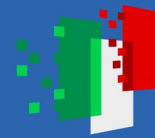




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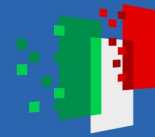
# The axion flavour connection

L.Darmé, E.Nardi, C.Smarra  
[arxiv.org/abs/2211.05796](https://arxiv.org/abs/2211.05796)

Speaker: Clemente Smarra

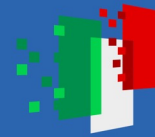
Kickoff Meeting AxionOrigins, Frascati, Italy,  
25/01/2024





## Outline

- Strong CP problem in one slide
- Benchmark axion solutions:
  - Focus on the origin and ~~quality~~ (pew...see Giacomo's talk) issues
- The Axion-Flavour connection
  - Cook our recipe: a simple (simplistic?) realisation



## Strong CP problem

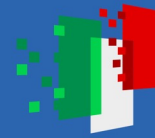
- The QCD Lagrangian contains a CP-violating term

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f (i\not{D} - m_f) \psi_f - \frac{1}{4} (G_{\mu\nu}, G^{\mu\nu}) + \theta \frac{1}{32\pi^2} (G_{\mu\nu}, \tilde{G}^{\mu\nu})$$

- However, experimental bounds indicate that, in strong interactions,
- How can we explain this ? Anthropic reasoning? No..
  - Mechanism that constrains  $\theta \lesssim 10^{-10}$  or drives it to 0.

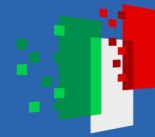
$$\theta \lesssim 10^{-10}$$





## Benchmark Axion Models

- Different ways of implementing the PQ mechanism can be organised in three large classes:
  - The Peccei-Quinn-Weinberg-Wilczek (PQWW) model.
  - The Dine-Fishler-Srednicki-Zhitnitsky (DFSZ) model;
  - The Kim-Shifman-Vainshtein-Zakharov (KSVZ) model.



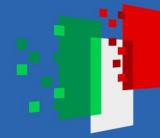
## PQWW Model

- In the PQWW model, the SM is extended by adding a new complex scalar, namely a second Higgs doublet. The Lagrangian contains:

$$\mathcal{L} \supset -y_u \bar{q}_L H_u u_R - y_d \bar{q}_L H_d d_R - V(H_u, H_d) + \text{h.c.}$$

$$V = \frac{\lambda_u}{4} \left( |H_u|^2 - \frac{v_u^2}{2} \right)^2 + \frac{\lambda_d}{4} \left( |H_d|^2 - \frac{v_d^2}{2} \right)^2 + \lambda_{ud} (H_u^\dagger H_d) (H_d^\dagger H_u) + \dots$$

- One could naively say that, since there are two Higgs fields, there are two independent symmetries, which can be redefined to obtain the hypercharge and an orthogonal accidental Peccei-Quinn symmetry. Is it true? NO!

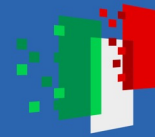


## PQWW Model

- The general two Higgs doublet model potential, in fact, can be written as

$$V(H_u, H_d) = m_u^2 H_u^\dagger H_u + m_d^2 H_d^\dagger H_d - \left[ m_{ud}^2 H_u^\dagger H_d + \text{h.c.} \right] + \frac{1}{2} \lambda_1 (H_u^\dagger H_u)^2 + \frac{1}{2} \lambda_2 (H_d^\dagger H_d)^2 + \lambda_3 (H_u^\dagger H_u) (H_d^\dagger H_d) + \lambda_4 (H_u^\dagger H_d) (H_d^\dagger H_u) + \left[ \frac{1}{2} \lambda_5 (H_u^\dagger H_d)^2 + \lambda_6 (H_u^\dagger H_u) (H_u^\dagger H_d) + \lambda_7 (H_d^\dagger H_d) (H_u^\dagger H_d) + \text{h.c.} \right],$$

- Therefore, if we want a PQ symmetry, we must *impose* that some terms are absent. *No accidental PQ symmetry*



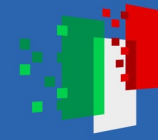
## ACHTUNG!

In general, all the afore-mentioned axion models DO NOT feature an accidental PQ symmetry.

But why do we focus on the fact that the PQ symmetry is not accidental? For two reasons:

- Global symmetries are widely believed not to be fundamental in QFT;
- Being anomalous,  $U(1)_{PQ}$  is not a symmetry of the quantum world.



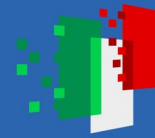


## Origin and quality of the PQ symmetry

Therefore, we should account for the origin of the PQ symmetry. It would be desirable that it came accidentally, as a result of imposing “sacred” principles: Lorentz and gauge invariance

Moreover, experimental bounds constrain  $\theta$ , so  $\theta < 10^{-10}$  symmetry must be highly protected. This is commonly referred to as the PQ quality issue.



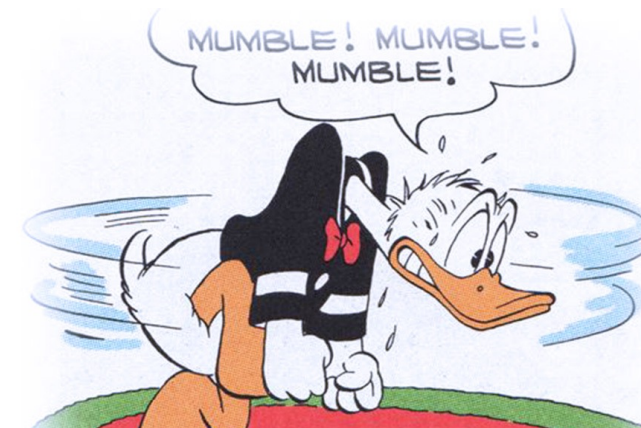


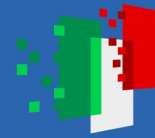
## Origin and quality of the PQ symmetry

Various constructions that enforce a high quality, accidental PQ symmetry have been proposed, but they all rely on imposing the dimension of the first PQ breaking operator by hand

Not satisfying..

Thus, we need a mechanism that enforces a high quality, accidental PQ symmetry without imposing any condition by hand

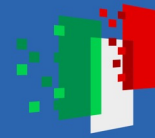




## The Flavour puzzle

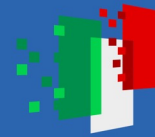
On a completely different note, the fermion mass hierarchy problem represents one among the most puzzling features of the Standard Model. In two lines, there is a 5 order of magnitude difference between the Yukawa couplings of the top quark and of the up quark.





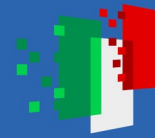
## The Axion Flavour connection: a top-down approach

- We will assume that the SM flavour pattern is generated by a SB flavour symmetry, which will be identified requiring that
  - It must automatically enforce an *accidental* global anomalous PQ symmetry
  - It must protect  $U(1)_{PQ}$  up to a sufficiently large operator dimension
- Then, we will analyse whether it reproduces, upon SSB, the observed pattern of quark mass hierarchies.



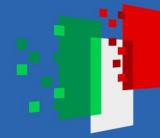
## The Axion Flavour Connection: Rectangular symmetries

- Let us consider the following examples of flavour symmetries:
  - :  $Y$  in the bifundamental,  $\det(Y)$  is PQ-violating;
  - $SU(N)_L \times SU(N)_R$  :  $Y$  in the bifundamental, no  $\det(Y)$
  - $SU(M)_L \times SU(N)_R$   $M \neq N$
- Therefore, rectangular symmetries are more effective, as for the *quality* issue



## The Axion Flavour Connection: Model Building

- *Simplicity*: simplest consistent gauge group and smallest number of fermions;
- *Phenomenology*:
  - Masses: top mass at tree level from renormalizable coupling to Higgs. Up and charm from effective operators
  - Mixings: field content sufficiently rich to generate all the masses and mixings of the quarks
- *Gauge anomalies*: gauge symmetries must be anomaly free
- *PQ origin and quality*: accidental and highly protected PQ symmetry



## Cook our recipe: a simple realisation

- The “simplest” model complying with all these requirements is a 2HDM containing 6 scalars ( $\lambda$ ,  $Y$ ,  $Z$ ,  $K$ ,  $H_u$ ,  $H_d$ ) and a 7x7 fermion mass matrix, transforming under the  $G_{SM} \times G_F \times U(1)_F$  gauge group, where  $G_F = SU(3) \times SU(2)$
- The most general  $G_F$  invariant mass matrix is

$$q_L \sim (\mathbf{3}, \mathbf{1}), \quad u_R \sim (\mathbf{1}, \mathbf{2}), \quad t_R \sim (\mathbf{1}, \mathbf{1}), \quad Q_L \sim (\mathbf{1}, \mathbf{1})$$

$$U_R \sim (\mathbf{3}, \mathbf{1}), \quad U_L \sim (\mathbf{1}, \mathbf{2}), \quad T_L \sim (\mathbf{1}, \mathbf{1}), \quad Q_R \sim (\mathbf{1}, \mathbf{1})$$

$$\mathcal{M}_u = \begin{pmatrix} & u_R & u_R & t_R & U_R & U_R & U_R & Q_R \\ \begin{pmatrix} 0 & 0 & 0 & v & 0 & 0 & z_1 \\ 0 & 0 & 0 & 0 & v & 0 & z_2 \\ 0 & 0 & 0 & 0 & 0 & v & z_3 \\ 0 & 0 & v & 0 & 0 & 0 & M \\ \Lambda_u & 0 & x_1^* & y_1^* & 0 & 0 & 0 \\ 0 & \Lambda_u & x_2^* & 0 & y_2^* & 0 & 0 \\ x_1 & x_2 & \Lambda_t & z_1^* & z_2^* & z_3^* & v \end{pmatrix} & \begin{matrix} q_L \\ q_L \\ q_L \\ Q_L \\ U_L \\ U_L \\ T_L \end{matrix} \end{pmatrix}$$



## How to construct a (failing) model

- Under  $G_F = SU(3) \times SU(2)$  we consider:

$$q_L \sim (\mathbf{3}, \mathbf{1}), \quad u_R \sim (\mathbf{1}, \mathbf{2}), \quad t_R \sim (\mathbf{1}, \mathbf{1}),$$

$$U_R \sim (\mathbf{3}, \mathbf{1}), \quad U_L \sim (\mathbf{1}, \mathbf{2}), \quad T_L \sim (\mathbf{1}, \mathbf{1}).$$

$$Y \sim (\mathbf{3}, \bar{\mathbf{2}}), \quad Z \sim (\mathbf{3}, \mathbf{1}), \quad X \sim (\mathbf{1}, \bar{\mathbf{2}})$$

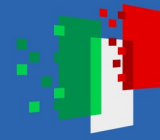
seesaw structure

3 light and 3 heavy eigenstates

non-vanishing determinant

$$\mathcal{M} = \begin{array}{cccccc} & u_R & u_R & t_R & U_R & U_R & U_R & \\ \left( \begin{array}{cccccc} 0 & 0 & 0 & v & 0 & 0 \\ 0 & 0 & 0 & 0 & v & 0 \\ 0 & 0 & 0 & 0 & 0 & v \\ \Lambda_u & 0 & x_1^* & y_1^* & 0 & 0 \\ 0 & \Lambda_u & x_2^* & 0 & y_2^* & 0 \\ x_1 & x_2 & \Lambda_t & z_1^* & z_2^* & z_3^* \end{array} \right) & \begin{array}{l} q_L \\ q_L \\ q_L \\ U_L \\ U_L \\ T_L \end{array} \end{array}$$

$$\det(\mathcal{M}) = v^3 \Lambda_u [ |X|^2 - \Lambda_t \Lambda_u ]$$



## How to construct a (failing) model

- From the determinant we can read off which operators must be allowed by  $U(1)_F$

- We can read off th  $\mathcal{L} \supset \bar{q}_L H_u U_R + \Lambda_u \bar{U}_L u_R + \begin{cases} \Lambda_t \bar{T}_L t_R & \text{or} \\ \bar{T}_L X u_R + \bar{U}_L X^\dagger t_R \end{cases}$

- and compute the  $q_L - U_R = H_u, U_L - u_R = 0, \begin{cases} T_L - t_R = 0 & \text{or} \\ T_L - u_R = t_R - U_L = X \end{cases}$

$$\mathcal{A} = 3q_L - 2u_R - t_R + 2U_L + T_L - 3U_R$$





## How to construct a (failing) model

- Substituting the charges, the anomaly yields

$$\mathcal{A} = \begin{cases} 3H_u + (T_L - t_R) = 3H_u & \text{or} \\ 3H_u + (T_L - u_R) + (U_L - t_R) = 3H_u \end{cases}$$

- If down-sector replicates the same structure, then

$$\mathcal{A} = 3(H_u + H_d)$$

- $U(1)_F$ :  $\mathcal{A}_F = 0$   $F_{H_u} = -F_{H_d}$
- $U(1)_{PQ}$ :  $\mathcal{A}_{PQ} \neq 0$   $\mathcal{X}_{H_u} + \mathcal{X}_{H_d} \neq 0$
- Therefore,  $H_u H_d$  gauge-allowed, but PQ-violating at D=2!

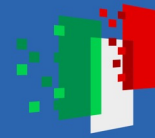


## Cook our recipe: a simple realization

- In the models analysed, we were able to retrieve compatible quark mass hierarchies from non-hierarchical (or mildly hierarchical) input parameters without imposing any number by hand.

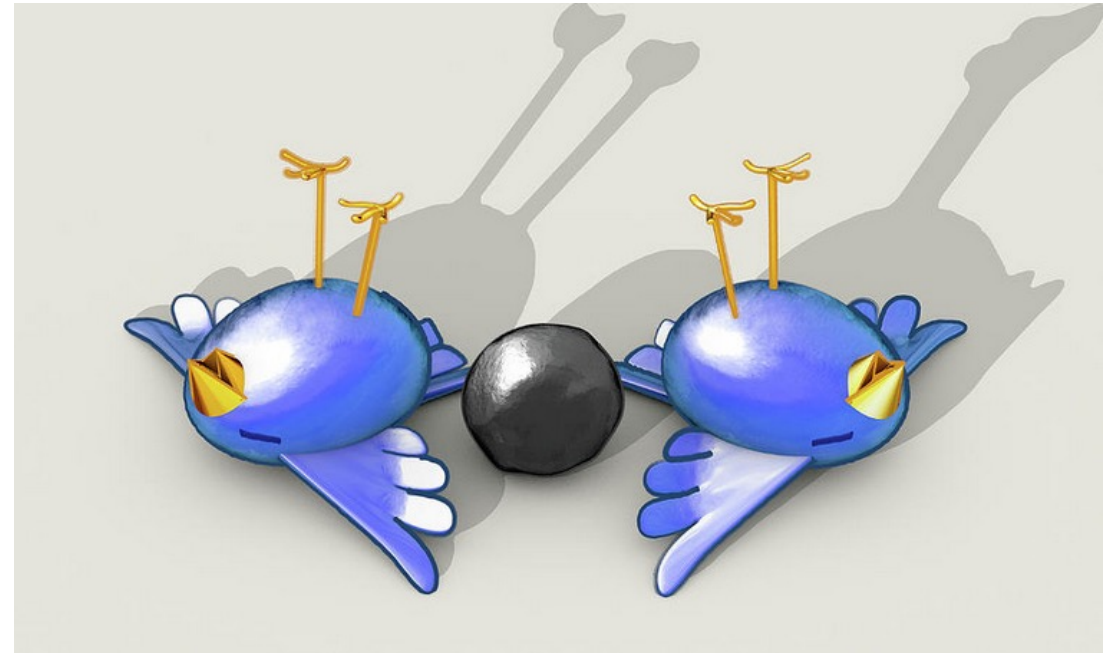
	model	experimental
$m_b(\text{GeV})$	1.5	1.5
$m_c(\text{GeV})$	0.5	0.4
$m_s(\text{MeV})$	20	30
$m_d(\text{MeV})$	0.5	1.5
$m_u(\text{MeV})$	0.3	0.7

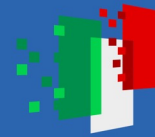
Numerical values of the quark masses and their experimental values evolved at the scale of  $10^8$  GeV.  $m_t = 102.5\text{GeV}$



## The Axion Flavour connection: conclusion and prospects

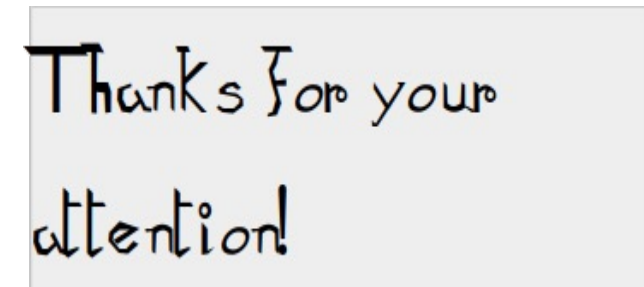
- As we have shown, the axion-flavour connection is a sensible ansatz to tackle the SM flavour puzzle and the strong CP problem in one fell swoop.





## The Axion Flavour connection: conclusions and prospects

- I have a dream! Flavour + Strong CP problem + CDM
- Research strategies:
  - Implement powerful minimisation routines (any suggestion accepted!!!) and try to obtain CKM + Hierarchies simultaneously
  - Extend the gauge symmetry (trade off with simplicity though)
  - Extend to the lepton sector

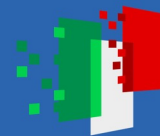




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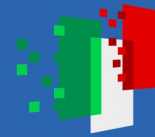
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**BACKUP SLIDES**



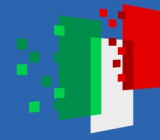
## Hierarchical structure of the VEVs

- The minimisation of the scalar potential yields hierarchical entries in the VEVs of the scalar fields.

$$|x_1|, |y_{1,2}|, |z_3|, |k_2| \sim O(\Lambda), \quad |x_2|, |z_{1,2}| \sim O(10^{-5}\Lambda), \quad |k_{1,3}| \sim O(10^{-6}\Lambda)$$

$$\Lambda = 10^{11} \text{ GeV}$$

- This intuitively explains how we can get hierarchies starting from non-hierarchical (or mildly hierarchical) input parameters



## PQ quality: the role of higher-dim operators

- In general, a PQ-breaking operator can be written as:

$$\eta \frac{\Phi^D}{M_P^{D-4}} + \text{h.c.} \approx \left( \frac{f_a}{M_P} \right)^{D-4} f_a^4 \cos \left( \frac{a}{f_a} + \xi_\eta \right)$$

- The total potential (QCD + PQ-break) would then be

$$V \supset -m_\pi^2 f_\pi^2 \cos \left( \frac{a}{f_a} \right) + \left( \frac{f_a}{M_P} \right)^{D-4} f_a^4 \cos \left( \frac{a}{f_a} + \xi_\eta \right)$$

- To comply with  $\theta \lesssim 10^{-10}$  then

$$\left( \frac{f_a}{M_P} \right)^{D-4} f_a^4 \lesssim 10^{-10} m_\pi^2 f_\pi^2 \rightarrow D \gtrsim 8 \text{ (13)} \Leftrightarrow f_a \lesssim 10^8 \text{ (10}^{13}\text{)}$$



## You cannot have your cake and eat it too: a no-go theorem

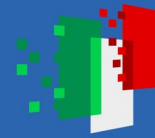
- The Yukawa Lagrangian can be written as

$$\mathcal{L}_Y = \sum_{i,j} \lambda_{ij} \bar{Q}_{m_i} Y_{m_i n_j} q_{n_j}$$

For which the mass matrix is

$$\mathcal{M} = \left( \begin{array}{c|c|c|c|c} \lambda_{m_1 n_1} Y_{m_1 n_1} & \lambda_{m_1 n_2} Y_{m_1 n_2} & \dots & \dots & \dots \\ \dots & \dots & \lambda_{m_2 n_3} Y_{m_2 n_3} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \lambda_{m_i n_j} Y_{m_i n_j} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{array} \right)$$





## You cannot have your cake and eat it too: a no-go theorem

- In order to have massive quarks, we need to require

$$\text{Det}(\mathcal{M}) = \varepsilon_{\alpha_1 \alpha_2 \dots \alpha_N} \mathcal{M}_{1\alpha_1} \mathcal{M}_{2\alpha_2} \dots \mathcal{M}_{N\alpha_N} \neq 0$$

- Such an operator has a charge proportional to the anomaly

$$\mathcal{A}_{PQ} \propto \sum_i m_i \chi_{Q_{m_i}} - \sum_j n_j \chi_{q_{n_j}} \neq 0$$

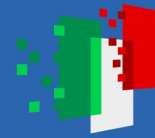
and thus breaks PQ at dimension N. Thus, if we want massive quarks and a PQ colour anomaly, we cannot help but break PQ at dimension N



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