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Effective One Body model for EMRIs

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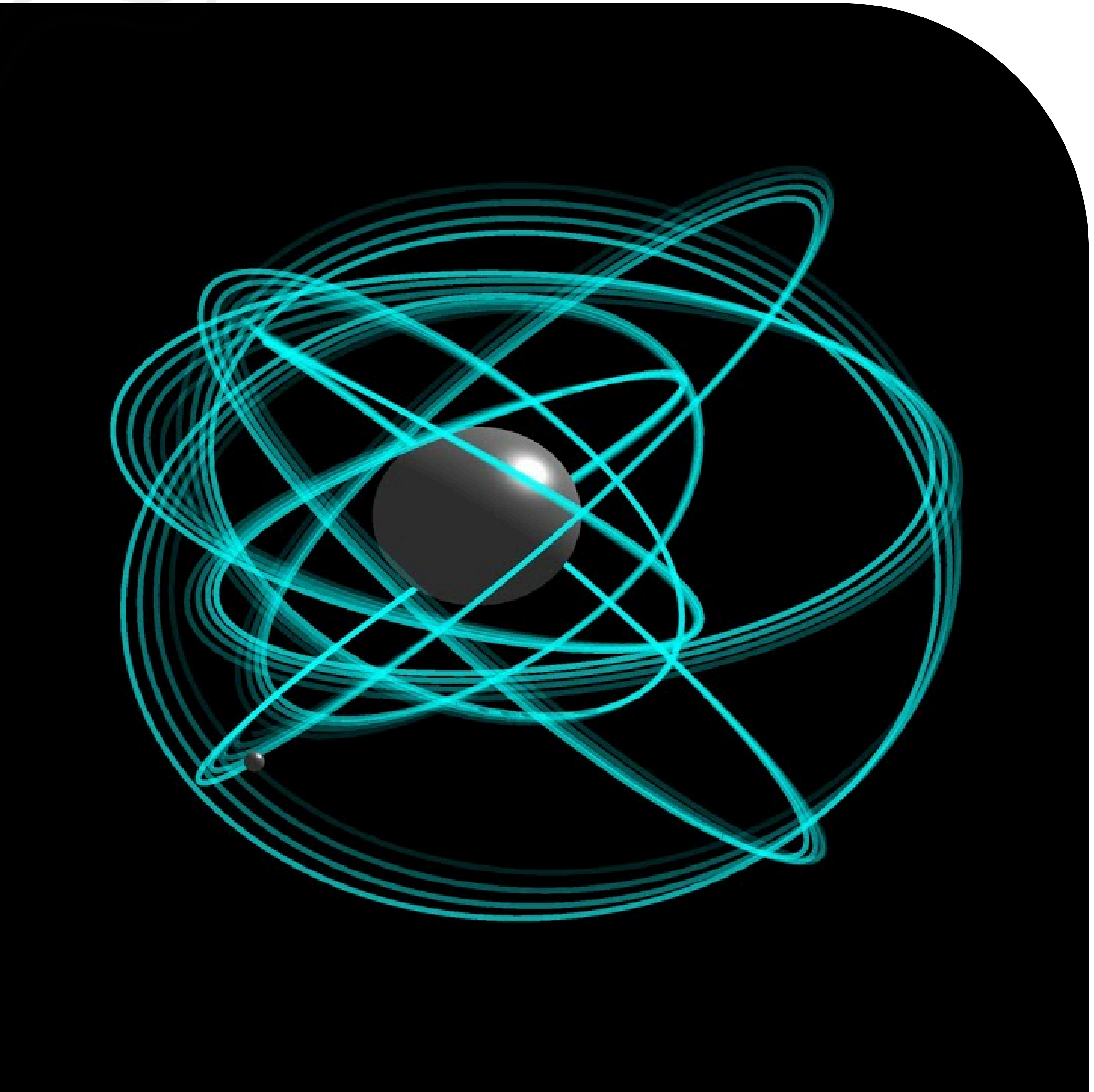
16/09/2024

1^st TEONGRAV Workshop, Rome

EMRIs



$$q \equiv \frac{m_1}{m_2} > 10^4$$

Extreme Mass Ratio Inspirals

- Two BH with very different masses
- Usually a SMBH and a Stellar Mass BH
- Year(s)-long inspiral
- Signal in the LISA band
- Superposed years long signals from different sources

EOB in a Nutshell

TECHNIQUES

EFFECTIVE METRIC

Deformation parameter

$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

ENERGY LEVEL MAPPING

Between the effective Hamiltonian and the real one

RESUMMATION

To extend the validity of the PN approximation

BUILDING BLOCKS

CONSERVATIVE DYNAMICS

The Hamiltonian we get from the metric with the three potentials A, D, and Q

THE WAVEFORM

From the PN approximated solution of Einstein Equation

RADIATION REACTION

Effect on the dynamics of the radiated energy and angular momentum

TEOBResumS

CALIBRATION AND BENCHMARK

- COMPARABLE MASSES → Numerical Relativity
- LARGE MASS RATIO → Gravitational Self Force

FEATURES

- Eccentricity
- Spin on both object
- Spin Precession

→ D. Chiaramello

- Dynamical Capture
- BH Scattering

→ S. Albanesi, P.Rettegno

- Binary Neutron Stars → R. Gamba

Gravitational Self Force

- Deviation from the test mass case due to the second object's gravitational field
- Nowdays 2GSF calculation : 2^nd order in $\epsilon = \frac{m_2}{m_1} \leq 1$

B.Wardell, 2023, 2112.12265

- Two timescale expansion → Slow: radiation reaction; → Fast: orbital dynamic

EOB Like

LMR Analytical Framework

$$p_{r_\star} \equiv \left(\frac{A}{B}\right)^{1/2} p_r$$

$$u \equiv \frac{1}{r}$$

$$\hat{H}_{\mathrm{EOB}} \equiv \frac{H_{\mathrm{EOB}}}{\mu} = \nu^{-1}\sqrt{1 + 2\nu(\hat{H}_{\mathrm{eff}} - 1)}$$

$$\hat{H}_{\mathrm{eff}} = \hat{H}_{\mathrm{orb}} + \hat{H}_{SO}$$

$$\hat{H}_{\mathrm{orb}} = \sqrt{A(u)\left[(1 + p_\varphi u^2) + Q(u, p_\star; \nu)\right] + p_{r_\star}}$$

$$\hat{H}_{SO} = p_\varphi \left(G_S\left(r, p_{r_\star}^2\right) S + G_{S_\star}\left(r, p_{r_\star}^2\right) S_\star\right)$$

EOB Potentials

$$A(u;\nu) = 1 - 2u + \nu a_{1\mathrm{SF}}(u) + O(\nu^2)$$

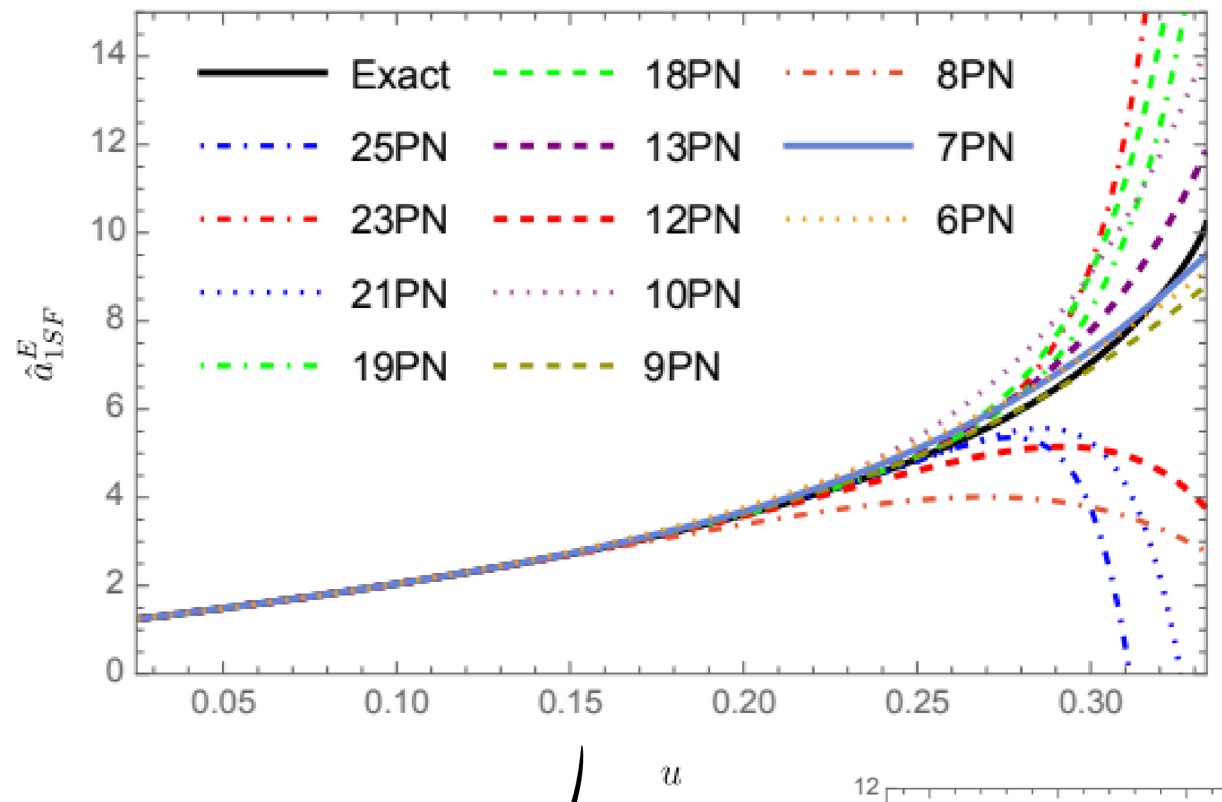
$$\bar{D}(u;\nu) = \frac{1}{AB} = 1 + \nu \bar{d}_{1\mathrm{SF}}(u) + O(\nu^2)$$

$$Q\left(u, p_{r_\star}; \nu\right) = \nu q_{1\mathrm{SF}}(u) p_{r_\star}^4 + O(\nu^2)$$

The SF functions are calculated analytically in D.Bini +, 2014, 1403.2366 and numerically in S. Akcay +, 2012, 1209.0964; S. Akcay +, 2013, 1308.5223; M. van de Meent +, 2015, 1506.04755 S. Akcay +, 2016, 1512.03392

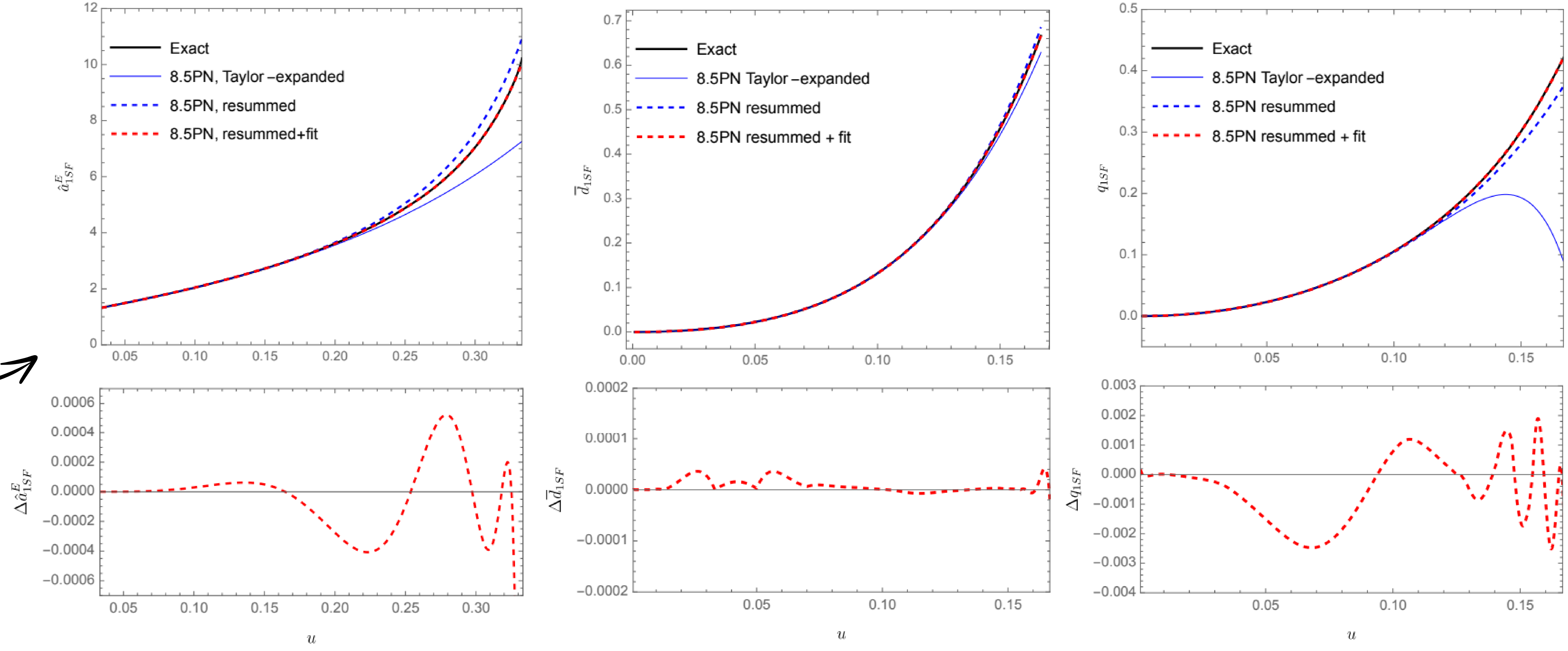
Potentials

EOB potentials resummed at 8.5 PN with GSF fit. A.Nagar&S. Albanesi, 2022.,2207.14002



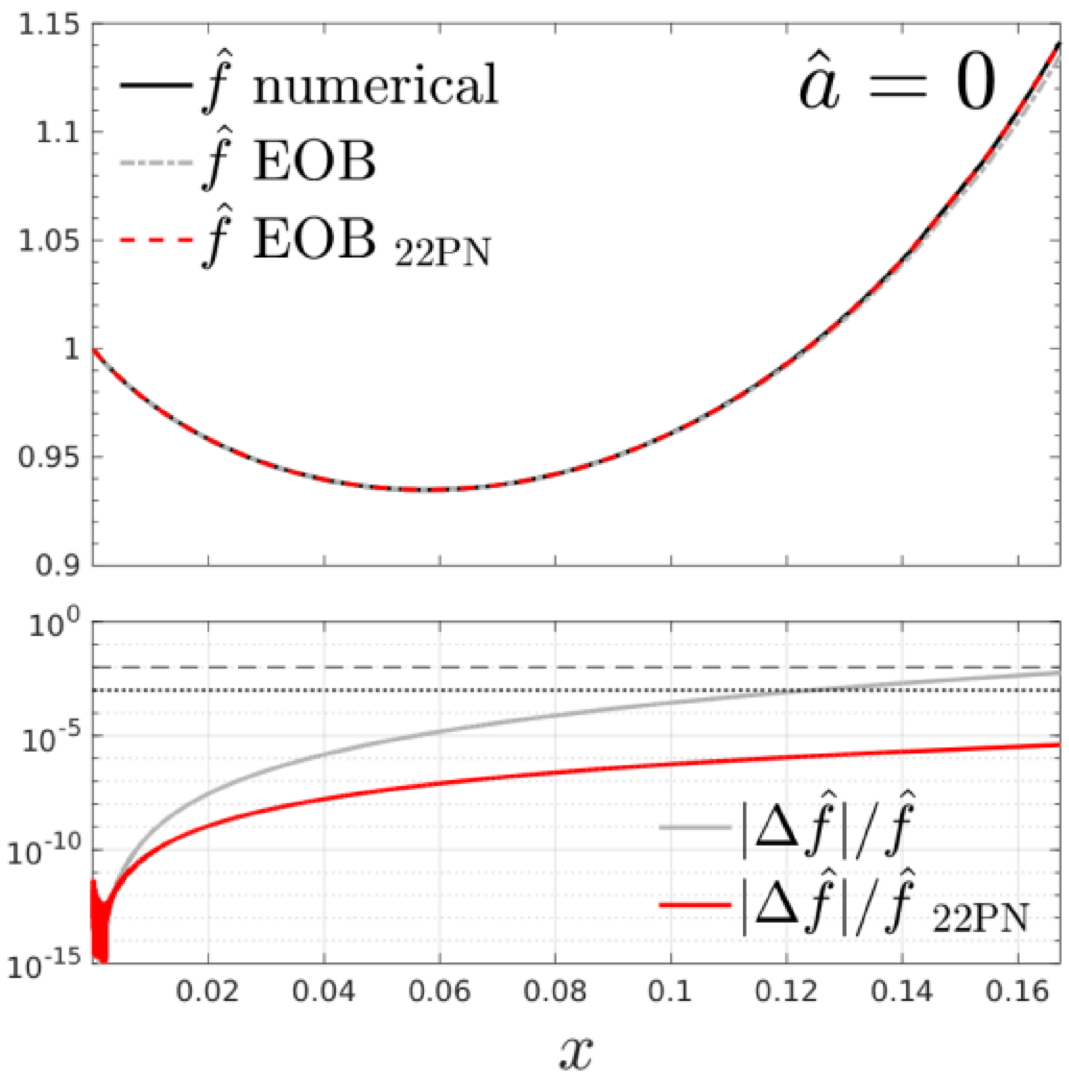
PN SERIES DIVERGE

RESUM AND FIT



Radiation Reaction

In test mass limit, Nagar&Albanesi, 2022. 2207.14002



$$F_{\ell m} = \frac{1}{8\pi} m^2 \Omega^2 |h_{\ell m}(x)|^2$$

$$\hat{f} = \sum_{\ell=2}^{\ell_{\max}} \sum_{m=1}^{\ell} \left(F_{22}^N\right)^{-1} F_{\ell m}, \quad \text{where} \quad F_{22}^N = \frac{32}{5}\nu^2 x^5.$$

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)} \hat{S}_{\text{eff}}^{(\epsilon)} \hat{h}_{\text{tail}} \rho_{\ell m}^{\ell}$$

Known in closed form

PN expanded residual, eventually factorized and resummed

22PN flux in non spinning test mass limit computed from R.Fujita, 2012, 1211.5535

EOB flux is mostly 6PN, with spinning contribution from F. Messina, 2018, 1801.02366

Analytical waveform

An example with eccentricity and spin on both objects

Nagar&Albanesi, 2022. 2207.14002

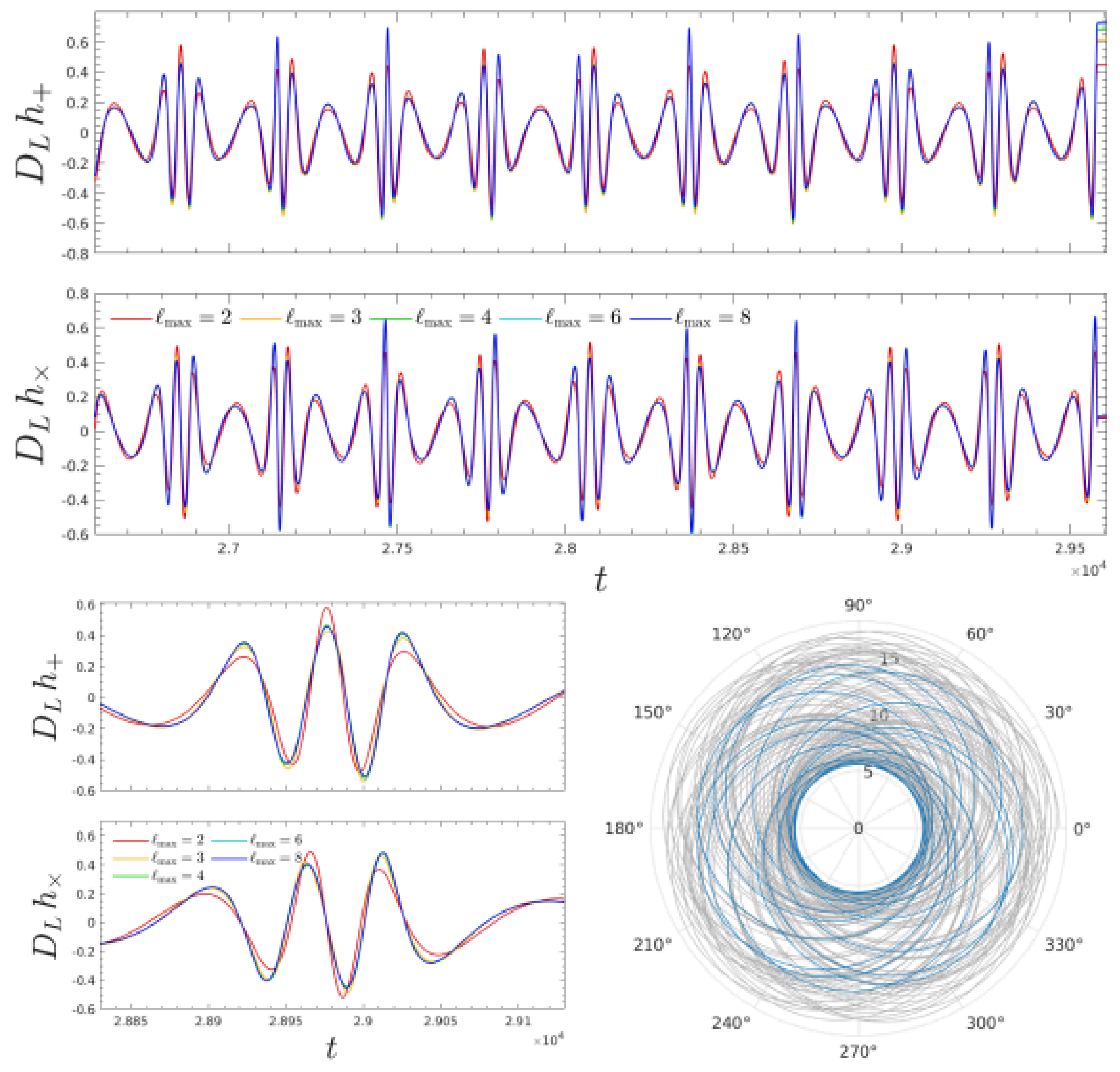
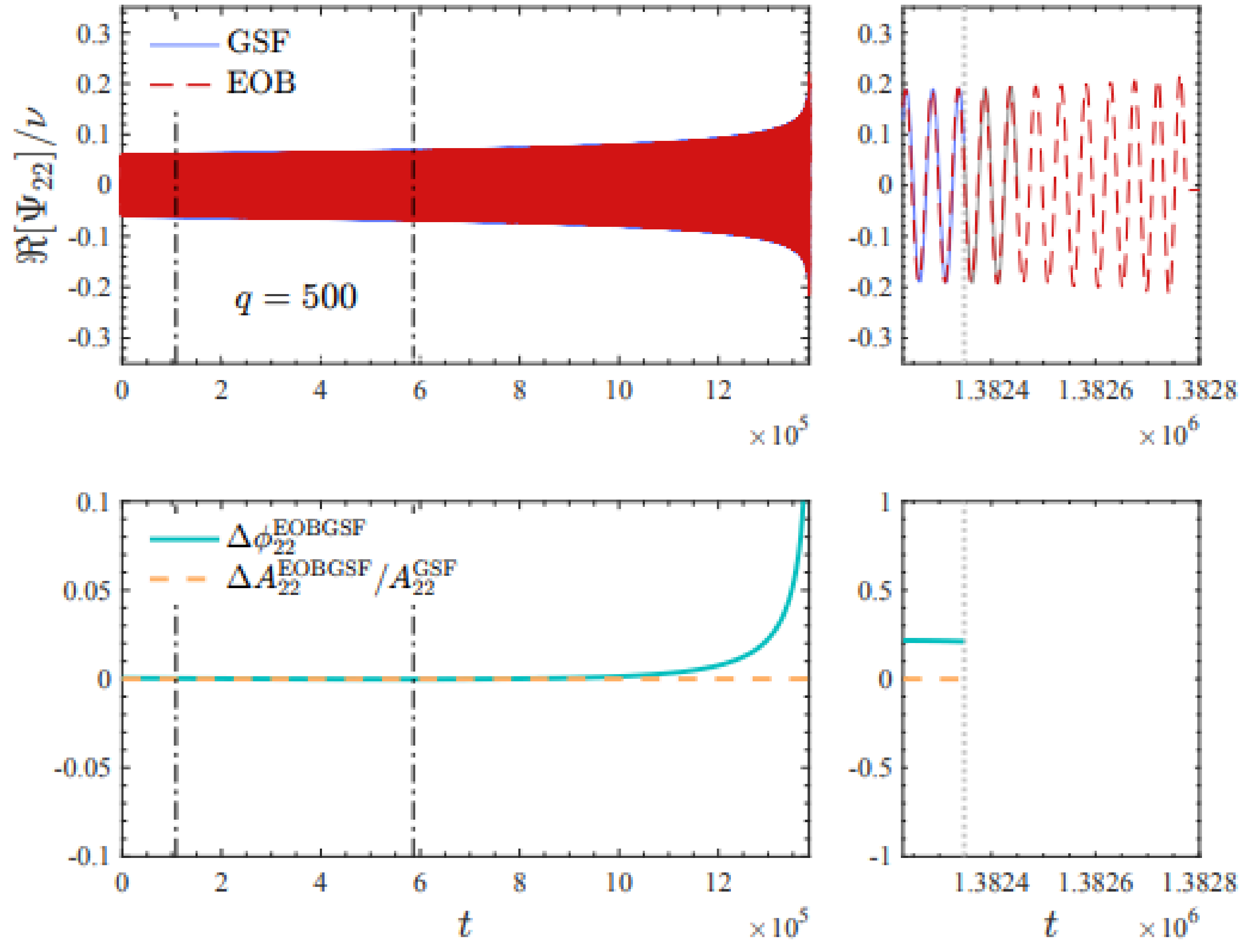


FIG. 5. Strain generated by a binary with $q = 10^3$, $e_0 = 0.5$, $\chi_1 = 0.3$, and $\chi_2 = 0.1$, as seen by an observer whose line of sight is inclined by 45° with the orbital plane. In order to highlight the relevance of the higher modes, we show the strain computed using different values for $l_{\max}$. We do not include $m = 0$ modes. We also show a zoom on a radial period (left bottom panel). The portion of the strain shown in the upper panel corresponds to the trajectory highlighted in blue in the right bottom panel.

EOB-GSF Comparison

Only 22 mode. A.Albertini + , 2022, 2208.02055



Time domain phasing: time and phase shift to minimize the rms over a chose interval

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\Delta\phi\left(t_i,\tau,\alpha\right)\right)^2}$$

$$\Delta\phi\left(t_i,\tau,\alpha\right) = \left(\phi_2(t_i-\tau)-\alpha\right)-\phi_1(t_1)$$

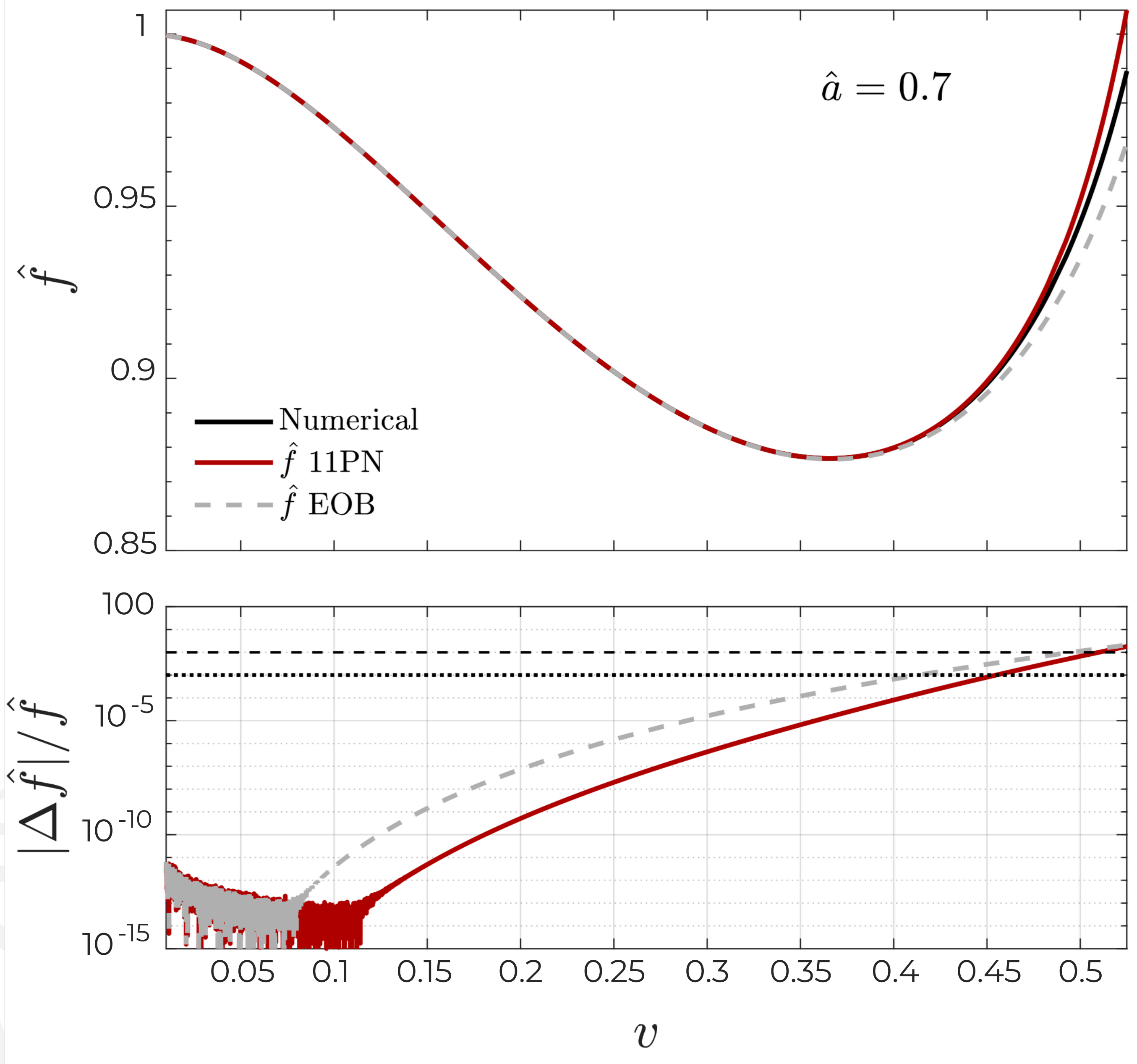
GSF $\quad \Psi_{22}^{1} = A_{22}^{1}(t_1)e^{-i\psi_1(t_1)}$

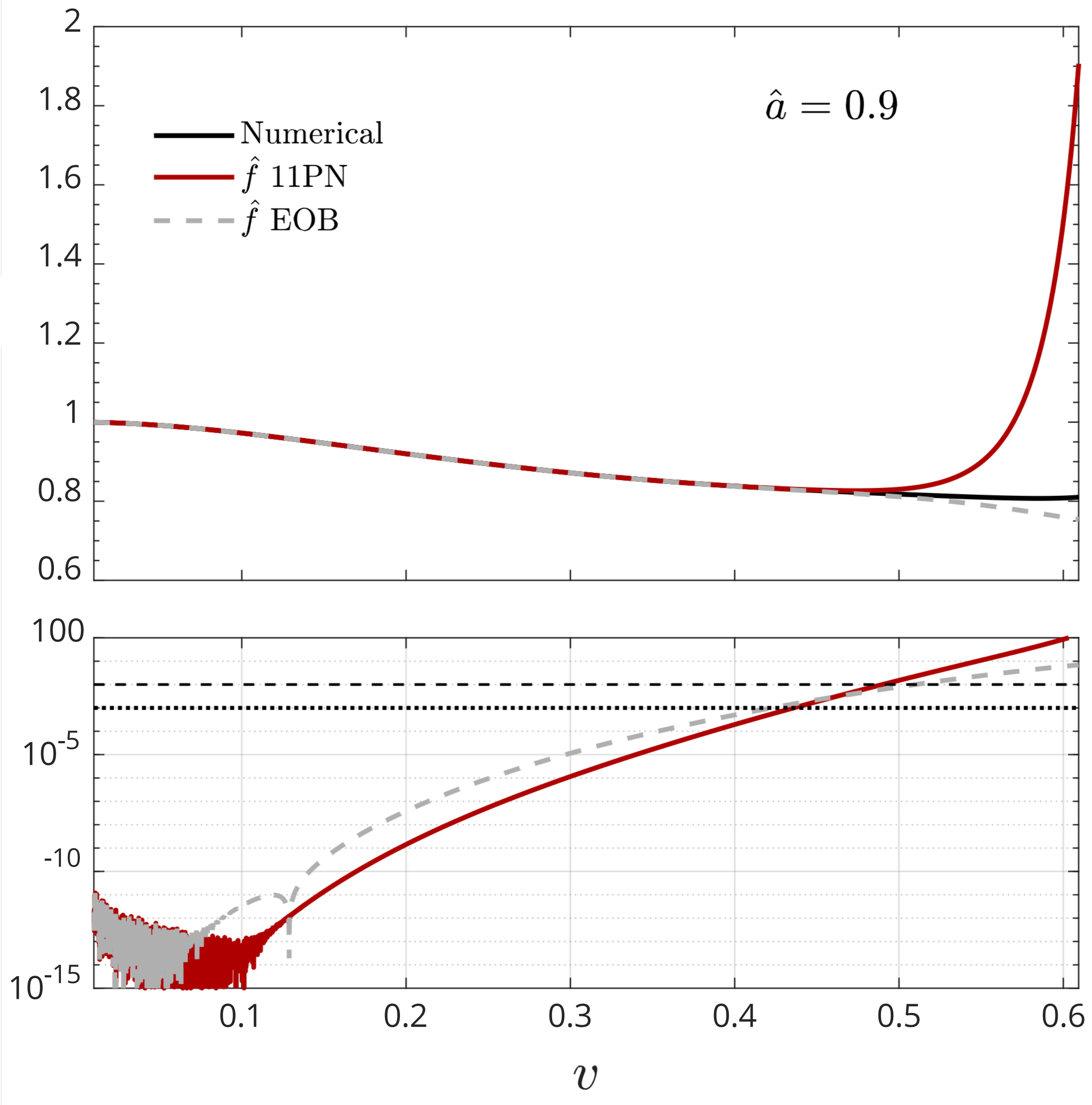
EOB $\quad \Psi_{22}^{2} = A_{22}^{2}(t_2-\tau)e^{-i[\psi_2(t_2-\tau)-\alpha]}$

Improving the resummation

- Wanting to include spin in the test mass limit
- 11PN computation (R. Fujita, 2015, 1412.5689)
- Numerical fluxes (S.Hughes +, 2013, 1305.2184)

DOES NOT WORK FOR HIGH SPIN!





New Resummation

Working on the $\rho_{\ell m}$ to choose the right PN order to Padè resum in a slightly different way

$$\rho_{\ell m} = \underbrace{1 + a_1 x + a_2 x^2 + \ldots}_{\rho_{\ell m}^{\mathrm{R}}} + b_1 x^3 \log x \,\underbrace{(1 + a_1' x + \ldots)}_{\rho_{\ell m}^{\log}} + \ldots$$

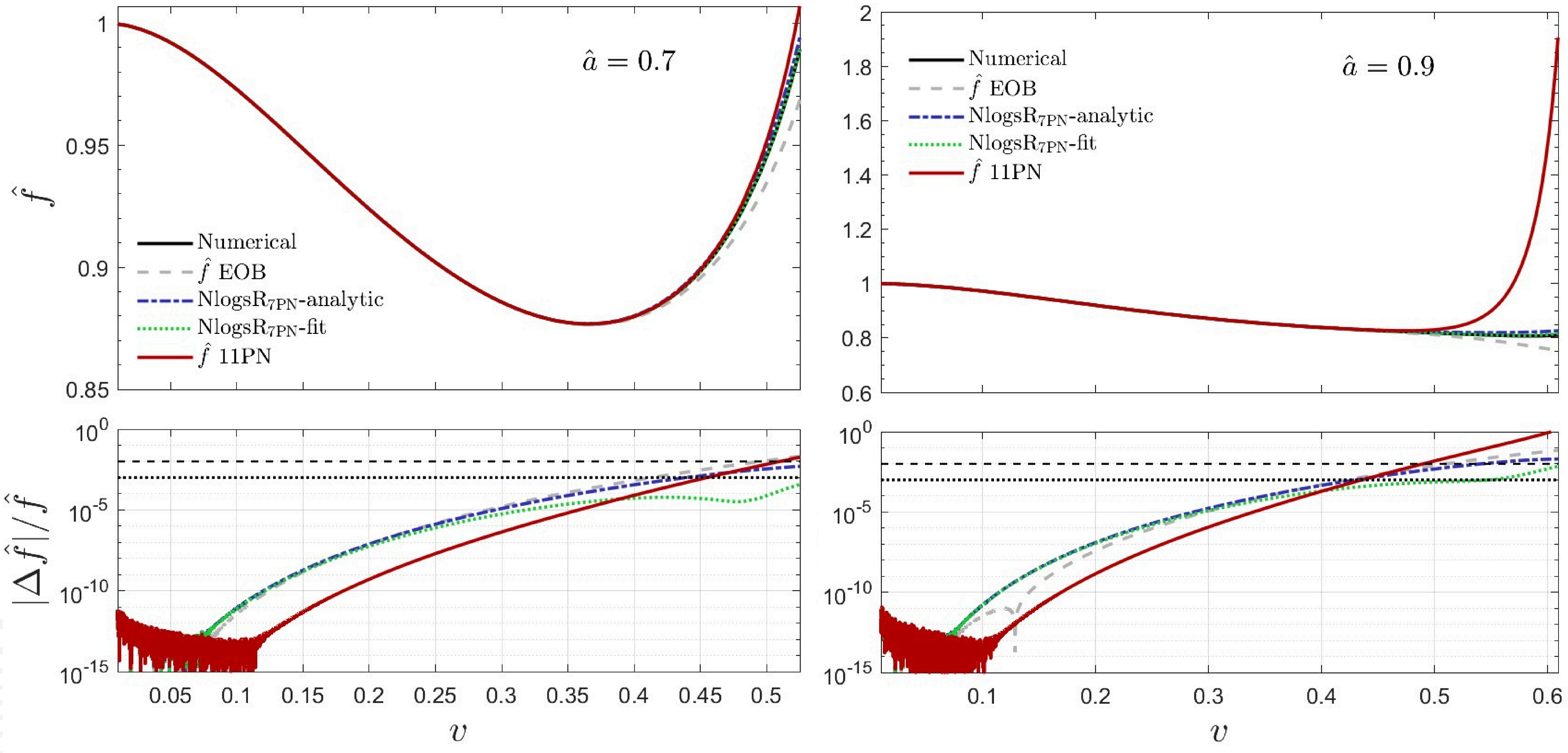
OLD → **Logarithmic terms as constant inside the coefficient for the Padè resummation**

NEW → **Resum only the rational coefficient and in case, separetly, the logarithmic coefficient**

BEST RESULT:

Working mostly at 6PN with the main modes (2,2),(2,1),(3,2),
(3,3),(4,4) taken at 7PN and fitted with a numerical test mass
result using a 7.5 and a 8 PN parameter

Results



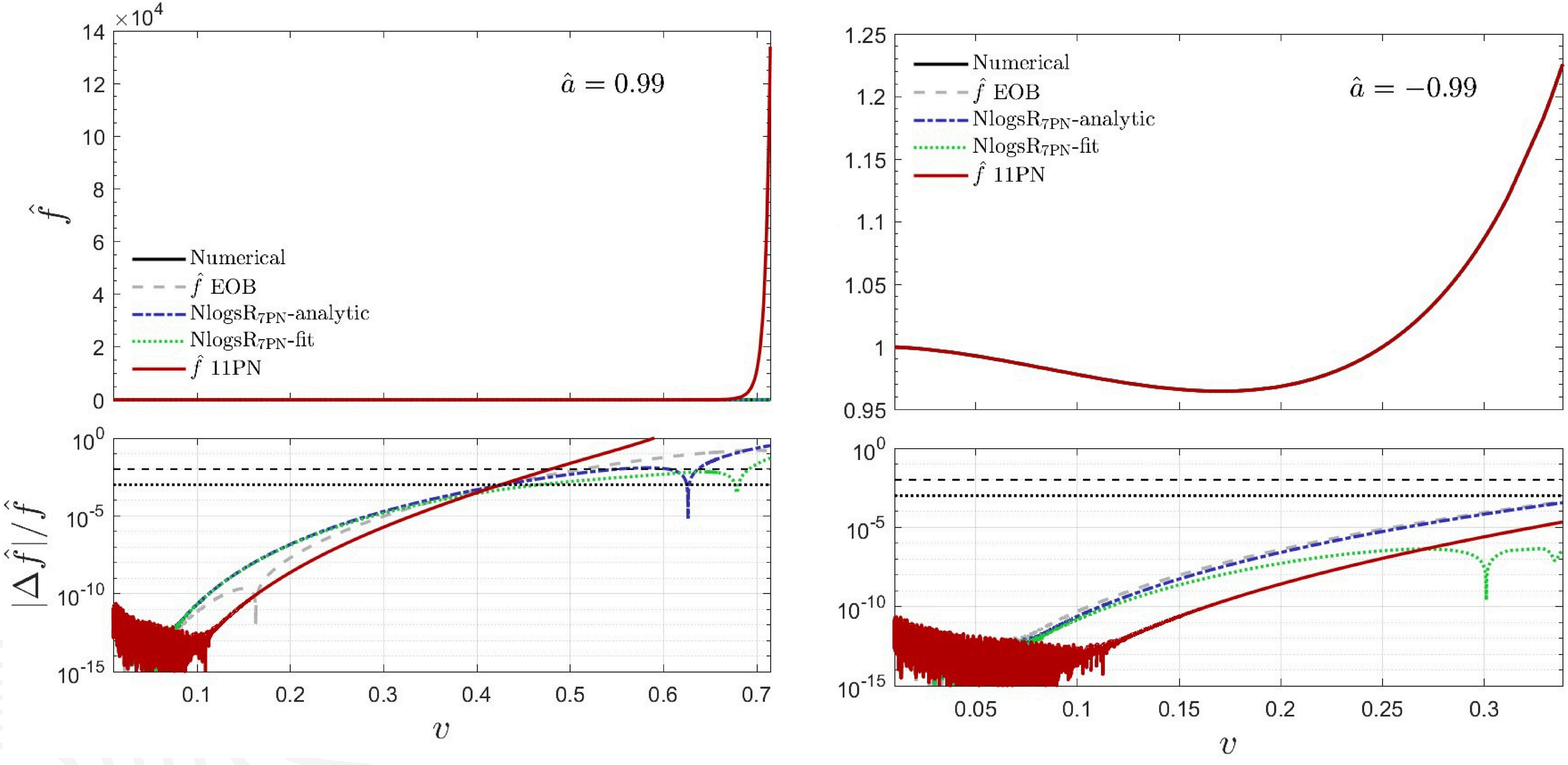
$\hat{a} = 0.7$
$\hat{a} = 0.9$
Numerical
$\hat{f}$ EOB
NlogsR$_{7PN}$-analytic
NlogsR$_{7PN}$-fit
$\hat{f}$ 11PN
$\hat{f}$
$|\Delta \hat{f}| / \hat{f}$
v
1
0.95
0.9
0.85
2
1.8
1.6
1.4
1.2
0.8
0.6
10^{0}
10^{-5}
10^{-10}
10^{-15}
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.6

Conclusions

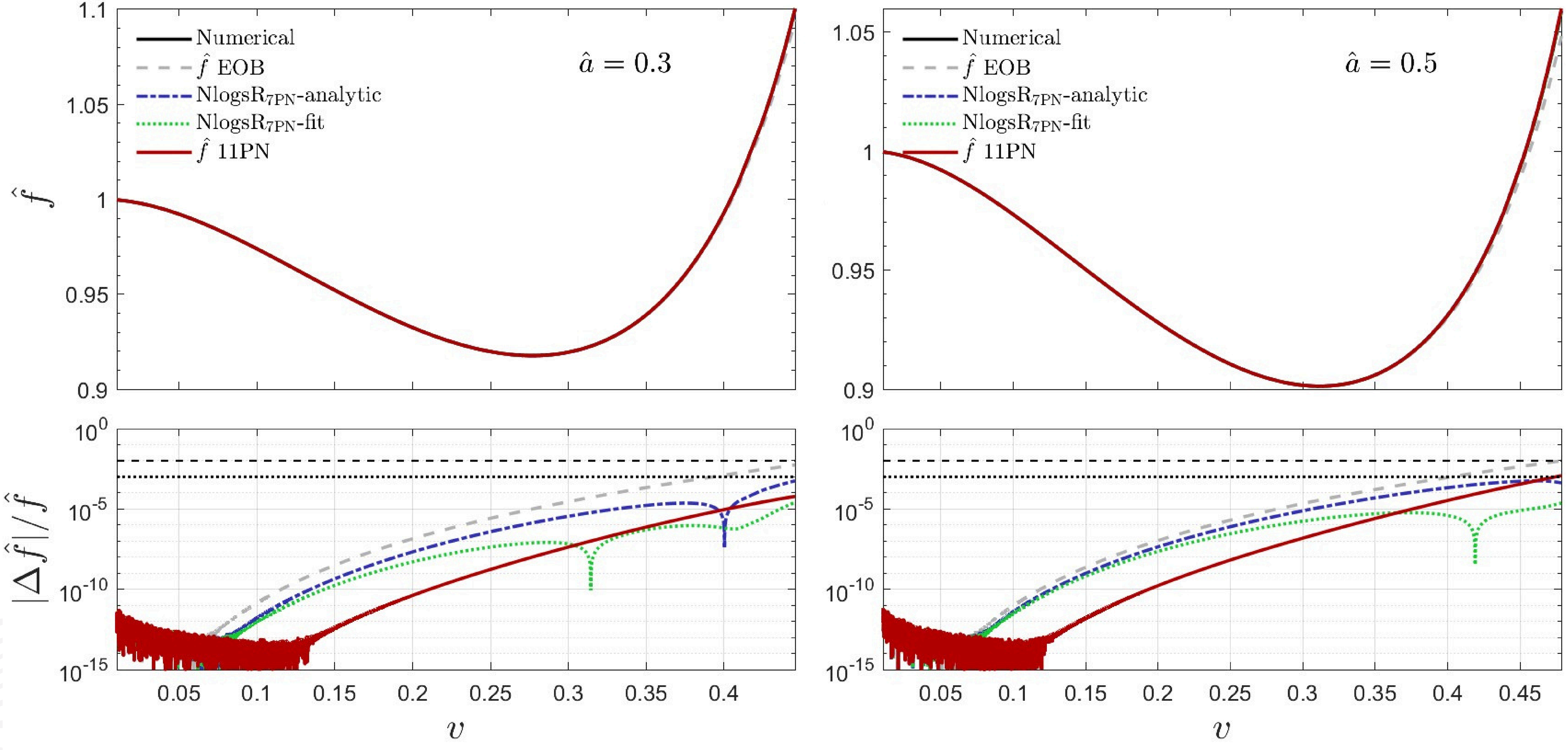
- EOB can generate accurate EMRIs waveform;
- EOB could be used for data analysis;
- The new resummation improves the flux.
- TEOBResumS is publicly avaliable at https://bitbucket.org/teobresums/teobresums/src

Thank you!

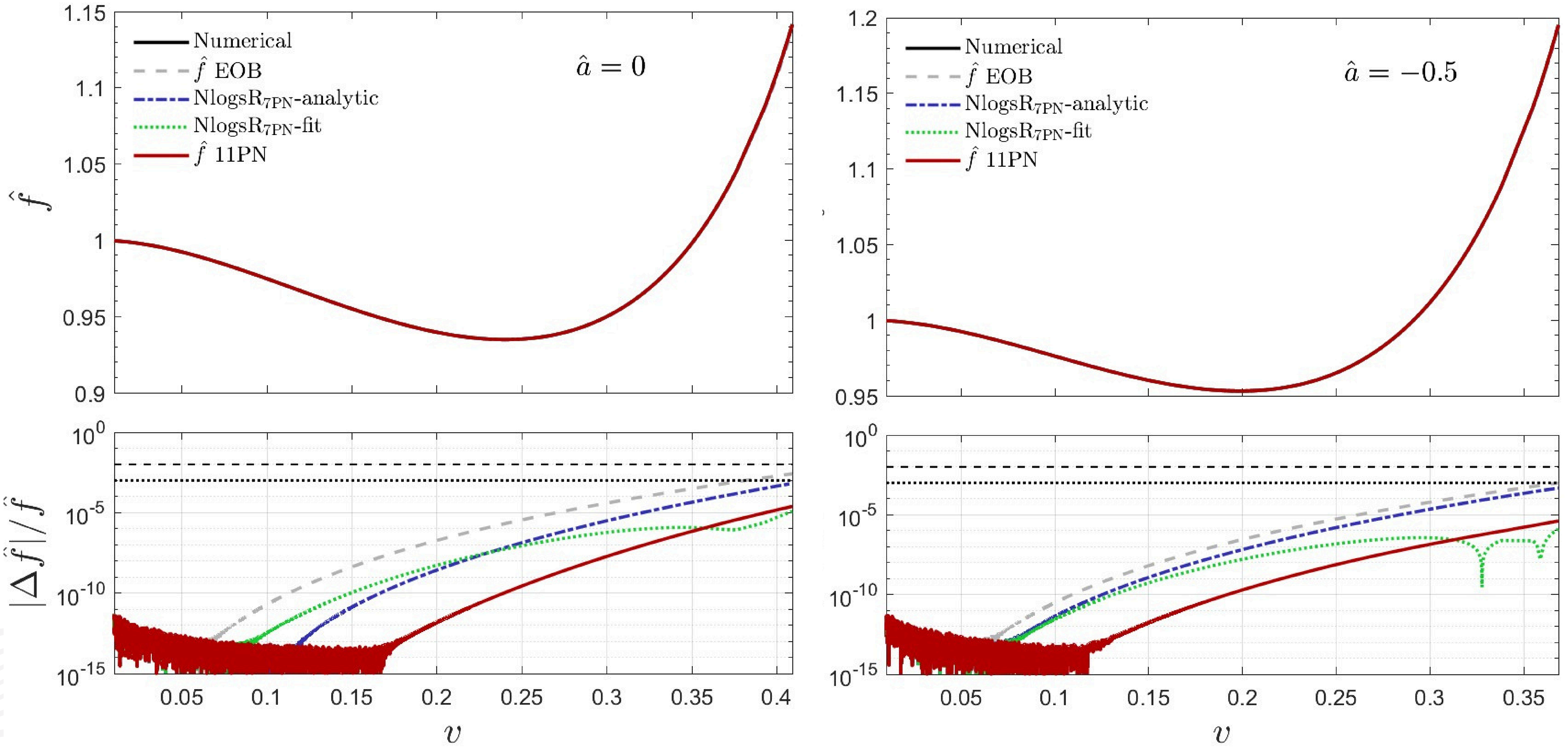
More results pt.1



More results pt.2



More results pt.3



Performance

Test for EOB for EMRIs PA approximation of the EOB model described in A.Nagar +, 2019, 1805.03891

$$\chi_{\mathrm{eff}} = \chi_1 \frac{m_1}{M} + \chi_2 \frac{m_2}{M}$$

Parameters

$$f_{\mathrm{in}} = 10^{-4}\mathrm{Hz} \qquad \frac{r_{\mathrm{fin}}}{M} = 5$$

$$M = 10^7 M_\odot$$

Albertini +, 2024, 2310.13578