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Grand Unified Theories:
A home to the axion?

Grand Unified Theories: a home to the axion?

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AxionOrigins: towards a complete theory for the origin of the axion

INFN, Laboratori Nazionali di Frascati

2024-01-25

Outline

- Motivation: what are GUTs and why consider them?

- Model building: possibilities and challenges
  - Choice of gauge group
  - Unification of gauge couplings
  - Symmetry breaking
  - Yukawa sector

- Some phenomenology [neutrinos, proton decay]

- Axion in GUTs

Motivation 1 — Unification of gauge couplings?

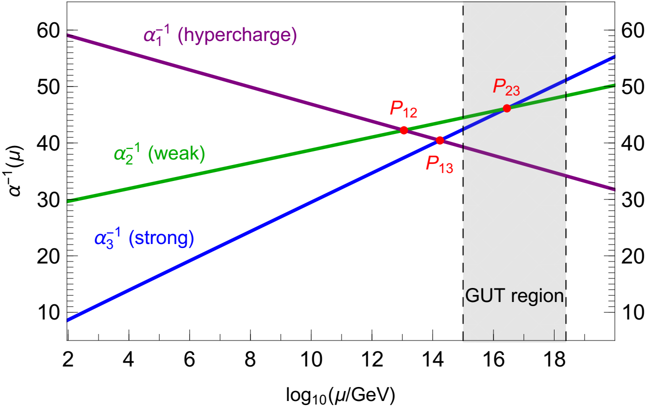
- RG running of gauge couplings in the Standard Model (SM):

• Proton decay ($\approx$ exp. limit):

$\mu_{\text{p-decay}} = 10^{15}\,\mathrm{GeV}$

• Gravity (*reduced Planck scale*):

$\mu_{\text{Pl}} = 2.4 \cdot 10^{18}\,\mathrm{GeV}$



- Do the SM forces unify in one point at high energy M_{GUT}?

  $\rightarrow$ GUT region: $\mu_{\text{p-decay}} < M_{\text{GUT}} < \mu_{\text{Pl}}$

  $\longrightarrow$ Consistent with picture (non-trivial indication!)

Motivation 2 — What is a GUT?

- **Grand Unified Theory** (GUT): a Yang-Mills theory with a *simple* gauge factor G, such that

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset G \tag{1}$$

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- Features of the GUT framework:
  - (a) All 3 SM gauge couplings unify at high energy scale in group G
  - (b) *Spontaneous symmetry breaking*: in one or multiple stages

$$G \to \ldots G_i \ldots \to G_{\mathrm{SM}} \equiv \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \tag{2}$$

  - (c) SM (in detail) should be recovered at low (EW) scale

Motivation 3 —Why GUT?

- Historical proposal [1974]: Georgi-Glashow $\mathrm{SU}(5)$ model

$$\text{fermions}: \quad 3 \times (10 \oplus \bar{5}) \tag{3}$$

$$\text{scalars}: \quad 24 \oplus 5 \tag{4}$$

→ Predictions at M_{GUT}: $\sin^2\theta_W = 3/8$, $\quad M_D = M_E^T$

→ ruled out long ago (proton decay, Yukawa fit)

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- Generic benefits of GUTs:

1. Unification of gauge interactions
2. Matter unification (at least partial)
3. Explain (hyper)charge quantization
4. Conceptually nicer/simpler than SM
5. Experimental sensitivity to super-high energies (e.g. proton decay)

Model building 1 — possibilities for unified group

- Requirements for a unified gauge group G:
  1. G is simple
  2. $G \supset G_{\mathrm{SM}}$
  3. G has complex representations (since SM is chiral)

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root system	name	comment	$\mathbb{C}$-irreps?
A_n	$\mathrm{SU}(n+1)$	rotations in $\mathbb{C}^{n+1}$	all n
B_n	$\mathrm{SO}(2n+1)$	rotations in $\mathbb{R}^{2n+1}$	/
C_n	$\mathrm{Sp}(2n)$	rotations in $\mathbb{H}^n$	/
D_n	$\mathrm{SO}(2n)$	rotations in $\mathbb{R}^{2n}$	odd n
$\mathrm{E}_6, \mathrm{E}_7, \mathrm{E}_8, \mathrm{F}_4, \mathrm{G}_2$		exceptional	E_6

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- Satisfying requirements:
  - (a) $\mathrm{SU}(n)$, $n \geq 5$
  - (b) $\mathrm{SO}(4n+2)$, $n \geq 2$
  - (c) E_6
- Minimal choices: $G_{\mathrm{SM}} \subset \mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6$

Model building 2 — embedding the SM fermions

- Each generation in Standard Model $[\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y]$:

$$\begin{aligned} Q &\sim (3, 2, +\tfrac{1}{6}) & u^c &\sim (\bar{3}, 1, -\tfrac{2}{3}) & d^c &\sim (\bar{3}, 1, +\tfrac{1}{3}) \\ L &\sim (1, 2, -\tfrac{1}{2}) & e^c &\sim (1, 1, +1) \end{aligned} \tag{5}$$

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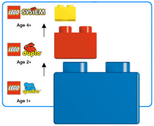

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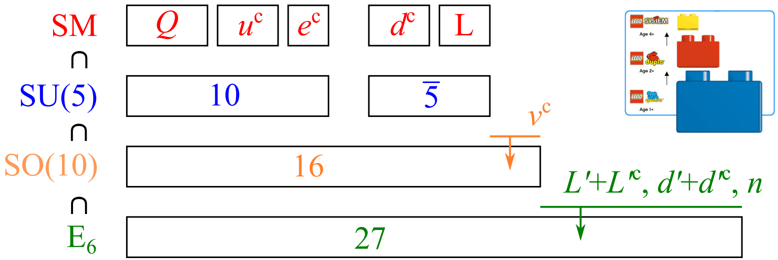

- SO(10): matter unification, R-neutrinos
- E_6: vector-like exotic fermions

Model building 3 — details of gauge coupling unification

- Coupling unification requires new states between M_{EW} and M_{GUT}

$$\tfrac{d}{dt}\,\alpha_i^{-1} = -\tfrac{1}{2\pi}\big(a_i + \tfrac{1}{4\pi} b_{ij}\alpha_j + \dots\big) \tag{6}$$

$$a_i = -\tfrac{11}{3} D_i(\mathcal{G}) + \tfrac{2}{3} D_i(\mathcal{F}) + \tfrac{1}{3} D_i(\mathcal{S}) \tag{7}$$

$\alpha_i := \frac{1}{4\pi} g_i^2$, D_i – Dynkin index;
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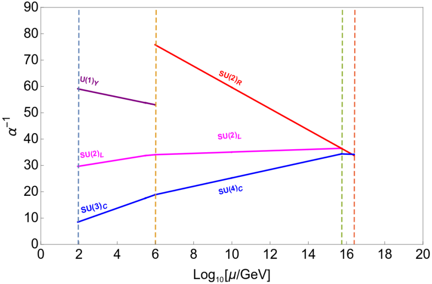

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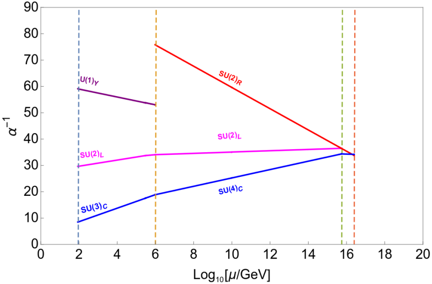

Properties:

- $\mathcal{G}$ push upward ↑
- $\mathcal{F}, \mathcal{S}$ push downward ↓
- $\alpha^{-1} = 0$: Landau pole
  limits matter content
- 1-loop: straight lines
- 2-loop: top-down viewpoint

- Typical addition of fields: multi-stage breaking or $\sim \mathrm{TeV}$ SUSY

Model building 4 — GUT symmetry breaking

- Usual mechanism for SSB $G \to \ldots \to G_{\mathrm{SM}}$: analogous to EW-symmetry breaking in SM (Higgs mechanism)
    - Symmetry broken by a VEV of the scalar field
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$$\text{example}: \quad \mathrm{SO}(10) \to 4_C\, 2_L\, 2_R \to 3_C\, 2_L\, 2_R\, 1_{B-L} \to G_{\mathrm{SM}} \tag{8}$$

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  - Independent of SUSY breaking (if SUSY is present)
- Which scalar irreps can acquire a SM-preserving VEV?
  $\to$ They must contain **SM-singlets**:

$$\mathrm{SU}(5): \quad 1, 5, 10, 15, 24, 35, 40, 45, 50, 70, 70', 75, \ldots \tag{9}$$

$$\mathrm{SO}(10): \quad 1, 10, 16, 45, 54, 120, 126, 144, 210, 210', \ldots \tag{10}$$

$$\mathrm{E}_6: \quad 1, 27, 78, 351, 351', 650, \ldots \tag{11}$$

Model building 5 — GUT symmetry breaking continued

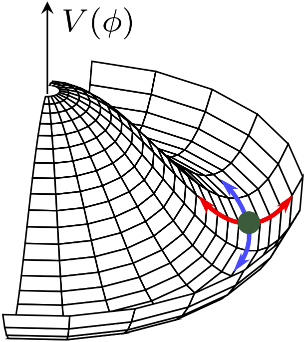
- Determining SSB for $G \to H$:

(a) Choose scalar representations ϕ of G

(b) Construct the most general scalar potential $V(\phi)$ using invariants of G

(c) Stationarity conditions (*singlet ansatz*): solve $\partial_\phi V = 0$ to get $\phi_0 = \langle \phi \rangle$

(d) Mass matrix $M = \partial^2_\phi V|_{\phi_0}$
→ positive mass: $\dim(\phi) - \dim(G/H)$
→ zero mass [WGBs]: $\dim(G/H)$



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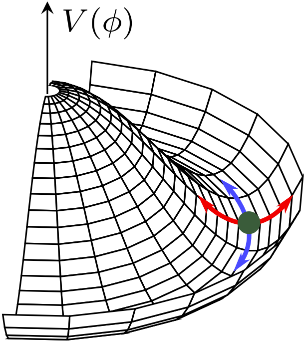

- Example: $\mathrm{SU}(5) \to G_{\mathrm{SM}}$ via $24 \sim \Sigma$

$$V(\Sigma) = -m^2 \operatorname{Tr} \Sigma^2 + m' \operatorname{Tr} \Sigma^3 + \lambda (\operatorname{Tr} \Sigma^2)^2 + \lambda' \operatorname{Tr} \Sigma^4 \tag{12}$$

$$\langle \Sigma \rangle = v \operatorname{diag}(2, 2, 2, -3, -3) \tag{13}$$

$$\text{WGB} \sim (3, 2, -\tfrac{5}{6}) \tag{14}$$

- Global minimum of V? In general a hard problem!

Model building 6 — Yukawa sector

- Fermions — 3 generations of each (usually flavor⊥GUT):

$$\mathrm{SU}(5): \mathbf{10}, \bar{\mathbf{5}} \qquad \mathrm{SO}(10): \mathbf{16} \qquad \mathrm{E}_6: \mathbf{27} \tag{15}$$

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$$\mathrm{SU}(5): \quad \bar{5} \otimes \bar{5} = \overline{15}_s \oplus \overline{10}_a \tag{16}$$

$$10 \otimes \bar{5} = 5 \oplus 45 \tag{17}$$

$$10 \otimes 10 = \bar{5}_s \oplus \overline{45}_a \oplus \overline{50}_s \tag{18}$$

$$\mathrm{SO}(10): \quad 16 \otimes 16 = 10_s \oplus 126_s \oplus 120_a \tag{19}$$

$$\mathrm{E}_6: \quad 27 \otimes 27 = \overline{27}_s \oplus 351'_s \oplus 351_a \tag{20}$$

Model building 7 — Yukawa sector continued

- Example 1: $\mathrm{SU}(5)$ with $\mathbf{5}$

$$\mathcal{L}_{Yuk} = {Y_{10}}^{ij}\ \underbrace{\mathbf{10}_i\, \mathbf{10}_j\, \mathbf{5}}_{Qu^c H + \ldots} \quad + \quad {Y_5}^{ij}\ \underbrace{\mathbf{10}_i\, \bar{\mathbf{5}}_j\, \mathbf{5}^*}_{Qd^c H^* + e^c L H^*} \tag{21}$$

$\rightarrow$ Yukawa relation $Y_D = {Y_E}^T$ (at M_{GUT})
NOT GOOD (ENOUGH)!

$\rightarrow$ Fix: new operators ($\mathbf{45}$ or non-renormalizable terms)

Model building 7 — Yukawa sector continued

- Example 1: $\mathrm{SU}(5)$ with 5

$$\mathcal{L}_{Yuk} = {Y_{10}}^{ij}\ \underbrace{10_i\, 10_j\, 5}_{Qu^cH+\ldots} \quad + \quad {Y_5}^{ij}\ \underbrace{10_i\, \bar{5}_j\, 5^*}_{Qd^cH^*+e^cLH^*} \tag{21}$$

- Example 2: $\mathrm{SO}(10)$ with $10_{\mathbb{C}} \oplus 126$

$$\mathcal{L}_{Yuk} = {Y_{10}}^{ij}\, 16_i\, 16_j\, 10 + {\tilde{Y}_{10}}^{ij}\, 16_i\, 16_j\, 10^* + {Y_{126}}^{ij}\, 16_i\, 16_j\, 126^* \tag{22}$$

Yukawa relations at M_{GUT}:

$$\begin{aligned} M_U &= Y_{10}\, v_{10}^u + \tilde{Y}_{10}\, {v_{10}^d}^* + Y_{126}\, v_{126}^u \\ M_D &= Y_{10}\, v_{10}^d + \tilde{Y}_{10}\, {v_{10}^u}^* + Y_{126}\, v_{126}^d \\ M_E &= Y_{10}\, v_{10}^d + \tilde{Y}_{10}\, {v_{10}^u}^* - 3\, Y_{126}\, v_{126}^d \end{aligned}$$

Fit: **works** already with Y_{10}, Y_{126}

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- SM Higgs: in $M^2(1, 2, \pm\frac{1}{2})$
  - one doublet at **EW** scale

    → fine-tuning or some mechanism

    → issue: doublet-triplet splitting
  - an admixture of flavor eigenstates

    (v's required by fermion fit)

Yukawa relations at M_{GUT}:

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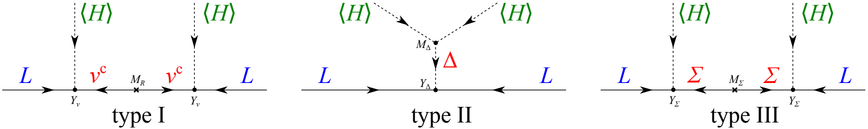

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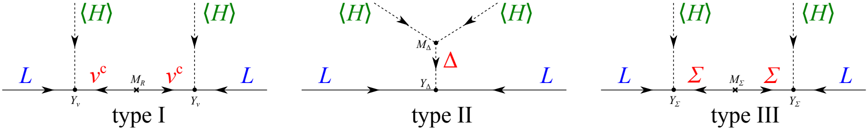

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$$Y_\nu v_{EW} = Y_{10} v_{10}^u + \tilde{Y}_{10} v_{10}^{d\,*} - 3\, Y_{126} v_{126}^u \qquad M_R = Y_{126}\,\langle 126\rangle \tag{24}$$

$$Y_\Delta = Y_{126} \qquad M_\Delta : \langle 126\rangle \cdot 126 \cdot 126^{*2} \tag{25}$$

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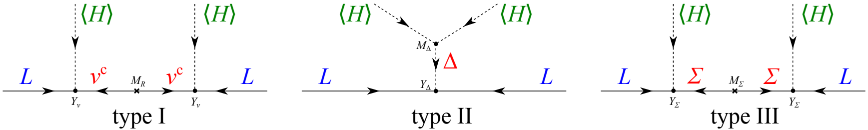

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$\nu^c \subset 16_F$, $\Delta \subset 126$

- See-saw scale $\sim 10^{12}\,\mathrm{GeV}$ from intermediate breaking stage

$$M_\nu = -{Y_\nu}^T {M_R}^{-1} Y_\nu {v_{EW}}^2 + Y_\Delta \,\langle \Delta \rangle_{\text{induced}} \tag{26}$$

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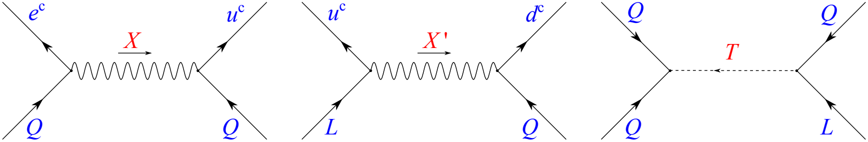
- Operators *qqql*: violate B and L number (preserve $B-L$)



$$X \sim (3,2,-\tfrac{5}{6}) \qquad X' \sim (3,2,+\tfrac{1}{6}) \qquad T \sim (3,1,-\tfrac{1}{3}) \quad (27)$$

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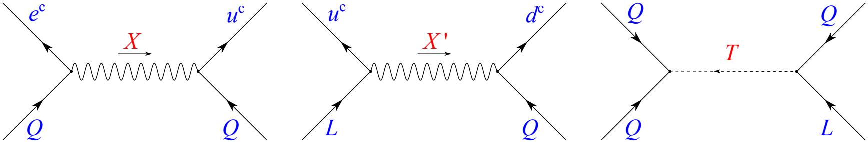

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- Gauge-mediated proton decay: **always** found in GUT adjoints
  $\to$ $X \subset$ 24 of $\mathrm{SU}(5)$; $X, X' \subset$ 45 of $\mathrm{SO}(10)$

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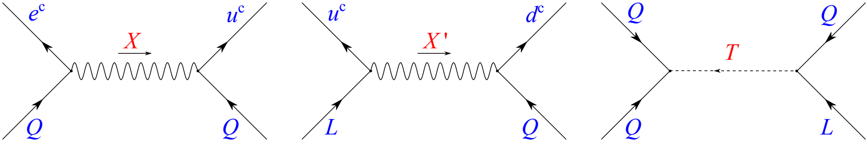

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- Gauge-mediated proton decay: **always** found in GUT adjoints
  $\to X \subset 24$ of $\mathrm{SU}(5)$; $\quad X, X' \subset 45$ of $\mathrm{SO}(10)$
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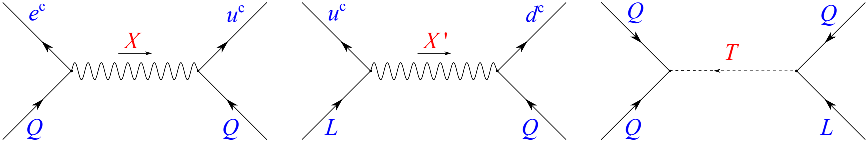
BSM pheno aspects 2 — proton decay

- Operators $qqql$: violate B and L number (preserve $B-L$)



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- Crude estimate:

$$\mathcal{A}_{\text{gauge}} \propto \frac{g^2}{M_X^2}, \qquad \mathcal{A}_{\text{scalar}} \propto \frac{y^2}{M_T^2}, \qquad \Gamma_p \approx \frac{g^4 m_p^5}{{M_{\text{GUT}}}^4} \qquad (28)$$

- SUSY: proton decay faster ($D=5$ operators)

BSM pheno aspects 3 — proton decay experimentally

- Process: proton $\rightarrow$ meson $+$ lepton

e.g. channels	τ_p limit
$p \rightarrow \pi^0\, e^+$	$2.4 \cdot 10^{34}\,\mathrm{yr}$
$p \rightarrow \pi^0\, \mu^+$	$1.6 \cdot 10^{34}\,\mathrm{yr}$
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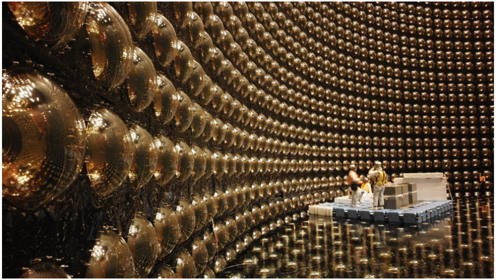

Super-K: water tank with photomultiplier tubes

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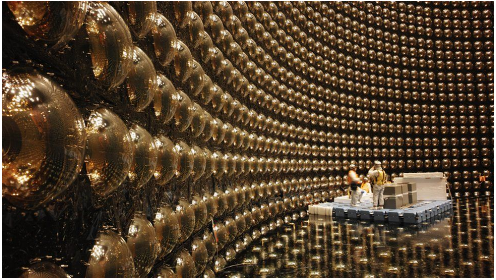
- Intuition:
  - 1 proton for $10^{33}\,\mathrm{yr}$ $\leftrightarrow$ 10^{33} protons for $1\,\mathrm{yr}$
  - 10^{27} protons $\sim 1\,\mathrm{kg}$ $\to$ 10^{33} protons $\sim 1\,\mathrm{kt}$
  - $1\,\mathrm{kt}$ water $\sim (10\,\mathrm{m})^3$
  - $10^{34}\,\mathrm{yr} \to M_{\mathrm{GUT}} \sim 10^{15.2}\,\mathrm{GeV}$



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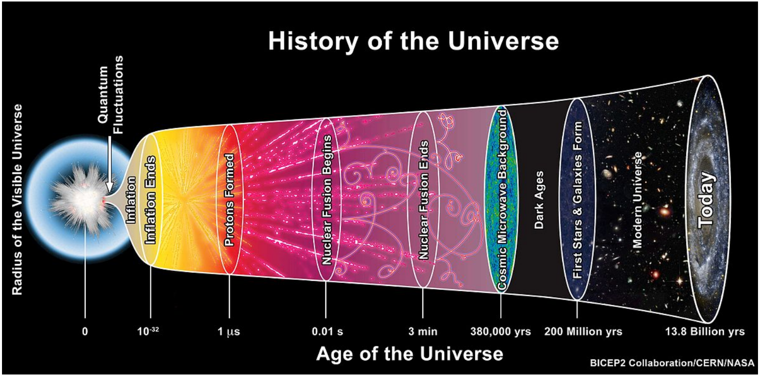
BSM pheno aspects 4 — Cosmology

- Early universe is hot: GUT era, cosmology implications



BICEP2 Collaboration/CERN/NASA

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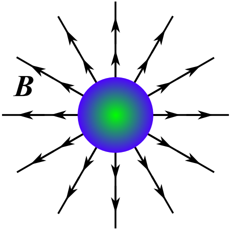
- Generic GUT prediction:

't Hooft–Polyakov monopoles



$\rightarrow$ $\pi_2(G/H) \neq \{1\}$
$\rightarrow$ core with "knotted" GUT gauge fields
$\rightarrow$ size $\sim M_{\text{GUT}}^{-1}$
$\rightarrow$ non-observation: # diluted by inflation

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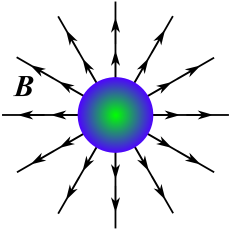

- Cosmology aspects addressed within GUTs:
  1. inflation (usually below M_{GUT})
  2. leptogenesis, baryogenesis
  3. dark matter
  4. symmetry breaking: phase transitions
     → formation of vacuum bubbles
     → formation of cosmic strings, domain walls, monopoles
     → gravitational wave signals

Intermezzo — a retrospective overview up to now

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5. Bottom up view [language of SM symmetry]:
   → new DOF that complete larger building blocks
   → couplings interrelated
   → will/can relate BSM phenomena (interesting!)

Axion in GUTs 1 — general considerations

- Axion: a solution to the strong-CP problem, DM candidate

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\,\frac{a}{f_a}\,G_{\mu\nu}\tilde{G}^{\mu\nu}, \qquad |\theta_{\text{eff}}| = \left|\frac{\langle a\rangle}{f_a}\right| \lesssim 10^{-10} \tag{29}$$

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- Axion a: pseudo-Goldstone of $\mathrm{U}(1)_{PQ}$, mass from QCD anomaly

$$m_a \approx m_\pi f_\pi / f_a \quad \sim 10\,\mathrm{neV}\cdot(10^{15}\,\mathrm{GeV}/f_a) \tag{31}$$

→ $f_a \sim \langle S\rangle$ …connection to scales in GUTs

→ relevant for experiment: via $g_{a\gamma\gamma}, g_{aD} \propto 1/f_a$

$$\mathcal{L} \supset g_{a\gamma\gamma}\,\frac{a}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{i}{2}\,g_{aD}\,a\,\overline{\Psi}_n\sigma_{\mu\nu}\gamma_5\Psi_n\,F^{\mu\nu} \tag{32}$$

Axion in GUTs 2 — examples in $\mathrm{SU}(5)$

- Example 1A: WGG (Wise-Georgi-Glashow) based on DFS Z
  - → In SM language: need 2 **weak doublets** and a **SM-singlet**
  - → $\mathrm{SU}(5) \times \mathrm{U}(1)_{PQ}$

$$3 \times 10_F(-\tfrac{1}{2}), \quad 3 \times \bar{5}_F(-\tfrac{1}{2}), \quad 5_1(1), \quad 5_2(-1), \quad 24_{\mathbb{C}}(-1) \tag{33}$$

$$\mathcal{L} \supset Y_{10}\, 10_F\, 10_F\, 5_1 + Y_5\, 10_F\, \bar{5}_F\, 5_2^* - V(5_1, 5_2, 24) \tag{34}$$

  - → VEVs: $\langle S \rangle \equiv \langle 24 \rangle_{\mathrm{GUT}}$, $\langle 5_1 \rangle_{\mathrm{EW}}$, $\langle 5_2 \rangle_{\mathrm{EW}}$
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- Realistic $\mathrm{SU}(5) \times \mathrm{U}(1)_{PQ}$ examples:

model	additions to 1A	ν mass	$m_a[\mathrm{neV}]$
1B	$24_F(-\frac{1}{2})$	seesaw I+III	$[4.8, 6.6]$
1C	$15_F(-\frac{1}{2}) \oplus \overline{15}_F(-\frac{1}{2}) \oplus 35(-1)$	at 1-loop	$[0.1, 4.7]$

Constraints on m_a: unification, proton decay, collider

Axion in GUTs 3 — examples in $\mathrm{SO}(10)$

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$$3 \times 16_F(-\tfrac{1}{2}), \quad 10(+1), \quad 126(-1), \quad 210(-2) \tag{35}$$

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- Some conceptual alternatives:
  - new in SO(10): relate f_a to intermediate scales → heavier axion
    → W is highest PQ-breaking VEV

$$\mathrm{SO}(10) \xrightarrow{V} G_1 \xrightarrow{W} G_2 \xrightarrow{Z} 3_C\,2_L\,1_Y \tag{38}$$

  - S is G-singlet: VEV at any scale → not predictive for axion mass

Conclusions

Grand Unified Theories: an attractive framework

(0) Tentative experimental indication: SM gauge couplings approximately unify in correct high energy window

(1) Conceptual/philosophical advantages over Standard Model

(2) Can incorporate and relate different SM or BSM phenomena (particle physics and cosmology)

(3) Offer possible windows to physics at super-high energies

  (3a) proton decay
  (3b) model-specific phenomena ...
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Thank you for your attention!