Beyond the Horizon
The status and future of gravitational-wave source modelling in general relativity

Geraint Pratten

TEONGRAV Workshop, Rome, Italy, 16th Sept. 2024

Introduction

- Aim of talk is to highlight some of the key developments in recent years
  - Impossible to cover all work by the community over the past N+ years
  - Provide a broad overview of the key approaches
  - Signpost some of the talks throughout this week
  - Stimulate discussion during the **coffee** breaks!

- Focus on inspiral-merger-ringdown (IMR) models
  - A lot of complementary work on perturbative approaches

Why Waveform Modelling?

Mining Astrophysical Information

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- **Key concept**: gravitational-wave signal encodes astrophysical information

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- **Bayes**ian inference key tool in inferring parameters

Likelihood   Priors

Posterior Probability
$$p(\theta \mid d) = \frac{\mathcal{L}(d \mid \theta)\pi(\theta)}{\mathcal{Z}}$$

Evidence

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$$\mathcal{L}(d \mid \theta) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2}\frac{|d - h(\theta)|^2}{\sigma^2}\right)$$

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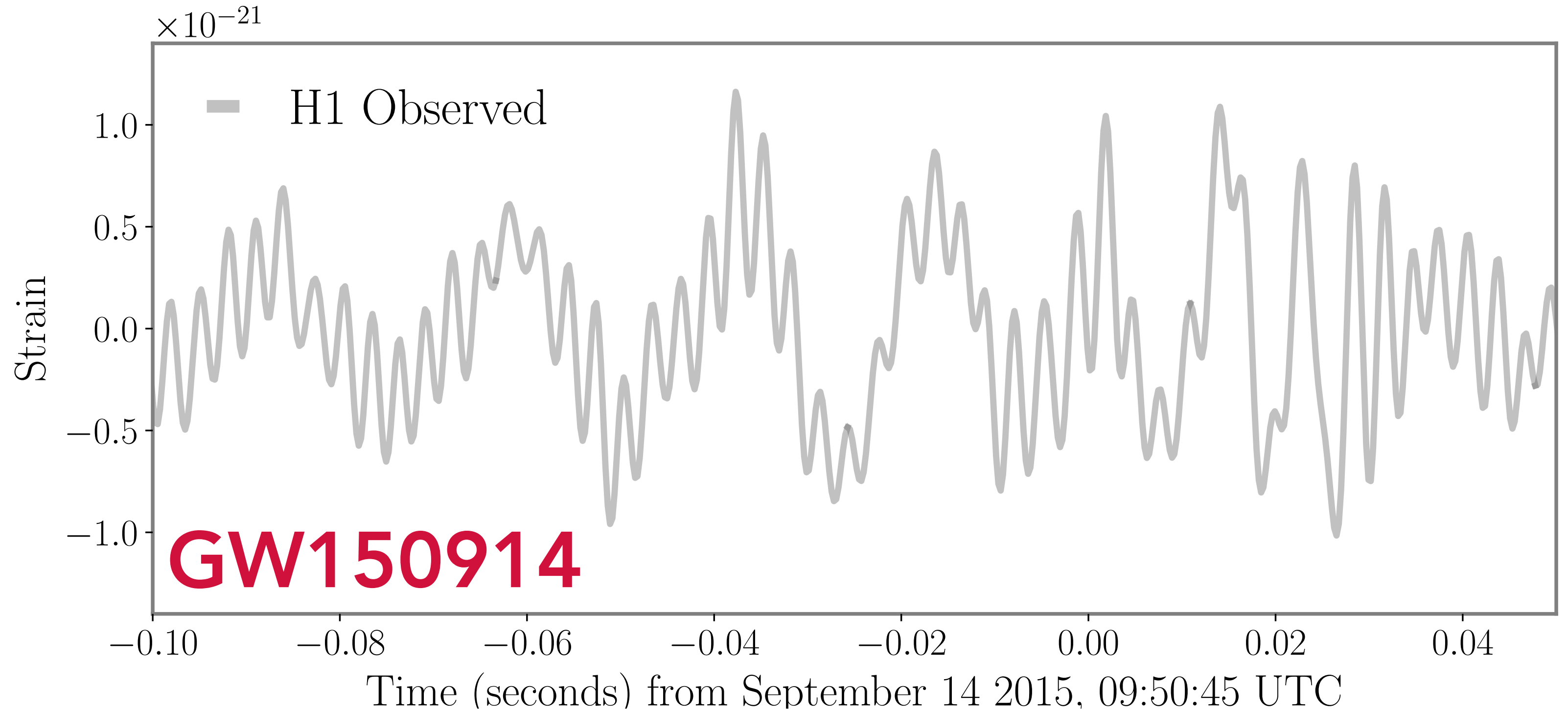
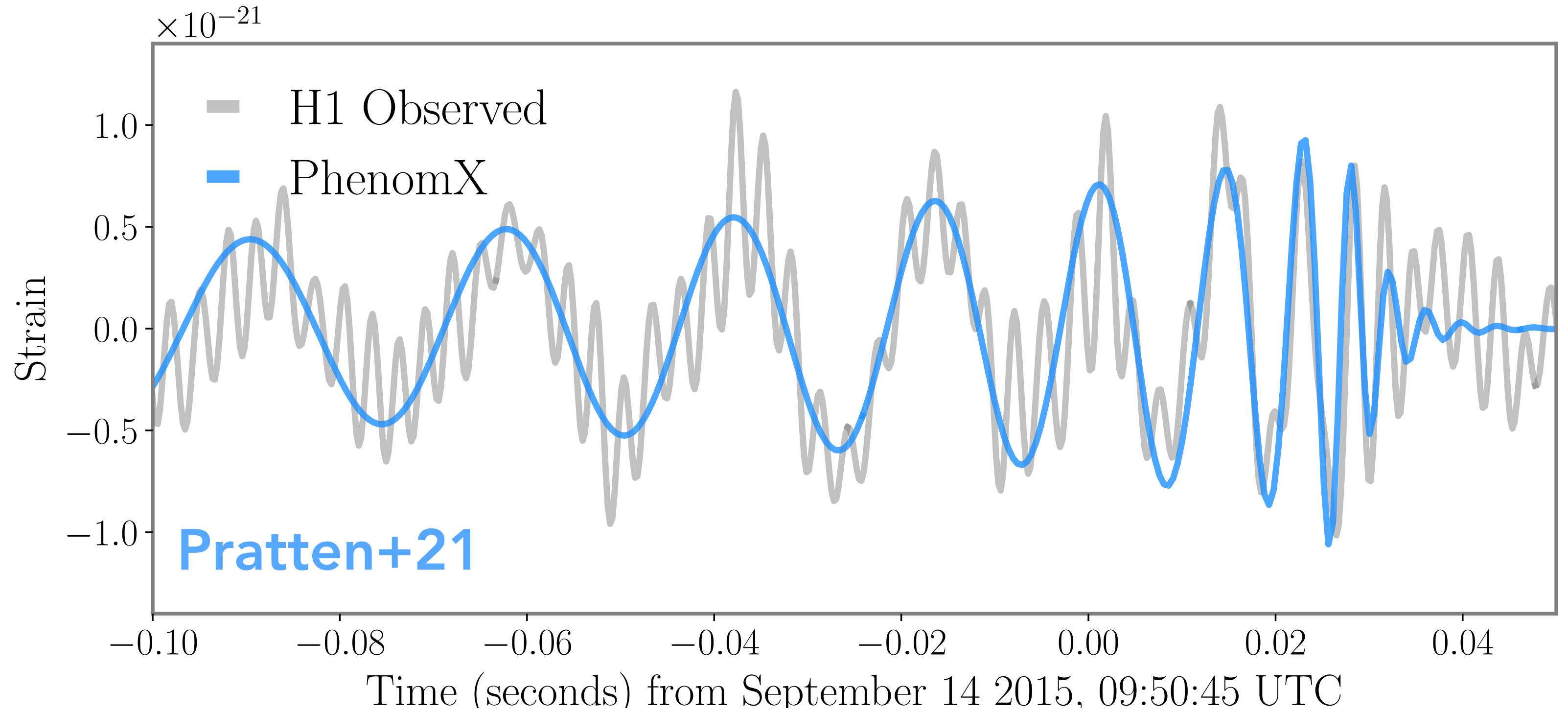
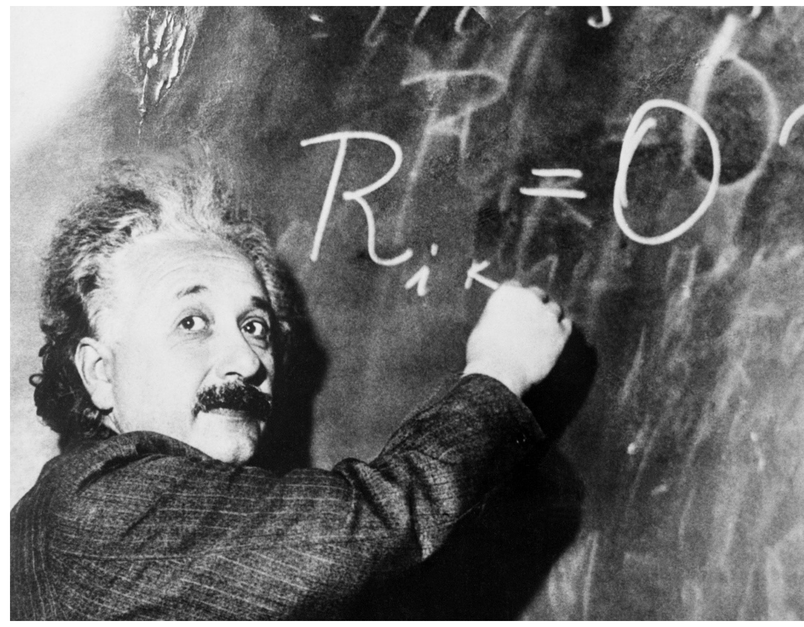
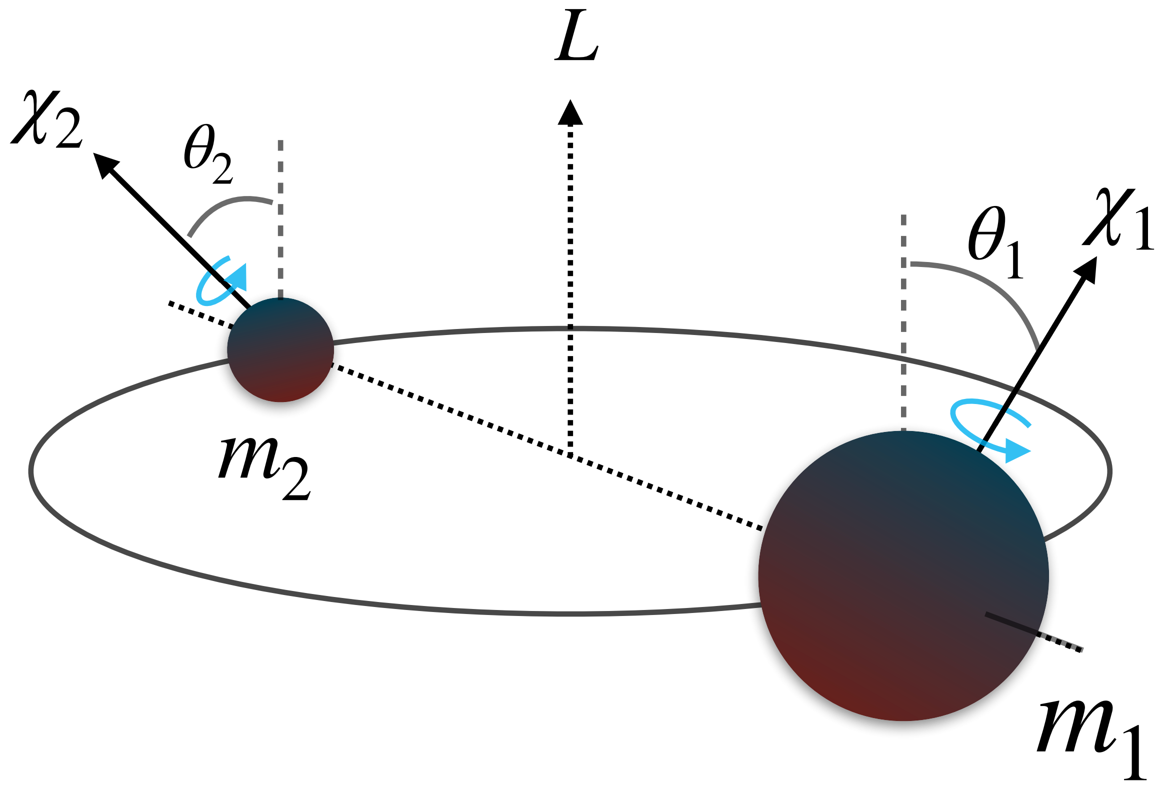


Signal observed by detectors: LIGO, Virgo



Compare signal to **theoretical models**





Mine **astrophysical information**: masses, spins, fundamental physics, …

Gravitational Waves

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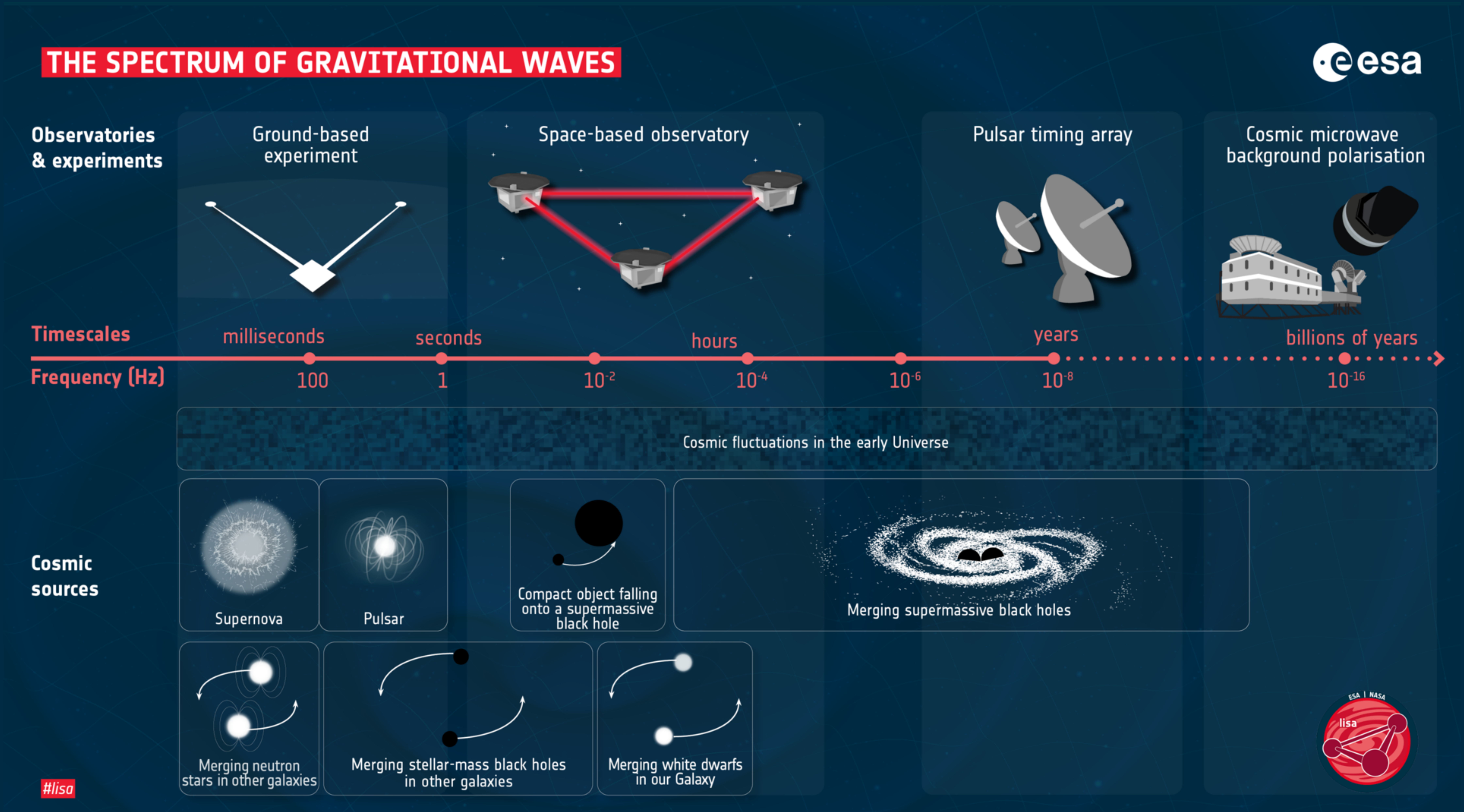

LISA Massive Black Hole Binaries

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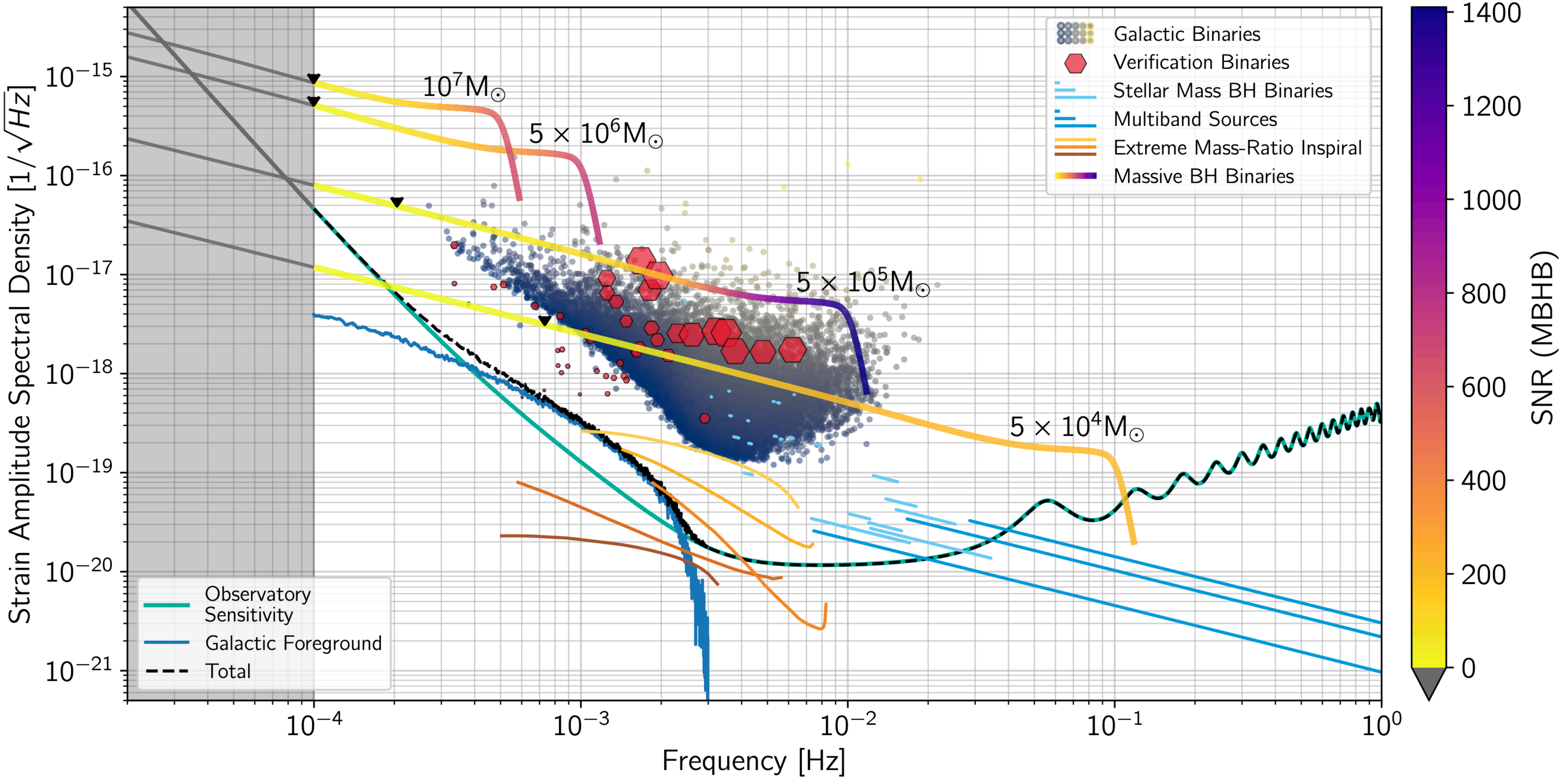
- Wealth of signals at low frequencies both distinct and complementary to ground-based detectors



The Morphology of Binary Black Hole Mergers

Anatomy of an Inspiral…

Anatomy of an Inspiral…

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Flux Balance:

$$\frac{dE_{\rm orbital}}{dt} = \mathcal{L}_{\rm GW} \approx \frac{32}{5}\frac{c^5}{G}\frac{(m_1 m_2)^2}{M^4}\left(\frac{v}{c}\right)^5$$

einstein toolkit

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Anatomy of an Inspiral…

post-Newtonian (MPM)

$$h^{\alpha\beta}_{(1)} = \sum_{\ell=0}^{+\infty} \partial_L \left(\frac{K_L^{\alpha\beta}(t - r/c)}{r} \right)$$

scattering amplitudes

$$\mathcal{M}^{\Delta+\nabla} = \frac{2\pi^2 G^2 \epsilon_1\cdot\epsilon_4 \epsilon_2\cdot\epsilon_3}{\sqrt{-q^2}} \sum_{n,i} \alpha^{(n,i)} \mathcal{O}^{(n,i)}$$

numerical relativity

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$

$$\partial_t K_{ij} = \cdots$$

BH perturbation theory

$$h = \sum C_{[p]\ell mn}\, e^{-i\tilde{\omega}_{[p]\ell mn} t}\, {}_{-2}S_{[p]\ell mn}(\iota, \varphi)$$

gravitational self-force

$$g_{\alpha\beta} + \sum_{m=-\infty}^{\infty} \left[q h^{1,m}_{\alpha\beta}(\Omega) + q^2 h^{2,m}_{\alpha\beta}(\Omega) \right] e^{-im\phi} + \mathcal{O}(q^3)$$

EFT

$$S_{\rm eff} = -\frac{1}{16\pi G} \int d^4x \sqrt{\bar{g}}\, R[\bar{g}_{\mu\nu}] + \sum_{i=1}^{\infty} C_i(r_s) \int d\sigma \mathcal{O}_i(\sigma)$$

post-Minkowski

$$h^{\alpha\beta} = \sum_{n=1} G^{(n)} h^{\alpha\beta}_{(n)}$$

effective one body

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

\+ other approaches

Inspiral

Analytical approximations begin to break down

Merger

Ringdown

Flux Balance:

$$\frac{dE_{\rm orbital}}{dt} = \mathcal{L}_{\rm GW} \approx \frac{32}{5}\frac{c^5}{G}\frac{(m_1 m_2)^2}{M^4}\left(\frac{v}{c}\right)^5$$

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Anatomy of an Inspiral…

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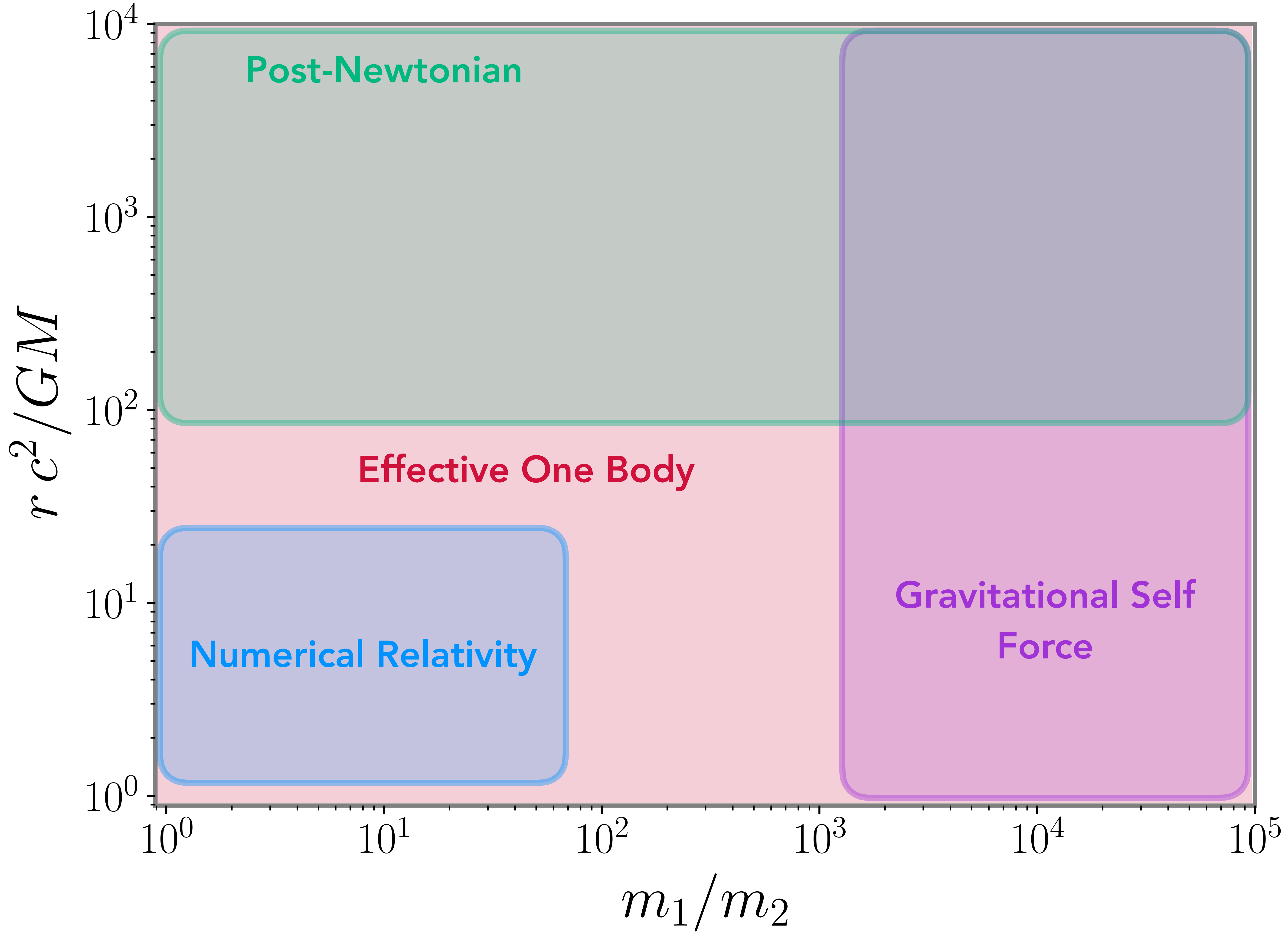
e.g. Buonanno & Sathyaprakash 14



each approach has a parameter-dependent domain of validity
Synergy!

Talk by Niels Warburton

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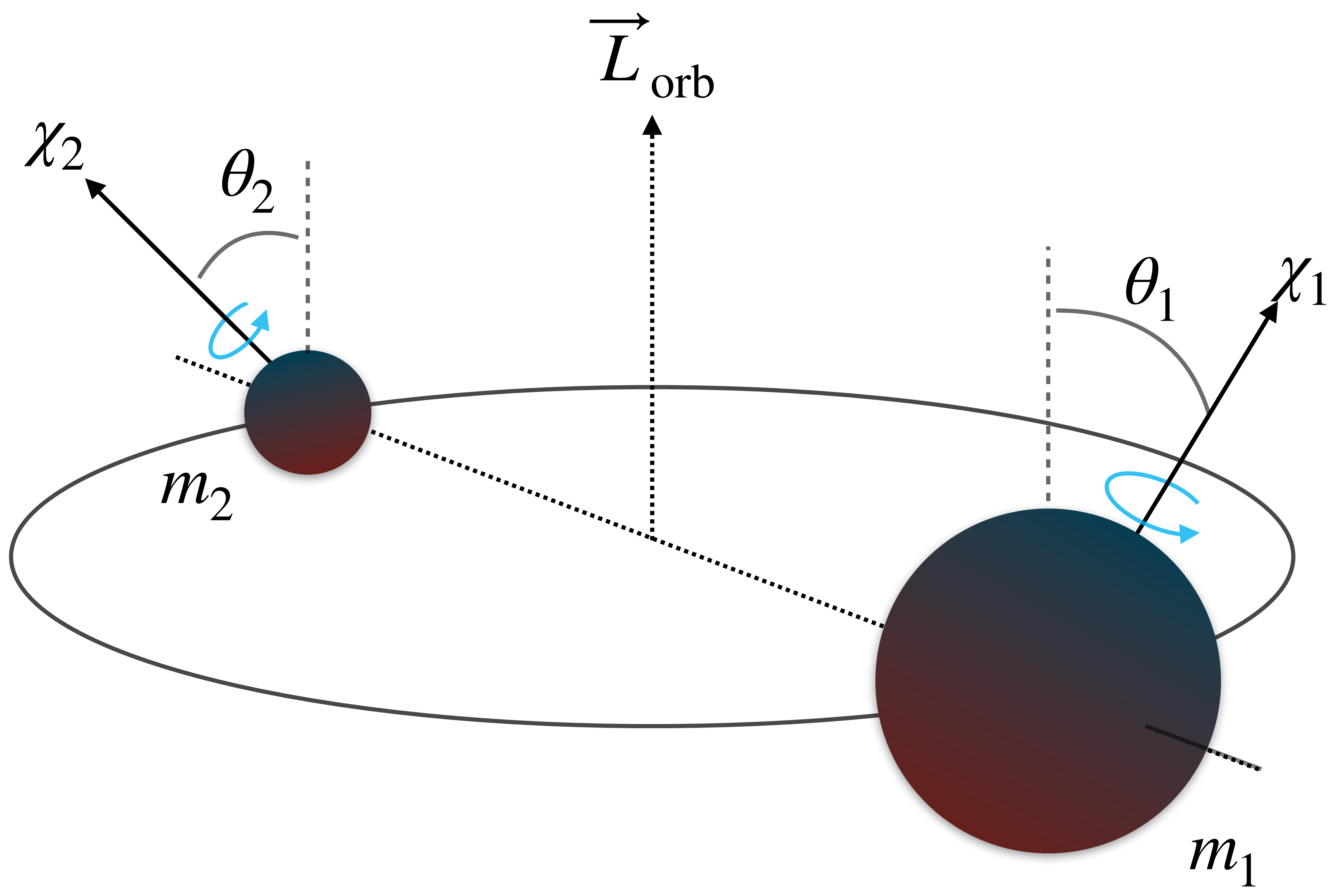
The Parameter Space?

- GWs efficient at circularising binary
- Significant effort on modelling quasi-circular binaries



The Parameter Space?

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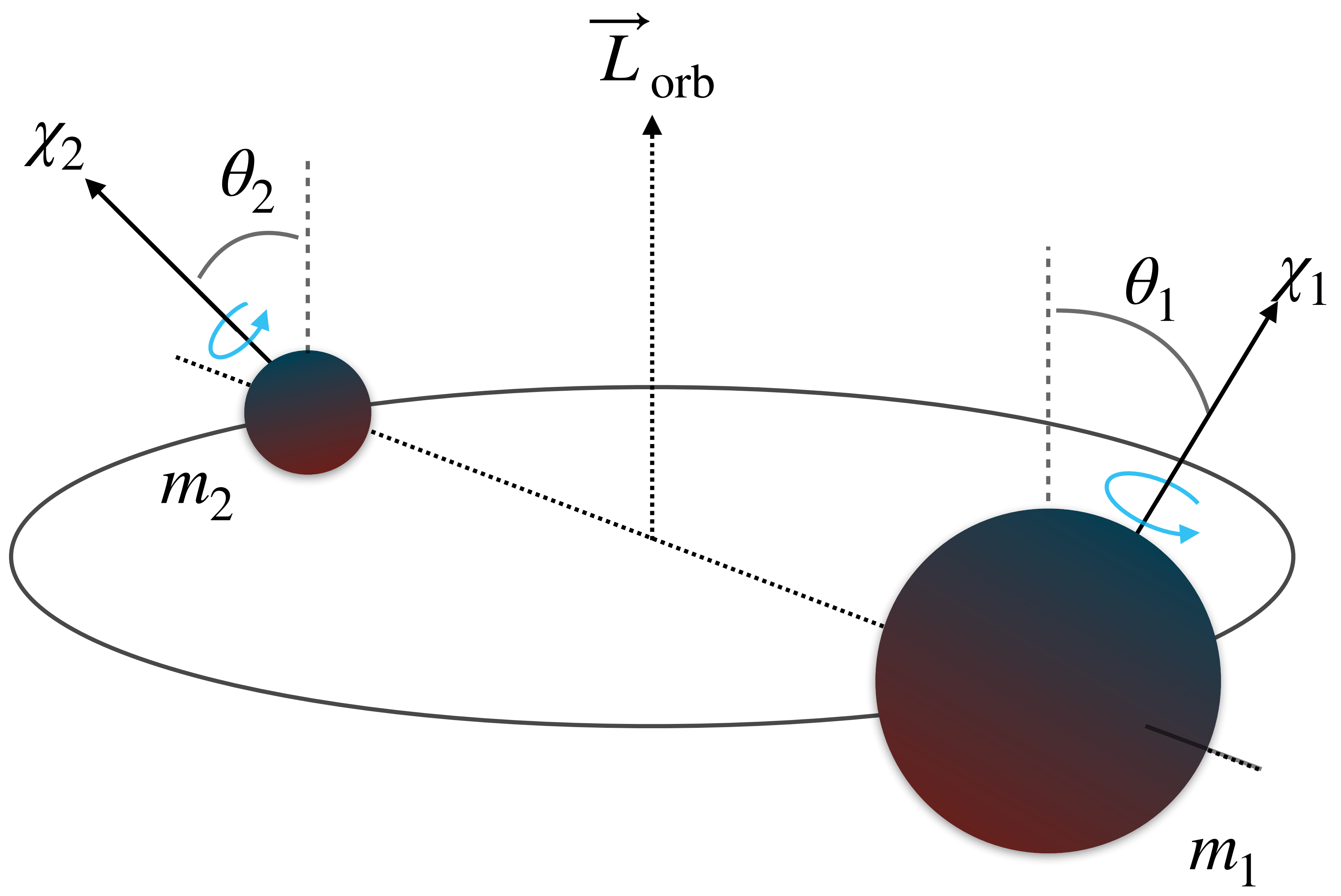
Recent efforts have been working to relax this assumption!
We will come back to this later…



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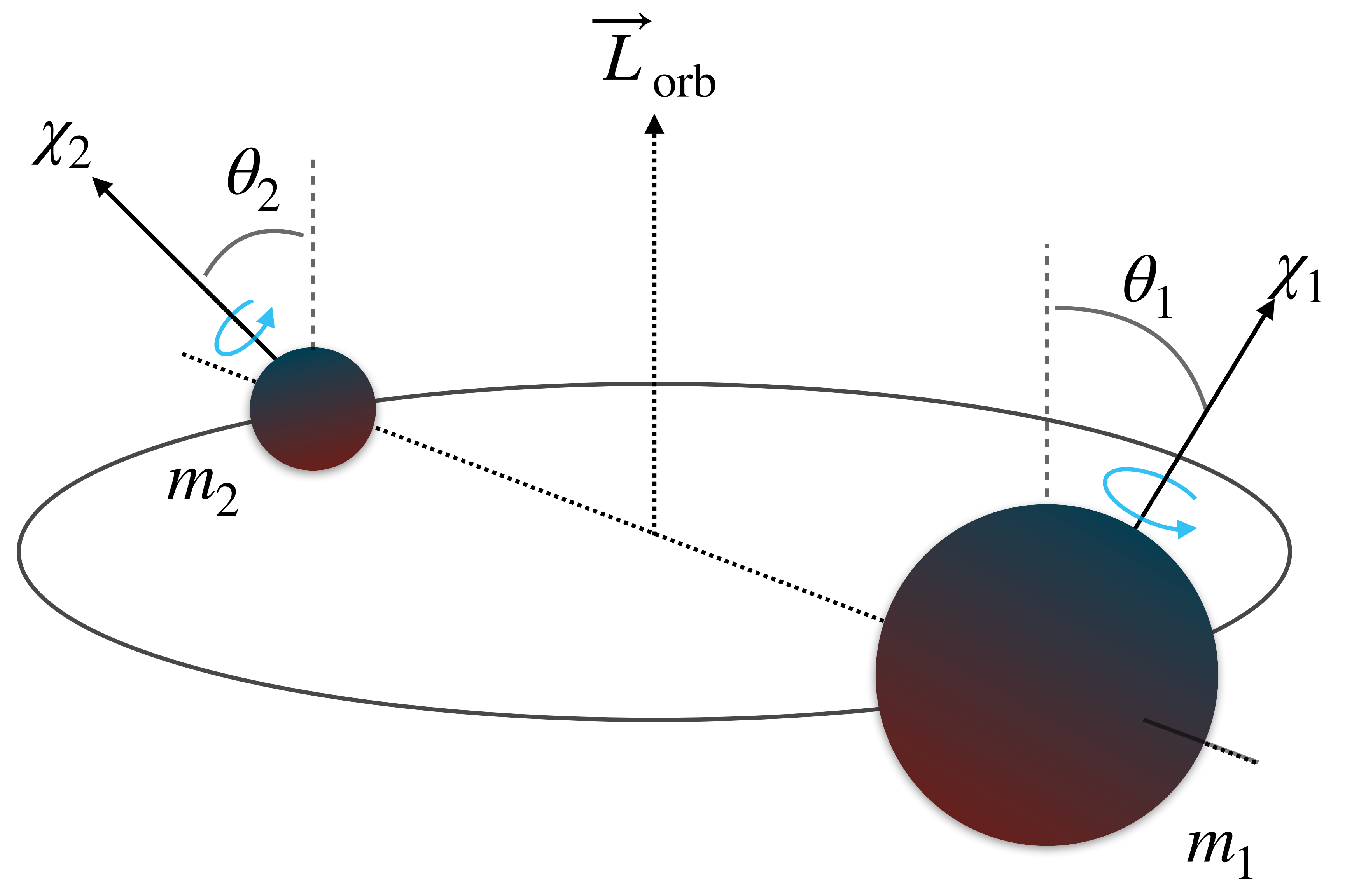
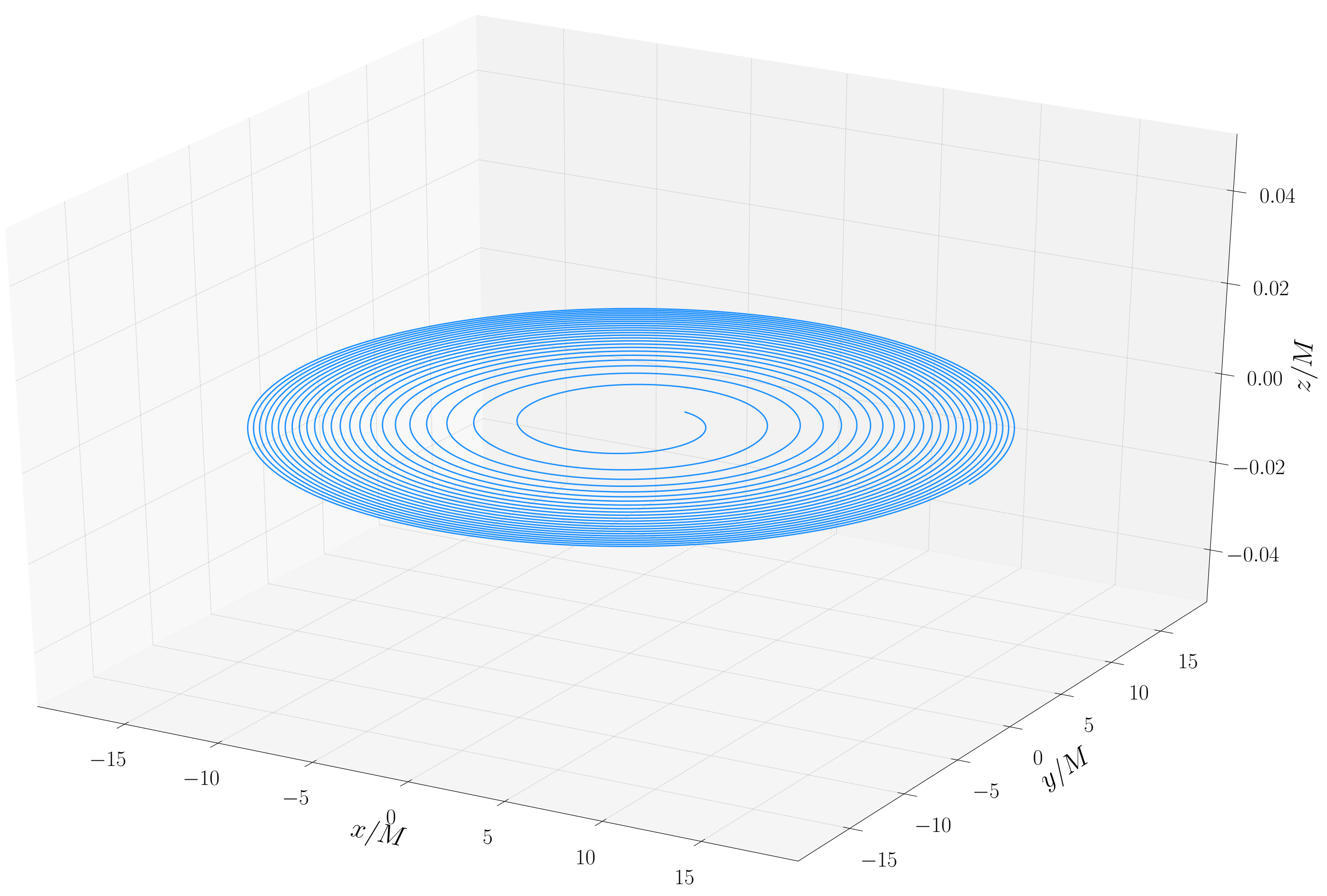
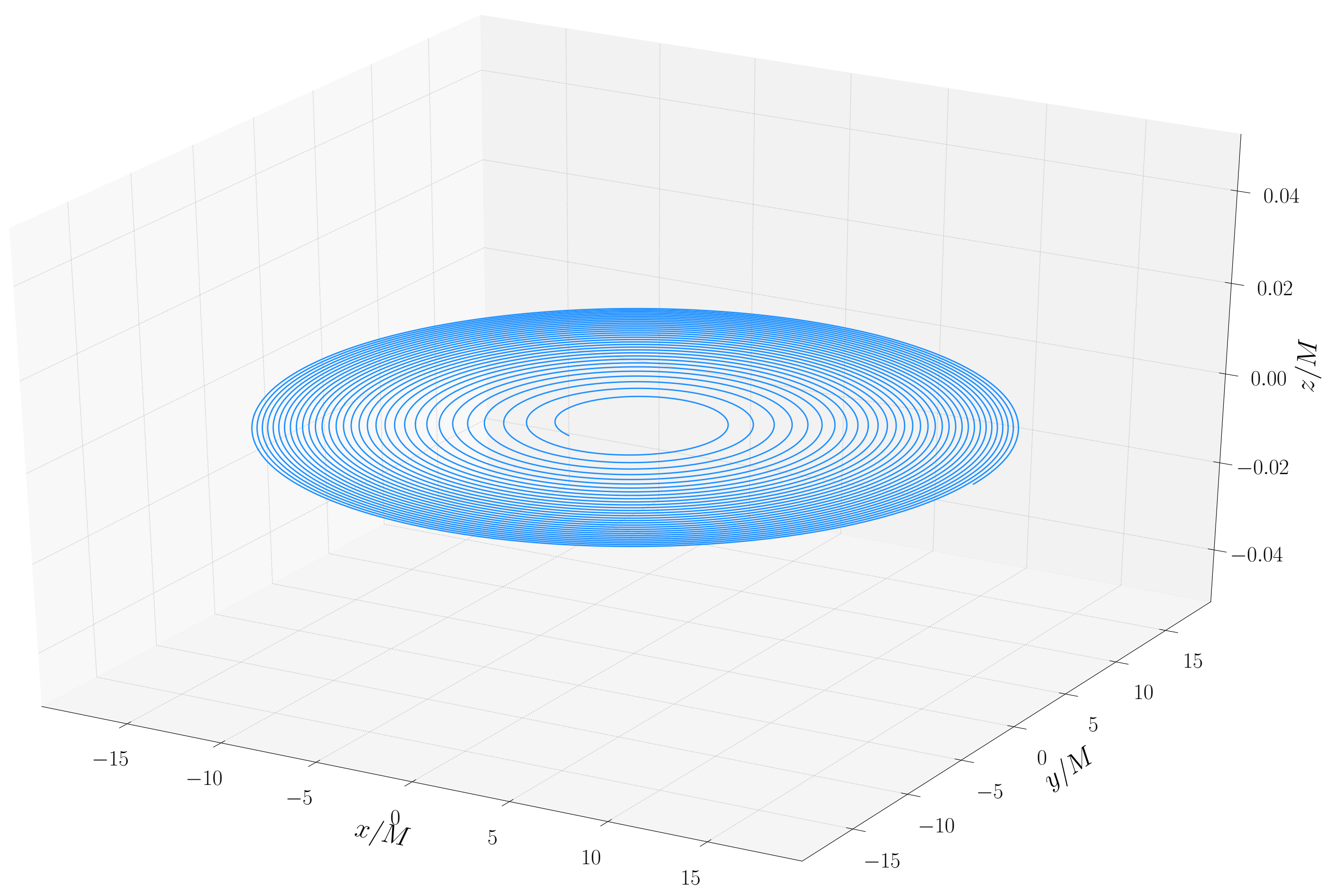


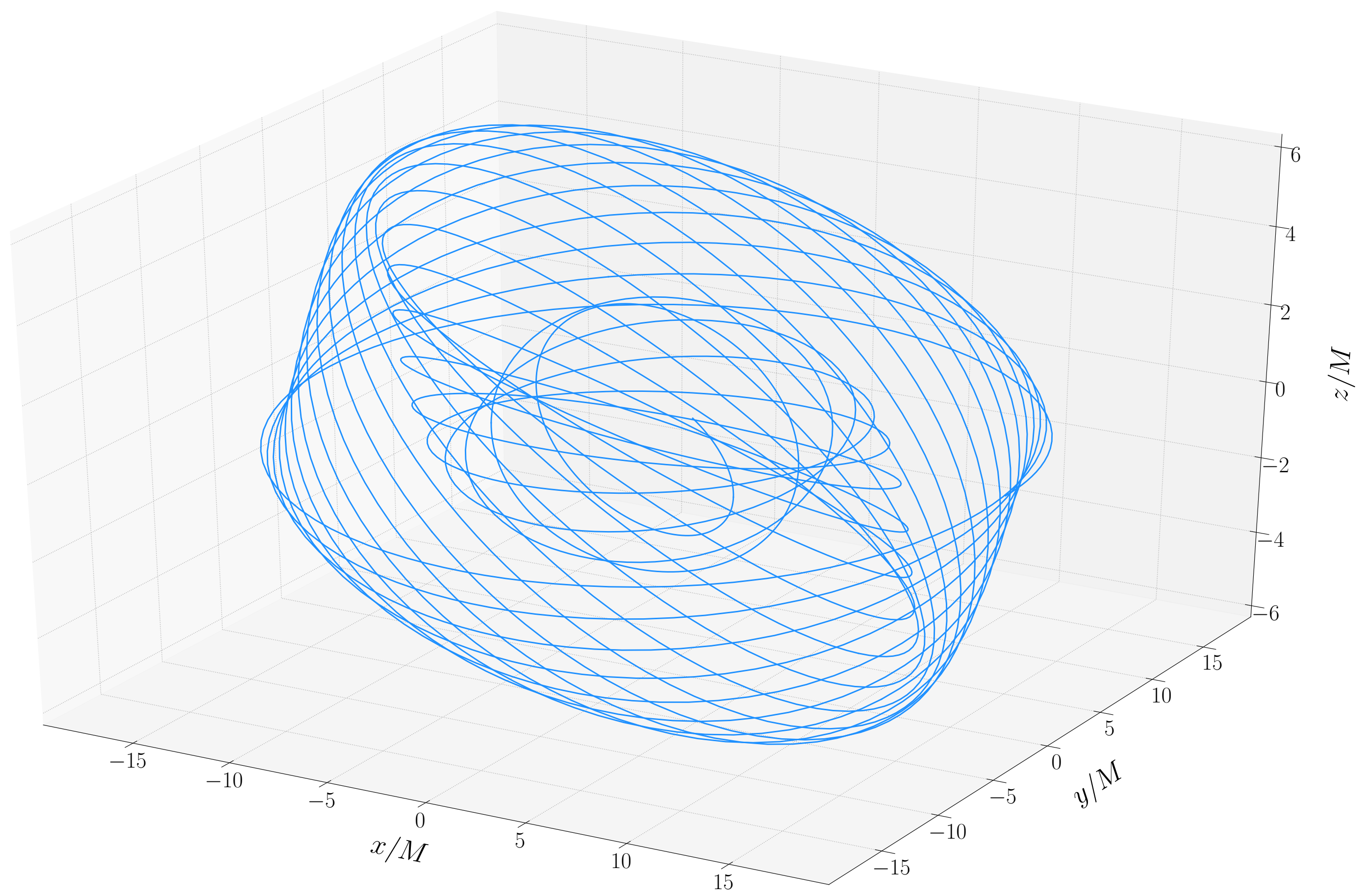
Non-Spinning



Aligned Spins



Precessing Spins



Gravitational Waves: Cosmic Fingerprints

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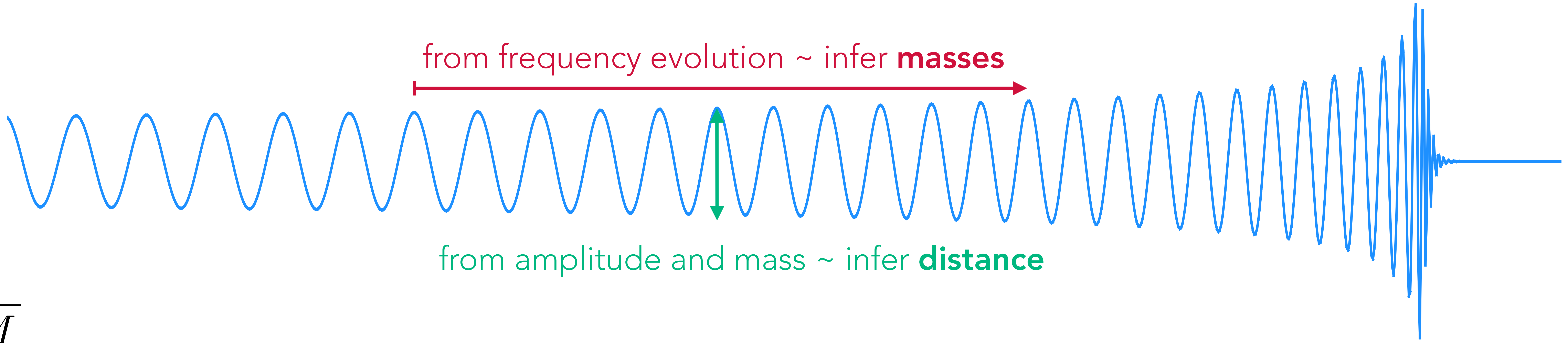
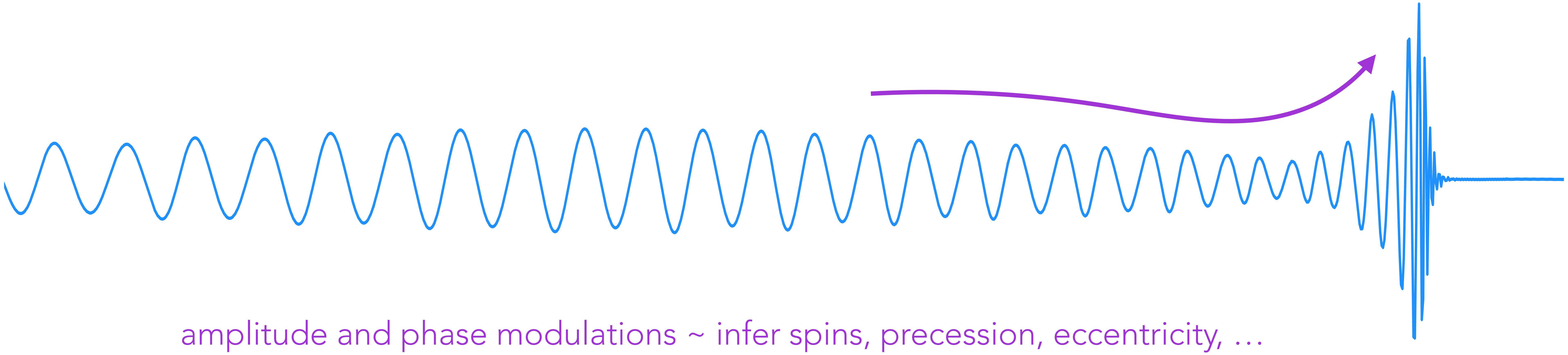


$$\omega_{\mathrm{orb}} = \sqrt{\frac{GM}{r^3}}$$

$$f_{\mathrm{GW}} = \frac{\omega_{\mathrm{orb}}}{\pi}$$

time of arrival, **amplitude** and **phase** at detectors ~ infer **sky location**



IMR Waveform Models

Breakdown of the Market?

- Flagship IMR models ~ grouped into 3 families
- Pros and cons to each family

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NR Surrogates

- Interpolate NR waveforms across parameter space
- Accuracy comparable to input NR
- Reasonably efficient waveform evaluation
- Limited by availability of NR
- Limited by NR duration but can hybridise with inspiral models

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Phenomenological

- Analytical + NR calibration model of GW signal
- Extremely efficient to evaluate
- Time- and frequency-domain models available
- Limited in calibration by availability of NR
- Less fundamental - harder to incorporate information

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Effective **O**ne **B**ody

- Hamiltonian framework for dynamics **and** GW signal
- Evolve system of ODEs - work needed to mitigate computational cost
- Limited in calibration by availability of NR
- Natural framework for incorporating additional physics (GSF, scattering, …)

Breakdown of the Market?

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- Flagship IMR models ~ grouped into 3 families
- Pros and cons to each family - cross-fertilisation of knowledge extremely successful

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Building and IMR Model...?

- Current models (broadly) follow the same schematic construction [e.g. Schmidt+ arXiv:1207.3088]

$$h_{\ell m}^{\text{inertial}} = \sum_{m'=-\ell}^{\ell} \mathcal{D}_{mm'}^{\ell *}(\alpha, \beta, \gamma)\, h_{\ell m}^{\text{copr}}$$

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Building and IMR Model…?

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Why this approach?

Break waveform model down into simple (smooth, weakly oscillatory) pieces:

vastly easier to model

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modes in a frame that tracks orbital plane

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approximate map [Schmidt+12] $h_{\ell m}^{\mathrm{copr}} \approx h_{\ell m}^{\mathrm{AS}}$

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modes in a frame that tracks orbital plane

approximate map [Schmidt+12] $h^{\mathrm{copr}}_{\ell m} \approx h^{\mathrm{AS}}_{\ell m}$

see also coorbital frame modes
[Boyle+, Blackman+, Varma+]

$$h^{\mathrm{coorb}}_{\ell m} = h^{\mathrm{copr}}_{\ell m}\, e^{i m \phi}$$

Building and IMR Model…?

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time dependent rotation operator

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time dependent rotation operator

Euler angles : $\{\alpha, \beta, \gamma\}$

quaternions : $\vec{q}$

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need a prescription for the final state

$$\chi_f = \chi_f(\eta, \vec{\chi}_1, \vec{\chi}_2)$$

$$M_f = M_f(\eta, \vec{\chi}_1, \vec{\chi}_2)$$

Surrogate Models

Surrogate Models

- Data driven approach that ~ interpolate data across parameter space [NR, Waveforms, Dynamics]
- Aims to reconstruct phenomenology of input data with no assumptions
- Reduced order modelling has been a leading paradigm in the construction of surrogate models

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- Reduced order modelling has been a leading paradigm in the construction of surrogate models

- So how does on *schematically* construct a surrogate model?

Surrogate Models: Step 1

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- Step 1: Build a *reduced basis* that represents function space in terms of N-dim orthonormal basis to a specified tolerance σ

orthonormal basis

$$h(t,\vec{\lambda}) \approx \sum_{i=1}^{N} c_i(\vec{\lambda})\, \hat{e}_i(t)$$

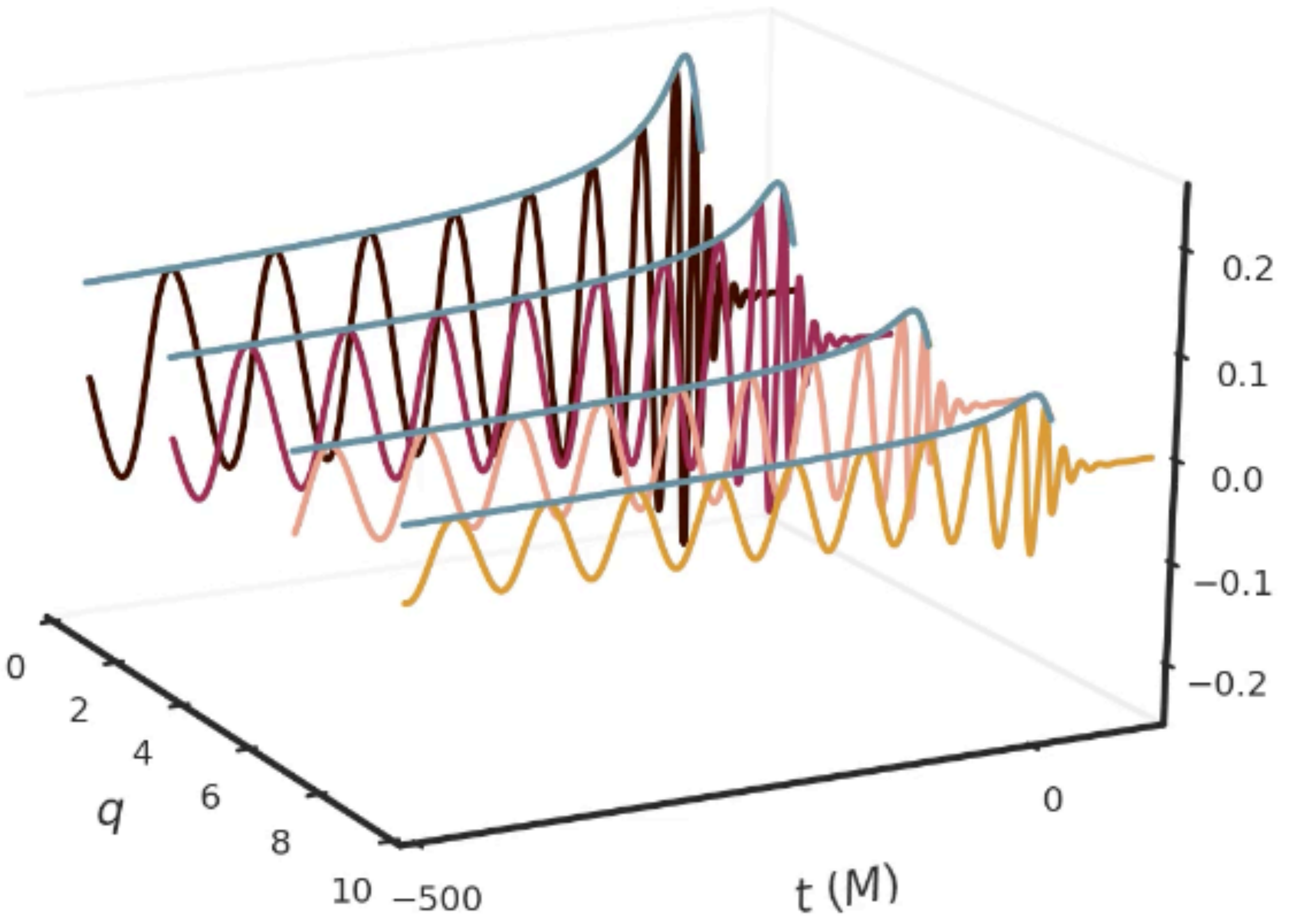
projection coefficients

maximum representation error

$$\max \epsilon = \max \left\| h(t;\vec{\lambda}) - \sum_{i=1}^{n} c_i(\vec{\lambda})\hat{e}_i(t) \right\|^2 \leq \sigma$$

e.g. L^2-norm

- Range of techniques to find orthonormal basis
  - Greedy algorithms, SVD, PCA, etc
  - Works best on smoothly/slowly varying data



Surrogate Models: Step 2

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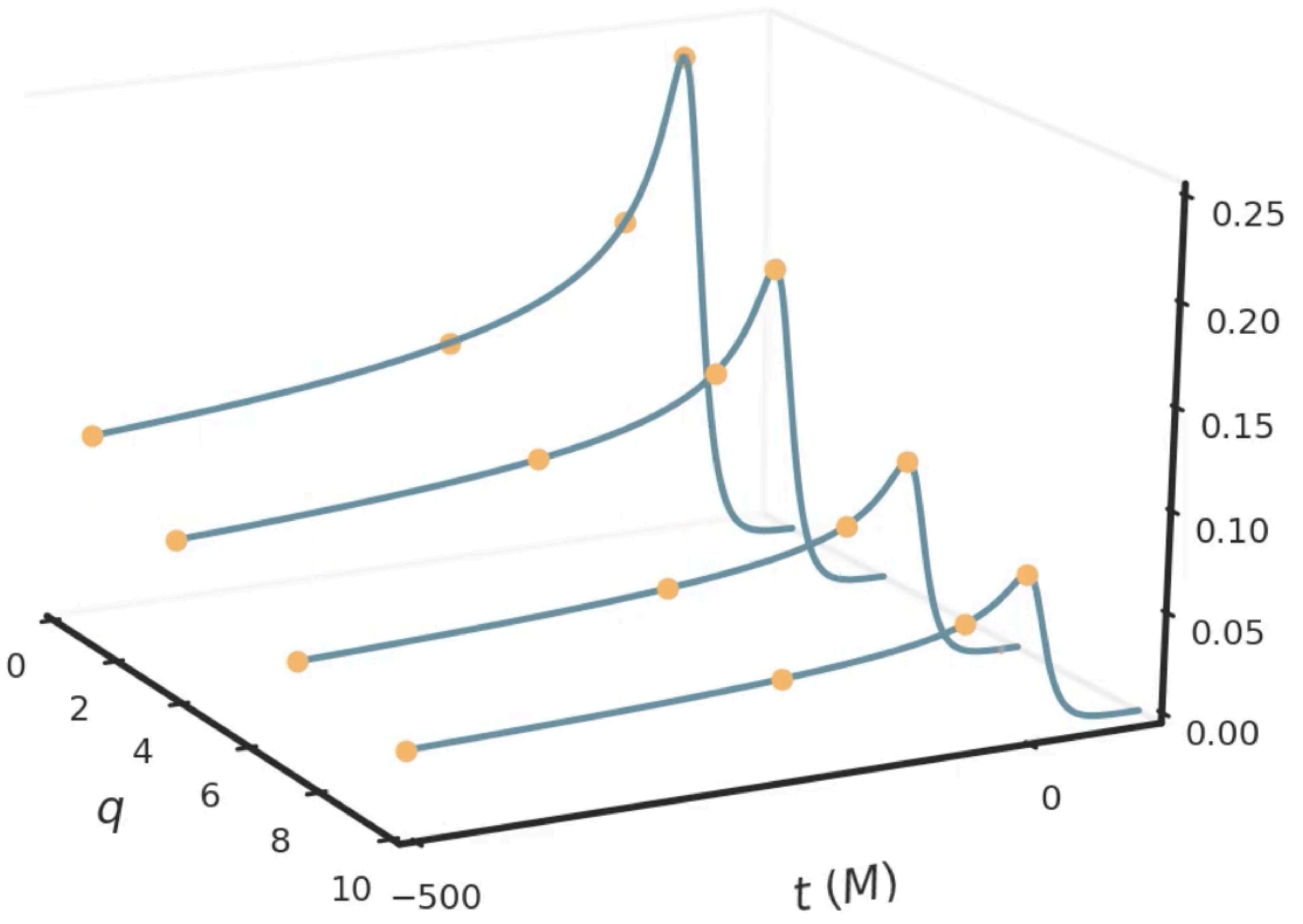
- Step 2: Build an *empirical interpolant* that compresses time/frequency dimension
- Picks out values that are most representative → more nodes when data rapidly changing

$$\mathrm{EI}[h](t,\vec{\lambda}) = \sum_{j=1}^{n} B_j(t)\, h(T_j;\vec{\lambda})$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \hat{e}_i(t)\, \left([\hat{e}_i(T_j)]^{-1}\right)_{ij}\, h(T_j;\vec{\lambda})$$



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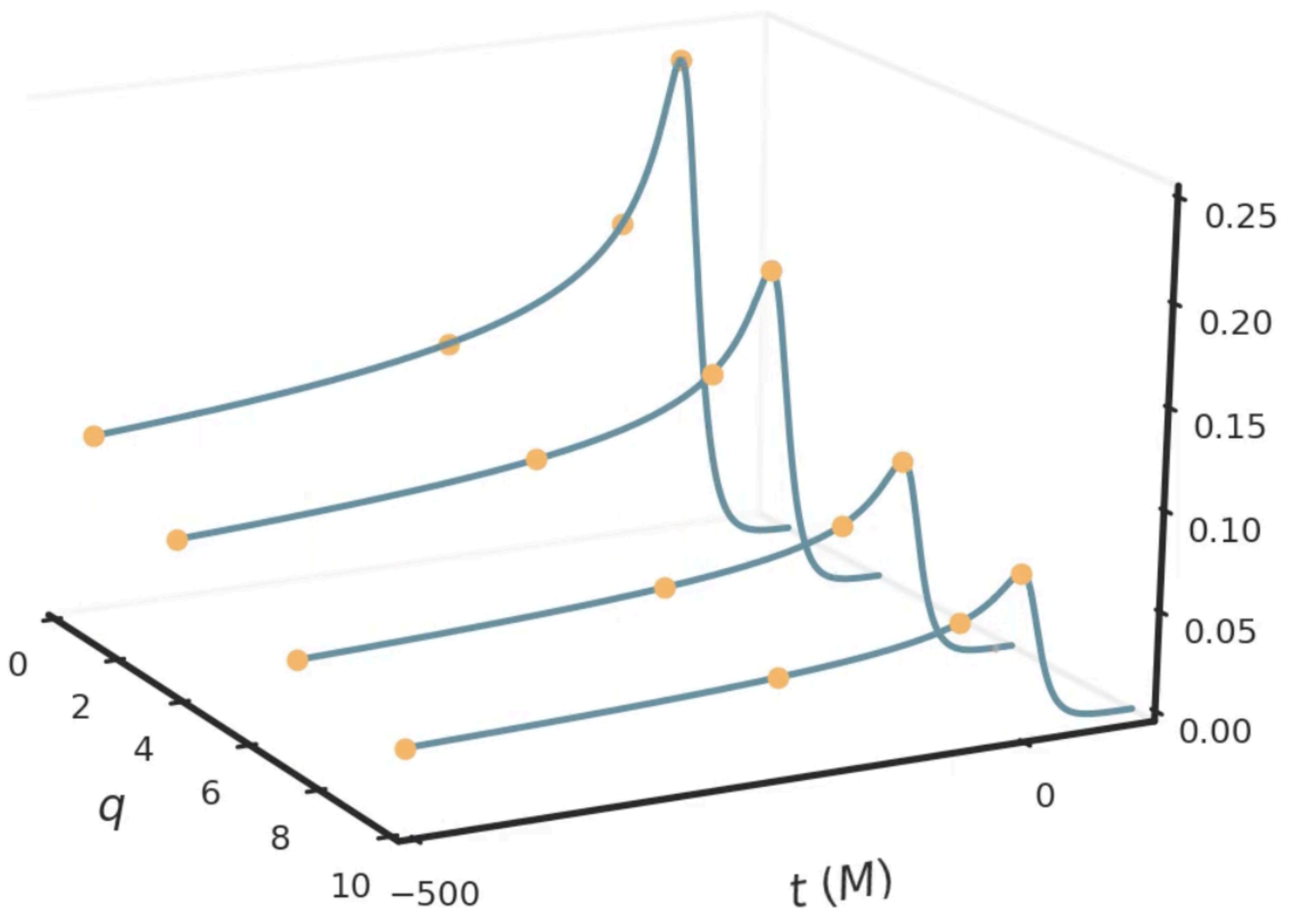
Calculate offline



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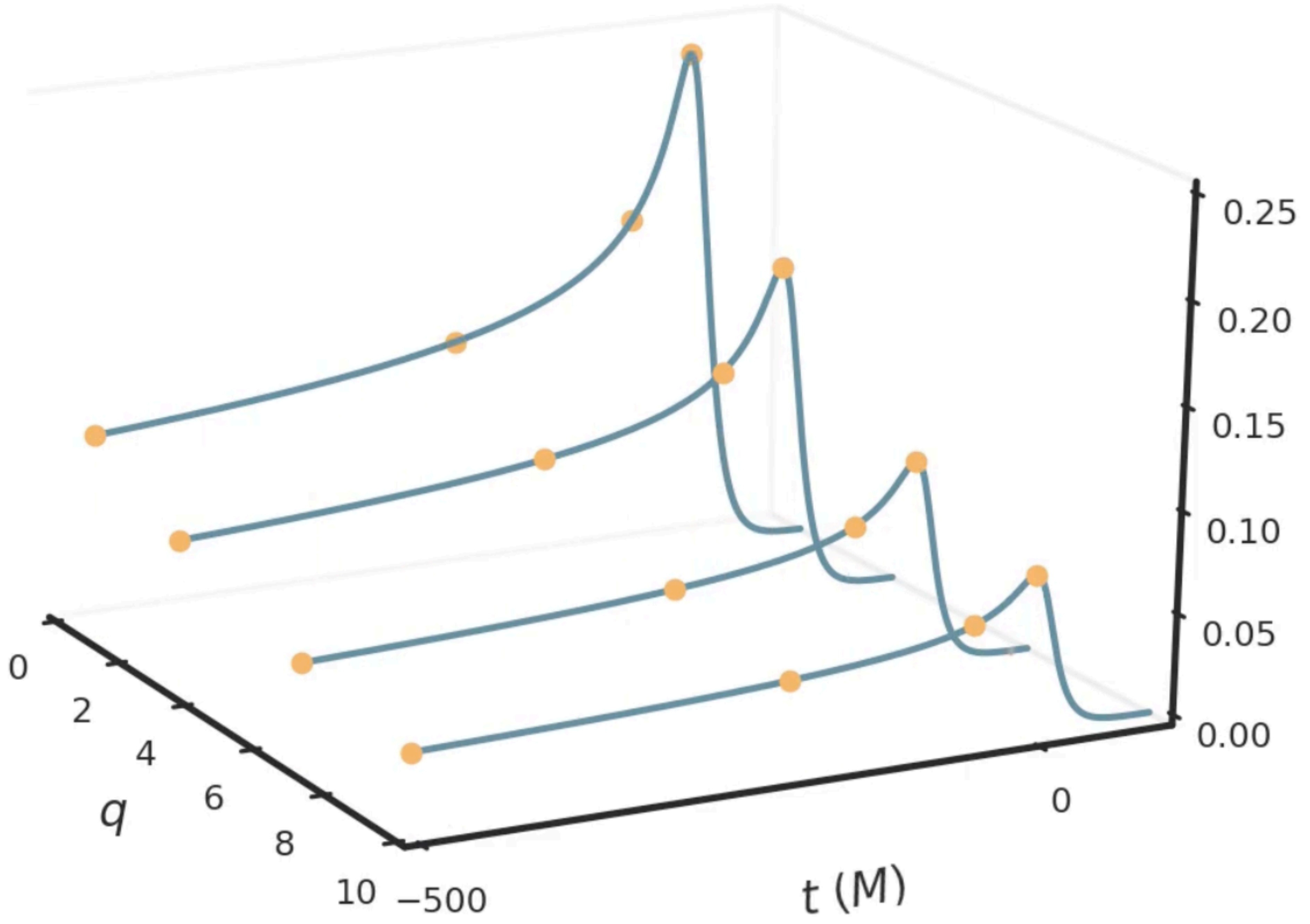
Need expression for this?

Calculate offline



Surrogate Models: Step 3

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- Step 3: Fit value of *waveform at empirical nodes* as function of parameters

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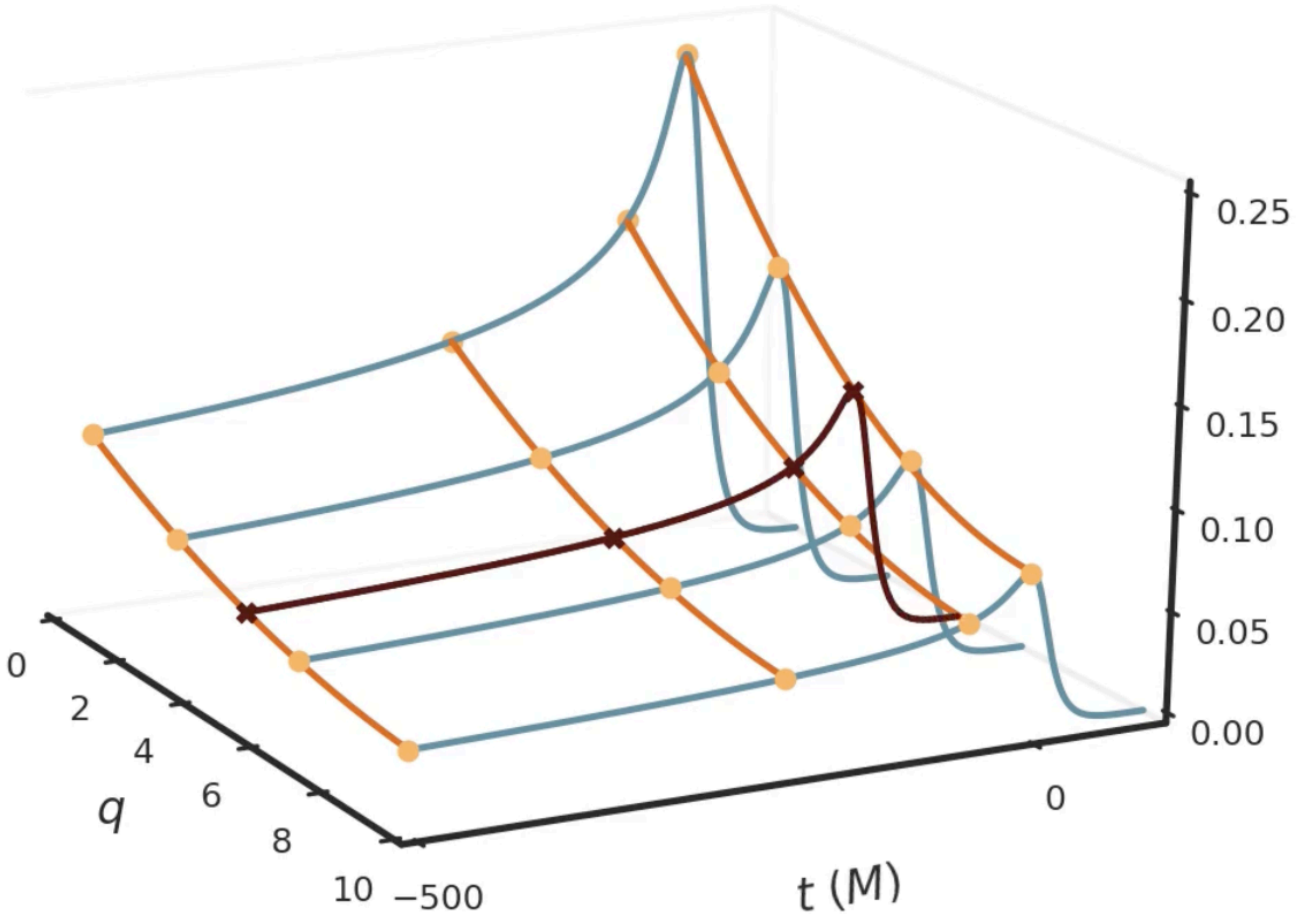
Fit it!



Surrogate Models: Step 3

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- Step 3: Fit value of *waveform at empirical nodes* as function of parameters
  - Parametric fit across parameter space [e.g. Varma+19]
  - Neural networks [e.g. Thomas+22 inc GP]
  - Gaussian process regression [e.g. Varma+19, Boschini+23]

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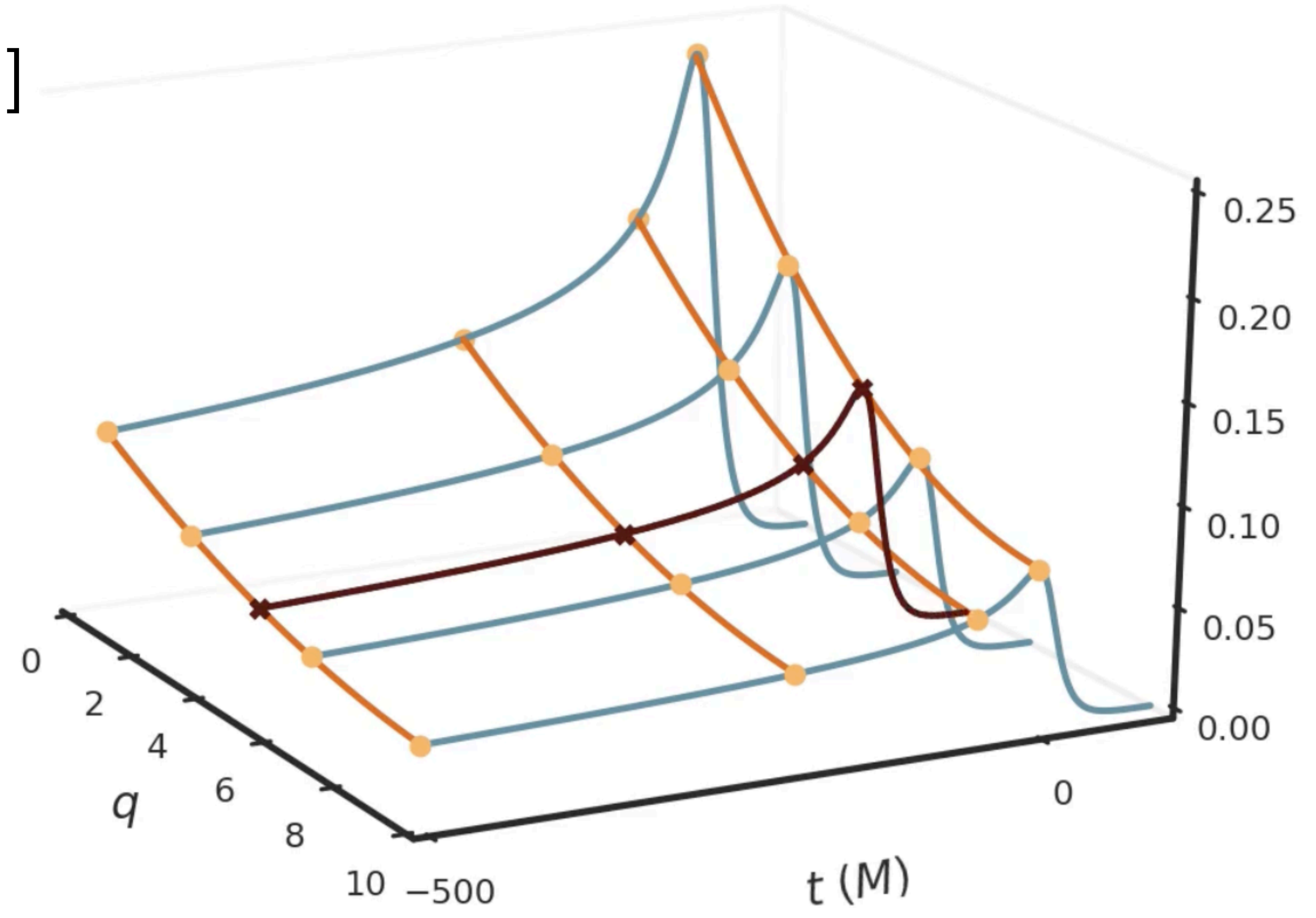
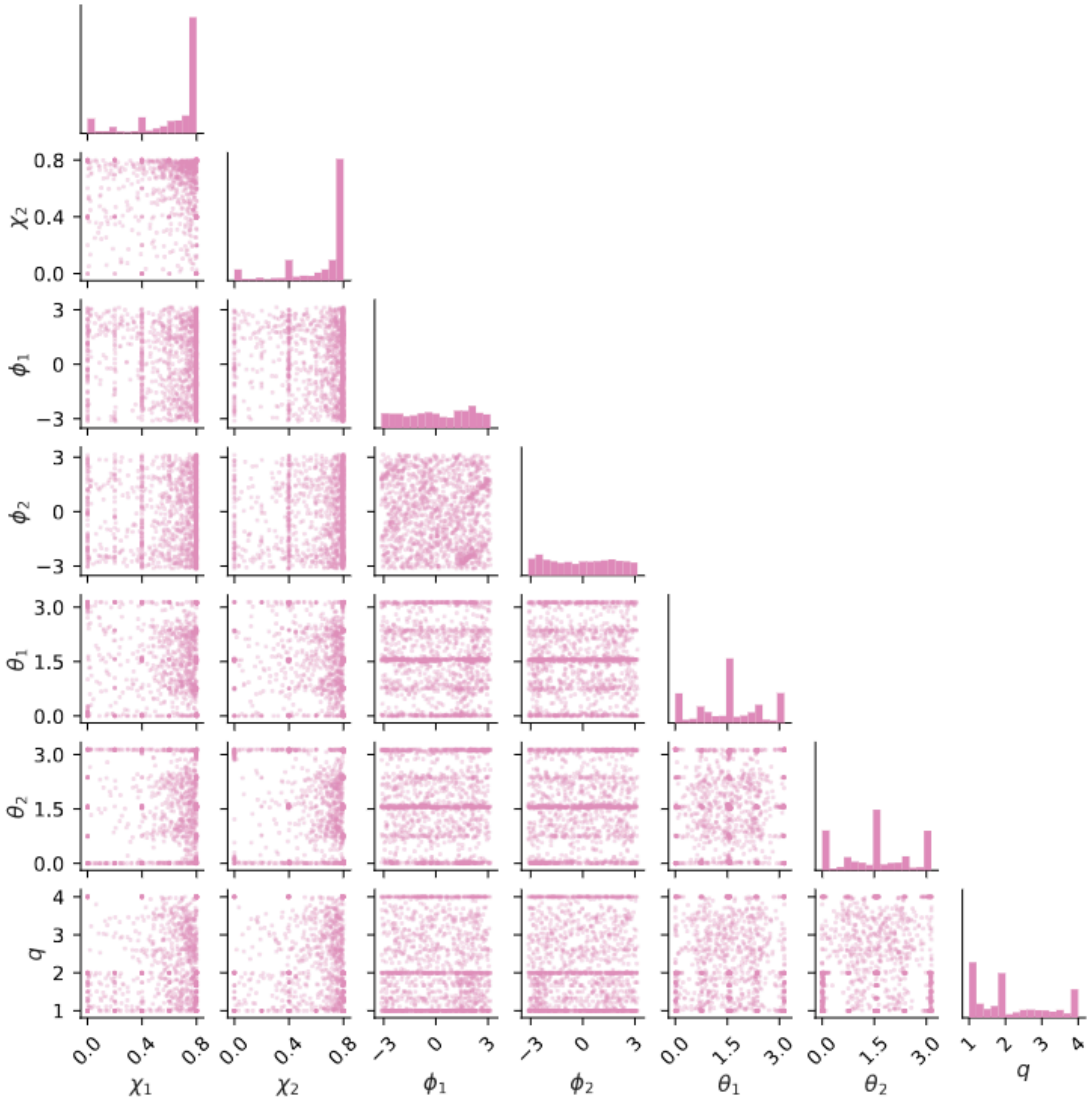


Surrogate Models

- State-of-the-art precessing surrogate still NRSur7dq4
- 1528 precessing NR simulations used to build surrogate
    - Calibrated to $q = 4$ and $|\chi_i| = 0.8$
    - But extrapolation up to $q \sim 6$ and $|\chi_i| \sim 0.99$



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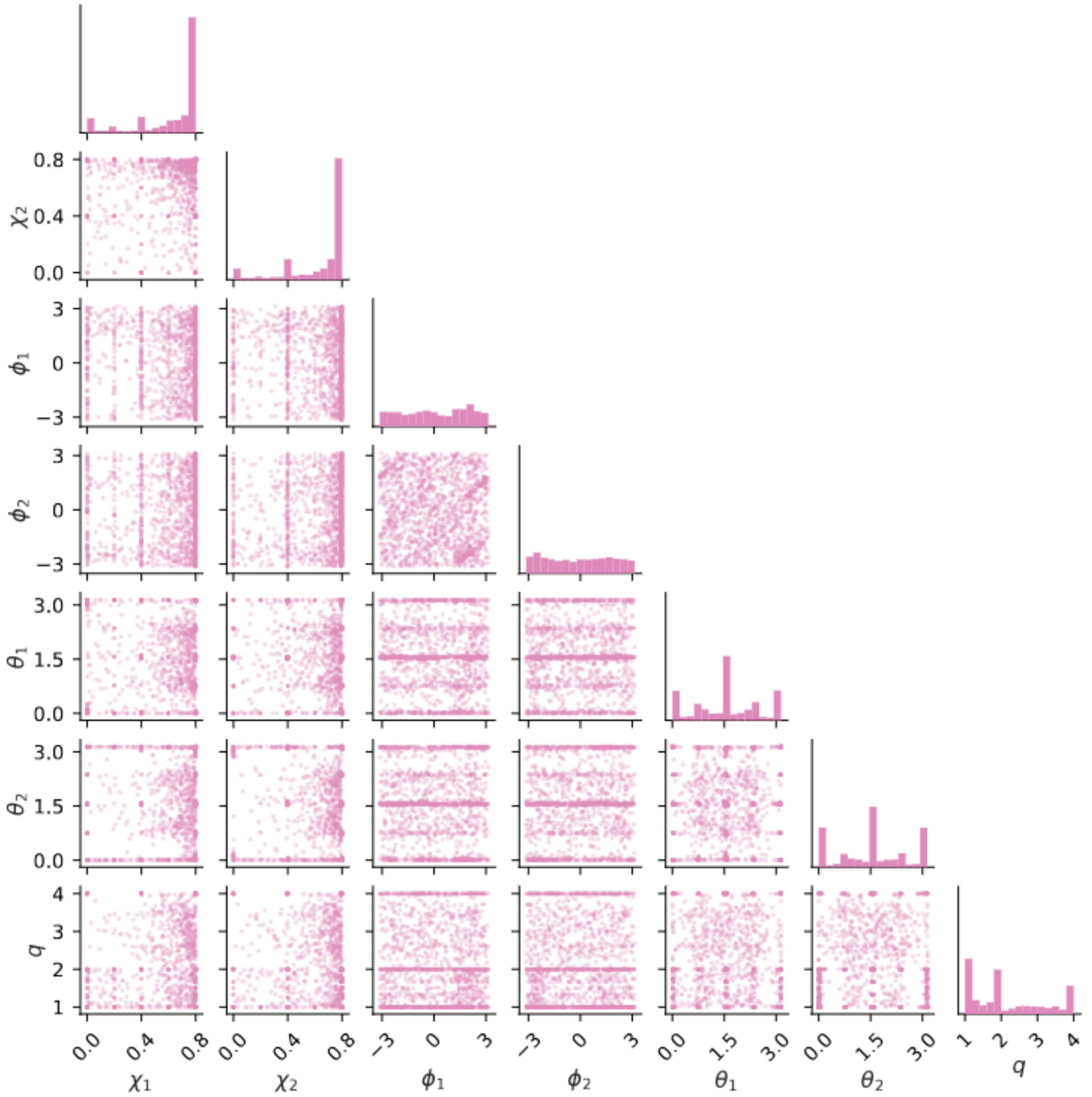
  - Calibrated to $q = 4$ and $|\chi_i| = 0.8$
  - But extrapolation up to $q \sim 6$ and $|\chi_i| \sim 0.99$
- *Examples* of recent surrogate models
  - Aligned-spin NR+PN/EOB [Varma+]
    - Memory effects using CCE waveforms [Yoo+23]
  - Extreme mass ratios [Islam+22]
  - Eccentric aligned-spin surrogate [Islam+22]
  - Remnant surrogate [Varma+19, Boschini+23]
  - Effective One Body [Thomas+22, Pompili+23]



Incorporating Numerical Information?

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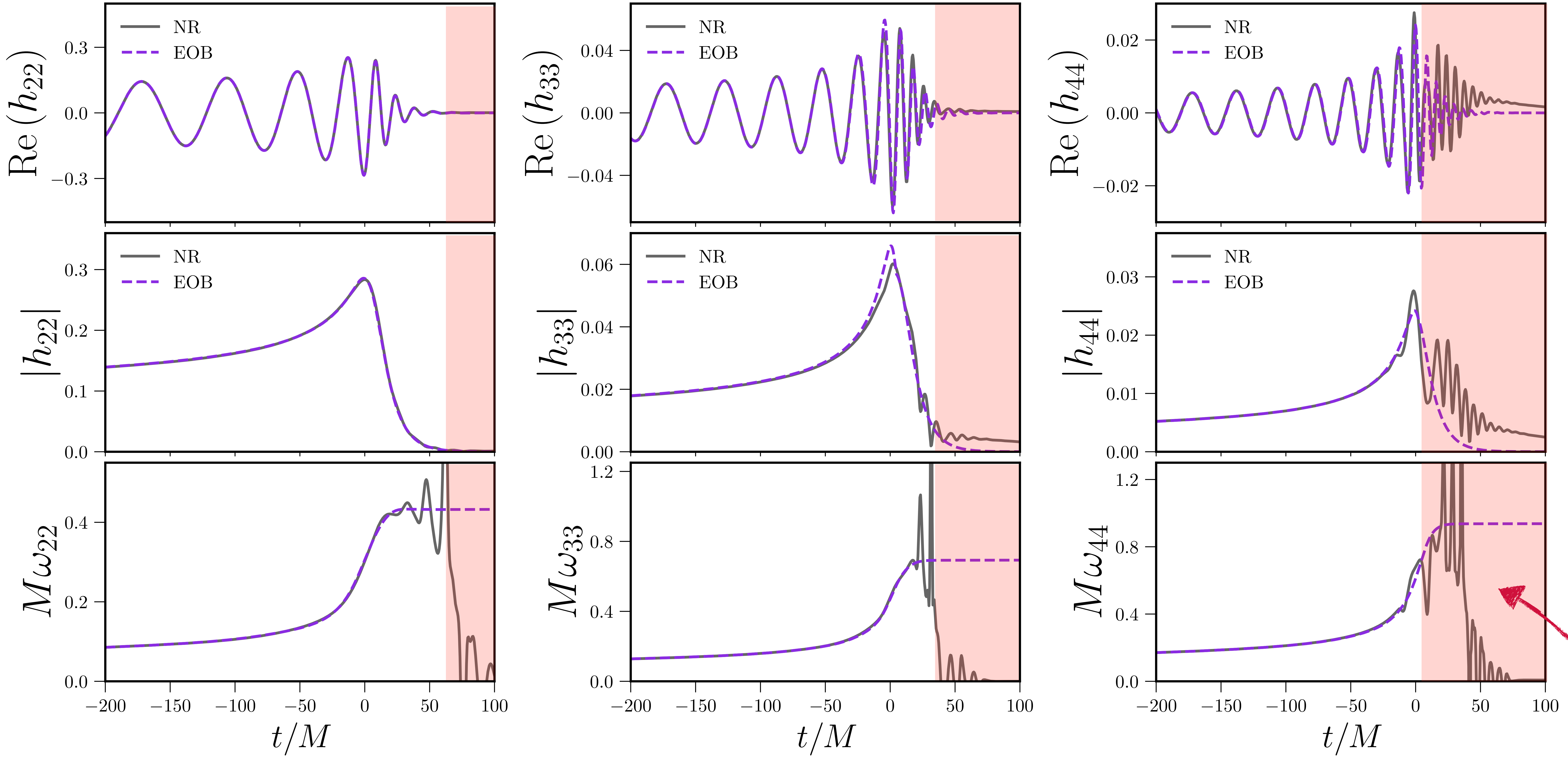
- Numerical relativity allows us to incorporate full non-perturbative information in strong-field regime
- Not free from systematics and couples to how models are informed and calibrated

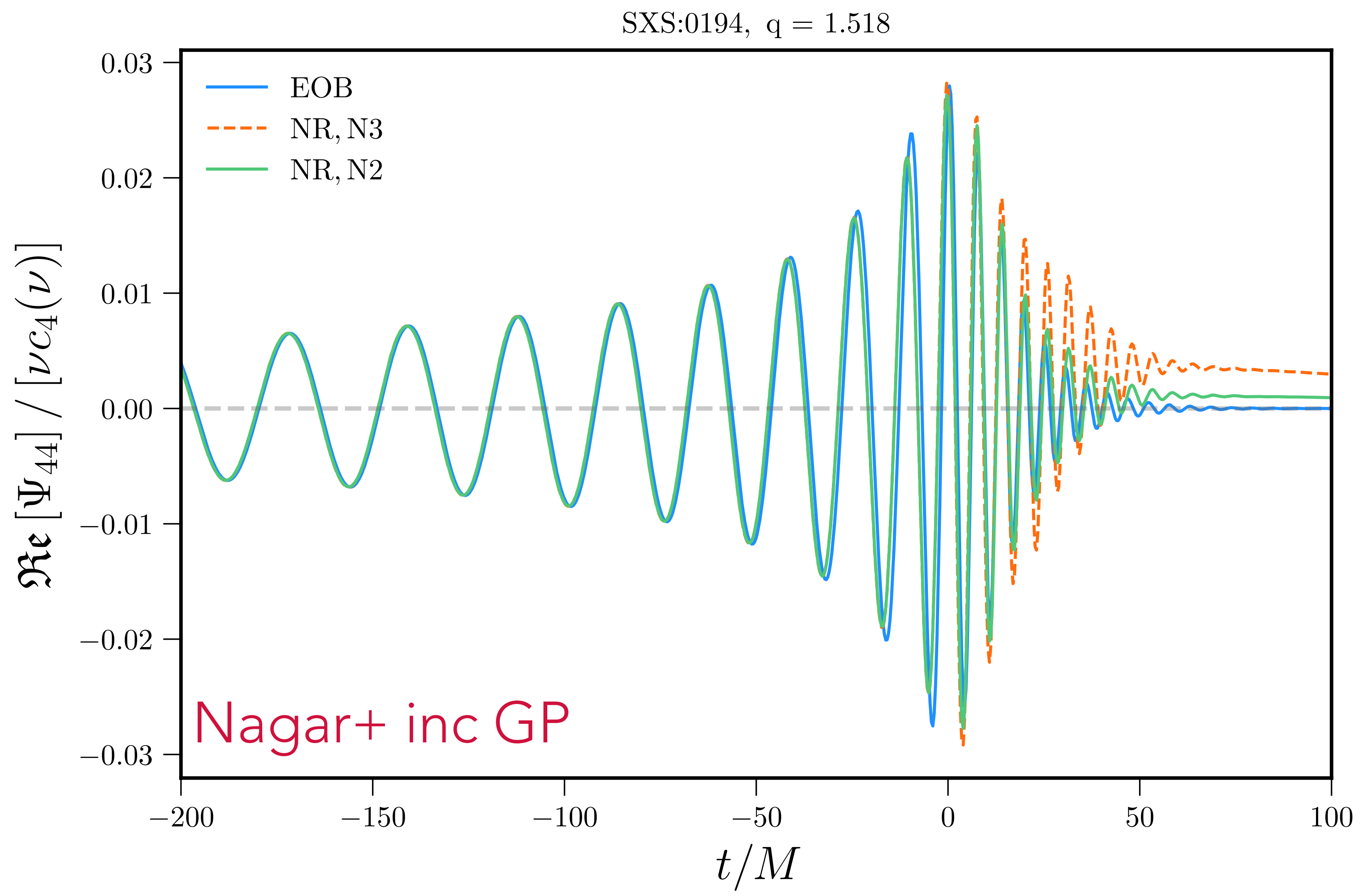
Nagar+ inc GP

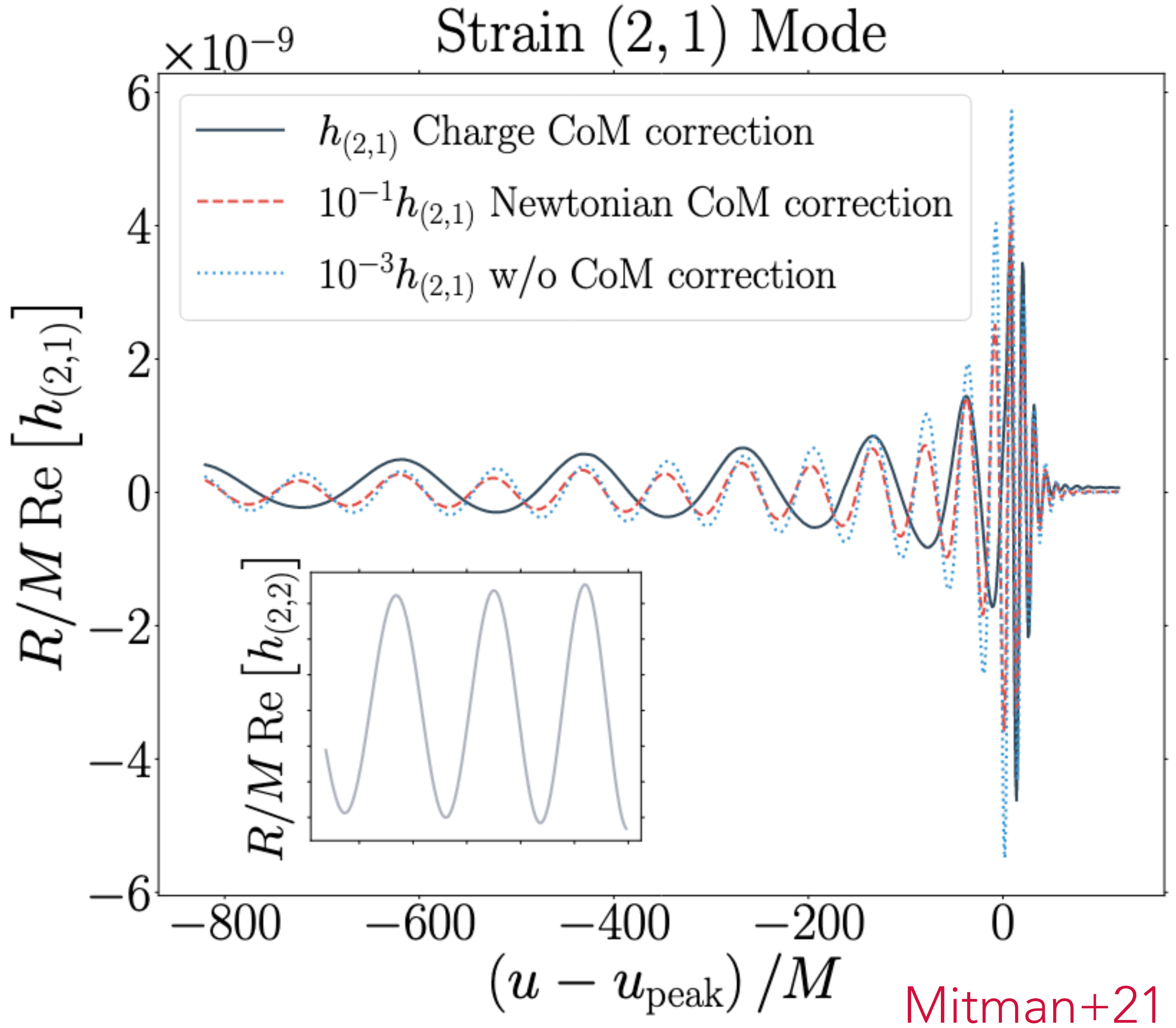


Unphysical behaviour in amplitude and phase

Incorporating Numerical Information?

- Extrapolation of waveforms to $\mathscr{I}^+$ can introduce unphysical features [Chu+, Boyle+, Nagar (inc GP)+]
- Mitigate with cauchy characteristic extrapolation (CCE) [Bishop+, Reisswig+, Taylor+, Barkett+, Moxon+]
  - Help reduce near-zone and gauge-effects on waveform
- Recent work to understand impact of frame choice on waveform
  - Fix Poincaré (by mapping to center-of-mass) frame [Boyle+, Woodford+]
  - Use Poincaré charges and super translation charges to fix BMS frame [Mitman+]
  - Methodology increasingly important to meet accuracy requirements





Phenomenological Waveform Models

Phenomenological Models

- Phenomenological waveforms follow a data-driven approach to directly model the GW signal
- Goal is an extreme compression of information into closed-form expressions
- Implementations in both the *time* and *frequency* domain

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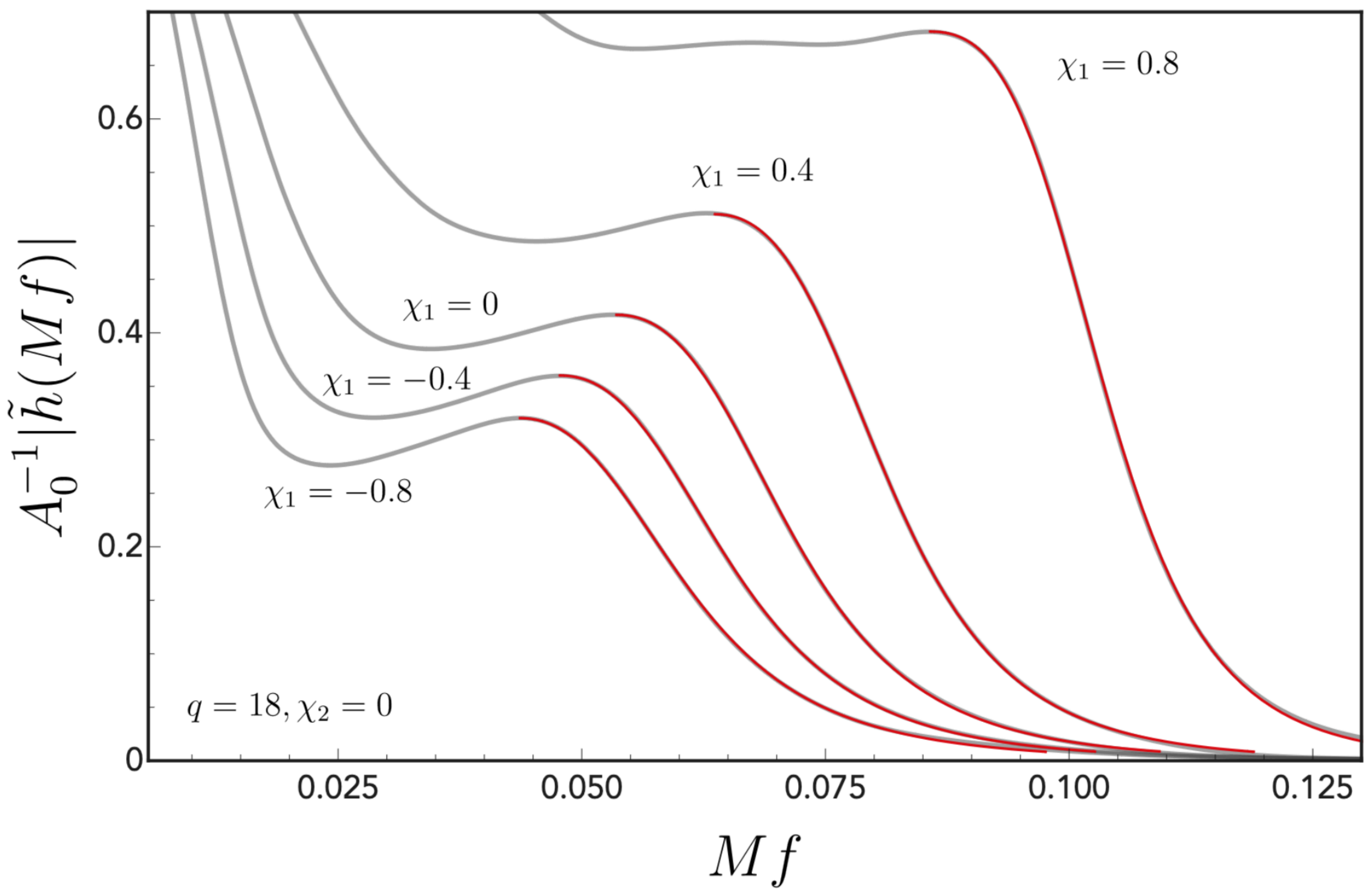
    - Fit ansatz to each dataset in a catalogue [local fit]

Pratten+21



Phenomenological Models

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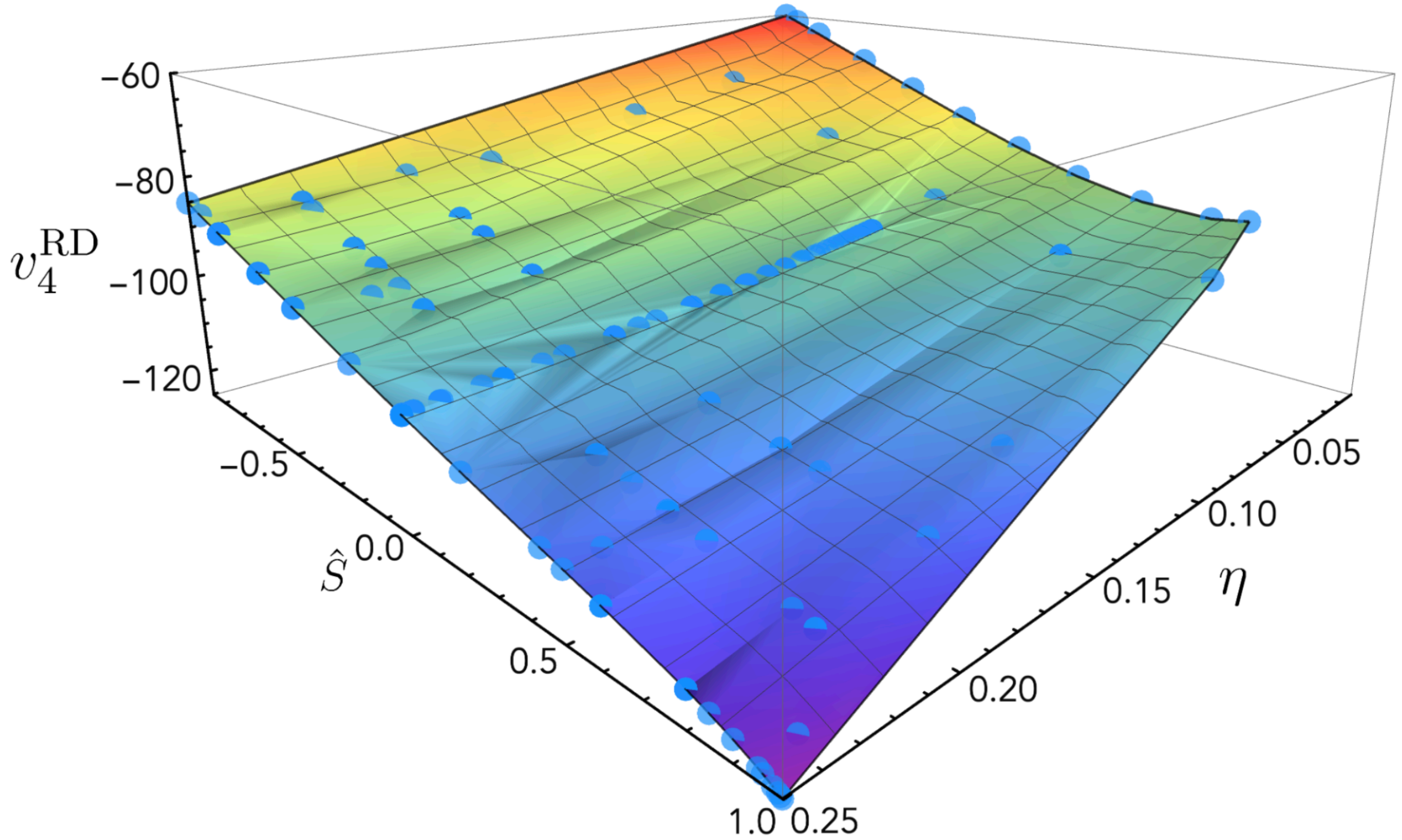

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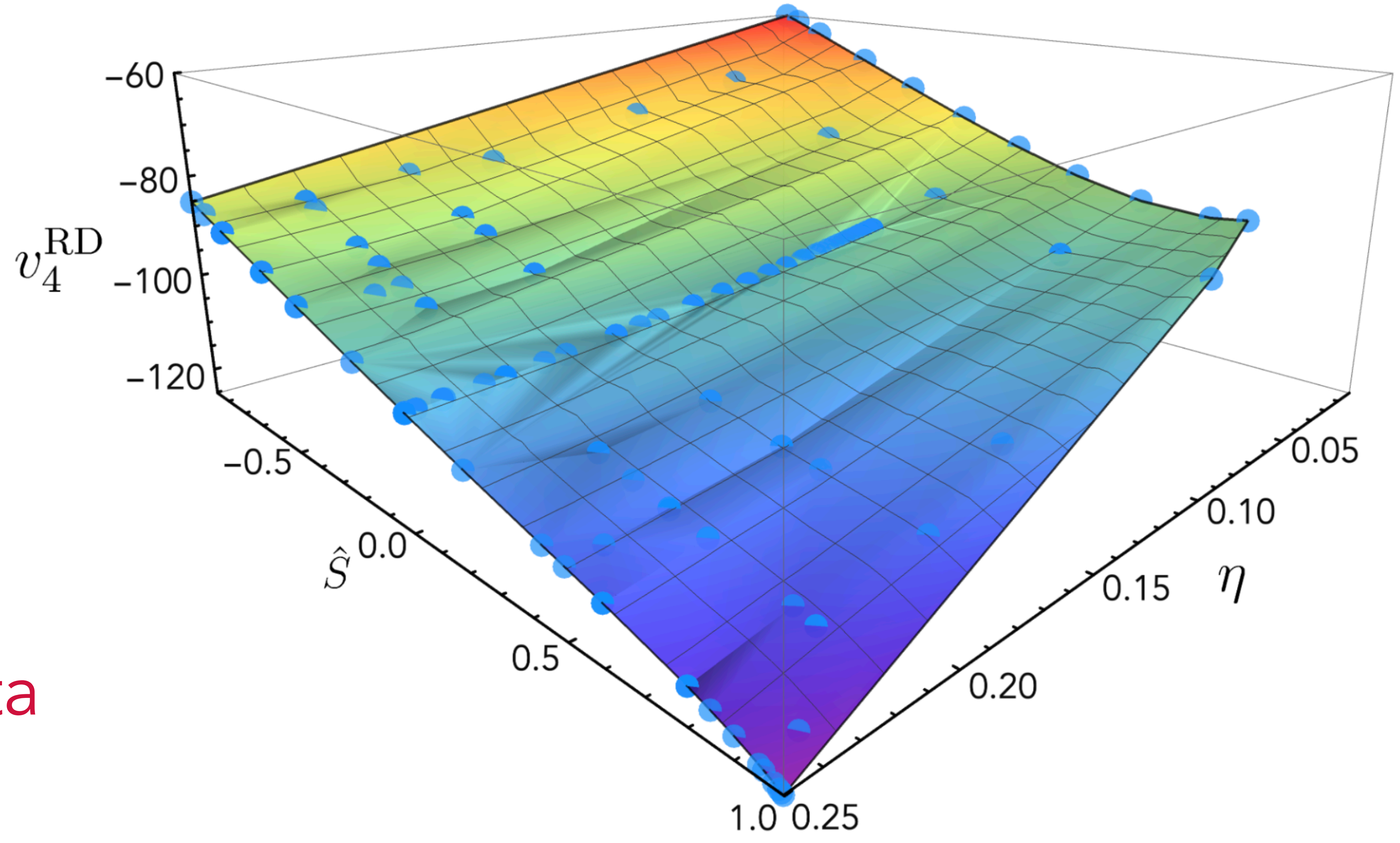
  - Methods to avoid under/over-fitting + deal with noisy data

Pratten+21



Phenomenological Models

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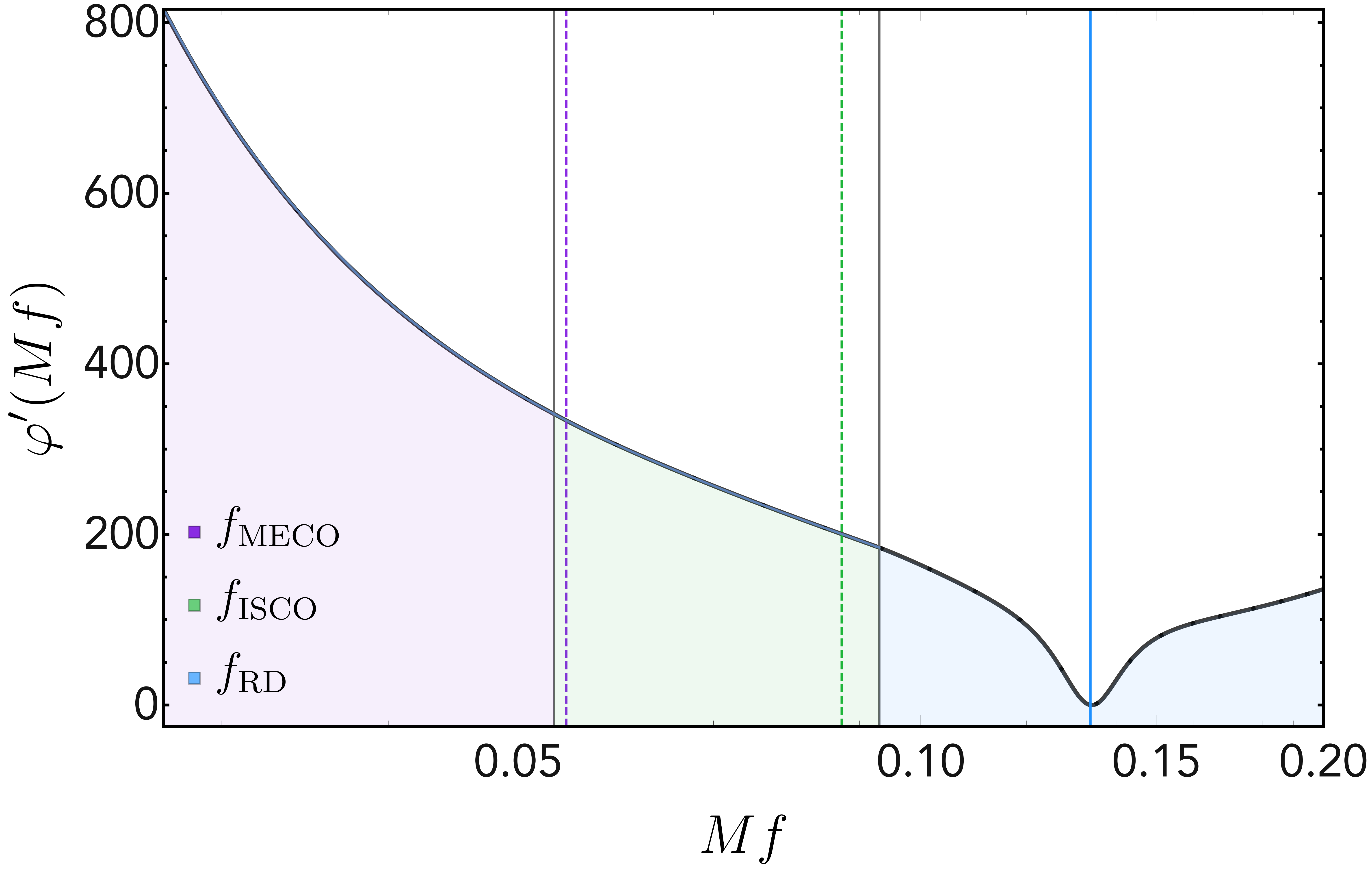
- Decompose GW signal into different regimes: inspiral, intermediate, merger-ringdown



Phenomenological Models

- Inspiral from post-Newtonian theory

$$E(v) = -\frac{\mu}{2}v^2 + \cdots$$

centre-of-mass energy (conservative dynamics)

$$\mathcal{F}(v) = \frac{32}{5}\nu^2\frac{c^5}{G}\left(\frac{v}{c}\right)^{10} + \cdots$$

GW luminosity (dissipative dynamics)

- Balance equation can be used to derive the GW phase

$$\frac{dE(\omega)}{dt} = -\mathcal{F}(\omega) \dashrightarrow \dot{\omega}(t) = -\frac{\mathcal{F}(\omega)}{dE(\omega)/d\omega} \dashrightarrow \varphi_{\rm GW}(t) = \frac{1}{\pi}\int^t \omega(t')\, dt'$$

- Cautionary note: care to distinguish between conservative and dissipative effects when comparing GW results to other experiments

Phenomenological Models

- Decompose GW signal into amplitude and phase

Phenomenological Models

- Decompose GW signal into amplitude and phase

$$h(f) = \mathcal{A}(f)\, e^{i\varphi(f)}$$

Phase

Amplitude

Phenomenological Models

- Decompose GW signal into amplitude and phase

Phase

$$h(f) = \mathcal{A}(f)\, e^{i\varphi(f)}$$

Amplitude

- Inspiral phase encodes a wealth of physics

$$\varphi(v) = \left(\frac{v}{c}\right)^{-5} \left[\varphi_0 + \varphi_2 \left(\frac{v}{c}\right)^2 + \cdots + \varphi_{5l} \ln\left(\frac{v}{c}\right)\left(\frac{v}{c}\right)^5 + \cdots + \varphi_{10}\left(\frac{v}{c}\right)^{10} + \cdots\right]$$

Phenomenological Models

- Decompose GW signal into amplitude and phase

Phase

$$h(f) = \mathcal{A}(f)\, e^{i\varphi(f)}$$

Amplitude

- Inspiral phase encodes a wealth of physics

0PN 1PN 2.5PN(l) 5PN

$$\varphi(v) = \left(\frac{v}{c}\right)^{-5} \left[\varphi_0 + \varphi_2 \left(\frac{v}{c}\right)^2 + \cdots + \varphi_{5l} \ln\left(\frac{v}{c}\right)\left(\frac{v}{c}\right)^5 + \cdots + \varphi_{10}\left(\frac{v}{c}\right)^{10} + \cdots\right]$$

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} \qquad q = \frac{m_2}{m_1}$$

Spin effects at 1.5PN

Tail terms ~ back reaction due to scattering

Tidal effects first enter here…

Phenomenological Models

- Decompose GW signal into amplitude and phase

$$h(f) = \mathcal{A}(f)\, e^{i\varphi(f)}$$

Phase

Amplitude

- Intermediate regime ~ phenomenological
- Merger-ringdown ~ black hole perturbation theory

$$\varphi_{\rm Int} \sim \frac{1}{\eta}\left(\beta_0 + \beta_1 f + \beta_2 \log(f) - \frac{\beta_3}{3} f^{-3}\right)$$

$$\varphi_{\rm MR} \sim \frac{1}{\eta}\left\{\alpha_0 + \alpha_1 f - \alpha_2 f^{-1} + \frac{4}{3}\alpha_3 f^{3/4} + \alpha_4 \tan^{-1}\left(\frac{f - \alpha_5 f_{\rm RD}}{f_{\rm damp}}\right)\right\}$$

Coefficients calibrated against NR but *not* expressed in parameters relevant to GR or any modified theory of gravity…

Phenomenological Models

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THE ROYAL SOCIETY

- Frequency domain: IMRPhenomXPHM [Pratten+, Garcia-Quiros+, Pratten+]
- Time domain: IMRPhenomTPHM [Estelles+]

Phenomenological Models

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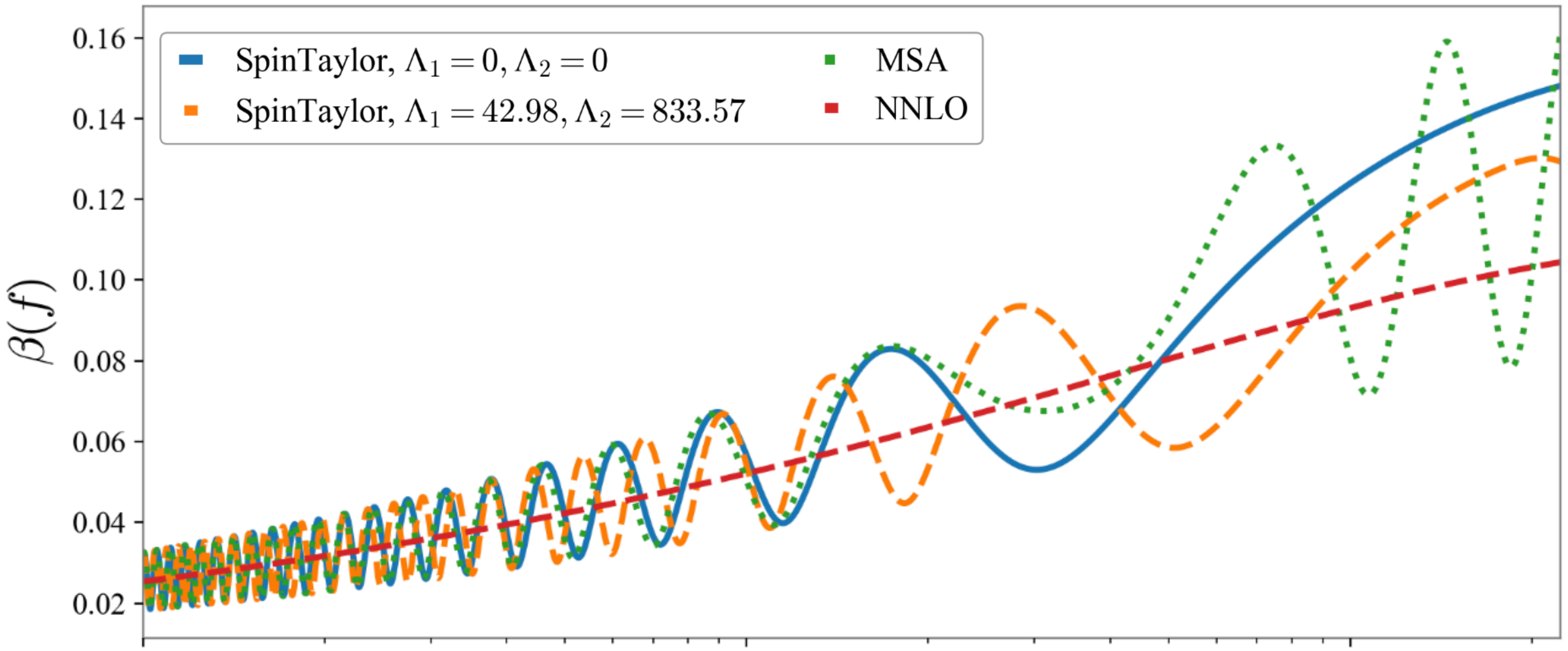
- Recent developments

Phenomenological Models

- Frequency domain: IMRPhenomXPHM [Pratten+, Garcia-Quiros+, Pratten+]
- Time domain: IMRPhenomTPHM [Estelles+]
- Recent developments
  - Implementation of PN spin dynamics following SpinTaylor [Colleoni+]

$$\frac{dv}{dt} = \cdots$$
$$\frac{d\boldsymbol{S}_1}{dt} = \boldsymbol{\Omega}_1 \times \boldsymbol{S}_1$$
$$\frac{d\boldsymbol{S}_2}{dt} = \boldsymbol{\Omega}_2 \times \boldsymbol{S}_2$$
$$\frac{d\mathbf{L}}{dt} = \left(\boldsymbol{\Omega}_L \times \hat{\boldsymbol{L}}\right) L + \frac{dL}{dt}\hat{\boldsymbol{L}}$$



Incorporates more PN information

Phenomenological Models

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THE ROYAL SOCIETY

- Frequency domain: IMRPhenomXPHM [Pratten+, Garcia-Quiros+, Pratten+]
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  - Mode asymmetries [Ghosh+, Kolitsidou+]

$$\tilde{h}_s^{NR}(f) = \frac{1}{2}(\tilde{h}_{2,2}^{CP} + \tilde{h}_{2,-2}^{*CP}),$$
$$\tilde{h}_a^{NR}(f) = \frac{1}{2}(\tilde{h}_{2,2}^{CP} - \tilde{h}_{2,-2}^{*CP}).$$

Capture precession-induced mode assymetry

Phenomenological Models

- Frequency domain: IMRPhenomXPHM [Pratten+, Garcia-Quiros+, Pratten+]
- Time domain: IMRPhenomTPHM [Estelles+]
- Recent developments
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  - Mode asymmetries [Ghosh+, Kolitsidou+]
  - Calibration of precession dynamics against (single-spin) NR [Hamilton+]

Phenomenological Models

Waveform Family	Domain	Waveform Model	Spins	Mode Content		Eccentricity	Calibration Region
4th generation		IMRPhenomXAS	✓		(2,±2)	in development	NR calibration: $q \lesssim 18$, $\lvert\chi_{1/2}\rvert \lesssim 0.99$ Teukolsky calibration: $q \lesssim 1000$
		IMRPhenomXP	✓✓	CP			
		IMRPhenomXHM	✓		(2,±2),(2,±1),(3,±2),(3,±3),(4,±4)		
		IMRPhenomXPHM	✓✓	CP			
	TD	IMRPhenomT	✓		(2,±2)	in development	NR calibration: $q \lesssim 18$, $\lvert\chi_{1/2}\rvert \lesssim 0.99$ Teukolsky calibration: $q \lesssim 1000$
		IMRPhenomTP	✓✓	CP			
		IMRPhenomTHM	✓		(2,±2),(2,±1),(3,±3),(4,±4),(5,±5)		
		IMRPhenomTPHM	✓✓	CP			

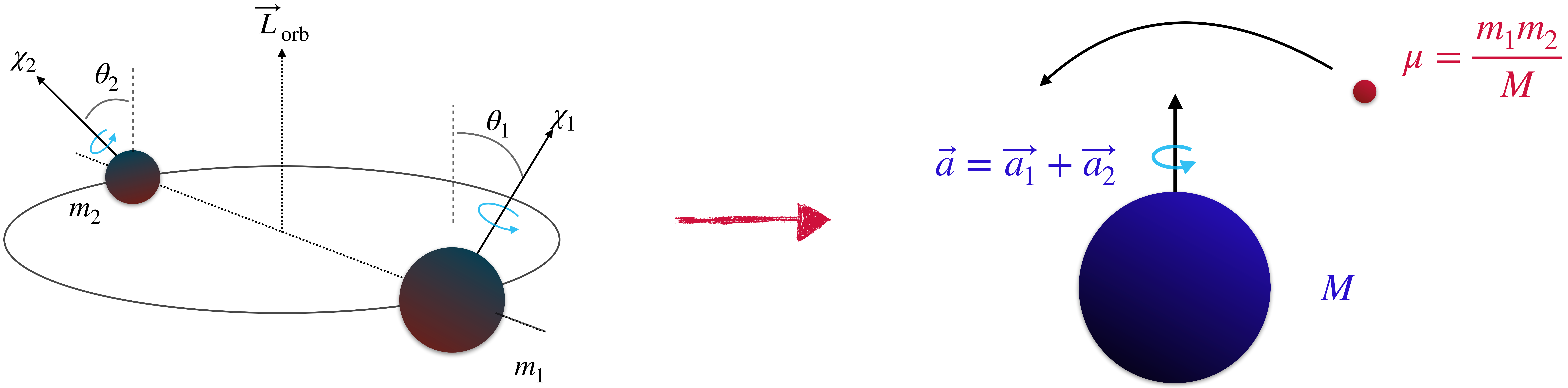
✗ no spins ✓ spins aligned with orbital angular momentum ✓✓ precessing spins CP mode content in co-precessing frame

https://arxiv.org/abs/2311.01300

Effective One Body

Effective One Body

- Map two-body dynamics onto dynamics of effective one body moving in deformed BH spacetime
- Natural deformation parameter is the symmetric mass ratio $\nu = \mu/M$



Buonanno and Damour 1999: arXiv:gr-qc/9811091
Buonanno and Damour 2000: arXiv:gr-qc/0001013

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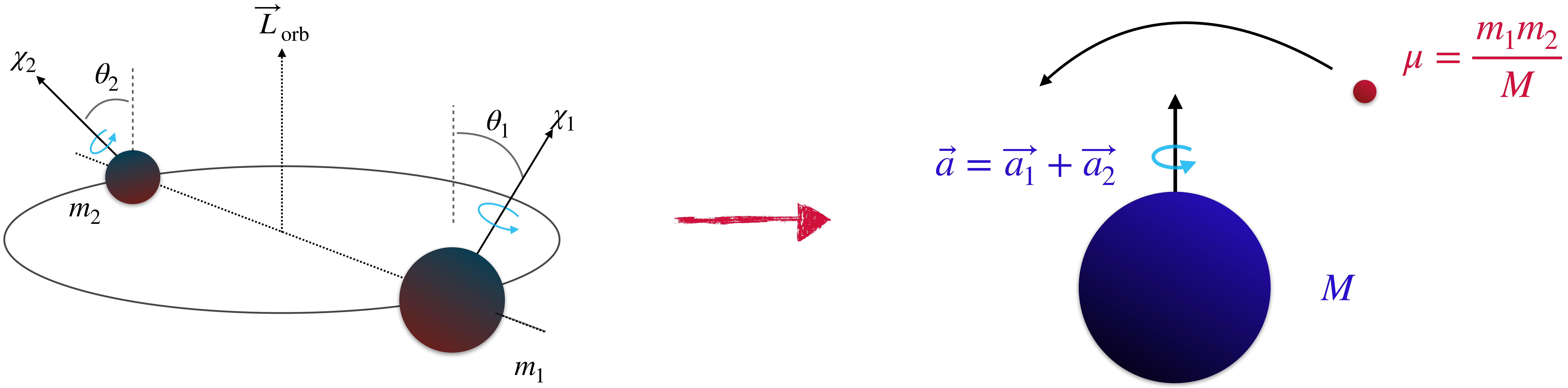

Significant freedom in construction of EOB model:

Hamiltonian, PN information, gauge, resummation, coupling to spins, etc

Buonanno and Damour 1999: arXiv:gr-qc/9811091
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Effective One Body:

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- An EOB is constructed from a number of key ingredients

Effective One Body:

- An EOB is constructed from a number of key ingredients
  - Hamiltonian encoding the conservative dynamics

$$H_{\mathrm{EOB}} = M\sqrt{1 + 2\nu\left(\frac{H_{\mathrm{eff}}}{\nu} - 1\right)}$$

Effective One Body:

- An EOB is constructed from a number of key ingredients
  - Hamiltonian encoding the conservative dynamics
  - Radiation reaction force to account for loss of energy and angular momentum via emission of GWs

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\nu} - 1\right)}$$

$$\mathcal{F} = \frac{\Omega}{16\pi}\frac{p}{L}\sum_{\ell m} m^2 |h_{\ell m}|^2$$

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Solve Hamilton-Jacobi Equations

$$\dot{\boldsymbol{r}} = \frac{\partial H_{\mathrm{EOB}}}{\partial \boldsymbol{p}}$$

$$\dot{\boldsymbol{p}} = -\frac{\partial H_{\mathrm{EOB}}}{\partial \boldsymbol{r}} + \boldsymbol{\mathcal{F}}$$

Effective One Body:

- An EOB is constructed from a number of key ingredients
  - Hamiltonian encoding the conservative dynamics
  - Radiation reaction force to account for loss of energy and angular momentum via emission of GWs
  - Waveform modes that describe the inspiral, merger, and bringdown

Solve Hamilton-Jacobi Equations

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\nu} - 1\right)}$$

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$$\dot{\boldsymbol{p}} = -\frac{\partial H_{\rm EOB}}{\partial \boldsymbol{r}} + \boldsymbol{\mathcal{F}}$$

$$h^{\rm IM}_{\ell m} = h^{F}_{\ell m} N_{\ell m}$$

$$h^{F}_{\ell m}(t) = h^{(N,\epsilon)}_{\ell m}\, S^{(\epsilon)}_{\rm eff}\, T_{\ell m}\, e^{i\delta_{\ell m}}\, (\rho_{\ell m})^{\ell}\, h^{\rm NQC}_{\ell m}$$

$$h_{\ell m}(t) = h^{\rm IM}_{\ell m}(t)\,\Theta\left(t^{\ell m}_{\rm match} - t\right) + h^{\rm R}_{\ell m}(t)\,\Theta\left(t - t^{\ell m}_{\rm match}\right)$$

Adapted from slides by: Buonanno, Pompili

Effective One Body: The Hamiltonian

- A key object is $H_{\rm eff}$ encoding how higher-order analytical information enters framework

$$H_{\rm eff}^{\nu} = \mu\sqrt{A_\nu(r)\left[\mu^2 + A_\nu(r)\bar{D}_\nu(r)p_r^2 + \frac{p_\varphi^2}{r^2} + Q_\nu(r,p_r)\right]}$$

In non-spinning $\mu \to 0$ limit reduces to Hamiltonian of test-particle in Schwarzschild background

Effective One Body: The Hamiltonian

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Differs from Schwarzschild due to PN corrections that depend on ν

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$$ds_{\text{eff}}^2 = -A_\nu(r)dt^2 + \frac{\bar{D}_\nu(r)}{A_\nu(r)}dr^2 + r^2 d\Omega^2$$

effective deformed metric

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effective deformed metric

- The dynamics is encoded in the potentials A_ν and D_ν

$$A_{\text{non-spin}}^{\text{Taylor}}(u) = 1 - 2u$$

Schwarzschild

Effective One Body: The Hamiltonian

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Differs from Schwarzschild due to PN corrections that depend on ν

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effective deformed metric

- The dynamics is encoded in the potentials A_ν and D_ν

$$A^{\rm Taylor}_{\rm non-spin}(u) = 1 - 2u + 2\nu u^3 + \nu \left(\frac{94}{3} - \frac{41\pi^2}{32} \right) u^4 + \left[\nu \left(\cdots \right) + \nu^2 \left(\cdots \right) + \frac{64}{5} \nu \ln u \right] u^5 + \left[\nu a_6 + \cdots \right] u^6$$

Schwarzschild

Effective One Body: state-of-the-art?

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- Currently two main EOB families: SEOBNR and TEOBResumS

TABLE II. Summary of the main differences of the **SEOBNRv5** Hamiltonian derived here, which builds on the results of Refs. [103, 104], compared to that of **SEOBNRv4** and **TEOBResumS**.

	SEOBNRv5	SEOBNRv4 [99, 100, 107, 111]	TEOBResumS [102, 112, 113]
nonspinning part	4PN with partial 5PN in A_{noS} and $\bar{D}_{\mathrm{noS}}$, 5.5PN in Q_{noS}	4PN in A_{noS}, 3PN in $\bar{D}_{\mathrm{noS}}$ and Q_{noS}	4PN with partial 5PN in A_{noS}, 3PN in $\bar{D}_{\mathrm{noS}}$ and Q_{noS}
A_{noS} resummation	(1,5) Padé	horizon factorization and log resummation	(1,5) Padé
$\bar{D}_{\mathrm{noS}}$ resummation	(2,3) Padé	log	Taylor expanded ($D_{\mathrm{noS}} \equiv 1/\bar{D}_{\mathrm{noS}}$ is inverse-Taylor resummed)
Hamiltonian in the $\nu \to 0$ limit	reduces to Kerr Hamiltonian for a *test mass* in a generic orbit	reduces to Kerr Hamiltonian for a *test spin*, to linear order in spin, in a generic orbit	the A potential reduces to Kerr, but not the full Hamiltonian
spin-orbit part	3.5PN, in (r, L^2) gauge, Taylor expanded	3.5PN, added in the spin map	3.5PN, in (r, p_r^2) gauge, inverse-Taylor resummed
higher-order spin information	NNLO SS (4PN), LO S^3 (3.5PN), LO S^4 (4PN)	LO SS (2PN)	NNLO SS (4PN) for circular orbits
precessing-spin Hamiltonian	yes	yes	no
spin-multipole constants included	yes	no	yes (in the SS contributions for circular orbits)

Pompili+, Khalil+, van de Meent+, Ramos-Buades+ + many others

Bernuzzi+, Damour+, Gamba+, Messina+, Nagar+, Rettegno+, + many others

Model Performance?

Asessing Accuracy?

- Mismatch as one way to gauge (point-wise) level of agreement between models (and/or NR)
- Overlap is the noise-weighted inner product - weighted by PSD of detector

$$\langle h_1, h_2 \rangle = 4\,\mathrm{Re} \int_{\mathrm{f_{low}}}^{f_{\mathrm{high}}} df \, \frac{\tilde{h}_1(f)\,\tilde{h}_2^*(f)}{S_n(f)}$$

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- Interested in the mismatch optimised over polarisation angle as well as time and phase (gauge)

$$\mathcal{M} \approx 1 - \max_{t_c, \varphi_0, \psi} \left[\frac{\langle h_1, h_2 \rangle}{\sqrt{\langle h_1, h_1 \rangle \langle h_2, h_2 \rangle}} \right]$$

- Treat as a measure of agreement between two waveforms at point in parameter space

Mismatches Across the Parameter Space?

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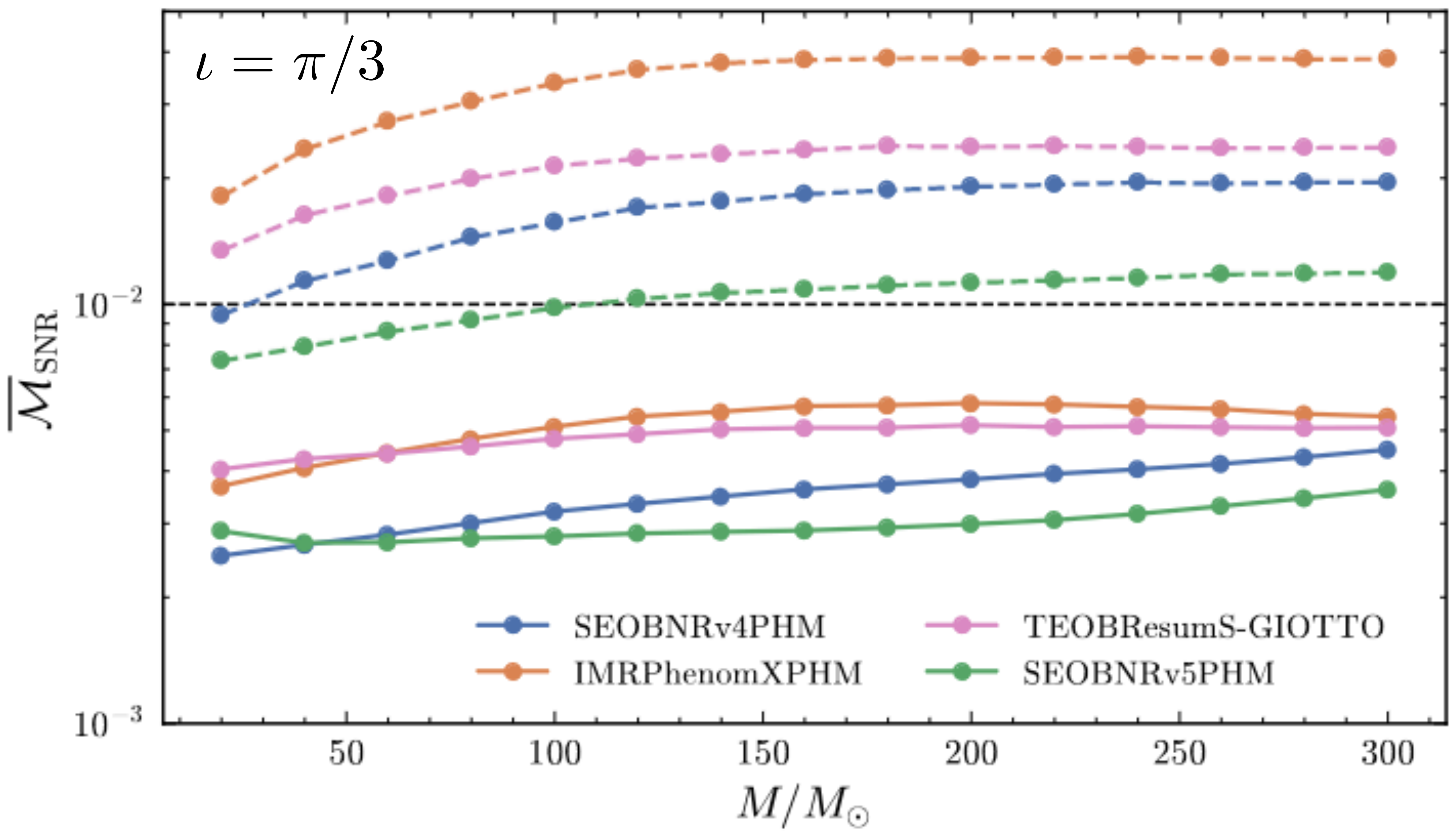
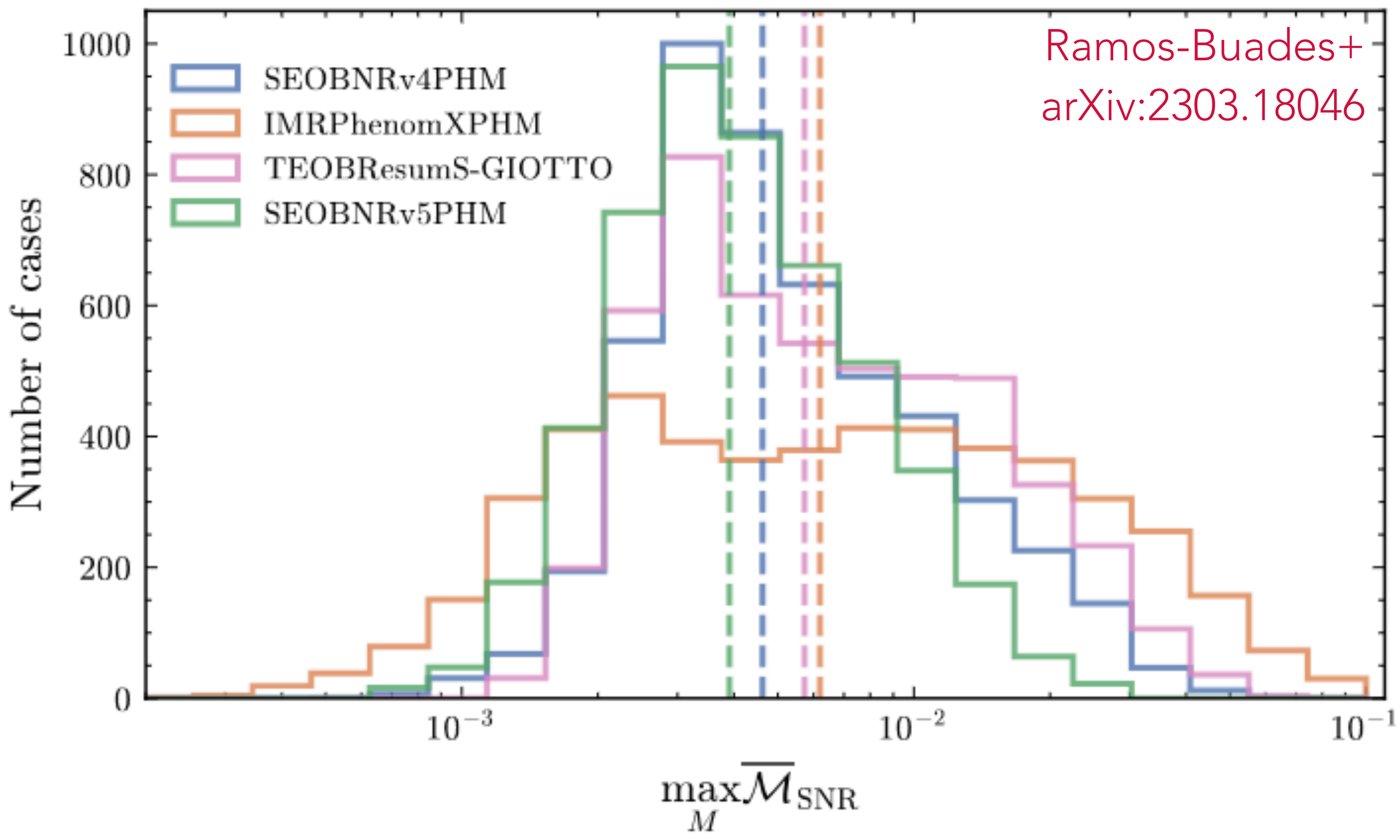
- Compare semi-analytical models against the NR surrogate



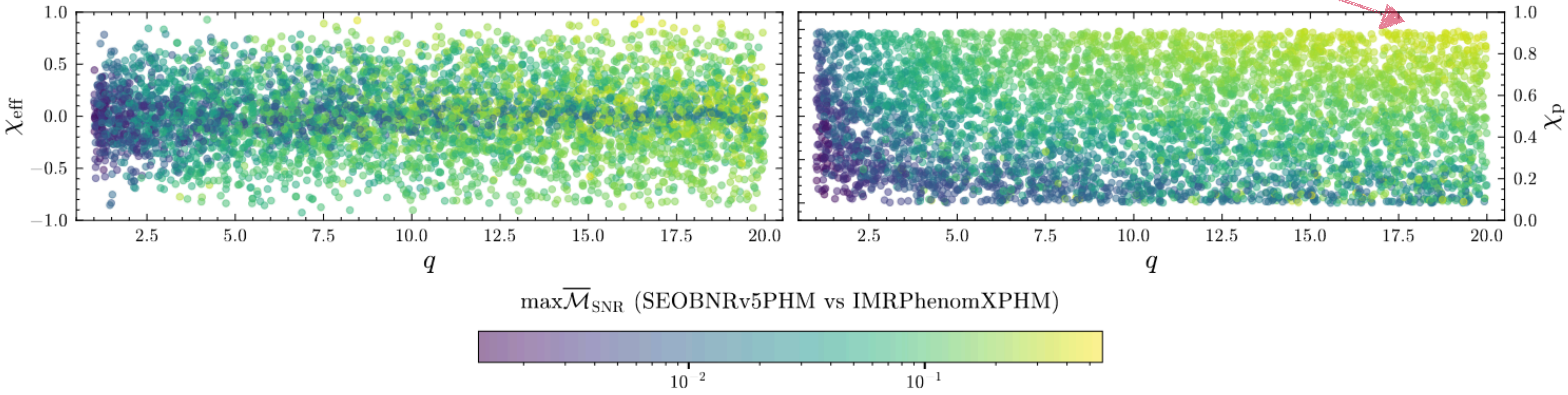


Ramos-Buades+
arXiv:2303.18046

Mismatches Across the Parameter Space?

Largest differences at high(er) mass ratios and precession

Ramos-Buades+ arXiv:2303.18046



Mismatches Across the Parameter Space?

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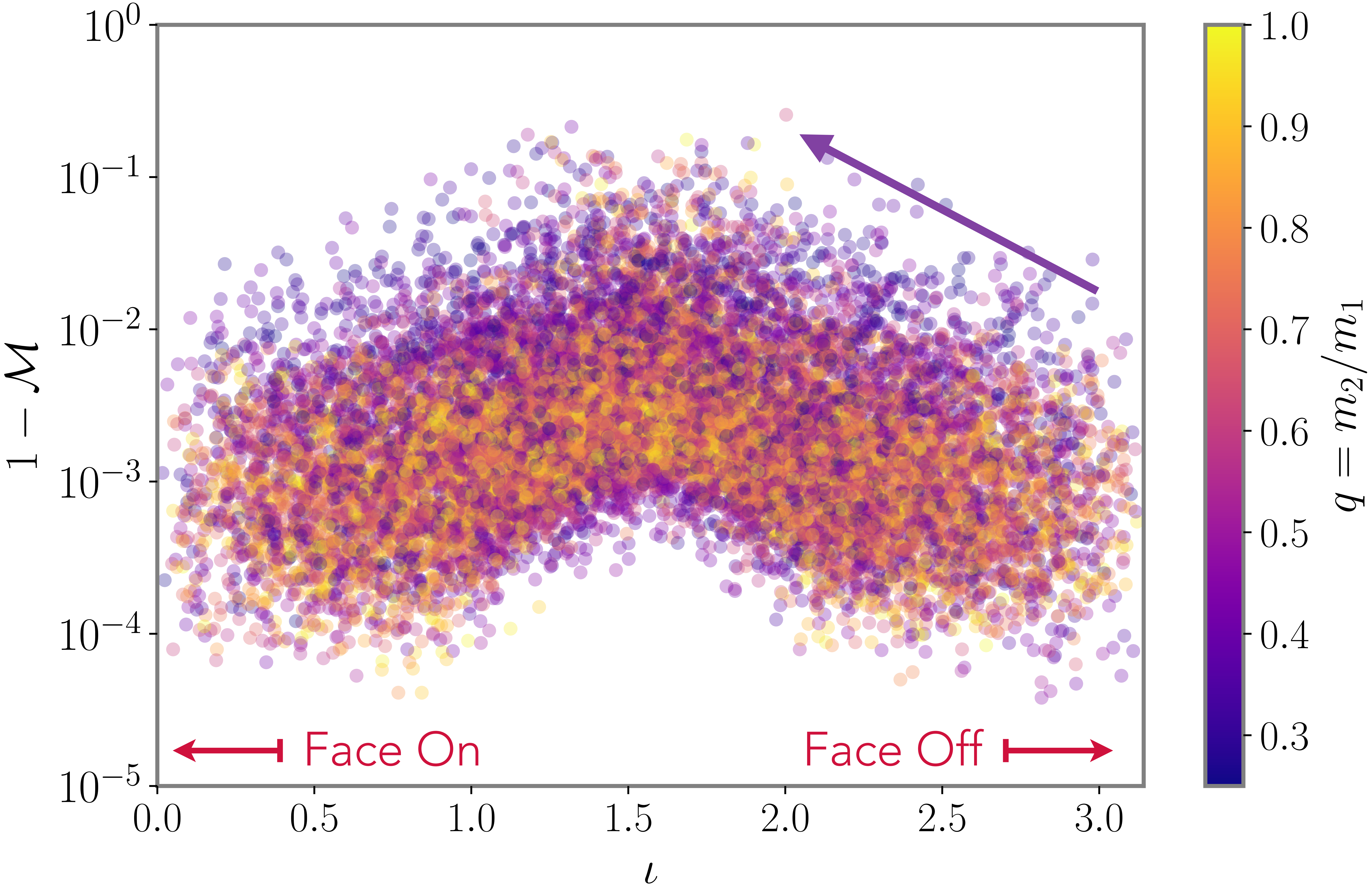
- Accuracy of models highly dependent on binary geometry, mass ratio, spins, etc

$\bar{\mathcal{M}} = \mathcal{M}_{t_0,\varphi_0,\psi_0}$



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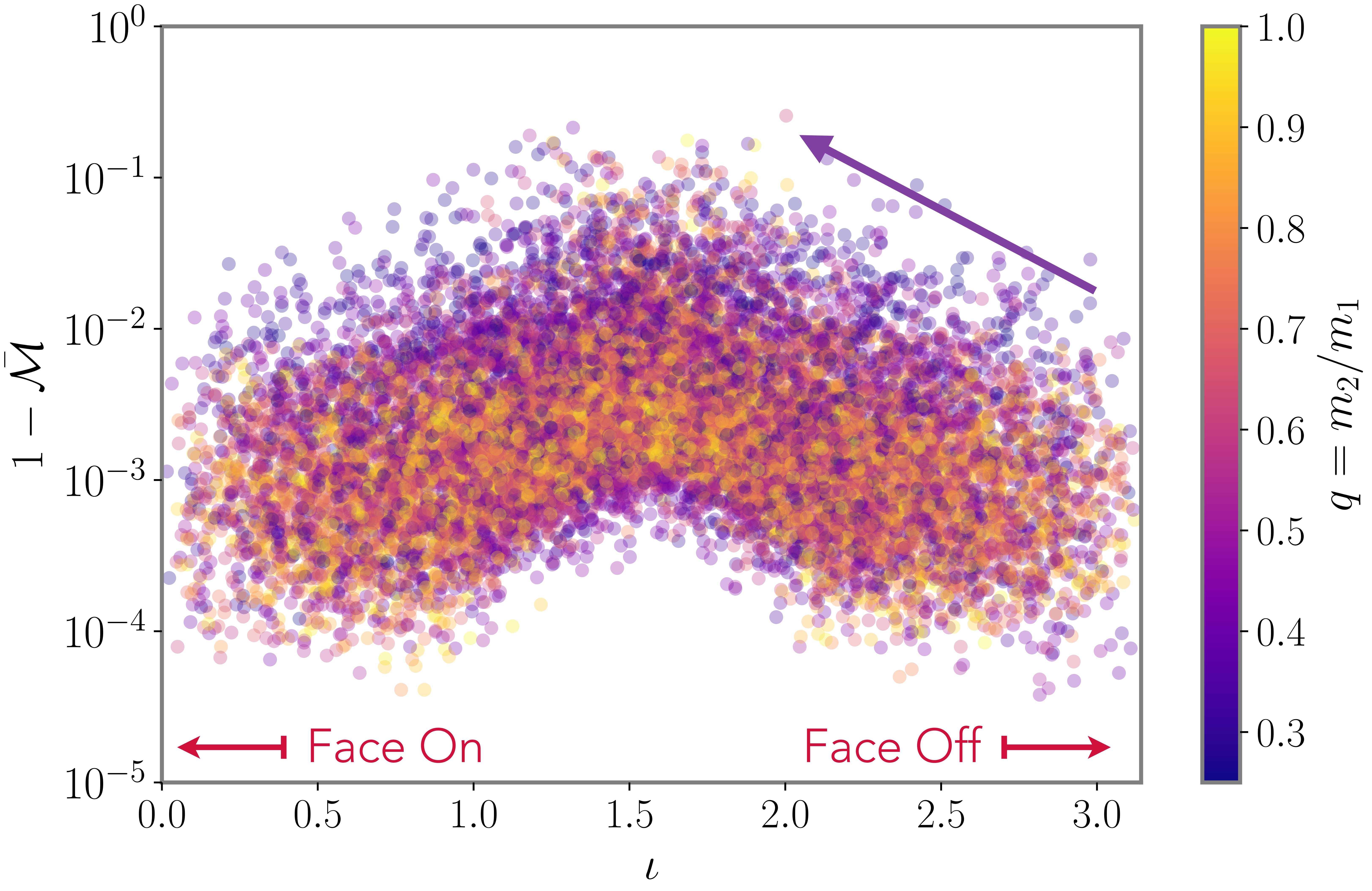

Detailed modelling of higher multipoles

Mismatches Across the Parameter Space?

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THE ROYAL SOCIETY

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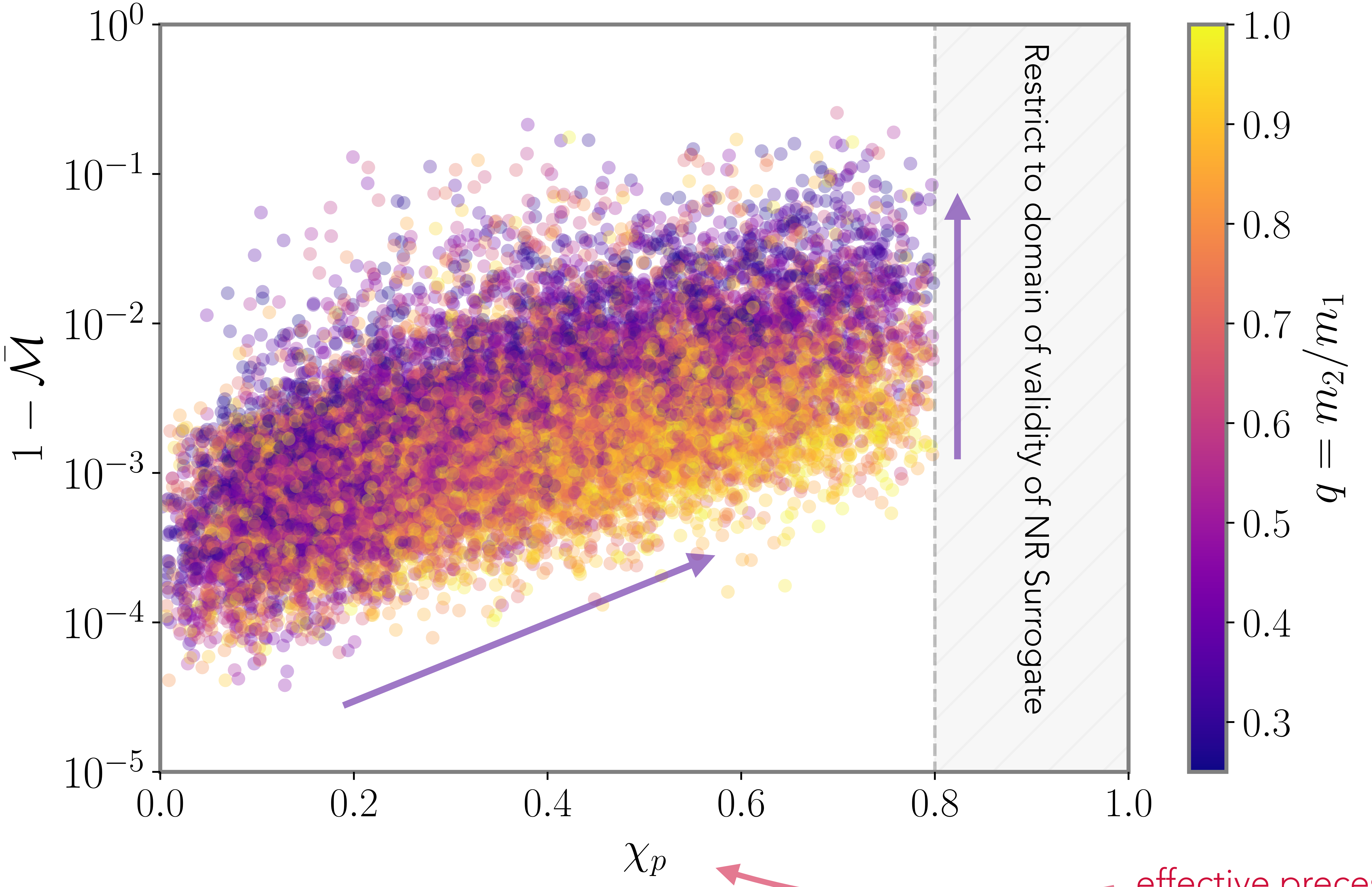

effective precessing spin [Schmidt+]

Mismatches Across the Parameter Space?

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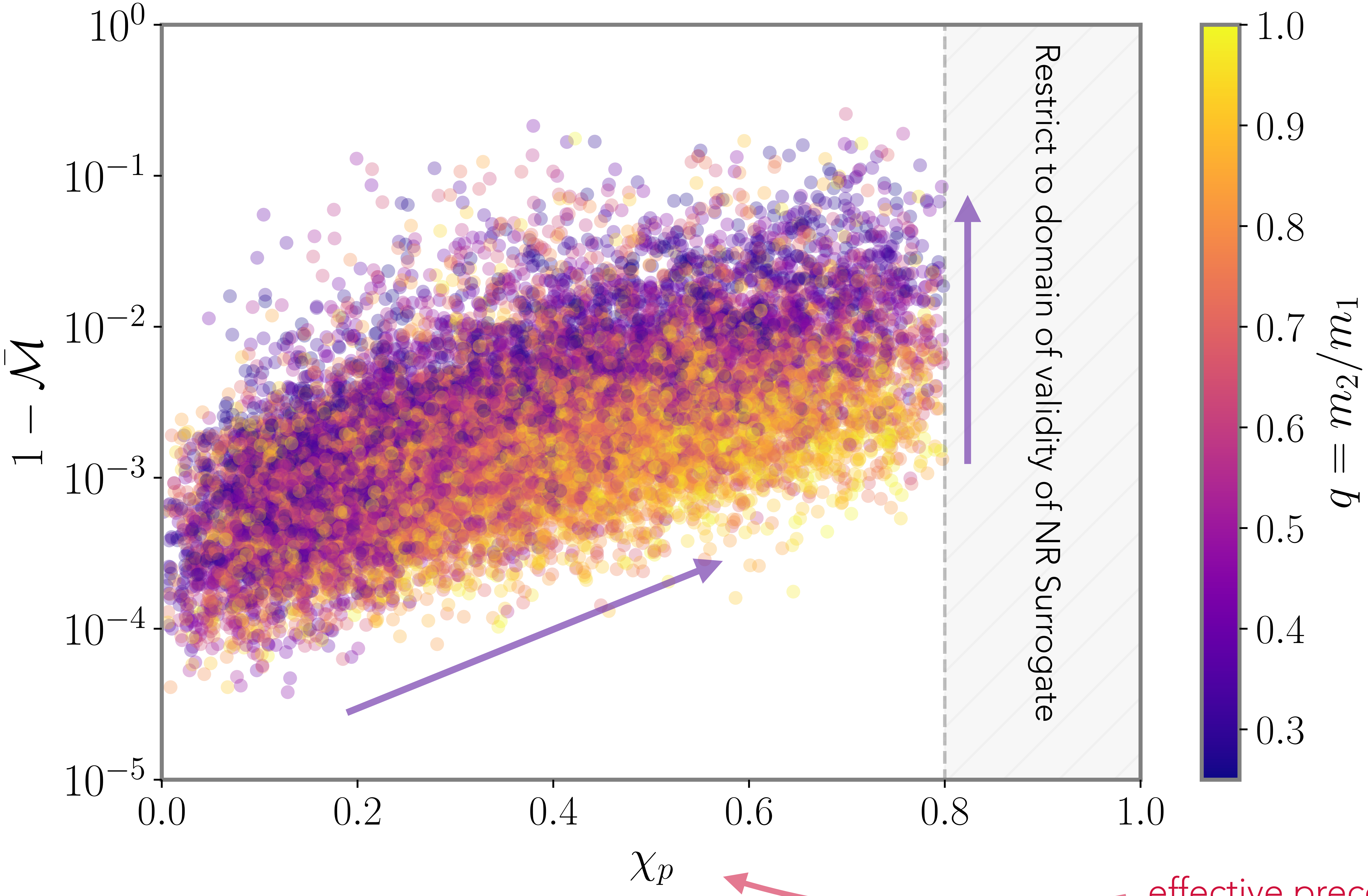

Detailed modelling of precession: mode asymmetries, co-precessing frame, NR tuning, higher multipoles, remnant, …

effective precessing spin [Schmidt+]

Highlighting Some Challenges and Progress?

Eccentricity

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Eccentricity

- Eccentricity as an excellent tracer for astrophysical formation channels

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- Incorrect inference of eccentricity can bias reconstruction of astrophysical channels [Fumagalli+24]

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Eccentricity

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+ see talk by Khun Sang Phukon on eccentric searches

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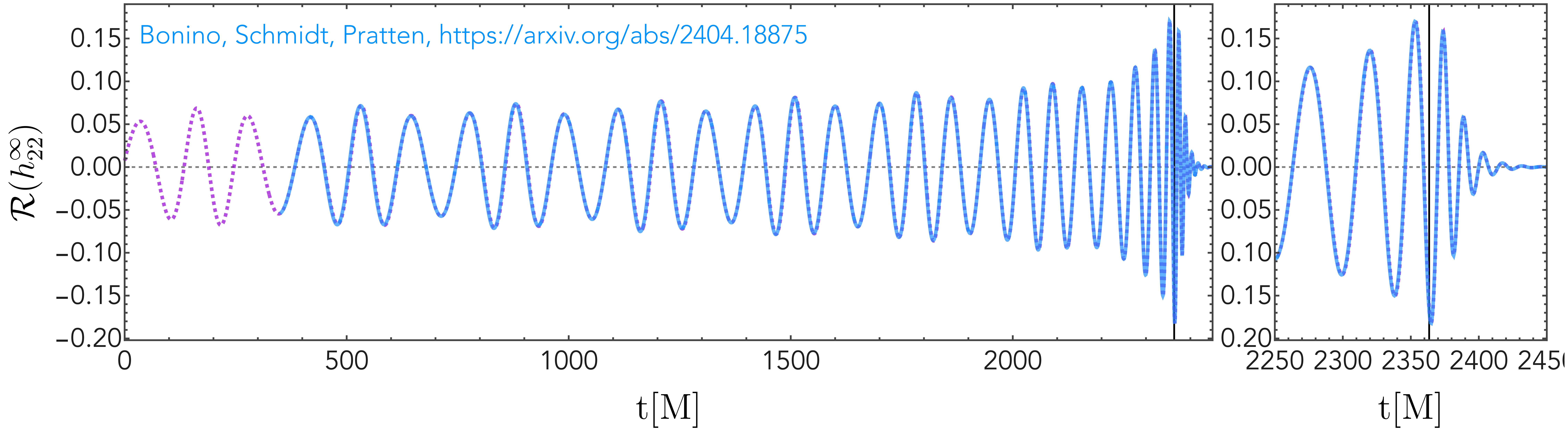

Comparison of TEOBResumS-DALI against NR
[Albertini, Albanesi, Bernuzzi, Chiaramello, Gamba, Nagar, Placidi Rettegno, + many others]

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See talks by: Danilo Chiaramello, Jacob Lange, Giulia Fumagalli

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See talk by Aldo Gamboa

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High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour

Phys. Rev. D **97**, 044038 – Published 26 February 2018

Article | References | Citing Articles (208) | PDF | HTML | Export Citation

ABSTRACT

A technique for translating the classical scattering function of two gravitationally interacting bodies into a corresponding (effective one-body) Hamiltonian description has been recently introduced [Phys. Rev. D **94**, 104015 (2016)]. Using this technique, we derive, for the first time, to second-order in Newton's constant (i.e. one classical loop) the Hamiltonian of two point masses having an arbitrary (possibly relativistic) relative velocity. The resulting (second post-Minkowskian) Hamiltonian is found to have a tame high-energy structure which we relate both to gravitational self-force studies of large mass-ratio binary systems, and to the ultra high-energy quantum scattering results of Amati, Ciafaloni and Veneziano. We derive several consequences of our second post-Minkowskian Hamiltonian: (i) the need to use special phase-space gauges to get a tame high-energy limit; and (ii) predictions about a (rest-mass independent) linear Regge trajectory behavior of high-angular-momenta, high-energy circular orbits. Ways of testing these predictions by dedicated numerical simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the two-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian.

https://arxiv.org/abs/1710.10599

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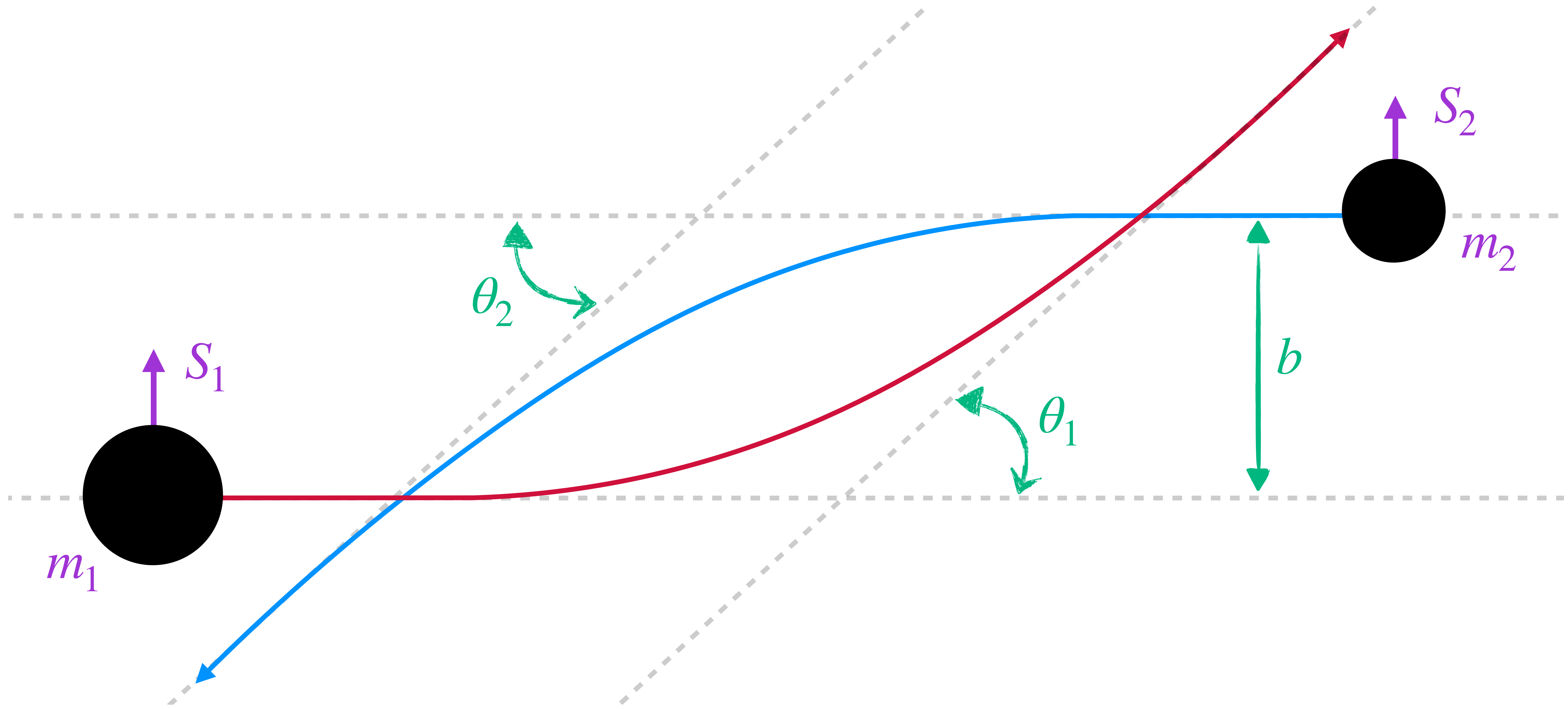

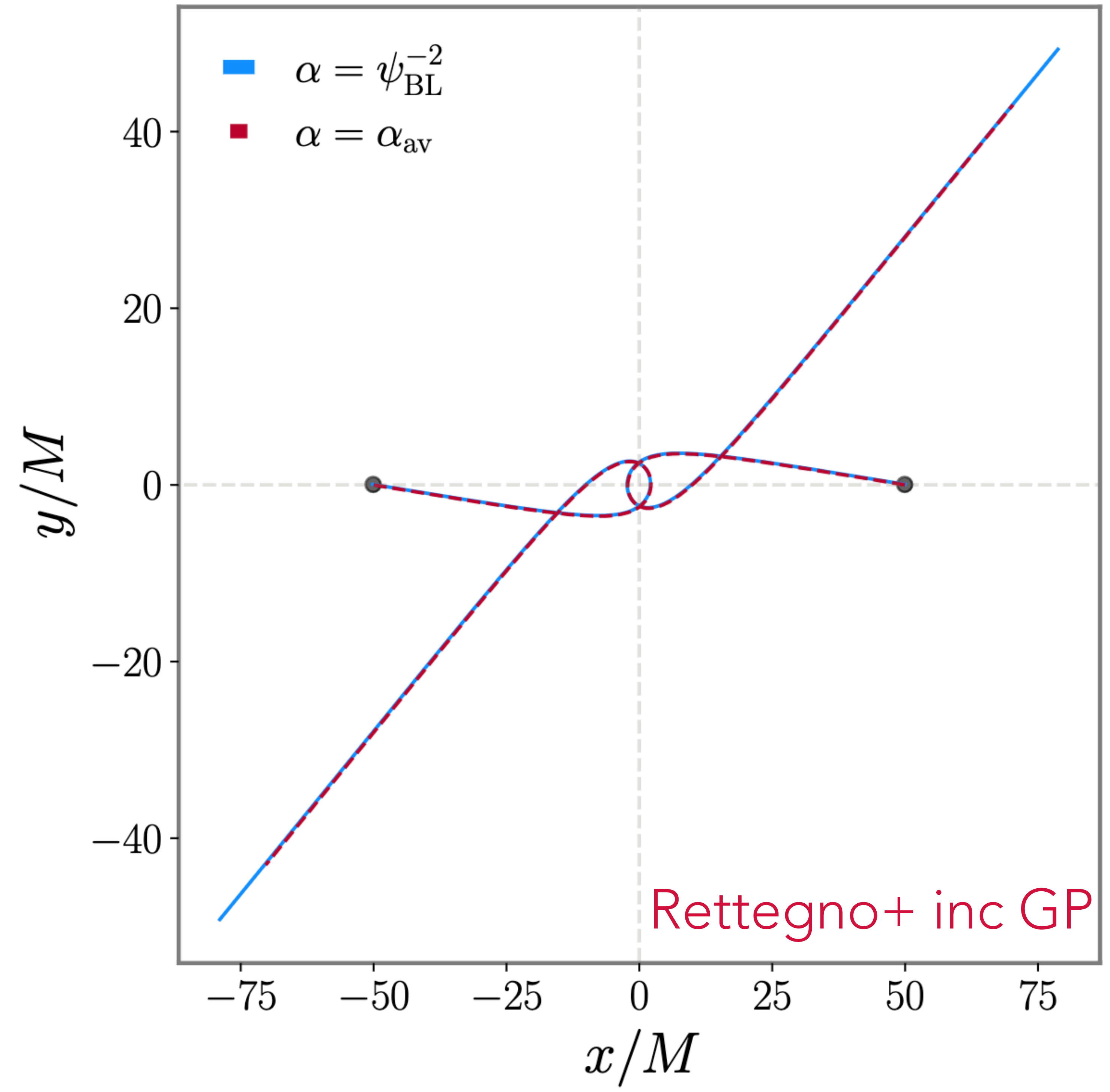

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A number of talks this conference:
Piero Rettegno, Simone Albanesi, Matteo Sergola, Saulo Soares de Albuquerque Filho