A new multipolar waveform model for eccentric, spin-aligned binary black holes within the effective-one-body formalism

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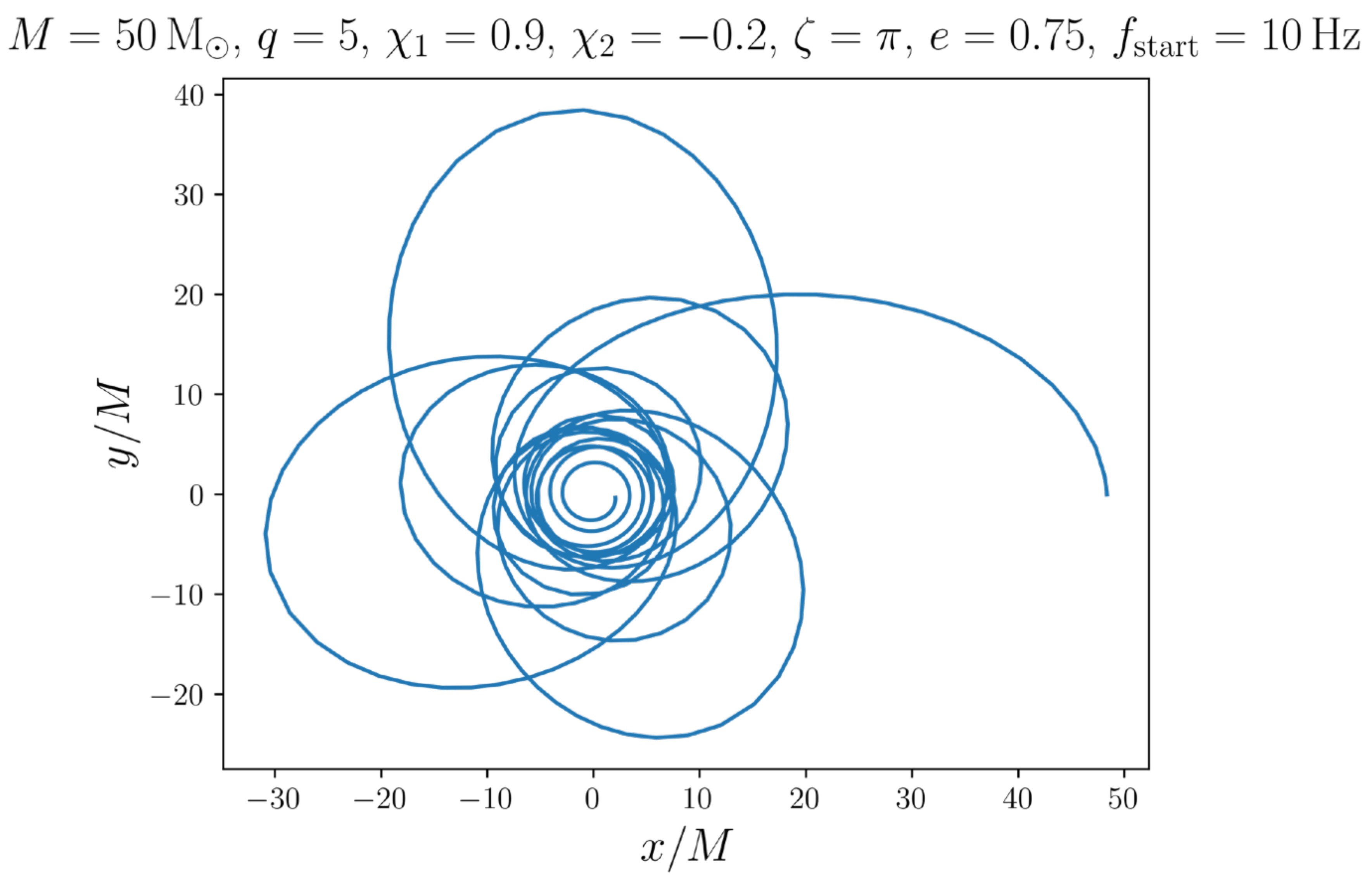
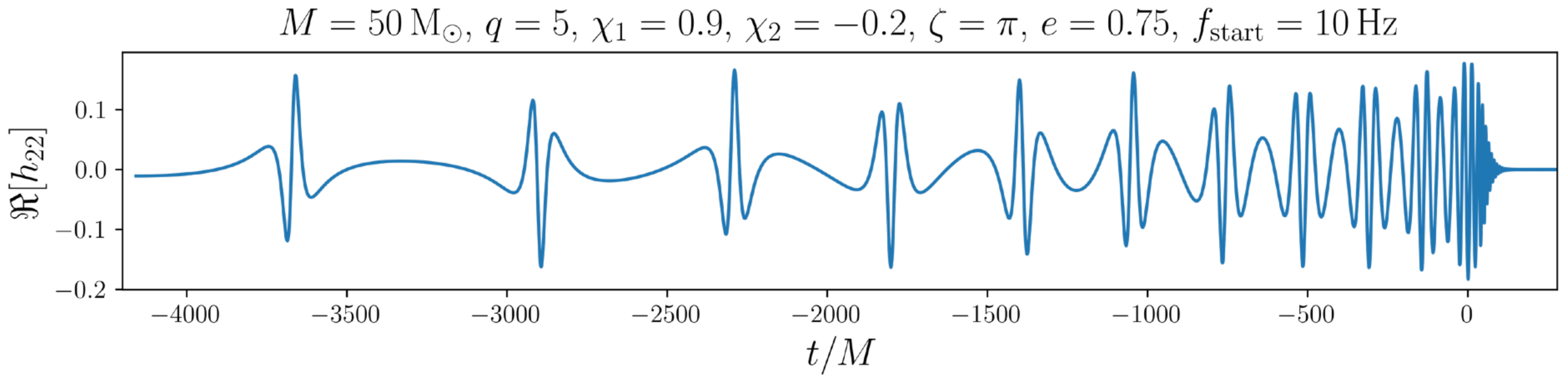
Eccentric binary black hole systems



- **Binary black hole (BBH) formation channels**: isolated binary evolution and dynamical formation.
- Gravitational waves (GWs) circularize binaries $\rightarrow$ only binaries which are **dynamically formed close to merger** or which are **not isolated** will have a detectable imprint of eccentricity [e.g. Zevin+, APJL 921, L43 (2021); Samsing, PRD 97, 103014 (2018)].
- Detecting (or not detecting) eccentricity will allow us to **constrain the populations** of BBHs and to **understand their origin and evolution** [e.g. LVK Collaboration, arXiv:2308.03822; Zeeshan & O'Shaughnessy, arXiv:2404.08185].
- Eccentricity is needed for a **correct estimation of BBH parameters** and to **avoid false violations of General Relativity** [e.g. Divyajyoti+, PRD 109, 043037 (2024); Gil Choi+, PRD 110, 024025 (2024); Saini+, PRD 109, 084056 (2024)].
- More sensitive GW detectors and improved data-analysis techniques $\longrightarrow$ **the detection or confirmation of an eccentric signal will happen very soon!** [e.g. Gupte+, arXiv:2404.14286; Romero-Shaw+, APJ 940, 171 (2022)].
- **Waveform models will play a fundamental role** in the analysis of eccentric GWs: TEOBResumS-Dalí, SEOBNRPE, SEOBNRv4EHM, SEOBNRv5EHM, ESIGMAHM, extensions of IMRPhenom, and more!



SEOBNR waveform family

- In the effective-one-body (EOB) formalism [Buonanno & Damour, PRD 59, 084006 (1999), PRD 62, 064015 (2000)], the dynamics of a compact binary is mapped to that of a test mass in a deformed Kerr background, with the deformation parameter being the symmetric mass ratio, $\nu = m_1 m_2/(m_1+m_2)^2$.

- The BBH dynamics is determined by an EOB Hamiltonian $H_{\rm EOB}$ supplemented by a radiation-reaction (RR) force $\mathcal{F}$:

$$H_{\rm EOB} = M\sqrt{1+2\nu\left(\frac{H_{\rm eff}}{\mu}-1\right)} \quad \longrightarrow \quad \dot{r} = \frac{\partial H_{\rm EOB}}{\partial p_r}, \qquad \dot{p}_r = -\frac{\partial H_{\rm EOB}}{\partial r} + \mathcal{F}_r,$$

$$\Omega = \frac{\partial H_{\rm EOB}}{\partial p_\phi}, \qquad \dot{p}_\phi = -\frac{\partial H_{\rm EOB}}{\partial \phi} + \mathcal{F}_\phi\,.$$

- $H_{\rm EOB}$ and $\mathcal{F}$ represent a *resummation* of analytical information of the gravitational two-body problem. $\mathcal{F}$ accounts for the losses of energy and angular momentum due to the emission of GWs.

- GWs are decomposed in terms of waveform modes $h_{\ell m}$ which satisfy: $\quad h_+ - i\,h_\times = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{m=\ell} {}_{-2}Y_{\ell m}(\iota,\varphi)\,h_{\ell m}(\Theta;t),$

  where ${}_{-2}Y_{\ell,m}$ are the -2 spin-weighted spherical harmonics which depend on the line of sight (ι,φ) measured in the source frame, and Θ are the *intrinsic* parameters of the binary.

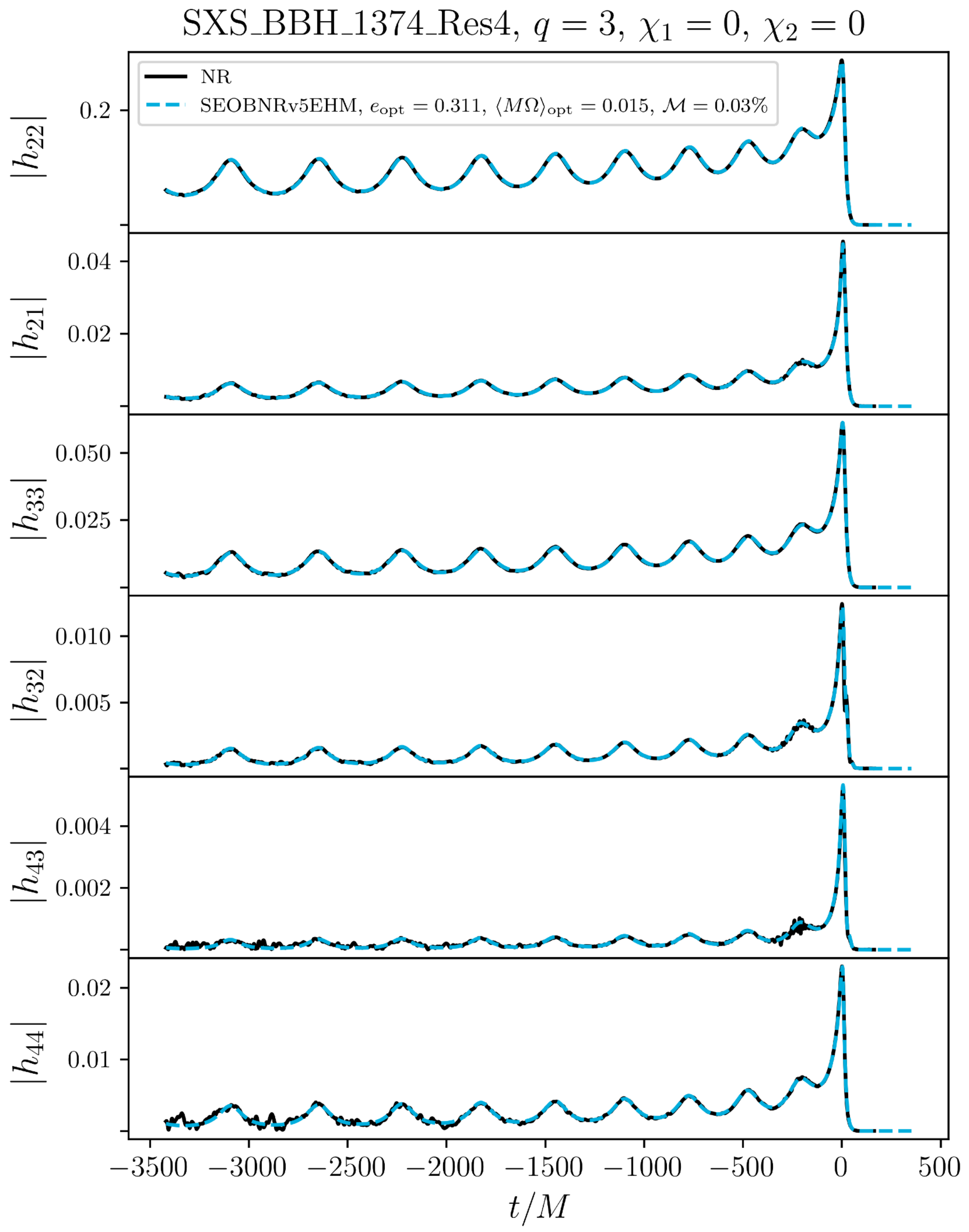
- The waveform modes $h_{\ell m}$ are computed with factorized formulas and physically-motivated ansatz, and are further improved with calibrations to numerical relativity (NR) and BH perturbation theory.

- In this way, SEOBNR $\equiv$ EOB dynamics + NR + BH perturbation theory $\longrightarrow$ GWs!

Overview of SEOBNRv5EHM

- A new time-domain inspiral-merger-ringdown multipolar waveform model for **aligned-spin, eccentric BBHs**, developed within the EOB formalism, with higher-order modes: (2,2), (2,1), (3,2), (3,3), (4,3), (4,4).
- Developed in the python pySEOBNR package, and built upon the quasi-circular (QC) waveform model SEOBNRv5HM.*
- Based on SEOBNR**v4**EHM [Khalil+, PRD 104, 024046 (2021), Ramos-Buades+, PRD 105, 044035 (2022)].
- Includes **new analytical results** for the non-spinning eccentricity contributions to the EOB waveform modes, RR force, and fluxes **at 3PN order** [Gamboa+, in prep. (2024); Henry & Khalil, PRD 108, 104016 (2023)].
- Faster, more accurate and more robust compared to SEOBNRv4EHM (developed in LALSuite) [Ramos-Buades+, PRD 105, 044035 (2022)].

*: SEOBNRv5 papers: Pompili+, PRD 108, 124035 (2023); Khalil+, PRD 108, 124036 (2023); Ramos-Buades+, PRD 108, 124037 (2023); van de Meent+, PRD 108, 124038 (2023); Mihaylov+, arXiv:2303.18203 (2023)



SEOBNRv5EHM equations of motion

- We employ the eccentricity e defined by the Keplerian parametrization:

$$r = \frac{1}{u_p\,(1 + e\cos\zeta)} \qquad u_p = u_p(e, x) \qquad x = \langle M\Omega\rangle^{2/3}$$

where u_p is the inverse semi-latus rectum, ζ is the relativistic anomaly and x is the orbit-averaged frequency.

- Equations of motion for $(r, \phi, p_r, p_\phi, e, \zeta)$:

$$\dot{r} = \frac{\partial H_{\rm EOB}}{\partial p_r} \qquad \Omega = \dot{\phi} = \frac{\partial H_{\rm EOB}}{\partial p_\phi} \qquad \dot{p}_r = -\frac{\partial H_{\rm EOB}}{\partial r} + \mathcal{F}_r(x, e, \zeta) \qquad \dot{p}_\phi = \mathcal{F}_\phi(x, e, \zeta)$$

$$\dot{e} = \frac{\nu e x^4}{M}\left[-\frac{\left(121 e^2 + 304\right)}{15\left(1 - e^2\right)^{5/2}} + \text{3PN expansion}\right] \qquad \dot{\zeta} = \frac{x^{3/2}}{M}\left[\frac{(1 + e\cos\zeta)^2}{\left(1 - e^2\right)^{3/2}} + \text{3PN expansion}\right]$$

$$x = (M\Omega)^{2/3}\left[\frac{1 - e^2}{\left(1 + e\cos\zeta\right)^{4/3}} + \text{3PN expansion}\right]$$

SEOBNRv5EHM radiation-reaction (RR) force and waveform modes

- Eccentric effects are included as multiplicative corrections to the modes and RR force:

$$\mathcal{F}_{\phi} = \mathcal{F}_{\text{modes}}\, \mathcal{F}_{\phi,\,\text{corr}}(x, e, \zeta) \qquad \mathcal{F}_{r} = \frac{p_r}{p_\phi}\, \mathcal{F}_{\text{modes}}\, \mathcal{F}_{r,\,\text{corr}}(x, e, \zeta)$$

$$\mathcal{F}_{\text{modes}} = -\frac{M\Omega}{8\pi} \sum_{\ell=2}^{8} \sum_{m=1}^{\ell} m^2 \left| d_L\, h^{\text{F}}_{\ell m} \right|^2$$

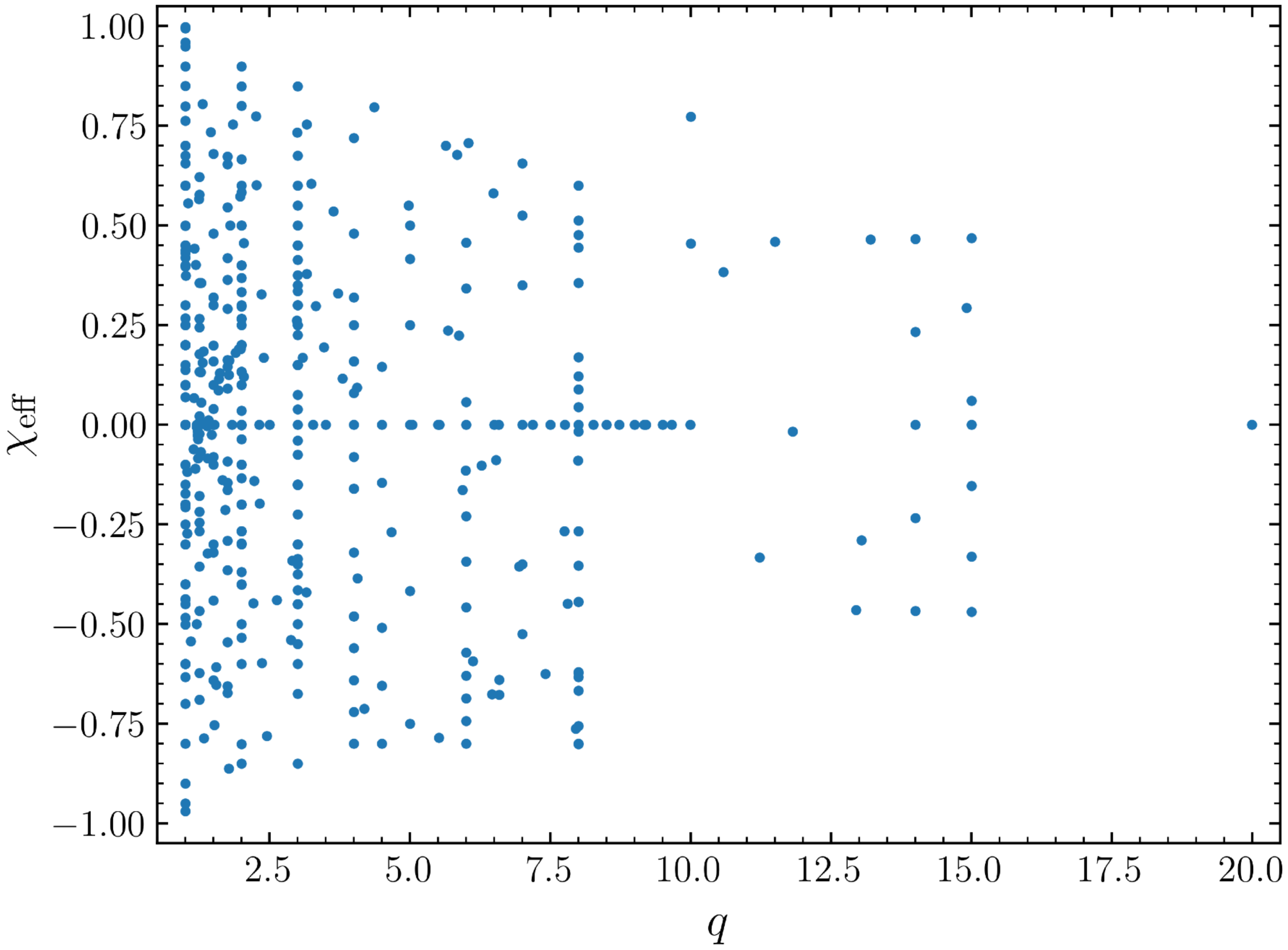
$$h^{\text{F}}_{\ell m} = h^{\text{N,\,qc}}_{\ell m}\, S_{\text{eff}}\, T^{\text{qc}}_{\ell m}\, f^{\text{qc}}_{\ell m}\, e^{i\delta^{\text{qc}}_{\ell m}}\, h^{\text{ecc,\,corr}}_{\ell m}(x, e, \zeta)$$

- This choice is ideal for recovering the underlying QC model in the $e \to 0$ limit:

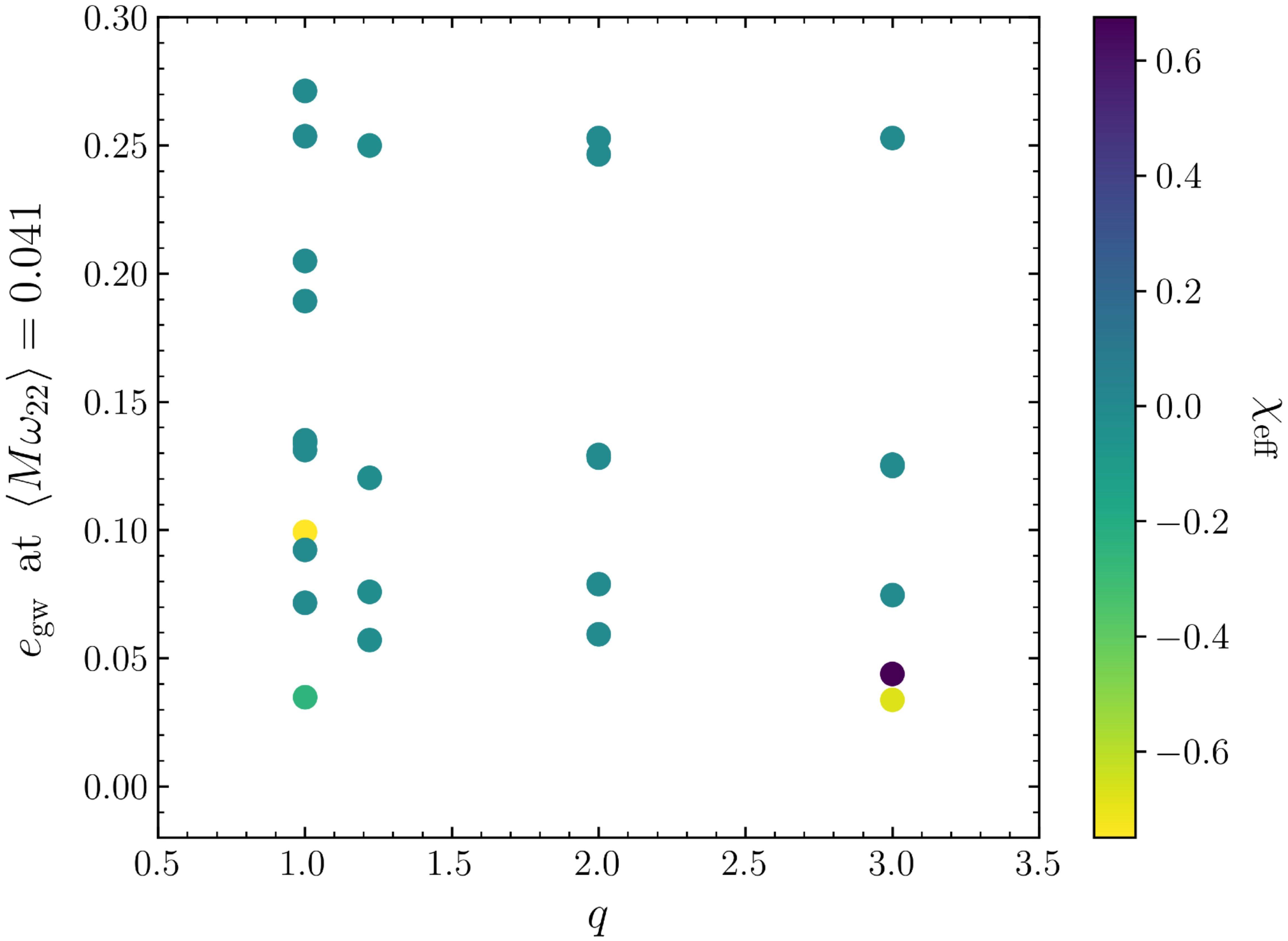
$$\mathcal{F}^{\text{qc}}_{\text{modes}} = -\frac{\Phi_E}{\Omega} = -\Phi_J\,, \qquad \mathcal{F}_{\phi,\,\text{corr}} = 1, \qquad \mathcal{F}_{r,\,\text{corr}} = 1, \qquad \text{and} \qquad h^{\text{ecc,\,corr}}_{\ell m}(x, e, \zeta) = 1$$

Validation against Numerical Relativity waveforms

441 public + private QC NR simulations from the SXS collaboration (see [Pompili+, PRD 108, 124035 (2023)] and references therein)



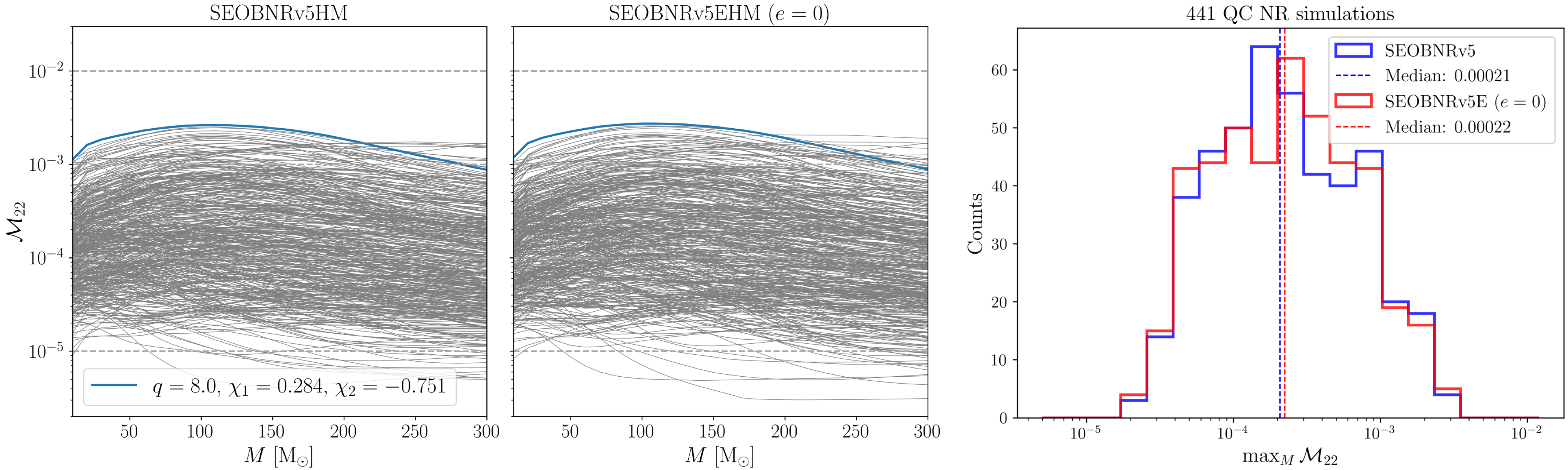
28 public eccentric NR simulations from the SXS collaboration [Hinder+, PRD 98, 044015 (2018)]



Quasi-circular (QC) limit of SEOBNRv5EHM

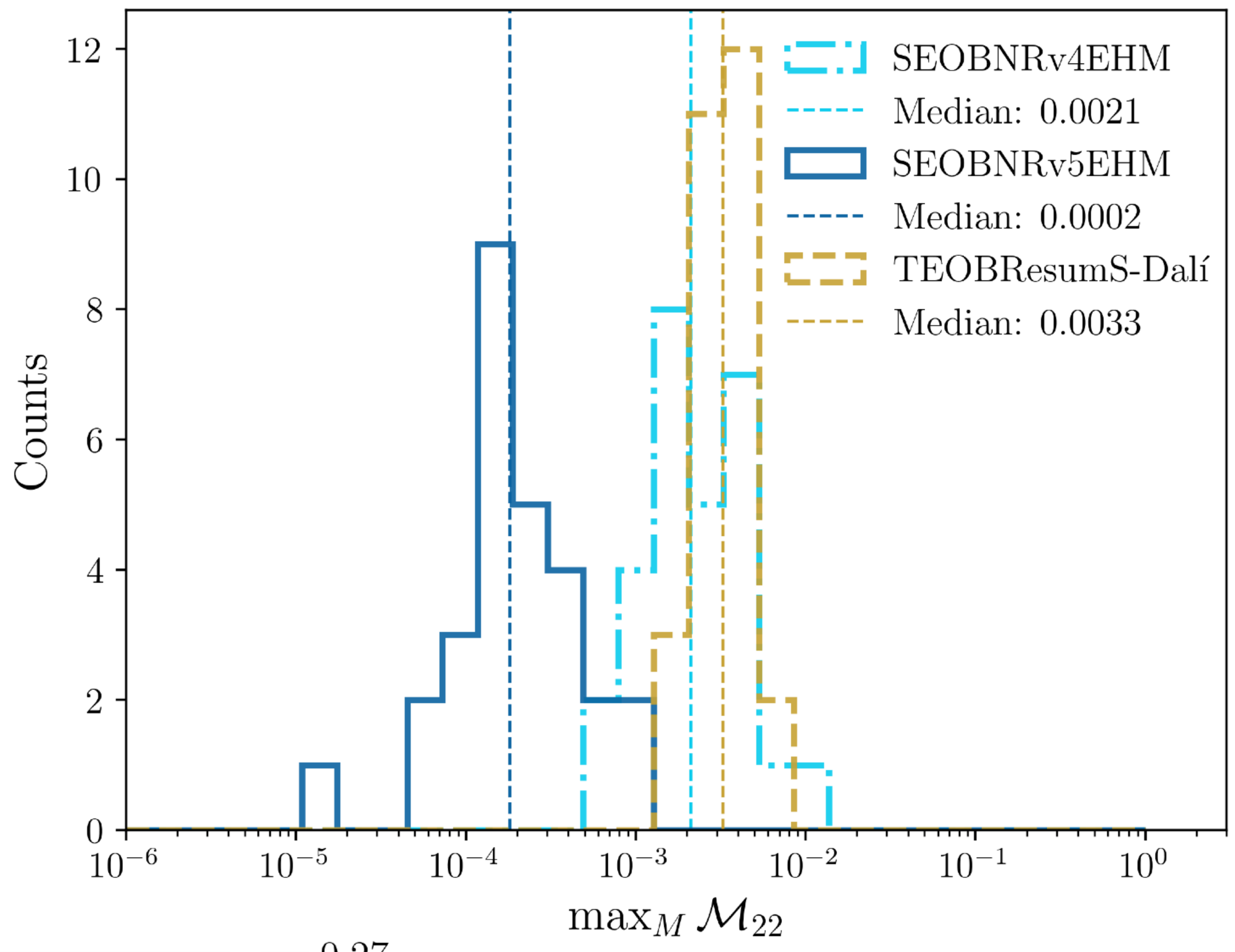
- Mismatch between the NR waveform h_s and the EOB waveform h_t: $\mathcal{M}(M) = 1 - \max_{t_c, \varphi_t} \left[\langle \tilde{h}_s | \tilde{h}_t \rangle |_{\Theta^{qc}_{s,0} = \Theta^{qc}_{t,0}} \right]$,

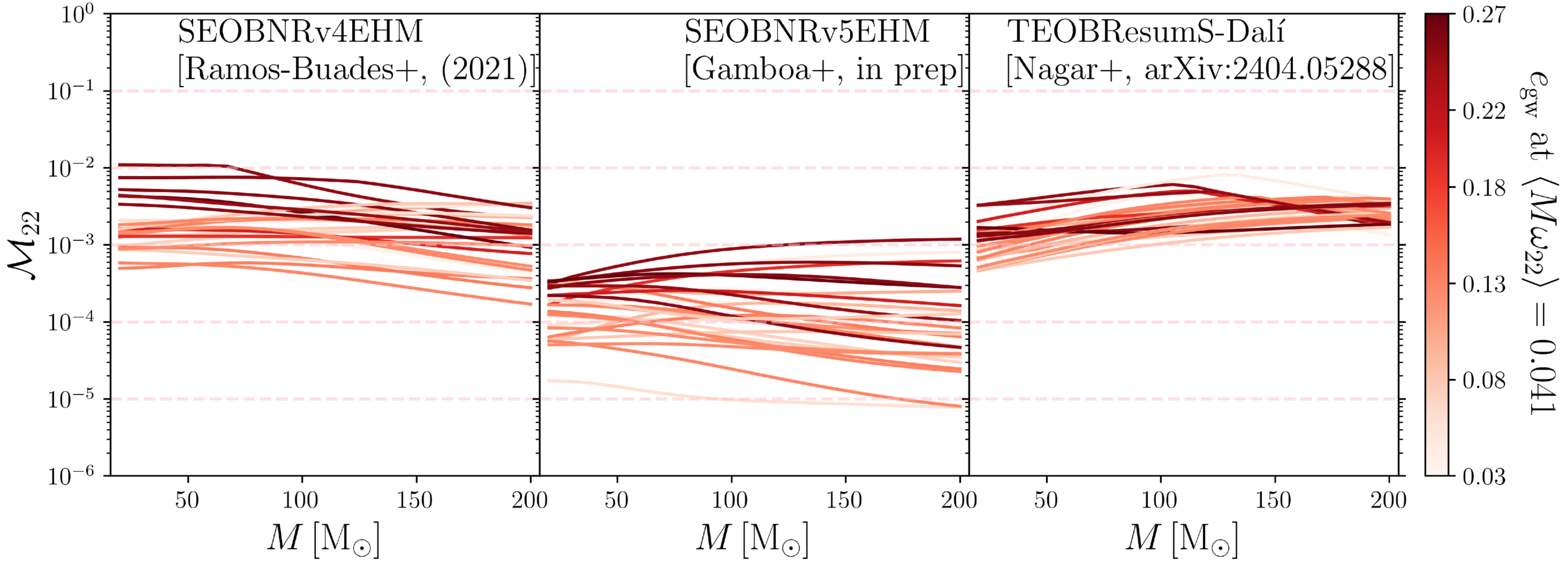
  where $\langle \tilde{h}_s | \tilde{h}_t \rangle$ denotes the noise-weighted (Advanced LIGO's power-spectral density) inner-product between the Fourier transforms of the waveforms.
- SEOBNRv5EHM has the **same accuracy** as SEOBNRv5HM in the QC limit: excellent agreement for all multipoles (ℓ, m).
- Important to avoid biases in PE [e.g. Bonino+, PRD 107, 064024 (2023), Ramos-Buades+, arXiv:2309.15528].



Eccentric waveforms: (2,2) mode

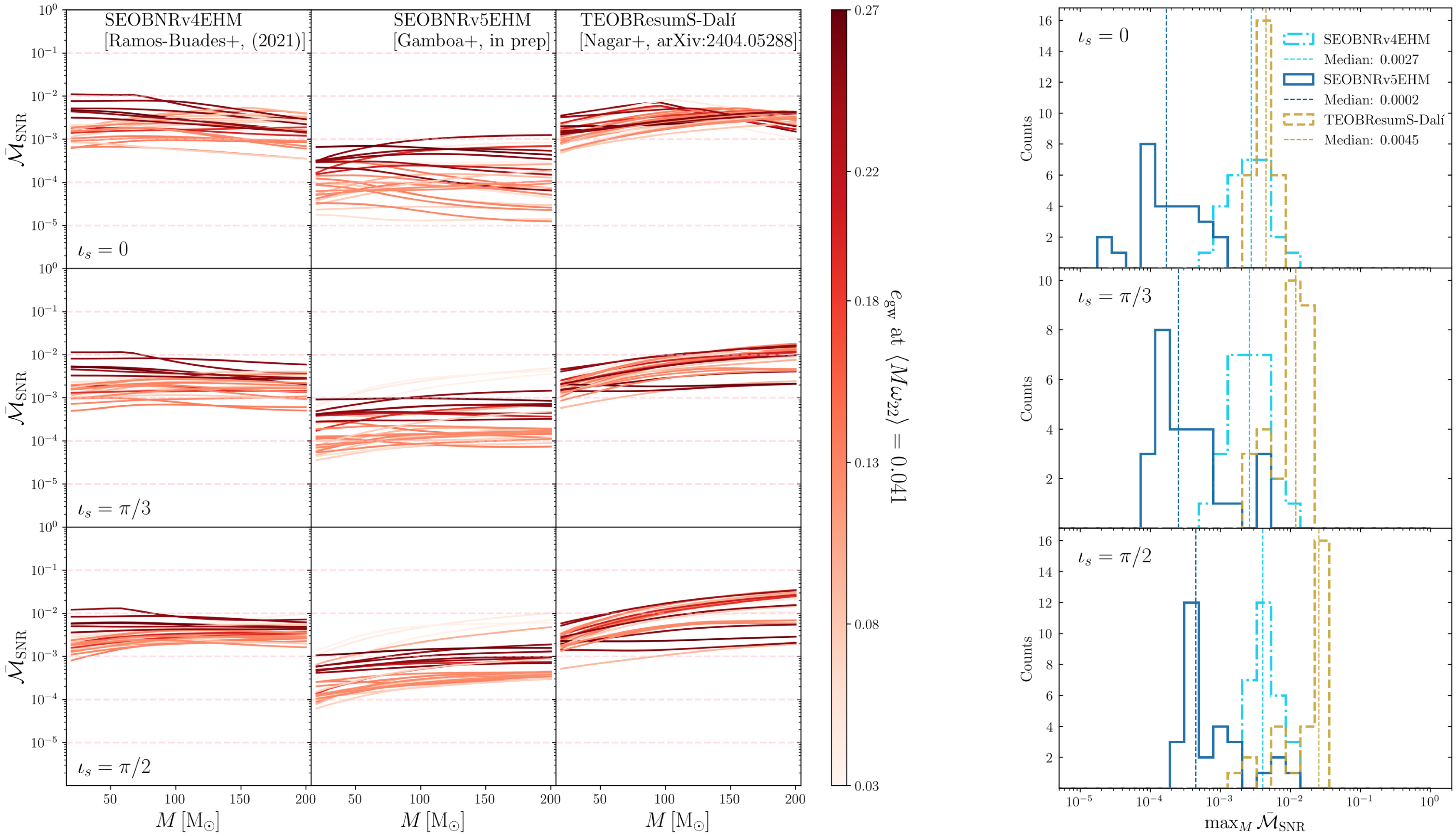
- Mismatches across an interval of total masses $M \in [20, 200]\,\mathrm{M_\odot}$.
- Computed by optimizing over starting frequency and eccentricity at fixed apastron [Ramos-Buades+, PRD 105, 044035 (2022)].
- Higher-order mode mismatches computed with the optimum values $(e_{\rm opt}, \langle M\Omega\rangle_{\rm opt})$ at $M = 20\,\mathrm{M_\odot}$.





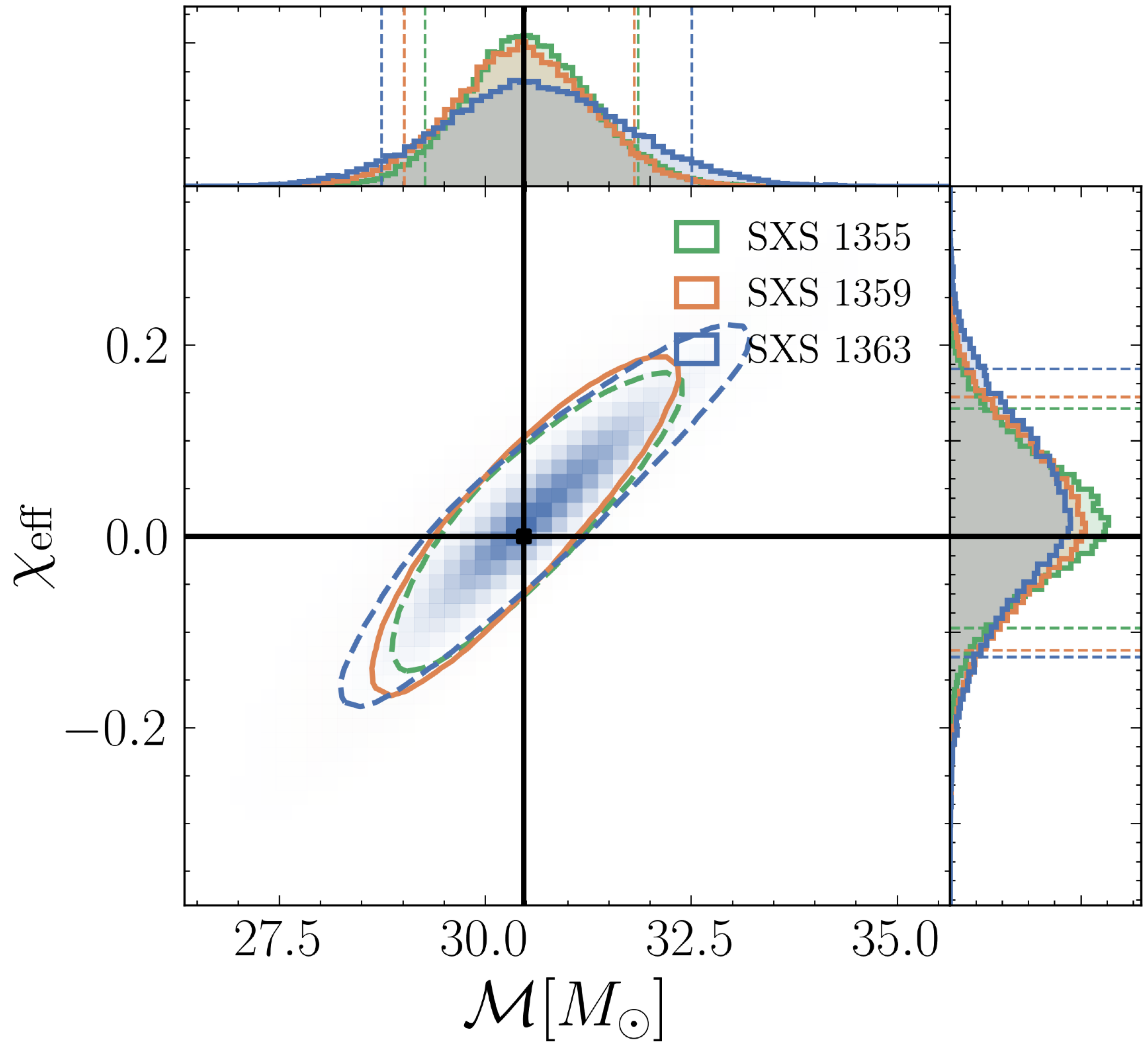
- Eccentricity $e_{\rm gw}$ defined as in [Shaikh+, PRD 108, 104007 (2023)].
- TEOBResumS-Dalí waveforms from the Dali branch in this repository.

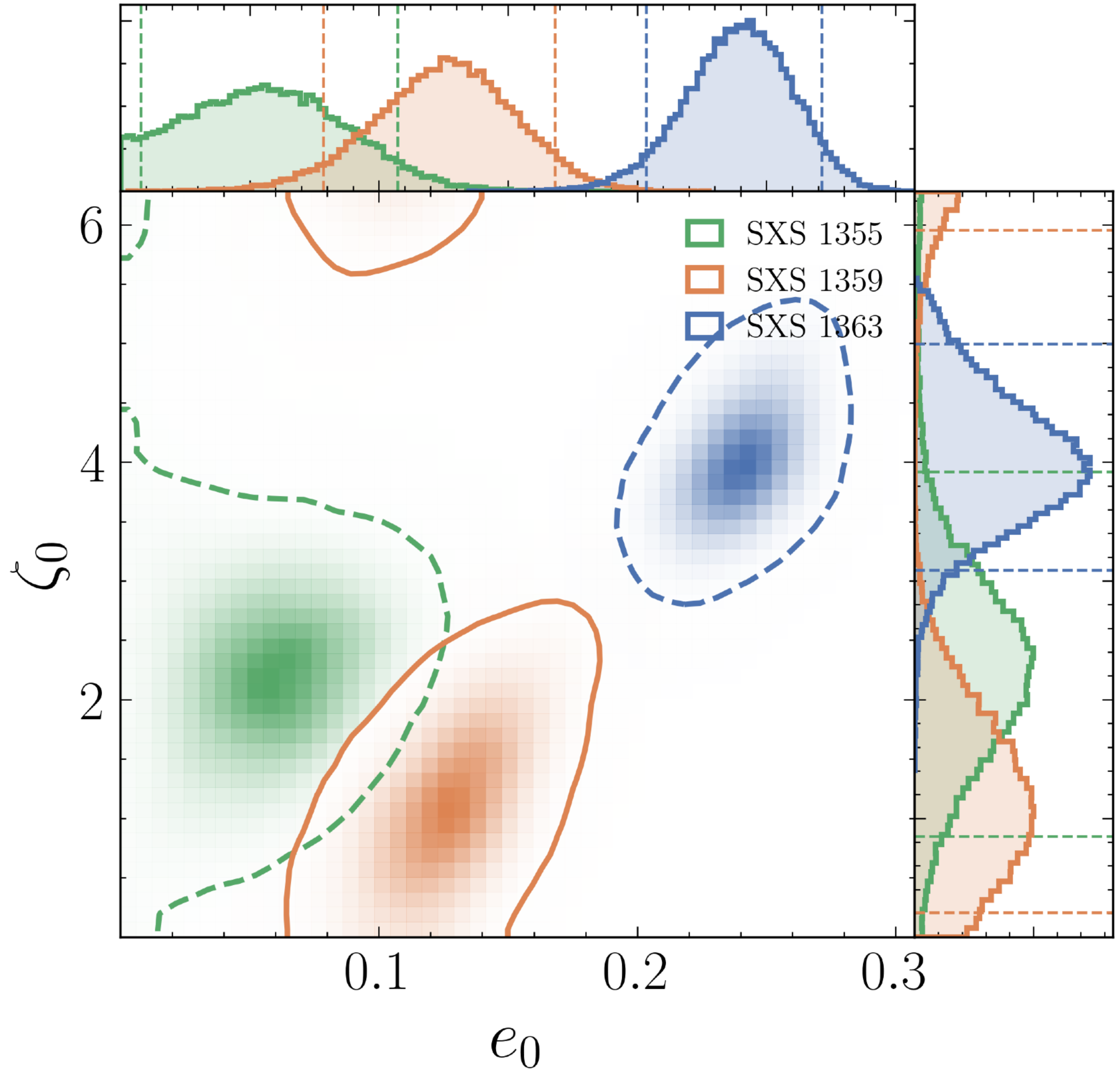
Eccentric waveforms: Higher-order modes



Parameter estimation: NR injections

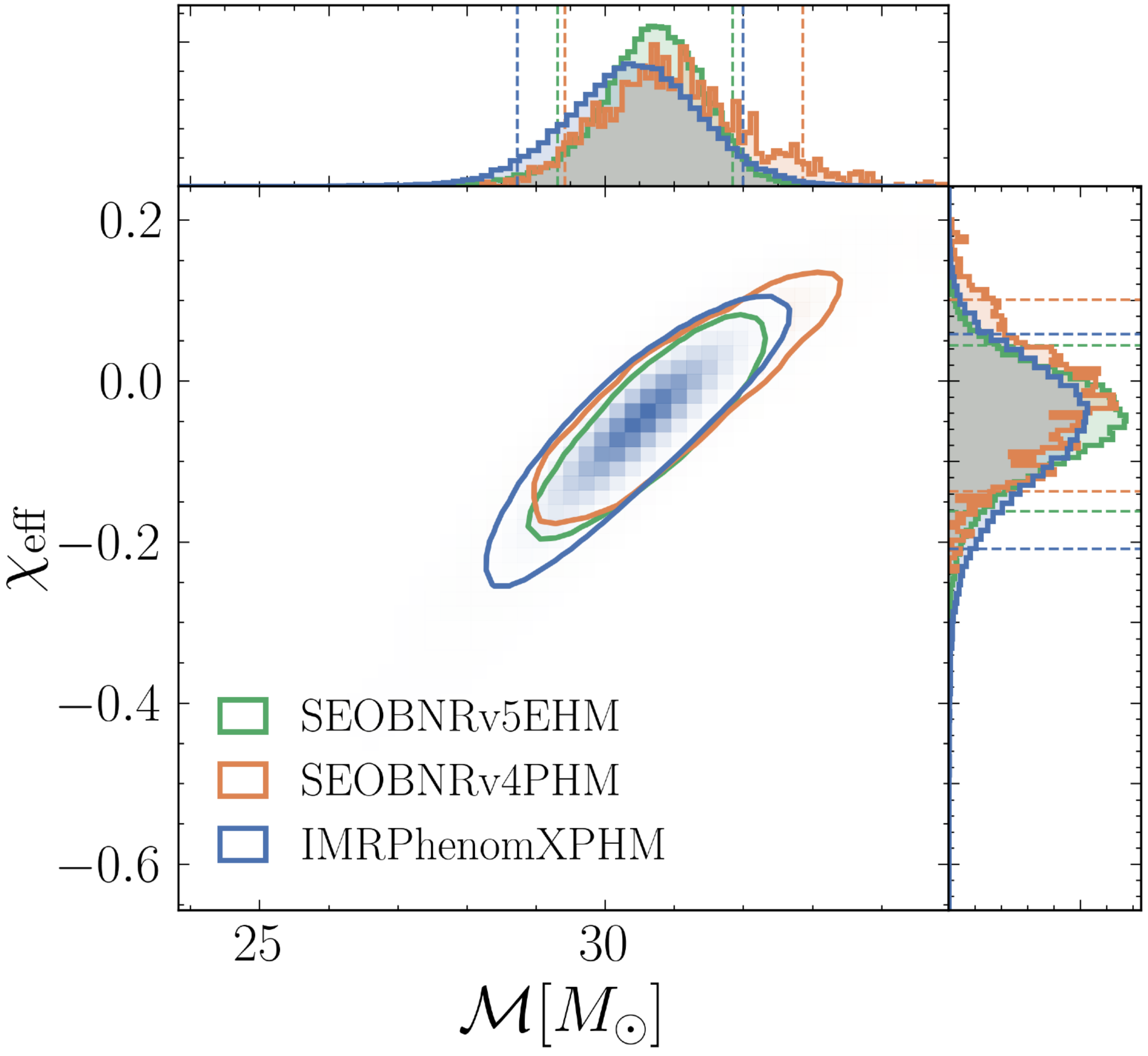
- We inject **three** equal-mass, non-spinning NR simulations with increasing eccentricity.

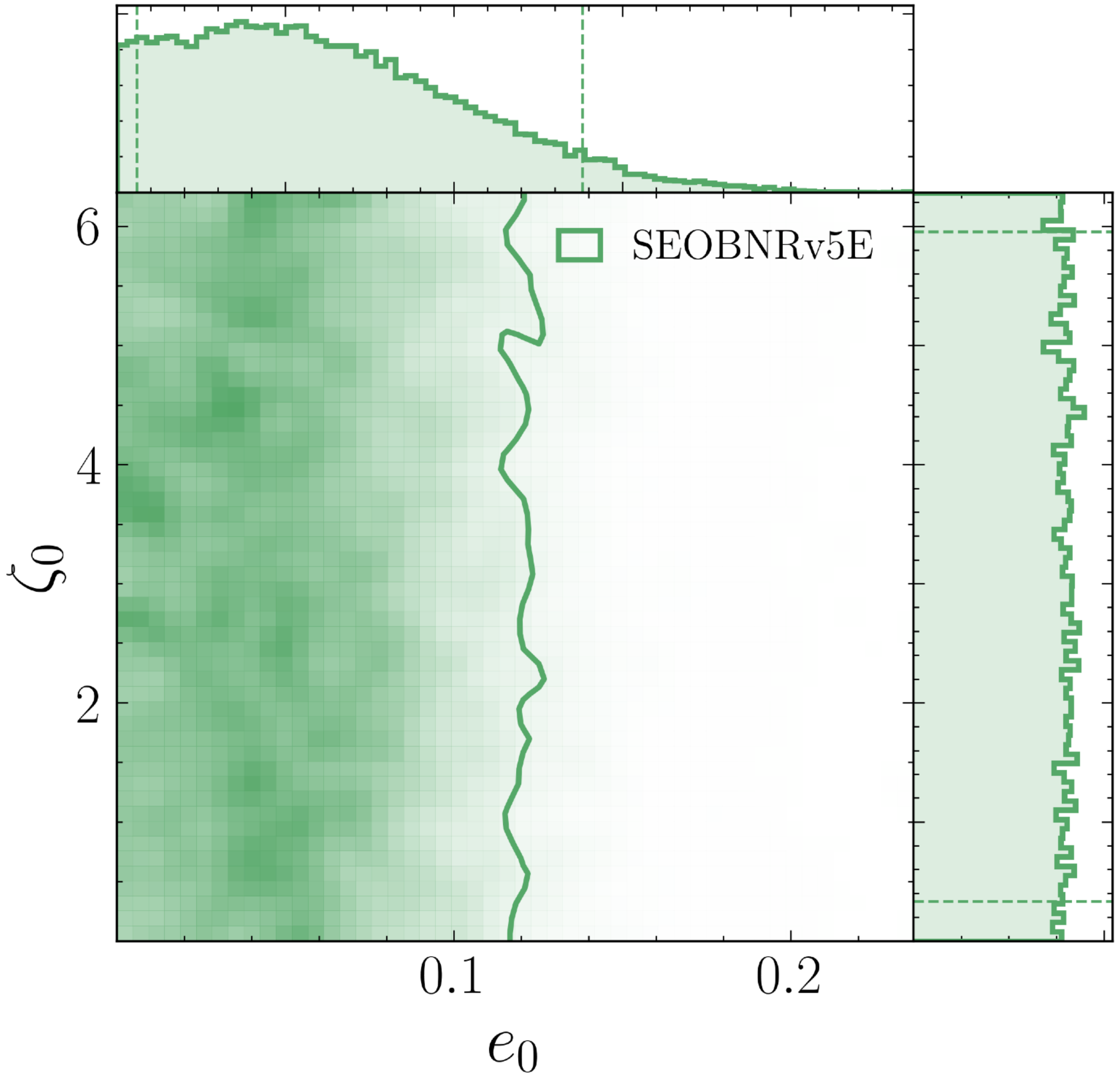




Parameter estimation: GW150914

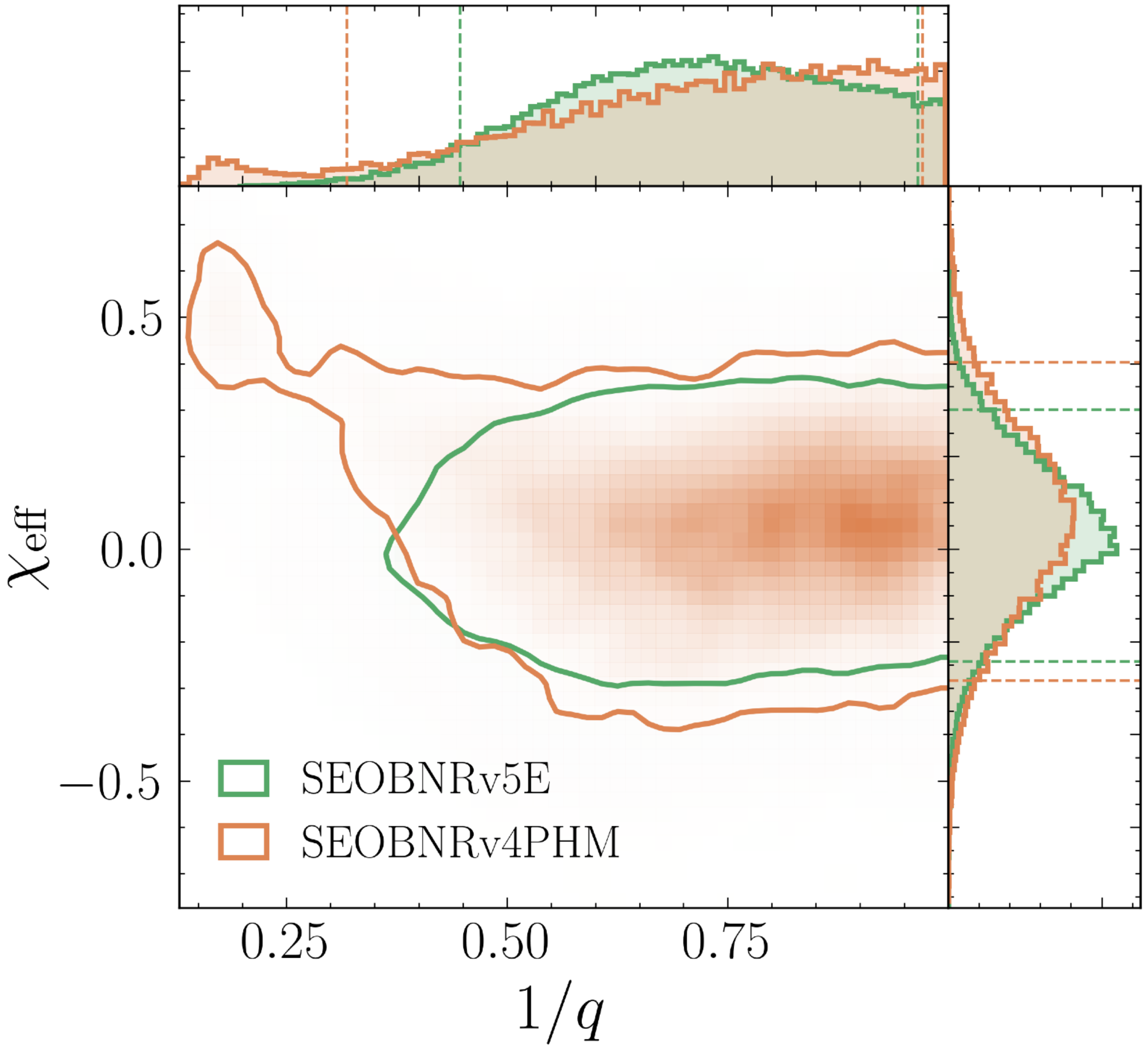
- Strong support in the zero eccentricity region, which is in agreement with other analyses of GW150914 with eccentric waveforms [e.g. Romero-Shaw+, MNRAS 490, 5210-5216 (2019); Bonino+, PRD 107, 064024 (2023); Ramos-Buades+, PRD 108, 124063 (2023)].

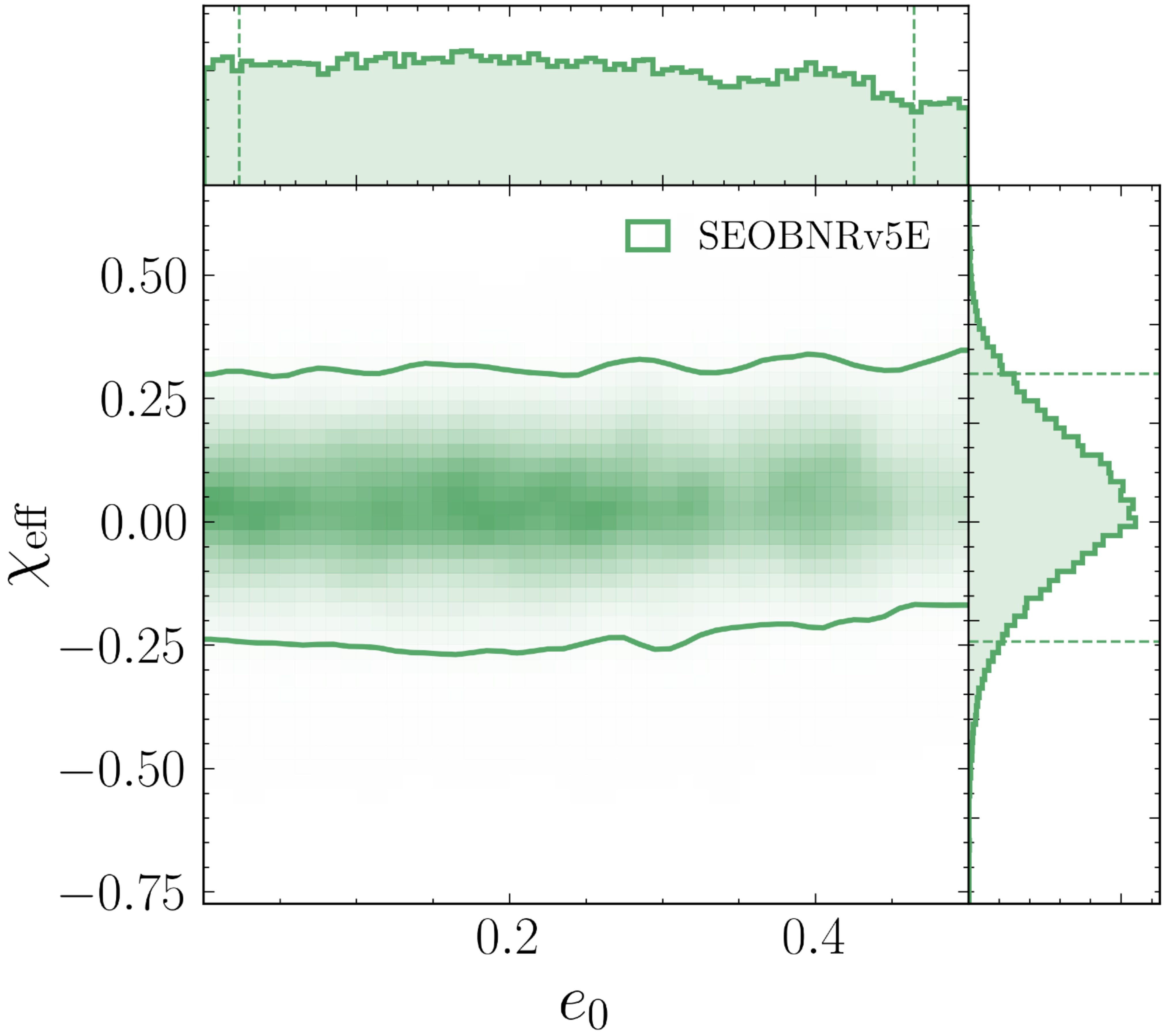




Parameter estimation: GW190521

- Controversial event due to its short duration [LVK Collaboration, PRL 125, 101102 (2020)].
- Some references find support for eccentricity [e.g. Romero-Shaw+, APJL 903, L5 (2020); Gamba+, Nature Astro. 7, 11 (2023)], whereas other references find no clear evidence for eccentricity [e.g. Gupte+, arXiv:2404.14286; Ramos-Buades et al. PRD 108, 124063 (2023)].

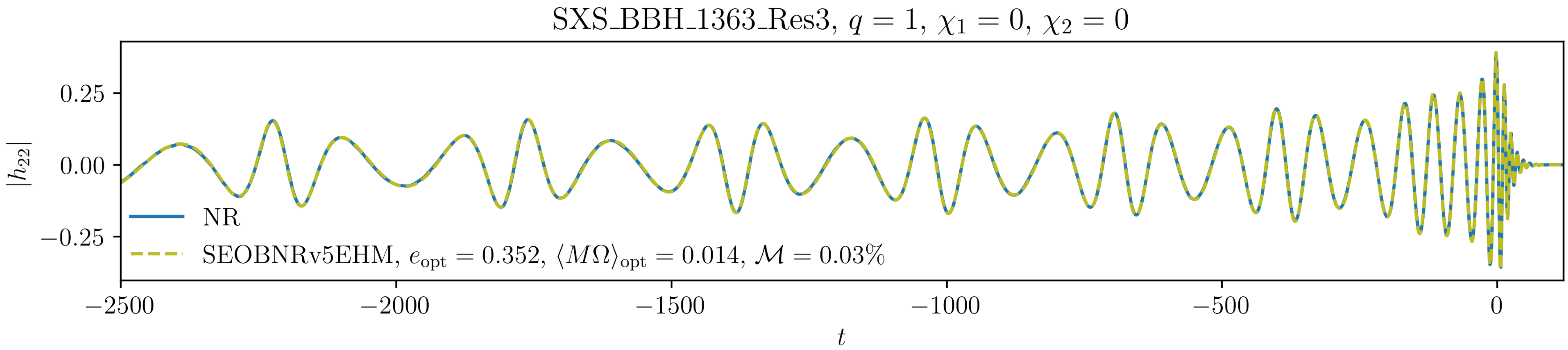




Conclusions

- Current detectors are already sensible to eccentric GW signals, and the expectations of observing eccentric GWs will increase significantly for upcoming LVK runs and next-generation detectors (e.g. LISA, Einstein Telescope, Cosmic Explorer).
- Eccentric GW signals will play a key role in the identification of binary formation channels.
- Ignoring the effects of eccentricity could lead to biases in parameter estimation (PE) and tests of General Relativity.
- Therefore, eccentric waveform models will be **fundamental** for current and future GW science.
- SEOBNRv5EHM: a new time-domain IMR multipolar waveform model for spin-aligned, eccentric BBHs, which incorporates new analytical results for the EOB formalism. It is being validated with NR simulations and thoroughly tested across parameter space.
- The accuracy, robustness, and speed of SEOBNRv5EHM are suitable for PE studies (e.g. analysis of GW events) and waveform systematics studies.
- Furthermore, SEOBNRv5EHM is being reviewed by the LVK collaboration. The review will finish soon!
- **Future work:**
    - We plan a PE study with DINGO of the O3 signals and possibly confirm the results found with SEOBNRv4EHM.
    - Improvement with calibration to eccentric NR simulations.
    - Spin-precession + eccentricity will be fundamental for the complete characterization of GWs [e.g. Liu+, arXiv:2310.04552; Gamba+, arXiv:2404.15408] $\longrightarrow$ SEOBNRv5EPHM.

Thanks for your attention!



Back-up slides

Unfaithfulness formulae

- Eccentric (2,2) mode unfaithfulness:

$$\mathcal{F}^{\rm ecc}_{22}(M) = \max_{t_{\rm ct}, \varphi_{0\rm t}, e_{\rm t}, \langle M\Omega\rangle_{\rm t}} \left[\left\langle \tilde{h}_{\rm s} \middle| \tilde{h}_{\rm t} \right\rangle \Bigg|_{\substack{\zeta_{\rm t} = \pi \\ \Theta^{\rm qc}_{\rm s,0} = \Theta^{\rm qc}_{\rm t,0}}} \right]$$

- Higher-order modes unfaithfulness:

$$\overline{\mathcal{F}}_{\rm SNR}(M, \iota_{\rm s}) = \sqrt[3]{\frac{\int_0^{2\pi} d\varphi_{0\rm s} \int_0^{2\pi} d\kappa_{\rm s}\ \mathcal{F}^3(M_{\rm s}, \iota_{\rm s}, \varphi_{0\rm s}, \kappa_{\rm s})\ {\rm SNR}^3(\iota_{\rm s}, \varphi_{0\rm s}, \kappa_{\rm s})}{\int_0^{2\pi} d\varphi_{0\rm s} \int_0^{2\pi} d\kappa_{\rm s}\ {\rm SNR}^3(\iota_{\rm s}, \varphi_{0\rm s}, \kappa_{\rm s})}},$$

where

$${\rm SNR}(\iota_{\rm s}, \varphi_{0\rm s}, \kappa_{\rm s}, \theta_{\rm s}, \phi_{\rm s}, D_L) = \sqrt{\langle h_{\rm s} | h_{\rm s} \rangle}$$

Problematic NR eccentric waveforms

- Some of the old NR waveforms employed (e.g. SXS:BBH:0089) show unphysical features which are particularly relevant for HoMs.
- SEOBNRv5EHM is highly accurate. Hence, the mismatch $\mathcal{O}(10^{-4})$ becomes affected by these features.
- This explains some relatively high mismatches ($\lesssim 10^{-2}$) in the HoM waveforms.