Correlated D decays at the $\Psi(3770)$

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We update the calculations of correlated $D^0 - \overline{D}^0$ decay rates presented at the Elba Collaboration Meeting. In particular, we present the equations for

- $(K^-\pi^+\pi^0, K^\mp e^\pm \nu)$, $(K^-\pi^+\pi^0, K^-K^+)$, $(K^-\pi^+\pi^0, K^-\pi^+)$
- $(K_S^0\pi^-\pi^+, K^\mp e^\pm \nu)$, $(K_S^0\pi^-\pi^+, K^-K^+)$, $(K_S^0\pi^-\pi^+, K^-\pi^+)$
- $(K^-\pi^+\pi^0, K^-\pi^+\pi^0)$
- $(K_S^0\pi^-\pi^+, K_S^0\pi^-\pi^+)$

We also note that

- the equations for $K_L^0\pi^-\pi^+$ in place of $K_S^0\pi^-\pi^+$ have exactly the same form. When observed in conjunction with fully reconstructed final states, CLEO observes more than twice as many $K_L^0\pi^-\pi^+$ decays as $K_S^0\pi^-\pi^+$ decays, so SuperB should benefit from these channels similarly.
- the formalism for correlated $K^-\pi^-\pi^+\pi^+$ decays is algebraically the same as that for $K^-\pi^+\pi^0$ decays except that the phase space is 5-dimensional rather than 2-dimensional

Forms of $\mathcal{M}$ and $|\mathcal{M}|^2$

For the Elba meeting we derived the following equations:

$$2\sqrt{2}\,\mathcal{M} = \left(\frac{q}{p}\overline{\mathcal{A}}_\alpha\overline{\mathcal{A}}_\beta - \frac{p}{q}\mathcal{A}_\alpha\mathcal{A}_\beta\right)[e_1(t_1)e_2(t_2) - e_1(t_2)e_2(t_1)] \quad + \quad (\mathcal{A}_\alpha\overline{\mathcal{A}}_\beta - \overline{\mathcal{A}}_\alpha\mathcal{A}_\beta)[e_1(t_1)e_2(t_2) + e_1(t_2)e_2(t_1)] \tag{1}$$

which has the form

$$2\sqrt{2}\,\mathcal{M} = X(e_{11}e_{22} - e_{12}e_{21}) \; + \; Y(e_{11}e_{22} + e_{12}e_{21})\,. \tag{2}$$

From this one calculates

$$8|\mathcal{M}|^2 = e^{-\Gamma(t_1+t_2)} \times \{ \quad XX^*(\cosh y\Gamma\Delta t - \cos x\Gamma\Delta t) - \quad 2\,\Re(XY^*)\ \sinh y\Gamma\Delta t + \quad 2\,\Im(XY^*)\ \sin x\Gamma\Delta t + \quad YY^*(\cosh y\Gamma\Delta t + \cos x\Gamma\Delta t\ \} \tag{3}$$

For $x\Gamma\Delta t$, $y\Gamma\Delta t \ll 1$ this can be approximated by

$$4|\mathcal{M}|^2 = e^{-\Gamma(t_1+t_2)} \times \left\{ \quad XX^*\left[\frac{(x^2+y^2)}{4}(\Gamma\Delta t)^2\right] - \quad \Re(XY^*)\,y\Gamma\Delta t + \Im(XY^*)\,x\Gamma\Delta t + \quad YY^*\left[1 + \frac{(y^2-x^2)}{4}(\Gamma\Delta t)^2\right]\right\} \tag{4}$$

- Y is the unmixed amplitude
- X is the mixing amplitude
- XY^* controls the interference terms in the mixing rate

Summary of Results Presented at Elba Meeting

We have made rough estimates of Super$B$ sensitivity to mixing assuming

- events rates scale from CLEO-c,
- Super$B$ integrated $\mathcal{L} = 500\,\mathrm{fb}^{-1}$,
- we can cleanly separate $|\Gamma\Delta t| > 2$ from $|\Gamma\Delta t| < 2$,
- $p/q \approx 1$,
- $y \approx 0.01$

channel	type of measurement	figure of merit
K^-K^+, $\pi^-\pi^+$ v K^-K^+, $\pi^-\pi^+$	integrated	$\|q/p - p/q\|^2 \times 6$ events
$K^-\pi^+$ v $K^-\pi^+$ + cc	integrated	46 events
$K^-e^+\nu$ v $K^-e^+\nu$ + cc	integrated	46 events
$K^-e^+\nu$ v K^-K^+, $\pi^-\pi^+$ + cc	TDA	1887 ± 247 events ($\sim 7\sigma$)
$K^-e^+\nu$ v $K^-\pi^+$, + cc	TDA	800 ± 40 events ($\sim 20\sigma$)
$K^-\pi^+$ v K^-K^+, $\pi^-\pi^+$ + cc	integrated	$\cos\delta_{K\pi} \sim \pm 2\%$
$K^-\pi^+$ v K^-K^+, $\pi^-\pi^+$ + cc	TDA	650 ± 156 events ($\sim 4\sigma$)

Sensitivity to mixing (and CP violation) is greatest when the interference term is as large as possible compared to the direct correlated decay term. This requires "same-sign" decays with a DCS amplitude interfering with a CF amplitude.

$(K^-\ell^+X, K^-\pi^+\pi^0)$

With the notation

$$
\begin{aligned}
\mathcal{A}(D^0 \to K^-\pi^+\pi^0) &= A_r\zeta(s_{12}, s_{13}) \\
\overline{\mathcal{A}}(\overline{D}^0 \to K^-\pi^+\pi^0) &= \overline{A}_r\overline{\zeta}(s_{12}, s_{13}) = k e^{i\delta_{K\pi\pi^0}} A_r\overline{\zeta}(s_{12}, s_{13})
\end{aligned}
\tag{5}
$$

The (small $y\Gamma\Delta t$, small $x\Gamma\Delta t$) limit for the $(K^-\ell^+X, K^-\pi^+\pi^0)$ decay rate is

$$
\begin{aligned}
|\mathcal{M}|^2 \quad &= \frac{1}{4} e^{-\Gamma(t_1+t_2)} |A_r|^2 |\mathcal{A}_\beta|^2 \times \\
&\left\{ \left|\frac{p}{q}\right|^2 \zeta(s_{12}, s_{13})\, \zeta^*(s_{12}, s_{13}) \left(\frac{x^2+y^2}{4}\right) (\Gamma\Delta t)^2 \right. \\
&- \left[\Re\left(\frac{p}{q}\, \zeta(s_{12}, s_{13})\overline{\zeta}^*(s_{12}, s_{13})\right) \cos\delta_{K\pi\pi^0} \right. \\
&\left. + \Im\left(\frac{p}{q}\zeta(s_{12}, s_{13})\overline{\zeta}^*(s_{12}, s_{13}))\right) \sin\delta_{K\pi\pi^0} \right] k\, y\Gamma\Delta t \\
&+ \left[\Im\left(\frac{p}{q}\zeta(s_{12}, s_{13})\overline{\zeta}^*(s_{12}, s_{13})\right) \cos\delta_{K\pi\pi^0} \right. \\
&\left. - \Re\left(\frac{p}{q}\zeta(s_{12}, s_{13})\overline{\zeta}^*(s_{12}, s_{13})\right) \sin\delta_{K\pi\pi^0} \right] k\, x\Gamma\Delta t \\
&\left. + k^2\, \overline{\zeta}(s_{12}, s_{13})\, \overline{\zeta}^*(s_{12}, s_{13}) \left[1 + \left(\frac{y^2-x^2}{4}\right)(\Gamma\Delta t)^2 \right] \right\} .
\end{aligned}
\tag{6}
$$

As a first approximation, the time-integrated rate is dominated by the doubly-Cabibbo suppressed rate associated with YY^*. To a lesser degree, the pure mixing rate propotional to the Cabibbo favored rate, XX^*, also contributes.

$(K^-K^+, K^-\pi^+\pi^0)$

In this case, the time-integrated rate will be dominated by

$$YY^* \ = \zeta\zeta^* + k^2\overline{\zeta\zeta}^* - 2k\left[\Re(\zeta\overline{\zeta}^*)\ \cos\delta_{K\pi\pi^0} + \Im(\zeta\overline{\zeta}^*)\ \sin\delta_{K\pi\pi^0}\right]\ . \tag{7}$$

The time-odd rate will depend on the real and imaginary parts of

$$\begin{aligned} XY^* \ &= \ -\frac{q}{p}k^2\overline{\zeta\zeta}^* - \frac{p}{q}\zeta\zeta^* \\ &+ \ k\left[\frac{q}{p}e^{i\delta_{K\pi\pi^0}}\overline{\zeta}\zeta^* + \frac{p}{q}e^{i\delta_{K\pi\pi^0}}\zeta\overline{\zeta}^*\right] \end{aligned} \tag{8}$$

In the limit $p/q = 1$,

$$XY^* \to -k^2\overline{\zeta\zeta}^* - \zeta\zeta^* + 2k\left[\Re(\zeta\overline{\zeta}^*)\cos\delta_{K\pi\pi^0} + \Im(\zeta\overline{\zeta}^*)\sin\delta_{K\pi\pi^0}\right] \tag{9}$$

which is purely real and equal in magnitude to YY^*. In this limit, the time-odd part of the rate is proportional only to $y\Gamma\Delta t$ and is independent of x.

$(K^-\pi^+, K^-\pi^+\pi^0)$

Here we will write

$$\begin{aligned}
\mathcal{A}_\alpha &= \mathcal{A}(D^0 \to K^-\pi^+) \\
\overline{\mathcal{A}}_\alpha &= k_1 e^{i\delta_1}\mathcal{A}_\alpha \\
\mathcal{A}_\beta &= \mathcal{A}(D^0 \to K^-\pi^+\pi^0) = A_r\zeta \\
\overline{\mathcal{A}}_\beta &= k^2 e^{i\delta_2}\, A_r\,\overline{\zeta}
\end{aligned} \tag{10}$$

so that

$$\begin{aligned}
X &= \left(\frac{q}{p}\, k_1 k_2\, e^{i(\delta_1+\delta_2)}\,\overline{\zeta} \;-\; \frac{p}{q}\,\zeta\right)\, A_r\mathcal{A}_\beta \\
Y &= \left(k_2\, e^{i\delta_2}\,\overline{\zeta} - k_1\, e^{i\delta_1}\,\zeta\right)\, A_r\mathcal{A}_\beta\,.
\end{aligned} \tag{11}$$

It follows that

$$\begin{aligned}
YY^* &= k_2^2\,\overline{\zeta}\overline{\zeta}^* + k_1^2\zeta\zeta^* \\
&\quad -2\,k_1k_2\left[\Re(\zeta\overline{\zeta}^*)\;\cos(\delta_1-\delta_2) - \Im(\zeta\overline{\zeta}^*)\;\sin(\delta_1-\delta_2)\right]\,|\mathcal{A}_\alpha|^2\,|A_r|^2\,,
\end{aligned} \tag{12}$$

and as a good first approximation,

$$\begin{aligned}
XY^* &= \frac{p}{q}\,\Big\{\,\Big[\Re(\zeta\overline{\zeta}^*)\,(k_1\cos\delta_1 - k_2\cos\delta_2) \\
&+\; \Im(\zeta\overline{\zeta}^*)(-k_1\sin\delta_1 + k_2\sin\delta_2)\Big] \\
&+\; i\,\Big[\Im(\zeta\overline{\zeta}^*)\,(k_1\cos\delta_1 + k_2\cos\delta_2) \\
&+\; \Re(\zeta\overline{\zeta}^*)(k_1\sin\delta_1 + k_2\sin\delta_2)\Big]\,\Big\}\,|\mathcal{A}_\alpha|^2\,|A_r|^2
\end{aligned} \tag{13}$$

Some Notation for $D^0 \to K^0_S\pi^-\pi^+$

At first sight, $K^0_S\pi^-\pi^+$, appears to be similar to $K^-\pi^+\pi^0$ as both are three-body decays whose amplitudes are often described using isobar models. However, in the limit of no direct CP violation in D decay, and ignoring the known CP violation in K^0_S decay, we can exploit the relationship

$$\mathcal{A}(\overline{D}^0 \to K^0_S\pi^-\pi^+)(s_{12},\, s_{13}) = \mathcal{A}(D^0 \to K^0_S\pi^-\pi^+)(s_{13},\, s_{12}) \tag{14}$$

Using the notation

$$\mathcal{A}(D^0 \to K^0_S\pi^-\pi^+)(s_{13},\, s_{12}) = A_r\zeta(s_{12}, s_{13}) \tag{15}$$

and assuming no direct CP violation, we have

$$\mathcal{A}_\alpha = A_r\zeta(s_{12}, s_{13})\,; \quad \overline{\mathcal{A}}_\alpha = A_r\zeta(s_{13}, s_{12})\,; \quad \overline{\mathcal{A}}_\beta = \mathcal{A}_\beta \tag{16}$$

It is sometimes useful to re-write $\zeta(s_{12}, s_{13})$ and $\zeta(s_{13}, s_{12})$ in terms of symmetric and antisymmetric functions

$$\begin{aligned} \zeta_S(s_{13}, s_{12}) &= \frac{1}{2}\left[\zeta(s_{12}, s_{13}) + \zeta(s_{13}, s_{12})\right] \\ \zeta_A(s_{13}, s_{12}) &= \frac{1}{2}\left[\zeta(s_{12}, s_{13}) - \zeta(s_{13}, s_{12})\right] \end{aligned} \tag{17}$$

so that

$$\begin{aligned} \zeta(s_{12}, s_{13}) &= \zeta_S(s_{13}, s_{12}) + \zeta_A(s_{13}, s_{12}) \\ \zeta(s_{13}, s_{12}) &= \zeta_S(s_{13}, s_{12}) - \zeta_A(s_{13}, s_{12})\,. \end{aligned} \tag{18}$$

Correlated $(K^-\ell^+\nu, K^0_S\pi^-\pi^+)$ decays

With the notation introduced for $\mathcal{A}_\alpha = \mathcal{A}(D^0 \to K^0_S\pi^-\pi^+)$,

$$
\begin{aligned}
X &= -\frac{p}{q}(\zeta_S + \zeta_A) A_r \mathcal{A}_\beta \\
Y &= -(\zeta_S - \zeta_A) A_r \mathcal{A}_\beta
\end{aligned} \tag{19}
$$

which gives

$$
\begin{aligned}
Y Y^* &= (2\,\zeta_S\,\zeta_S^* + 2\,\zeta_A\,\zeta_A^* - \zeta\,\zeta^*)\,|A_r|^2\,|\mathcal{A}_\beta|^2 \\
X Y^* &= \frac{p}{q}\left[\zeta_S\,\zeta_S^* - \zeta_A\,\zeta_A^* - 2\,i\,\Im(\zeta_S\,\zeta_A^*)\right]\,|A_r|^2\,|\mathcal{A}_\beta|^2
\end{aligned} \tag{20}
$$

so that

$$
\begin{aligned}
\Re(XY^*) &= \left[\Re\left(\frac{p}{q}\right)(\zeta_S\,\zeta_S^* - \zeta_A\,\zeta_A^*) + 2\,\Im\left(\frac{p}{q}\right)\Im(\zeta_S\,\zeta_A^*)\right]\,|A_r|^2\,|\mathcal{A}_\beta|^2 \\
\Im(XY^*) &= \left[-2\,\Re\left(\frac{p}{q}\right)\,\Im(\zeta_S\,\zeta_A^*) + \Im\left(\frac{p}{q}\right)\,(\zeta_S\,\zeta_S^* - \zeta_A\,\zeta_A^*)\right]\,|A_r|^2\,|\mathcal{A}_\beta|^2
\end{aligned} \tag{21}
$$

Correlated $(\boldsymbol{K^-K^+, K_S^0\pi^-\pi^+})$ decays

As usual, the time integrated rate is dominated by

$$\boldsymbol{YY^*} = 4\,\zeta_A(\boldsymbol{s_{12}, s_{13}})\zeta_A^*(\boldsymbol{s_{12}, s_{13}})\,|\boldsymbol{A_r}|^2\,|\boldsymbol{\mathcal{A}}|^2 \tag{22}$$

which we can identify as the antisymmetric rate. Were we to consider $K_S^0\pi^-\pi^+$ produced in conjunction with a pure CP odd eigenstate rather than CP even, YY^* would be the symmetric rate instead. The time-odd rates are proportional to the real and imaginary parts of XY^* which is

$$\boldsymbol{XY^*} = \boldsymbol{2}\left[\zeta_S\,\zeta_A^*\left(\frac{\boldsymbol{p}}{\boldsymbol{q}} - \frac{\boldsymbol{q}}{\boldsymbol{p}}\right) + \zeta_A\,\zeta_A^*\left(\frac{\boldsymbol{p}}{\boldsymbol{q}} + \frac{\boldsymbol{q}}{\boldsymbol{p}}\right)\right]\,|\boldsymbol{A_r}|^2\,|\boldsymbol{\mathcal{A}}|^2\,. \tag{23}$$

In the limit $p = q$, $XY^* \to (1/2)\,YY^*$. Were we to consider $K_S^0\pi^-\pi^+$ produced in conjunction with a pure CP odd eigenstate rather than CP even, XY^* becomes

$$\boldsymbol{XY^*} = \boldsymbol{2}\left[(\zeta_S\,\zeta_A^*)^*\left(\frac{\boldsymbol{p}}{\boldsymbol{q}} - \frac{\boldsymbol{q}}{\boldsymbol{p}}\right) + \zeta_S\,\zeta_S^*\left(\frac{\boldsymbol{p}}{\boldsymbol{q}} + \frac{\boldsymbol{q}}{\boldsymbol{p}}\right)\right]\,|\boldsymbol{A_r}|^2\,|\boldsymbol{\mathcal{A}}|^2\,. \tag{24}$$

The roles of ζ_S and ζ_A are interchanged. If $p \neq q$ the $\Re(\zeta_S\,\zeta_A^*) = \Re(\zeta_S\,\zeta_A^*)^*$ but the $\Im(\zeta_S\,\zeta_A^*) = -\Im(\zeta_S\,\zeta_A^*)^*$ so the time-odd asymmetries will differ and the difference of the two as a function of position in the Dalitz plot will provide additional senstivity to the real and imaginary parts of p/q.

Correlated $(K^-\pi^+, K^0_S\pi^-\pi^+)$ decays

With the same type of notation as used earlier,

$$\begin{aligned} YY^* &= \zeta'\,\zeta'^* + k^2\,\zeta\,\zeta^* - 2\,k\,e^{i\delta}\,\Re(\zeta\,\zeta'^*) \\ &= \zeta'\,\zeta'^* + k^2\,\zeta\,\zeta^* - 2k\,\cos\delta\,\Re(\zeta\,\zeta'^*) + 2k\,\sin\delta\,\Im(\zeta\,\zeta'^*)\,. \end{aligned} \tag{25}$$

We can identify the real and imaginary parts of $\zeta\,\zeta'^*$ with ζ_S and ζ_A writing

$$\zeta\,\zeta'^* = \zeta_S\,\zeta_S^* - \zeta_A\,\zeta_A^* - 2i\Im(\zeta_S\,\zeta_A^*) \tag{26}$$

from which we find

$$\Re(\zeta\,\zeta'^*) = \zeta_S\,\zeta_S^* - \zeta_A\,\zeta_A^*\,; \quad \Im(\zeta\,\zeta'^*) = -2\Im(\zeta_S\,\zeta_A^*)\,. \tag{27}$$

This gives

$$YY^* = \zeta'\,\zeta'^* + k^2\,\zeta\,\zeta^* - 2k\,\cos\delta\,(\zeta_S\,\zeta_S^* - \zeta_A\,\zeta_A^*) - 4k\,\sin\delta\,\Im(\zeta_S\,\zeta_A^*)\,. \tag{28}$$

The time-odd rate is proportional the real and imaginary parts of

$$\begin{aligned} XY^* &= \frac{q}{p}\,k^2\,(\zeta\,\zeta'^*)^* + \frac{p}{q}(\zeta\,\zeta'^*) \\ &+ k\left[\frac{q}{p}\,e^{i\delta}\,(\zeta'\,\zeta'^*) - \frac{p}{q}\,e^{-i\delta}\,(\zeta\,\zeta^*)\right] \end{aligned} \tag{29}$$

In the limit $p/q \to 1$, these become

$$\begin{aligned} \Re(XY^*) &= (1+k^2)\,[\zeta_S\zeta_S^* - \zeta_A\,\zeta_A^*] + \cos\delta\,[\zeta\zeta^* - k\,\zeta'\zeta'^*] \\ \Im(XY^*) &= \sin\delta\,[\zeta\,\zeta^* + k\,\zeta'\zeta'^*] - (1-k^2)\,\Im(\zeta_S\zeta_A^*) \end{aligned} \tag{30}$$

Correlated $(K^-\pi^+\pi^0, K^-\pi^+\pi^0)$ decays

s correlated final state we will use the notation

$$\begin{aligned}
\mathcal{A}_\alpha(s_{12}, s_{13}) &= A_r \zeta(s_{12}, s_{13}) \qquad (31)\\
\overline{\mathcal{A}}_\alpha(s_{12}, s_{13}) &= A_r \overline{\zeta}(s_{12}, s_{13}) = \kappa(s_{12}, s_{13})\, e^{i\epsilon(s_{12},s_{13})}\, A_r \zeta(s_{12}, s_{13})\\
\mathcal{A}_\beta(s'_{12}, s'_{13}) &= A_r \zeta'(s'_{12}, s'_{13})\\
\overline{\mathcal{A}}_\beta(s'_{12}, s'_{13}) &= A_r \overline{\zeta}'(s'_{12}, s'_{13}) = \kappa'(s'_{12}, s'_{13})\, e^{i\epsilon(s'_{12},s'_{13})}\, A_r \zeta'(s'_{12}, s'_{13})\,.
\end{aligned}$$

The real functions κ, κ', ϵ, and ϵ' are chosen so that κ and κ' are positive definite.

$$YY^* = \zeta\,\zeta^*\,\overline{\zeta}'\,\overline{\zeta}'^* + \overline{\zeta}\,\overline{\zeta}^*\,\zeta'\,\zeta'^* + 2\Re\left[(\zeta\,\overline{\zeta}^*\,)\,(\overline{\zeta}'\,\zeta'^*)\right]|A_r|^4\,. \qquad (32)$$

The first two terms are the products of the Cabibbo favored rate for one decay and the doubly-Cabibbo suppressed rate for the other. The last term is the product of two Cabibbo favored, doubly-Cabibbo suppressed interference rates. As a good approximation, we can find calculate the interference term ignoring the doubly-Cabibbo suppressed term in X:

$$XY^* \approx \frac{p}{q}\left[(\zeta\,\zeta^*)\,(\zeta'\,\overline{\zeta}'^*) - (\zeta\,\overline{\zeta}^*)\,(\zeta'\,\zeta'^*)\right]|A_r|^4\,. \qquad (33)$$

Here, each term is the product of a Cabibbo favored rate for one decay and the interference of amplitudes for the other. Events will populate a four-dimensional phase space corresponding to the two Dalitz plot positions (s_{12}, s_{13}) and (s'_{12}, s'_{13}). Furthermore, this interference term is antisymmetric under the interchange of the ζ and ζ'. This is evident algebraically from the form of Eqn. (33). Physically, it corresponds to identifying one or the other Dalitz plot position as that of the first D to decay. As the interference rate is time-odd, XY^* must be antisymmetric when the two Dalitz plot positions are interchanged.

Correlated $(K^-\pi^+\pi^0, K^-\pi^+\pi^0)$ decays - II

We can also expand the interference term using the functions κ, κ', ϵ, and ϵ' defined in Eqn. (31):

$$XY^* \;=\; \frac{p}{q}(\zeta\,\zeta^*)\,(\zeta'\,\zeta'^*)\;\left[(\kappa'\cos\epsilon' - \kappa\cos\epsilon)\;\; - i(\kappa'\sin\epsilon' - \kappa\sin\epsilon)\right] \tag{34}$$

which manifests the (anti)symmetry very explicitly. In the band of the Dalitz plot where the DCS amplitude is dominant, $\kappa^{(\prime)}$ will be large while in the band of the Dalitz plot where the CF amplitude is large it will be small. The magnitude of XY^* can be large if either, (a) one of the Dalitz plot points is in a DCS dominated region and the other is in a CF dominated dominated region, or (b) both Dalitz points are in DCS dominated regions but the phases differ significantly. In the same way, we can write

$$YY^* \;=\; \zeta\,\zeta^*\,\overline{\zeta}'\,\overline{\zeta}'^* + \overline{\zeta}\,\overline{\zeta}^*\,\zeta'\,\zeta'^* + 2\kappa\kappa'(\zeta\,\zeta^*)\,(\zeta'\,\zeta'^*)\;\cos(\epsilon' - \epsilon)\,. \tag{35}$$

To extract the mixing parameters x and y from the data in this channel, we can divide the 4-dimensional phase space of the two Dalitz plot positions into a finite number of bins according to the real and imaginary parts of XY^* and fit the time difference asymmetries in these bins. The statistical significance of the assymmetry will also depend inversely on the total expected rate, so we should use this as part of the metric for creating bins.

Correlated $K^0_S\pi^-\pi^+$, $K^0_S\pi^-\pi^+$ decays

Here, we use notation here more similar to that used for $K^-\pi^+\pi^0$, $K^-\pi^+\pi^0$ than for $K^0_S\pi^-\pi^+$, $K^-\pi^+$:

$$
\begin{aligned}
\mathcal{A}_\alpha(s_{12}, s_{13}) &= \zeta(s_{12}, s_{13})\, A_r &= (\zeta_S + \zeta_A)\, A_r \\
\overline{\mathcal{A}}_\alpha(s_{12}, s_{13}) &= \overline{\zeta}(s_{13}, s_{12})\, A_r &= (\zeta_S - \zeta_A)\, A_r \\
\mathcal{A}_\beta(s'_{12}, s'_{13}) &= \zeta(s'_{12}, s'_{13})\, A_r &= (\zeta'_S + \zeta'_A)\, A_r \\
\overline{\mathcal{A}}_\beta(s'_{12}, s'_{13}) &= \overline{\zeta}(s'_{13}, s'_{12})\, A_r &= (\zeta'_S - \zeta'_A)\, A_r
\end{aligned}
\tag{36}
$$

The superscript $'$ distinguishes the amplitudes associated with the two Dalitz plot positions of the $K^0_S\pi^-\pi^+$ decays rather than the amplitudes associated with direct D^0 and $\overline{D}^0$ decay. With this notation

$$
\begin{aligned}
YY^* = 4\, &\Big\{\zeta_A\zeta_A^*\,\zeta'_S\zeta'^*_S + \zeta_S\zeta_S^*\,\zeta'_A\zeta'^*_A \\
&-2\,\Big[\Re(\zeta_S\zeta_A^*)\,\Re(\zeta'_S\zeta'^*_A) + \Im(\zeta_S\zeta_A^*)\,\Im(\zeta'_S\zeta'^*_A)\Big]\Big\}
\end{aligned}
\tag{37}
$$

In the limit $p = q$, the mixing amplitude becomes

$$
\begin{aligned}
X &= (\zeta_S - \zeta_A)\,(\zeta'_S - \zeta'_A) - (\zeta_S + \zeta_A)\,(\zeta'_S + \zeta'_A) \\
&= -2\,[\zeta_A\,\zeta'_S + \zeta_S\,\zeta'_A]
\end{aligned}
\tag{38}
$$

in which case

$$
\begin{aligned}
XY^* &= -4\,(\zeta_A\,\zeta'_S + \zeta_S\,\zeta'_A)\,\,(\zeta_A^*\,\zeta'^*_S - \zeta_S\zeta'^*_A) \\
&= -4\Big[\zeta_A\,\zeta_A^*\,\zeta'_S\,\zeta'^*_S - \zeta_S\,\zeta_S^*\,\zeta'_A\,\zeta'^*_A + 2\,i\,\Im(\zeta_S\zeta_A^*\zeta'_A\zeta'^*_S)\Big] \\
&= -4\Big[\zeta_A\,\zeta_A^*\,\zeta'_S\,\zeta'^*_S - \zeta_S\,\zeta_S^*\,\zeta'_A\,\zeta'^*_A \\
&\qquad +2\,i\,\Big(-\Re(\zeta_S\,\zeta_A^*)\,\Im(\zeta'_S\,\zeta'^*_A) + \Re(\zeta'_S\,\zeta'^*_A)\,\Im(\zeta_S\,\zeta_A^*)\Big)\Big]
\end{aligned}
\tag{39}
$$

Summary

In the limit of no CP violation in mixing ($p/q = 1$),

- The formulas presented at the Elba meeting and here allow one to write correlated events generators for the modes studied. These almost certainly include the most important modes for studying mixing and CP violation in mixing (*i.e.*, $p/q \neq 1$). This will allow us to determine efficiency and Δt resolution for possible machine and detector configurations ($\beta\gamma$, B-field strength, etc.).
- The rates associated with direct decays can be extracted from data independently of models using time-integrated measurements. Mixing perturbs these determinations at the 10^{-4} level if one integrates over all decay times, and even less if one integrates over limited ranges of Δt.
- Relative (strong interaction) phases between Cabibbo favored and doubly-Cabibbo suppressed decays to the same final states can be extracted from time-integrated measurements.
- The interaction rate terms can generally be extracted from data independently of models using a multiplicity of time-integrated measurements if relative strong phases are kind to us. If not, a multiplicity of time-dependent measurements allows these rates as well as the mixing parameters to be extracted with no need for an model.
- Numerical studies using the correlated formulas can be made without full Monte Carlo simulation to determine nominal sensitivities to mixing and CP violation in mixing.