1st TEONGRAV international workshop on the Theory of Gravitational Waves





Modeling the full spectrum of Gravitational waves from BNS

Rossella Gamba Rome, 20/09/2024





Outline

• Gravitational waves from BNS Mergers

2

- Matter effects
- Inspiral models
- Post-merger models

• What's next?

- Challenges
- Developments

Part 1: Gravitational waves from BNSs Matter effects, models and all that

Phenomenology of a merger



Phenomenology of a merger



Phenomenology of a merger: GWs



Phenomenology of a merger: GWs







BBH + (some kind of correction) = **BNS**?

Corrections = Matter effects. They are what distinguish NS from **point particles**.

- Tidal effects:
 - "Adiabatic" tidal effects $(\Lambda_\ell, \Sigma_\ell)_{\text{[Damour1983, Flanagan+2007, Damour+2008, Vines+2010, Henry+2020, Mandal+24]}}$
 - \circ "Dynamical" tidal effects $(ar{\omega}_f)$ [Lai+1994, Hinderer+2016, Steinhoff+2016, Steinhoff+2021]
- Spin induced effects $(C_q, C_{oct}, C_{hex}, ...)$ [Poisson1998, Krishnendu+2017]
- **Other effects**: nonlinear mode couplings, other modes resonances and excitations, crust shattering, dissipative effects ... [Ho+1999, Tsang+2013, Andersson+2017, Ripley+24, ...]

All of the above coefficients depend on the Equation of State (EOS):

 $(m, \mathrm{EOS})
ightarrow (\Lambda_\ell, \Sigma_\ell, \dots)$ Talk to Micaela

What we'd like: $(m, \Lambda_{\ell}, \Sigma_{\ell}, \dots) \to \text{EOS}$

Various matter effects are not (entirely) independent from one-another: **quasi-universal** relations

$$\Lambda_2 \to (\Lambda_\ell, \Sigma_\ell, C_q, C_{\text{oct}}, \bar{\omega}_f, \dots)$$



$$\Psi(x) = 2\pi f t_c - \phi_c - \pi/4 + \frac{3}{128\nu} x^{-5/2} \Big[1 + \dots + x^2 C_q c_{MQ} + \dots + x^5 \tilde{\Lambda} c_{\Lambda} + \dots + x^8 \frac{c_{dyn}}{\Omega_{A,f}^2} + \dots \Big]$$

0 PN

- Matter effects are a high-frequency correction:
 - Tidal effects are important above ~300 Hz
 - f-mode (most) relevant above 1kHz
- Current detectors are not very sensitive in those regions
 - \rightarrow Hard to measure





BBH + (some kind of correction) = **BNS**?



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Inspiral models

Breakdown of the Market?

- Flagship IMR models ~ grouped into 3 families
- Pros and cons to each family

NR Surrogates

- Interpolate NR waveforms across parameter space
- Accuracy comparable to input NR
- Reasonably efficient waveform evaluation
- Limited by availability of NR
- Limited by NR duration but can hybridise with inspiral models

Phenomenological

- Analytical + NR calibration model of **GW** signal
- Extremely efficient to evaluate
- Time- and frequency-domain models available
- Limited in calibration by availability of NR
- Less fundamental harder to incorporate information

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THE ROYAL SOCIETY

- The BBH situation . Hamiltonian framework for dynamics and GW signal
- Evolve system of ODEs work needed to mitigate computational cost
- Limited in calibration by availability of NR
- Natural framework for incorporating additional physics (GSF, scattering, ...)

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Inspiral models

- Post Newtonian approximants (PN) [Krishnendu+2017, Henry+2020, Schmidt+2021]
 - Analytical and fast
 - Examples: TaylorF2, TaylorT4
- **Phenomenological** approximants (Phenom)

[Dietrich+2017,Kawaguchi+2018,Dietrich+2019,Gamba+23,Williams+2024,Abac+2024]

- Fits to EOB+NR hybrid waveforms
- Fast
- Examples: (any BBH inspiral model) + NRTidalv3, Kawaguchi+, PhenomGSF,...
- Effective One Body approximants (EOB) [Bini+2012, Akcay+2018, Lackey+2018, Tissino+22, Gamba+2023]
 - Semi-analytical, resummed PN + NR
 - Not-as-fast, generally, but there exist acceleration techniques (PA, SPA)
 - Examples: TEOBResumS, SEOBNRv4T (& related surrogates)

Inspiral models: Phenom

Simple idea:

- Choose your target BNS waveform (EOB + NR, pure EOB, ...)
- 2. Choose your BBH baseline (EOB, NR, Phenom, NR surrogate, ...)
- 3. Separate matter contributions from BBH baseline (both phase and amplitude):

$$\begin{split} \Psi_{\rm BNS} &\sim \Psi_{\rm BBH} + \Delta \Psi_{\rm matter} \\ \Delta \Psi_{\rm matter} &\sim \Psi_{\rm BNS} - \Psi_{\rm BBH} \end{split}$$

4. Directly fit the matter contributions and Adu $\Delta \Psi_{\text{matter}} \sim \Delta \Psi_{\text{ad.tides}} + \Delta \Psi_{\text{dyn.tides}} + \Delta \Psi_{MQ} + \dots$



Inspiral models: EOB

Two body problem \rightarrow test particle around (deformed) Kerr. Three ingredients:

Hamiltonian

$$H_{\rm EOB} = M\sqrt{1 + 2\nu(\hat{H}_{\rm eff} - 1)},$$

$$\hat{H}_{\rm eff} = \sqrt{p_{r_*}^2 + A(r)\left(1 + \frac{p_{\varphi}^2}{r^2} + 2\nu(4 - 3\nu)\frac{p_{r_*}^4}{r^2}\right)}$$

Waveform

$$\begin{aligned} \text{Waveform} \qquad & h_{\ell m} = h_{\ell m}^{(N,\epsilon)} \hat{S}_{\text{eff}}^{(\epsilon)} \hat{h}_{\ell m}^{\text{tail}} f_{\ell m} \hat{h}_{\ell m}^{\text{NQC}} \\ & h_{\ell m} = h_{\ell m}^{0} + h_{\ell m}^{T} = h_{\ell m}^{\text{Newt}} (\hat{h}_{\ell m}^{0} + \hat{h}_{\ell m}^{T}) \\ & h_{\ell m} = \hat{F}_{\varphi} , \\ \text{Radiation Reaction} \\ & \dot{p}_{r_{*}} = \sqrt{\frac{A}{B}} \left(-\partial_{r} \hat{H}_{\text{EOB}} + \hat{\mathcal{F}}_{r} \right) \end{aligned}$$



Inspiral models: in a nutshell

	PN	TEOBResumS	SEOBNRv4T	PhenomGSF	PhenomNRTv3
Adiabatic tides	Cons.3PN Diss. 2PN	Cons. 2.5 PN Diss. 2PN	Cons. 2.5 PN	2.5PN	2.5PN
Dynamic tides	Yes	yes	yes	no(t yet)	yes
self-spin	NNLO	NNLO (resummed)	NNLO (resummed)	no(t yet)	NNLO (PN)
Additional notes	Many more effects	GSF-resummation, ell=2,,8 electric contributions; ell=2 magnetic contributions; Higher modes in wf; eccentricity; precession	BBH NQC corrections; ell=2,3 electric contributions;	Fits of phase calibrated to TEOBResumS, residual w.r.t. 7.5PN	Fits for phase and amplitude calibrated to EOBNR hybrids, Padé resummed, 55 simulations

/20

Inspiral models: performance



Inspiral models: performance



Tides	Mass ratio	Spins	Error @ merger			
Moderate (<1000)	equal mass	X	< 1 rad (0.2-0.5)			
Large	equal mass	X	~ 1 rad			
Moderate (<1000)	Unequal mass	X	~ 1 rad (0.5-1)			
Large	Unequal mass	X	> 1 rad			
Madarata	Fault		> 1 red lorge for			
 Models perform the worst when for large tidal parameters/large spins Typical NR error ~1 rad at merger (large!) 						

Phenomenology of a merger: GWs



Post-merger: common features

Very high frequency emission (> 1kHz) \rightarrow **Even harder to measure than inspiral!** Additionally: complicated post-Merger (B fields, neutrinos, hydro, ...)



 \rightarrow Look for robust, common features and model those

Post-merger: quasi-universality



25

Post-merger: "models"

- Unmodeled [Chatziioannou+2017, Wijngaarden+22]
 - Detects PM even with (very) low SNRs
 - Identifies unexpected features in the waveform
 - Cannot directly be joined to inspiral WFs
- Phenomenological models [Tsang+19, Breschi+19, Soultanis+2021, Breschi+22]
 - Models the most robust features of the PM (beyond f2)
 - Usually, lower fitting factors than unmodeled
 - Can be immediately joined to inspiral waveforms
- NR-based [Clark+2015, Easter+2018]
 - Statistical representation of NR waveforms (reduced basis, PCA)...
 - Good fitting factors w/ NR
 - Retains all of NR uncertainty, less "flexible" than phenomenological wfs

Post-merger: performance





Post-merger: performance



Part 2: What's next? Future developments and challenges

Future detectors



	BNS			BBH		
Cosmic rate	4	4.7×10^{5}			1.2×10^{5}	
SNR ρ	≥ 10	≥ 30	≥ 100	≥ 10	≥ 30	≥ 100
HLVKI+	190	6	1	6,100	240	e
VK+HLIv	2,000	71	2	33,000	2,900	74
HLKI+E	41,000	1,700	45	97,000	31,000	2,100
VKI+C	110,000	6,000	160	110,000	53,000	6,400
KI+EC	160,000	9,400	250	120,000	69,000	9,500
ECS	240,000	20,000	550	120,000	87,000	17,000

Next gen: large sensitivity improvement:

- Low frequency → improved masses measurement
- High frequency → improved matter effects measurement

Few very loud signals (SNR > 100)

- Some loud signals also with current gen!
- \rightarrow What does this mean for our models?

Waveform systematics

"What's the biggest challenge for numerical relativity and waveform modeling for next generation detectors?"



Waveform systematics

and injected Lambda

recovered

Relative difference between



Lambdas "Cumulative" difference between recovered and injected

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Waveform systematics

Relative difference between



How do you beat systematics?

• "Marginalize" over model uncertainty, at the expense of measurement precision

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- "Marginalize" over model uncertainty, at the expense of measurement precision [Read+23]
 - EOB and Phenom: sample over NR-fits error during PE
 - Do the same whenever quasi-universal relations are employed
 - Hypermodel approach [Puecher+24]

How do you beat systematics?

- "Marginalize" over model uncertainty, at the expense of measurement precision [Read+23]
 - EOB and Phenom: sample over NR-fits error during PE
 - Do the same whenever quasi-universal relations are employed
 - Hypermodel approach [Puecher+24]
- "Just" build better models

• Inspiral:

- Inspiral:
 - Folding in some information from NR in EOB models



- Inspiral:
 - Folding in some information from NR in EOB models
 - Include some of the recent 3PN tidal effect terms, **not just circular**[Mandal+24]

$ \widetilde{\mathcal{H}}_{\rm 3PN}^{\rm AT} = \widetilde{\lambda}_{(1)} \left\{ \widetilde{L}^6 \left(-\frac{45\nu^3}{32\widetilde{r}^{12}} - \frac{15\nu^2}{4\widetilde{r}^{12}} - \frac{33\nu}{32\widetilde{r}^{12}} \right) + \widetilde{L}^4 \left(\widetilde{p}_r^2 \left(-\frac{189\nu^3}{32\widetilde{r}^{10}} + \frac{99\nu^2}{8\widetilde{r}^{10}} + \frac{711\nu}{32\widetilde{r}^{10}} \right) - \frac{9\nu^2}{\widetilde{r}^{11}} + \frac{15\nu}{4\widetilde{r}^{11}} \right) \right\} $
$+ \widetilde{L}^2 \left[\widetilde{p}_r^2 \left(-\frac{9\nu^3}{2\widetilde{r}^9} - \frac{2775\nu^2}{16\widetilde{r}^9} - \frac{72\nu}{\widetilde{r}^9} \right) + \widetilde{p}_r^4 \left(-\frac{99\nu^3}{32\widetilde{r}^8} + \frac{108\nu^2}{\widetilde{r}^8} - \frac{1431\nu}{32\widetilde{r}^8} \right) \right.$
$+\frac{3(117600\ \bar{\kappa}_{(1)}+1584771)\nu}{19600\tilde{r}^{10}}+\frac{93\nu^2}{2\tilde{r}^{10}}\biggr]+\tilde{p}_r^2\left(\frac{9993\nu^2}{56\tilde{r}^8}-\frac{3(117600\ \bar{\kappa}_{(1)}+626121)\nu}{9800\tilde{r}^8}\right)$
$+ \widetilde{p}_r^4 \left(-\frac{45\nu^3}{\widetilde{r}^7} - \frac{729\nu^2}{8\widetilde{r}^7} + \frac{777\nu}{16\widetilde{r}^7} \right) + \widetilde{p}_r^6 \left(\frac{1485\nu^3}{32\widetilde{r}^6} - \frac{465\nu^2}{8\widetilde{r}^6} + \frac{465\nu}{32\widetilde{r}^6} \right) - \frac{3(29400\bar{\kappa}_{(1)} + 429119)\nu}{9800\widetilde{r}^9} \\$
$+\frac{1}{q}\Bigg[\widetilde{L}^6\left(-\frac{15\nu^3}{8\widetilde{r}^{12}}-\frac{75\nu^2}{8\widetilde{r}^{12}}-\frac{99\nu}{16\widetilde{r}^{12}}+\frac{33}{32\widetilde{r}^{12}}\right)+\widetilde{L}^4\left(\widetilde{p}_r^2\left(-\frac{9\nu^3}{\widetilde{r}^{10}}-\frac{171\nu^2}{8\widetilde{r}^{10}}-\frac{27\nu}{16\widetilde{r}^{10}}+\frac{9}{32\widetilde{r}^{10}}\right)$
$+\frac{273\nu^2}{16\tilde{r}^{511}}+\frac{1545\nu}{16\tilde{r}^{511}}+\frac{495}{16\tilde{r}^{511}}\Big)+\tilde{L}^2\left(\tilde{p}_r^4\left(-\frac{99\nu^3}{8\tilde{r}^8}+\frac{315\nu^2}{8\tilde{r}^8}+\frac{27\nu}{16\tilde{r}^8}-\frac{9}{32\tilde{r}^8}\right)$
$\left. + \widetilde{p}_r^2 \left(-\frac{9\nu^3}{2\widetilde{r}^9} - \frac{831\nu^2}{16\widetilde{r}^9} + \frac{567\nu}{8\widetilde{r}^9} - \frac{99}{8\widetilde{r}^9} \right) + \frac{867\nu^2}{28\widetilde{r}^{10}} + \frac{3\left(8384 + 63\pi^2\right)\nu}{512\widetilde{r}^{10}} - \frac{1335}{8\widetilde{r}^{10}} \right) \right.$
$+ \tilde{p}_r^2 \left(\frac{2097\nu^2}{16\tilde{r}^8} - \frac{3\left(20576 + 63\pi^2\right)\nu}{256\tilde{r}^8} + \frac{261}{8\tilde{r}^8} \right) + \tilde{p}_r^4 \left(-\frac{45\nu^3}{\tilde{r}^7} - \frac{105\nu^2}{8\tilde{r}^7} - \frac{327\nu}{16\tilde{r}^7} + \frac{99}{16\tilde{r}^7} \right)$
$\left. + \widetilde{p}_{r}^{6} \left(\frac{99\nu^{3}}{4\widetilde{r}^{6}} - \frac{69\nu^{2}}{8\widetilde{r}^{6}} - \frac{45\nu}{16\widetilde{r}^{6}} + \frac{15}{32\widetilde{r}^{6}} \right) - \frac{1599\nu^{2}}{56\widetilde{r}^{9}} + \frac{\left(6376 - 945\pi^{2}\right)\nu}{64\widetilde{r}^{9}} + \frac{519}{4\widetilde{r}^{9}} \right] \right\}$
$+ (1 \leftrightarrow 2). \tag{6.4}$

- Inspiral:
 - Folding in some information from NR in EOB models
 - Include some of the recent 3PN tidal effect terms, **not just circular**[Mandal+24]
 - Study the effect of eccentricity: dynamical tidal effects → excitations of the modes after close passages [Hang+24, Takatsi+24]



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 - Spin-induced effects: spin-tidal couplings?[Abdelsahin+18,Castro+22]
- Post-merger:
 - Numerical Relativity: understand muons, Pions,B-fields, neutrinos, thermal effectsresolution,... [Gieg+24, Pajkos+24]



Conclusions

- Modeling the inspiral is """easy""", but we need to do it extremely well to avoid systematics
- Modeling the post-merger is harder, current models capture just few features
- NR is going to be fundamental to model the beyond-contact regime

Backup slides

Next Gen detectors: post-merger

Quasi-universality breaking: \rightarrow phase transitions, thermal effects, magnetic fields, muons, ...?



Current GW observations

Confidently detected systems with at least one NS:

Name	Kind	SNR	Tides?	EM?
GW170817	BNS	~30	Upper limits	
GW190425	BNS	~14	(weak) upper limits	×
GW200105	BHNS	~14	Uninformative	×
GW200115	BHNS	~12	Uninformative	×

Current GW observations

From GW alone (using a spectral parameterization of the EOS):



Current EOS constraints: GW + KN (+ NICER)

[Breschi+24]



 $R_{1.4M_{\odot}}$ between 12-14 km or 11-13 km, depending on NICER

Current EOS constraints: GW + KN (+ NICER +...)



 Set A: Chiral EFT, pQCD, radio timing, NICER, GW170817

Set B: Heavy-Ion collisions, qLMXBe, Black Widow, GW170817 + AT2017gfo + GRB170817A

Set C: PREX, CREX, Burster, Hess, GW190425, GRB211211A, GW170817 (postmerger)

Next Gen detectors: inspiral

- Generate fake data: GW injection & recovery with ET
- SNR ~ 540 • q = 1.5, M = 2.8 Prior SFHo $M_{\text{max}}^{\text{TOV}} \ge 2.09$ MPA1 $\circ \Lambda_1 = 600$ тма GW (next gen) MS1 \circ Λ_2 =1000 ENG • EOB model + ROQ APR4 ---- NL3 PSR 10952-0607 • Single tidal parameters meas ₩² $\begin{tabular}{l} \circ & \Lambda_1 = 600^{+130}_{-160} \\ \circ & \Lambda_2 = 970^{+730}_{-600} \end{tabular} \end{tabular}$ • Very tight constraints! $\pm 200m_{1}$ 10 12 14 16

/50

R[km]

Next Gen detectors: post-merger

Post-merger detectable (on its own) if SNR(PM) > 8 \rightarrow SNR(inspiral) O(100). Constraints on the "max TOV" properties of the (cold) EOS



NR simulations: physics & accuracy (PM)



HY = pure hydro LK = Leakage VM0 = Viscosity + M0 $\Delta x_{LR} \approx 247 \text{ m}, \Delta x_{SR} \approx 185 \text{ m}, \Delta x_{HR} \approx 123 \text{ m}.$

- Up to merger: little to no difference due to microphysics
- "Early" times: small differences
- "Late" times: large differences, especially for SR/HR simulations

52

Post-merger detectability



- Post-merger only
- Simulated ET signal, analyzed with NRPMw
- Locating the detector in Virgo's place (with typical triangular configuration)
- 10 different noise realizations

Current EOS constraints: GW + KN (+ NICER)

"Joint and coherent" analysis of GW170817 + AT2017gfo:

• Joint likelihood

$$p(\boldsymbol{d}_{gw}, \boldsymbol{d}_{kn} | \boldsymbol{\theta}_{gw}, \boldsymbol{\theta}_{kn}) = p(\boldsymbol{d}_{gw} | \boldsymbol{\theta}_{gw}) p(\boldsymbol{d}_{kn} | \boldsymbol{\theta}_{kn})$$

- Common parameters:
 - Luminosity distance
 - merger time
 - (inclination, if anisotropic model)
 - NR fits: link some KN parameters with GW ones

$$M_{\rm ej}^d, v^d, m_{\rm disk}$$

 Re-sampling of the posterior to determine EOS from a prior set of ~10M EOSs, adding also NICER results

Accuracy requirements (on the back of an envelope)

[Puecher+22]

-* • NO-PM 225Mpc ---NO-PM 135 Mpc NO-PM 68Mpc -* - QU-PM 225Mpc QU-PM 68Mpc Zero noise - $Source2_{[QU-PM]}$ 2.00 600 1.75 500 1.50 \tilde{V}_{00000}^{400} 1.25 de la 1.00 0.75 200 0.50 100 -0.25 20 120 40 60 80 100 0 ΔÃ

 $\Delta \tilde{\Lambda} = 100 \rightarrow \Delta \phi \sim 2 \text{rad}$ $\Delta \tilde{\Lambda} = 25 \rightarrow \Delta \phi \sim 0.5 \text{rad}$