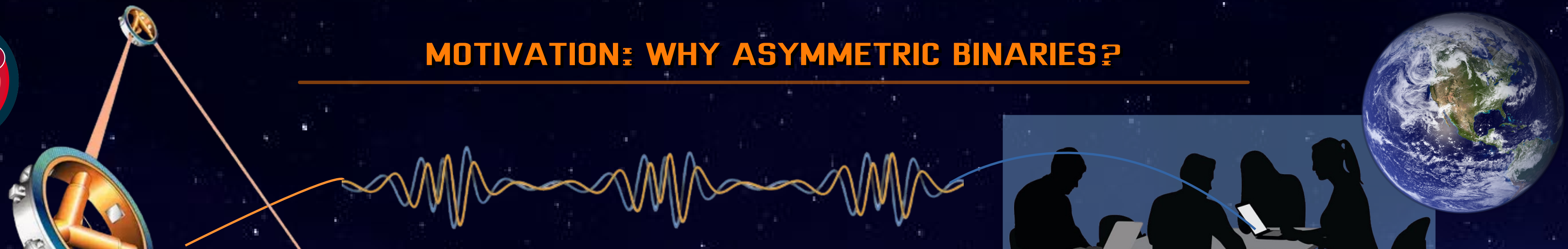
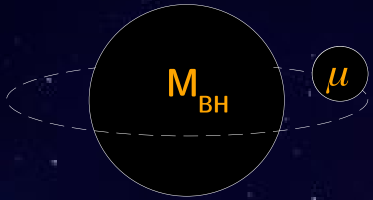
MOTIVATION: WHY ASYMMETRIC BINARIES?





LISA

→ milli-Hz frequency band

→ New families of sources (coalescing binaries with large mass asymmetries)

EXTREME MASS-RATIO INSPIRALS

Golden probes: inspiral phase of EMRIs

$$M_{BH} = (10^5 - 10^7)\, M_\odot$$
$$\mu = (1 - 100)\, M_\odot$$

$$q = \frac{\mu}{M_{BH}} \qquad N_{cycles} \simeq 1/q \simeq 10^5$$

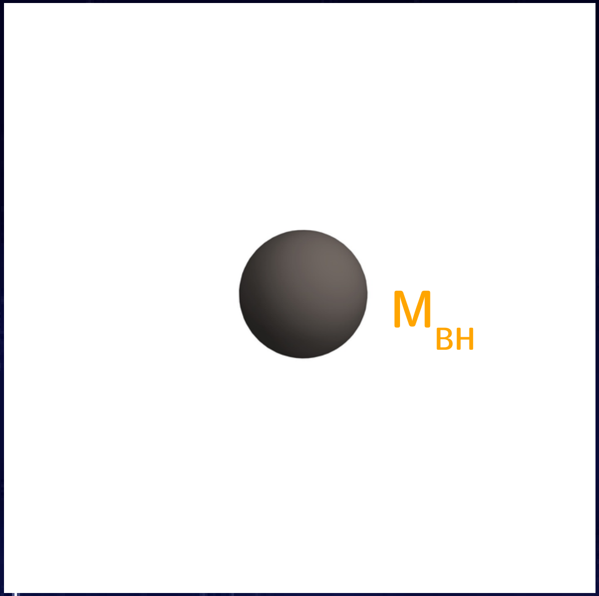
→ precise measurement of source parameters

SCIENCE CASE

→ Study of the environment

→ Infer dark matter properties

BACKGROUND SPACETIME: GEOMETRY CONTENT

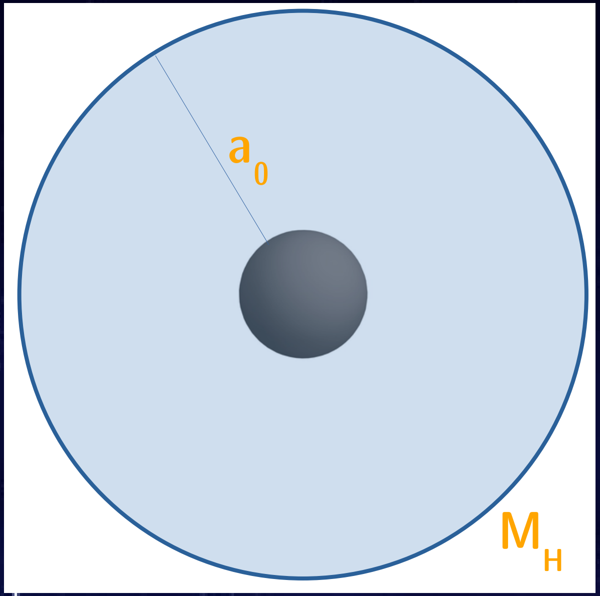


Einstein Equations:

$$G^{(0)}_{\mu\nu} = 8\pi T^{(0)\,\mathrm{env}}_{\mu\nu}$$

Primary Schwarzschild Black Hole

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -a(r)\,dt^2 + \frac{dr^2}{1-2m(r)/r} + r^2 d\Omega^2$$



Einstein equations:

$$G^{(0)}_{\mu\nu} = 8\pi T^{(0)\,env}_{\mu\nu}$$

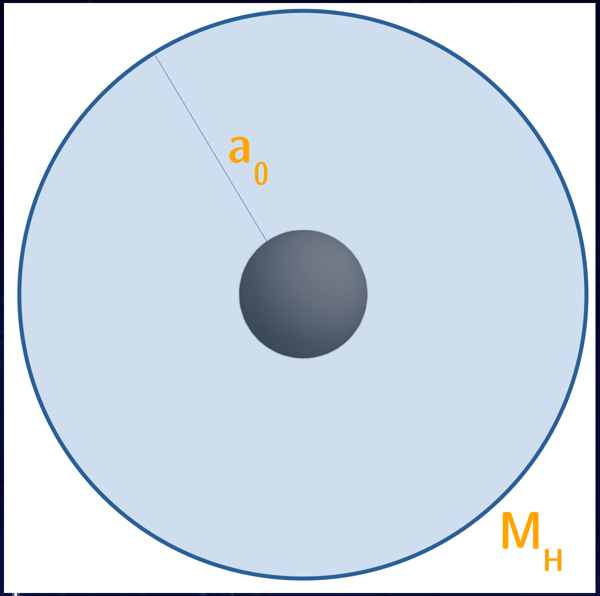
Dark Matter Halo (Hernquist Profile)

Stress-energy tensor

$$T^{(0)\,env}_{\mu\nu} = \mathrm{diag}\left(-\rho(r), 0, P_t(r), P_t(r)\right)$$

Density profile

$$\rho(r) = \frac{M_H (a_0 + 2M_{BH})}{2\pi r (r + a_0)^3} \left(1 - 4\frac{M_{BH}}{r}\right)$$



Compactness $C = \frac{M_H}{a_0} \simeq 10^{-4}$

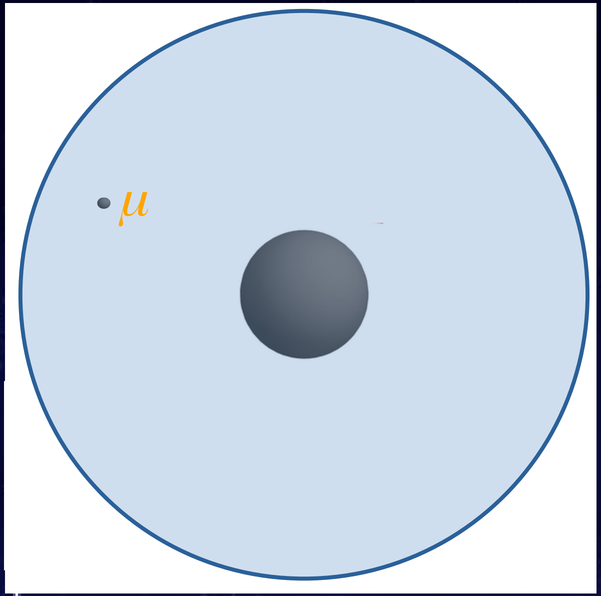
Einstein equations:

$$G^{(0)}_{\mu\nu} = 8\pi T^{(0)\,env}_{\mu\nu}$$

solution at $O(M_H)$

$$m(r) = M_{BH} + M_H\left(\frac{a_0}{a_0 + 2M_{BH}} - \frac{(a_0 + 2M_{BH})(a_0 - 4M_{BH} + 2r)}{(a_0 + r)^2}\right) + O(M_H^2)$$

$$a(r) = 1 - \frac{2M_{BH}}{r} + \frac{2M_H(2M_{BH} - r)(a_0^2 - 4M_{BH})(a_0 + r)\log(a_0 + r) + 4M_{BH}(a_0 + r)\log(r - 2M_{BH}) + 6a_0 M_{BH} + 8M_{BH}^2}{r(a_0 + 2M_{BH})^2(a_0 + r)} + O(M_H^2)$$



Secondary Stellar-Mass Compact Object

moving along circular equatorial orbits

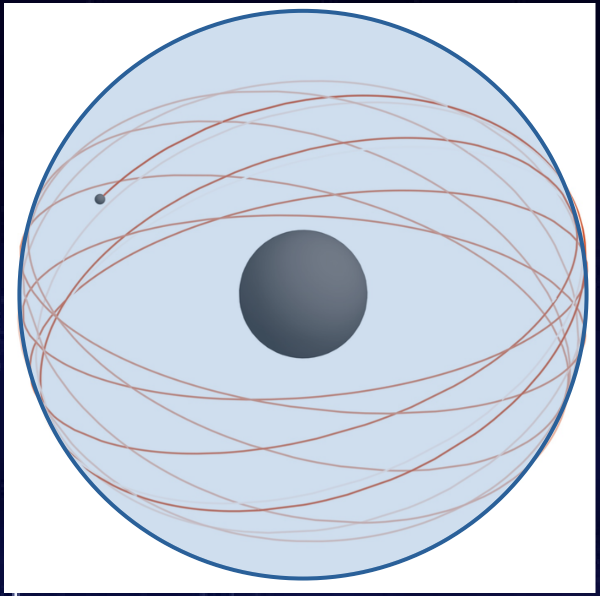
↓

perturbation of the background spacetime

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu} \qquad T^{env}_{\mu\nu} = T^{(0)\,env}_{\mu\nu} + T^{(1)\,env}_{\mu\nu}$$

$$G^{(1)}_{\mu\nu} = 8\pi T^{(1)\,env}_{\mu\nu} + 8\pi T^{p}_{\mu\nu}$$

ADIABATIC ORBITAL EVOLUTION



Radial evolution

$$\frac{dr}{dt} = -\dot{E}\,\frac{dr}{dE_p}$$

Phase evolution

$$\frac{d\Phi}{dt} = \omega_p$$

$\dot{E}$ energy flux (GW emission)

E_p particle orbital energy

ω_p particle orbital angular frequency

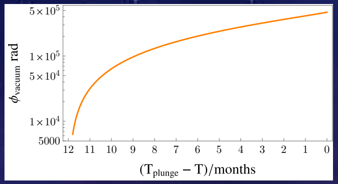
fixed observation time

$t \in (t_0, T)$

$r(t_0) = r_0$

$r(T) = r_{plunge} = 6\,M_{BH}$

$\Rightarrow r(t), \Phi(t)$



Spherical symmetry of the background → decomposition of $g_{\mu\nu}$ and $T_{\mu\nu}$ into axial and polar modes

$l + m = \text{odd}$ $\qquad$ $l + m = \text{even}$

$$\Rightarrow \dot{E}^{tot} = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \dot{E}^{axial}_{lm} + \dot{E}^{polar}_{lm}$$

$$\dot{E}^{axial}_{lm} = \frac{1}{8\pi} \frac{(l+2)!}{(l-2)!} |\Psi(r_{obs})|^2$$

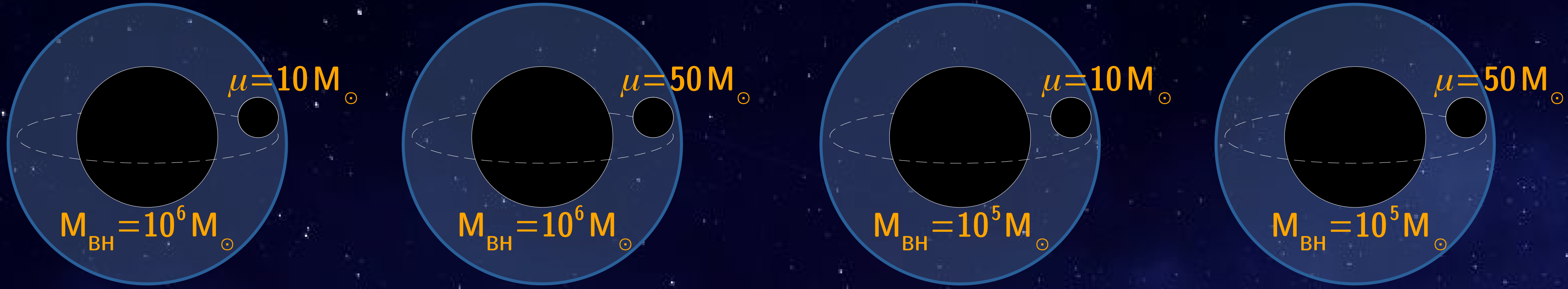
$$\frac{d^2 \Psi^{ax}_{lm}(r)}{dt^2} + (\omega^2 - V^{ax}_l(r)) \Psi^{ax}_{lm}(r) = S^{ax}_{lm}(r)$$

$$\dot{E}^{polar}_{lm} = \lim_{r \to r_{obs}} \frac{1}{32\pi} \frac{(l+2)!}{(l-2)!} |K(r)|^2$$

$$\frac{d\vec{\Psi}^{pol}}{dr} - \hat{\alpha} \vec{\Psi}^{pol} = \vec{S}^{pol}$$

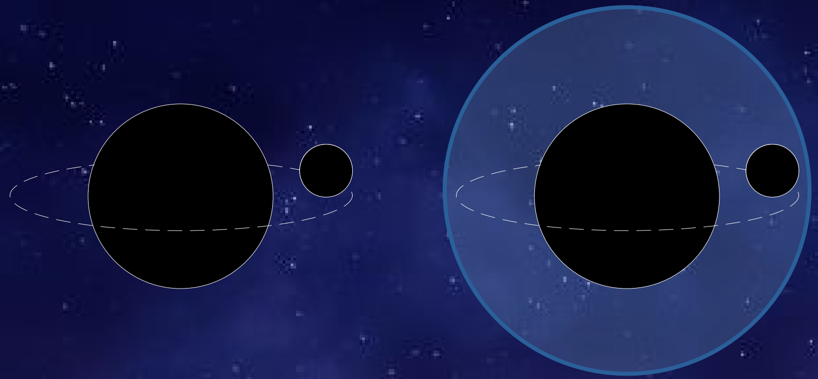
$$\vec{\Psi}^{pol} = \{K(r), H_0(r), H_1(r), W(r), \delta\rho(r)\}$$

SYSTEM CONFIGURATIONS



surrounded by three different halos: $C = (10^{-3}, 10^{-4}, 10^{-5})$

$\forall$ configuration:



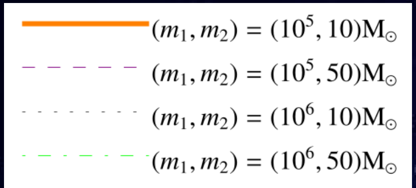
in vacuum   in a dark matter halo

$\Longrightarrow$ Detectability:

- dephasing
- faithfulness

$2\left(\Phi_{vacuum} - \Phi_{matter}\right)$

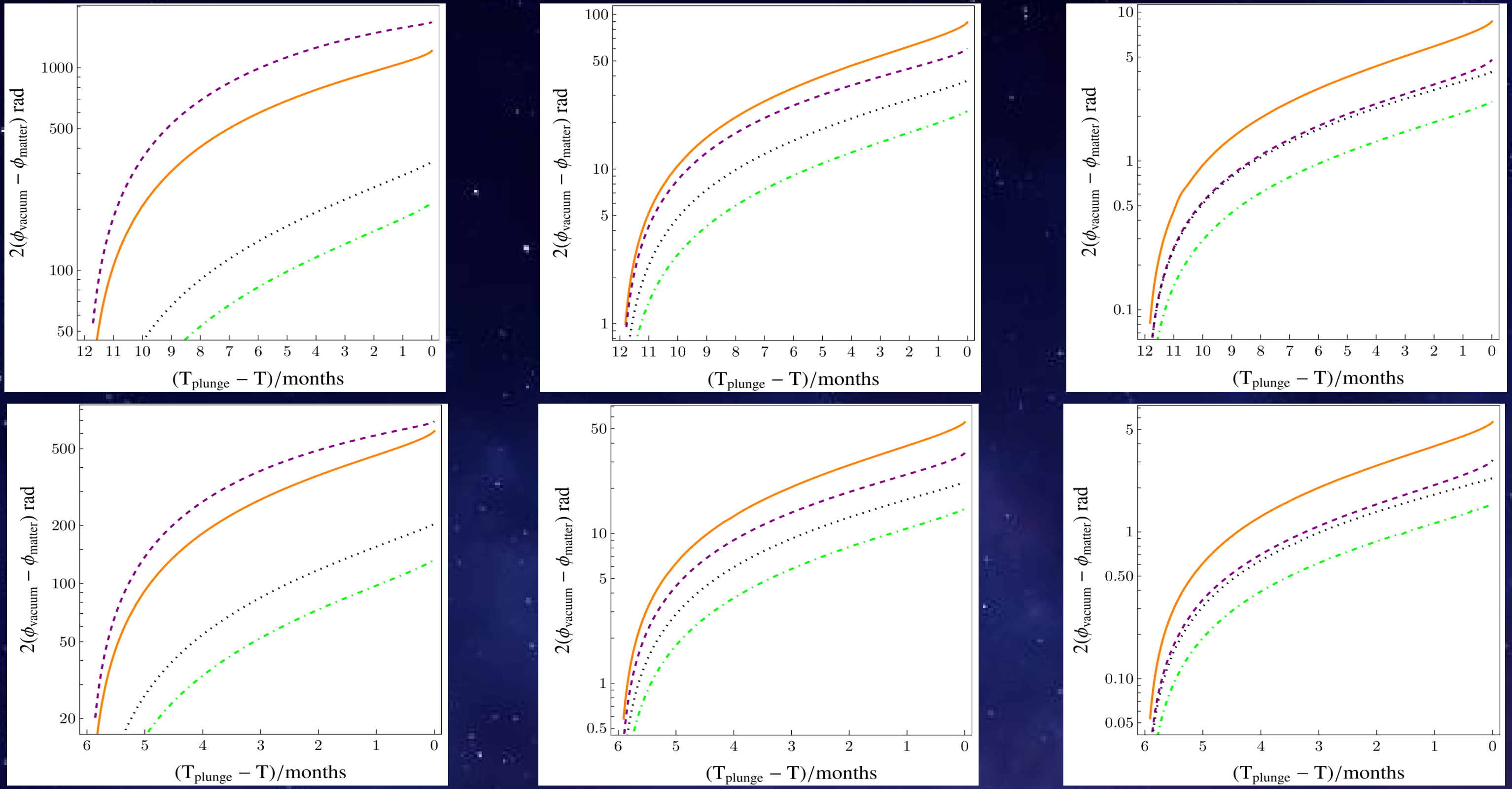
DEPHASING



C = 10^{-3} | C = 10^{-4} | C = 10^{-5}

1 year observation

6 months observation



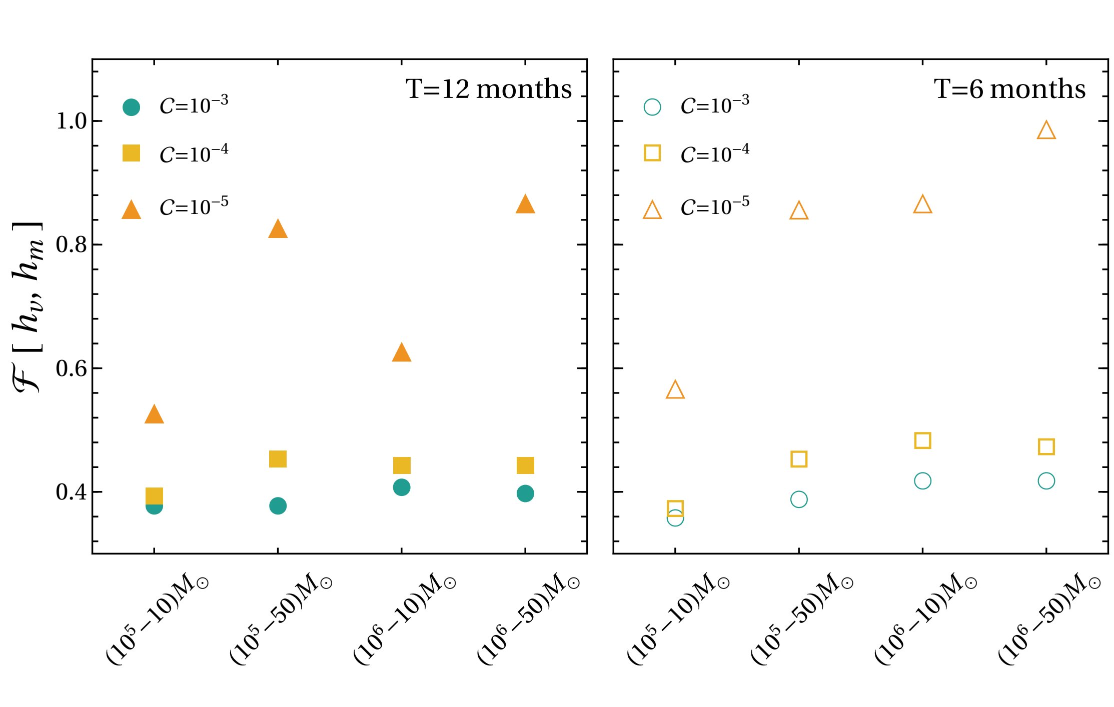
FAITHFULNESS

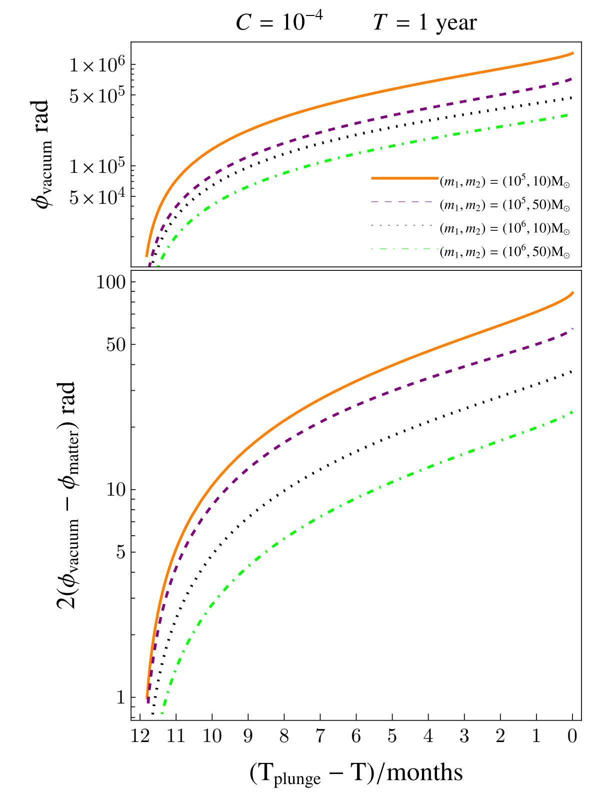
$$F[h_1, h_2] = \max_{\{t_c, \phi_c\}} \frac{\langle h_1 | h_2 \rangle}{\sqrt{\langle h_1 | h_1 \rangle \langle h_1 | h_1 \rangle}}$$

$$\langle h_1 | h_2 \rangle = 4\,\Re \left[\int_{f_{min}}^{f_{max}} \frac{\tilde{h}_1(f)\,\tilde{h}_2(f)}{S_n(f)} df \right]$$

h_1 = GW template of EMRI in vacuum

h_2 = GW template of EMRI in dark matter





$\Longrightarrow$

- Generalizing to eccentric orbits (currently working on it!)

- Performing a more quantitative analysis
- Studying other dark matter profiles
- Adding the spin to the primary black hole

BACK-UP SLIDES

Expansion for small M_H

Exact solution:

$$a(r) = 1 - \frac{2M_{BH}}{r}\exp(Y)$$

$$Y = -\pi\sqrt{M_H/\xi} + 2\sqrt{M_H/\xi}\arctan\frac{r + a_0 - M_H}{\sqrt{M_h\xi}}$$

$$\xi = 2a_0 - M_H + 4M_{BH}$$

Expansion for small M_H

First/second order comparison

	l=2 m=1	l=3 m=2	l=4 m=1	l=4 m=3
$\mathcal{O}(M_{\rm H})$	1.1784×10^{-6}	3.8461×10^{-7}	1.3005×10^{-12}	9.3385×10^{-8}
$\mathcal{O}(M_{\rm H}^2)$	1.1784×10^{-6}	3.8461×10^{-7}	1.3005×10^{-12}	9.3385×10^{-8}

axial sector

	l=2 m=2	l=3 m=1	l=3 m=3	l=4 m=2	l=4 m=4
$\mathcal{O}(M_{\rm H})$	2.2905×10^{-4}	3.1509×10^{-9}	3.6194×10^{-5}	3.8573×10^{-9}	7.1152×10^{-6}
$\mathcal{O}(M_{\rm H}^2)$	2.2904×10^{-4}	3.1440×10^{-9}	3.6194×10^{-5}	3.8397×10^{-9}	7.1151×10^{-6}

polar sector

Same compactness, but different values of M_H and a_0

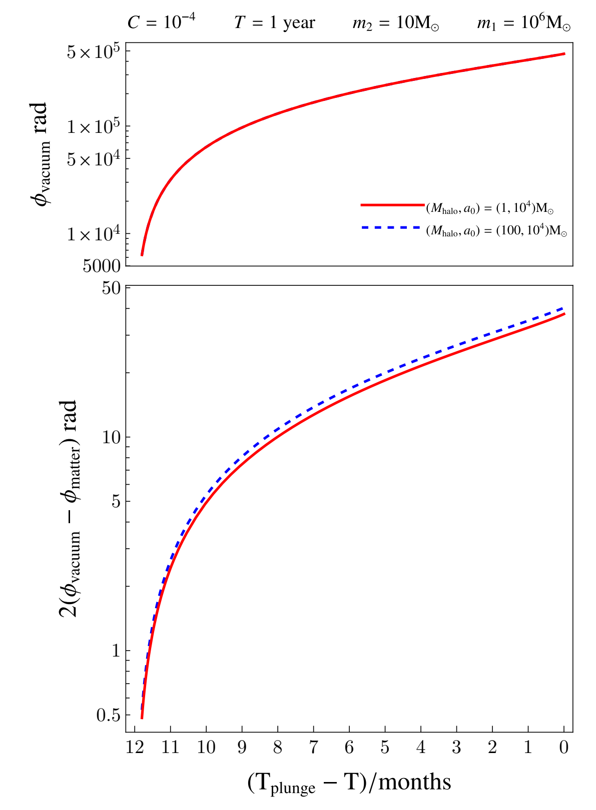
	l=2 m=1	l=3 m=2	l=4 m=1	l=4 m=3
$a_0 = 10^4 \quad M_{\rm H} = 1$	1.1784×10^{-6}	3.8461×10^{-7}	1.3005×10^{-12}	9.3385×10^{-8}
$a_0 = 10^5 \quad M_{\rm H} = 10$	1.1784×10^{-6}	3.8461×10^{-7}	1.3005×10^{-12}	9.3385×10^{-8}

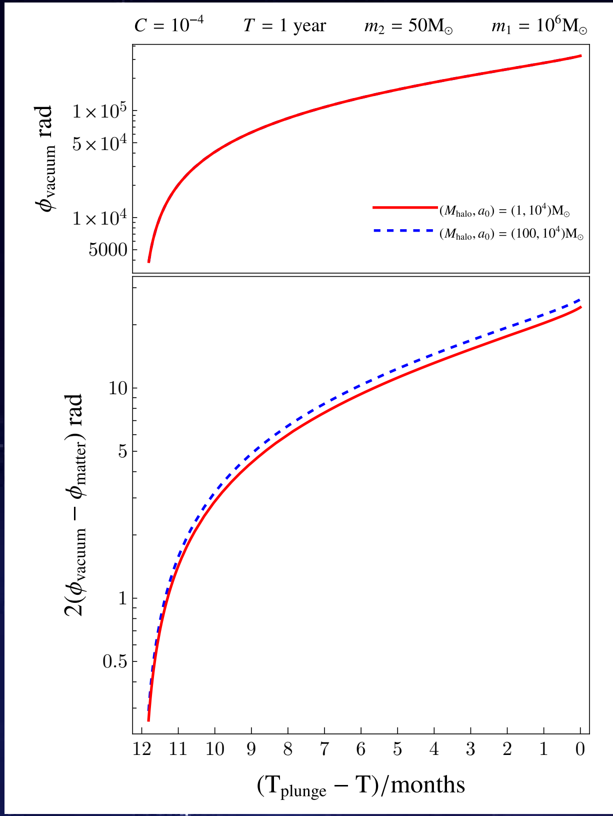
axial sector

	l=2 m=2	l=3 m=1	l=3 m=3	l=4 m=2	l=4 m=4
$a_0 = 10^4 \quad M_{\rm H} = 1$	2.2905×10^{-4}	3.1509×10^{-9}	3.6194×10^{-5}	3.8573×10^{-9}	7.1152×10^{-6}
$a_0 = 10^5 \quad M_{\rm H} = 10$	2.2905×10^{-4}	3.1554×10^{-9}	3.6201×10^{-5}	3.8419×10^{-9}	7.1158×10^{-6}

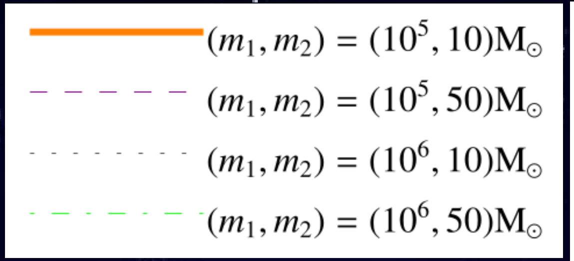
polar sector

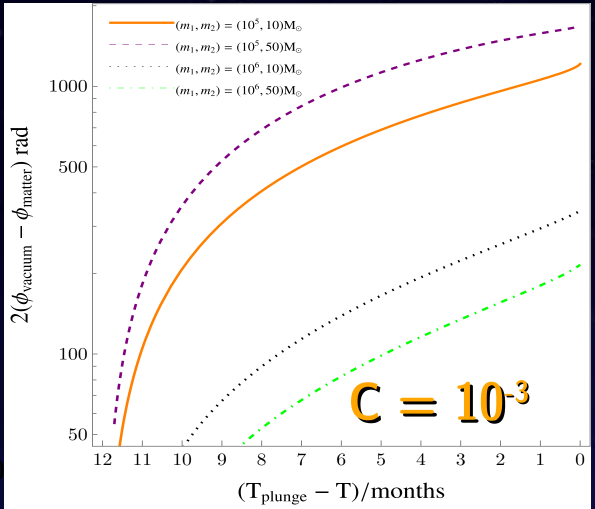
Dependence on the Compactness

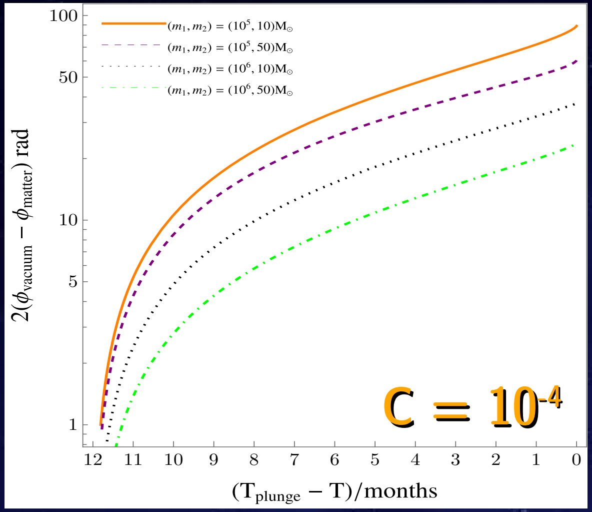


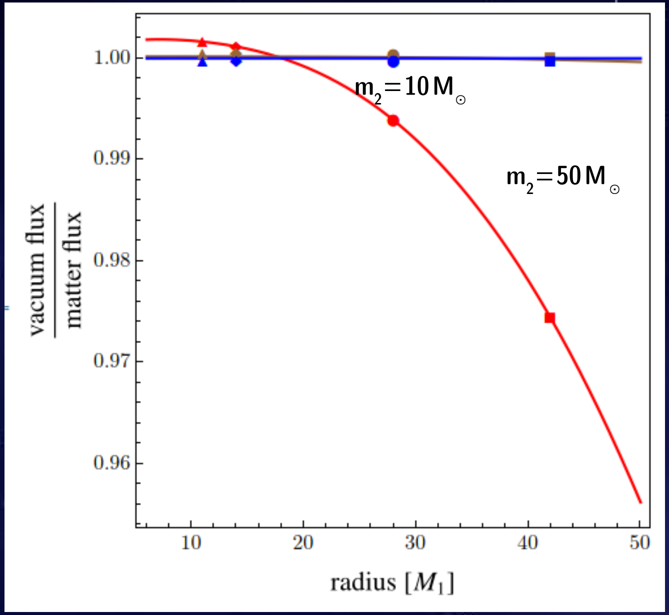


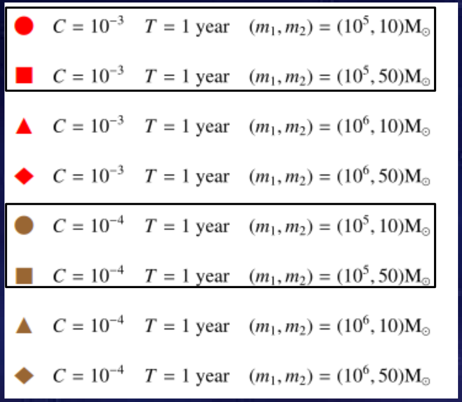
Initial Radius and Flux Ratio











Total number of cycles:

$$(m_1, m_2) = (10^5, 10) \rightarrow N \sim 102 \times 10^3 \text{ orbits}$$
$$(m_1, m_2) = (10^5, 50) \rightarrow N \sim 58 \times 10^3 \text{ orbits}$$
$$(m_1, m_2) = (10^6, 10) \rightarrow N \sim 37 \times 10^3 \text{ orbits}$$
$$(m_1, m_2) = (10^6, 50) \rightarrow N \sim 26 \times 10^3 \text{ orbits}$$

Stress-Energy Tensor of the environment

Einstein Cluster approach:

Way of modeling a stationary BH surrounded by a collection of gravitation masses

$$\langle T_{\mu\nu}\rangle = \frac{n}{m_p}\langle P_\mu P_\nu\rangle$$

Equivalent to define a anisotropic stress-energy tensor

$$T^{(0)\,env}_{\mu\nu} = \mathrm{diag}\left(-\rho(r), 0, -P_t(r), -P_t(r)\right)$$

Other DM profiles

$$\rho(r) = \rho(r/a_0)^{-\gamma}[1+(r/a_0)^{\alpha}]^{(\gamma-\beta)/\alpha}$$

α: dependence of the profile at large distances

β: dependence of the profile at small distances

γ: Sharpness of the transition

$(\alpha,\beta,\gamma) = (1,4,1) \rightarrow$ Hernquist (closed analytical form)

$(\alpha,\beta,\gamma) = (1,3,1) \rightarrow$ Navarro-Frenk-White

$\rho(r) = \rho_e \exp\{-d_n[(r/r_e)^{1/n} - 1]\}$ Einasto

ρ(r) must vanish at $2R_s$: $\rho(r) \rightarrow \rho(r)(1 - 4M_{BH}/r)$

Spikes in the dark matter density

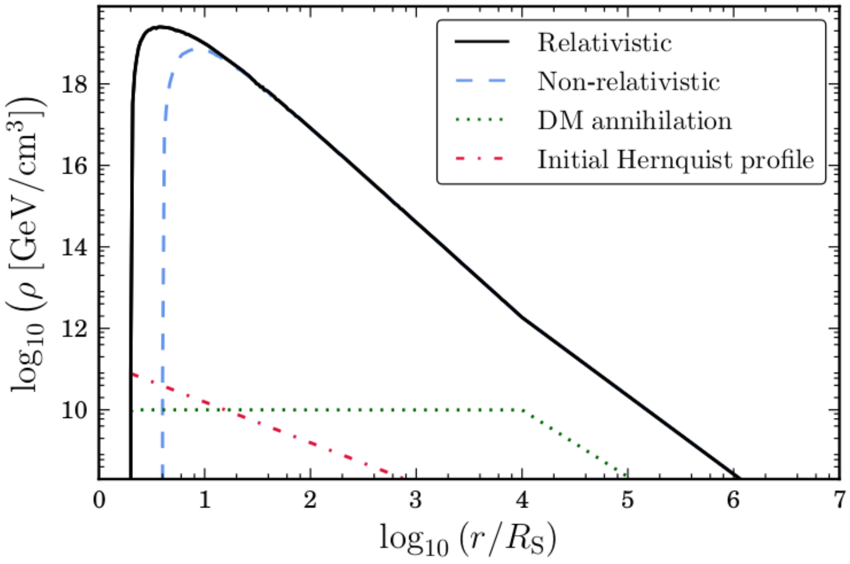


FIG. 3: Effect of the adiabatic growth of the supermassive black hole at the center of the galaxy on a Hernquist dark matter profile. Shown are the results of the fully relativistic calculation, and the effects of dark matter annihilations. The dashed line (blue in color version) shows the non-relativistic approximation using the GS method.

Sadeghian et all. 2013