Theory of Neutrino Masses Beyond the Standard Model

S. T. Petcov

SISSA/INFN, Trieste, Italy, and
Kavli IPMU, University of Tokyo, Japan

15th International Neutrino Summer School (INSS 2024)

Department of Physics, University of Bologna

Bologna, Italy

June 3 - 14 (June 5, 6 and 10), 2024

Plan of the Lectures

Lectures 1 and 2

1. Introduction.

2. Massive Neutrinos, Neutrino Mixing: Overview of the Current Status.

3. Open Questions in the Physics of Massive Neutrinos;
Goals of Future research. The Lepton Flavour Problem.

4. The Nature of Massive neutrinos I:
Massive Majorana versus Massive Dirac Neutrinos.

5. PMNS Neutrino Mixing Matrix: Dirac and Majorana CP Violation.

6. The Nature of Massive Neutrinos II: Origins of Dirac and Majorana Massive Neutrinos.

7. The Seesaw Mechanisms of Neutrino Mass Generation.

8. Radiative Mechanisms of Neutrino Mass Generation.

Lecture 3

9. Determining the Nature of Massive Neutrinos: Neutrinoless Double Beta Decay (Elementary Particle Aspects of the Theory)

10. Aspects of the Theory of Neutrino Oscillations:
– Describing Analytically the Solar Neutrino Transitions in the Sun.
– Oscillations of Neutrino Crossing the Earth Core.
– Determining the Neutrino Mass Spectrum in JUNO Experiment with Reactor $\overline{\nu}_e$.

11. Conclusions

Basic Literature

1. S. M. Bilenky and S. T. Petcov,
"Massive Neutrinos and Neutrino Oscillations,"
Reviews of Modern Physics 59 (1987) 671,
doi:10.1103/RevModPhys.59.671.

2. R. N. Mohapatra and P. B. Pal, "Massive neutrinos in physics and astrophysics. Second edition," World Sci. Lect. Notes Phys. 60 (1998) 1-397.

3. C. Giunti and C.W. Kim,
"Fundamentals of Neutrino Physics and Astrophysics", Oxford University Press, 2007.

4. S. T. Petcov, "The Nature of Massive Neutrinos," Adv. High Energy Phys. 2013 (2013) 852987 [arXiv:1303.5819 [hep-ph]].

5. K. Nakamura and S.T. Petcov,
"Neutrino Masses, Mixing and Oscillations",
in M. Tanabashi *et al.* **[Particle Data Group], "Review of Particle Physics,"**
Phys. Rev. D 98 (2018) 030001, pp. 251 - 286,
doi:10.1103/PhysRevD.98.030001.

Determining the Nature of Massive Neutrinos

Determining the status of lepton charge conservation and the nature - Dirac or Majorana - of massive neutrinos is one of the most challenging and pressing problems in present day elementary particle physics.

ν_j- Dirac or Majorana particles, **fundamental problem**

ν_j-Dirac: **conserved lepton charge exists,**
$L = L_e + L_\mu + L_\tau$, $\nu_j \neq \bar{\nu}_j$

ν_j-Majorana: **no lepton charge is exactly conserved**,
$\nu_j \equiv \bar{\nu}_j$

The observed patterns of $\nu-$mixing and of $\Delta m^2_{\rm atm}$ and $\Delta m^2_\odot$ can be related to Majorana ν_j and a new fundamental (approximate) symmetry (Lectures 4 and 5).

See-saw mechanisms: ν_j- Majorana

Establishing that the total lepton charge $L = L_e + L_\mu + L_\tau$ is not conserved in particle interactions by observing the $(\beta\beta)_{0\nu}-$decay would be a fundamental discovery (similar to establishing baryon number nonconservation (e.g., by observing proton decay)).

Establishing that ν_j are Majorana particles would be of fundamental importance, as important as the discovery of $\nu-$ oscillations, and would have far reaching implications.

Dirac CP-Nonconservation: δ in $U_{\rm PMNS}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ , \qquad l,l' = e,\mu,\tau$$

- not sensitive to Majorana CPVP $\alpha_{21},\ \alpha_{31}$

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{l'j} e^{-i(E_j t - p_j x)} U^{\dagger}_{jl}$$

$$U = VP\ :\ \ P_j e^{-i(E_j t - p_j x)} P^*_j = e^{-i(E_j t - p_j x)}$$

P **- diagonal matrix of Majorana phases.**
The result is valid also for oscillations in matter:

ν_l **oscillations are not sensitive to the nature of** ν_j.

If ν_j- Majorana particles, $U_{\rm PMNS}$ contains (3-ν mixing)

δ-Dirac, α_{21}, α_{31} - Majorana **physical CPV phases**

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- **are not sensitive to the nature of ν_j,**

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- **provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.**

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$
$$\mu^- + \text{(A,Z)} \rightarrow \mu^+ + \text{(A,Z}-2)$$

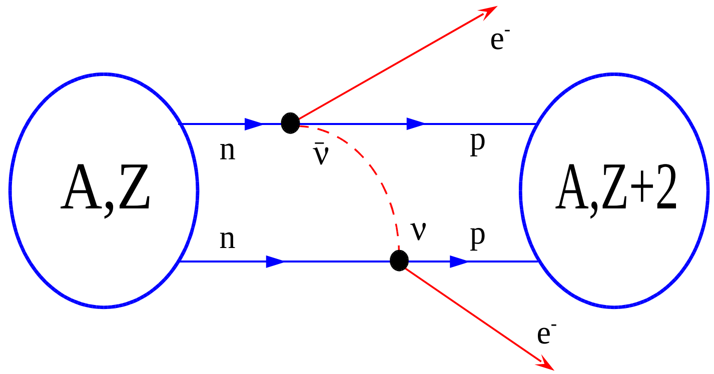
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$\text{(A,Z)} \rightarrow \text{(A,Z}+2) + e^- + e^-$$

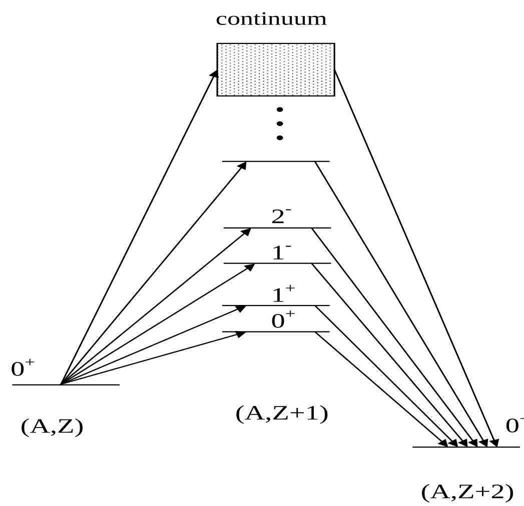
of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process
$dd \to uue^- e^- (\overline{\nu}_e \overline{\nu}_e)$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

Figure due to V. Rodin

$(\beta\beta)_{0\nu}$−Decay Experiments:

- $L-$**nonconservation, Majorana nature of ν_j.**
- **Type of $\nu-$mass spectrum (NH, IH, QD)**
- **Absolute neutrino mass scale**

^{3}H β-**decay, cosmology:** min(m_i) **(QD, IH)**

- **CPV due to Majorana CPV phases**

$A(\beta\beta)_{0\nu} \sim G_F^2 < m >$ **M(A,Z),** **M(A,Z) - NME,**

$$|<m>| = |\sum_k U_{ek}^2 m_k| = \left|m_1|U_{e1}|^2 + m_2|U_{e2}|^2\; e^{i\alpha_{21}} + m_3|U_{e3}|^2\; e^{i\alpha_{31}}\right|$$

$$= \left|m_1\; c_{12}^2\; c_{13}^2 + m_2\; s_{12}^2\; c_{13}^2\; e^{i\alpha_{21}} + m_3\; s_{13}^2\; e^{i\alpha_{31}}\right|,\;\; \theta_{12} \equiv \theta_\odot,\; \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} $((\alpha_{31} - 2\delta) \rightarrow \alpha_{31})$ **- the two Majorana CPVP of the PMNS matrix.**

$$|<m>| = |<m>|\; (\mathrm{min}(\mathrm{m_j}), \alpha_{21,31}, \mathrm{MO})$$

CP-invariance: $\alpha_{21} = 0, \pm\pi$, $\alpha_{31} = 0, \pm\pi$;

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;
S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;
B. Kayser, 1984.

$(\beta\beta)_{0\nu}$−Decay Effective Lagrangian:

$$\mathcal{L}^{CC}(x) = \frac{G_F}{\sqrt{2}} 2\, \bar{e_L}(x)\gamma_\alpha\, \nu_{e\mathrm{L}}(x)\, j_h^\alpha(x) + \text{h.c.},$$

$$\nu_{e\mathrm{L}}(x) = \sum_{j=1}^{3} U_{ek}\, \chi_{k\mathrm{L}}(x), \quad \chi_{k\mathrm{L}}(x):\ m_k \neq 0\,.$$

$j_h^\alpha(x)$: $(A, Z) \rightarrow (A, Z+2)$, **nuclear matrix element (NME).**

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathsf{T}} = \xi_k \chi_k(x),\ |\xi_k|^2 = 1,\ \xi = 1,\ C^{-1}\gamma_\mu C = -\gamma_\mu^T\ (C^T = -C)$$

$(\beta\beta)_{0\nu}-$**Decay can occur in 2nd order of perturbation theory in the weak interaction, i.e., in** G_F**. The following term in the** $S-$**matrix gives the contribution to the matrix elelemen of the** $(\beta\beta)_{0\nu}-$**Decay,** $(\mathrm{A},\mathrm{Z}) \rightarrow (\mathrm{A},\mathrm{Z}+2) + e^- + e^-$**:**

$$S^{(2)} = -\frac{i^2}{2} 4 \left(\frac{G_F}{\sqrt{2}}\right)^2 \int dx_1 dx_2 N \left[\bar{e_L}(x_1)\gamma_\alpha \nu_{eL}(x_1) \left(\nu^T_{eL}(x_2)\gamma^T_\beta \bar{e_L}^T(x_2)\right)^T\right]$$

$$T\left[j^\alpha_h(x_1)\, j^\beta_h(x_2)\, exp\left(i\int \mathcal{L}_{str}(x)\, dx\right)\right]$$

$\bar{e_L}(x_2)\gamma_\beta\,\nu_{eL}(x_2) = -\,\left(\nu^T_{eL}(x_2)\gamma^T_\beta \bar{e}^T_L(x_2)\right)^T = -\,\nu^T_{eL}(x_2)\gamma^T_\beta \bar{e}^T_L(x_2)\,,$ **since there is a sum over all Lorentz indices.**

$$< 0|T(\nu_{eL}(x_1)(\nu^T_{eL}(x_2))|0 >= \sum_{k,j} U_{ek}\,U_{ej}\,P_L < 0|T(\chi_k(x_1)\chi^T_j(x_2))|0 > P^T_L\,, P_L = \frac{1-\gamma_5}{2}\,,$$

$$\chi^T_j(x_2) = \bar{\chi}_j(x_2)\,C^T\,, \quad C^T = -\,C\,.$$

$$-\,P_L < 0|T(\chi_k(x_1)\bar{\chi}_j(x_2))|0 > C\,P^T_L = -\,\delta_{kj}P_L S^F_k(x_1 - x_2)P_L\,C\,, \quad C\gamma^T_5 = \gamma_5\,C\,.$$

$$P_L S^F_k(x_1 - x_2)P_L = \frac{1-\gamma_5}{2}\frac{i}{(2\pi)^4}\int \frac{\hat{q} + m_k I}{q^2 - m^2_k}\,e^{i(x_1-x_2)q}\,d^4q\,\frac{1-\gamma_5}{2}$$

$$= m_k\,\frac{i}{(2\pi)^4}\int \frac{e^{i(x_1-x_2)q}\,d^4q}{q^2 - m^2_k}\,P_L\,,\ \ \hat{q} = q^\kappa\gamma_\kappa\,, \quad |q^2| \sim (200\ \text{MeV})^2 >> m^2_k\,.$$

Thus,

$$< 0|T(\nu_{eL}(x_1)(\nu^T_{eL}(x_2))|0 >= <m> \frac{-\,i}{(2\pi)^4}\int \frac{e^{i(x_1-x_2)q}\,d^4q}{q^2}\,P_L\,C\,, \quad q^2 - m^2_k \cong q^2\,.$$

$A(\beta\beta)_{0\nu} \sim G_F^2 < m > $ M(A,Z), M(A,Z) - NME,

$$|< m >| \cong \left| \sqrt{\Delta m^2_{21}} \sin^2\theta_{12} e^{i\alpha_{21}} + \sqrt{\Delta m^2_{31}} \sin^2\theta_{13} e^{i\alpha_{31}} \right| , \; m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|< m >| \cong \sqrt{m_3^2 + \Delta m^2_{23}} \left| \cos^2\theta_{12} + e^{i\alpha_{21}} \sin^2\theta_{12} \right| , \; m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|< m >| \cong m \left| \cos^2\theta_{12} + e^{i\alpha_{21}} \sin^2\theta_{12} + \sin^2\theta_{13} e^{i\alpha_{31}} \right| ,$$
$m_{1,2,3} \cong m \gtrsim 0.10$ eV (QD).

CP-invariance: $\alpha_{21} = 0, \pm\pi$, $\alpha_{31} = 0, \pm\pi$;

$$1.31 \times 10^{-3} \lesssim |< m >| \lesssim 5.41 \times 10^{-3} \text{ eV, NH } (3\sigma);$$

$$\sqrt{\Delta m^2_{23}} \cos 2\theta_{12} \cong 0.015 \text{ eV} \lesssim |< m >| \lesssim \sqrt{\Delta m^2_{23}} \cong 0.051 \text{ eV, IH } (3\sigma);$$

$$m \cos 2\theta_{12} \lesssim |< m >| \lesssim m, \; m \gtrsim 0.03 \text{ eV, QD } (3\sigma).$$

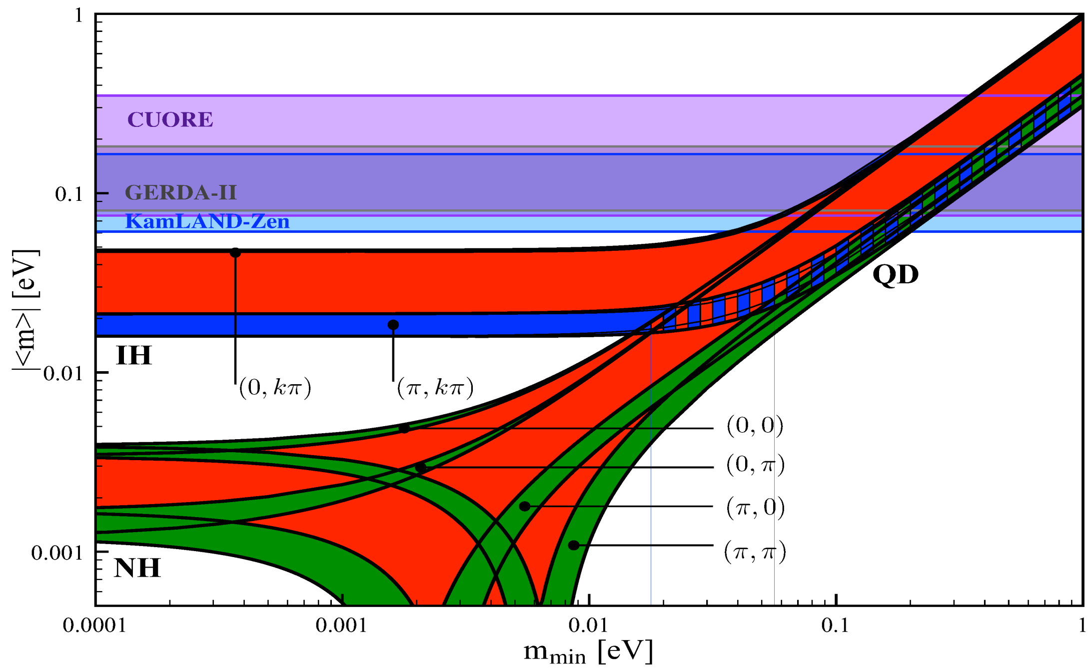
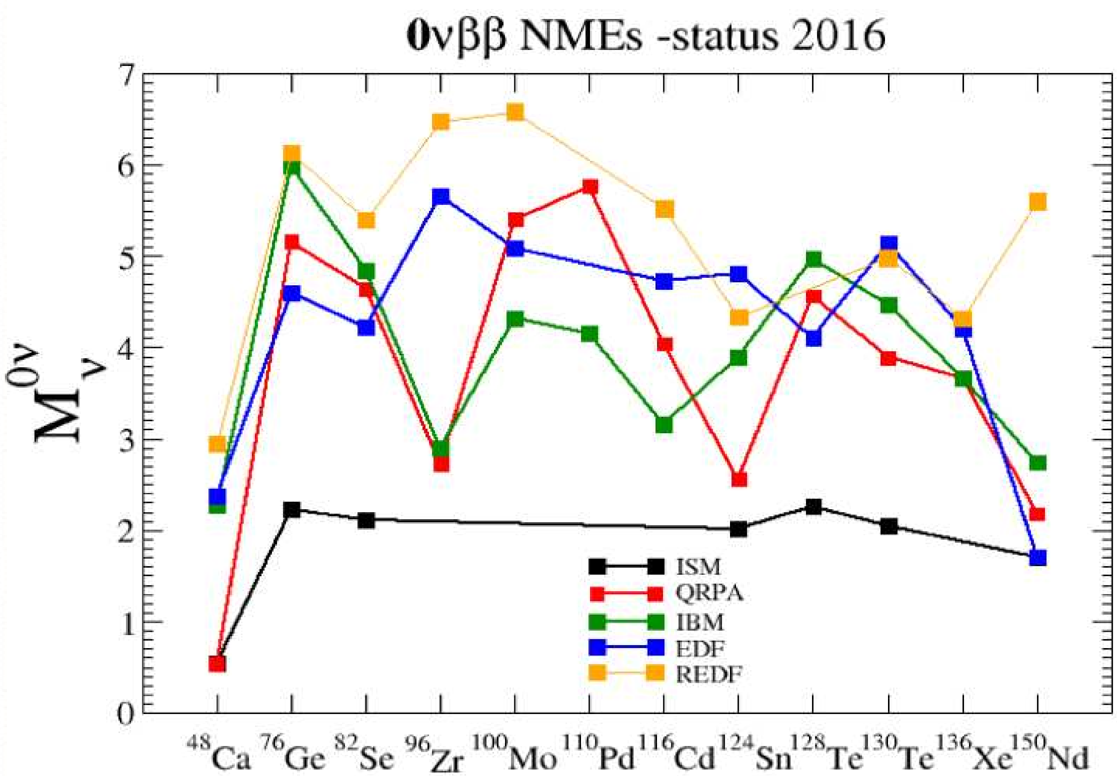


Figure by S. Pascoli, 2020

The figure is obtained using the b.f.v. and the 1σ ranges of allowed values of Δm^2_{21}, $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $|\Delta m^2_{31(32)}|$ from F. Capozzi et al., arXiv:2003.08511, propagated to $<m>$ and then taking a 2σ uncertainty. α_{21} and $(\alpha_{31}-2\delta)$ are varied in the interval [0,2π]. The predictions for the NH, IH and QD spectra as well as the CUORE, GERDA-II and KamLAND-Zen limits are indicated. The black lines determine the ranges of values of $|<m>|$ for the CP conserving values $(\alpha_{21},\alpha_{31}-2\delta)=(0,0)$, $(0,\pi)$, $(\pi,0)$ and (π,π). The red regions correspond to α_{21} and/or $(\alpha_{31}-2\delta)$ having a CP violating value. (Update by S. Pascoli of a figure from S.Pascoli, STP, Phys. Rev. D77 (2008) 113003.)

NMEs for Light ν Exchange



	mean field meth.	ISM	IBM	QRPA
Large model space	yes	no	yes	yes
Constr. Interm. States	no	yes	no	yes
Nucl. Correlations	limited	all	restricted	restricted

F. Simkovic, 2017

Table 1: Compilation of $M_{\alpha i}^{\text{long}}$ values for light Majorana neutrino exchange calculated with different nuclear models from [2]. These results have been obtained by assuming the bare value of the axial coupling constant $g_A^{\text{free}} = 1.27$. Each model is identified through an index, given in the second column.

Nuclear Model	Index [Ref.]	^{76}Ge	^{82}Se	^{100}Mo	^{130}Te	^{136}Xe
NSM	N1 [25]	2.89	2.73	-	2.76	2.28
	N2 [25]	3.07	2.90	-	2.96	2.45
	N3 [26]	3.37	3.19	-	1.79	1.63
	N4 [26]	3.57	3.39	-	1.93	1.76
	N5 [27, 28]	2.66	2.72	2.24	3.16	2.39
QRPA	Q1 [29]	5.09	-	-	1.37	1.55
	Q2 [30]	5.26	3.73	3.90	4.00	2.91
	Q3 [31]	4.85	4.61	5.87	4.67	2.72
	Q4 [32]	3.12	2.86	-	2.90	1.11
	Q5 [32]	3.40	3.13	-	3.22	1.18
	Q6 [33]	-	-	-	4.05	3.38
EDF	E1 [34]	4.60	4.22	5.08	5.13	4.20
	E2 [35]	5.55	4.67	6.59	6.41	4.77
	E3 [36]	6.04	5.30	6.48	4.89	4.24
IBM	I1 [37]	5.14	4.19	3.84	3.96	3.25
	I2 [13]	6.34	5.21	5.08	4.15	3.40

M. Agostini et al., arXiv:2202.01787

It has been noticed in 2019 that in addition to the known long-range light virtual Majorana neutrino exchange contribution to the $(\beta\beta)_{0\nu}$-decay amplitude, also a short-range contribution to amplitude appears when one combines SM effective field theory method with chiral perturbation theory for taking into account low-energy strong interaction effects.

This new contribution is not related to heavy lepton-number violating beyond-standard model physics inducing $(\beta\beta)_{0\nu}$-decay, but appears already in the minimal "standar" scenario, with only three light Majorana neutrinos with non-zero masses being present.

V. Cirigliano et al., 1907.11254 and 2107.13354

Taking into account this additional contribution one has:

$$A(\beta\beta)_{0\nu} \propto G_F^2 \left[g_A^2\, M^{0\nu}_{GT} + g_\nu^{NN}\, m_\pi^2\, M^{0\nu}_{cont} \right] \frac{<m>}{m_e}\,.$$

g_ν^{NN} is unknown constant, m_π is the pion mass and $M^{0\nu}_{cont} > 0$ is the NME of the new contact term. The estimated uncertainty in the knowledge of $M^{0\nu}_{cont}$ is the same as that for the "standrad" mechanism NME $M^{0\nu}_{GT}$, namely, a factor of $\sim (2-3)$.

Information about the constant g_ν^{NN} can be obtained, in particular, from QCD calculations on the lattice. Results of calculations using different nuclear physics techniques (shell model, QRPA, etc.) obtained for some of the nuclei of interest, ^{48}Ca, ^{76}Ge, ^{130}Te and ^{136}Xe, suggest that g_ν^{NN} is positive and that the contact term can enhance by a factor of ∼(20% - 40%) the magnitude of $A(\beta\beta)_{0\nu}$ (increasing the sensitivity rearch of the experiments.

See arXic:2207.05108, arXiv:2210.05809, arXiv:2112.08146 and references quoted therein

The searches for $(\beta\beta)_{0\nu}-$ decay have a long history.

See, e.g., S.T.P., arXiv:1910.09331 and A. Barabash, arXiv:1104.2714

Results from IGEX (^{76}Ge), NEMO3 (^{100}Mo), CUORICINO+CUORE-0 (^{130}Te):

IGEX ^{76}Ge: $|<m>| \ < (0.33 - 1.35)$ eV **(90% C.L.).**

Data from NEMO3 (^{100}Mo):

$T(^{100}Mo) > 1.1 \times 10^{24}$ **yr,** $|<m>| \ <$**(0.3–0.6) eV;**

Best Sensitivity Results

$$\mathbf{T(^{136}Xe)} > 1.6 \times 10^{25}\text{yr at 90\% C.L., EXO}$$

$$\mathbf{T(^{136}Xe)} > 2.3 \times 10^{26}\text{yr at 90\% C.L., KamLAND} - \text{Zen}$$

$$|<m>| \; < (0.036 - 0.156) \text{ eV}\,.$$

S. Abe et al., arXiv:2203.02139 (PRL 130 (2023) 051801)

$$\mathbf{T(^{76}Ge)} > 1.8 \times 10^{26}\text{yr at 90\% C.L., GERDA II}\,.$$

$$|<m>| \; < (0.079 - 0.182) \text{ eV}\,.$$

S. Calgora, Talk at NuTel, October 2023

$$\mathbf{T(^{130}Te)} > 3.3 \times 10^{25}\text{yr at 90\% C.L.}\,,$$

$$|<m>| \; < (0.075 - 0.255) \text{ eV}\,, \text{ CUORE}\,.$$

A. Ressa, talk at NuTel, October 2023

Constraints on min(m_j)

$$\mathbf{T(^{136}Xe)} > 2.3 \times 10^{26}\text{yr at } 90\% \text{ C.L., KamLAND} - \text{Zen}$$

$$|<m>| \ < (0.036 - 0.156) \text{ eV}\,.$$

$$\min(m_j) < \frac{0.156 \text{ eV}}{\cos 2\theta_{12}} \cong 0.53 \text{ eV } (\cos 2\theta_{12} \gtrsim 0.31,\ 3\sigma)$$

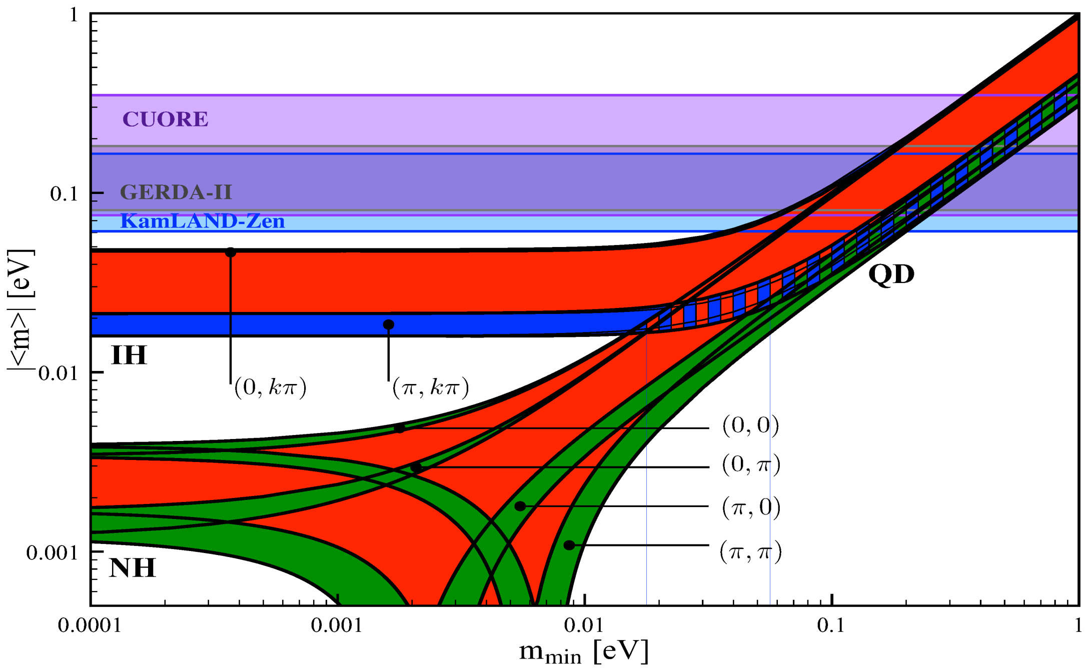
The GERDA II result on $|<m>|$ imply a similar upper limit on min(m_j).

Large number of experiments: $|<m>|$ $\sim$ (0.01-0.05) eV

CUORE - ^{130}Te;
***CUPID - ^{100}Mo (250 kg; CUPID-IT: 1000 kg (8×10^{27} yr);**
GERDA-II - ^{76}Ge; MAJORANA - ^{76}Ge;
***LEGEND - ^{76}Ge (LEGEND-200; LEGEND-1000);**
KamLAND-ZEN - ^{136}Xe (750 kg, 4.6×10^{26} yr);
***nEXO - ^{136}Xe (5 tons, 9×10^{27} yr);**
NEXT-100 - ^{136}Xe (NEXT-HD (10^{27} yr); NEXT-BOLD (10^{28} yr));
SNO+ - ^{130}Te (800 tons of LS loaded with (0.5%) Te, 33.8% ^{130}Te);
AMoRE - ^{100}Mo (200 kg of XMoO4 crystals);
PANDAX-III - ^{136}Xe (200 kg of Xe enriched at 90% in ^{136}Xe);
CANDLES - ^{48}Ca;
***SuperNEMO - ^{82}Se (100 kg of ^{82}S, 20 modules);**
DCBA - ^{82}Se, ^{150}Nd;
XMASS - ^{136}Xe;
ZICOS - ^{96}Zr;
MOON - ^{100}Mo;
...

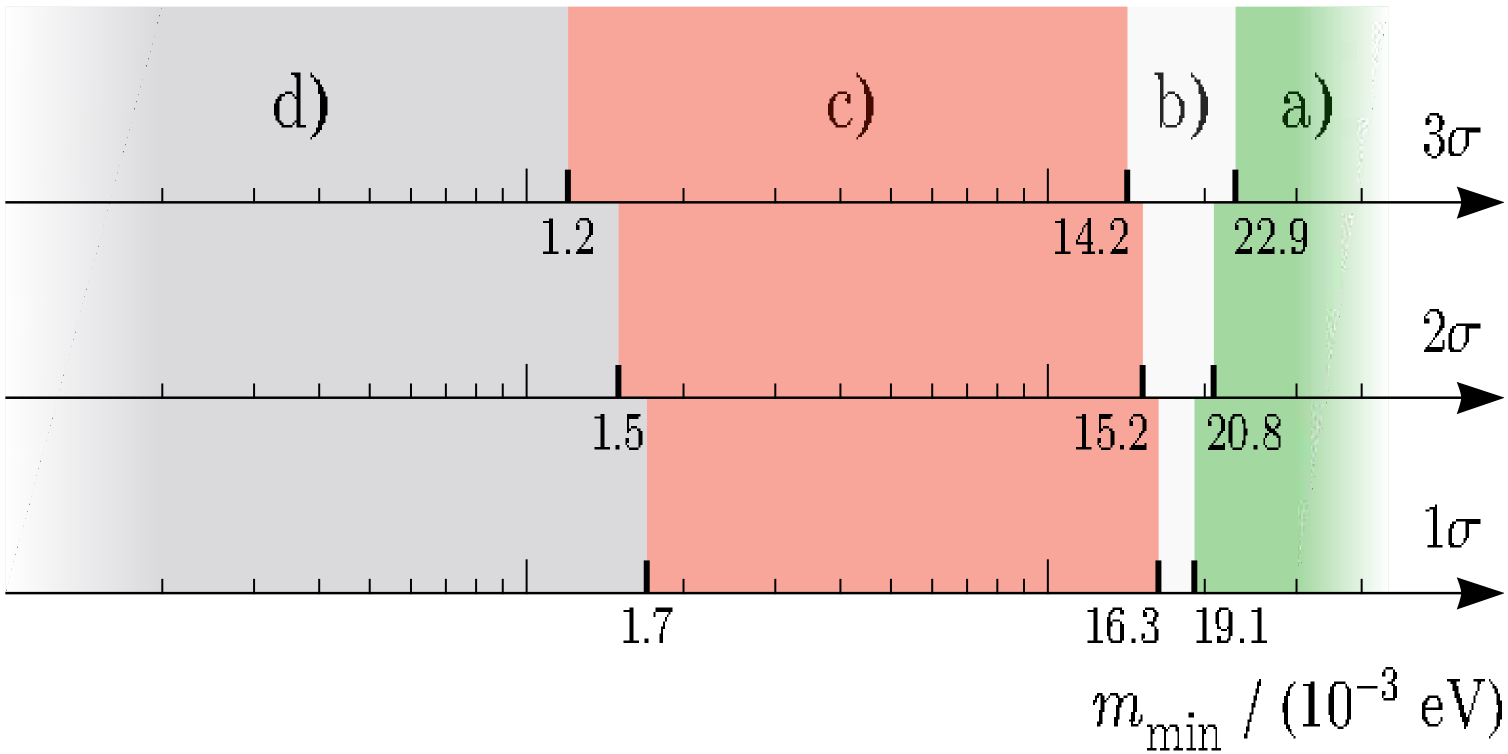
See, e.g., A. Giuliani et al., 1910.04688; M. Agostini et al., 2202.01787

If $|<m>|\ < 0.01$ eV, the next frontier will be $|<m>| \sim(1.0 - 5.0)\times10^{-3}$ eV
Experiments: nEXO, LEGEND-1000, CUPID-IT, NEXT-BOLD



Under what conditions $|<m>|\ \geq 1.0\ (5.0) \times 10^{-3}$ eV,
or how not to "fall" in the "well of non-obsevability".

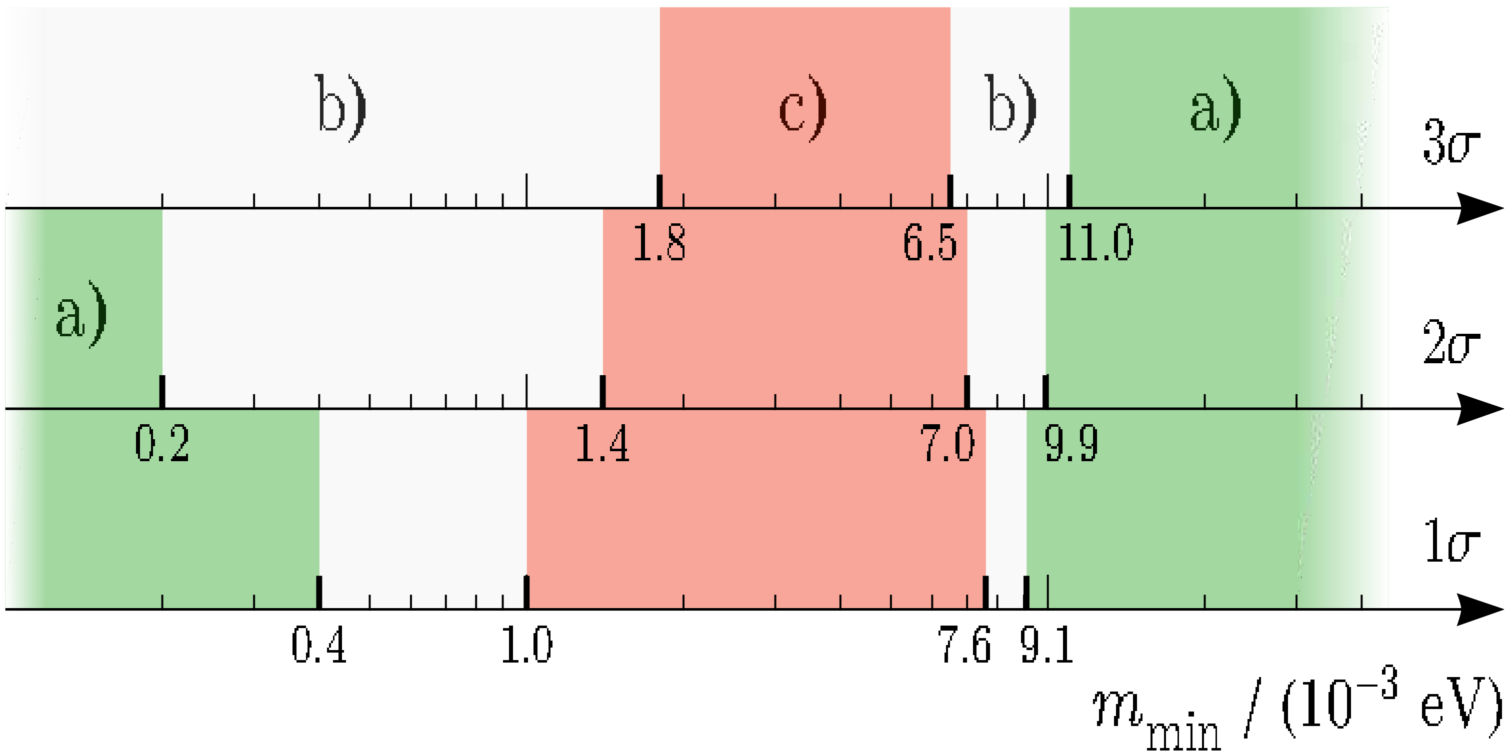
S. Pascoli, STP, arXiv:0711.4993; J. Penedo, STP, PL B786 (2018) 410



Ranges of $m_{\rm min}$ for a NO spectrum and for oscillation parameters inside their $n\sigma$ ($n = 1, 2, 3$) intervals for which: in green, a) $> 5 \times 10^{-3}$ eV for all values of θ_{ij}, Δm^2_{ij}, and $\alpha^{(\prime)}_{ij}$ from the corresponding allowed or defining intervals; in grey, b) there exist values of θ_{ij}, Δm^2_{ij} from the 1σ, 2σ and 3σ allowed intervals and values of $\alpha^{(\prime)}_{ij}$ such that $< 5 \times 10^{-3}$ eV and in red, c) for all values of θ_{ij} and Δm^2_{ij} from the corresponding allowed intervals there exist values of the phases α_{21} and α'_{31} for which $< 5 \times 10^{-3}$ eV. In darker grey d), ranges of $m_{\rm min}$ for which one has $< 5 \times 10^{-3}$ eV independently of the values of oscillation parameters and CPV phases.

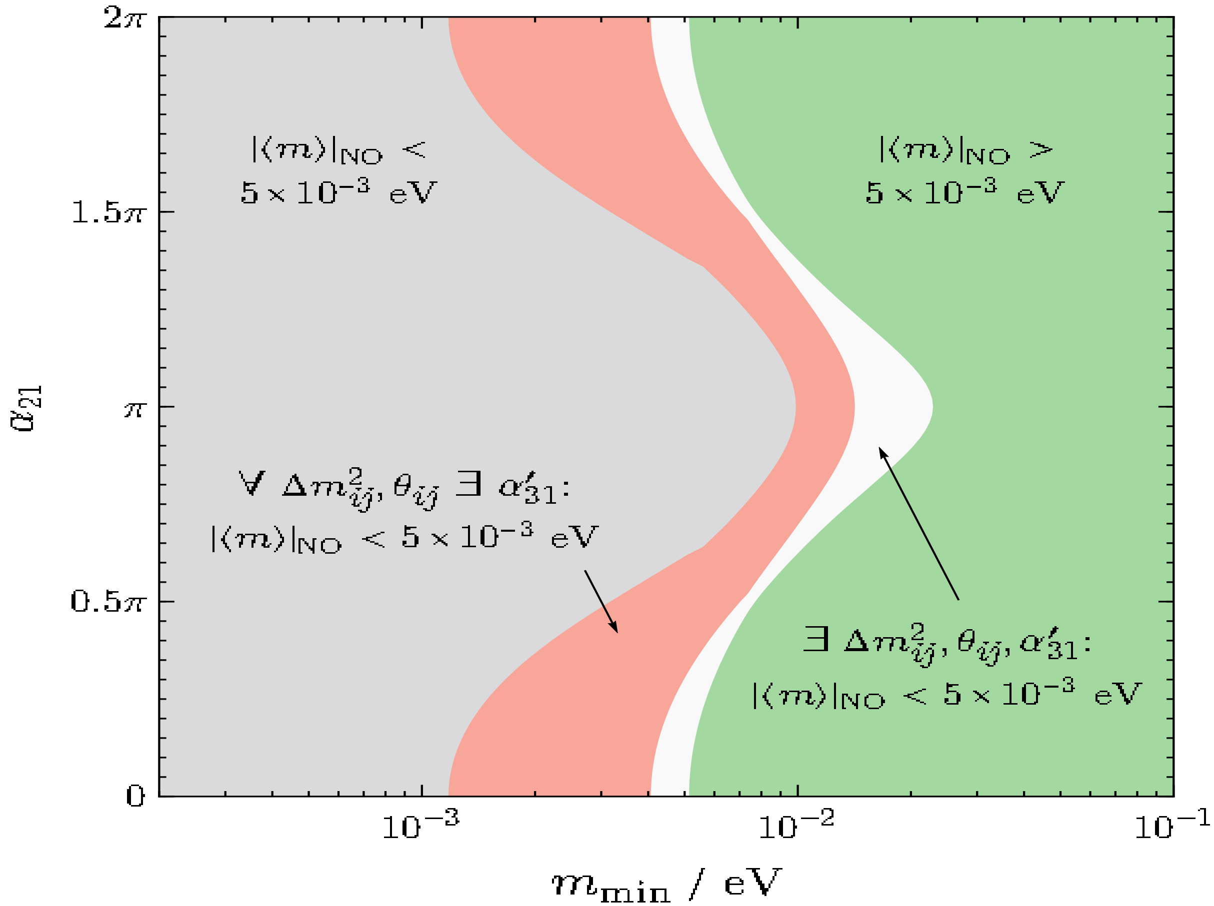
J. Penedo, STP, PL B786 (2018) 410

$>5\times10^{-3}$ eV if $m_{\rm min} > 0.0229$ eV, or if $\sum m_j \geq 0.1$ eV (3σ).



Ranges of $m_{\rm min}$ for a NO spectrum and for oscillation parameters inside their $n\sigma$ ($n = 1,2,3$) intervals for which: in green, a) $>= 10^{-3}$ eV for all values of θ_{ij}, Δm^2_{ij}, and $\alpha^{(\prime)}_{ij}$ from the corresponding allowed or defining intervals; in grey, b) there exist values of θ_{ij}, Δm^2_{ij} from the 1σ, 2σ and 3σ allowed intervals and values of $\alpha^{(\prime)}_{ij}$ such that $<= 10^{-3}$ eV; and in red, c) for all values of θ_{ij} and Δm^2_{ij} from the corresponding allowed intervals there exist values of the phases α_{21} and α'_{31} for which $<= 10^{-3}$ eV.

$> 10^{-3}$ eV if $m_{\rm min} >$ 0.011 eV, or if $\sum m_j \geq$ 0.08 eV (3σ).



J. Penedo, STP, PL B786 (2018) 410

$\sum m_j \geq 0.1$ **eV implies** $|<m>| \geq 5 \times 10^{-3}$ **eV (3σ);**

$\sum m_j \geq$ **0.08 eV implies** $|<m>| \geq 10^{-3}$ **eV (3σ).**

If it is experimentally established that

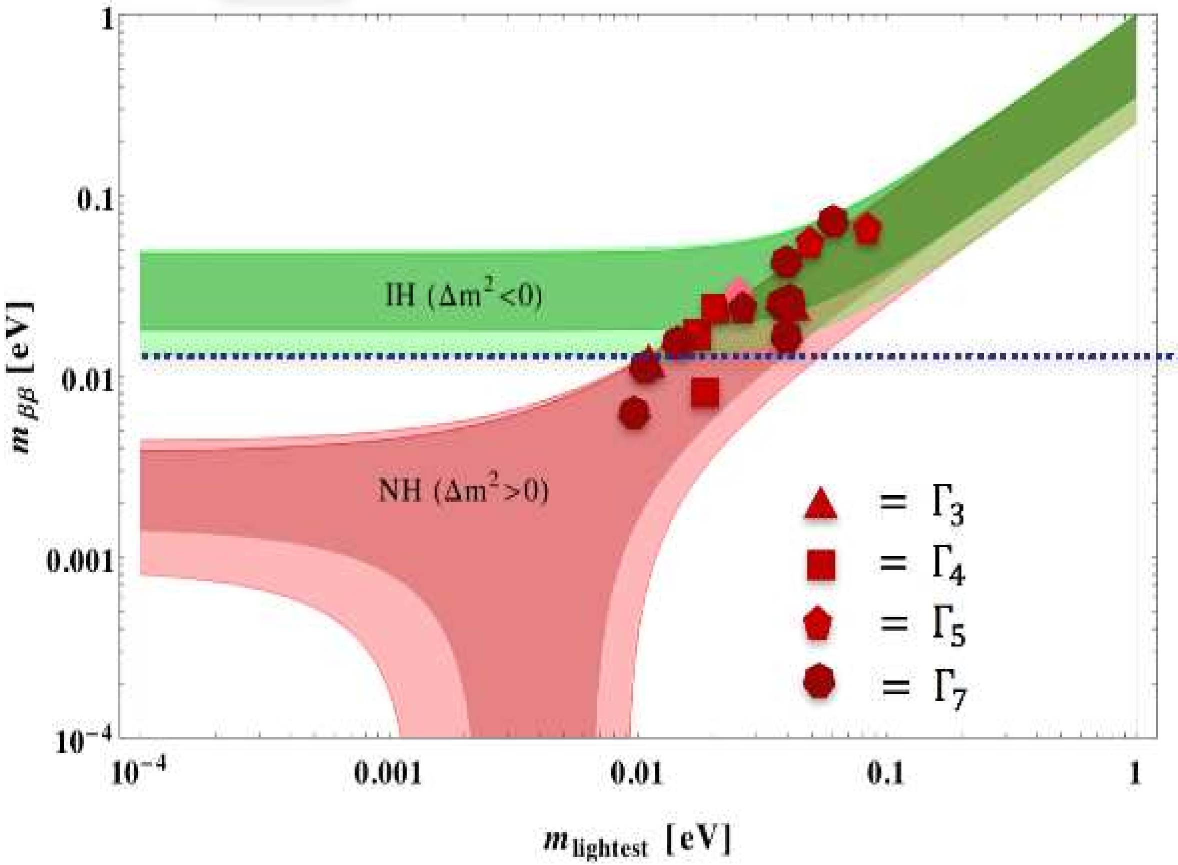
$$\sum_j m_j \gtrsim 0.10 \text{ eV},$$

this would imply for NO ν mass spectrum that

$$|<m>| \geq 5 \times 10^{-3} \text{ eV}$$

J.T. Penedo, STP, arxiv:1806.03203

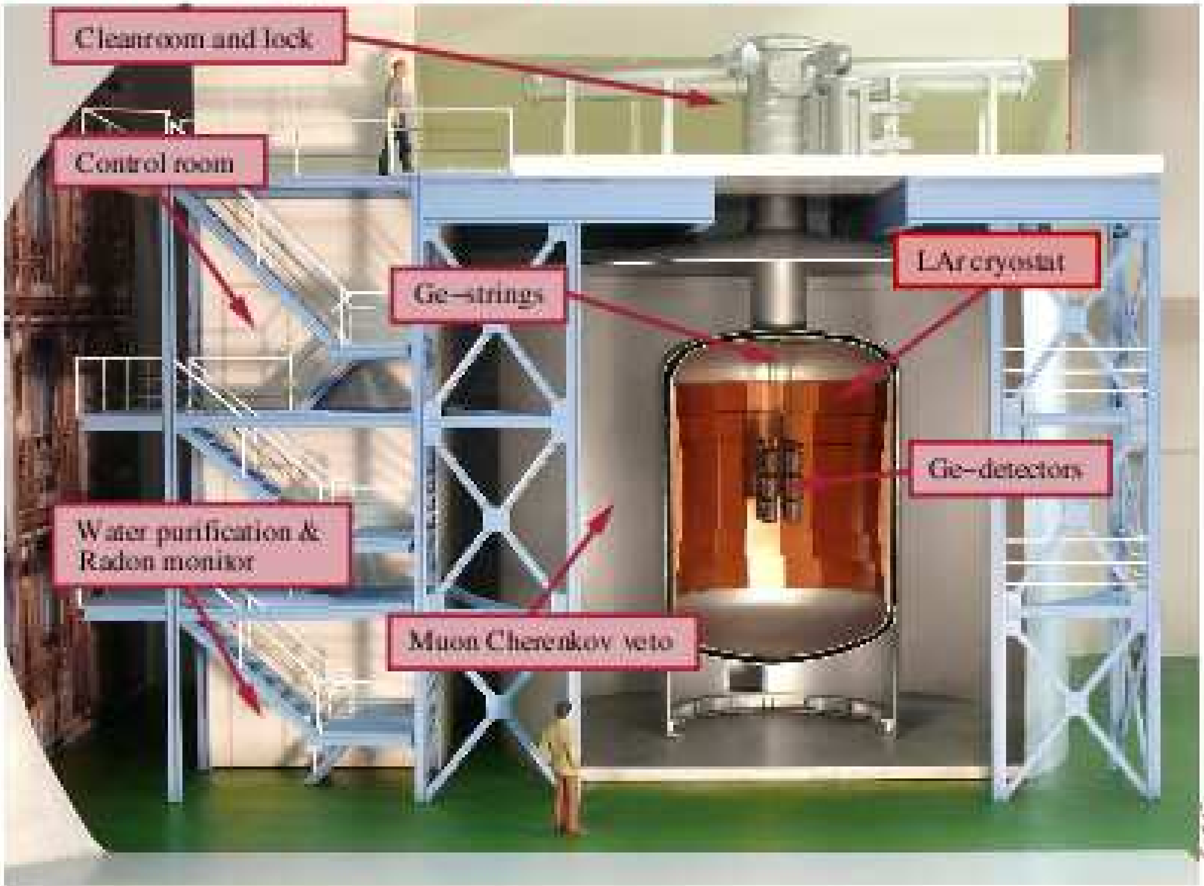
Predictions of modular invariant theories of lepton flavour



F. Feruglio, talk at Bethe Colloquium, 18/06/2020

New approach to neutrino and charged lepton masses, lepton (neutrino) mixing and leptonic CP violation (for a review see, e.g., F. Feruglio and A. Romanino, arXiv:1912.06028). Has been intensively developed in the last two years. Models typically predict $m_1 > 0.01$ eV for NO spectrum (see, e.g., J. Penedo, STP, arXiv:1806.1040, P. Novichkov et al., arXiv:1811.04933).

GERDA: Experimental Setup



Majorana CPV Phases and $|<m>|$

CPV can be established provided

– $|<m>|$ measured with $\Delta \lesssim 15\%$;

– $\Delta m^2_{\rm atm}$ (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;

– $\xi \lesssim 1.5$;

– α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;

– $\tan^2\theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002

New Physics and $(\beta\beta)_{0\nu}$-Decay

Light Sterile Neutrinos and $(\beta\beta)_{0\nu}$-Decay

One Sterile Neutrino: the $3+1$ Model

$$|<m>| \;= \left| m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha} + m_3|U_{e3}|^2 e^{i\beta} + m_4|U_{e4}|^2 e^{i\gamma} \right| .$$

$$U_{e1} = c_{12}c_{13}c_{14}, \quad U_{e2} = e^{i\alpha/2} c_{13}c_{14}s_{12},$$

$$U_{e3} = e^{i\beta/2} c_{14}s_{13}, \quad U_{e4} = e^{i\gamma/2} s_{14}\,,$$

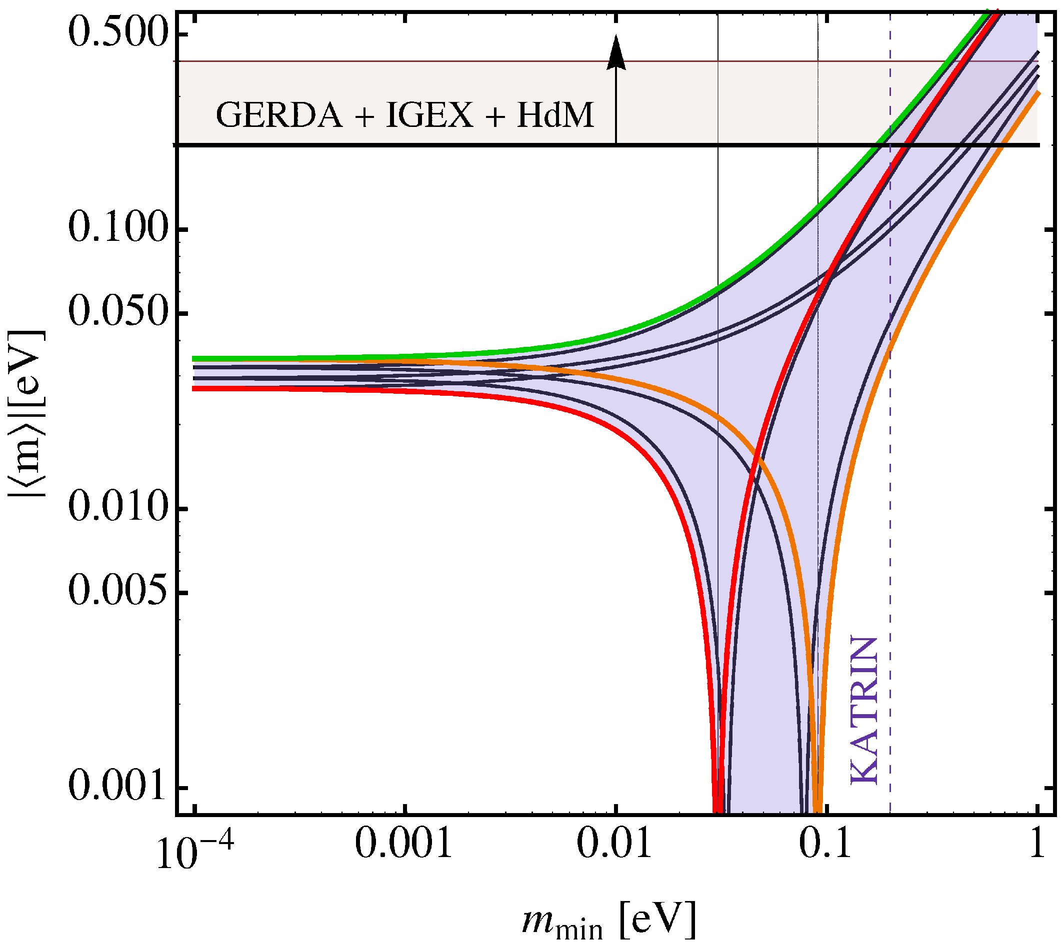
For $m_4 \sim$ 1 eV and $|U_{e4}|^2 \sim \text{few} \times 10^{-2}$, significant contribution to $|<m>|$ – can modify the "standard" 3-ν mechanism NO and IO predictions.

J. Barry et al., arXiv:1007.5217; S.M. Bilenky et al., hep-ph/0104218; I. Girardi et al., arXiv:1308.5802

Eample:

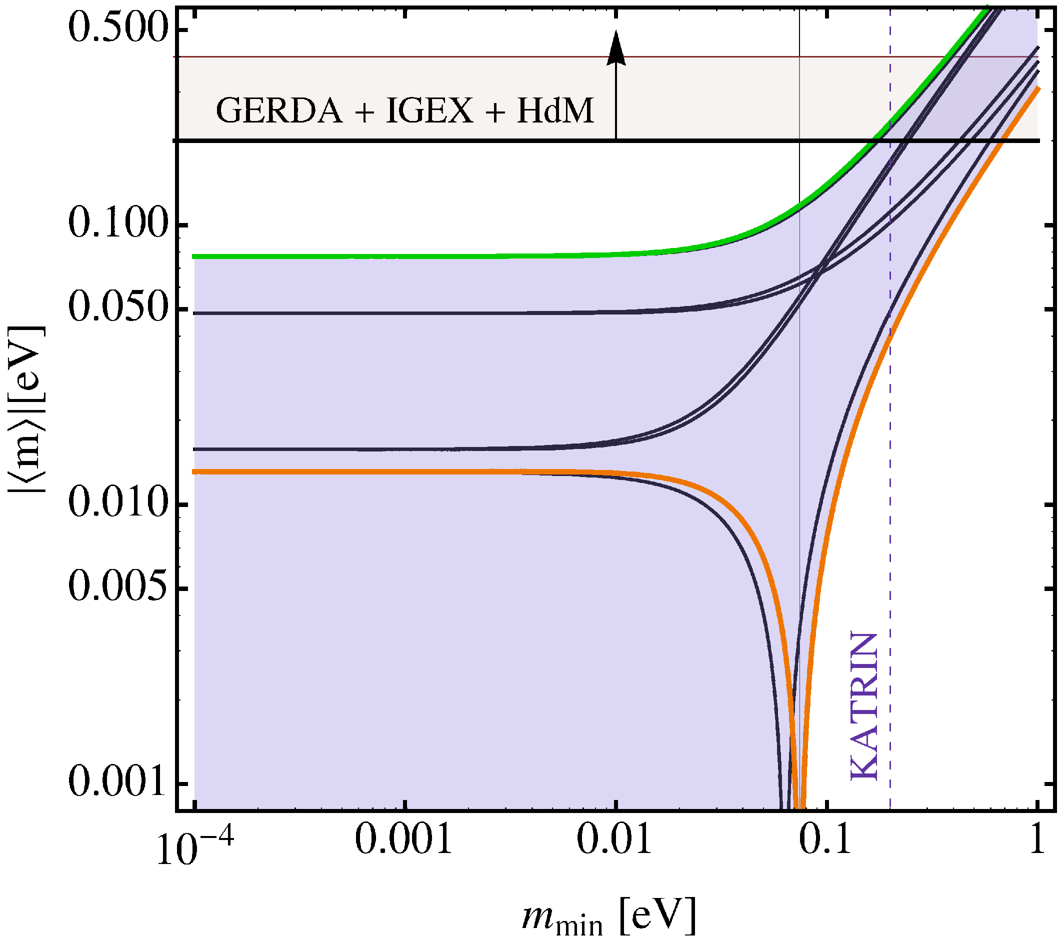
$$\sin^2\theta_{14} = 0.023\ (0.028)[0.019]\,,\ \Delta m^2_{41(43)} = 1.78\ (1.60)[1.70]\ \text{eV}^2\,.$$

S. Gariazzo et al., arXiv:1703.00860; M. Archidiacono et al., arXiv:2006.12885



I. Girardi A. Meroni, S.T.P., 2013

NO spectrum; green, red and orange lines: $(\alpha, \beta, \gamma) = (0,0,0), (0,0,\pi), (\pi,\pi,\pi)$; five gray lines: the other five sets of CP conserving values.
$\Delta m^2_{41} = 1.78$ eV^2, $\sin\theta_{14} = 0.15$.

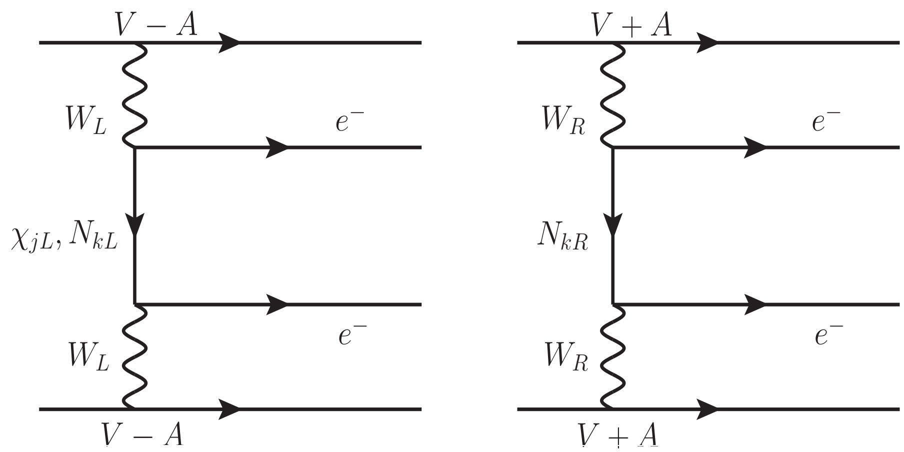


I. Girardi A. Meroni, S.T.P., 2013

IO spectrum; green and orange lines: $(\alpha, \beta, \gamma) = (0, 0, 0), (\pi, \pi, \pi)$; six gray lines: the other six sets of CP conserving values.
$\Delta m_{43}^2 = 1.78$ eV^2, $\sin\theta_{14} = 0.15$.

Different Mechanisms of $(\beta\beta)_{0\nu}$-Decay

Heavy Majorana Neutrino Exchange Mechanisms



Light Majorana Neutrino Exchange

$$\eta_\nu = \frac{<m>}{m_e}\,.$$

Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction, LH N_k, $M_k \gtrsim 10$ GeV:

$\eta^L_N \;=\; \sum_k^{heavy}\; U^2_{ek}\frac{m_p}{M_k}$, m_p - proton mass , U_{ek} - CPV .

Heavy Majorana Neutrinos N_k - from Low (Weak) Scale Type I Seesaw Mechanism.

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
T. Yanagida, 1979;
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
R. Mohapatra, G. Senjanovic, 1980.

- Explain the smallness of $\nu-$masses.

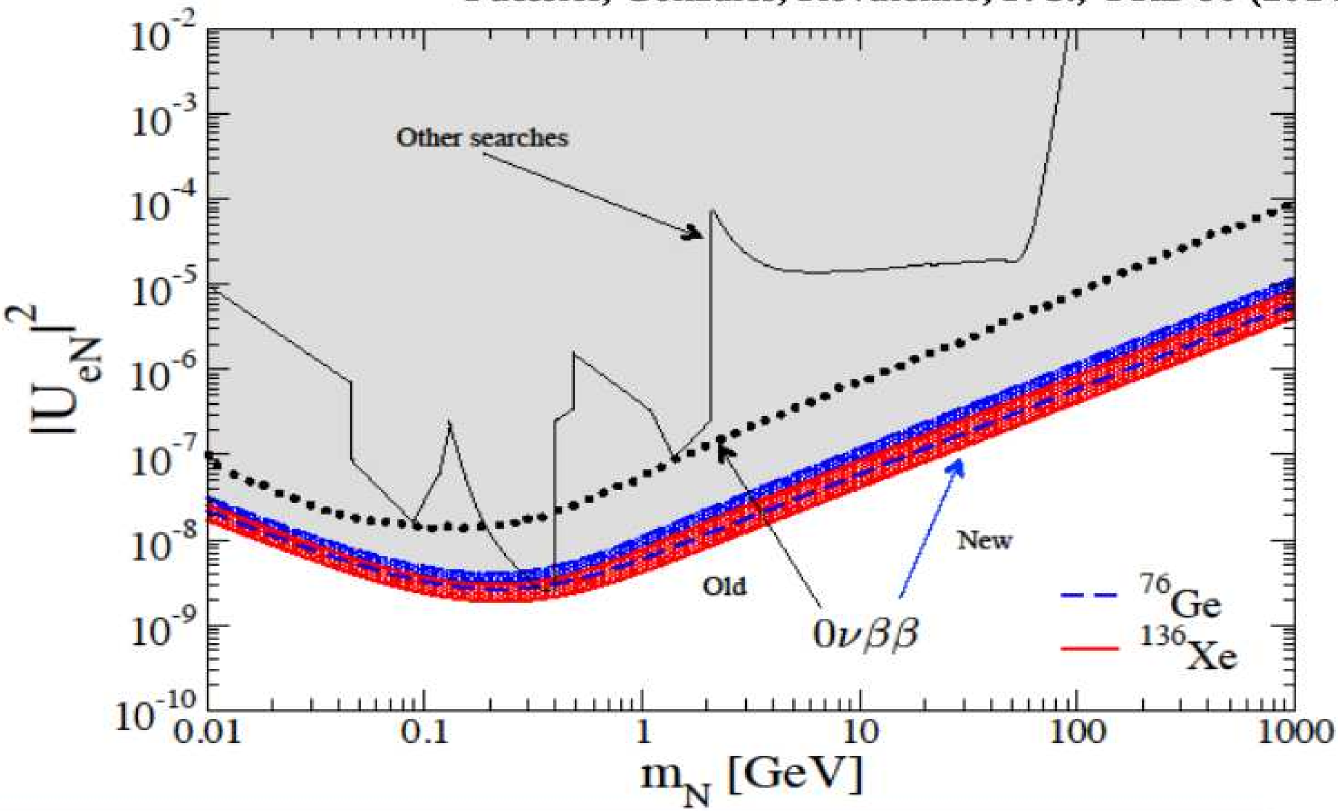
- Through leptogenesis theory link the $\nu-$mass generation to the generation of baryon asymmetry of the Universe.

S. Fukugita, T. Yanagida, 1986.

Exclusion plot in $|U_{eN}|^2 - m_N$ plane

$T^{0\nu}_{1/2}(^{76}Ge) \geq 3.0\ 10^{25}$ yr
$T^{0\nu}_{1/2}(^{136}Xe) \geq 3.4\ 10^{25}$ yr

Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]



Improvements: i) QRPA (constrained Hamiltonian by 2νββ half-life, self-consistent treatment of src, restoration of isospin symmetry ...), ii) More stringent limits on the 0νββ half-life

F. Simkovic, October 2017

T(^{136}Xe)>1.07×10^{26} yr, T(^{76}Ge)>1.8×10^{26} yr: limits on $|U_{eN}|^2$ by $\sim\sqrt{3}$ and $\sqrt{6}$ better.

(V+A) Weak Interaction, RH N_k, $M_k \gtrsim 10$ GeV:

$\eta^R_N \; = \; \left(\frac{M_W}{M_{WR}}\right)^4 \sum_k^{heavy} \; V_{ek}^2 \frac{m_p}{M_k}$; V_{ek}: $N_k - e^-$ in the CC .

$M_W \cong 80$ GeV; $M_{WR} \gtrsim 2.5$ TeV; V_{ek} - CPV, in general.

A comment.

(V-A) CC Weak Interaction:

$\bar{e}(1+\gamma_5)e^c \equiv 2\bar{e_L}\,(e^c)_R\,,\; e^c = C(\bar{e})^T\,,$

C - **the charge conjugation matrix.**

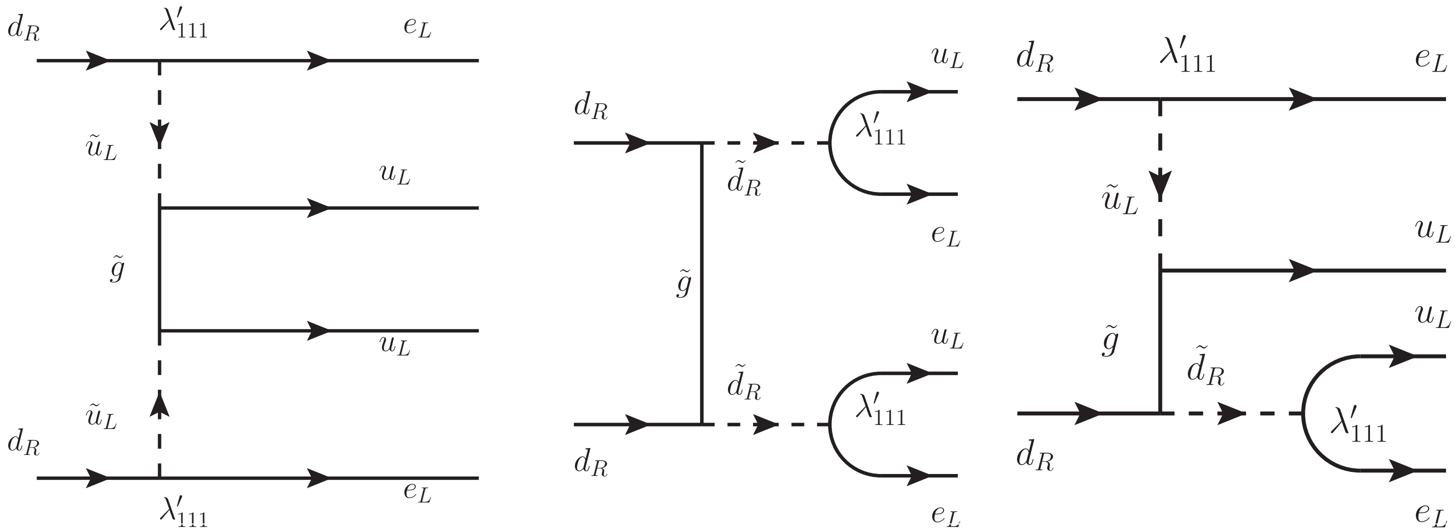
(V+A) CC Weak Interaction:

$\bar{e}(1-\gamma_5)e^c \equiv 2\bar{e_R}\,(e^c)_L\,.$

The interference term: $\propto m_e$, **suppressed.**

A. Halprin, S.T.P., S.P. Rosen, 1983

SUSY Models with R-Parity Non-conservation



$$\mathcal{L}_{R_p} = \lambda'_{111}\Big[(\bar{u}_L\ \bar{d}_L)\begin{pmatrix} e^c_R \\ -\nu^c_{eR}\end{pmatrix}\tilde{d}_R + (\bar{e}_L\ \bar{\nu}_{eL})d_R\begin{pmatrix}\tilde{u}^*_L \\ -\tilde{d}^*_L\end{pmatrix}$$

$$+\ (\bar{u}_L\ \bar{d}_L)d_R\begin{pmatrix}\tilde{e}^*_L \\ -\tilde{\nu}^*_{eL}\end{pmatrix}\Big] + h.c.$$

The Gluino Exchange Dominance Mechanism

$$\eta_{\lambda'} = \frac{\pi\alpha_s}{6}\frac{\lambda'^2_{111}}{G_F^2 m^4_{\tilde{d}_R}}\frac{m_p}{m_{\tilde{g}}}\left[1 + \left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^2\right]^2 ,$$

G_F - the Fermi constant, $\alpha_s = g_3^2/(4\pi)$, g_3 - the SU(3)$_c$ gauge coupling constant, $m_{\tilde{u}_L}$, $m_{\tilde{d}_R}$ and $m_{\tilde{g}}$ - the masses of the LH u-squark, RH d-squark and gluino.

The Squark-Neutrino Mechanism

$$\eta_{\tilde{q}} = \sum_k \frac{\lambda'_{11k}\lambda'_{1k1}}{2\sqrt{2}G_F} \sin 2\theta^d_{(k)} \left(\frac{1}{m^2_{\tilde{d}_1(k)}} - \frac{1}{m^2_{\tilde{d}_2(k)}}\right) ,$$

$d_{(k)} = d, s, b$; θ^d: $\tilde{d}_{kL} - \tilde{d}_{kR}$ - mixing (3 light Majorana neutrinos assumed).

The $2e^-$ current in both mechanisms:

$\bar{e}(1+\gamma_5)e^c \equiv 2\bar{e}_L\,(e^c)_R$, as in the "standard" mechanism.

Effective operator approach to $(\beta\beta)_{0\nu}$-decay developed in, e.g.:

F. Bonnet et al., JHEP 2013 (arXiv:1212.3045);
F. Bonnet et al., JHEP 2014 (arXiv:212.3045v2);
J.C. Helo et al., arXiv:1502.05188 (scalar mediator).

Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$-Decay

If the decay $(A,Z) \to (A,Z+2) + e^- + e^-$ ($(\beta\beta)_{0\nu}$-decay) will be observed, the question will inevitably arise:

Which mechanism is triggering the decay?

How many mechanisms are involved?

"Standard Mechanism": light Majorana ν exchange.

Fundamental parameter - the effective Majorana mass:

$$< m > = \sum_j^{light} \left(U_{ej}\right)^2 m_j \; , \text{ all } m_j \geq 0 \; ,$$

U - **PMNS neutrino mixing matrix,** m_j - **the light Majorana neutrino masses,** $m_j \lesssim 1$ **eV.**

U - **CP violating, in general:** $(U_{ej})^2 = |U_{ej}|^2 \, e^{i\alpha_{j1}}$**,** $j = 2,3$**,** α_{21}, α_{31} - **Majorana CPV phases.**

S.M. Bilenky, J. Hosek, S.T.P.,1980

A number of different mechanisms possible.

For a given mechanism κ we have in the case of $(A,Z) \to (A,Z+2) + e^- + e^-$:

$$\frac{1}{T^{0\nu}_{1/2}} = |\eta^{LNV}_{\kappa}|^2 \; G^{0\nu}(E_0,Z) |M'^{0\nu}_{\kappa}|^2 \,,$$

η^{LNV}_{κ} **- the fundamental LNV parameter characterising the mechanism κ,**

$G^{0\nu}(E_0,Z)$ **- phase-space factor (includes $g_A^4 = (1.25)^4$,**
as well as $R^{-2}(A)$, $R(A) = r_0 A^{1/3}$ with $r_0 = 1.1\ fm$),

$M'^{0\nu}_{\kappa} = (g_A/1.25)^2 M^{0\nu}_{\kappa}$ **- NME**
(includes $R(A)$ as a factor).

The problem of distinguishing between different sets of multiple (e.g., two) mechanisms being operative in $(\beta\beta)_{0\nu}$-decay was studied in

1. A. Faessler et al., "Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$-Decay", arXiv:1103.2434, Phys. Rev. D83 (2011) 113003.

2. A. Faessler et al., "Multi-Isotope Degeneracy of Neutrinoless Double Beta Decay Mechanisms in the Quasi-Particle Random Phase Approximation", arXiv:1103.2504, Phys. Rev. D83 (2011) 113015.

3. A. Meroni et al., "Multiple CP Non-conserving Mechanisms of bb0nu-Decay and Nuclei with Largely Different Nuclear Matrix Elements", arXiv:1212.1331, JHEP 1302 (2013) 025.

Earlier studies include:
A. Halprin et al., "Effects of Mixing of Light and Heavy Majorana Neutrinos in Neutrinoless Double Beta Decay", Phys. Lett. 125B (1983) 335).

Conclusions on $(\beta\beta)_{0\nu}$−decay part

Determining the nature - Dirac or Majorana, of massive neutrinos is of fundamental importance for understanding the origin of neutrino masses.

The $(\beta\beta)_{0\nu}$−decay experiments:

– **Are testing the status of L conservation, can establish the Majorana nature of ν_j;**

– **Can provide unique information on the ν mass spectrum;**

– **Can provide unique information on the absolute scale of ν masses;**

– **Can provide information on the Majorana CPV phases;**

– **If $|<m>|\ < 0.01$ eV, next frontier will be $|<m>|\ \sim$(1.0 - 5.0)$\times 10^{-3}$ eV.**

– **"Standard mechanism": $\sum m_j \geq$ 0.08 (0.10) eV implies $|<m>|\ \geq 10^{-3}$ (5×10^{-3}) eV (3σ).**

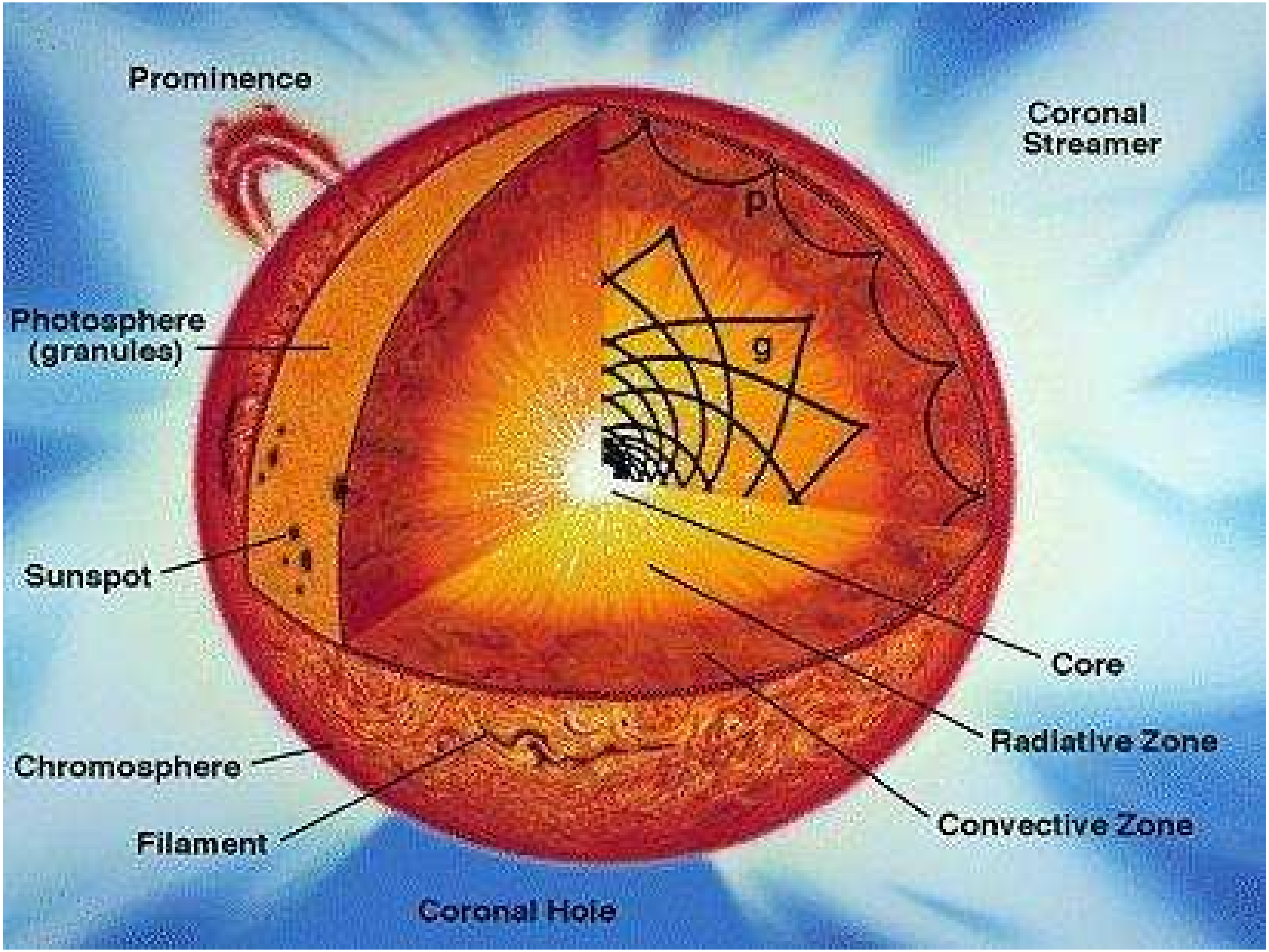
–

– **Provide crtitical tests of neutrino-related BSM theoretical ideas.**
$T^{0\nu}_{1/2} = 10^{25}$ **yr probes** $|<m>|\ \sim 0.1$ **eV;**
$T^{0\nu}_{1/2} = 10^{25}$ **yr probes** $\Lambda_{LNV} \sim 1$ **TeV.**

– **Synergy with searches of BSM physics at LHC.**

Selected Topics in Neutrino Oscillations

Oscillations, or Flavour Conversion, of Solar Neutrinos


Prominence
Coronal
Streamer
p
g
Photosphere
(granules)
Sunspot
Core
Radiative Zone
Chromosphere
Convective Zone
Filament
Coronal Hole

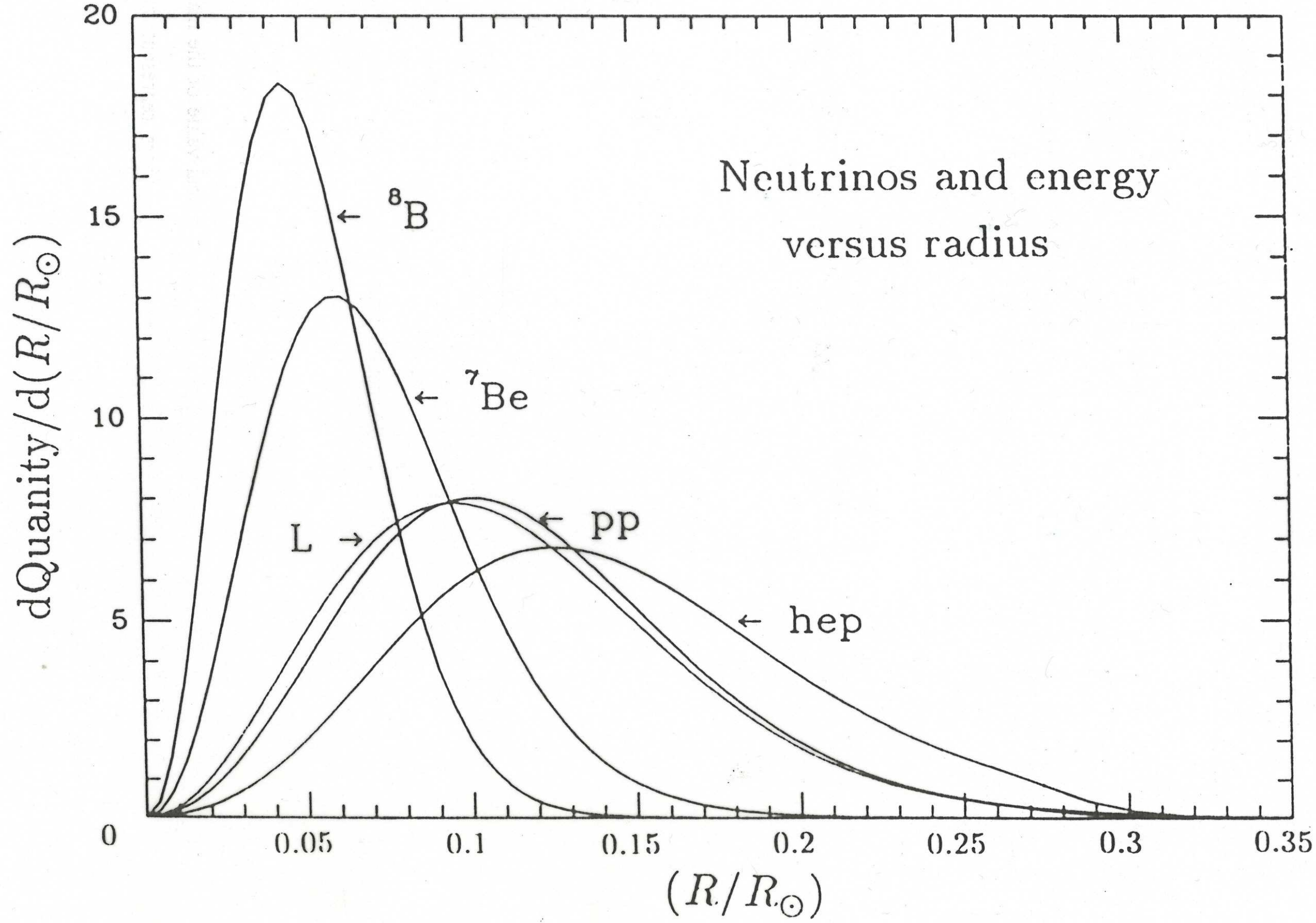
Solar Neutrino Production: pp Chain

REACTION	TERM. (%)	ν ENERGY (MeV)
$p + p \rightarrow {}^2H + e^+ + \nu_e$	(99.96)	≤ 0.420
or		
$p + e^- + p \rightarrow {}^2H + \nu_e$	(0.44)	1.442
${}^2H + p \rightarrow {}^3He + \gamma$	(100)	
${}^3He + {}^3He \rightarrow \alpha + 2\,p$	(85)	
or		
${}^3He + {}^4He \rightarrow {}^7Be + \gamma$	(15)	
${}^7Be + e^- \rightarrow {}^7Li + \nu_e$	(15)	0.861 90% 0.383 10%
${}^7Li + p \rightarrow 2\,\alpha$		
or		
${}^7Be + p \rightarrow {}^8B + \gamma$	(0.02)	
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$		< 15
${}^8Be^* \rightarrow 2\,\alpha$		
or		
${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$	(0.000004)	18.8

$$4\text{p} \rightarrow {}^4\text{He} + 2\text{e}^+ + 2\nu_e.$$

- pp neutrinos, $E \leq 0.420$ MeV, $\bar{E} = 0.265$ MeV,
- ^{7}Be neutrinos, E=0.862 MeV (89.7% of the flux), 0.384 MeV (10.3%) ,
- ^{8}B neutrinos, $E \leq 14.40$ MeV, $\bar{E} = 6.71$ MeV,
- pep neutrinos, E=1.442 MeV,
- of ^{13}N, $E \leq 1.199$ MeV, $\bar{E} = 0.707$ MeV,
- of ^{15}O, $E \leq 1.732$ MeV, $\bar{E} = 0.997$ MeV.

J. N. Bahcall


Neutrinos and energy versus radius
dQuanity/d(R/R⊙)
(R/R⊙)
⁸B
⁷Be
L
pp
hep
20
15
10
5
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Solar Neutrinos ν_e, $E \sim$1 MeV: B. Pontecorvo 1946

$$\nu_e + ^{37}Cl \rightarrow ^{37}Ar + e^-$$

R. Davis et al., 1967 - 1996: 615 t C_2Cl_4; 0.5 Ar atoms/day, exposure 60 days.

$$\nu + e^- \rightarrow \nu + e^-$$

Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 -);

$$\nu_e + D \rightarrow e^- + p + p \,, \qquad \text{SNO}$$

$$\nu_l + D \rightarrow \nu_l + n + p \,,\; l = e, \mu, \tau \qquad \text{SNO}$$

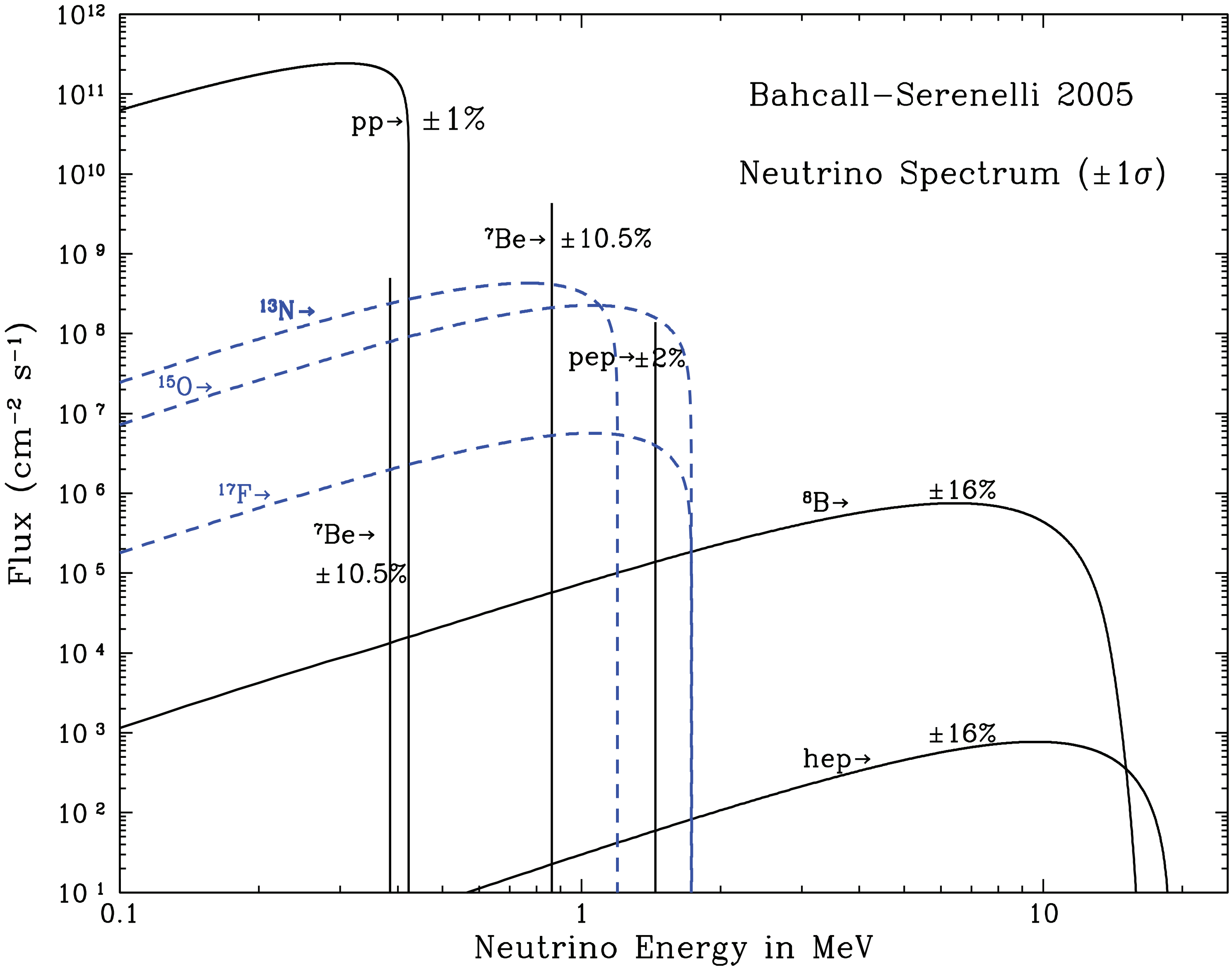
Super-Kamiokande: 50000t ultra-pure water;

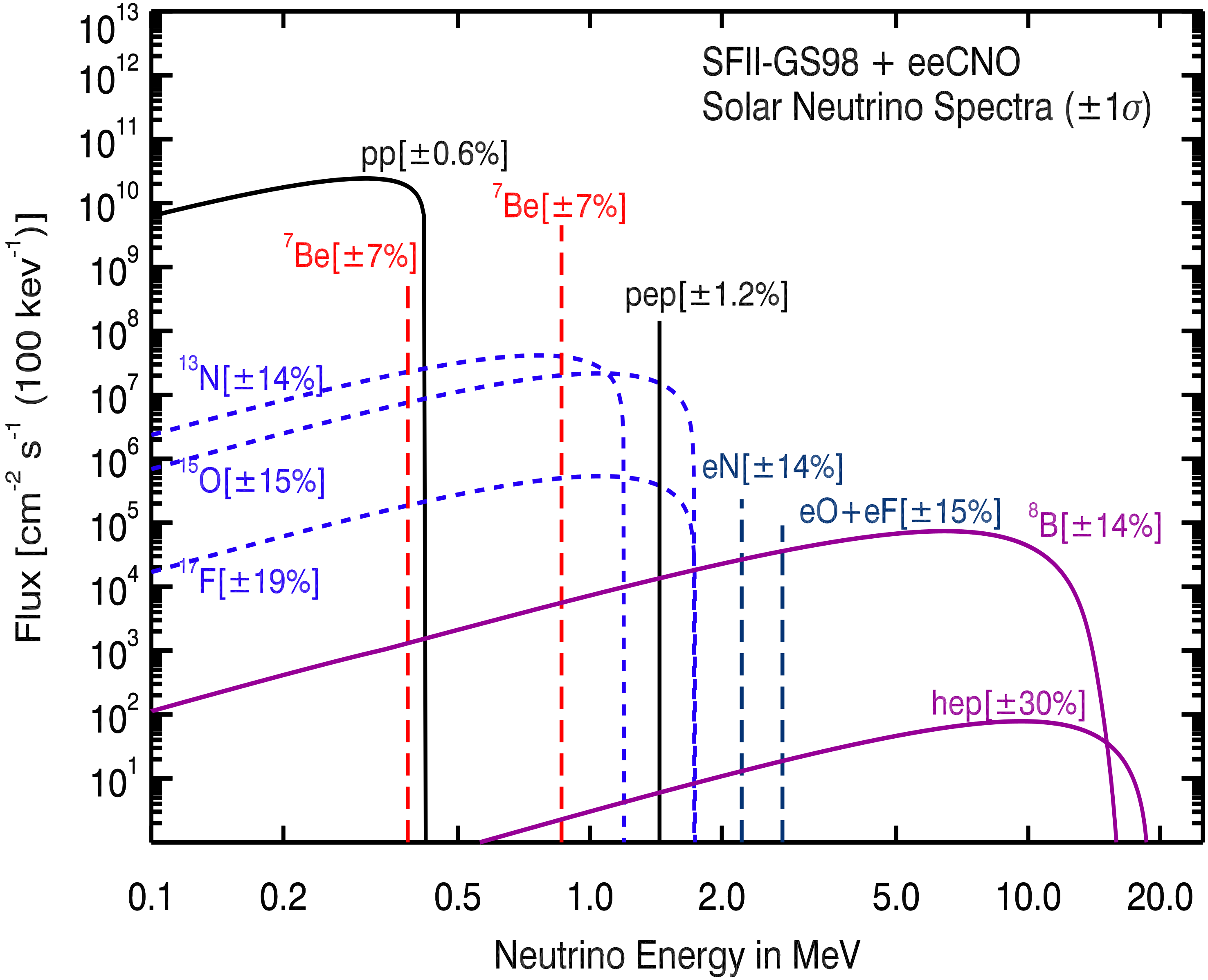
SNO: 1000 t heavy water (D_2O)

R. Davis: Nobel Prize for Physics in 2002 for the observation of $\nu_\odot$.

$$\nu_e +^{71} Ga \rightarrow^{71} Ge + e^-$$

SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-2003


Bahcall−Serenelli 2005
Neutrino Spectrum (±1σ)
Flux (cm⁻² s⁻¹)
Neutrino Energy in MeV
pp→ ±1%
⁷Be→ ±10.5%
¹³N→
¹⁵O→
¹⁷F→
⁷Be→
±10.5%
pep→±2%
⁸B→ ±16%
hep→ ±16%
0.1
1
10


SFII-GS98 + eeCNO
Solar Neutrino Spectra (±1σ)
pp[±0.6%]
7Be[±7%]
7Be[±7%]
pep[±1.2%]
13N[±14%]
15O[±15%]
17F[±19%]
eN[±14%]
eO+eF[±15%]
8B[±14%]
hep[±30%]
Flux [cm-2 s-1 (100 kev-1)]
Neutrino Energy in MeV
0.1
0.2
0.5
1.0
2.0
5.0
10.0
20.0

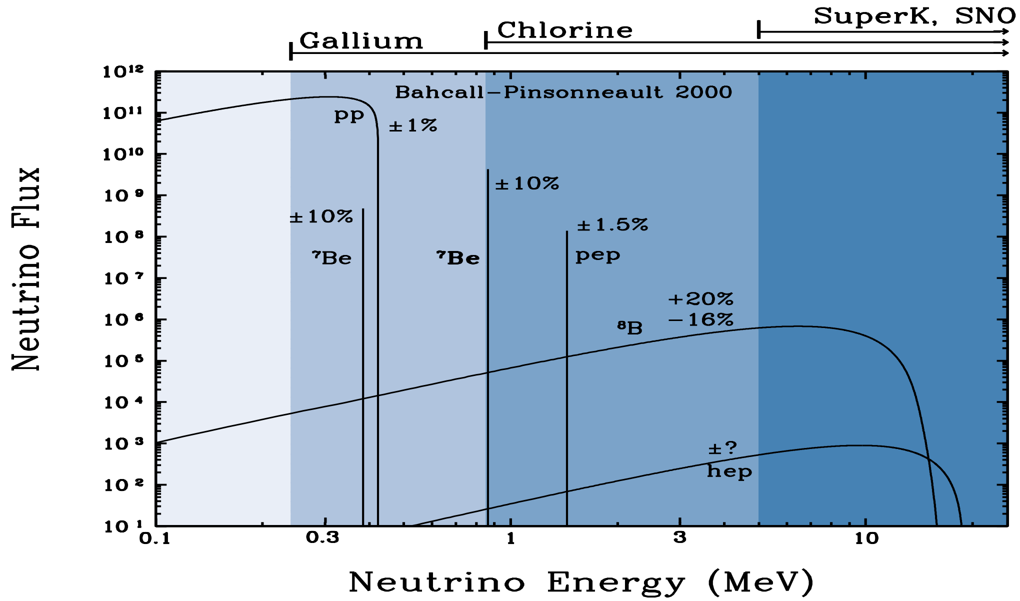


Figure 2: Differential Standard Solar Model neutrino fluxes [14].

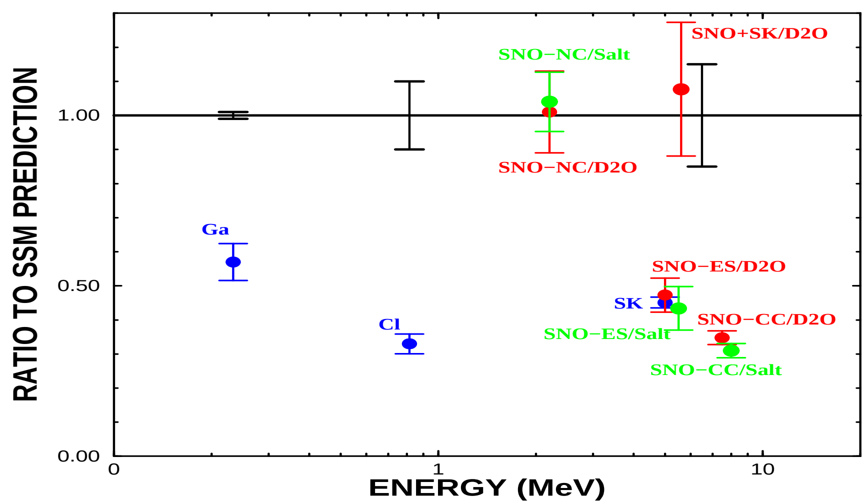


Figure 3: Comparison of measurements to Standard Solar Model predictions.

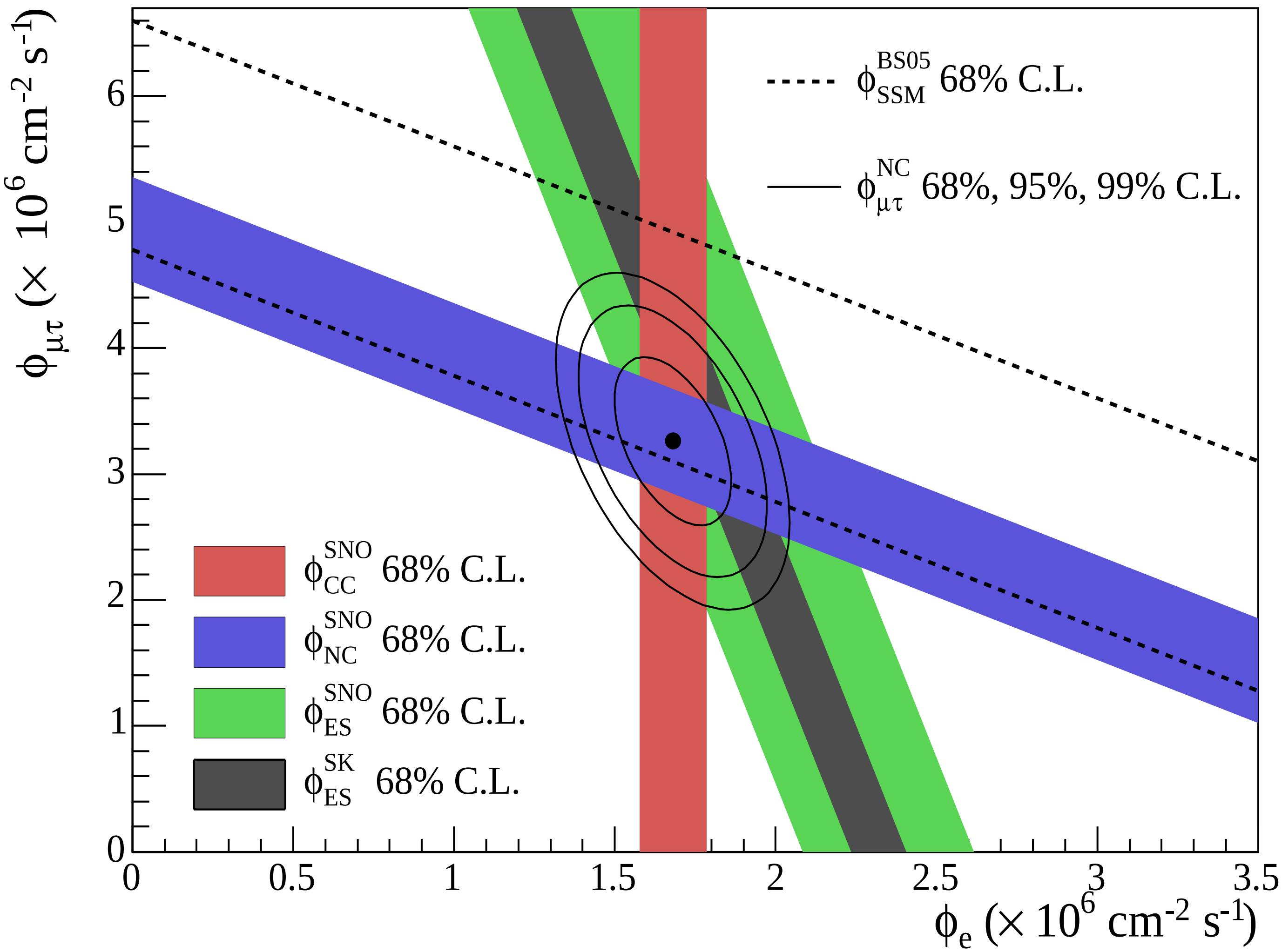
Flux	BP'00	Cl–Ar	Ga–Ge
$\Phi_{pp} \times 10^{-10}$	$5.95(1\ ^{+0.01}_{-0.01})$	0.00	69.7
$\Phi_{pep} \times 10^{-8}$	$1.40(1\ ^{+0.01}_{-0.01})$	0.22	2.8
$\Phi_{Be} \times 10^{-9}$	$4.77(1\ ^{+0.09}_{-0.09})$	1.15	34.2
$\Phi_{B} \times 10^{-6}$	$5.93(1\ ^{+0.14}_{-0.15})$	6.76	14.2
$\Phi_{N} \times 10^{-8}$	$5.48(1\ ^{+0.19}_{-0.13})$	0.09	3.4
$\Phi_{O} \times 10^{-8}$	$4.80(1\ ^{+0.22}_{-0.15})$	0.33	5.5
Total		$8.55\ ^{+1.1}_{-1.2}$	$129.8\ ^{+9}_{-7}$

The contributions to the rates of $Cl-Ar$ and $Ga-Ge$ reactions are in Solar Neutrino Units (SNU).

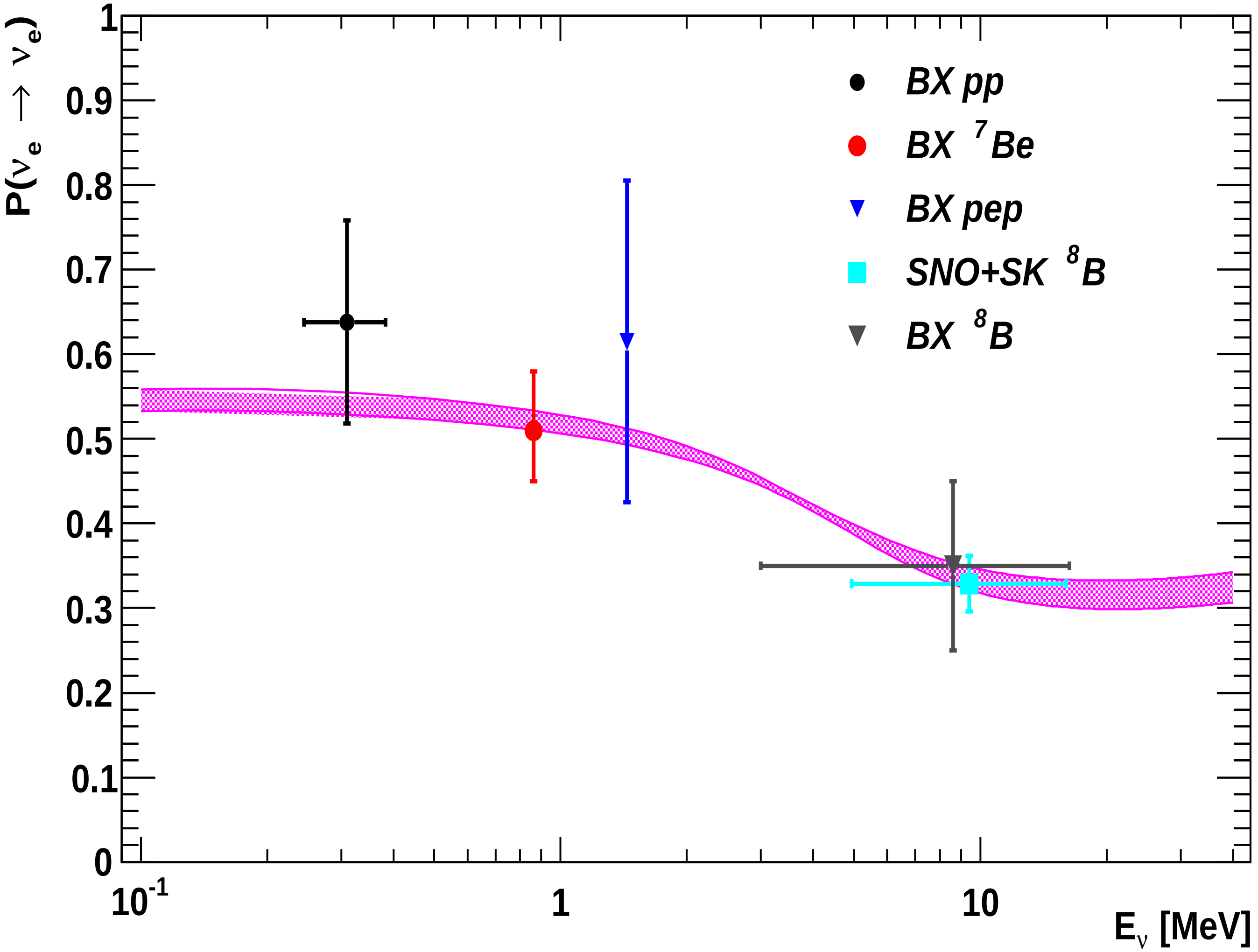
1 SNU = 10^{-36} ν_e captures per target atom per second.

Experiment	Observed rate/BP04 prediction	Predicted Rate at global best-fit	Predicted R at solar best
Ga	0.52 ± 0.029	**0.555**	**0.540**
Cl	0.301 ± 0.027	**0.356**	**0.345**
SK(ES)	0.406 ± 0.014	**0.394**	**0.395**
SNO(CC)	0.274 ± 0.019	**0.289**	**0.289**
SNO(ES)	0.38 ± 0.052	**0.386**	**0.386**
SNO(NC)	0.895 ± 0.08	**0.889**	**0.908**

The observed rates w.r.t predictions from the Standard Solar Model BP04. Shown are also the predicted rates for the best fit values of Δm^2_{21} and $\sin^2\theta_{12}$, obtained in the analysis of the i) global solar neutrino data, and ii) global solar neutrino +KamLAND data.


ϕ^{BS05}_{SSM} 68% C.L.
$\phi^{NC}_{\mu\tau}$ 68%, 95%, 99% C.L.
ϕ^{SNO}_{CC} 68% C.L.
ϕ^{SNO}_{NC} 68% C.L.
ϕ^{SNO}_{ES} 68% C.L.
ϕ^{SK}_{ES} 68% C.L.
$\phi_{\mu\tau}$ ($\times$ 10^6 cm^{-2} s^{-1})
ϕ_e ($\times$ 10^6 cm^{-2} s^{-1})

Results from BOREXINO



MSW Transitions of Solar Neutrinos in the Sun and the Hydrogen Atom

$$i\frac{d}{dt}\left(\begin{array}{c} A_{\alpha}(t,t_0) \\ A_{\beta}(t,t_0) \end{array}\right) = \left(\begin{array}{cc} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{array}\right)\left(\begin{array}{c} A_{\alpha}(t,t_0) \\ A_{\beta}(t,t_0) \end{array}\right)$$

where $\alpha = \nu_e$, $\beta = \nu_{\mu(\tau)}$,

$\epsilon(t) = \frac{1}{2}\,[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e(t)]$,

$\epsilon'(t) = \frac{\Delta m^2}{4E}\sin 2\theta$, with $\Delta m^2 = m_2^2 - m_1^2$.

- Standard Solar Models

$N_e(t) \;=\; N_e(t_0)\,\exp\left\{-\frac{t-t_0}{r_0}\right\}$, $r_0 \sim 0.1 R_{\odot}$, $R_{\odot} = 6.96 \times 10^5$km

The region of $\nu_{\odot}$ production: $r \lesssim 0.2 R_{\odot}$

$20\ N_A\ cm^{-3} \lesssim N_e(x_0) \lesssim 100\ N_A\ cm^{-3}$

Suppose $N_e(x_0) \gg N_e^{res}$: $|\nu_e\rangle \ \cong \ \ |\nu_2^m\rangle$.

Possible evolution:

The system stays at this level; at the surface: $|\nu_2^m\rangle = |\nu_2\rangle$

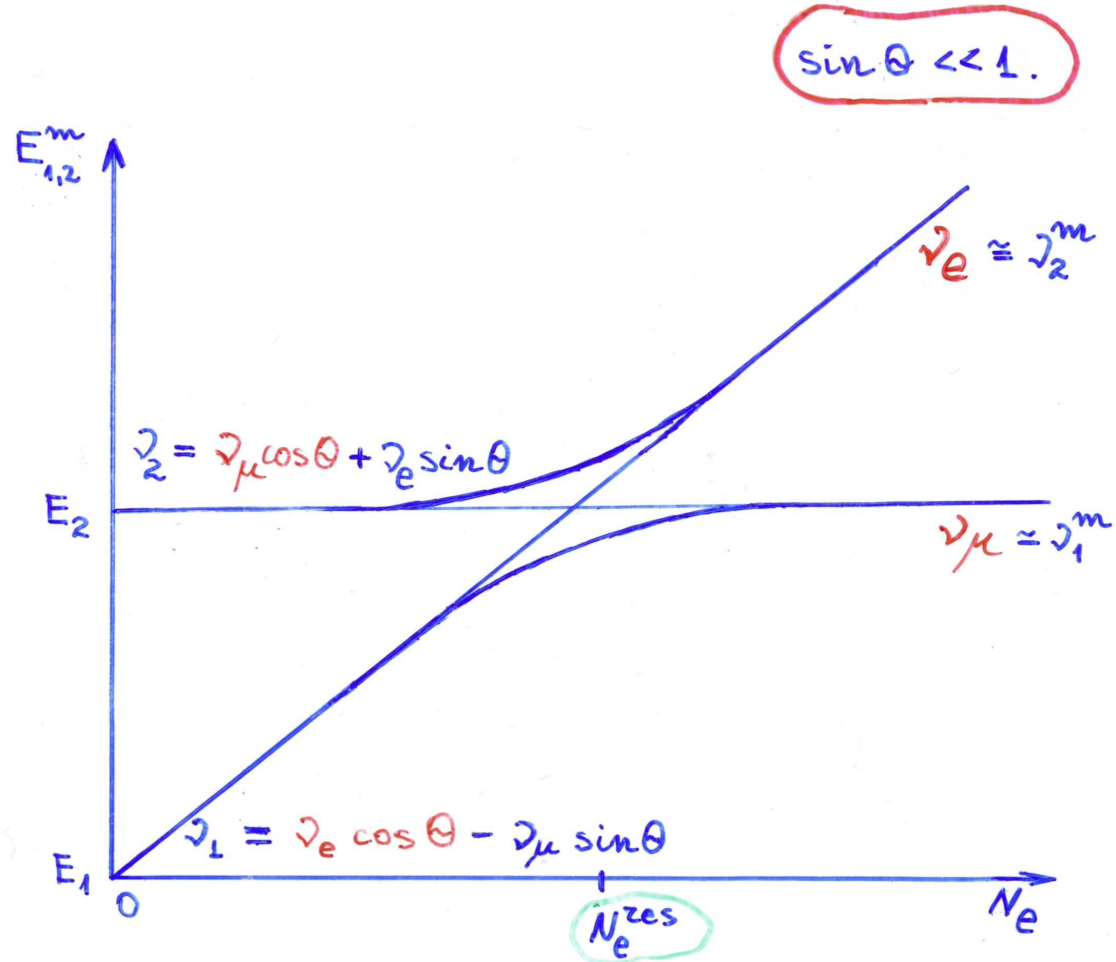
$P(\nu_e \to \nu_e) \cong |\langle\nu_e|\nu_2\rangle|^2 = \sin^2\theta\,,$ Adiabatic

At $N_e = N_e^{res}$, where $E_2^m - E_1^m$ is minimal, the system jumps to lower level $|\nu_1^m\rangle$; at the surface: $|\nu_1^m\rangle = |\nu_1\rangle$

$P(\nu_e \to \nu_e) \cong |\langle\nu_e|\nu_1\rangle|^2 = \cos^2\theta\,,$ Nonadiabatic

Type of transition: $P' \equiv P(\nu_2^m(t_0) \to \nu_1)\,,$ jump probability

4.



1. $P(\nu_2^m \to \nu_1; t_\odot, t_0) \equiv P'$ - negligible: adiabatic transition

2. P' - nonnegligible: nonadiabatic

Introducing the dimensionless variable
$Z = ir_0\sqrt{2}G_F N_e(t_0)e^{-\frac{t-t_0}{r_0}}, \quad Z_0 = Z(t = t_0),$

and making the substitution

$$A_e(t,t_0) = (Z/Z_0)^{c-a}\ e^{-(Z-Z_0)+i\int_{t_0}^{t}\epsilon(t')dt'}\ A'_e(t,t_0),$$

$A'_e(t,t_0)$ **satisfies the confluent hypergeometric equation (CHE):**

$$\left\{\ Z\frac{d^2}{dZ^2} + (c-Z)\ \frac{d}{dZ} - a\right\} A'_e(t,t_0) = 0,$$

where

$$a = 1 + ir_0\ \frac{\Delta m^2}{2E}\sin^2\theta, \qquad c = 1 + ir_0\ \frac{\Delta m^2}{2E}.$$

The confluent hypergeometric equation describing the ν_e oscillations in the Sun, coincides in form with the Schroedinger (energy eigenvalue) equation obeyed by the radial part, $\psi_{kl}(r)$, of the non-relativistic wave function of the hydrogen atom,

$\Psi(\vec{r}) = \frac{1}{r}\psi_{kl}(r) Y_{lm}(\theta', \phi')$,

r, θ' and ϕ' are the spherical coordinates of the electron in the proton's rest frame, l and m are the orbital momentum quantum numbers ($m = -l, ..., l$), k is the quantum number labeling (together with l) the electron energy (the principal quantum number is equal to $(k + l)$), E_{kl} ($E_{kl} < 0$), and $Y_{lm}(\theta', \phi')$ are the spherical harmonics. The function

$\psi'_{kl}(Z) = Z^{-c/2}\ e^{Z/2}\ \psi_{kl}(r)$

$$Z = 2\,\frac{r}{a_0}\sqrt{-E_{kl}/E_I},\ a \equiv a_{kl} = l{+}1{-}\sqrt{-E_I/E_{kl}},\ c \equiv c_l = 2(l{+}1)$$

$a_0 = \hbar/(m_e e^2)$ is the Bohr radius and $E_I = m_e e^4/(2\hbar^2) \cong 13.6\ eV$ is the ionization energy of the hydrogen atom.

Quite remarkably, the behavior of such different physical systems as solar neutrinos undergoing MSW transitions in the Sun and the non-relativistic hydrogen atom are governed by one and the same differential equation.

Any solution - linear combination of two linearly independent solutions:

$\Phi(a,c;Z)$, $Z^{1-c}\ \Phi(a-c+1,2-c;Z)$; $\Phi(a',c';Z=0)=1$, $a',c'\neq 0,-1,-2,....$

$$A(\nu_e \rightarrow \nu_{\mu(\tau)}) = \frac{1}{2}\ \sin 2\theta\ \left\{ \Phi(a-c,2-c;Z_0) - e^{i(t-t_0)\frac{\Delta m^2}{2E}}\ \Phi(a-1,c;Z_0)\ \right\}.$$

Sun: $N_e(x) \cong N_e(x_0)e^{-\frac{x}{r_0}}$, $r_0 \cong 0.1R_\odot$, $R_\odot \cong 7\times 10^5$ **km**

The region of $\nu_\odot$ production:

$20\ N_A\ cm^{-3} \lesssim N_e(x_0) \lesssim 100\ N_A\ cm^{-3}$: $|Z_0| > 500$ **(!)**

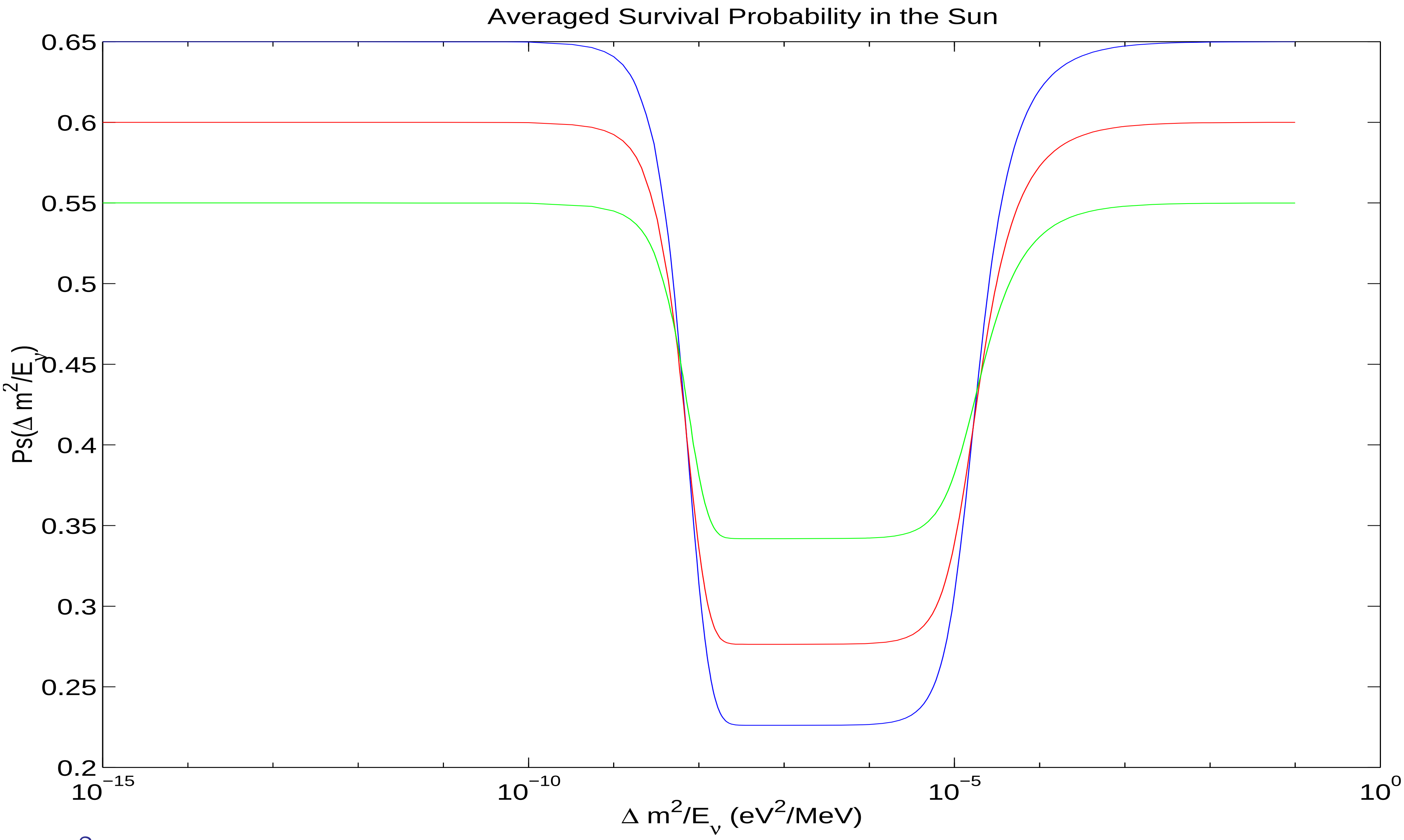
The solar ν_e survival probability:

$\bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + (\frac{1}{2} - P')\cos 2\theta_m^0 \cos 2\theta$,

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}\sin^2\theta} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

S.T.P., 1988

$\nu_e \rightarrow \nu_e$



$\sin^2 2\theta = 0.7$, **0.8, 0.9.**

The solar ν_e survival probability:

$\bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + (\frac{1}{2} - P') \cos 2\theta_m^0 \cos 2\theta$,

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E} \sin^2\theta} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

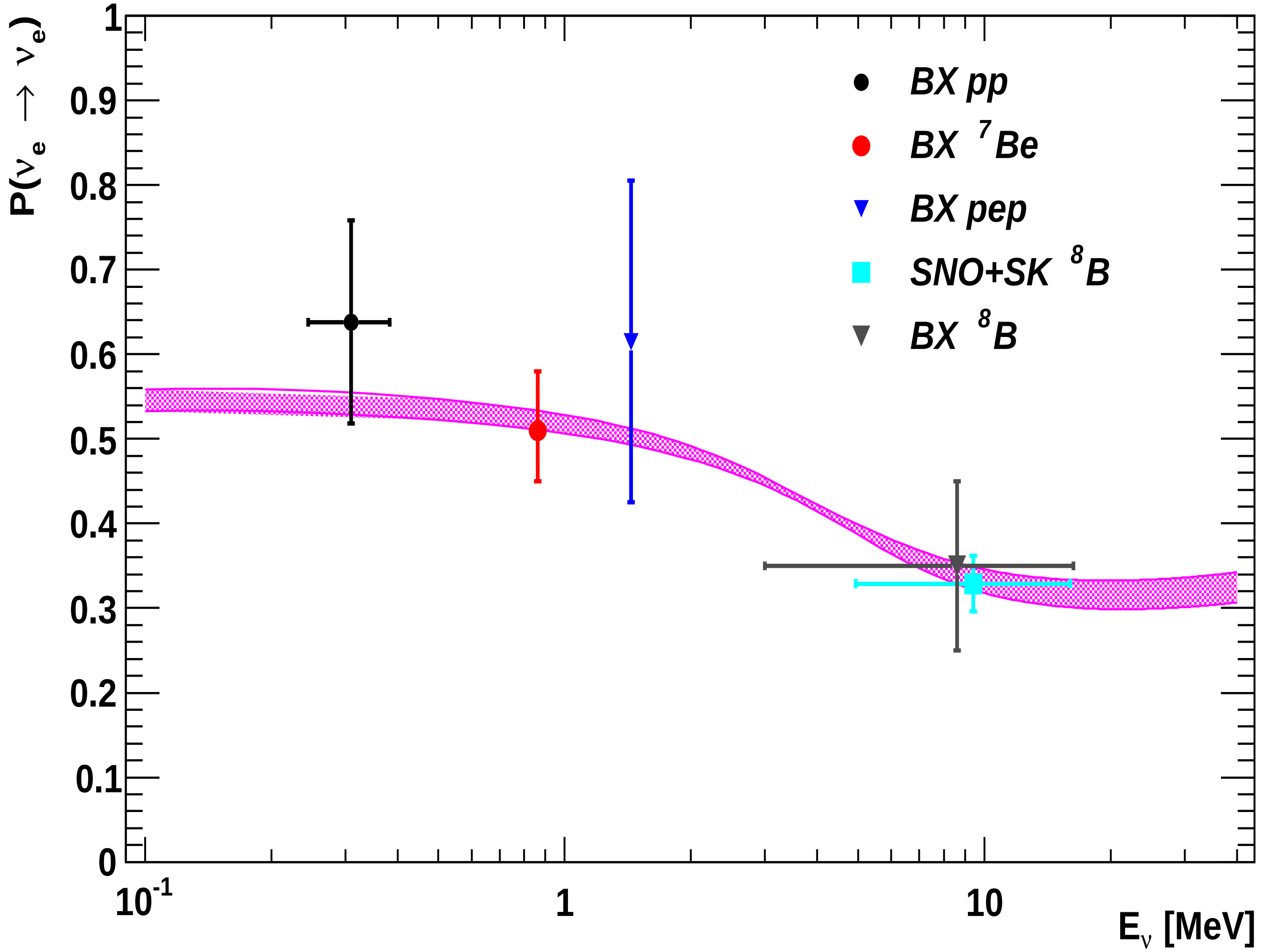
Case 1: $\cos 2\theta_m^0 = -1$, $P' = 0$, $\bar{P} = \frac{1}{2}(1 - \cos 2\theta)$.

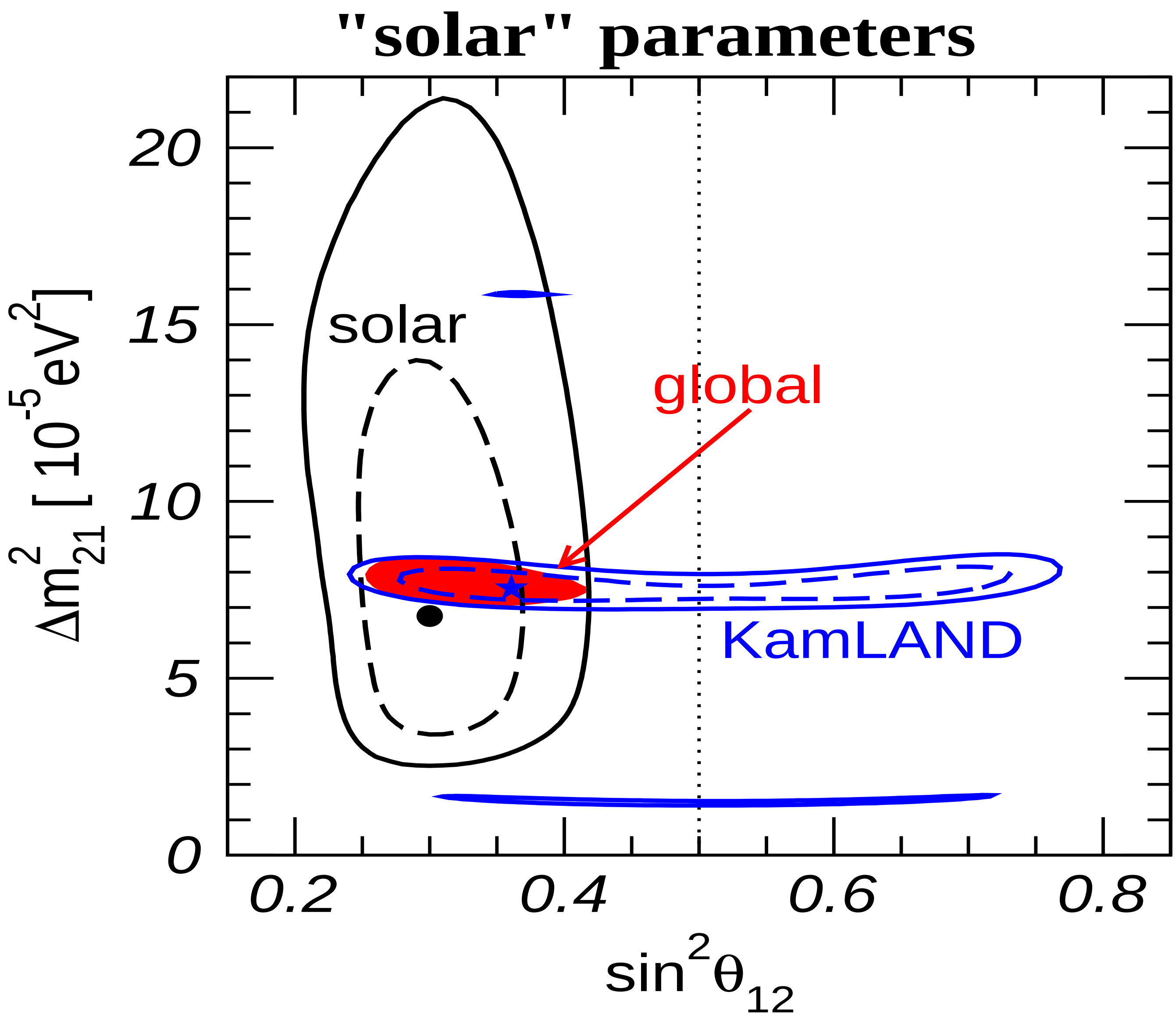
Case 2: $\theta_m^0 = \theta$, $P' = 0$, $\bar{P}(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta$

Case 1: SNO, Super Kamiokande; $\bar{P} \cong 0.3$: $\cos 2\theta > 0$!

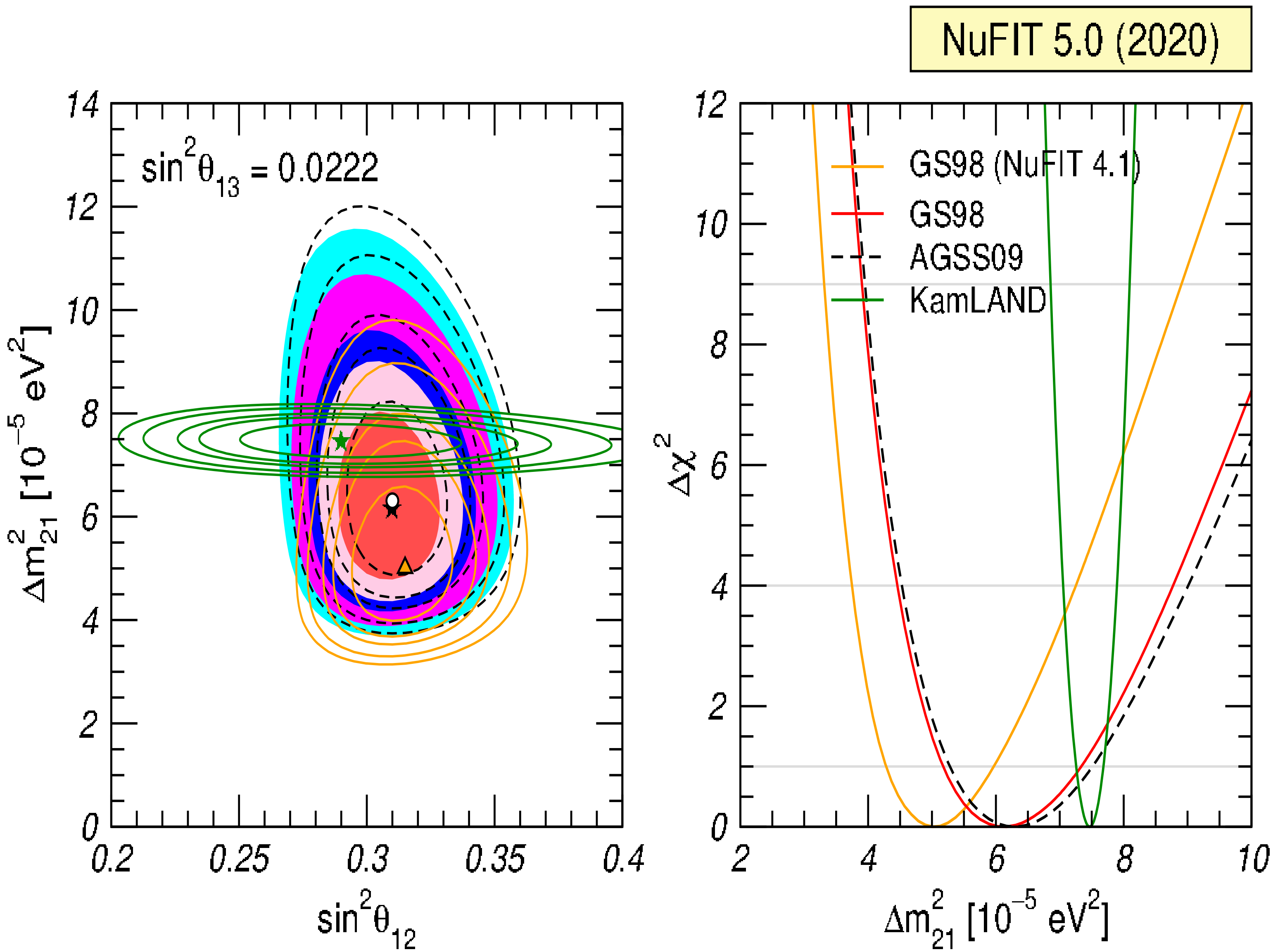
Case 2: pp **neutrinos.**

Results from BOREXINO





T. Schwetz, arXiv:0710.5027[hep-ph]



I. Esteban et al., arXiv:2007.14792 [hep-ph]

J.N. BAHCALL, M.C. GONZALEZ-GARCIA, C. PEÑA GARAY '01

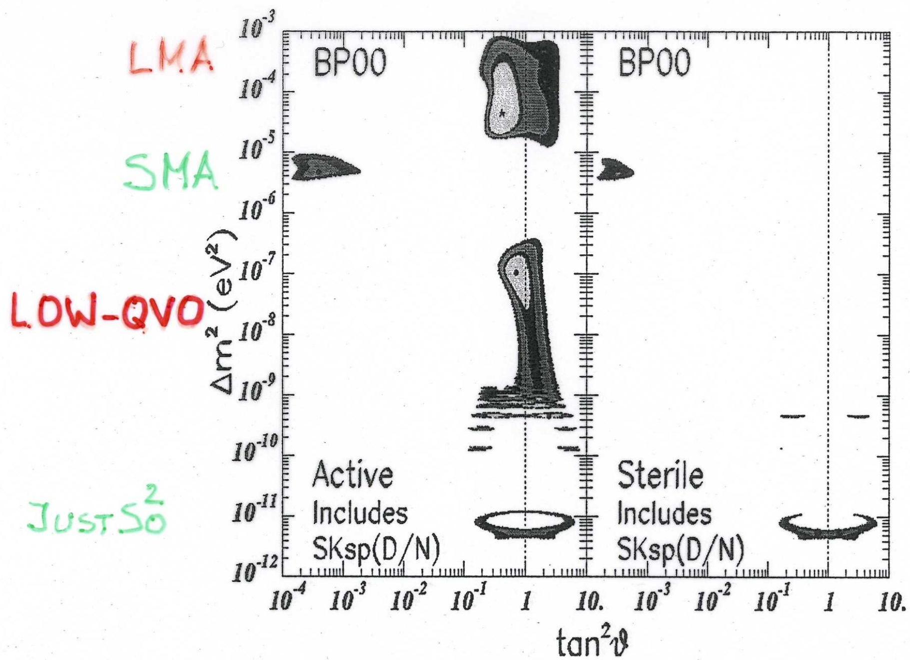
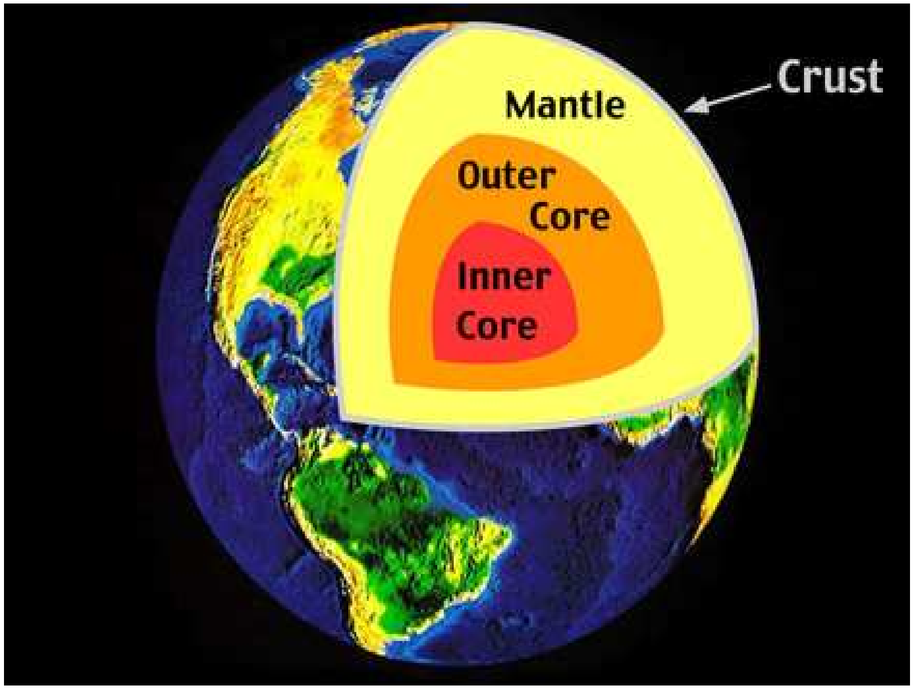
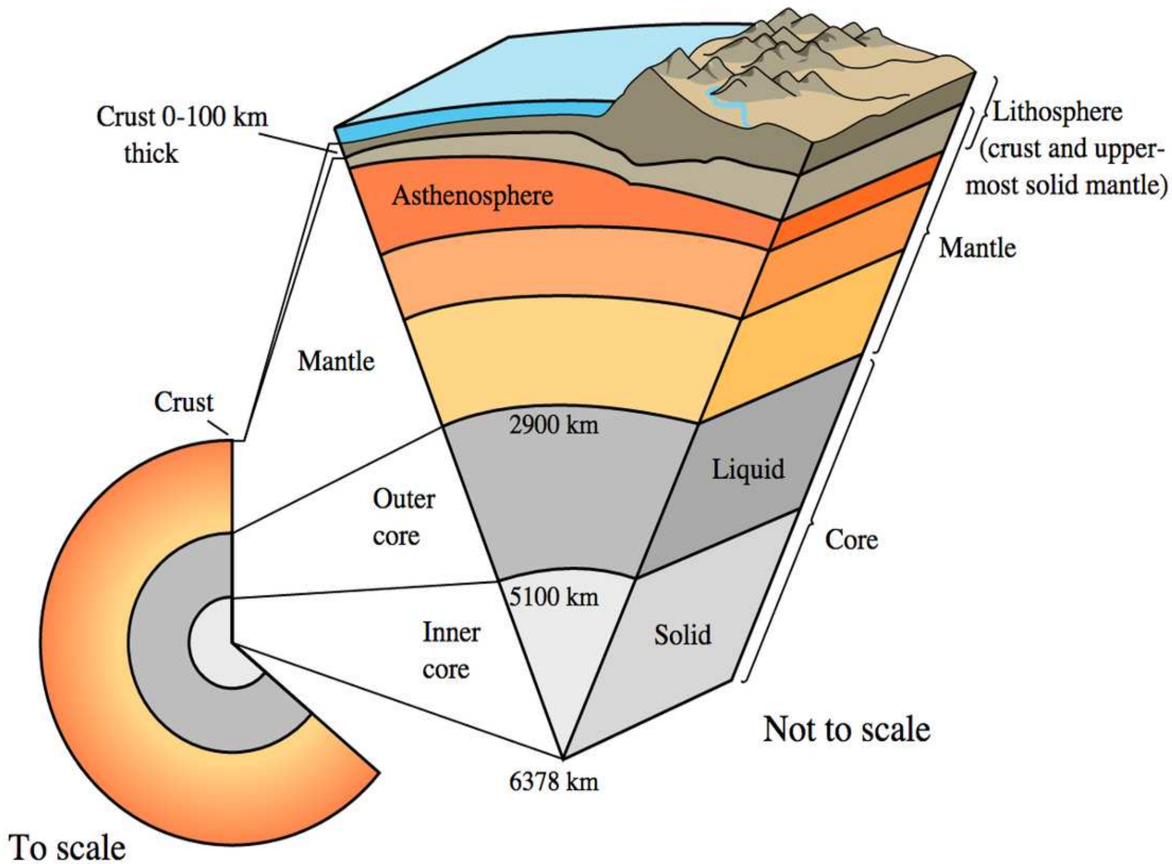
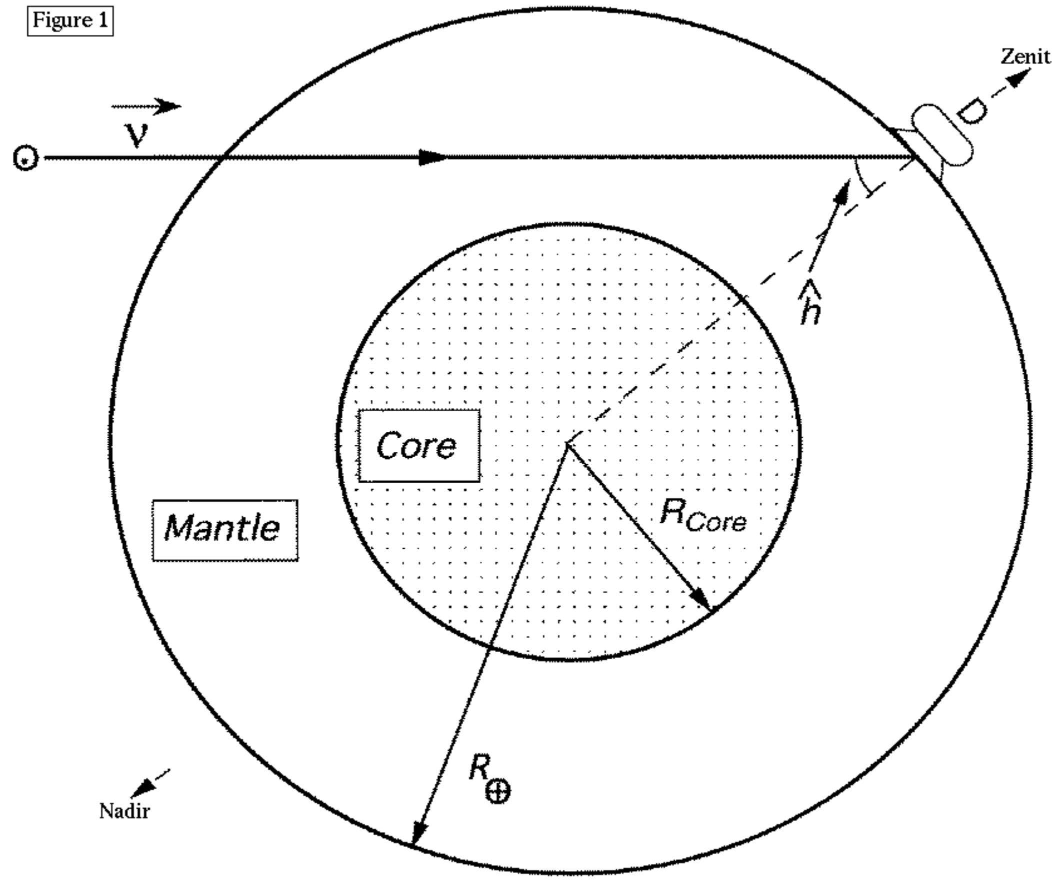


Figure 1: Global solutions including all available solar neutrino data. The input data include the total rates from the Chlorine [2], Gallium (averaged) [3, 5, 4], Super-Kamiokande [6], and SNO [1] experiments, as well as the recoil electron energy spectrum measured by Super-Kamiokande during the day and separately the energy spectrum measured at night. The C.L. contours shown in the figure are 90%, 95%, 99%, and 99.73% (3σ). The allowed regions are cutoff below 10^{-3}eV^2 by the Chooz reactor measurements [22]. The local best-fit points are marked by dark circles. The theoretical errors for the BP2000 neutrino fluxes are included in the analysis.

The Earth



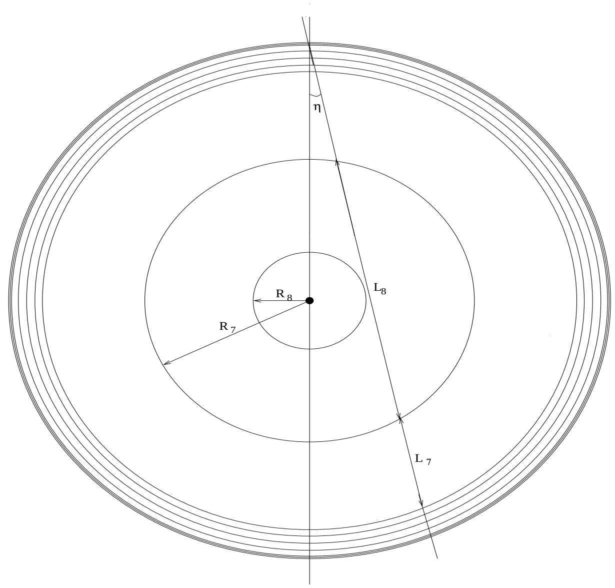

Crust 0-100 km
thick
Lithosphere
(crust and upper-
most solid mantle)
Asthenosphere
Mantle
Mantle
Crust
2900 km
Liquid
Outer
core
Core
5100 km
Inner
core
Solid
Not to scale
6378 km
To scale



Earth: $R_{core} = 3446$ **km,** $R_{mant} = 2885$ **km**

Earth: $\bar{N}_e^{mant} \sim 2.3\ N_A\ cm^{-3}$**,** $\bar{N}_e^{core} \sim 5.7\ N_A\ cm^{-3}$

The Earth



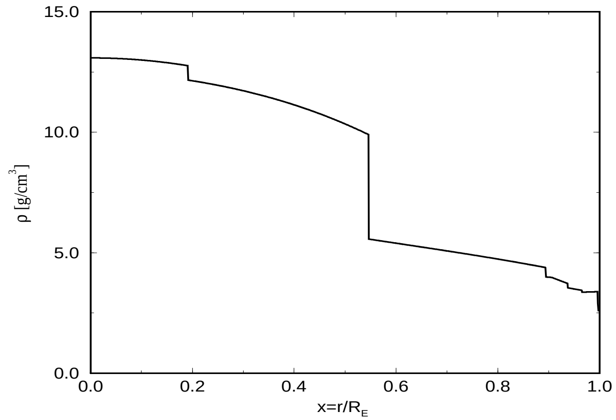
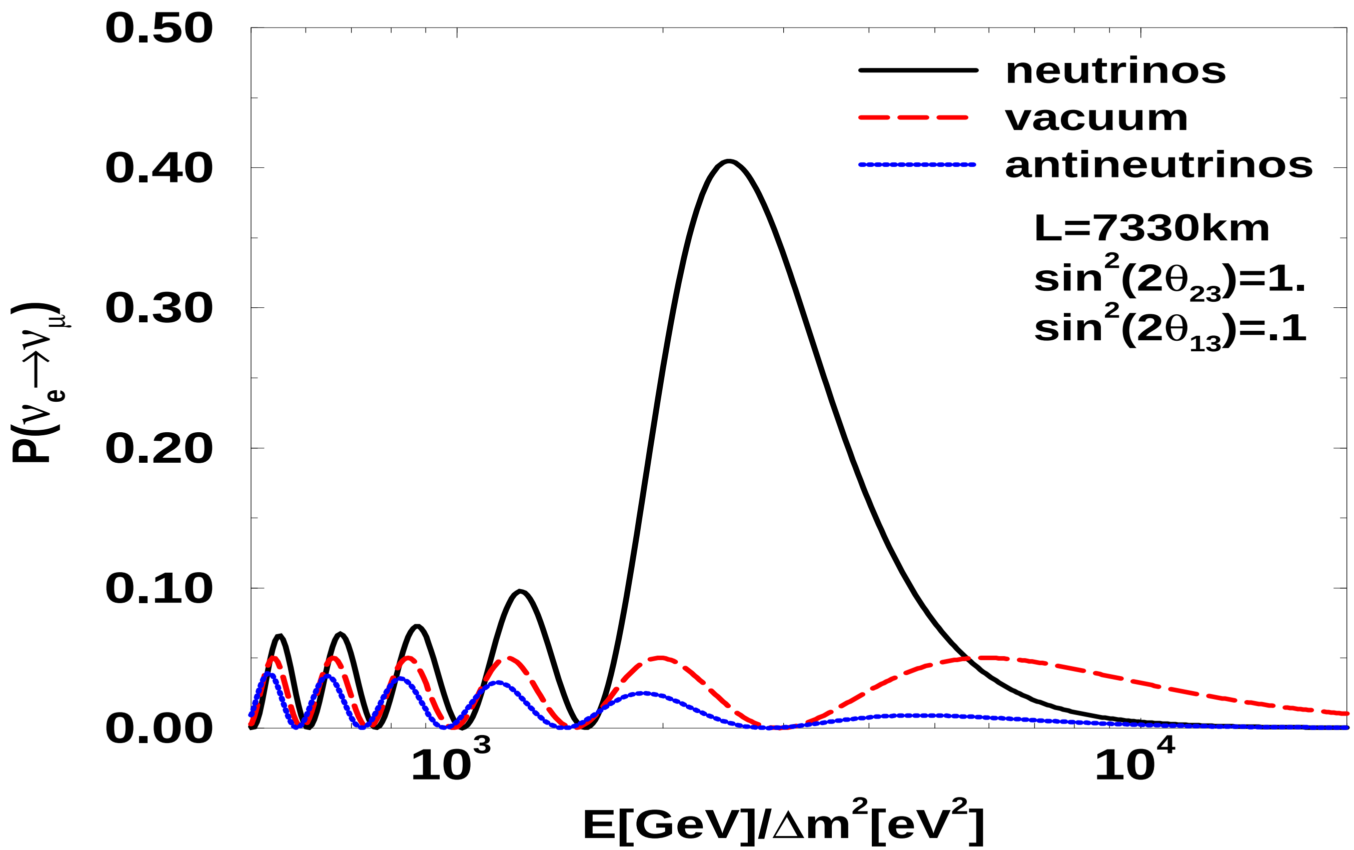


FIG. 1. Density profile of the Earth.

$R_c = 3446$ **km,** $R_m = 2885$ **km;** $\bar{N}_e^{mant} \overset{16}{\sim} 2.3\ N_A\ cm^{-3}$**,** $\bar{N}_e^{core} \sim 5.7\ N_A\ cm^{-3}$

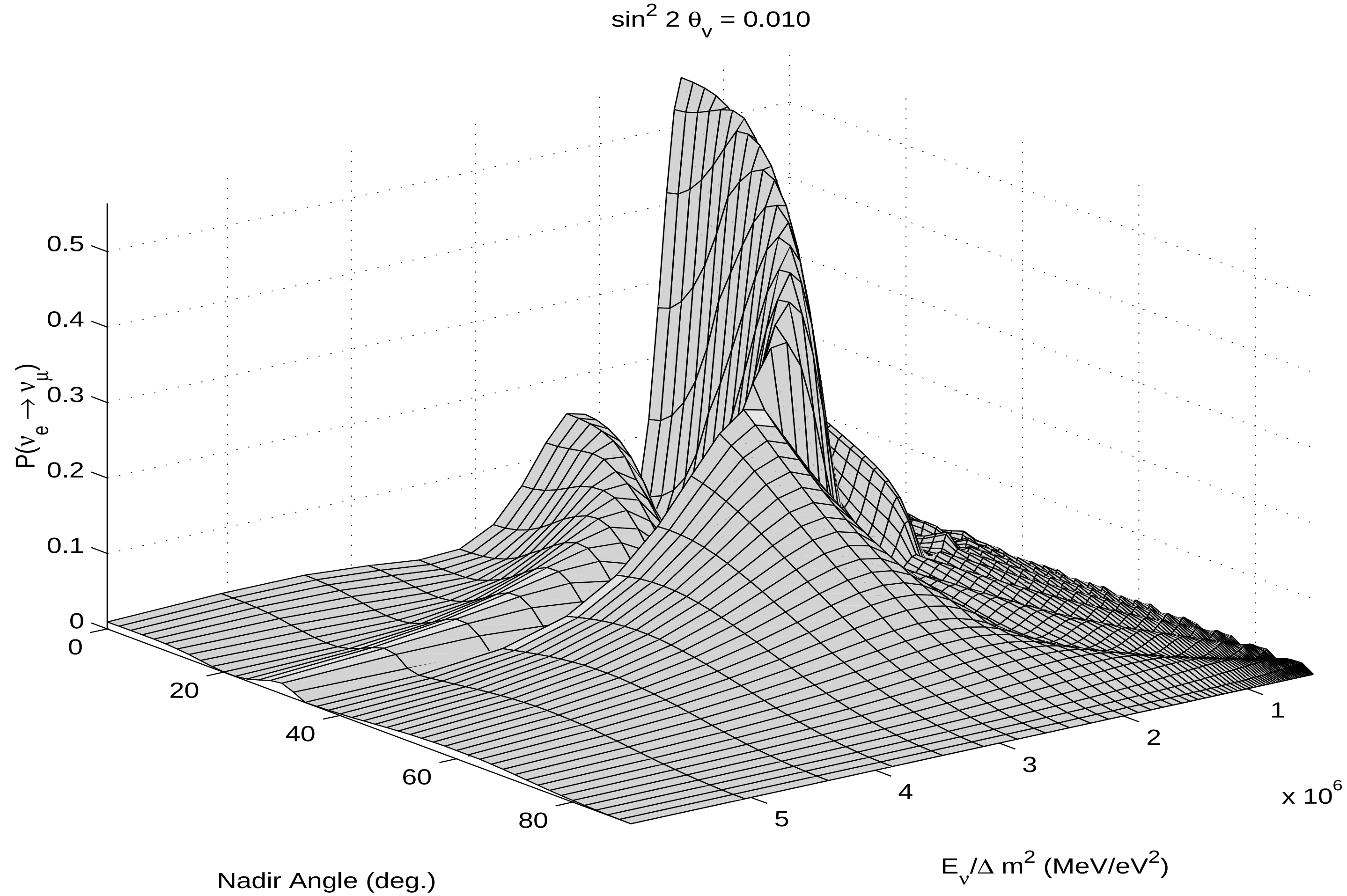
Earth matter effect in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (MSW)



$\Delta m^2 = 2.5 \times 10^{-3}$ eV^2, $E^{res} = 6.25$ **GeV;** $P^{3\nu} = \sin^2\theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu}$;

$N_e^{res} \cong 2.3$ cm^{-3} N$_A$; $L_m^{res} = L^v / \sin 2\theta_{13} \cong 6250/0.32$ **km;** $2\pi L/L_m \cong 0.75\pi (\neq \pi)$.

Earth matter effects in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (NOLR)

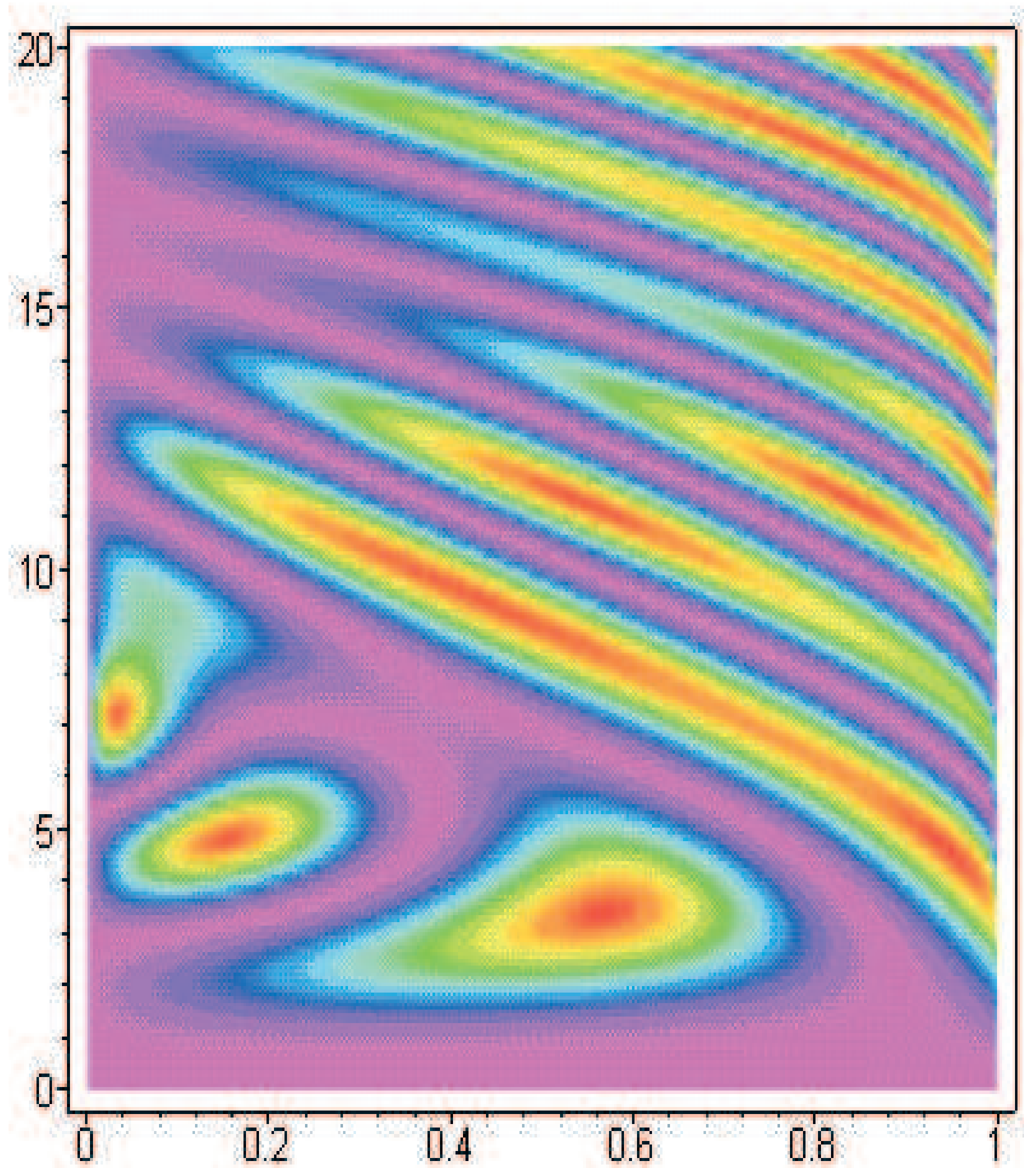


S.T.P., 1998;
M. Chizhov, M. Maris, S.T.P., 1998; M. Chizhov, S.T.P., 1999

$P(\nu_e \to \nu_\mu) \equiv P_{2\nu} \equiv (s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)})$, $\theta_\mathrm{v} \equiv \theta_{13}$, $\Delta m^2 \equiv \Delta m^2_\mathrm{atm}$;

Absolute maximum: Neutrino Oscillation Length Resonance (NOLR);

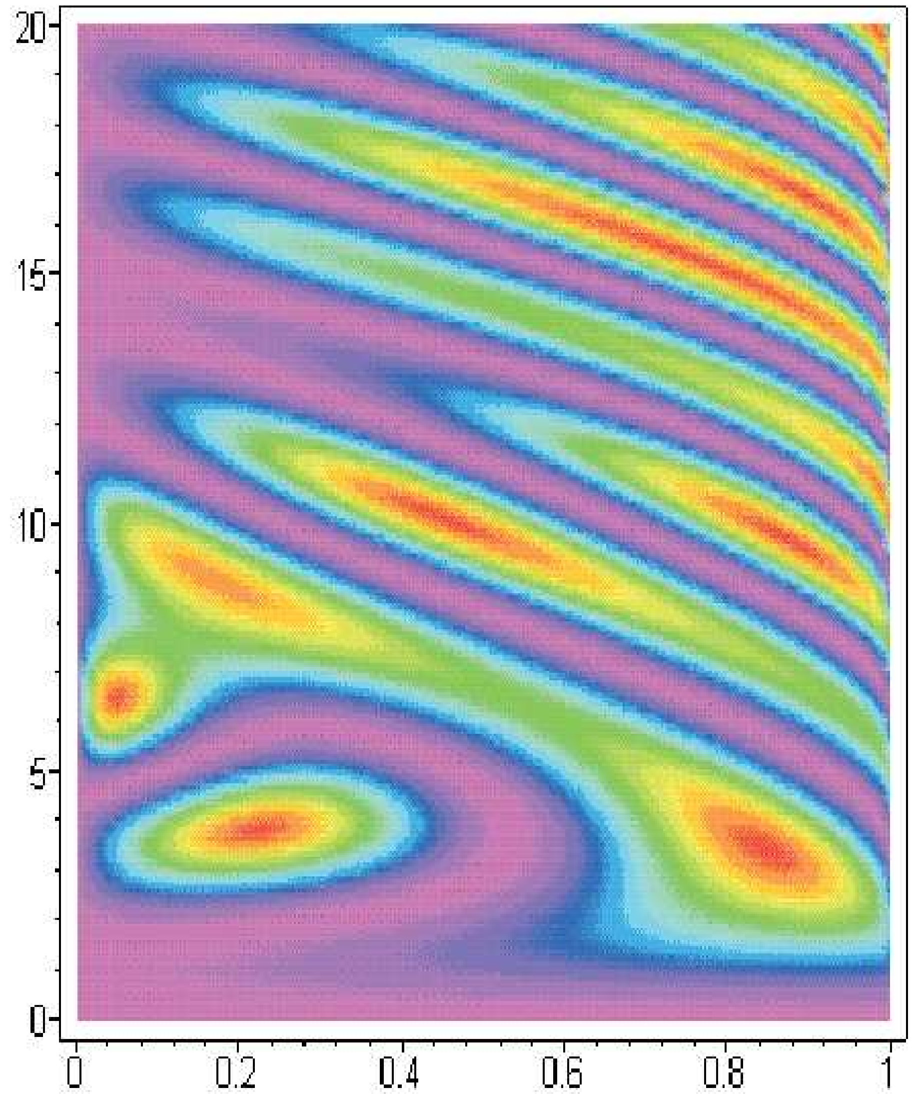
Local maxima: MSW effect in the Earth mantle or core.



$(s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}) \equiv P_{2\nu}$; **NOLR: "Dark Red Spots"**, $P_{2\nu} = 1$;

Vertical axis: $\Delta m^2/E\ [10^{-7} eV^2/MeV]$; **horizontal axis:** $\sin^2 2\theta_{13}$; $\theta_n = 0$

M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)



$(s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}) \equiv P_{2\nu}$; **NOLR: "Dark Red Spots"**, $P_{2\nu} = 1$;
Vertical axis: $\Delta m^2/E$ $[10^{-7} eV^2/MeV]$; **horizontal axis:** $\sin^2 2\theta_{13}$; $\theta_n = 23^\circ$
M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)

• **For Earth center crossing ν's ($\theta_n = 0$) and, e.g.** $\sin^2 2\theta_{13} = 0.01$, **NOLR occurs at** $E \cong 4$ **GeV ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV^2).**

S.T.P., hep-ph/9805262

• **For the Earth core crossing ν's:** $P_{2\nu} = 1$ **due to NOLR when**

$$\tan \Phi^{\text{man}}/2 \equiv \tan \phi' = \pm \sqrt{\frac{-\cos 2\theta''_m}{\cos(2\theta''_m - 4\theta'_m)}},$$

$$\tan \Phi^{\text{core}}/2 \equiv \tan \phi'' = \pm \frac{\cos 2\theta'_m}{\sqrt{-\cos(2\theta''_m)\cos(2\theta''_m - 4\theta'_m)}}$$

Φ^{man} (Φ^{core}) - **phase accumulated in the Earth mantle (core),**
θ'_m (θ''_m) - **the mixing angle in the Earth mantle (core).**

$P_{2\nu} = 1$ **due to NOLR for** $\theta_n = 0$ **(Earth center crossing ν's) at,**
e.g. $\sin^2 2\theta_{13} = 0.034;\ 0.154$, $E \cong 3.5;\ 5.2$ **GeV ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV^2).**

M. Chizhov, S.T.P., Phys. Rev. Lett. 83 (1999) 1096 (hep-ph/9903399); Phys. Rev. Lett. 85 (2000) 3979 (hep-ph/0504247); Phys. Rev. D63 (2001) 073003 (hep-ph/9903424).

JUNO

20 kt LS detector of reactor $\bar{\nu}_e$ via IBD
$\bar{\nu}_e + p \to n + e^+$; $E_{res} = 3\%/\sqrt{E}$; $L \cong 53$ **km;**
thermal power of the used reactors: 26.6 GW;
Sphere with a diameter of 38 m.
Cost: 300×10^6 **US Dollars.**
Built by international collaboration of more than 700 scientists from 74 Institutions in 17 countries/regions. Expected to start data-taking at the beginning of 2025.
After 6 years of operation: NMO at 3σ **(using reactor** ν **data only). Adding** ν_{atm} **data can improve the sensitivity by** $(0.8 - 1.4)\sigma$**.**

The idea put forward in S.T.P., M. Piai, PLB 553 (2002) 94 (hep-ph/0112074).

Based on: $P_{NO}(\bar{\nu}_e \to \bar{\nu}_e) \neq P_{IO}(\bar{\nu}_e \to \bar{\nu}_e)$

$$P^{NO}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{atm}\,L}{2\,E_\nu}\right) - \frac{1}{2}\cos^4\theta_{13}\sin^2 2\theta_{12}\left(1 - \cos\frac{\Delta m^2_{\odot}\,L}{2\,E_\nu}\right)$$

$$+ \frac{1}{2}\sin^2 2\theta_{13}\sin^2\theta_{12}\left(\cos\left(\frac{\Delta m^2_{atm}\,L}{2\,E_\nu} - \frac{\Delta m^2_{\odot}\,L}{2\,E_\nu}\right) - \cos\frac{\Delta m^2_{atm}\,L}{2\,E_\nu}\right), \quad \Delta m^2_{\odot} \equiv \Delta m^2_{21},$$

$$P^{IO}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{atm}\,L}{2\,E_\nu}\right) - \frac{1}{2}\cos^4\theta_{13}\sin^2 2\theta_{12}\left(1 - \cos\frac{\Delta m^2_{\odot}\,L}{2\,E_\nu}\right)$$

$$+ \frac{1}{2}\sin^2 2\theta_{13}\cos^2\theta_{12}\left(\cos\left(\frac{\Delta m^2_{atm}\,L}{2\,E_\nu} - \frac{\Delta m^2_{\odot}\,L}{2\,E_\nu}\right) - \cos\frac{\Delta m^2_{atm}\,L}{2\,E_\nu}\right).$$

$\Delta m^2_{\textbf{atm}} = \Delta m^2_{31(32)}(NO)$, $\Delta m^2_{\textbf{atm}} = \Delta m^2_{32(31)}(IO)$,

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

Spectrum of e^+ - sensitive to the difference between $P^{NO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ and $P^{IO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ - can be used to determine neutrino mass ordering. Optimal $L \sim 60$ km.

S.T.P., M. Piai, 2001

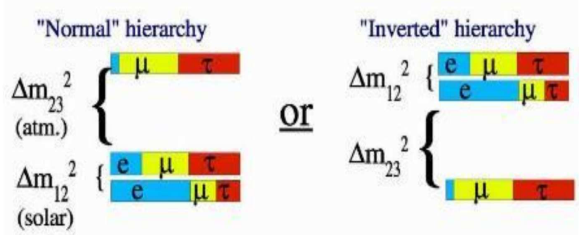
JUNO (China, International collaboration)

S. Choubey, S.T.P., M. Piai, PRD 68 (2003) 113006 ((hep-ph/0306017):
can measure $\sin^2\theta_{12}$, Δm^2_{21} and Δm^2_{31} with exceptionally high precision.
After 6 years of datakingt:
$\sin^2\theta_{12}$: 0.5%; Δm^2_{21}: 0.3%; Δm^2_{31}: 0.2% (1σ)
(Y. Wang, talk given at CERN on March 20, 2024).

Wide program of research: atmospheric ν oscillations, solar neutrinos, SN neutrinos, geo-neutrinos, nucleon decay; distant future: $(\beta\beta)_{0\nu}$ decay.

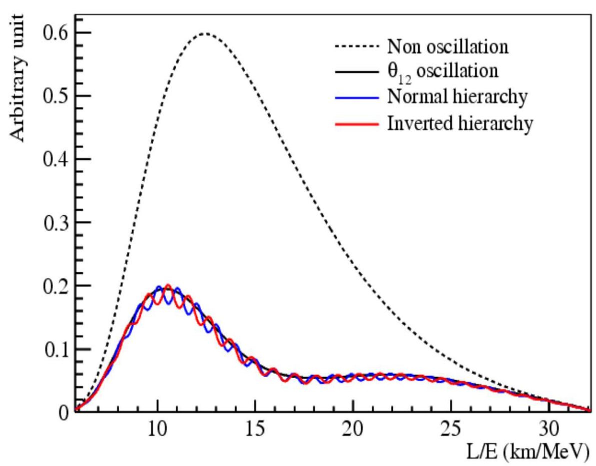

Huizhou
Guangzhou
Guang Zhou
Dongguan
Zhaoqing
CNS
2.5 h drive
Shen Zhen
Huizhou NPP
Lufeng NPP
Daya Bay
NPP
Zhongshan
Zhu Hai
Zhujiang River Estuary
Hong Kong
Hong Kong
Macau
JUNO site
Macau
53 km
53 km
Taishan NPP
Yangjiang NPP

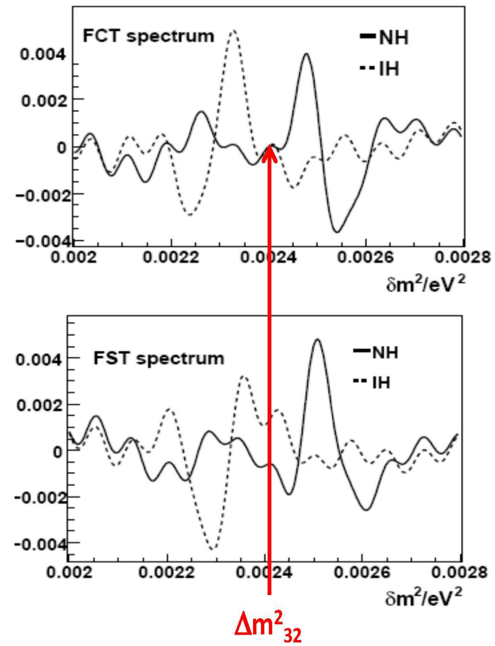
Mass Ordering by Reactor Neutrinos



$$\Delta m^2_{31} = \Delta m^2_{32} + \Delta m^2_{21}$$
$$\text{NH}: \; |\Delta m^2_{31}| = |\Delta m^2_{32}| + |\Delta m^2_{21}|$$
$$\text{IH}: \; |\Delta m^2_{31}| = |\Delta m^2_{32}| - |\Delta m^2_{21}|$$

$$\frac{\Delta m^2_{21}}{|\Delta m^2_{32}|} \sim 3\%$$



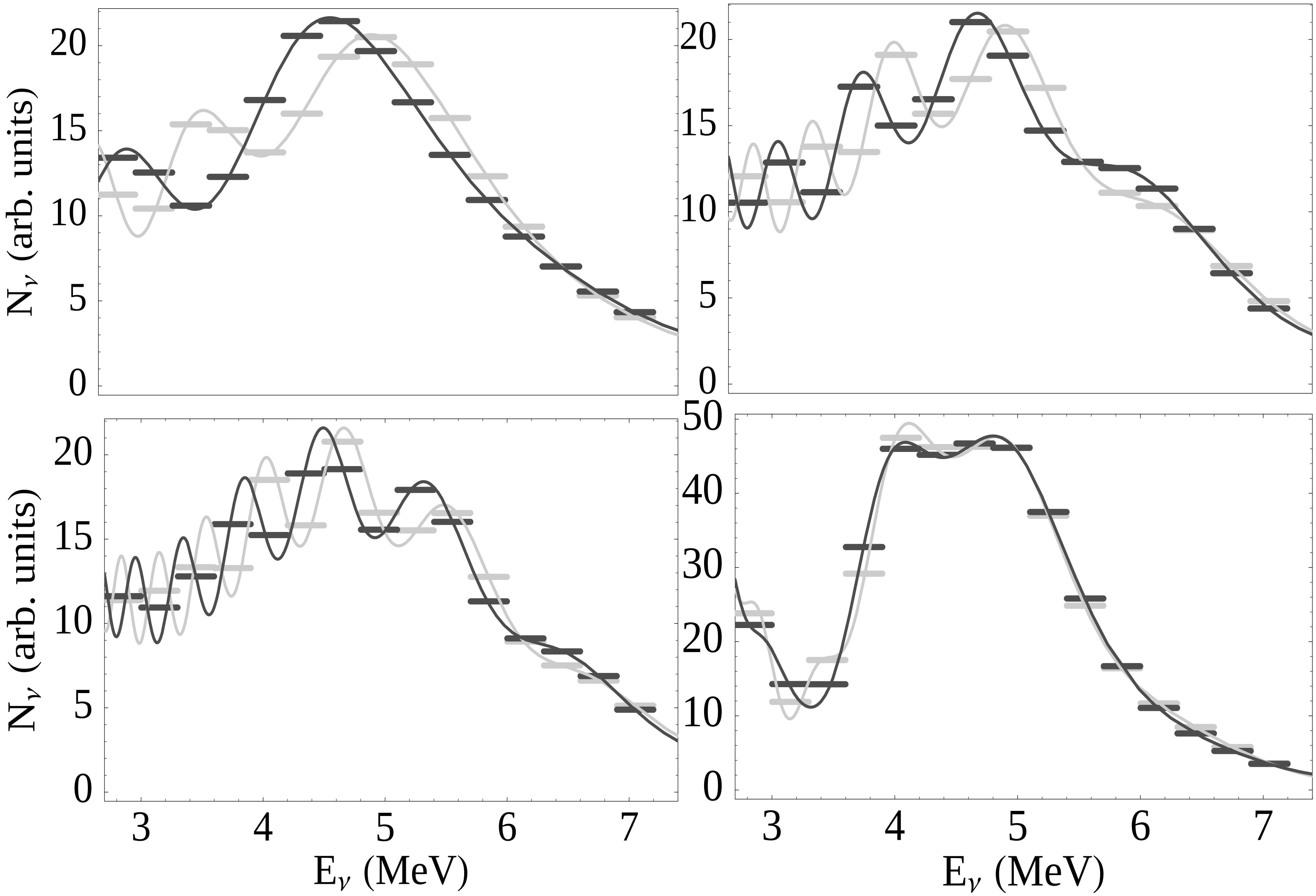


$$P_{ee}(L/E) = 1 - P_{21} - P_{31} - P_{32}$$
$$P_{21} = \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2(\Delta_{21})$$
$$P_{31} = \cos^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{31})$$
$$P_{32} = \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{32})$$

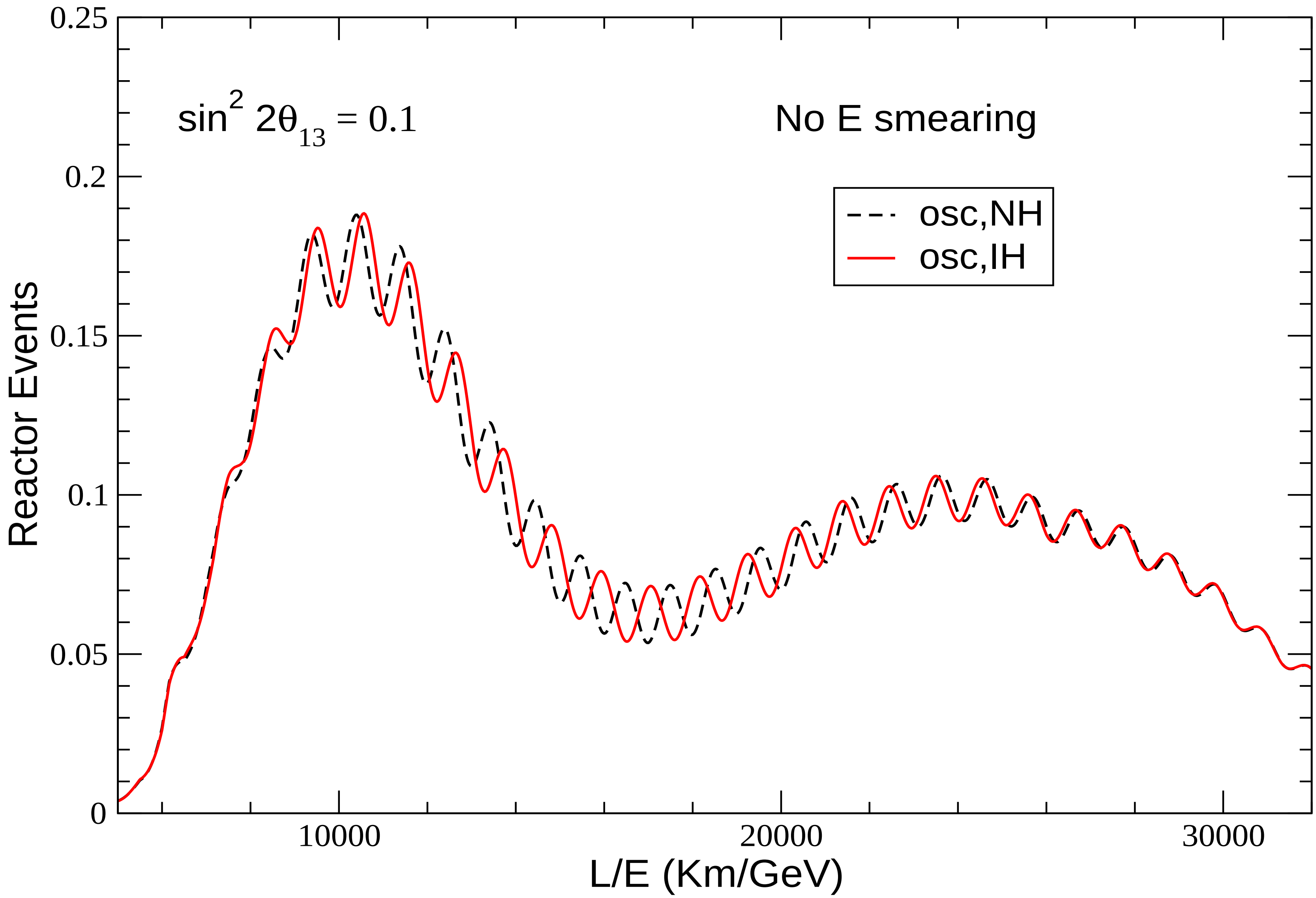
S. Petcov and Piai, Phys. Lett. B 553, 94-106(2002)
J. Learned et al., PRD 78(2008)071302
L. Zhan, YFW et al., PRD 78(2008)111103

5

Y. Wang, talk given at CERN on March 20, 2024



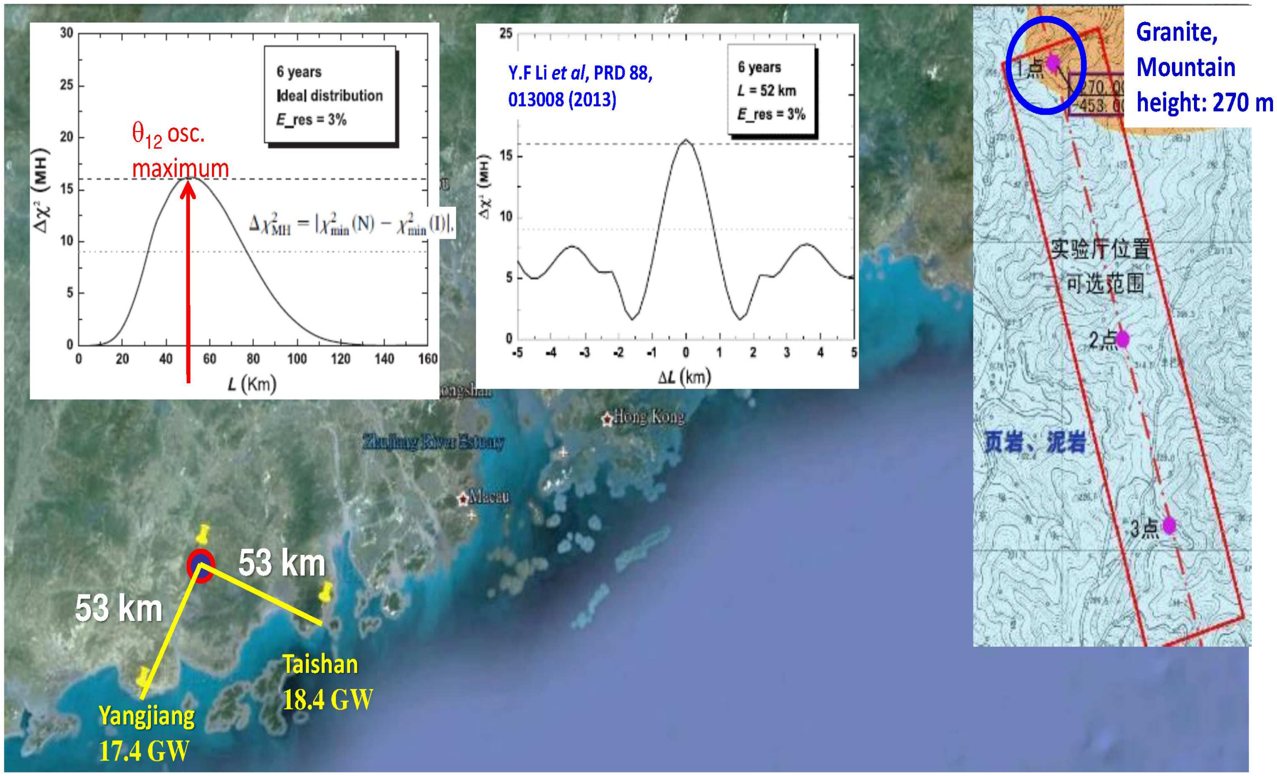
S.T.P., M. Piai, PLB 553 (2002) 94 (hep-ph/0112074)



P. Ghoshal, S.T.P., JHEP 03 (2011) 058 (arXiv:1011.1646)

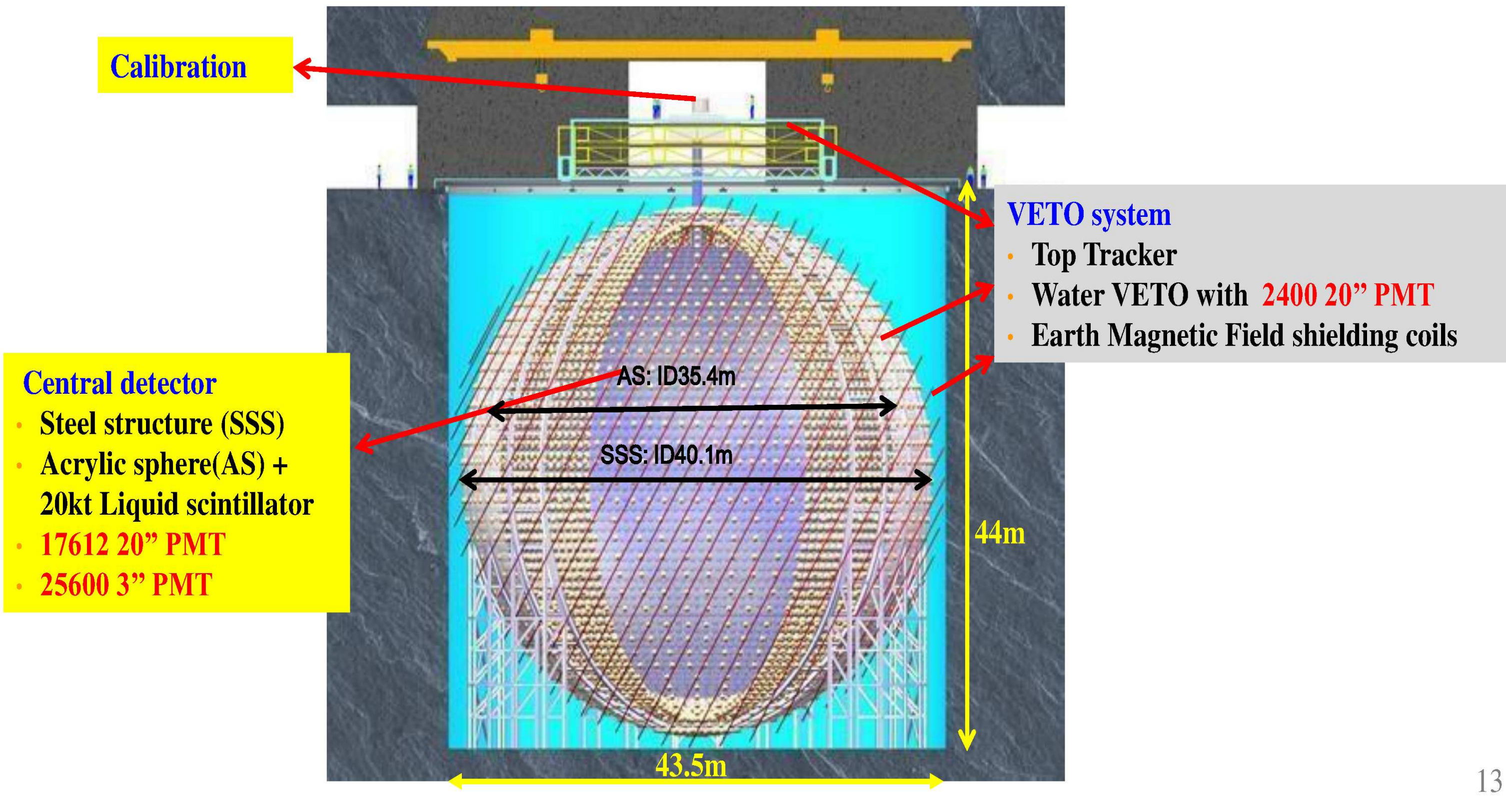
Optimum Baseline and Site Selection

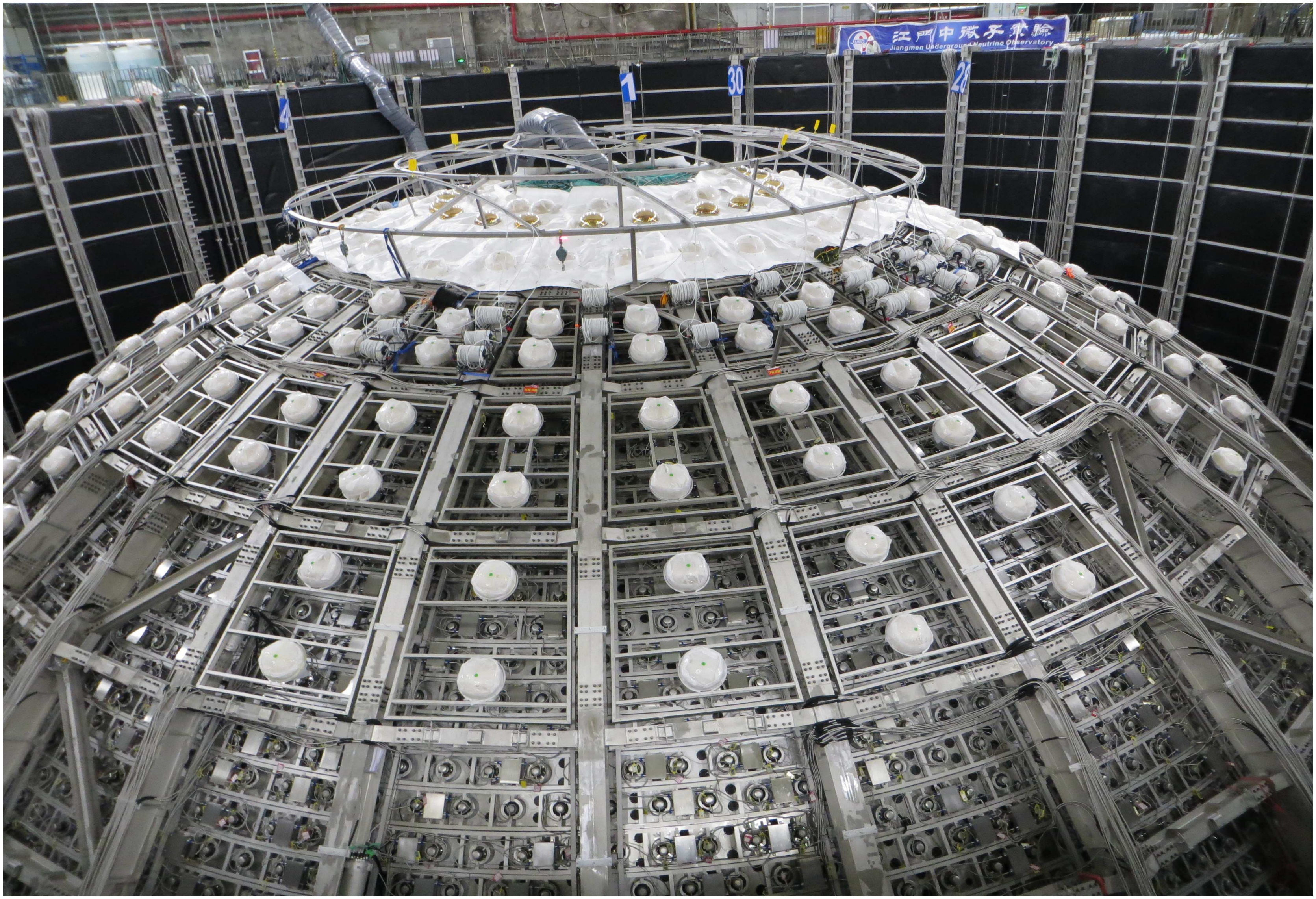
- Optimum sensitivity at the oscillation maximum of θ_{12}
- Multiple baseline reactors may wash out the oscillation structure
  - Baseline difference should be < 500 m



Concept of JUNO for Mass Ordering

- Two-layer structure for simplicity and cost: stainless steel frame + Acrylic tank
- Water as VETO and Buffer ➔ radiopurity control of water




JUNO
江門中微子實驗
Jiangmen Underground Neutrino Observatory


江門中微子實驗
Jiangmen Underground Neutrino Observatory
8
4
1
30
28

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'} \; , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'} \; , \qquad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

CP-invariance:

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P.,1980;
V. Barger, S. Pakvasa et al.,1980.

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \; , \quad l \neq l' = e, \mu, \tau$$

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l' : \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \; l \neq l'$$

3ν—mixing:

$$A^{(l,l')}_{\text{CP}} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \; , \quad l \neq l' = e, \mu, \tau$$

$$A^{(l,l')}_{\text{T}} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l), \; l \neq l'$$

$$A^{(e,\mu)}_{\text{T(CP)}} = A^{(\mu,\tau)}_{\text{T(CP)}} = -A^{(e,\tau)}_{\text{T(CP)}}$$

P.I. Krastev, S.T.P., 1988

In vacuum: $A^{(e,\mu)}_{\rm CP(T)} = 4\, J_{\rm CP} F^{vac}_{osc}$

$$J_{CP} = \text{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$$F^{vac}_{osc} = \sin(\frac{\Delta m^2_{21}}{2E} L) + \sin(\frac{\Delta m^2_{32}}{2E} L) + \sin(\frac{\Delta m^2_{13}}{2E} L)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

CP : $P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$

CPT : $P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$

P. Langacker et al., 1987

Can conserve the T-invariance (Earth)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l),\; l \neq l'$$

In matter with constant density: $\text{A}^{(\text{e},\mu)}_{\text{T}} = \text{J}^{\text{mat}}_{\text{CP}} \text{F}^{\text{mat}}_{\text{osc}}$

$$\text{J}^{\text{mat}}_{\text{CP}} = \text{J}^{\text{vac}}_{\text{CP}}\; \text{R}_{\text{CP}}$$

R_{CP} **does not depend on** θ_{23} **and** δ**;** $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \ll 1$ **and** $\sin^2\theta_{13} \ll 1$**:**

$$P^{3\nu\ man}_{m}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,,$$

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta),$$

$$P_{\sin\delta} = -\,\alpha\,\frac{8\,J_{CP}}{A(1-A)}(\sin\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}(\cos\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m^2_{31}\,L}{4E}\,,\;\; A = \sqrt{2}G_{\mathrm{F}}N^{man}_{e}\frac{2E}{\Delta m^2_{31}}.$$

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$**:** $\delta,\;\; A \rightarrow (-\delta),\;\; (-A)$

Conclusions

We are heading to a period in which some of the fundamental questions in neutrino physics, and more genereally, in particle and astroparticle physics - the status of CP symmetry in the lepton sector and its possible implications for the generation of BAU, the type of spectrum neutrino masses obey, the origin of the patterns of neutrino masses and mixing (symmetry?), the value of the absolute neutrino mass scale, and possibly the nature - Dirac or Majorana - of massive neutrinos, will be answered. This will have profound implications for particle and astroparticle physics, for astrophysics and cosmology.

The program of research in neutrino physics extends beyond 2035.

The future of neutrino physics is bright.