INFN - Sezione di Torino

ECCENTRICITY AND SPIN-PRECESSION

An Effective-One-Body model informed by Post-Newtonian studies

R. Gamba, D. Chiaramello, S. Neogi

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16/09/2024

1st TEONGRAV International Workshop on the
Theory of Gravitational Waves

Motivation

PHYSICAL COMPLETENESS

Gravitational Wave (GW) models have expanded to cover the CBC parameter space, including up to:

- Spin-precession on quasi-circular (QC) orbits
- Non-circular planar orbits (eccentric, hyperbolic-like)

Study of the combination of the two effects limited

SEOBNRE: Liu+ [2310.04552]

GW measurements rely on accurate theoretical modeling to extract real signals from noise

⟶ Unmodeled sources much harder to detect

FORMATION CHANNELS

- Orbital eccentricity and spin-precession are degenerate in their effects in some cases Romero-Shaw+ [2211.07528]
- Models including both effects key to understand real signals
- Relevant for information on binary formation channels

The Effective-One-Body approach

Two-body problem in GR

$$H = H_{\rm N} + \frac{1}{c^2} H_{\rm 1PN} + \dots$$

$$\xrightarrow{\textbf{EOB}}$$

Buonanno, Damour [gr-qc/9811091]

Motion of an effective
particle in an effective
metric

Key features:

- **Hamiltonian**:

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\hat{H}_{\rm eff} - 1\right)}$$

$$\downarrow$$

ν-deformation of Schwarzschild/Kerr

Resummed potentials and spin-orbit couplings

$$\hat{H}_{\rm eff} = \sqrt{p_{r_*}^2 + A\left(1 + \frac{p_\varphi^2}{r_c^2} + Q\right)} + p_\varphi\left(G_S S + G_{S_*} S_*\right)$$

$$\chi_{1,2} = \frac{S_{1,2}}{m_{1,2}^2} \qquad \nu = \frac{\mu}{M} = \frac{m_1 m_2}{(m_1 + m_2)^2} \qquad q = \frac{m_1}{m_2} \geq 1$$

- **Waveform model**: factorization and resummation of each multipole

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)} \hat{h}_{\ell m}^{(\epsilon)} = h_{\ell m}^{(N,\epsilon)} \hat{S}_{\rm eff}^{(\epsilon)} \hat{h}_{\ell m}^{\rm tail} \rho_{\ell m}^{\ell} \hat{h}_{\ell m}^{\rm NQC}$$

- **Radiation reaction**: inherits waveform structure

$$\dot{p}_\varphi = \hat{\mathcal{F}}_\varphi = -\frac{32}{5}\nu r_\omega^4 \Omega^5 \sum_{\ell m} \left|\frac{h_{\ell m}}{h_{22}^{\rm N}}\right|^2$$

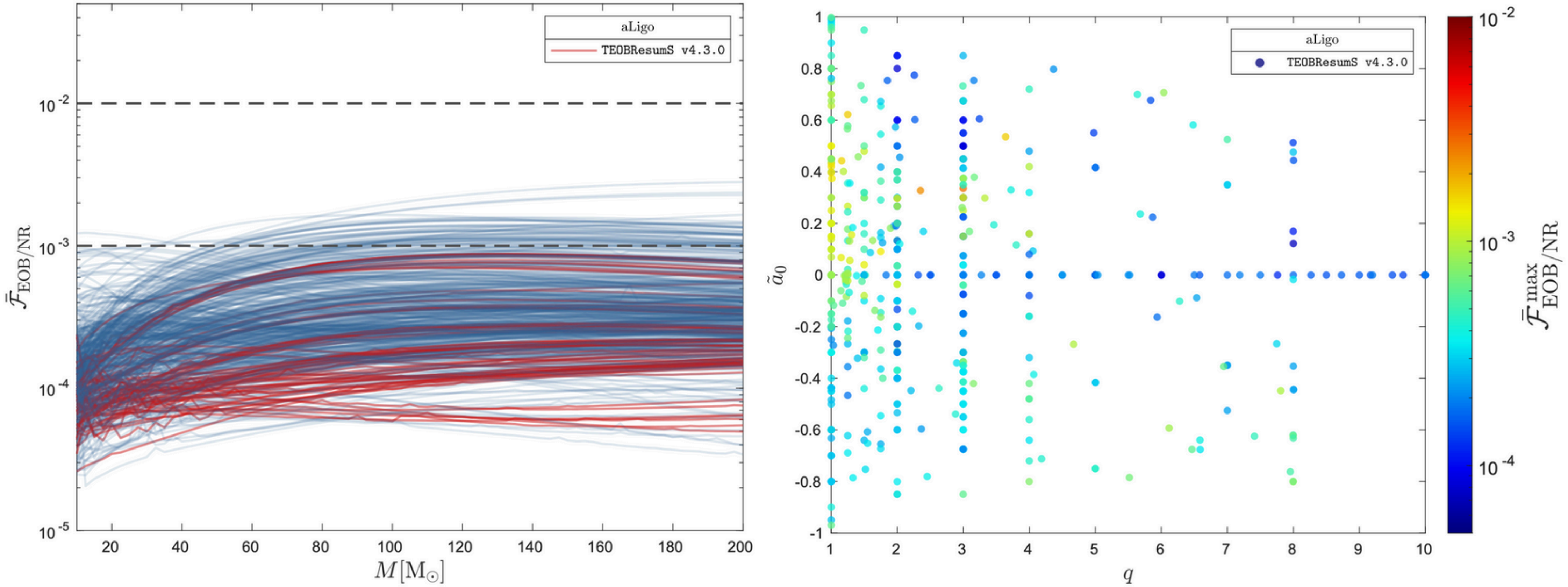
(+ horizon flux)

TEOBResumS

GIOTTO

Nagar+ [2304.09662]
Akcay+ [2005.05338]
Gamba+ [2111.03675]
Riemenschneider+ [2104.07533]
Nagar, Bernuzzi, Del Pozzo+ [1806.01772]
Damour, Nagar [1506.08457]

- Quasi-circular BBH/BNS/BHNS
- Inspiral-merger-ringdown
- Fast waveform generation with Post-Adiabatic (PA) evolution



EOB/NR (SXS) unfaithfulness

Common features

- **NR calibration:** one function each in non-spinning ($a_6^c(\nu)$) and spinning ($c_{30}(\nu, \chi_{1,2})$) sectors determined through time-domain phasing comparisons with QC data
- Accurate phenomenological **ringdown model informed by (QC) NR**
- Thorough validation with **EOB-NR unfaithfulness** and comparison of scattering angles (Dalí)

TEOBResumS

Dalí

Nagar+ [2407.04762]
Nagar+ [2404.05288]
Chiaramello, Nagar [2001.11736]

- Generic-orbit BBH/BNS/BHNS
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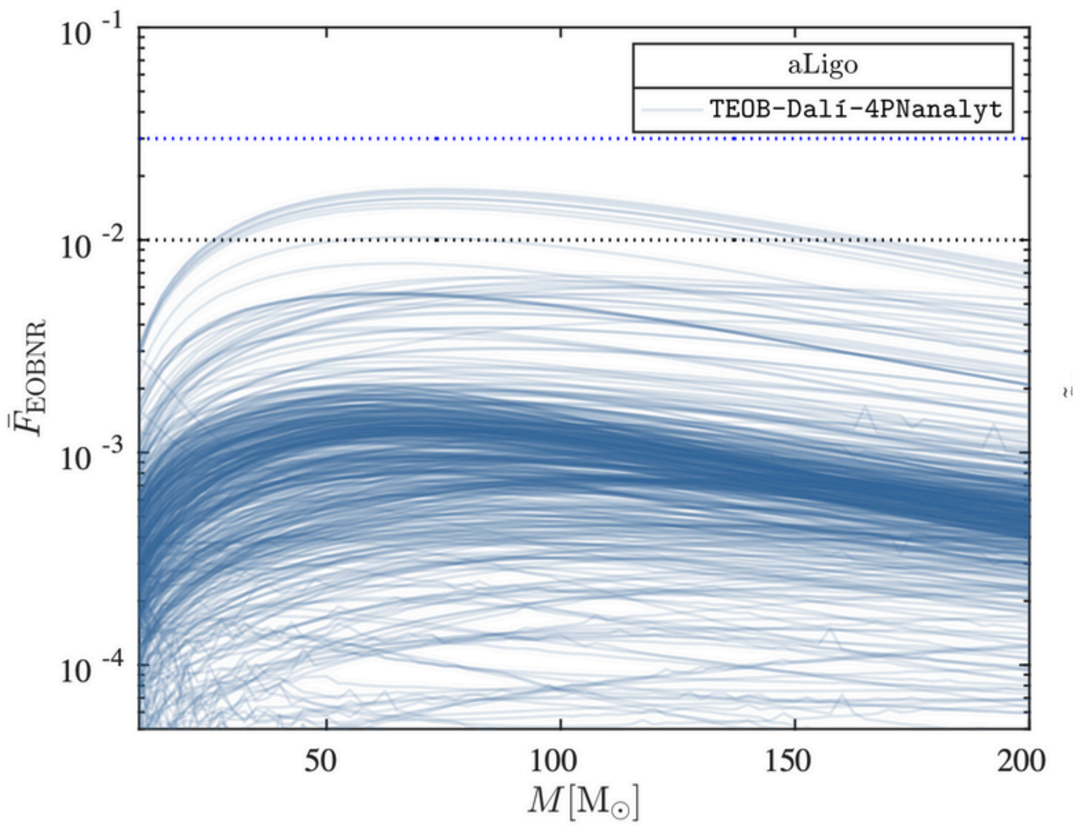
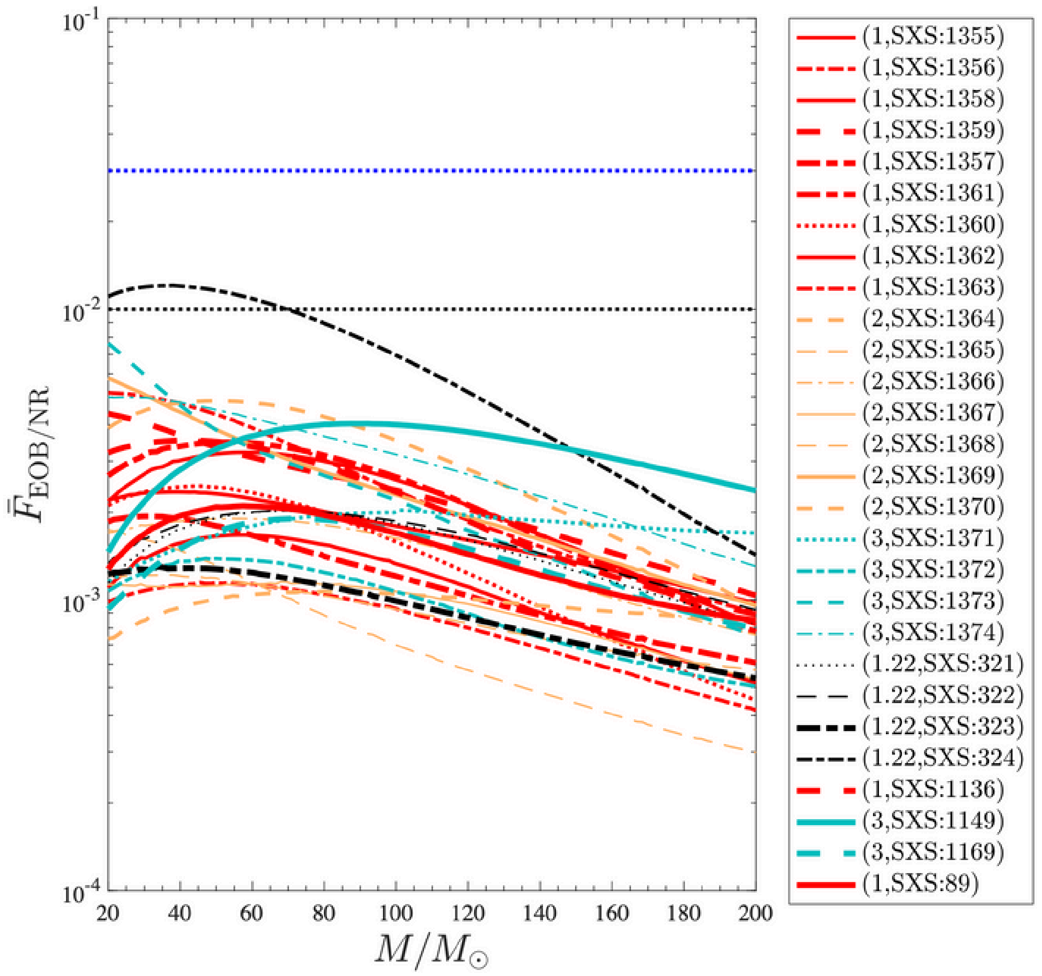


EOB/NR (SXS) unfaithfulness - QC



EOB/NR (SXS) unfaithfulness - ecc

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(Tentative name)

Caravaggio?

Gamba, Chiaramello, Neogi
[2404.15408]

- Dalí + spin-precession

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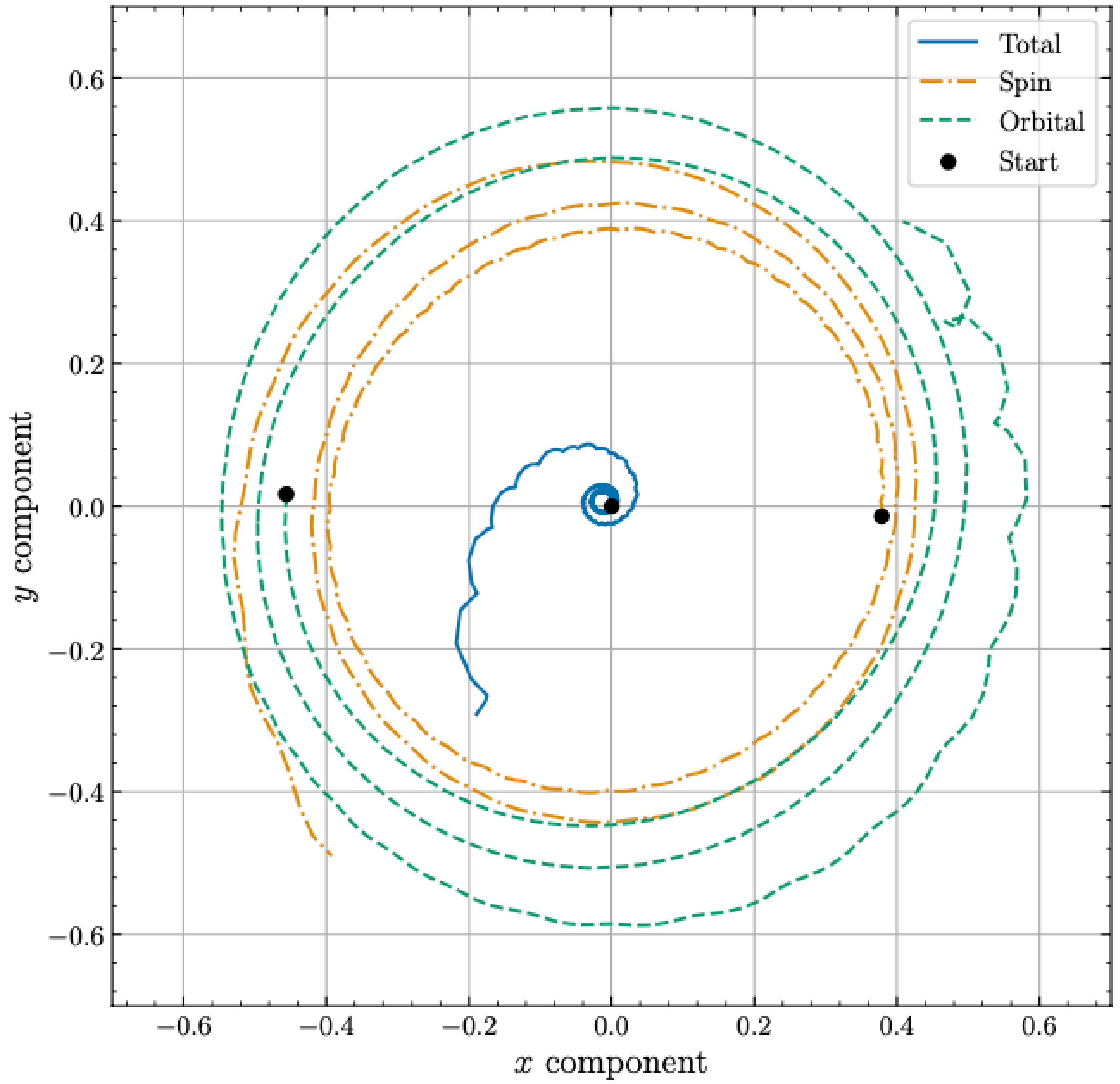
Spin-precession

When the BH spins $\boldsymbol{S_1}$, $\boldsymbol{S_2}$ are **not parallel to the orbital angular momentum $\boldsymbol{L}$**, the three vectors change over time

→

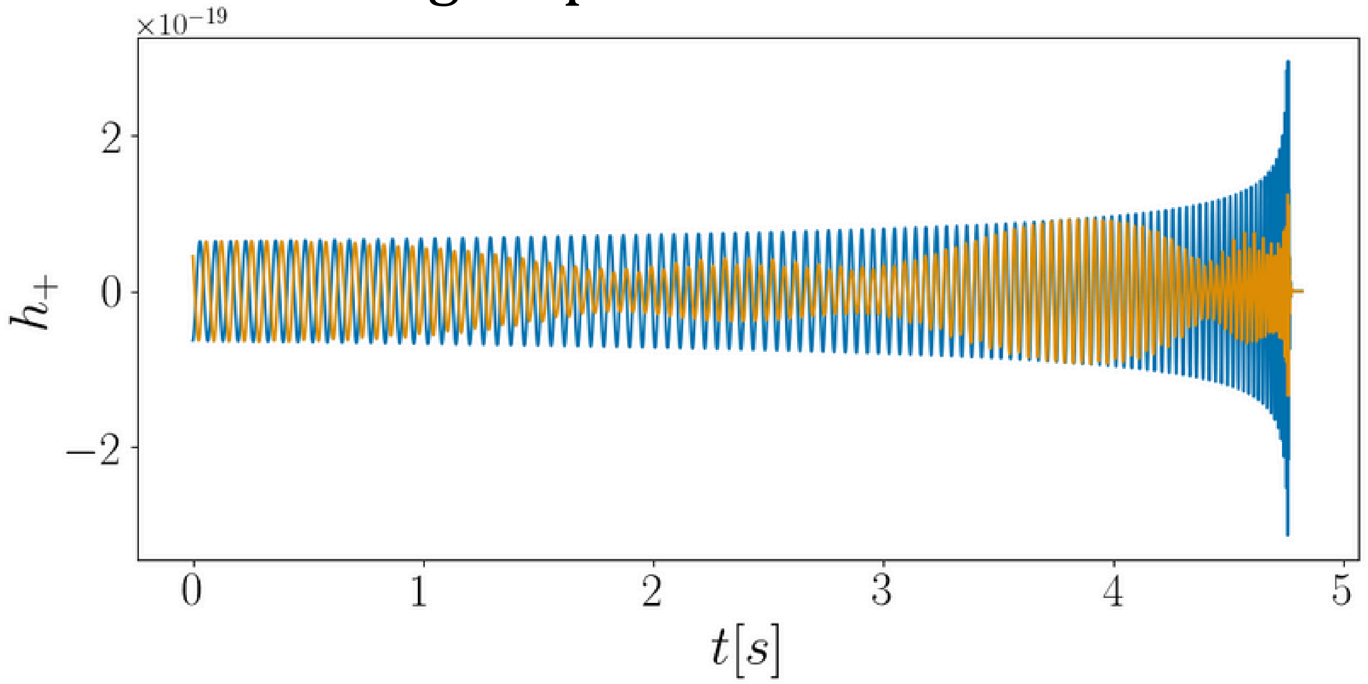
They precess about the total $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S_1} + \boldsymbol{S_2}$ **and the orbital plane shifts**

Typical phenomenology:



$\boldsymbol{J}$ only approximately conserved because of radiation reaction

Inducing amplitude modulations:



Separation of timescales

$$T_{\text{orb}} \sim v^{-3} \ll T_{\text{prec}} \sim v^{-5} \ll T_{\text{RR}} \sim v^{-8}$$

Key parameters:

$$\chi_{\text{eff}} = \frac{m_1 \chi_{1,z} + m_2 \chi_{2,z}}{m_1 + m_2} \qquad \chi_p = \max \left\{ |\boldsymbol{\chi}_{1,\perp}|, \frac{4+3q}{4q^2+3q} |\boldsymbol{\chi}_{2,\perp}| \right\}$$

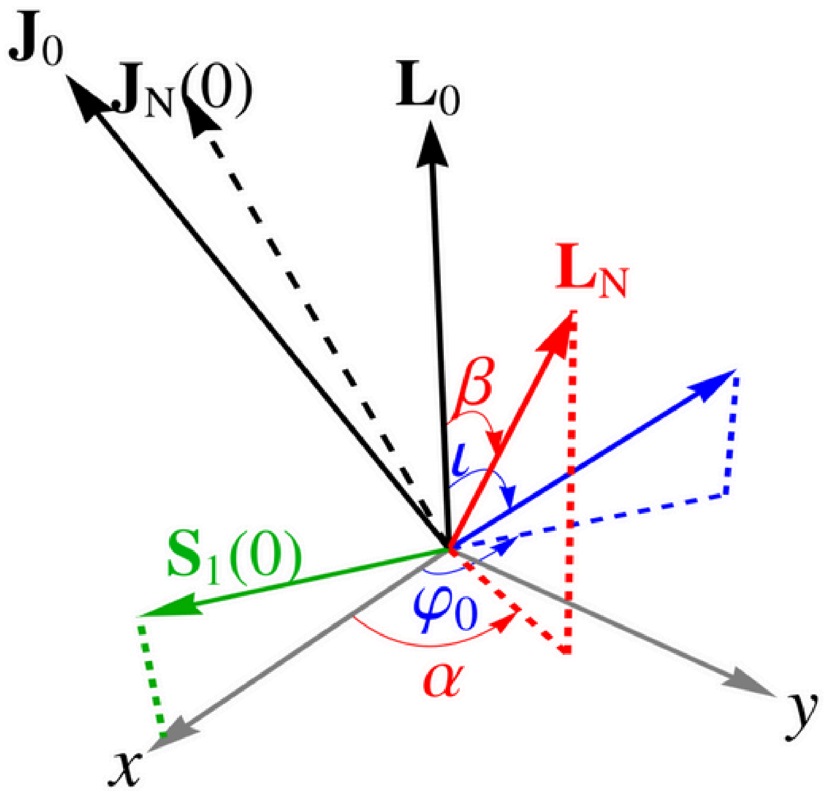
Here’s the twist

In a **co-precessing frame** tied to the orbital angular momentum, the GW signal is very close to that of a spin-aligned system.

Schmidt+ [1012.2879], Buonanno+ [gr-qc/0211087]

“The twist”:

1. Evolve a spin-aligned system and compute the waveform in the co-precessing frame
2. Evolve the spin and orbital angular momentum vectors, finding Euler angles for the rotation to the inertial frame
3. Rotate the waveform



Akcay+ [2005.05338], Gamba+ [2111.03675]:

TEOBResumSP: spin-precessing model for circularized BBHs using the twist

- Spins and **L:** N4LO (2PN) orbit-averaged, QC evolution equations
- 3.5PN TaylorT4-resummed $\dot{\omega}$ in spin evolution equations
- Validated against precessing NR simulations:

$$\bar{F}^{\iota=0,\pi/3}_{\rm median} \lesssim 7\cdot 10^{-3}$$

Post-Newtonian theory

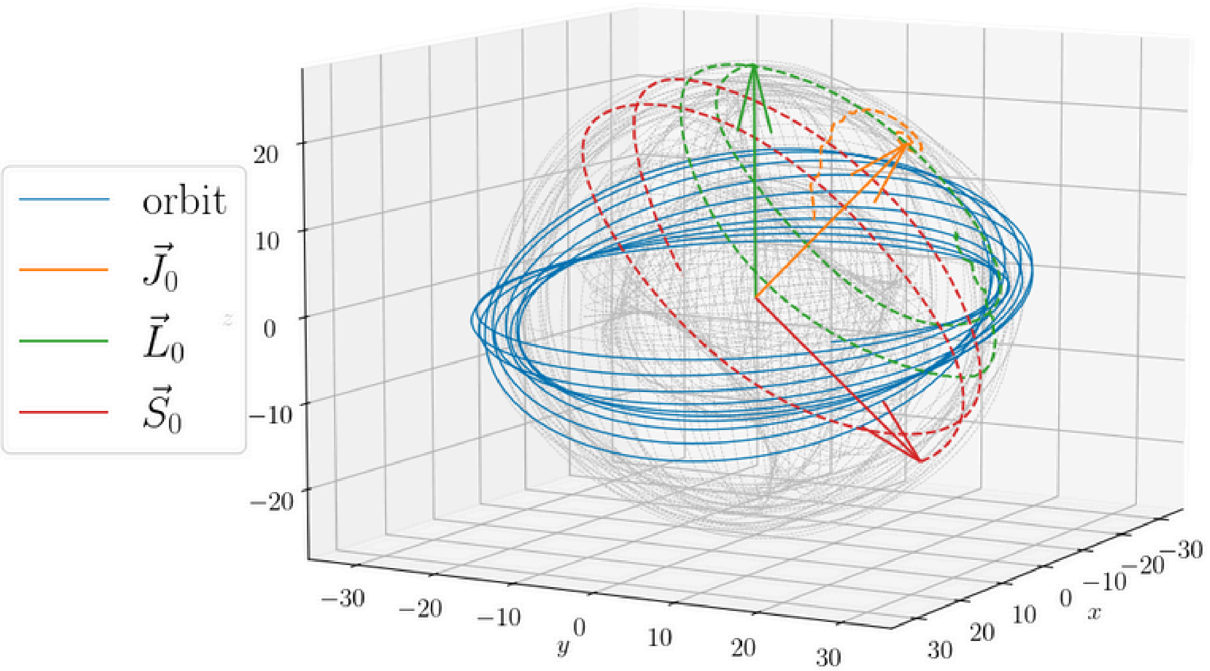
Orbital plane shifts because of a term here ∝ **L**

- 3PN orbital and spin dynamics in modified harmonic coordinates

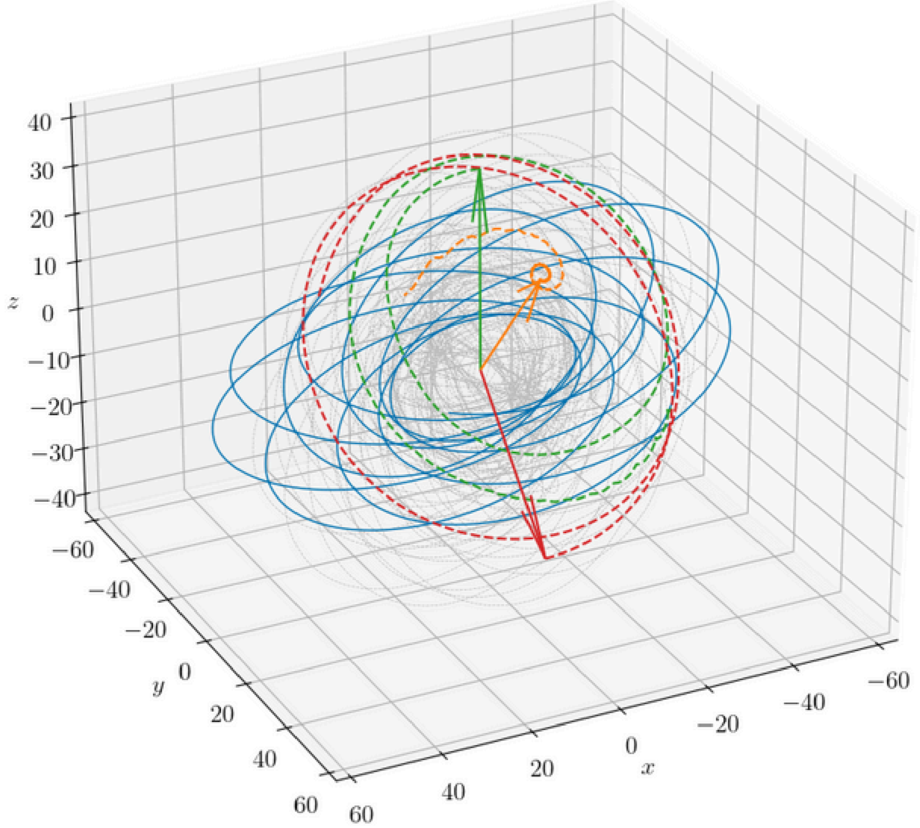
Blanchet [1310.1528], Bohe [1212.5520]

- Integration of bound and unbound (scattering) orbits
- Tracking Euler angles linking **L**(t) and **L**(0)

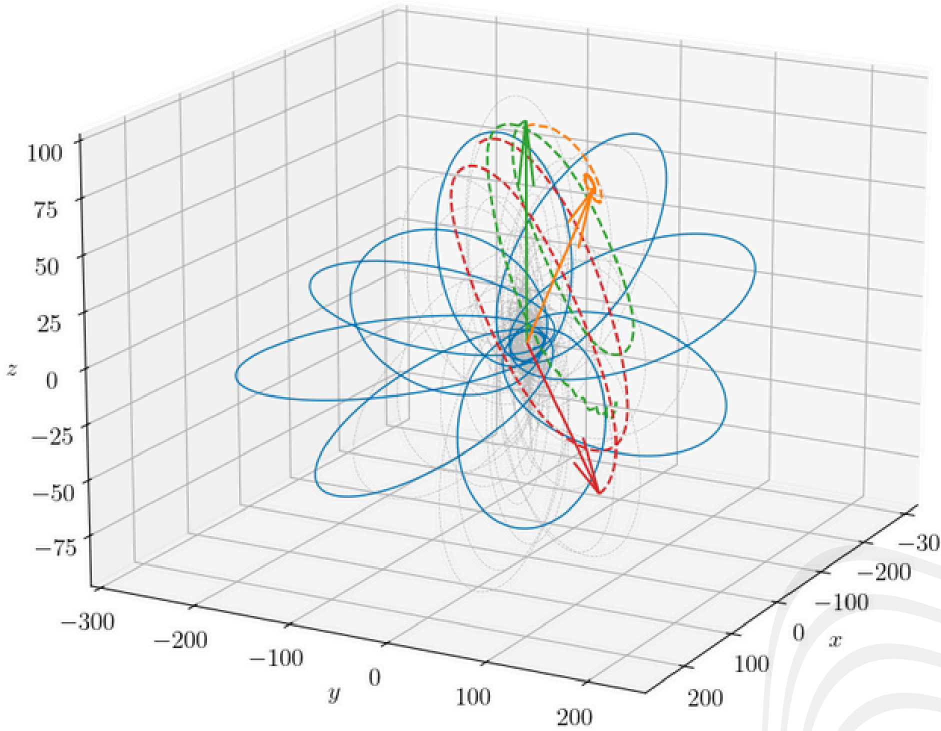
$$
\begin{cases}
\dfrac{\mathrm{d}\mathbf{r}}{\mathrm{dt}} &= \mathbf{v} \\
\dfrac{\mathrm{d}\mathbf{v}}{\mathrm{dt}} &= -\dfrac{Gm}{r^2}\Big[\left(1+\mathcal{A}\right)\mathbf{n}+\mathcal{B}\,\mathbf{v}\Big]+\dfrac{\mathrm{d}\mathbf{v}_{\mathrm{S}}}{\mathrm{dt}}+\mathcal{O}\left(\frac{1}{c^7}\right) \\
\dfrac{\mathrm{d}\mathbf{S}}{\mathrm{dt}} &= \left(X_1\boldsymbol{\Omega}_1+X_2\boldsymbol{\Omega}_2\right)\times\mathbf{S}+\nu\left(\boldsymbol{\Omega}_2-\boldsymbol{\Omega}_1\right)\times\boldsymbol{\Sigma} \\
\dfrac{\mathrm{d}\boldsymbol{\Sigma}}{\mathrm{dt}} &= \left(X_2\boldsymbol{\Omega}_1+X_1\boldsymbol{\Omega}_2\right)\times\boldsymbol{\Sigma}+\left(\boldsymbol{\Omega}_2-\boldsymbol{\Omega}_1\right)\times\mathbf{S} \\
\boldsymbol{\Omega}_i &= \ell\left[\frac{1}{c^2}\alpha^{(i)}_{1\mathrm{PN}}+\frac{1}{c^4}\alpha^{(i)}_{2\mathrm{PN}}+\frac{1}{c^6}\alpha^{(i)}_{3\mathrm{PN}}+\mathcal{O}\left(\frac{1}{c^7}\right)\right]
\end{cases}
$$



e = 0.1

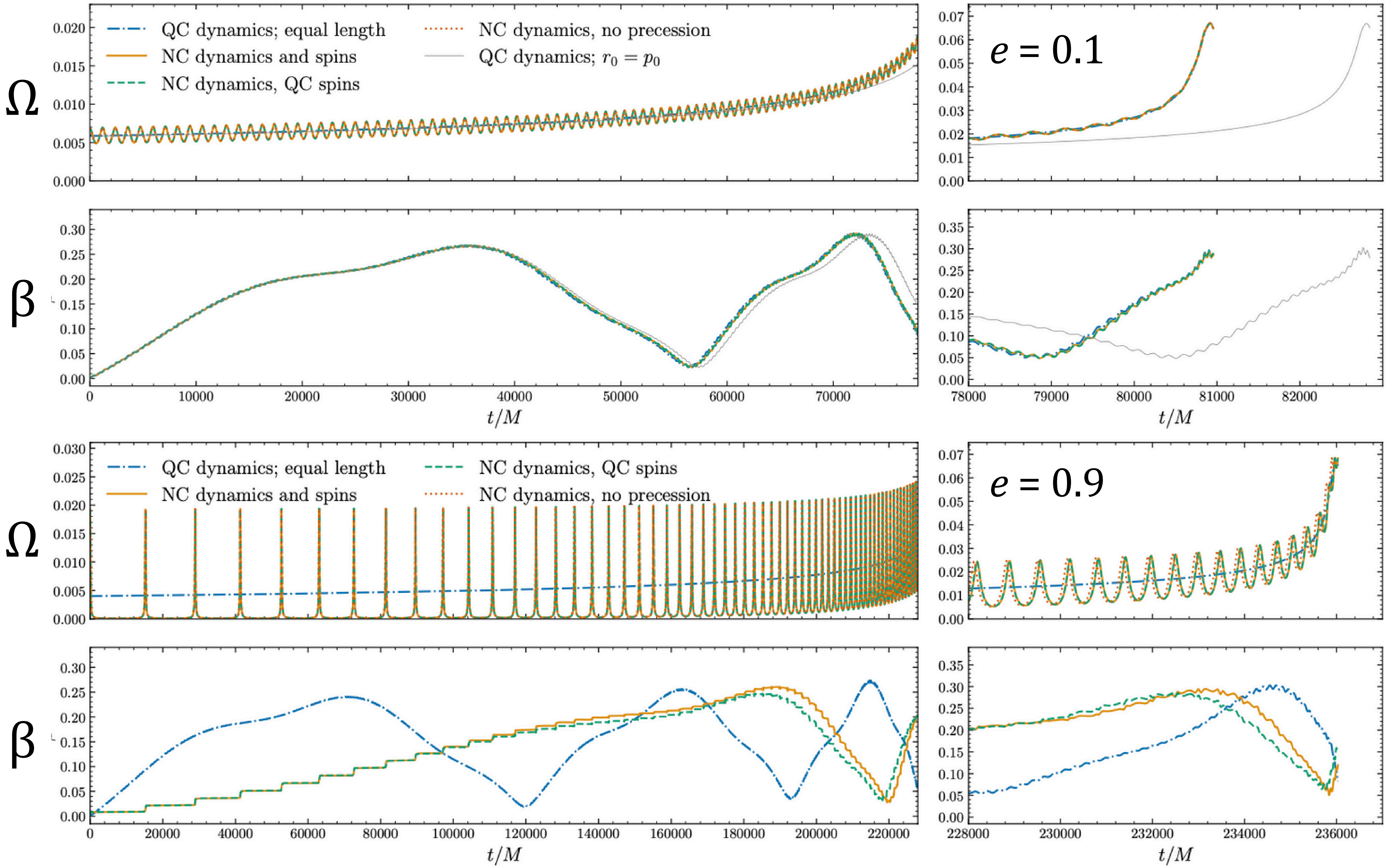


e = 0.5



e = 0.9

PN precessing dynamics



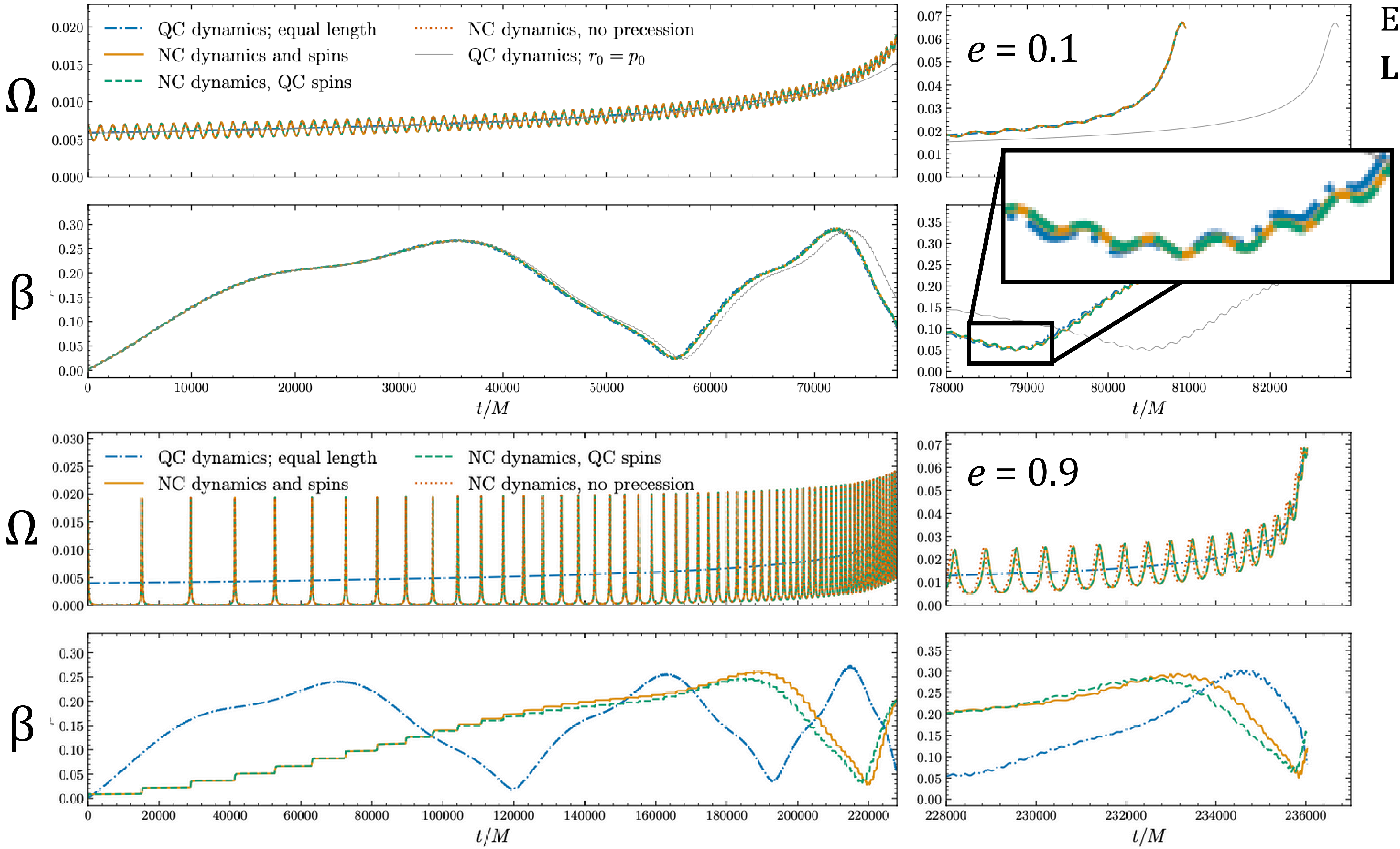
$$q = 1, \quad \boldsymbol{\chi_1} = (0, 0.2, 0.1), \quad \boldsymbol{\chi_2} = (0.4, 0, 0.2)$$

Evolution of β (= angle between **L**(t) and **L**(0)):

1. **Full NC dynamics**
2. **QC spin evolution**
3. **Non-precessing**
4. **QC orbital dynamics**

- **NC terms in spin evolution have small effect outside high eccentricity**
- Precession doesn't strongly impact orbital dynamics (length of orbit, orbital frequency)

PN precessing dynamics



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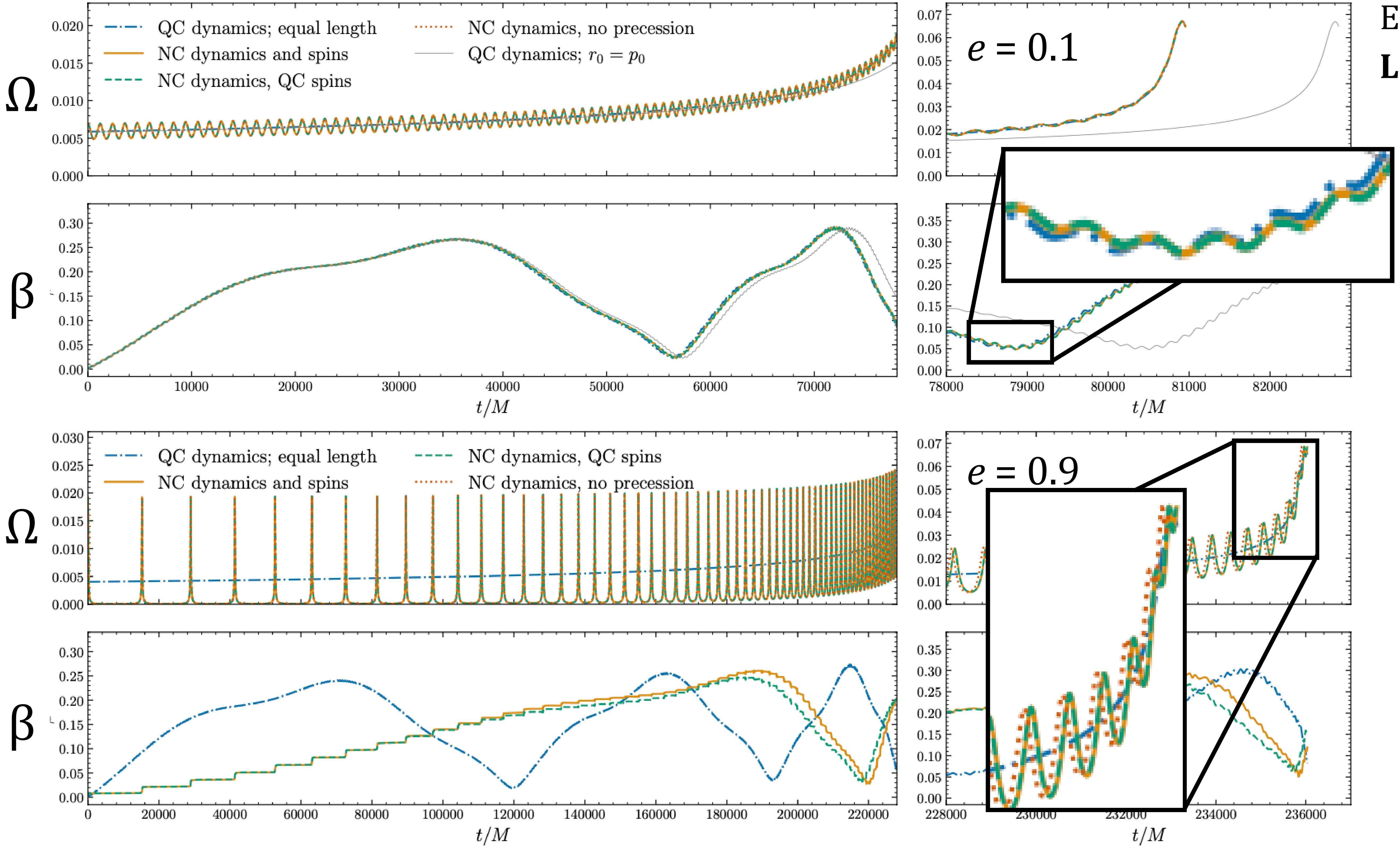

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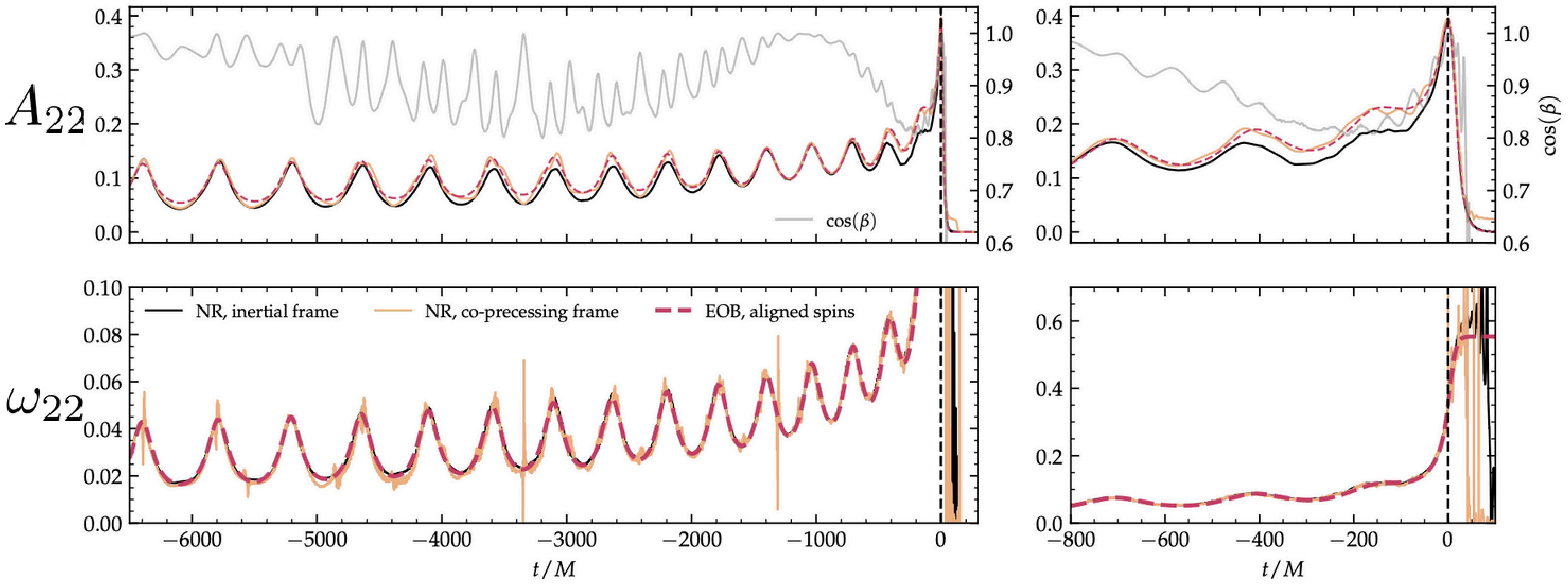
Numerical relativity

RIT:eBBH:1632

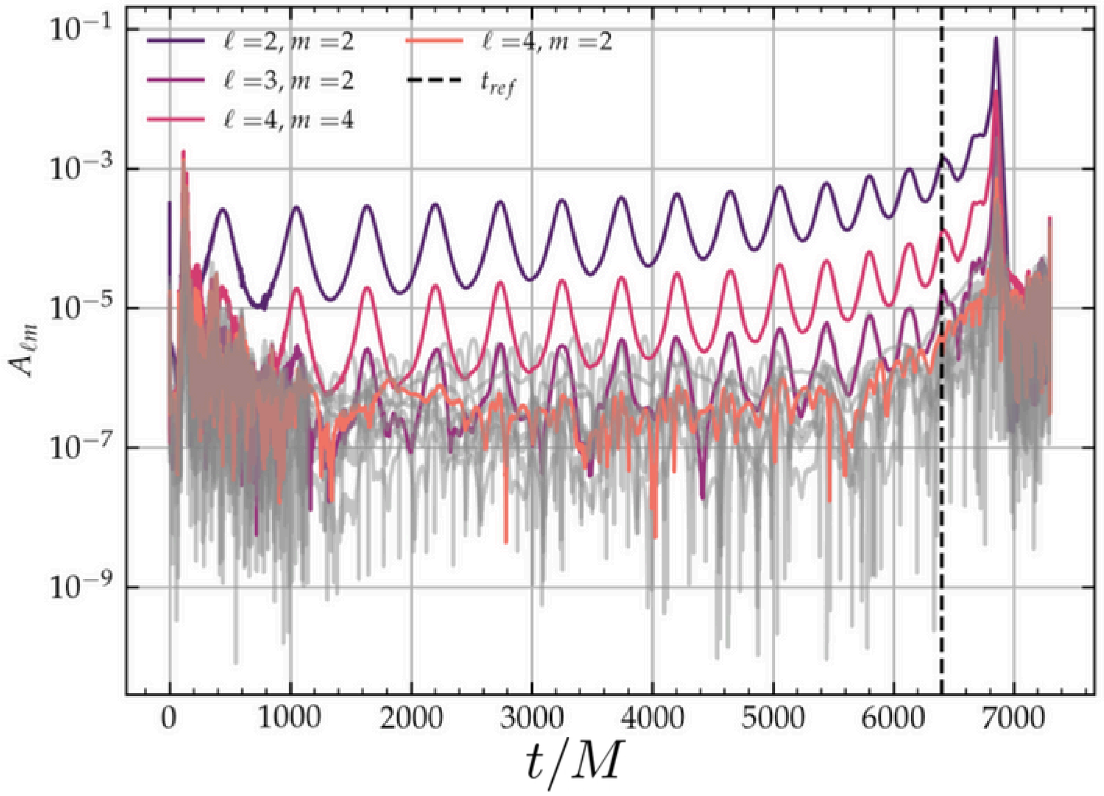
Healy+ [2202.00018]

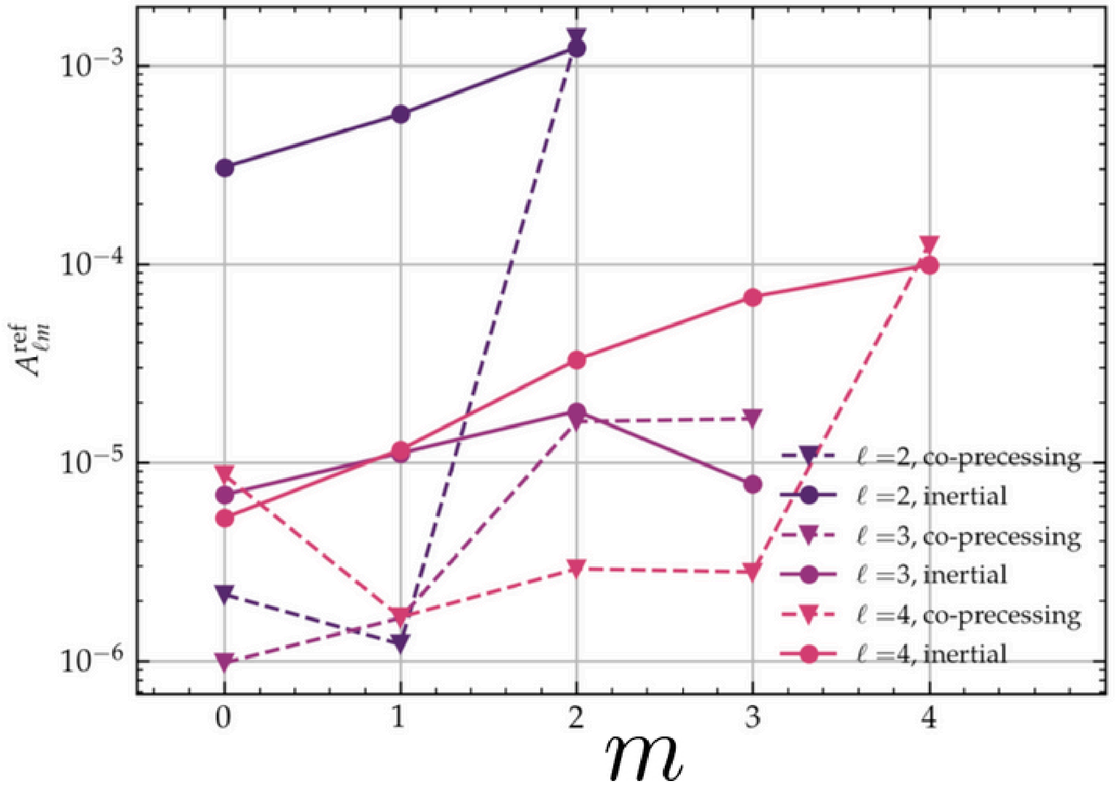


Does the twist work for non-circular systems?

- Co-precessing frame can be identified from wave multipoles
- Rotate NR waveform into co-precessing frame and compare with aligned-spins TEOBResumS

Amplitude and frequency modulations match!





- Compare relative strength of (sufficiently resolved) mulipoles in both frames

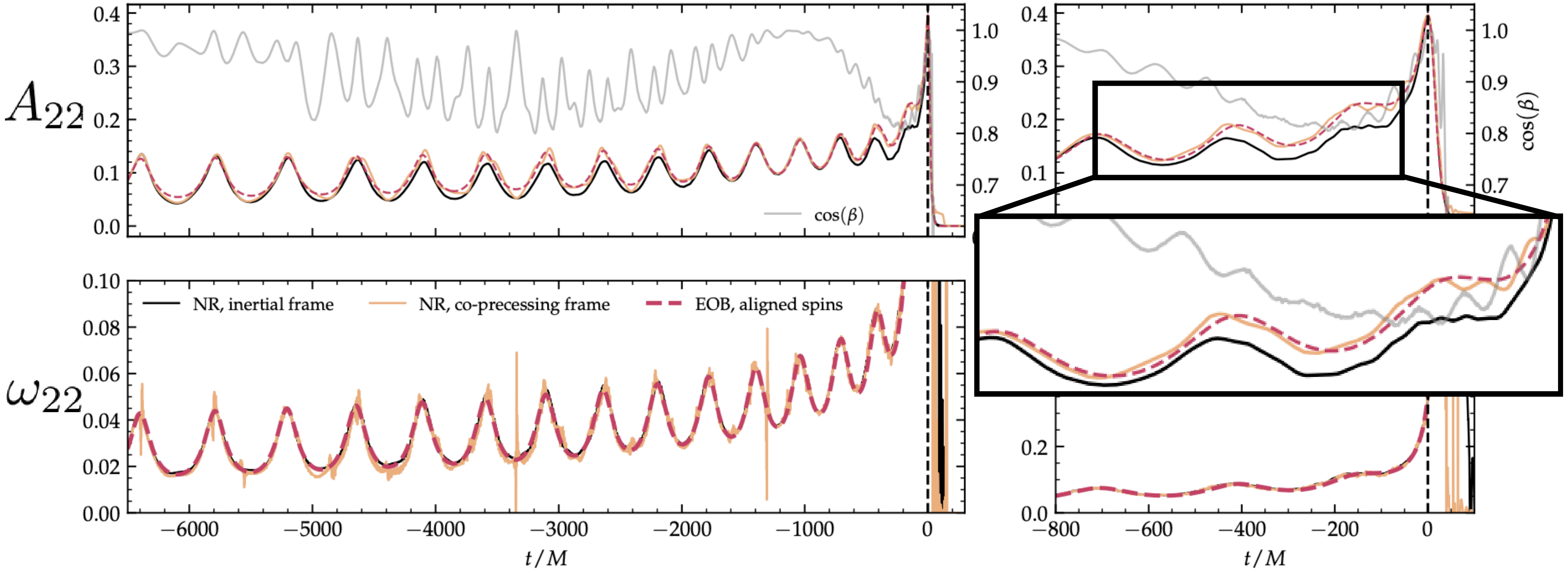
Odd-m multipoles weaker in co-precessing frame

↓

Evidence that twist technique is applicable

Numerical relativity

RIT:eBBH:1632



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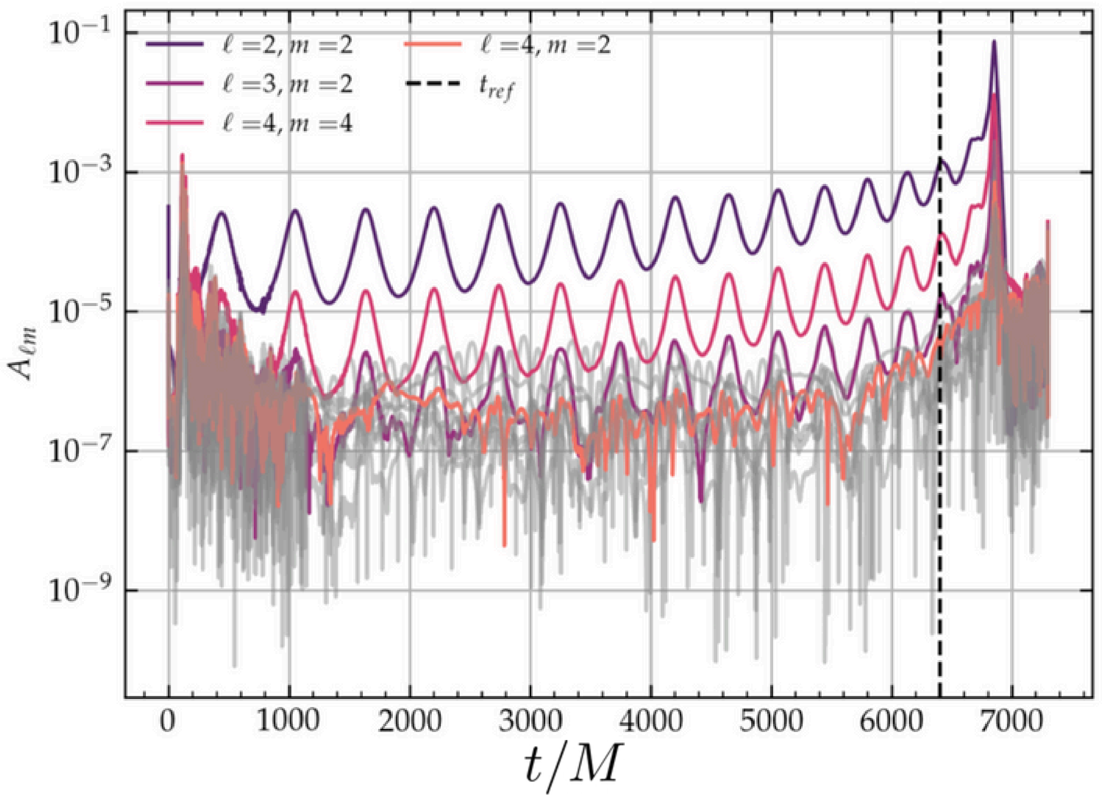

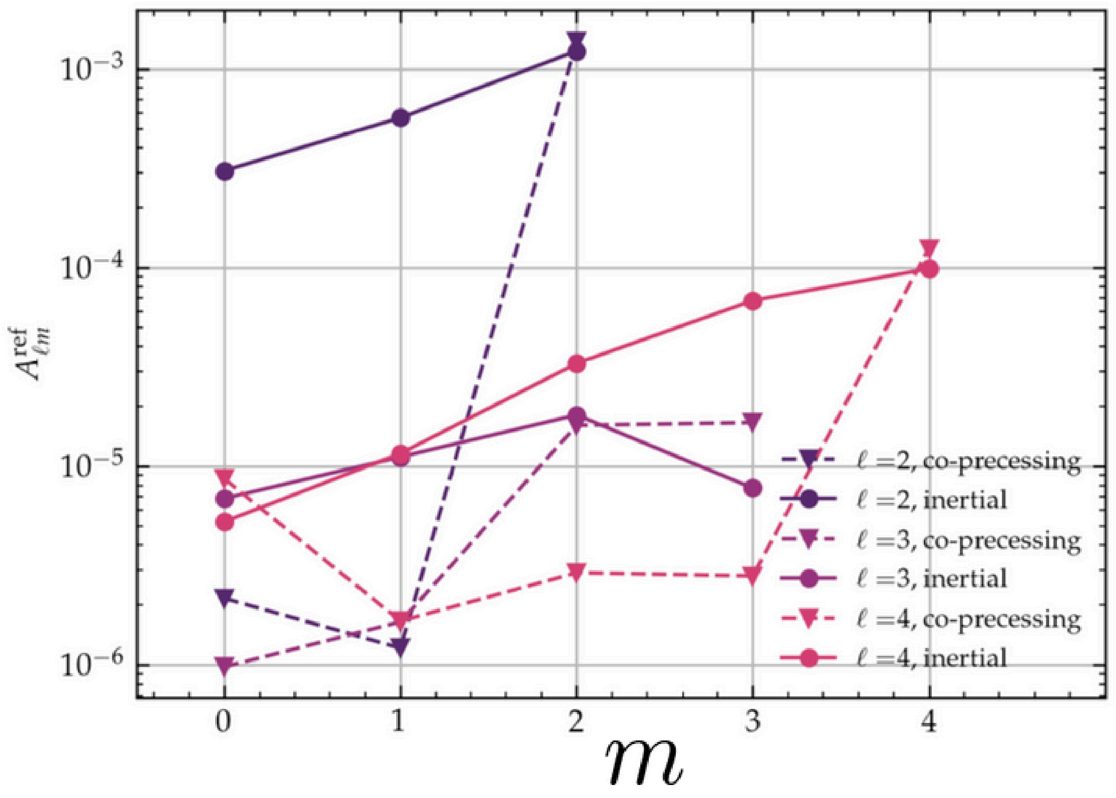

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↓

Evidence that twist technique is applicable

Eccentricity and spin-precession in TEOBResumS

Simple prescription:

- Solve aligned-spin non-circular orbit (bound or unbound) ⟶ **$\Omega(t)$, co-precessing waveform**
- Use Ω to evolve spins, orbital angular momentum with 2PN orbit-averaged **QC** equations up to merger
- Compute Euler angles from $\mathbf{L}(t)$, set them to constant values after merger
- Twist co-precessing waveform into inertial frame

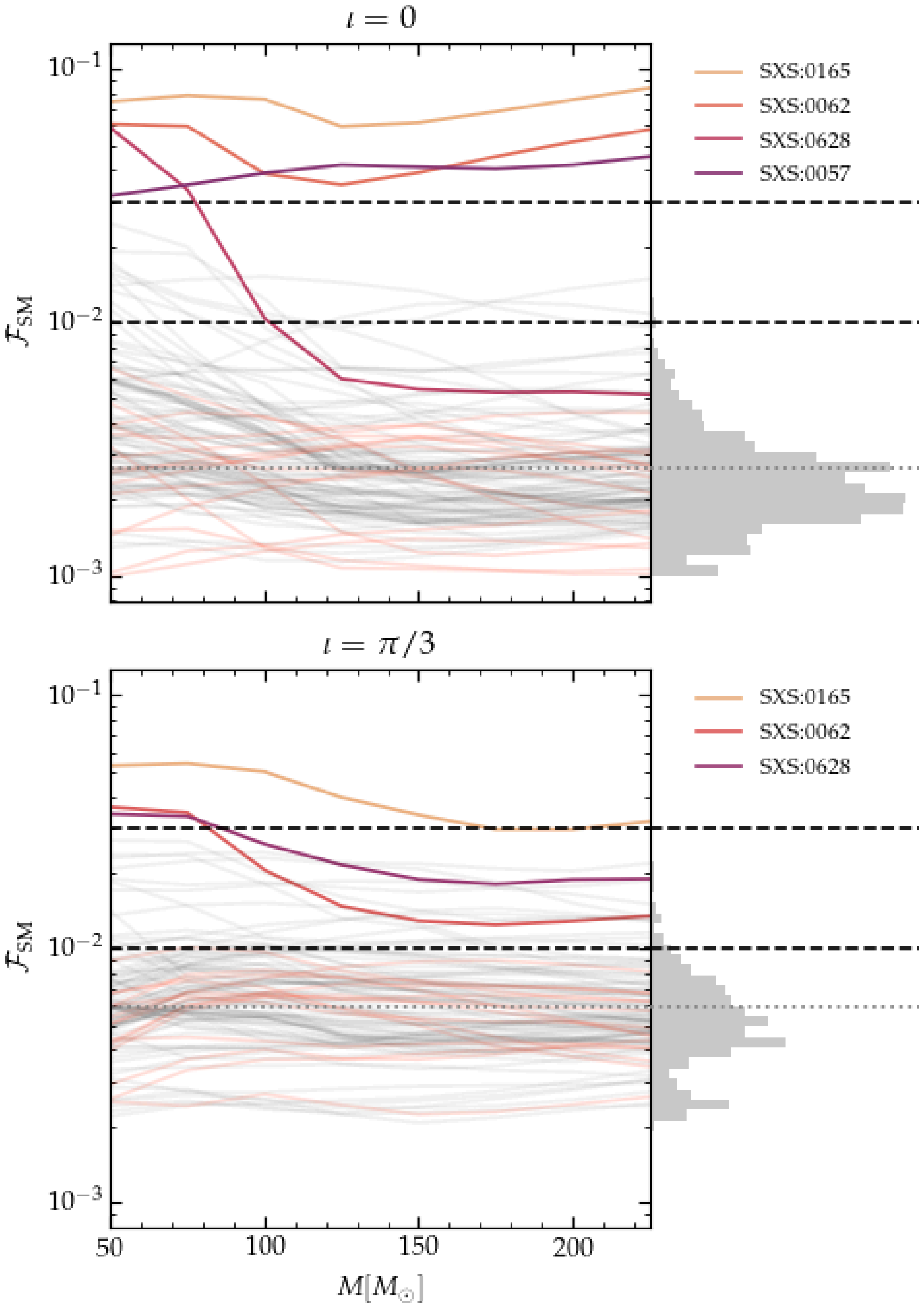
Pros:

- Straightforward in idea and practice
- Naturally applicable to any orbit geometry
- Supported by PN finding that NC terms in spin equations can be neglected

Cons:

- No varying spins in orbital dynamics
- NC terms in spin equations can't be ignored on long, very eccentric orbits

Validation: quasi-spherical limit



Noise-weighted inner product

EOB-NR unfaithfulness: $\bar{\mathcal{F}}_{\rm SM} = 1 - \max_{t_0^h,\varphi_0^h,\kappa^h,\xi_0} \dfrac{(h^{\rm NR}, h^{\rm EOB})}{\sqrt{(h^{\rm NR}, h^{\rm NR})(h^{\rm EOB}, h^{\rm EOB})}}$

- 99 **lvcnr** simulations with $q \leq 6, \chi_p \leq 0.89, \chi_{\rm eff} \in [-0.45, 0.65]$
- 21 “long” **SXS** simulations with $q \leq 4, \chi_p \leq 0.49, \chi_{\rm eff} \in [-0.2, 0.3]$
- Modes: $(\ell, |m|) = (2,2), (2,1), (3,3), (4,4)$

Performance comparable with other state-of-the-art models:

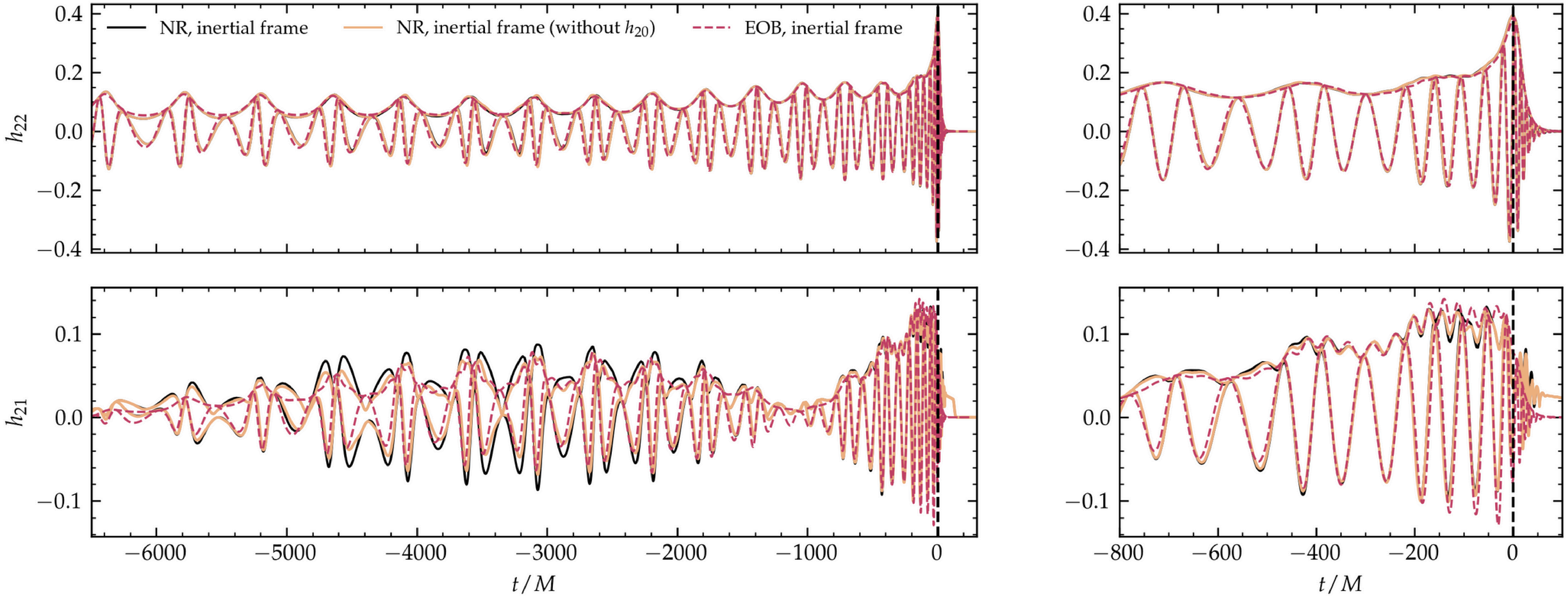
- Mismatch below 3% for all but three (high q, strongly precessing) data sets
- Median unfaithfulness:

$$\iota = 0 \rightarrow 0.003^{+0.009}_{-0.001} \qquad \iota = \pi/3 \rightarrow 0.006^{+0.010}_{-0.003}$$

Compare with QC model: $\bar{F}^{\iota=0,\pi/3}_{\rm median} \lesssim 7 \cdot 10^{-3}$

Eccentric, precessing NR comparison

RIT:eBBH:1632 $q = 1$ $\chi_1 = (-0.7, 0, 0)$ $\chi_2 = (-0.7, 0, 0)$ $e = 0.28$



- Precession does not affect (2, 2) mode greatly; good agreement in amplitude and phase modulation
- (2, 1) mode reproduced less accurately; amplitude envelope and phase overall captured
- Unusual features in NR waveform prevent more quantitative evaluation of model performance

Conclusions

- Important step towards physical completeness in BBH models
- PN, NR studies points to simplifications:
  - ⟶ Explicit NC terms in spin evolution equations mostly negligible
  - ⟶ Time-varying spins largely unimportant in orbital dynamics
  - ⟶ Twist viable for NC systems
- Implementation in TEOBResumS: 2PN QC orbit-averaged spin equations drawing Ω from EOB
- Performance equivalent to existing models in quasi-spherical limit (NR mismatches)

Future prospects:

- Thorough NR-validation with upcoming new simulations
- Inclusion of (2, 0) mode
- Non-circular, precessing post-merger model
- Non-circular horizon flux model

Chiaramello, Gamba [2408.15322]

Backup slides

NR calibration in TEOBResumS

A potential in the Hamiltonian: $A_{5\mathrm{PN}} = 1 - 2u + 2\nu u^3 + \nu a_4 u^4 + \nu \left(a_5^c + a_5^{\log} \ln u \right) u^5 + \nu \left(\boldsymbol{a_6^c} + a_6^{\log} \ln u \right) u^6$

Spin-orbit coupling: $G_S = 2uu_c^2 \left(1 + c_{10} u_c + c_{20} u_c^2 + \boldsymbol{c_{30}} u_c^3 + c_{02} p_{r_*}^2 + c_{12} u_c p_{r_*}^2 + c_{04} p_{r_*}^4 \right)^{-1}$

$$u^{-1} = r; \qquad u_c^{-1} = r_c = \text{centrifugal radius} = \sqrt{r^2 + a^2 + O(r^{-1})}$$

$a_6^c(\nu)$ and $c_{30}(\nu, \chi_{1,2})$ determined by calibrating to quasi-circular NR simulations:

1. Align waveforms in time-domain
2. Adjust coefficients for flat phase difference during inspiral and monotonic dephasing during merger
3. Fit against ν, spins

$a_6^c(\nu) \longrightarrow$ 4 non-spinning SXS datasets

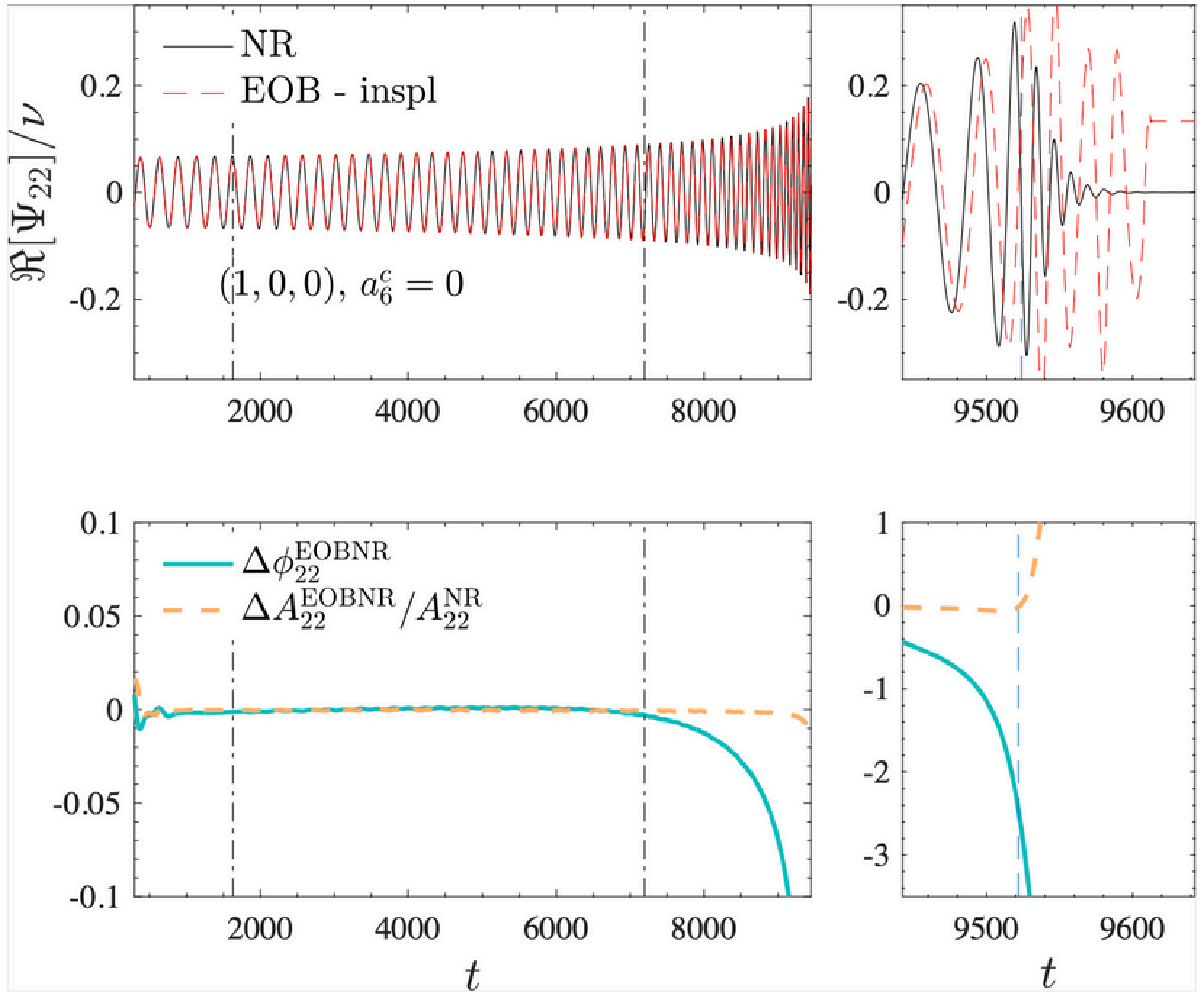
$c_{30}(\nu, \chi_{1,2}) \longrightarrow$ 59 spinning SXS datasets

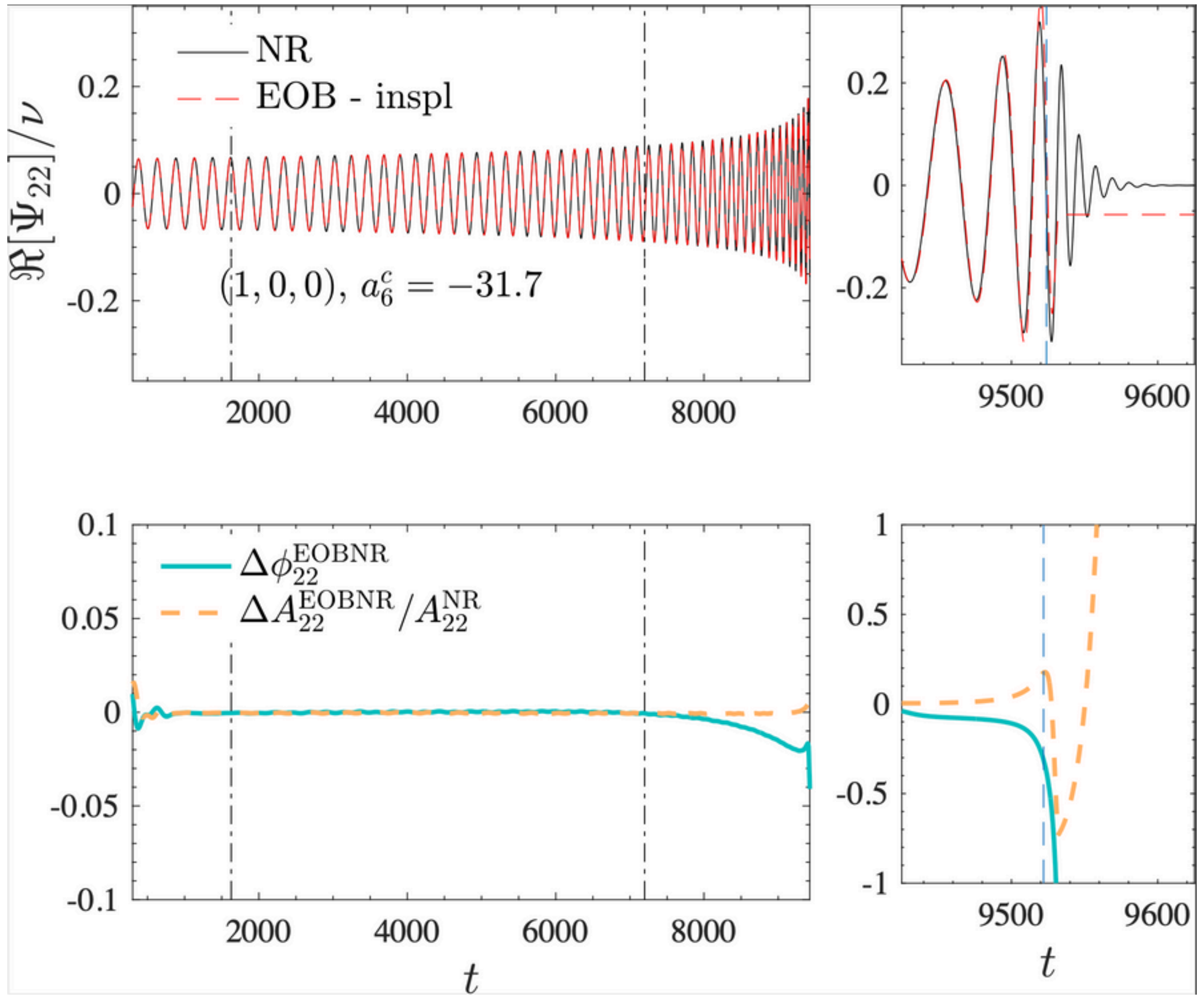
Next-to-Quasi-Circular (NQC) corrections: $\hat{h}_{\ell m}^{\mathrm{NQC}} = \left(1 + a_1^{\ell m} n_1^{\ell m} + a_2^{\ell m} n_2^{\ell m} \right) e^{\mathrm{i}\left(b_1^{\ell m} n_3^{\ell m} + b_2^{\ell m} n_4^{\ell m} \right)}$

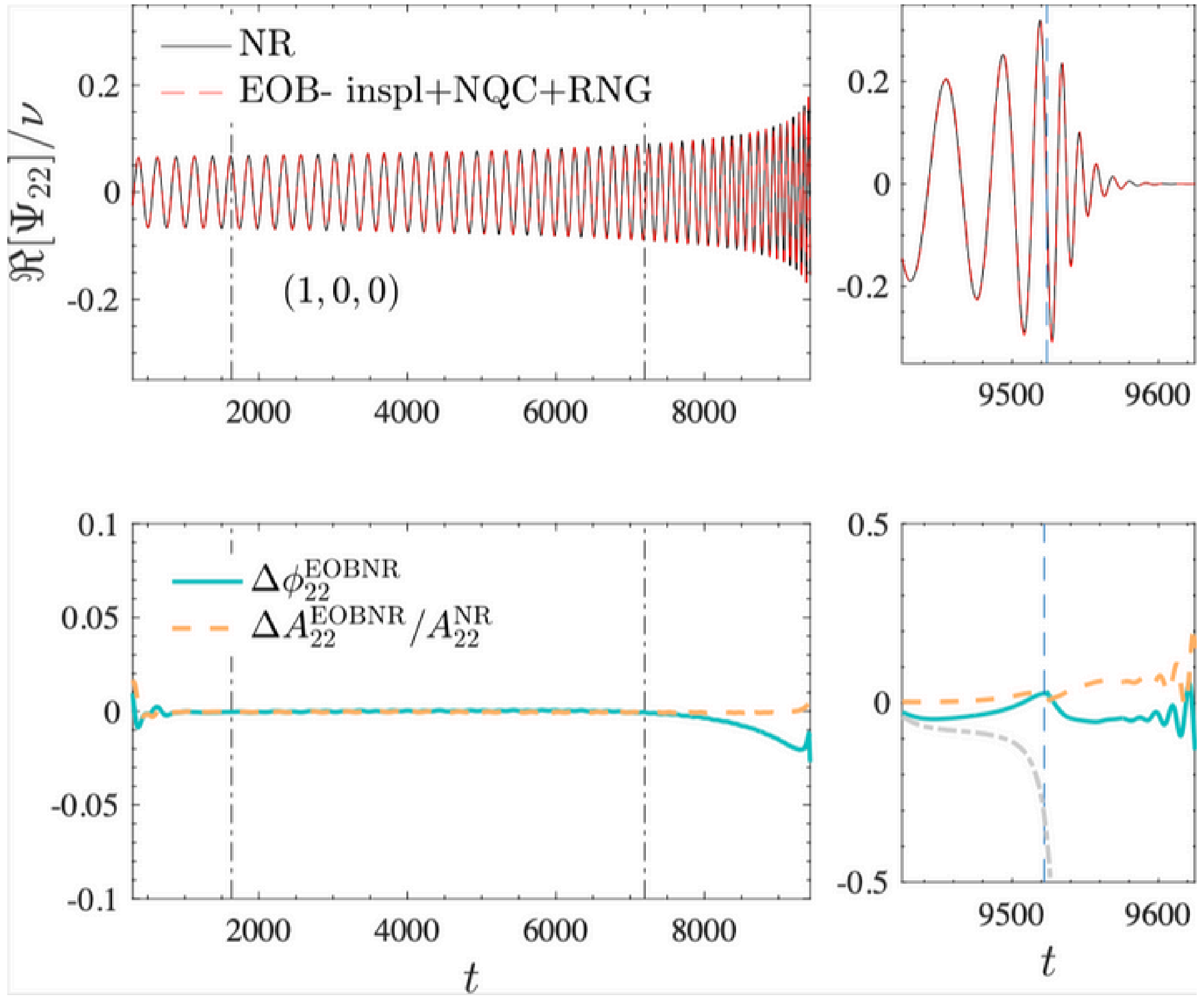
Correcting factor to each multipole explicitly depending on radial momentum and acceleration through the functions $n_k^{\ell m}$.

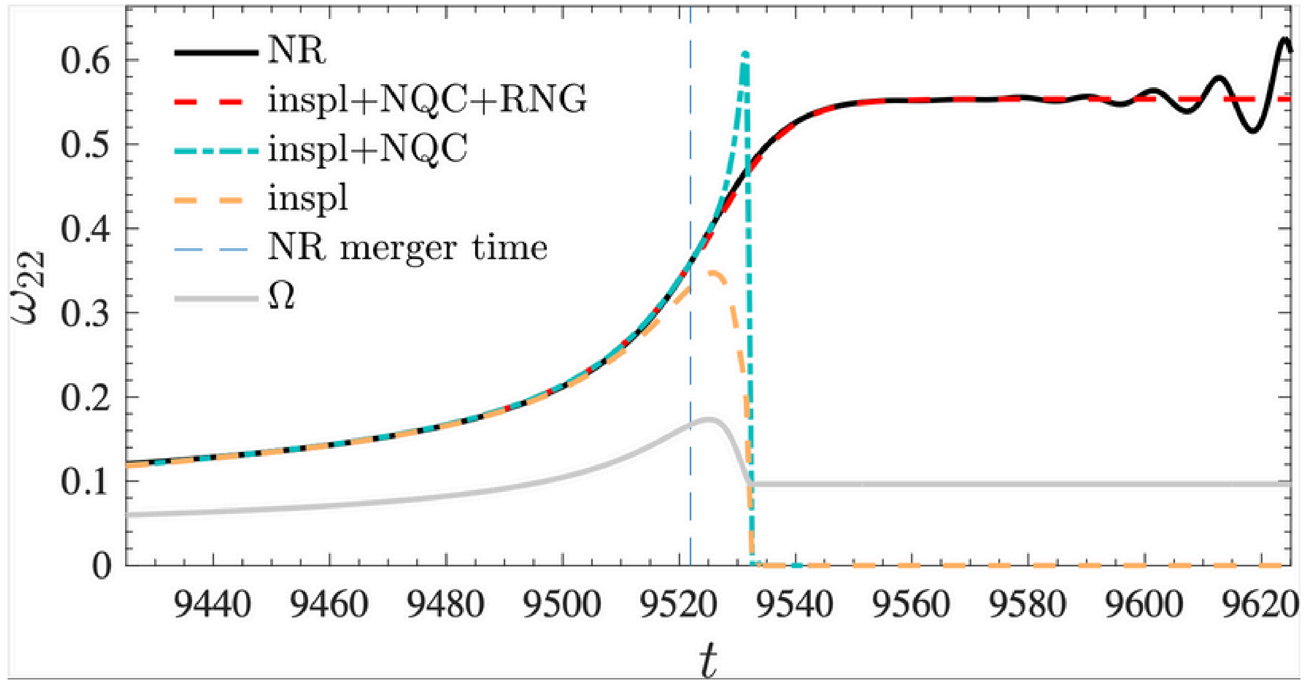
Coefficients $a_k^{\ell m}$ and $b_k^{\ell m}$ determined by requiring C^2 EOB-NR match slightly before merger.

NR calibration in TEOBResumS

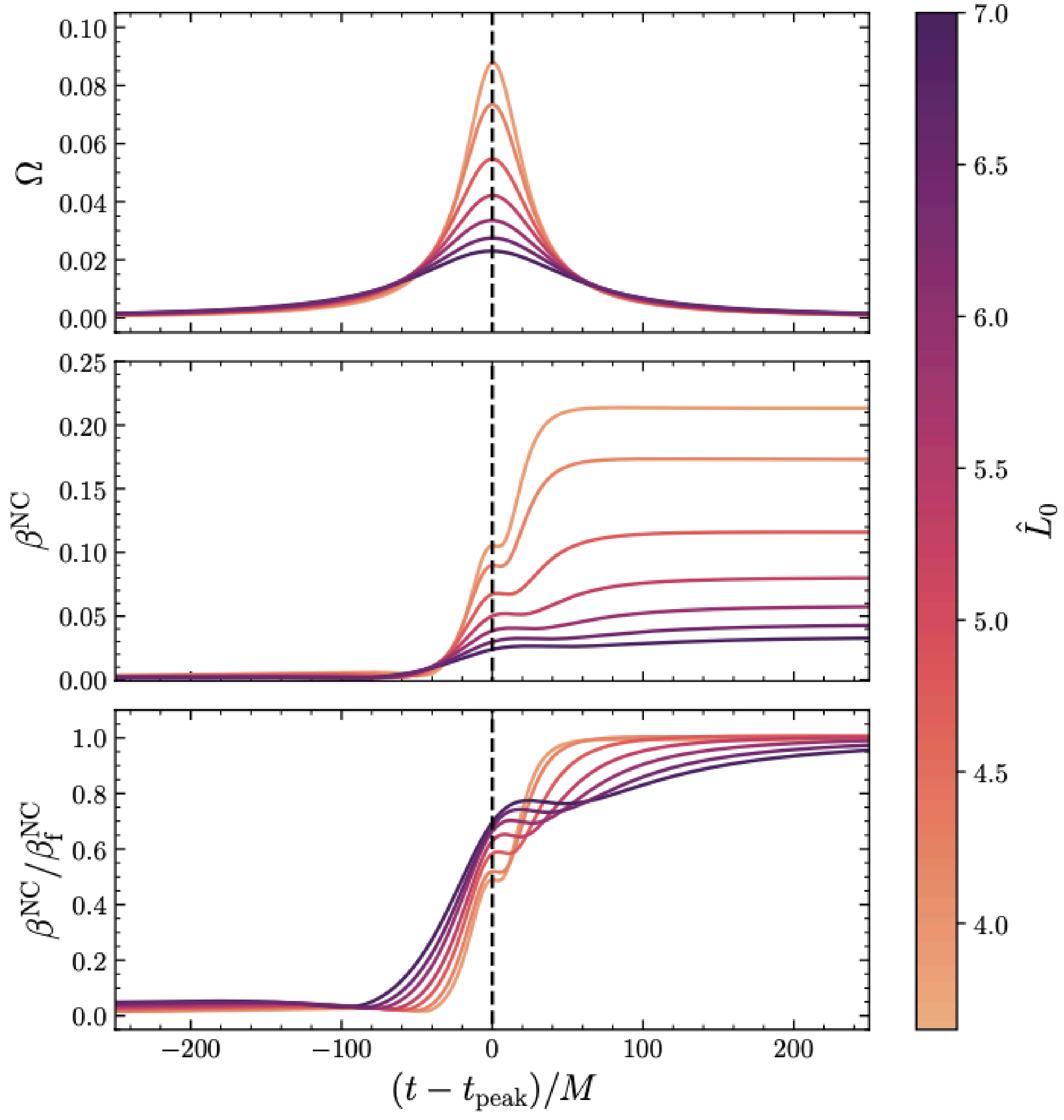


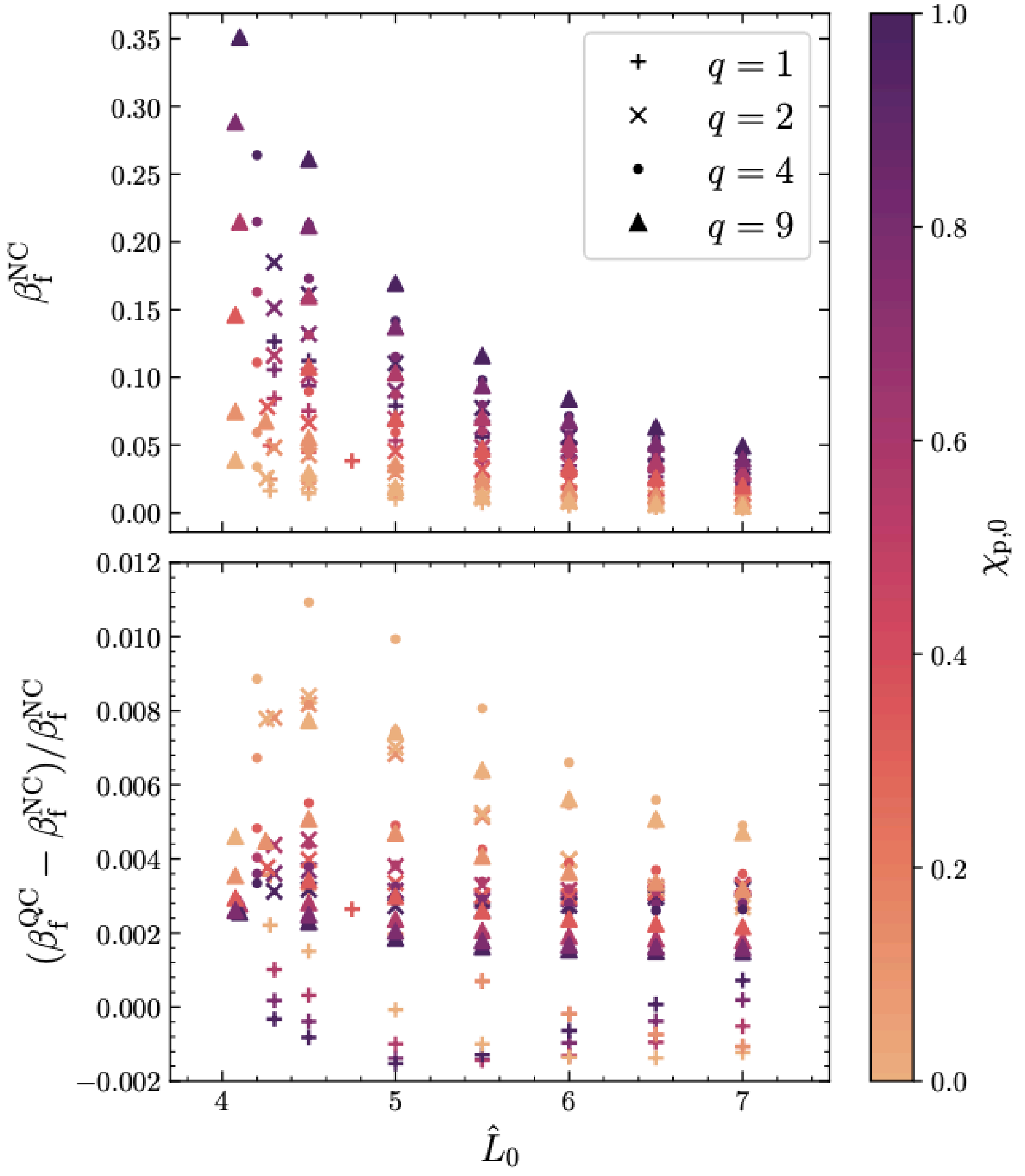


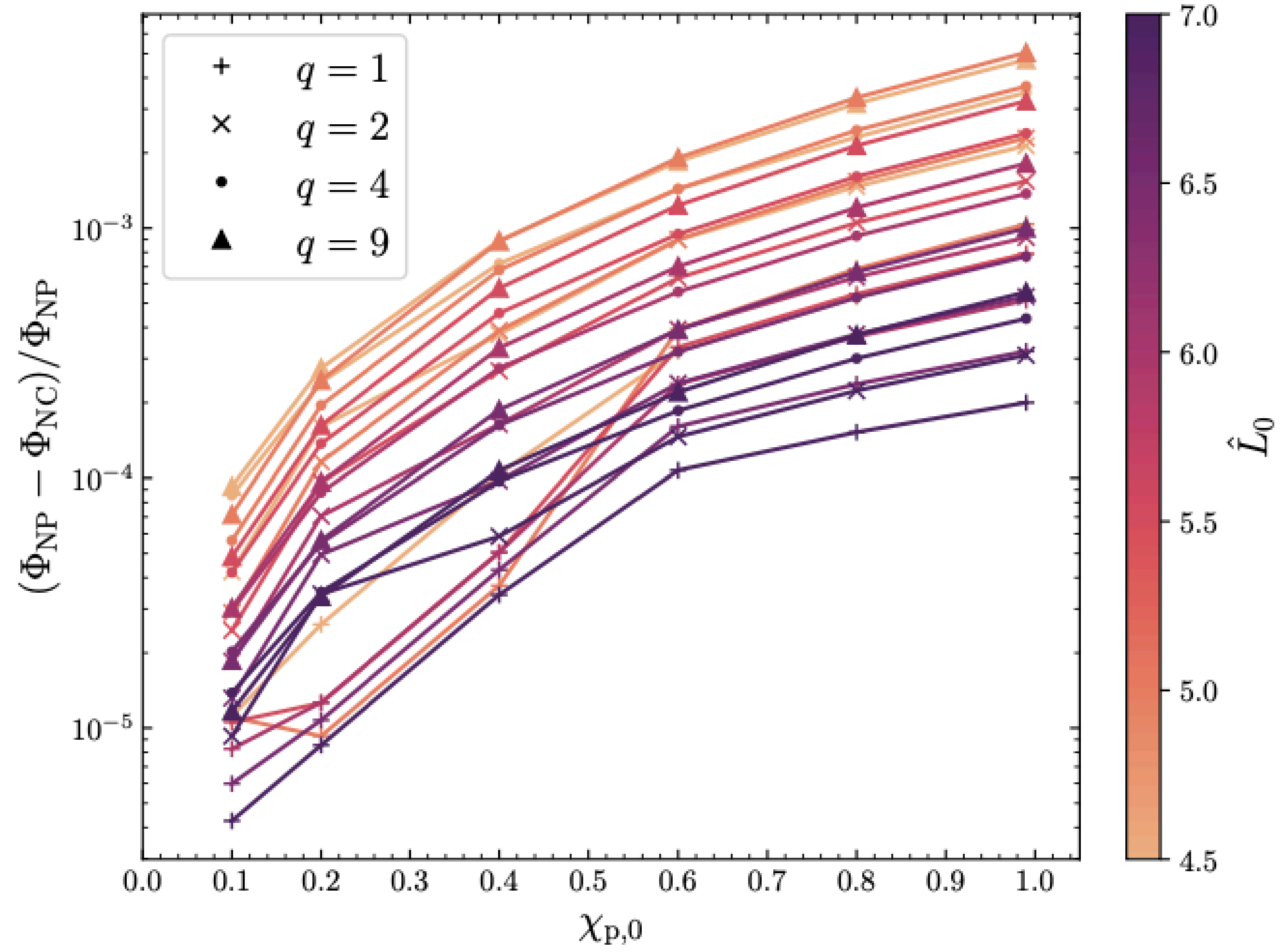




PN precession and scattering







$$\Phi = \varphi_{\rm final} - \varphi_0 - \pi$$

- Euler angle β shifts abruptly at periastron; final value correlated with χ_p
- Azimuthal scattering angle strongly correlated with L, $\chi_{\rm eff}$; weaker (repulsive) effect from χ_p