The standard model and neutrinos

Eligio Lisi
(INFN, Bari, Italy)

This 5-hour lecture course is intended for a broad audience of PhD students and postdocs working in different areas (theo/pheno/expt) of particle physics, astrophysics, cosmology.

The main goal is to “get you (more) interested” in ν oscillations, by moving from basic neutrino properties and phenomena to more advanced topics at the current frontier of the field (4 hours*).

Several exercises are also proposed, especially on ν oscillation probabilities (with worked-out solutions).

I have also been asked to start the lecture course with a general introduction (1 hour, this lecture)

(*) Lectures also given at the ISAPP-SIF 2023 Varenna School

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INFN Istituto Nazionale di Fisica Nucleare

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People interested in further reading can usefully browse the "Neutrino Unbound" website: www.nu.to.infn.it , or just email me for advice about specific topics: eligio.lisi@ba.infn.it

Outline: SM ν …

Lecture “0”
Standard model and neutrinos

Outline: SM ν ...

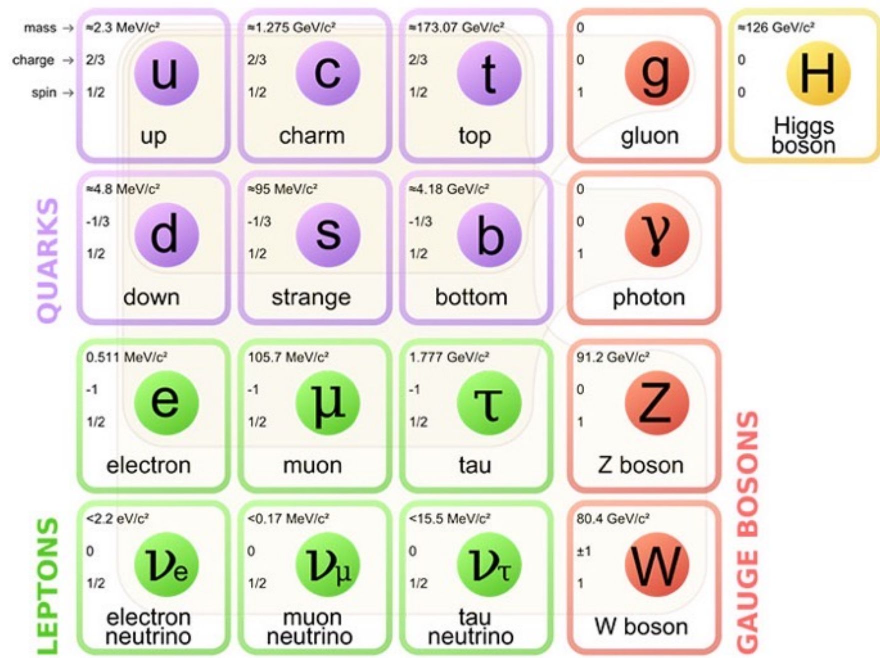
Lecture "0"
Standard model and neutrinos

... and ν oscillations

Lecture I
Pedagogical introduction + warm-up exercise
Lecture II
3ν osc. in vacuum and matter: notation and basic math
Lecture III
2ν approximations of phenomenological interest
Lecture IV
Back to 3ν oscillations: Status and Perspectives

Feel free to stop me and ask questions at any time!

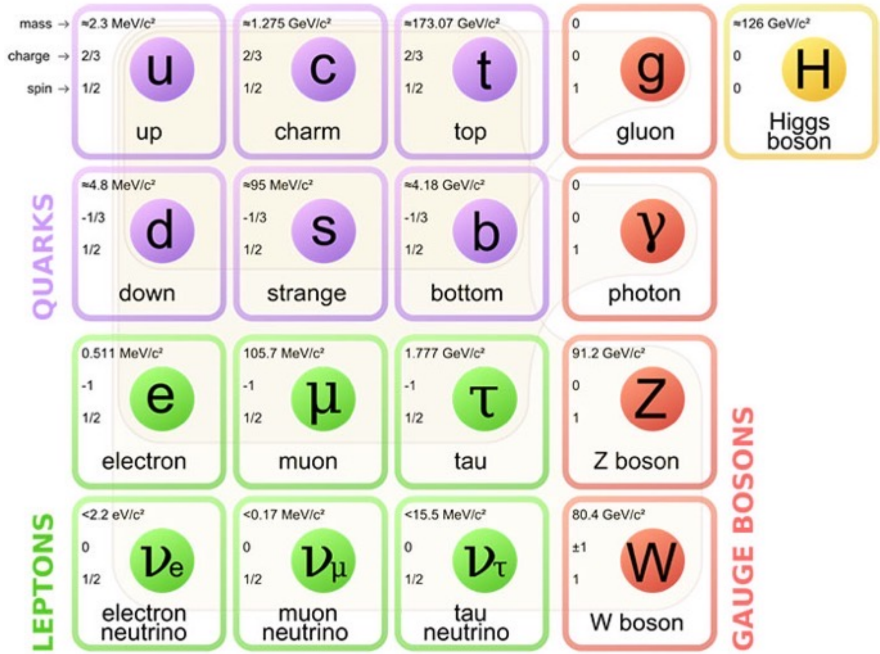
Standard Model and neutrinos: A tale about symmetry…



Standard Model	→	**Neutrinos**
Lorentz symmetry		Free fermion fields
Gauge symmetry		Interactions
Symmetry breaking		Masses and mixings

Image credit: Wikimedia

Standard Model and neutrinos: A tale about symmetry…



Standard Model → **Neutrinos**

Lorentz symmetry — **Free fermion fields**

Gauge symmetry — Interactions

Symmetry breaking — Masses and mixings

Lorentz symmetry and free fermion fields

Let us briefly (re)visit some consequences of Lorentz invariance (units c=1)

- 4-component vectors transform via rotation+boost
- 2-component spinors transform either as RH or as LH
- RH, LH 2-spinors form a 4-spinor obeying Dirac equation*
- Lorentz invariants involve Dirac spinor and its adjoint

**Exercise*

4-vector rotation x'=Λx with angle ω

- Around x axis:
  (c_ω=cosω, s_ω=sinω)

$$\begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_\omega & s_\omega \\ 0 & 0 & -s_\omega & c_\omega \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}$$

- Exponential form:
  (J = generator)

$$\Lambda = e^{i\omega J_1} \quad , \quad J_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & +i & 0 \end{bmatrix}$$

- Around generic axis:

$$\Lambda = e^{i\vec{\omega}\vec{J}} \quad , \quad \vec{J} = (J_1,\ J_2,\ J_3)$$

- Algebra (commutators):

$$[J_i,\ J_j] = i\epsilon_{ijk}J_k$$

2-spinor rotation $\xi' = \Lambda\xi$ with angle ω

The **<u>same</u> rotation algebra** applies to 2-component objects (spinors with "up/down" components) by replacing ***J*** → σ/2 (halved Pauli matrices):

- Same algebra:

$$\left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\epsilon_{ijk}\frac{\sigma_k}{2}$$

- Spinor rotations:

$$\Lambda = e^{i\vec{\omega}\frac{\vec{\sigma}}{2}} \quad , \quad \vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$$

- Rotation group named:

$$SU(2)$$

- Spin operator:

$$\vec{s} = \frac{1}{2}\vec{\sigma}$$

Note: rotation algebra involves 3 generators (J or σ). What about boosts?

4-vector boost x'=Λx with velocity v (and rapidity u), c=1

- Velocity along x-axis
  [β=v, $\gamma^2=1/(1-\beta^2)$]

$$\begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}$$

- Exponential form:
  "rapidity" u=tgh(v)

$$\Lambda = e^{iuK_1} \quad , \quad K_1 = \begin{bmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Generic boost axis:

$$\Lambda = e^{i\vec{u}\vec{K}} \quad , \quad \vec{K} = (K_1,\, K_2,\, K_3)$$

- Rotation + boost:
  (general Lorentz transform.)

$$\Lambda = e^{i\vec{\omega}\vec{J}+i\vec{u}\vec{K}}$$

- Algebra:

$$[K_i,\, K_j] = -i\epsilon_{ijk}J_k$$
$$[J_i,\, K_j] = i\epsilon_{ijk}K_k$$

2-spinor rotation + boost: A surprise and a question…

Surprise: the **<u>same</u> boost algebra** applies to spinors, but **in two options…**
…obtained by replacing ***K*** → ∓ iσ/2 (- right-handed RH, + left-handed LH)

$$\xi_R' = e^{i\vec{\omega}\frac{\vec{\sigma}}{2}+\vec{u}\frac{\vec{\sigma}}{2}}\xi_R$$

$$\xi_L' = e^{i\vec{\omega}\frac{\vec{\sigma}}{2}-\vec{u}\frac{\vec{\sigma}}{2}}\xi_L$$

rotation boost

2-spinor rotation + boost: A surprise and a question…

Surprise: the **<u>same</u> boost algebra** applies to spinors, but **in two options…**
…obtained by replacing ***K*** → ∓ iσ/2 (- right-handed RH, + left-handed LH)

$$\xi'_R = e^{i\vec{\omega}\frac{\vec{\sigma}}{2}+\vec{u}\frac{\vec{\sigma}}{2}}\xi_R$$

$$\xi'_L = e^{i\vec{\omega}\frac{\vec{\sigma}}{2}-\vec{u}\frac{\vec{\sigma}}{2}}\xi_L$$

Consider a **massive spinor ξ at rest (v=0)** → undefined handedness.
Now, boost to a new reference frame with **v>0**.

Question: When seen in motion, would ξ be RH or LH?

Answer: **We don't know, so we must admit both cases…**

…but not as a "linear combination" of RH+LH (not Lorentz invariant)

- Build a 4-dof object with both RH, LH: $$\psi = \begin{pmatrix} \xi_R \\ \xi_L \end{pmatrix}$$

- Get $\xi_{R,L}$ by boosting rest-frame ξ $$\xi_R = e^{+\vec{u}\frac{\vec{\sigma}}{2}}\xi$$ $$\xi_L = e^{-\vec{u}\frac{\vec{\sigma}}{2}}\xi$$

- Eliminate ξ and get Dirac equation (eq. of a free fermion field)

$$\begin{bmatrix} -m & E+\vec{p}\vec{\sigma} \\ E-\vec{p}\vec{\sigma} & -m \end{bmatrix} \begin{bmatrix} \xi_R \\ \xi_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

… where p=momentum, E=energy (See Exercise at the end).

For nonzero fermion mass **m**, the RH and LH components are **coupled**!

Helicity, chirality, notation

- Helicity:
  (spin along momentum)

$$h = \frac{\vec{p}\,\vec{\sigma}}{E} \rightarrow \begin{bmatrix} -m/E & 1+h \\ 1-h & -m/E \end{bmatrix} \begin{bmatrix} \xi_R \\ \xi_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- Limit m/E→0:

$$h\,\xi_{R,L} = \pm\xi_{R,L} + O(m/E)$$

helicity ~ chirality, but only in this limit!

Helicity, chirality, notation

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- **Gamma matrices:** (in "Weyl basis")

$$\gamma^\mu = (\gamma^0, \vec{\gamma}) \;,\; \gamma^0 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \;,\; \vec{\gamma} = \begin{bmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{bmatrix}$$

- **Momentum + QM:**

$$p_\mu = (E, -\vec{p}) \;\rightarrow\; i\partial_\mu$$

- **Get "usual" form**

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

Ending on notation…

- Chiral components: $$\psi_{R,L} = \frac{1 \pm \gamma^5}{2}\psi$$ $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$

- Adjoint Dirac spinor: $$\overline{\psi} = \psi^\dagger \gamma^0$$ *subject to inverse Lorentz transform…*

$$\overline{\psi}(\ldots)\psi$$ ← *…providing bilinear invariants*

$$\overline{\psi}\psi \quad \overline{\psi}\gamma^\mu\psi \quad \overline{\psi}\gamma^5\gamma^\mu\psi \quad \ldots$$

Scalar *vector V* *axial vector A*

- Conjugate spinor: $$C(\psi) = \psi^c = i\gamma^2\psi^*$$ ← *"antiparticle"*

Physical content: The four components of a Dirac field can be interpreted as LH and RH states of a ½-spin particle + antiparticle. Can build Lorentz-invariant quantities from field + adjoint field *(proofs omitted).*

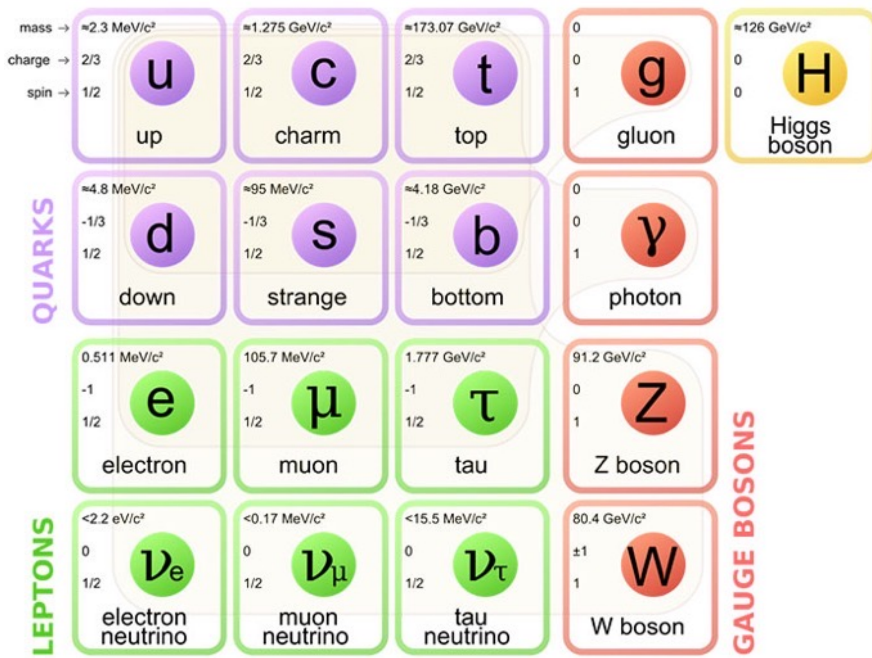
Ending on notation…

Of particular importance is the Lorentz scalar invariant that describes the fermion mass term in a lagrangian approach:

$$m\overline{\psi}\psi = m(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L)$$

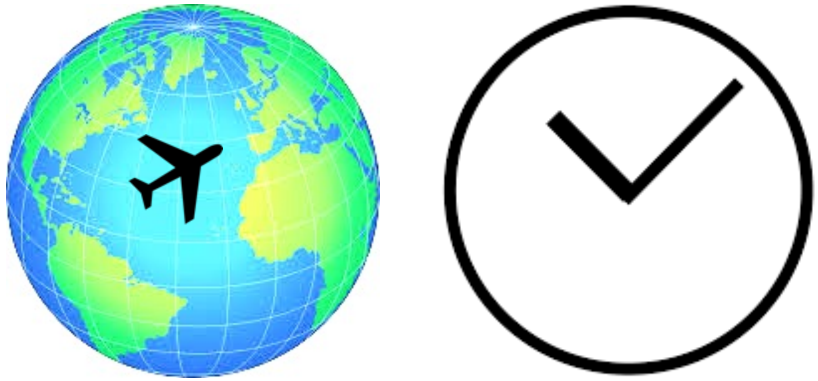
Not surprisingly: mass term couples LH and RH components.

Standard Model and neutrinos: A tale about symmetry…

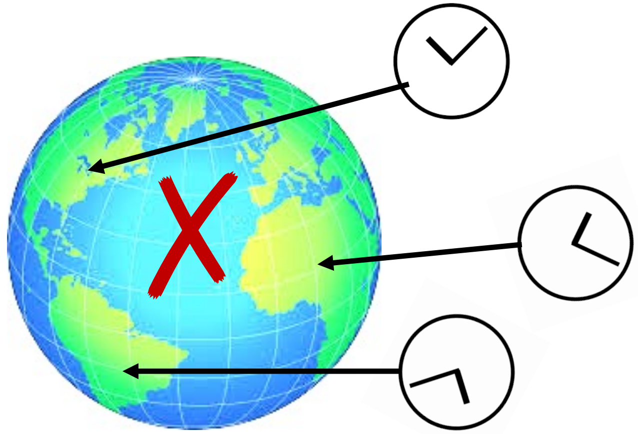


Standard Model	→	Neutrinos
Lorentz symmetry		Free fermion fields
Gauge symmetry		**Interactions (EW)**
Symmetry breaking		Masses and mixings

From space-time to gauge (internal) symmetries

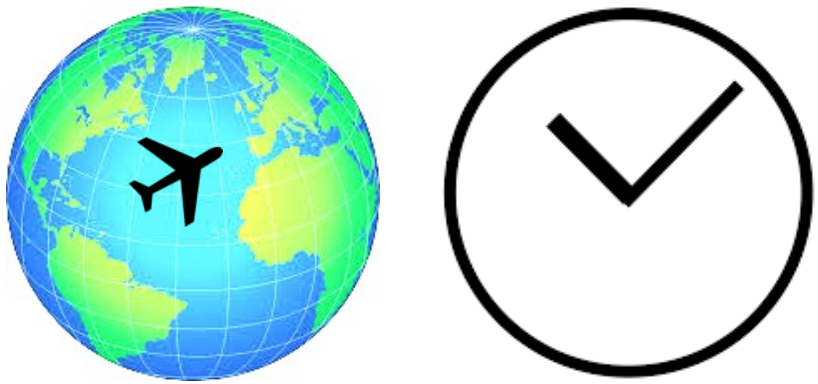


Time zones are conventional.
E.g., airline traffic invariant under under global shift of clock “phase” everywhere
→ **U(1) global invariance**



But if you ask the freedom to change any clock phase locally, airline traffic would be disrupted…

From space-time to gauge (internal) symmetries



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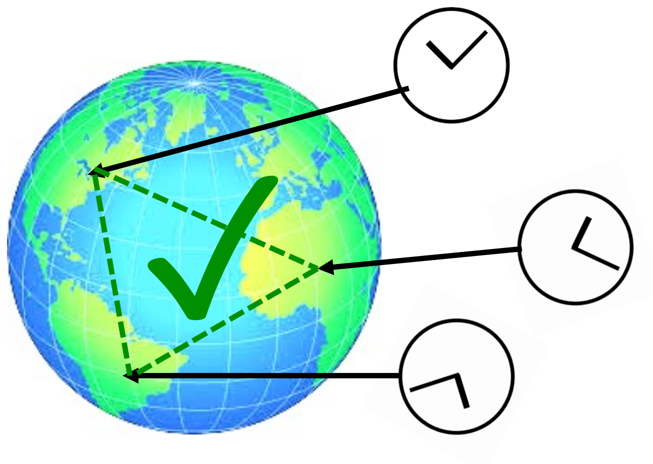


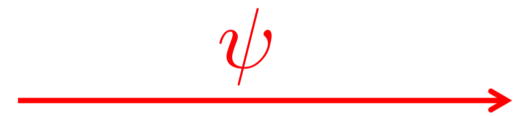
But if you ask the freedom to change any clock phase locally, airline traffic would be disrupted…

…unless change is communicated everywhere at maximal speed!
→ **Need a carrier of information to ensure U(1) local invariance**

~100 y ago: Freedom for local phase of ψ leads to photon (carrier) & QED:

Physics of free-fermion equation invariant under global $\psi \to e^{iq\alpha}\psi$

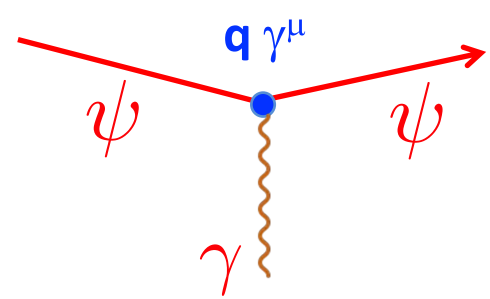
$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$



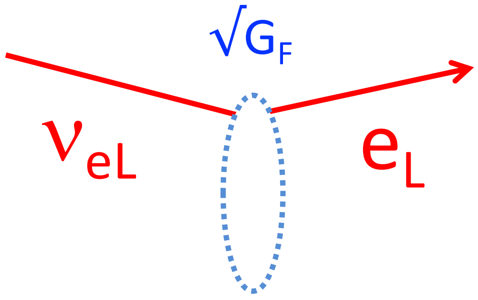
To ensure local invariance $\alpha \to \alpha(x)$, need interaction with massless vector field A_μ + gauge transform.
→ Interaction with photon γ via electr. charge q

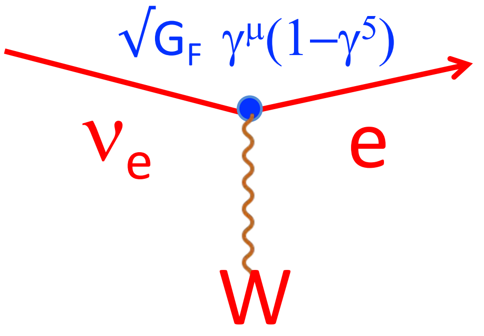
$$\psi \to e^{iq\alpha(x)}\psi$$
$$\partial_\mu \to \partial_\mu + iqA_\mu$$
$$A_\mu \to A_\mu + \partial_\mu \alpha(x)$$

$$(i\gamma^\mu(\partial_\mu + iqA_\mu) - m)\,\psi = 0$$



From QED to weak charged-current (CC) interactions with V-A (LH) structure





Idea: Maybe charged bosons $W^{\pm}$ act on LH fermion pairs (doublets)…

… and the $\nu_e \rightarrow e$ "change" stems from some gauge transformation?

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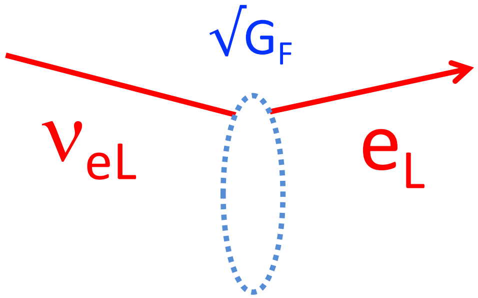
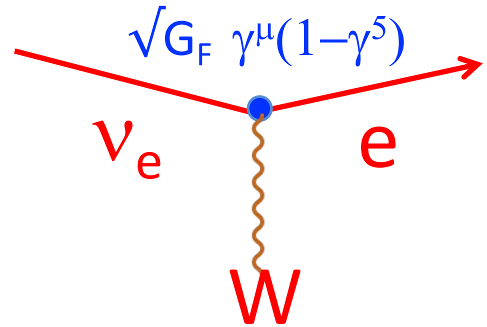
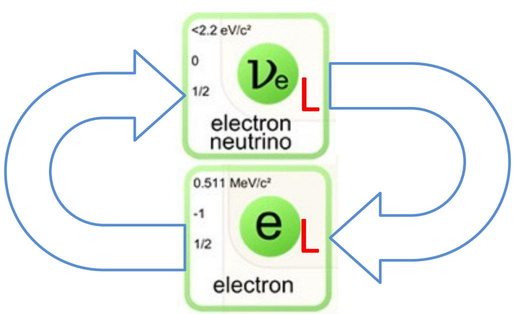




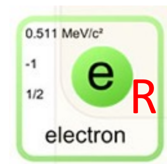
Idea: Maybe charged bosons $W^{\pm}$ act on LH fermion pairs (doublets)…
… and the $\nu_e \rightarrow e$ "change" stems from some gauge transformation?

A possibility was to think about SU(2) local rotations, not in real space, but in the "abstract" space of fermion LH doublets:

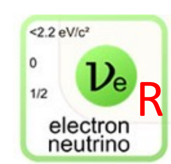


LH doublet "rotation"

$SU(2)_L$



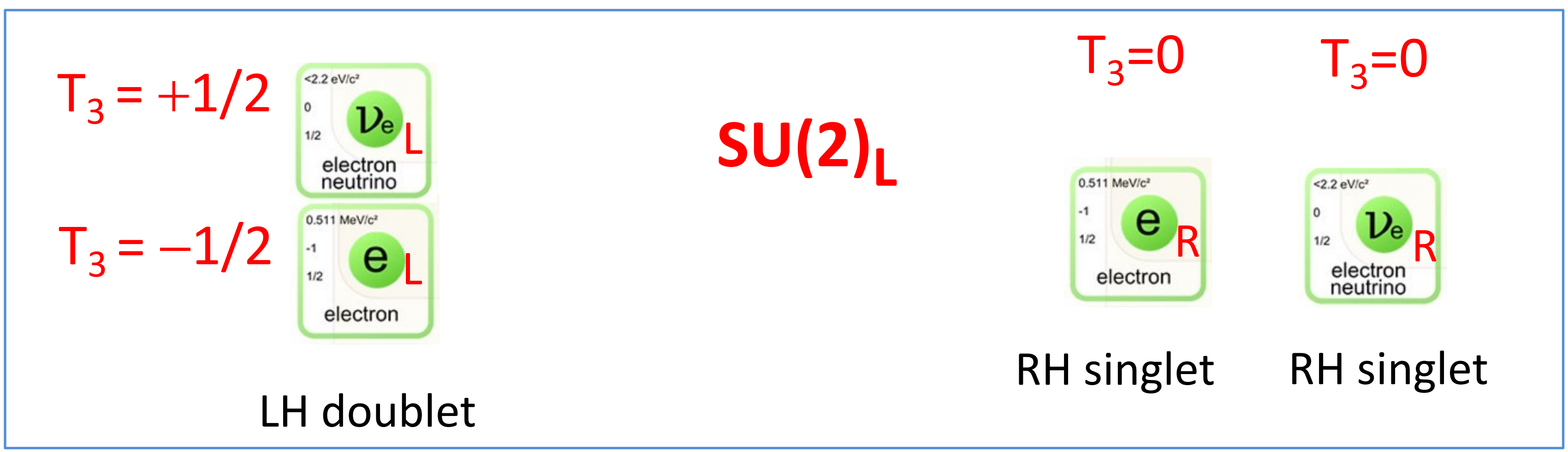
RH singlet



RH singlet
(absent if m_ν=0)

Call $SU(2)_L$ generators T_i ("weak isospin") to avoid confusion with $\sigma_i/2$ ("spin")

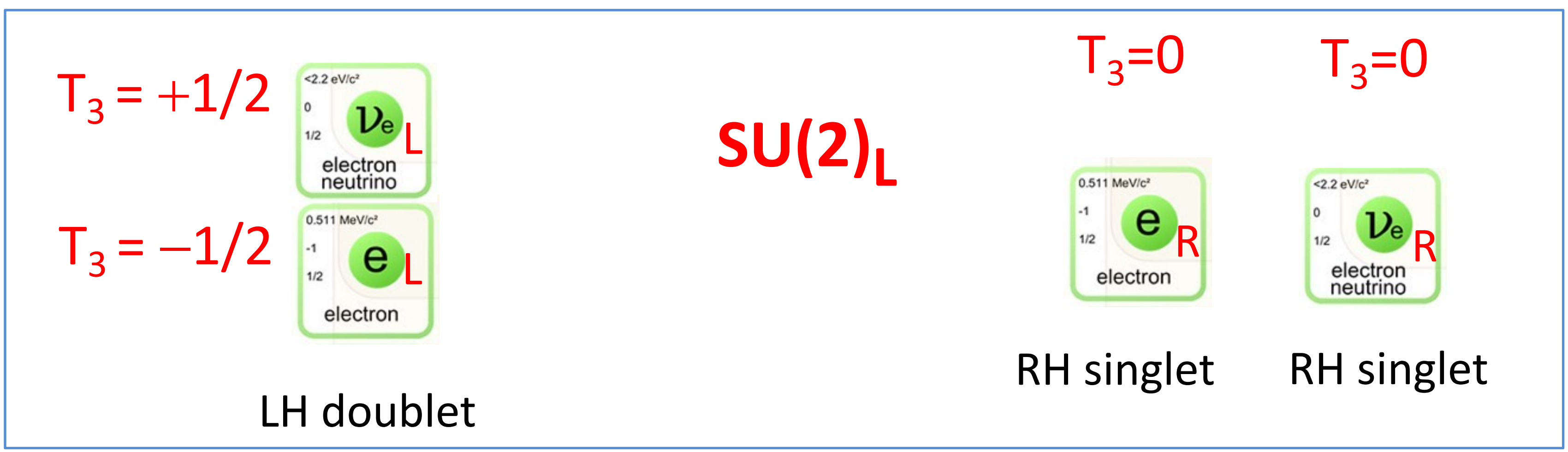
Q. Can the three bosons A_μ, $W_\mu^\pm$ be associated with the three generators T_i?



Call $SU(2)_L$ generators T_i ("weak isospin") to avoid confusion with $\sigma_i/2$ ("spin")

Q. Can the three bosons A_μ, $W_\mu^\pm$ be associated with the three generators T_i?

A. No: cannot get electric charge operator Q as linear combination of T_i



Extension: add “weak hypercharge” $Y/2 = Q - T_3 \rightarrow$ **$SU(2)_L \otimes U(1)_Y$**

$T_3 = +1/2$
$Y = -1$

$T_3 = -1/2$
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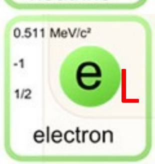
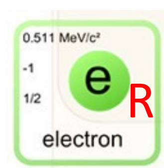


LH doublet

$SU(2)_L \otimes U(1)_Y$

$T_3=0$
$Y=-2$



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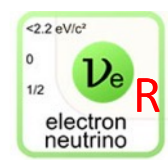

RH singlet

Extension: add "weak hypercharge" $Y/2 = Q-T_3 \rightarrow$ **$SU(2)_L \otimes U(1)_Y$**

Gauge transformation involves, for SU(2) and U(1):

- two couplings *g* and *g'*
- four vector boson carriers (A^1, A^2, A^3) and B

Dirac equation modified by **interactions with boson carriers:**

$$\partial_\mu \rightarrow \partial_\mu - ig\vec{T}\vec{A}_\mu - ig'\frac{Y}{2}B_\mu$$

where only the latter term matters for RH singlets

$T_3 = +1/2$
$Y = -1$

$T_3 = -1/2$
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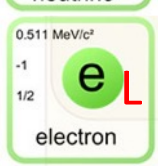

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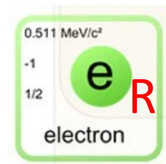

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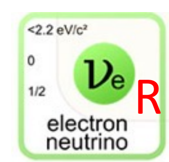

RH singlet

Extension: add "weak hypercharge" $Y/2 = Q - T_3$ → $SU(2)_L \otimes U(1)_Y$

Linear combinations of the four carriers can be associated to:

- photon (EM interactions)
- charged $W^{\pm}$ (CC interactions)
- neutral Z (NC interactions) ← *bonus: A triumph of SM construction!*

$$\partial_\mu \to \partial_\mu - ig\vec{T}\vec{A}_\mu - ig'\frac{Y}{2}B_\mu$$

$T_3 = +1/2$
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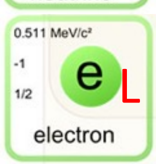

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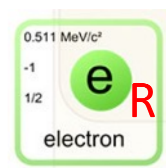

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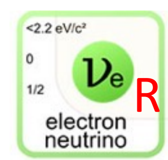
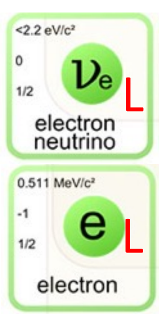


RH singlet

Problem: all these fields (fermions, vector bosons) are **massless.**

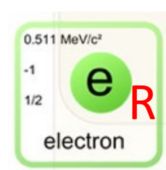
Masses prohibited by (unbroken) gauge invariance.

Vector bosons: all propagate with v=c, but you want this only for the photon…
Fermions: cannot get an invariant mass term out of LH doublet & RH singlet…



LH doublet

$$SU(2)_L \otimes U(1)_Y$$



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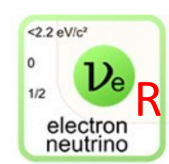
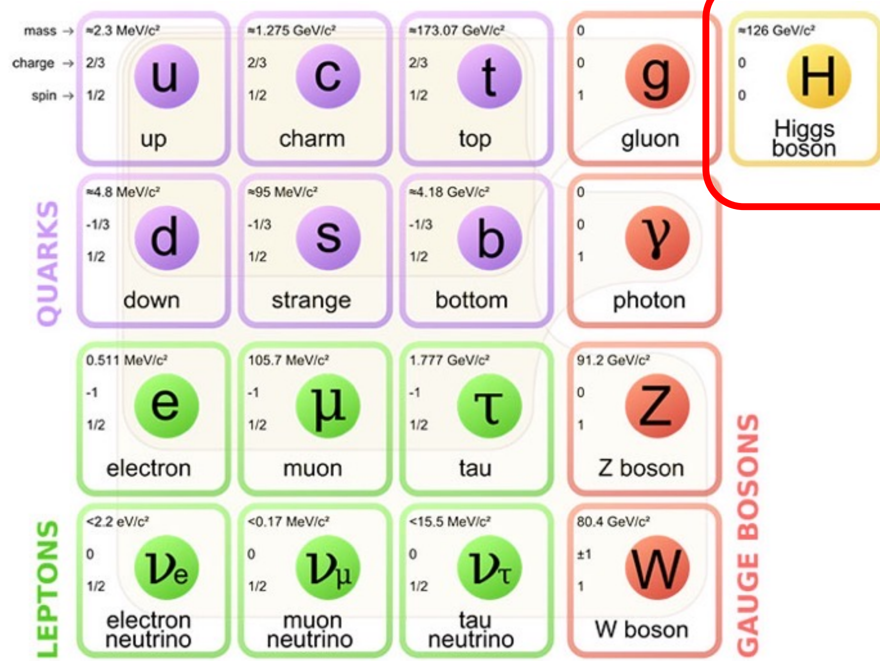


RH singlet

Standard Model and neutrinos: A tale about symmetry…



Higgs and SSB

Standard Model	→	**Neutrinos**
Lorentz symmetry		Free fermion fields
Gauge symmetry		Interactions (EW)
Symmetry breaking		**Masses and mixings**

CERN mug with SM lagrangian

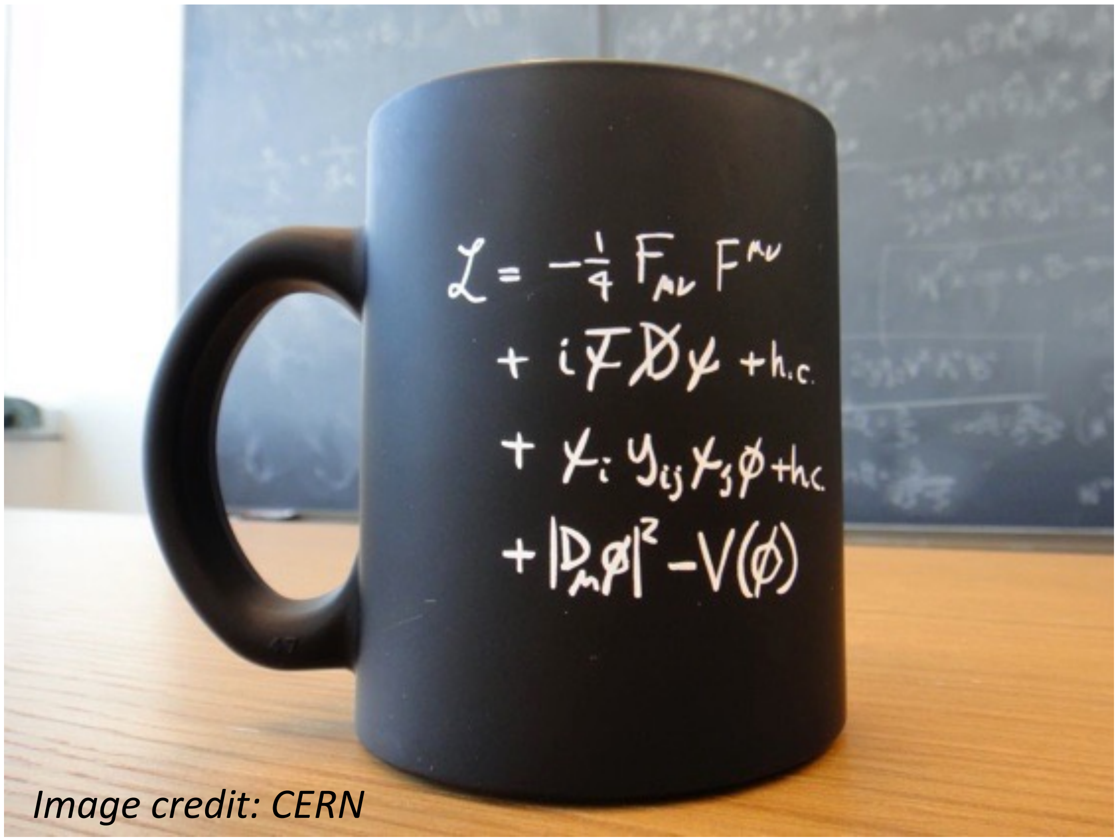
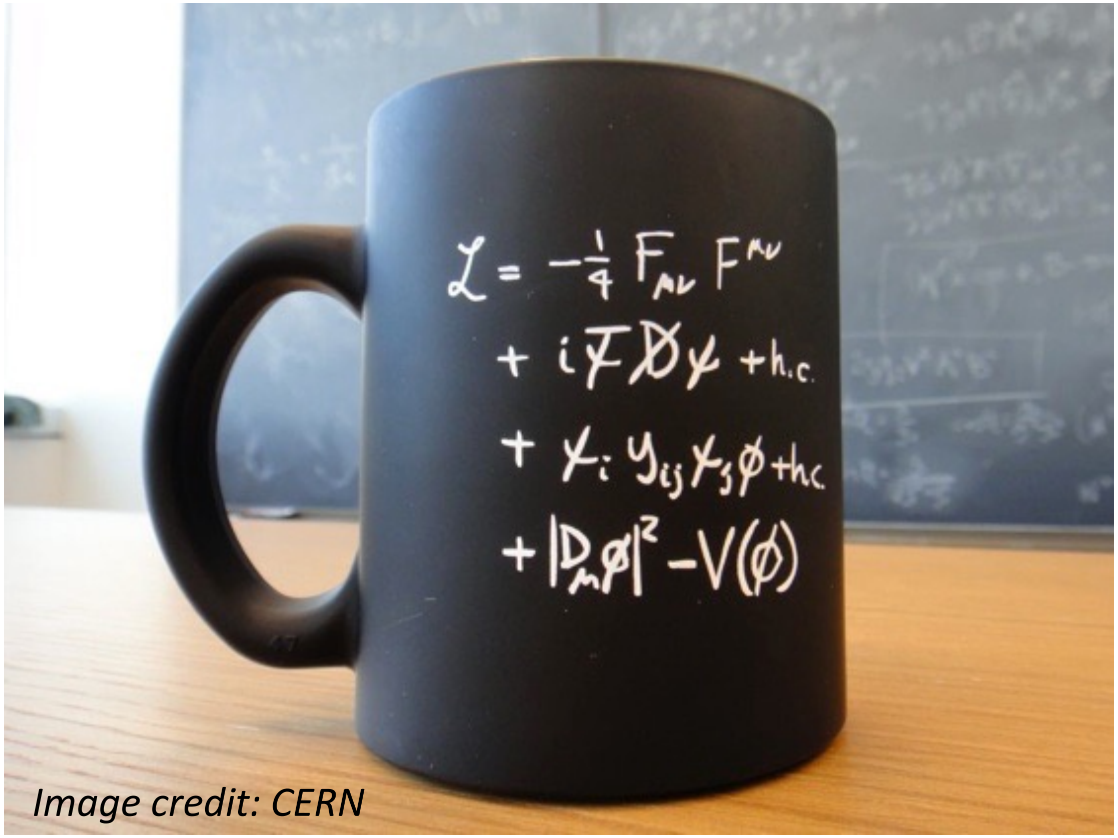


Image credit: CERN

So far, two lines:

← vector bosons and…
← …fermion interactions

CERN mug with SM lagrangian



Now, Higgs Φ and more lines:

← vector bosons and…
← …fermion interactions

← Higgs & fermions
← Higgs & vector bosons
(+ Higgs self-interaction potential V)

To make short a long story… $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM}$

Introduce a doublet of complex scalar fields, both charged and neutral → Higgs Φ (4 dof)

$$\phi = \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix}$$

Potential $V(\Phi)$ minimized for an neutral ground state with $U(1)_{EM}$ symmetry → physical H (1 dof)

$$\phi \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ H \end{pmatrix}$$

v.e.v. Higgs

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v.e.v. Higgs

In ground state:

The photon remains massless.

The extra 3 dof give mass to Z, $W^{\pm}$

The Higgs also gets mass.

$$m_Z = \frac{v}{2}\sqrt{g^2 + g'^2}$$

$$m_W = \frac{v}{2} g$$

Useful bookkeeping parameter:

$$cos\theta_W = m_W / m_Z$$

Three measurable quantities fix three gauge parameters (tree level):

Electric charge*	**e**	$\simeq$ 0.3	=	$gg'/\sqrt{(g^2+g'^2)}$
Fermi constant	$\mathbf{G_F}$	$\simeq$ 1.17 x 10^{-5} GeV $^{-2}$	=	$1/(\sqrt{2}v^2)$
Z mass	$\mathbf{m_Z}$	$\simeq$ 91 GeV	=	$\frac{v}{2}\sqrt{g^2+g'^2}$

**Fine structure constant in natural units:* $\alpha = e^2/4\pi \simeq 1/137$

Fix: **v** $\simeq$ 246 GeV, **g** $\simeq$ 0.64, **g'** $\simeq$ 0.34

Also: $\mathbf{m_W} \simeq$ 80 GeV $\mathbf{\sin^2\theta_W} \simeq$ 0.22

What about fermion masses?

Remind…

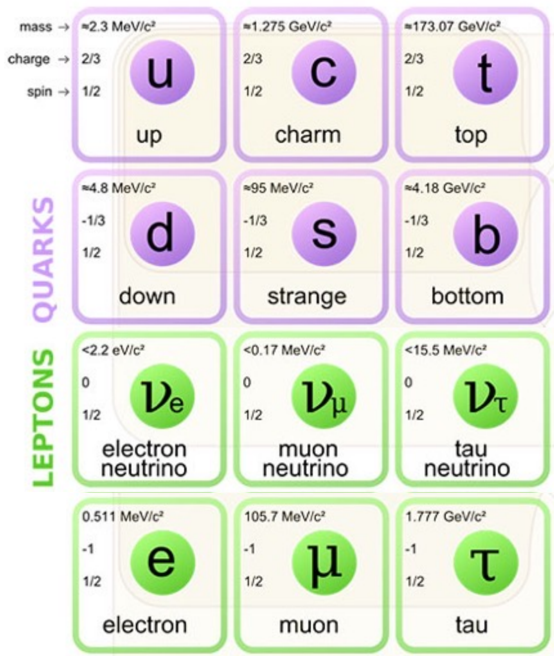
Fermion doublets (LH): *(three generations, four "types")*

$Q = +2/3$

$Q = -1/3$

$Q = 0$

$Q = -1$



$T_3 = +1/2$ "up" quarks

$T_3 = -1/2$ "down" quarks

$T_3 = +1/2$ "up" leptons

$T_3 = -1/2$ "down" leptons

Fermion singlets (LH): $T_3 = 0$

… fermion mass terms must couple LH and RH states

To make short another long story… $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM}$

Couple LH doublets with RH singlets via Higgs doublet → Four 3x3 matrices of arbitrary Yukawa couplings y_{ij} (one matrix for each fermion type)

$$y_{ij} \begin{bmatrix} \bullet & \bullet \end{bmatrix} \begin{bmatrix} \bullet \\ \bullet \end{bmatrix} \begin{bmatrix} \bullet \end{bmatrix}$$

$i,j = 1,2,3$ L_i Higgs R_j

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$$y_{ij} \; \begin{pmatrix} \bullet & \bullet \end{pmatrix} \begin{pmatrix} \bullet \\ \bullet \end{pmatrix} \begin{pmatrix} \bullet \end{pmatrix}$$

$i,j = 1,2,3$ $\quad L_i$ $\quad$ Higgs $\quad R_j$

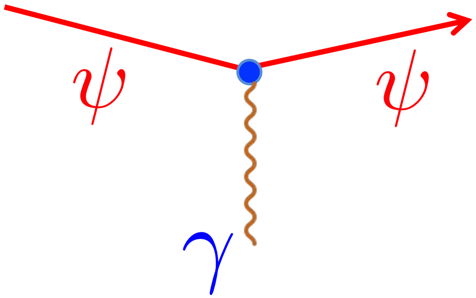
After symmetry breaking (ground state), get four 3x3 fermion mass matrices ∝ v.e.v.

$$M_{ij} = y_{ij}\frac{v}{\sqrt{2}}$$

Diagonalize and rewrite interactions in terms of fields with definite mass: γ, W, Z and massive fermions

Results →

EM

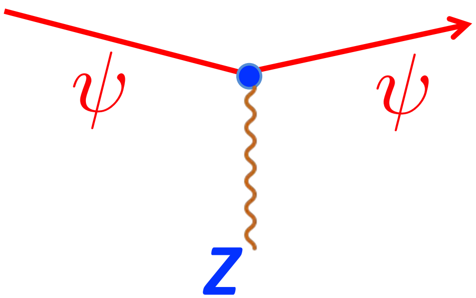


Photon couples to each fermion (LH, RH) via

Q

(Q=0 only for neutrinos)

NC



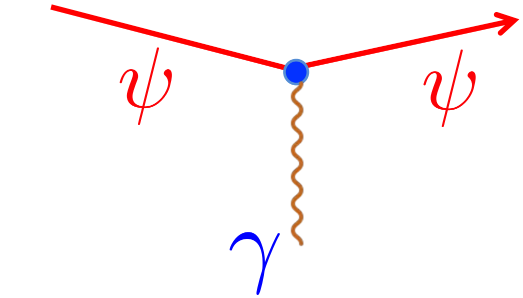
Z boson couples to each fermion (LH, RH) via

$\mathbf{T_3 - Q \sin^2\theta_w}$

(T_3=0 for RH fermions)

Note: No flavor-changing neutral currents (FCNC)

EM



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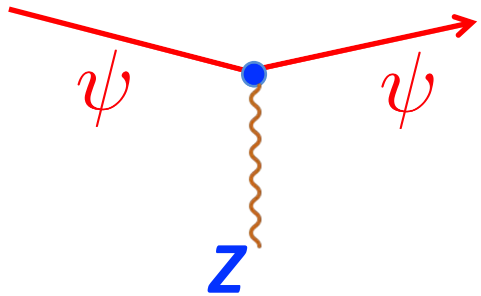

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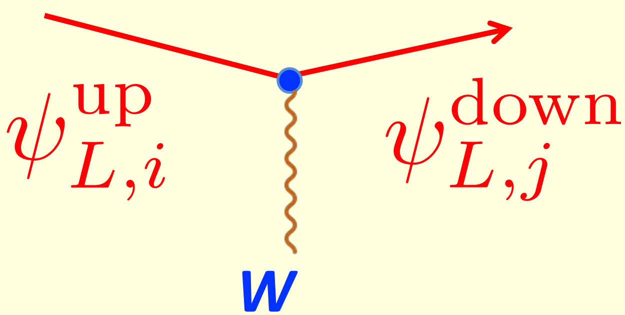


W boson **flips up/down LH** fermions via

$\mathbf{V_{ij}}$

(Mixing matrices of quarks and leptons)

Quarks: $\mathbf{V = V_{CKM}}$ Cabibbo-Kobayashi-Maskawa

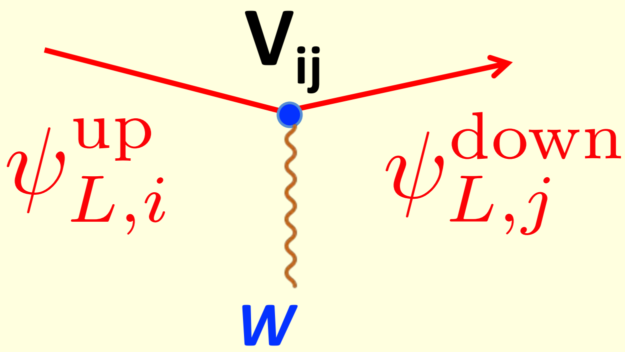
Leptons: $\mathbf{V = U_{PMNS}}$ Pontecorvo-Maki-Nakagawa-Sakata

Unitary matrix V appears in the “up-down” CC vertex:

not a property of “down” or “up” fermions separately

(although it may be conventionally used in this way)

CC



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(Mixing matrices of quarks and leptons)

If leptons and quarks get masses and mixing in similar ways…

…what's so special about neutrino masses in the SM (and beyond) ?

- Historically, in the original SM constructions, RH neutrino fields were omitted → no neutrino masses, no lepton mixing (a prejudice!)

- To get neutrino masses <0.1 eV from v~O(100) GeV, their Yukawa couplings must be $<10^{-12}$ → (more) unnatural wrt other fermions.

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- More fundamentally, neutrinos are the only fermions that may admit another mass term, not violating gauge symmetry and not originated by the Higgs doublet via symmetry breaking … if nu's are Majorana:

$\nu = \overline{\nu}$ or, more precisely: $\psi = \psi^c$ (up to a phase)

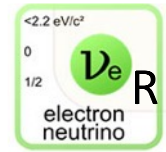
No charge of any kind!
Only 2 dof, not 4 as for Dirac

(You will see much more in other courses at this School!)

In particular, a RH ν has no SM charges and is decoupled from W, Z, γ (“sterile”), unlike LH ν coupled to W, Z (“active”)

$T_3=0$

$Y=0$



$$\psi_R$$

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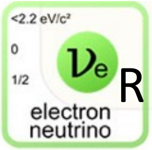

$$\psi_R$$

(1) May build a Majorana neutrino out of it + extra mass term, not coming from Higgs:

Not forbidden by SM symmetries, and fundamentally different from other fermions!

$$\psi = \psi_R + \psi_R^c$$

LH

$$m_R \overline{\psi}\psi =$$

$$m_R(\overline{\psi}_R \psi_R^c + \overline{\psi}_R^c \psi_R)$$

New mass scale(s) m_R?

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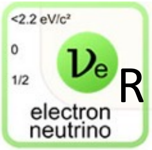

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← *LH*

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New mass scale(s) m_R?

(2) Also notice: number of sterile ν states is unrestricted, might be > 3. Most general Majorana ν mass matrix leads to new states +"active-sterile" mixing after diagonalization

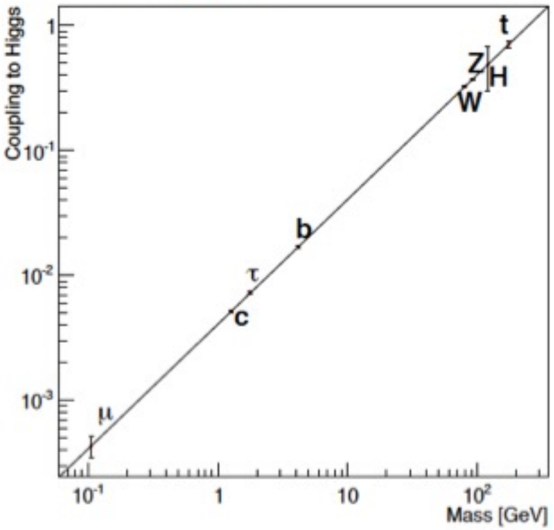
3 active + N_S sterile ?

Worldwide effort to test if some options are realized in nature…

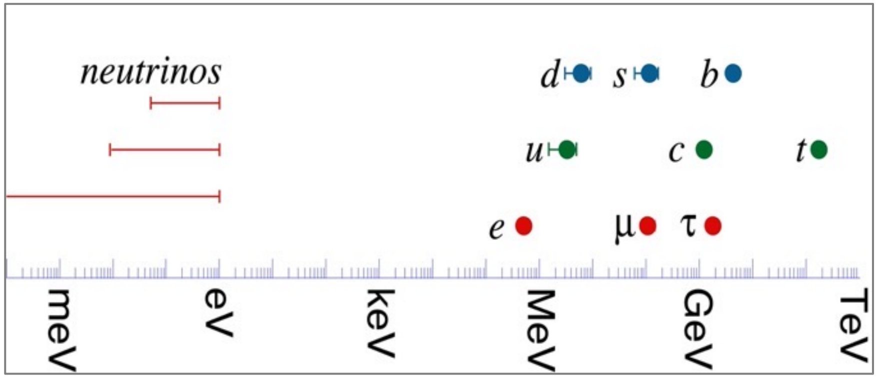
We are still at beginning of our understanding of ν masses!

Neutrino mass and nature in a wider context: Linking two research programs

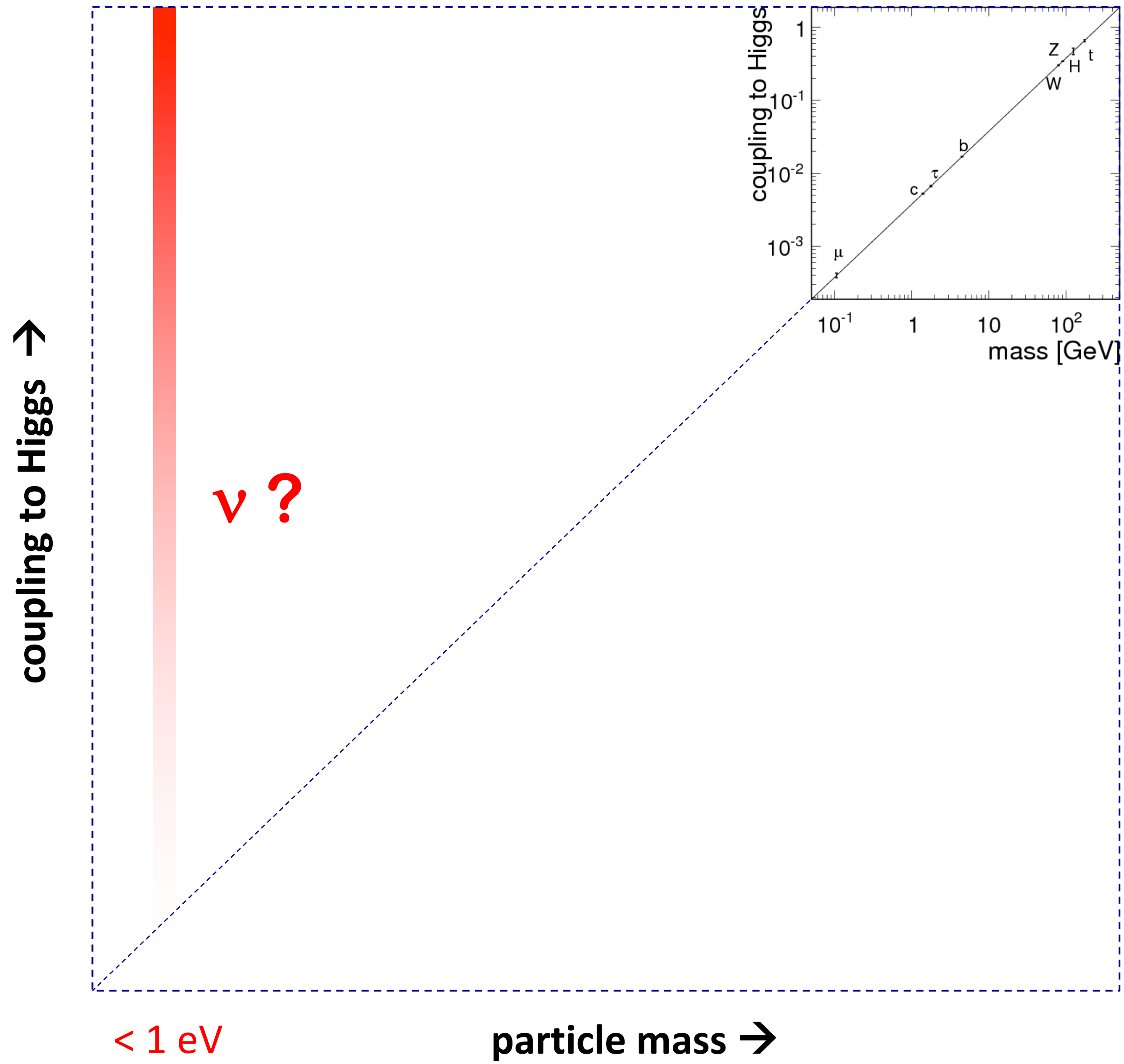
1. Test Higgs sector



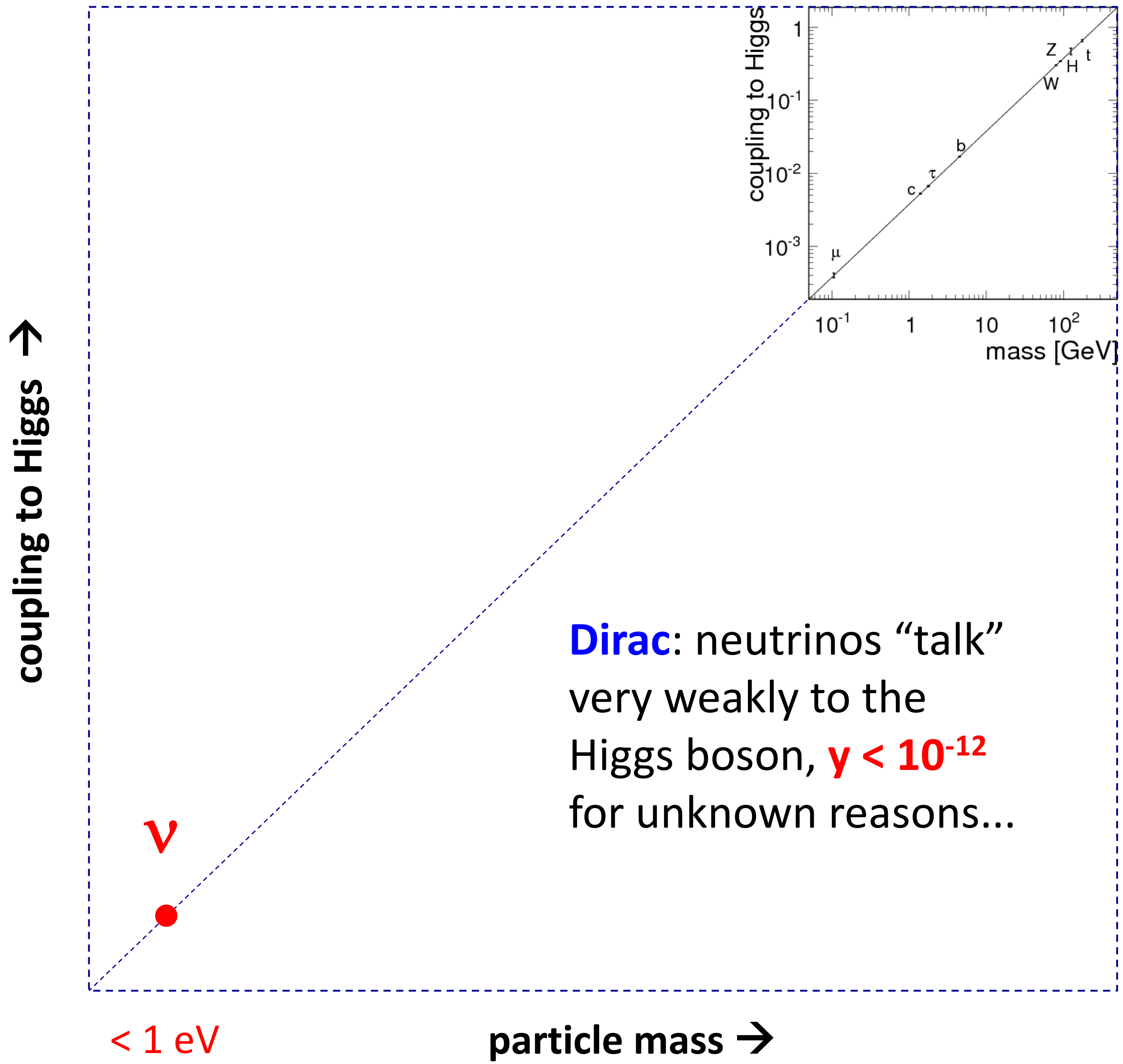
2. Probe ν masses



1 + 2 Where are the ν's on this plot? Why are they so light?



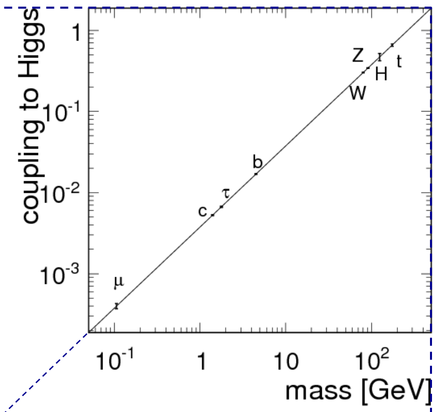
Options:



Dirac: neutrinos "talk" very weakly to the Higgs boson, **$y < 10^{-12}$** for unknown reasons...

Options:

ν



coupling to Higgs →

Majorana: neutrinos talk "normally" to the Higgs, but also to other (much) **higher scale(s) M**

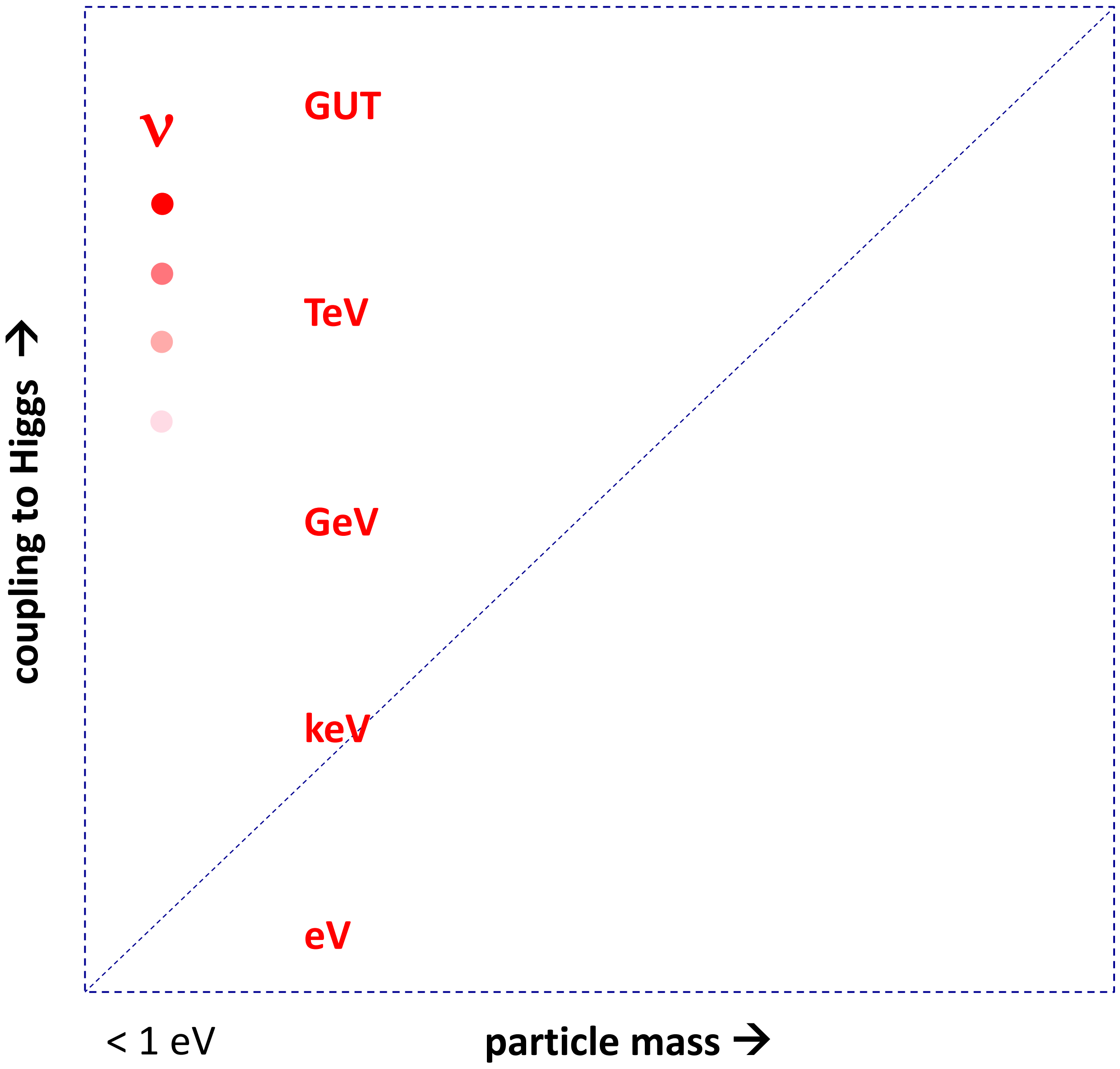
→ suppression **yM_H/M**

(so-called see-saw)

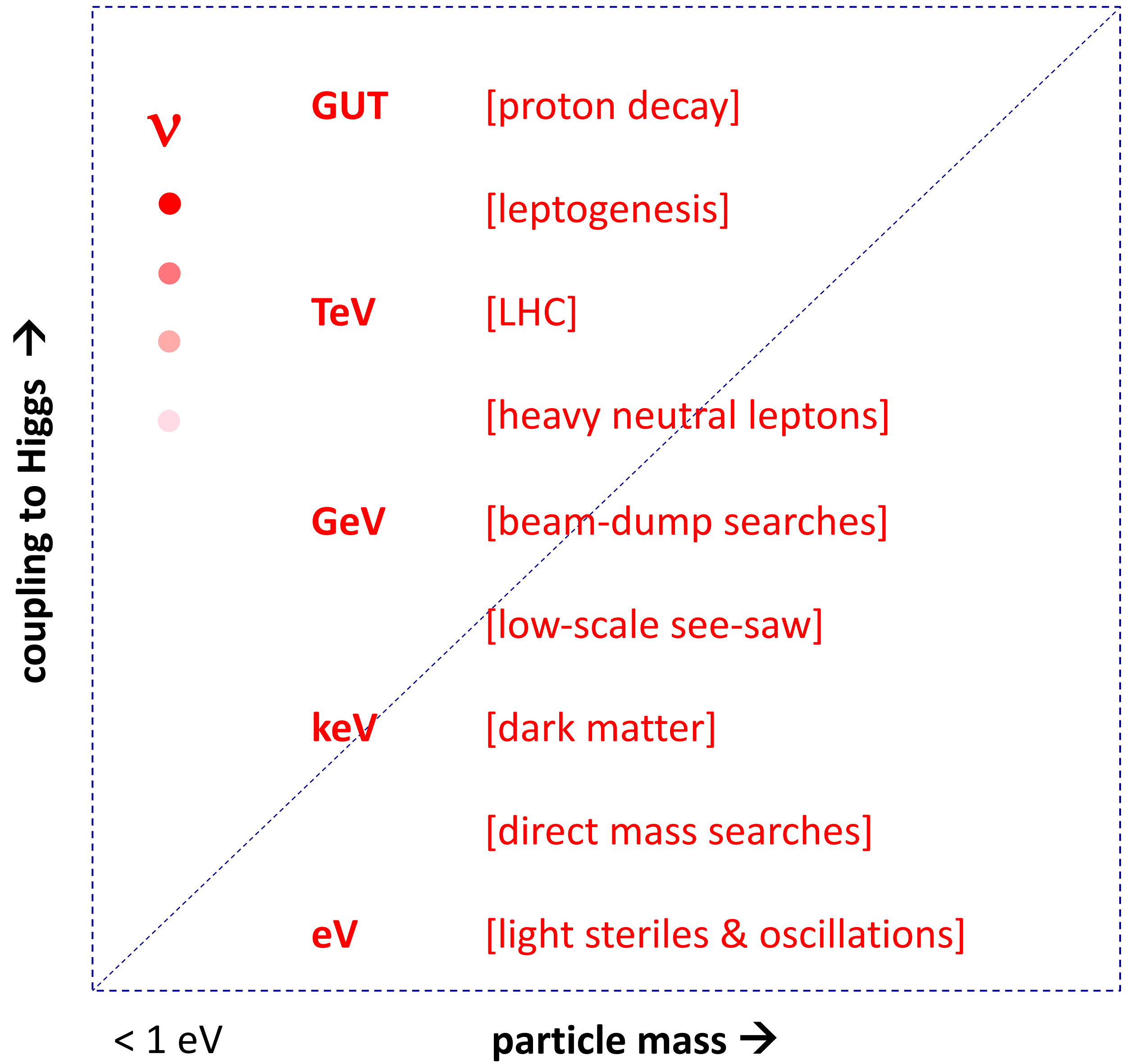
< 1 eV

particle mass →

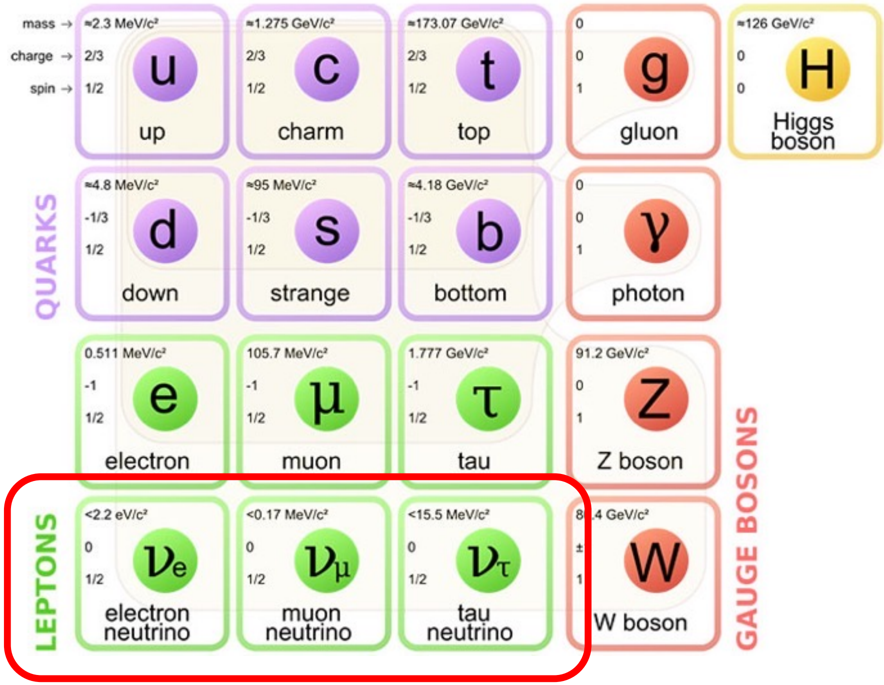
...New ν mass states could emerge at one or more scales ...



… and contribute to a wide research program with HE/LE links…



ν: A tale about (broken) symmetries…



… and a portal to new physics!

Exercise (Dirac eq.)

Find the equation connecting ξ_R and ξ_L by eliminating ξ from:

$$\xi_{R,L} = e^{\pm \vec{u}\cdot\frac{\vec{\sigma}}{2}}\,\xi$$

Notation and useful properties for Pauli matrices:

- $\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $\qquad \left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\,\epsilon_{ijk}\frac{\sigma_k}{2}$

  $\Rightarrow (\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b} + i\,\vec{\sigma}\,(\vec{a}\times\vec{b})$

- Boost with velocity $\vec{v} = v\,\hat{v}$ $(|\hat{v}|=1)$, $v = \mathrm{tgh}\,u$, $\vec{u} = u\,\hat{v}$

  $\Rightarrow e^{\vec{u}\cdot\frac{\vec{\sigma}}{2}} = \cosh\left(\frac{u}{2}\right) + \hat{v}\cdot\vec{\sigma}\,\sinh\left(\frac{u}{2}\right)$

- Rotation with angle ω around $\hat{n}$ $(|\hat{n}|=1)$, $\vec{\omega} = \omega\,\hat{n}$

  $\Rightarrow e^{i\vec{\omega}\cdot\frac{\vec{\sigma}}{2}} = \cos\left(\frac{\omega}{2}\right) + i\,\hat{n}\cdot\vec{\sigma}\,\sin\left(\frac{\omega}{2}\right)$ $\qquad$ (← not used in this exercise)

• First, simplify the exponential:

$$\exp\left(\pm \vec{u}\cdot\frac{\vec{\sigma}}{2}\right) = \cosh\left(\frac{u}{2}\right) \pm \hat{v}\cdot\vec{\sigma}\,\sinh\left(\frac{u}{2}\right)$$

$$= \left(\frac{\cosh u+1}{2}\right)^{\frac{1}{2}} \pm \hat{v}\cdot\vec{\sigma}\left(\frac{\cosh u-1}{2}\right)$$

$$= \left(\frac{\gamma+1}{2}\right)^{\frac{1}{2}} \pm \hat{v}\cdot\vec{\sigma}\left(\frac{\gamma-1}{2}\right)^{\frac{1}{2}}$$

$$= \left(\frac{E+m}{2m}\right)^{\frac{1}{2}} \pm \hat{v}\cdot\vec{\sigma}\left(\frac{E-m}{2m}\right)^{\frac{1}{2}}$$

$$= \frac{E+m}{\sqrt{2m(E+m)}} \pm \hat{v}\cdot\vec{\sigma}\,\frac{p}{\sqrt{2m(E+m)}}$$

$$= \frac{E+m \pm \vec{p}\cdot\vec{\sigma}}{\sqrt{2m(E+m)}}$$

$\leftarrow \begin{cases} \cosh\frac{u}{2} = \left(\frac{\cosh u+1}{2}\right)^{\frac{1}{2}} \\ \sinh\frac{u}{2} = \left(\frac{\cosh u-1}{2}\right)^{\frac{1}{2}} \end{cases}$

$\leftarrow \cosh u = \gamma$

$\leftarrow \gamma = m/E$

$\leftarrow$ times $\frac{E+m}{E+m}$ inside $(\ldots)$ with $E^2 - m^2 = p^2$

$\leftarrow \vec{p} = \hat{v}\,p$

• Thus the boost $\xi \to \xi_{R,L}$ is: $\quad \xi_{R,L} = \dfrac{E+m \pm \vec{p}\cdot\vec{\sigma}}{\sqrt{2m(E+m)}}\,\xi$

and the inverse $\xi_{R,L} \to \xi$ is: $\quad \xi = \dfrac{E+m \mp \vec{p}\cdot\vec{\sigma}}{\sqrt{2m(E+m)}}\,\xi_{R,L} = \dfrac{E+m \pm \vec{p}\cdot\vec{\sigma}}{\sqrt{2m(E+m)}}\,\xi_{L,R}$

$(\vec{p} \to -\vec{p})$ $\qquad$ swapped

- Elimination of ξ:

$$\xi_{R,L} = \frac{E+m\pm\vec{p}\cdot\vec{\sigma}}{\sqrt{2m(E+m)}}\,\xi$$

$$= \frac{(E+m\pm\vec{p}\cdot\vec{\sigma})^2}{2m(E+m)}\,\xi_{L,R}$$

$$= \frac{E^2+2mE+m^2\pm 2(E+m)\,\vec{p}\cdot\vec{\sigma}+(\vec{p}\cdot\vec{\sigma})^2}{2m(E+m)}\,\xi_{L,R} \qquad \leftarrow (\vec{p}\cdot\vec{\sigma})^2=(\vec{p})^2$$

$$= \frac{2E^2+2mE\pm 2(E+m)\,\vec{p}\cdot\vec{\sigma}}{2m(E+m)}\,\xi_{L,R} \qquad \leftarrow p^2+m^2=E^2$$

$$= \frac{E\pm\vec{p}\cdot\vec{\sigma}}{m}\,\xi_{L,R} \qquad \leftarrow \text{drop } \frac{2(E+m)}{2(E+m)}$$

$$\Rightarrow\quad -m\,\xi_{R,L}+(E\pm\vec{p}\cdot\vec{\sigma})\,\xi_{L,R}=0$$

$$\Rightarrow\quad \begin{bmatrix} -m & E+\vec{p}\cdot\vec{\sigma} \\ E-\vec{p}\cdot\vec{\sigma} & -m \end{bmatrix}\begin{bmatrix}\xi_R\\ \xi_L\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix} \qquad \leftarrow \text{Dirac eq. in momentum space}$$

Describes the motion of a free 1/2-spin particle
(analogous to $\vec{v}$ = const for a classical free point particle)

- Note that:

- By construction, the equation behaves correctly under Lorentz boosts.
- It also behave correctly under rotations, since $\xi_{L,R}$ rotate in the same way
  → Desired covariant equation
- the $\xi_{R,L}$ are naturally organized in a 4-component Dirac spinor $\psi = \begin{bmatrix} \xi_R \\ \xi_L \end{bmatrix}$
  (in other conventions $\psi = \begin{bmatrix} \xi_L \\ \xi_R \end{bmatrix}$)
- The $\xi_{R,L}$ components decouple in the $m \to 0$ limit:

$$(p + \vec{p}\cdot\vec{\sigma})\,\xi_L = 0$$
$$(p - \vec{p}\cdot\vec{\sigma})\,\xi_R = 0$$

← "Weyl" equations for $m=0$

where $\xi_{R,L}$ become eigenstates of the helicity $h = \vec{p}\cdot\vec{\sigma}/p$

$h\,\xi_{R,L} = \pm 1$ → spin (anti) parallel with $\vec{p}$

which explains the L,R terminology

- Notice however that chirality and helicity coincide only for $m=0$.
  They start to differ at $\mathcal{O}(m/E)$ for $m>0$.