Topics at the intersection of Particle Physics and Cosmology

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Based on previous works and ongoing collaborations with:

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Giacomo Ferrante
Lorenzo Frosina
Antonio J. Iovino

Primordial Black Holes (PBHs)

[hypothetical black holes that could have formed in the early universe through the gravitational collapse of high-density regions]

Viable dark matter candidate

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Gravitational waves (GWs) from PBH mergers

[If primordial black holes exist, **their mergers could generate detectable gravitational waves**. Advanced gravitational wave detectors, such as LIGO and Virgo, are actively searching for these signals. The detection of gravitational waves from primordial black hole mergers would provide valuable insights into their properties and distribution.]

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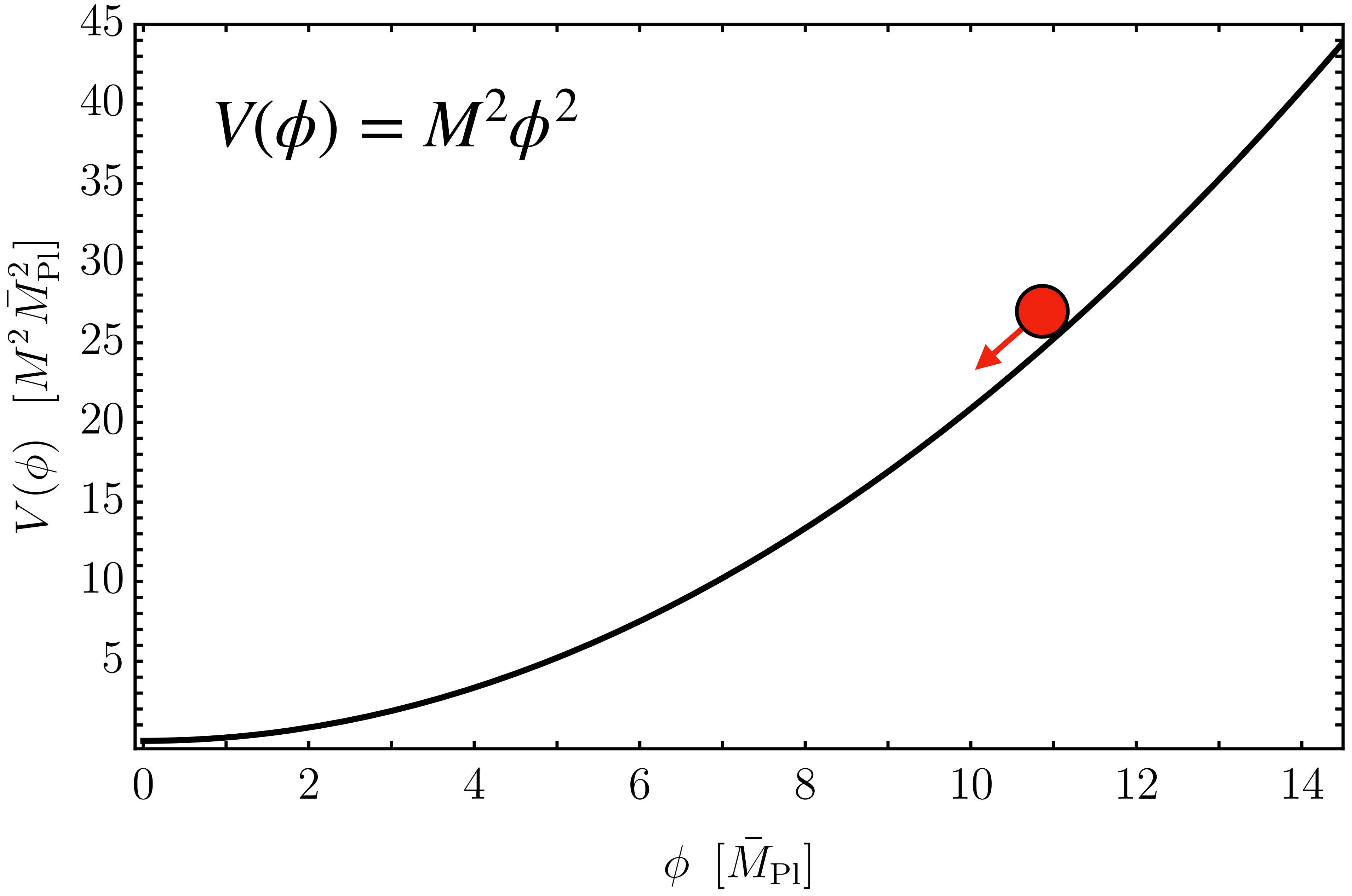
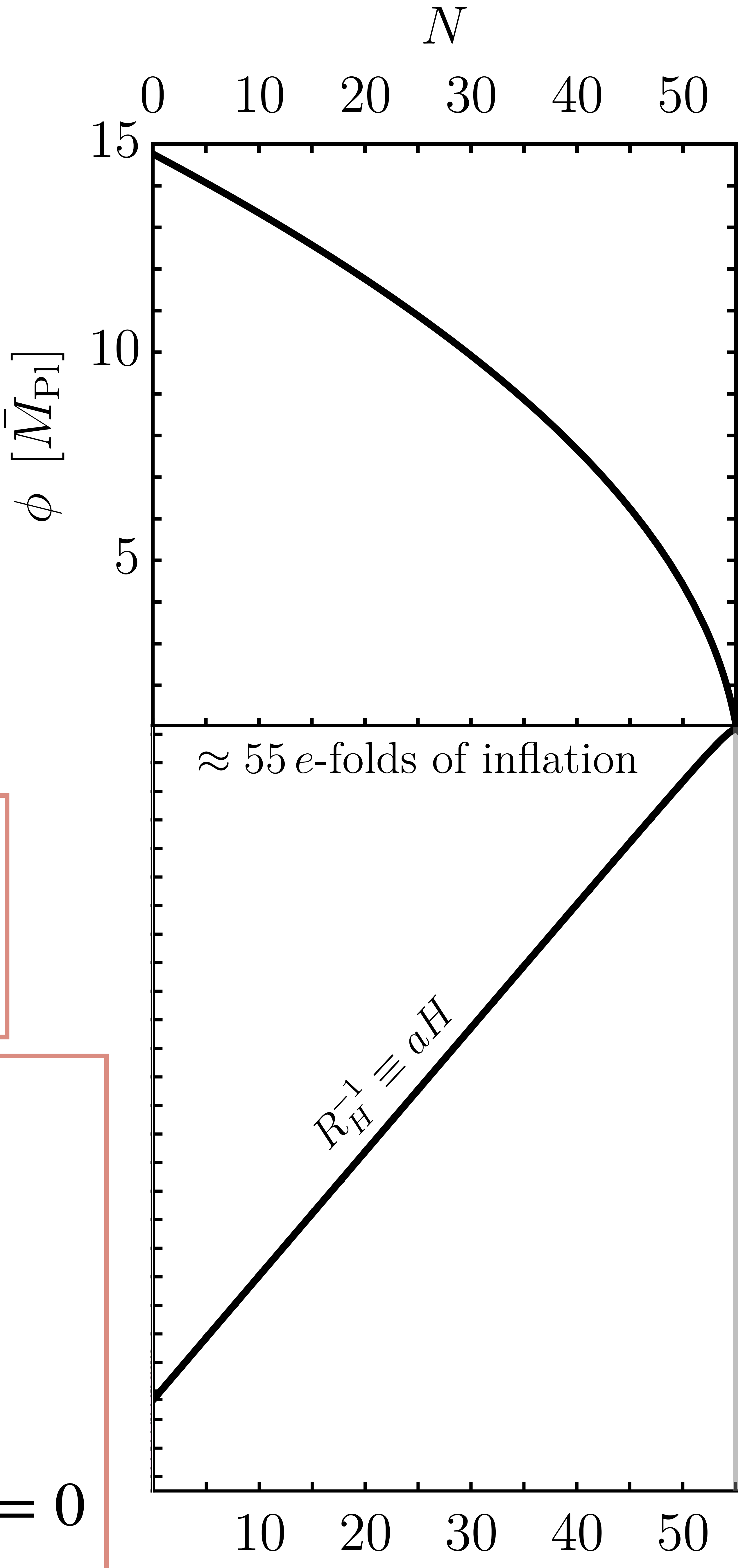




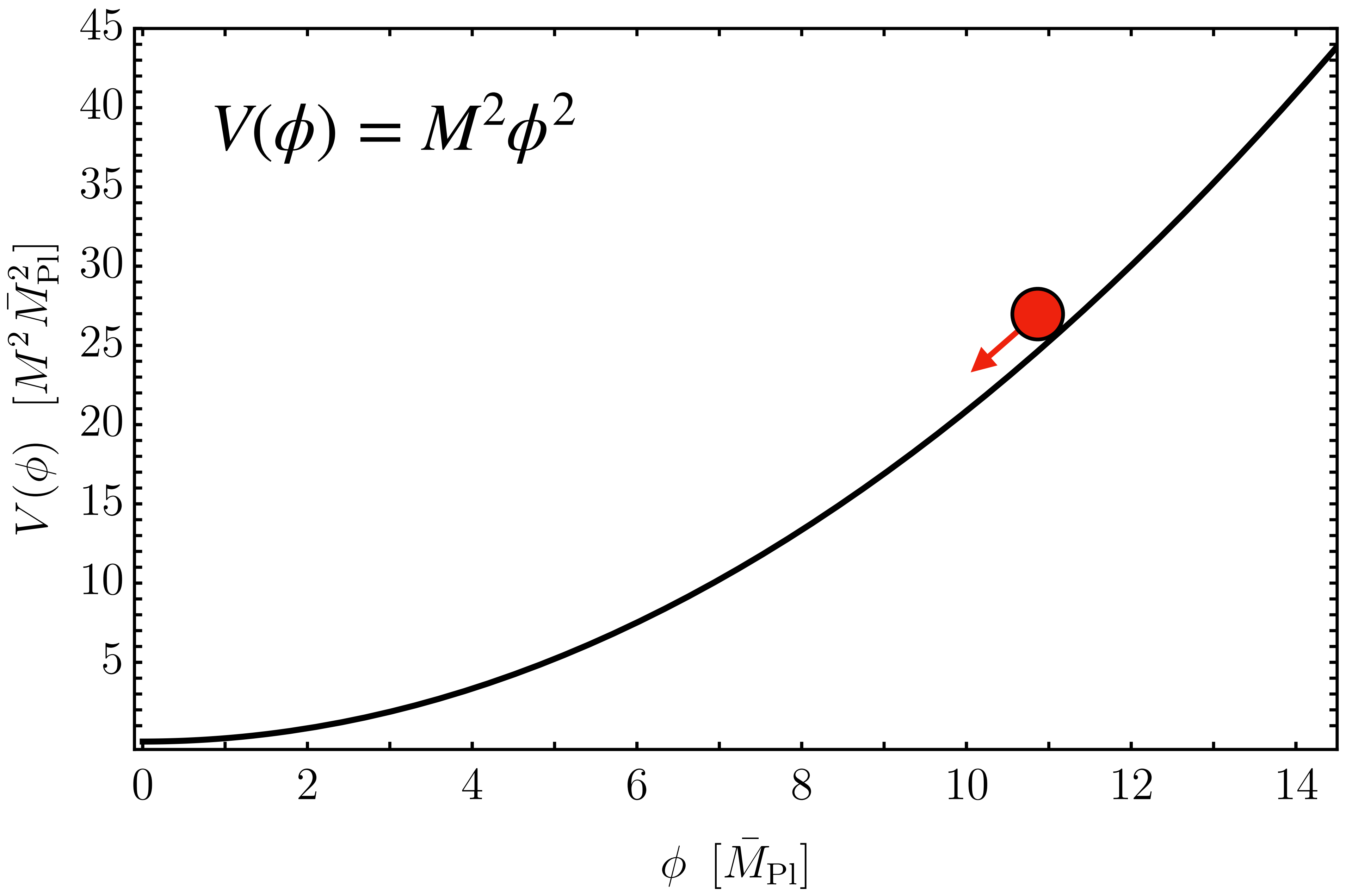
$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$$

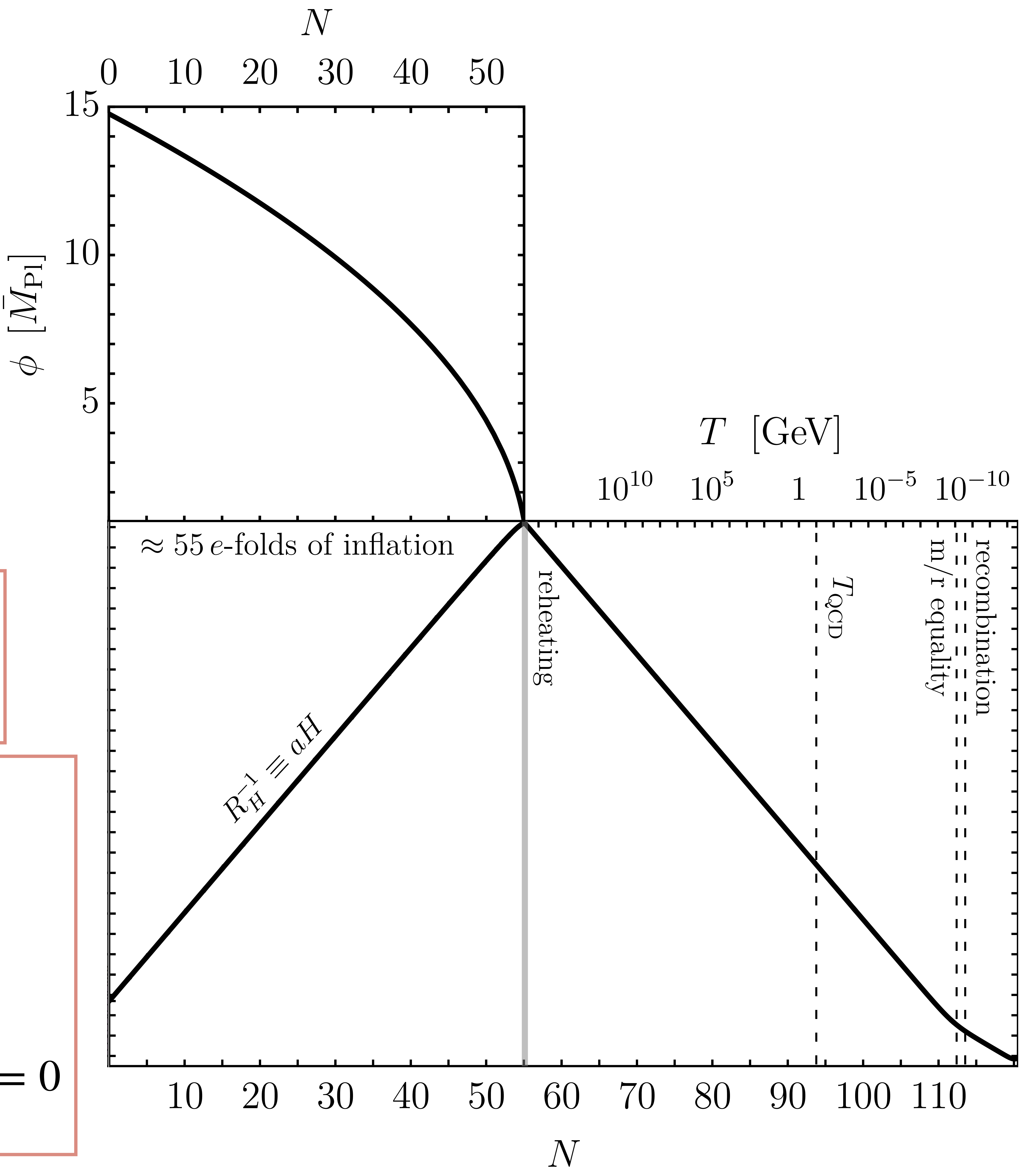
$$H = \dot{a}(t)$$

$$(3-\epsilon)\bar{M}_{\rm Pl}^2 H^2 = V(\phi)$$

$$\epsilon = \frac{1}{2\bar{M}_{Pl}^2}\left(\frac{d\phi}{dN}\right)^2$$

$$\frac{d^2\phi}{dN^2} + \left[3 - \frac{1}{2\bar{M}_{\rm Pl}^2}\left(\frac{d\phi}{dN}\right)^2\right]\left[\frac{d\phi}{dN} + \bar{M}_{\rm Pl}^2\frac{d\log V(\phi)}{d\phi}\right] = 0$$





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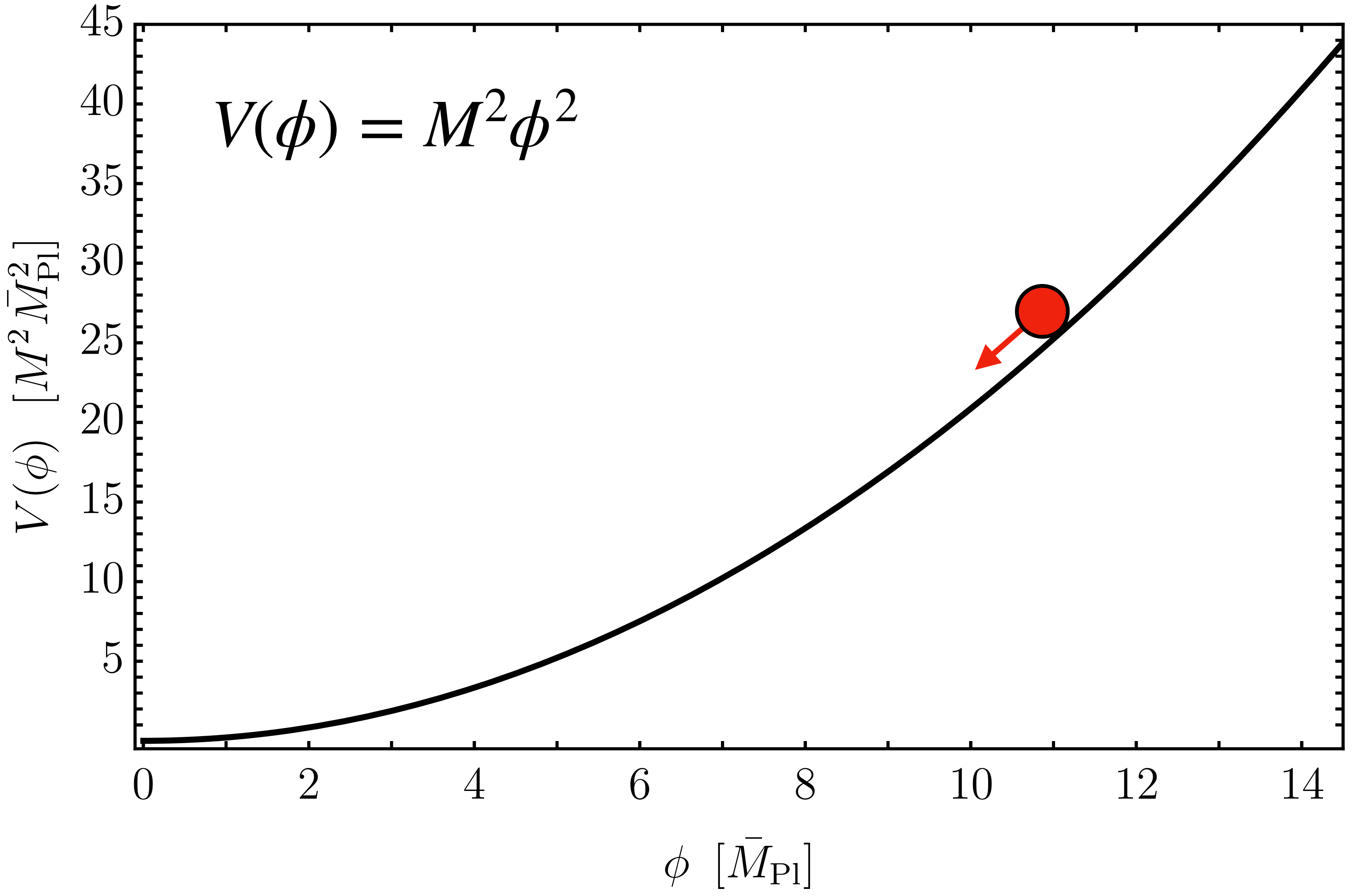
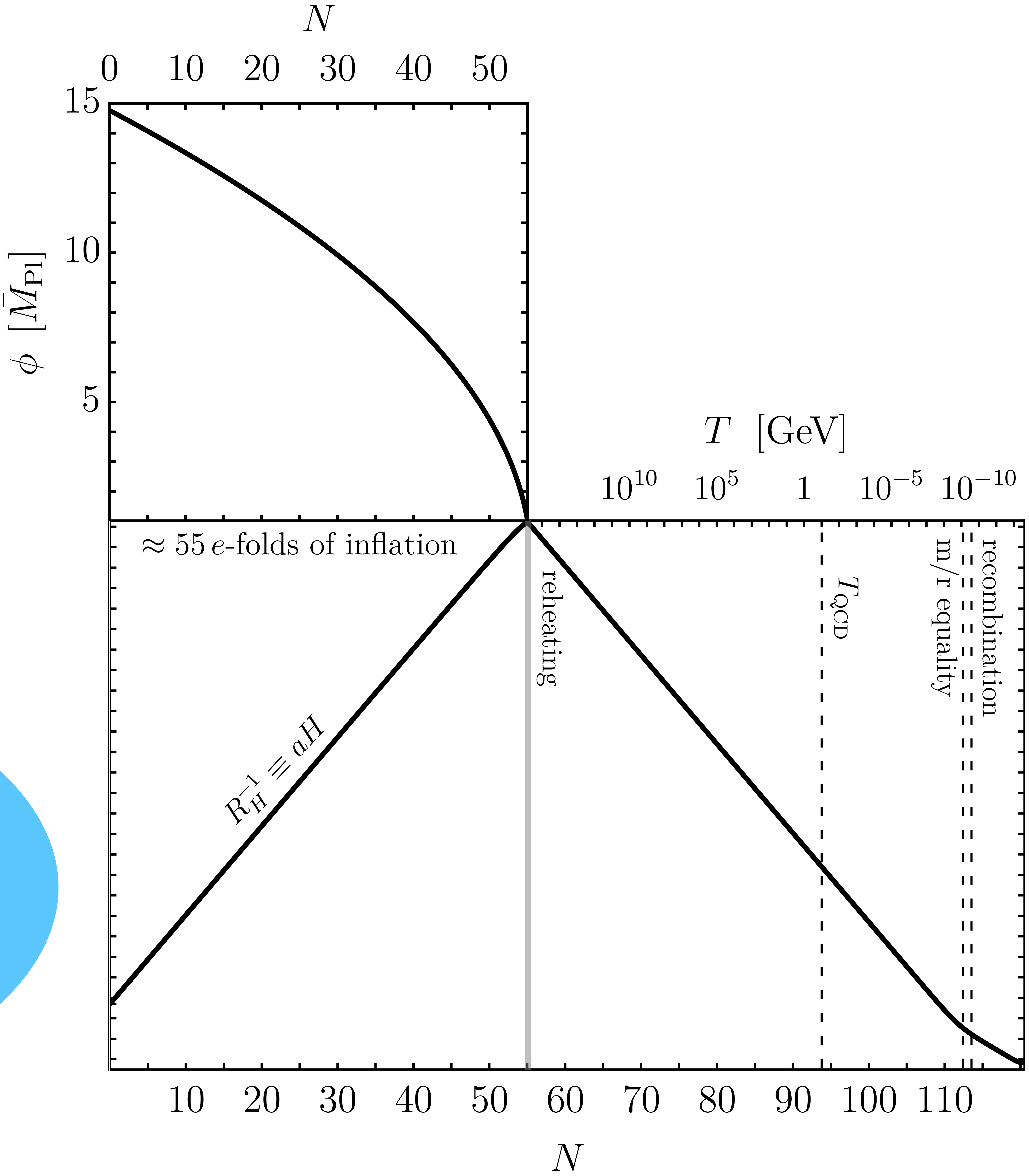
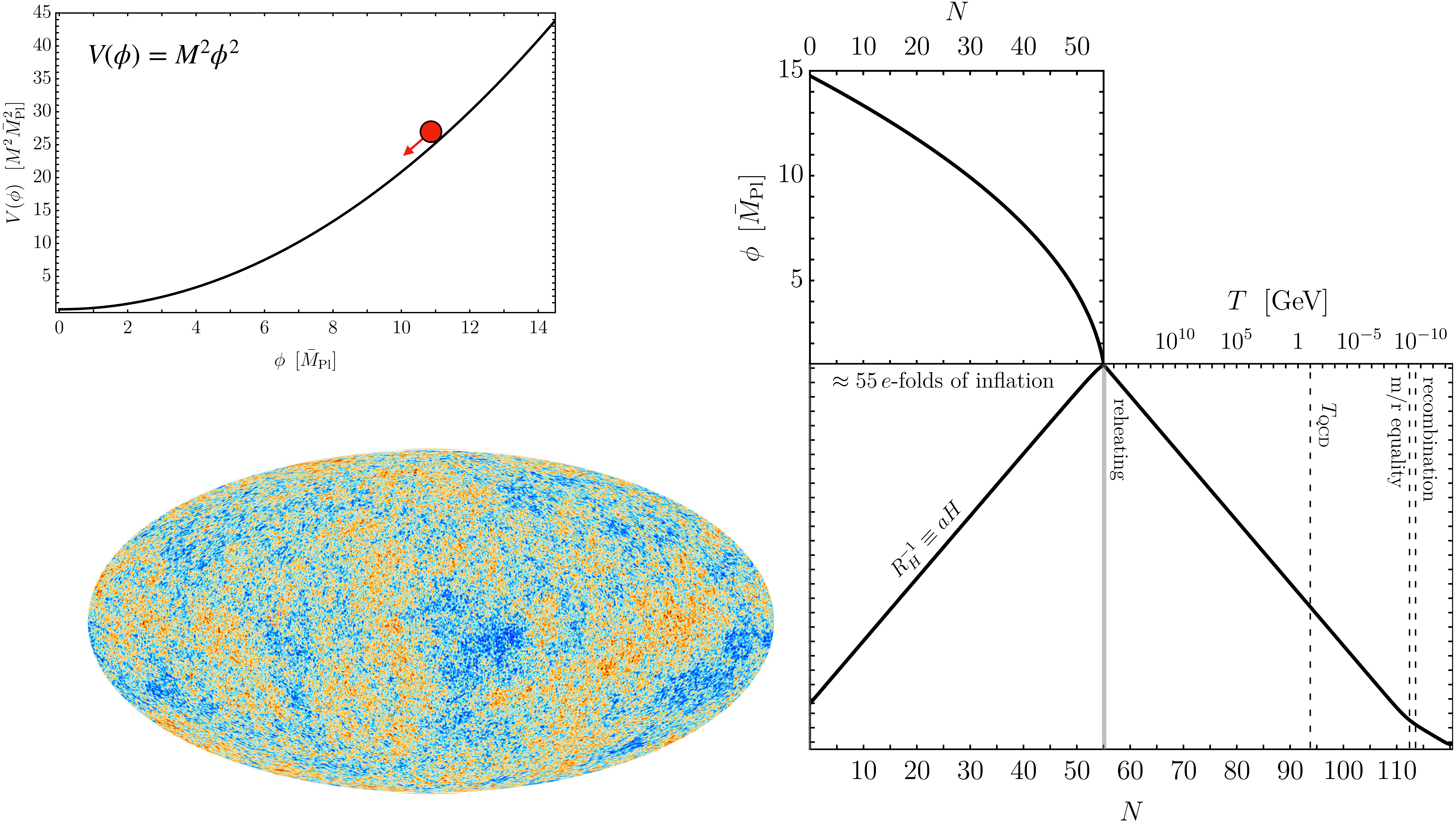


$$T_0 = 2.725\,K = 2.35 \times 10^{-13}\,\text{GeV}$$





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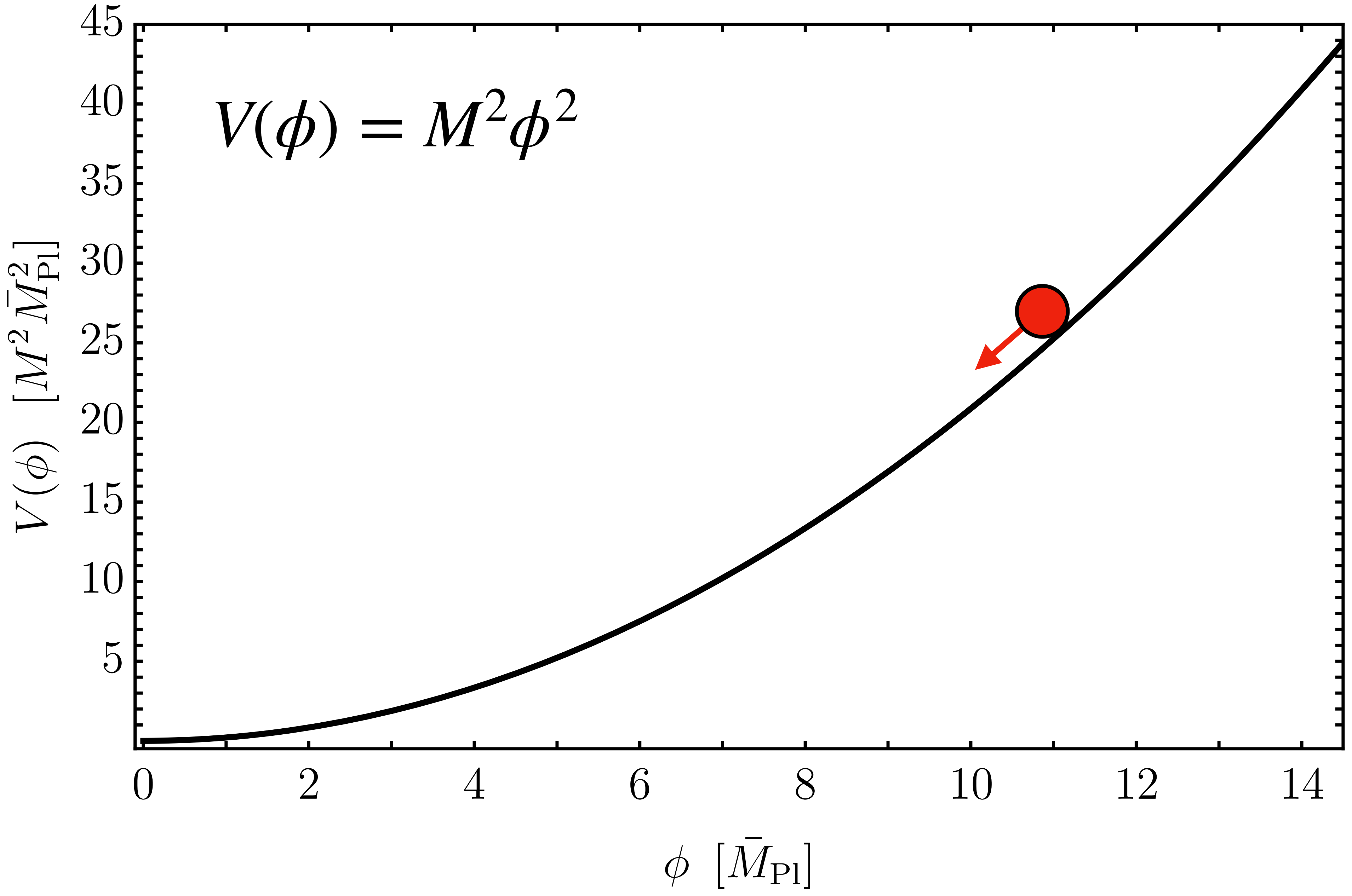

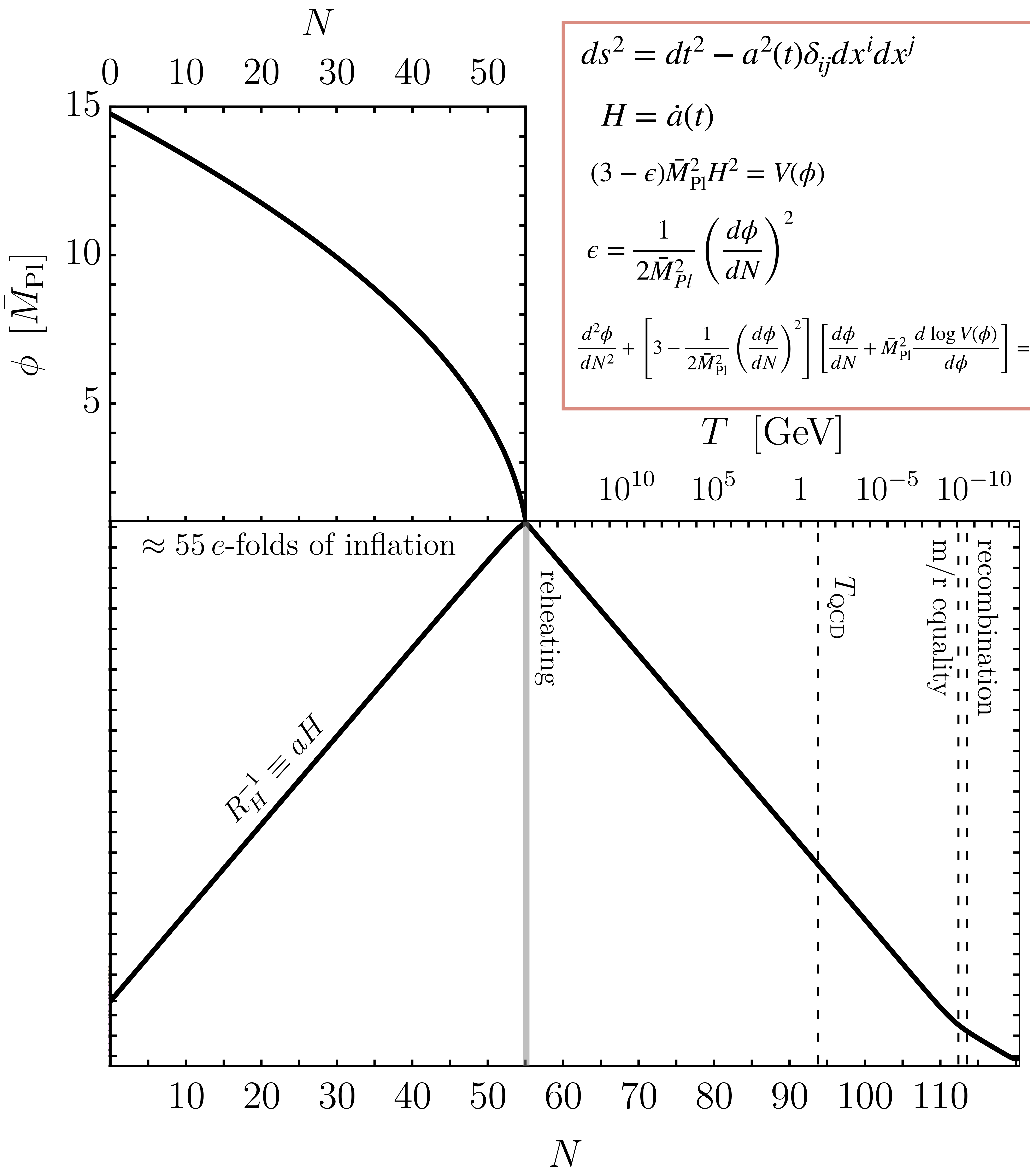

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One very appealing property of inflation is that a tiny amount of spatial dependence is inevitable.
These tiny, **primordial variations in the space-time**, which are naturally present in inflation, provide the **initial inhomogeneities** needed to explain the beginnings of the structures that are observed today. Their **origin in the theory lies in the quantum behavior of both the field and the space-time**.

$$ds^2 = N^2 dt^2 - a^2(t) e^{2\zeta(\vec{x},t)} \delta_{ij} (N^i dt + dx^i)(N^j dt + dx^j)$$

$$\delta\phi(\vec{x}, t) = 0$$

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The field $\zeta(\vec{x}, t)$ is the only independent scalar degree of freedom since N and N^i are Lagrange multipliers subject to the momentum and Hamiltonian constraints.

$$\mathcal{S}_2 = \int d^4x \, \epsilon \, a^3 \left[\dot{\zeta}^2 - \frac{(\partial_k \zeta)(\partial^k \zeta)}{a^2} \right]$$

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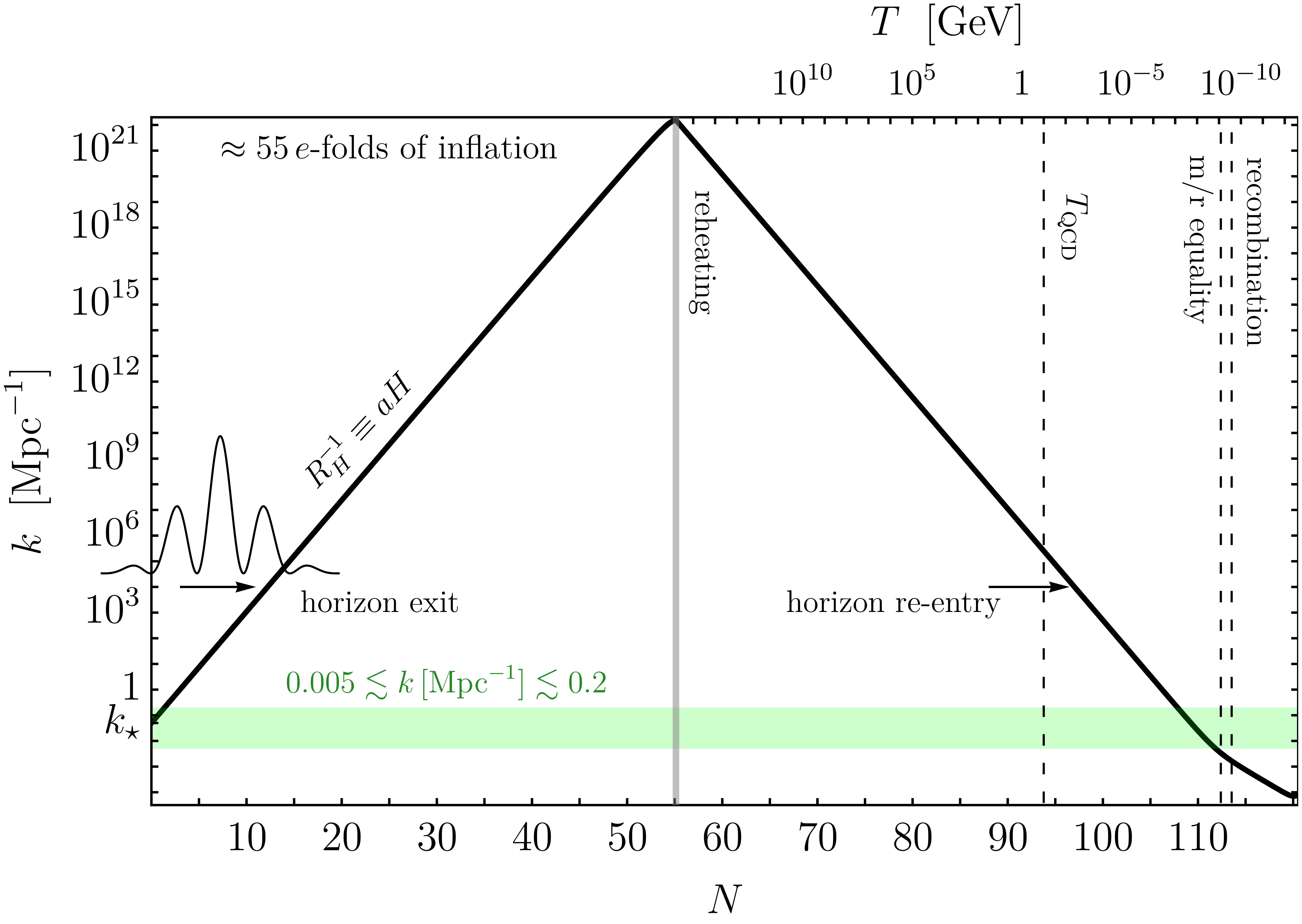
$$\hat{\zeta}(\vec{x}, \tau) = \int \frac{d^3\vec{k}}{(2\pi)^3} \hat{\zeta}(\vec{k}, \tau) e^{i\vec{x}\cdot\vec{k}}$$

$$\hat{\zeta}(\vec{k}, \tau) = \zeta_k(\tau) a_{\vec{k}} + \zeta_k^*(\tau) a^\dagger_{-\vec{k}}$$

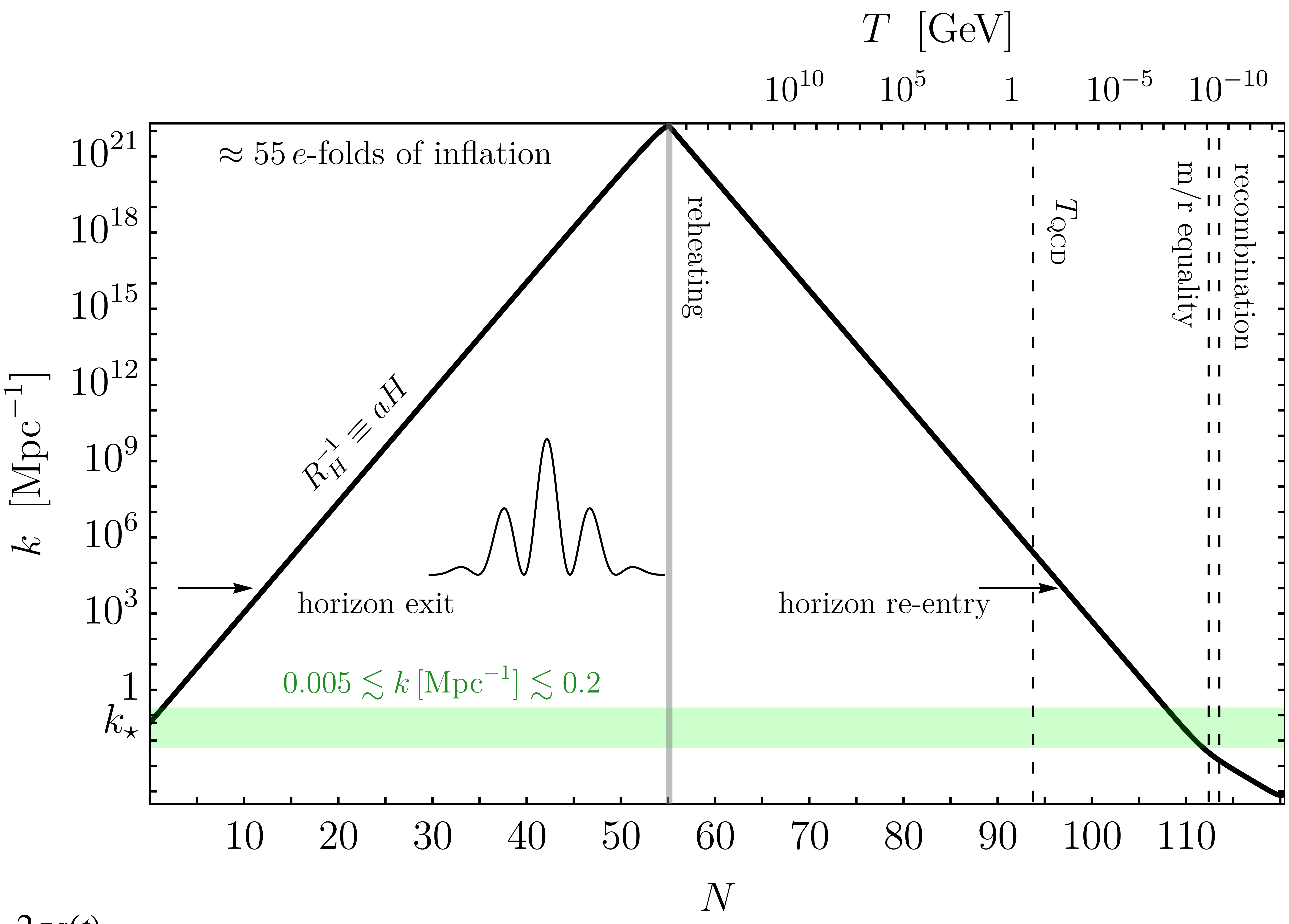
Note that the Fourier transformation is defined with respect to the comoving coordinates. So, ***k* is the comoving wavenumber**

$$[a_{\vec{k}}, a_{\vec{k}'}] = [a^\dagger_{\vec{k}}, a^\dagger_{\vec{k}'}] = 0 \,, \quad [a_{\vec{k}}, a^\dagger_{\vec{k}'}] = (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}') \,, \quad a_{\vec{k}} |0\rangle = 0$$

Space-time itself fluctuates quantum mechanically about a background that is expanding at an accelerating rate.

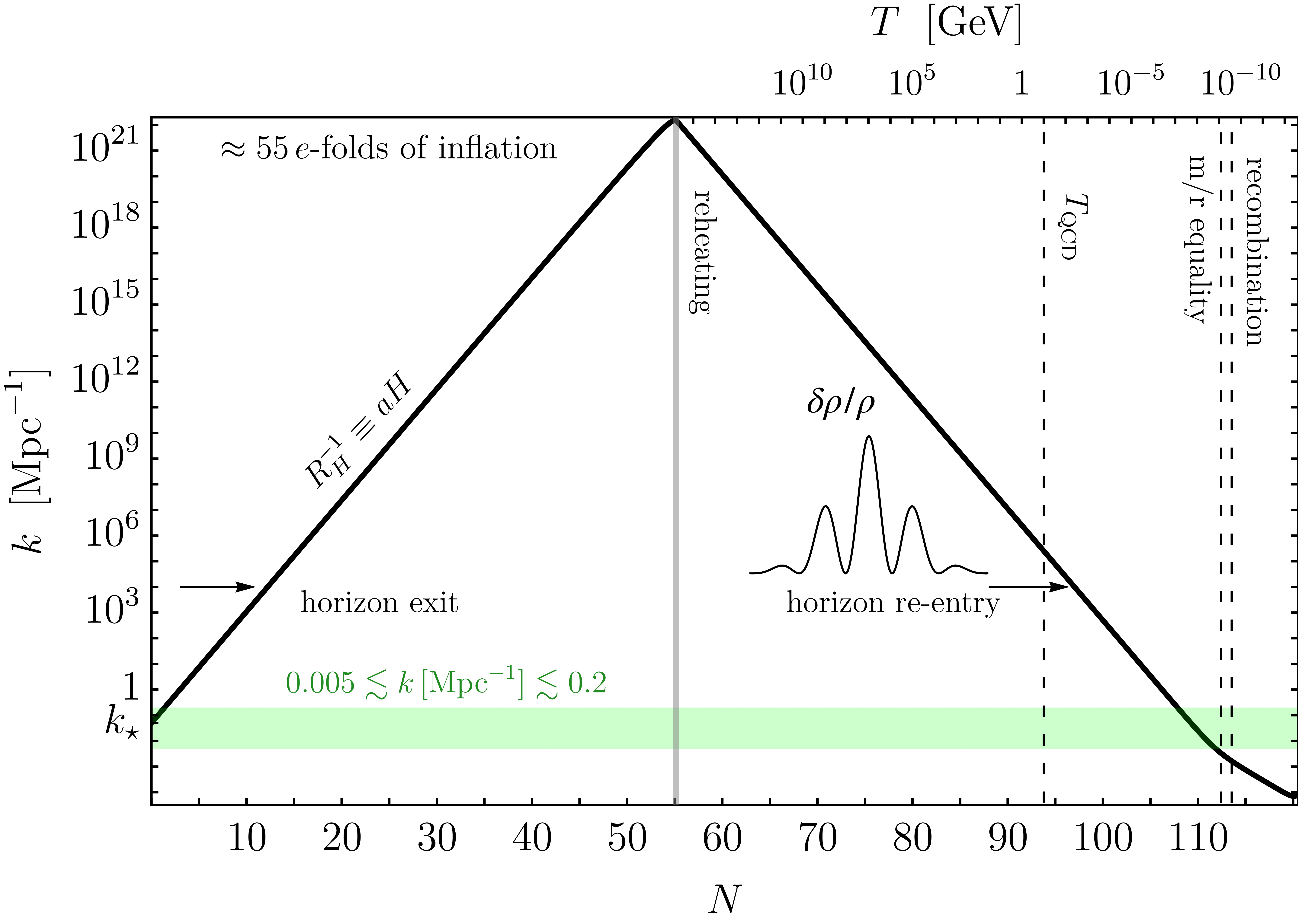


This extreme expansion spreads the fluctuations, which begin with a tiny spatial extent, throughout a vast region of the universe, where they eventually become small classical fluctuations in the space-time curvature — or equivalently, small spatial variations in the strength of gravity.

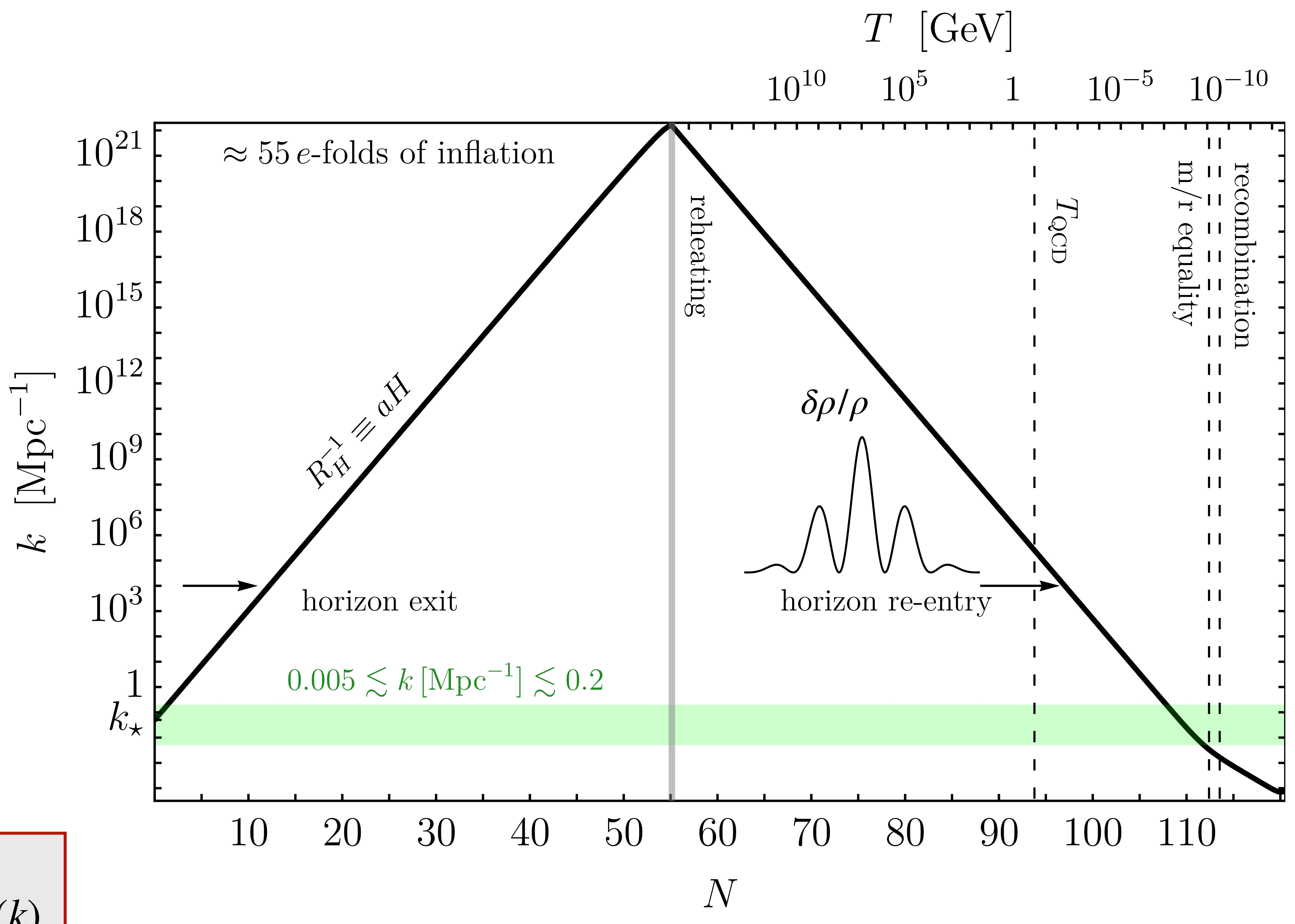


The physical wavelength is $\lambda(t) = \frac{2\pi a(t)}{k}$

Since everything in the universe feels the influence of gravity, these fluctuations in the gravitational field
are transferred to the matter and radiation fields, creating slightly over-dense and under-dense regions. The resulting matter fluctuations then become the 'initial conditions' that start the process of collapse which forms the stars and galaxies of later epochs.



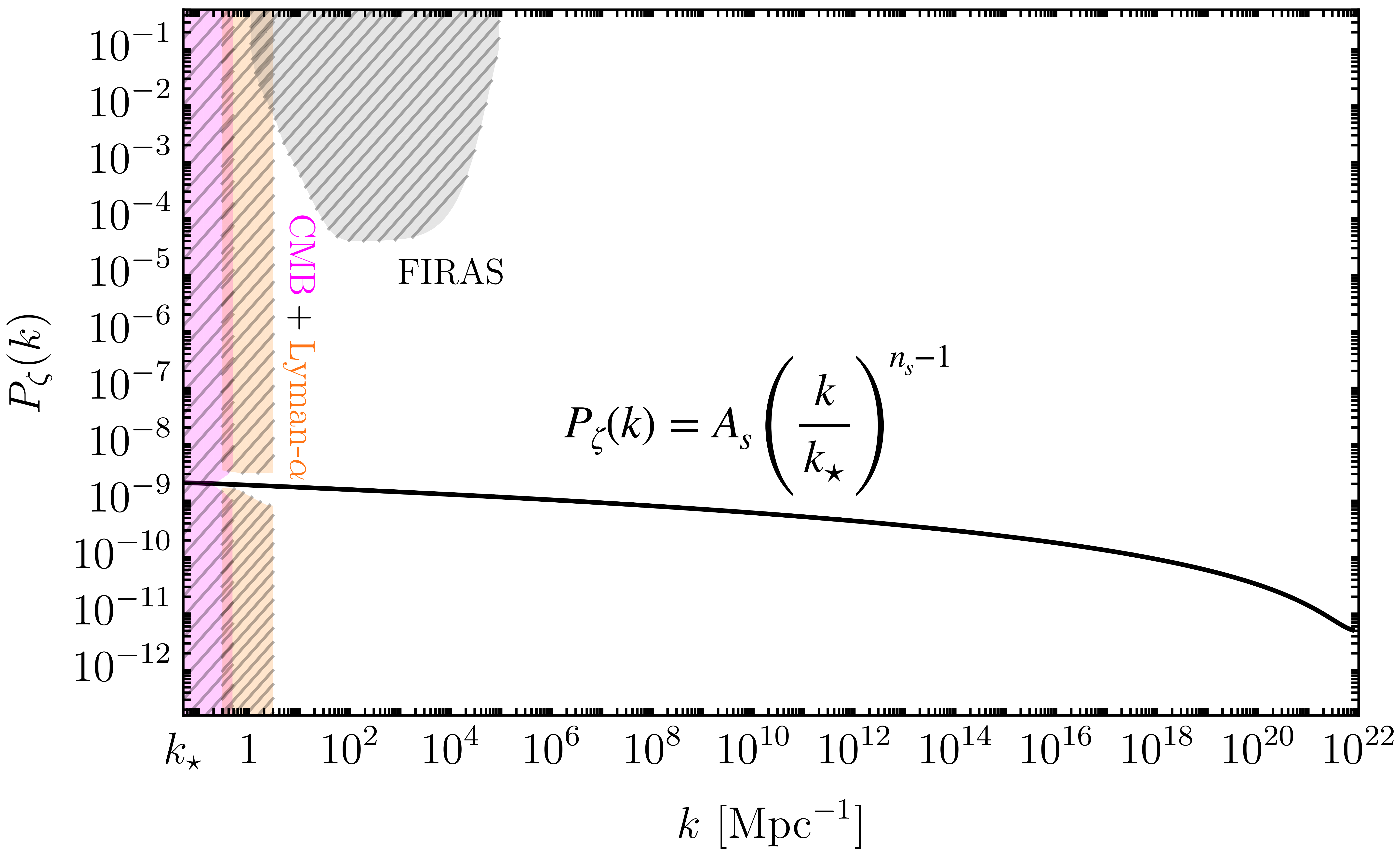
To test whether this picture is correct, it is necessary to describe very accurately the properties of the pattern generated by inflation for the original, primordial fluctuations in space-time which can then be compared with what is inferred from observations.



$$\lim_{\tau \to 0^-} \langle \hat{\zeta}(\vec{x}, \tau) \hat{\zeta}(\vec{x}, \tau) \rangle = \int \frac{dk}{k} P_\zeta(k)$$

The power spectrum of primordial fluctuations during inflation is a way to describe how the amplitude of density fluctuations in the early universe varies with different comoving scales.

A scale-invariant or nearly scale-invariant spectrum implies that fluctuations have a relatively constant amplitude across different scales.



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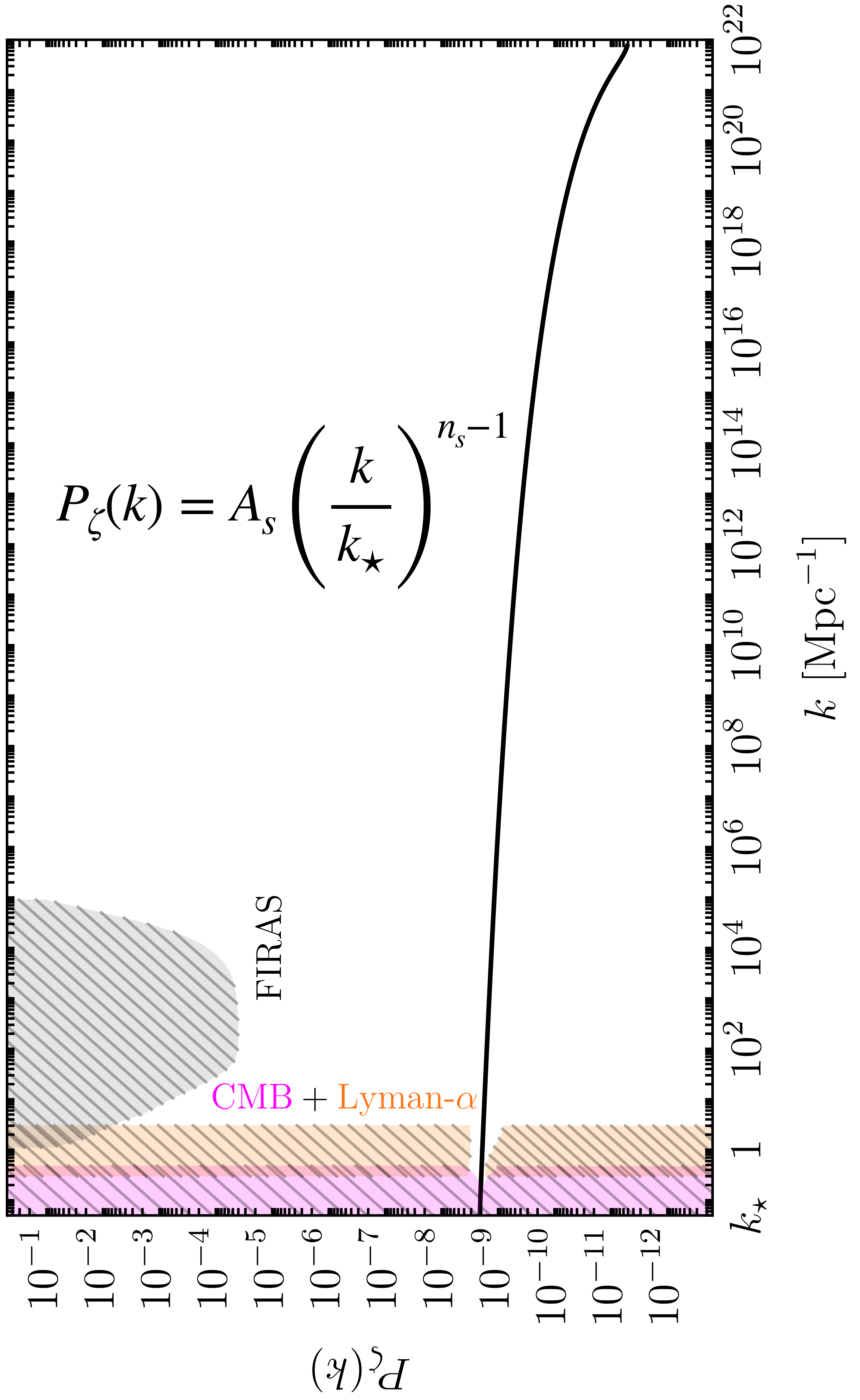
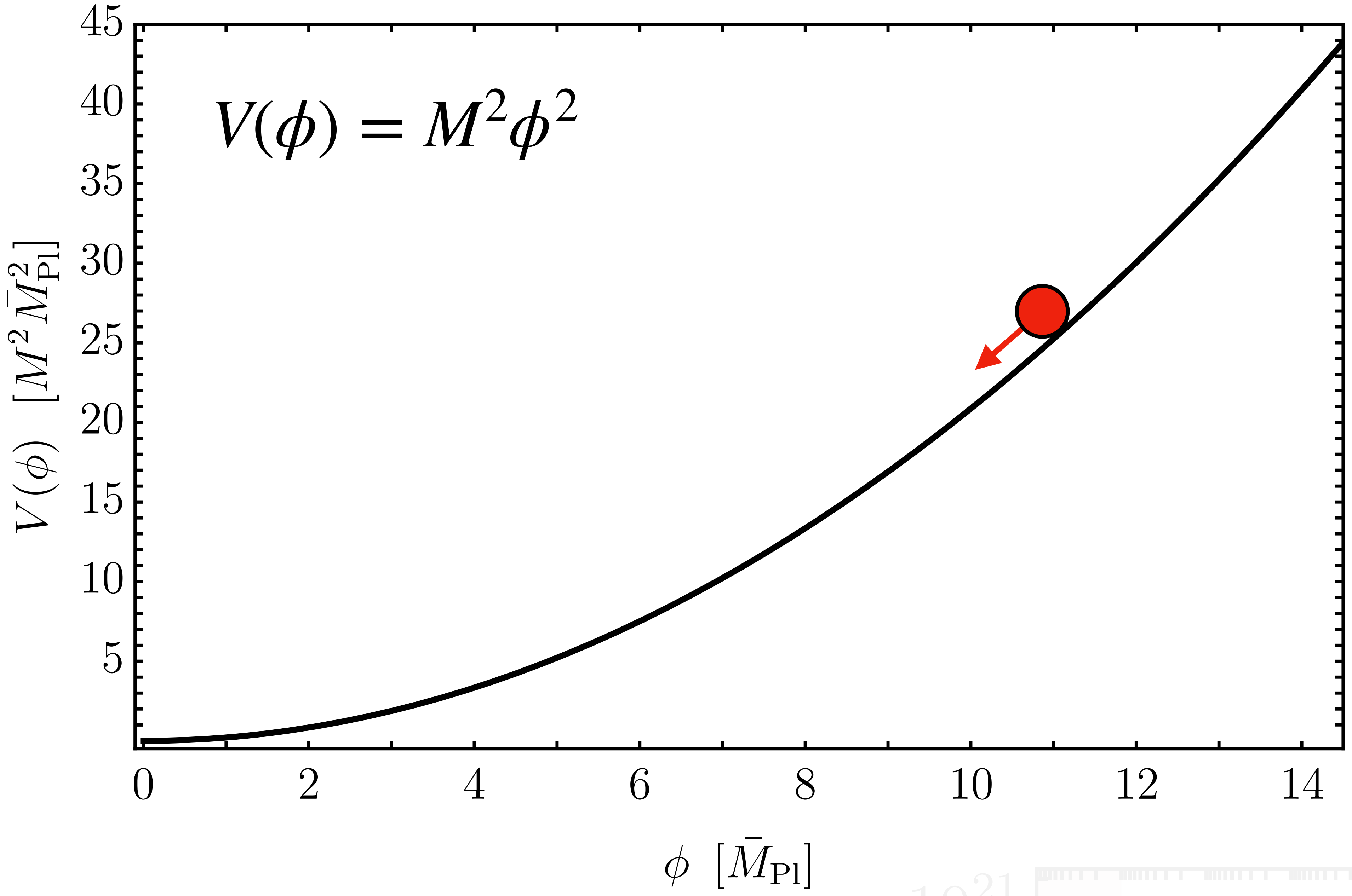


$V(\phi) = M^2\phi^2$
$V(\phi)$ $[M^2\bar{M}_{\rm Pl}^2]$
ϕ $[\bar{M}_{\rm Pl}]$
$A_s = \frac{(1+2N)^2M^2}{24\pi^2\bar{M}_{\rm Pl}^2}$
$n_s = 1 - \frac{4}{1+2N}$
N
ϕ $[\bar{M}_{\rm Pl}]$
T [GeV]
10^{10} 10^5 1 10^{-5} 10^{-10}
$P_\zeta(k) = A_s\left(\frac{k}{k_\star}\right)^{n_s-1}$
k $[\mathrm{Mpc}^{-1}]$
$k_\star$
≈ 55 e-folds of inflation
reheating
$T_{\rm QCD}$
m/r equality
recombination
$R_H^{-1} \equiv aH$
horizon exit
horizon re-entry
$0.005 \lesssim k\,[\mathrm{Mpc}^{-1}] \lesssim 0.2$
$P_\zeta(k)$
N

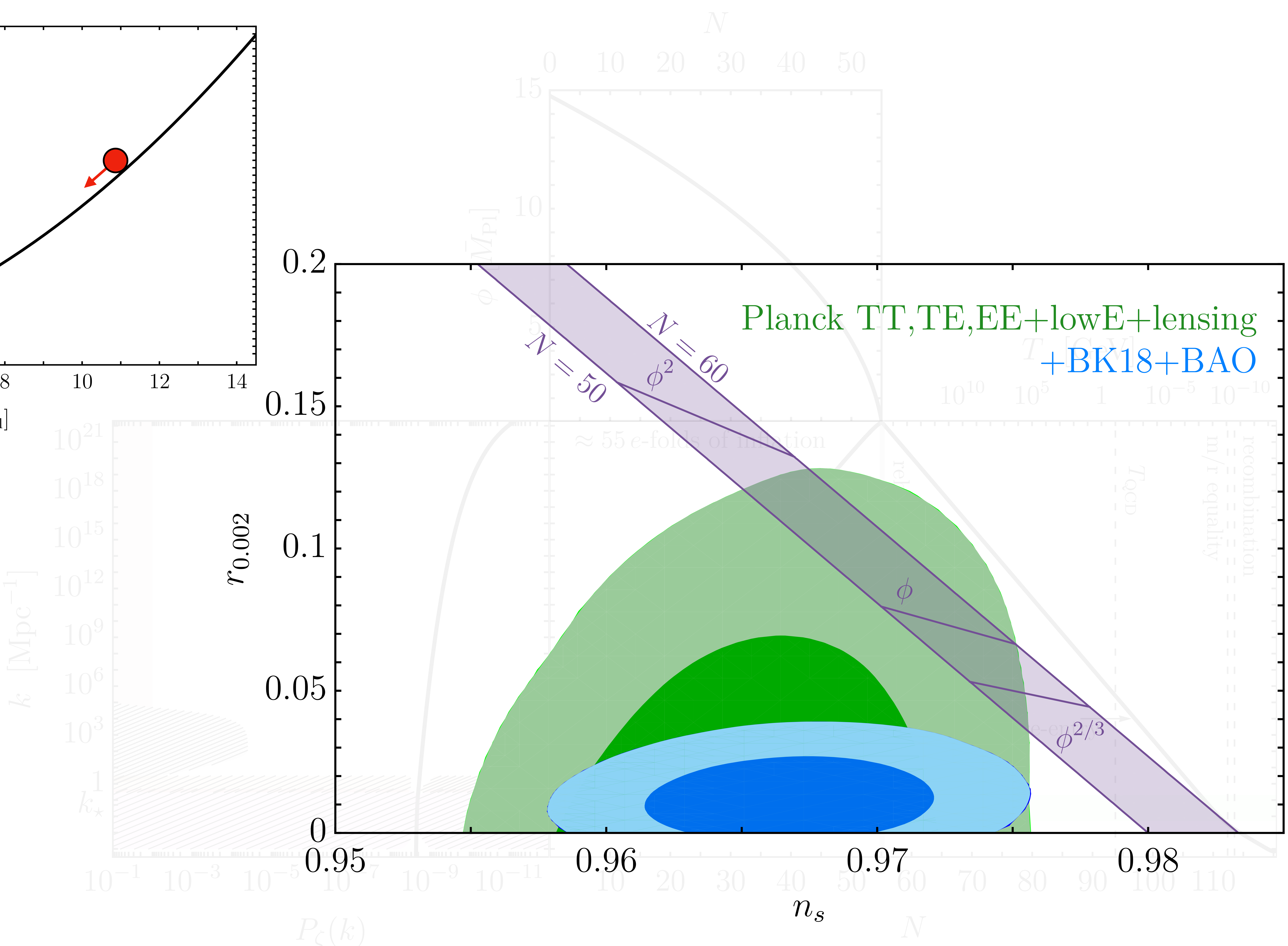
V(φ) = M²φ²
V(φ) [M² M̄²Pl]
φ [M̄Pl]
N
φ [M̄Pl]
T [GeV]
10^10
10^5
1
10^-5
10^-10
≈ 55 e-folds of inflation
reheating
T_QCD
m/r equality
recombination
R_H^-1 ≡ aH
horizon exit
horizon re-entry
0.005 ≲ k [Mpc^-1] ≲ 0.2
k [Mpc^-1]
k⋆
P_ζ(k) = A_s (k/k⋆)^(n_s−1)
P_ζ(k)
N
A_s = (1 + 2N)² M² / (24π² M̄²Pl)
n_s = 1 − 4/(1 + 2N)
M ≈ g_s M̄Pl



$$A_s = \frac{(1+2N)^2 M^2}{24\pi^2 \bar{M}_{\rm Pl}^2}$$

$$n_s = 1 - \frac{4}{1+2N}$$

$$r = \frac{16}{1+2N}$$



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$V(\phi)$ $[M^2\bar{M}_{\rm Pl}^2]$
ϕ $[\bar{M}_{\rm Pl}]$
0
2
4
6
8
10
12
14
5
10
15
20
25
Planck TT,TE,EE+lowE+lensing
+BK18+BAO
$N = 60$
$N = 50$
ϕ^2
ϕ
$\phi^{2/3}$
ϕ_ξ^4
$r_{0.002}$
n_s
0
0.05
0.1
0.15
0.2
0.95
0.96
0.97
0.98

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k $[{\rm Mpc}^{-1}]$
$k_\star$
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N

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$V(\phi)$ $[M^2\bar{M}_{Pl}^2]$
ϕ $[\bar{M}_{Pl}]$
$M_H(t) \simeq 17 \times \left(\frac{g_*}{10.75}\right)^{-1/6} \left(\frac{k}{10^6\,\mathrm{Mpc}^{-1}}\right)^{-2} M_\odot$
N
$P_\zeta(k) = A_s \left(\frac{k}{k_\star}\right)^{n_s-1}$
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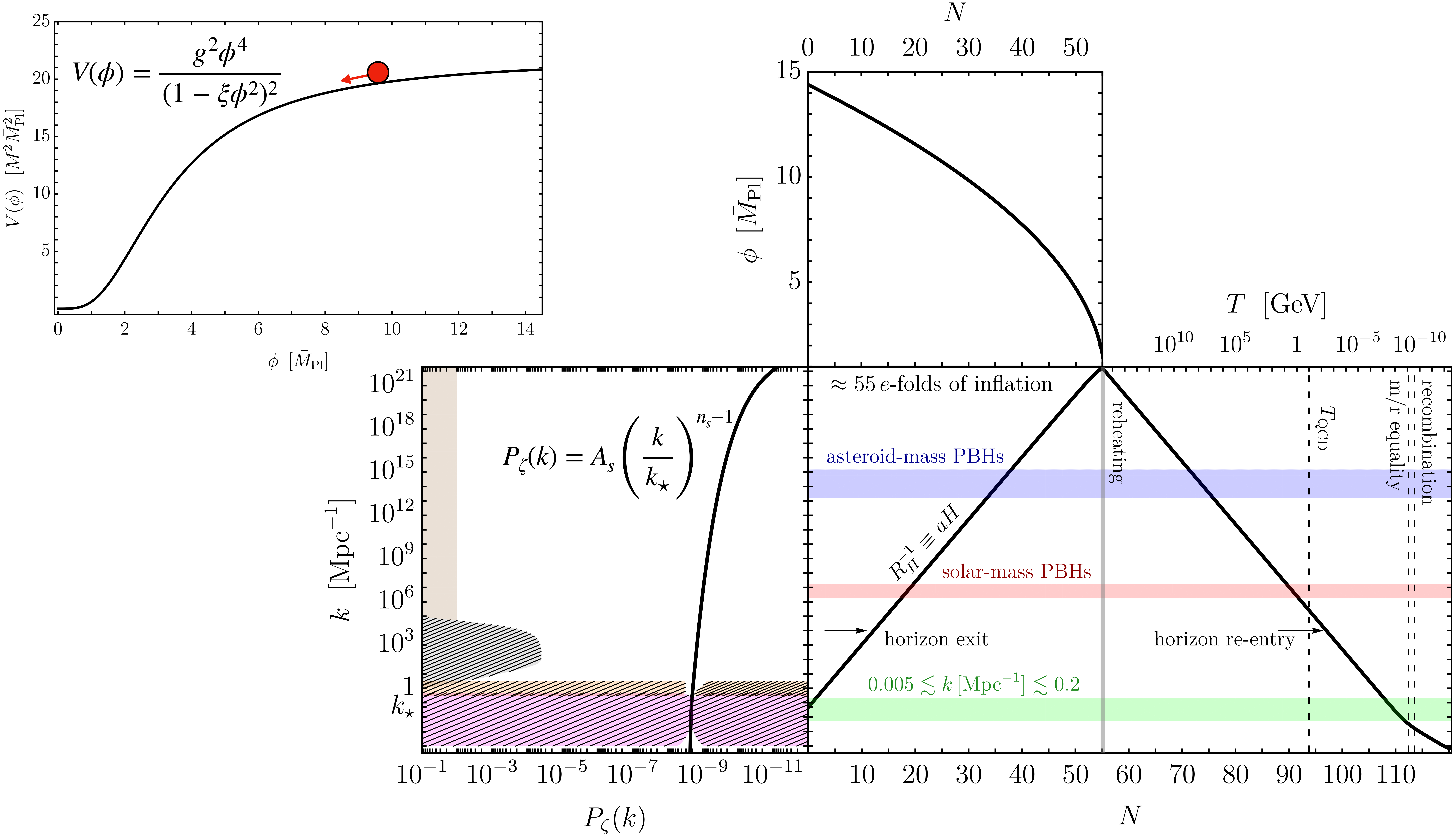

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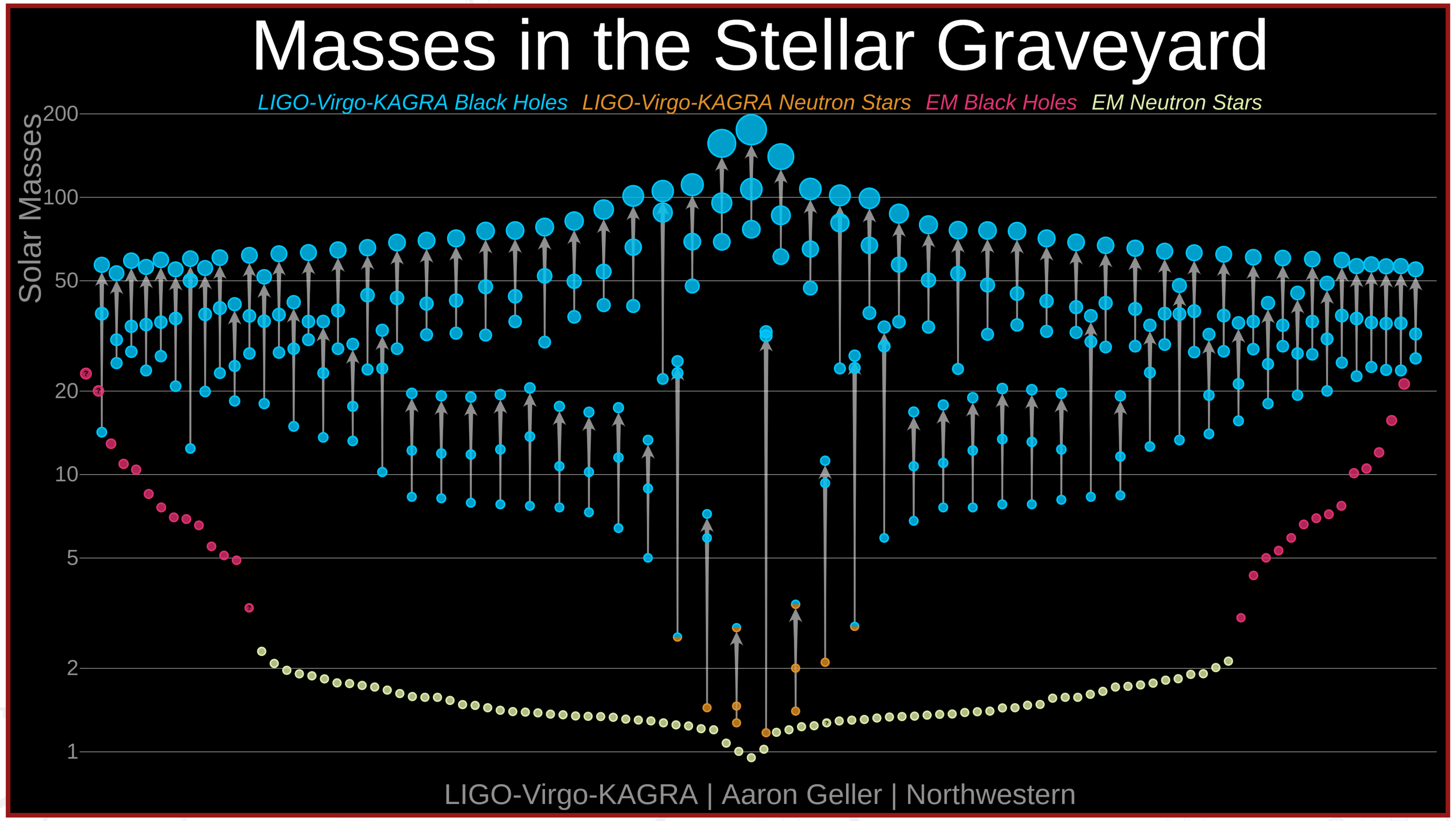
$P_\zeta(k) = \frac{H^2}{8\pi^2\bar{M}_{\rm Pl}^2\epsilon}$

$f_{\rm PBH}(M_{\rm PBH})$
$M_{\rm PBH}\ [M_\odot]$
Ev
HSC
E
O
sub
I
L
CMB

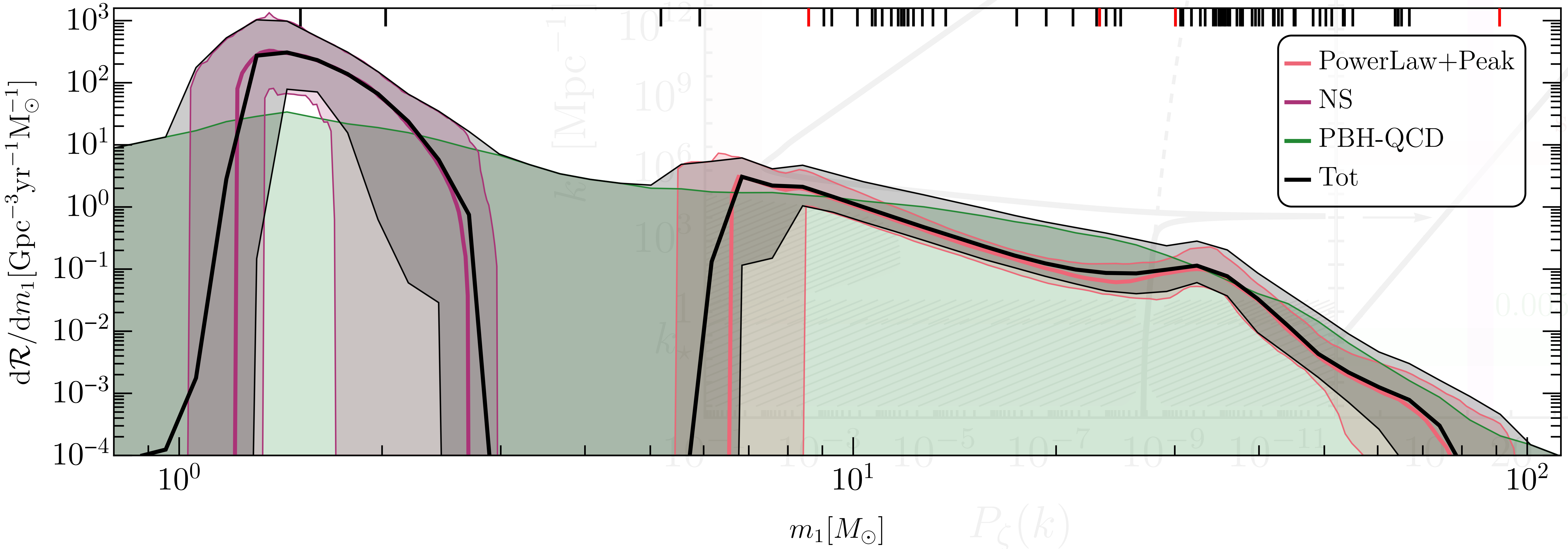
Gravitational-wave detections made by the LIGO-Virgo-KAGRA (LVK) Collaborations to date, arXiv:2111.03634

GWTC-3 dataset contains 69 binary BH events and 7 potential NS-involving binaries (which are characterised by at least one object with mass below 3 solar mass)



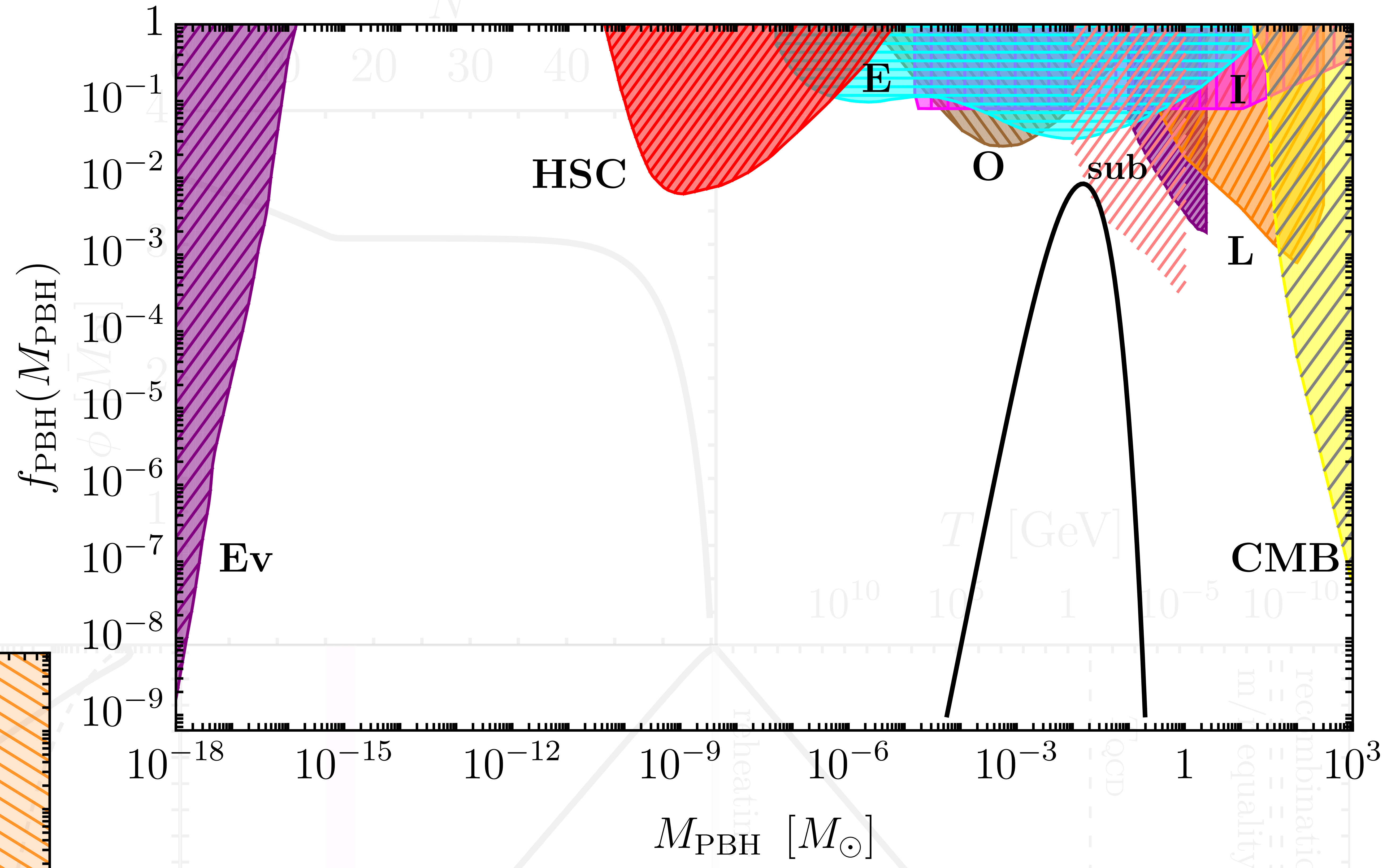
Only GW170817 is confidently regarded as a NS binary due to the observation of the electromagnetic counterpart

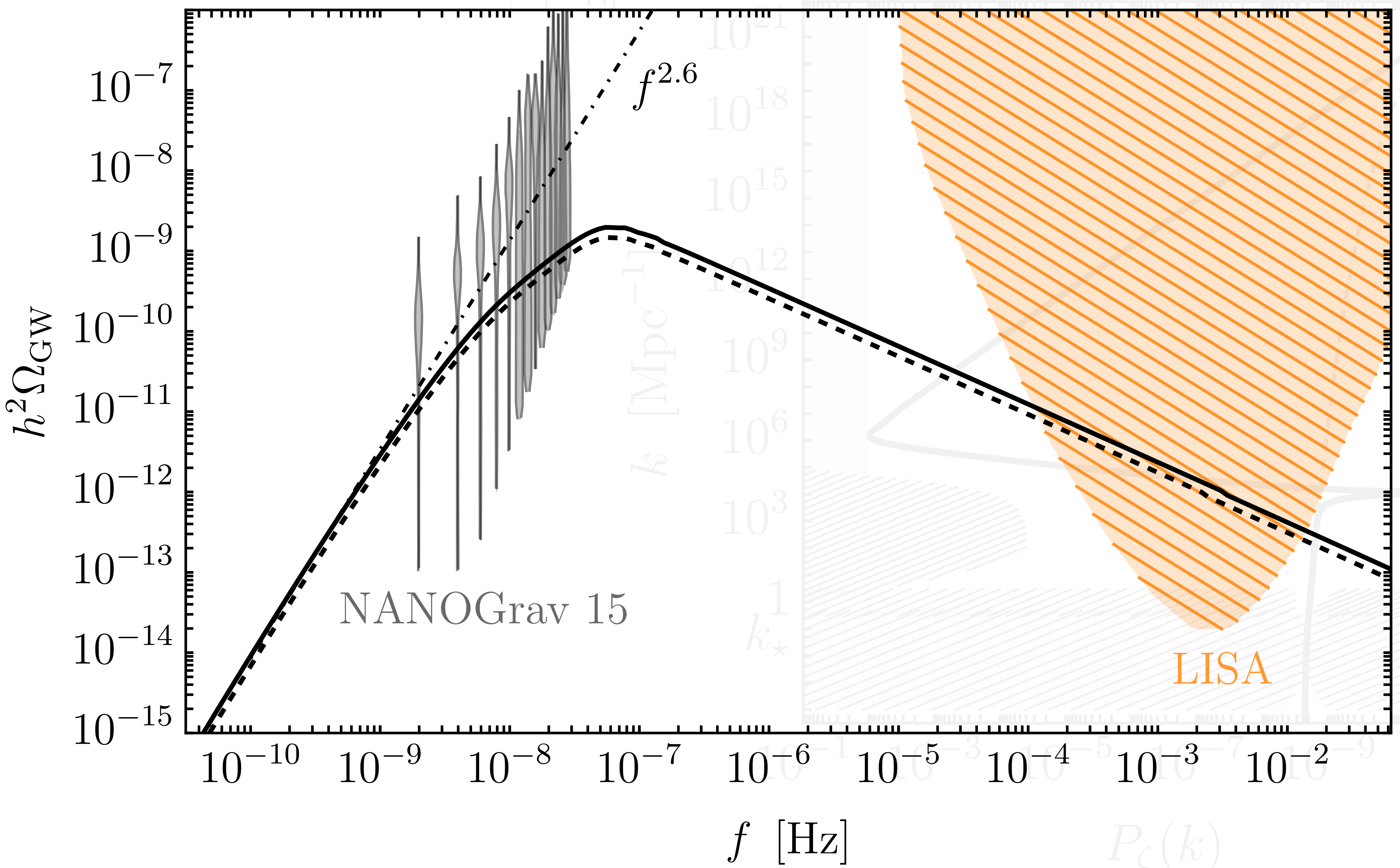
"Lower" mass gap: $\approx [2.2 \div 6]\ M_\odot$
"Upper" mass gap: $\gtrsim 50\ M_\odot$



GW event	$m_1[M_\odot]$	$m_2[M_\odot]$
GW190814	$23.2^{+1.1}_{-1.0}$	$2.59^{+0.08}_{-0.09}$
GW190924_021846	$8.9^{+7.0}_{-2.0}$	$5.0^{+1.4}_{-1.9}$

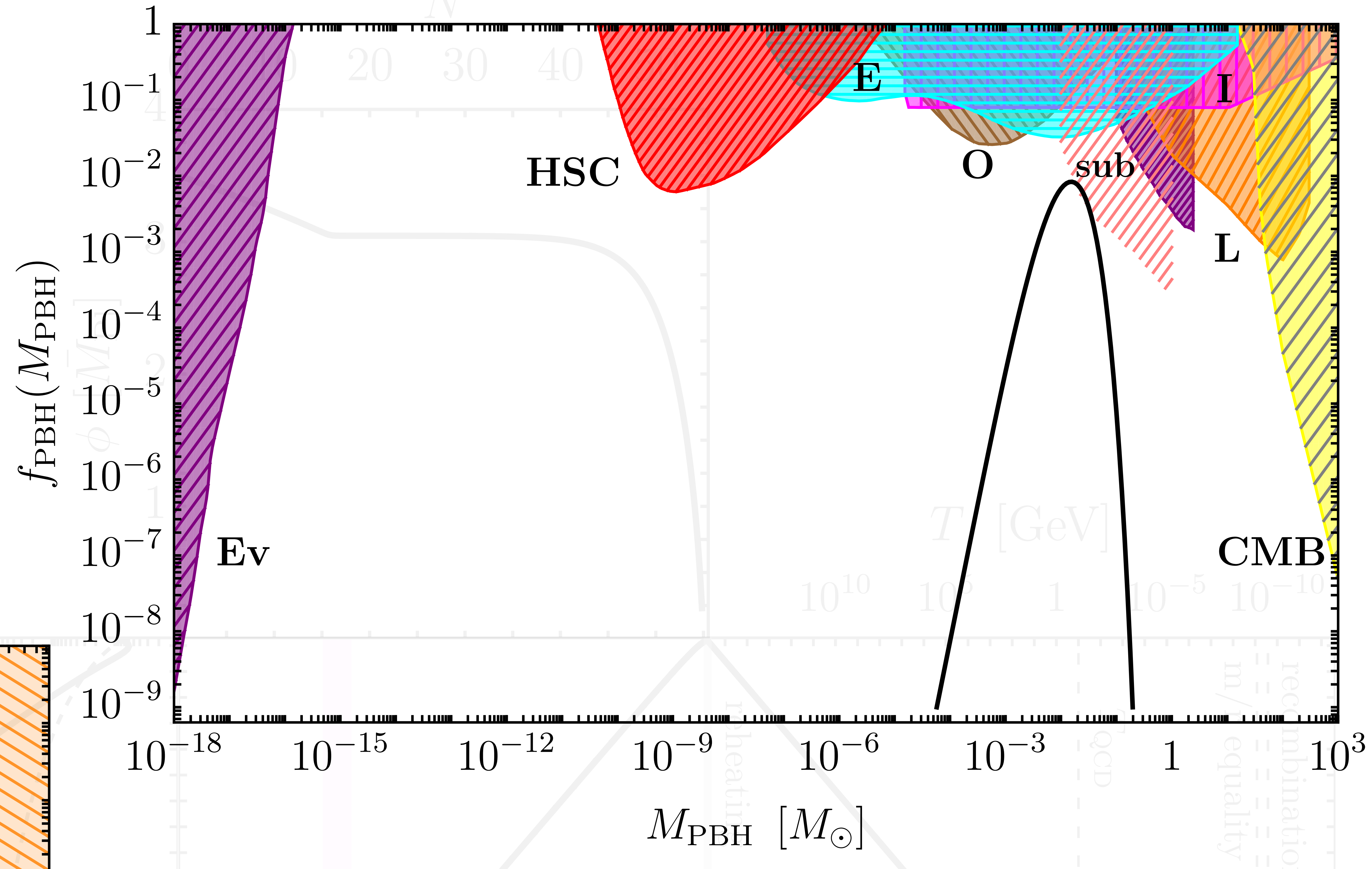
Intriguingly, our ab-initio PBH distribution allows us to make some falsifiable predictions: if some of the GWTC-3 events are primordial, then the merger rates in the subsolar mass range and in the lower mass gap are high enough to be detectable by future LVK runs. In particular, the absence of subsolar mergers in O5 would automatically exclude the primordial origin of the light events within GWTC-3.

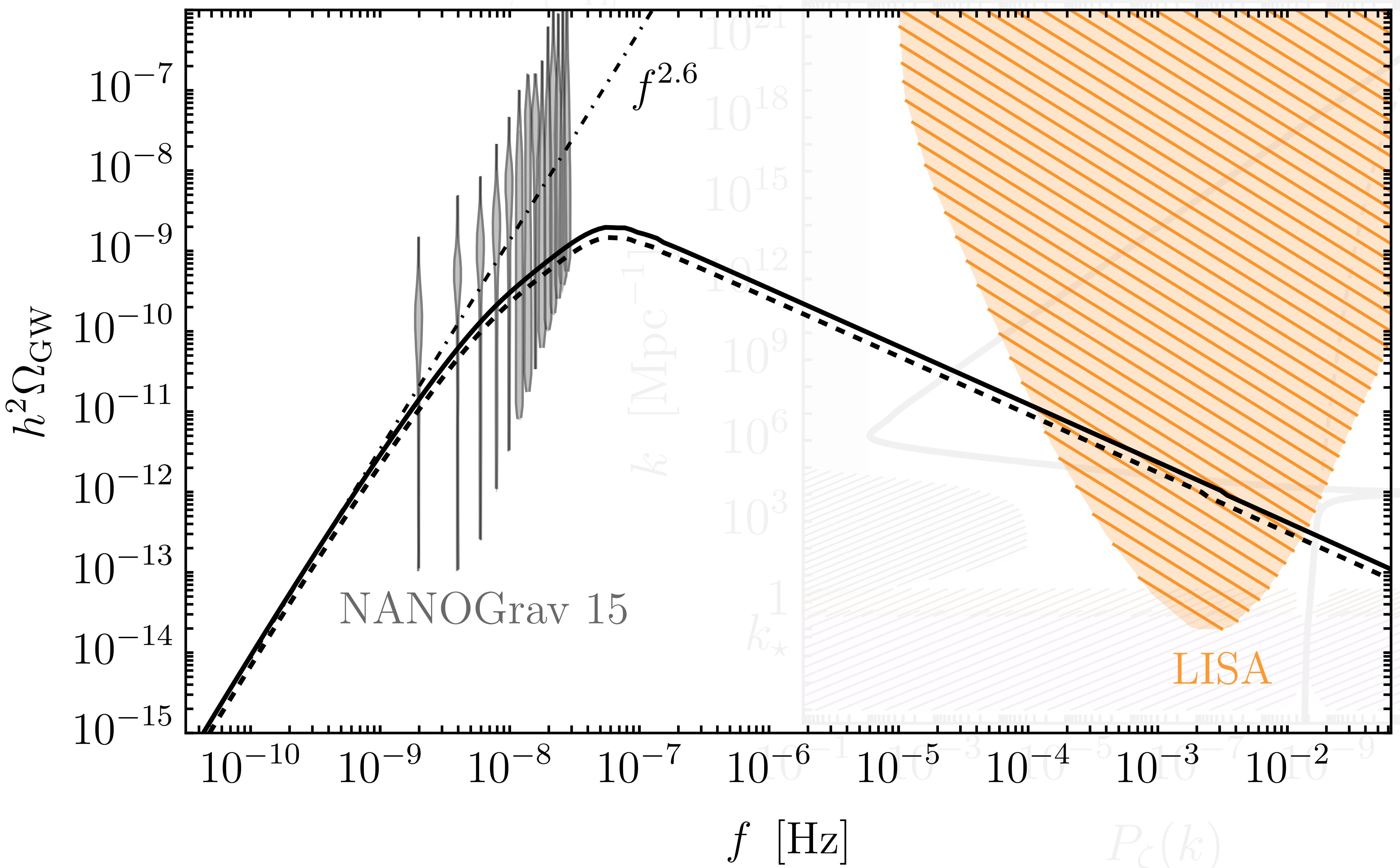




$$f \simeq 1.55 \left(\frac{k}{10^6\,\mathrm{Mpc}^{-1}} \right) \mathrm{nHz}$$

I interpret this tension not as a problem but as an opportunity: it means that we need to have full control over how we calculate the abundance of PBHs to ensure that the predicted signal of gravitational waves has the right amplitude.





$$f \simeq 1.55 \left(\frac{k}{10^6\,\mathrm{Mpc}^{-1}} \right) \mathrm{nHz}$$

Uncertainties in the theoretical computation of the PBH abundance arise from various aspects related to the early universe's physics, inflationary models, and the dynamics of structure formation. Addressing and understanding these uncertainties are crucial for accurate predictions and comparisons with observational data.

Equation of State during Radiation and Matter Domination:

- The behavior of the cosmic fluid (radiation or matter) during different epochs influences the collapse of overdense regions into black holes. Variations in the equation of state can affect the PBH abundance.

Critical Overdensity for Collapse:

- The critical threshold for collapse, which determines when an overdense region collapses to form a black hole, can be subject to uncertainties, particularly if non-Gaussian features in the primordial perturbations are considered.

Numerical Simulations:

- Numerical simulations used to study PBH formation are subject to limitations and assumptions, and uncertainties can arise from the numerical methods employed.

Non-Gaussianity:

- Deviations from Gaussian statistics in the primordial perturbations can introduce additional complexities in predicting the PBH abundance.

Do theoretical arguments exist against PBHs?

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + i\overline{N_R}\gamma^{\mu}(\partial_{\mu}N_R) - \left[\overline{N_R}Y_{\nu}\tilde{H}^{\dagger}L + \frac{1}{2}\,\overline{N_R}M_R N_R^{\,C} + h.c.\right]$$

$$\frac{dm_{\mathrm{R}}^2}{d\log\mu} = \beta^{\mathrm{SM}}_{m^2,1-\mathrm{loop}} m_{\mathrm{R}}^2 + \frac{2}{(4\pi)^2}\mathrm{Tr}\left(Y_\nu^\dagger Y_\nu\right) m_{\mathrm{R}}^2 + \frac{16}{(4\pi)^2}\mathrm{Tr}\left[\left(Y_\nu^\dagger M_R\right)\left(M_R^\dagger Y_\nu\right)\right]$$

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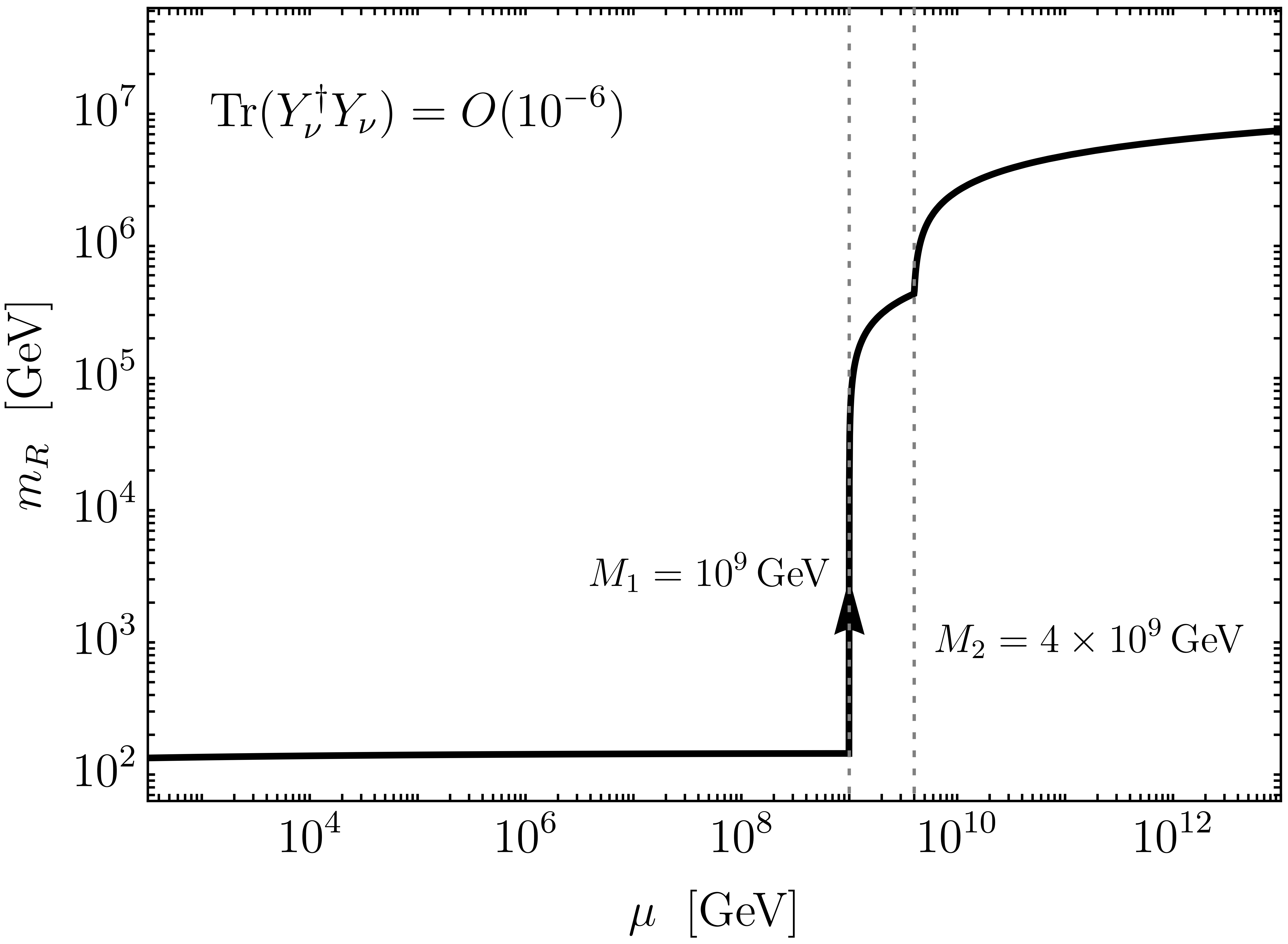
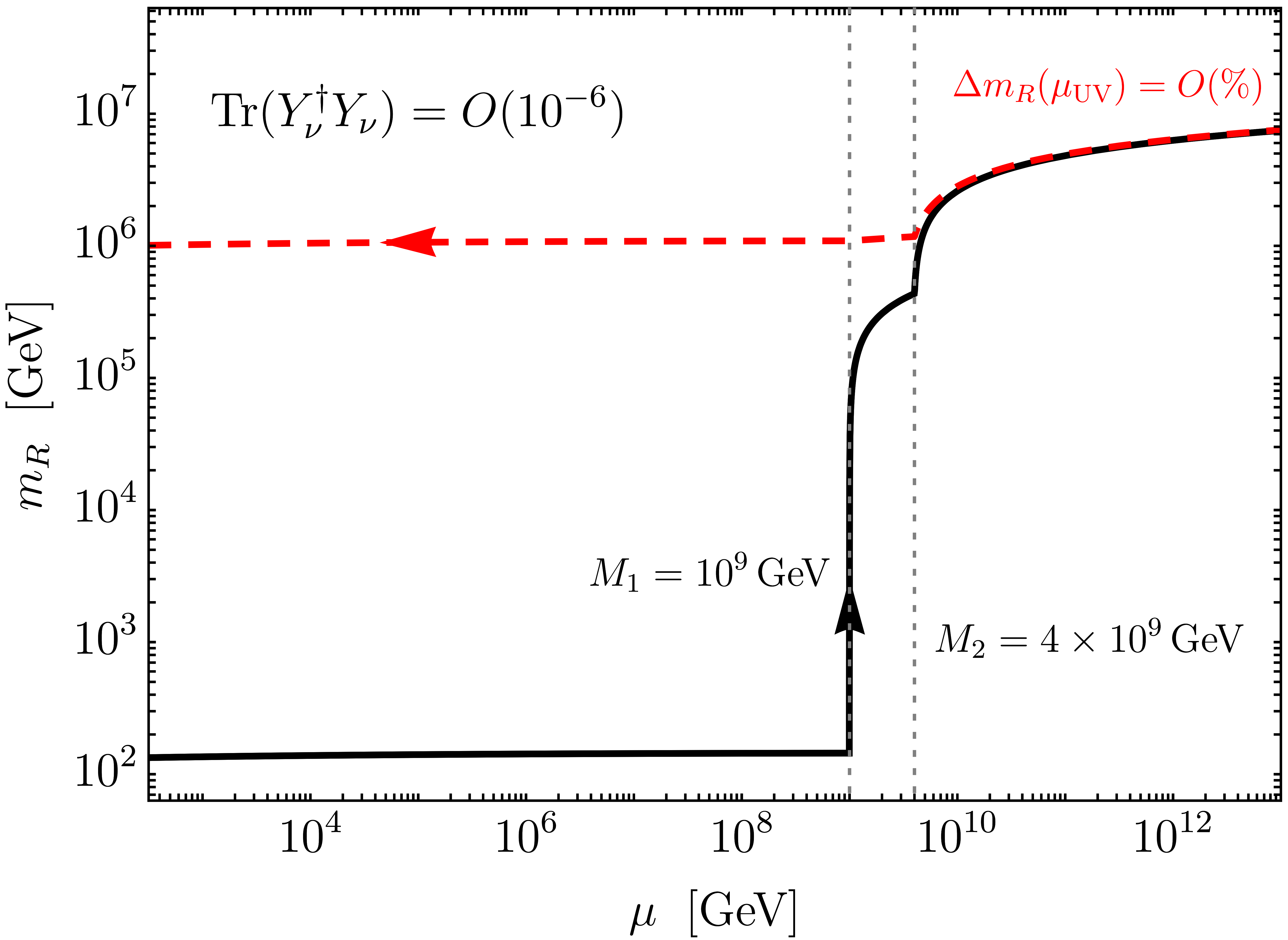
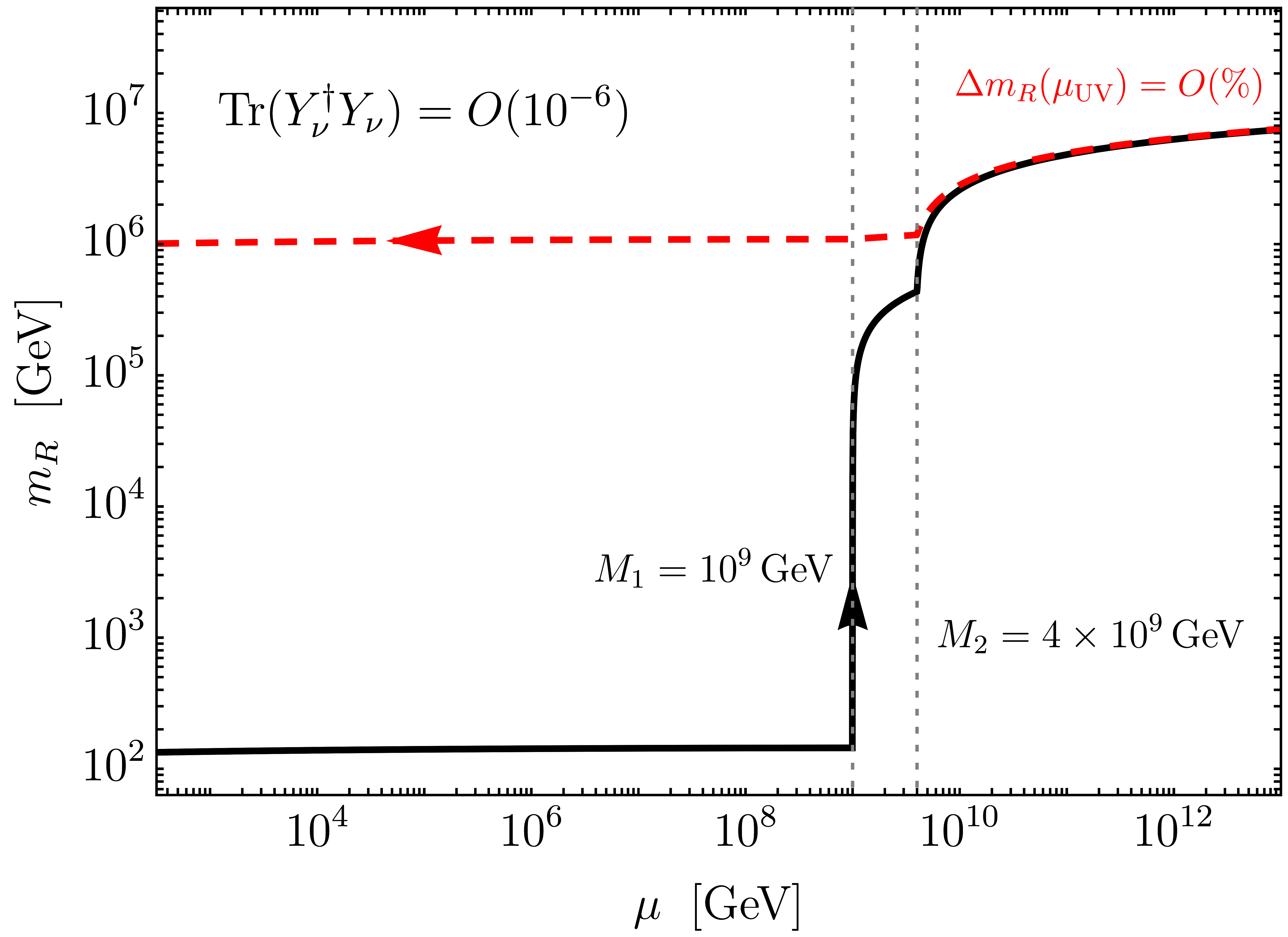
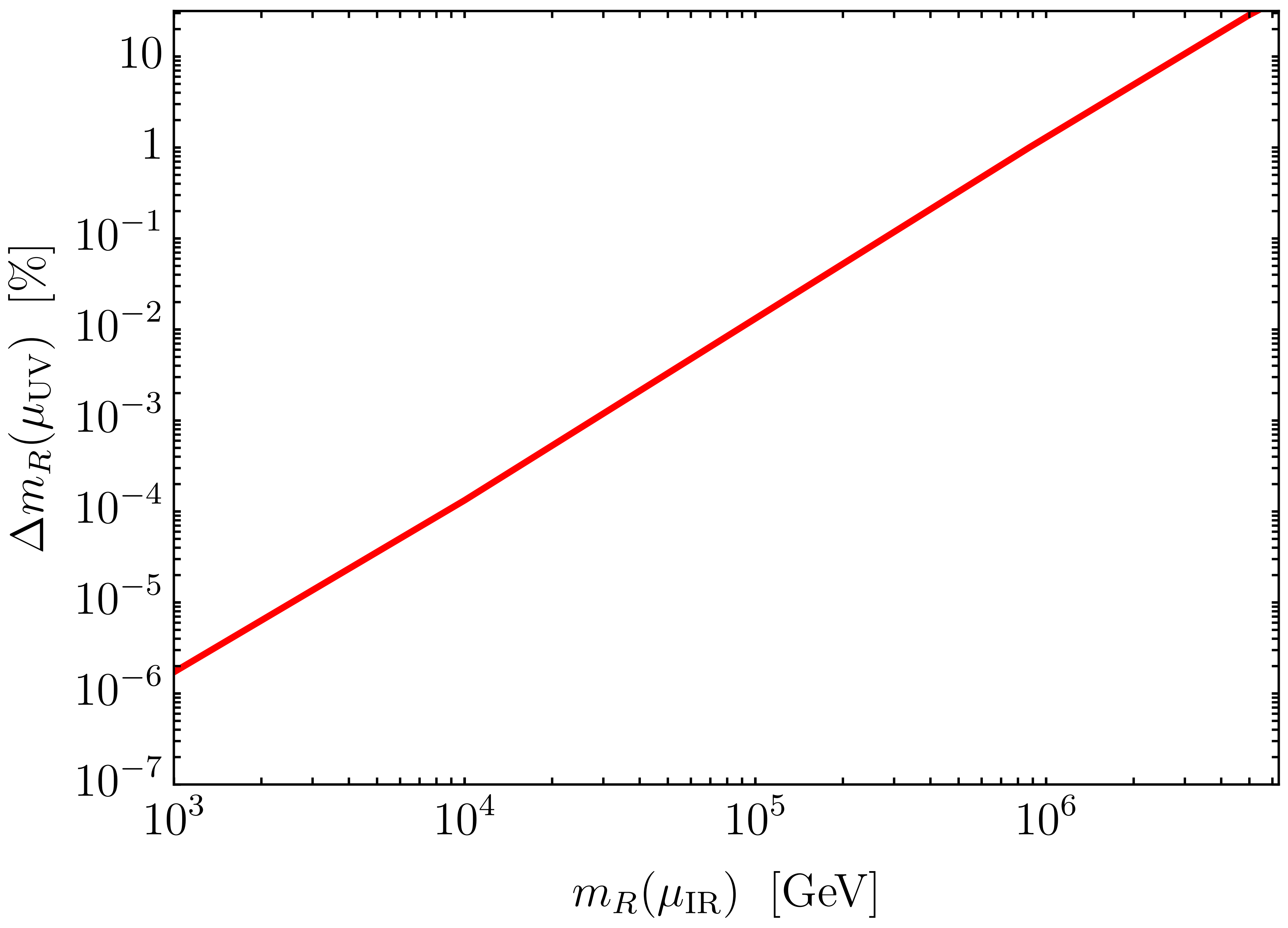


$$\frac{dm_{\rm R}^2}{d\log\mu} = \beta^{\rm SM}_{m^2,1-\rm loop} m_{\rm R}^2 + \frac{2}{(4\pi)^2}{\rm Tr}\left(Y_\nu^\dagger Y_\nu\right) m_{\rm R}^2 + \frac{16}{(4\pi)^2}{\rm Tr}\left[\left(Y_\nu^\dagger M_R\right)\left(M_R^\dagger Y_\nu\right)\right]$$



$$\frac{dm_R^2}{d\log\mu} = \beta^{SM}_{m^2,1-loop} m_R^2 + \frac{2}{(4\pi)^2}\mathrm{Tr}\left(Y_\nu^\dagger Y_\nu\right) m_R^2 + \frac{16}{(4\pi)^2}\mathrm{Tr}\left[\left(Y_\nu^\dagger M_R\right)\left(M_R^\dagger Y_\nu\right)\right]$$





Finite naturalness criterium: the Higgs mass is naturally small as long as there are no heavier particles that give large finite contributions to the Higgs mass.

Let's consider the case of a **heavy mass scale not coupled to the Higgs**.
Is there a finite naturalness bound?

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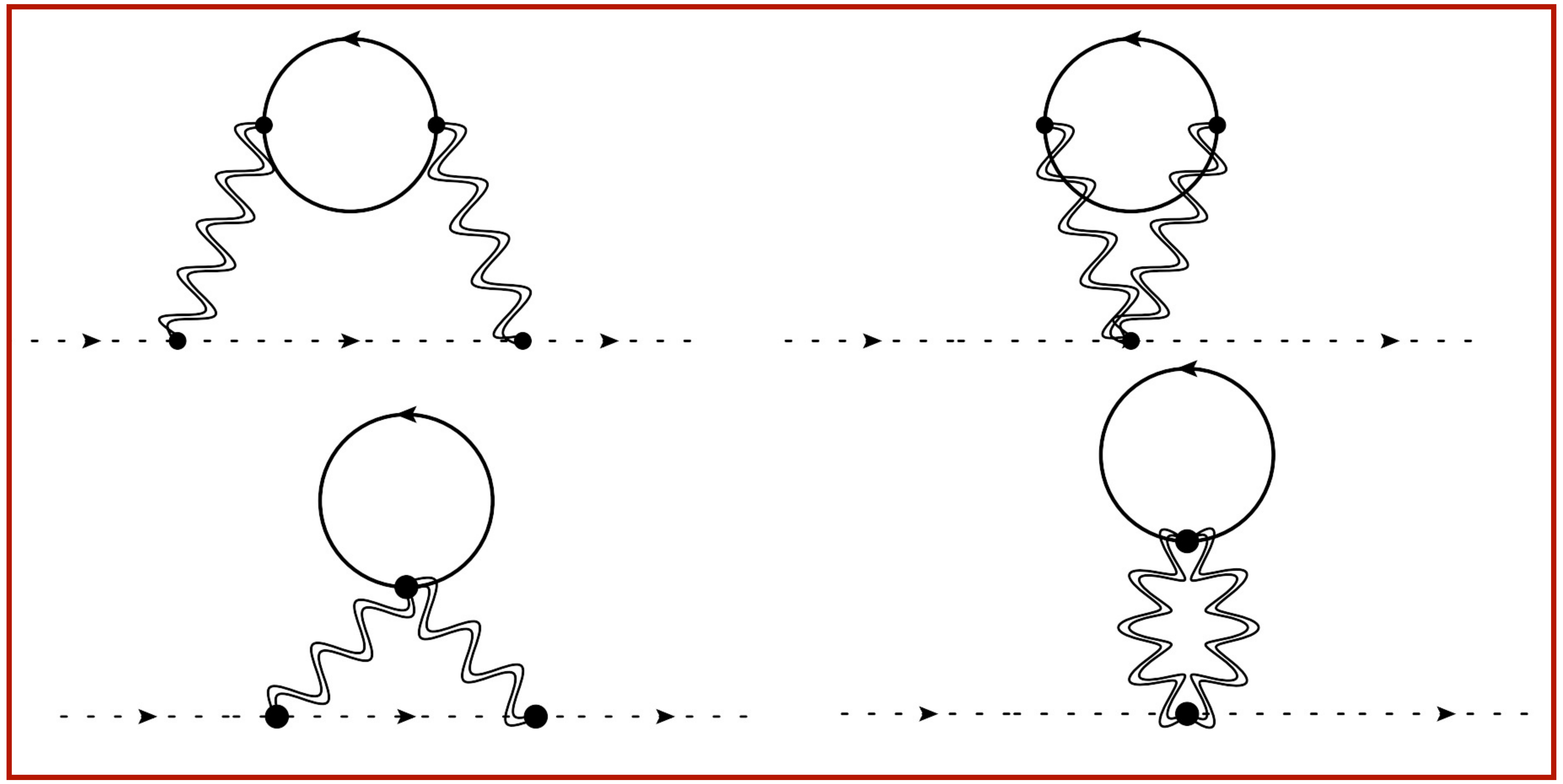
Is there a finite naturalness bound?



$$\delta m_H^2 \sim \frac{1}{(16\pi^2)^2}\frac{M_\Psi^4}{M_{Pl}^4} \times m_H^2$$

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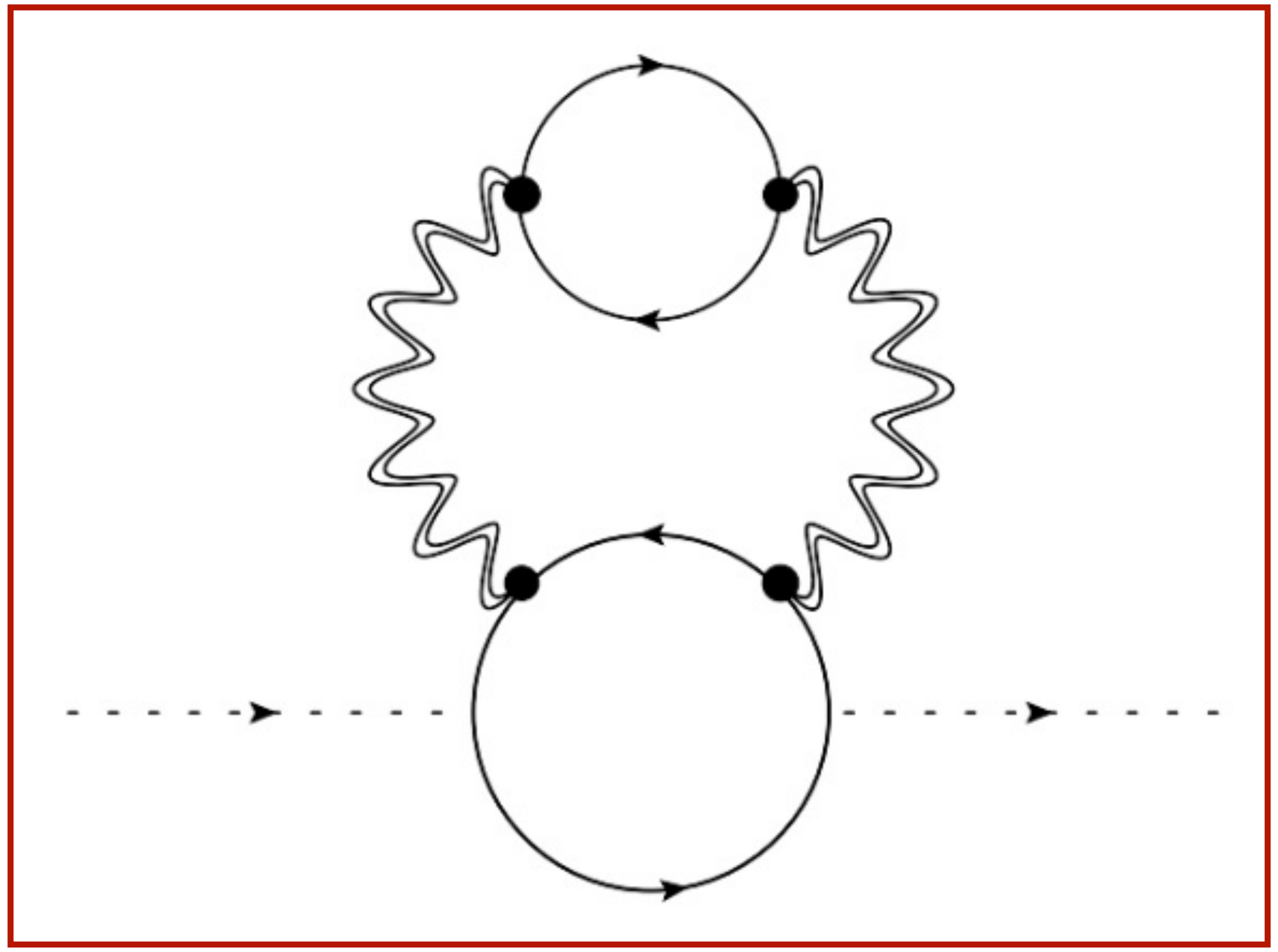
Is there a finite naturalness bound?

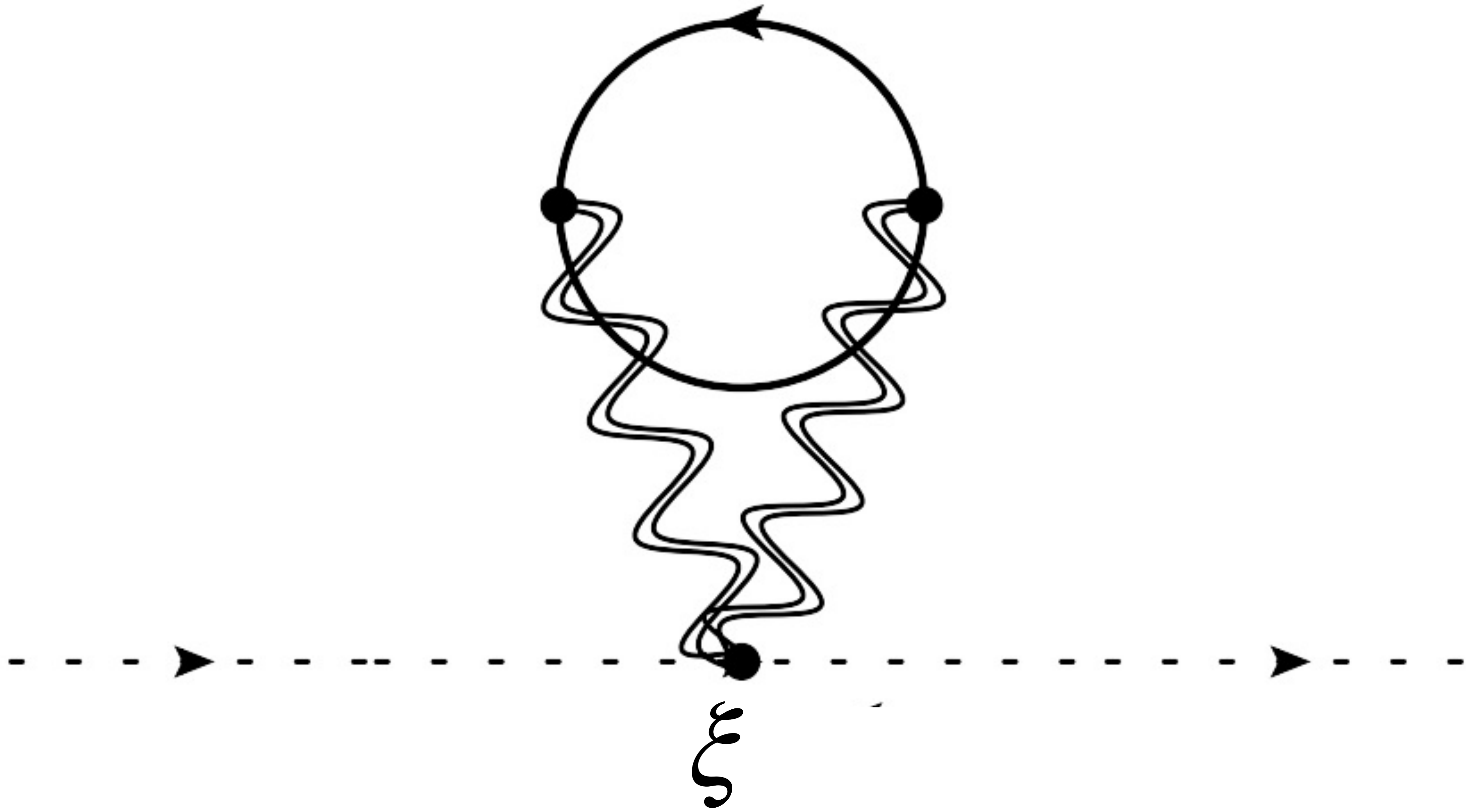


$$\delta m_H^2 \sim \frac{y_t^2}{(16\pi)^3}\frac{M_\Psi^4}{M_{\rm Pl}^4} \times M_\Psi^2$$

$$M_\Psi \lesssim 10^{14}\,\mathrm{GeV}$$

Let's consider the case of a **heavy mass scale not coupled to the Higgs.**
Is there a finite naturalness bound?

$$\mathcal{S}_{\mathrm{EH}+H} = \int d^4x\sqrt{-g}\left[\left(\frac{1}{2}\bar{M}_{\mathrm{Pl}}^2 + \xi H^\dagger H\right)R + (D_\mu H)^\dagger(D^\mu H) - V(H^\dagger H)\right]$$



$$M_\Psi \lesssim 10^{10}\,\xi^{-1/4}\,\mathrm{GeV}$$

Let's consider the case of a **heavy mass scale not coupled to the Higgs.** Is there a finite naturalness bound?

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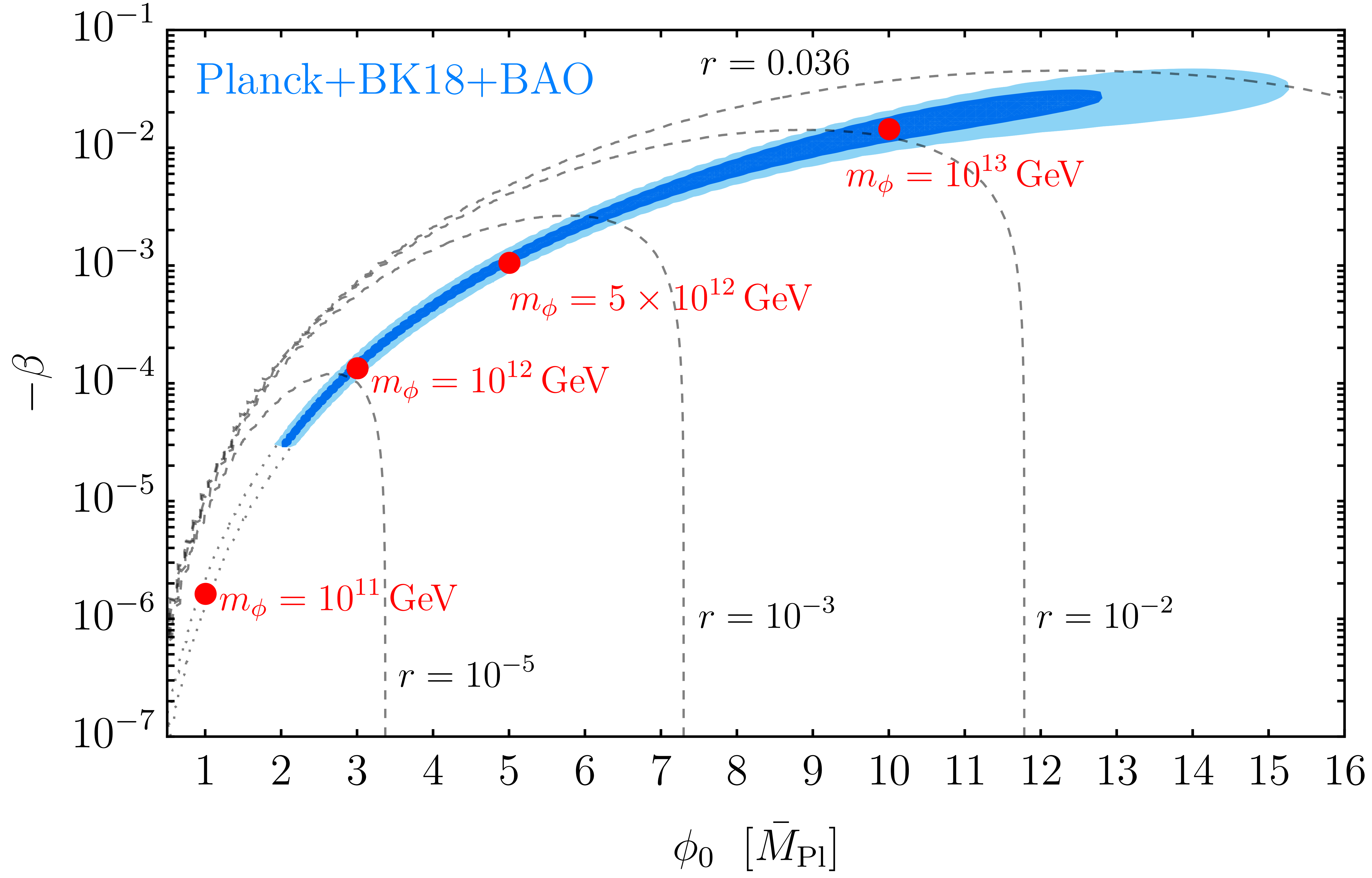

$$M_\Psi \lesssim 10^{10}\,\xi^{-1/4}\,\mathrm{GeV}$$

The criterion of finite naturalness disfavors large-field inflationary models with super-Planckian field excursions.

Do theoretical arguments exist against PBHs?

$$ds^2 = N^2 dt^2 - a^2(t) e^{2\zeta(\vec{x},t)} \delta_{ij} (N^i dt + dx^i)(N^j dt + dx^j)$$

$$\delta\phi(\vec{x}, t) = 0$$

The field $\zeta(\vec{x}, t)$ is the only independent scalar degree of freedom since N and N^i are Lagrange multipliers subject to the momentum and Hamiltonian constraints.

$$\mathcal{S}_2 = \int d^4x \, \epsilon \, a^3 \left[\dot{\zeta}^2 - \frac{(\partial_k \zeta)(\partial^k \zeta)}{a^2} \right]$$

$$ds^2 = N^2 dt^2 - a^2(t) e^{2\zeta(\vec{x},t)} \delta_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

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$$\mathcal{S} = \int d^3\vec{x} dt \underbrace{\mathcal{L}[\zeta(\vec{x}, t), \dot{\zeta}(\vec{x}, t), \partial_k \zeta(\vec{x}, t)]}_{\equiv \mathcal{L}[\zeta(\vec{x},t)]} = \underbrace{\int d^3\vec{x} dt\, \mathcal{L}_2(\vec{x}, t)}_{\equiv \mathcal{S}_2} + \underbrace{\int d^3\vec{x} dt\, \mathcal{L}_3[\zeta(\vec{x}, t)]}_{\equiv \mathcal{S}_3} + \underbrace{\int d^3\vec{x} dt\, \mathcal{L}_4[\zeta(\vec{x}, t)]}_{\equiv \mathcal{S}_4} + \ldots$$

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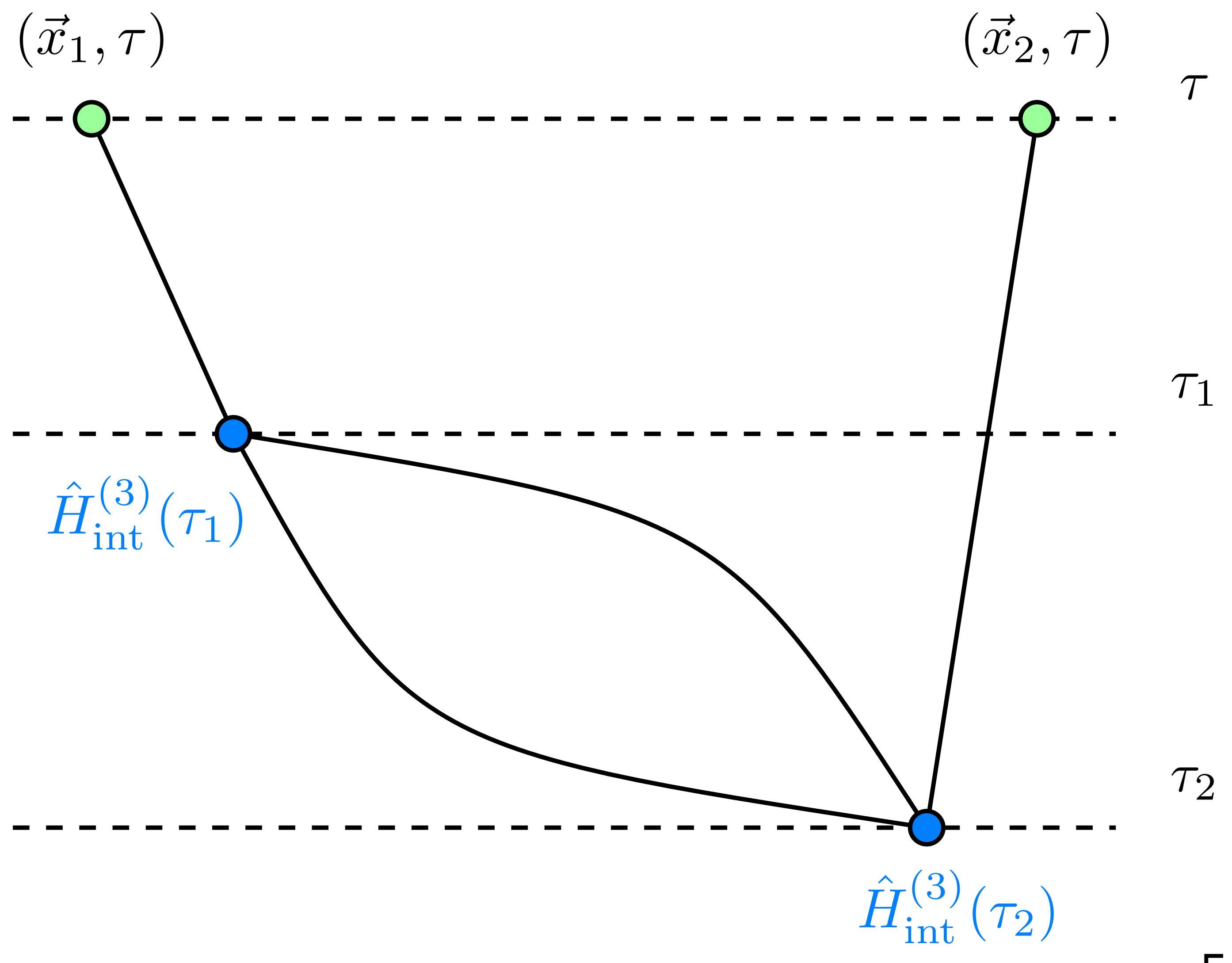
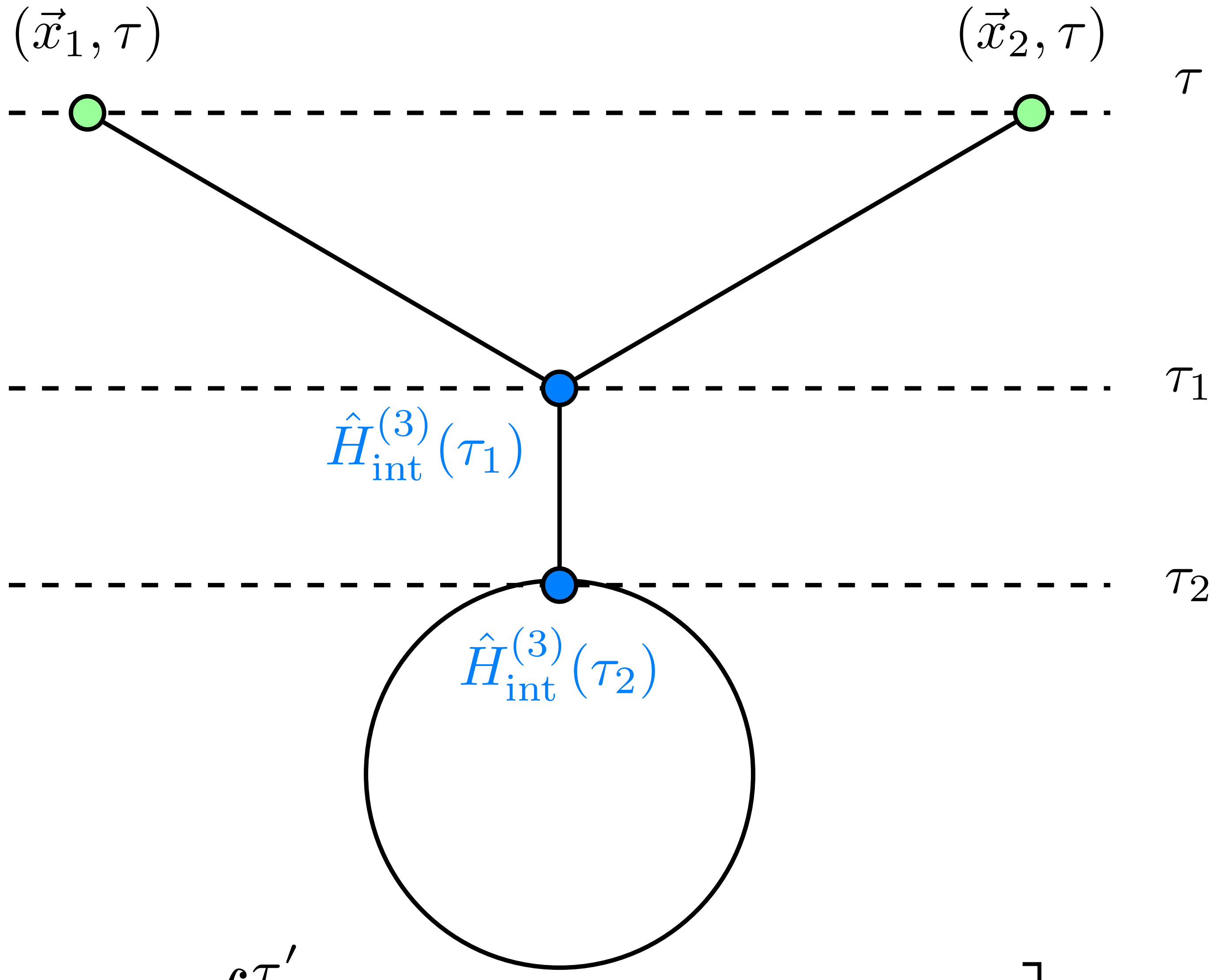
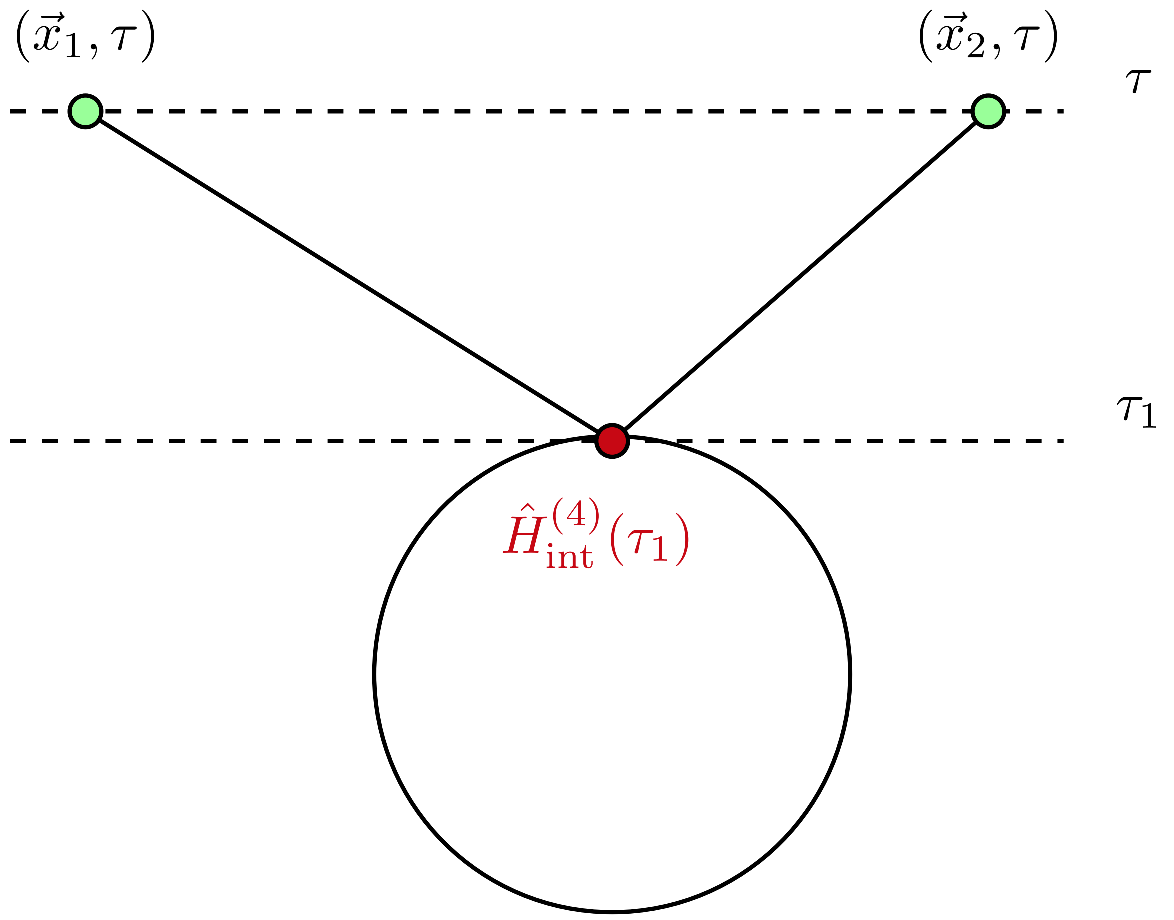
$$\mathcal{S}_2 = \int d^4x\, \epsilon\, a^3 \left[\dot{\zeta}^2 - \frac{(\partial_k\zeta)(\partial^k\zeta)}{a^2}\right]$$

$$\mathcal{S}_3 = \int d^4x \Big\{ \epsilon^2 a^3 \dot{\zeta}^2 \zeta + \epsilon^2 a \zeta (\partial_k\zeta)(\partial^k\zeta) - 2\epsilon^2 a^3 \dot{\zeta}(\partial_k\zeta)\partial^k(\partial^{-2}\dot{\zeta}) + \frac{\epsilon\dot{\epsilon}_2}{2} a^3 \dot{\zeta}\zeta^2 - \frac{a^3\epsilon^3}{2}[\dot{\zeta}^2\zeta - \zeta\partial_k\partial_l(\partial^{-2}\dot{\zeta})\partial^k\partial^l(\partial^{-2}\dot{\zeta})]$$
$$+ \left[\frac{d}{dt}\left(\epsilon a^3 \dot{\zeta}\right) - \epsilon a \partial_k\partial^k\zeta\right]\Big[\frac{\epsilon_2}{2}\zeta^2 + \frac{2}{H}\dot{\zeta}\zeta - \frac{1}{2a^2H^2}(\partial_k\zeta)(\partial^k\zeta) + \frac{1}{2a^2H^2}\partial^{-2}\partial_k\partial_l(\partial^k\zeta\partial^l\zeta)$$
$$+ \frac{\epsilon}{H}(\partial_k\zeta)\partial^k(\partial^{-2}\dot{\zeta}) - \frac{\epsilon}{H}\partial^{-2}\partial_k\partial_l\partial^k\zeta\partial^l(\partial^{-2}\dot{\zeta})\Big]\Big\}.$$



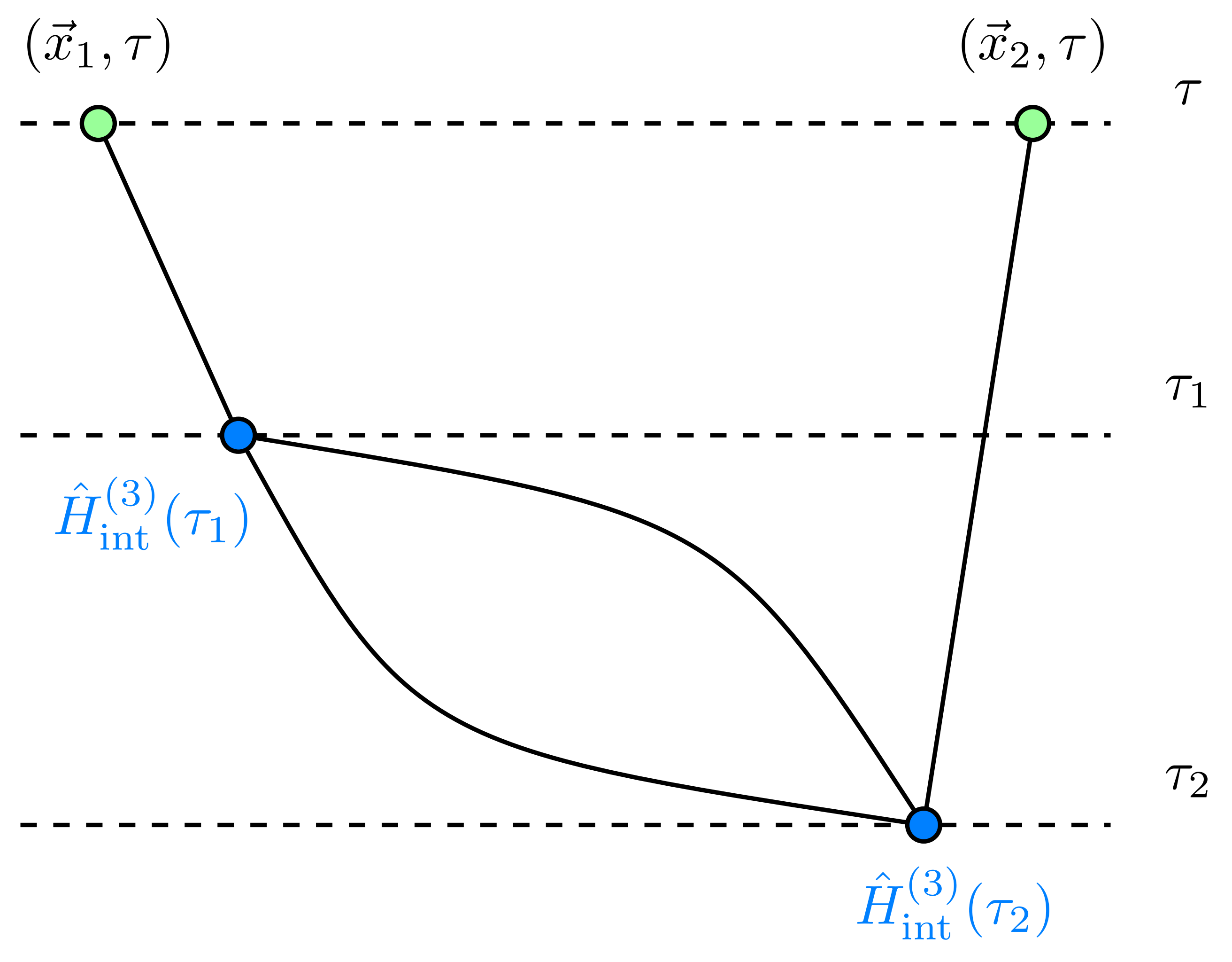


$$
\begin{aligned}
\langle\hat{\zeta}(\vec{x}_1,\tau)\hat{\zeta}(\vec{x}_2,\tau)\rangle_{2^{\rm nd}} = &\langle 0|\hat{\zeta}_I(\vec{x}_1,\tau)\hat{\zeta}_I(\vec{x}_2,\tau)\Big[-\int_{-\infty_-}^{\tau} d\tau' \int_{-\infty_-}^{\tau'} d\tau'' \hat{H}^{(3)}_{\text{int}}(\tau')\hat{H}^{(3)}_{\text{int}}(\tau'')\Big]|0\rangle \\
&+\langle 0|\Big[-\int_{-\infty_+}^{\tau} d\tau' \int_{-\infty_+}^{\tau'} d\tau'' \hat{H}^{(3)}_{\text{int}}(\tau'')\hat{H}^{(3)}_{\text{int}}(\tau')\Big]\hat{\zeta}_I(\vec{x}_1,\tau)\hat{\zeta}_I(\vec{x}_2,\tau)|0\rangle \\
&+\langle 0|\Big[i\int_{-\infty_+}^{\tau} d\tau' \hat{H}^{(3)}_{\text{int}}(\tau')\Big]\hat{\zeta}_I(\vec{x}_1,\tau)\hat{\zeta}_I(\vec{x}_2,\tau)\Big[-i\int_{-\infty_-}^{\tau} d\tau'' \hat{H}^{(3)}_{\text{int}}(\tau'')\Big]|0\rangle
\end{aligned}
$$



$$
\begin{aligned}
&\langle\hat{\zeta}(\vec{x}_1,\tau)\hat{\zeta}(\vec{x}_2,\tau)\rangle_{1^{\rm st}} = \\
&\langle 0|\hat{\zeta}_I(\vec{x}_1,\tau)\hat{\zeta}_I(\vec{x}_2,\tau)\Big[-i\int_{-\infty_-}^{\tau} d\tau' \hat{H}^{(4)}_{\text{int}}(\tau')\Big]|0\rangle + \langle 0|\Big[i\int_{-\infty_+}^{\tau} d\tau' \hat{H}^{(4)}_{\text{int}}(\tau')\Big]\hat{\zeta}_I(\vec{x}_1,\tau)\hat{\zeta}_I(\vec{x}_2,\tau)|0\rangle
\end{aligned}
$$

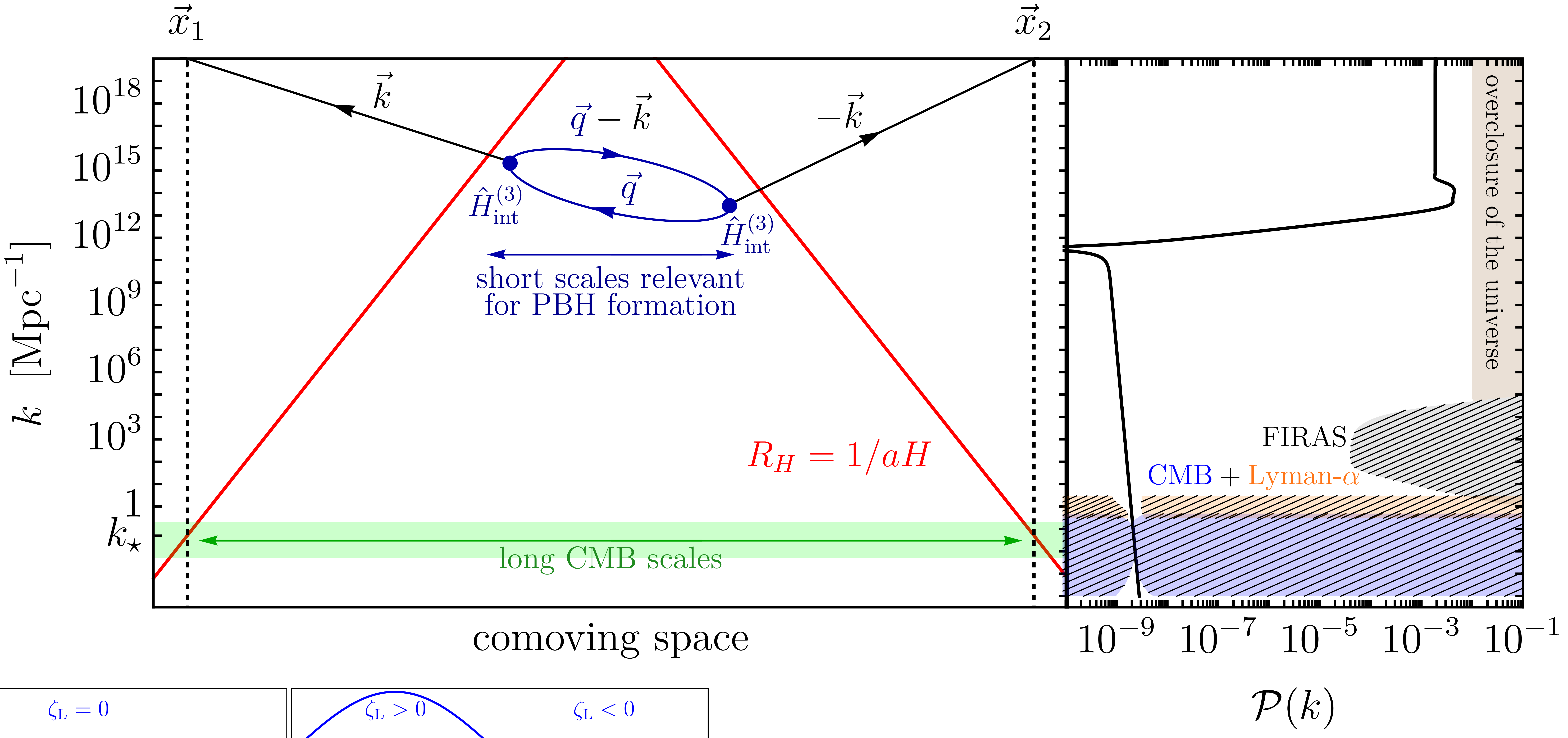
J.Kristiano and J.Yokoyama,
"Ruling Out Primordial Black Hole Formation From Single-Field Inflation,"
[arXiv:2211.03395[hep-th]].

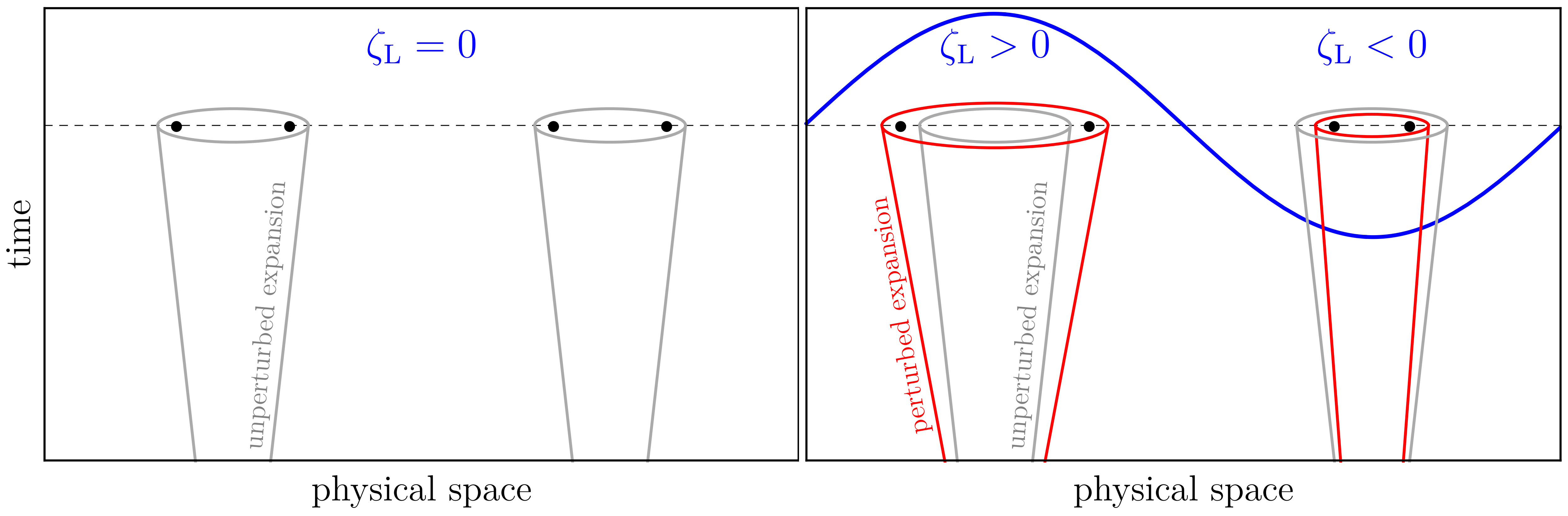


$$P(k) \equiv P_{\rm tree}(k)\left[1 + \Delta P_{1-{\rm loop}}(k)\right] \quad \Longrightarrow \quad \Delta P_{1-{\rm loop}}(k) \overset{!}{<} 1$$

$$\lim_{\delta N \to 0} \Delta P_{1-{\rm loop}}(k_*) \approx \left(\frac{H^2}{8\pi^2 \epsilon_{\rm ref}}\right) \eta_{\rm II}^2 \left(\frac{k_{\rm end}}{k_{\rm in}}\right)^6 \left[1 + \log\left(\frac{k_{\rm end}}{k_{\rm in}}\right)\right]$$

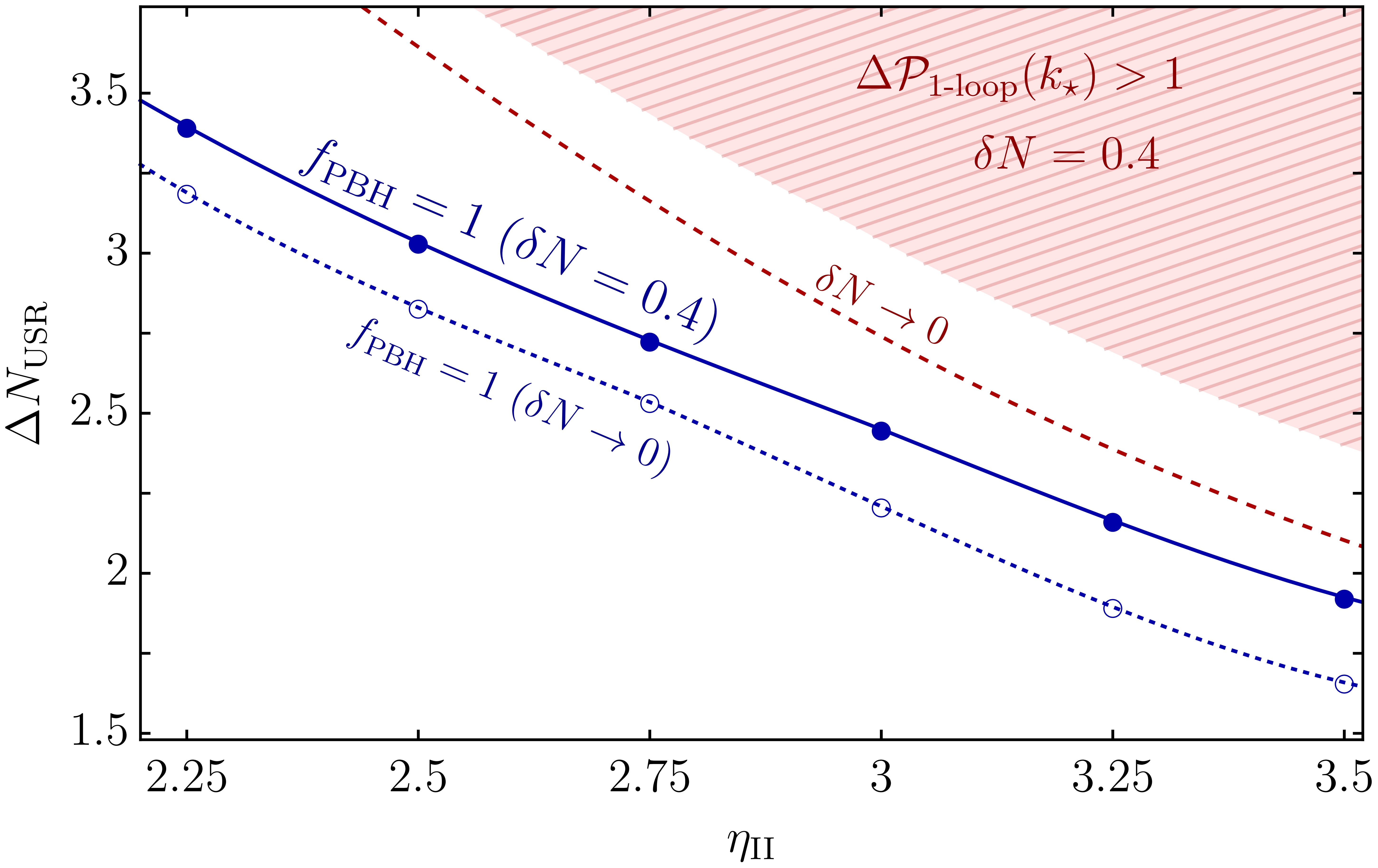
"We have shown that models realizing appreciable amount of PBH formation with the enhanced small-scale spectrum by USR inflaton dynamics inevitably induces nonperturbative coupling to the power spectrum on CMB scale. We therefore conclude that PBH formation from cosmological perturbation theory single-field inflation with an USR dynamics is ruled out."





$$\Delta P_{1-\text{loop}}(k) \sim P(k)\int \frac{dq}{q}\,P(q)\,\frac{d\log P}{d\log q}$$

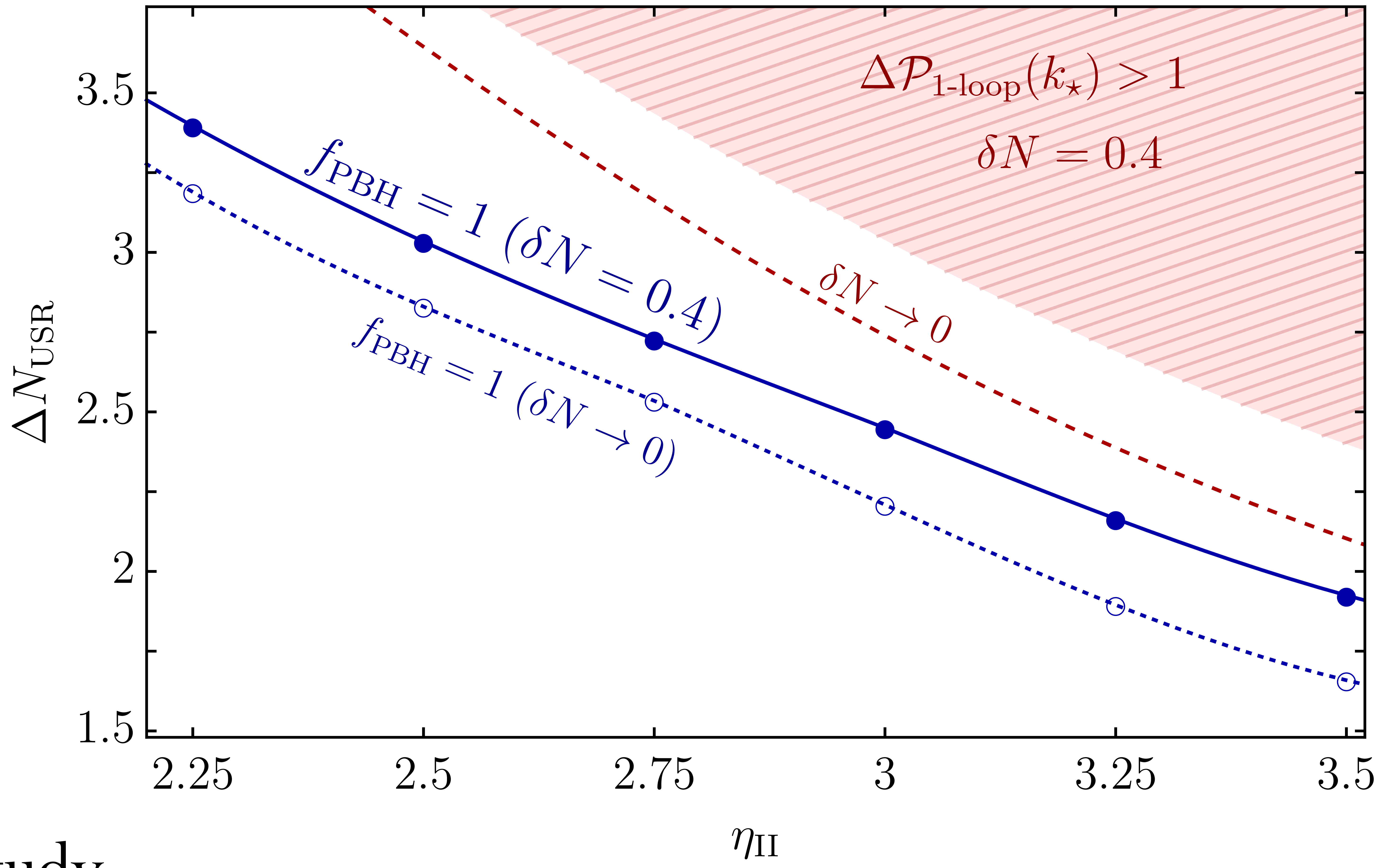
Sizable abundance of PBHs in USR single-field inflation is **not in conflict** with perturbativity constraints.



On the other hand, one-loop corrections are sizable, and the contribution of short wavelengths to the power spectrum at large scales **does not decouple**. This suggests that theoretical constraints dictated by the requirement of perturbativity might be important.

G.L.Pimentel, L.Senatore and M.Zaldarriaga,
"On Loops in Inflation III:
Time Independence of zeta in Single Clock Inflation,"
JHEP **07** (2012), 166 [arXiv:1203.6651 [hep-th]].

L.Senatore and M.Zaldarriaga,
"On Loops in Inflation,"
JHEP **12** (2010), 008
[arXiv:0912.2734 [hep-th]]



I am convinced that a more in-depth study
in the presence of non-single-clock dynamics is necessary.

Conclusions?

I haven't included the conclusions, for good luck.
Maybe at the next meeting, I will be able to update you on the things I presented or discuss new ones.