Nonlinearities in BH ringdown

ADRIEN KUNTZ (SISSA)

With B. Bucciotti, S. Yi, E. Trincherini, E. Barausse, E. Berti…

SISSA
INFN Istituto Nazionale di Fisica Nucleare
ifpU
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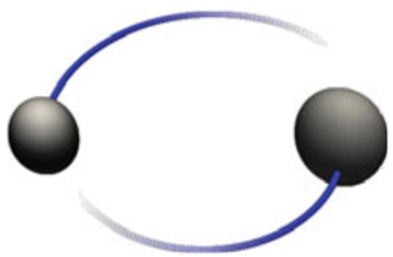
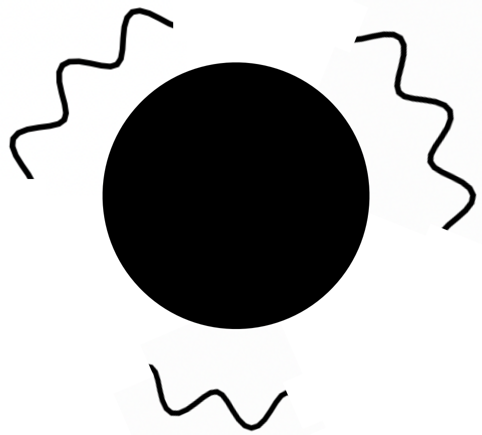
What are QNMs?



$$\bar{g}_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = -f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\phi^2\right)$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu}$$

$$\Rightarrow G^{(1)}_{\mu\nu}\left[\mathbf{h}^{(1)}\right] = 0$$



Using background symmetries and fixing gauge:

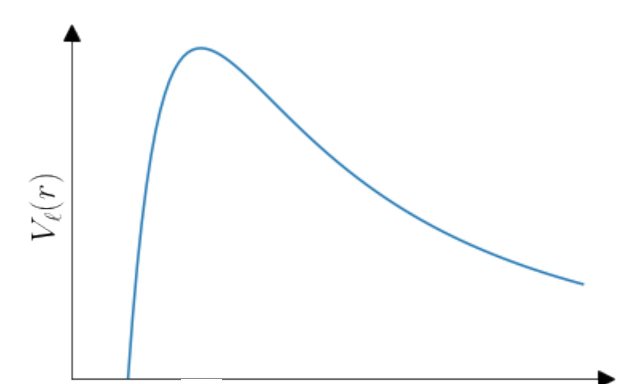
$$h^{(1)} \sim \sum_{\ell,m,\omega,\pm} e^{-i\omega t}\psi^{(1),\ell m}_{\pm}(r)Y^{\ell m}_{\pm}(\theta,\phi) + \mathrm{c.c.}$$

Equatorial (reflection) symmetry: $\pm = (-1)^{\ell+m}$

$$\frac{\mathrm{d}^2\psi^{(1)}}{\mathrm{d}r_*^2} + (\omega^2 - V_\ell(r))\psi^{(1)} = 0$$

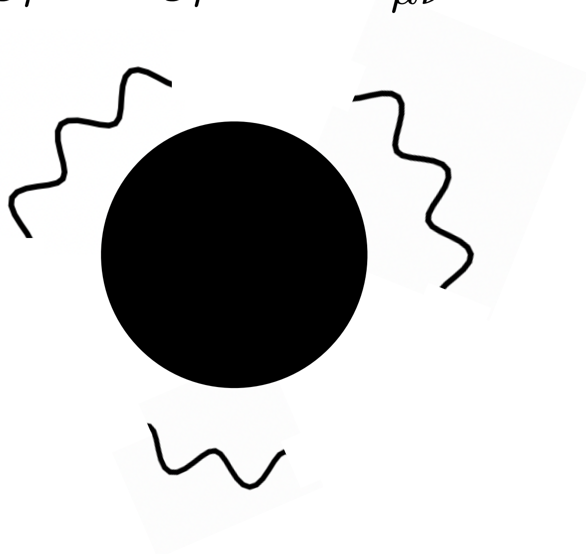
$$r_* = r + 2GM\log\left(r/(2GM) - 1\right)$$

$$V_\ell(r) = \left(1 - \frac{2GM}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} - \frac{6GM}{r^3}\right)$$



What are QNMs?

$g_{\mu\nu} = \bar{g}_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu}$



$$\frac{\mathrm{d}^2\psi^{(1)}}{\mathrm{d}r_*^2} + (\omega^2 - V_\ell(r))\psi^{(1)} = 0$$

A QNM is a solution with:

$$\psi^{(1)}(r) \propto e^{-i\omega r_*} \,, \quad r_* \to -\infty$$
$$\psi^{(1)}(r) \propto e^{i\omega r_*} \,, \quad r_* \to \infty$$

This is possible only if the frequency is quantised:

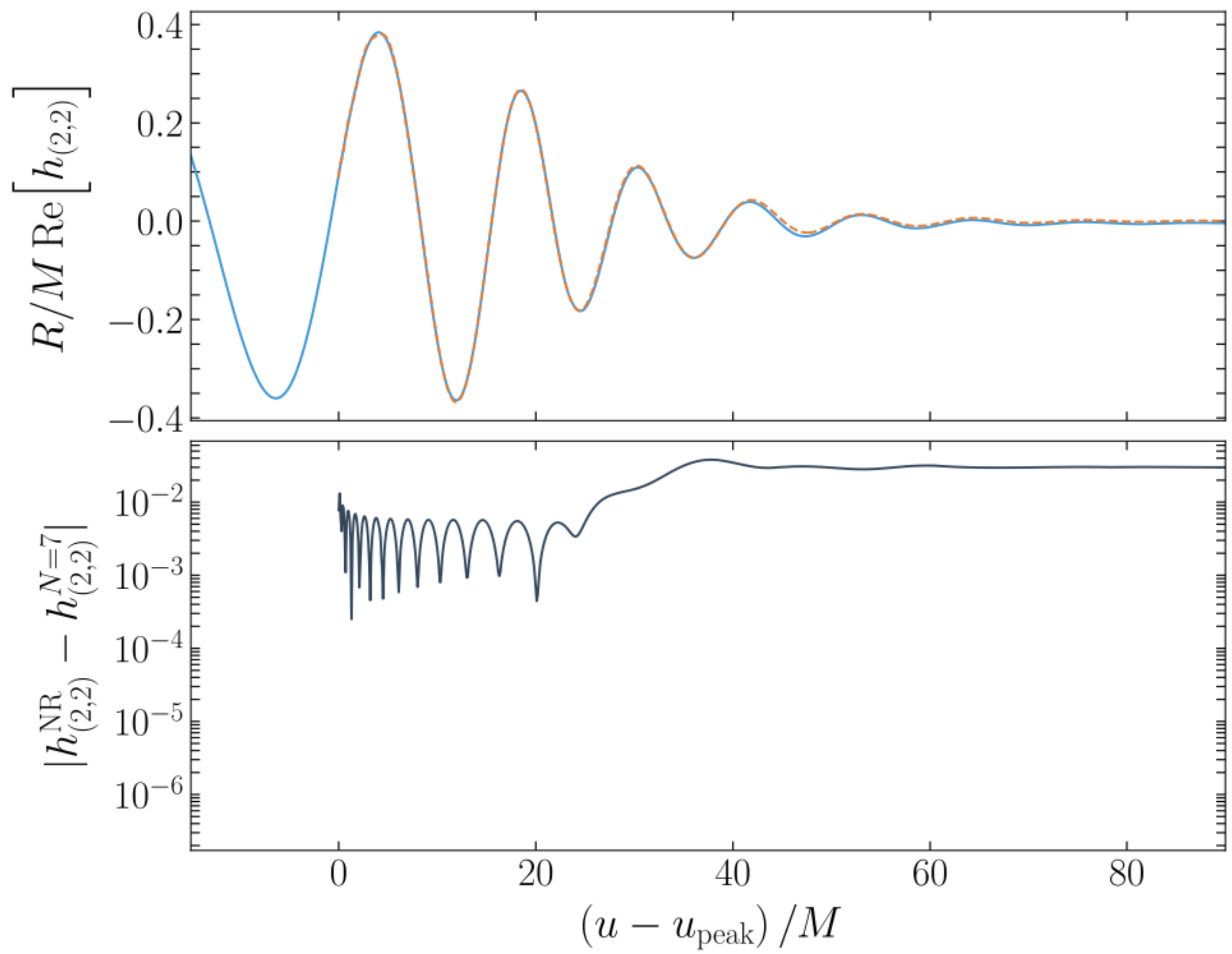
$$\omega = \omega_{\ell m n}$$

ℓ	n	$\omega_{\ell m n}$
2	0	$0.37367 + 0.08896i$
	1	$0.34671 + 0.27391i$
	2	$0.30105 + 0.47828i$
3	0	$0.59944 + 0.09270i$
	1	$0.58264 + 0.28130i$
	2	$0.55168 + 0.47909i$

The amplitude is arbitrary in this approach

→ It MUST be fitted to NR

A REMARKABLE FIT



Zertuche et al. (2022)

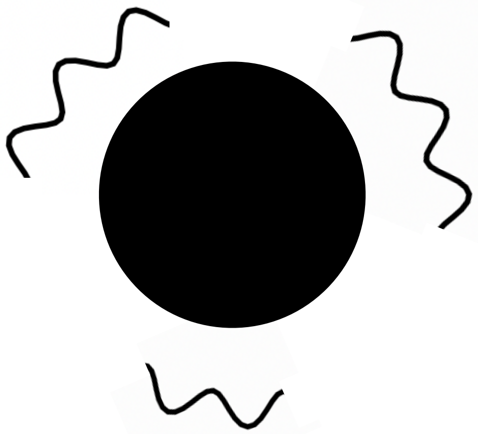
SXS:BBH:0305 ⇔ GW150914

$$h = \sum_{\ell m n} \mathcal{A}_{\ell m n} e^{-i\omega_{\ell m n}(t - r_*)} Y^{\ell m}(\theta, \phi)$$

$\ll 1$

NONLINEARITIES?

$g_{\mu\nu} = \bar{g}_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu} + \varepsilon^2 h^{(2)}_{\mu\nu}$



$$G^{(1)}_{\mu\nu}\left[\mathbf{h}^{(2)}\right] = -G^{(2)}_{\mu\nu}\left[\mathbf{h}^{(1)}, \mathbf{h}^{(1)}\right]$$

Same operator! Source term

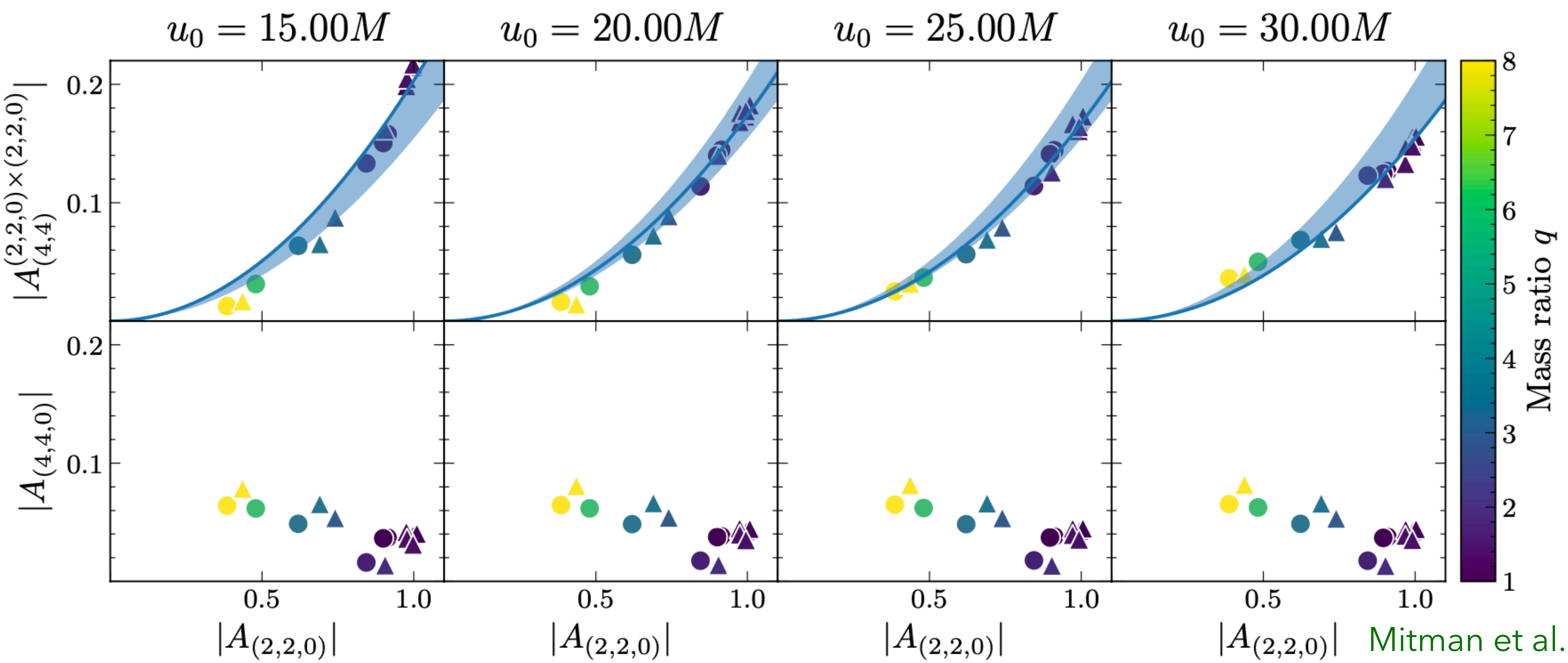
$$h^{(1)} \times h^{(1)} \sim \left(\sum_{\ell_1, m_1, n_1} e^{-i\omega_1 t}\psi^{(1),\ell_1 m_1} Y^{\ell_1 m_1} + \text{c.c.}\right) \times \left(\sum_{\ell_2, m_2, n_2} e^{-i\omega_2 t}\psi^{(1),\ell_2 m_2} Y^{\ell_2 m_2} + \text{c.c.}\right)$$

« QUADRATIC » MODES:

Nakano&Ioka(2007), London&al(2017), Lagos&Hui(2023), Bucciotti AK&al (2024)….

- Frequency $\omega_1 + \omega_2$ or $\omega_1 - (\omega_2)^*$
- Angular momentum $|\ell_1 - \ell_2| \leq \ell \leq \ell_1 + \ell_2$ and $m = m_1 + m_2$
- Parity $(-1)^{\ell+m}$
- Amplitude is quadratic in linear modes

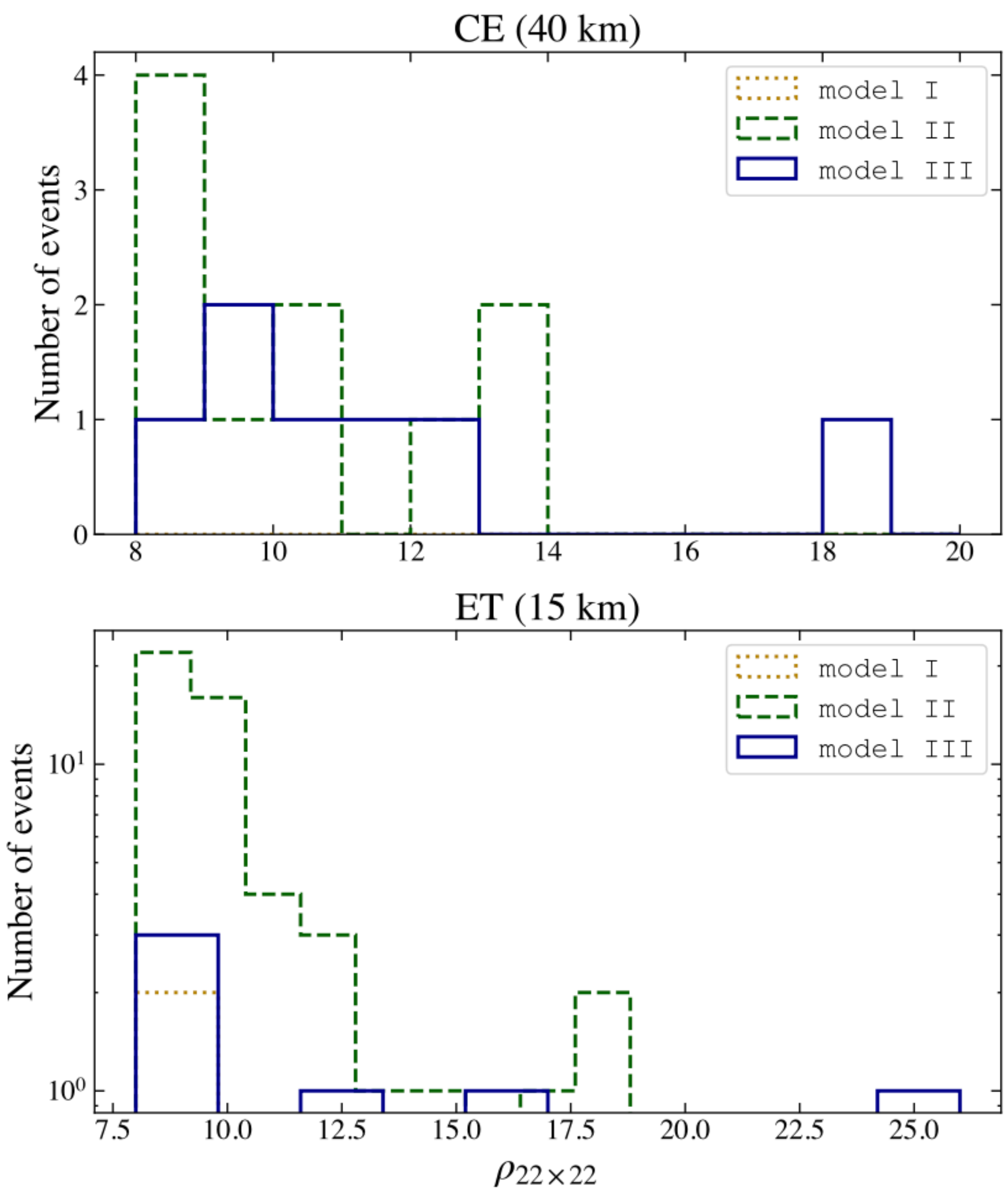
NONLINEARITIES!



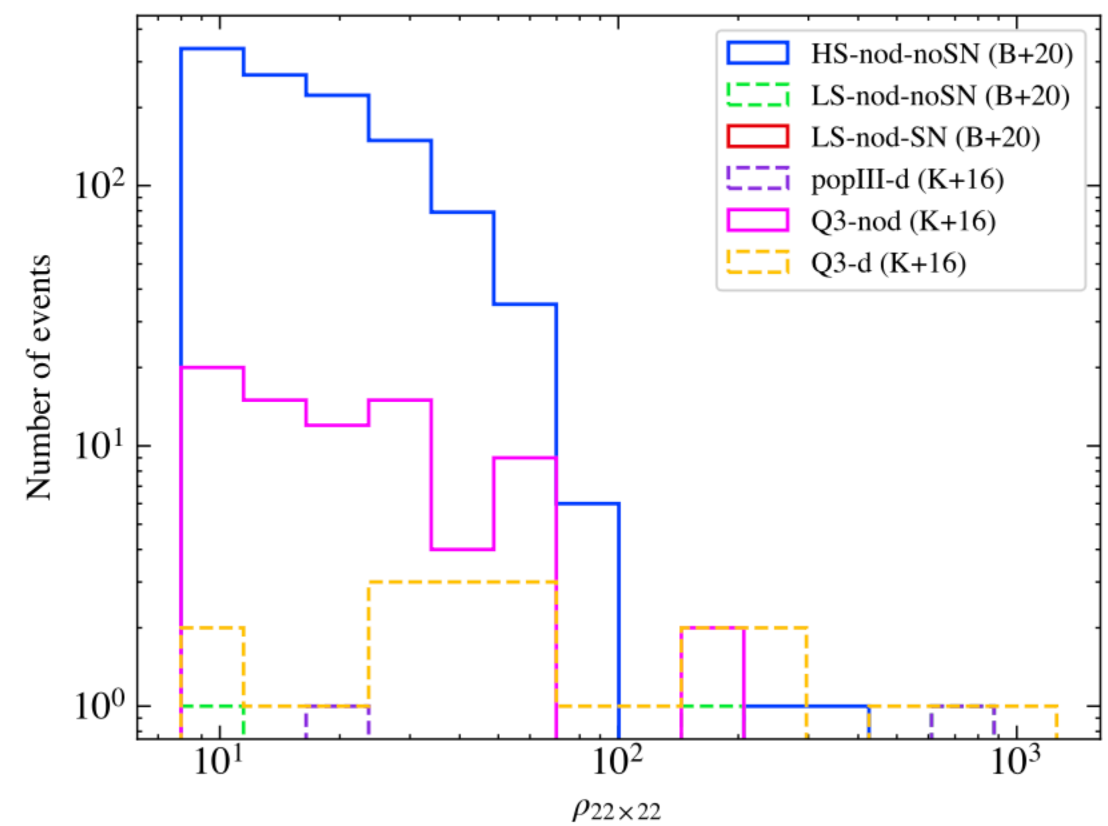
Mitman et al. (2023)

$$h = \sum_{\ell mn} \mathcal{A}_{\ell mn} e^{-i\omega_{\ell mn}(t-r_*)} Y^{\ell m}(\theta,\phi) + \mathcal{A}^{220\times 220}_{44} e^{-2i\omega_{220}(t-r_*)} Y^{44}(\theta,\phi)$$

OBSERVABILITY



S. Yi, AK & al (2024)



Can we have a better understanding of these nonlinearities?

$$\frac{\mathcal{A}_{44}^{220\times220}}{(\mathcal{A}_{220})^2} = ?$$

Second-order RW

$$G^{(1)}_{\mu\nu}[\mathbf{h}^{(2)}] = -G^{(2)}_{\mu\nu}[\mathbf{h}^{(1)},\mathbf{h}^{(1)}] \quad \longrightarrow \quad \frac{\mathrm{d}^2\psi^{(2)}}{\mathrm{d}r_*^2} + (\omega^2 - V_\ell(r))\psi^{(2)} = S(r)$$

A toy-model: $S(r) = f(r)\psi_1^{(1)}(r)\psi_2^{(1)}(r)$

Homogeneous solution: Renormalization of linear amplitudes

Particular solution: Amplitude fixed by $S(r)$!

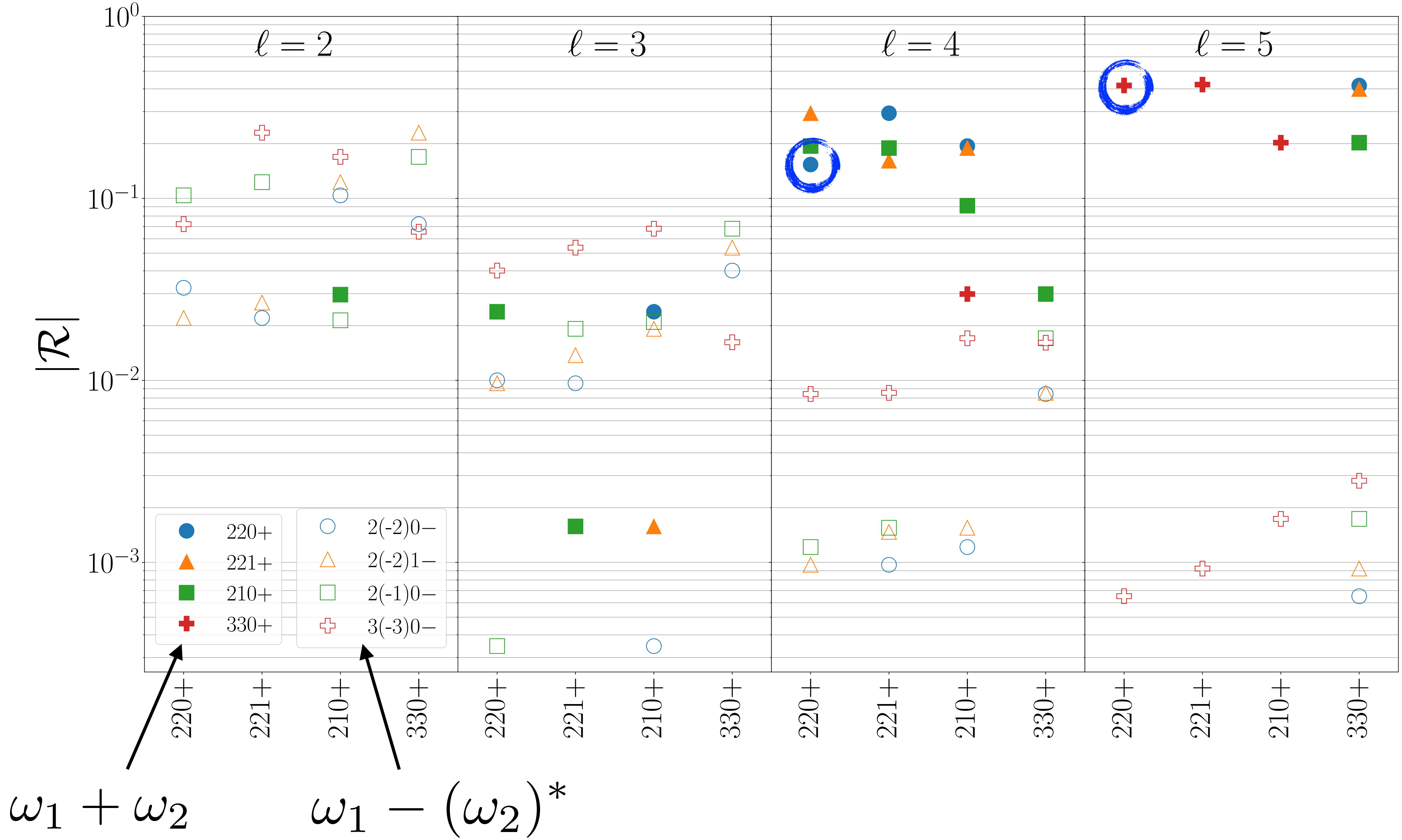
A QNM is a solution with:

$$\psi^{(2)}(r) \propto e^{-i\omega r_*} \,, \quad r_* \to -\infty$$
$$\psi^{(2)}(r) \propto e^{i\omega r_*} \,, \quad r_* \to \infty$$

LEAVER SOLUTION

Bucciotti, Juliano, AK, Trincherini (2024)

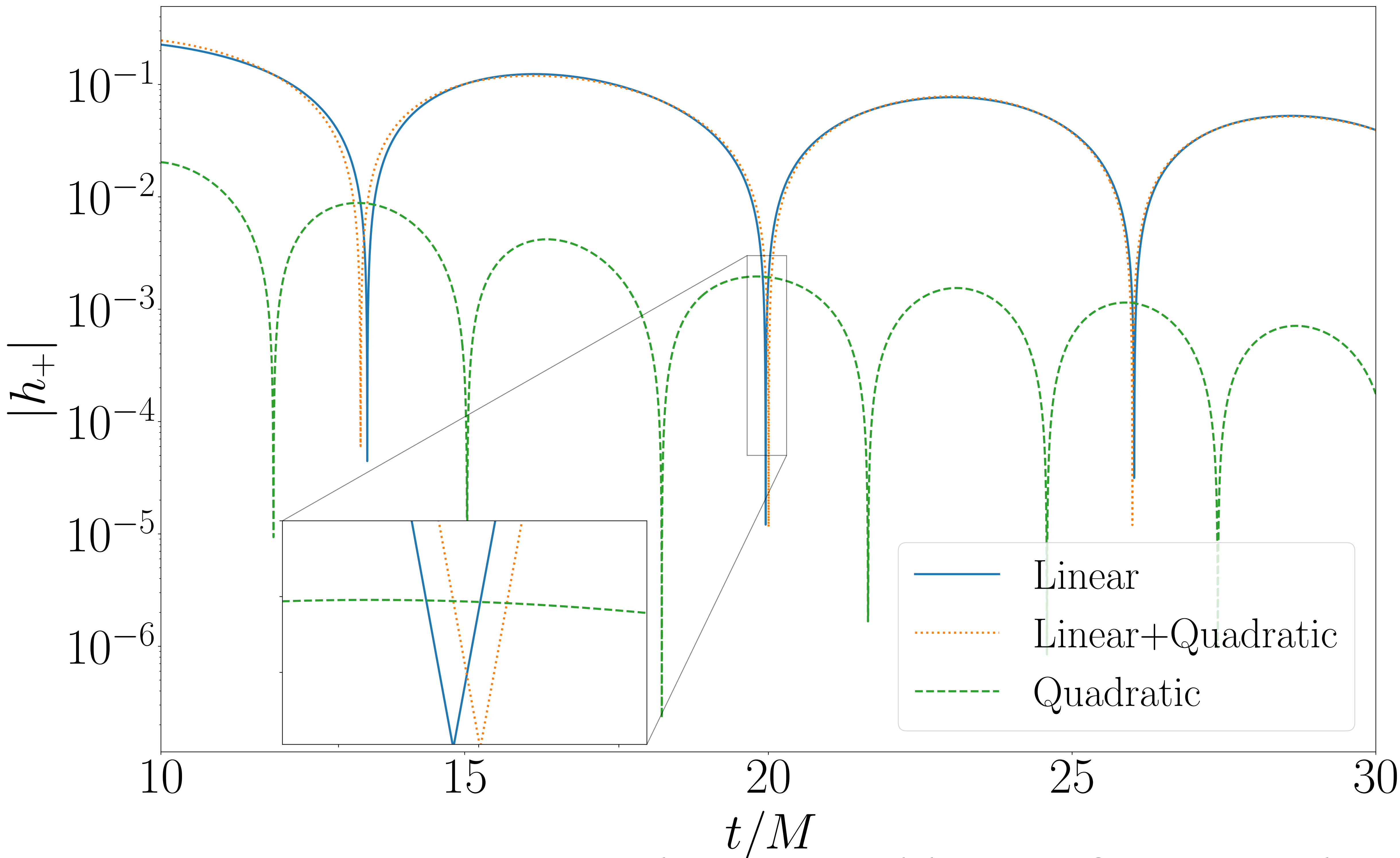
$$\mathcal{R} = \mathcal{A}^{(2)} / (\mathcal{A}_1^{(1)} \mathcal{A}_2^{(1)})$$



Nonlinear ringdown

Bucciotti, Juliano, AK, Trincherini (2024)

GW190521-like merger



(Neglecting spin and deviations from equatorial symmetry)

Conclusions

- As precision improves, we should aim to model nonlinearities in ringdown
- Quadratic modes will be observable by ET/CE/LISA
- Amplitude of quadratic modes is fully determined by:
    - Amplitudes of linear modes
    - mass and spin of final BH

Available at:

https://github.com/akuntz00/QuadraticQNM

- Applications:
    - Improvement of ringdown waveforms for free
    - Tests of GR

BACKUP

Deviations to reflection symmetry

Parity is not given by reflection symmetry

Polarization parameter:

$$\kappa_{\ell m} = \frac{\mathcal{A}_{+,\ell m} + i\mathcal{A}_{-,\ell m}}{\mathcal{A}_{+,\ell m} - i\mathcal{A}_{-,\ell m}}$$

Total ratio:

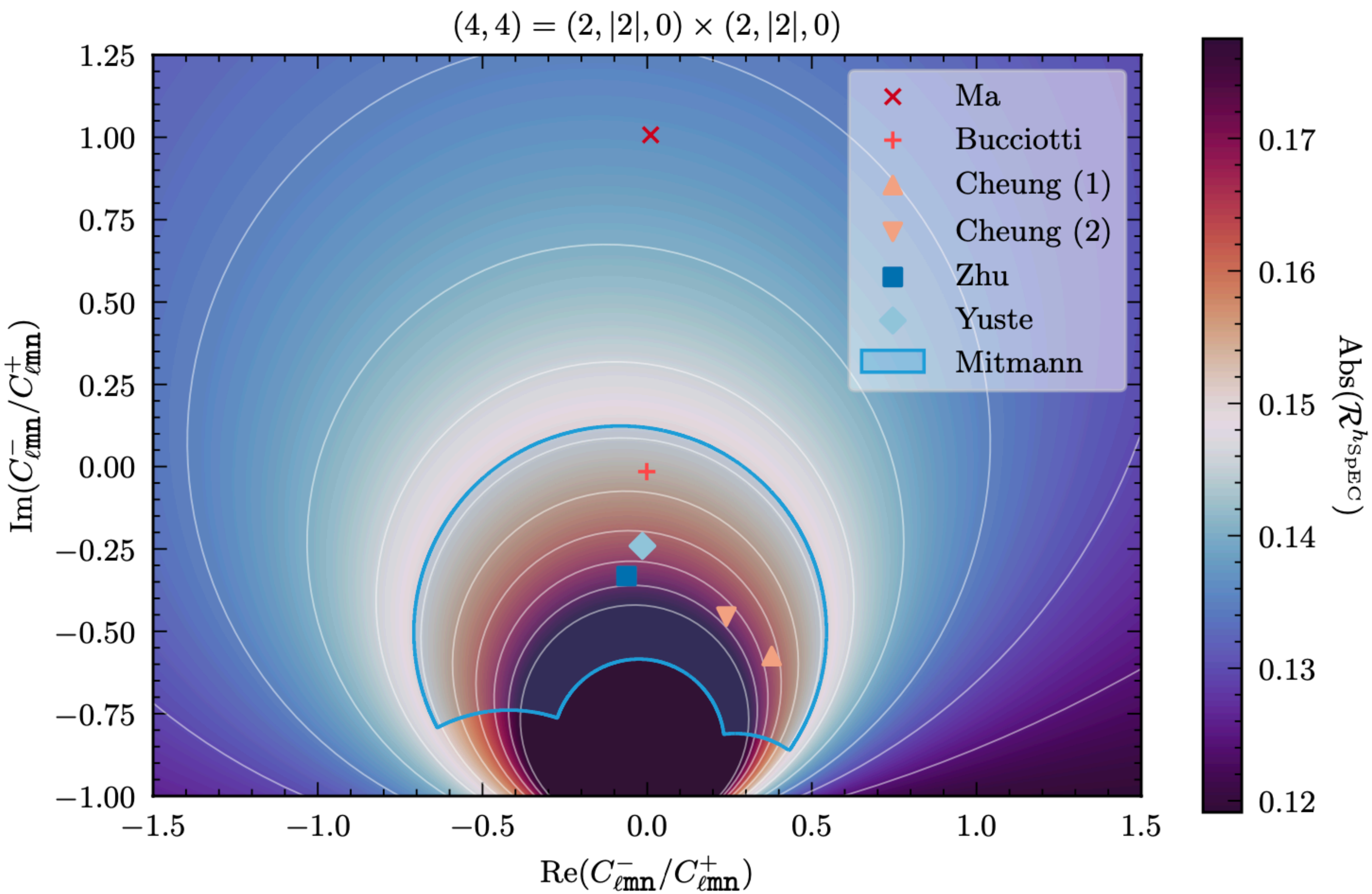
$$\frac{\mathcal{A}^{(2)}}{\mathcal{A}_1^{(1)}\mathcal{A}_2^{(1)}} = \frac{1}{4}\left[\alpha_+ + \beta_+\right] , \quad \kappa^{(2)} = \frac{\alpha_+ - \beta_+}{\alpha_+ + \beta_+} ,$$

$$\alpha_+ = \mathcal{R}_{--\to+}\left(1-\kappa_1^{(1)}\right)\left(1-\kappa_2^{(1)}\right) + \mathcal{R}_{++\to+}\left(1+\kappa_1^{(1)}\right)\left(1+\kappa_2^{(1)}\right) ,$$

$$\beta_+ = \mathcal{R}_{+-\to-}\left(1+\kappa_1^{(1)}\right)\left(1-\kappa_2^{(1)}\right) + \mathcal{R}_{-+\to-}\left(1-\kappa_1^{(1)}\right)\left(1+\kappa_2^{(1)}\right)$$

DEVIATIONS TO REFLECTION SYMMETRY

Bourg&al (2024)



Selections rules

Bucciotti, Juliano, AK, Trincherini (2024)

$$\begin{pmatrix} \ell_1 & \ell_2 & \ell \\ m_1 & m_2 & -m \end{pmatrix} = (-1)^{\ell_1+\ell_2+\ell} \begin{pmatrix} \ell_2 & \ell_1 & \ell \\ m_2 & m_1 & -m \end{pmatrix}$$

Rule 1. Quadratic modes vanish if $\ell_1 = \ell_2$, $m_1 = m_2$ and ℓ is odd

Rule 2. Quadratic modes vanish if $\ell_1 = \ell_2$, $n_1 = n_2$, $\mathfrak{m}_1 = \mathfrak{m}_2$, $\kappa_1 = \kappa_2$ and ℓ is odd

REGULARIZATION

$$\frac{\mathrm{d}^2\psi^{(2)}}{\mathrm{d}r_*^2} + (\omega^2 - V_\ell(r))\psi^{(2)} = S(r)$$

A QNM is a solution with:

$$\psi^{(2)}(r) \propto e^{-i\omega r_*} \;, \quad r_* \to -\infty$$
$$\psi^{(2)}(r) \propto e^{i\omega r_*} \;, \quad r_* \to \infty$$

Issue with the asymptotics!

Gleiser&al (95,98,99)

Brizuela&al (06,07,09)

Nakano&Ioka (07)

We want $\frac{\mathrm{d}^2\psi^{(2)}}{\mathrm{d}r_*^2} + \omega^2\psi^{(2)} = \mathcal{O}\left(\frac{e^{i\omega r_*}}{r^2}\right)$ We have $S = \mathcal{O}(e^{i\omega r_*})$

REGULARIZATION

Bucciotti, Juliano, AK, Trincherini (2024)

$$\Psi^{(2)}(r) = \psi^{(2)}(r) + \Delta(r)\psi_1^{(1)}\psi_2^{(1)}$$

$$\longrightarrow \quad \frac{\mathrm{d}^2\Psi^{(2)}}{\mathrm{d}r_*^2} + (\omega^2 - V_\ell(r))\Psi^{(2)} = \tilde{S}(r)$$

$$\tilde{S}(r) \propto \left(1 - \frac{2M}{r}\right)e^{-i\omega r_*}\,, \quad r_* \to -\infty$$

$$\tilde{S}(r) \propto \frac{e^{i\omega r_*}}{r^2}\,, \quad r_* \to \infty$$

(Similar to $V_\ell(r)$)

The $\Delta(r)$ cancels when going from RW gauge to TT gauge