Scattering and dynamical capture of two black holes: synergies between numerical and analytical methods

Based on 2405.20398:

S. Albanesi, A. Rashti, F. Zappa, R. Gamba, W. Cook, B. Daszuta, S. Bernuzzi, A. Nagar, and D. Radice

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1st TEONGRAV international workshop

Simone Albanesi

Introduction

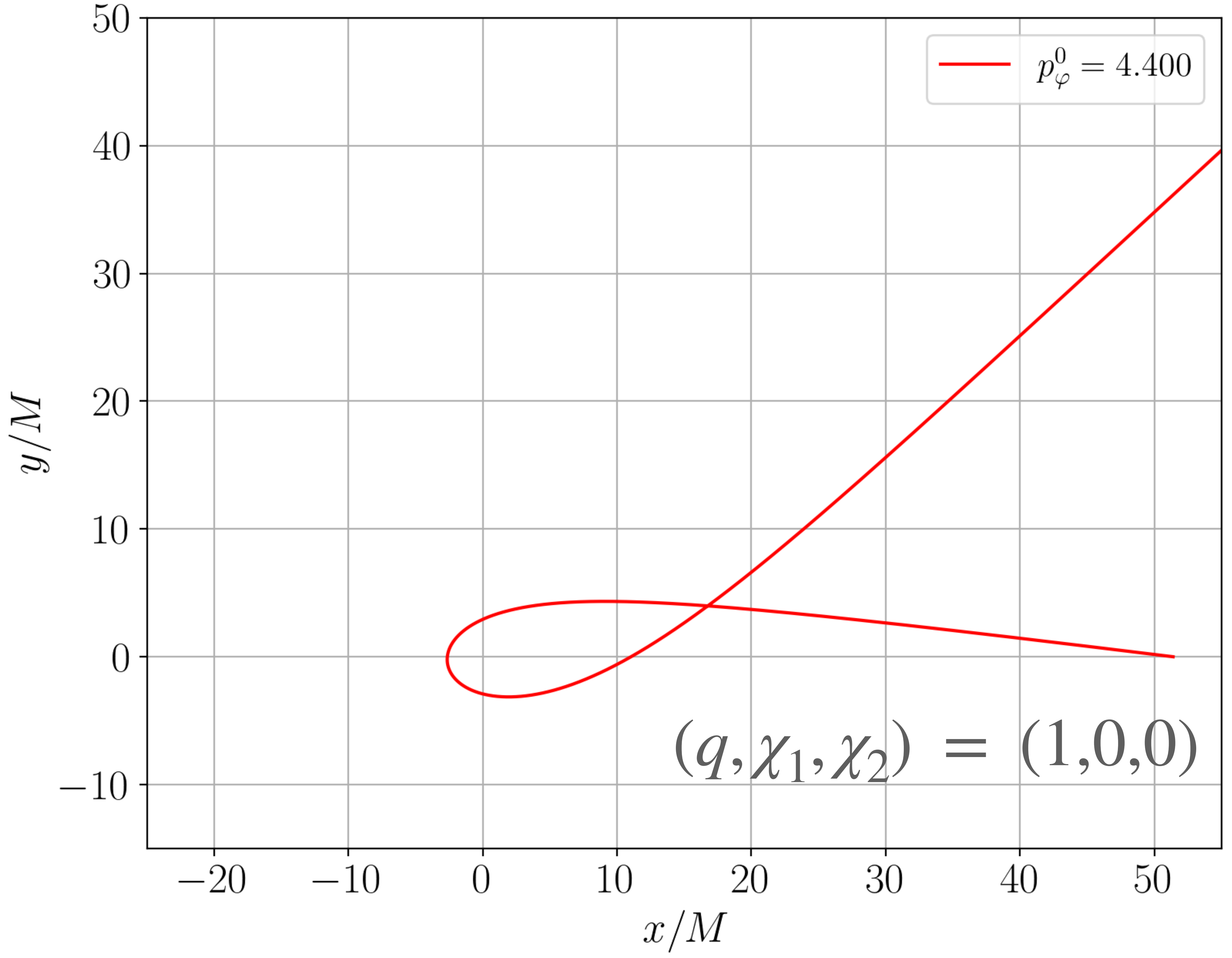
- Detectable signals from binaries with residual eccentricity: need of GW models for non-circularized orbits
- Focus on **hyperbolic systems** and their **transition to bound** orbits
  - scatterings → dynamical captures
- Astrophysical motivations:
  - Detection rates are highly uncertain, but populations of these systems are expected in dense environments such as globular clusters and AGN
  - **GW190521** is compatible with a dynamical capture scenario [1]
- Theoretical motivations:
  - Captures are the most delicate systems to model → useful insights for eccentric (elliptic) models
  - Post-Minkowskian descriptions for scatterings can be tested with NR simulations

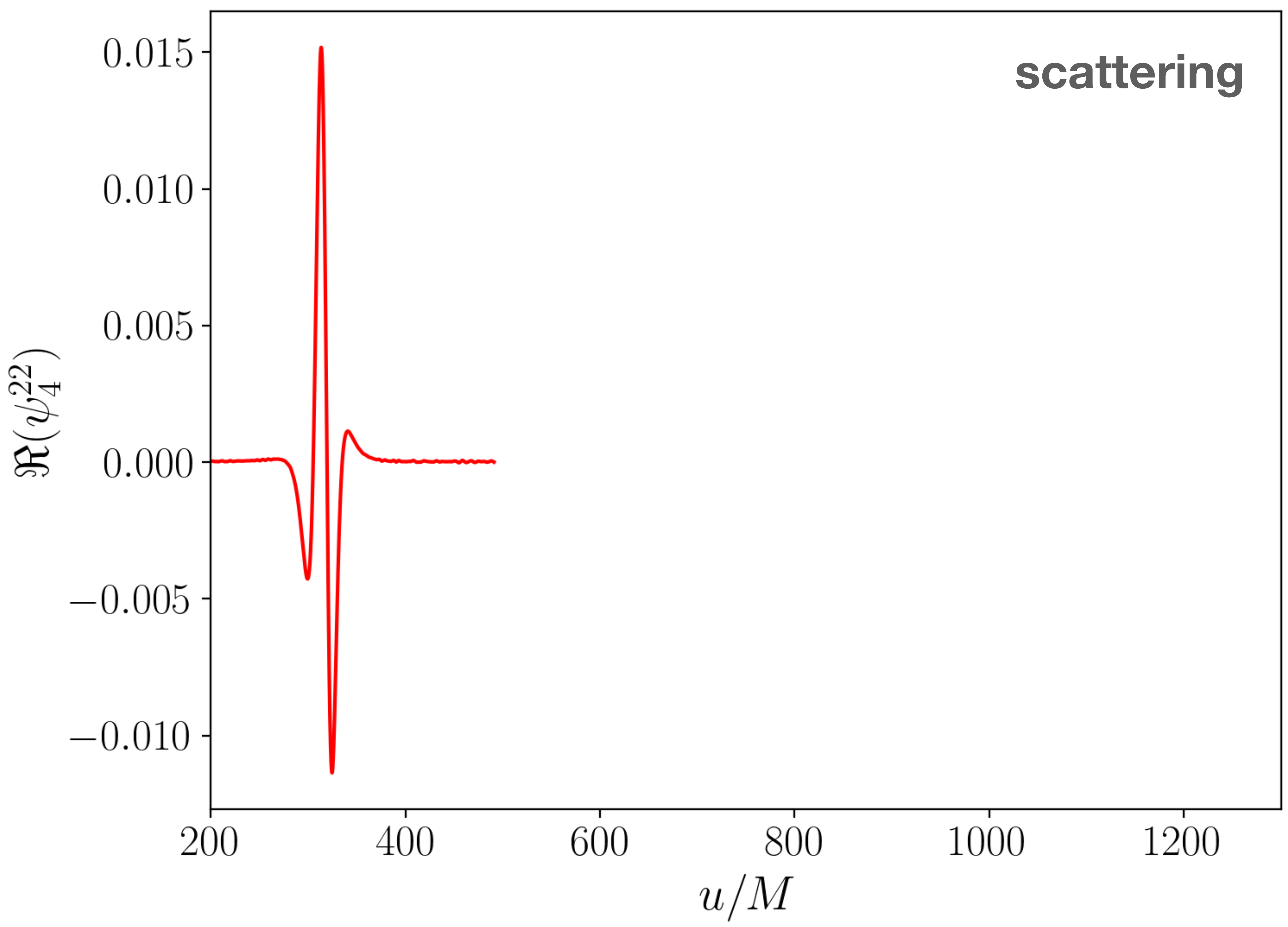
[1] Gamba+:2106.05575

Hyperbolic systems: scatterings and dynamical captures

[2] Daszuta+:2101.08289

- **Hyperbolic systems**: initial positive binding energy ($E_{\rm ADM} > M$)
- Some numerical relativity simulations, `GR-Athena++` [2]
- Series of runs with $E_0 = 1.011\, M$ and decreasing angular mom $p_\varphi^0 \equiv J_0/(\mu M)$





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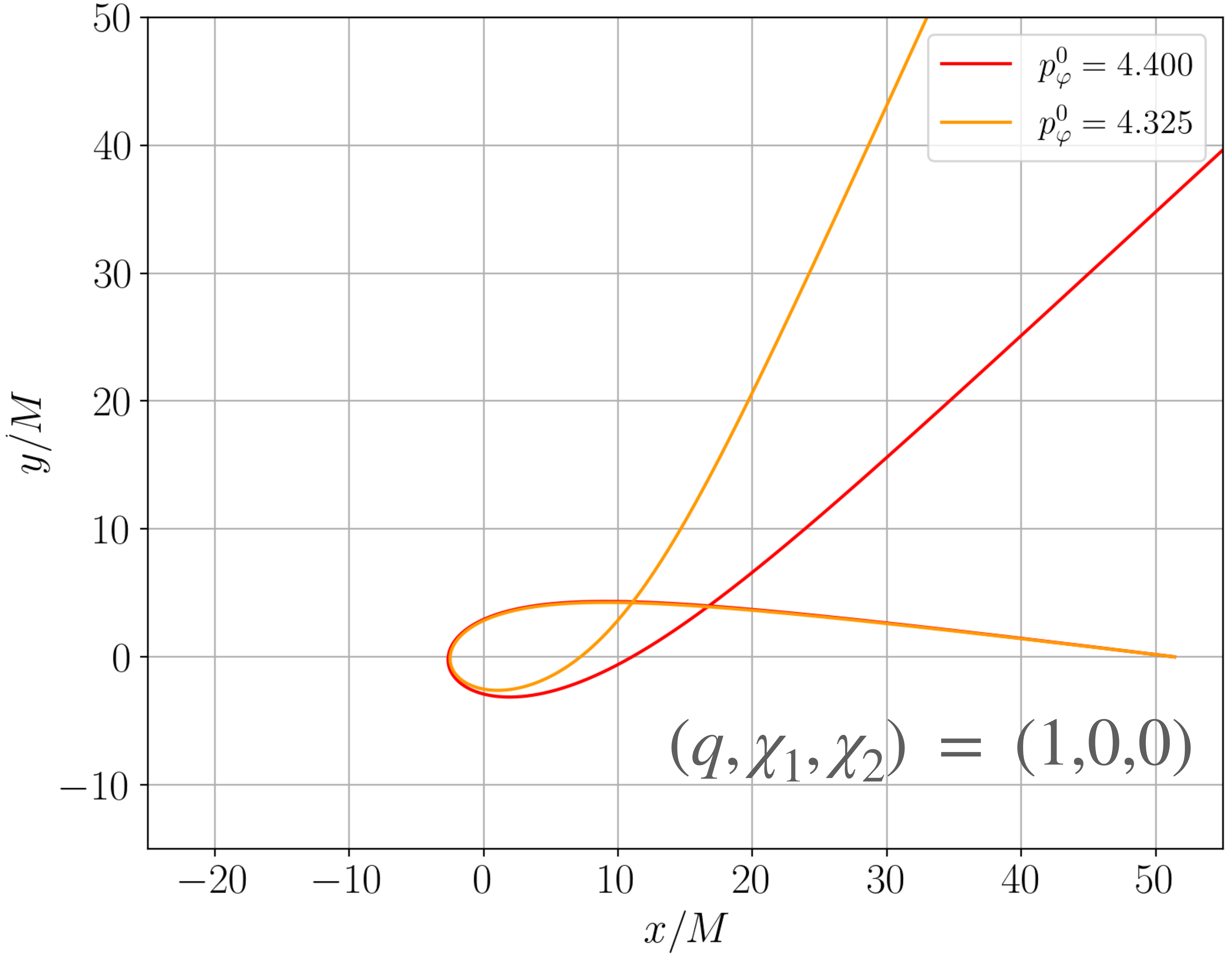

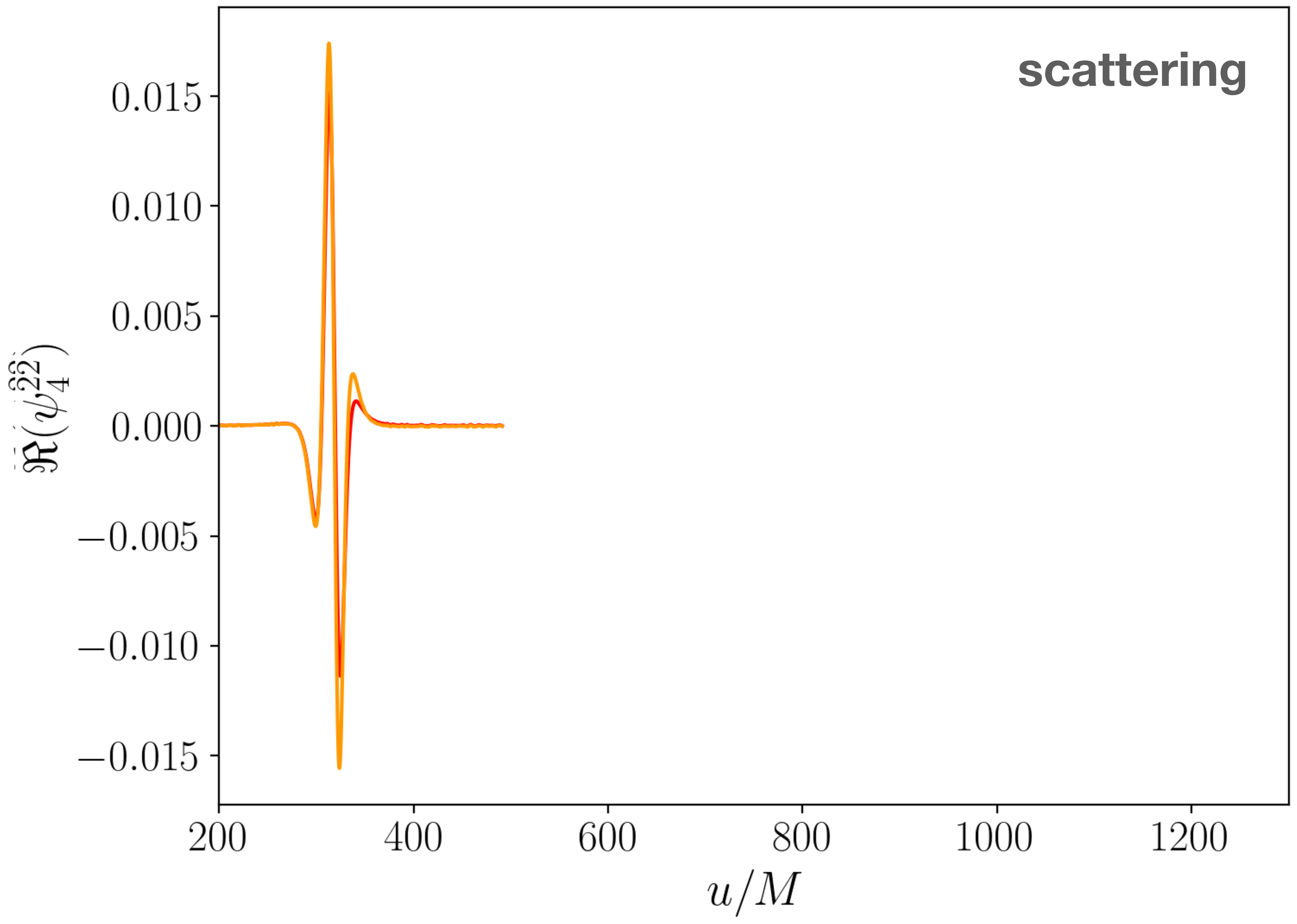

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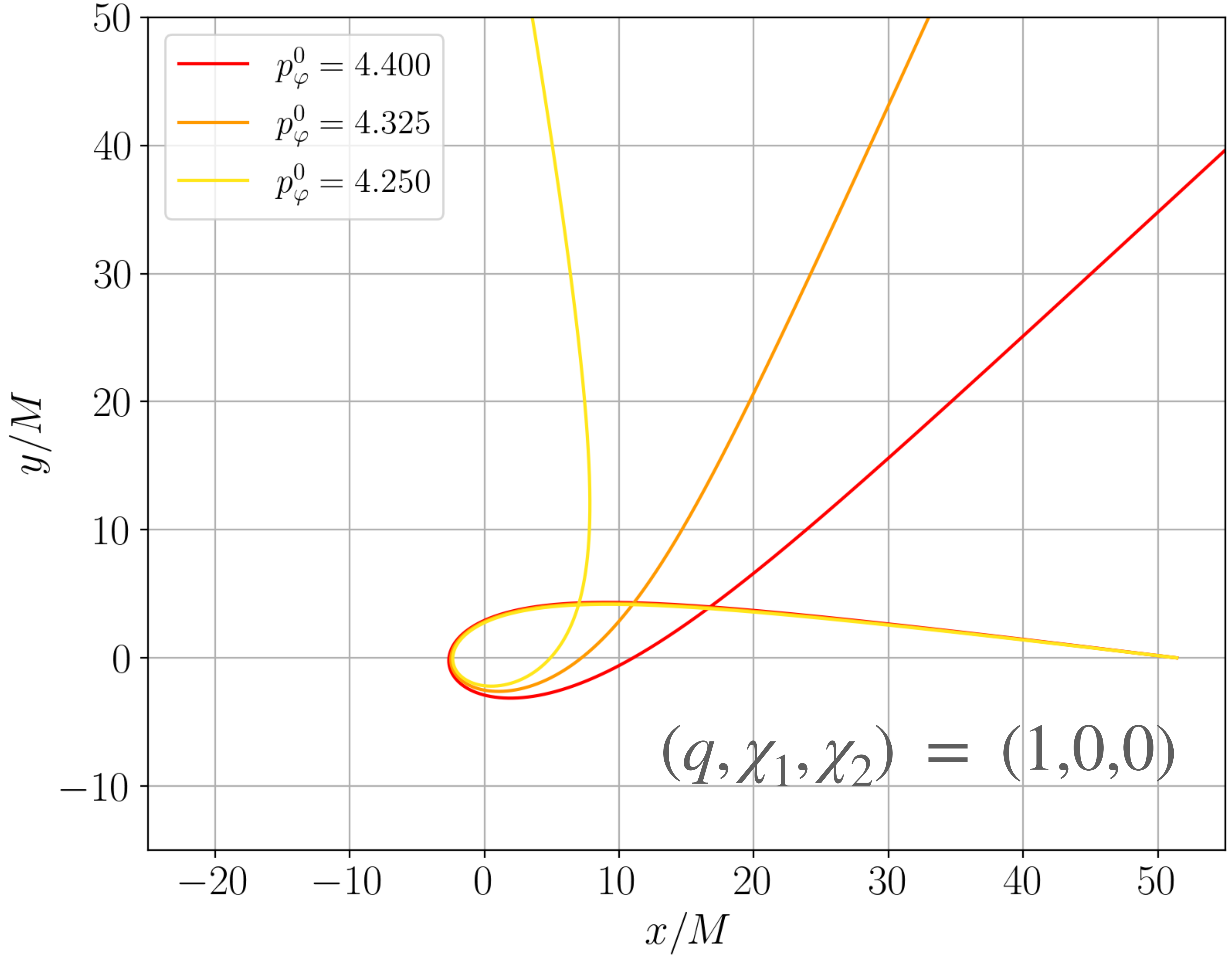

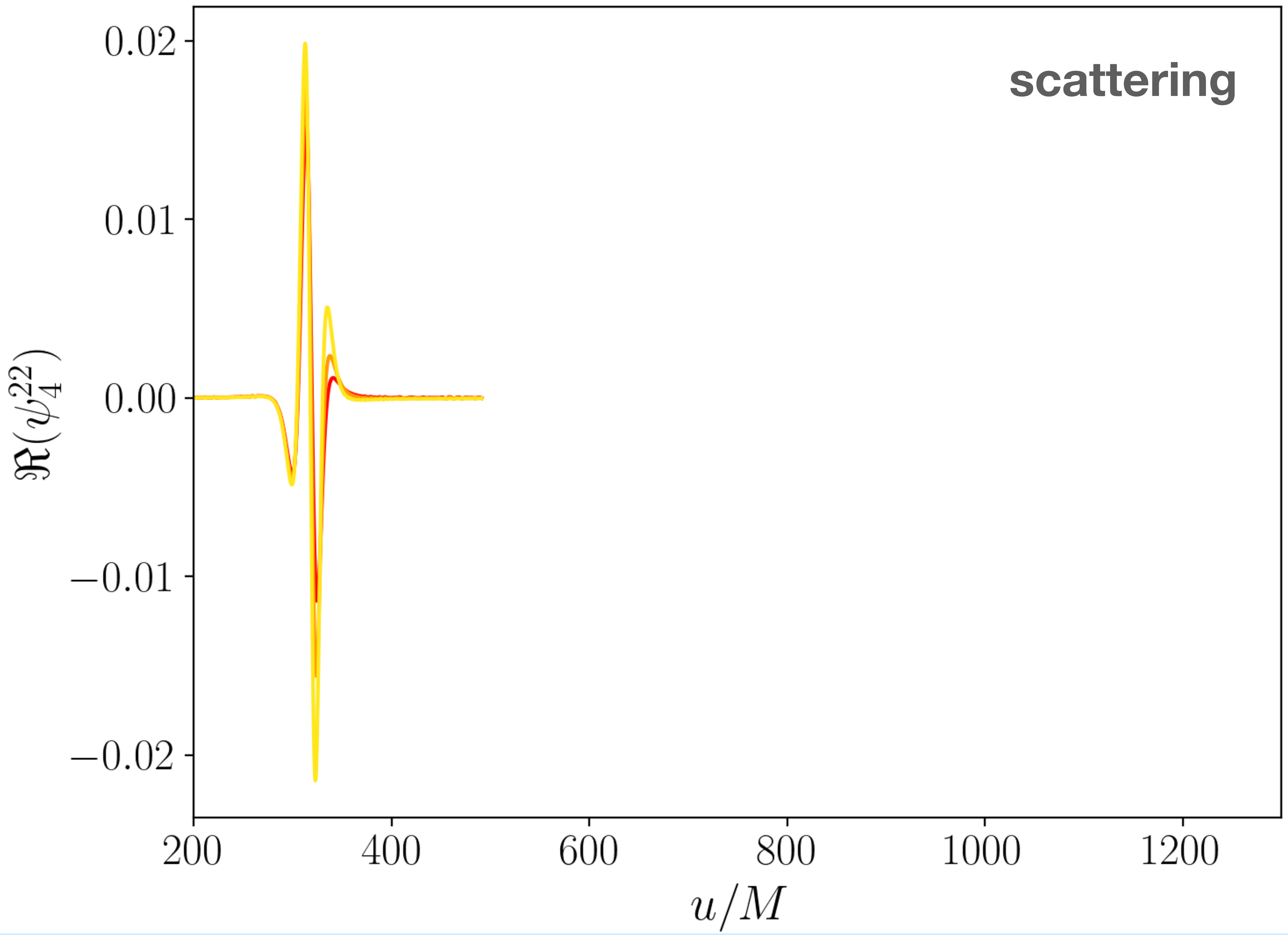

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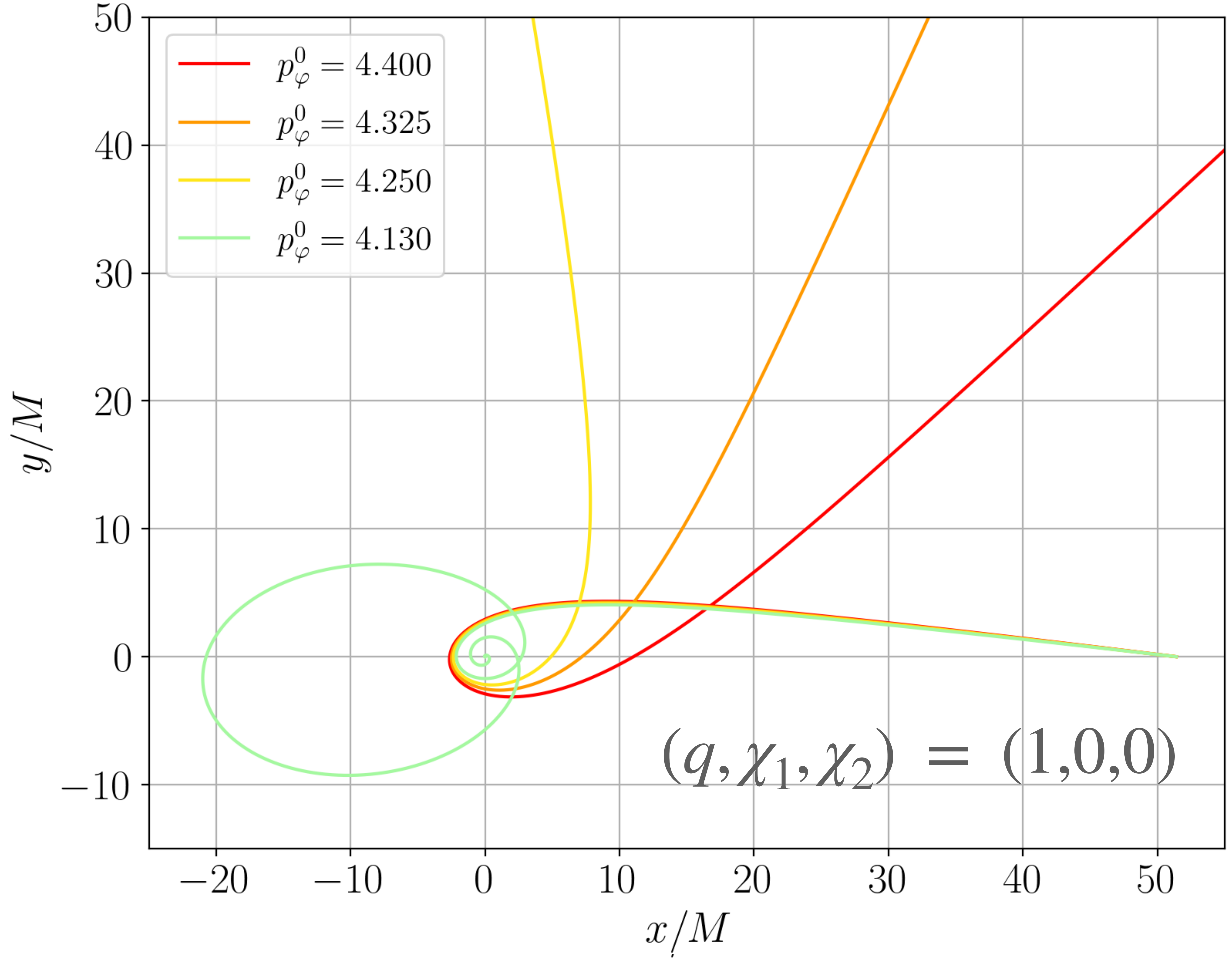

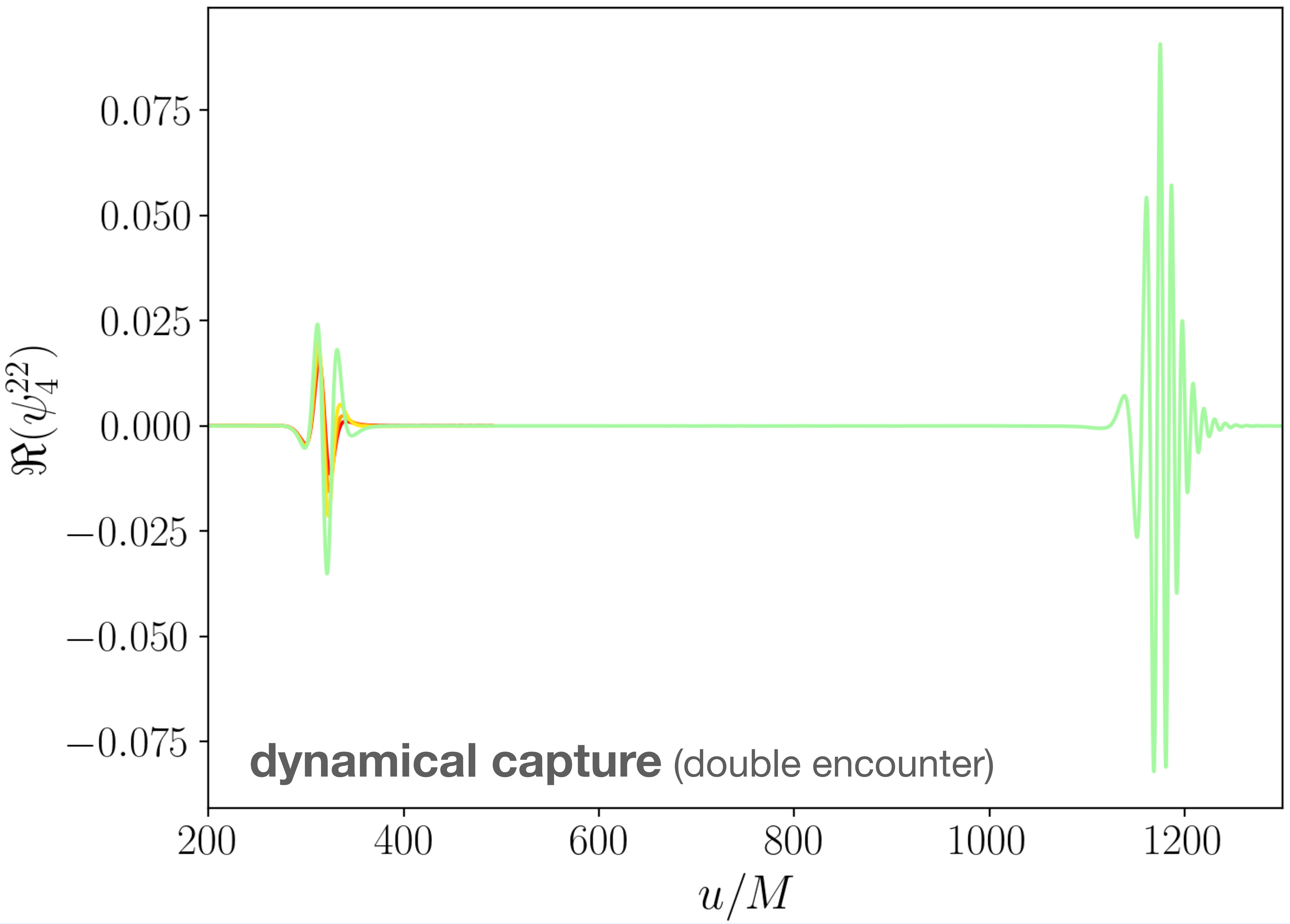

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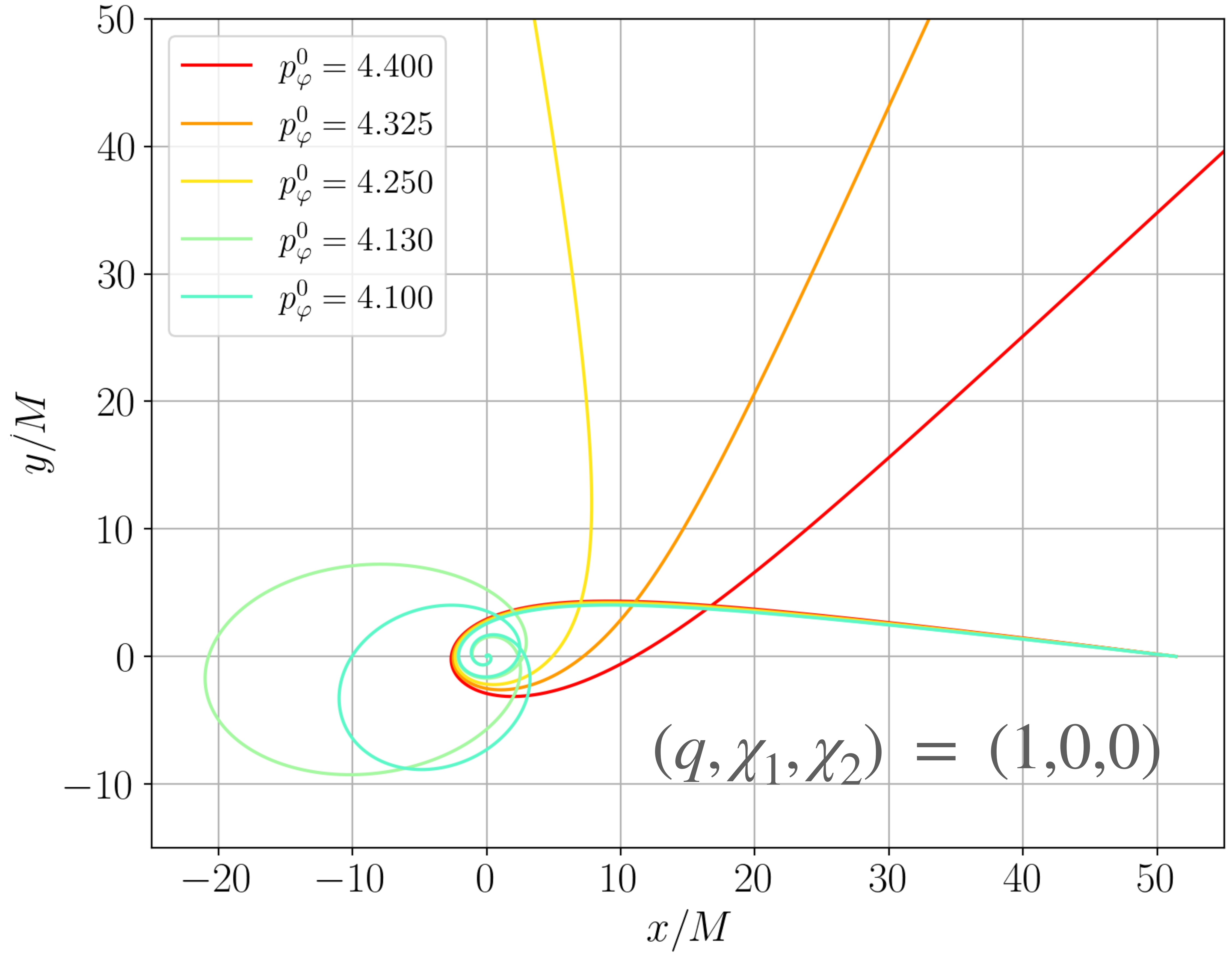

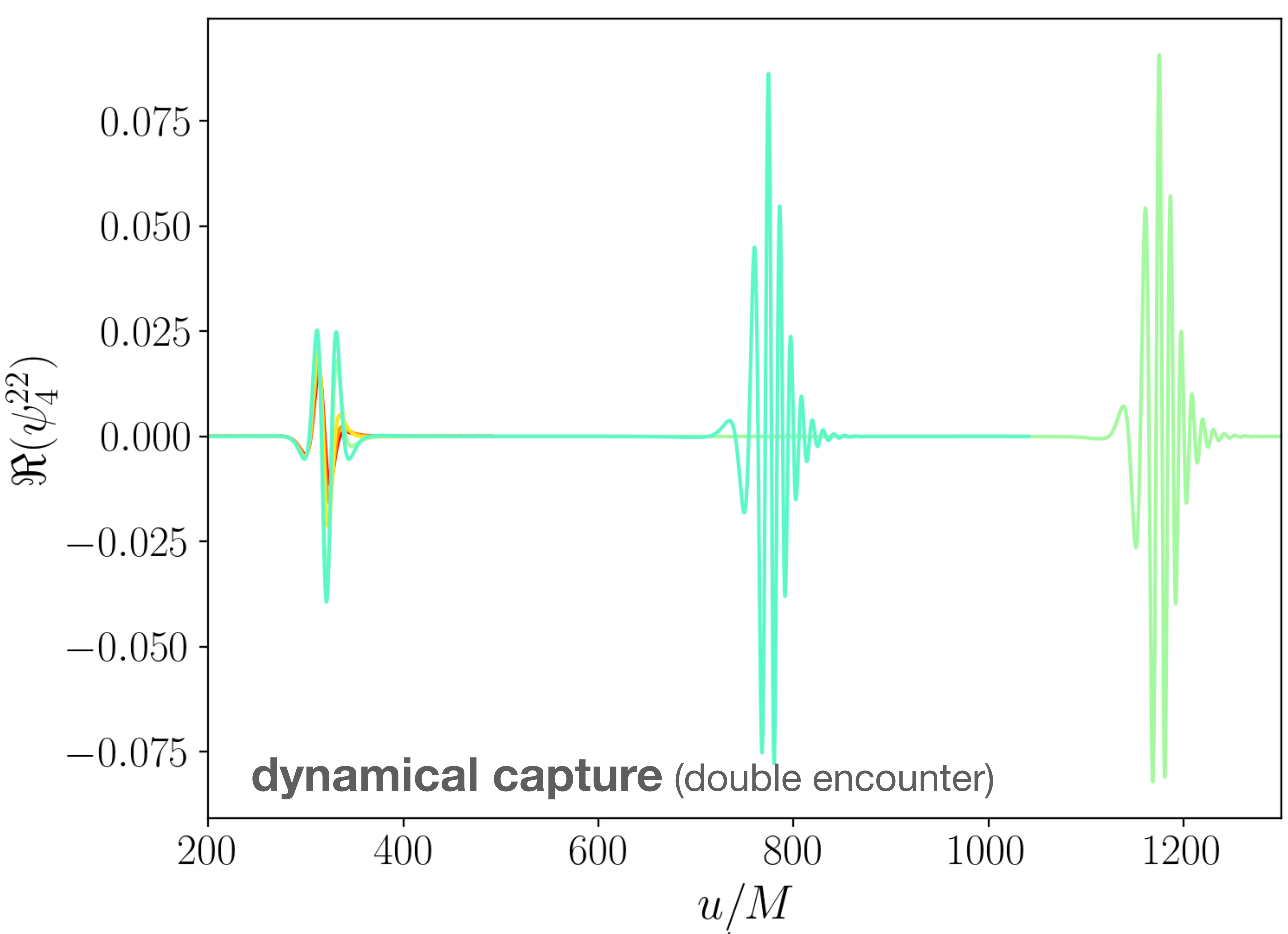

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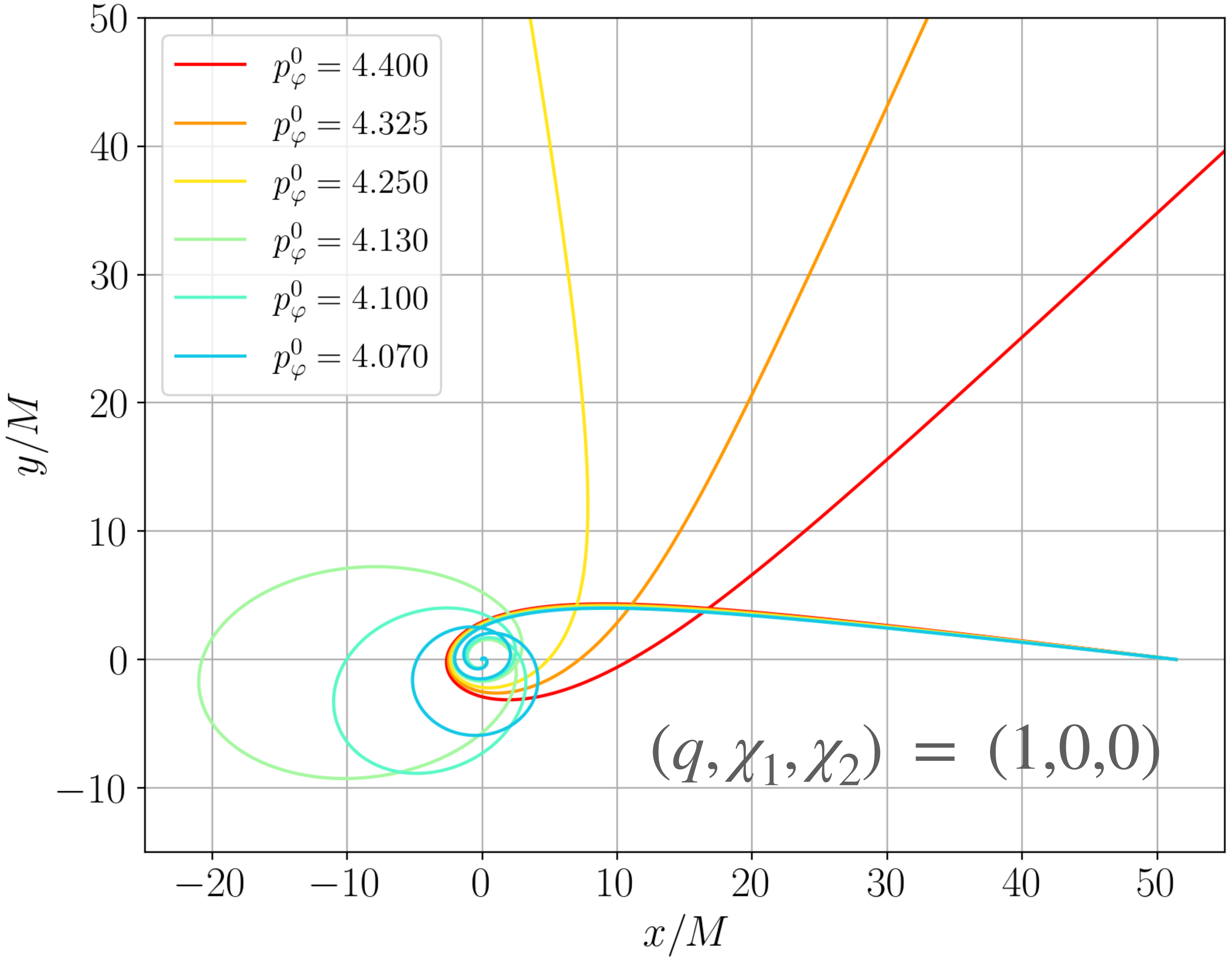

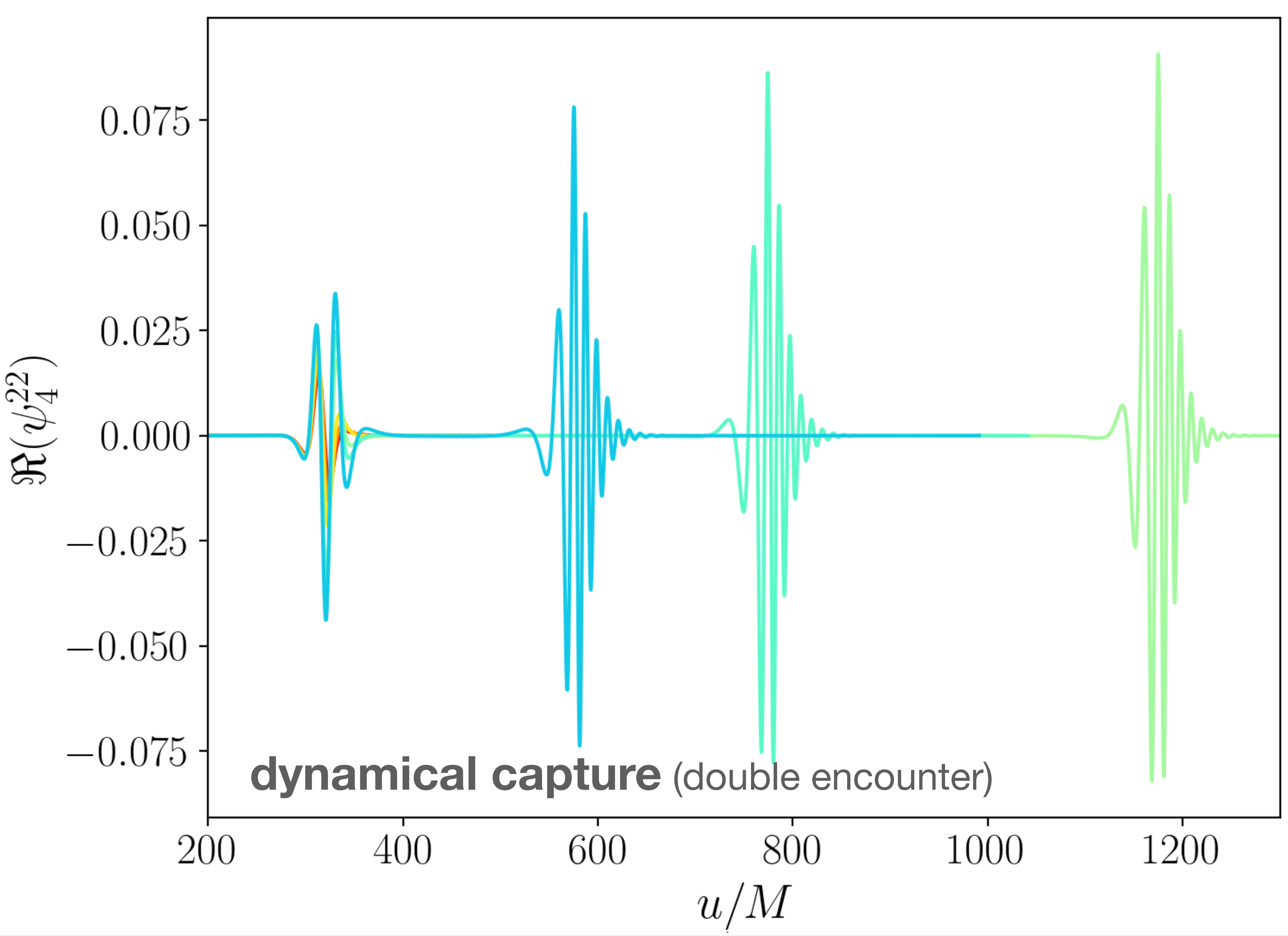

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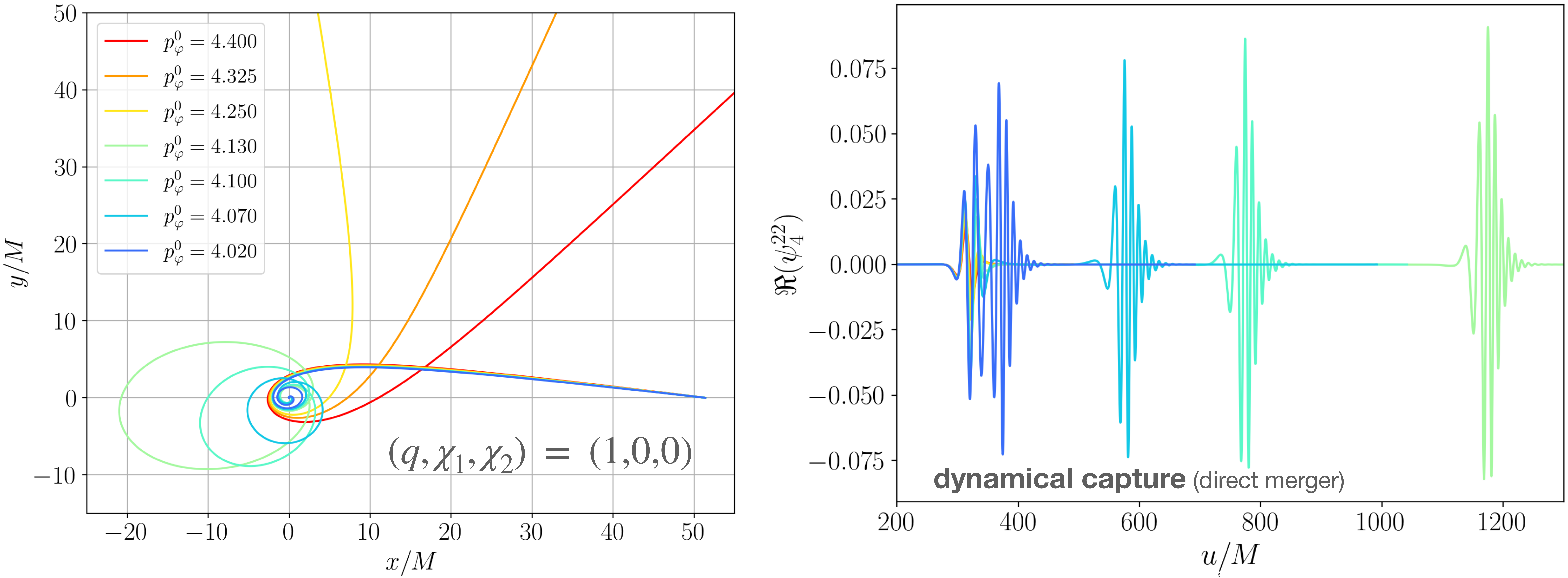

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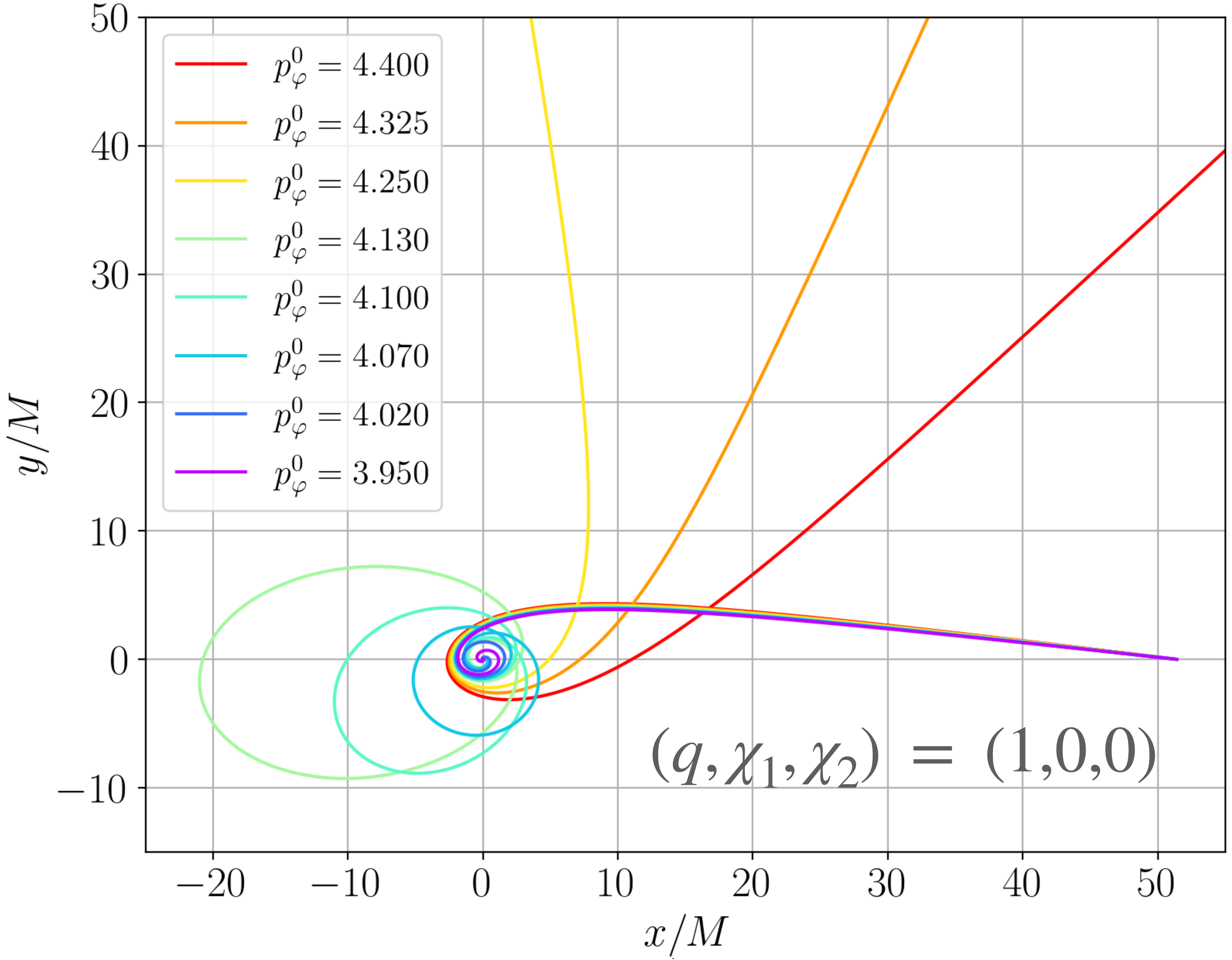
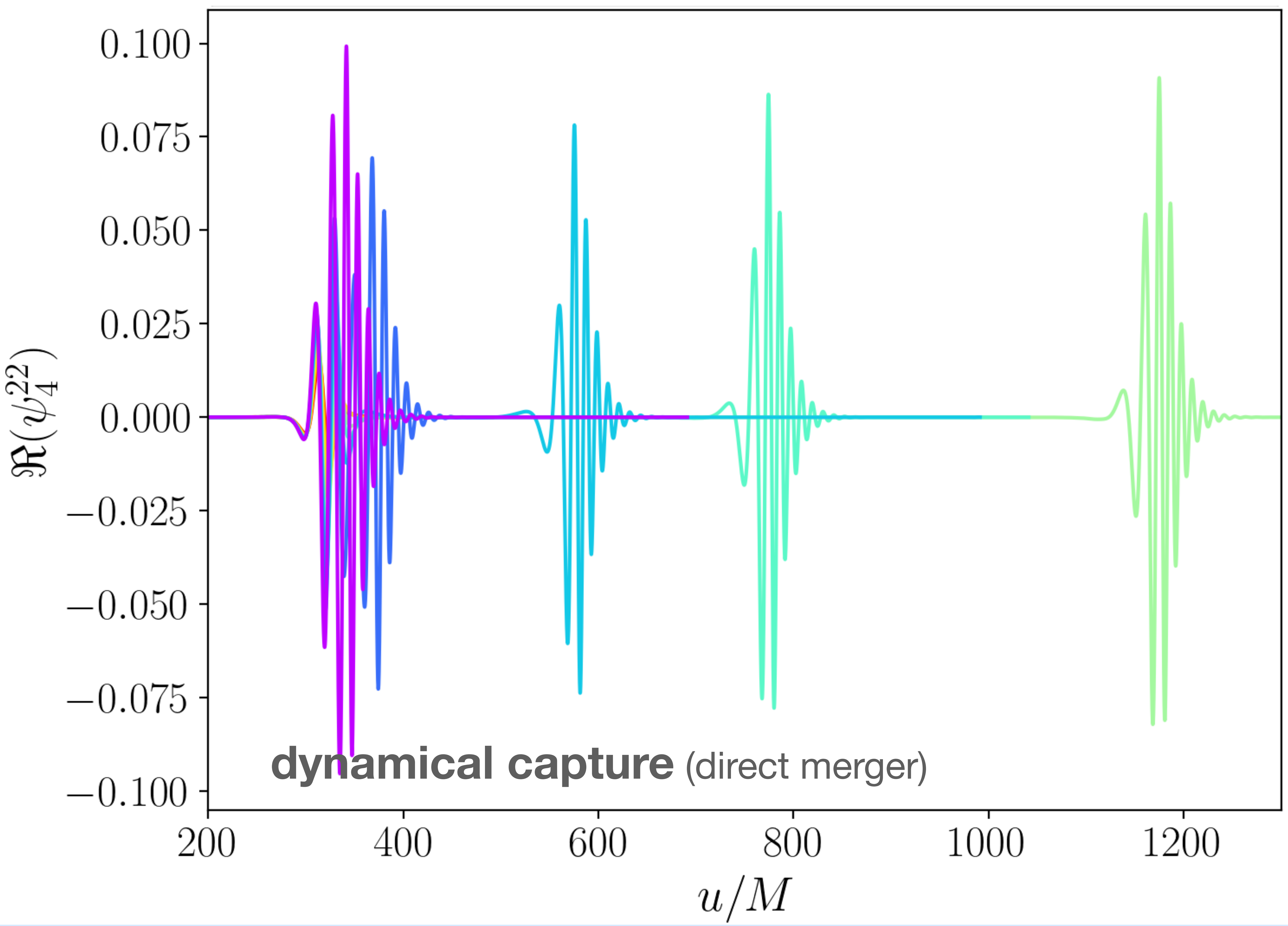
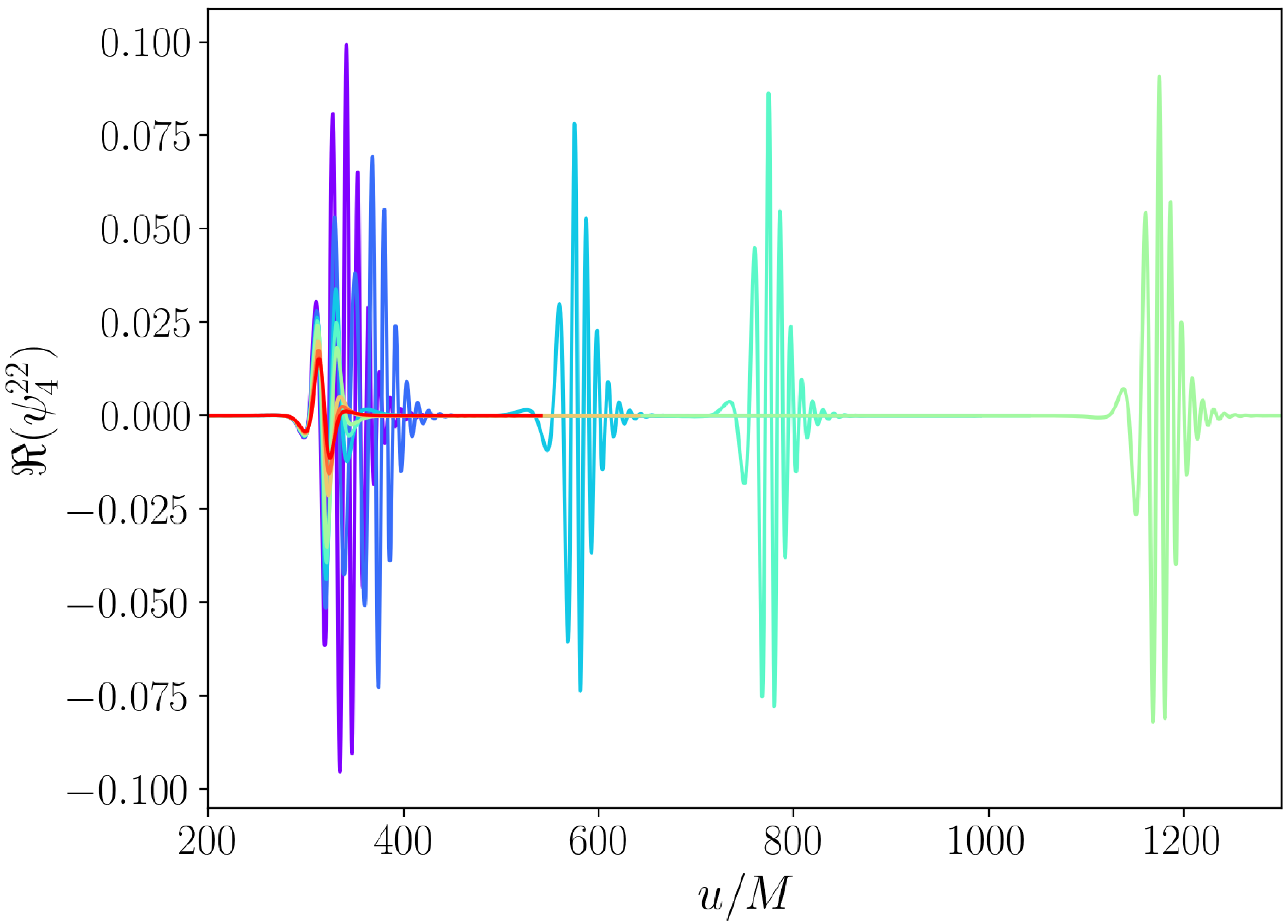
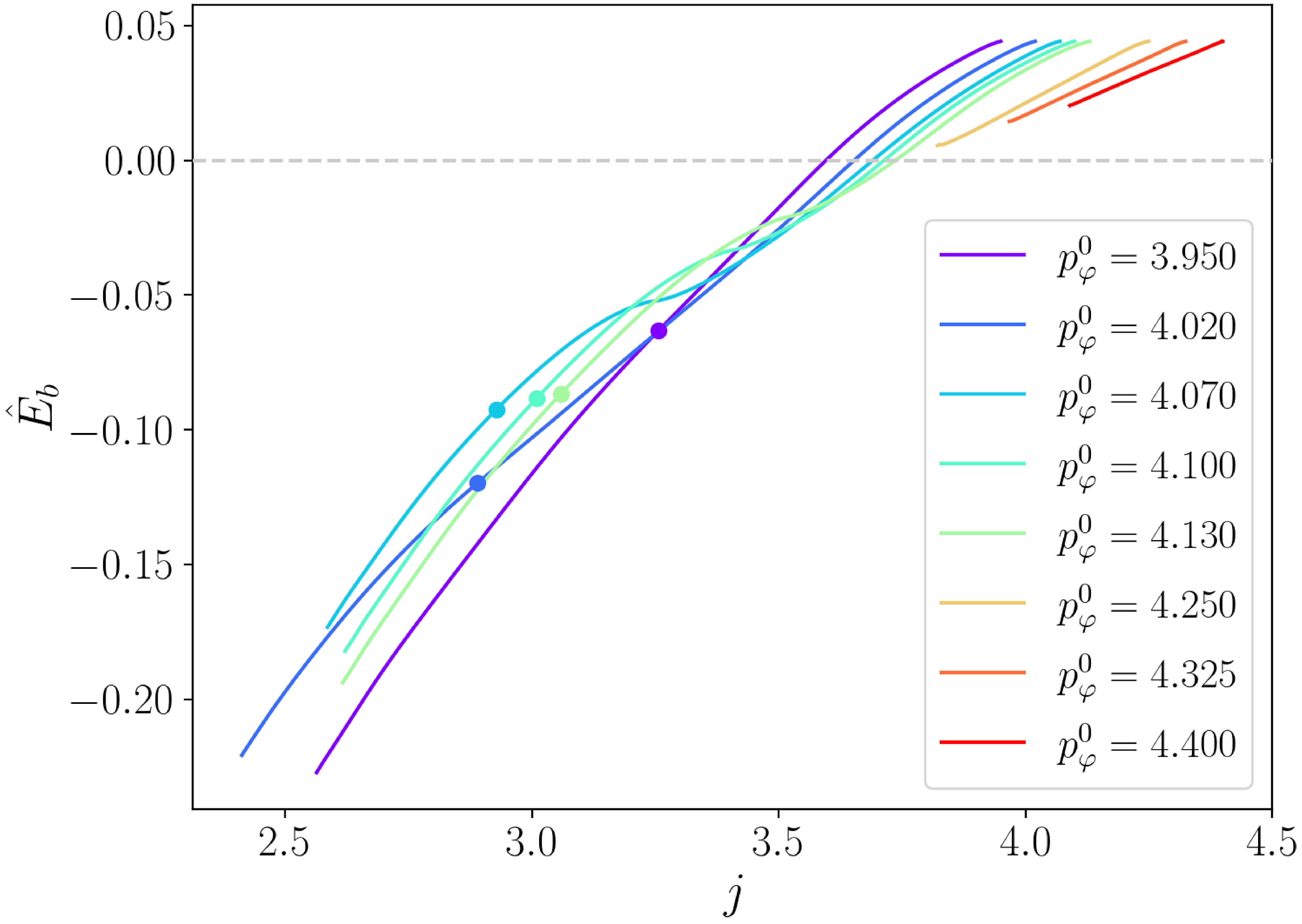




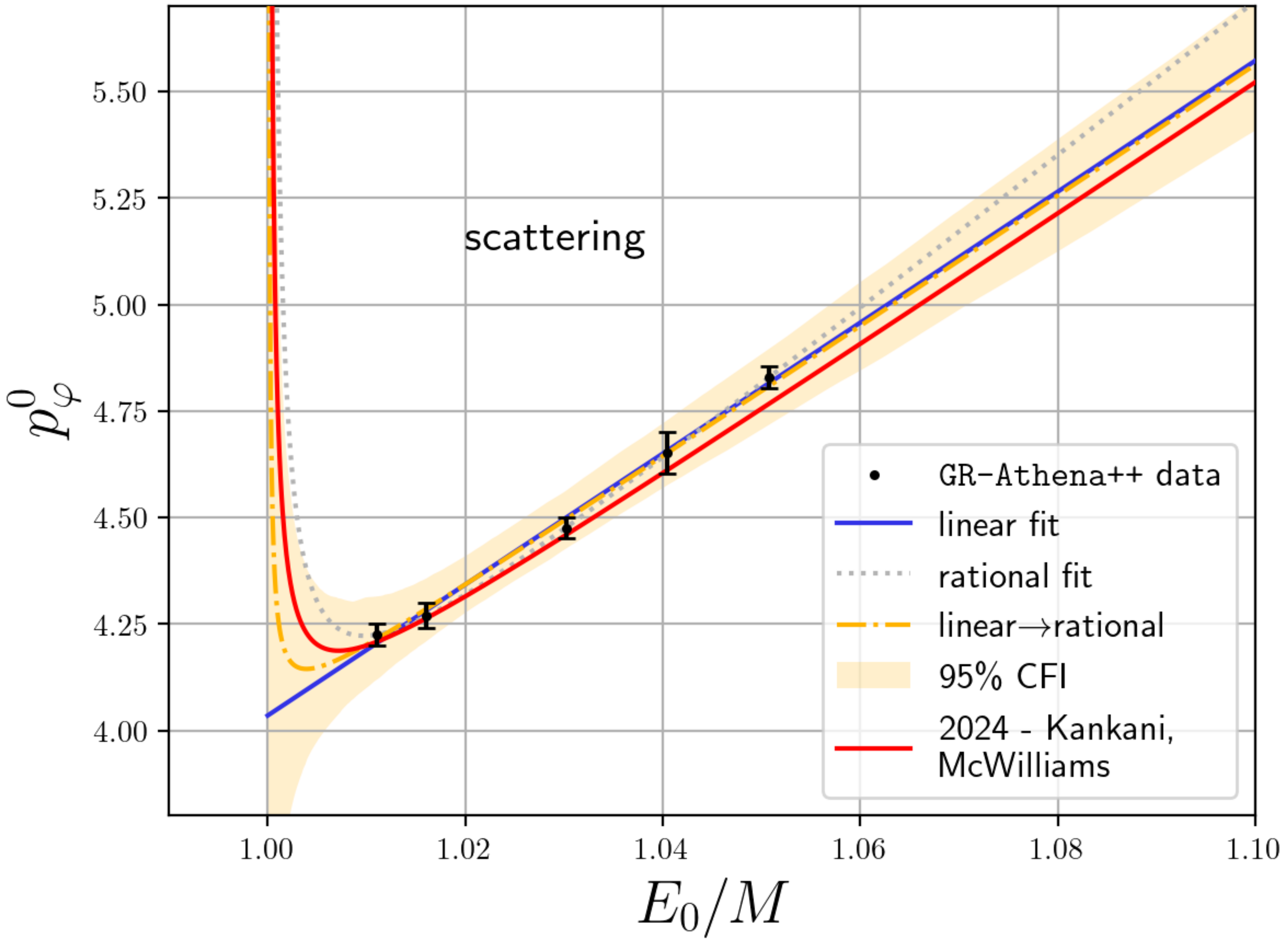
Distinguishing scatterings and captures

- Same simulations as before with $E_0 = 1.011\ M$: **scatterings**, **double encounters**, and **direct captures**
- **Are scatterings really scatterings?** Trivial to determined most of the time, but tricky at the capture threshold
- Solution: **energy balance argument** (i.e. subtract fluxes from initial energy), but be careful with integration





Transition from unbound to bound



- Transition from scattering to merger
- Points/errors from scattering with lowest p_φ^0 and merger with highest p_φ^0
- How to distinguish scatterings from double encounters with huge apastra?
  → check radiated energy*
- Similar **fits from GR-Athena++** and **Kankani-McWilliams fit** [3] but some differences

* if computed from fluxes, require integration of ψ_4

[3] Kankani,McWilliams 2404.03607

Orbital parameter space

- How to sample/explore the orbital parameter space?
- **Eccentricity** (typically) defined only for elliptic-like orbits
  - dynamics-eccentricity cannot be defined in a gauge-invariant way
  - waveform-eccentricity cannot be generalized to single/short bursts
- Alternative: initial energy (ADM) and μ-rescaled angular momentum (“ADM”) of the system, $(\mathbf{E_0}, \mathbf{p}_\varphi^\mathbf{0})$
  - used to sample the hyperbolic PS for the analysis of GW190521 as a dynamical capture [1]
- Preliminary exploration of the PS using **TEOBResumS-Dalí** [4]

[1] Gamba+:2106.05575 [4] Nagar+:2009.12857

Effective-one-body models

- Map from 2-body PN equations of motion to motion of a particle in an effective metric [5]
- The effective metric is a ν-deformation of Schwarzschild/Kerr, being $\nu = m_1 m_2/(m_1 + m_2)^2$ the symmetric mass ratio

[5] Buonanno,Damour:gr-qc/9811091

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- Three building blocks:
  - Hamiltonian: $H_{\rm EOB} = M\sqrt{1 + 2\nu(\hat{H}^{\rm eff}_\nu - 1)}$

non-spinning case, Schwarzschild ν-deformation

$$\hat{H}^{\rm eff}_\nu = \sqrt{A_\nu\left(1 + p_\varphi^2 u^2 + Q_\nu(r, p_{r_*})\right) + p_{r_*}^2}$$

$$A_\nu = 1 - 2u + 2\nu u^3 + \dots$$

  - Radiation reaction $\mathcal{F}_{\varphi,r}$

    $\rightarrow$ solve Hamilton's equations

  - Waveform $h^{\rm inspl}_{\ell m} = h^{(\epsilon,{\rm N})_{\rm c}}_{\ell m} \hat{h}^{(\epsilon,{\rm N})_{\rm nc}}_{\ell m} \hat{h}^{\rm c}_{\ell m} \hat{h}^{\rm nc}_{\ell m}$

$$h_{\ell m} = \theta(t - t^{\rm match}_{\ell m}) h^{\rm inspl}_{\ell m} \hat{h}^{\rm NQC}_{\ell m} + \theta(t^{\rm match}_{\ell m} - t) h^{\rm ringdown}_{\ell m}$$

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- What has to be changed for **non-circularized** inspirals?
  - Hamiltonian ok-ish modulo non-local terms and calibration
  - Radiation has to be generalized: **generic Newtonian prefactor** [6] and PN noncircular corrections

[5] Buonanno,Damour:gr-qc/9811091
[6] Chiaramello,Nagar:2001.11736

Effective potentials

- Define effective potential as $V = H_{\rm EOB}(r, p_\varphi, p_{r_*} = 0; \nu)$
- Some equal mass non-spinning cases:

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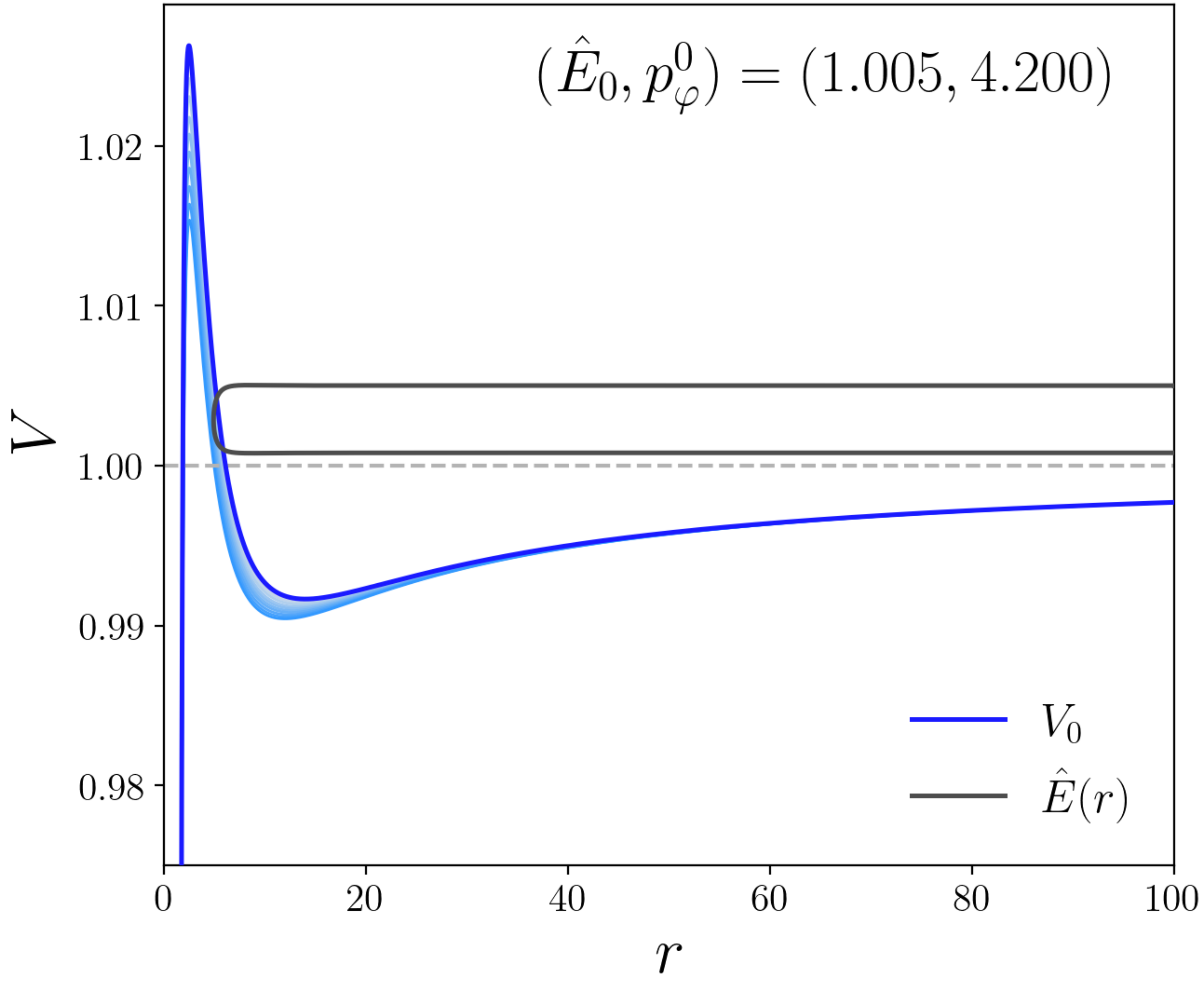

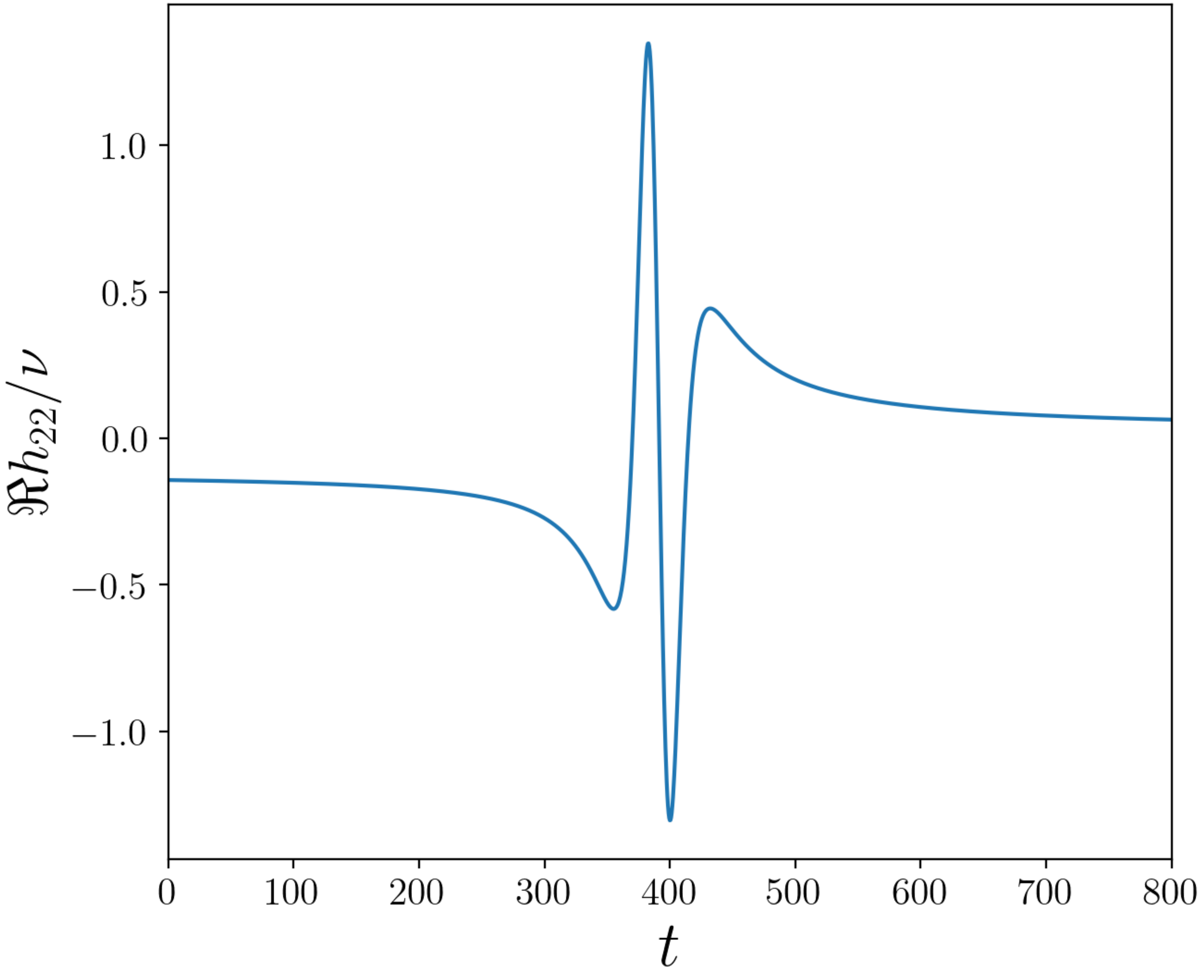

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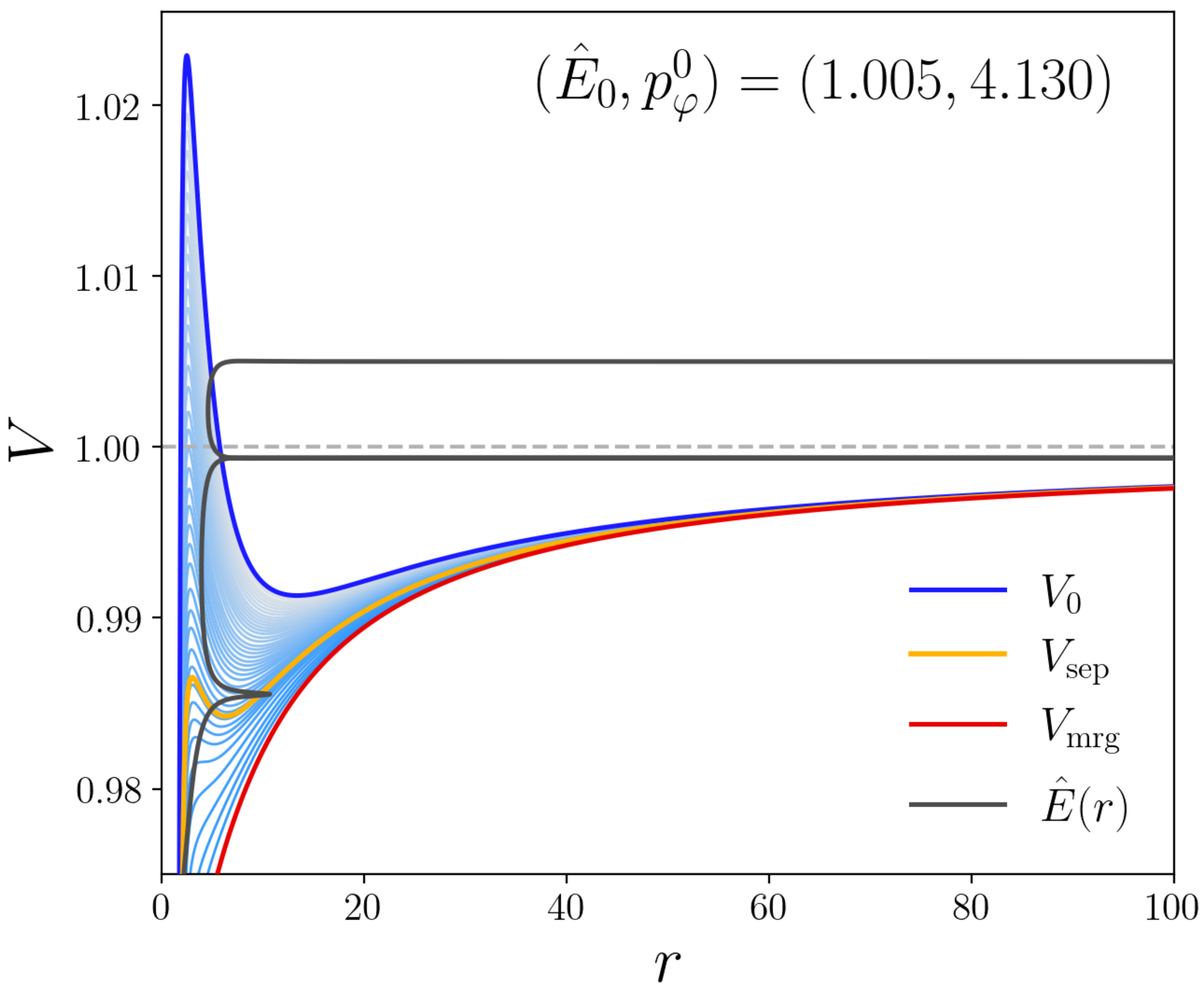

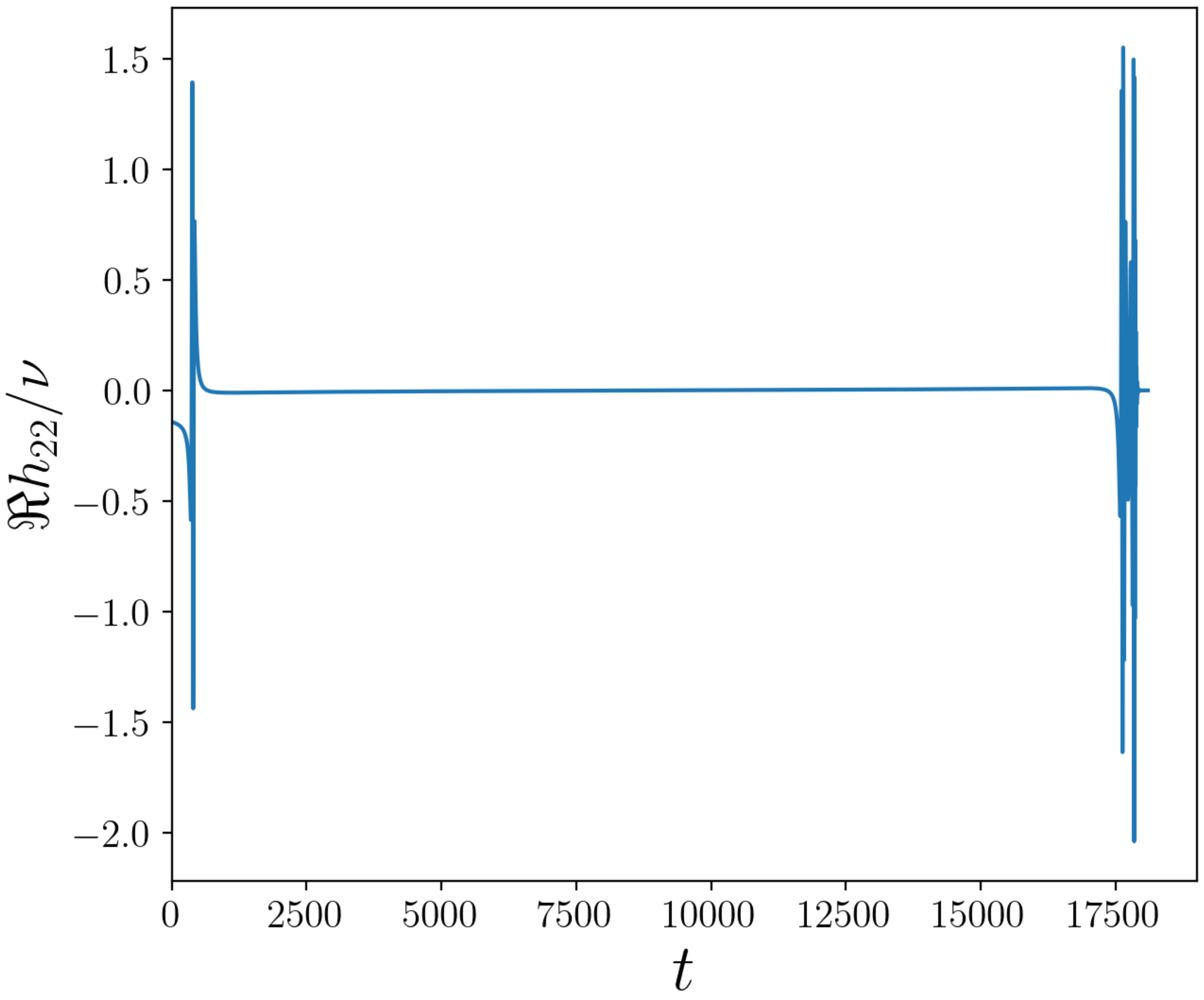

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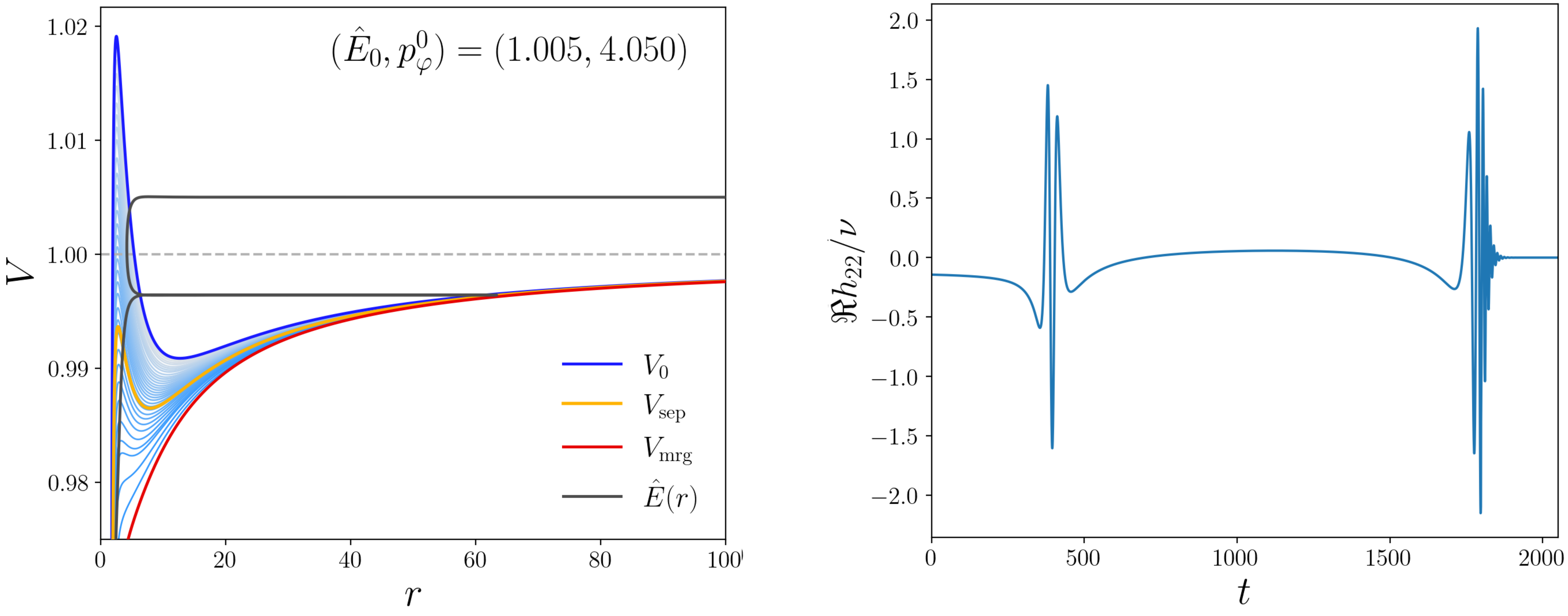

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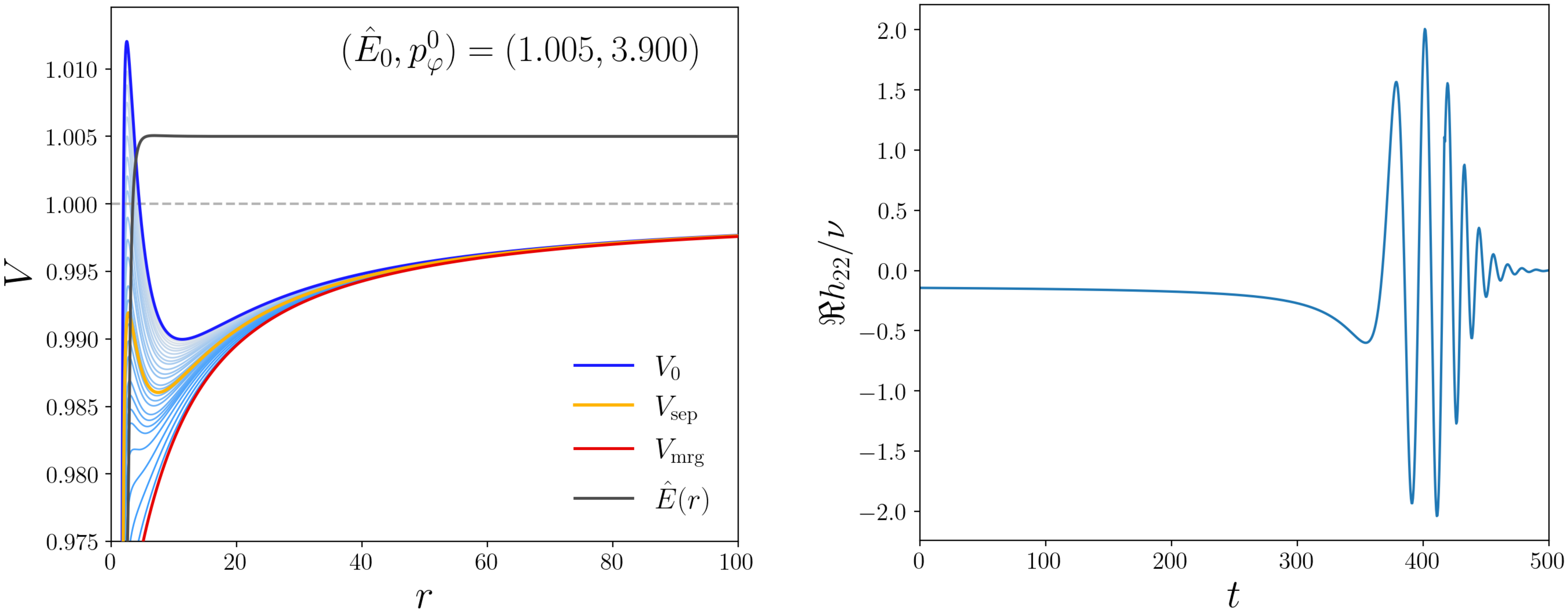
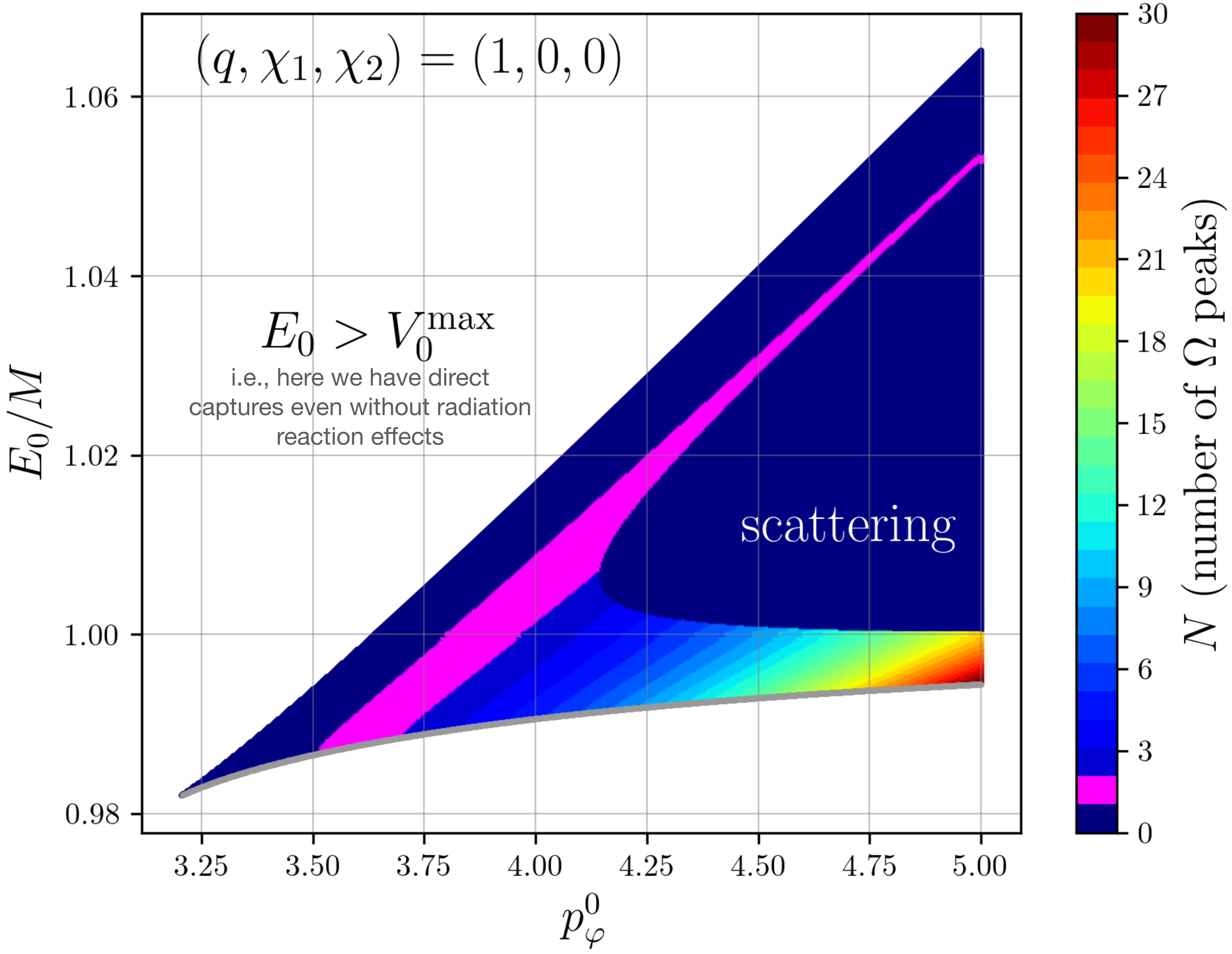
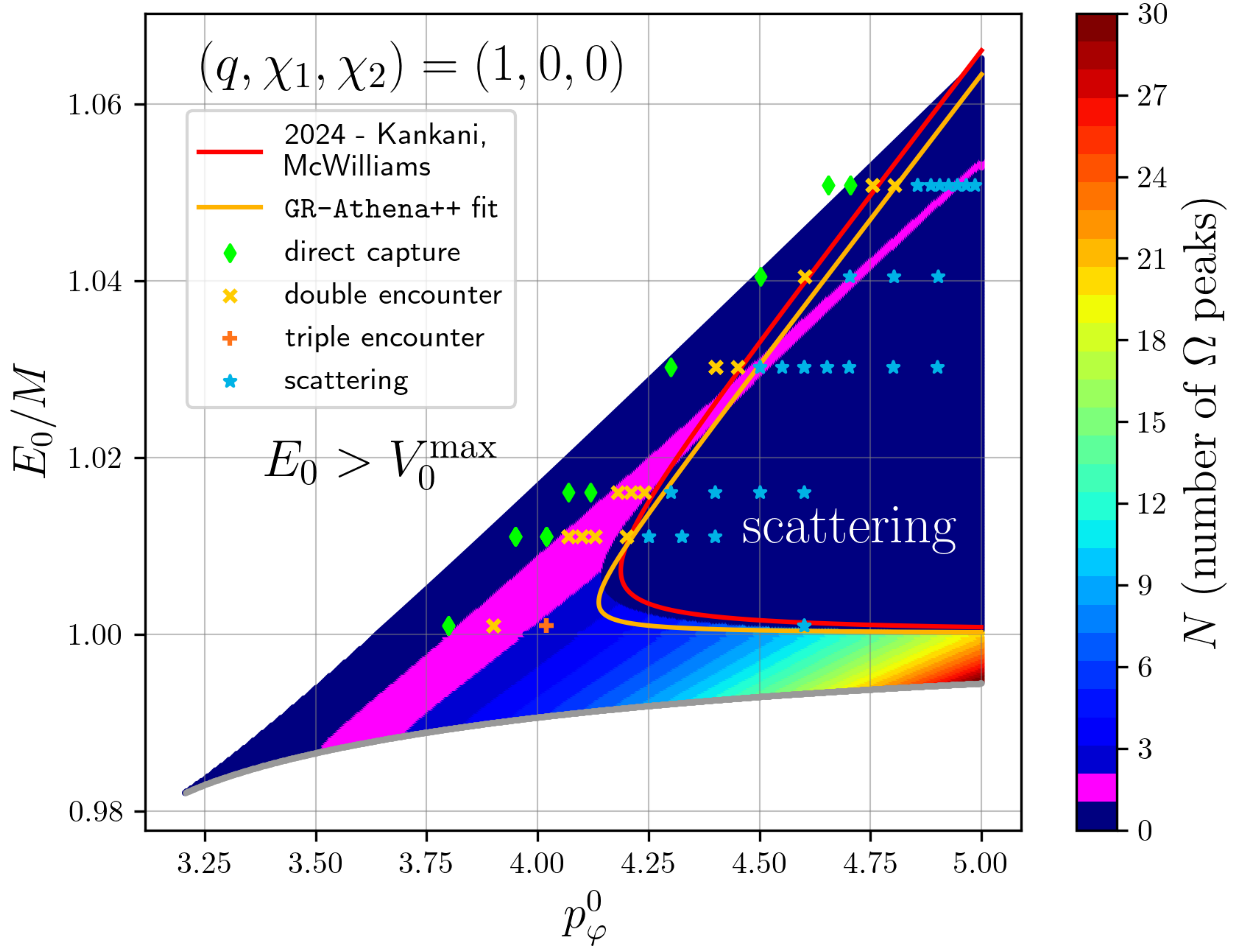


Parameter space: TEOBResumS-Dalí



- Number of orbital frequency peaks N as a proxy for the number of encounters
- Parameter space dominated by **scatterings** and **direct captures** $(N = 1)$
- **Many encounters** if the initial energy is close to the parabolic limit $(E_0 = M)$. **Double encounters** also permitted for higher energies: **transition from scattering to capture**
- Rich phenomenology produced by Dalí, but how accurate is this picture?

Parameter space: TEOBResumS-Dalí and NR

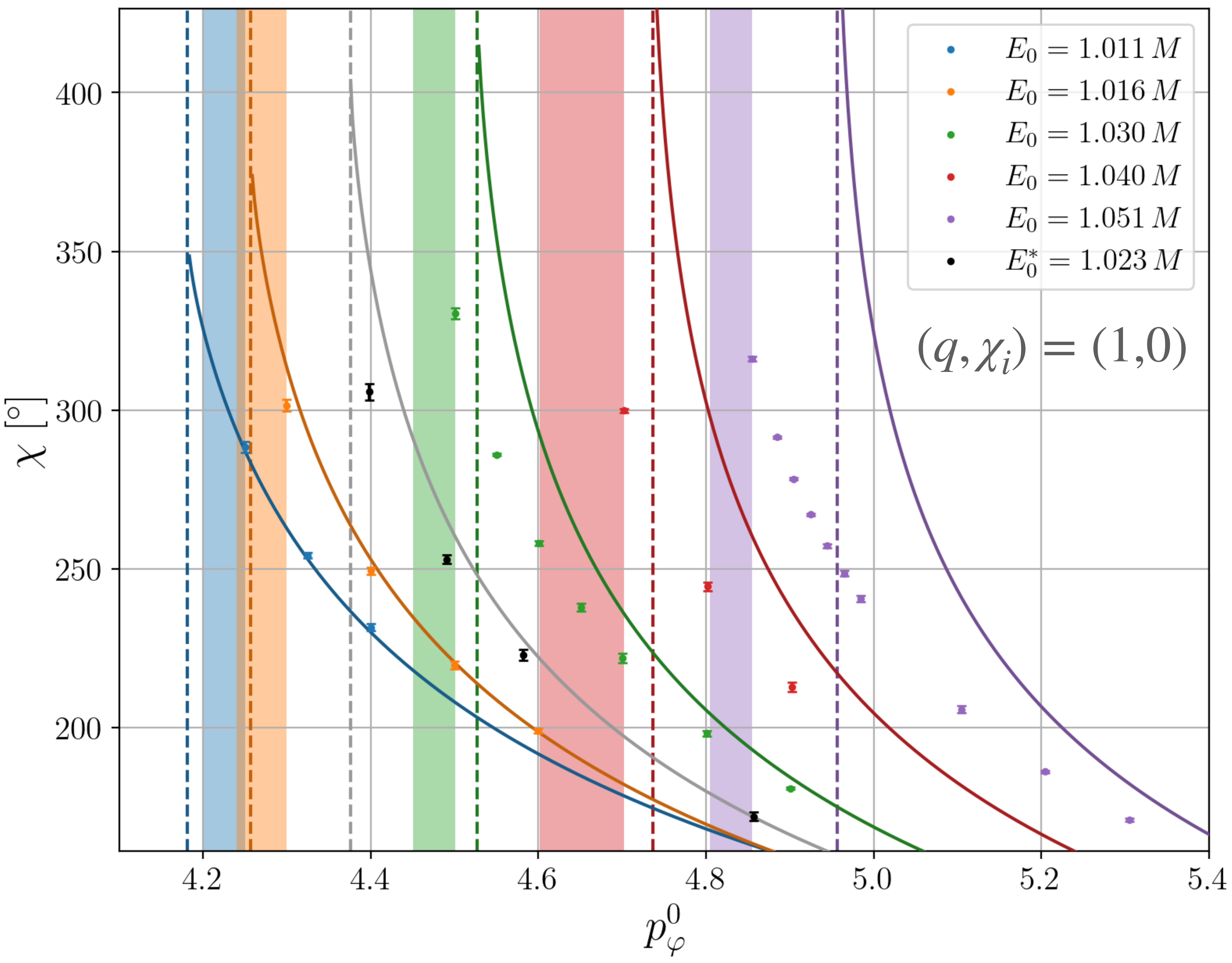


- Numerical relativity simulations performed with `GR-Athena++`
- EOB/NR phenomenologies agrees for $E_0 \lesssim 1.02\ M$ ($v_{\rm cm} \lesssim 0.2$)
- Dalí can **continuously span** the parameter space from quasi-circular binaries to hyperbolic systems
- EOB/NR deviations increase with the for higher energy/velocity (cfr. NR **fits** of the scattering-capture transition vs EOB one)
- Dalí built on PN results and informed only on quasi-circular NR simulations: **large room for improvement**

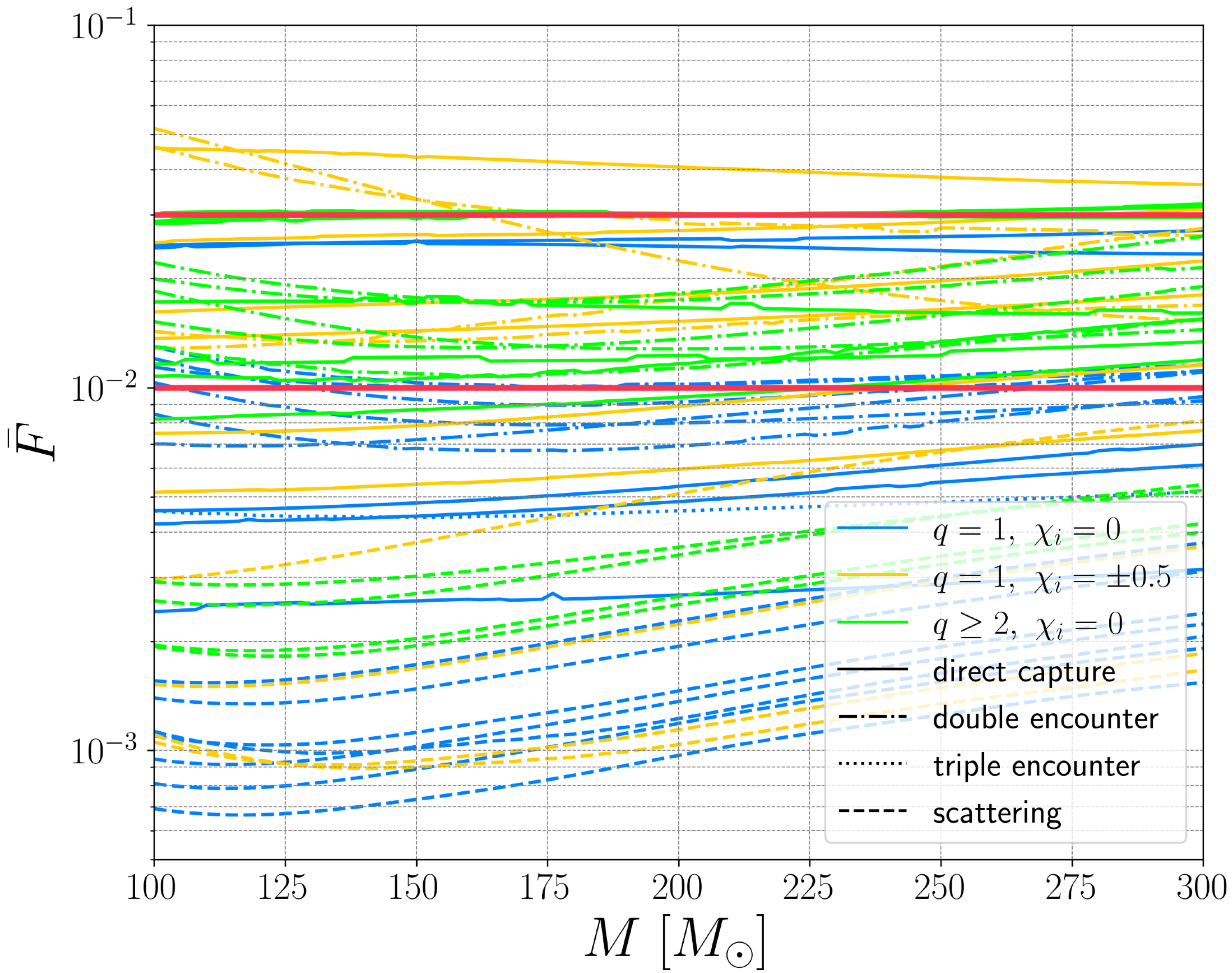
Scatterings

- **Scattering angles** computed extrapolating relative tracks
- Each color corresponds to an energy series: markers for NR, lines for EOB
- Scattering-capture transition marked by vertical bands (NR) and dashed lines (EOB)
- As before, the EOB/NR disagreement increases for higher energies
- Agreement restored for high angular momentum (weak field)
- We also considered scatterings of unequal mass binaries (not shown in this plot)

$E_0 = 1.023\,M$ from Damour+:1402.7307



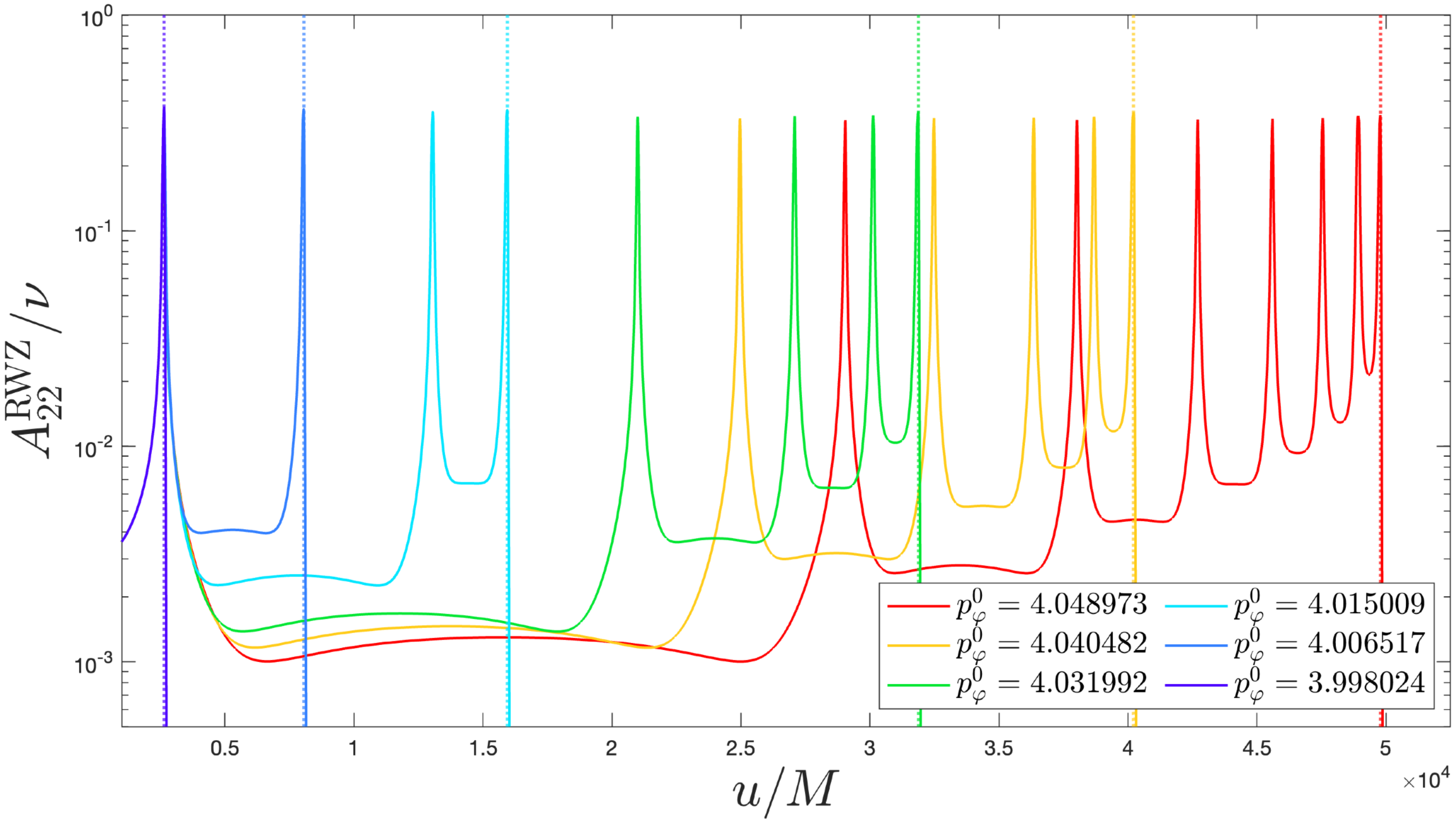
EOB/NR mismatches

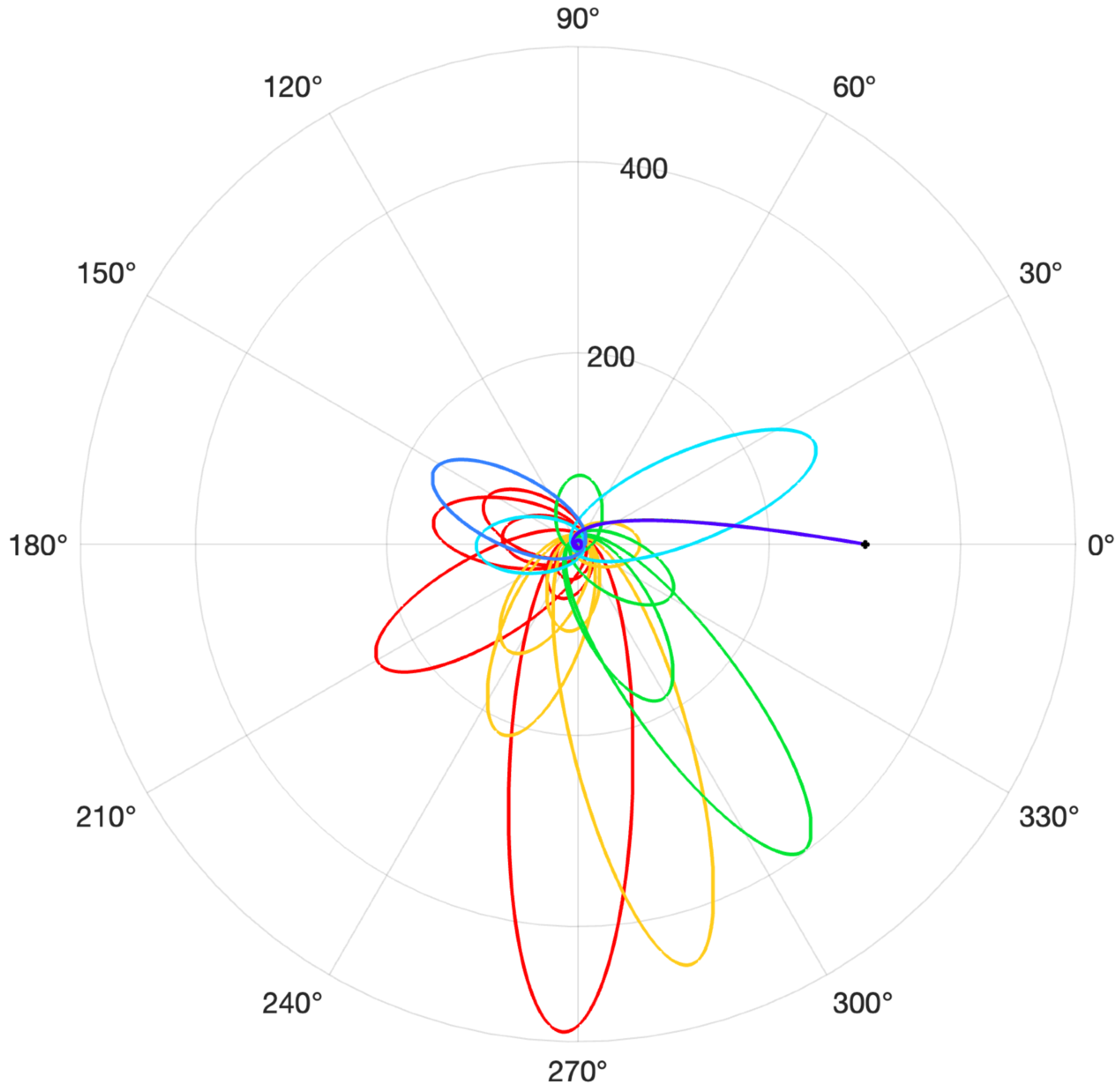


- (2,2) mismatches useful to quantify the accuracy of the model
- Configurations with $E_0 \lesssim 1.02\ M$
- Initial data optimization on a small region
- **Nonspinning** and **spin-aligned** ($|\chi_i| = 0.5$) equal mass + **higher mass ratios** up to $q = 3$ (nonspinning)
- Below/around 1% for most cases, some around 3%
- NR waveforms are far from being perfect! Issues when integrating the Weyl scalars. CCE / metric-perturbation-extraction?
- In the paper we also studied energetic curves

Test-mass: useful insights

- We studied a few test-mass cases: **dynamics** driven by a PN-radiation reaction
- Numerical **waveform** obtained by solving RWZ eqs. with `RWZHyp` [7,8]





[7] Bernuzzi,Nagar:1003.0597
[8] Bernuzzi,Nagar,Zenginoglu:1107.5402

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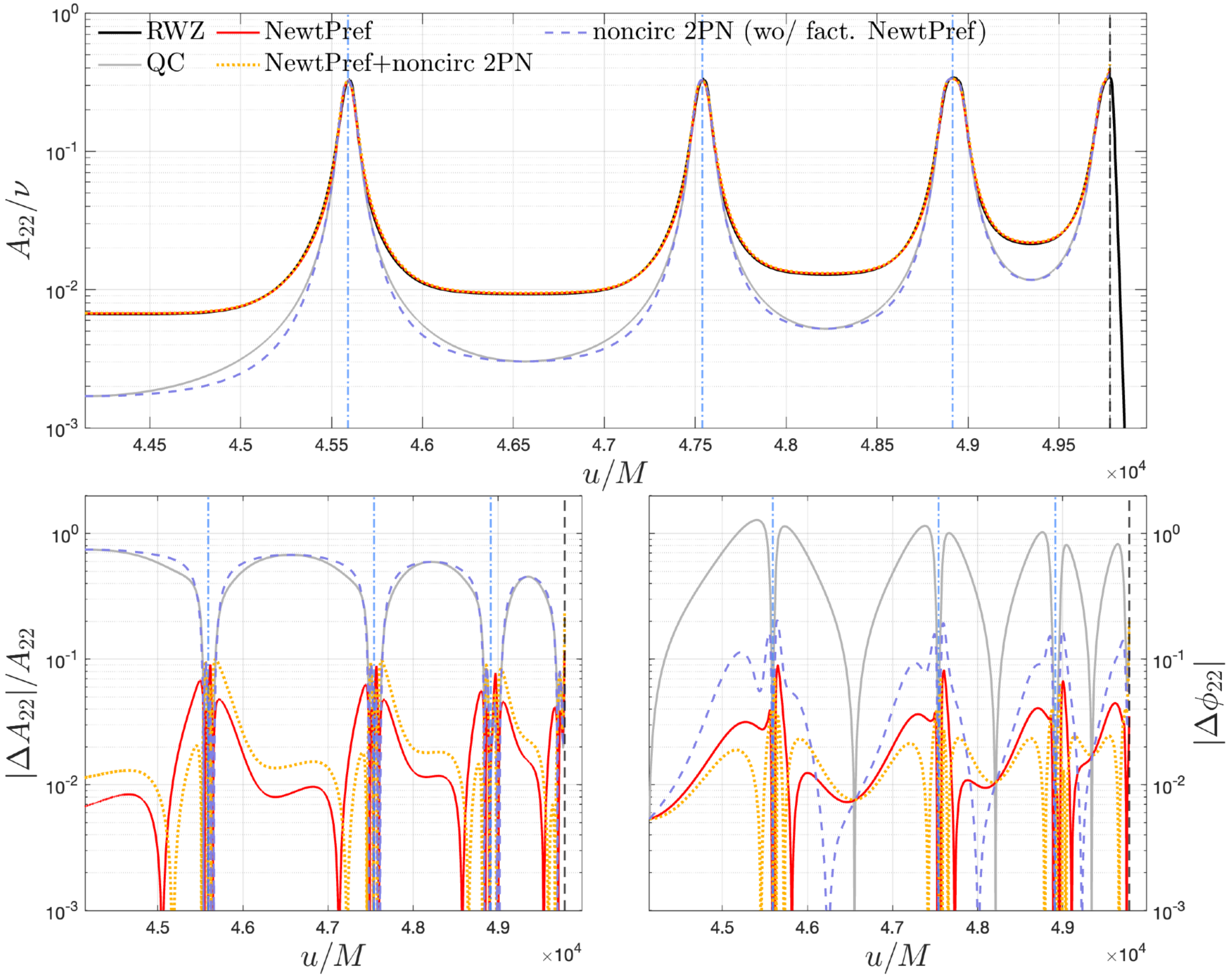
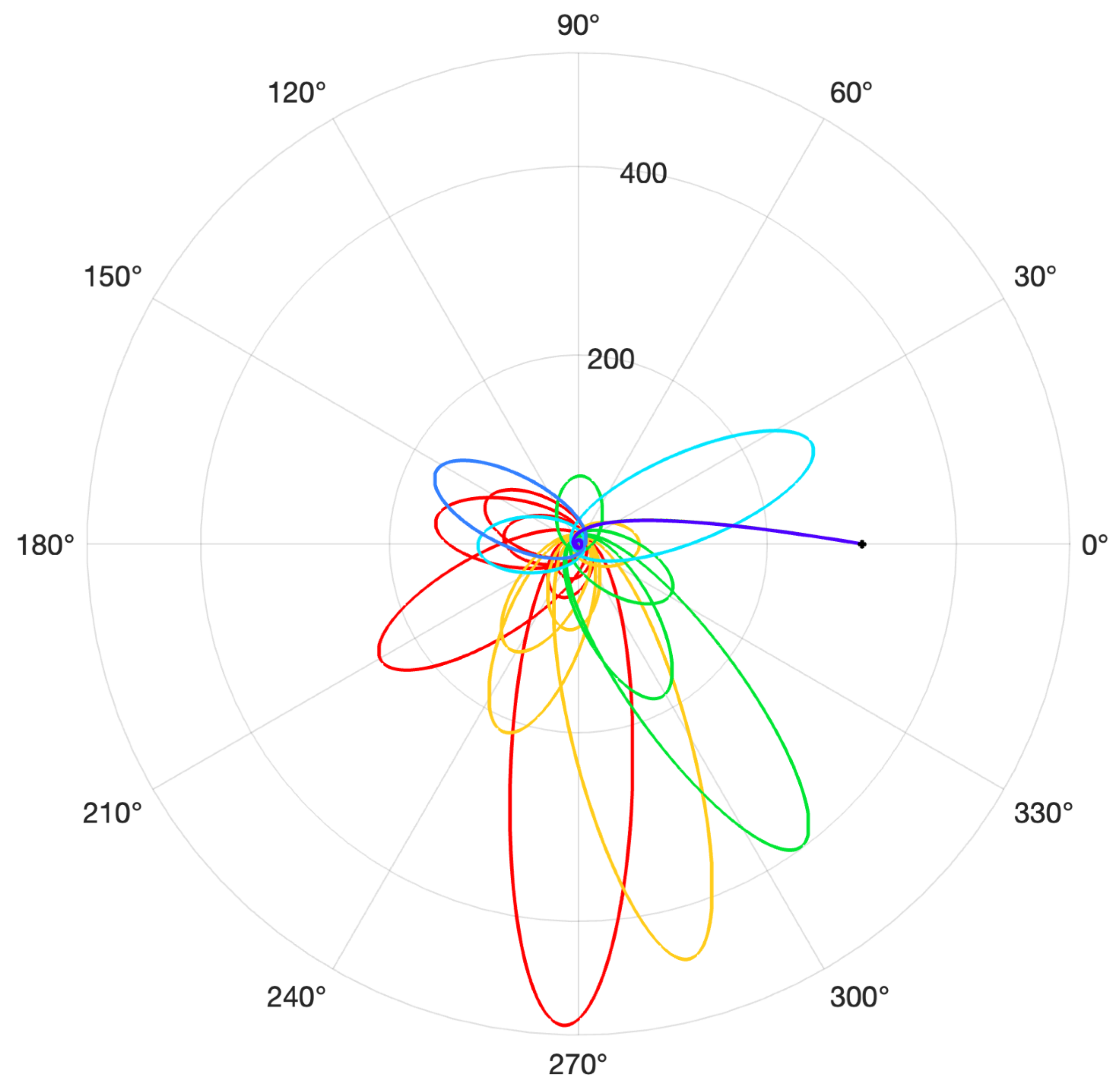


waveform associated to the last four close passages of the red trajectory



- We studied a few test-mass cases: **dynamics** driven by PN-radiation reaction
- **Numerical waveforms** obtained by solving RWZ eqs. in the time-domain
- Different **EOB waveforms** computed on the same dynamics
- Noncircular corrections with **explicit derivatives** (not PN-expanded in the Newtonian contribution) seem more reliable. In particular, they catch the low-frequency signal generated at large separations

Conclusions

- Exploration of **hyperbolic systems** with **NR** and **TEOBResumS-Dalí**
- EOB models can be used to describe these systems, with some caveats:
  - avoid explicit dependance on eccentricity
  - low-energy regime ($E_0 \lesssim 1.02\, M$) or large angular momentum (weak-field)
  - not too-high spins (to quantify), inherited from QC case
  - phenomenological ringdown model still calibrated on QC data
- Next steps to **increase/extend EOB accuracy**:
  - inclusion of post-Minkowskian results in the dynamical sector (Lagrangian EOB?)
  - NR-information from non-circularized binaries (e.g. ringdown)
- **Test-mass limit** always useful to gain insights on EOB models, but also on full-NR simulations

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Thank you for your attention!

Backup slides

Analytical corrections to radiation

- How can we describe noncircularized binaries?
  1. **Hamiltonian**: already generic, nothing to do (modulo calibration/non local terms)
  2. **Radiation reaction** $(\mathcal{F}_\varphi, \mathcal{F}_r)$: noncircular corrections in $\mathcal{F}_\varphi$
  3. **Waveform**: noncircular corrections in each multipole

- Noncircular terms:
  - $\mathcal{F}_r$ from generic energy/momentum balance eqs. [6]
  - Generic "Newtonian" prefactor [7,8], e.g. $\hat{h}_{22}^{(N,0)_{\rm nc}} = 1 - \frac{\ddot{r}}{2r\Omega^2} - \frac{\dot{r}^2}{2r^2\Omega^2} + \frac{2{\rm i}\dot{r}}{r\Omega} + \frac{{\rm i}\dot{\Omega}}{2\Omega^2}$ **(crucial thing: $\dot{r}, \dot{\Omega}$,... are not PN-expanded)**
  - Extended noncircular corrections up to 2PN [9-12]: improvement of the waveform

$$h_{\ell m}^{\rm inspl} = h_{\ell m}^{(\epsilon,{\rm N})_{\rm c}} \hat{h}_{\ell m}^{\rm c} \hat{h}_{\ell m}^{(\epsilon,{\rm N})_{\rm nc}} \hat{h}_{\ell m}^{\rm nc},$$

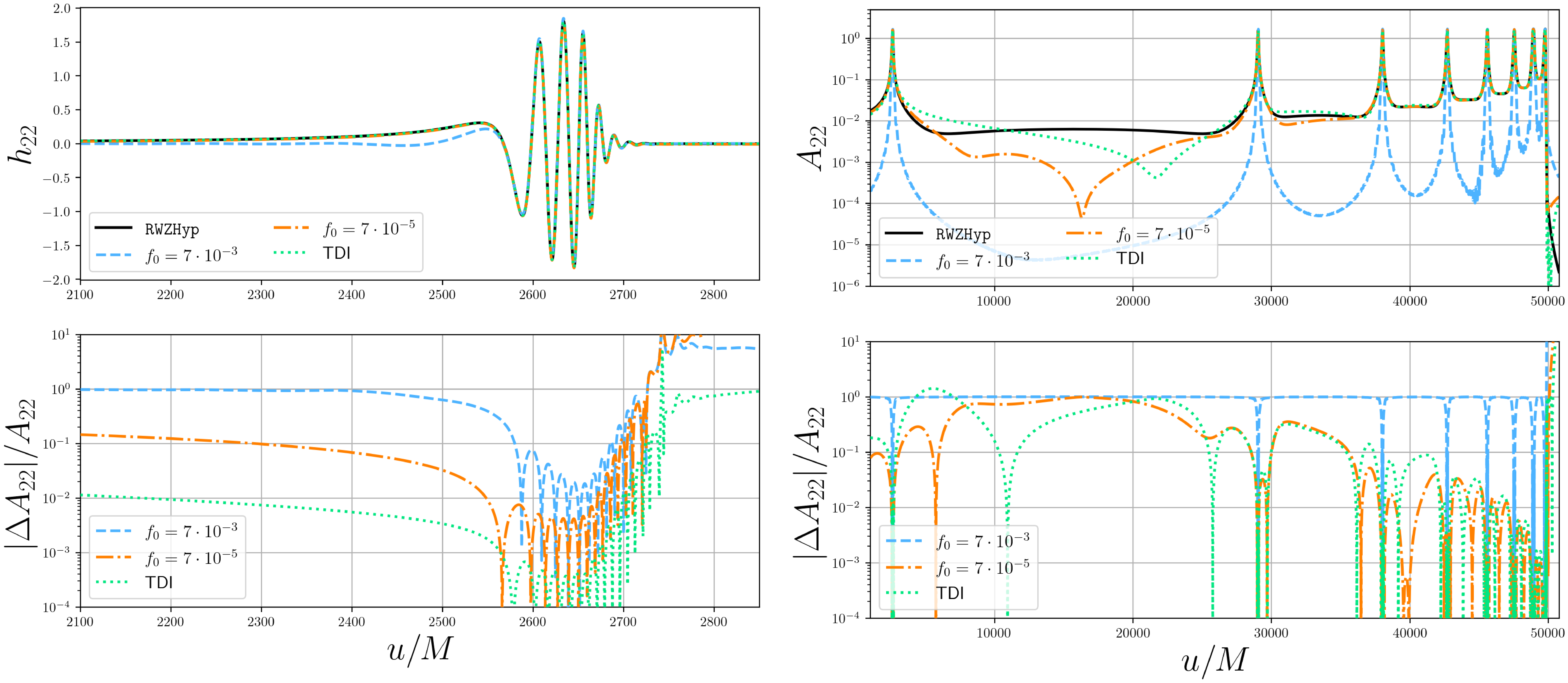
$$\mathcal{F}_\varphi = -\frac{32}{5}\nu r_\Omega^4 \Omega^5 \hat{f}_{{\rm nc}_{22}},$$

$$\hat{h}_{\ell m}^{\rm nc} = \hat{h}_{\ell m}^{\rm inst} \hat{h}_{\ell m}^{\rm hered}$$

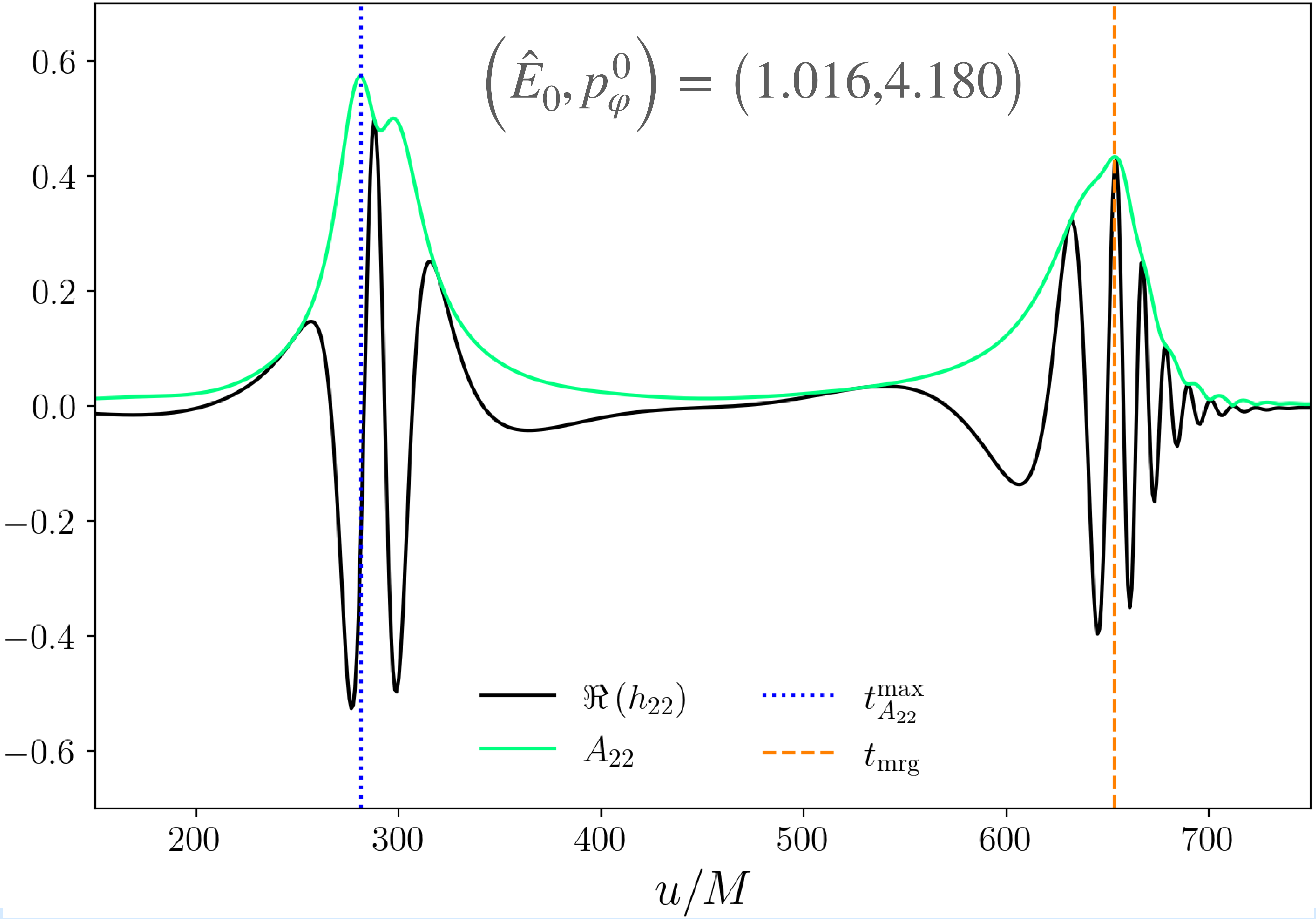
$$\hat{f}_{{\rm nc}_{22}} = \hat{F}_{22} \hat{f}_{\varphi,22}^{\rm N_{nc}} + \hat{F}_{21} + \sum_{\ell=3}^{8} \sum_{m=1}^{\ell} \hat{F}_{\ell m}$$

[6] Bini,Damour:1210.2834
[7] Chiaramello,Nagar : 2001.11736
[8] Albanesi+ : 2104.10559
[9] Khalil+:2104.11705
[10] Placidi+ : 2112.05448
[11] Albanesi+ : 2202.10063
[12] Albanesi+ : 2203.16286

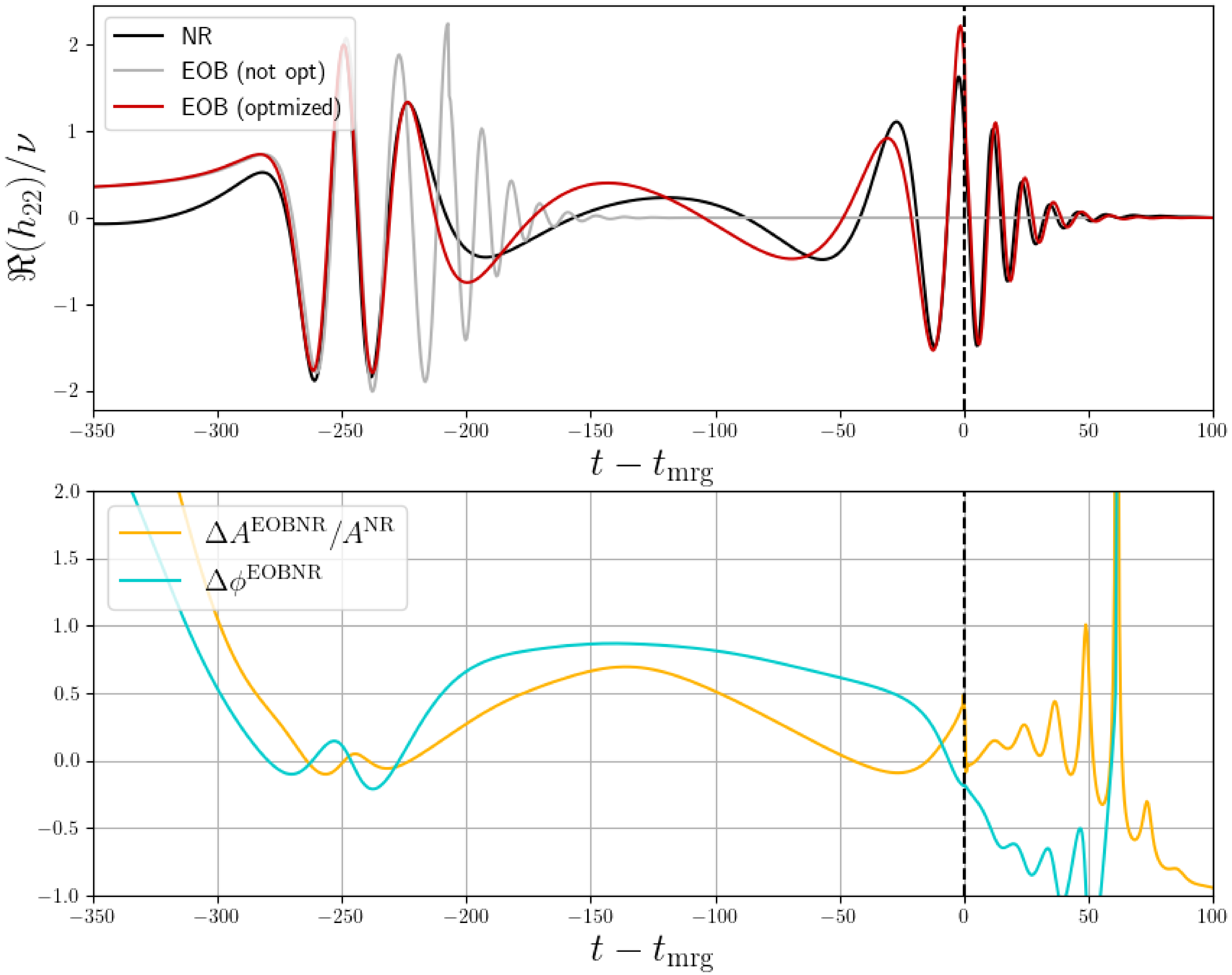
Test: integration of test-mass waveforms



NR: integration from ψ_4



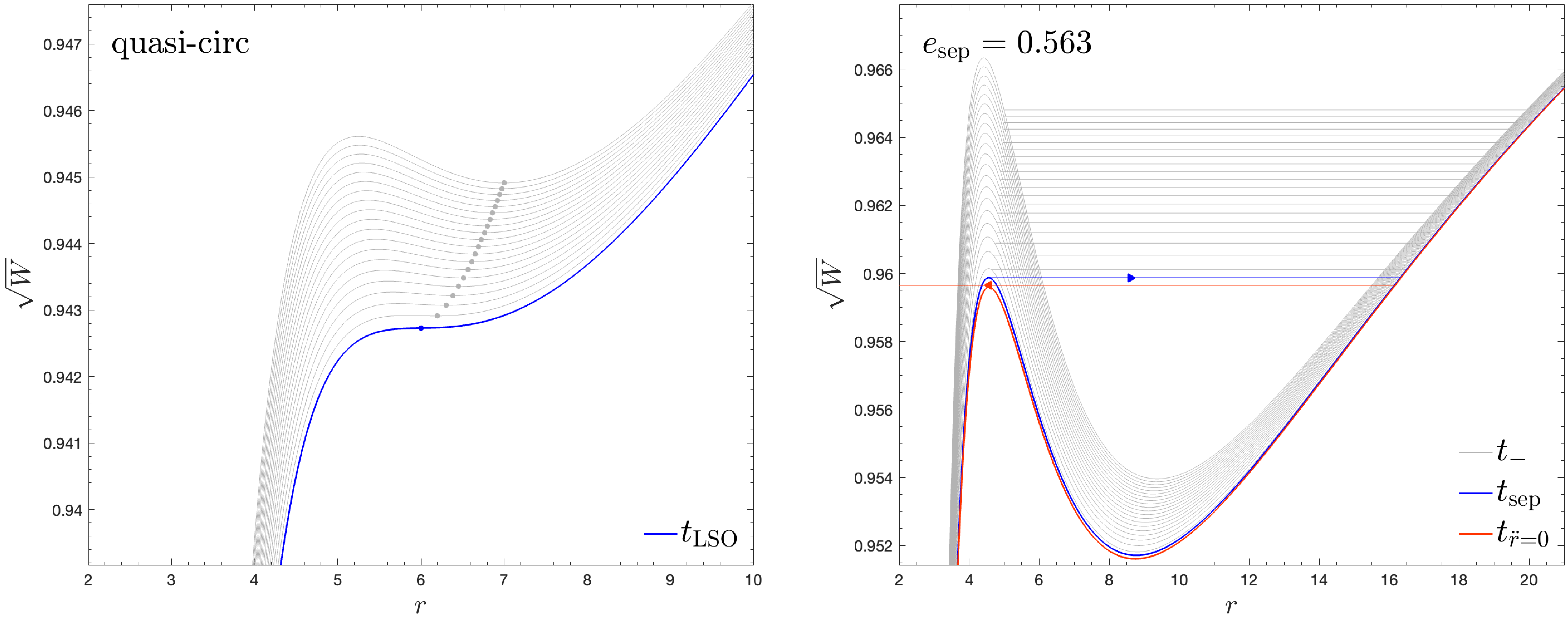
EOB/NR time-domain comparison



- Initial conditions in an ideal world: same $(E_0/M, J_0/M^2)$ for both EOB/NR, $r_0^{\rm EOB}$ from EOB/ADM 2PN coords transformation
- Practical issues, e.g. junk-radiation, introduce small variations and may lead to completely different phenomenologies
- Optimize EOB ICs to minimize unfaithfulness (mismatch), conceptually similar to what is done for elliptic binaries
- NR calibration on QC simulations (also for the ringdown)
- Artifacts in the NR waveform due to integration

Potentials for QC and elliptic cases

- Evolution of Schwarzschild potentials under the effect of EOB radiation reaction



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