Adam Falkowski

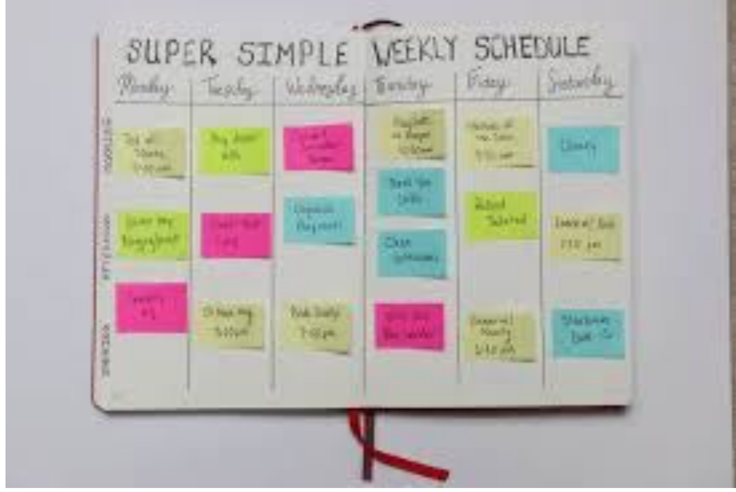
Effective Field Theories

(EFTs)

Lectures given at the Invisibles'24 school in Bologna

27-28 June 2024

Timetable



- Lecture 1
  Effective toy story or an EFT of a single scalar

- Lecture 2
  EFT in action or an illustrated philosophy of EFT

- Lecture 3
  SMEFT et al. or effective theory above the electroweak scale

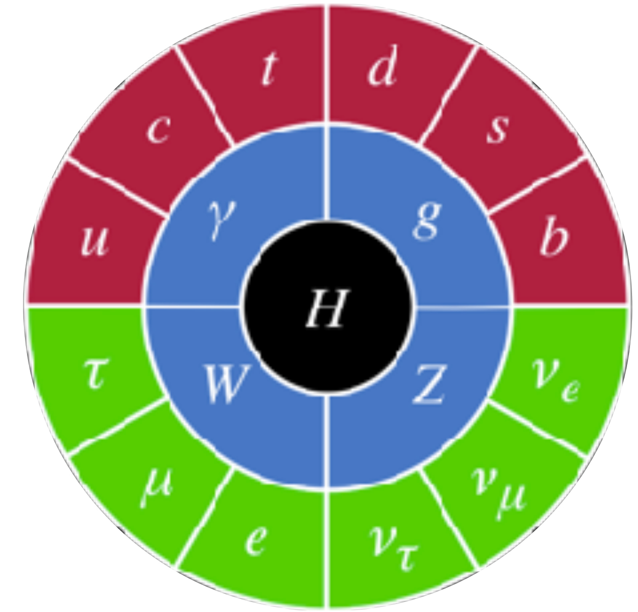
Motivation to go beyond the Standard Model

- The Standard Model has been totally successful in describing all collider and low-energy experiments. Discovery of the 125 GeV Higgs boson was the last piece of puzzle to fall into place

- On the other hand, we know for a fact that physics beyond the SM exists (neutrino masses, dark matter, inflation, baryon asymmetry). There are also some theoretical hints for new physics (strong CP problem, flavor hierarchies, gauge coupling unification, naturalness problem)

- But there isn't one model or class of models that is strongly preferred, at this moment. We need to keep an open mind on many possible forms of new physics that may show up in experiment. This requires a model-independent approach

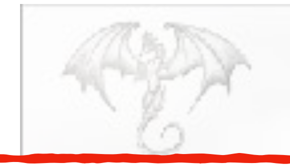
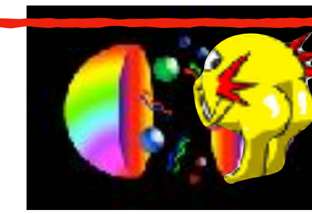
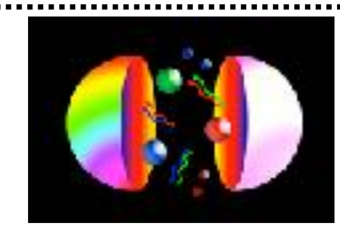
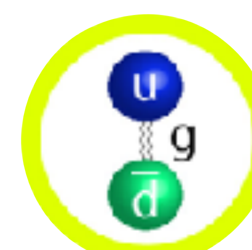
- Currently, the leading model-independent tool to parametrize the possible effects of heavy new physics is effective field theory



Lecture 3

EFT above the electroweak scale, or SMEFT et al

EFT worldview

UV	100 TeV ?	**Dragons** 
SMEFT	100 GeV	$\gamma, g, W, Z, \nu_i, e, \mu, \tau$ + **u, d, s, c, b, t** + **h** 
WEFT5	5 GeV	$\gamma, g, \nu_i, e, \mu, \tau$ + **u, d, s, c, b** 
WEFT4	2 GeV	$\gamma, g, \nu_i, e, \mu, \tau$ + **u, d, s, c** 
ChRT	1 GeV	γ, ν_i, e, μ + **hadrons** 
ChPT	100 MeV	$\gamma, \nu_i, e, \mu, \pi, K, p^{(*)}$ 
QED+	1 MeV	$\gamma, \nu_i, e, p^{(*)}$ 
EH+	0.01 eV	$\gamma, \nu_i, p^{(*)}, e^{(*)}$ $\gamma, p^{(*)}, e^{(*)}$ 

SMEFT

SMEFT is an effective theory for these degrees of freedom:

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Name	Spin	Dimension
G^a_μ	$\mathbf{8}$	$\mathbf{1}$	0	Gluon	1	1
W^k_μ	$\mathbf{1}$	$\mathbf{3}$	0	Weak SU(2) bosons	1	1
B_μ	$\mathbf{1}$	$\mathbf{1}$	0	Hypercharge boson	1	1
Q	$\mathbf{3}$	$\mathbf{2}$	1/6	Quark doublets	1/2	3/2
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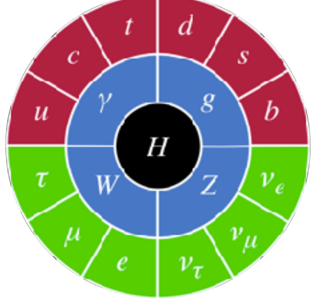
E^c	$\mathbf{1}$	$\mathbf{1}$	1	Charged anti-leptons	1/2	3/2
H	$\mathbf{1}$	$\mathbf{2}$	1/2	Higgs field	0	1

incorporating certain physical assumptions:

1. **Usual relativistic QFT: locality, unitarity, Poincaré symmetry**
2. **Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale**
3. **Gauge symmetry: local $SU(3)_C \times SU(2)_W \times U(1)_Y$ symmetry strictly respected by all interactions and spontaneously broken to $SU(3)_C \times U(1)_{\rm em}$ by a VEV of the Higgs field**

SMEFT power counting



1. Locality, unitarity, Poincaré symmetry
2. Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale
3. Gauge symmetry: local SU(3)xSU(2)xU(1) symmetry strictly respected by all interactions

We can organize the SMEFT Lagrangian in a dimensional expansion:

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

Each $\mathscr{L}_D$ is a linear combination of SU(3)xSU(2)xU(1) invariant interaction terms (operators) where D is the sum of canonical dimensions of all the fields entering the interaction

Since Lagrangian has mass dimension $[\mathscr{L}] = 4$, by dimensional analysis the couplings (Wilson coefficients) of interactions in $\mathscr{L}_D$ have mass dimension $[C_D] = 4 - D$

Standard SMEFT power counting: $C_D \sim \frac{c_D}{\Lambda^{D-4}}$ where $c_D \sim 1$,
and Λ is identified with the mass scale of the UV completion of the SMEFT,

In the spirit of EFT, each $\mathscr{L}_D$ should include a <u>complete</u> and <u>non-redundant</u> set of interactions

Dimensional analysis

Using the unit system where $c = \hbar = 1$. Then all objects can be assigned mass dimension

$$[m] = [E] = \text{mass}^1 \;\Rightarrow\; [x] = [t] = \text{mass}^{-1} \;\Rightarrow\; [\partial_\mu] \equiv \left[\frac{\partial}{\partial x^\mu}\right] = \text{mass}^1$$

Canonical dimension of fields follow from canonically normalized action:

$$S = \int d^4x \mathscr{L} = \int d^4x \left\{ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi - \frac{1}{2}[\partial_\mu A_\nu - \partial_\nu A_\mu]\partial^\mu A^\nu \right\}$$

Action is dimensional
(because path integral contains $e^{iS/\hbar}$) $\Rightarrow$

$$[\phi] = \text{mass}^1$$

$$[\psi] = \text{mass}^{3/2}$$

$$[A] = \text{mass}^1$$

These rules allows one to determine dimensions of any interaction term, e.g.

$$\mathscr{L} \supset \lambda|H|^4 + C_H|H|^6 + C_\psi(\psi\psi)(\bar{\psi}\bar{\psi}) + \ldots \;\Rightarrow\; [\lambda] = \text{mass}^0 \quad [C_H] = \text{mass}^{-2} \quad [C_\psi] = \text{mass}^{-2}$$

SMEFT at dimension 2

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

Only a single D=2 operator can be constructed from the SM fields:

$$\mathscr{L}_{D=2} = \mu_H^2 H^\dagger H$$

Philosophy of EFT: $\mu_H \sim \Lambda \gtrsim 1\ \text{TeV}$

Experiment: $\mu_H \sim 100\ \text{GeV}$

Unsolved mystery why $\mu_H^2 \ll \Lambda^2$,
which is called the hierarchy problem

From the point of view of EFT, the hierarchy problem is a breakdown of dimensional analysis

SMEFT at dimension 3

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

$$\mathscr{L}_{D=3} = 0$$

Simply, no gauge invariant operators made of SM fields exist at canonical dimension D=3

The absence of D=3 operators is a feature of SMEFT, but not a law of nature!
(see in a couple of minutes)

SMEFT at dimension 4

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

D=4 is special because it doesn't contain an explicit scale (marginal interactions)

$$\mathscr{L}_{D=4} = -\frac{1}{4}\sum_{V\in B,W^i,G^a} V_{\mu\nu}V^{\mu\nu} + \sum_{f\in Q,L} i\bar{f}\bar{\sigma}^\mu D_\mu f + \sum_{f\in U,D,E} if^c\sigma^\mu D_\mu \bar{f}^c + D_\mu H^\dagger D^\mu H$$
$$-\left(U^c Y_u \tilde{H}^\dagger Q + D^c Y_d H^\dagger Q + E^c Y_e H^\dagger L + \text{h.c.}\right) - \lambda (H^\dagger H)^2$$
$$+\tilde{\theta} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu},$$

$$\tilde{H}_a = \epsilon^{ab}H^*_b$$

$$V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g f^{abc}V^b_\mu V^c_\nu$$

$$D_\mu f = \partial_\mu f + i g_s G^a_\mu T^a f + i g_L W^i_\mu \frac{\sigma^i}{2} f + i g_Y B_\mu Y f$$

$$\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta\ a}$$

$$U^c = \begin{pmatrix} u^c \\ c^c \\ t^c \end{pmatrix} \qquad D^c = \begin{pmatrix} d^c \\ s^c \\ b^c \end{pmatrix} \qquad E^c = \begin{pmatrix} e^c \\ \mu^c \\ \tau^c \end{pmatrix}$$

$$Q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} \\ \begin{pmatrix} c \\ s \end{pmatrix} \\ \begin{pmatrix} t \\ b \end{pmatrix} \end{pmatrix} \qquad L = \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix} \\ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \\ \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \end{pmatrix}$$

Note on fermion conventions

I am using the 2-component spinor formalism

A Dirac fermion is described by a pair of spinor fields f and $\bar{f}^c$ **with the kinetic and mass terms**

$$\mathscr{L} = i\bar{f}\bar{\sigma}^\mu D_\mu f + i f^c \sigma^\mu D_\mu \bar{f}^c - m f^c f - m\bar{f}\bar{f}^c$$

$$\sigma^\mu = (1, \boldsymbol{\sigma})$$
$$\bar{\sigma}^\mu = (1, -\boldsymbol{\sigma})$$
$$\bar{f} \equiv f^*$$

To translate to 4-component Dirac notation use

$$F = \begin{pmatrix} f \\ \bar{f}^c \end{pmatrix}, \qquad \bar{F} = \begin{pmatrix} f^c & \bar{f} \end{pmatrix}, \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\bar{F} \equiv F^\dagger \gamma^0$$

For example

$$\bar{f}\bar{\sigma}^\mu \partial_\mu f = \bar{F}_L \gamma^\mu \partial_\mu F_L$$
$$f^c \sigma^\mu \partial_\mu \bar{f}^c = \bar{F}_R \gamma^\mu \partial_\mu F_R$$
$$f^c f = \bar{F}_R F_L$$
$$\bar{f}\bar{f}^c = \bar{F}_L F_R$$

See the spinor bible [arXiv:0812.1594] for more details

SMEFT at dimension 4

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{D=2} + \mathcal{L}_{D=3} + \mathcal{L}_{D=4} + \mathcal{L}_{D=5} + \mathcal{L}_{D=6} + \mathcal{L}_{D=7} + \mathcal{L}_{D=8} + \dots$$

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$$-\left(U^c Y_u \tilde{H}^\dagger Q + D^c Y_d H^\dagger Q + E^c Y_e H^\dagger L + \mathrm{h.c.}\right) - \lambda(H^\dagger H)^2$$
$$+\tilde{\theta} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu},$$

Experiment: all these interactions at D=4 above have been observed, except for $\tilde{\theta}$

Strictly speaking, λ has not been observed directly. Its value is known within SM hypothesis, but not within SMEFT, without additional assumptions. A precision measurement of double Higgs production (receiving contributions from cubic Higgs coupling) will be a direct proof that λ is present in the Lagrangian.

Note that $\theta_B B_{\mu\nu}\tilde{B}_{\mu\nu}$ has no physical consequences, while $\theta_W W^k_{\mu\nu}\tilde{W}^k_{\mu\nu}$ can be eliminated by chiral rotation

Standard SMEFT power counting works ok for λ, but the Yukawa matrices contain clear structures, hinting at additional selection rules. For $\tilde{\theta}$ the EFT power counting fails completely

SMEFT at dimension-5

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

Weinberg (1979)
Phys. Rev. Lett. 43, 1566

$$H \to \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix}$$

$$\mathscr{L}_{D=5} = (LH)C(LH) + \mathrm{h.c.} \to \frac{1}{2} \sum_{J,K=e,\mu,\tau} \mathrm{v}^2 C_{JK}(\nu_J \nu_K) + \mathrm{h.c.}$$

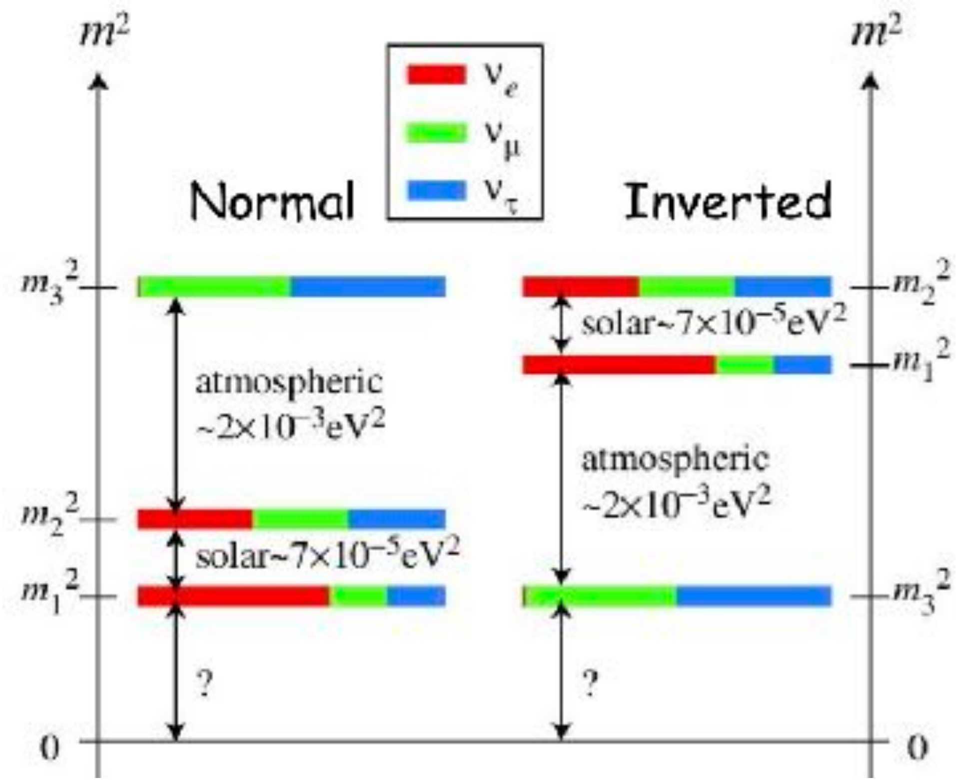
- At dimension 5, the only gauge-invariant operators one can construct are the so-called Weinberg operators, which break the lepton number
- After electroweak symmetry breaking they give rise to mass terms for the SM (left-handed) neutrinos with the mass matrix $M = -\mathrm{v}^2 C$. In the SMEFT scenario, neutrinos are purely Majorana.
- Neutrino oscillation experiments strongly suggest that these operators are present (unless new degrees of freedom exist at low energy scale , see later)

This is a huge success of the SMEFT paradigm:
corrections to the SM Lagrangian predicted at the next order in the EFT expansion, are indeed the ones observed in experiment!

SMEFT at dimension-5

$$\mathscr{L}_{\rm SMEFT} \supset -\frac{1}{2}(\nu M \nu) + \text{h.c.} \qquad M = -\,\mathrm{v}^2 C$$

Neutrino masses or most likely in the 0.01 eV - 0.1 eV ballpark (though the lightest neutrino may even be massless)



It follows that the dimension-5 Wilson coefficient is of order $C \sim \frac{1}{\Lambda}$ with $\Lambda \sim 10^{15}$ **GeV**

SMEFT paradigm points to an existence of a large scale in physics, independent of the Planck scale !

Digression on nu-SMEFT

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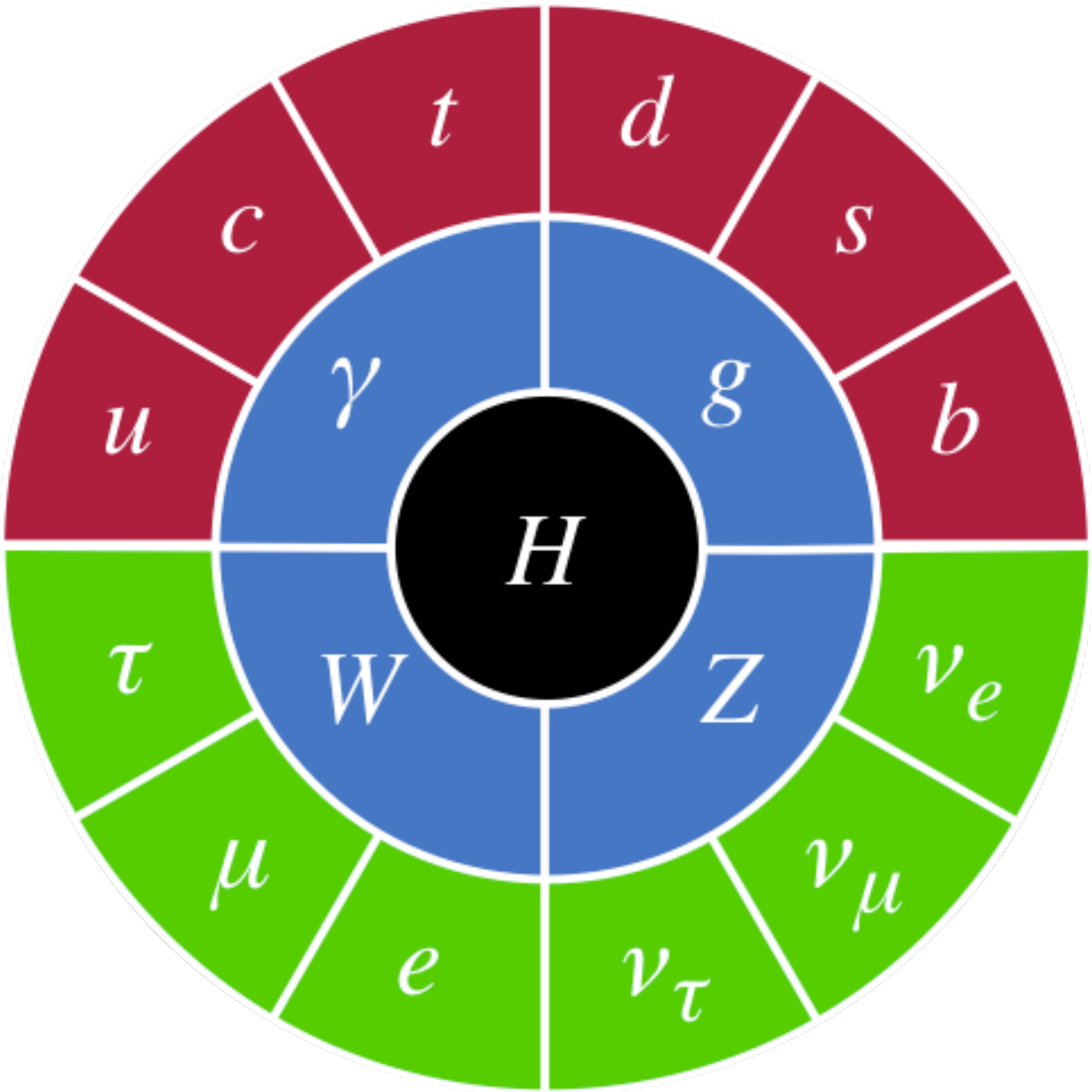

nu-SMEFT

nu-SMEFT is an effective theory for these degrees of freedom:

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Q	$\mathbf{3}$	$\mathbf{2}$	1/6	Quark doublets	1/2	3/2
U^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	-2/3	Up-type anti-quarks	1/2	3/2
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E^c	$\mathbf{1}$	$\mathbf{1}$	1	Charged anti-leptons	1/2	3/2
H	$\mathbf{1}$	$\mathbf{2}$	1/2	Higgs field	0	1
ν^c	**1**	**1**	**0**	**Singlet neutrinos**	**1/2**	**3/2**

incorporating certain physical assumptions:

1. **Usual relativistic QFT: locality, unitarity, Poincaré symmetry**
2. **Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale**
3. **Gauge symmetry: local $SU(3)_C \times SU(2)_W \times U(1)_Y$ symmetry strictly respected by all interactions and spontaneously broken to $SU(3)_C \times U(1)_{\rm em}$ by a VEV of the Higgs field**

SMEFT at dimension 3

$$\mathscr{L}_{\nu\text{SMEFT}} = \mathscr{L}^{\nu\text{SMEFT}}_{D=2} + \mathscr{L}^{\nu\text{SMEFT}}_{D=3} + \mathscr{L}^{\nu\text{SMEFT}}_{D=4} + \mathscr{L}^{\nu\text{SMEFT}}_{D=5} + \mathscr{L}^{\nu\text{SMEFT}}_{D=6} + \ldots$$

In the presence of singlet (right-handed) neutrinos, one can write down their mass term at D=3:

$$\mathscr{L}^{\nu\text{SMEFT}}_{D=3} = \frac{1}{2}\nu^c M_\nu \nu^c + \text{h.c.}$$

Here M_ν is a 3x3 symmetric matrix containing a new mass scale

Standard power counting suggests $M_\nu \sim \Lambda \gg \text{v}$, but if that is the case, then we can integrate out the singlet neutrinos and return to SMEFT

nu-SMEFT is worth considering only assuming $M_\nu \leq \text{v}$, creating another violation of natural EFT power counting

nu-SMEFT at dimension 4

$$\mathcal{L}_{\nu\text{SMEFT}} = \mathcal{L}^{\nu\text{SMEFT}}_{D=2} + \mathcal{L}^{\nu\text{SMEFT}}_{D=3} + \mathcal{L}^{\nu\text{SMEFT}}_{D=4} + \mathcal{L}^{\nu\text{SMEFT}}_{D=5} + \mathcal{L}^{\nu\text{SMEFT}}_{D=6} + \dots$$

D=4 is special because it doesn't contain an explicit scale (marginal interactions)

$$\mathcal{L}^{\nu\text{SMEFT}}_{D=4} = -\frac{1}{4}\sum_{V\in B,W^i,G^a} V_{\mu\nu}V^{\mu\nu} + \sum_{f\in Q,L} i\bar{f}\bar{\sigma}^\mu D_\mu f + \sum_{f\in U,D,E} i f^c \sigma^\mu D_\mu \bar{f}^c$$
$$-\left(U^c Y_u \tilde{H}^\dagger Q + D^c Y_d H^\dagger Q + E^c Y_e H^\dagger L + \nu^c Y_\nu \tilde{H}^\dagger L + \text{h.c.}\right)$$
$$+ D_\mu H^\dagger D^\mu H - \lambda (H^\dagger H)^2 + \tilde{\theta} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu},$$

In nu-SMEFT at D=4 there are additional Yukawa interactions with right-handed neutrinos
Together with the D=3 term, it gives neutrino masses

$$\mathcal{L}_{\nu\text{SMEFT}} \supset \frac{1}{2}\nu^c M_\nu \nu^c - \frac{\text{v}}{\sqrt{2}}\nu^c Y_\nu \nu + \text{h.c.}$$

As a result, neutrinos are generically mixed Majorana-Dirac

However, in the nu-SMEFT scenario the smallness of the neutrino masses does not have a natural explanation, and it only adds to mysteries of the SM (why are M_ν and Y_ν small) ?

nu-SMEFT at dimension 5

There are qualitatively new effects at D=5 in nu-SMEFT...

$$\mathscr{L}^{\nu\text{SMEFT}}_{D=5} \supset (\nu^c C_{NNH} \nu^c) H^\dagger H + (\nu^c C_{NNB} \sigma^{\mu\nu} \nu^c) B_{\mu\nu}$$

Another contribution to neutrino masses

Might also affect Higgs decays

Magnetic and electric Majorana dipole moment of neutrinos

Leads also to neutrino radiative decay

$$(\nu^c_J \sigma^{\mu\nu} \nu^c_K) B_{\mu\nu} = (\nu^c_K \sigma^{\nu\mu} \nu^c_J) B_{\mu\nu} = -(\nu^c_K \sigma^{\mu\nu} \nu^c_J) B_{\mu\nu}$$

Therefore Majorana dipole moment involves necessarily 2 different neutrino flavours

The more usual Dirac dipole moment arises only at D=6 in nu-SMEFT:

$$\mathscr{L}^{\nu\text{SMEFT}}_{D=6} \supset (\nu^c C_{\nu B} \tilde{H}^\dagger L) B_{\mu\nu} + (\nu^c C_{\nu B} \tilde{H}^\dagger \sigma^k L) W^k_{\mu\nu} + \mathrm{h.c.}$$

and in this case the dipole moments can also be flavor diagonal

Digression on HEFT

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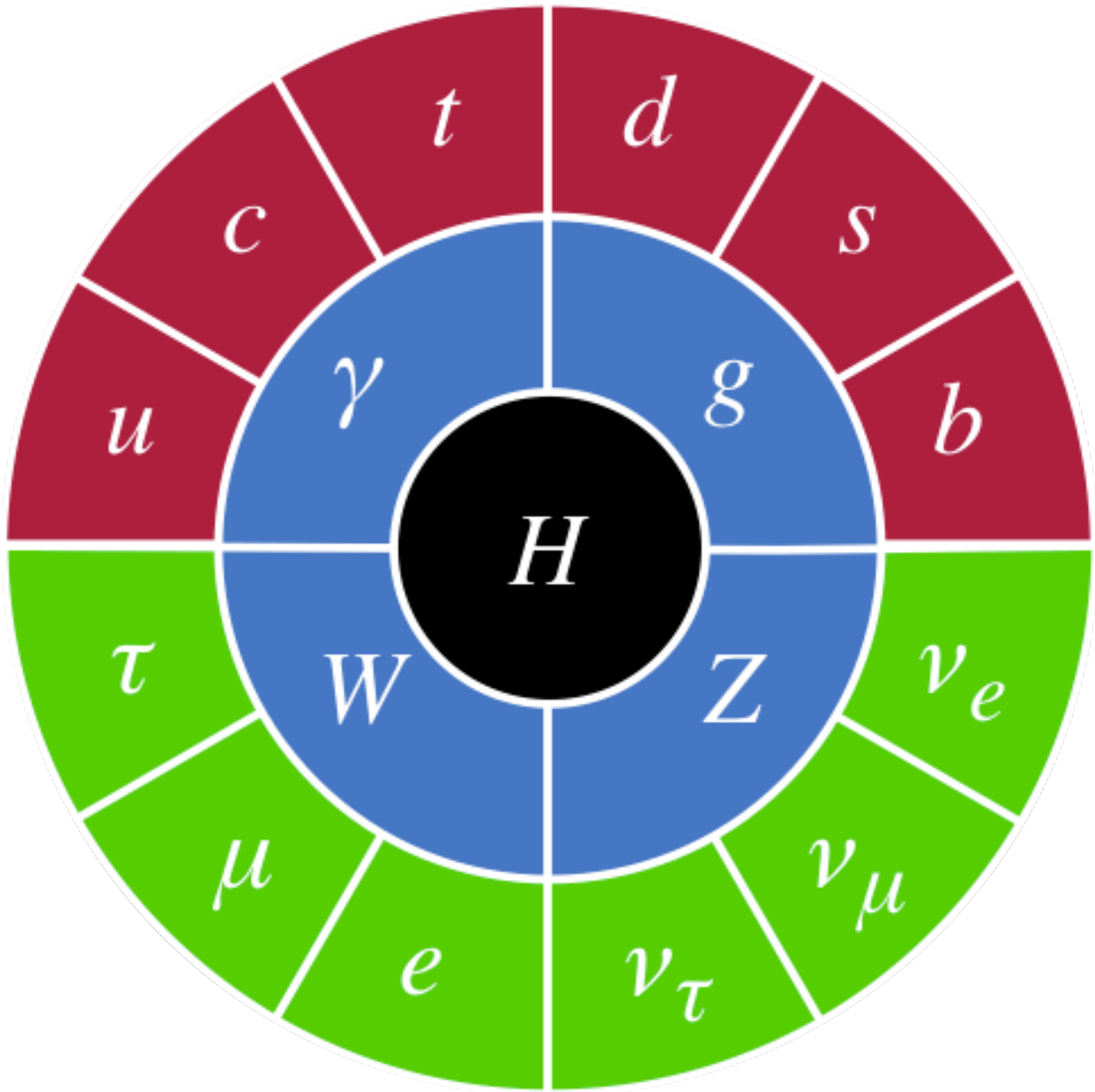

SMEFT

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incorporating certain physical assumptions:

1. **Usual relativistic QFT: locality, unitarity, Poincaré symmetry**
2. **Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale** $\mathrm{v} = 246$ **GeV**
3. **Gauge symmetry: local** $SU(3)_C \times U(1)_{\mathrm{em}}$ **symmetry strictly respected by all interactions.** $SU(3)_C \times SU(2)_W \times U(1)_Y$ **realised non-linearly**

Linear vs non-linear

Two mathematical formulations for effective theories with SM spectrum

Linearly realized electroweak symmetry

$SU(3)_C \times SU(2)_L \times U(1)_Y$

Non-linearly realized electroweak symmetry

$SU(3)_C \times U(1)_{em}$

$$L \in SU(2)_L \quad R \in U(1)_Y$$

$$H \to LH$$

$$U \to LUR^\dagger \qquad h \to h$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} iG_1 + G_2 \\ \mathrm{v} + h + iG_3 \end{pmatrix}$$

125 GeV Higgs boson

Goldstone bosons eaten by W and Z

$$U = \exp\left(\frac{i\pi^a\sigma^a}{\mathrm{v}}\right)$$

The two formulations lead to two distinct effective theories

Higgs VEV
$v \approx 246$ GeV

SMEFT $\subset$ **HEFT**

Expansion parameter
$v \approx 246$ GeV

Linear vs non-linear: Higgs self-couplings

In the SM self-coupling completely fixed…

$$\mathscr{L}_{\text{SM}} \supset m^2 |H|^2 - \lambda |H|^4$$

$$\to -\frac{1}{2} m_h^2 h^2 - \frac{m_h^2}{2\text{v}} h^3 - \frac{m_h^2}{8\text{v}^2} h^4$$

…but they can be deformed by BSM effects

SMEFT

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} - \frac{c_6}{\Lambda^2} |H|^6 + \mathscr{O}(\Lambda^{-4})$$

$$\mathscr{L}_{\text{SMEFT}} \supset -\frac{m_h^2}{2\text{v}}(1+\delta\lambda_3)h^3 - \frac{m_h^2}{8\text{v}^2}(1+\delta\lambda_4)h^4 - \frac{\lambda_5}{\text{v}} h^5 - \frac{\lambda_6}{\text{v}^2} h^6$$

$$\delta\lambda_3 = \frac{2c_6\text{v}^4}{m_h^2\Lambda^2},\ \delta\lambda_4 = \frac{12c_6\text{v}^4}{m_h^2\Lambda^2},\ \lambda_5 = \frac{3c_6\text{v}^2}{4\Lambda^2},\ \lambda_6 = \frac{c_6\text{v}^2}{8\Lambda^2}$$

SMEFT: Predicts correlations between self-couplings as long as $\Lambda \gg \text{v}$, that is to say, higher-dimensional operators can be neglected

HEFT

$$\mathscr{L}_{\text{HEFT}} \supset -c_3 \frac{m_h^2}{2\text{v}} h^3 - c_4 \frac{m_h^2}{8\text{v}^2} h^4 - \frac{c_5}{\text{v}} h^5 - \frac{c_6}{\text{v}^2} h^6 + \ldots$$

HEFT: no correlations between self-couplings

Linear vs non-linear

- Choosing SMEFT vs HEFT implicitly entails an assumption about a class of BSM theories that we want to characterize
- SMEFT is appropriate to describe BSM theories which can be parametrically decoupled, that is to say, where the mass scale of the new particles depends on a free parameter(s) that can be taken to infinity
- Conversely, HEFT is appropriate to describe non-decoupling BSM theories, where the masses of the new particles vanish in the limit $v \to 0$

Example: cubic Higgs deformation

Consider a toy EFT model where Higgs cubic (and only that) deviates from the SM

$$\mathscr{L} = \mathscr{L}_{\rm SM} - \Delta_3 \frac{m_h^2}{2v} h^3$$

$$V(h) = \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v}\left(1+\Delta_3\right) h^3 + \frac{m_h^2}{8v^2} h^4$$

This EFT belongs to the HEFT but not SMEFT parameter space

HEFT = Non-analytic Higgs potential

$$V(h) = \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2\mathrm{v}} \left(1 + \Delta_3\right) h^3 + \frac{m_h^2}{8\mathrm{v}^2} h^4 \quad \textbf{(1)}$$

Given a Lagrangian for Higgs boson h, one can always uplift it to a manifestly SU(2)xU(1) invariant form by replacing

$$h \to \sqrt{2H^\dagger H} - \mathrm{v}$$

After this replacement, Higgs potential contains terms non-analytic at H=0

$$V(H) = \frac{m_h^2}{8\mathrm{v}^2} \left(2H^\dagger H - \mathrm{v}^2\right)^2 + \Delta_3 \frac{m_h^2}{2\mathrm{v}} \left(\sqrt{2H^\dagger H} - \mathrm{v}\right)^3 \quad \textbf{(2)}$$

(1) and (2) are equal in the unitary gauge

$$H \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mathrm{v} + h \end{pmatrix}$$

<u>**Thus, (1) and (2) describe the same physics**</u>

Non-analytic Higgs potential

$$V(H) = \frac{m_h^2}{8\mathrm{v}^2}\left(2H^\dagger H - \mathrm{v}^2\right)^2 + \Delta_3 \frac{m_h^2}{2\mathrm{v}}\left(\sqrt{2H^\dagger H} - \mathrm{v}\right)^3$$

In the unitary gauge, the Higgs potential looks totally healthy and renormalizable…

Going away from the unitary gauge:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} iG_1 + G_2 \\ \mathrm{v} + h + iG_3 \end{pmatrix} \quad \Rightarrow \quad V \supset \Delta_3 \frac{m_h^2}{2\mathrm{v}}\left(\sqrt{(h+\mathrm{v})^2 + G^2} - \mathrm{v}\right)^3$$

$$G^2 \equiv \sum_i G_i^2$$

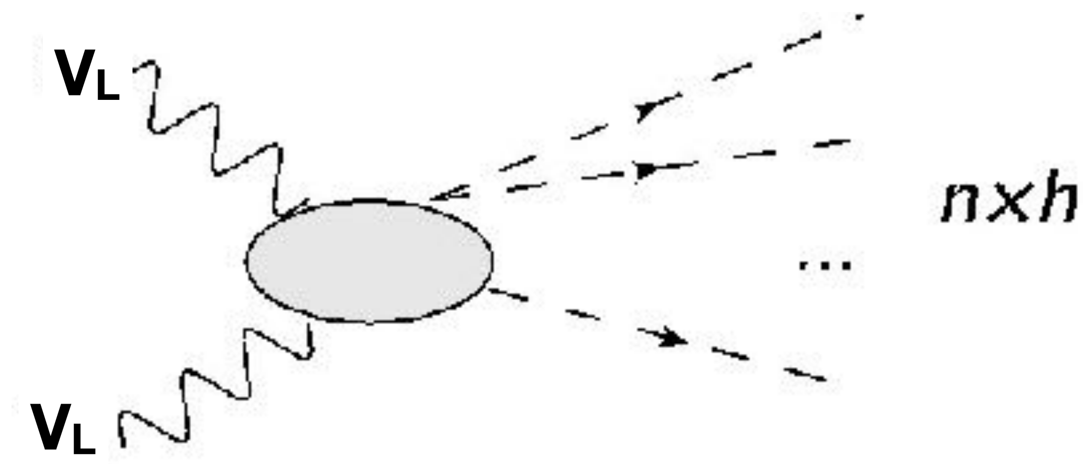
Away from the unitary gauge, it becomes clear that the Higgs potential contains non-renormalizable interactions suppressed only by the EW scale v

$$V \supset \Delta_3 \frac{3m_h^2}{4\mathrm{v}}\frac{G^2 h^2}{h+\mathrm{v}} + \mathcal{O}(G^4) = \Delta_3 \frac{3m_h^2}{4} G^2 \sum_{n=2}^{\infty}\left(\frac{-h}{\mathrm{v}}\right)^n + \mathcal{O}(G^4)$$

Multi-Higgs production

Consider VBF production of n ≥ 2 Higgs bosons: $V_L V_L \to n \times h$

By the equivalence theorem, at high energies the same as $GG \to n \times h$



Expanded potential contains interactions

$$V \supset = \Delta_3 \frac{3m_h^2}{4} G^2 \sum_{n=2}^{\infty} \left(\frac{-h}{\mathrm{v}} \right)^n$$

leading to interaction vertices with arbitrary number of Higgs bosons

$$\mathcal{M}(GG \to \underbrace{h\ldots h}_{n}) \sim \Delta_3 \frac{n! m_h^2}{\mathrm{v}^n}$$

Amplitudes for multi-Higgs production in W/Z boson fusion are only suppressed by the scale v and do not decay with growing energy, leading to unitarity loss at some scale right above v

Unitarity primer

S matrix unitarity $S^\dagger S = 1$

symmetry factor for n-body final state

implies relation between forward scattering amplitude, and elastic and inelastic production cross sections

$$2\mathrm{Im}\mathscr{M}(p_1p_2 \to p_1p_2) = S_2 \int d\Pi_2 \,|\, \mathscr{M}^{\mathrm{elastic}}(p_1p_2 \to k_1k_2)\,|^2 + \sum S_n \int d\Pi_n \,|\, \mathscr{M}^{\mathrm{inelastic}}(p_1p_2 \to k_1\ldots k_n)\,|^2$$

Equation is "diagonalized" after initial and final 2-body state are projected into partial waves

$$a_l(s) = \frac{S_2}{16\pi}\sqrt{1-\frac{4m^2}{s}}\int_{-1}^{1} d\cos\theta P_l(\cos\theta)\mathcal{M}(s,\cos\theta),$$

$$2\mathrm{Im}a_l = a_l^2 + \sum S_n \int d\Pi_n \,|\, \mathscr{M}_l^{\mathrm{inelastic}}\,|^2$$

This can be rewritten as the Argand circle equation

$$(\mathrm{Re}a_l)^2 + (\mathrm{Im}a_l - 1)^2 = R_l^2, \qquad R_l^2 = 1 - \sum S_n \int d\Pi_n \,|\, \mathscr{M}_l^{\mathrm{inelastic}}\,|^2$$

Unitarity primer

Argand circle equation

$$(\mathrm{Re}a_l)^2 + (\mathrm{Im}a_l - 1)^2 = R_l^2, \qquad R_l^2 = 1 - \sum S_n \int d\Pi_n |\mathscr{M}_l^{\text{inelastic}}|^2$$

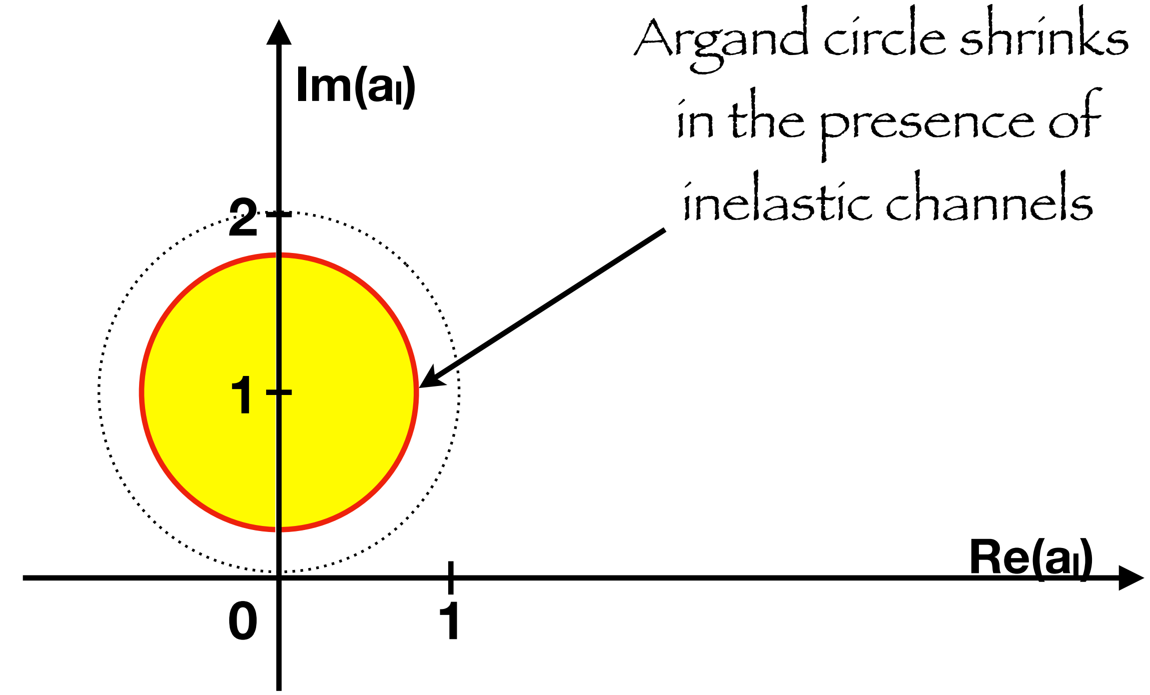
implies constraints on <u>both</u> elastic and inelastic amplitudes

Often used

$$|\mathrm{Re}a_l| \leq 1$$

$$\sum S_n \int d\Pi_n |\mathscr{M}_l^{\text{inelastic}}|^2 \leq 1$$

Often forgotten



Unitarity constraints on inelastic channels

Unitarity (strong coupling) constraint on inelastic multi-Higgs production

$$\sum_{n=2}^{\infty} \frac{1}{n!} \int d\Pi_n \, |\mathcal{M}(GG \to h^n)|^2 = \sum_{n=2}^{\infty} \frac{1}{n!} V_n(\sqrt{s}) \, |\mathcal{M}(GG \to h^n)|^2 \lesssim \mathcal{O}(1)$$

Volume of phase space in the massless limit:

$$V_n(\sqrt{s}) = \int d\Pi_n = \frac{s^{n-2}}{2(n-1)!(n-2)!(4\pi)^{2n-3}} \sim \frac{s^{n-2}}{(n!)^2 (4\pi)^{2n}}$$

In a fundamental theory,
2 → n amplitude must decay as $1/s^{n/2-1}$
in order to maintain unitarity up to arbitrary high scales

Process	*Unitarity limit*
2 → 2	**1**
2 → 3	**$1/s^{1/2}$**
2 → 4	**$1/s$**
...	...

Unitarity constraints on HEFT

Unitarity equation

$$\sum_{n=2}^{\infty} \frac{1}{n!} V_n(\sqrt{s})\, |\mathcal{M}(GG \to h^n)|^2 \lesssim \mathcal{O}(1)$$

Our amplitude

$$\mathcal{M}(GG \to \underbrace{h\ldots h}_{n}) \sim \Delta_3 \frac{n! m_h^2}{\mathrm{v}^n}$$

$$\mathcal{O}(1) \gtrsim \sum_{n=2}^{\infty} \frac{1}{n!} V_n(\sqrt{s})\, |\mathcal{M}(GG \to h^n)|^2 \sim \sum_{n=2}^{\infty} \frac{1}{n!} \frac{s^{n-2}}{(n!)^2 (4\pi)^{2n}} \Delta_3^2 \frac{(n!)^2 m_h^4}{\mathrm{v}^{2n}} \sim \frac{\Delta_3^2 m_h^4}{s^2} \exp\left[\frac{s}{(4\pi \mathrm{v})^2}\right]$$

In model with deformed Higgs cubic, multi-Higgs amplitude do not decay with energy leading to unitarity loss at a finite value of energy

$$\Lambda \lesssim (4\pi \mathrm{v}) \log^{1/2}\left(\frac{4\pi \mathrm{v}}{m_h |\Delta_3|^{1/2}}\right)$$

AA, Rattazzi [arXiv:1902.05936]

Unless Δ_3 is unobservably small, unitarity loss happens at the scale $4\pi \mathrm{v} \sim 3$ TeV!

Linear vs non-linear summary

- EFT with non-linearly realized electroweak symmetry (aka HEFT) is equivalent to EFT with linearly realized electroweak symmetry but whose Lagrangian is a non-polynomial function of the Higgs field that is non-analytic at H=0

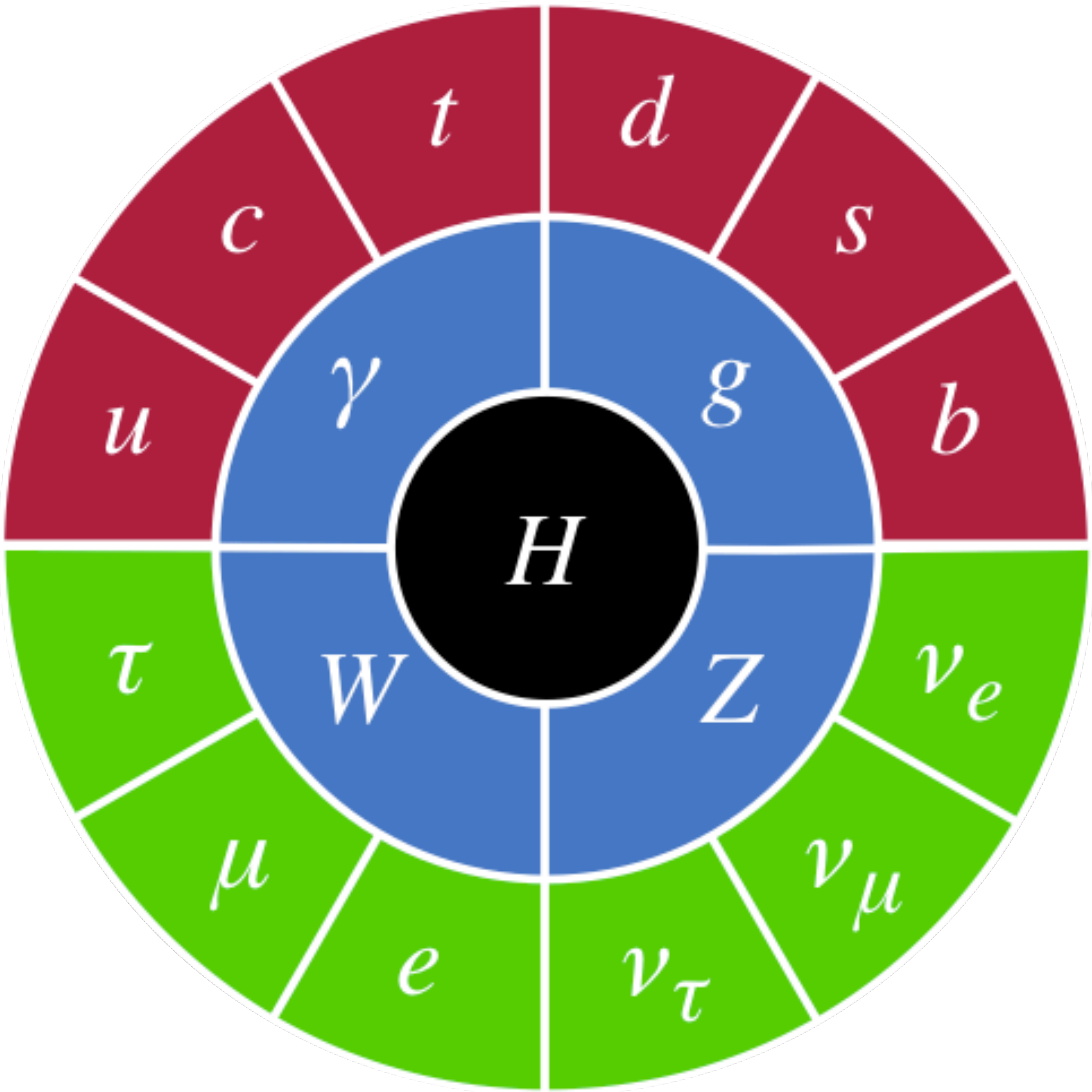
- This non-analyticity leads to explosion of multi-Higgs amplitudes at the scale 4 π v . For this reason, the validity regime of HEFT is limited below the scale of order $4\pi v \sim 3$ TeV

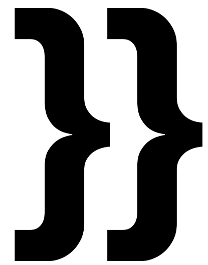
- HEFT is useful to approximate BSM theories where new particles' masses vanish in the limit v → 0, e.g. SM + a 4th generation of chiral fermion
  See Banta et al. [arXiv:2110.02967] for more examples

- On the other hand, an EFT with linearly realized electroweak symmetry and the Lagrangian polynomial in the Higgs field (aka SMEFT) is useful to approximate BSM theories where new particles' masses do not vanish in the limit v → 0, and thus can be parametrically larger than the electroweak scale, e.g. SM + vector-like fermions

- In the following we forget HEFT and focus on SMEFT

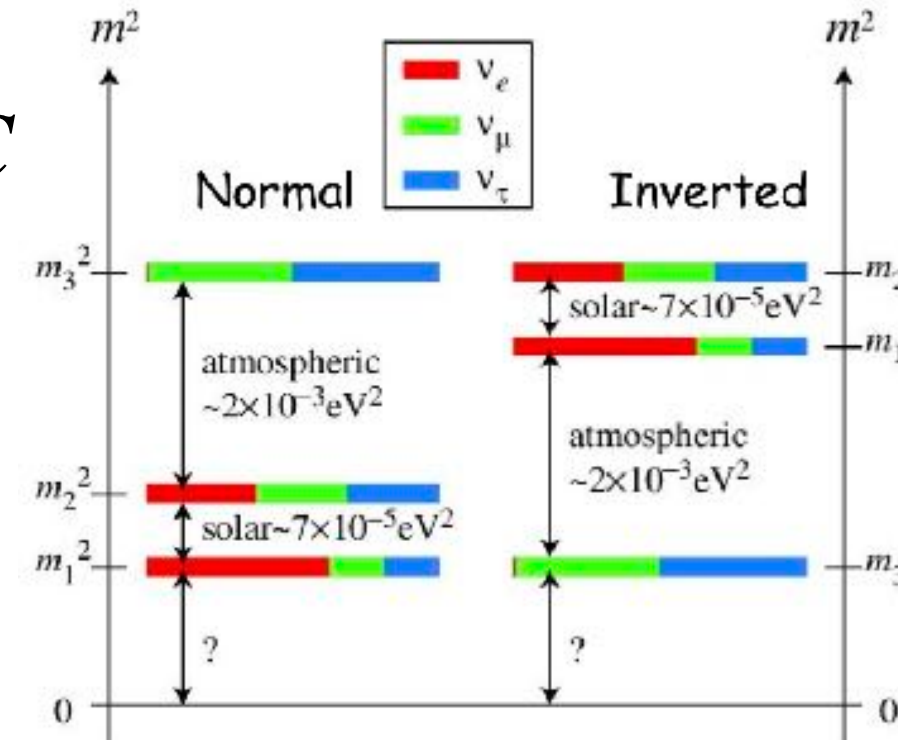
Back to SMEFT





Scales in SMEFT

$$\mathscr{L}_{\rm SMEFT} \supset -\frac{1}{2}(\nu M \nu) + \text{h.c.} \quad M = -\text{v}^2 C$$



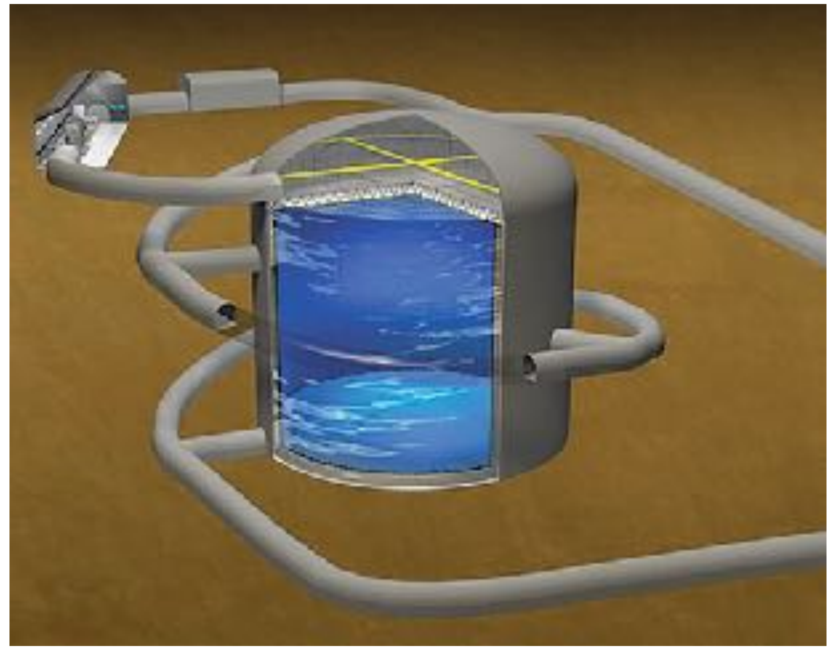
However, $\Lambda \sim 10^{15}$ GeV leads to a *psychological* problem

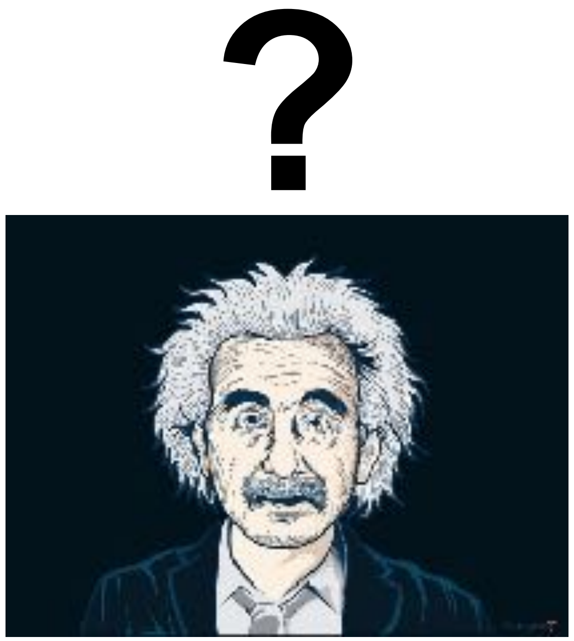
$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

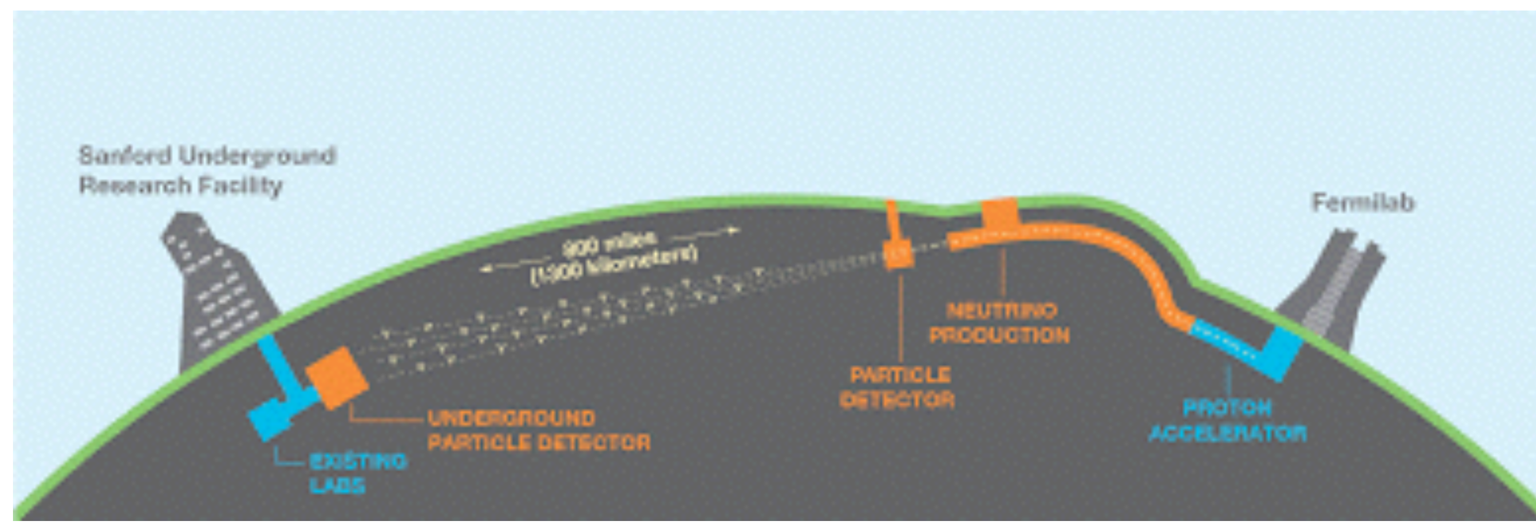
If $\mathscr{L}_{D=5} \sim \frac{1}{\Lambda}$ then naive SMEFT counting suggest $\mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}$, $\mathscr{L}_{D=7} \sim \frac{1}{\Lambda^3}$, and so on

If this is really the correct estimate, then we will never see any other effects of higher-dimensional operators, except possibly of the baryon-number violating ones :/

Career opportunities

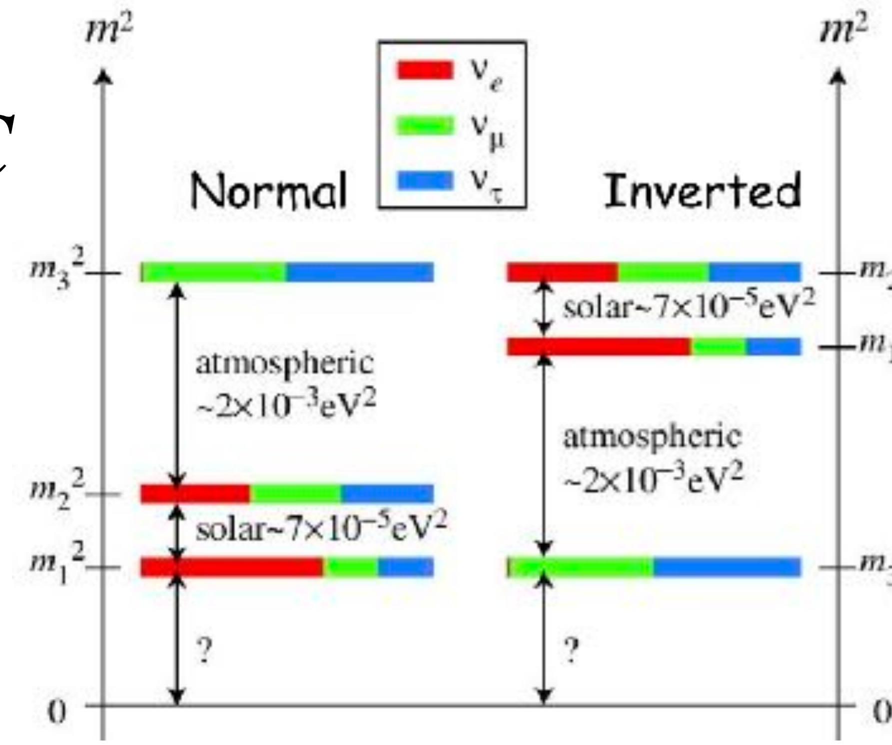






SMEFT at dimension-5

$$\mathscr{L}_{\mathrm{SMEFT}} \supset -\frac{1}{2}(\nu M \nu) + \mathrm{h.c.} \quad M = -\mathrm{v}^2 C$$



$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

If $\mathscr{L}_{D=5} \sim \frac{1}{\Lambda}$ **then naive SMEFT counting suggest**

$$\mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}, \mathscr{L}_{D=7} \sim \frac{1}{\Lambda^3}, \dots$$

However, this conclusion is not set in stone

It is possible that the true new physics scale is not far from TeV, but its coupling to the lepton sector is very small

Alternatively, it is possible (and likely) that there is more than one mass scale of new physics

Dimension-5 interactions are special because they violate lepton number L.

More generally, all odd-dimension SMEFT operators violate B-L

If we assume that the mass scale of new particles with B-L-violating interactions is Λ_L**, and there is also B-L-conserving new physics at the scale** $\Lambda \ll \Lambda_L$ **, then the estimate is**

$$\mathscr{L}_{D=5} \sim \frac{1}{\Lambda_L}, \mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}, \mathscr{L}_{D=7} \sim \frac{1}{\Lambda_L^3}, \mathscr{L}_{D=8} \sim \frac{1}{\Lambda^4},$$ **and so on**

SMEFT at dimension-6

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

Grządkowski et al
arXiv:1008.4884

At dimension-6 all hell breaks loose

$$\begin{aligned}
\mathscr{L}_{D=6} = {} & C_H (H^\dagger H)^3 + C_{H\Box}(H^\dagger H)\Box(H^\dagger H) + C_{HD}|H^\dagger D_\mu H|^2 \\
& + C_{HWB} H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu} + C_{HG} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu} + C_{HW} H^\dagger H\, W^k_{\mu\nu} W^k_{\mu\nu} + C_{HB} H^\dagger H\, B_{\mu\nu} B_{\mu\nu} \\
& ++ C_W \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} + C_G f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \\
& + C_{H\widetilde{G}} H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu} + C_{H\widetilde{W}} H^\dagger H\, \widetilde{W}^k_{\mu\nu} W^k_{\mu\nu} + C_{H\widetilde{B}} H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu} + C_{H\widetilde{W}B} H^\dagger \sigma^k H\, \widetilde{W}^k_{\mu\nu} B_{\mu\nu} \\
& + C_{\widetilde{W}} \epsilon^{klm} \widetilde{W}^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} + C_{\widetilde{G}} f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \\
& + H^\dagger H(\bar{L} H C_{eH} \bar{E}^c) + H^\dagger H(\bar{Q} \tilde{H} C_{uH} \bar{U}^c) + H^\dagger H(\bar{Q} H C_{dH} \bar{D}^c) \\
& + i H^\dagger \overleftrightarrow{D}_\mu H (\bar{L} C^{(1)}_{Hl} \bar{\sigma}^\mu L) + i H^\dagger \sigma^k \overleftrightarrow{D}_\mu H (\bar{L} C^{(3)}_{Hl} \bar{\sigma}^\mu \sigma^k L) + i H^\dagger \overleftrightarrow{D}_\mu H (E^c C_{He} \sigma^\mu \bar{E}^c) \\
& + i H^\dagger \overleftrightarrow{D}_\mu H (\bar{Q} C^{(1)}_{Hq} \bar{\sigma}^\mu Q) + i H^\dagger \sigma^k \overleftrightarrow{D}_\mu H (\bar{Q} C^{(3)}_{Hq} \bar{\sigma}^\mu \sigma^k Q) + i H^\dagger \overleftrightarrow{D}_\mu H (U^c C_{Hu} \sigma^\mu \bar{U}^c) \\
& + i H^\dagger \overleftrightarrow{D}_\mu H (D^c C_{Hd} \sigma^\mu \bar{D}^c) + \Big\{ i \tilde{H}^\dagger D_\mu H (U^c C_{Hud} \sigma^\mu \bar{D}^c) \\
& + (\bar{Q} \sigma^k \tilde{H} C_{uW} \bar{\sigma}^{\mu\nu} \bar{U}^c) W^k_{\mu\nu} + (\bar{Q} \tilde{H} C_{uB} \bar{\sigma}^{\mu\nu} \bar{U}^c) B_{\mu\nu} + (\bar{Q} \tilde{H} C_{uG} T^a \bar{\sigma}^{\mu\nu} \bar{U}^c) G^a_{\mu\nu} \\
& + (\bar{Q} \sigma^k H C_{dW} \bar{\sigma}^{\mu\nu} \bar{D}^c) W^k_{\mu\nu} + (\bar{Q} H C_{dB} \bar{\sigma}^{\mu\nu} \bar{D}^c) B_{\mu\nu} + (\bar{Q} H C_{dG} T^a \bar{\sigma}^{\mu\nu} \bar{D}^c) G^a_{\mu\nu} \\
& + (\bar{L} \sigma^k H C_{eW} \bar{\sigma}^{\mu\nu} \bar{E}^c) W^k_{\mu\nu} + (\bar{L} H C_{eB} \bar{\sigma}^{\mu\nu} \bar{E}^c) B_{\mu\nu} + \mathrm{h.c.} \Big\} + \mathscr{L}^{4-\text{fermion}}_{D=6}
\end{aligned}$$


$|H|^2 B_{\mu\nu}\widetilde{B}_{\mu\nu}$
$|H|^2 G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}$
$|H|^2 W^a_{\mu\nu}\widetilde{W}^a_{\mu\nu}$
$|H|^2 W^a_{\mu\nu}W^a_{\mu\nu}$
$|H|^2 B_{\mu\nu}B_{\mu\nu}$
$G^a_{\mu\nu}G^a_{\nu\rho}\widetilde{G}^a_{\rho\mu}$
$|H|^6$
$|H|^2 G^a_{\mu\nu}G^a_{\mu\nu}$

SMEFT at dimension-6

Bosonic operators

$$\mathscr{L}_{\text{SMEFT}} \supset \sum_X C_X O_X$$

$$O_H = (H^\dagger H)^3$$

$$O_{H\Box} = (H^\dagger H)\Box(H^\dagger H)$$

$$O_{HD} = |H^\dagger D_\mu H|^2$$

$$O_{HG} = H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu} \qquad O_{H\widetilde{G}} = H^\dagger H\, G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu}$$

$$O_{HW} = H^\dagger H\, W^k_{\mu\nu} W^k_{\mu\nu} \qquad O_{H\widetilde{W}} = H^\dagger H\, W^k_{\mu\nu} \widetilde{W}^k_{\mu\nu}$$

$$O_{HB} = H^\dagger H\, B_{\mu\nu} B_{\mu\nu} \qquad O_{H\widetilde{B}} = H^\dagger H\, B_{\mu\nu} \widetilde{B}_{\mu\nu}$$

$$O_{HWB} = H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu} \qquad O_{HW\widetilde{B}} = H^\dagger \sigma^k H\, W^k_{\mu\nu} \widetilde{B}_{\mu\nu}$$

$$O_W = \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} \qquad O_{\widetilde{W}} = \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} \widetilde{W}^m_{\rho\mu}$$

$$O_G = f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \qquad O_{\widetilde{G}} = f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} \widetilde{G}^c_{\rho\mu}$$

These are mostly relevant for Higgs physics and certain electroweak precision observables. The CP-odd ones, affect important CP observables via loop effects, such as e.g. EDMs

SMEFT at dimension-6

$$\mathcal{L}_{\text{SMEFT}} \supset \sum_{I,J=1}^{3} [O_{fH}]_{IJ}[C_{fH}]_{IJ} + \text{h.c.}$$

Yukawa-like operators

$$O_{eH} = H^\dagger H(\bar{L}H\bar{E}^c)$$

$$O_{uH} = H^\dagger H(\bar{Q}\tilde{H}\bar{U}^c)$$

$$O_{dH} = H^\dagger H(\bar{Q}H\bar{D}^c)$$

These affect single Higgs boson couplings to SM fermions. Bounds depends on the flavor but typically don't exceed $|C| \lesssim \frac{1}{(1\ \text{TeV})^2}$

SMEFT at dimension-6

Vertex-like operators

$$O_{Hl}^{(1)} = iH^\dagger \overleftrightarrow{D}_\mu H(\bar{L}\bar{\sigma}^\mu L)$$

$$O_{Hl}^{(3)} = iH^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{L}\bar{\sigma}^\mu \sigma^k L)$$

$$O_{He} = iH^\dagger \overleftrightarrow{D}_\mu H(E^c \sigma^\mu \bar{E}^c)$$

$$O_{Hq}^{(1)} = iH^\dagger \overleftrightarrow{D}_\mu H(\bar{Q}\bar{\sigma}^\mu Q)$$

$$O_{Hq}^{(3)} = iH^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{Q}\bar{\sigma}^\mu \sigma^k Q)$$

$$O_{Hu} = iH^\dagger \overleftrightarrow{D}_\mu H(U^c \sigma^\mu \bar{U}^c)$$

$$O_{Hd} = iH^\dagger \overleftrightarrow{D}_\mu H(D^c \sigma^\mu \bar{D}^c)$$

$$O_{Hud} = i\tilde{H}^\dagger D_\mu H(U^c \sigma^\mu \bar{D}^c)$$

These affect electroweak precision observables (W boson mass, Z branching fractions), which are measured at per-mille level at LEP

Bounds of order $|C| \lesssim \frac{1}{(10\ \mathrm{TeV})^2}$

SMEFT at dimension-6

$$\mathcal{L}^{\rm dipole}_{D=6} = (\bar{Q}\sigma^k \tilde{H} C_{uW} \bar{\sigma}^{\mu\nu} \bar{U}^c) W^k_{\mu\nu} + (\bar{Q}\tilde{H} C_{uB} \bar{\sigma}^{\mu\nu} \bar{U}^c) B_{\mu\nu} + (\bar{Q}\tilde{H} C_{uG} T^a \bar{\sigma}^{\mu\nu} \bar{U}^c) G^a_{\mu\nu}$$
$$+ (\bar{Q}\sigma^k H C_{dW} \bar{\sigma}^{\mu\nu} \bar{D}^c) W^k_{\mu\nu} + (\bar{Q} H C_{dB} \bar{\sigma}^{\mu\nu} \bar{D}^c) B_{\mu\nu} + (\bar{Q} H C_{dG} T^a \bar{\sigma}^{\mu\nu} \bar{D}^c) G^a_{\mu\nu}$$
$$+ (\bar{L}\sigma^k H C_{eW} \bar{\sigma}^{\mu\nu} \bar{E}^c) W^k_{\mu\nu} + (\bar{L} H C_{eB} \bar{\sigma}^{\mu\nu} \bar{E}^c) B_{\mu\nu} + \text{h.c.} \quad ($$

These affect anomalous magnetic and electric moments of SM particles at tree level
Bounds depend on flavor and can be very strong, especially for the first generation

$$\sigma_{\mu\nu} = \frac{i}{2}\left[\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu\right] \qquad \bar{\sigma}_{\mu\nu} = \frac{i}{2}\left[\bar{\sigma}_\mu \sigma_\nu - \bar{\sigma}_\nu \sigma_\mu\right]$$

SMEFT at dimension-6

4-fermion operators

$$\begin{aligned}
\mathscr{L}^{4-\text{fermion}}_{D=6} = & (\bar{L}\bar{\sigma}^{\mu}L)C_{ll}(\bar{L}\bar{\sigma}_{\mu}L) + (E^c\sigma_{\mu}\bar{E}^c)C_{ee}(E^c\sigma_{\mu}\bar{E}^c) + (\bar{L}\bar{\sigma}^{\mu}L)C_{le}(E^c\sigma_{\mu}\bar{E}^c) \\
& + (\bar{L}\bar{\sigma}^{\mu}L)C^{(1)}_{lq}(\bar{Q}\bar{\sigma}_{\mu}Q) + (\bar{L}\bar{\sigma}^{\mu}\sigma^k L)C^{(3)}_{lq}(\bar{Q}\bar{\sigma}_{\mu}\sigma^k Q) \\
& + (E^c\sigma_{\mu}\bar{E}^c)C_{eu}(U^c\sigma_{\mu}\bar{U}^c) + (E^c\sigma_{\mu}\bar{E}^c)C_{ed}(D^c\sigma_{\mu}\bar{D}^c) \\
& + (\bar{L}\bar{\sigma}^{\mu}L)C_{lu}(U^c\sigma_{\mu}\bar{U}^c) + (\bar{L}\bar{\sigma}^{\mu}L)C_{ld}(D^c\sigma_{\mu}\bar{D}^c) + (E^c\sigma_{\mu}\bar{E}^c)C_{eq}(Q\bar{\sigma}_{\mu}Q) \\
& + \left\{ (\bar{L}\bar{E}^c)C_{ledq}(D^cQ) + \epsilon^{kl}(\bar{L}^k\bar{E}^c)C^{(1)}_{lequ}(\bar{Q}^l\bar{U}^c) + \epsilon^{kl}(\bar{L}^k\bar{\sigma}^{\mu\nu}\bar{E}^c)C^{(3)}_{lequ}(\bar{Q}^l\bar{\sigma}^{\mu\nu}\bar{U}^c) + \mathrm{h.c.} \right\} \\
& + (\bar{Q}\bar{\sigma}^{\mu}Q)C^{(1)}_{qq}(\bar{Q}\bar{\sigma}_{\mu}Q) + (\bar{Q}\bar{\sigma}^{\mu}\sigma^k Q)C^{(3)}_{qq}(\bar{Q}\bar{\sigma}_{\mu}\sigma^k Q) \\
& + (U^c\sigma_{\mu}\bar{U}^c)C_{uu}(U^c\sigma_{\mu}\bar{U}^c) + (D^c\sigma_{\mu}\bar{D}^c)C_{dd}(D^c\sigma_{\mu}\bar{D}^c) \\
& + (U^c\sigma_{\mu}\bar{U}^c)C^{(1)}_{ud}(D^c\sigma_{\mu}\bar{D}^c) + (U^c\sigma_{\mu}T^a\bar{U}^c)C^{(8)}_{ud}(D^c\sigma_{\mu}T^a\bar{D}^c) \\
& + (Q^c\sigma_{\mu}\bar{Q}^c)C^{(1)}_{qu}(U^c\sigma_{\mu}\bar{U}^c) + (Q^c\sigma_{\mu}T^a\bar{Q}^c)C^{(8)}_{qu}(U^c\sigma_{\mu}T^a\bar{U}^c)] \\
& + (Q^c\sigma_{\mu}\bar{Q}^c)C^{(1)}_{qd}(D^c\sigma_{\mu}\bar{D}^c) + (Q^c\sigma_{\mu}T^a\bar{Q}^c)C^{(8)}_{qd}(D^c\sigma_{\mu}T^a\bar{D}^c) \\
& + \left\{ \epsilon^{kl}(\bar{Q}^k\bar{U}^c)C^{(1)}_{quqd}(\bar{Q}^l\bar{D}^c) + \epsilon^{kl}(\bar{Q}^kT^a\bar{U}^c)C^{(1)}_{quqd}(\bar{Q}^lT^a\bar{D}^c) + \mathrm{h.c.} \right\} \\
& + \left\{ (D^cU^c)C_{duq}(\bar{Q}\bar{L}) + (QQ)C_{qqu}(\bar{U}^c\bar{E}^c) + (QQ)C_{qqq}(QL) + (D^cU^c)C_{duu}(U^cE^c) + \mathrm{h.c.} \right\}.
\end{aligned}$$

These affect a wide range of physics.
Bounds can be very strong, especially for baryon-number violating operators and for certain flavor- or lepton-flavor-violating operators

From operators to observables

(roughly) three kinds of effects

dimension-6 SMEFT operators

New vertices not present in SM Lagrangian

New Lorentz structures for vertices present in SM Lagrangian

Corrections to strength of SM interactions

New vertices

Most spectacular SMEFT effects, when new vertices violate exact symmetries of SM

Example: baryon number violation

$$\mathscr{L}_{D=6} \supset C_{duu}(d^c u^c)(u^c e^c) + \mathrm{h.c.} \qquad C_{duu} \equiv [C_{duu}]_{1111}$$

This contributes to proton decay (in the limit $m_e \to 0$:

$$\Gamma(p \to e^+\pi^0) = \frac{|C_{duu}|^2 m_p W_0^2}{32\pi}\left(1 - \frac{m_{\pi_0}^2}{m_p^2}\right)^2$$

Yoo et al. [arXiv:2111.01608]

where $W_0 \approx 0.15\ \mathrm{GeV}^2$ is a lattice fudge factor known with a roughly 20% error

Experimental limits on proton decay constrain corresponding Wilson coefficient

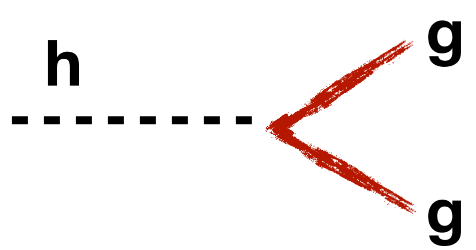
$$\Gamma(p \to e^+\pi^0) \leq 1.3 \times 10^{-66}\ \mathrm{GeV} \quad \Rightarrow \quad |C_{duu}| \leq \left(\frac{1}{3.5 \times 10^{15}\ \mathrm{GeV}}\right)^2$$

New vertices

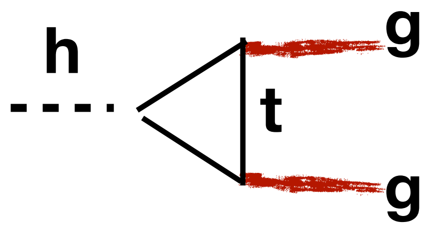
Less spectacular, when new vertices do not violate SM symmetries.
Then the process that in the SM would occur at loop level, in SMEFT appears at tree level

Example: Higgs to gluon coupling

$$\mathscr{L}_{D=6} \supset C_{HG} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu} \quad \rightarrow \quad \mathrm{v}^2 C_{HG} \frac{h}{\mathrm{v}} G^a_{\mu\nu} G^a_{\mu\nu}$$



In SM (and SMEFT), this coupling appears at one loop dominantly due to top quark loops



Thanks to the gained loop factor in SMEFT, decent bounds on the corresponding Wilson coefficient can be obtained:

$$|\,C_{HG}\,| \lesssim \frac{1}{(17\ \mathrm{TeV})^2}$$

Ellis et al. [arXiv:2012.02779]

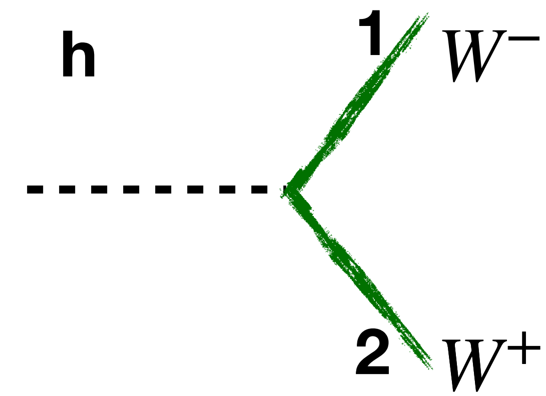
New Lorentz structures

Another class of SMEFT effects is when dimension-6 operators contribute to a vertex that appears already in SM, but with a different Lorentz structure

Example:

$$\frac{2i}{v}\left(m_W^2 \eta^{\mu\nu} + \mathrm{v}^2 C_{HW}\left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - p_1 p_2 \eta^{\mu\nu}\right] + \ldots \right)$$

SM has $\frac{h}{\mathrm{v}} 2 m_W^2 W_\mu^+ W_\mu^-$



SMEFT contains

$$\mathscr{L}_{D=6} \supset C_{HW} |H|^2 W_{\mu\nu}^k W_{\mu\nu}^k \to 2\mathrm{v}^2 C_{HW} \frac{h}{\mathrm{v}} W_{\mu\nu}^+ W_{\mu\nu}^- + \ldots$$

One way to differentiate between the two is too look at high-energy behaviour of Higgs production

$$\mathscr{M}(q\bar{q}' \to W^\pm h) \sim \#_0 + \#_2 C_{HW} E^2$$

Another way is to study differential distributions of $h \to W^+ W^- \to 2\ell 2\nu$ **decays**

Both of these currently lead to weak constraints, $|C_{HW}| \lesssim \frac{1}{\mathrm{TeV}^2}$

(stronger constraints on C_{HW} **can be obtained thanks to its contribution to** $h \to \gamma\gamma$**)**

New Lorentz structures

Spectacular examples of new Lorentz structures are anomalous magnetic and electric moments

$$\mathscr{L}_{D=6} \supset C_{eB}(\bar{l}_1 H \bar{\sigma}^{\mu\nu} \bar{e}^c) B_{\mu\nu} + \mathrm{h.c.} \qquad C_{eB} \equiv [C_{eB}]_{ee}$$

In the presence of these operator

$$\mathscr{L}_{\mathrm{SMEFT}} \supset i\bar{e}\bar{\sigma}^\mu \partial_\mu e + ie^c \sigma^\mu \partial_\mu \bar{e}^c - \left[m_e e^c e + \mathrm{h.c.}\right] \qquad q_e = -1$$

$$-q_e e A_\mu(\bar{e}\bar{\sigma}^\mu e) - q_e e A_\mu(e^c \sigma^\mu \bar{e}^c) - \left\{ \frac{\Delta\mu_e - i d_e}{4} F_{\mu\nu}(e^c \sigma^{\mu\nu} e) + \mathrm{h.c.} \right\}$$

such that **or** $\frac{g_e - 2}{2} = \frac{g_e^{\mathrm{SM}} - 2}{2} + \Delta\mu_e \frac{m_e}{q_e e}$ $\quad \vec{\mu}_e = \left(\frac{q_e e}{m_e} + \Delta\mu_e\right)\vec{s}, \qquad \vec{d}_e = d_e \vec{s}$

The anomalous moments are related to the D=6 operator as

$$\Delta\mu_e = -2\sqrt{2}\mathrm{v}\cos\theta_W \mathrm{Re} C_{eB}$$

$$d_e = -2\sqrt{2}\mathrm{v}\cos\theta_W \mathrm{Im} C_{eB}$$

New Lorentz structures

To constrain the real part of the Wilson coefficients, we need SM prediction for g_e^{SM}

This depends on the low-energy value of the electromagnetic constant $\alpha(0)$

There are two recent measurements

$$1/\alpha(0) = 137.035999206(11)$$

Morel et al.
Nature 588 (2020)

$$1/\alpha(0) = 137.035999046(27)$$

Parker et al.
Science 360 (2018) 191
[arXiv:1812.04130]

They differ by more than 5 sigma :(Combine a-la PDG blowing up errors by S=5.5

$$1/\alpha(0) = 137.035999183(56)$$

Then

$$g_e^{\mathrm{SM}}/2 = 1.00115965218045(48)$$

Experiment

$$g_e/2 = 1.00115965218059(13)$$

Fan et al.
Phys. Rev. Lett. 130 (2023)
[arXiv:2209.13084]

It follows

$$|\,C_{eB}\,| \lesssim \frac{1}{(940\ \mathrm{TeV})^2}$$

Modified interaction strength

There are 2 ways higher-dimensional operators may modify SM interaction strength

1. **Directly:** after electroweak symmetry breaking, an operator contributes to a gauge or Yukawa interaction already present in the SM

2. **Indirectly**: after electroweak symmetry breaking, an operator contributes to a kinetic term of a SM field or to an experimental observable from which some SM parameter is extracted, thus effectively shifting the strength of all interactions of that field

Modified interaction strength: directly

Example: $\mathscr{L}_{D=6} \supset iC_{He} e^c \sigma^\mu \bar{e}^c (H^\dagger D_\mu H - D_\mu H^\dagger H)$ $\quad C_{He} \equiv [C_{He}]_{ee}$

After electroweak symmetry breaking $i(H^\dagger D_\mu H - D_\mu H^\dagger H) \to -\frac{\text{v}^2}{2}\sqrt{g_L^2 + g_Y^2} Z_\mu + \ldots$

$$\mathscr{L}_{\text{SMEFT}} \supset -C_{He} \frac{\text{v}^2 \sqrt{g_L^2 + g_Y^2}}{2} (e^c \sigma^\mu \bar{e}^c) Z_\mu$$

This adds up to the weak interaction in the SM $\sqrt{g_L^2 + g_Y^2} \left(T_f^3 - \sin^2\theta_W Q_f + \delta g^{Zf}\right) \bar{f} \gamma^\mu f Z_\mu$

$$\boxed{\delta g_R^{Ze} = -C_{He} \frac{\text{v}^2}{2}}$$

Thus C_{He} can be constrained, e.g., form LEP-1 Z-pole data

Current constraints: $|C_{He}| \lesssim \frac{1}{(10\ \text{TeV})^2}$

Modified interaction strength: indirectly

Example: $$\mathscr{L}_{D=6} \supset C_{H\Box}(H^\dagger H)\Box(H^\dagger H)$$

This contributes to the kinetic term of the Higgs boson

$$\mathscr{L}_{\text{SMEFT}} \supset -C_{H\Box}\text{v}^2(\partial_\mu h)^2$$

Together with the SM kinetic term:

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{1}{2}(\partial_\mu h)^2\left(1 - 2C_{H\Box}\text{v}^2\right)$$

To restore canonical normalization, we need to rescale the Higgs boson field:

$$h \to h\left(1 + C_{H\Box}\text{v}^2\right)$$

This restores canonical normalization of the Higgs boson field, up to terms of order $1/\Lambda^4$, which we ignore here

Modified interaction strength: indirectly

$$h \to h\left(1 + C_{H\Box}\mathrm{v}^2\right)$$

After this rescaling, the dimension-6 contribution vanishes from the Higgs boson kinetic term

However, it resurfaces in all Higgs boson couplings present in the SM !

$$\frac{h}{\mathrm{v}}\left[2m_W^2 W_\mu^+ W_\mu^- + m_Z^2 Z_\mu Z_\mu\right] \to \frac{h}{\mathrm{v}}\left(1 + C_{H\Box}\mathrm{v}^2\right)\left[2m_W^2 W_\mu^+ W_\mu^- + m_Z^2 Z_\mu Z_\mu\right]$$

$$\frac{h}{\mathrm{v}} m_f \bar{f} f \to \frac{h}{\mathrm{v}}\left(1 + C_{H\Box}\mathrm{v}^2\right) m_f \bar{f} f$$

Hence, the Higgs boson interaction strength predicted by the SM is universally shifted

LHC measurements of the Higgs signal strength provide a bound on the Wilson coefficient

$$\mu = 1.09 \pm 0.11$$

$$C_{H\Box}\mathrm{v}^2 = 0.09 \pm 0.11$$

or, equivalently

$$C_{H\Box} = \frac{1}{(820\mathrm{GeV})^2} \pm \frac{1}{(740\mathrm{GeV})^2}$$

Higgs measurements only probe new physics scale of order a TeV

Modified interaction strength: indirectly

Consider the dimension-6 operator $\mathscr{L}_{D=6} \supset C_{HD} |H^\dagger D_\mu H|^2$

After electroweak symmetry breaking:

$$\mathscr{L}_{\rm SMEFT} \supset \frac{C_{HD} \mathrm{v}^2}{2} \frac{(g_L^2 + g_Y^2)\mathrm{v}^2}{8} Z_\mu Z_\mu + \ldots$$

Thus it modifies the Z boson mass: $m_Z^2 = \frac{(g_L^2 + g_Y^2)\mathrm{v}^2}{4}\left(1 + \frac{C_{HD}\mathrm{v}^2}{2}\right)$

We have this very precise O(10^{-4}) measurement of the Z boson mass

$$m_Z = (91.1876 \pm 0.0021) \text{ GeV}$$

From which we find the very stringent constraint

$$\frac{(g_L^2 + g_Y^2)\mathrm{v}^4}{8} C_{HD} \leq 0.0021 \text{ GeV} \quad \Rightarrow \quad |C_{HD}| \lesssim \frac{1}{(26 \text{ TeV})^2}$$

Modified interaction strength: indirectly

Consider the dimension-6 operator $\mathscr{L}_{D=6} \supset C_{HD} |H^\dagger D_\mu H|^2$

After electroweak symmetry breaking:

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{C_{HD} \text{v}^2}{2} \frac{(g_L^2 + g_Y^2)\text{v}^2}{8} Z_\mu Z_\mu + \dots$$

Thus it modifies the Z boson mass: $m_Z^2 = \frac{(g_L^2 + g_Y^2)\text{v}^2}{4} \left(1 + \frac{C_{HD}\text{v}^2}{2}\right)$

We have this very precise O(10^{-4}) measurement of the Z boson mass

$$m_Z = (91.1876 \pm 0.0021) \text{ GeV}$$

No!

From which we find the very stringent constraint

Ni!

$$\frac{(g_L^2 + g_Y^2)\text{v}^2}{8} C_{HD} \leq 0.0021 \text{ GeV} \Rightarrow |C_{HD}| \lesssim \frac{1}{(26 \text{ TeV})^2}$$

Non! Нет!

Nein! Nie!

Modified interaction strength: indirectly

Consider the dimension-6 operator

$$\mathscr{L}_{D=6} \supset C_{HD} |H^\dagger D_\mu H|^2$$

After electroweak symmetry breaking:

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{C_{HD}\text{v}^2}{2}\frac{(g_L^2 + g_Y^2)\text{v}^2}{8} Z_\mu Z_\mu + \ldots$$

Thus it modifies the Z boson mass:

$$m_Z^2 = \frac{(g_L^2 + g_Y^2)\text{v}^2}{4}\left(1 + \frac{C_{HD}\text{v}^2}{2}\right)$$

We cannot use the Z-boson mass measurement to constrain new physics because, it is one of the inputs to determine the electroweak parameters of the SM

In the SM:

$$G_F = \frac{1}{\sqrt{2}\text{v}^2} = 1.1663787(6) \times 10^{-5}\ \text{Gev}^{-2}$$

$$\alpha(m_Z) = \frac{g_L^2 g_Y^2}{4\pi(g_L^2 + g_Y^2)} = 7.81549(55) \times 10^{-3}$$

$$m_Z^2 = \frac{(g_L^2 + g_Y^2)\text{v}^2}{4} 91.1876(21)\ \text{GeV}$$

$$g_L = 0.648457(10)$$

$$g_Y = 0.357968(18)$$

$$\text{v} = 246.219651(63)\ \text{GeV}$$

Modified interaction strength: indirectly

$$|H^\dagger D_\mu H|^2$$

In the presence of our dimension-6 operators, the relation between electroweak couplings and observables is disrupted

$$G_F = \frac{1}{\sqrt{2}\text{v}^2} \qquad \alpha = \frac{g_L^2 g_Y^2}{4\pi(g_L^2 + g_Y^2)} \qquad m_Z^2 = \frac{(g_L^2 + g_Y^2)\text{v}^2}{4}\left(1 + \frac{C_{HD}\text{v}^2}{2}\right)$$

Now we cannot assign numerical values to the electroweak parameters, because they depend on c_{HD}

A useful trick is to get rid of the dimension-6 pollution in the input equations by redefining the SM electroweak parameters

$$g_L \to \tilde{g}_L\left(1 - \frac{C_{HD} g_L^2 \text{v}^2}{4(g_L^2 - g_Y^2)}\right) \qquad g_Y \to \tilde{g}_Y\left(1 + \frac{C_{HD} g_Y^2 \text{v}^2}{4(g_L^2 - g_Y^2)}\right)$$

For the twiddle electroweak parameter, we can now assign numerical values

$$G_F = \frac{1}{\sqrt{2}\text{v}^2}$$

$$\alpha = \frac{\tilde{g}_L^2 \tilde{g}_Y^2}{4\pi(\tilde{g}_L^2 + \tilde{g}_Y^2)}$$

$$m_Z^2 = \frac{(\tilde{g}_L^2 + \tilde{g}_Y^2)\text{v}^2}{4}$$

$$\tilde{g}_L = 0.648457(10)$$

$$\tilde{g}_Y = 0.357968(18)$$

$$\text{v} = 246.219651(63)\ \text{GeV}$$

same as in the SM

Modified interaction strength: indirectly

Z mass cannot be used to constrain new physics, because it was already used to set numerical values for the twiddle electroweak parameter

But new physics emerges now in other observables, e.g. in the W mass

$$m_W = \frac{g_L \mathrm{v}}{2} = \frac{\tilde{g}_L \mathrm{v}}{2}\left(1 - \frac{C_{HD} g_L^2 \mathrm{v}^2}{4(g_L^2 - g_Y^2)}\right) = \frac{\tilde{g}_L \mathrm{v}}{2}\left(1 - \frac{C_{HD} \tilde{g}_L^2 \mathrm{v}^2}{4(\tilde{g}_L^2 - \tilde{g}_Y^2)}\right)$$

We can now use the experimental measurement of the W mass and the SM prediction

$$m_W = \left(80.369 \pm 0.013\right)\ \mathrm{GeV} \qquad m_W^{\mathrm{SM}} = \left(80.361 \pm 0.006\right)\ \mathrm{GeV}$$

(without CDF)

to constrain the Wilson coefficients

$$-\frac{1}{(8.8\ \mathrm{TeV})^2} \leq C_{HD} \leq \frac{1}{(16.6\ \mathrm{TeV})^2}$$

at 1 sigma

Numerically a different constraint than what one would (incorrectly) obtain from Z mass!

(Somewhat futile) exercise: what constraint on C_{HD} is obtained using CDF measurement of W mass?

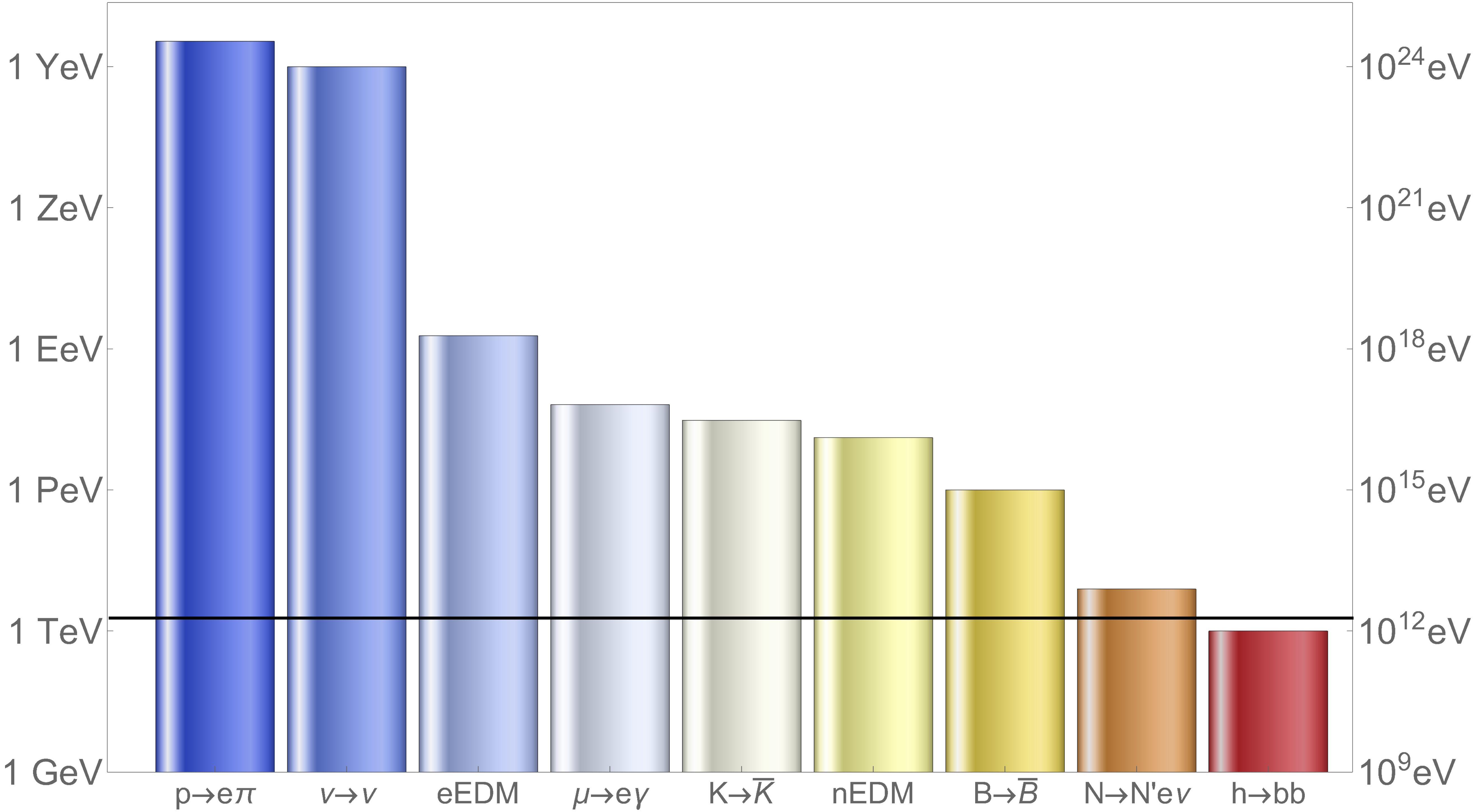
Modified interaction strength: indirectly

Corollary: relation between Wilson coefficients and interaction strength in the Lagrangian depends on the input scheme

Sector	Electroweak	Flavor
SM parameters	$g_L\, g_Y\, \mathrm{v}\, \lambda$	$\lambda\, A\, \rho\, \eta$
Example Input	$G_F\, \alpha(0)\, m_Z\, m_h$	$\Gamma(K \to \mu\nu_\mu)/\Gamma(\pi \to \mu\nu_\mu), \quad \Gamma(B \to \tau\nu_\tau), \quad \Delta M_d, \quad \Delta M_s.$

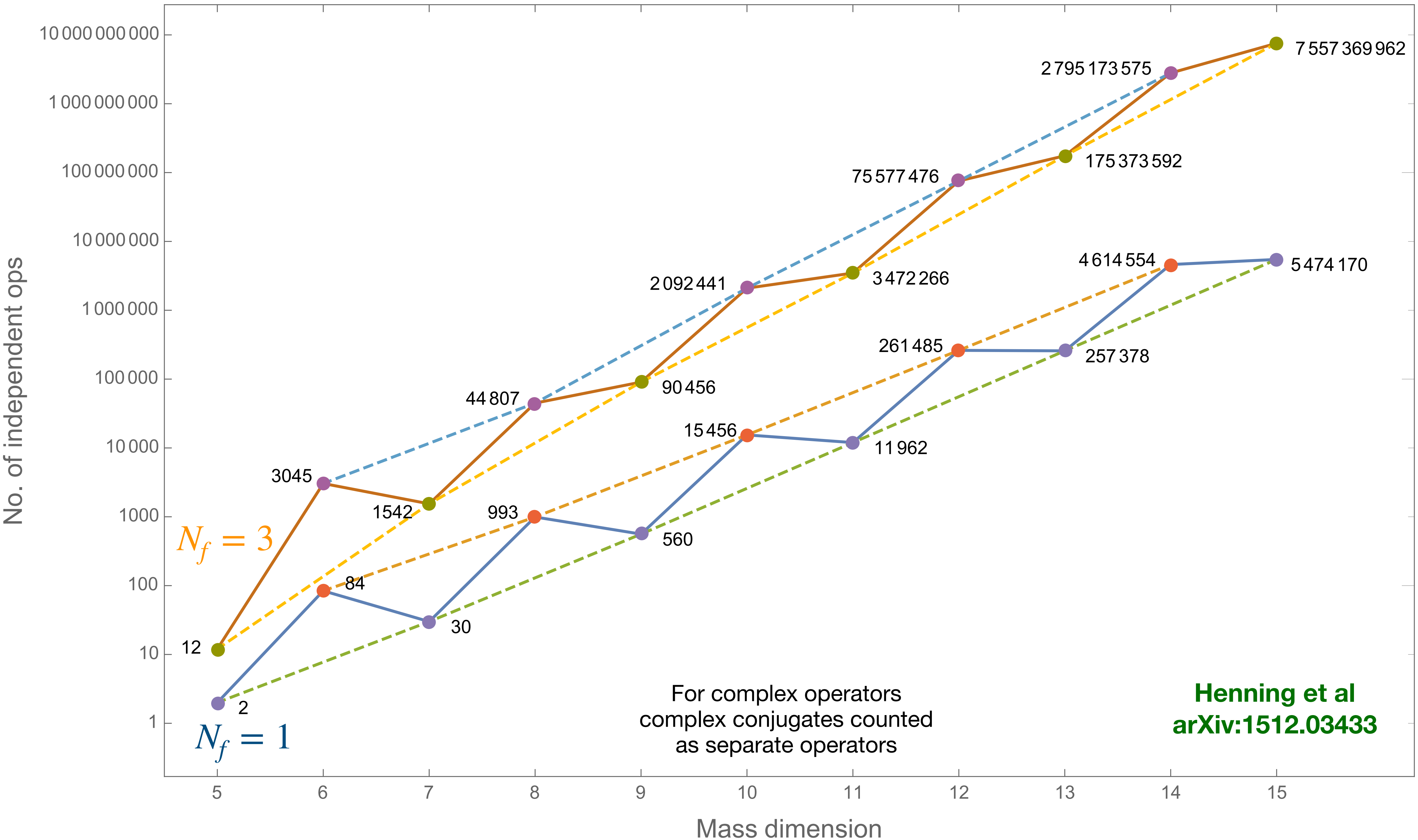
SMEFT up to dimension-6

SMEFT Lagrangian up to dimension-6 provides a convenient framework for a bulk of precision physics happening today.
In particular, it allows one to quantify the strength of different observables



SMEFT at higher dimensions

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$



Exponential growth of the number of operators with the canonical dimension D

SMEFT at higher dimensions

SMEFT at dimension-5: Weinberg (1979) Phys. Rev. Lett. 43, 1566

SMEFT at dimension-6: Grzadkowski et al arXiv: 1008.4884

SMEFT at dimension-7: Lehman arXiv: 1410.4193

SMEFT at dimension-8: Li et al arXiv: 2005.00008

SMEFT at dimension-9: Li et al arXiv: 2012.09188

SMEFT at dimension-10,11,12: Harlander, Kempkens, Schaaf arXiv: 2305.06832

Code to generate a basis at arbitrary dimension in SMEFT: Li et al arXiv:2201.04639

Beyond dimension-6

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

You need to be aware of the existence of higher-dimensional operators, whenever you need to argue validity of the EFT description

Moreover, a qualitatively new phenomenon may arise at higher dimensions

Electric and magnetic Majorana dipole moments of left-handed neutrinos arise at dimension-7

$$\mathscr{L}_{D=7} \supset (LH)\sigma^{\mu\nu}(LH)B_{\mu\nu} + \ldots$$

At tree level, light-by-light scattering receives contribution from dimension-8, which in some situations may with lower order loop contributions

$$\mathscr{L}_{D=8} \supset (B_{\mu\nu}B_{\mu\nu})^2 + \ldots$$

Neutron-antineutron oscillations arise at dimension-9

$$\mathscr{L}_{D=9} \supset \epsilon_{abc}\epsilon_{def}(\bar{d}_a\bar{d}_d)(q_b q_e)(q_c q_f) + \ldots$$

In all such cases however, you need to argue validity of your EFT and why you don't expect any larger effects of new physics from operators of lower dimensions

Beyond dimension-6

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

You need to be aware of the existence of higher-dimensional operators, whenever you need to argue validity of the EFT description

Moreover, a qualitatively new phenomenon may arise at higher dimensions

If experiment pinpoints a coefficient of some operators of dimension-6, then subleading dimension-8 operators will provide precious information

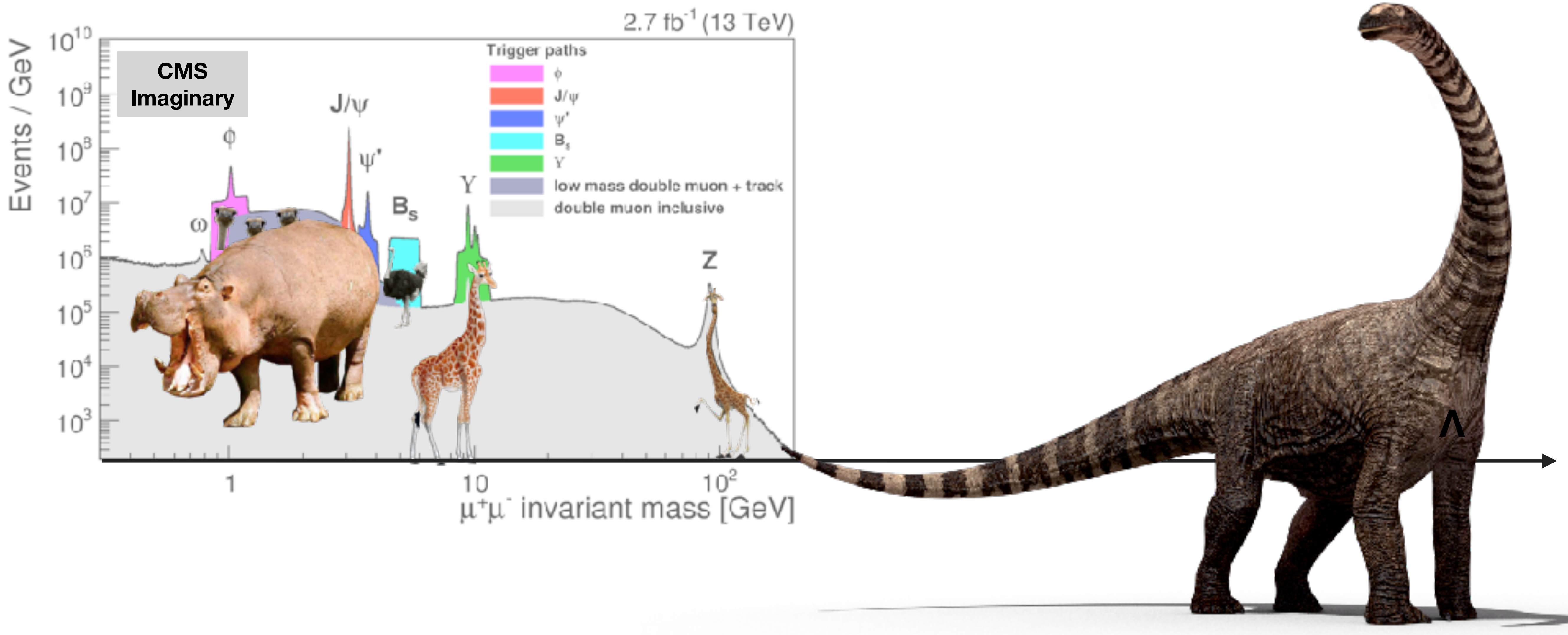
$$C_6 \sim \frac{g_*^2}{M^2}$$

Only determines coupling over mass scala of new physics

$$C_8 \sim \frac{g_*^2}{M^4}$$

May allow disentangle coupling and mass

Fantastic Beasts and Where To Find Them



THANK YOU