Geometric template bank for low-mass compact binaries with moderately eccentric orbits

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Introduction

- Modelled search analyses are very successful in detecting gravitational waves from compact binaries and involve
    - Matched filtering using sets of template waveforms or banks
    - Consistency checking with expected GW signals and significance estimation
- Status of template bank generation
    - Largely restricted to four dimensions: mass parameters in detectors' frame and spins along orbital angular momentum (*see for generic binaries:* McIssac et al. 2023, Schmidt et al. 2024, Wang et al. 2023)
    - Relies on a) geometric, b) stochastic, and c) combination of both
- Eccentricity is often excluded in template bank generations
    - Is this assumption sufficient to cover any possible binary system (with a focus on BNS/NSBH) ?
- Present a geometric template bank placement for eccentric, spinning binary sources

Geometric Template Bank

- Template banks are sets of waveforms in to matched-filter GW data

$$\{h_b^M(\vec{\theta})\}$$

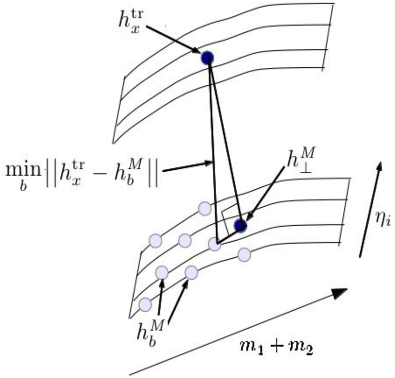
- Signal detection is minimizing distance between template and true GW signal h_x^{tr}

$$||h_x^{\mathrm{tr}} - h_b^M|| = 1 - \mathcal{M}(h_x^{\mathrm{tr}}, h_b^M) = 1 - \langle h_x^{\mathrm{tr}} \mid h_b^M \rangle$$

- Match between waveforms with small parameter offset gives a metric tensor in $\vec{\theta}$

$$ds^2 = \mathrm{mismatch} = 1 - \langle h(\vec{\theta} + \Delta\theta) \mid h(\vec{\theta}) \rangle = -\frac{1}{2}\frac{\partial^2 \mathcal{M}}{\partial\theta^i \partial\theta^j}\Delta\theta^i \Delta\theta^i = g_{ij}\Delta\theta^i \Delta\theta^i$$

- Metric defined in a flat or nearly flat space is used to guide grid placement methods (lattice placement) to build geometric bank



Kumar et al., PRD, 2024

Effectualness of QC bank: QC & eccentric injections

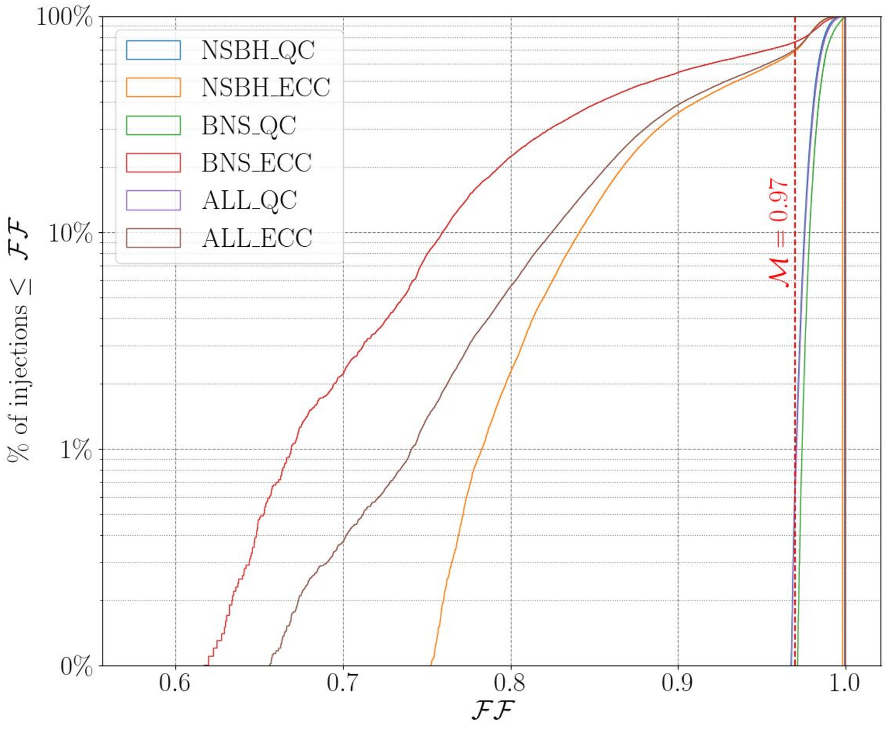
Effectualness is presented via fitting factor: $\mathcal{FF}(h_i) = \max_{\lambda_b} \mathcal{M}(h_i, h_{\lambda_b})$

Quasi-circular (QC) bank details:

- Metric: TaylorF2 metric (pycbc_geom_aligned_bank); upto 3.5PN order
- Placement: 3D A^*_n lattice (2D hexagonal lattice is sufficient for spinning qc binaries)
- Parameters:
  - m_1 : [1 - 7] Msun, m_2 : [1 - 3] Msun
  - χ_{1max} (χ_{2max}): 0.9(0.05)
  - f_{low}:15Hz , Noise curve: aligo_O4high.txt (Optimistic)
- Bank size: 566 974

Injection params: Uniform for masses and spins

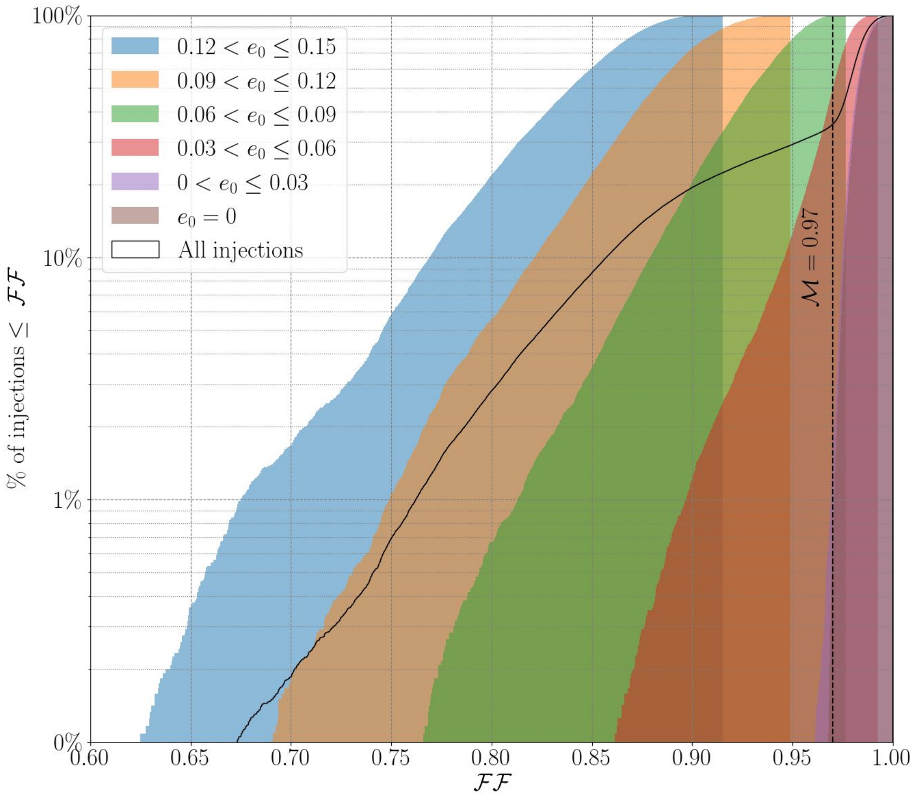
- e_0 (15Hz) : U[0-0.15]
- waveforms: TaylorF2Ecc (ecc inj), TaylorF2 (qc inj and temp)



QC bank is highly ineffectual in recovering eccentric injections

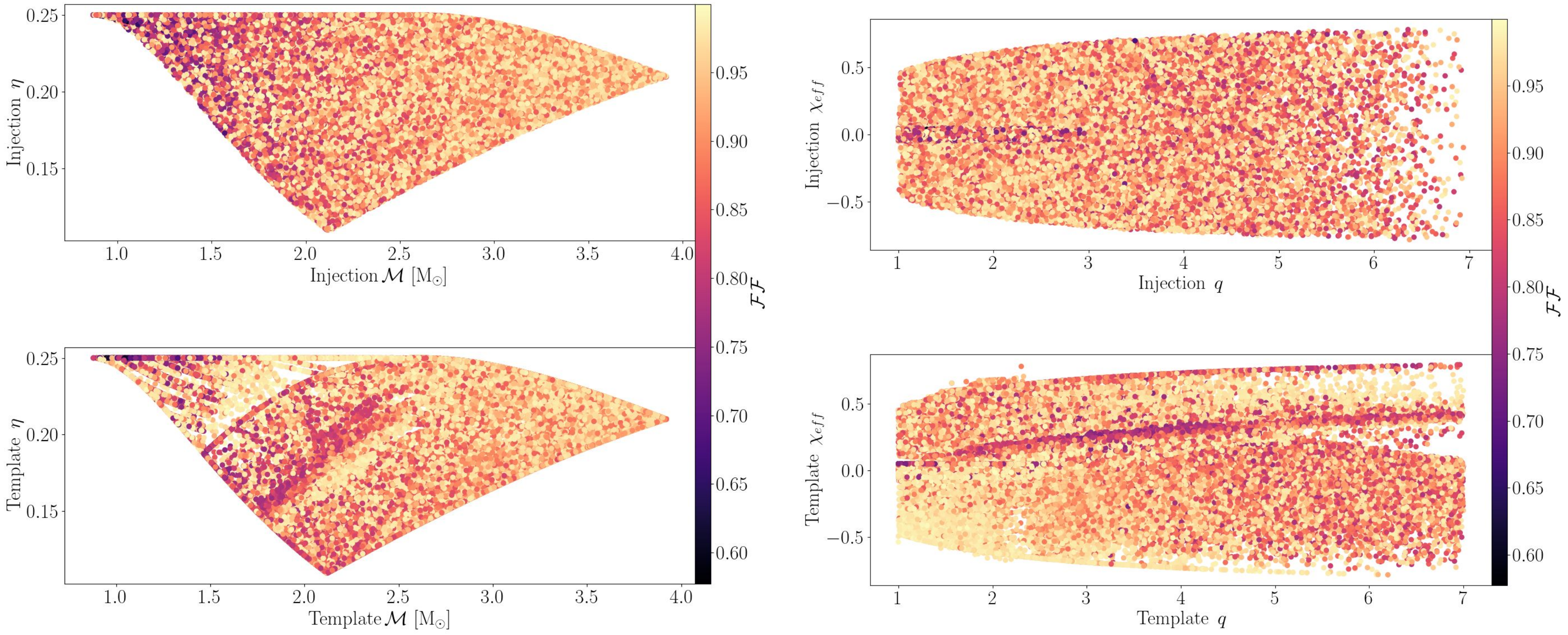
Effectualness of QC bank: fitting-factor (FF) vs eccentricity

e_0 range	Min FF	Max FF	% signals < minimal match (0.97)
0 - 0.03	0.95	1	1%
0.03 - 0.06	0.80	0.99	5%
0.06-0.09	0.72	0.98	99%
0.09-0.12	0.64	0.95	100%
0.12-0.15	0.58	0.92	100%



Effectualness of QC bank: injections, best matched templates

Significant fraction of interior/bulk injections have bad fitting-factor



Top panels: injections' parameters, Bottom panels: best match templates' parameters

Constructing eccentric template bank: TaylorF2Ecc metric

$$\Psi_{\mathrm{TF2Ecc}}(f;\vec{\theta}) = 2\pi f t_0 + \phi_0 + \sum_{i=0}^{7} \varphi_i(\vec{\theta}_{int}) f^{(-5+i)/3} + \sum_{i=5}^{6} \varphi_i^{\ell}(\vec{\theta}_{int}) \log f f^{(-5+i)/3} + \sum_{i,j=0}^{4} \varepsilon_{ij}(\vec{\theta}_{int}) f_{ecc}^{(19+3j)/9} f^{(-34+3i)/9}$$

$$\Psi_{\mathrm{TF2Ecc}}(f;\vec{\theta}) = \sum_{i=0}^{8} \varphi_i(\vec{\theta}) f^{(-5+i)/3} + \sum_{i=5}^{6} \varphi_i^{\ell}(\vec{\theta}_{int}) \log f f^{(-5+i)/3} + \sum_{i=0}^{4} \varepsilon_i'(\vec{\theta}_{int}, f_{ecc}) f^{(-34+3i)/9}$$

QC(Ecc) terms: n=i (n=i+j) corresponds n/2 PN order

Eccentric terms are truncated at 2PN order

$$\varepsilon_i'(\vec{\theta}_{int}, f_{ecc}) = \sum_{j=0}^{4} \varepsilon_{ij}(\vec{\theta}_{int}) f_{ecc}^{(19+3j)/9}$$

Match in physical or PN coefficient space:

$$\mathcal{M}(\vec{\theta}, \vec{\theta}+\Delta\vec{\theta}) := \max_{\phi_0, t_0} 4\mathcal{R} \int df \frac{\tilde{h}^*(f:\vec{\theta})\tilde{h}(f:\vec{\theta}+\Delta\vec{\theta})}{S_n(f)} \equiv 1 - g_{ij}\Delta\theta_i\Delta\theta_j$$

Match in PCA space:

$$\mathcal{M}(\vec{\theta}, \vec{\theta}+\Delta\vec{\theta}) \equiv 1 - \sum_{i=1}^{3} \Delta\xi_i^2$$

Globally flat metric in PN coefficient space: $\{ \varphi_i, \varphi^{\ell}_i, \varepsilon'_i \}$

Flat metric using 3 principal components: $\{\xi_1, \xi_2, \xi_3\}$

Samples in the principal coordinate space with a minimal match 0.97

3-d A_n^* lattice

Templates in Physical coordinates: $\theta_{int} = \{m_1, m_2, s_{1z}, s_{2z}, e_0\}$

Moore et al., Phys. Rev. D **93**, 124061 (2016)
Brown et al., Phys. Rev. D 86, 084017 (2012)
Ohme et al., Phys. Rev. D 88, 042002 (2013)
Harry et al., Phys. Rev. D 89, 024010, (2014)

Constructing eccentric template bank: TaylorF2Ecc metric

$$\Psi_{\rm TF2Ecc}(f;\vec{\theta}) = 2\pi f t_0 + \phi_0 + \sum_{i=0}^{7} \varphi_i(\vec{\theta}_{int}) f^{(-5+i)/3} + \sum_{i=5}^{6} \varphi_i^{\ell}(\vec{\theta}_{int}) \log f f^{(-5+i)/3} + \sum_{i,j=0}^{4} \varepsilon_{ij}(\vec{\theta}_{int}) f_{ecc}^{(19+3j)/9} f^{(-34+3i)/9}$$

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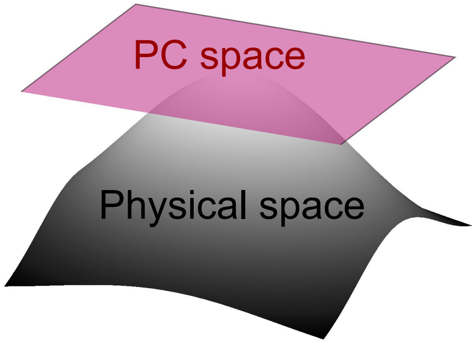
Globally flat metric in PN coefficient space: $\{ \varphi_i, \varphi^{\ell}_i, \varepsilon'_i \}$

Flat metric using 3 principal components: $\{\xi_1, \xi_2, \xi_3\}$

Samples in the principal coordinate (PC) space with a minimal match 0.97

3-d A_n^* lattice

Templates in Physical coordinates: $\theta_{int} = \{m_1, m_2, s_{1z}, s_{2z}, e_0\}$

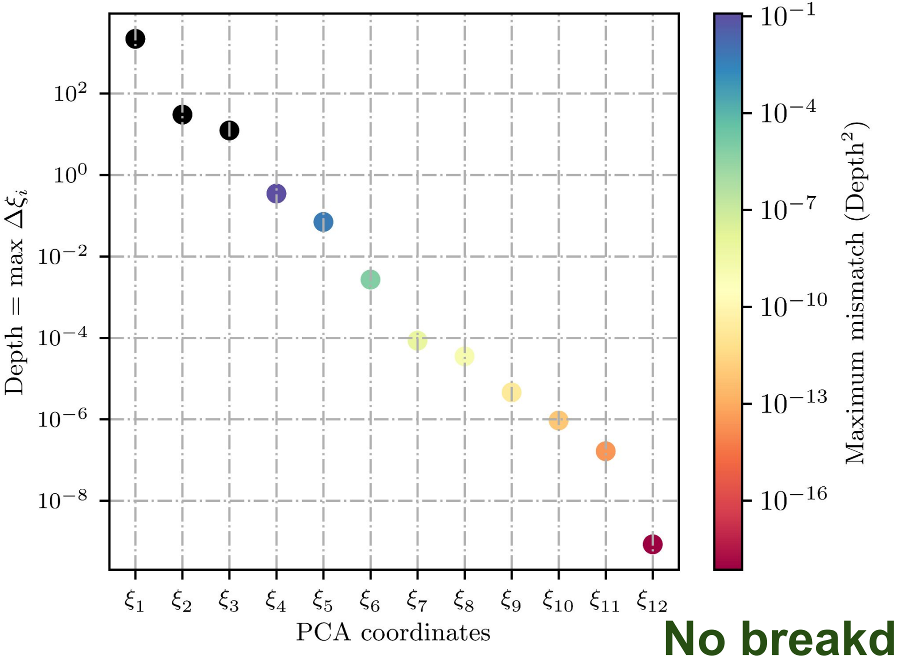


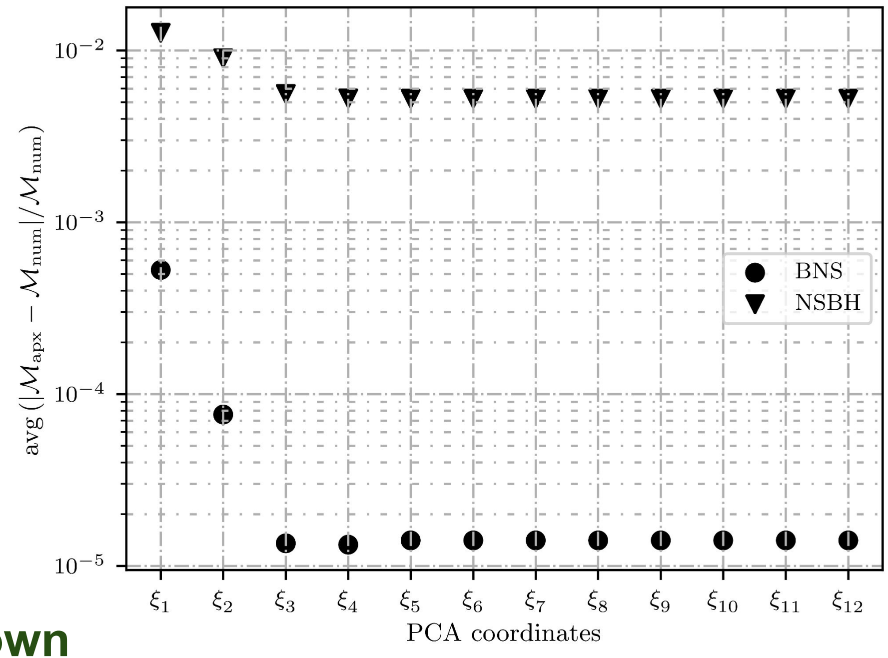
Metric approximation in principal components space

The choice of first three principal coordinates is validated by drawing millions of samples of binary parameters and computing maximum mismatch along each principal direction

$$\mathcal{M}(\vec{\theta}, \vec{\theta} + \Delta\vec{\theta}) := \max_{\phi_0, t_0} 4\mathcal{R} \int df \frac{\tilde{h}^*(f : \vec{\theta})\tilde{h}(f : \vec{\theta} + \Delta\vec{\theta})}{S_n(f)} \equiv 1 - g_{ij}\Delta\theta_i\Delta\theta_j$$

$$\mathcal{M}(\vec{\theta}, \vec{\theta} + \Delta\vec{\theta}) \equiv 1 - \sum_{i=1}^{3} \Delta\xi_i^2$$



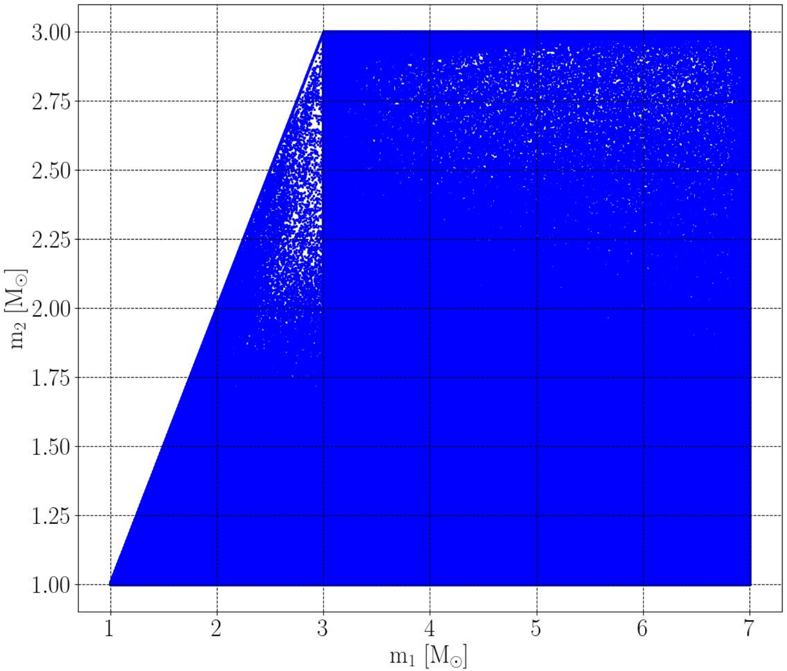


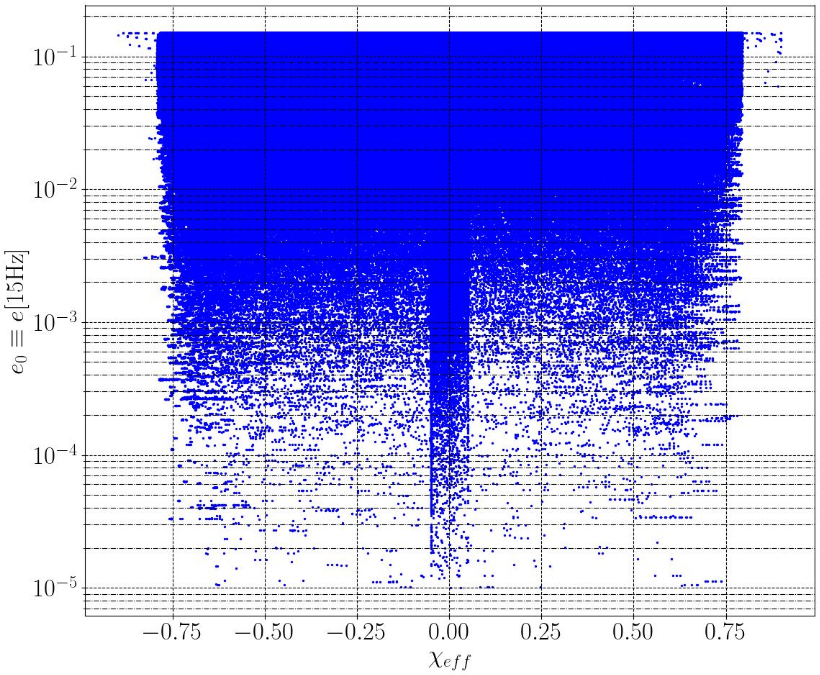
No breakdown

Eccentric template bank

m_1 : [1 - 7] Msun, m_2 : [1 - 3] Msun, χ_{1max} (χ_{2max}):0.9(0.05), e_o: [1E-5 - 0.15] , $f_{low/ecc}$:15Hz

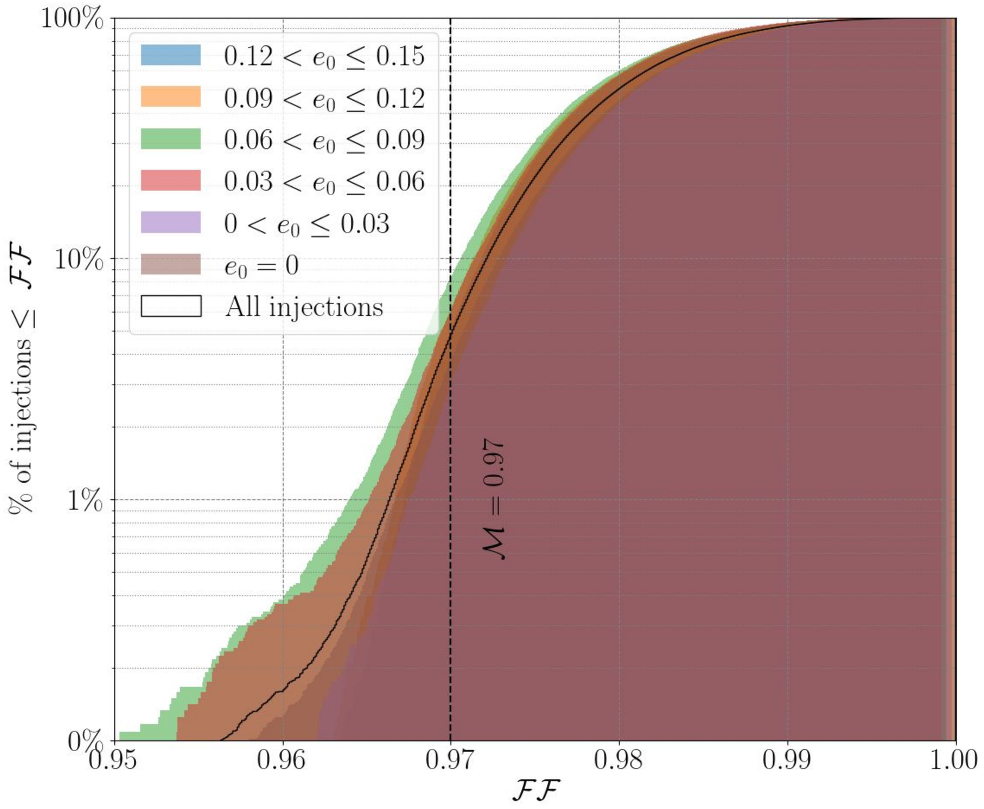
Noise curve: aligo_O4high.txt, Number of templates: 4,781,475





Effectualness of the eccentric template bank

We use the same injections as for the QC bank



Eccentric vs. QC template bank: recovery of eccentric injections

$$\mathcal{FF}_{\text{eff}} = \left(\frac{\sum_{i=1}^{N} \mathcal{FF}_i^3 \sigma_i^3}{\sum_{i=1}^{N} \sigma_i^3} \right)^{1/3}$$

A. Buonanno, Y. Chen, and M. Vallisneri, Phys. Rev. D **67**, 104025, (2003)

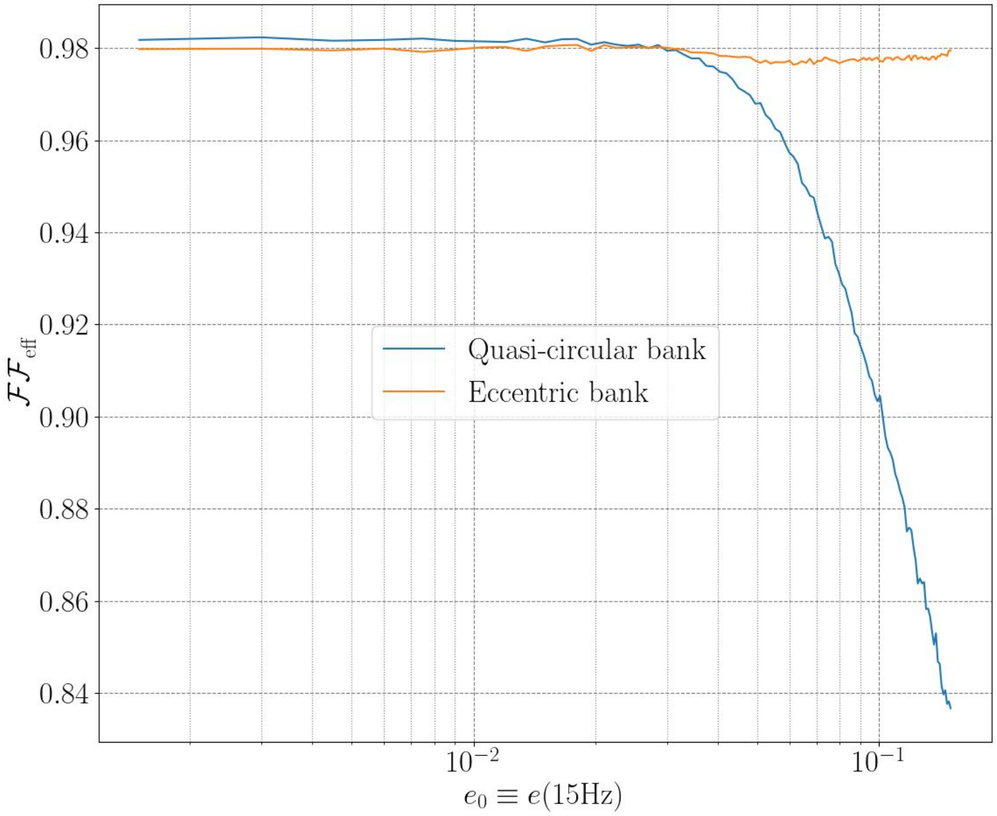
σ: optimal SNR of an injection

$\mathcal{FF}_{\text{eff}}$: computed for injections in 100 eccentricity bins

Eccentricity bin size Δe_o = 0.00149997

	QC bank	Ecc bank
BNS ecc pop	0.90	0.98
NSBH ecc pop	0.93	0.98
Full ecc pop	0.93	0.98

Table for $\mathcal{FF}_{\text{eff}}$



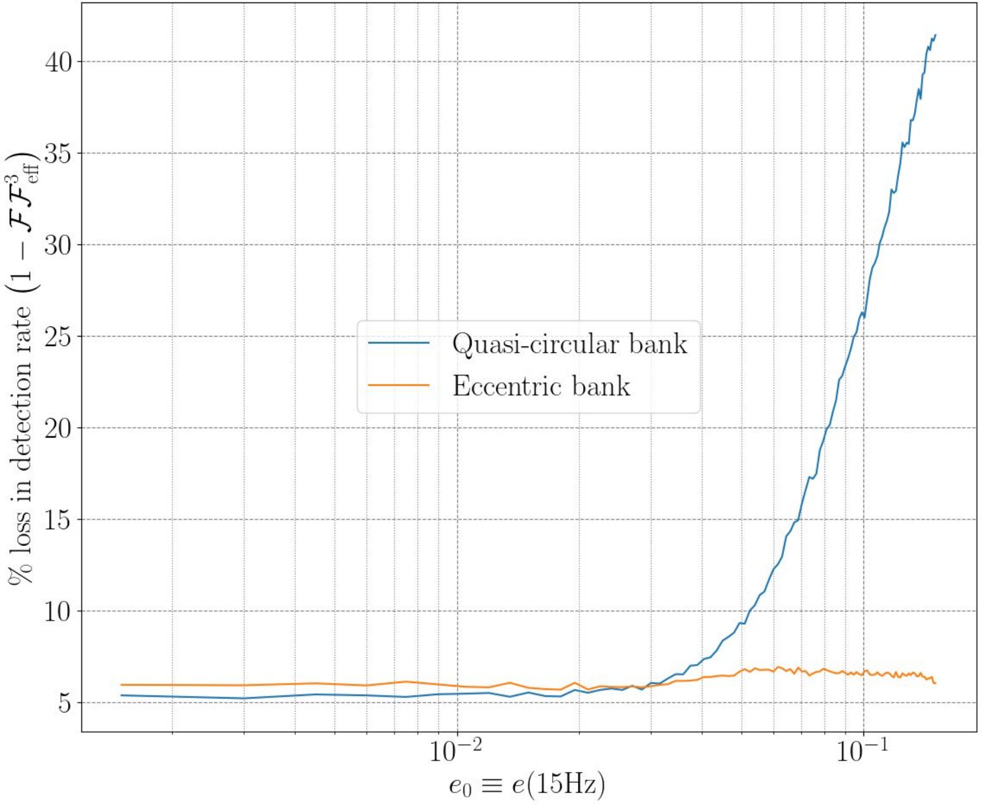
Eccentric vs. QC template bank: recovery of eccentric injections

Detection rate

$$\mathcal{R}_{\rm detect} = \mathcal{FF}^3_{\rm eff}\mathcal{R}_{\rm optimal}$$

Fraction of signal loss:

	QC bank	Ecc bank
BNS ecc pop	0.27	0.06
NSBH ecc pop	0.20	0.06
Full ecc pop	0.20	0.06



Conclusions

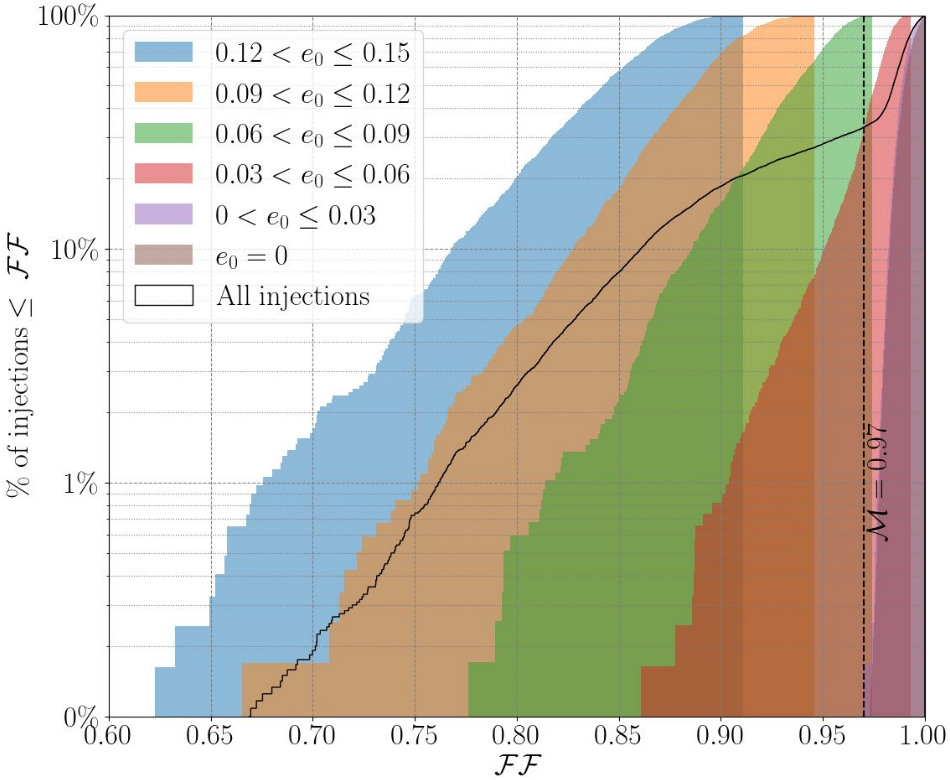
- A geometric 3.5 PN aligned spin, quasi-circular template bank is not effectual for detecting eccentric compact binaries.
- We present TaylorF2Ecc-based metric for geometric aligned spin, eccentric template bank.
- The eccentric bank is developed using TaylorF2Ecc-based metric and 3-D lattice placement algorithm.
- The eccentric bank is ~ 8, 19, 7 times larger than quasi-circular bank in the full, BNS, NSBH parameter space.
- The eccentric bank is highly effectual for BNS, NSBH signals with non-zero or zero eccentricity.
- This bank will enhance discovery potential of new types of binaries in GW observations

Backup Slides

Is the high effectualness due to higher density of templates?

We use the same eccentric injections, but zeroed eccentricities of the eccentric bank

Denser placement cannot compensate for non-inclusion of eccentricity effects



Eccentric template bank