Numerical relativity for strong field tests of gravity

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Credit: Noora Ghadiri

Today’s story line:

Act 1: Observing the gravitational universe

Act 2: Connecting theory and observations

Act 3: Quadratic gravity in a nutshell

Act 4: Black hole in quadratic gravity

Act 5: Black hole mergers in quadratic gravity

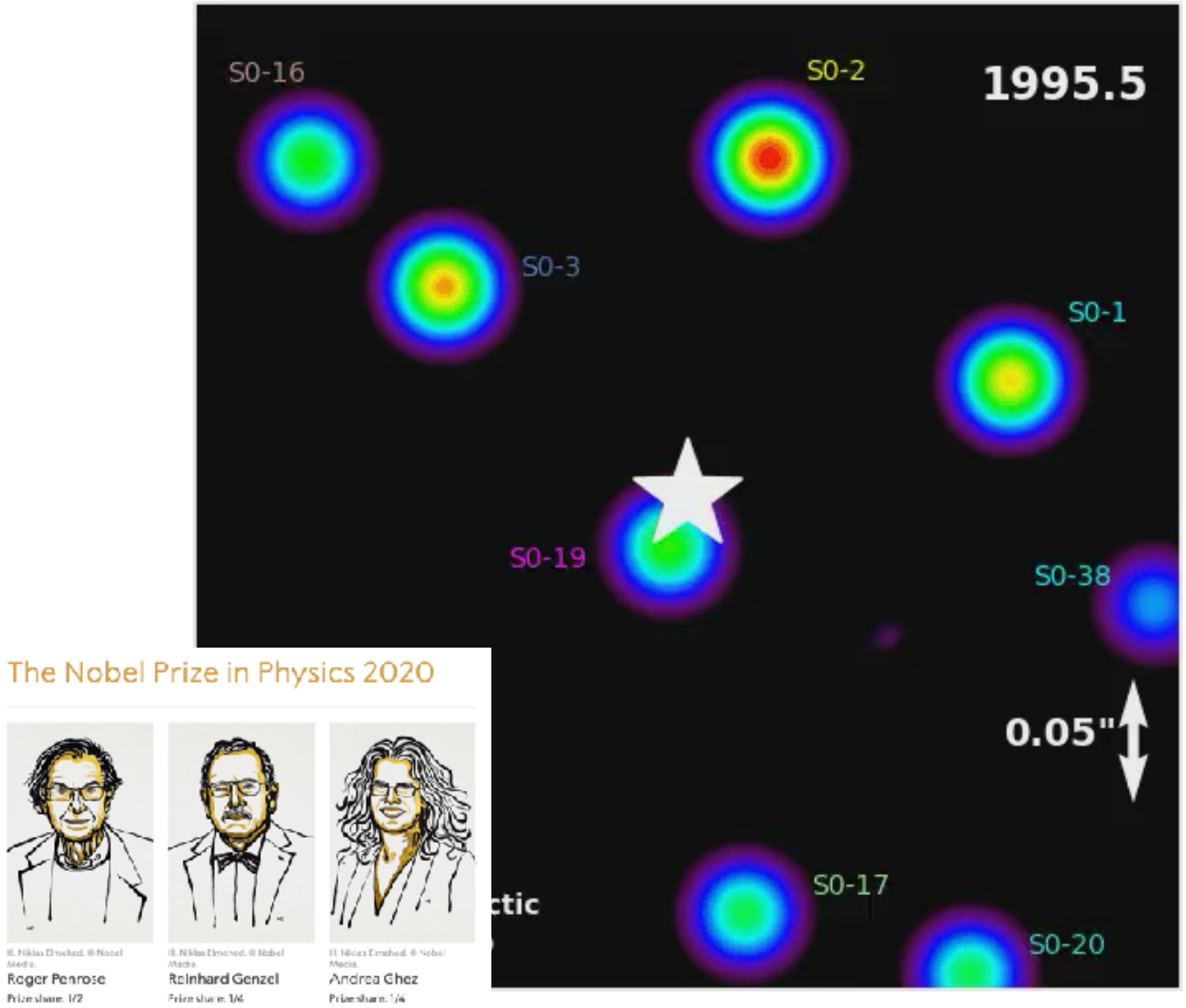
Final musings

Act 1:

Observing the gravitational universe … a best of

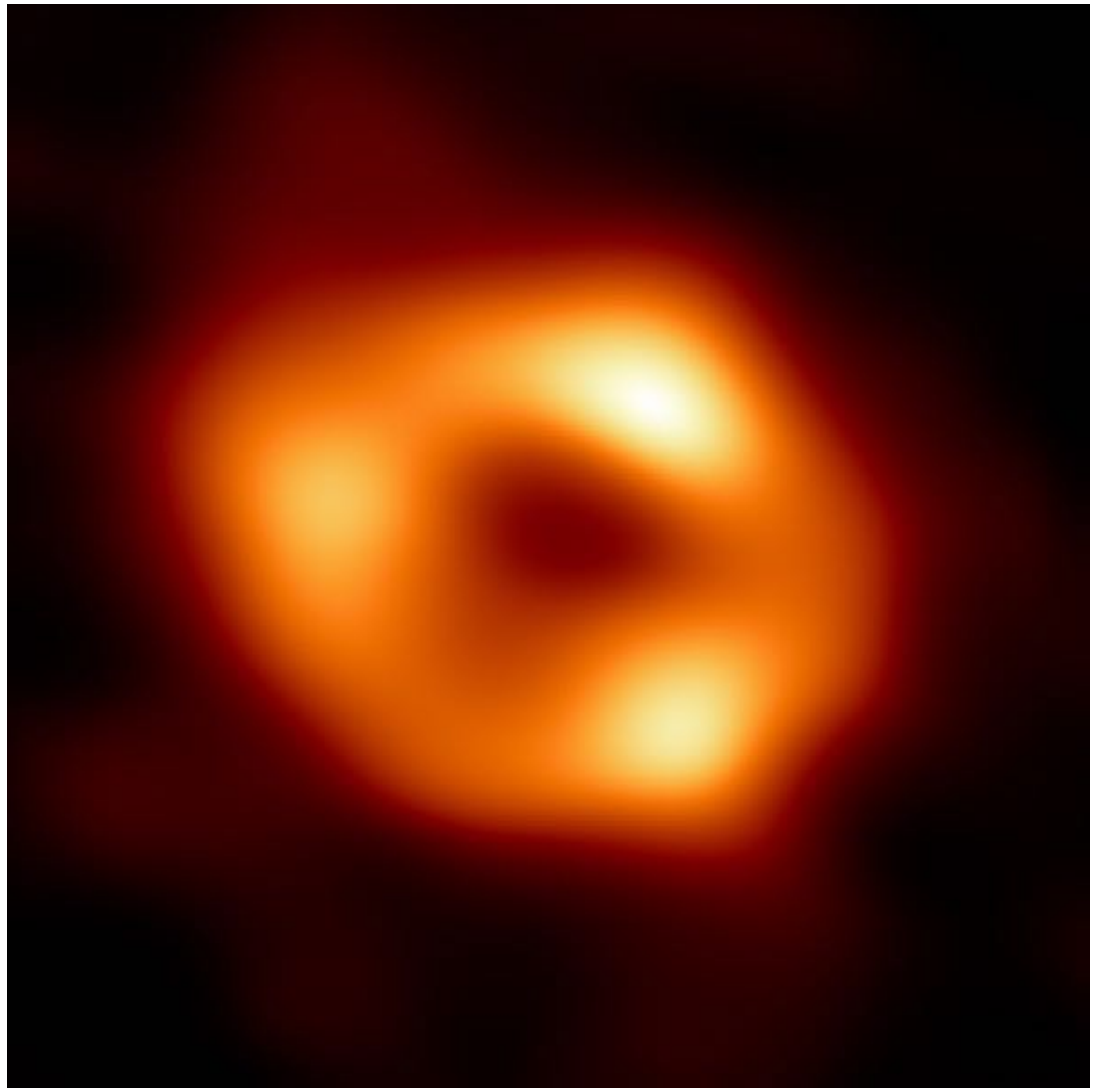
Act 1: The gravitational universe in electromagnetic waves

Recording
the center of the Milky Way
Sag A* — a black hole weighing $4 \cdot 10^6 M_\odot$



Credit: UCLA and Keck observatory

Imaging
the center of the Milky Way
Shadow of Sag A*



Credit: Event Horizon Telescope Collaboration

Act 1: The gravitational universe in gravitational waves

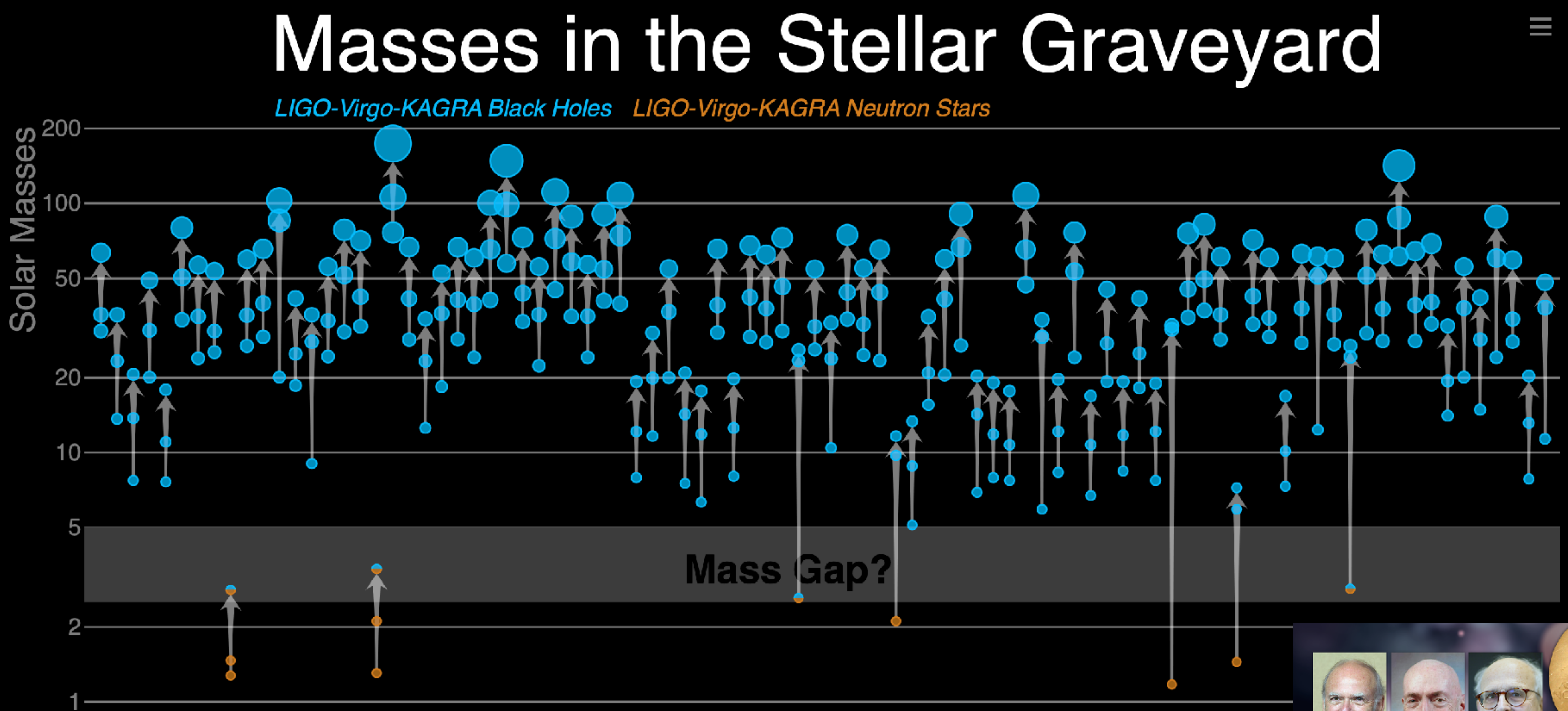
Stats of GW 150914:

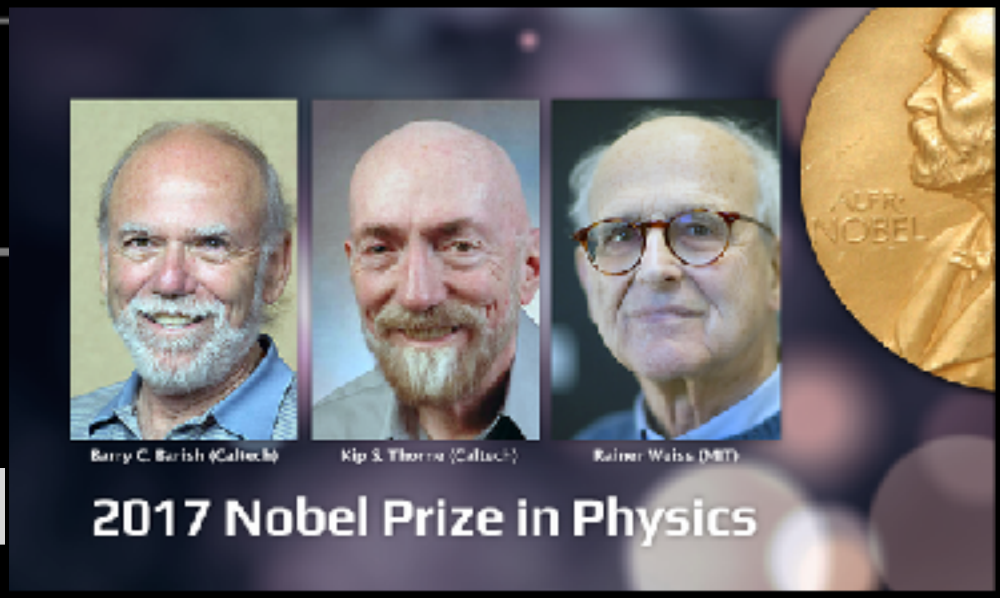
$36\,M_\odot + 29\,M_\odot \rightarrow 62\,M_\odot + E_{GW}$

$E_{GW} = 3\,M_\odot c^2 \sim 10^{41}\,\mathrm{kWh}$

Credit: Deborah Ferguson (UIUC)

Act 1: Status of gravitational wave detections



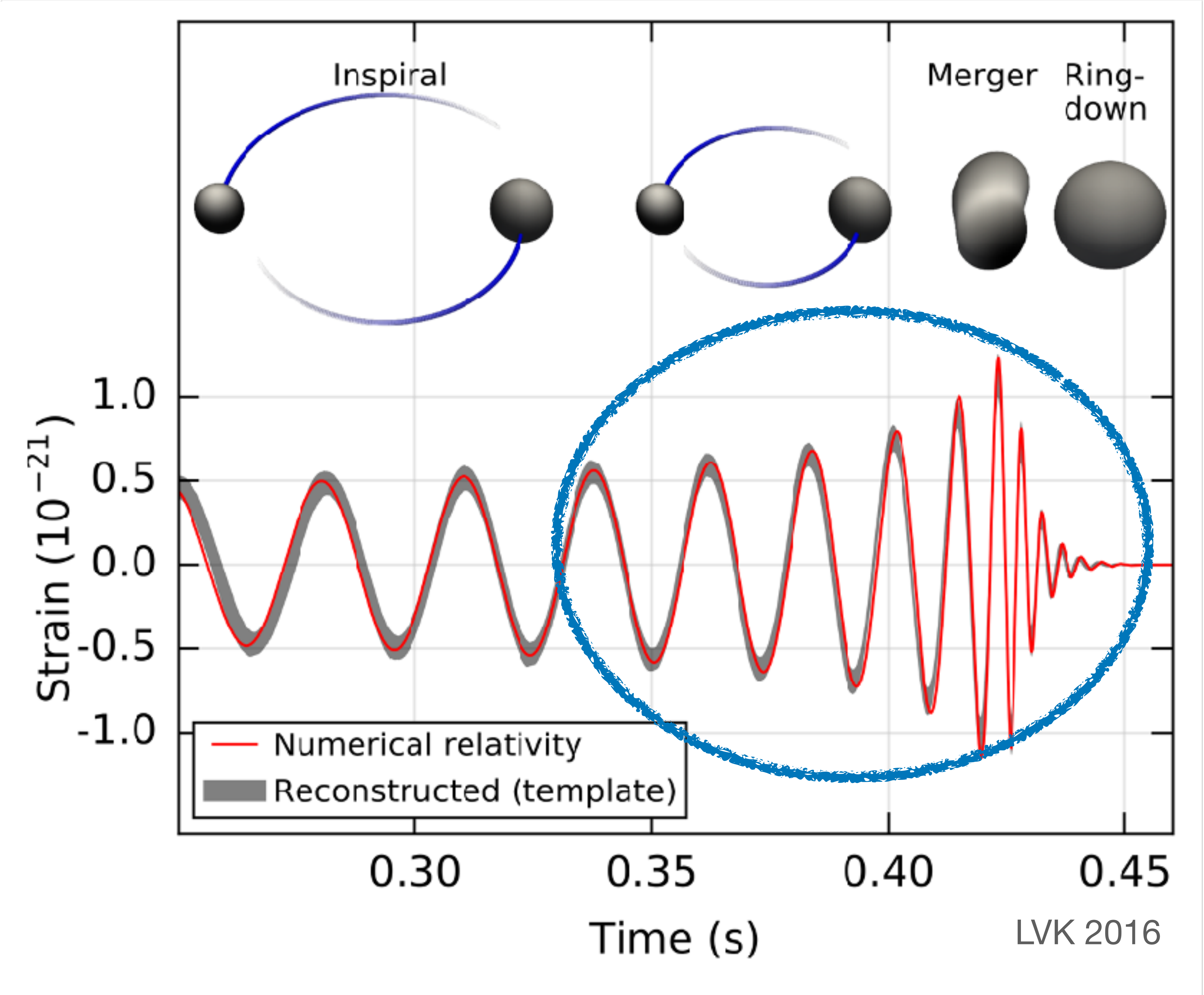


First detection: 2015

Daily alerts for 4th (current) observing run: https://gracedb.ligo.org/superevents/publ

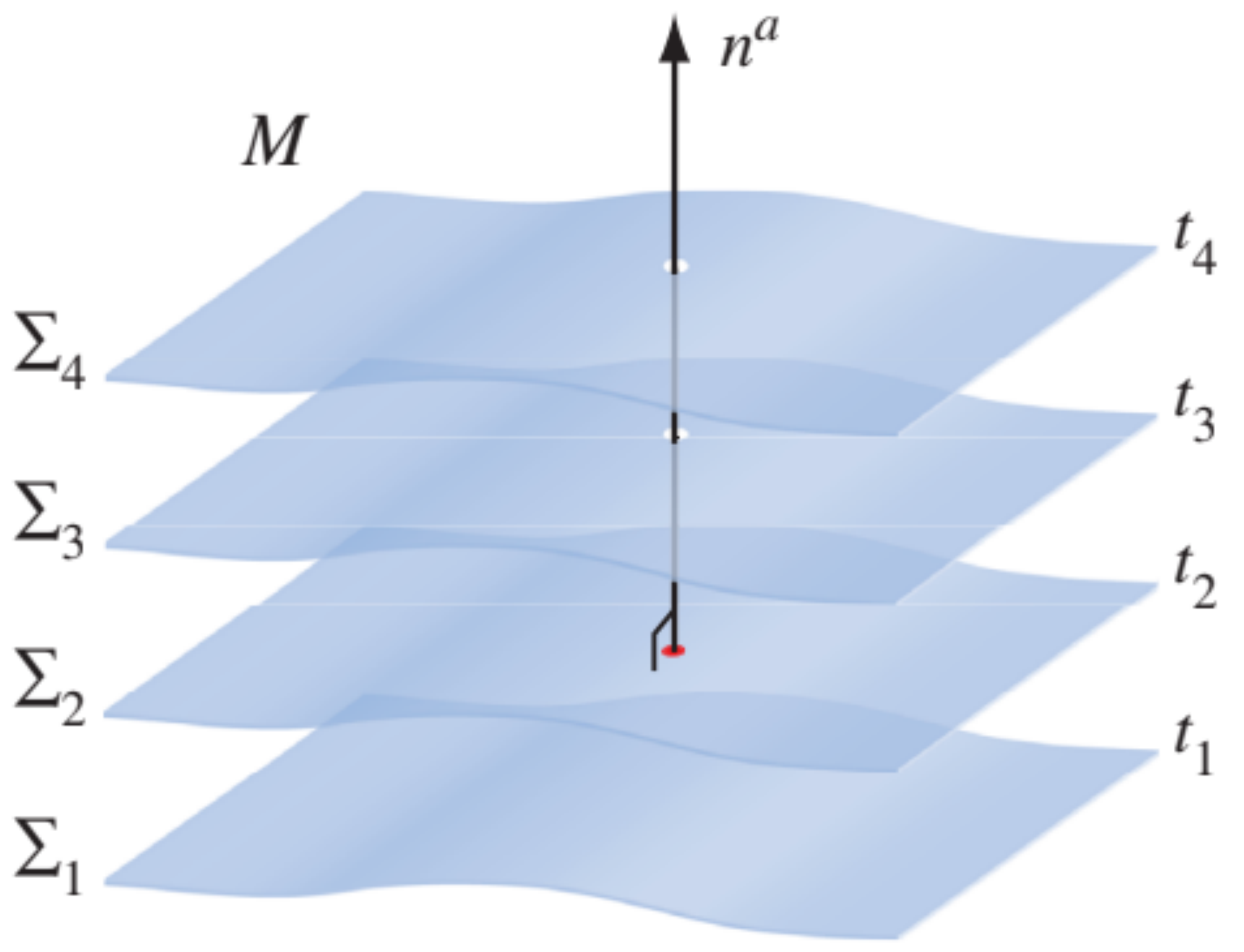
Act 2: Connecting theory and observations

Act 2:
Connecting theory and observations



- Formation and (stability) properties of black holes and compact objects
- Waveform templates crucial for signal characterisation
- Construct consistent inspiral-merger-ringdown waveform models in and beyond General Relativity

Act 2: Numerical relativity in a nutshell



Baumgarte & Shapiro “Numerical Relativity”

Einstein’s equations as initial value problem

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

Geometry
(Gravitational field)

Matter

Constraint equations
- Elliptic PDEs
- Initial conditions

Time evolution equations
- Hyperbolic (causal) PDEs
- Good gauge conditions (coordinate choice)
- *Well-posed initial value formulation*

Diagnostic tools
- Wave extraction
- (Apparent) horizon finder
- Matter density, fluxes, …

First stable simulations of quasi-circular black hole mergers: Pretorius ’05; Baker+ ’06; Campanelli+ ‘06

PDE — partial differential equation

Act 2: Landscape of Numerical Relativity for fundamental physics

In four spacetime dimensions the only divergence-free symmetric rank-2 tensor constructed solely from the metric and its derivatives up to second order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological constant. [Lovelock ’69-‘72]

Extra fields

- Scalar-tensor theory (Healy+’11, Barausse+ ’12, Shibata+ ’13, Berti et al ’13, Palenzuela+ ’15, Gerosa+ ’16, Sperhake+ ’17, Sagunski+’17, Rosca-Mead+ ’19, ’20, Mendes+ ’21)
- Horndeski gravity (Ripley+ ’19, Bernard+ ’19, Kovacs+ 20, Figueras+ ’20, ’21, Dima+ '21, Bezares+ ’20, ’21, Lara+ 22; Review: Ripley ’22)
- Boson stars (Liebling & Palenzuela ’12, Palenzuela+ ’17, Helfer+ ’18, ’19, ’21, Clough+ ’18, Bezares+ ’18-’22, Alcubierre+ ’19, Di Giovanni+’21, Jaramillo+ ’22, Siemonsen+ ’23, …)
- Ultralight fields (**HW+ ’12, Okawa+ ’14, Boscovic+ ’19** Bamber+ ’22, Aurrekoetxea ‘23, **Cheng+ in prep., Zilhao+15**, East+ ‘18, Siemonsen+ ‘22, **Clough+ ’22,** East+ ‘23)

Higher curvature

- Effective field theory of gravity (Held+ ’21, Held+ ’23, Cayuso+ ’23, Figueras+ ‘24)
- Semi-classical gravity (Benitez+ ’20)
- Black holes with near-horizon fluctuations (Liebling+ ’17)
- Scalar Gauss–Bonnet gravity (**Benkel+ ’16**; Ripley+ ’19, ‘20, Ramazanoglu ’19, Dima+ ’20, **Hegade+ ’22; HW+ ’18**, Okounkova ’20, East+ ‘21, **Silva+’21; Elley+ ’22,** Franchini+ 22, Salo+ ’22, Corman+ ’22, Thaalba+ ’23, Doneva+ ’23, Corman+ ’24, Nee+ ’24)
- Dynamical Chern-Simons gravity (Okounkova+ ’17 - ’19, Doneva+ ’20, ’21, **Richards+ ’23**)

Higher D gravity

- Black hole collisions (**Zilhao+ ’10, HW+ ’10 - ’14**, Cook+ ’17, Sperhake+ ’19, Andrade+ ‘22)
- Black hole stability (Lehner+Pretorius ’11, Yoshino+ ’09-’11, Figueras+ ’15-‘22)

Lorentz violations

- Einstein-æther theory (Garfinkle+ ’07; Barausse, Sarbach+ ’19, Adam+ ’22 Rubio+ ’23)
- Bimetric gravity (Torsello+ ’19, Kocic+ ’20)
- Massive gravity (de Rham+ ’22)

My team and collaborators

Credit: Bill Wiegland for UIUC Physics

Witek Gravity Group, May 2024

C.-H. Cheng, A. Dima (-> Sapienza), M. Elley, D. Ferguson, G. Ficarra (-> Calabria), D. Garzon, N. Ghadiri, L. Gualtieri, R. Haas (NCSA), F. Hancock, A. Hegade, T. Hinderer, H. Kogan (-> PhD at Stanford), E. Kopteva, E. R. Most, S. Nissanke, J. Noronha, U. Ohri, N. Ortiz, P. Pani, F. Pardoe, C. Richards, B. Shiralilou, H. O. Silva, T. Sotiriou, N. Yunes, Y. Zhang

Act 2: our software: Einstein Toolkit and Canuda

Einstein Toolkit

Community-driven,
Open-source
numerical relativity code
for computational astrophysics

DOI 10.5281/zenodo.7942541

Web: einsteintoolkit.org

Einstein Toolkit

428 members in 49 countries

Canuda

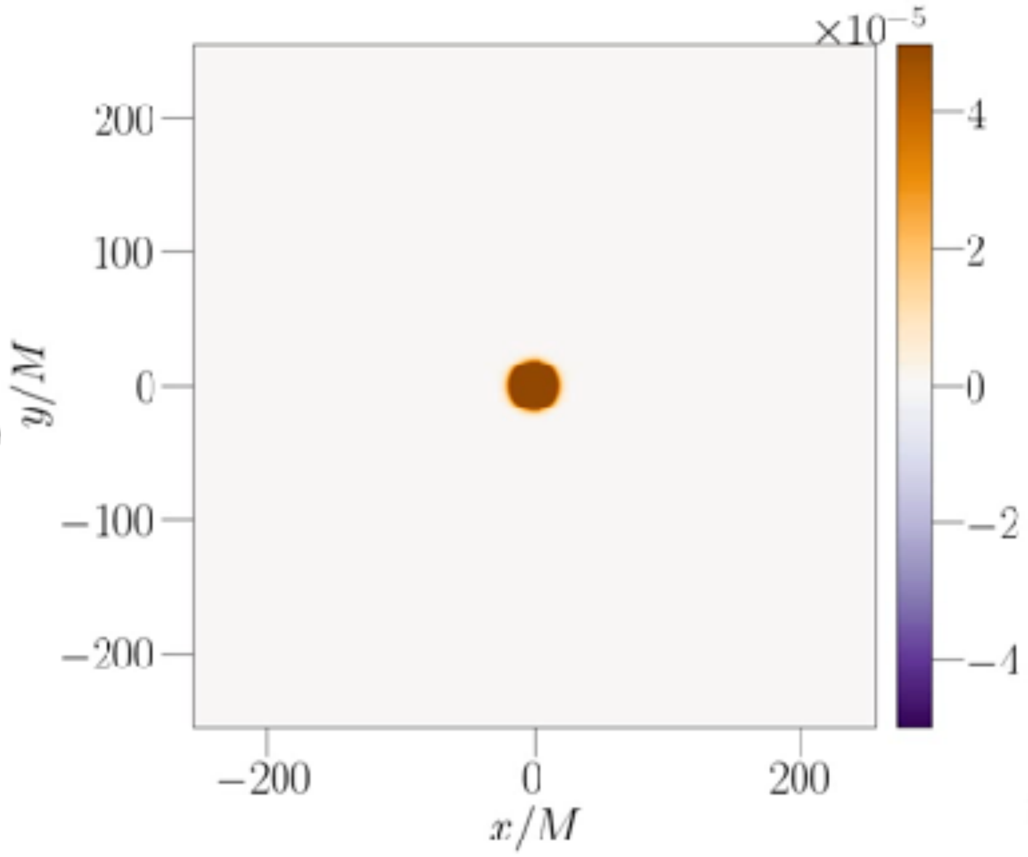
Open-source numerical relativity code
for fundamental physics

Witek, Helvi; Zilhao, Miguel; Bozzola, Gabriele; Cheng, Cheng-Hsin; Dima, Alexandru; Elley, Matthe Ficarra, Giuseppe; Ikeda, Taishi; Luna, Raimon; Richards, Chloe; Sanchis-Gual, Nicolas; Silva, Hector

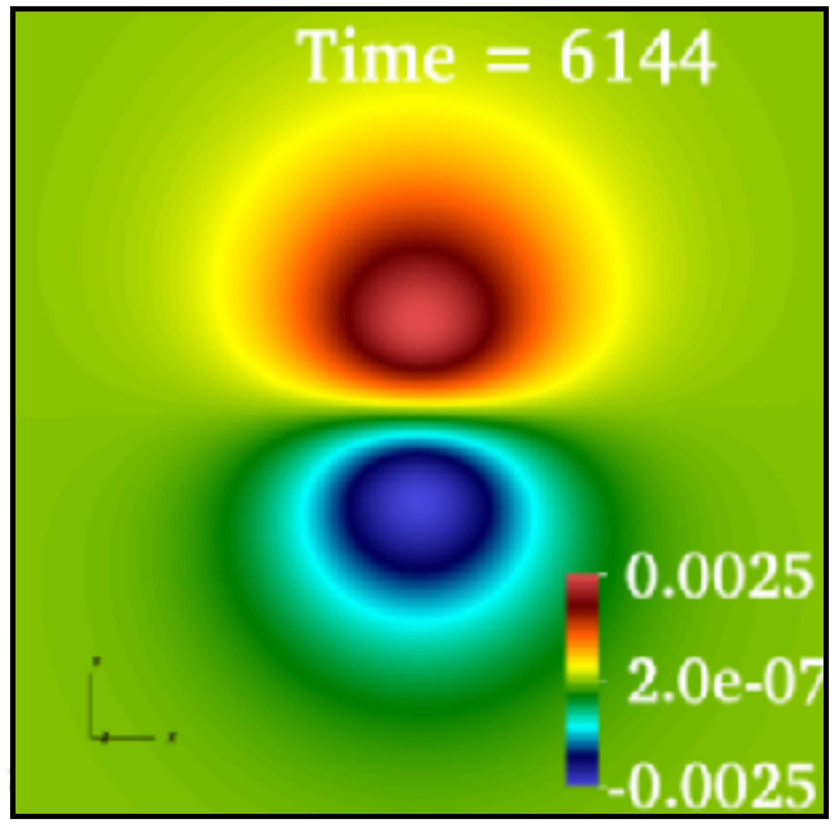
DOI 10.5281/zenodo.7791842

Web: https://bitbucket.org/canuda/

Canuda Numerical Relativity

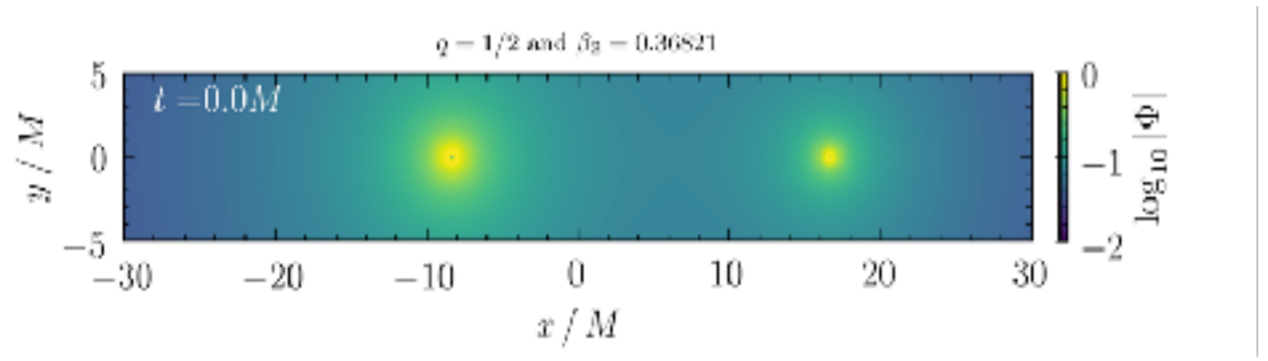


Binary black holes in scalar clouds
***Cheng**, **Ficarra** & **HW**, in prep.*

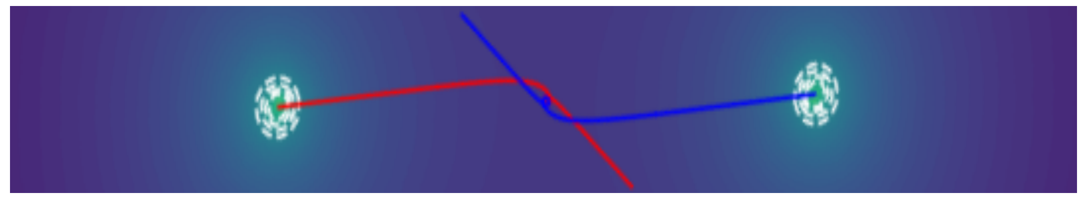


Multi-field boson stars,
Sanchis-Gual et al. (2022)

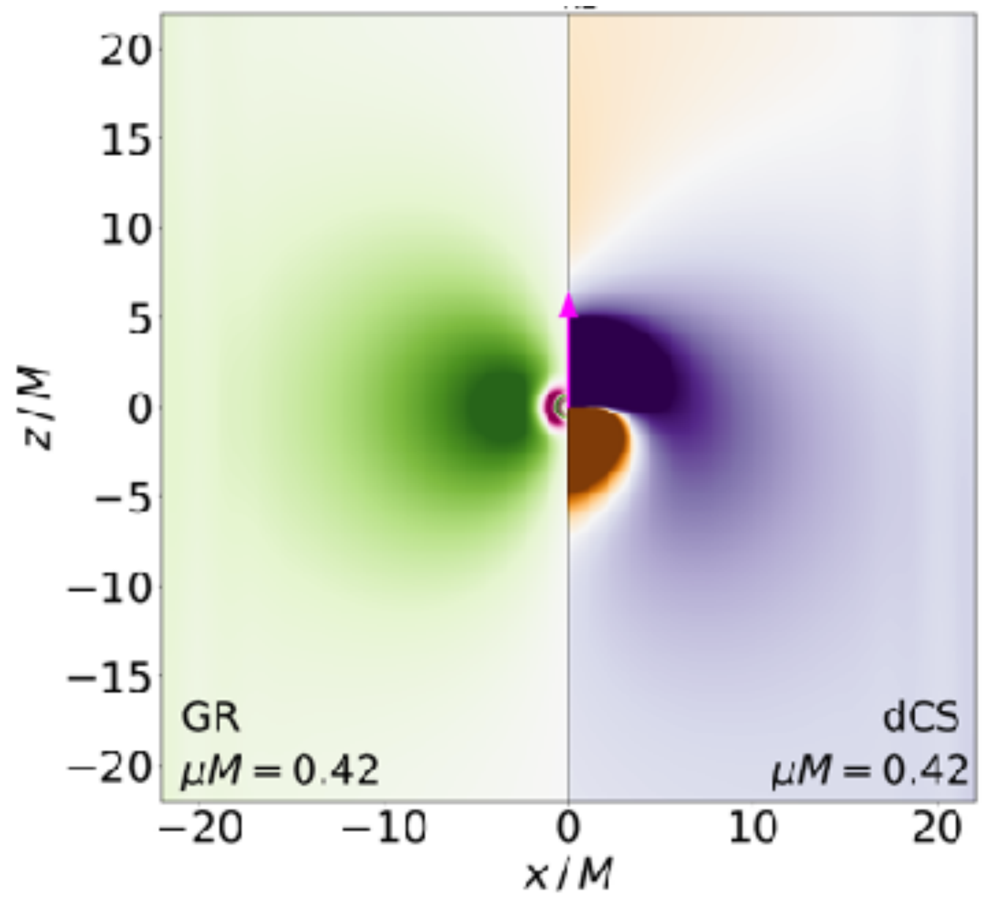
Our "Lab": Supercomputers like
Frontera (TACC) or Delta (NCSA)



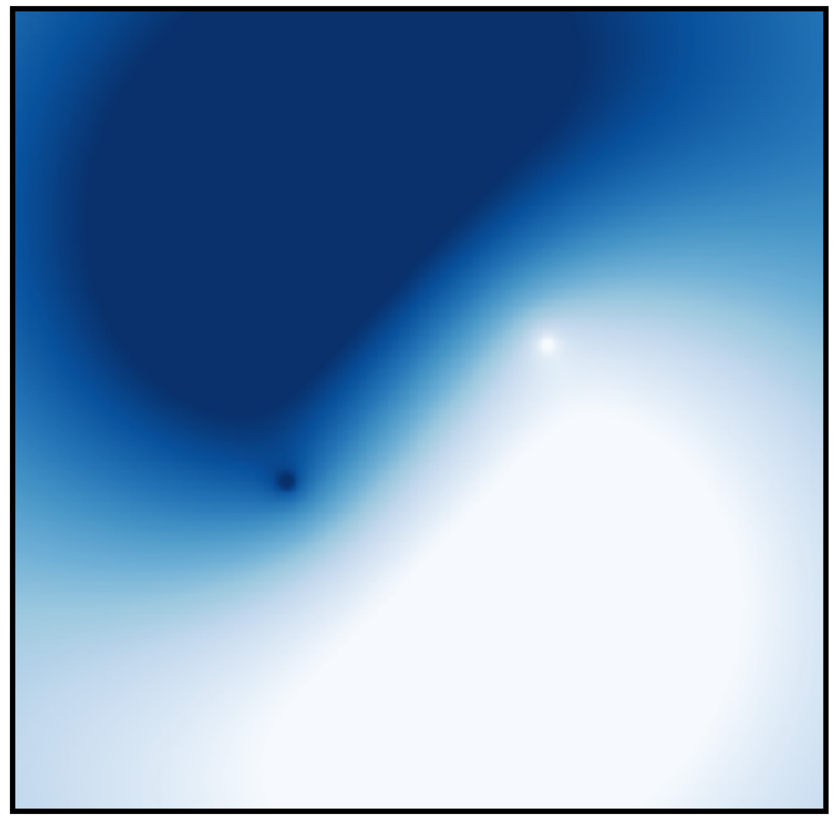
Dynamical descalarization in black hole mergers.
*Silva, **HW**, Elley, & Yunes (PRL 2021)*



Scattering black holes
***Kogan**, **Pardoe** & **HW**, in prep.*



Massive hair in dCS gravity
***Richards**, **Dima** & **HW** 2023*



Gravitational "molecules",
Ikeda et al. (2021)

[In Italy: Bruno Giacomazzo, Alexandre Dima, Giuseppe Ficarra, …]

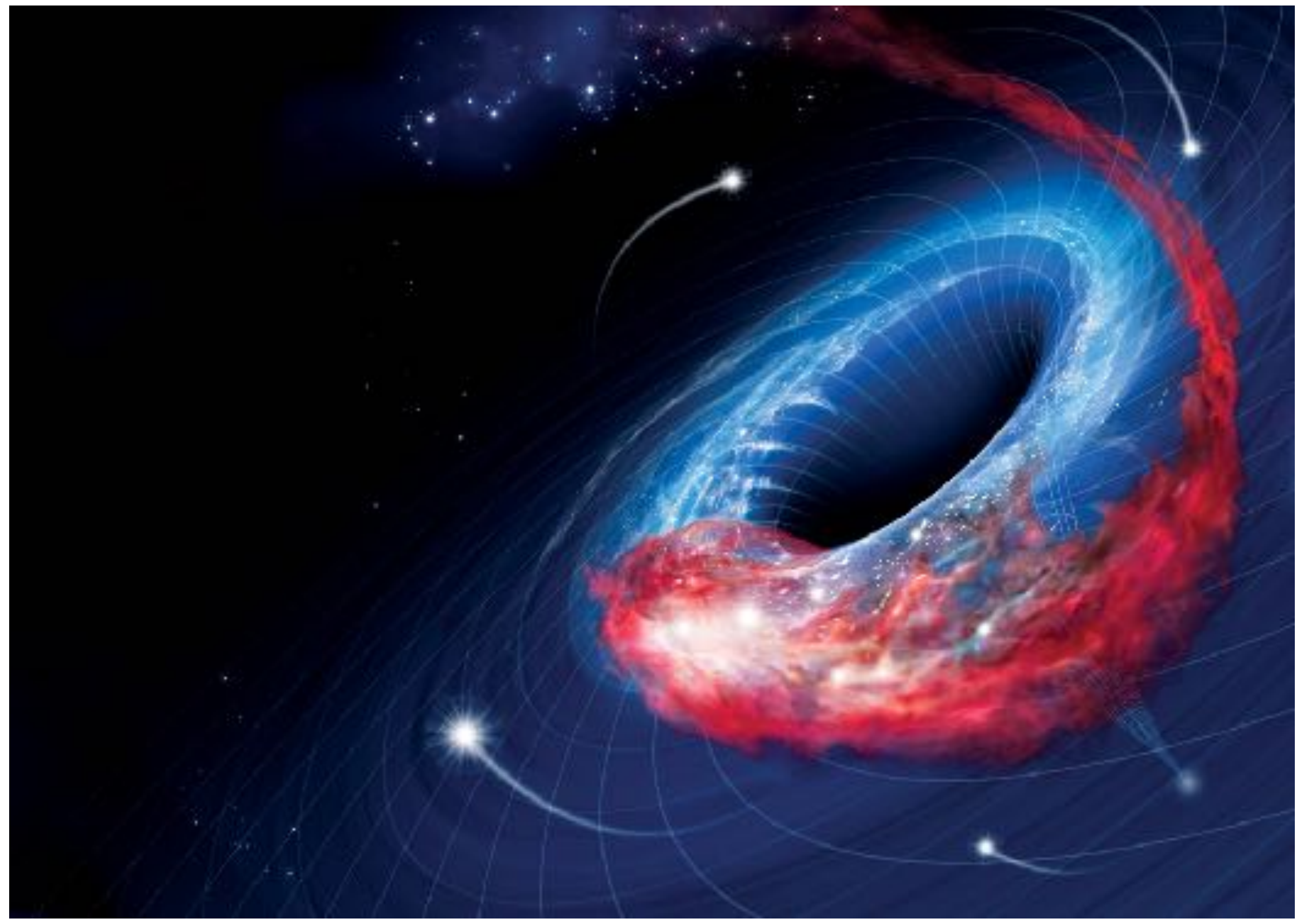
Act 3:

Quadratic gravity

in a nutshell

Act 3: Quadratic Gravity in a nutshell

- Higher curvature corrections relevant in strong-curvature regime
- Bottom-up: series expansion in curvature
- Top-down: inspired by low-energy limit of quantum gravity candidates, e.g., string theories
(Gross & Sloan '87, Kanti et al '95, Cano et al '21)



$$\mathscr{L} = R + \frac{\alpha_{GB}}{4} f(\Phi)\mathscr{G} - \frac{1}{2}(\nabla\Phi)^2 - V(\Phi) - g^2(\Phi)\left(\frac{1}{2}(\nabla\Theta)^2 + V(\Theta)\right) + \frac{\alpha_{CS}}{4} h(\Theta)\,{}^*RR$$

Einstein-Hilbert

Dilaton

Scalar Gauss-Bonnet (sGB)

$$\mathscr{G} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$$

Dynamical Chern-Simons (dCS)

Axion

Axi-dilaton gravity: $f(\Phi) = g(\Phi)^{-1} = e^{-\Phi}$ **and** $h(\Theta) = \Theta$

How to formulate higher derivative gravity theories?

- Field equations may have (i) mixed hyperbolic-elliptic character, (ii) higher time derivatives,…
- Causal and well-posed initial value formulation necessary for numerical stability

Order-by-order expansion

- Expand metric and scalars in coupling parameter

$$G^{(0)}_{\mu\nu}[g^{(n)}] \sim T^{eff}_{\mu\nu}(g^{(n-1)}, \ldots, \Phi^{(n-1)}, \ldots)$$

$$\Box^{(0)}\Phi^{(n)} \sim F(\mathscr{R}^{(n-1)}, \ldots, \Phi^{(n-1)}, \ldots)$$

- Easy to implement
- BUT: secular effects
  - Consider Renormalization group flow [Ghersi & Stein '21]

Fixing a la Israel-Stewart

- Inspired by relativistic hydrodynamics (Cayuso, Ortiz, Lehner '17; Allwright, Lehner '18,…)
- Toy model: $\Box\,\Phi \sim \lambda\partial_x^4\Phi$
- Superluminal modes
- Fix:

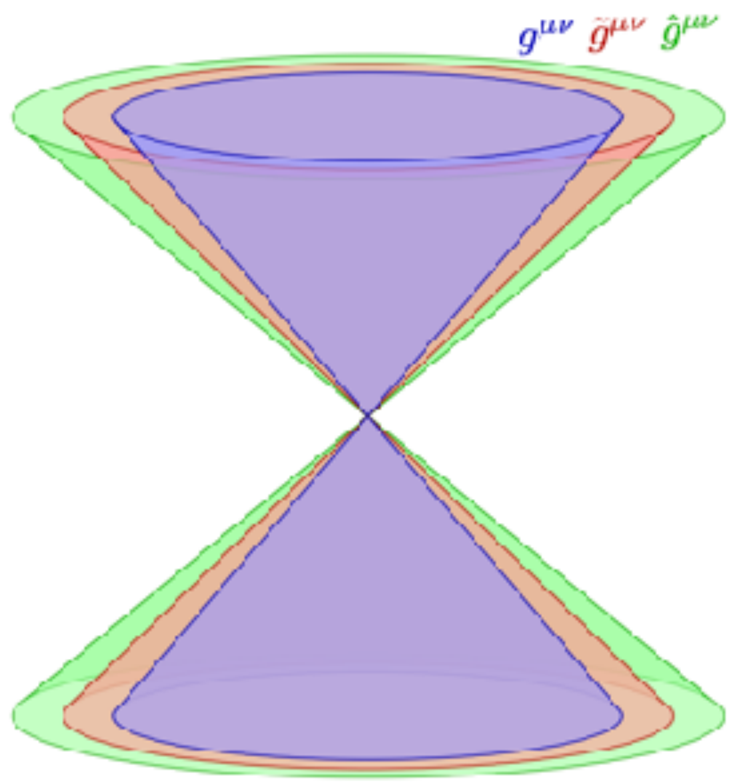
$$\Pi = \partial_x^2\Phi \rightarrow \Box\,\Phi \sim \lambda\partial_x^2\Pi$$

$$\sigma\Box\Pi \sim \partial_x^2\Phi - \tau\partial_t\Pi - \Pi$$

[see R. Cayuso et al '23 for EFT of gravity]

Modified gauge conditions

- Modified generalized harmonic gauge (Kovacs, Reall '20)
- Gauge degrees of freedom propagate along light cone of auxiliary metric



[see also Kovacs, Held & Figueras '24 for new formulation in Effective field theories of gravity]

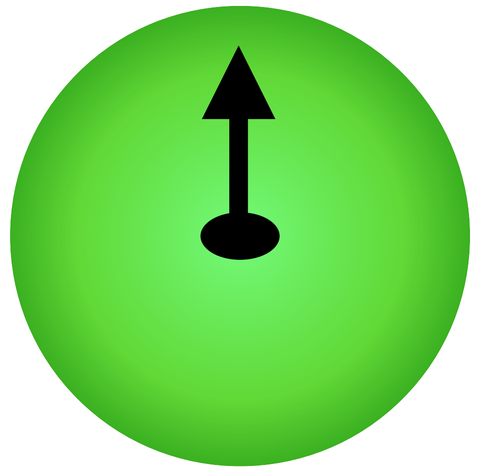
Act 4:
Black holes
in quadratic gravity

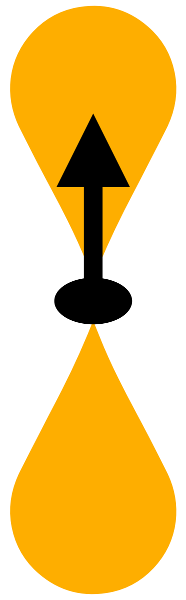
Growing monopole and dipole hair in quadratic gravity
(*Abhishek Hegade*, E.R. Most, J. Noronha, HW, N. Yunes '21, '22)

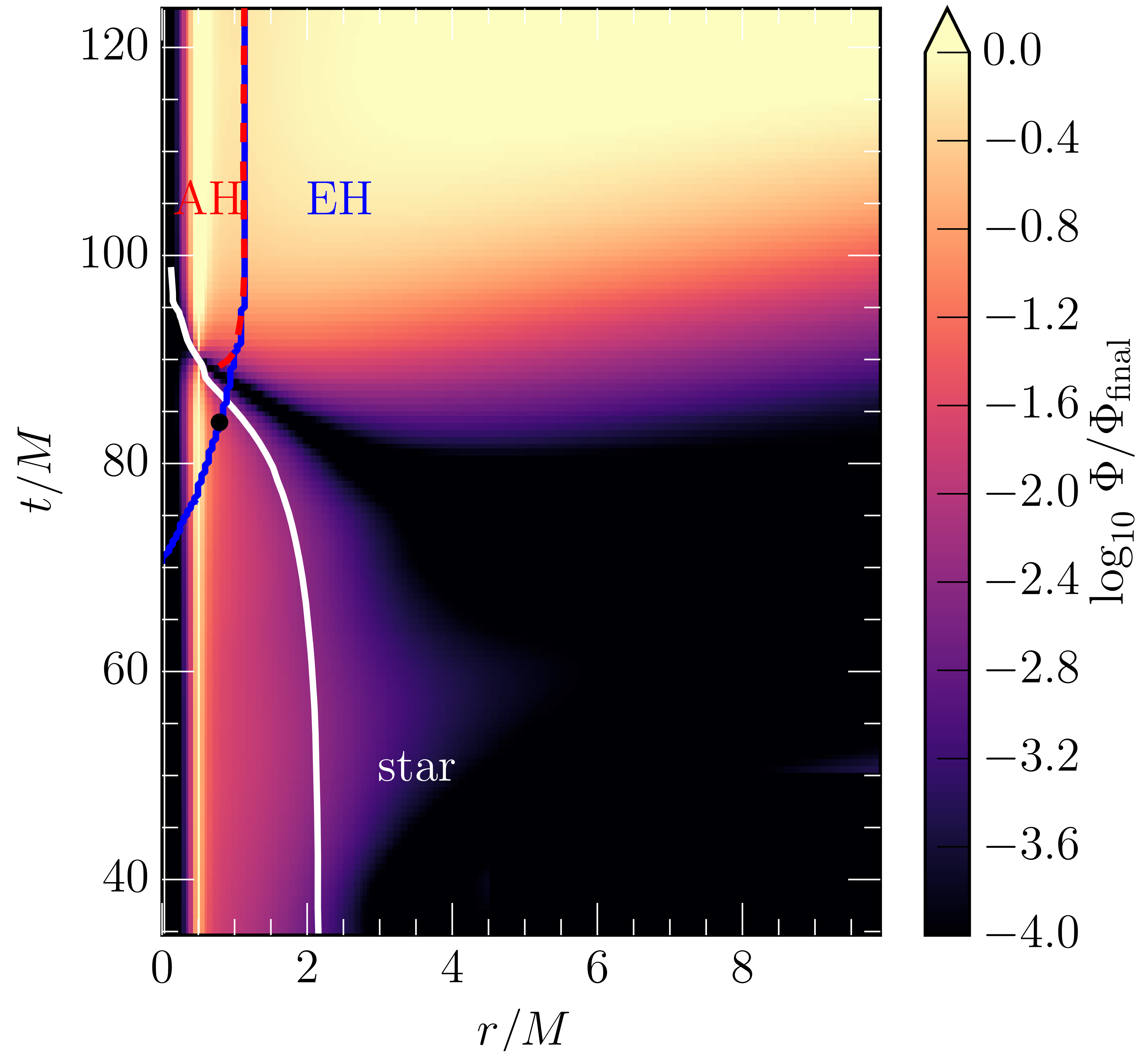
- Model: $f(\Phi) = \Phi$, $h(\Theta) = \Theta$ and fields decoupled $(g(\Phi) = 1)$

$$\Box\Phi + \frac{\alpha_{GB}}{4}\mathscr{G} = 0 \qquad \Box\Theta + \frac{\alpha_{CS}}{4}{}^{*}RR = 0$$

- Monopole or dipole black-hole hair (but none for neutron stars)







How does the black hole hair form in stellar collapse?

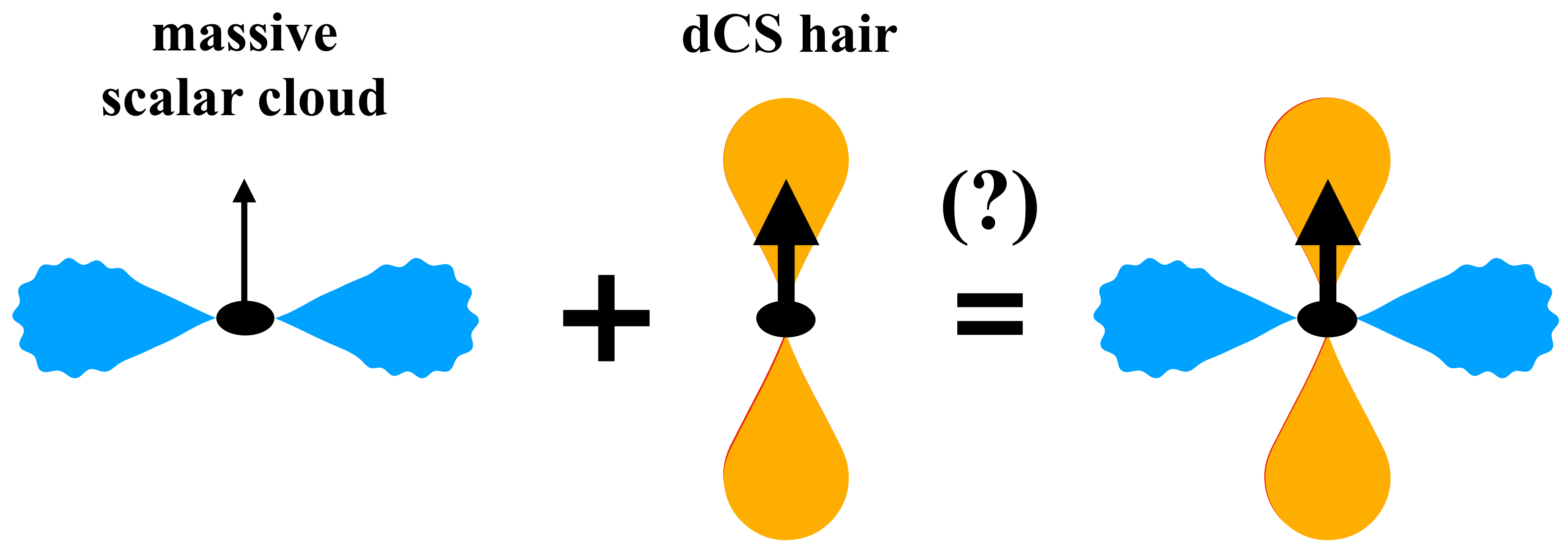
- triggered by formation of event horizon

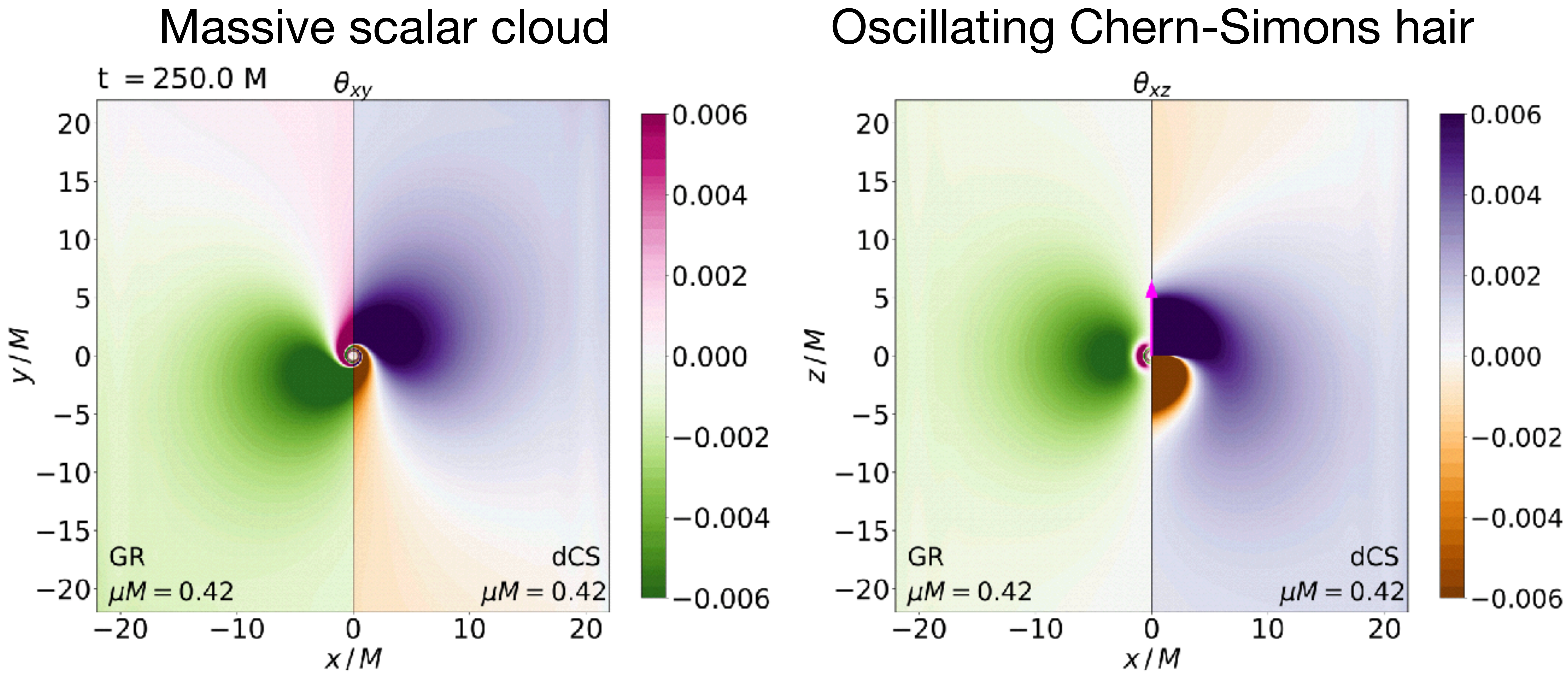
(BHs: Kanti et al '95, Torii et al '96, Pani et al '09, '11, Yunes & Stein '11, Sotiriou & Zhou '14, Ayzenberg et al '14, Maselli et al '15, Kunz et al '11, '14
BH formation: Benkel, Sotiriou, **HW** '16, **HW** et al '19, Ripley & Pretorius '19, Doneva et al '21,…)

Scalar clouds non minimally coupled to gravity
(C. Richards, A. Dima, HW '23)

- Evolving massive scalar cloud around single black hole
- Non minimally coupled to gravity via Pontryagin density

$$\Box\Theta + \frac{\alpha_{CS}}{4} R^*R - \mu^2\Theta = 0$$





BH Spin: a/M = 0.99
Particle mass
$M\mu = 0.42$

See: massive dCS scalar around Schwarzschild (Macedo '18), Superadiance of massive scalar around dCS black hole (Alexander et al '22)

Axidilaton gravity: parametrized numerical relativity

(A. Dima, C. Richards, D. Ferguson, HW in prep.)

- Quadratic gravity — scalar equations of motion:

$$\Box\Phi - V'(\Phi) + \frac{\alpha_{GB}}{4} f'(\Phi)\mathcal{G} - g'(\Phi)g(\Phi)\left[(\nabla\Theta)^2 + 2V(\Theta)\right] = 0$$

$$\Box\Theta - \dot{V}(\Theta) + \frac{\alpha_{CS}}{4}\frac{\dot{h}(\Theta)}{g(\Phi)^2}{}^*RR + 2\frac{g'(\Phi)}{g(\Phi)}\nabla_\mu\Phi\nabla^\mu\Theta = 0$$

→ **Parameterized numerical relativity code for quadratic gravity**

(Implemented in Canuda, open-source w/ paper)

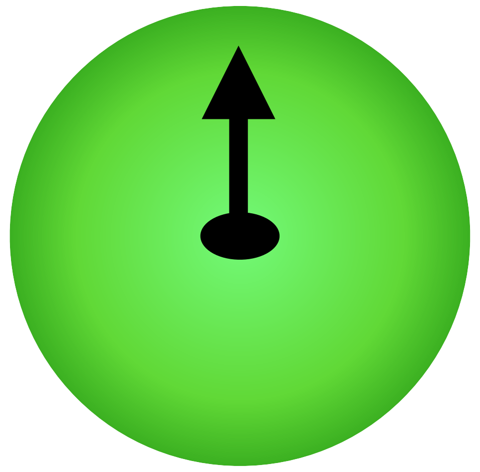
- Parameter selection for scalar Gauss-Bonnet, dynamical Chern-Simons, axidilaton gravity, …

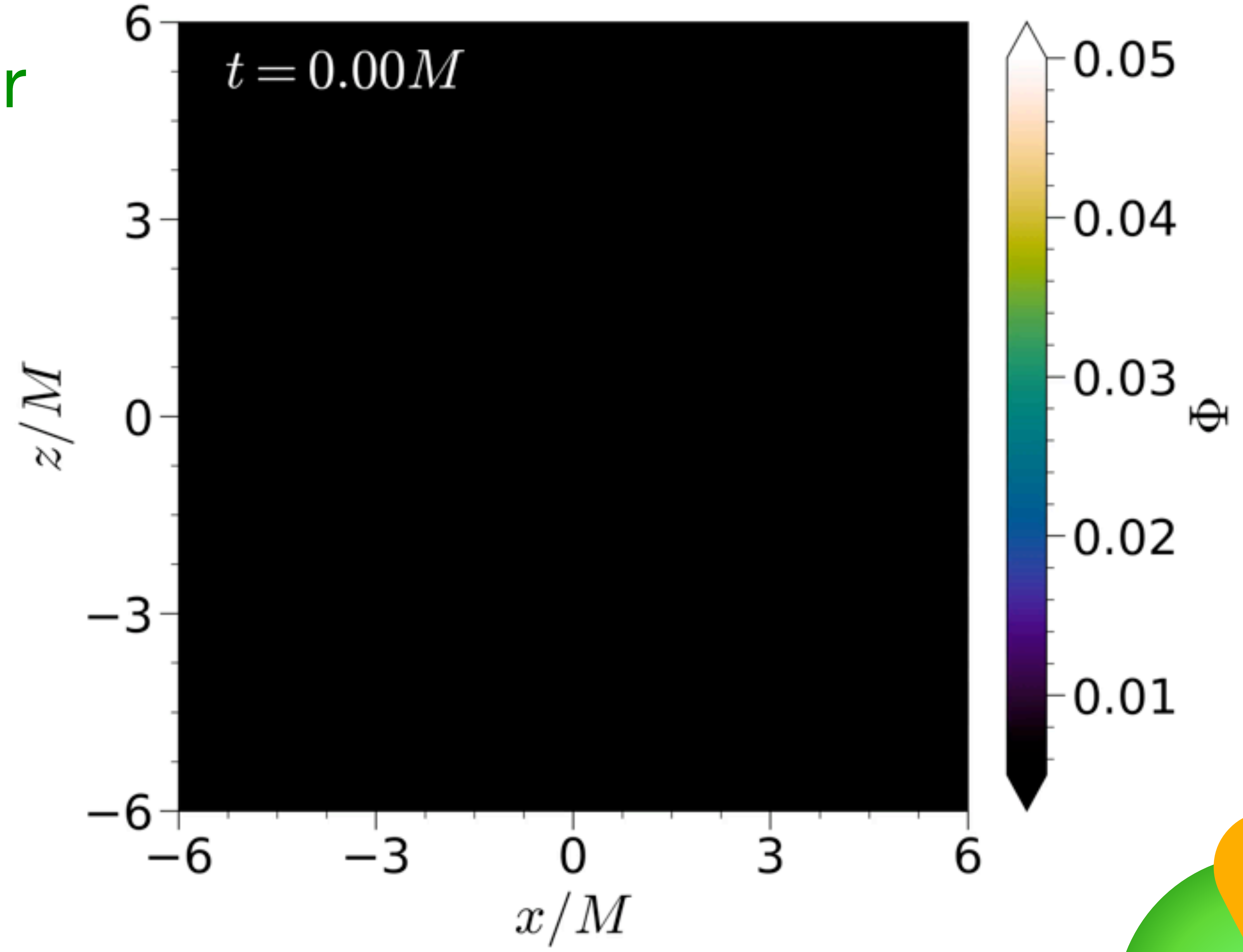
Axidilaton gravity: a tale of two hairs

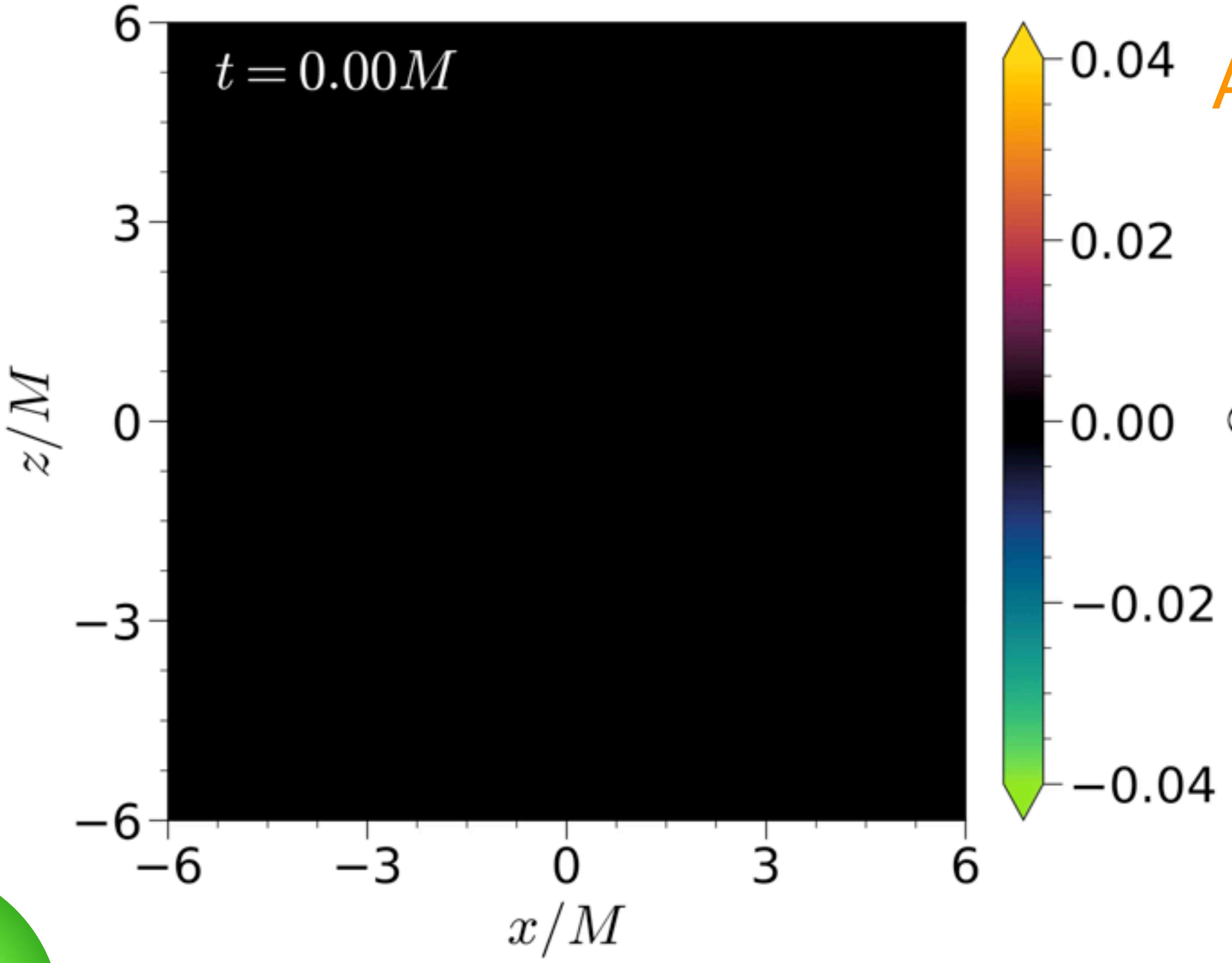
(A. Dima, C. Richards, D. Ferguson, HW in prep.)

- Axidilaton gravity: $f(\Phi) = e^{\lambda\Phi}$, $h(\Theta) = \Theta$, $g(\Phi) = e^{-\lambda\Phi} = 1/f(\Phi)$, $\alpha_{GB}/M^2 = \alpha_{CS}/M^2 = 0.1$
- Background: Kerr in quasi-isotropic coordinates with $a/M = 0.9$
- Zero initial scalar fields —> formation of two-hair black hole

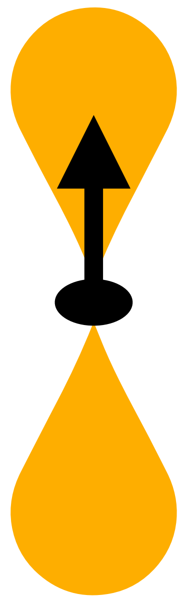
Dilaton hair

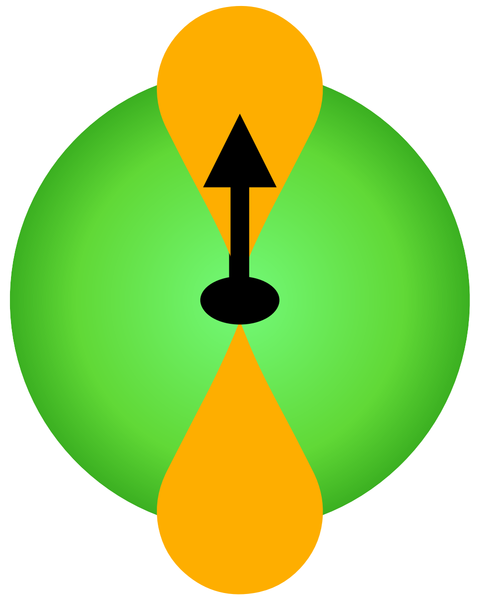






Axion hair





See: analytical (small-spin, small coupling) solution (Cano+ ’21)

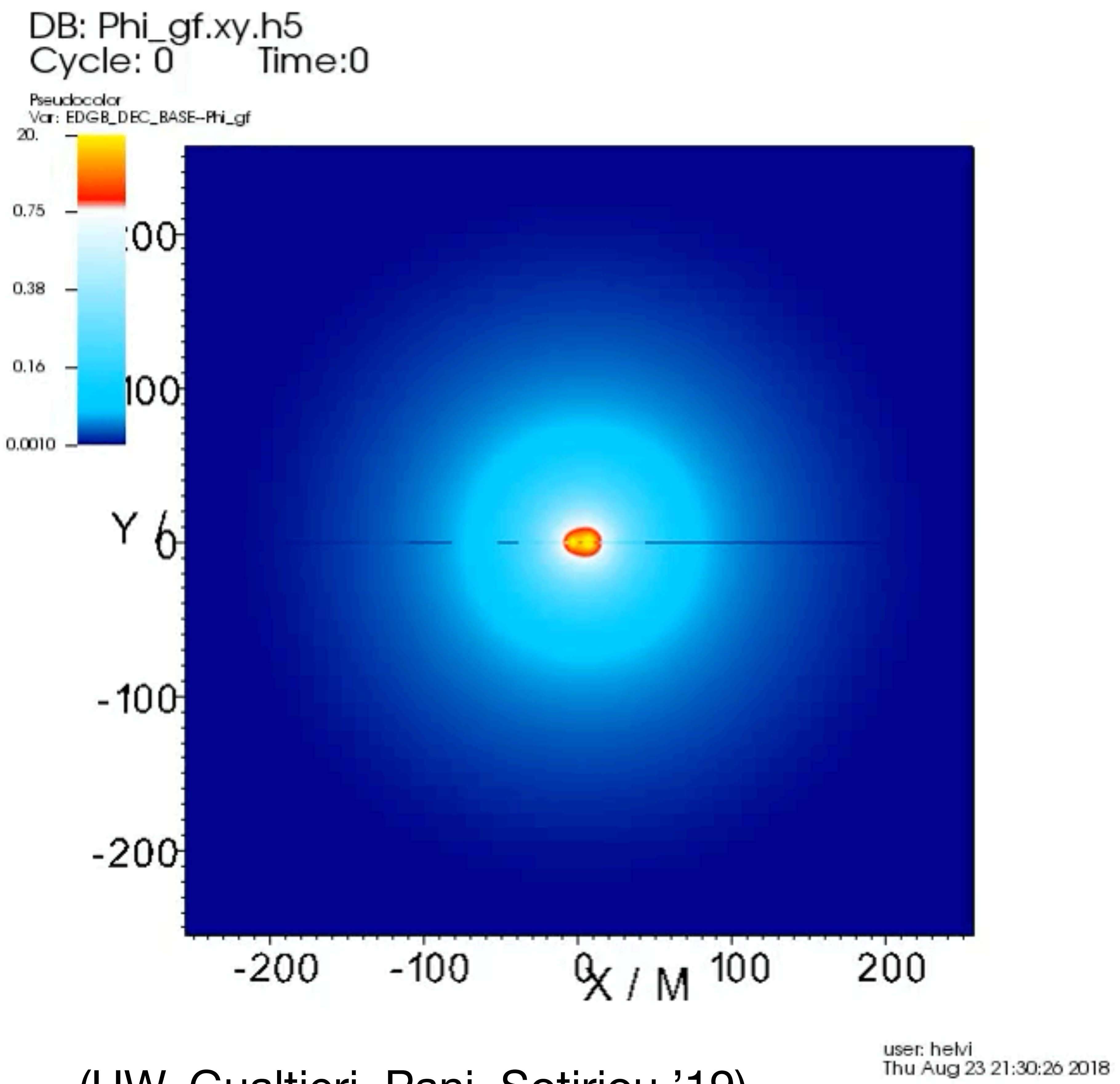
Act 4:
Black-hole mergers
in quadratic gravity

Scalar Gauss-Bonnet gravity I
(HW, Gualtieri, Pani, Sotiriou '19; Shiralilou, Hinderer, Nissanke, Ortiz, HW '20, '21)

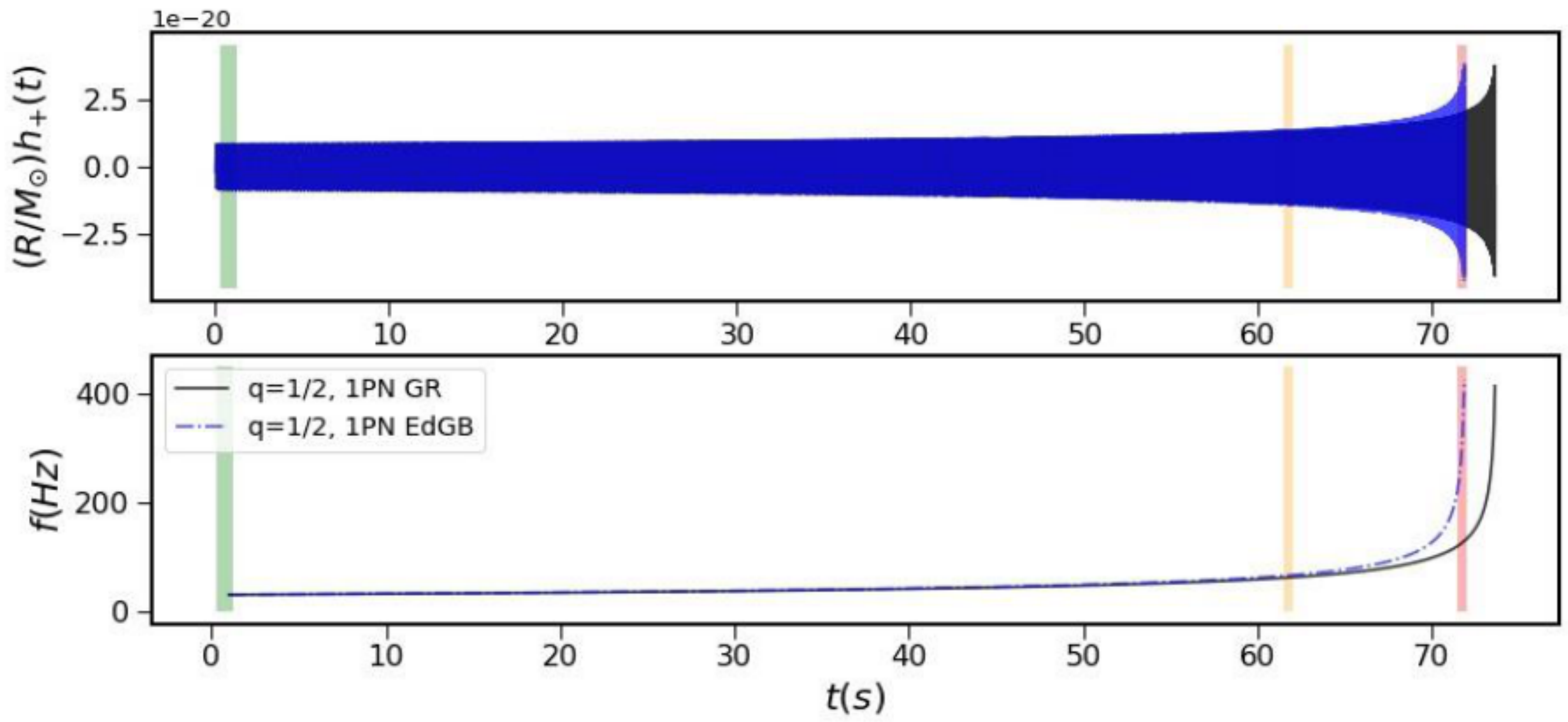
- Coalescence of black holes with scalar hair (`charge")
    - scalar dipole radiation at decoupling
    - Gravitational wave phase shift

Linear coupling: $f(\Phi) \sim \Phi$

$$\Box\Phi + \frac{\alpha_{GB}}{4}\mathcal{G} = 0$$



(HW, Gualtieri, Pani, Sotiriou '19)



PN Waveform (Shiralilou, Hinderer, Nissanke, Ortiz, HW '20, '21)

EoB model and observational bound $\sqrt{\alpha_{GB}} \lesssim 0.3$ km

[Julie, *Lorenzo Pompili,* Buonanno '24]

[Equivalent numerical approach in dCS: Okounkova et al '17-'19] [Constraints: Yagi '12, Nair+ '19, Wang+ '21, Perkins+ '21, Lyu+ '22]

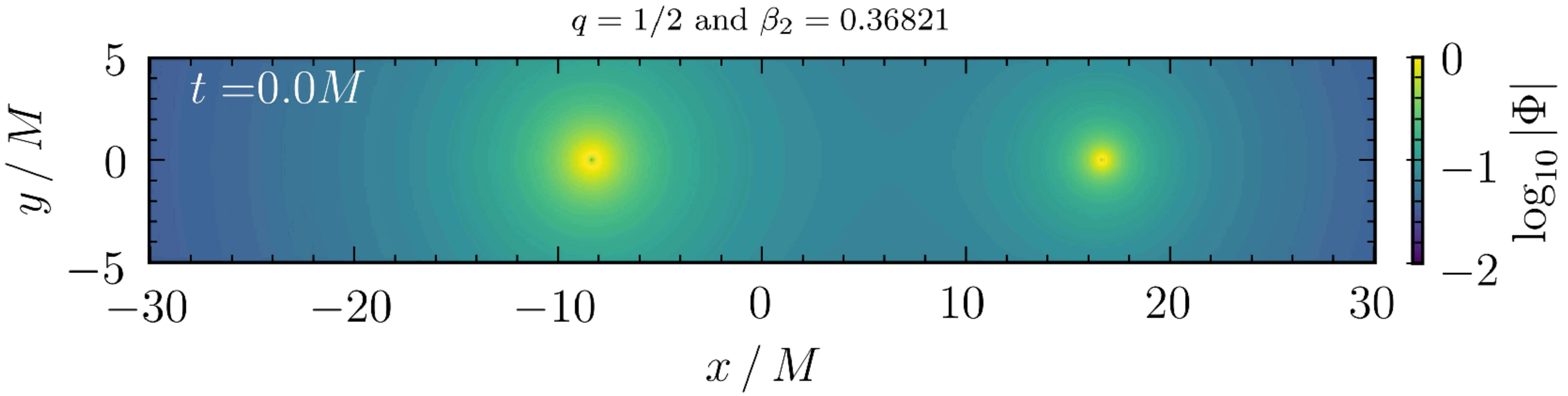
Scalar Gauss-Bonnet gravity II: dynamical (de-)scalarization

(Silva, Elley, HW, Yunes PRL '21, '22)

$$(\Box - \mu^2_{eff})\Phi = 0 \quad \mu^2_{eff} \sim -\beta_2 \mathscr{G}$$

- Consider quadratic coupling $f(\Phi) \sim \beta_2 \Phi^2$
- if $\mu^2_{eff} = -\beta_2 \mathscr{G} > 0$ Kerr black hole exists (no scalarization)
- If $\mu^2_{eff} = -\beta_2 \mathscr{G} < 0$ Tachyonic instability —> black hole scalarizes

Dynamical descalarization



[Single BHs: Silva+ '17, Doneva+ '17, Antoniou+ '17, Blazquez-Salcedo '18, Macedo+'19, Ripley & Pretorius '20; Spin-induced scalarization: Dima+ '20, Hod '20, Doneva et al '20, Herdeiro+ '20, Berti+ '20, Pombo+ '23 Binaries: East, Ripley '21, Doneva+ '23, '24; see Khalil+ '22 in EFTs, See review "Scalarization" by Doneva, Silva et al '22, …]

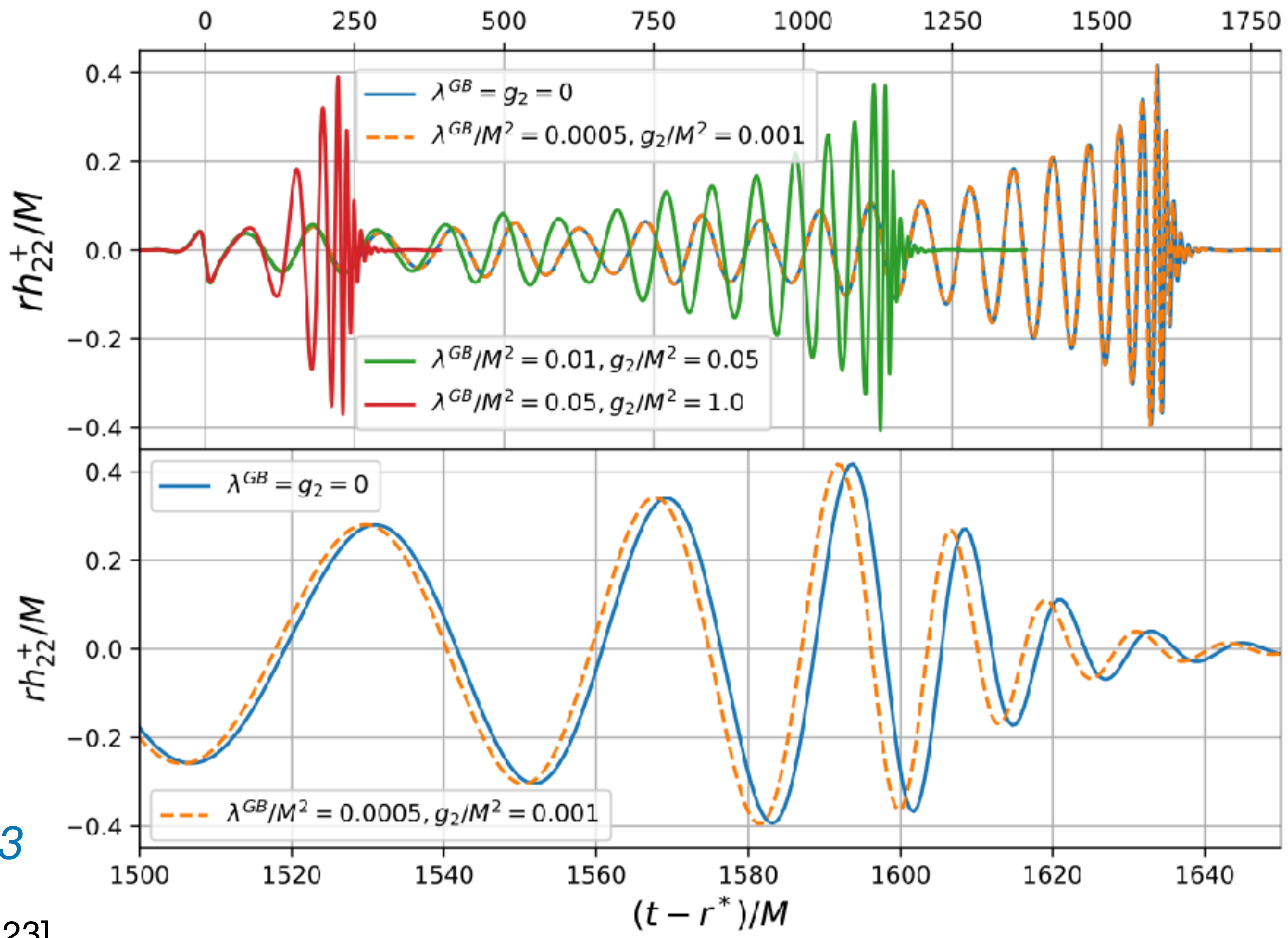
[FYI 2: In binary neutron stars in scalar-tensor theory: dynamical scalarization [Barausse+ '12, Shibata+ '13, Palenzuela+ '13]]

Four derivative scalar-tensor theory
(Salo, Clough, Figueras ’22)

- General scalar-tensor theory w/ up to four derivatives

$$\mathscr{L} = R - \frac{1}{2}(\nabla\Phi)^2 + \frac{\lambda_{GB}}{4}\Phi\mathscr{G} + g_2(\Phi)(\nabla\Phi)^4$$

- Using ccZ4 w/ modified puncture gauge (inspired by modified generalized harmonic gauge)
- Binary black holes w/ q=1, d=11M



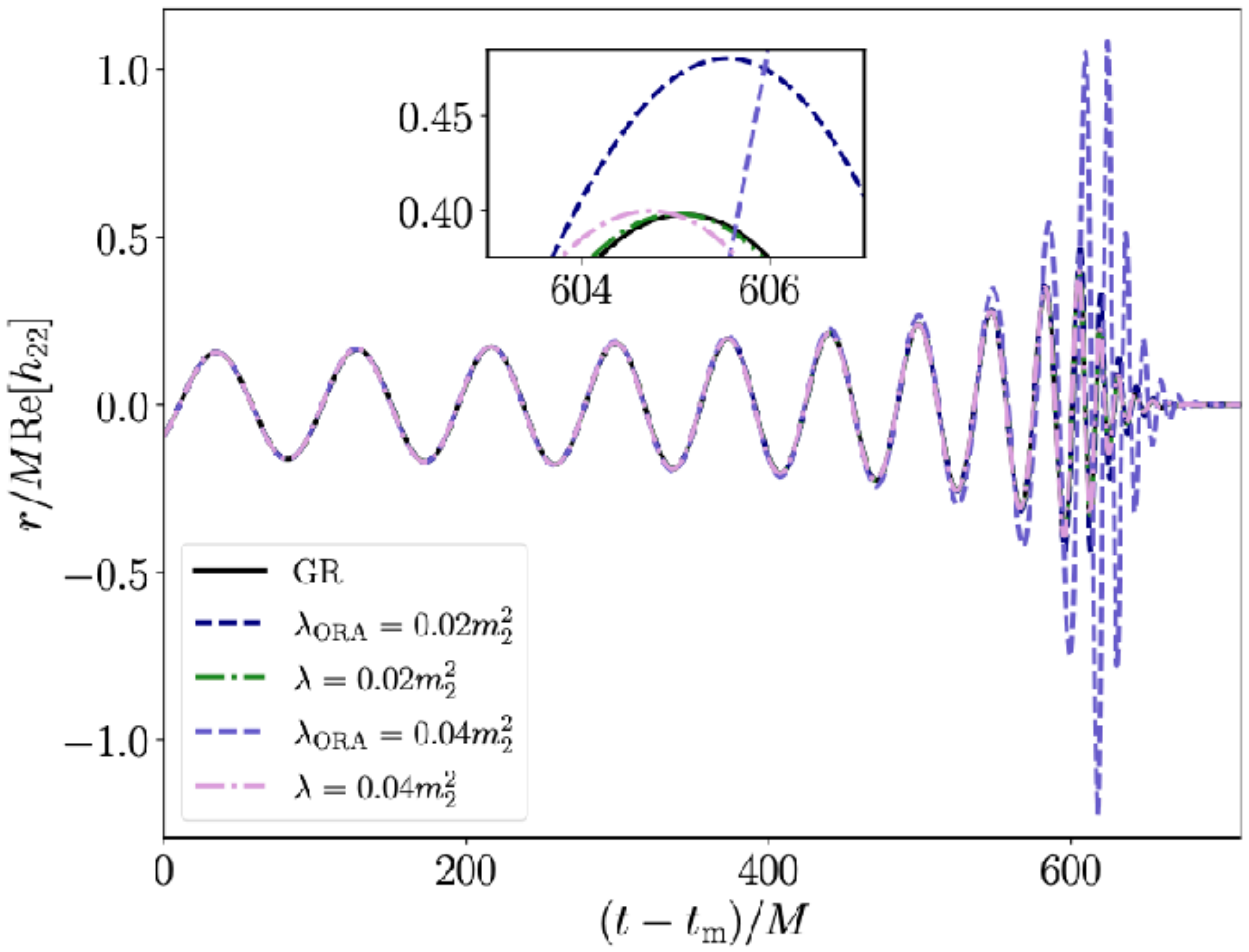
Open-source code: GRFolres (w/ GRChombo)
Areste-Salo, Brady, Clough, Doneva, Evstafyeva et al 2023

[See also Kovacs & Reall ’20, East & Ripley ’21, ’22, Corman+ 23, Doneva+ ‘23]

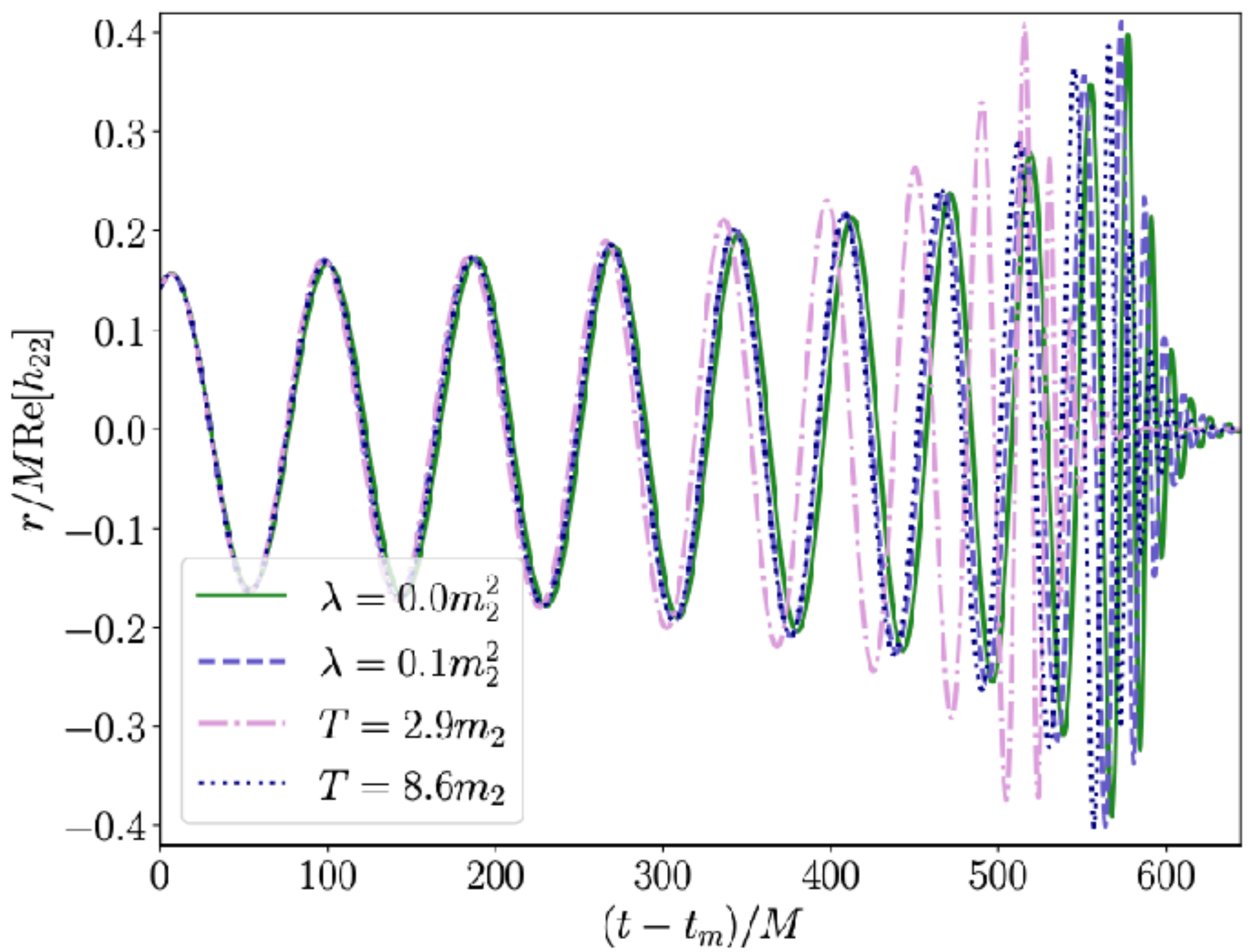
Comparing different approaches for scalar Gauss-Bonnet gravity
(Corman, Lehner, East, Diderot ’24)

- Simulate GW150914-like binary black hole
- Strong numerical parameter dependence of order-by-order and fixing equations approach
- Most accurate for “full” approach using modified gauge

Order-by-order expansion



Fixing a la Israel-Stewart



[See also Kovacs & Reall ’20, East & Ripley ’21, ’22, Areste-Salo+ ’22, Corman+ 23, Doneva+ ’23, G. Lara+ in prep.]

Summary

Status update in Numerical Relativity beyond GR (focus: quadratic gravity)

- Proof-of-principle simulations of compact binary coalescence in classes of beyond GR model
- Scalar charges typical yields scalar (dipole/quadrupole) radiation
- Dephasing of gravitational wave signal
- Surprisingly simple merger signal — not as nonlinear as may have been expected?
- Open-source codes: Canuda (w/ Einstein Toolkit), GRResFol (w/ GRChombo), …

Where do we go from here?

- Theory-specific tests of gravity with inspiral-merger-ringdown models within reach!
- Waveform catalogs needed? How many/few are sufficient?
- Waveform quality:
  benchmark from different approaches [Corman+ '24], initial data, numerical accuracy, degeneracies
- Full IMR models and GW analysis [see EOB model by Julie, ***Lorenzo Pompili***, Buonanno '24]

Thank you!

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