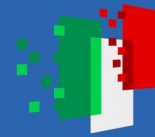




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Missione 4 Istruzione e Ricerca

Giacomo Landini (IFIC, Valencia)

Axion quality:
problems and possible solutions

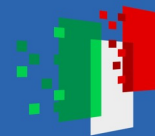
25/01/2024



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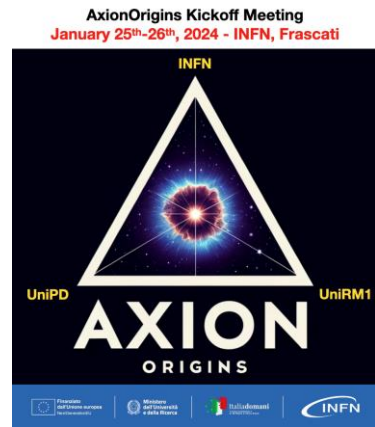


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Axion quality: problem and solutions

partially based on

M.Ardu, L.Di Luzio, GL, A.Strumia, D.Teresi, J.W.Wang

arXiv [2007.12663]



Giacomo Landini
INFN-LNF, Frascati

25/01/24

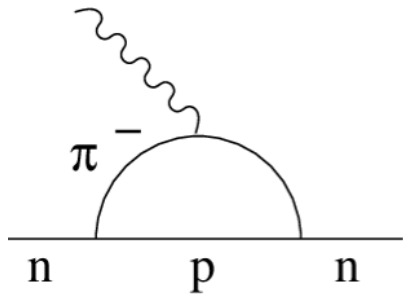


The Strong CP problem

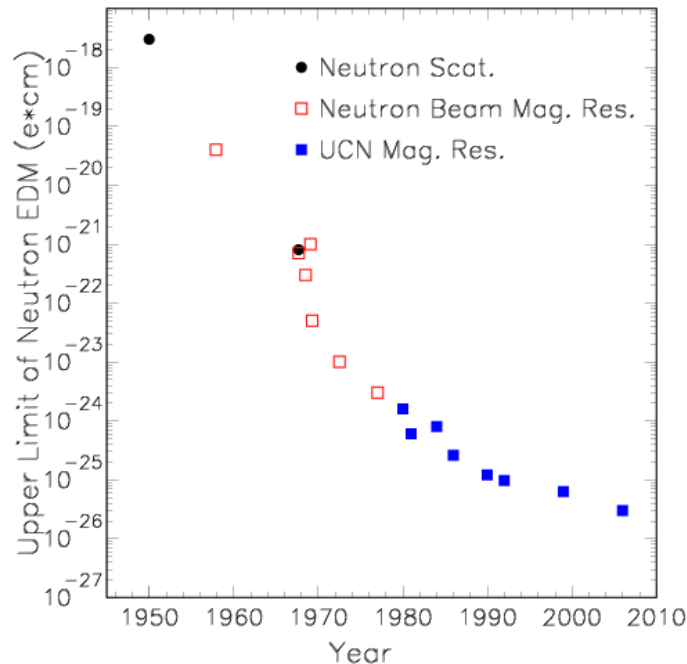
The QCD Lagrangian violates CP symmetry

$$\mathcal{L}_{\text{QCD}} \supset \bar{\theta} \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} \longrightarrow \begin{array}{l} \text{Gauge-invariant + renormalizable} \\ \text{Non-perturbative QCD (instantons)} \end{array} \longrightarrow \begin{array}{l} \text{Natural expectation} \\ \bar{\theta} \sim \mathcal{O}(1) \end{array}$$

Prediction: electric dipole moment for the neutron



$$d_n \approx 10^{-16} |\bar{\theta}| e \text{ cm}$$



Upper bound

$$\bar{\theta} \leq 10^{-10}$$

Why so small??

Peccei-Quinn mechanism

A new *global chiral* $U(1)$ symmetry + scalar field Φ

Peccei-Quinn mechanism

A new *global chiral* U(1) symmetry + scalar field Φ

1) Spontaneously broken \longrightarrow a new Goldstone boson: the *axion*

$$\Phi(x) \simeq f_a e^{ia(x)/f_a}$$

$$a(x) \xrightarrow{PQ} a(x) + \gamma f_a$$

Peccei-Quinn mechanism

A new *global chiral* U(1) symmetry + scalar field Φ

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2) QCD anomaly

Colored fermions with *chiral* U(1) charges

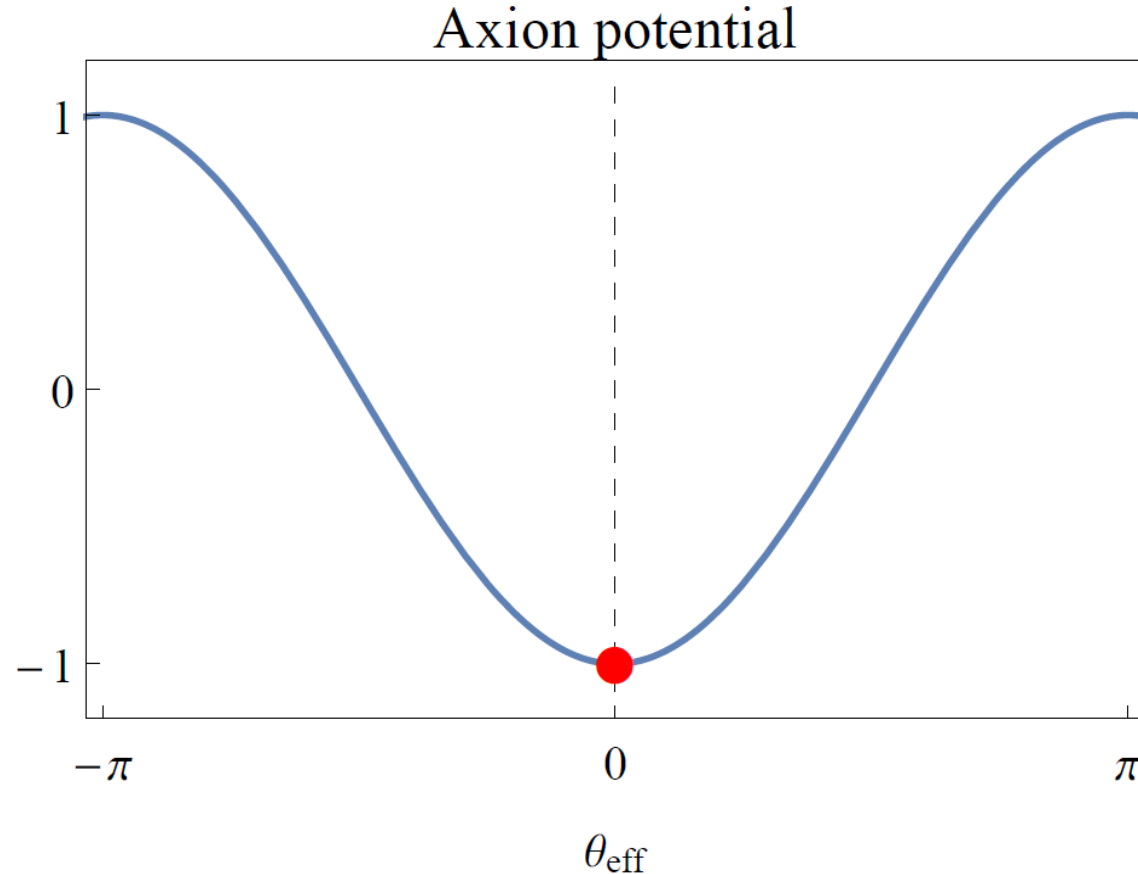
\longrightarrow generates a potential for the axion
(shift-symmetry is broken!)

$$\mathcal{L}_{\text{QCD}}^{\text{PQ}} \supset \theta_{\text{eff}}(x) \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}$$

$$\theta_{\text{eff}}(x) \equiv \bar{\theta} - \frac{a(x)}{f_a}$$

Peccei-Quinn mechanism

The QCD potential relaxes to the CP-conserving minimum



$$V \approx -m_a^2 f_a^2 \cos \theta_{\text{eff}}$$

$$m_a \sim \Lambda_{\text{QCD}}^2 / f_a$$

$$\langle \bar{\theta}_{\text{eff}} \rangle = 0$$

Vafa-Witten theorem

$$E(\theta) \geq E(0)$$

Global symmetries

Global symmetries are not fundamental but *accidental*

Gauge theory perspective

The Lagrangian must be gauge-invariant



Accidental global symmetries at the renormalizable level

$$\mathcal{L} = \mathcal{L}^{(4)} \quad \text{Ex: baryon/lepton number in the SM}$$



Broken by higher-dimensional operators (UV physics)

$$\Delta\mathcal{L}_{\text{UV}} \sim \frac{1}{\Lambda_{\text{UV}}^{d-4}} \mathcal{O}^{[d]}$$

Global symmetries

Global symmetries are not fundamental but accidental

Quantum Gravity conjectures, based on Black Hole physics



No global symmetry can exist in a theory of Quantum Gravity

[D. Harlow, H. Ooguri '18]

The quality problem

PQ symmetry arises accidentally at low-energy (*How?*)

↳ New gauge symmetries?

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UV physics (gravity?) violates PQ symmetry

The quality problem

PQ symmetry arises accidentally at low-energy (*How?*)

↳ New gauge symmetries?

↓
UV physics (gravity?) violates PQ symmetry

↓
This generates Peccei-Quinn breaking operators at low-energy

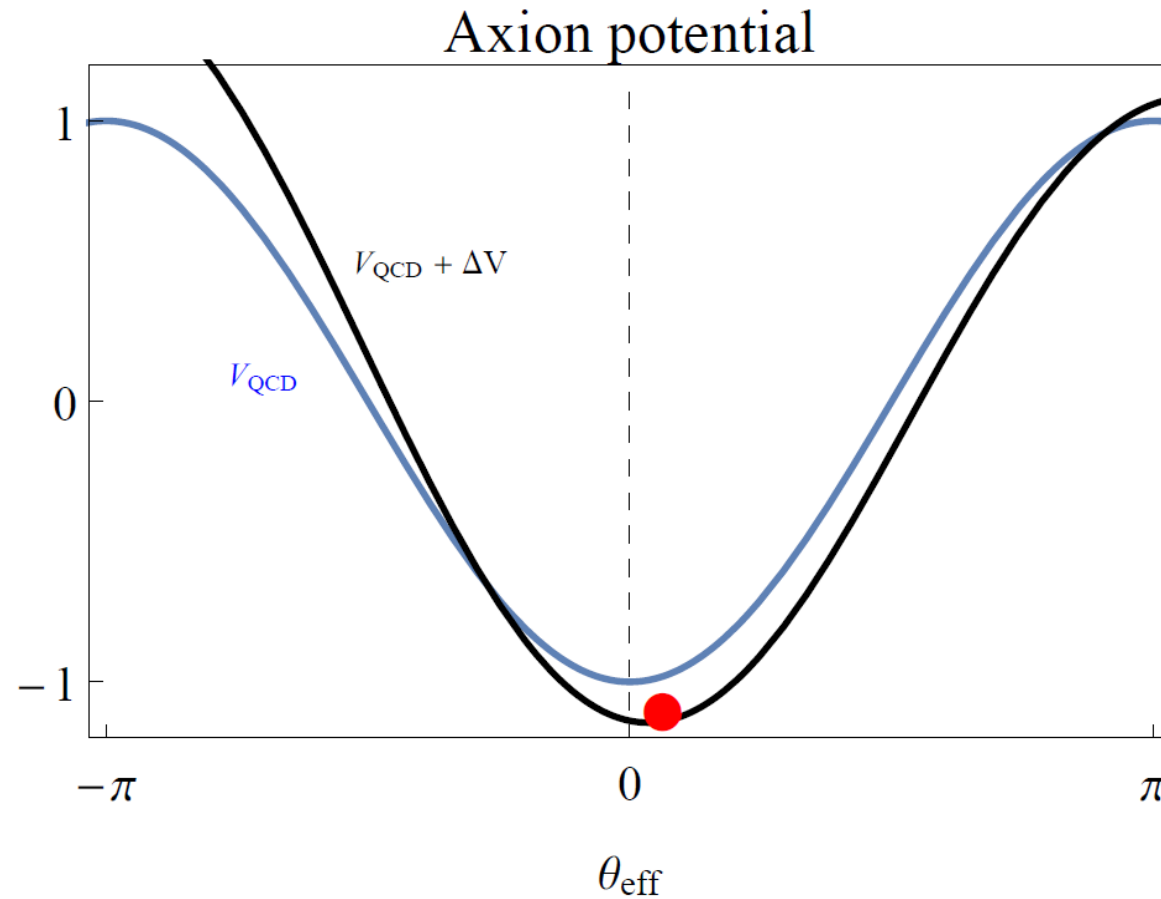
$$\Delta\mathcal{L}_{UV} \sim \frac{1}{\Lambda_{UV}^{d-4}} \mathcal{O}^{[d]}$$

↓
New contribution to the axion potential (shift-symmetry is broken)

$$\Delta V_{UV}(\theta_{\text{eff}}) \sim \Lambda_{UV}^4 \left(\frac{f_a}{\Lambda_{UV}} \right)^d$$

The quality problem

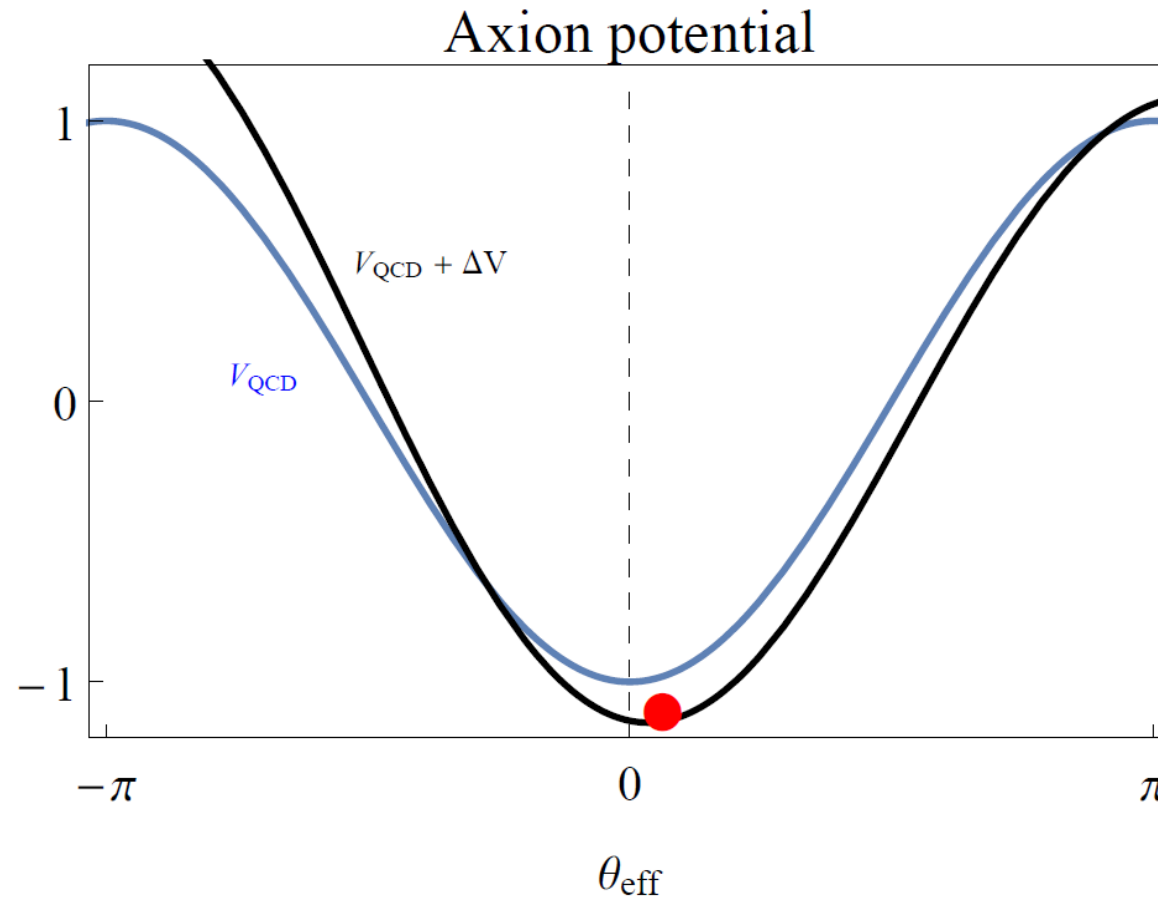
The minimum of the potential is shifted



$\langle \theta_{\text{eff}} \rangle \neq 0 \longrightarrow$ CP – violating minimum!

The quality problem

The minimum of the potential is shifted



PQ breaking by UV physics \longrightarrow induced θ_{eff} at low energy

The quality problem

We solve the Strong CP problem only if

$$\langle \theta_{\text{eff}} \rangle < 10^{-10} \quad (\text{neutron EDM})$$

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UV PQ-breaking physics Experimental bound

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$$f_a \ll \Lambda_{\text{UV}}$$

The lowest-dimensional PQ-breaking operators are the most dangerous!

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For physically well-motivated scales

Dark Matter $f_a \sim 10^{9-11} \text{ GeV}$

Gravity $\Lambda_{\text{UV}} \sim M_{\text{Pl}}$



PQ must be **preserved** up to operators of dimension $\mathbf{d} \geq 10 - 12$
(high-quality symmetry)

The quality problem

We solve the Strong CP problem only if

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For physically well-motivated scales

Dark Matter $f_a \sim 10^9 - 11 \text{ GeV}$

Gravity $\Lambda_{\text{UV}} \sim M_{\text{Pl}}$



PQ-breaking operator with $d < 10 - 12$ must be **forbidden**

Solutions to the quality problem

$$\left(\frac{f_a}{\Lambda_{\text{UV}}}\right)^{d-4} f_a^4 \lesssim 10^{-10} \Lambda_{\text{QCD}}^4$$

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Low-scale f_a

Challenge: evade astrophysical bounds $f_a \gtrsim 10^8$ GeV

Solutions to the quality problem

$$\left(\frac{f_a}{\Lambda_{\text{UV}}}\right)^{d-4} f_a^4 \lesssim 10^{-10} \Lambda_{\text{QCD}}^4$$

Low-scale f_a

Challenge: evade astrophysical bounds $f_a \gtrsim 10^8$ GeV

↓ possible realization

Modified $m_a - f_a$ relation

extra contribution to m_a from hidden gauge sector with PQ anomaly $\Lambda_D \gg \Lambda_{\text{QCD}}$

May arise in mirror SM (or mirror QCD or mirror GUT) models

[Rubakov '97, Berezhiani, Gianfagna, Giannotti '01,
Gianfagna Giannotti Nesti '04, Gaillard, Gavela et al. '18, ...]

Solutions to the quality problem

$$\lambda \left(\frac{f_a}{\Lambda_{\text{UV}}} \right)^{d-4} f_a^4 \lesssim 10^{-10} \Lambda_{\text{QCD}}^4$$

Low-scale f_a

Gravitational suppression of coupling

If PQ-breaking operators are generated by non-perturbative gravitational effects

$$\lambda \Delta \mathcal{L}_{\text{UV}}, \quad \lambda \sim e^{-S}$$

Studied in the context of Euclidian Wormholes

[Lee '88, Giddings, Strominger '88] $S \sim M_{\text{Pl}}/f_a$ No quality problem

[Abbott, Wise '88, Alvey, Escudero '20] $S \sim \log M_{\text{Pl}}/f_a$ Quality problem

Solutions to the quality problem

$$\left(\frac{f_a}{\Lambda_{\text{UV}}}\right)^{d-4} f_a^4 \lesssim 10^{-10} \Lambda_{\text{QCD}}^4$$

Low-scale f_a

Gravitational suppression of coupling

Gauge protection

Low-dimensional PQ-breaking operators are forbidden by gauge invariance

1. New gauge group *unrelated* to SM flavor
2. New gauge group *related* to SM flavor

Discrete/Abelian gauge symmetry

$$\mathbb{Z}_n : \Phi \rightarrow e^{i\frac{2\pi}{n}} \Phi \quad \longrightarrow \quad \Delta\mathcal{L}_{UV} = \frac{\Phi^n}{\Lambda_{UV}^{n-4}} \quad \longrightarrow \quad \text{We need } n \geq 10$$



It can emerge from a U(1) gauge symmetry:

[Krauss and Wilczek '89, Dias et al. '03]

[Carpenter et al. '09, Harigaya et al. '13]

$$\Phi \rightarrow e^{i\alpha} \Phi$$

$$S \rightarrow e^{in\alpha} S$$

$$\langle S \rangle : U(1) \rightarrow \mathbb{Z}_n$$

$$\alpha = \frac{2\pi}{n}$$

$$\text{Abelian gauge U(1) : } \Phi_1 \rightarrow e^{iq_1\alpha}$$

[Barr and Seckel '92]

$$\Phi_2 \rightarrow e^{iq_2\alpha}$$



$$\Delta\mathcal{L}_{UV} = \frac{(\Phi_1^\dagger)^{q_2} (\Phi_2)^{q_1}}{\Lambda_{UV}^{q_1+q_2-4}}$$

\longrightarrow We need $q_1 + q_2 \geq 10$

Non Abelian gauge symmetry

[Di Luzio, Ubaldi and Nardi '17]

$$G = SU(N) \otimes SU(N) + \text{scalar field } Y \sim (N, \bar{N})$$

Renormalizable potential $V(Y) = f(\text{Tr}[Y^\dagger Y], \text{Tr}[Y^\dagger Y Y^\dagger Y])$
 $N \geq 4$

→ Accidental global U(1) $Y \rightarrow e^{i\alpha} Y$

→ promoted to a PQ symmetry introducing new colored fermions charged under G

PQ broken by $\Delta\mathcal{L}_{UV} = \frac{\det Y}{\Lambda_{UV}^{N-4}} \propto \epsilon^{i_1, \dots, i_N} \epsilon_{j_1, \dots, j_N} Y_{i_1}^{j_1} \dots Y_{i_N}^{j_N}$
→ $N \geq 10$

Dangerous heavy colored relics in post-inflationary scenario

→ See related discussion later

PQ and flavor?

A new gauge group G acts on scalars which couple to (exotic or SM) quarks

If (some of the) SM quarks are charged under G

G is a flavor symmetry

likely Non-Abelian

Spontaneously broken

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A new gauge group G acts on scalars which couple to (exotic or SM) quarks

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Progammm: determine G and breaking patterns which

- (1) predict an accidental U(1) symmetry **origin of PQ**
- (2) protect the U(1) symmetry from UV-breaking sources **PQ quality**
- (3) reproduce the fermion masses/mixing of the SM **Flavor**

➔ *multiple scalar vevs to provide high-scale PQ and SM flavor scales*

Axion quality from flavor

[Darmé, Nardi '21]

[Darmé, Nardi, Smarra '22]

Rectangular gauge groups

We saw before that

$$G = SU(N) \otimes SU(N) + \text{scalar field } Y \sim (N, \bar{N})$$

provides an accidental PQ broken by

$$\Delta\mathcal{L}_{UV} = \frac{\det Y}{\Lambda_{UV}^{N-4}} \propto \epsilon^{i_1, \dots, i_N} \epsilon_{j_1, \dots, j_N} Y_{i_1}^{j_1} \dots Y_{i_N}^{j_N}$$

solving the quality problem for

$$N \geq 10$$

Axion quality from flavor [Darmé, Nardi '21]

[Darmé, Nardi, Smarra '22]

Rectangular gauge groups

Observation

$$G_F = SU(m) \otimes SU(n) + \text{scalar field } Y \sim (m, \bar{n}) \quad m \neq n$$

All operators $\propto \epsilon^{i_1, \dots, i_N} \epsilon_{j_1, \dots, j_N} Y_{i_1}^{j_1} \dots Y_{i_N}^{j_N}$ vanish identically

➔ Exact accidental U(1) $Y \rightarrow e^{i\alpha} Y$

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➡ Exact accidental U(1) $Y \rightarrow e^{i\alpha} Y$

- (i) We need to add additional scalars to avoid massless colored fermions
- (ii) The accidental U(1) is not exact but...
- (iii) ...we can protect it up to D=10 avoiding a too large number of chiral new quarks

➡ keep $(m, n) \leq 3$ to interpret G_F as a flavor symmetry!

Axion quality from flavor [Darmé, Nardi '21]

[Darmé, Nardi, Smarra '22]

Rectangular flavor gauge groups

$$G_F \otimes U(1) \text{ with } G_F = SU(m) \otimes SU(n) \quad m \neq n$$

$$Y \sim (m, \bar{n}) + \text{additional scalars/chiral quarks}$$

SM quarks are charged under the flavor gauge group

 *Example*

$$SU(3) \otimes SU(2) \otimes U(1)_F$$

$$q_L \sim (\mathbf{3}, \mathbf{1}), u_R \sim (\mathbf{1}, \mathbf{2}), t_R \sim (\mathbf{1}, \mathbf{1}), Q_L \sim (\mathbf{1}, \mathbf{1}), \quad Y \sim (\mathbf{3}, \bar{\mathbf{2}}), \quad Z \sim (\mathbf{3}, \mathbf{1}), \quad X \sim (\mathbf{1}, \bar{\mathbf{2}}). \\ U_R \sim (\mathbf{3}, \mathbf{1}), U_L \sim (\mathbf{1}, \mathbf{2}), T_L \sim (\mathbf{1}, \mathbf{1}), Q_R \sim (\mathbf{1}, \mathbf{1}).$$

Possible signatures: light flavored gauge bosons

See next talk by Clemente Smarra!

A GUT + flavor solution

[Di Luzio '20]

$SO(10)$ GUT

$$\mathcal{L}_{Y=0} + \mathcal{L}_Y = y_{10} \psi_{16} \psi_{16} \phi_{10} + \tilde{y}_{10} \psi_{16} \psi_{16} \phi_{10}^* + y_{1\bar{2}6} \psi_{16} \psi_{16} \phi_{1\bar{2}6} + h.c$$



SM fermions + RHNs

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SM fermions + RHNs

Flavor symmetry



If $y_i \rightarrow 0$

Accidental global $U(3) = SU(3)_f \otimes U(1)_{PQ}$

$$PQ(\phi_{10}) = PQ(\phi_{1\bar{2}6}) = -2PQ(\psi_{16})$$



Anomalous under QCD
SSB due to $\phi_{16}, \phi_{1\bar{2}6}$

A GUT + flavor solution

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Anomalous under QCD

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Gauging of flavor symmetry $SU(3)_f$

$U(1)_{PQ}$ is promoted to an (accidental) symmetry of the full renormalizable Lagrangian

A GUT + flavor solution

[Di Luzio '20]

Field	Lorentz	SO(10)	\mathbb{Z}_4	$SU(3)_f$	\mathbb{Z}_3	$U(1)_{PQ}$
ψ_{16}	(1/2, 0)	16	i	3	$e^{i2\pi/3}$	1
$\psi_1^{1,\dots,16}$	(1/2, 0)	1	1	$\bar{3}$	$e^{i4\pi/3}$	0
ϕ_{10}	(0, 0)	10	-1	$\bar{6}$	$e^{i2\pi/3}$	-2
ϕ_{16}	(0, 0)	16	i	$\bar{3}$	$e^{i4\pi/3}$	-1
$\phi_{\overline{126}}$	(0, 0)	$\overline{126}$	-1	$\bar{6}$	$e^{i2\pi/3}$	-2
ϕ_{45}	(0, 0)	45	1	1	1	0

$SU(3)_f$ provides protection from UV sources

PQ-breaking operators allowed by $SO(10) \otimes SU(3)_f$

$$\phi_{10}^6 \quad (d = 6)$$

$$\phi_{\overline{126}}^6 \quad (d = 6)$$

$$\phi_{16}^6 \phi_{10}^3 \quad (d = 9)$$

$$\phi_{16}^6 \phi_{\overline{126}}^3 \quad (d = 9)$$

$$\phi_{16}^{12} \quad (d = 12)$$

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PQ-breaking operators allowed by $SO(10) \otimes SU(3)_f$

ϕ_{10} can only develop
electroweak VEV



Negligible contribution to θ_{eff}

$$\begin{array}{ll}
 \leftarrow \phi_{10}^6 & (d = 6) \\
 \leftarrow \phi_{\overline{126}}^6 & (d = 6) \\
 \leftarrow \phi_{16}^6 \phi_{10}^3 & (d = 9) \\
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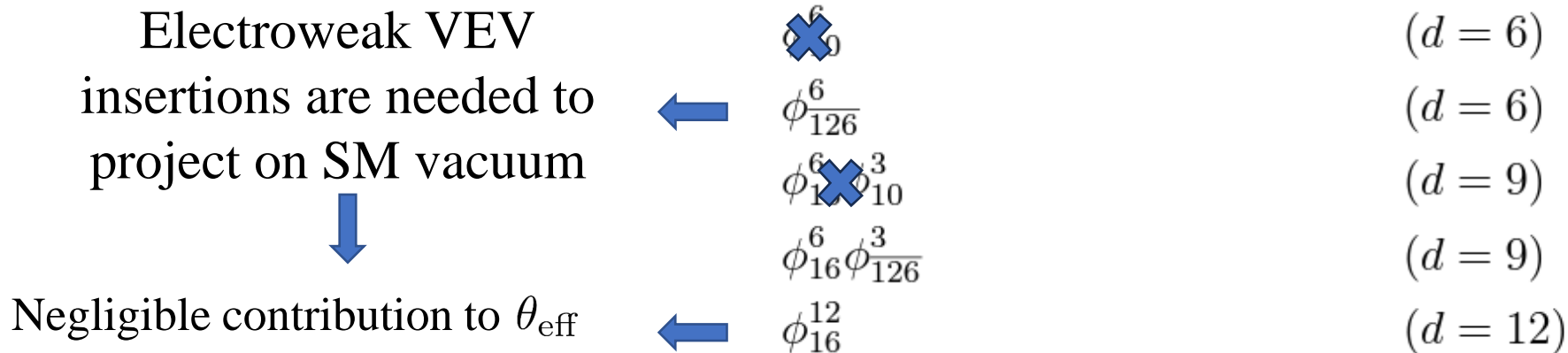
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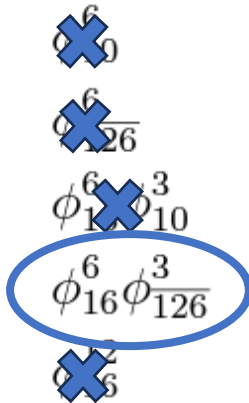
PQ-breaking operators allowed by $SO(10) \otimes SU(3)_f$

First PQ-breaking operator

$$\theta_{\text{eff}} \equiv \frac{\langle a \rangle}{f_a} \approx \frac{|k| V_{16}^6 V_{\overline{126}}^3 \sin \delta}{8\sqrt{2} M_{\text{Pl}}^5 \Lambda_{\text{QCD}}^4}$$



$$f_a \lesssim 3.0 \times 10^8 \text{ GeV}$$



$$(d = 6)$$

$$(d = 6)$$

$$(d = 9)$$

$$(d = 9)$$

$$(d = 12)$$

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Interesting feature of the model



Exotic *flavored* fermions
(anomalons)

Use the same scalar vevs to break simultaneously $SO(10) \otimes U(1)_{PQ} \otimes SU(3)_f$

$$SU(3)_f \xrightarrow{\phi_{16}} SU(2)_f \xrightarrow{\phi_{\bar{126}}} \mathbf{1}$$

A GUT + flavor solution

[Di Luzio '20]

Field	Lorentz	SO(10)	\mathbb{Z}_4	$SU(3)_f$	\mathbb{Z}_3	$U(1)_{PQ}$
ψ_{16}	(1/2, 0)	16	i	3	$e^{i2\pi/3}$	1
$\psi_1^{1,\dots,16}$	(1/2, 0)	1	1	$\bar{3}$	$e^{i4\pi/3}$	0
ϕ_{10}	(0, 0)	10	-1	$\bar{6}$	$e^{i2\pi/3}$	-2
ϕ_{16}	(0, 0)	16	i	$\bar{3}$	$e^{i4\pi/3}$	-1
$\phi_{\overline{126}}$	(0, 0)	$\overline{126}$	-1	$\bar{6}$	$e^{i2\pi/3}$	-2
ϕ_{45}	(0, 0)	45	1	1	1	0

Interesting feature of the model



Exotic *flavored* fermions
(anomalons)

Use the same scalar vevs to break simultaneously $SO(10) \otimes U(1)_{PQ} \otimes SU(3)_f$

$$SU(3)_f \xrightarrow{\phi_{16}} SU(2)_f \xrightarrow{\phi_{1\bar{2}6}} \mathbf{1}$$

→ massless ψ_1

→ they get a mass from

$$\frac{1}{M_{\text{Pl}}^2} \psi_1 \psi_1 \phi_{16}^2 \phi_{\overline{126}}$$

→ Dark Matter?
Dark radiation?
Flavor puzzle?

Light exotic fermions could provide new testable dynamics directly related to the solution of PQ quality problem and the flavor puzzle

Remarks

Solutions to the quality problem typically need new physics at high scale
(new gauge dynamics, GUT, flavor,...)



How can we test/differentiate the possible models?



We should prefer models with characteristic low-energy signatures
and/or
which can explain additional problems (e.g SM flavor structure, Dark Matter)

➔ exotic flavored fermions/gauge bosons, Dark Matter from gauge confinement,...

The symmetric model

[Ardu, Di Luzio, GL, Strumia, Teresi, Wang '20]

Idea similar to
[Di Luzio, Ubaldi and Nardi '17]

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(\mathcal{N})$	$U(1)_{PQ}$	$U(1)_Q$	$U(1)_\mathcal{L}$
\mathcal{S}	0	0	1	1	$\mathcal{N}\mathcal{N}$	+1	0	0
Q_L	1/2	$+Y_Q$	1	3	\mathcal{N}	+1/2	+1	0
Q_R	1/2	$-Y_Q$	1	$\bar{3}$	\mathcal{N}	+1/2	-1	0
$\mathcal{L}_L^{1,2,3}$	1/2	$+Y_\mathcal{L}$	1	1	$\bar{\mathcal{N}}$	-1/2	0	+1
$\mathcal{L}_R^{1,2,3}$	1/2	$-Y_\mathcal{L}$	1	1	$\bar{\mathcal{N}}$	-1/2	0	-1

New dark $SU(\mathcal{N})$ gauge group

+

A new scalar field in the symmetric representation of $SU(\mathcal{N})^*$



Accidental $U(1)$ global symmetry at the renormalizable level

$$V_4(\text{Tr}[\mathcal{S}^\dagger \mathcal{S}], \text{Tr}[\mathcal{S}^\dagger \mathcal{S} \mathcal{S}^\dagger \mathcal{S}])$$

* it also works for the anti-symmetric representation

The symmetric model

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(\mathcal{N})$	$U(1)_{PQ}$	$U(1)_Q$	$U(1)_\mathcal{L}$
\mathcal{S}	0	0	1	1	$\mathcal{N}\mathcal{N}$	+1	0	0
\mathcal{Q}_L	1/2	$+Y_Q$	1	3	\mathcal{N}	+1/2	+1	0
\mathcal{Q}_R	1/2	$-Y_Q$	1	$\bar{3}$	\mathcal{N}	+1/2	-1	0
$\mathcal{L}_L^{1,2,3}$	1/2	$+Y_\mathcal{L}$	1	1	$\bar{\mathcal{N}}$	-1/2	0	+1
$\mathcal{L}_R^{1,2,3}$	1/2	$-Y_\mathcal{L}$	1	1	$\bar{\mathcal{N}}$	-1/2	0	-1

New colored fermions $\mathcal{Q}_{L,R}$ provide the QCD anomaly



$U(1)$ is a Peccei-Quinn symmetry

Extra color-singlet fermions $\mathcal{L}_{L,R}$ cancel gauge anomalies

The symmetric model

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(\mathcal{N})$	$U(1)_{PQ}$	$U(1)_Q$	$U(1)_\mathcal{L}$
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New colored fermions $\mathcal{Q}_{L,R}$ provide the QCD anomaly



$U(1)$ is a Peccei-Quinn symmetry

The new fermions have EW charge $Y_Q \pm Y_\mathcal{L} \neq 0$

different from [Di Luzio, Ubaldi and Nardi '17]

The symmetric model

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(\mathcal{N})$	$U(1)_{PQ}$	$U(1)_Q$	$U(1)_\mathcal{L}$
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SU(N) gauge structure



Additional accidental \mathbb{Z}_2 symmetry

Parity reflection in SU(N) group space $\mathcal{S}_{ij} \rightarrow (-1)^{\delta_{1i} + \delta_{1j}} \mathcal{S}_{ij}$

(details in backup slides) $Q_i \rightarrow (-1)^{\delta_{1i}} Q_i$

The symmetric model

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(\mathcal{N})$	$U(1)_{PQ}$	$U(1)_Q$	$U(1)_\mathcal{L}$
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$\mathcal{L}_R^{1,2,3}$	1/2	$-Y_\mathcal{L}$	1	1	$\bar{\mathcal{N}}$	-1/2	0	-1

Gauge invariance forbids PQ-breaking operators below dimension N

↓ PQ-breaking

$$\frac{\det \mathcal{S}}{\Lambda_{UV}^{N-4}} \propto \epsilon^{i_1, \dots, i_N} \epsilon^{j_1, \dots, j_N} \mathcal{S}_{i_1 j_1} \dots \mathcal{S}_{i_N j_N}$$

High-Quality if $N \geq 10$

The symmetric model

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(\mathcal{N})$	$U(1)_{PQ}$	$U(1)_Q$	$U(1)_\mathcal{L}$
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Interestingly, this protects at the same time the \mathbb{Z}_2 symmetry

Decay of colored relics

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(\mathcal{N})$	$U(1)_{PQ}$	$U(1)_Q$	$U(1)_\mathcal{L}$
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\mathcal{Q}_R	1/2	$-Y_Q$	1	$\bar{3}$	\mathcal{N}	+1/2	-1	0
$\mathcal{L}_L^{1,2,3}$	1/2	$+Y_\mathcal{L}$	1	1	$\bar{\mathcal{N}}$	-1/2	0	+1
$\mathcal{L}_R^{1,2,3}$	1/2	$-Y_\mathcal{L}$	1	1	$\bar{\mathcal{N}}$	-1/2	0	-1

EW charges allow the decay of heavy colored fermions in the Early Universe

Gauge invariance allows dimension-6 operators

$$(q_R \mathcal{Q})(e_R \mathcal{L}) \quad (q_R \mathcal{Q})(q'_R \mathcal{L})$$

$$\text{if } Y_Q \pm Y_\mathcal{L} = \{-1/3, 2/3, -4/3\}$$

$$\text{Decays before BBN: } f_a > \frac{1}{y_Q} \sqrt{\frac{10}{\mathcal{N}}} \left(\frac{\Lambda_{UV}}{M_{Pl}} \right)^{4/5} \times 10^{11} \text{ GeV}$$

Decay of colored relics

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
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$$\text{if } Y_Q \pm Y_\mathcal{L} = \{-1/3, 2/3, -4/3\}$$

Post-inflationary PQ-breaking is viable!

Gauge dynamics and Dark Matter

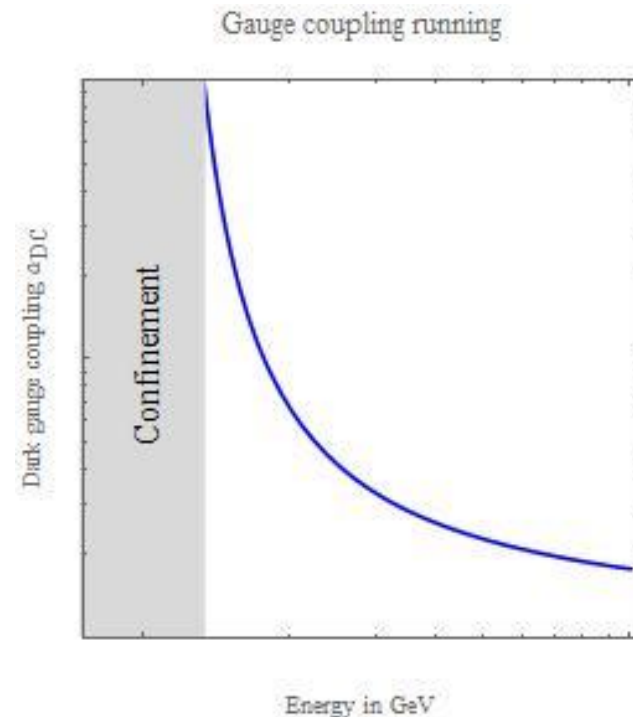
$$\mathcal{S}(x) = \left(\sqrt{N/2} f_a + \dots \right) e^{ia(x)/f_a} \quad SU(N) \otimes U(1)_{\text{PQ}} \xrightarrow{f_a} SO(N) \quad (\mathbb{Z}_2 \text{ is preserved})$$

$SO(N)$ dynamics *confines* at lower energy

$$\Lambda_{\text{SO}} \approx f_a \exp\left[-\frac{6\pi}{11(N-2)\alpha_{\text{DC}}(f_a)}\right]$$

2 energy scales in the model

→ $\Lambda_{\text{SO}} \ll f_a$



Asymptotic states must be $SO(N)$ gauge singlets

Gauge dynamics and Dark Matter

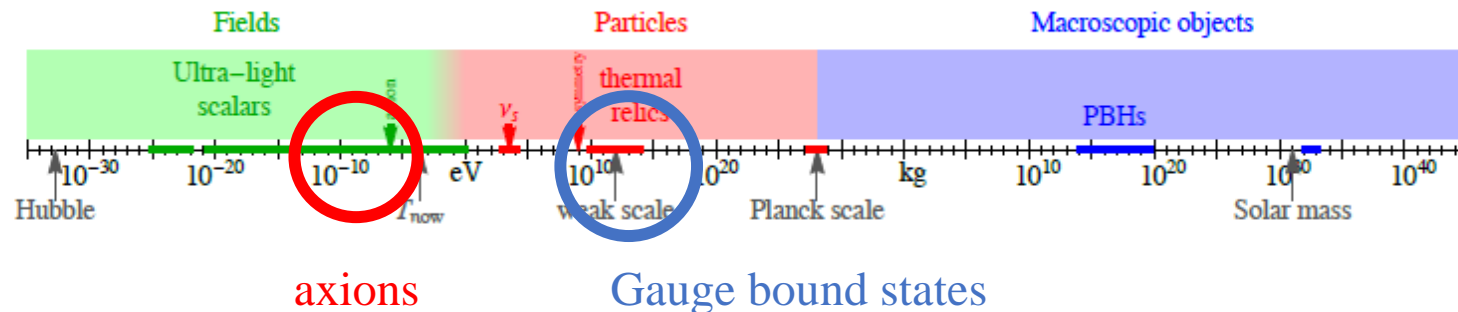
$$SU(N) \otimes U(1)_{\text{PQ}} \xrightarrow{f_a} SO(N) \quad (\mathbb{Z}_2 \text{ is preserved})$$

Asymptotic states must be $SO(N)$ gauge singlets

1. Axions (cosm. **stable**)
2. Gauge bound states even under \mathbb{Z}_2 (unstable)
3. Gauge bound states odd under \mathbb{Z}_2 (cosm. **stable**)

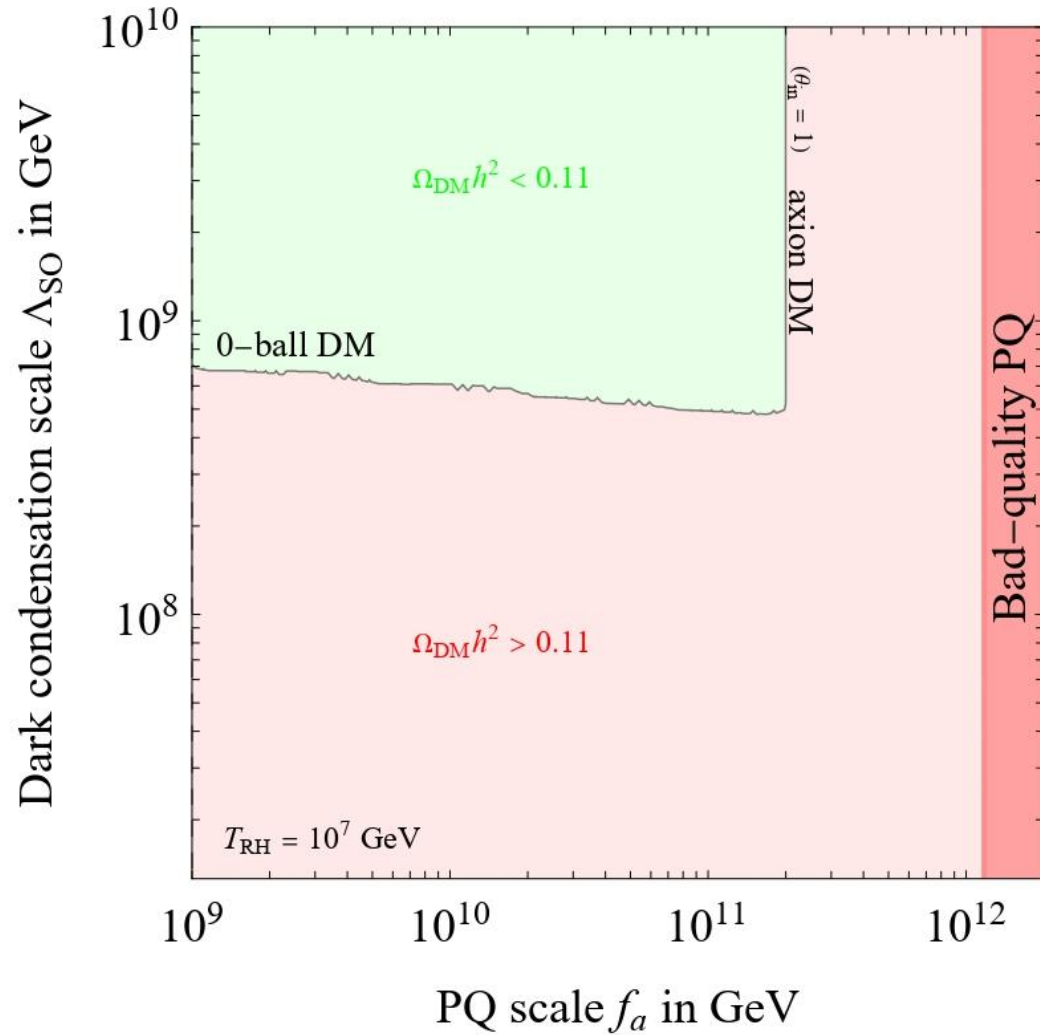
$$\longrightarrow M_{BS} \sim \Lambda_{SO} \sim 10^{8,9} \text{ GeV}$$

Dark Matter is made by **axion** and/or heavy **gauge bound states**

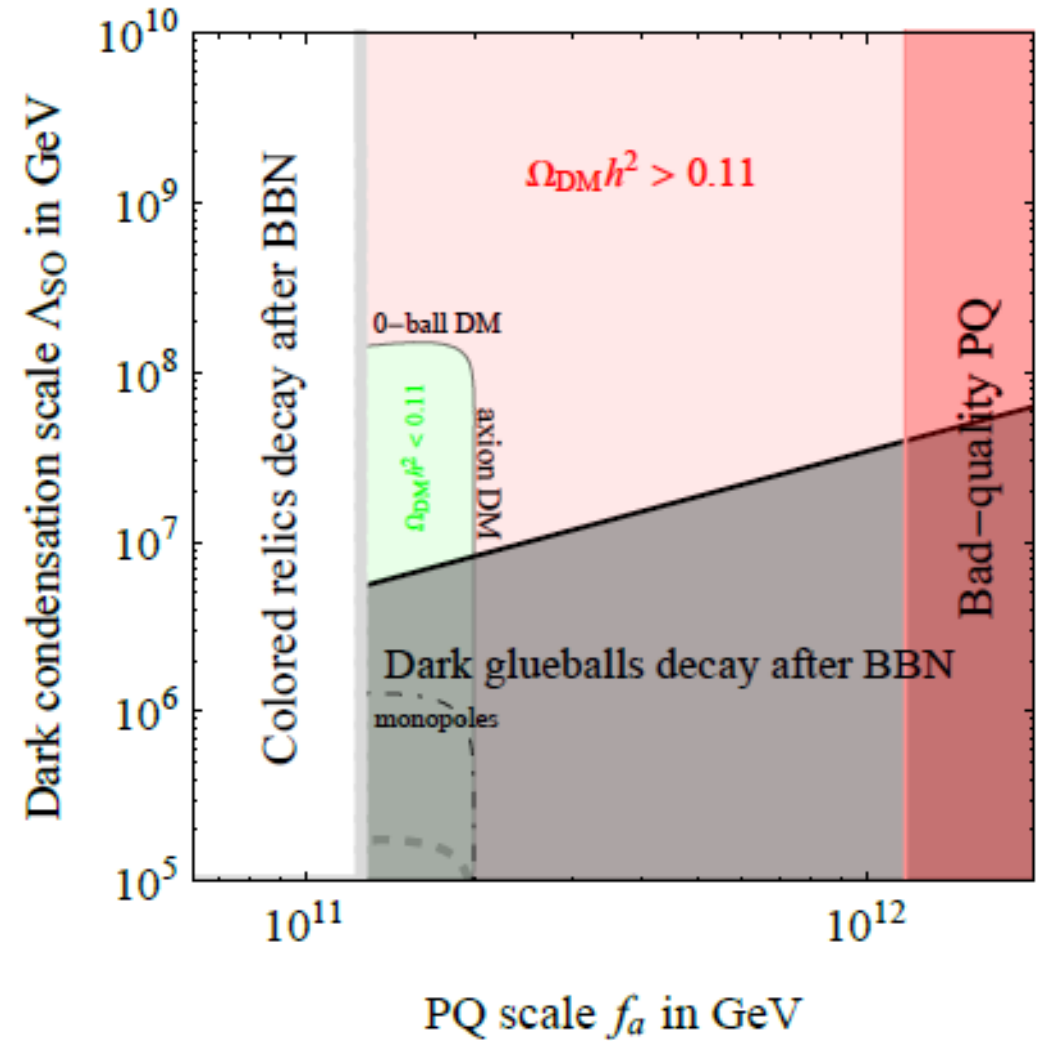


Dark Matter

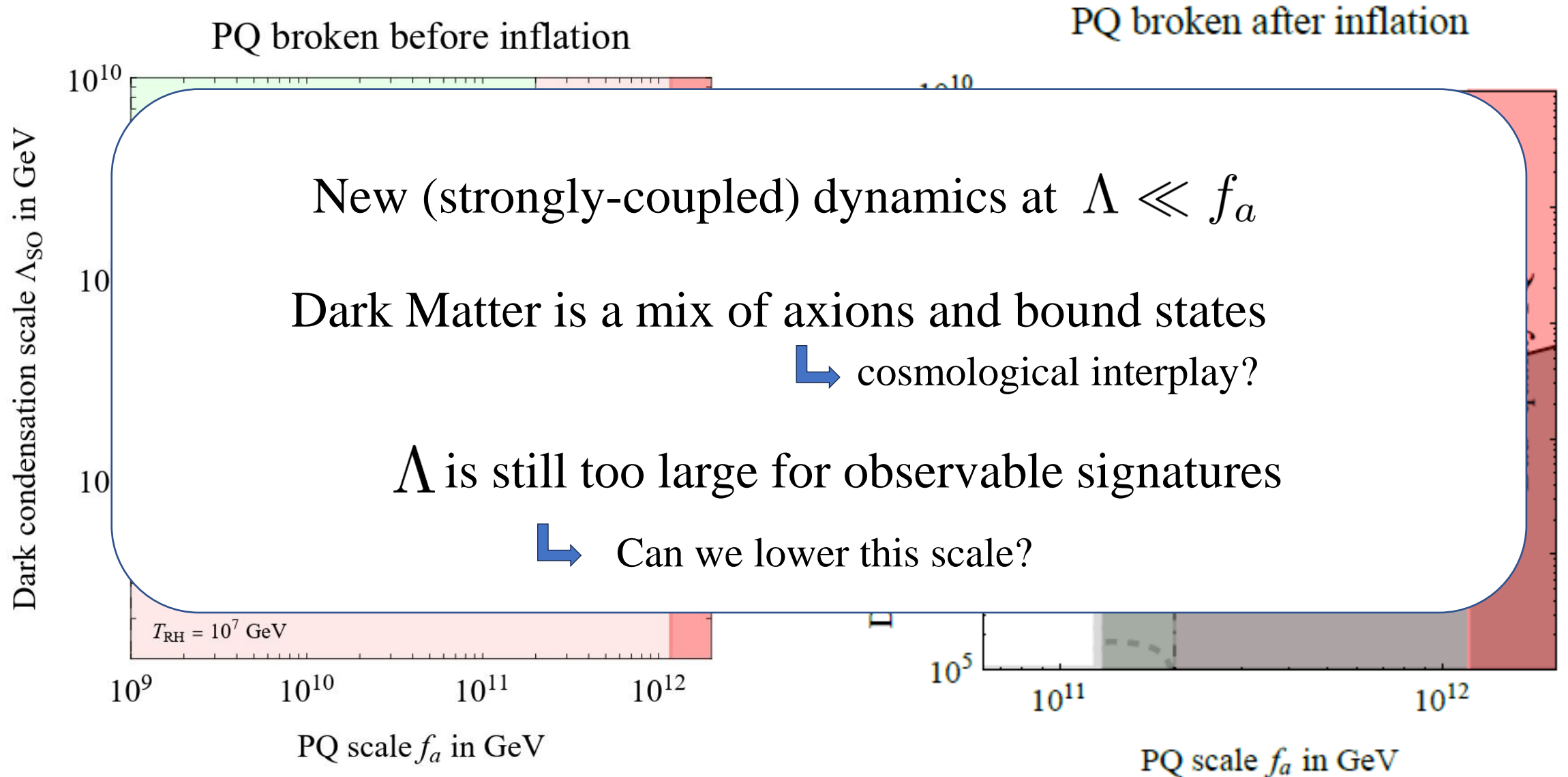
PQ broken before inflation



PQ broken after inflation



Dark Matter



Conclusions

The Peccei-Quinn solution has a problem of *UV sensitivity*



New gauge symmetry can protect from Peccei-Quinn breaking



We should systematically investigate directions which provide characteristic features and signatures

axion-flavor connection, Dark Matter, GUT, light flavoured particles, new dynamics at low scale, ...

Backup slides

SU(N) reminder (1)

SU(N) gauge group



Representations

Fundamental (N)

$$\phi_{\underline{i}} \rightarrow U_i^j \phi_j$$

Anti-fundamental (\bar{N})

$$\psi^{\bar{i}} \rightarrow U_j^i \psi^j$$

$$i, j = 1, \dots, N$$

Symmetric (NN)

$$S_{ij} \rightarrow U_i^k U_j^l S_{kl}$$

SU(N) reminder (2)

SU(N) gauge group

↓ Invariant tensors

Symmetric δ_i^j

Antisymmetric $\epsilon_{i_1 \dots i_N}$

↓ Gauge invariant operators

$$\phi^\dagger \phi = \delta_i^j \phi_i (\phi^*)^j$$

$$\epsilon_N \phi^N = \epsilon_{i_1, \dots, i_N} \phi_{i_1} \dots \phi_{i_N}$$

...

N-ality rule (1)

What are the dangerous operators?

Gauge-invariant

$$\delta_i^j, \epsilon_{i_1 \dots i_N}$$

PQ breaking

N-ality rule (1)

What are the dangerous operators?

Gauge-invariant

$$\delta_i^j, \epsilon_{i_1 \dots i_N}$$

PQ breaking

PQ charge is proportional to *N-ality*

$$\text{PQ}[\Phi_{\{i\}}^{\{j\}}] \propto \#\{i\} - \#\{j\}$$

N-ality rule (2)

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(\mathcal{N})$	$U(1)_{PQ}$	$U(1)_Q$	$U(1)_\mathcal{L}$
\mathcal{S}	0	0	1	1	$\mathcal{N}\mathcal{N}$	+1	0	0
\mathcal{Q}_L	1/2	$+Y_Q$	1	3	\mathcal{N}	+1/2	+1	0
\mathcal{Q}_R	1/2	$-Y_Q$	1	$\bar{3}$	\mathcal{N}	+1/2	-1	0
$\mathcal{L}_L^{1,2,3}$	1/2	$+Y_\mathcal{L}$	1	1	$\bar{\mathcal{N}}$	-1/2	0	+1
$\mathcal{L}_R^{1,2,3}$	1/2	$-Y_\mathcal{L}$	1	1	$\bar{\mathcal{N}}$	-1/2	0	-1

$$PQ[\mathcal{S}_{ij}] = \frac{1}{2} \times 2 = 1 \quad PQ[\mathcal{Q}_i] = \frac{1}{2} \times 1 = \frac{1}{2} \quad PQ[\mathcal{L}^i] = \frac{1}{2} \times (-1) = -\frac{1}{2}$$

N-ality rule (3)

What are the dangerous operators?

Gauge-invariant

$$\delta_i^j, \epsilon_{i_1 \dots i_N}$$

PQ breaking

PQ breaking = N-ality breaking

$$\#\{i\} - \#\{j\} \neq 0$$

PQ charge is proportional to *N-ality*

$$\text{PQ}[\Phi_{\{i\}}^{\{j\}}] \propto \#\{i\} - \#\{j\}$$

N-ality rule (3)

What are the dangerous operators?

Gauge-invariant

$$\delta_i^j, \epsilon_{i_1 \dots i_N}$$

PQ breaking

PQ breaking = N-ality breaking

$$\#\{i\} - \#\{j\} \neq 0$$

PQ charge is proportional to *N-ality*

$$\text{PQ}[\Phi_{\{i\}}^{\{j\}}] \propto \#\{i\} - \#\{j\}$$

Operators with indices contracted with δ_i^j tensors only can not break PQ

N-ality rule (3)

What are the dangerous operators?

Gauge-invariant

$$\delta_i^j, \epsilon_{i_1 \dots i_N}$$

PQ breaking

PQ breaking = N-ality breaking

$$\#\{i\} - \#\{j\} \neq 0$$

PQ charge is proportional to *N-ality*

$$\text{PQ}[\Phi_{\{i\}}^{\{j\}}] \propto \#\{i\} - \#\{j\}$$

PQ is only broken by operators containing at least one $\epsilon_{i_1 \dots i_N}$ tensor

(1) Solving the quality problem

What are the dangerous operators?

Gauge-invariant

PQ breaking

Lowest dimensional
has dimension N

$$\longrightarrow \det \mathcal{S} = \epsilon_{\mathcal{N}} \epsilon_{\mathcal{N}} \mathcal{S}^N / \Lambda_{UV}^{N-4}$$

If N is large enough the PQ quality problem is solved!

$$N \geq 12 \quad \text{for} \quad f_a \sim 10^{11} \text{ GeV}$$

Gauge invariance forbids PQ breaking below dimension N
and allows for *arbitrary high protection*

Decay of colored relics

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(\mathcal{N})$	$U(1)_{PQ}$	$U(1)_Q$	$U(1)_\mathcal{L}$
\mathcal{S}	0	0	1	1	$\mathcal{N}\mathcal{N}$	+1	0	0
Q_L	1/2	$+Y_Q$	1	3	\mathcal{N}	+1/2	+1	0
Q_R	1/2	$-Y_Q$	1	$\bar{3}$	\mathcal{N}	+1/2	-1	0
$\mathcal{L}_L^{1,2,3}$	1/2	$+Y_\mathcal{L}$	1	1	$\bar{\mathcal{N}}$	-1/2	0	+1
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EW charges allow the decay of heavy colored fermions in the Early Universe

Fermion number symmetries

$$U(1)_Q \otimes U(1)_\mathcal{L}$$

$$\mathcal{L}_{\text{yuk}} = y_Q Q_L \mathcal{S}^\dagger Q_R + y_\mathcal{L}^{ij} \mathcal{L}_L^i \mathcal{S} \mathcal{L}_R^j + h.c.$$

Decay of colored relics

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(\mathcal{N})$	$U(1)_{PQ}$	$U(1)_Q$	$U(1)_\mathcal{L}$
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\mathcal{Q}_L	1/2	$+Y_Q$	1	3	\mathcal{N}	+1/2	+1	0
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EW charges allow the decay of heavy colored fermions in the Early Universe

Fermion number symmetries

$$U(1)_Q \otimes U(1)_\mathcal{L} \rightarrow U(1)_{Q-\mathcal{L}}$$

Broken by dimension-6 operators

Decay of colored relics

Gauge invariance allows dimension-6 operators

$$(q_R \mathcal{Q})(e_R \mathcal{L}) \quad (q_R \mathcal{Q})(q'_R \mathcal{L})$$

$$\text{if } Y_{\mathcal{Q}} \pm Y_{\mathcal{L}} = \{-1/3, 2/3, -4/3\}$$

We assume that $m_{\mathcal{Q}} > m_{\mathcal{L}}$ ($m_{\mathcal{Q},\mathcal{L}} = y_{\mathcal{Q},\mathcal{L}} f_a$)

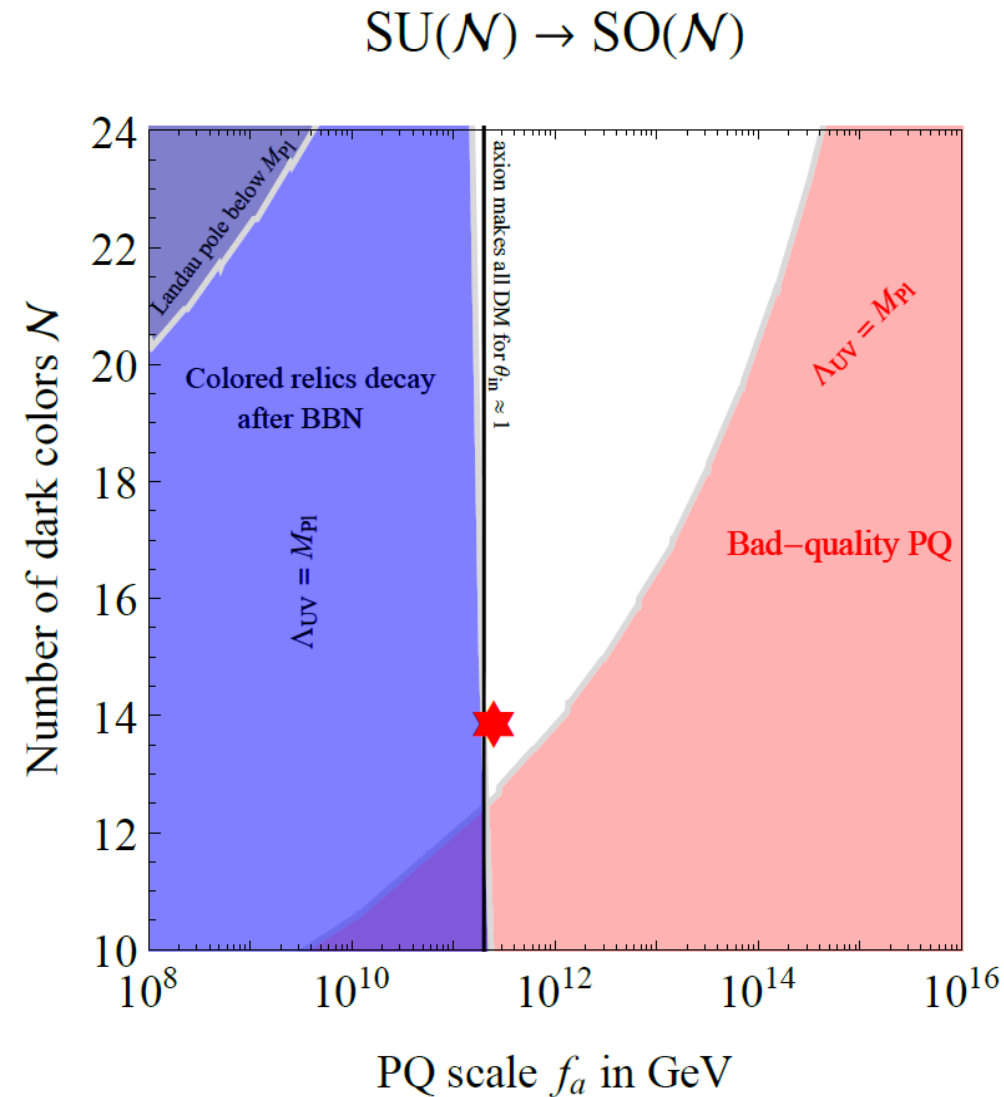
We require that decays of colored fermions occur before BBN

$$\Gamma_{\mathcal{Q}} \simeq \frac{1}{13 \text{ sec}} \left(\frac{m_{\mathcal{Q}}}{2 \times 10^{11} \text{ GeV}} \right)^5 \left(\frac{M_{\text{Pl}}}{\Lambda_{\text{UV}}} \right)^4 \quad f_a > \frac{1}{y_{\mathcal{Q}}} \sqrt{\frac{10}{\mathcal{N}}} \left(\frac{\Lambda_{\text{UV}}}{M_{\text{Pl}}} \right)^{4/5} \times 10^{11} \text{ GeV}$$

Parameter space

- Solution to the quality problem
- **Colored relics decay before BBN**

$$f_a > \frac{1}{y_Q} \sqrt{\frac{10}{\mathcal{N}}} \left(\frac{\Lambda_{\text{UV}}}{M_{\text{Pl}}} \right)^{4/5} \times 10^{11} \text{ GeV}$$



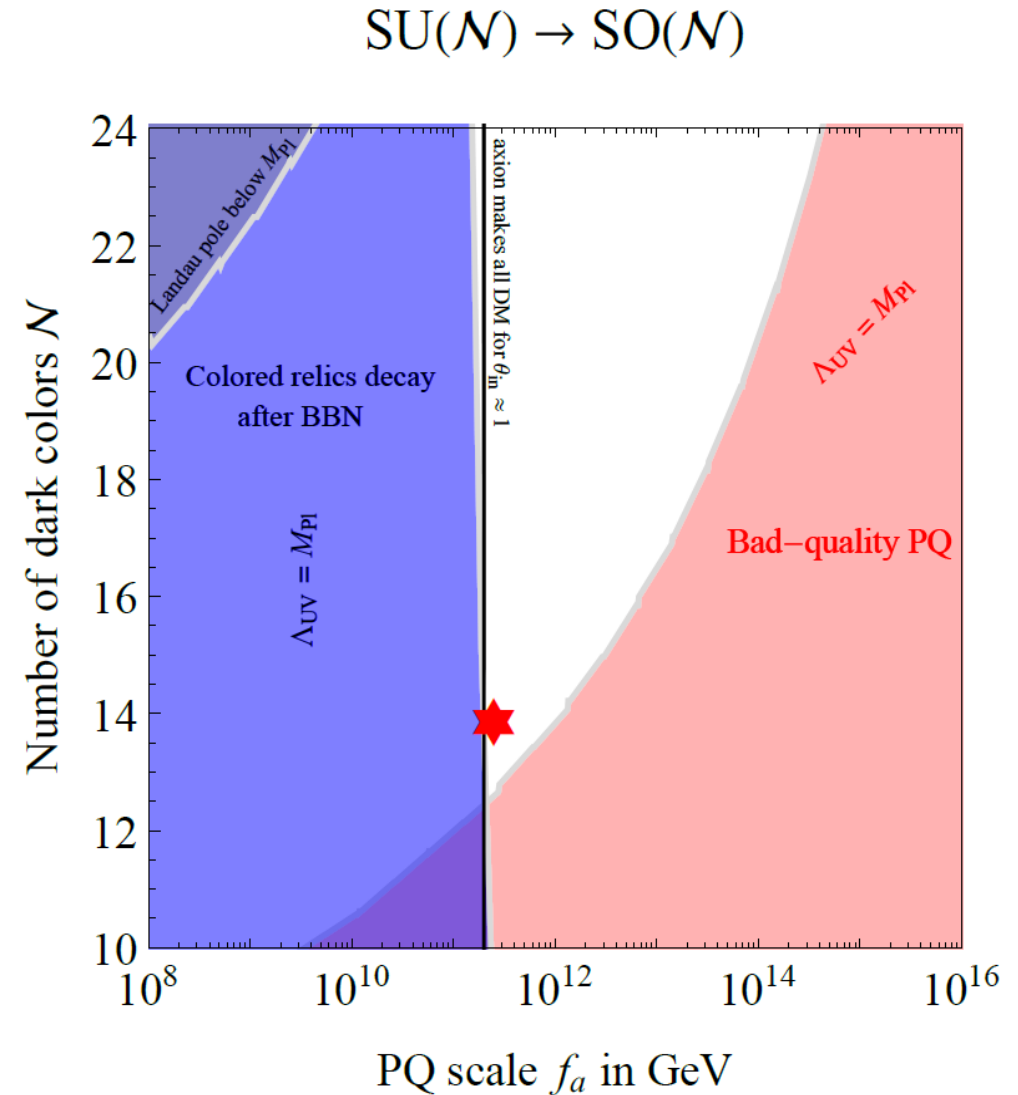
Parameter space

- Solution to the quality problem
- **Colored relics decay before BBN**

$$f_a > \frac{1}{y_Q} \sqrt{\frac{10}{\mathcal{N}}} \left(\frac{\Lambda_{\text{UV}}}{M_{\text{Pl}}} \right)^{4/5} \times 10^{11} \text{ GeV}$$

The bound only apply if PQ is broken after inflation

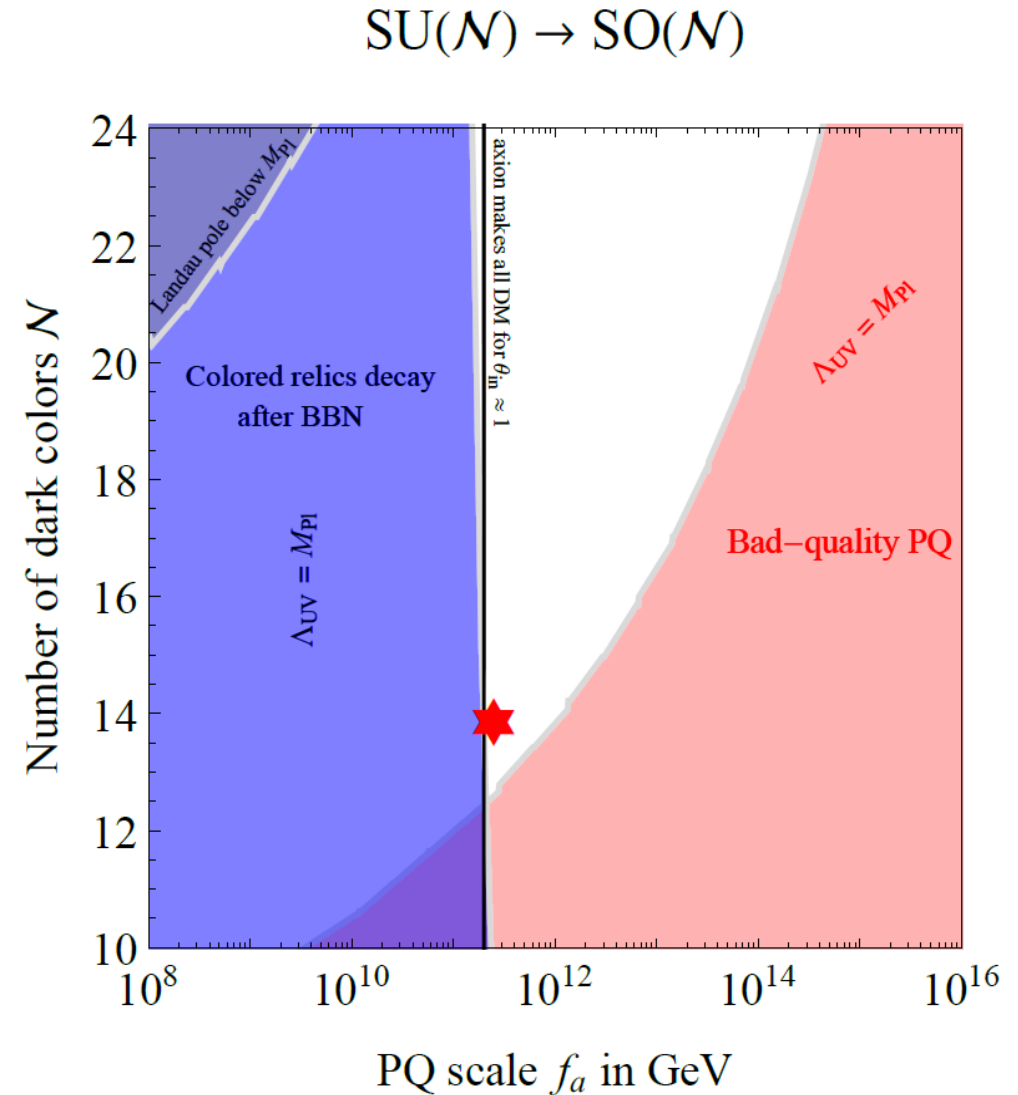
If PQ is broken before the colored relics are diluted by inflation! $m_Q \sim f_a$



Parameter space

- Solution to the quality problem
- Colored relics decay before BBN
- No Landau Poles below the cut-off scale

Fast decays of colored relics allow to enlarge the parameter space to the post-inflationary scenario



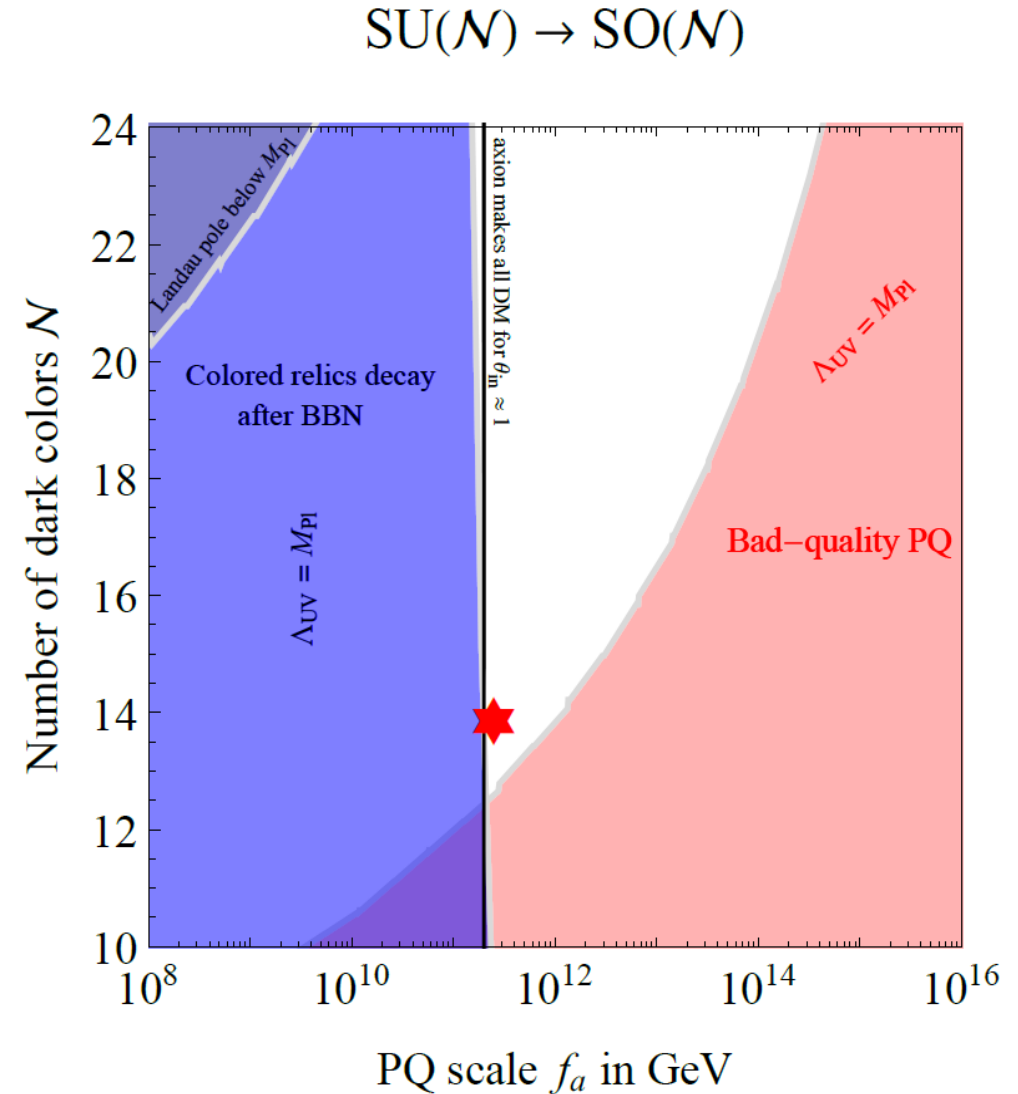
Parameter space

- Solution to the quality problem
- Colored relics decay before BBN
- No Landau Poles below the cut-off scale

$$\frac{dg_3^2}{d \ln \mu^2} = \frac{g_3^4}{(4\pi)^2} \left(-7 + \frac{2}{3}N \right)$$

$$\frac{dg_1^2}{d \ln \mu^2} = \frac{g_1^4}{(4\pi)^2} \left(\frac{41}{10} + \frac{12N}{5} (Y_{\mathcal{L}}^2 + Y_{\mathcal{Q}}^2) \right)$$

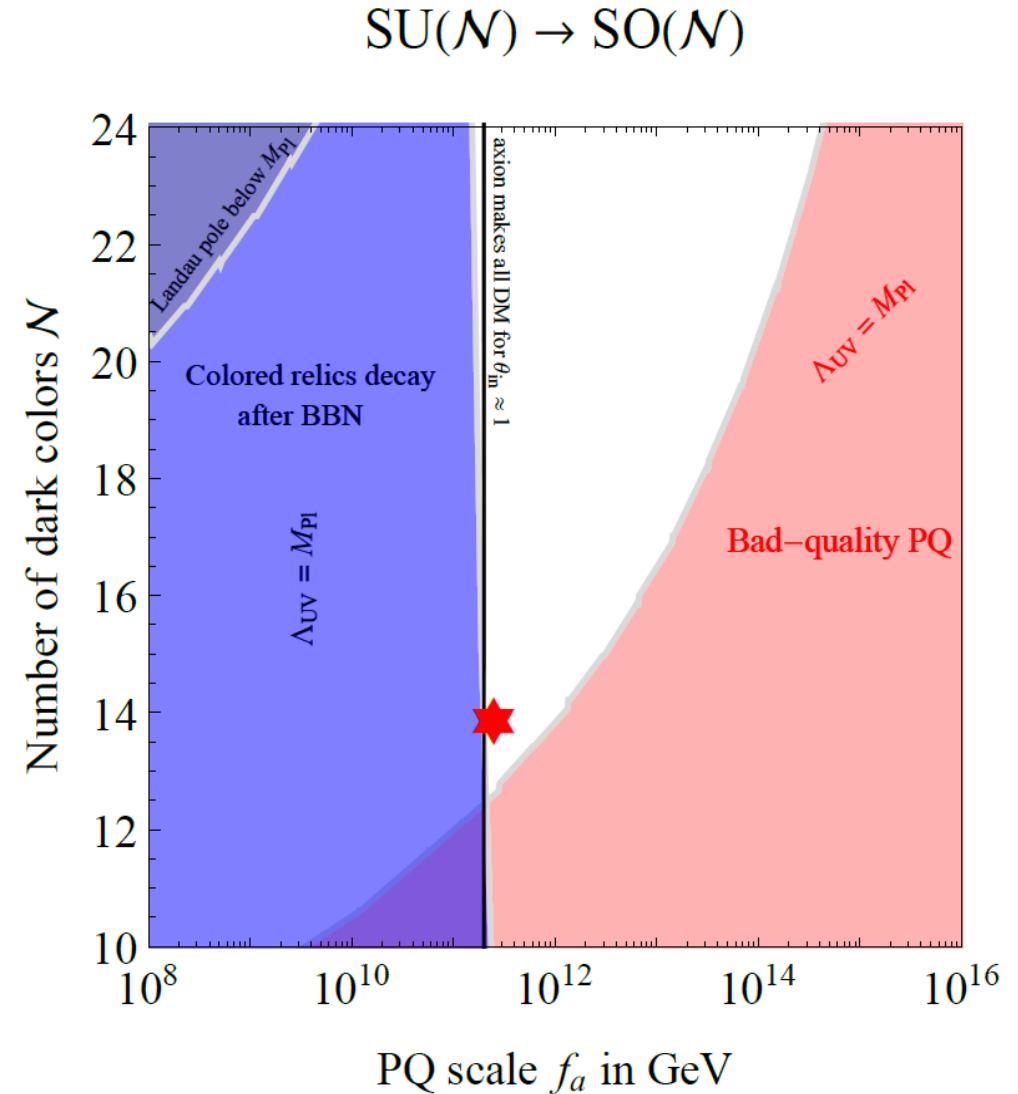
$$\mu > m_{\mathcal{Q}}, m_{\mathcal{L}}$$



Parameter space

- Solution to the quality problem
- Colored relics decay before BBN
- No Landau Poles below the cut-off scale

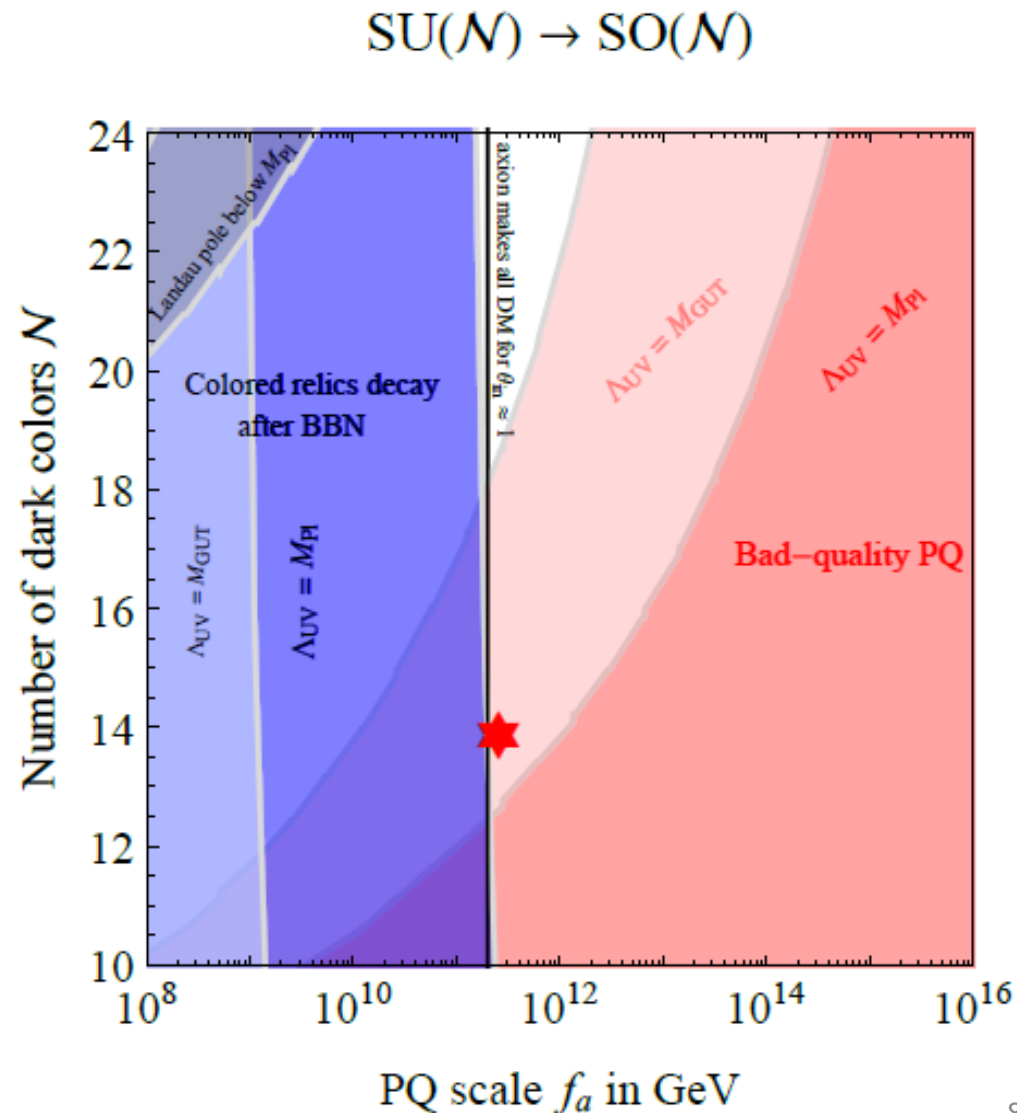
- We need $N > 12$ if $\Lambda_{UV} = M_{Pl}$



Parameter space

- Solution to the quality problem
- Colored relics decay before BBN
- No Landau Poles below the cut-off scale


- We need $N > 12$ if $\Lambda_{UV} = M_{Pl}$
- Larger values of N if the cut-off is lower



Group parity

Renormalizable SU(N) / SO(N) theories are invariant under a discrete symmetry

$$\mathcal{S}_{ij} \rightarrow (-1)^{\delta_{1i} + \delta_{1j}} \mathcal{S}_{ij} \quad \mathcal{A}_{ij} \rightarrow (-1)^{\delta_{1i} + \delta_{1j}} \mathcal{A}_{ij} \quad \mathcal{Q}_i \rightarrow (-1)^{\delta_{1i}} \mathcal{Q}_i$$


indices in group space

Reflection in group space along an arbitrary direction

Ex:

$$\begin{cases} \mathcal{Q}_1 \rightarrow -\mathcal{Q}_1 \\ \mathcal{Q}_j \rightarrow \mathcal{Q}_j \end{cases} \quad \begin{cases} \mathcal{A}_{11} \rightarrow \mathcal{A}_{11} \\ \mathcal{A}_{1j} \rightarrow -\mathcal{A}_{1j} \\ \mathcal{A}_{ij} \rightarrow \mathcal{A}_{ij} \end{cases}$$

Similar to parity but in group space (instead of space-time)

Group parity

Renormalizable $SU(N)$ / $SO(N)$ theories are invariant under a discrete symmetry

$$\mathcal{S}_{ij} \rightarrow (-1)^{\delta_{1i} + \delta_{1j}} \mathcal{S}_{ij} \quad \mathcal{A}_{ij} \rightarrow (-1)^{\delta_{1i} + \delta_{1j}} \mathcal{A}_{ij} \quad \mathcal{Q}_i \rightarrow (-1)^{\delta_{1i}} \mathcal{Q}_i$$

Reflection in group space along an arbitrary direction

The symmetry is broken by operators of dimension (at least) N



High protection for large N (similar to PQ)

Group parity

Renormalizable $SU(N)$ / $SO(N)$ theories are invariant under a discrete symmetry

$$\mathcal{S}_{ij} \rightarrow (-1)^{\delta_{1i} + \delta_{1j}} \mathcal{S}_{ij} \quad \mathcal{A}_{ij} \rightarrow (-1)^{\delta_{1i} + \delta_{1j}} \mathcal{A}_{ij} \quad \mathcal{Q}_i \rightarrow (-1)^{\delta_{1i}} \mathcal{Q}_i$$

Reflection in group space along an arbitrary direction

We can build $SO(N)$ singlets which are odd under group-parity

$$\mathbf{Ex:} \quad \epsilon_{\mathcal{N}} \mathcal{A}^{\mathcal{N}/2} = \epsilon_{i_1 \dots i_{\mathcal{N}}} \overbrace{\mathcal{A}_{i_1 i_2}}^{\text{SO(N) gauge boson}} \cdots \mathcal{A}_{i_{\mathcal{N}-1} i_{\mathcal{N}}}$$

It acts as a Z_2 symmetry : $\epsilon_{\mathcal{N}} \mathcal{A}^{\mathcal{N}/2} \rightarrow - \epsilon_{\mathcal{N}} \mathcal{A}^{\mathcal{N}/2}$

Stable states

The lightest bound states odd under group-parity are stable

	stable state
if N is odd	$\epsilon_N \mathcal{A}^{(N-1)/2} \mathcal{L}$.
if N is even	$\epsilon_N \mathcal{A}^{N/2}$

The typical mass is $M_{BS} \sim \Lambda_{SO}$

Production of composite DM

$$M_{\text{BS}} \approx \Lambda_{\text{SO}} \approx 10^8 \text{ GeV}$$

$$\frac{\Omega_{\text{DM}} h^2}{0.12} \approx \left(\frac{M_{\text{BS}}}{100 \text{ TeV}} \right)^2 \text{ if thermally produced}$$

The model provides 2 efficient suppression mechanisms!



Confinement after reheating

- BS are in thermal equilibrium
- Period of early matter domination (SO(N) glueballs)
- Late glueballs decay \rightarrow entropy injection

\rightarrow huge dilution of BS abundance



Confinement before reheating

Freeze-in production of BS is exponentially suppressed as

$$e^{-M_{\text{BS}}/T_{\text{RH}}}$$

Dark Matter

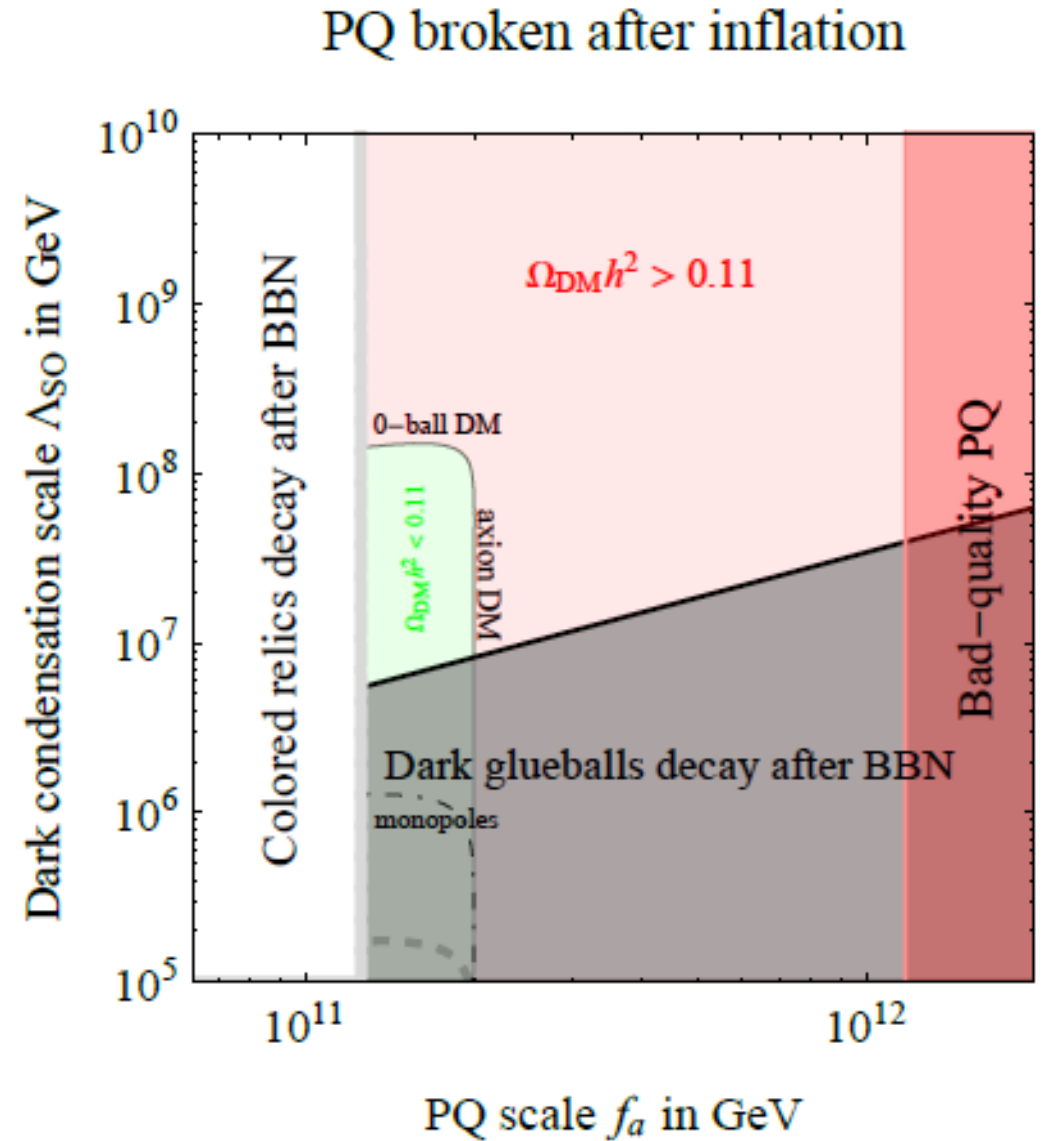
if axions are all DM

$$\theta_{\text{in}} \simeq 2.2 \quad \longrightarrow \quad f_a \simeq 2 \times 10^{11} \text{ GeV}$$

Long-lived glueballs dilute the stable bound states

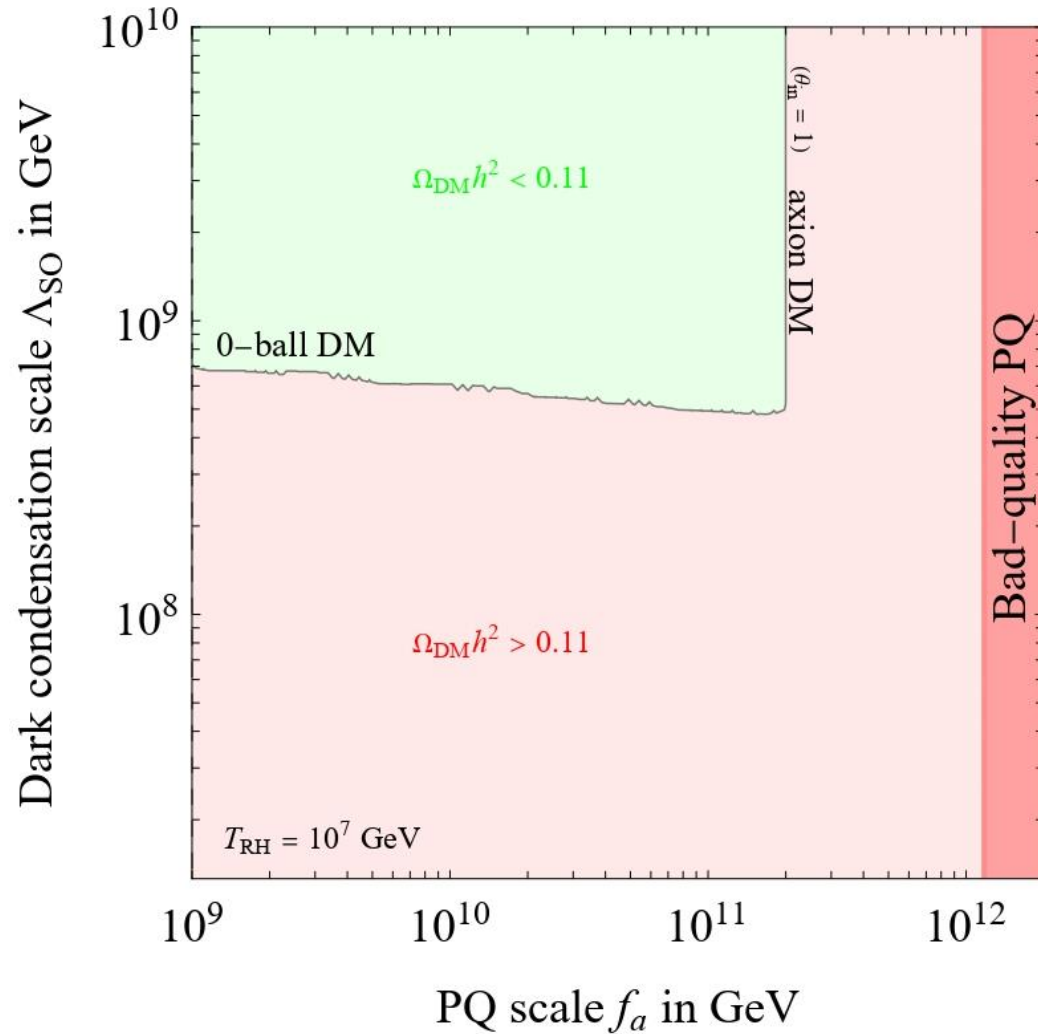
$$\frac{\Omega_{\text{DM}} h^2}{0.12} \approx \left(\frac{M_{\text{BS}}}{100 \text{ TeV}} \right)^2 \times D_{\text{DG}} \quad \text{if BS are all DM} \quad \longrightarrow \quad M_{\text{BS}} \simeq 10^8 \text{ GeV}$$

$$D_{\text{DG}} \sim \sqrt{\Gamma_{\text{DG}} M_{\text{Pl}}} / \Lambda_{\text{SO}}$$



Dark Matter

PQ broken before inflation



The axion abundance depends on initial condition

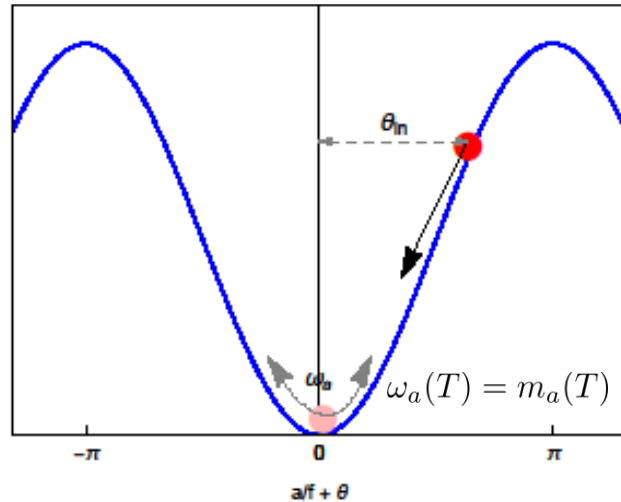
if axions are all DM $\longrightarrow f_a \simeq \theta_{\text{in}}^{12/7} \times 10^{11}$ GeV

Bound states are produced by freeze-in

if BS are all DM
 $Y_{\text{BS}} \propto e^{-2M_{\text{BS}}/T_{\text{RH}}} \longrightarrow M_{\text{BS}} \simeq 10^8$ GeV
 $T_{\text{RH}} = 10^7$ GeV

Axion production

$$\ddot{\theta}_{\text{eff}} + 3H\dot{\theta}_{\text{eff}} + m_a^2(T)\theta_{\text{eff}} = 0$$



$$H \propto T^2/M_{\text{Pl}}$$

$$m_a(T) \simeq m_a(T_c/T)^4$$

$$T_c \simeq 160 \text{ MeV}$$

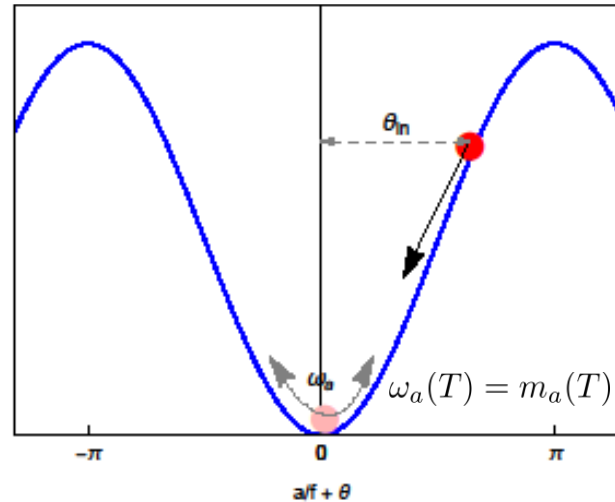
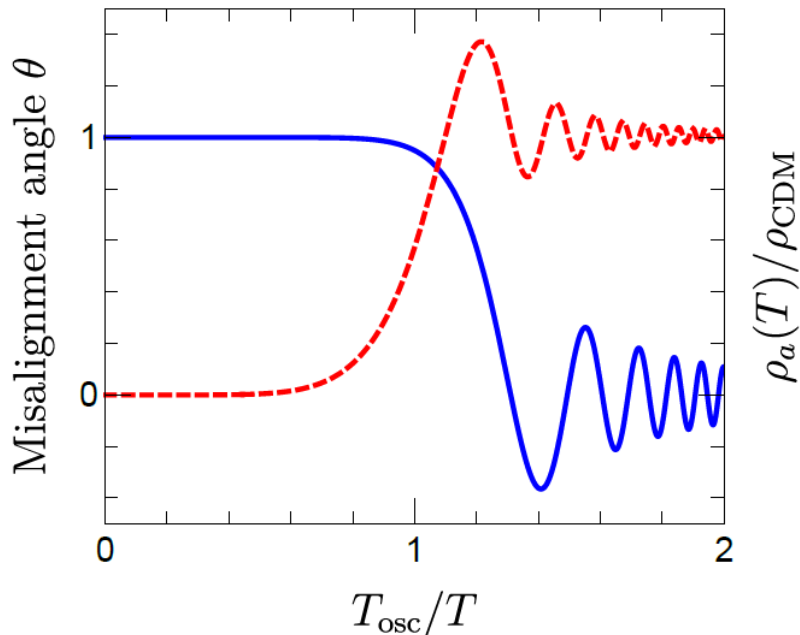
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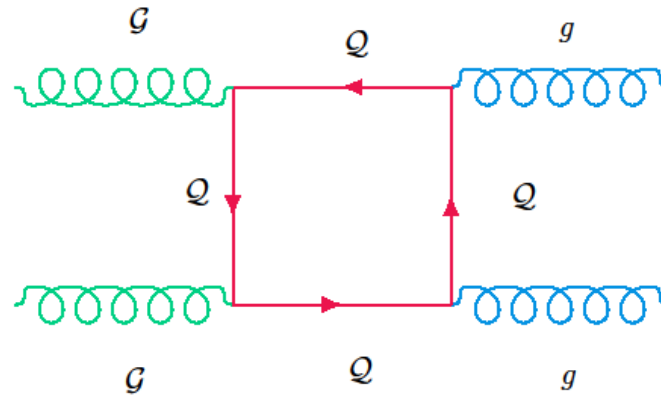
$$\frac{\Omega_a h^2}{0.12} \approx \theta_{\text{in}}^2 \left(\frac{f_a}{2 \times 10^{11} \text{ GeV}} \right)^{7/6}$$

SO(N) Glue-balls

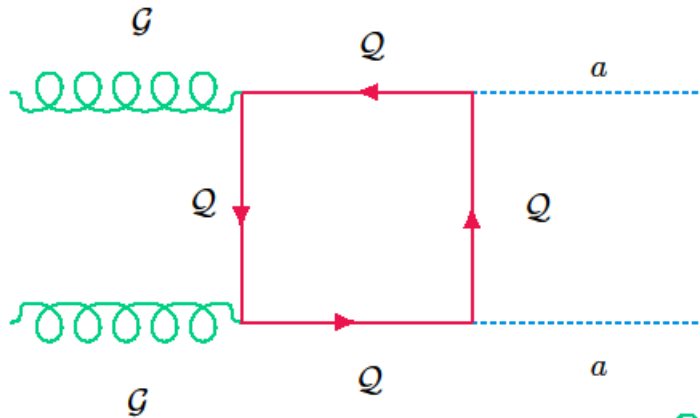
$$\text{Tr}[\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}]$$

$$M_{\text{DG}} = \mathcal{O}(1) \times \Lambda_{\text{SO}}$$

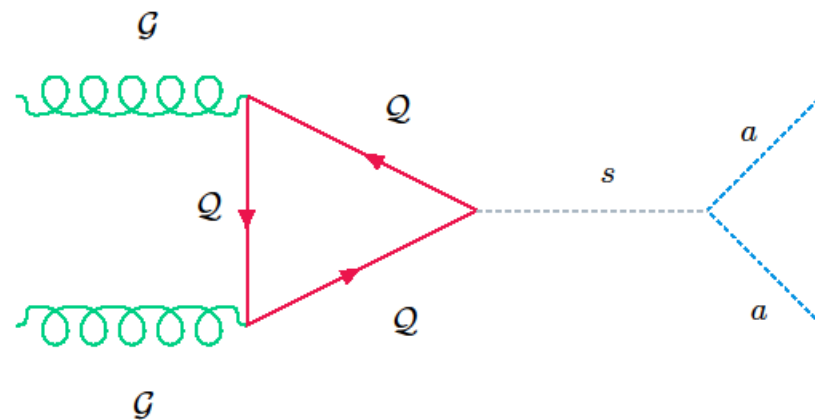
$$g_{\text{DG}} = N(N-1)$$



$$\Gamma_{\text{DG} \rightarrow gg} \simeq \frac{\alpha_{\text{DC}}^2 \alpha_3^2 M_{\text{DG}}^9}{m_Q^8}$$



$$\Gamma_{\text{DG} \rightarrow aa} \simeq \frac{\alpha_{\text{DC}}^2 M_{\text{DG}}^9}{m_{Q,\mathcal{L}}^8} \left(\frac{m_{Q,\mathcal{L}}}{4\pi f_a} \right)^2$$



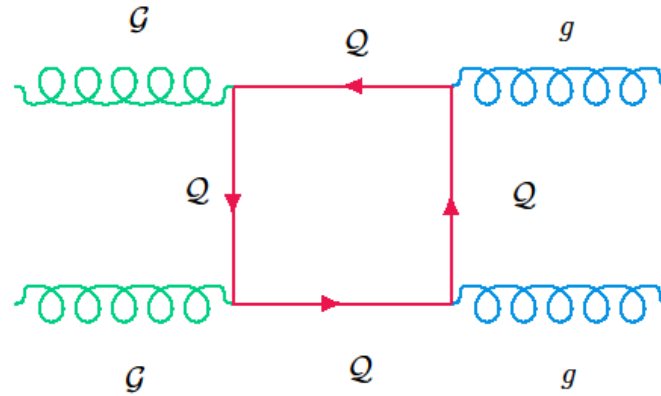
$$\Gamma_{\text{DG} \rightarrow aa} \simeq \frac{(18-7N)^2 \alpha_{\text{DC}}^2 M_{\text{DG}}^9}{512\pi^3 M_s^4 f_a^4}$$

SO(N) Glue-balls

$$\text{Tr}[\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}]$$

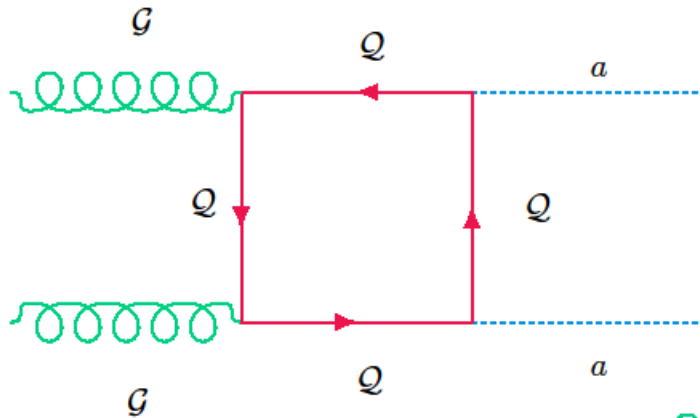
$$M_{\text{DG}} = \mathcal{O}(1) \times \Lambda_{\text{SO}}$$

$$g_{\text{DG}} = N(N-1)$$

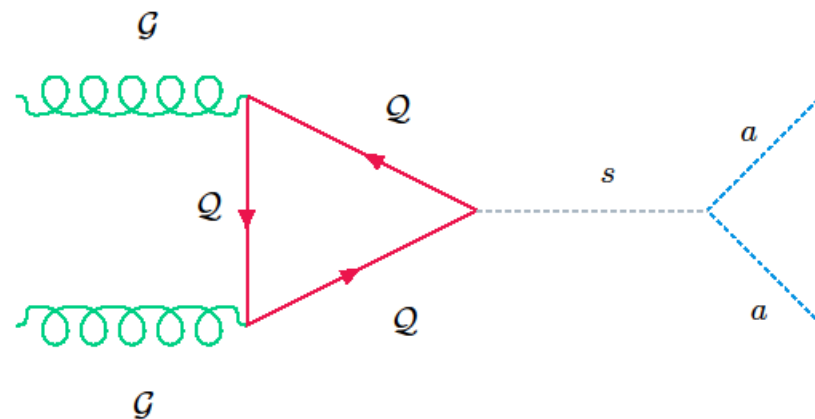


$$T_{\text{decay}} \sim (\Gamma_{\text{DG}}^2 M_{\text{Pl}}^2 / \Lambda_{\text{SO}})^{1/3}$$

$$T'_{\text{RH}} \sim T_{\text{decay}} (\Lambda_{\text{SO}} / T_{\text{decay}})^{1/4}$$



$$D_{\text{DG}} = \left(\frac{T_{\text{decay}}}{T'_{\text{RH}}} \right)^3 = \left[1 + \frac{g_{\text{DG}}}{g_{\text{SM}}} \left(\frac{\Lambda_{\text{SO}}^2}{\Gamma_{\text{DG}} M_{\text{Pl}}} \right)^{2/3} \right]^{-3/4}$$



↓

$$D_{\text{DG}} \sim \sqrt{\Gamma_{\text{DG}} M_{\text{Pl}} / \Lambda_{\text{SO}}}$$

Axion photon coupling

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_{\text{em}} C_{a\gamma}}{8\pi f_a} a F^{\mu\nu} \tilde{F}_{\mu\nu},$$

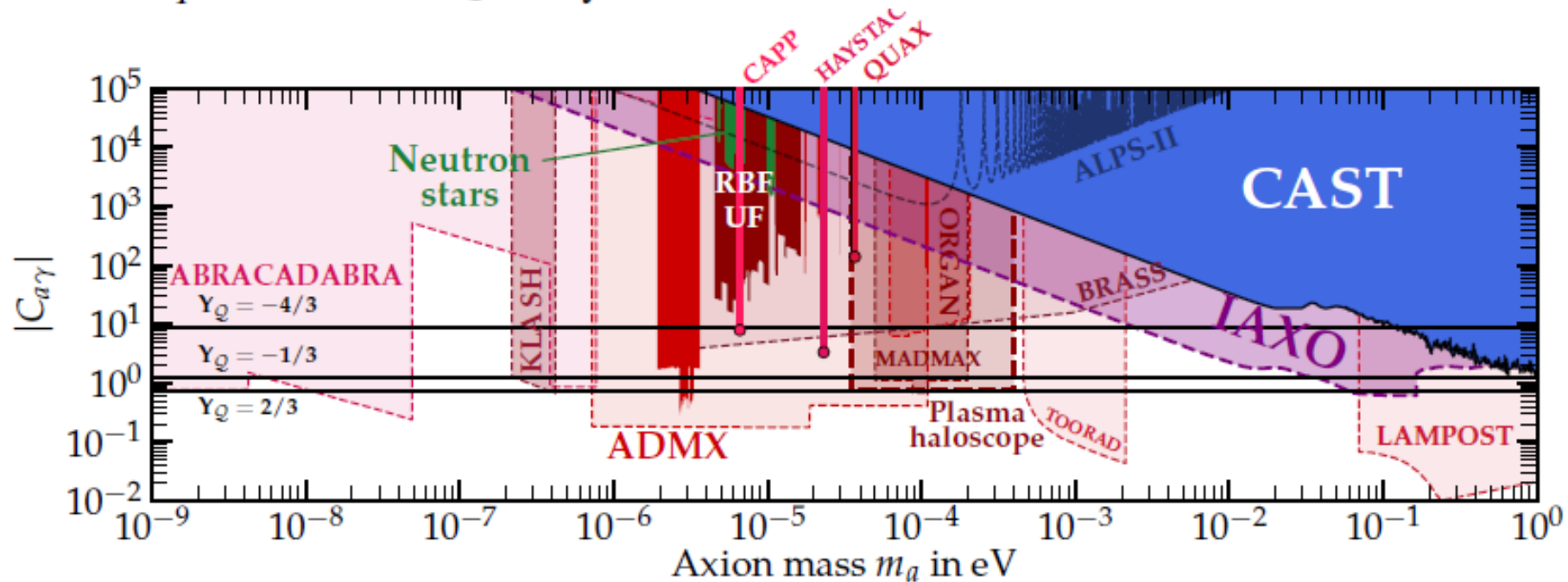
$$C_{a\gamma} = 6(Y_Q^2 - Y_L^2) - 1.92(4)$$

\downarrow
 model dependent
 $U(1)_{\text{PQ}} U(1)_Y^2$

\downarrow
 model independent
 $\pi_0 - a$ mixing

$Y_L = 0$ is needed to avoid charged relics

$Y_Q = \frac{\{2, -1, -4\}}{3}$ is required to allow Q decays

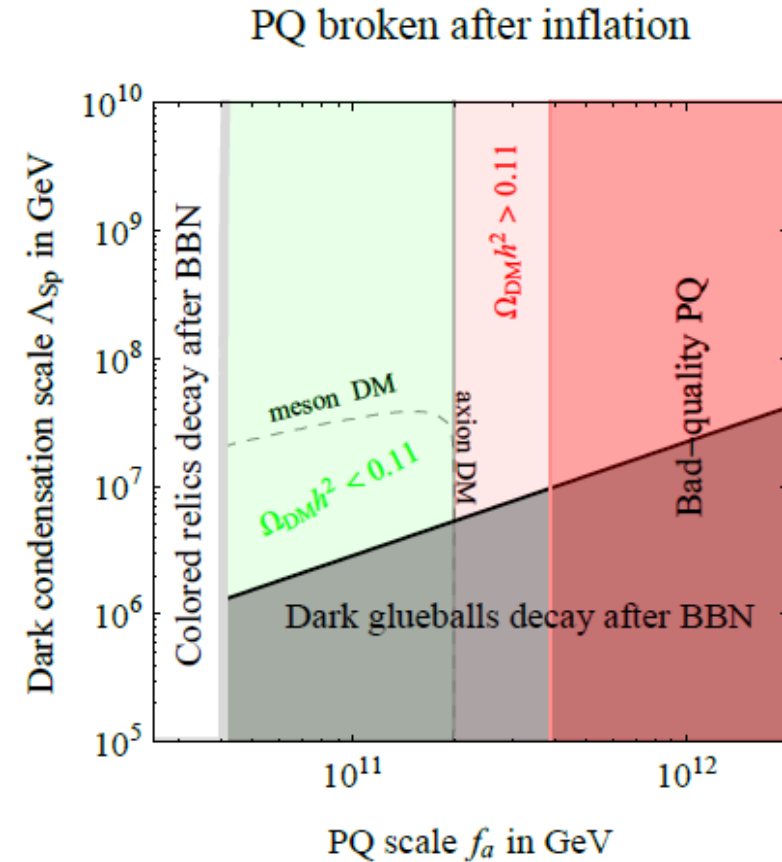
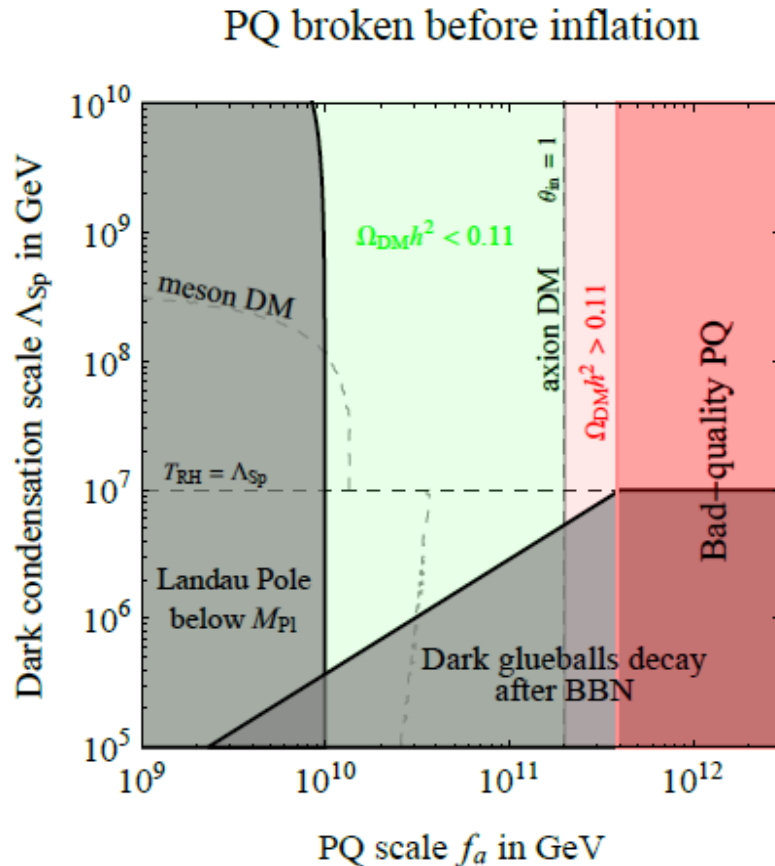


Scalar in antisymmetric of $SU(N)$

Same idea but $SU(N)$ is broken to $Sp(N)$

PQ is broken by dimension $N/2$ operator $\epsilon_{\mathcal{N}} S^{\mathcal{N}/2}$

DM: axions (+ mesons $\mathcal{L}_{\gamma\mathcal{N}}\mathcal{L}$)



Lagrangian and symmetries

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{yuk}} - V(\mathcal{S})$$

$$\mathcal{L}_{\text{yuk}} = y_Q Q_L \mathcal{S}^\dagger Q_R + y_{\mathcal{L}}^{ij} \mathcal{L}_L^i \mathcal{S} \mathcal{L}_R^j + h.c.$$



$$U(1)_Q$$

$$\text{if } Y_{\mathcal{L}} \neq 0$$

$$U(1)_{\mathcal{L}}$$

$$Q_{L(R)} \rightarrow e^{(-)i\alpha} Q_{L(R)}$$

$$\mathcal{L}_{L(R)} \rightarrow e^{(-)i\beta} \mathcal{L}_{L(R)}$$

$$V(\mathcal{S}) = M_S^2 \text{Tr} [\mathcal{S}^\dagger \mathcal{S}] + \lambda_S \text{Tr} [(\mathcal{S}^\dagger \mathcal{S})^2] + \lambda'_S \text{Tr} [\mathcal{S}^\dagger \mathcal{S} \mathcal{S}^\dagger \mathcal{S}] - \lambda_{HS} (H^\dagger H) \text{Tr} [\mathcal{S}^\dagger \mathcal{S}]$$

Lagrangian and symmetries

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{yuk}} - V(\mathcal{S})$$

$$\mathcal{L}_{\text{yuk}} = y_Q Q_L \mathcal{S}^\dagger Q_R + y_{\mathcal{L}}^{ij} \mathcal{L}_L^i \mathcal{S} \mathcal{L}_R^j + h.c.$$



$$U(1)_Q$$

$$\text{if } Y_{\mathcal{L}} = 0$$

$$Z_2$$

No distinction between L and R

$$Q_{L(R)} \rightarrow e^{(-)i\alpha} Q_{L(R)}$$

$$\mathcal{L} \rightarrow -\mathcal{L}$$

$$V(\mathcal{S}) = M_S^2 \text{Tr} [\mathcal{S}^\dagger \mathcal{S}] + \lambda_S \text{Tr} [(\mathcal{S}^\dagger \mathcal{S})^2] + \lambda'_S \text{Tr} [\mathcal{S}^\dagger \mathcal{S} \mathcal{S}^\dagger \mathcal{S}] - \lambda_{HS} (H^\dagger H) \text{Tr} [\mathcal{S}^\dagger \mathcal{S}]$$

Spectrum

$$\mathcal{S} = \left[\left(w + \frac{s}{\sqrt{2\mathcal{N}}} \right) \text{diag}(1, \dots, 1) + (\tilde{s}^b + i\tilde{a}^b) T_{\text{real}}^b \right] e^{\frac{ia}{\sqrt{2\mathcal{N}}w}}$$

- $\mathcal{N}(\mathcal{N} - 1)/2$ massless vectors \mathcal{A}^a in the adjoint of $\text{SO}(\mathcal{N})$.
- $\mathcal{N}(\mathcal{N} + 1)/2 - 1$ vectors \mathcal{W}^b in the traceless symmetric of $\text{SO}(\mathcal{N})$, that acquire a squared mass $M_{\mathcal{W}}^2 = 4\mathfrak{g}^2 w^2$ eating the Goldstone bosons \tilde{a}^b .
- The massless scalar a , singlet under $\text{SO}(\mathcal{N})$. In view of its QCD anomalies it can be called axion and its decay constant will be $f_a = w/\sqrt{\mathcal{N}/2}$.
- the scalar s , singlet under $\text{SO}(\mathcal{N})$. If symmetry breaking arises through the Coleman-Weinberg mechanism it is light with squared mass $M_s^2 = 6(\mathcal{N}\lambda_S + \lambda'_S)w^2$.
- $\mathcal{N}(\mathcal{N} + 1)/2 - 1$ scalars \tilde{s}^b with squared mass $M_{\tilde{s}}^2 = 2(\mathcal{N}\lambda_S + 3\lambda'_S)w^2$.
- One colored Dirac quark $\Psi_Q = (\mathcal{Q}_L, \bar{\mathcal{Q}}_R)^T$ with mass $M_Q = y_Q w$ in the fundamental representation of $\text{SO}(\mathcal{N})$ charged under the accidental global $\text{U}(1)_Q$.
- Three Dirac leptons $\Psi_{\mathcal{L}}^i = (\mathcal{L}_L^i, \bar{\mathcal{L}}_R^i)^T$ with masses $M_{\mathcal{L}^i} = y_{\mathcal{L}}^i w$ in the fundamental of $\text{SO}(\mathcal{N})$ charged under the accidental global $\text{U}(1)_{\mathcal{L}_i}$ if $Y_{\mathcal{L}} \neq 0$. If $Y_{\mathcal{L}} = 0$ one instead gets six Majorana leptons $\Psi_{\mathcal{L}}^i = (\mathcal{L}^i, \bar{\mathcal{L}}^i)^T$ with masses $M_{\mathcal{L}^i} = y_{\mathcal{L}}^i w$ in the fundamental of $\text{SO}(\mathcal{N})$ which transform as $\Psi_{\mathcal{L}}^i \rightarrow -\Psi_{\mathcal{L}}^i$ under the accidental \mathbb{Z}_2 symmetry.

Spectrum

$$\Lambda_{\text{SO}} \approx f_a \exp \left[-\frac{6\pi}{11(\mathcal{N} - 2)\alpha_{\text{DC}}(f_a)} \right]$$

- If \mathcal{N} is even the lightest baryon is the 0-ball $\epsilon_{\mathcal{N}}\mathcal{A}^{N/2}$ made of SO gluons only, stable thanks to U-parity (see e.g. [18]). On the other hand, the lighter baryons containing fermions

$$\epsilon_{\mathcal{N}}\mathcal{A}^{(\mathcal{N}-2)/2}\mathcal{Q}\mathcal{Q}, \quad \epsilon_{\mathcal{N}}\mathcal{A}^{(\mathcal{N}-2)/2}\mathcal{L}\mathcal{L}, \quad \epsilon_{\mathcal{N}}\mathcal{A}^{(\mathcal{N}-2)/2}\mathcal{Q}\mathcal{L} \quad (35)$$

can decay respecting U-parity into the 0-ball plus the corresponding lighter mesons $\mathcal{Q}\mathcal{Q}$, $\mathcal{L}\mathcal{L}$, $\mathcal{Q}\mathcal{L}$. Such mesons are stable in the limit of exact $U(1)_{\mathcal{Q},\mathcal{L}}$ symmetries.

- If \mathcal{N} is odd the lightest baryons contain one fermion (and thereby dubbed 1-ball)

$$\epsilon_{\mathcal{N}}\mathcal{A}^{(\mathcal{N}-1)/2}\mathcal{Q}, \quad \epsilon_{\mathcal{N}}\mathcal{A}^{(\mathcal{N}-1)/2}\mathcal{L} \quad (36)$$

are stable if the fermion \mathcal{Q} and/or \mathcal{L} is stable.

Stable states

Protected by U-parity or/and global U(1)

	if $Y_{\mathcal{L}} \neq 0$	if $Y_{\mathcal{L}} = 0$
if N is odd	$\epsilon_N \mathcal{A}^{(N-1)/2} \mathcal{L}$ and $\mathcal{L}\mathcal{L}$	$\epsilon_N \mathcal{A}^{(N-1)/2} \mathcal{L}$.
if N is even	$\epsilon_N \mathcal{A}^{N/2}$ and $\mathcal{L}\mathcal{L}$	$\epsilon_N \mathcal{A}^{N/2}$

To avoid charged relics we only consider

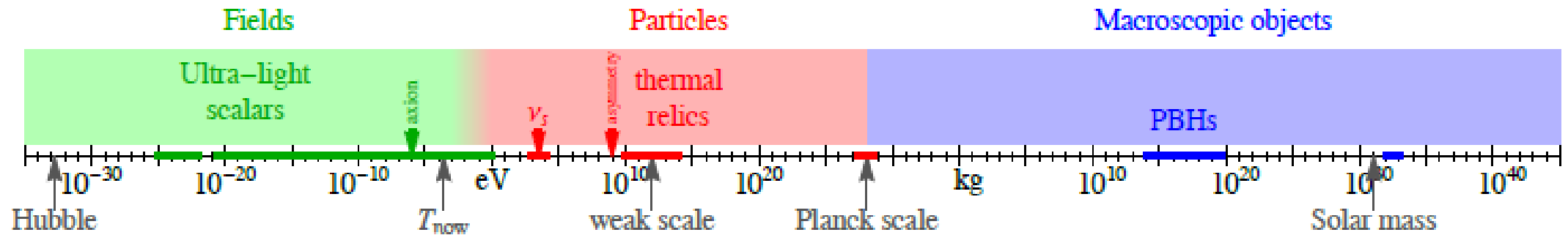
$$Y_{\mathcal{L}} = 0$$

With this choice the global U(1) is replaced by

$$Z_2 : \mathcal{L} \rightarrow -\mathcal{L}$$

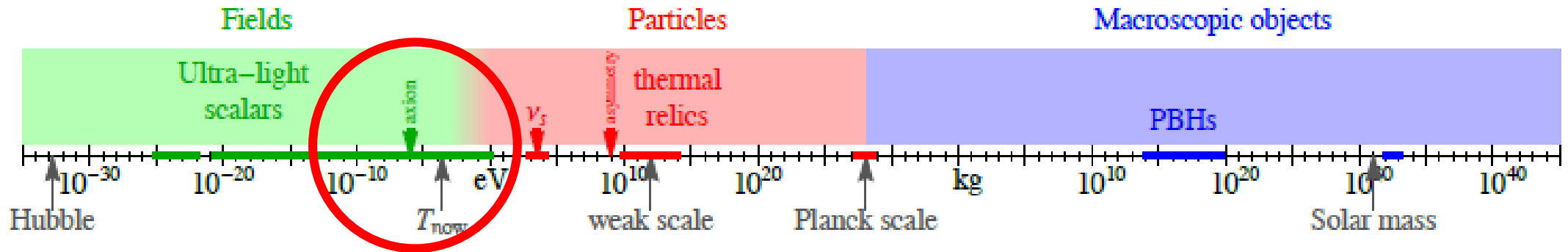
Dark Matter

The mass range of DM candidates spans over 80 order of magnitudes



Dark Matter

The mass range of DM candidates spans over 80 order of magnitudes



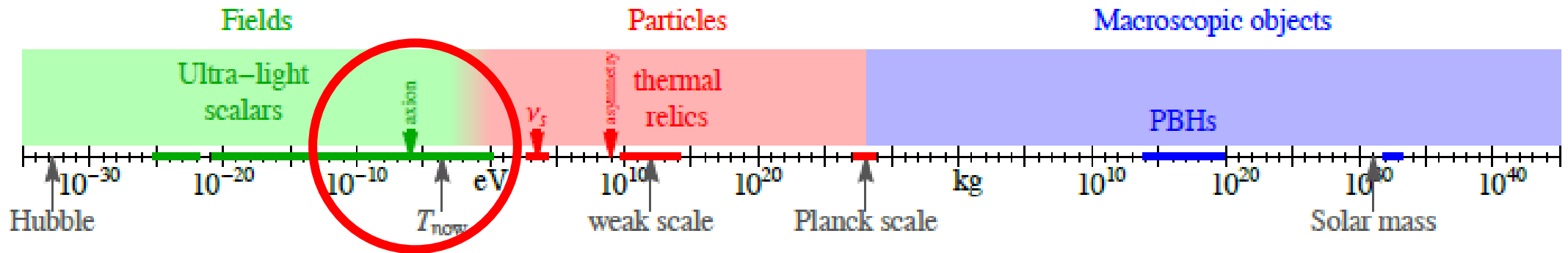
Axions are excellent ultra-light DM candidates

→ production: vacuum misalignment mechanism

$$f_a \simeq 10^{11} \text{ GeV}$$

Dark Matter

The mass range of DM candidates spans over 80 order of magnitudes



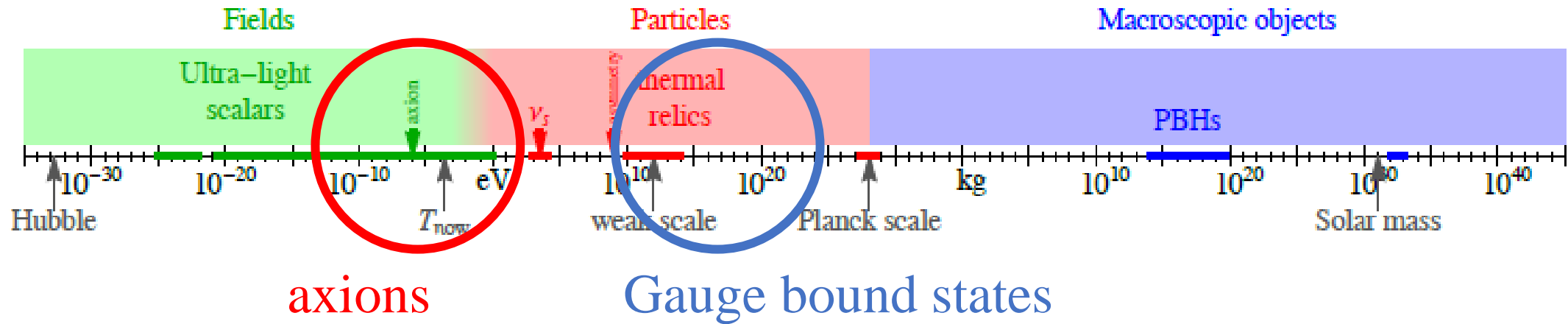
Axions are excellent ultra-light DM candidates

→ production: vacuum misalignment mechanism

$$f_a \simeq 10^{11} \text{ GeV}$$

Axion are DM...but there are other stable states which contribute!

Dark Matter



Dark Matter is either dominated by one state (**single-component**)
or a mixture of the two (**multi-component**)