1st TEONGRAV international workshop on the Theory of Gravitational Waves

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Rossella Gamba Rome, 20/09/2024

Outline

● Gravitational waves from BNS Mergers

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- Matter effects
- Inspiral models
- Post-merger models

● What's next?

- Challenges
- Developments

Part 1: Gravitational waves from BNSs Matter effects, models and all that

Phenomenology of a merger

Phenomenology of a merger

Phenomenology of a merger: GWs

Phenomenology of a merger: GWs

BBH + (some kind of correction) = **BNS ?**

Corrections = Matter effects. They are what distinguish NS from **point particles**.

- **Tidal effects:**
	- \circ \bullet "Adiabatic" tidal effects $(\Lambda_\ell, \Sigma_\ell)_{\text{\tiny{[Damour1983,Flanagan+2007,Damour+2008,Vines+2010,Henrv+2020,Mandal+24]}}}$ $(\Lambda_\ell, \Sigma_\ell)_{\text{\tiny{[Damour1983,Flanagan+2007,Damour+2008,Vines+2010,Henrv+2020,Mandal+24]}}}$
	- \circ "Dynamical" tidal effects $(\bar{\omega}_f)$ *[Lai+1994,](https://arxiv.org/abs/astro-ph/9404062) [Hinderer+2016](https://arxiv.org/abs/1602.00599), [Steinhoff+2016](https://arxiv.org/abs/1608.01907), [Steinhoff+2021](https://arxiv.org/abs/2103.06100)*]
- **•** Spin induced effects $(C_{\alpha}, C_{\text{oct}}, C_{\text{hex}}, ...)$ [Poisson1998](https://arxiv.org/abs/gr-qc/9709032), [Krishnendu+2017\]](https://arxiv.org/abs/1701.06318)
- **Other effects:** nonlinear mode couplings, other modes resonances and excitations, crust shattering, dissipative effects ... [\[Ho+1999](https://arxiv.org/pdf/astro-ph/9812116.pdf), [Tsang+2013,](https://arxiv.org/abs/1307.3554) [Andersson+2017](https://arxiv.org/abs/1710.05950), [Ripley+24](https://arxiv.org/abs/2312.11659), ...]

All of the above coefficients depend on the **Equation of State** (EOS):

 $(m, \text{EOS}) \rightarrow (\Lambda_\ell, \Sigma_\ell, \dots)$: **Talk to Micaela**

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What we'd like: $(m, \Lambda_\ell, \Sigma_\ell, \dots) \to \textup{EOS}$

Various matter effects are not (entirely) independent from one-another: **quasi-universal relations**

$$
\Lambda_2 \to (\Lambda_\ell, \Sigma_\ell, C_{\mathrm{q}}, C_{\mathrm{oct}}, \bar{\omega}_f, \dots)
$$

$$
\Psi(x) = 2\pi f t_c - \phi_c - \pi/4 + \frac{3}{128\nu} x^{-5/2} \left[1 + \dots + \frac{x^2 C_q c_{MQ}}{1 + \dots + \frac{x^5 \tilde{\Lambda} c_{\Lambda}}{1 + \dots + \frac{x^8 C_{dyn}}{1 + \
$$

0 PN

- Matter effects are a high-frequency correction:
	- Tidal effects are important above ~300 Hz
	- f-mode (most) relevant above 1kHz
- Current detectors are not very sensitive in those regions
	- → **Hard to measure**

BBH + (some kind of correction) = **BNS ?**

Inspiral models

Breakdown of the Market?

- Flagship IMR models ~ grouped into 3 families
- Pros and cons to each family

NR Surrogates

- Interpolate NR waveforms across parameter space
- Accuracy comparable to input NR
- · Reasonably efficient waveform evaluation
- Limited by availability of NR
- Limited by NR duration but can hybridise with inspiral models

Phenomenological

- · Analytical + NR calibration model of **GW** signal
- · Extremely efficient to evaluate
- · Time- and frequency-domain models available
- · Limited in calibration by availability of **NR**
- · Less fundamental harder to incorporate information

The BBH^{Situation</sub>} **(stolen from Geraint)**
Effective One Body

- · Hamiltonian framework for dynamics and GW signal
- · Evolve system of ODEs work needed to mitigate computational cost
- · Limited in calibration by availability of **NR**
- Natural framework for incorporating additional physics (GSF, scattering, ...)

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Inspiral models

- **Post Newtonian** approximants (PN) [\[Krishnendu+2017](https://arxiv.org/abs/1701.06318)[,Henry+2020,](https://arxiv.org/abs/2005.13367)[Schmidt+2021](https://arxiv.org/abs/1905.00818)]
	- Analytical and fast
	- Examples: TaylorF2, TaylorT4
- **Phenomenological** approximants (Phenom)

[\[Dietrich+2017,](https://arxiv.org/abs/1706.02969)[Kawaguchi+2018,](https://arxiv.org/abs/1802.06518)[Dietrich+2019,](https://arxiv.org/abs/1905.06011)[Gamba+23,](https://inspirehep.net/literature/2683195)[Williams+2024,](https://inspirehep.net/literature/2806886)[Abac+2024\]](https://inspirehep.net/literature/2722030)

- Fits to EOB+NR hybrid waveforms
- Fast
- Examples: (any BBH inspiral model) + NRTidalv3, Kawaguchi+, PhenomGSF,...
- **Effective One Body** approximants (EOB) [\[Bini+2012](https://arxiv.org/abs/1202.3565), [Akcay+2018](https://arxiv.org/abs/1812.02744), [Lackey+2018,](https://arxiv.org/abs/1812.08643) [Tissino+22,](https://inspirehep.net/literature/2172995) [Gamba+2023](https://inspirehep.net/literature/2683195)]
	- Semi-analytical, resummed PN + NR
	- Not-as-fast, generally, but there exist acceleration techniques (PA, SPA)
	- Examples: [TEOBResumS](https://bitbucket.org/eob_ihes/teobresums/src/master/), SEOBNRv4T (& related surrogates)

Inspiral models: Phenom

Simple idea:

- 1. Choose your target BNS waveform (EOB + NR, pure EOB, …)
- 2. Choose your BBH baseline (EOB, NR, Phenom, NR surrogate, …)
- 3. Separate matter contributions from BBH baseline (both phase and amplitude):

 $\Psi_{\rm RNS} \sim \Psi_{\rm BRH} + \Delta \Psi_{\rm matter}$ $\Delta\Psi_{\rm matter} \sim \Psi_{\rm BNS} - \Psi_{\rm BBH}$

4. Directly **fit** the matter contributions $\Delta\Psi_{\rm matter} \sim \Delta\Psi_{\rm ad. tides} + \Delta\Psi_{\rm dyn. tides} + \Delta\Psi_{MQ} + \ldots$

Inspiral models: EOB

Two body problem \rightarrow test particle around (deformed) Kerr. Three ingredients:

● **Hamiltonian**

$$
H_{\rm EOB} = M \sqrt{1 + 2\nu(\hat{H}_{\rm eff} - 1)},
$$

$$
\hat{H}_{\text{eff}} = \sqrt{\hat{p}_{r_{*}}^{2} \frac{1}{1 + 1} A(r) \left(1 + \frac{p_{\varphi}^{2}}{1 \, r^{2}} \frac{1}{1} + 2 \nu (4 - 3 \nu) \frac{p_{r_{*}}^{4}}{r^{2}} \right)}
$$

● **Waveform**

Waveform
\n
$$
h_{\ell m} = h_{\ell m}^{(N,\epsilon)} \hat{S}_{\text{eff}}^{(\epsilon)} \hat{h}_{\ell m}^{\text{tail}} f_{\ell m} \hat{h}_{\ell m}^{NQC}
$$
\n
$$
h_{\ell m} = h_{\ell m}^{0} + \frac{\Gamma_{\ell m}^{-1}}{2} - \frac{\Gamma_{\ell m}^{-1}}{2} - \frac{\Gamma_{\ell m}^{-1}}{2}
$$
\nRadiation Reaction

\n
$$
\dot{p}_{\varphi} = \hat{\mathcal{F}}_{\varphi},
$$
\n
$$
\dot{p}_{r*} = \sqrt{\frac{A}{B}} \left(-\partial_r \hat{H}_{\text{EOB}} + \hat{\mathcal{F}}_r \right)
$$

Inspiral models: in a nutshell

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Inspiral models: performance

Inspiral models: performance

Phenomenology of a merger: GWs

Post-merger: common features

Very high frequency emission (> 1kHz) → **Even harder to measure than inspiral!** Additionally: complicated post-Merger (B fields, neutrinos, hydro, ...)

 \rightarrow Look for robust, common features and model those

Post-merger: quasi-universality

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Post-merger: "models"

- **Unmodeled** [\[Chatziioannou+2017,](https://arxiv.org/pdf/1711.00040.pdf) [Wijngaarden+22](https://arxiv.org/pdf/2202.09382.pdf)]
	- Detects PM even with (very) low SNRs
	- Identifies unexpected features in the waveform
	- Cannot directly be joined to inspiral WFs
- **● Phenomenological models** [\[Tsang+19](https://arxiv.org/abs/1907.02424), [Breschi+19,](https://arxiv.org/pdf/1908.11418.pdf) [Soultanis+2021,](https://arxiv.org/pdf/2111.08353.pdf) [Breschi+22\]](https://arxiv.org/abs/2205.09979)
	- Models the most robust features of the PM (beyond f2)
	- Usually, lower fitting factors than unmodeled
	- Can be immediately joined to inspiral waveforms
- **● NR-based** [\[Clark+2015](https://arxiv.org/abs/1509.08522) , [Easter+2018](https://arxiv.org/abs/1811.11183)]
	- Statistical representation of NR waveforms (reduced basis, PCA)...
	- Good fitting factors w/ NR
	- Retains all of NR uncertainty, less "flexible" than phenomenological wfs

Post-merger: performance

Post-merger: performance

For more models see e.g. [Hotokezaka+2011](https://arxiv.org/abs/1307.5888), [Clark+2015,](https://arxiv.org/abs/1509.08522) [Chatziioannou+2017,](https://arxiv.org/abs/1711.00040) [Easter+2018,](https://arxiv.org/abs/1811.11183) [Bose+2017,](https://arxiv.org/abs/1705.10850) [Tsang+2019,](https://arxiv.org/abs/1907.02424) [Easter+2020,](https://inspirehep.net/literature/1799930) [Soultanis+2021,](https://arxiv.org/abs/2111.08353) [Wijngaarden+2022](https://arxiv.org/abs/2202.09382)

Part 2: What's next?

Future developments and challenges

Future detectors

Next gen: large sensitivity improvement:

- Low frequency \rightarrow improved masses measurement
- High frequency \rightarrow improved matter effects measurement

Few very loud signals (SNR > 100)

- Some loud signals also with current gen!
- \rightarrow What does this mean for our models?

Waveform systematics

"What's the biggest challenge for numerical relativity and waveform modeling for next generation detectors?"

Waveform systematics

Relative difference between

Relative difference between

and injected Lambda

recovered

"Cumulative" difference between "Cumulative" difference between Lambdas recovered and injected Lambdas and injected recovered

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]

Waveform systematics

Relative difference between

Relative difference between

How do you beat systematics?

● "Marginalize" over model uncertainty, at the expense of measurement precision

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	- EOB and Phenom: sample over NR-fits error during PE
	- Do the same whenever quasi-universal relations are employed
	- Hypermodel approach [\[Puecher+24\]](https://inspirehep.net/literature/2706493)

How do you beat systematics?

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	- EOB and Phenom: sample over NR-fits error during PE
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	- Hypermodel approach [\[Puecher+24\]](https://inspirehep.net/literature/2706493)
- **● "Just" build better models**

● Inspiral:

<u>and</u>

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- Post-merger:
	- Numerical Relativity: understand muons, Pions,B-fields, neutrinos, thermal effectsresolution,.. **[Gieg+24,](https://arxiv.org/abs/2409.04420) [Pajkos+24](https://arxiv.org/pdf/2409.09147)**]

 $\frac{1}{42}$

Conclusions

- Modeling the inspiral is """easy""", but we need to do it extremely well to avoid systematics
- Modeling the post-merger is harder, current models capture just few features
- NR is going to be fundamental to model the beyond-contact regime

Backup slides

Next Gen detectors: post-merger

Quasi-universality breaking: \rightarrow phase transitions, thermal effects, magnetic fields, muons, ...?

Current GW observations

Confidently detected systems with at least one NS:

Current GW observations

From GW alone (using a spectral parameterization of the EOS):

Current EOS constraints: GW + KN (+ NICER)

[\[Breschi+24](https://arxiv.org/pdf/1908.11418.pdf)]

 $R_{1.4M_{\odot}}$ between 12-14 km or 11-13 km, depending on NICER

Current EOS constraints: GW + KN (+ NICER +...)

Set A: Chiral EFT, pQCD, radio timing, NICER, GW170817

● Set B: Heavy-Ion collisions, qLMXBe, Black Widow, GW170817 + AT2017gfo + GRB170817A

Set C: PREX, CREX, Burster, Hess, GW190425, GRB211211A, GW170817 (postmerger)

Next Gen detectors: inspiral

• Generate fake data: GW injection & recovery with ET

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Next Gen detectors: post-merger

Post-merger detectable (on its own) if $SNR(PM) > 8 \rightarrow SNR$ (inspiral) O(100). Constraints on the "max TOV" properties of the (cold) EOS

NR simulations: physics & accuracy (PM)

HY = pure hydro LK = Leakage VM0 = Viscosity + M0 $\Delta x_{LR} \approx 247$ m, $\Delta x_{SR} \approx 185$ m, $\Delta x_{HR} \approx 123$ m.

- Up to merger: little to no difference due to microphysics
- "Early" times: small differences
- "Late" times: large differences, especially for SR/HR simulations

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Post-merger detectability

- Post-merger only
- Simulated ET signal, analyzed with NRPMw
- Locating the detector in Virgo's place (with typical triangular configuration)
- 10 different noise realizations

Current EOS constraints: GW + KN (+ NICER)

"Joint and coherent" analysis of GW170817 + AT2017gfo:

Joint likelihood

$$
p(d_{gw}, d_{kn}|\theta_{gw}, \theta_{kn}) = p(d_{gw}|\theta_{gw}) p(d_{kn}|\theta_{kn})
$$

- Common parameters:
	- Luminosity distance
	- merger time
	- (inclination, if anisotropic model)
	- NR fits: link some KN parameters with GW ones

$$
M_{\rm ej}^d, v^d, m_{\rm disk}
$$

Re-sampling of the posterior to determine EOS from a prior set of ~10M EOSs, adding also NICER results

Accuracy requirements (on the back of an envelope)

[\[Puecher+22\]](https://arxiv.org/pdf/2210.09259.pdf) $\rightarrow \cdot$ NO-PM 225Mpc $-\bullet$ NO-PM 135 Mpc \cdot . NO-PM 68Mpc \rightarrow QU-PM 225Mpc \rightarrow QU-PM 135 Mpc QU-PM 68Mpc \cdot . Zero noise - Source $2_{\text{[QU-PM]}}$ 2.00 600 1.75 500 1.50 $\frac{400}{2000}$ 1.25 ϕ 1.00 0.75 $200 -$ 0.50 $100 0.25$ Ω 20 40 120 Ω 60 80 100 ΔÃ

> $\Delta \tilde{\Lambda} = 100 \rightarrow \Delta \phi \sim 2 \text{rad}$ $\Delta \tilde{\Lambda} = 25 \rightarrow \Delta \phi \sim 0.5$ rad