Astrophysical and cosmological aspects of gravitational waves

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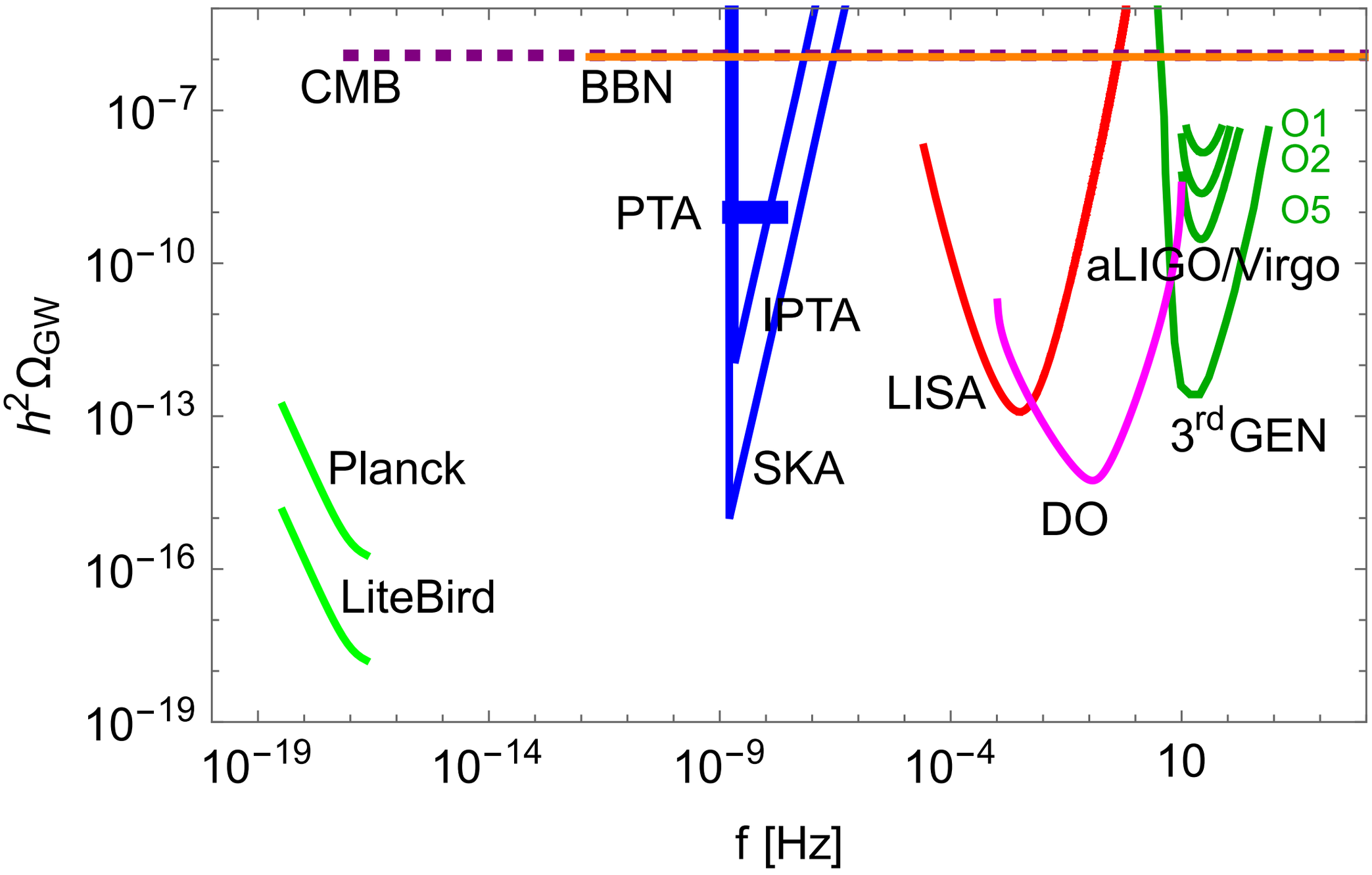
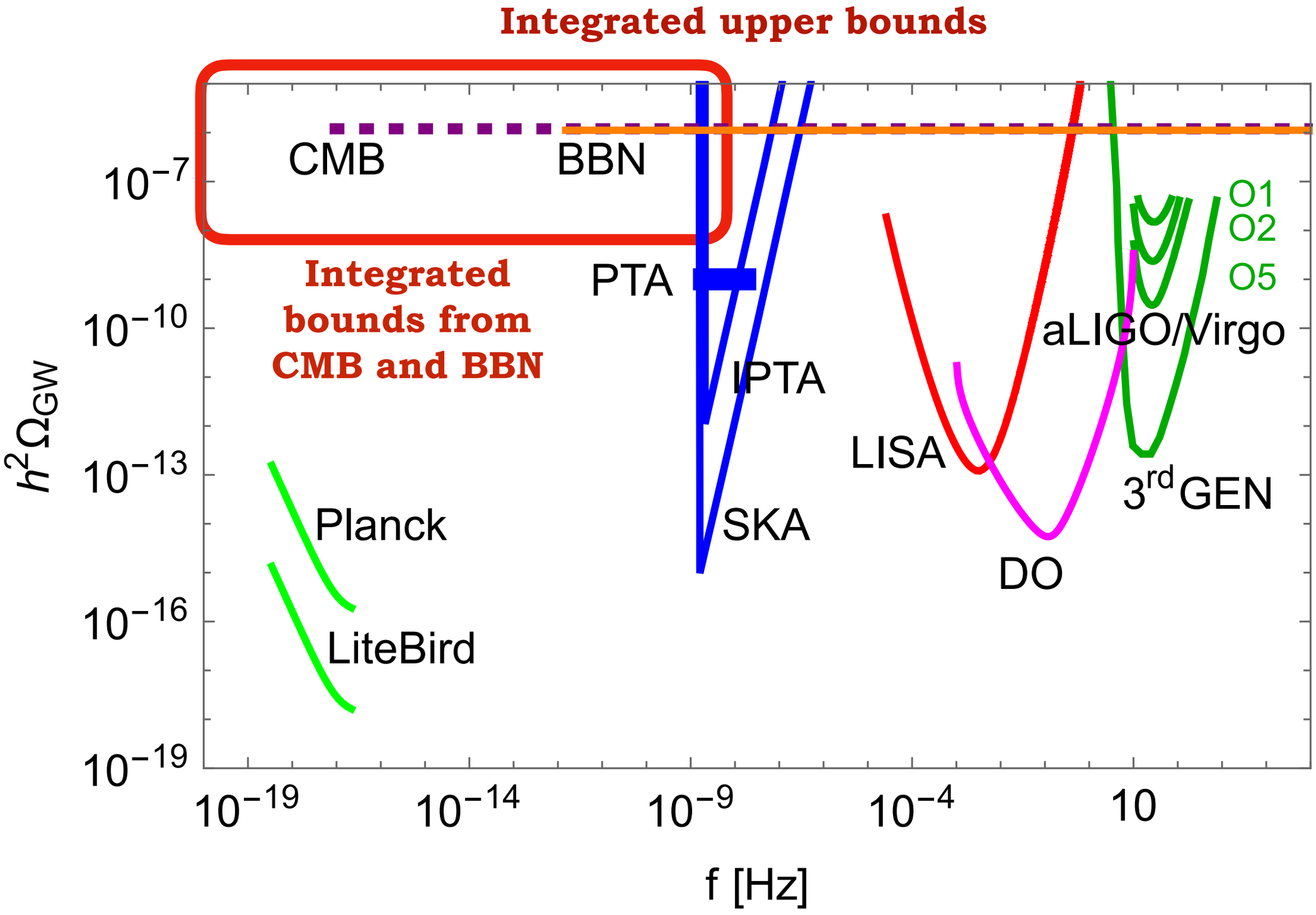
What is/will be known about the SGWB



What is/will be known about the SGWB



What is/will be known about the SGWB

- GW contribute to the energy density in the universe and change its background evolution

$$H^2(T) = \frac{8\pi G}{3}\Sigma_i \rho_i(T)$$

- The abundances of elements produced at Big Bang Nucleosynthesis (BBN) depend on the relative abundance of neutrons and protons, which depends on the Hubble scale at T ~ MeV

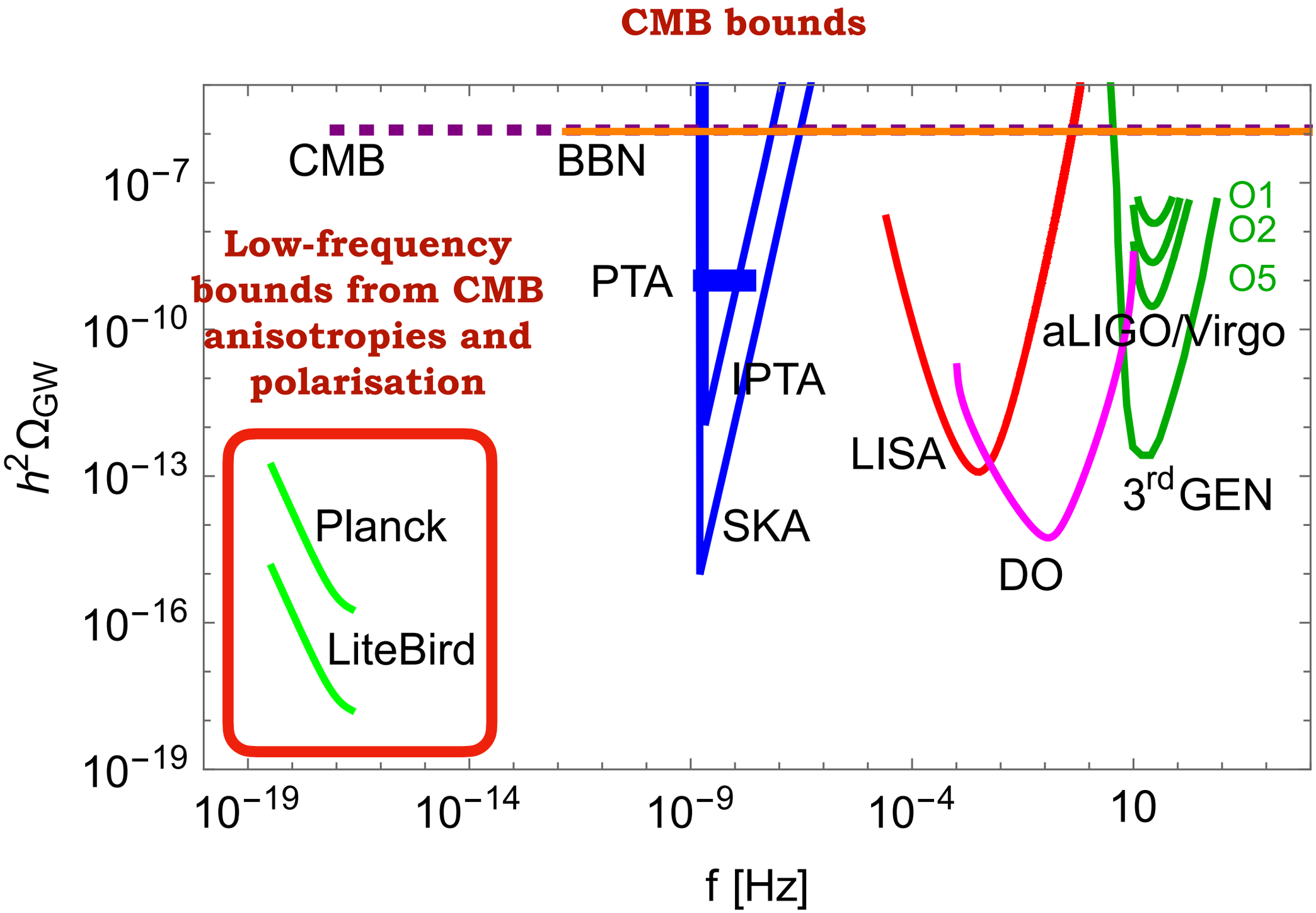
- The Cosmic Microwave Background (CMB) monopole and anisotropy spectrum depend on the Hubble scale at decoupling T ~ 0.3 eV, on the matter-radiation equality…

- Bounds on the *integrated GW energy density* at/previous to the BBN and CMB epochs

$$\left(\frac{\rho_{\rm GW}}{\rho_c}\right)_0 = \int \frac{df}{f}\,\Omega_{\rm GW}(f) = \Omega_\gamma^0 \left(\frac{g_S(T_0)}{g_S(T)}\right)^{4/3} \left(\frac{\rho_{\rm GW}}{\rho_\gamma}\right)_T$$

CAREFUL! Plot “wrong”…

What is/will be known about the SGWB



Cosmic microwave background

frequency range of detection: 10^{-18} Hz < f < 10^{-16} HZ

- temperature anisotropy: limit by Planck

$$\frac{\delta T}{T} = -\int_{t_{\rm dec}}^{t_0} \dot{h}_{ij}\, n^i n^j\, dt$$

$$P_h(k) = A_t(k_0)\,(k\,\eta_0)^{n_t}$$

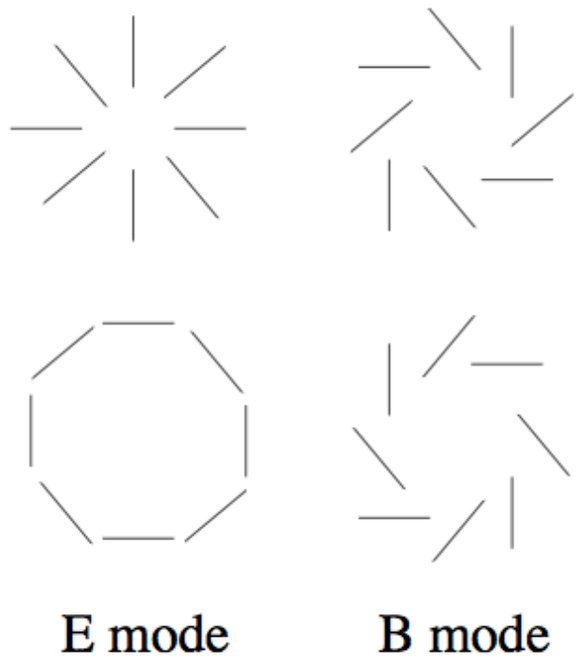
$$C_{\ell,T}^{\Theta\Theta} \simeq \frac{\sqrt{\pi}}{3}\, A_t(k_0)\, \frac{\ell(\ell+2)!}{(\ell-2)!}\, \frac{\Gamma\left[\frac{7-n_t}{2}\right]\Gamma\left[\ell+\frac{n_t}{2}\right]}{\Gamma\left[4-\frac{n_t}{2}\right]\Gamma\left[\ell+7-\frac{n_t}{2}\right]}$$

$$\propto\ \ell^{n_t-2} \qquad \text{for}\ \ 1 \ll \ell \lesssim 60$$

- polarisation: BB spectrum measured by BICEP2 and Planck generated at photon decoupling time, from Thomson scattering of electrons by a quadrupole temperature anisotropy in the photons

generated by primordial scalar and tensor perturbations

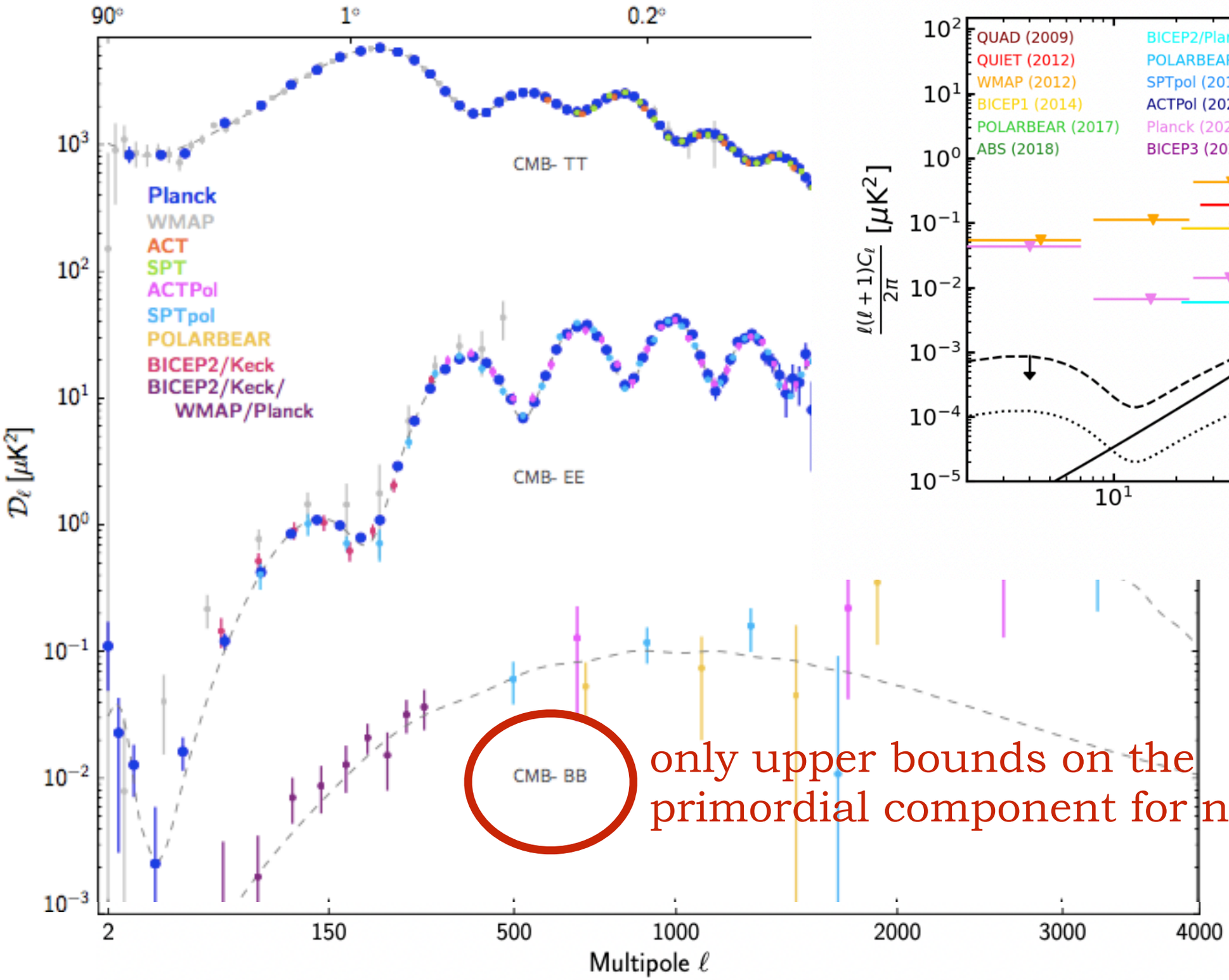
polarisation patterns



generated only by primordial tensor perturbations or by foregrounds

Cosmic microwave background

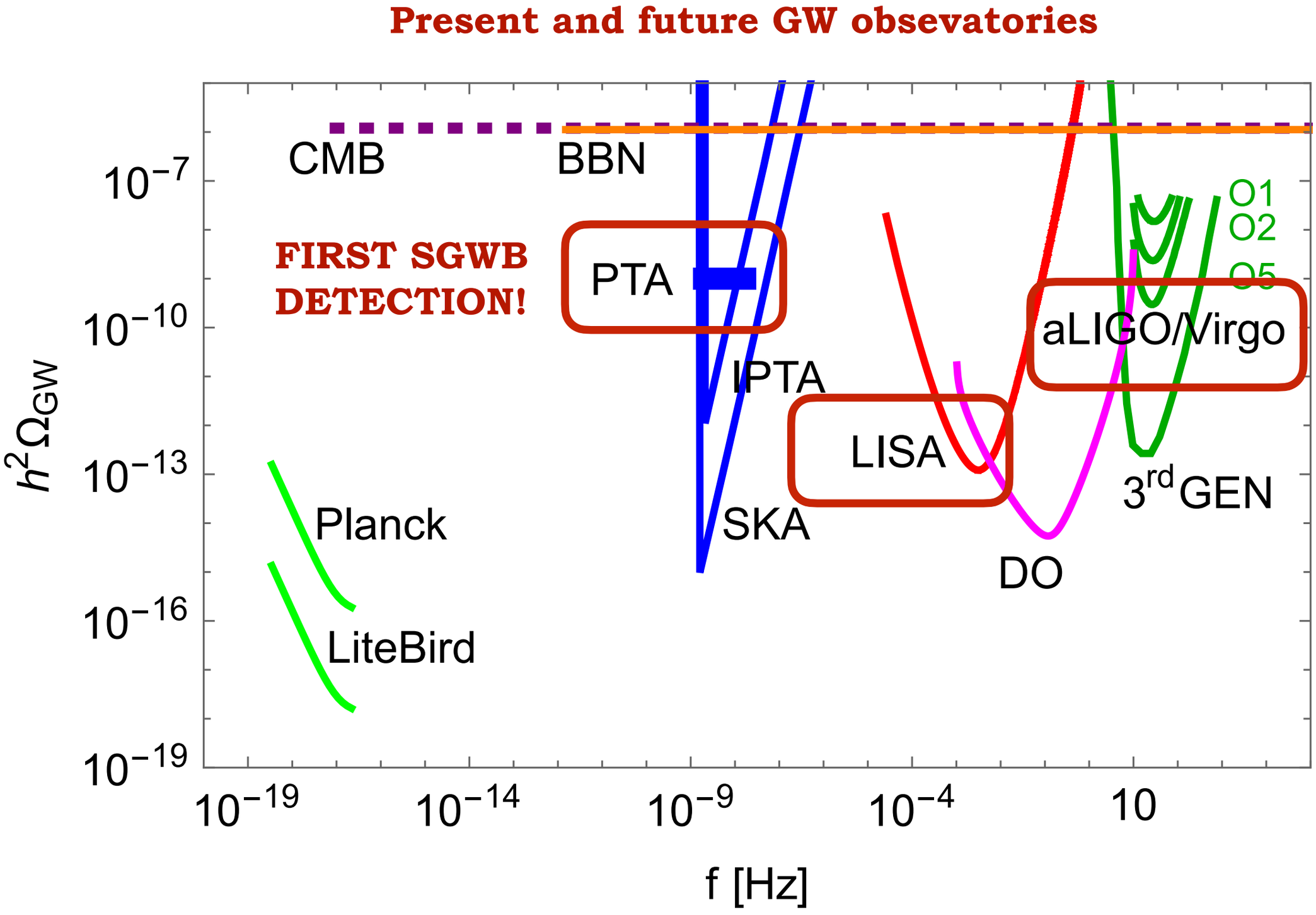
Galloni et al: arXiv:2208.00188



only upper bounds on the primordial component for now

Planck collaboration: arXiv:1807.06205

What is/will be known about the SGWB



Earth-based interferometers

aLIGO/aVirgo (operating)

arm length L = 4 km

frequency range of detection:
10 Hz < f < 5kHZ

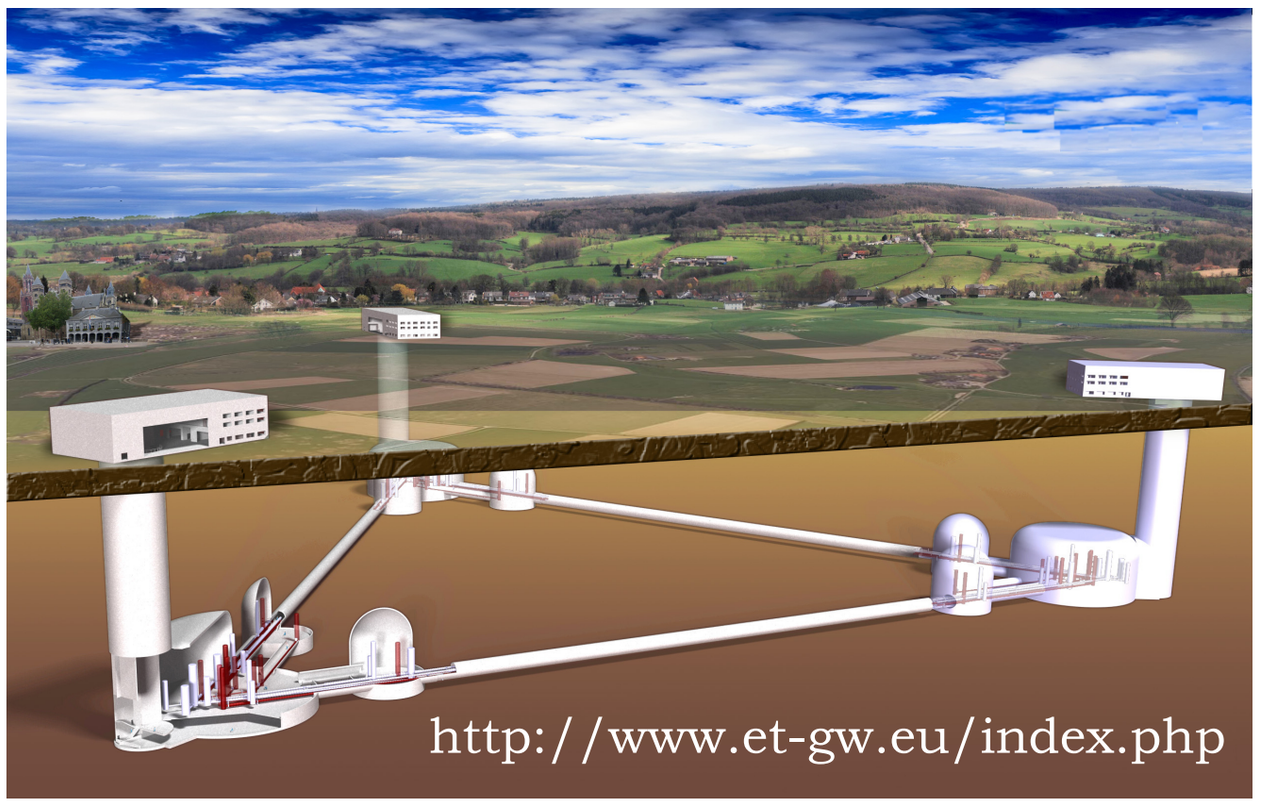
3rd generation (ET, CE… coming next)

arm length L = 3 km

frequency range of detection:
1 Hz < f < 10^4 HZ

DETECTION TARGETS:

- Black hole coalescing binaries of masses few to hundred solar masses (BHBs)
- Neutron Star and NS-BH binaries / SN explosions
- Stochastic GW background



Earth-based interferometers

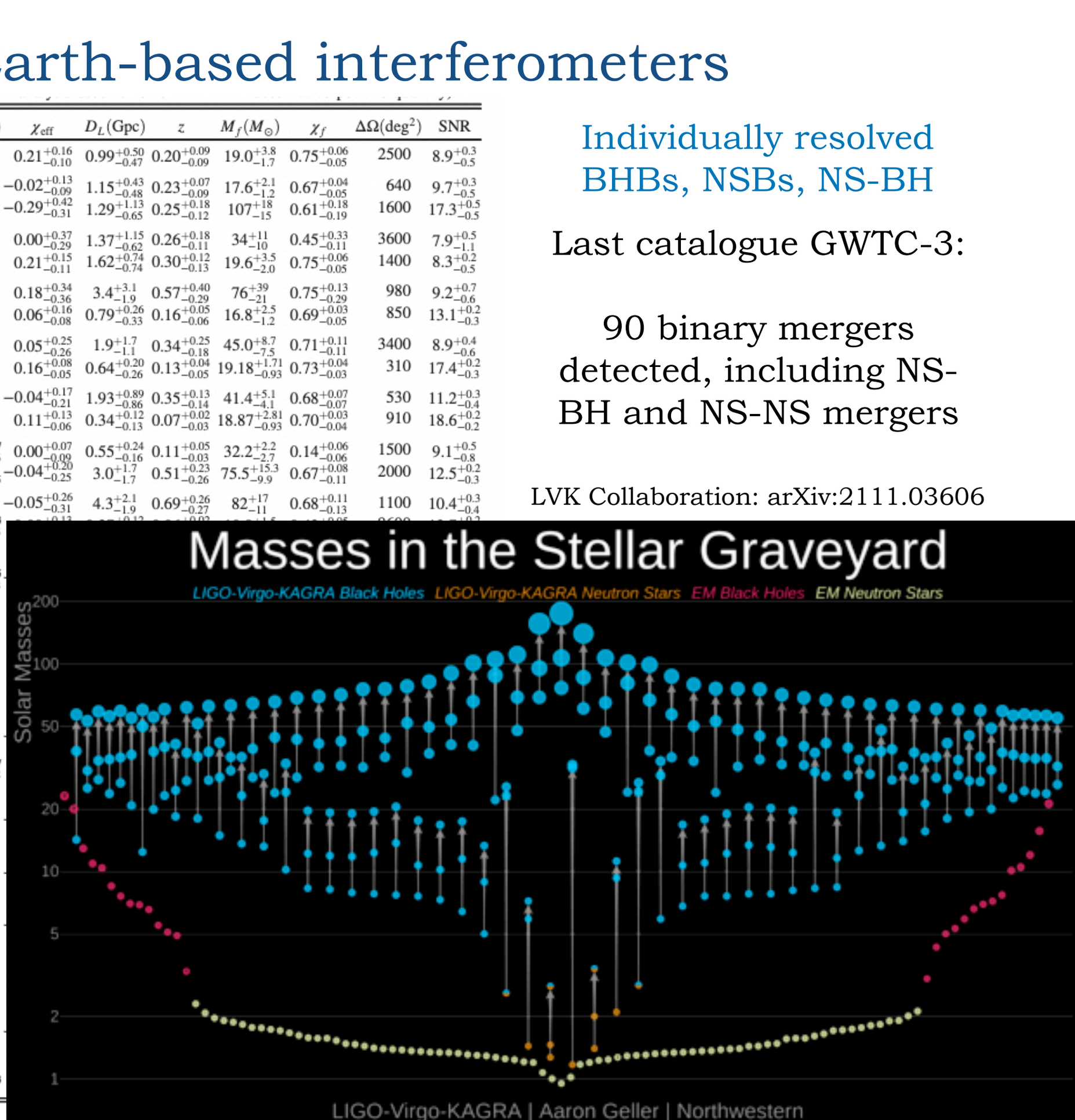
Candidate	$M(M_\odot)$	$\mathcal{M}(M_\odot)$	$m_1(M_\odot)$	$m_2(M_\odot)$	$\chi_{\rm eff}$	D_L(Gpc)	z	$M_f(M_\odot)$	χ_f	$\Delta\Omega$(deg^2)	SNR
GW191103_012549	$20.0^{+3.7}_{-1.8}$	$8.34^{+0.66}_{-0.57}$	$11.8^{+6.2}_{-2.2}$	$7.9^{+1.7}_{-2.4}$	$0.21^{+0.16}_{-0.10}$	$0.99^{+0.50}_{-0.47}$	$0.20^{+0.09}_{-0.09}$	$19.0^{+3.8}_{-1.7}$	$0.75^{+0.06}_{-0.05}$	2500	$8.9^{+0.3}_{-0.5}$
GW191105_143521	$18.5^{+2.1}_{-1.3}$	$7.82^{+0.61}_{-0.45}$	$10.7^{+3.7}_{-1.6}$	$7.7^{+1.4}_{-1.9}$	$-0.02^{+0.13}_{-0.09}$	$1.15^{+0.43}_{-0.48}$	$0.23^{+0.07}_{-0.09}$	$17.6^{+2.1}_{-1.2}$	$0.67^{+0.04}_{-0.05}$	640	$9.7^{+0.3}_{-0.5}$
GW191109_010717	112^{+20}_{-16}	$47.5^{+9.6}_{-7.5}$	65^{+11}_{-11}	47^{+15}_{-13}	$-0.29^{+0.42}_{-0.31}$	$1.29^{+1.13}_{-0.65}$	$0.25^{+0.18}_{-0.12}$	107^{+18}_{-15}	$0.61^{+0.18}_{-0.19}$	1600	$17.3^{+0.5}_{-0.5}$
GW191113_071753	$34.5^{+10.5}_{-9.8}$	$10.7^{+1.1}_{-1.0}$	29^{+12}_{-14}	$5.9^{+4.4}_{-1.3}$	$0.00^{+0.37}_{-0.29}$	$1.37^{+1.15}_{-0.62}$	$0.26^{+0.18}_{-0.11}$	34^{+11}_{-10}	$0.45^{+0.33}_{-0.11}$	3600	$7.9^{+0.5}_{-1.1}$
GW191126_115259	$20.7^{+3.4}_{-2.0}$	$8.65^{+0.95}_{-0.71}$	$12.1^{+5.5}_{-2.2}$	$8.3^{+1.9}_{-2.4}$	$0.21^{+0.15}_{-0.11}$	$1.62^{+0.74}_{-0.74}$	$0.30^{+0.12}_{-0.13}$	$19.6^{+3.5}_{-2.0}$	$0.75^{+0.06}_{-0.05}$	1400	$8.3^{+0.2}_{-0.5}$
GW191127_050227	80^{+39}_{-22}	$29.9^{+11.7}_{-9.1}$	53^{+47}_{-20}	24^{+17}_{-14}	$0.18^{+0.34}_{-0.36}$	$3.4^{+3.1}_{-1.9}$	$0.57^{+0.40}_{-0.29}$	76^{+39}_{-21}	$0.75^{+0.13}_{-0.29}$	980	$9.2^{+0.7}_{-0.6}$
GW191129_134029	$17.5^{+2.4}_{-1.2}$	$7.31^{+0.43}_{-0.28}$	$10.7^{+4.1}_{-2.1}$	$6.7^{+1.5}_{-1.7}$	$0.06^{+0.16}_{-0.08}$	$0.79^{+0.26}_{-0.33}$	$0.16^{+0.05}_{-0.06}$	$16.8^{+2.5}_{-1.2}$	$0.69^{+0.03}_{-0.05}$	850	$13.1^{+0.2}_{-0.3}$
GW191204_110529	$47.1^{+9.1}_{-7.8}$	$19.8^{+3.6}_{-3.2}$	$27.3^{+10.8}_{-5.9}$	$19.2^{+5.5}_{-6.0}$	$0.05^{+0.25}_{-0.26}$	$1.9^{+1.7}_{-1.1}$	$0.34^{+0.25}_{-0.18}$	$45.0^{+8.7}_{-7.5}$	$0.71^{+0.11}_{-0.11}$	3400	$8.9^{+0.4}_{-0.6}$
GW191204_171526	$20.19^{+1.64}_{-0.95}$	$8.56^{+0.41}_{-0.28}$	$11.7^{+3.3}_{-1.7}$	$8.4^{+1.3}_{-1.7}$	$0.16^{+0.08}_{-0.05}$	$0.64^{+0.20}_{-0.26}$	$0.13^{+0.04}_{-0.05}$	$19.18^{+1.71}_{-0.93}$	$0.73^{+0.04}_{-0.03}$	310	$17.4^{+0.2}_{-0.3}$
GW191215_223052	$43.3^{+5.3}_{-4.3}$	$18.4^{+2.2}_{-1.7}$	$24.9^{+7.1}_{-4.1}$	$18.1^{+3.8}_{-4.1}$	$-0.04^{+0.17}_{-0.21}$	$1.93^{+0.89}_{-0.86}$	$0.35^{+0.13}_{-0.14}$	$41.4^{+5.1}_{-4.1}$	$0.68^{+0.07}_{-0.07}$	530	$11.2^{+0.3}_{-0.4}$
GW191216_213338	$19.80^{+2.70}_{-0.93}$	$8.33^{+0.22}_{-0.19}$	$12.1^{+4.6}_{-2.2}$	$7.7^{+1.6}_{-1.9}$	$0.11^{+0.13}_{-0.06}$	$0.34^{+0.12}_{-0.13}$	$0.07^{+0.02}_{-0.03}$	$18.87^{+2.81}_{-0.93}$	$0.70^{+0.03}_{-0.04}$	910	$18.6^{+0.2}_{-0.2}$
GW191219_163120	$32.3^{+2.2}_{-2.7}$	$4.31^{+0.12}_{-0.17}$	$31.1^{+2.2}_{-2.8}$	$1.17^{+0.07}_{-0.06}$	$0.00^{+0.07}_{-0.09}$	$0.55^{+0.24}_{-0.16}$	$0.11^{+0.05}_{-0.03}$	$32.2^{+2.2}_{-2.7}$	$0.14^{+0.06}_{-0.06}$	1500	$9.1^{+0.5}_{-0.8}$
GW191222_033537	79^{+16}_{-11}	$33.8^{+7.1}_{-5.0}$	$45.1^{+10.9}_{-8.0}$	$34.7^{+9.3}_{-10.5}$	$-0.04^{+0.20}_{-0.25}$	$3.0^{+1.7}_{-1.7}$	$0.51^{+0.23}_{-0.26}$	$75.5^{+15.3}_{-9.9}$	$0.67^{+0.08}_{-0.11}$	2000	$12.5^{+0.2}_{-0.3}$
GW191230_180458	86^{+19}_{-12}	$36.5^{+8.2}_{-5.6}$	$49.4^{+14.0}_{-9.6}$	37^{+11}_{-12}	$-0.05^{+0.26}_{-0.31}$	$4.3^{+2.1}_{-1.9}$	$0.69^{+0.26}_{-0.27}$	82^{+17}_{-11}	$0.68^{+0.11}_{-0.13}$	1100	$10.4^{+0.3}_{-0.4}$
GW200105_162426	$11.0^{+1.5}_{-1.4}$	$3.42^{+0.08}_{-0.08}$	$9.1^{+1.7}_{-1.7}$	$1.91^{+0.33}_{-0.24}$							
GW200112_155838	$63.9^{+5.7}_{-4.6}$	$27.4^{+2.6}_{-2.1}$	$35.6^{+6.7}_{-4.5}$	$28.3^{+4.4}_{-5.9}$							
GW200115_042309	$7.4^{+1.7}_{-1.7}$	$2.43^{+0.05}_{-0.07}$	$5.9^{+2.0}_{-2.5}$	$1.44^{+0.85}_{-0.28}$							
GW200128_022011	75^{+17}_{-12}	$32.0^{+7.5}_{-5.5}$	$42.2^{+11.6}_{-8.1}$	$32.6^{+9.5}_{-9.2}$							
GW200129_065458	$63.3^{+4.5}_{-3.4}$	$27.2^{+2.1}_{-2.3}$	$34.5^{+9.9}_{-3.1}$	$29.0^{+3.3}_{-9.3}$							
GW200202_154313	$17.58^{+1.78}_{-0.67}$	$7.49^{+0.24}_{-0.20}$	$10.1^{+3.5}_{-1.4}$	$7.3^{+1.1}_{-1.7}$							
GW200208_130117	$65.3^{+8.1}_{-6.8}$	$27.7^{+3.7}_{-3.1}$	$37.7^{+9.3}_{-6.2}$	$27.4^{+6.3}_{-7.3}$							
GW200208_222617	63^{+100}_{-26}	$19.8^{+10.5}_{-5.2}$	51^{+103}_{-30}	$12.3^{+9.2}_{-5.5}$							
GW200209_085452	$62.6^{+13.9}_{-9.4}$	$26.7^{+6.0}_{-4.2}$	$35.6^{+10.5}_{-6.8}$	$27.1^{+7.8}_{-7.8}$							
GW200210_092254	$27.0^{+7.1}_{-4.3}$	$6.56^{+0.38}_{-0.40}$	$24.1^{+7.5}_{-4.6}$	$2.83^{+0.47}_{-0.42}$							
GW200216_220804	81^{+20}_{-14}	$32.9^{+9.3}_{-8.5}$	51^{+22}_{-13}	30^{+14}_{-16}							
GW200219_094415	$65.0^{+12.6}_{-8.2}$	$27.6^{+5.6}_{-3.8}$	$37.5^{+10.1}_{-6.9}$	$27.9^{+7.4}_{-8.4}$							
GW200220_061928	148^{+55}_{-33}	62^{+23}_{-15}	87^{+40}_{-23}	61^{+26}_{-25}							
GW200220_124850	67^{+17}_{-12}	$28.2^{+7.3}_{-5.1}$	$38.9^{+14.1}_{-8.6}$	$27.9^{+9.2}_{-9.0}$							
GW200224_222234	$72.3^{+7.2}_{-5.3}$	$31.1^{+3.3}_{-2.7}$	$40.0^{+6.7}_{-4.5}$	$32.7^{+4.8}_{-7.2}$							
GW200225_060421	$33.5^{+3.6}_{-3.0}$	$14.2^{+1.5}_{-1.4}$	$19.3^{+5.0}_{-3.0}$	$14.0^{+2.8}_{-3.5}$							
GW200302_015811	$57.8^{+9.6}_{-6.9}$	$23.4^{+4.7}_{-3.0}$	$37.8^{+8.7}_{-8.5}$	$20.0^{+8.1}_{-5.7}$							
GW200306_093714	$43.9^{+11.8}_{-7.5}$	$17.5^{+3.5}_{-3.0}$	$28.3^{+17.1}_{-7.7}$	$14.8^{+6.5}_{-6.4}$							
GW200308_173609*	92^{+169}_{-48}	34^{+44}_{-18}	60^{+166}_{-29}	24^{+36}_{-13}							
GW200311_115853	$61.9^{+5.3}_{-4.2}$	$26.6^{+2.4}_{-2.0}$	$34.2^{+6.4}_{-3.8}$	$27.7^{+4.1}_{-5.9}$							
GW200316_215756	$21.2^{+7.2}_{-2.0}$	$8.75^{+0.62}_{-0.55}$	$13.1^{+10.2}_{-2.9}$	$7.8^{+2.0}_{-2.9}$							
GW200322_091133*	50^{+132}_{-22}	$15.0^{+29.5}_{-4.0}$	38^{+130}_{-22}	$11.3^{+24.3}_{-6.0}$							

Individually resolved BHBs, NSBs, NS-BH

Last catalogue GWTC-3:

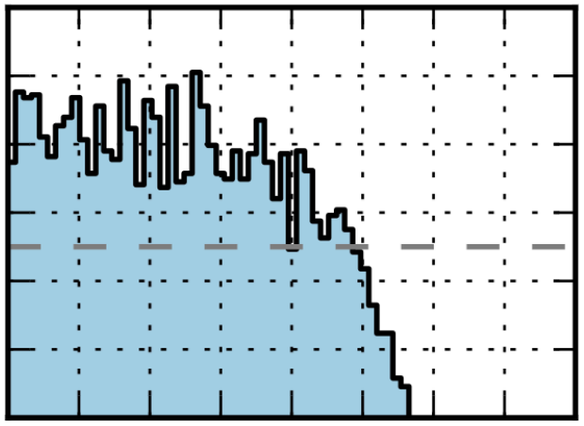
90 binary mergers detected, including NS-BH and NS-NS mergers

LVK Collaboration: arXiv:2111.03606



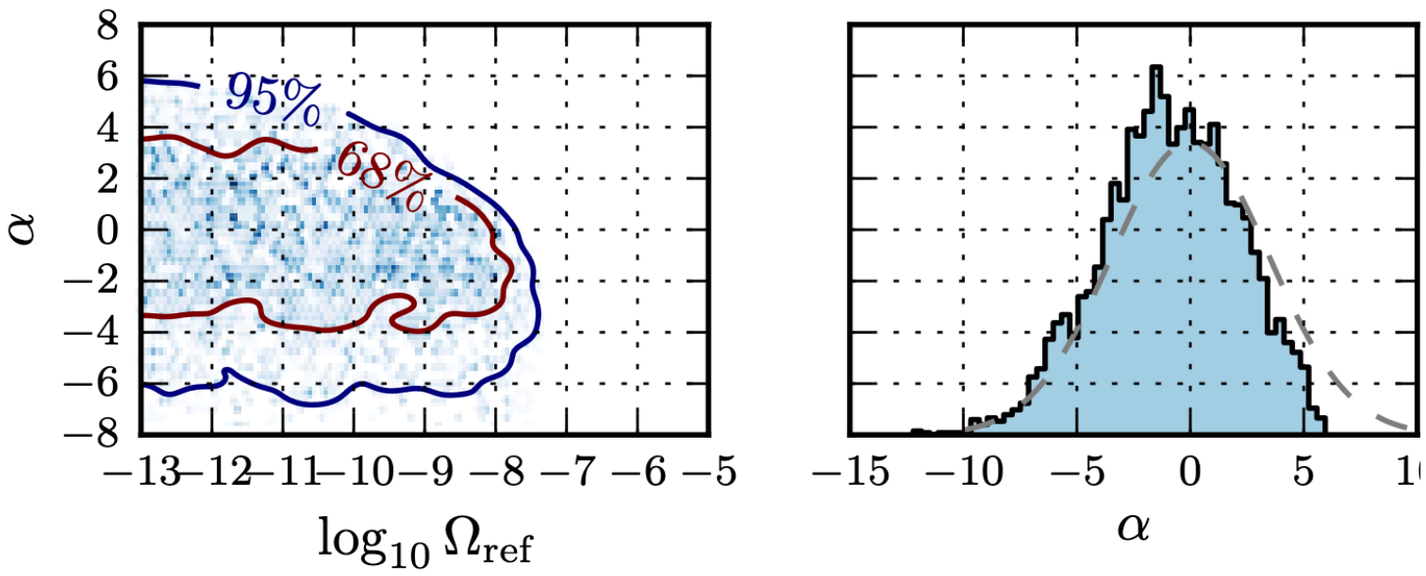
Earth-based interferometers

Stochastic GW background: for now, only upper bounds



Upper bound on generic

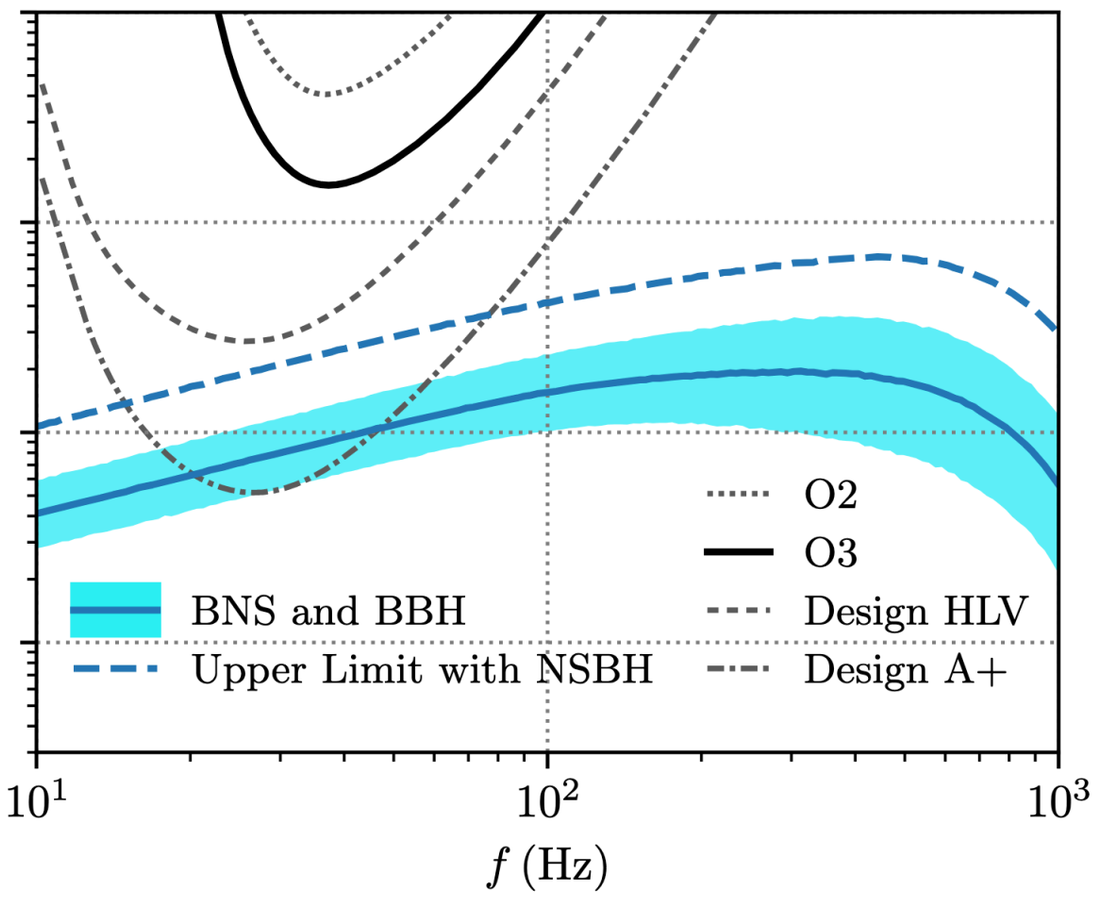
$$\Omega_{\rm GW}(f) = \Omega_{\rm ref} \left(\frac{f}{25\,{\rm Hz}} \right)^{\alpha}$$



Detection via cross-correlation of signals from different detectors

LVK Collaborations, arXiv:2101.12130

SGWB from BHBs and NSBs



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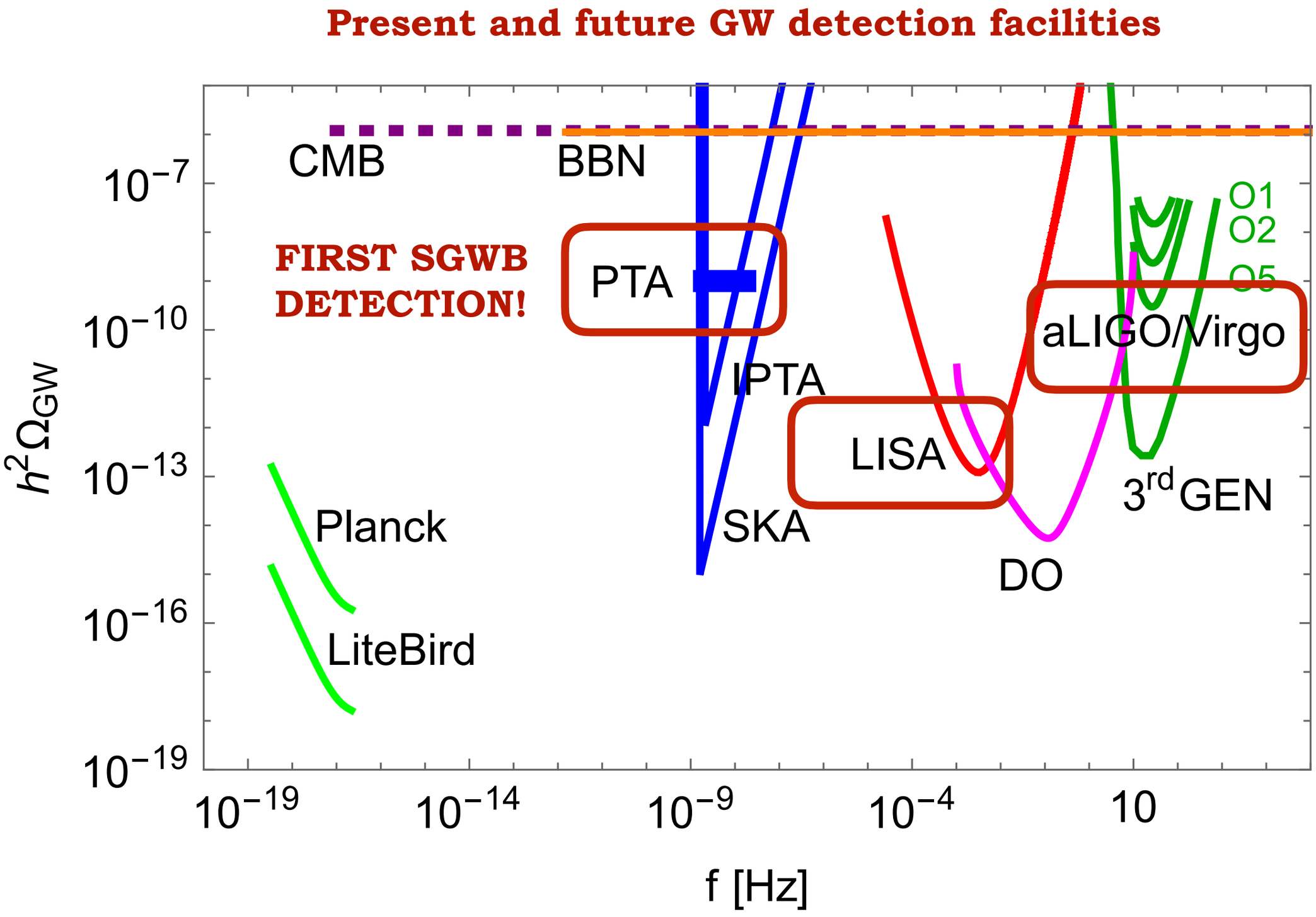


Space-based interferometers: LISA

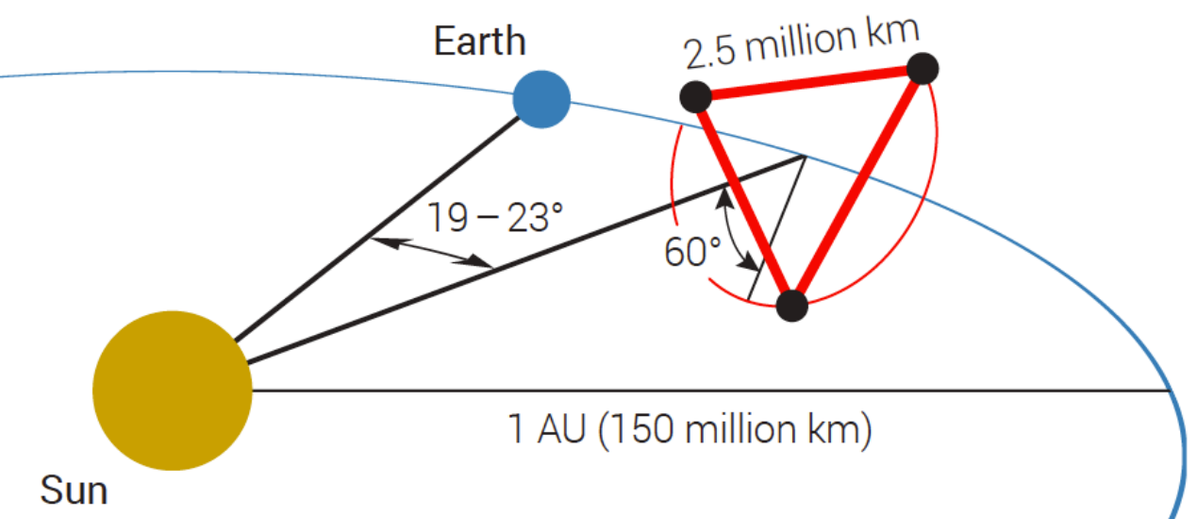
LISA: Laser Interferometer Space Antenna

- no seismic noise
- much longer arms than on Earth

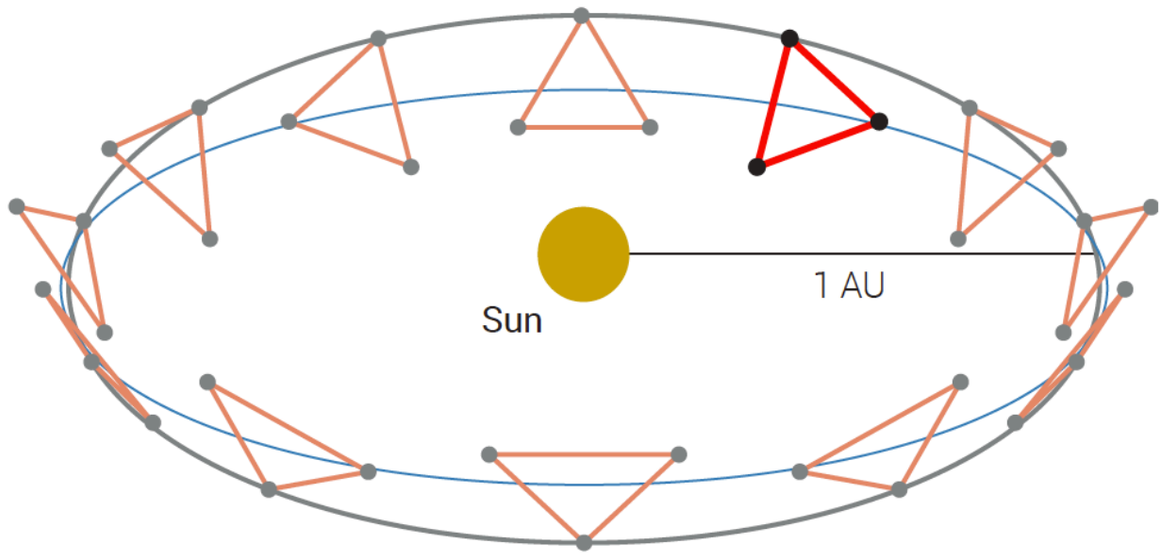
frequency range of detection:

$$10^{-4}\,\mathrm{Hz} < f < 1\,\mathrm{Hz}$$

- Launch in ~2034
- two masses in free fall per spacecraft
- 2.5 million km arms
- picometer displacement of masses

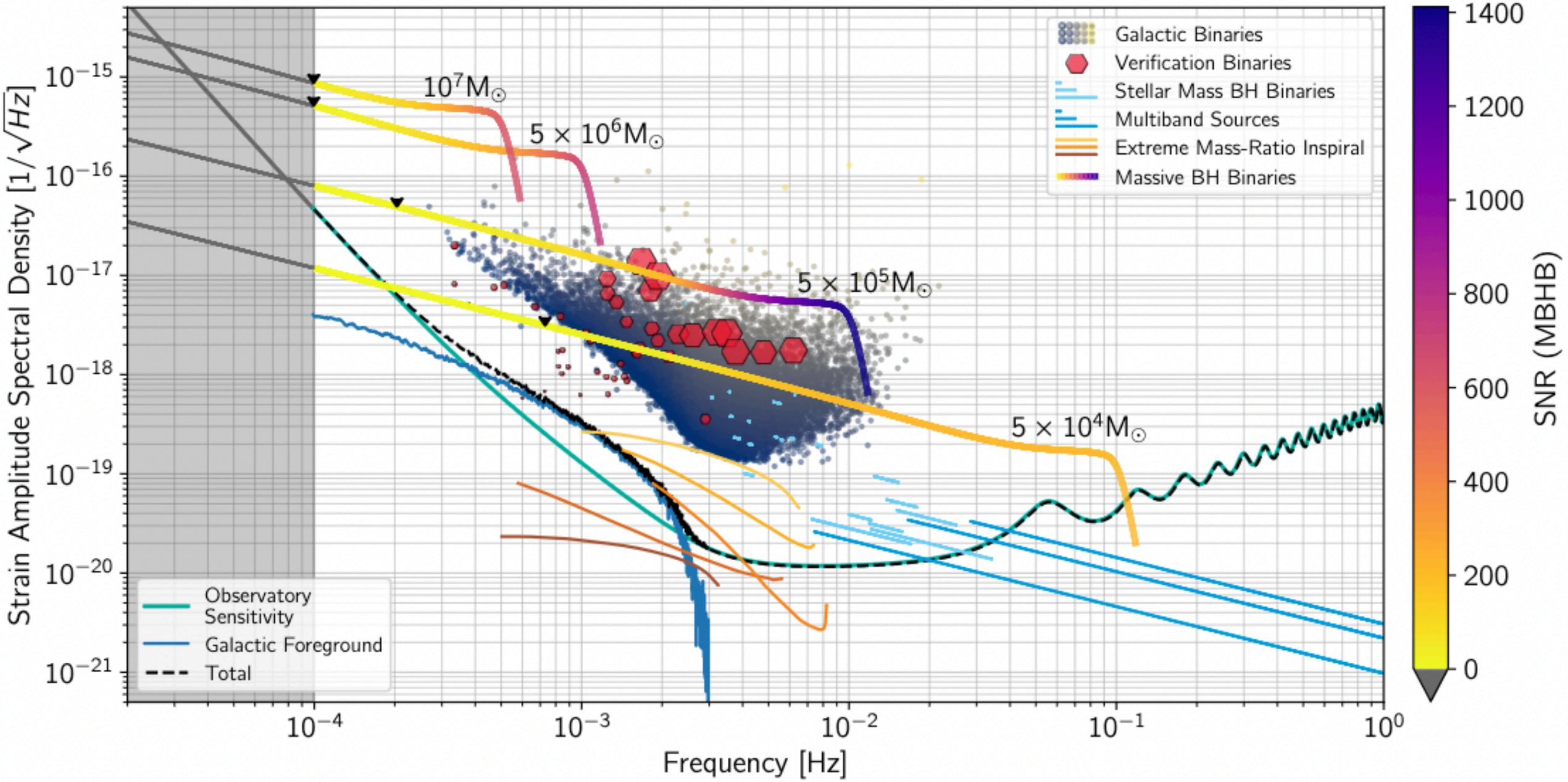


LISA Consortium arXiv:1702.00786



Space-based interferometers: LISA

DETECTION TARGETS:



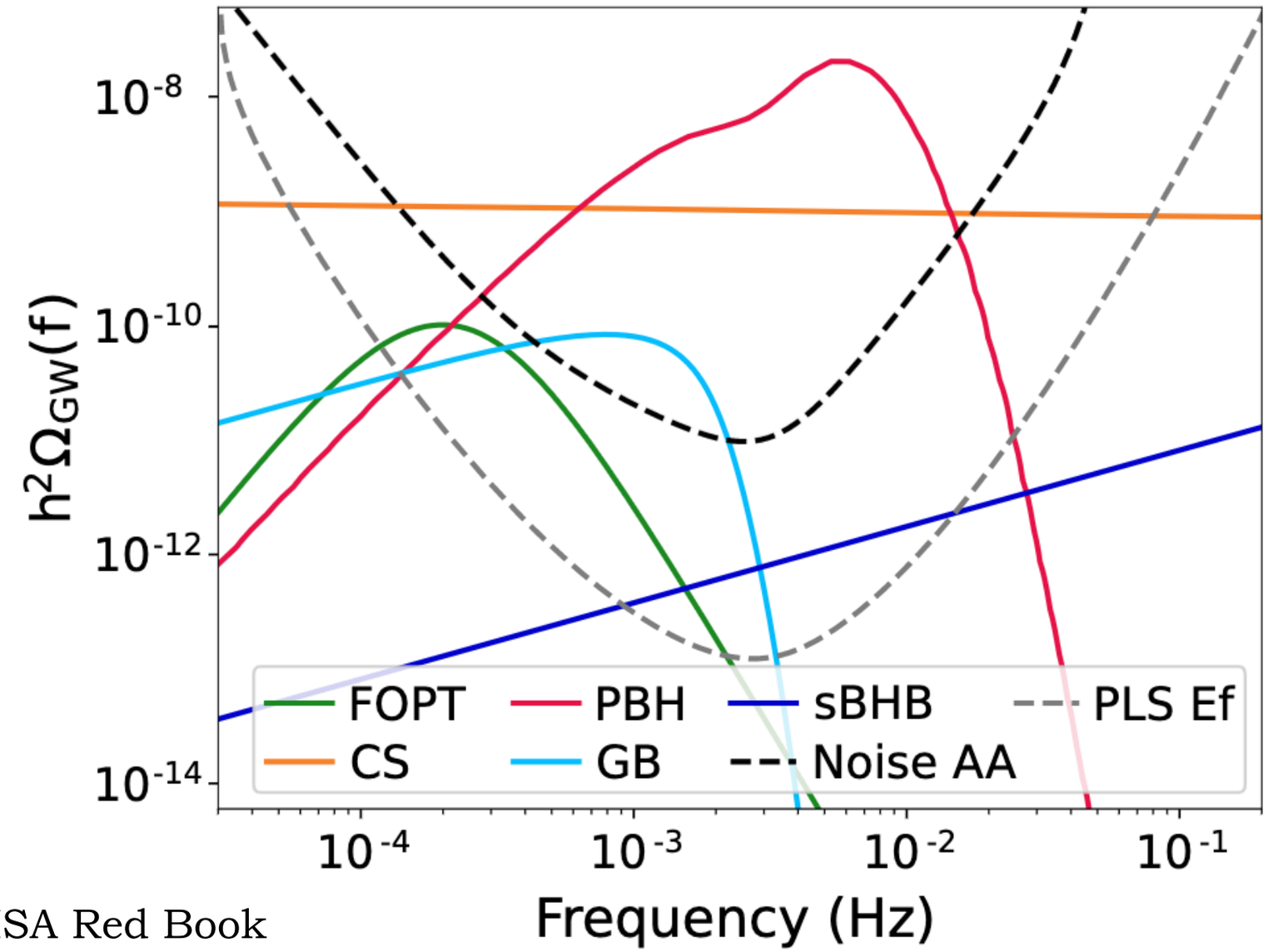
M. Colpi et al, arXiv:2402.07571 - LISA Red Book

Space-based interferometers: LISA

Stochastic GW background

- Confusion noise from the binaries in the Galaxy (mainly WD binaries)
- Confusion noise from extra-galactic binaries binaries (WD binaries and BHBs)
- Candidates from the early universe, in particular at the EW scale

Detecting a SGWB with LISA is challenging: no cross-correlation, need to assume knowledge of the noise (possibility of null channels)



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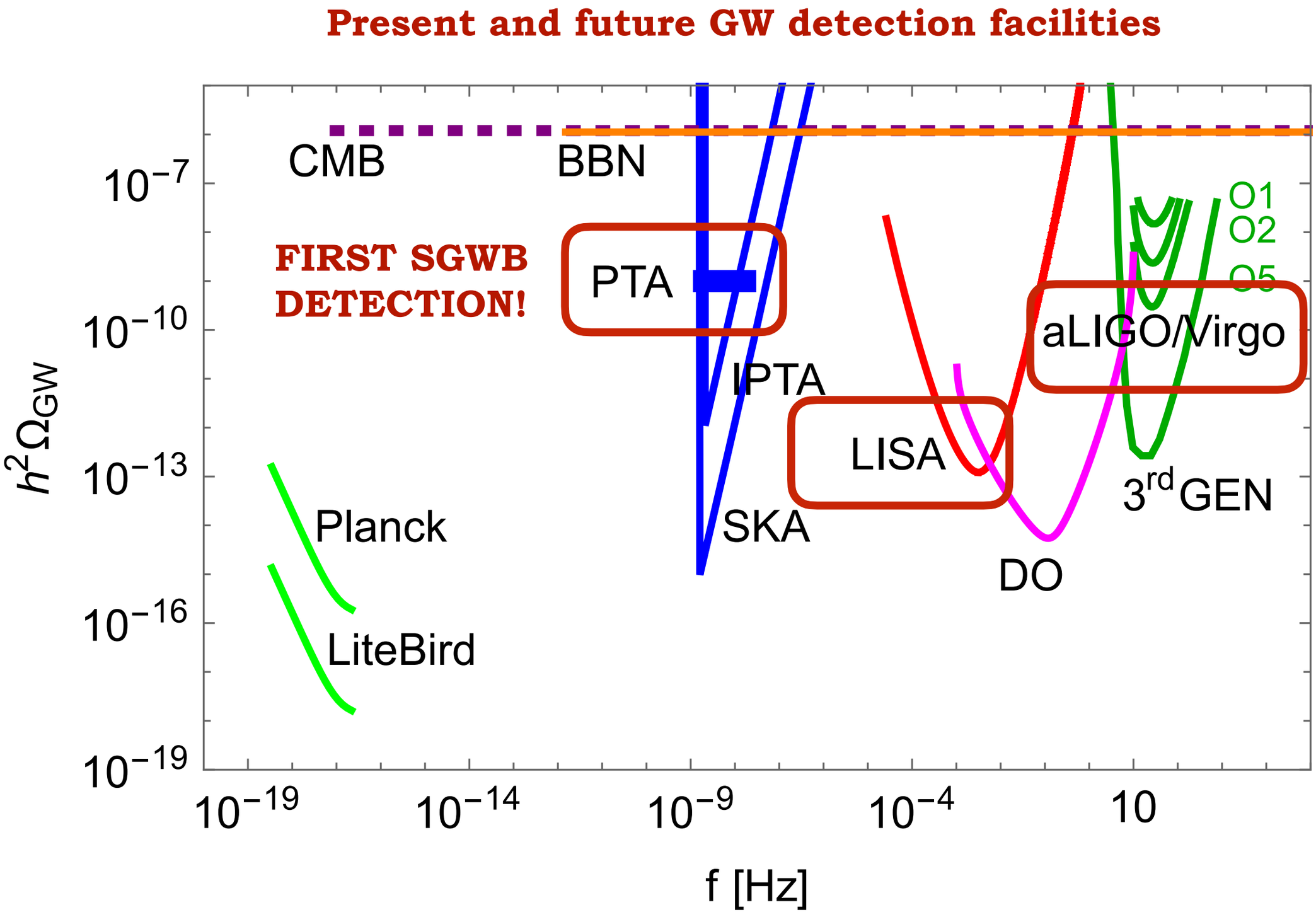


Pulsar timing arrays

CPTA, EPTA, NANOGrav, PPTA -> IPTA

frequency range of detection: 10^{-9} Hz < f < 10^{-7} HZ

- rotating, magnetised neutron stars emitting periodic radio-frequency EM pulses -> can be used as clocks in the sky

- the radio pulses are emitted at very regular time intervals, but their arrival times can be altered by a GW passing between the pulsar and the Earth

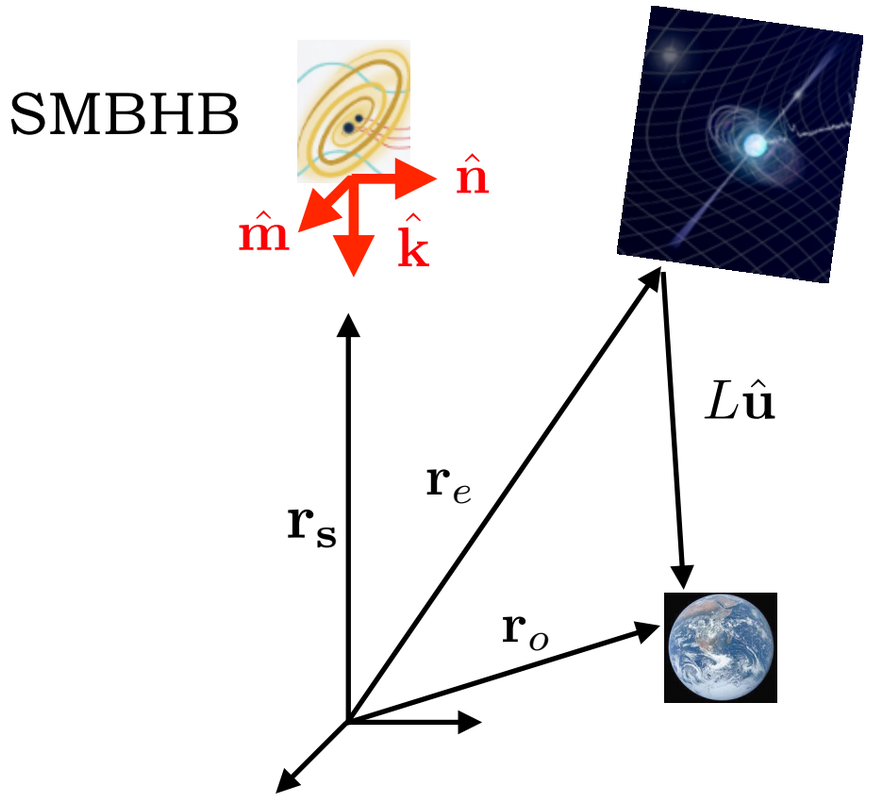
- First a timing model of the pulsar is constructed, which is then compared to observations to infer the *timing residuals* where the GW effect is looked for

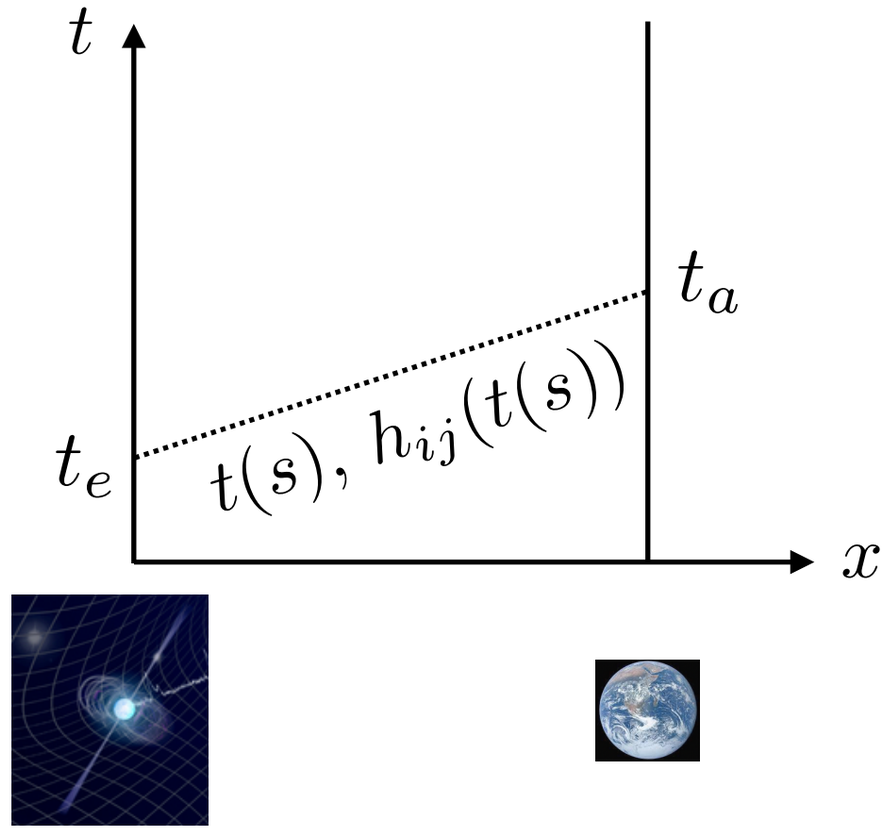
DETECTION TARGETS:

Individual emission and stochastic background from inspiralling Super Massive Black Hole Binaries (SMBHBs) with masses ~10^9 $M_\odot$ at the centre of galaxies

Pulsar timing arrays

Principle of the measurement: gravitational redshift caused by gravitational waves emitted by far-away sources and travelling through spacetime between the pulsars and the Earth





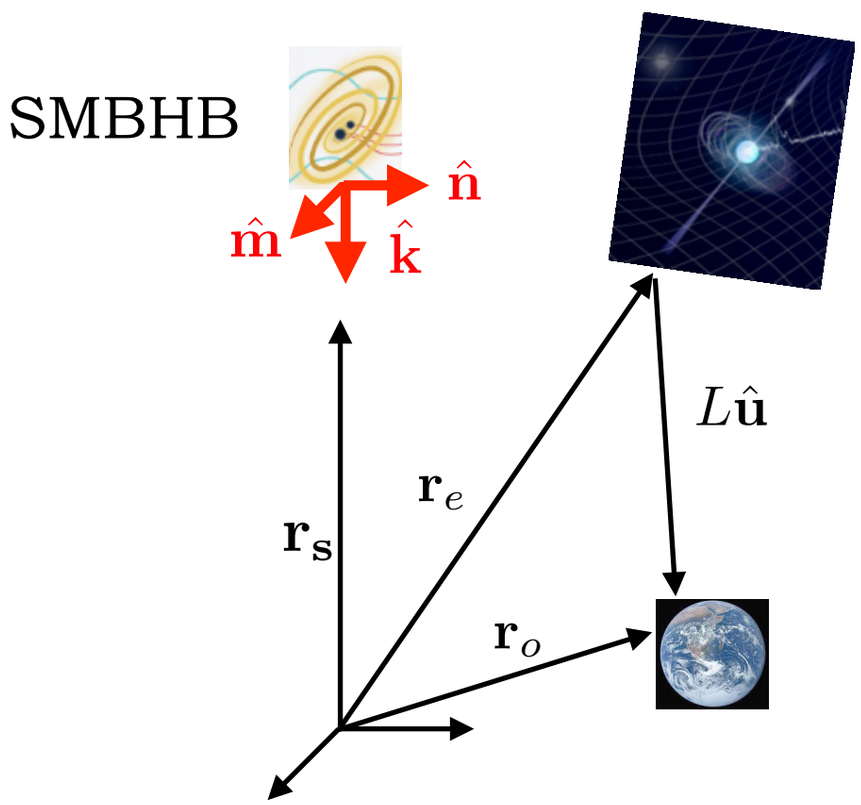
Photon from pulsar:

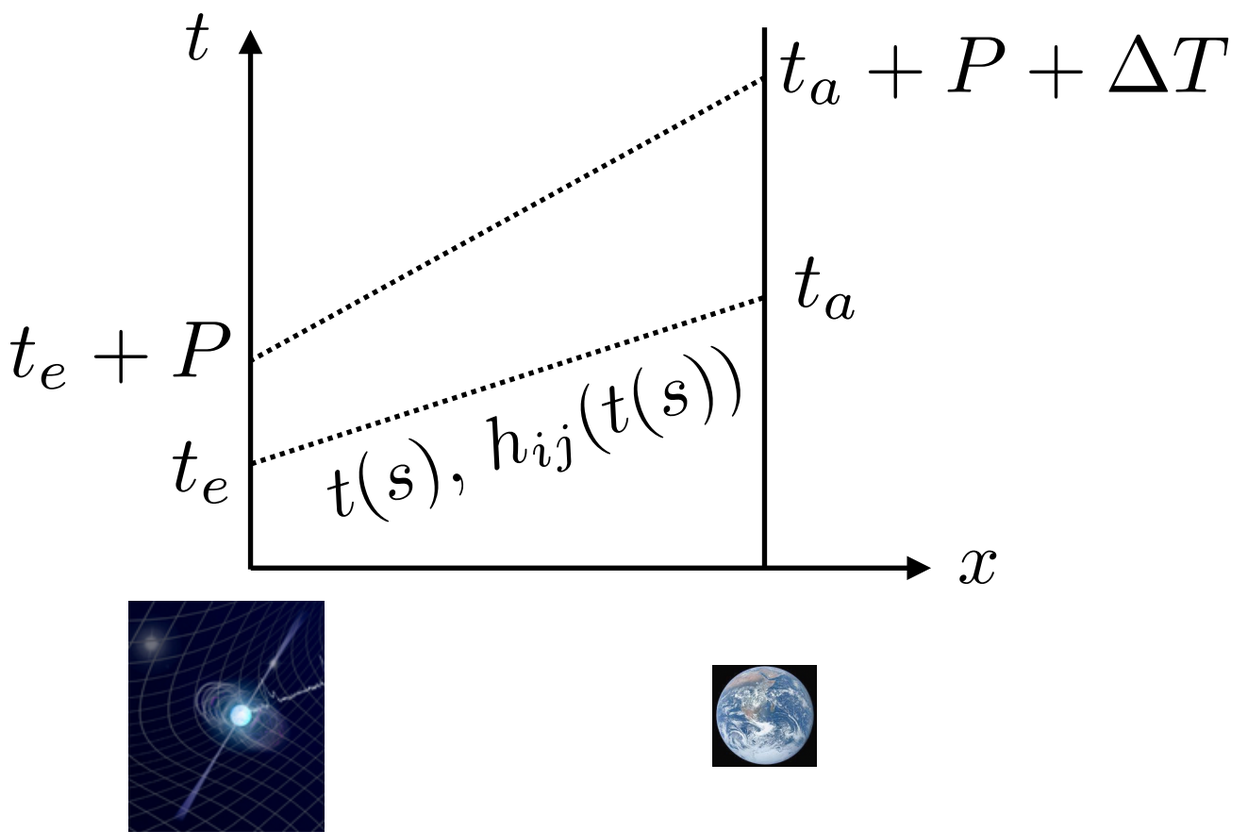
$$dt = \pm\sqrt{(\delta_{ij} + h_{ij})\mathrm{d}x^i \mathrm{d}x^j} \qquad t_a - t_e = L + \frac{1}{2}\hat{u}^i\hat{u}^j \int_0^L \mathrm{d}s\, h_{ij}(t_e + s, \mathbf{r_e} + s\hat{\mathbf{u}})$$

Effect of the metric perturbation on a single beam:
not measurable unless one knows the reference quantities

Pulsar timing arrays

Principle of the measurement: compare with the next pulse





$$\Delta T = \frac{1}{2}\hat{u}^i\hat{u}^j \int_0^L \mathrm{d}s \left[h_{ij}(t_e + P + s, \mathbf{r_e} + s\hat{\mathbf{u}}) - h_{ij}(t_e + s, \mathbf{r_e} + s\hat{\mathbf{u}})\right]$$

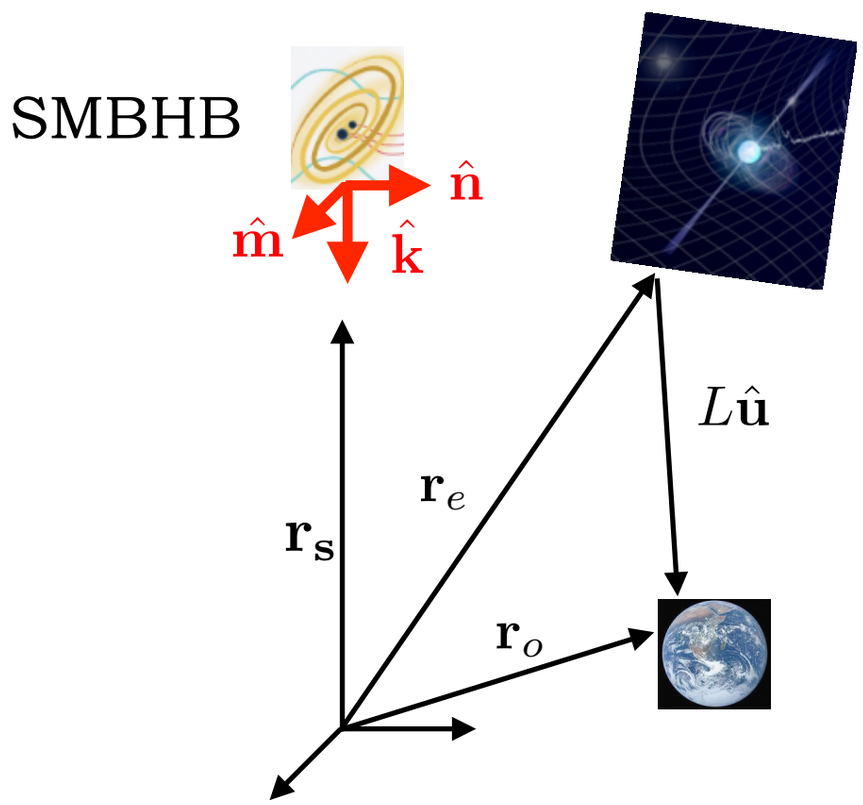
This effect is present only because the gravitational wave is time-dependent

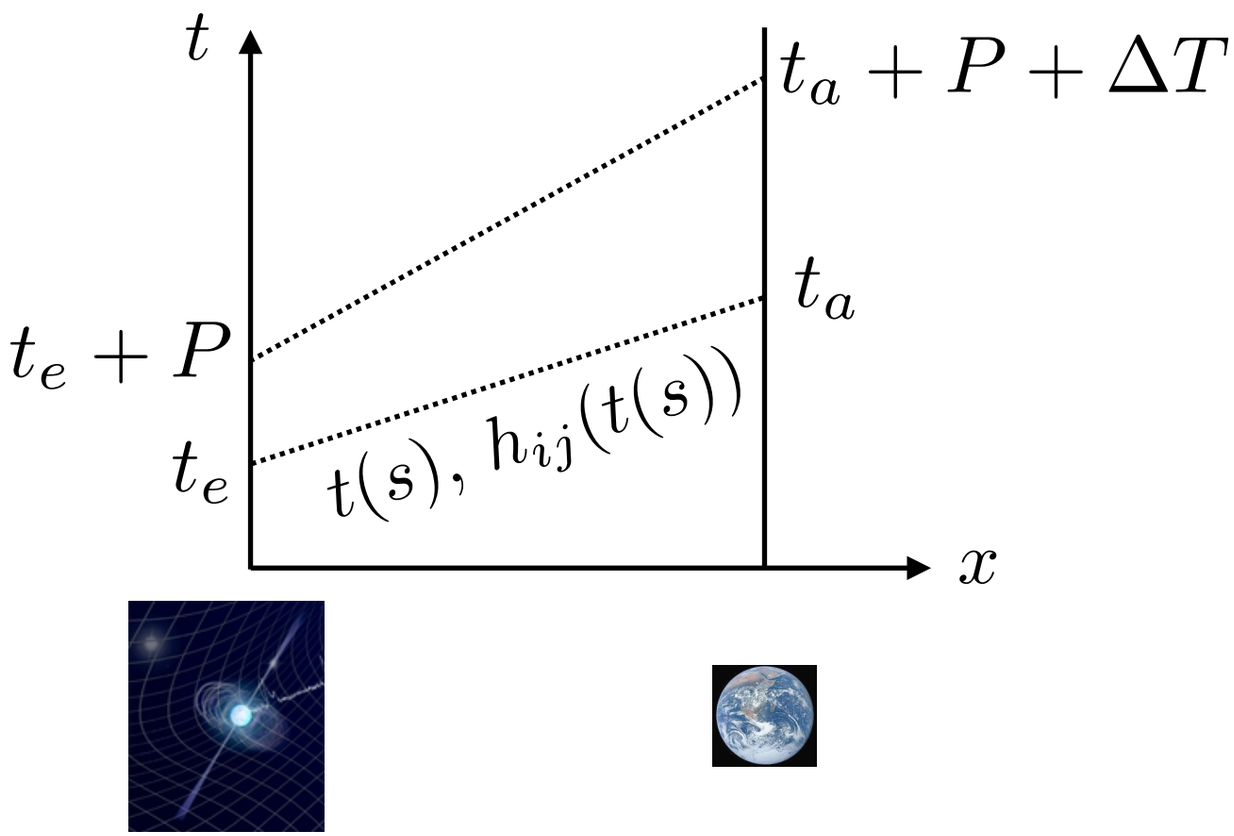
"Gravitational Waves Vol 2",
M. Maggiore, Oxford University Press 2018

S. Taylor, arXiv:2105.13270

Pulsar timing arrays

Principle of the measurement: compare with the next pulse





$$\Delta T = \frac{1}{2}\frac{\hat{u}^i\hat{u}^j}{1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}}\int_{t_e-\hat{\mathbf{k}}\cdot\mathbf{r_e}}^{t_e+L-\hat{\mathbf{k}}\cdot\mathbf{r_e}-L\hat{\mathbf{k}}\cdot\mathbf{u}} \mathrm{d}X\,[h_{ij}(X+P)-h_{ij}(X)]$$

Wave propagating in k direction: $h_{ij}(X)$ with $X = t_e + s - \hat{\mathbf{k}}\cdot\mathbf{r_e} - s\,\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}$

Pulsar timing arrays

Supposing the source is the inspiral of a super massive black hole binary:
what is the typical scale of the time variation of the metric perturbation?

$$f(\tau) = \frac{1}{\pi}\left(\frac{5}{256}\right)^{3/8} \frac{1}{(G\,M_c)^{5/8}\,\tau^{3/8}} \simeq 10^{-8}\,\mathrm{Hz} \qquad \text{for} \qquad \begin{array}{r} M_c \simeq 10^9\,M_\odot \\ \tau = 4\times 10^4\,\mathrm{yrs} \end{array}$$

Time to coalescence

Chirp mass

$$M_c = \frac{(m_1\,m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

GW varies on a scale of about 3 years

Period of the pulsar: millisecond

$$f_{\mathrm{GW}} P \ll 1$$

$$\Delta T = \frac{1}{2}\frac{\hat{u}^i\hat{u}^j}{1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}} \int_{t_e-\hat{\mathbf{k}}\cdot\mathbf{r_e}}^{t_e+L-\hat{\mathbf{k}}\cdot\mathbf{r_e}-L\hat{\mathbf{k}}\cdot\mathbf{u}} \mathrm{d}X\,[h_{ij}(X+P)-h_{ij}(X)]$$

Taylor expand

Relative change in the rate of the pulses measured on Earth because of the GW passing by:

$$\frac{\Delta T}{P} \simeq \frac{1}{2}\frac{\hat{u}^i\hat{u}^j}{1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}}\,[h_{ij}(t_e+L,\mathbf{r_o})-h_{ij}(t_e,\mathbf{r_e})]$$

Earth term — Pulsar term

Pulsar timing arrays

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Taylor expand

NB: this is the change in the frequency of the pulses due to the GWs, calculated between two successive geodesics, and NOT the redshift experienced by a photon on the same geodesic (usual gravitational redshift, depending on $\dot{h}_{ij}$)

However, the two expressions become the same in the limit of infinitesimal P

Pulsar timing arrays

Relative change in the rate of the pulses measured on Earth because of the GW passing by:

$$\frac{\Delta T}{P} \simeq \frac{1}{2}\frac{\hat{u}^i\hat{u}^j}{1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}}\left[h_{ij}(t_e+L,\mathbf{r_o})-h_{ij}(t_e,\mathbf{r_e})\right]$$

$$\sim 7\cdot 10^{-23}\,\frac{\mathrm{pc}}{d_L}\left(\frac{M_c}{M_\odot}\right)^{5/3}\left(\frac{f_{\mathrm{GW}}}{10^{-8}\,\mathrm{Hz}}\right)^{2/3} \simeq 7\cdot 10^{-16}$$

$M_c \simeq 10^9\, M_\odot$

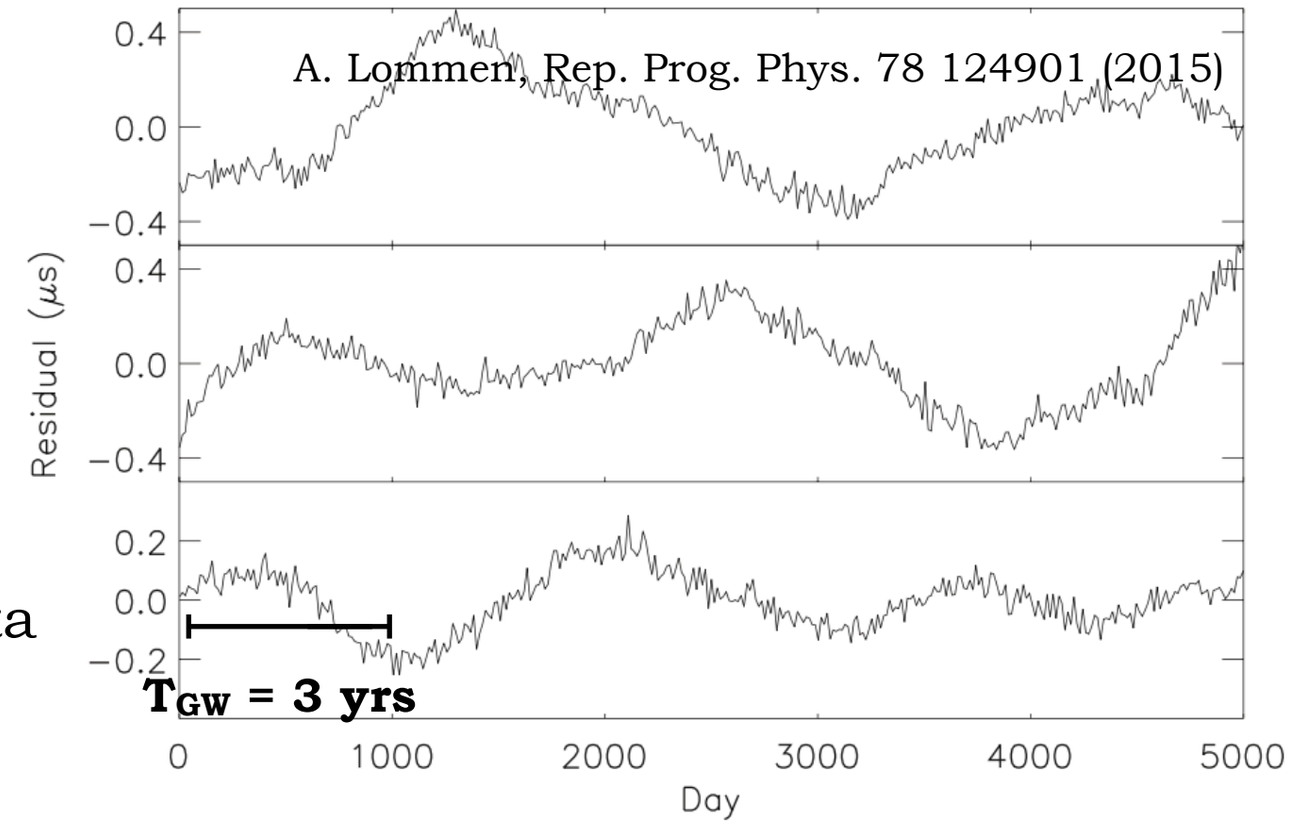
$d_L = 100\,\mathrm{Mpc}$

Timing residuals: $R(T) = \int_{t_{\mathrm{ref}}}^{t_{\mathrm{ref}}+T} \mathrm{d}t\,\frac{\Delta T}{P}$

$$R(T_{\mathrm{GW}} = 3\,\mathrm{yrs}) \simeq 60\,\mathrm{nsec}$$

A GW with period of a few years induces a timing residual of order 100 nsec, the precision of pulsar monitoring!

This renders the measurement possible, provided one has at least a few years of data

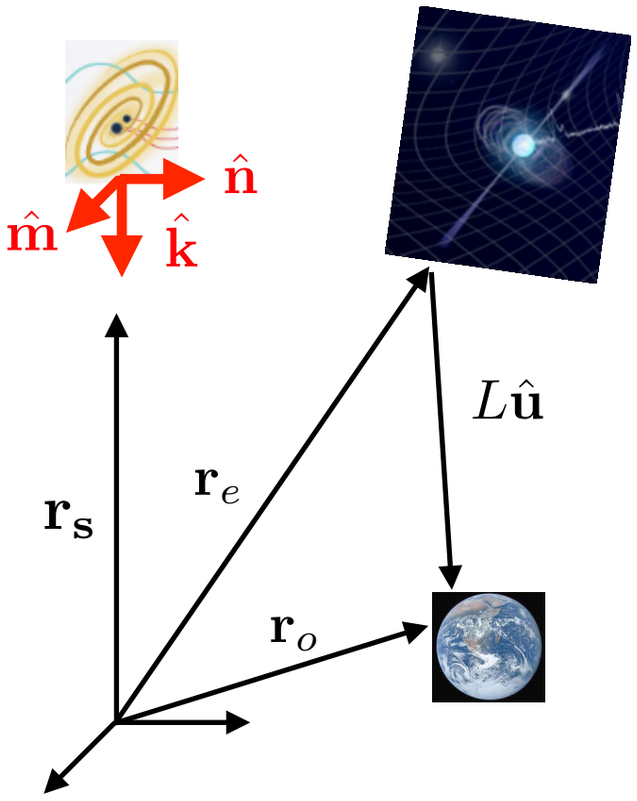


Pulsar timing arrays

HOWEVER! The signal from a single pulsar is very noisy: varying morphology of the pulses, propagation noise due to the dispersion by the interstellar medium, time referencing (time standards and solar system barycentre)…

Correlation between many pulsars to beat down the noise

$$\frac{\Delta T}{P} \simeq \frac{1}{2}\frac{\hat{u}^i\hat{u}^j}{1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}}\left[h_{ij}(t_e+L,\mathbf{r_o})-h_{ij}(t_e,\mathbf{r_e})\right]$$



Earth term:
GW left the source at time

$$t_e + L - |\mathbf{r_o} - \mathbf{r_s}|$$

Pulsar term:
GW left the source at time

$$t_e - |\mathbf{r_e} - \mathbf{r_s}|$$

This term is different for each pulsar

In the correlation the Earth term in general dominates, but the pulsar term can create noise (unless one can determine the delay of each pulsar)

Pulsar timing arrays

Response of a pair of pulsars to a stochastic GW background

$$\langle R_a(T) R_b(T) \rangle = \int_{t_{\rm ref}}^{t_{\rm ref}+T} {\rm d}t' \int_{t_{\rm ref}}^{t_{\rm ref}+T} {\rm d}t'' \langle \left. \frac{\Delta T}{P}(t') \right|_a \left. \frac{\Delta T}{P}(t'') \right|_b \rangle$$

$$\left. \frac{\Delta T}{P}(t') \right|_a = \sum_r \int \frac{{\rm d}\mathbf{k}^3}{(2\pi)^3} h_r(\mathbf{k}) F_a^r(\hat{\mathbf{k}}) e^{-ik(t' - \hat{\mathbf{k}}\cdot\mathbf{r}_o)} \left[1 - e^{ikL_a(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}_a)} \right]$$

"Detector response" $F_a^r(\hat{\mathbf{k}}) = \dfrac{\hat{u}_a^i \hat{u}_a^j e_{ij}^r(\hat{\mathbf{k}})}{2(1 - \hat{\mathbf{k}}\cdot\hat{\mathbf{u}})}$

put Earth at origin to simplify

Earth term

Pulsar term

Use SGWB power spectrum

$$\langle h_r(\mathbf{k},\eta)\, h_p^*(\mathbf{q},\eta) \rangle = \frac{8\pi^5}{k^3}\, \delta^{(3)}(\mathbf{k}-\mathbf{q})\, \delta_{rp}\, h_c^2(k,\eta)$$

The Earth term dominates

$$\left[1 - e^{ikL_a(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}_a)} \right] \left[1 - e^{-ikL_b(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}_b)} \right] \simeq 1$$

$a \neq b$

and SMBHB not in pulsar direction

$$kL_a = \mathcal{O}(2\pi \cdot 10^{-8}\,{\rm Hz} \cdot 500\,{\rm pc}) = \mathcal{O}(3000) \gg 1$$

Pulsar timing arrays

Response of a pair of pulsars to a stochastic GW background

$$\langle R_a(T) R_b(T) \rangle = \int_{t_{\rm ref}}^{t_{\rm ref}+T} {\rm d}t' \int_{t_{\rm ref}}^{t_{\rm ref}+T} {\rm d}t'' \langle \left. \frac{\Delta T}{P}(t') \right|_a \left. \frac{\Delta T}{P}(t'') \right|_b \rangle$$

θ_{ab} angle between pulsars

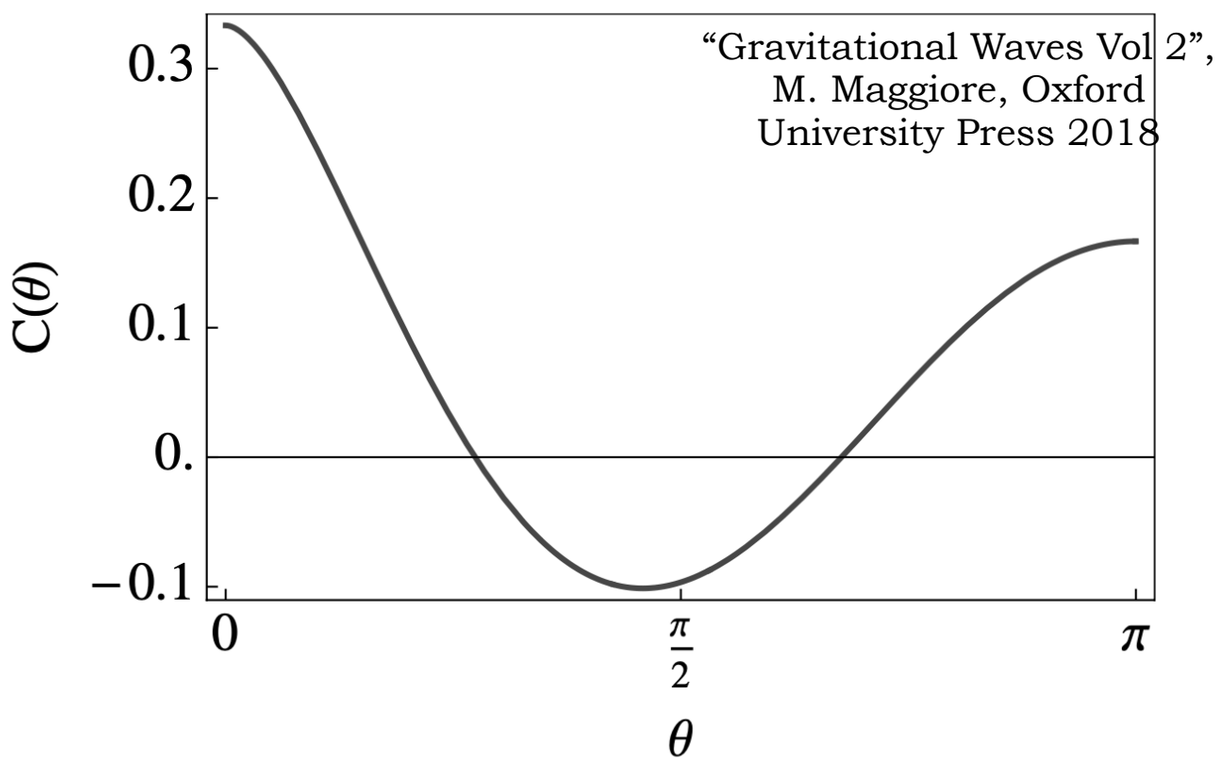
$$= \mathcal{C}(\theta_{ab}) \int_0^\infty {\rm d}f \, \frac{h_c^2(f)}{(2\pi)^2 f^3} \left[1 + \cos(2\pi f (T - t_{\rm ref})\right]$$

Hellings and Downs curve, characteristic of a GW signal because consequence of the quadrupolar nature of GWs

$$\mathcal{C}(\theta_{ab}) = \int \frac{{\rm d}\hat{\mathbf{k}}}{4\pi} \sum_r F_r^a(\hat{\mathbf{k}}) F_r^b(\hat{\mathbf{k}})$$

$$= \frac{1}{3} - \frac{1}{6} x_{ab} + x_{ab} \log(x_{ab})$$

$$x_{ab} = \frac{1}{2}(1 - \cos\theta_{ab})$$



Pulsar timing arrays

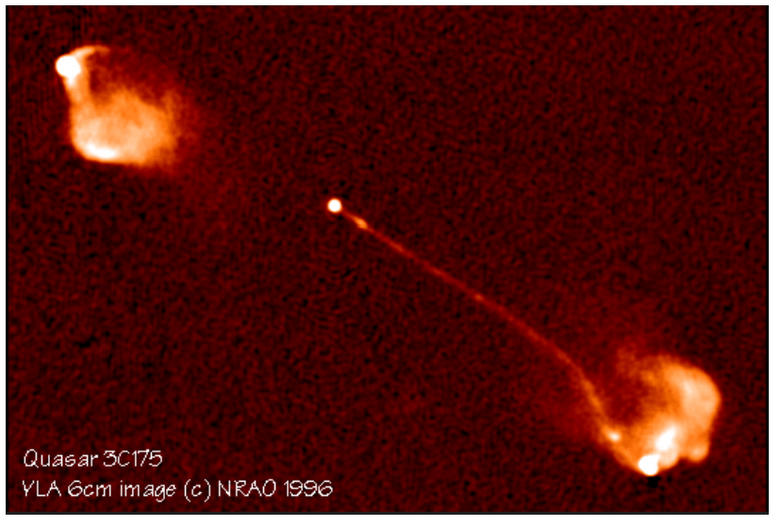
Observation of the *Hellings and Downs curve* is **smoking gun evidence** of GW detection

(not only SGWB but also from a single SMBHB - Cornish & Sesana arXiv:1305.0326)

time referencing errors generate a correlated noise but:

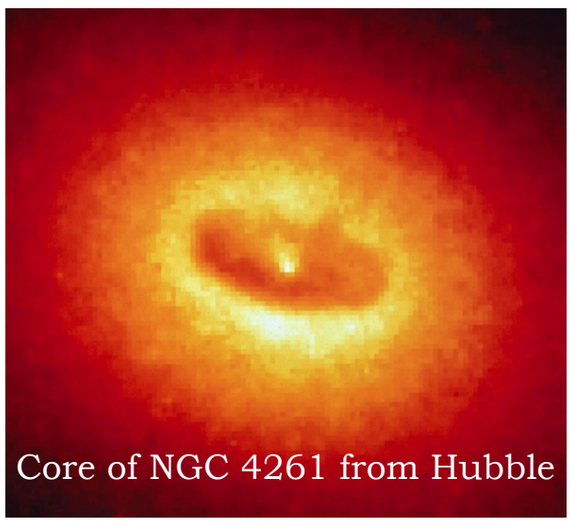
- noise from uncertainties on the time standards on Earth is independent on the pulsar angles
- noise from uncertainties on the solar system barycentre position take the form of a (rotating) dipole (dependent on the cosinus of the angle) -> can contaminate the quadrupole

A SGWB from SMBHBs is the best candidate source in PTA frequency band



What are SMBHBs?

- They have been observed in the core of galaxies and are the central engine of active galactic nuclei
- They can originate from the collapse of massive stars (~100 $M_\odot$) or gas clouds (~ 10^4 $M_\odot$), and then grow in mass through gas accretion and/or mergers following the collision of their host galaxies (but their origin is still to be confirmed, they can also be primordial…)
- JWST sees SMBHs up to very high redshift z ~ 11
- Their presence is linked to the formation of galaxies and matter structure in the Universe

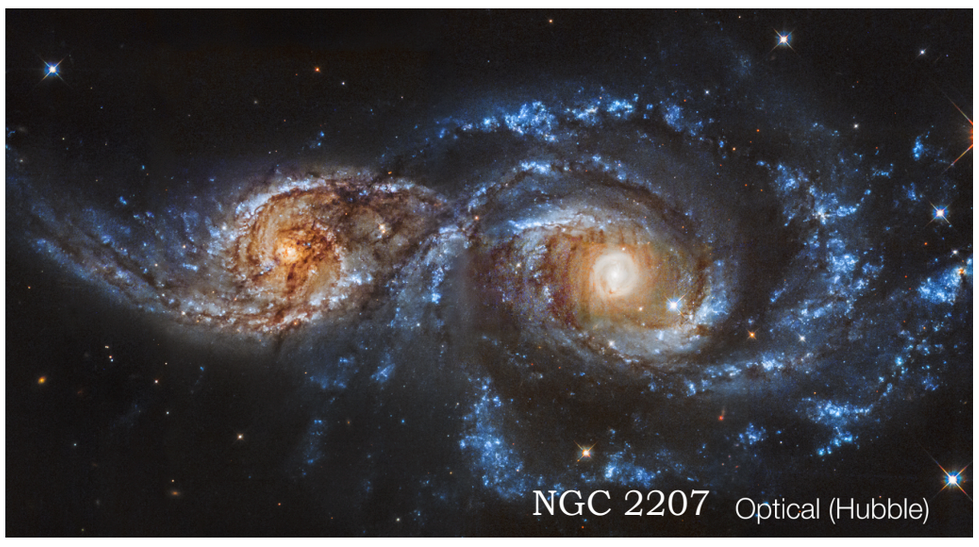


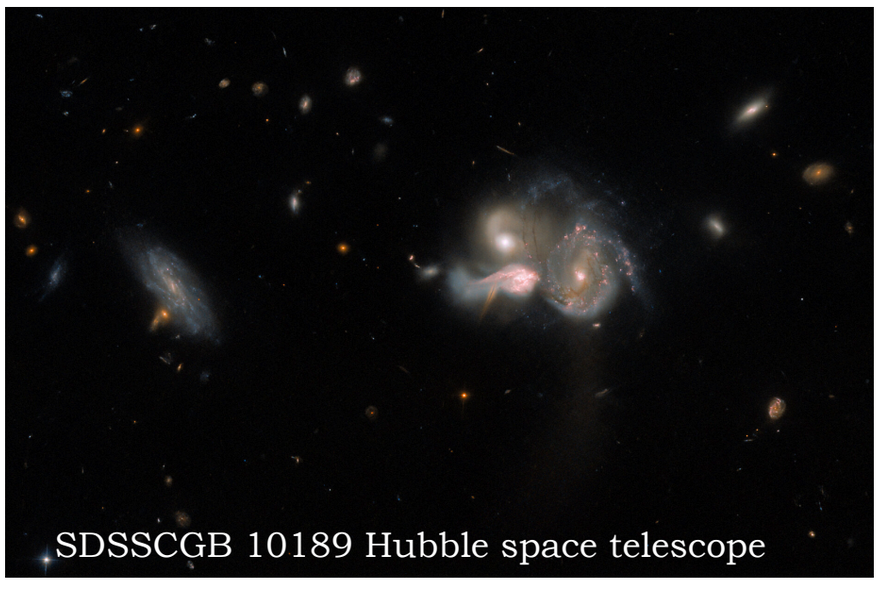
Pulsar timing arrays

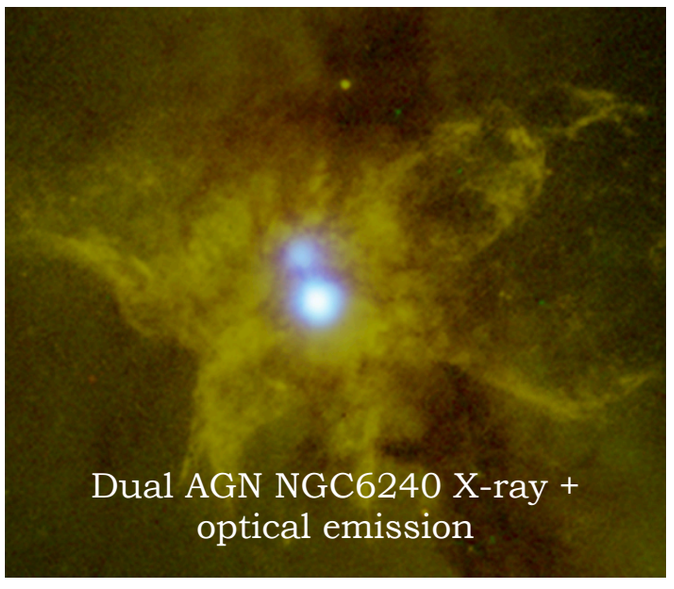
HOWEVER!

- To emit GWs, SMBHs must be paired in gravitationally bound binaries in the GW emitting regime: separation of ~ 0.01-0.001 pc
- Binaires can be formed after the collision of two galaxies: the MBH previously at the centre of galaxies get to ~ kpc separation (X-ray evidence from dual AGNs)
- Dynamical friction drives the two MBH towards the centre of the new galaxy until they form a bound binary
- 3-body interaction with the surrounding stars subsequently shrinks the binary to pc separation
- How to get them to the millipc separation necessary for GW emission and merger within one Hubble time? "LAST PARSEC PROBLEM"
- maybe more stars arrive, or there is gas drag from interaction with a circumbinary disk, and/or another MBH arrives…

If PTAs observe the SGWB from SMBHBs it means that SMBHBs exist and merge in the universe!







Pulsar timing arrays

Prediction from SMBHBs formation scenarios

How does the SGWB from SMBHBs look like?

Characteristic strain: red spectrum

$$h_c(f) = A\left(\frac{f}{f_{\rm ref}}\right)^{-\alpha} \text{ with } \alpha = \frac{2}{3}$$

$A = \mathcal{O}(10^{-15})$ at $f_{\rm ref} = 1\,\mathrm{yr}^{-1}$

Circular binary

Timing residuals power spectral density: Also red spectrum

$$S_{ab}(f) = \mathcal{C}(\theta_{ab})\Phi(f)$$

$$\Phi(f) = \frac{A^2}{(2\pi)^2} f_{\rm ref}^{-3}\left(\frac{f}{f_{\rm ref}}\right)^{-\gamma} \quad \text{with} \quad \gamma = 2\alpha + 3 = \frac{13}{3}$$

The features of the SGWB power spectrum (amplitude A, slope α…) depend on the population characteristics such as the binary merger rate, its dependence with mass and redshift, the surrounding stellar density, the initial binary eccentricity…

Pulsar timing arrays

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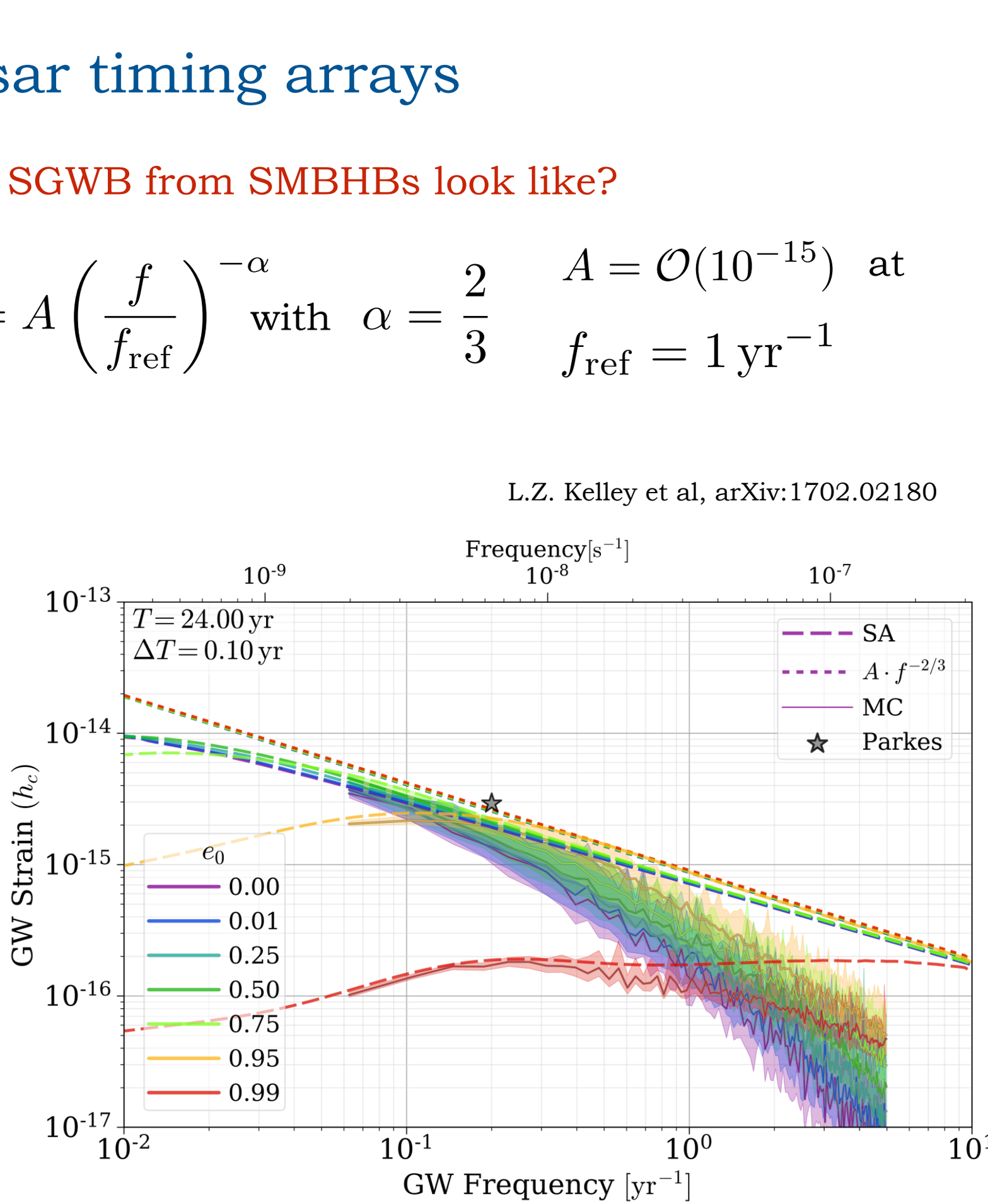
$A = \mathcal{O}(10^{-15})$ at $f_{\rm ref} = 1\,{\rm yr}^{-1}$

- The assumption of homogeneous and isotropic SGWB isn't justified at high frequency: SMBHBs are less numerous, the SGWB slope is steeper, and discreteness starts to appear with spikes due to the loudest SMBHBs
- Interactions with the binary environment makes hardening stronger and suppresses SGWB power at low frequency
- Eccentricity enhances GW emission at higher frequencies

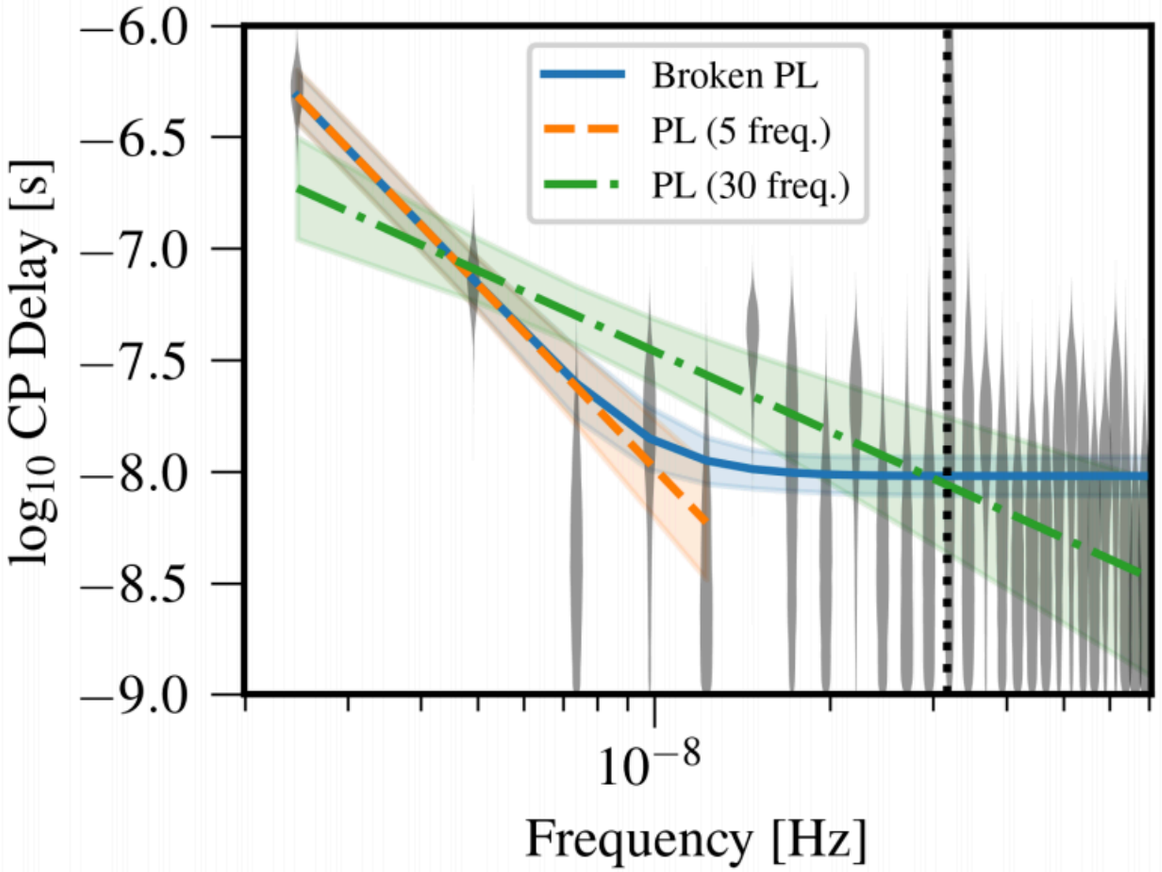
L.Z. Kelley et al, arXiv:1702.02180

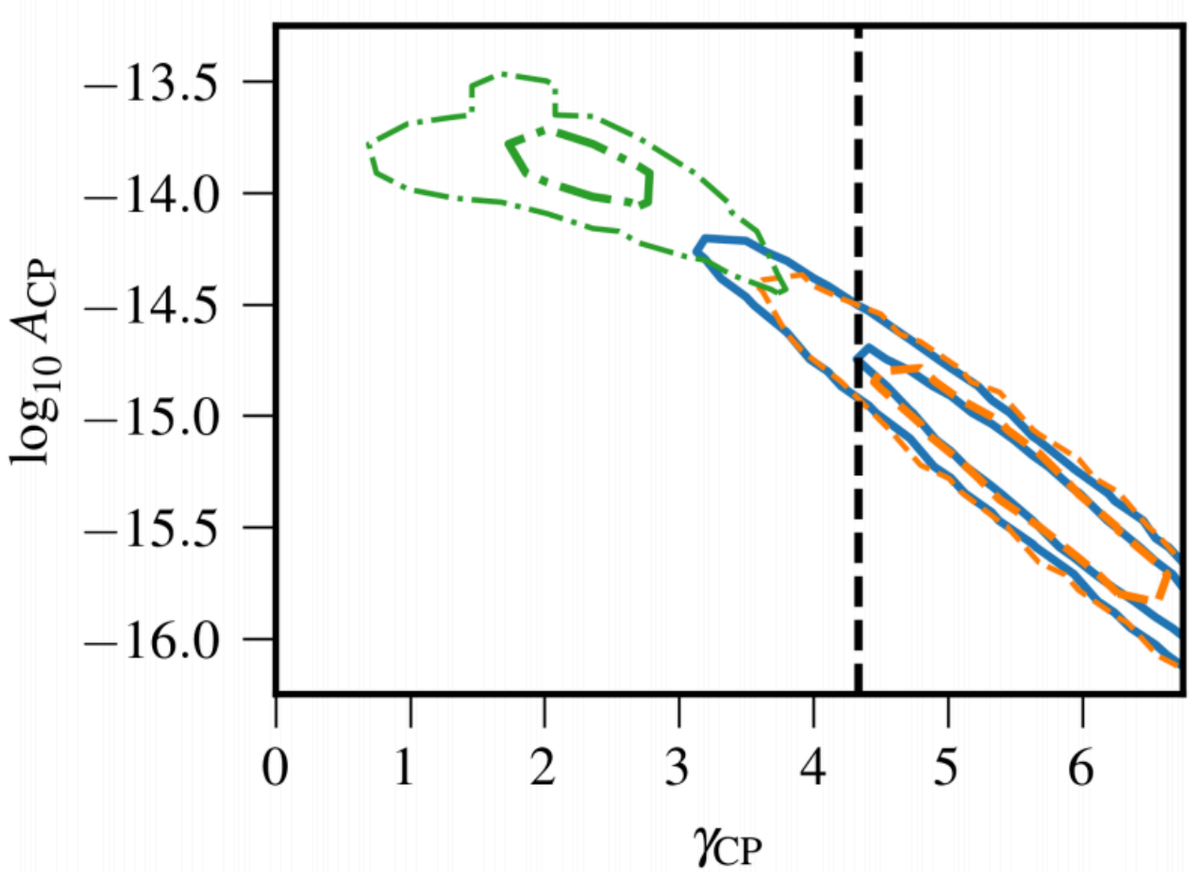


Pulsar timing arrays

In 2020, NANOGrav (followed by EPTA and PPTA) has announced the presence of a common red noise in their 12.5 years data

$$\Phi(f) = \frac{A^2}{(2\pi)^2} f_{\rm ref}^{-3} \left(\frac{f}{f_{\rm ref}}\right)^{-\gamma} \quad \text{with} \quad \gamma = 2\alpha + 3 = \frac{13}{3}$$



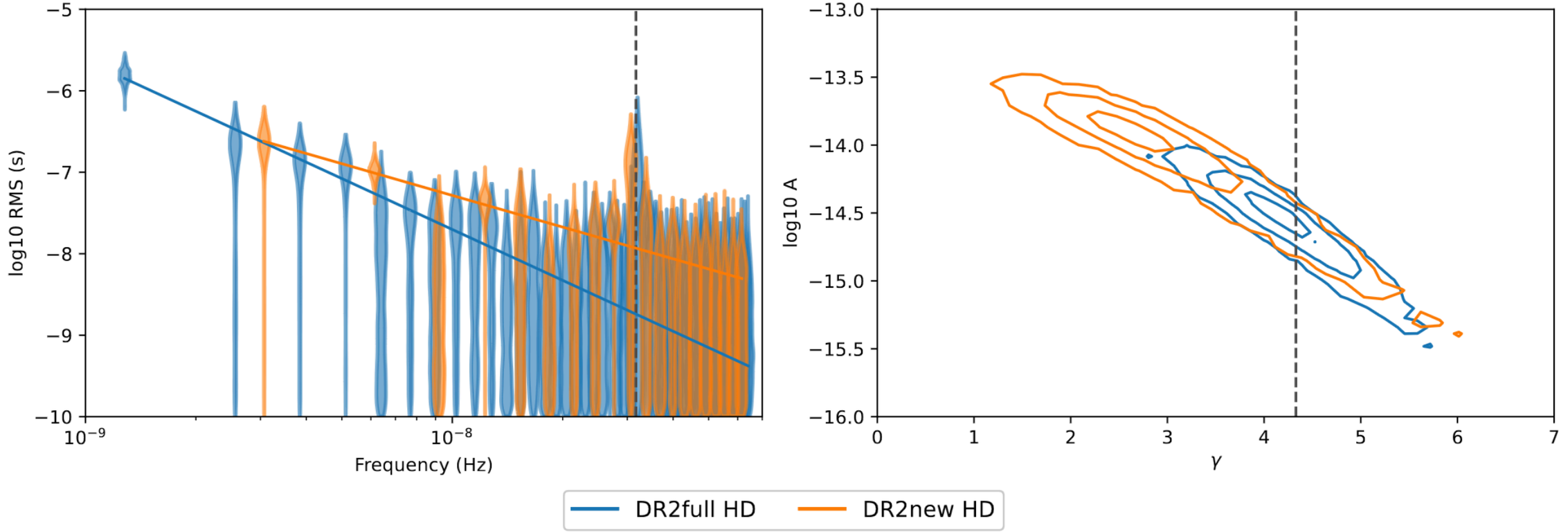


NANOGrav collaboration: arXiv:2009.04496

Pulsar timing arrays

Last year, all PTAs have confirmed the observation of a common red noise supplemented by evidence for the Hellings-Downs correlation

EPTA results:

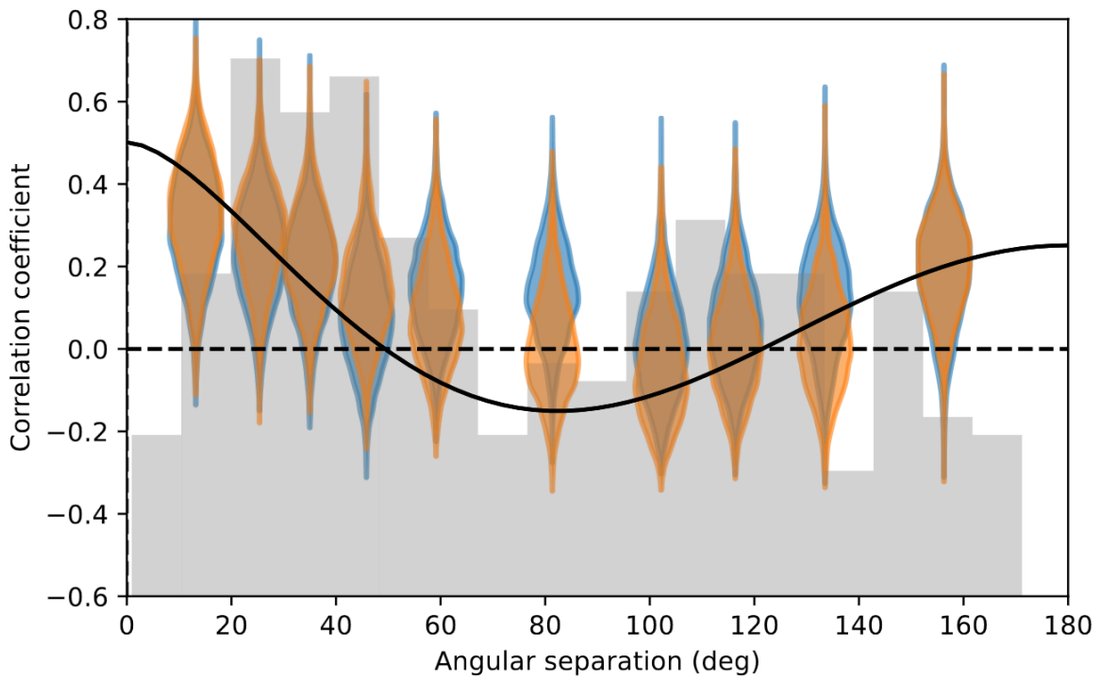


DR2new (10.3 yrs):

$$\log A = -13.94^{+0.23}_{-0.48} \quad \gamma = 2.71^{+1.18}_{-0.73} \quad \mathcal{B} = 60$$

DR2full (25 yrs):

$$\log A = -14.54^{+0.28}_{-0.41} \quad \gamma = 4.19^{+0.73}_{-0.63} \quad \mathcal{B} = 4$$



J. Antoniadis et al, arXiv:2306.16214

Pulsar timing arrays

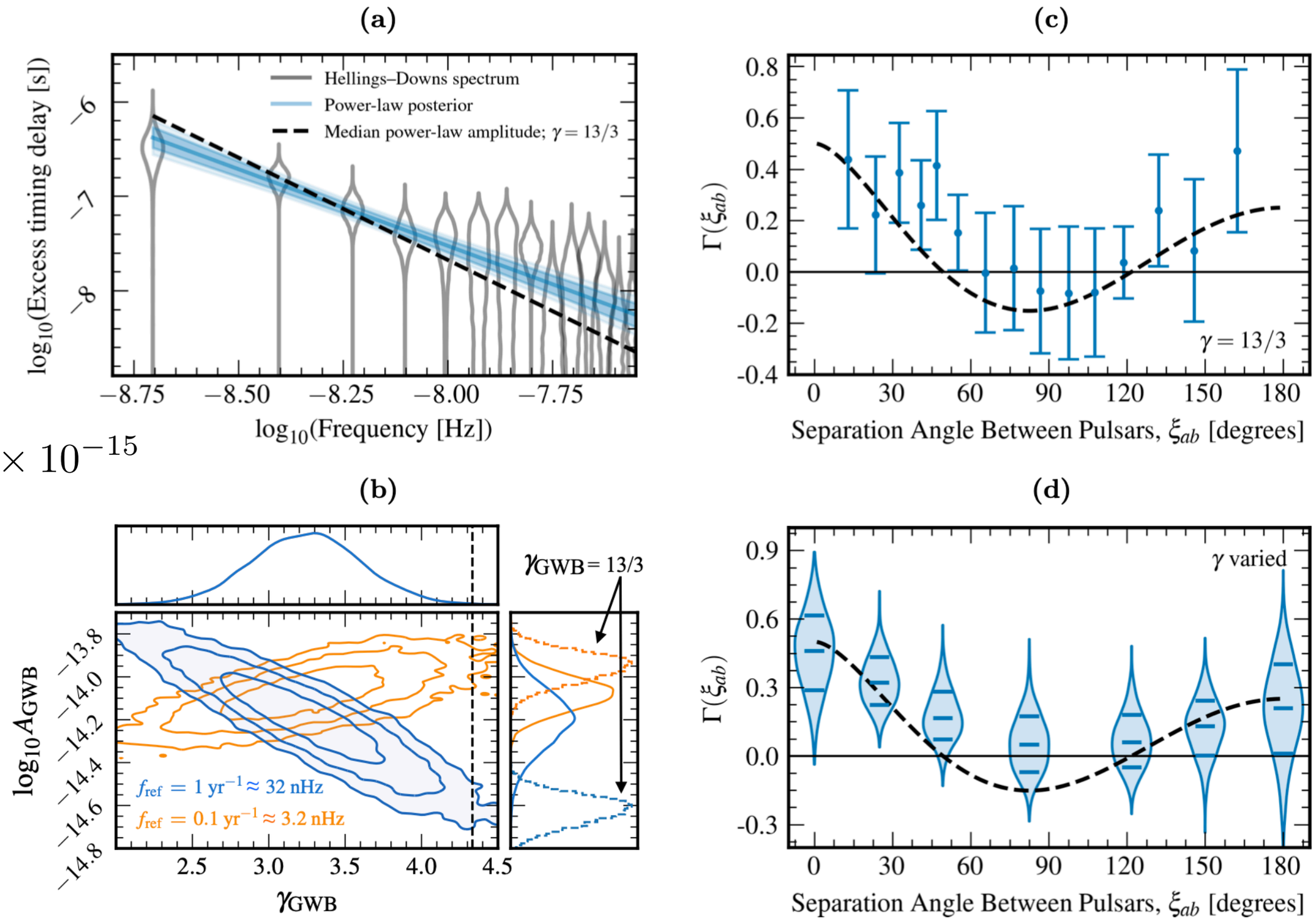
Last year, all PTAs have confirmed the observation of a common red noise supplemented by evidence for the Hellings-Downs correlation

NANOGrav results:

$$A = -6.4^{+4.2}_{-2.7} \times 10^{-15}$$

$$\gamma = 3.2^{+0.6}_{-0.6}$$

$$\mathcal{B} = 226$$

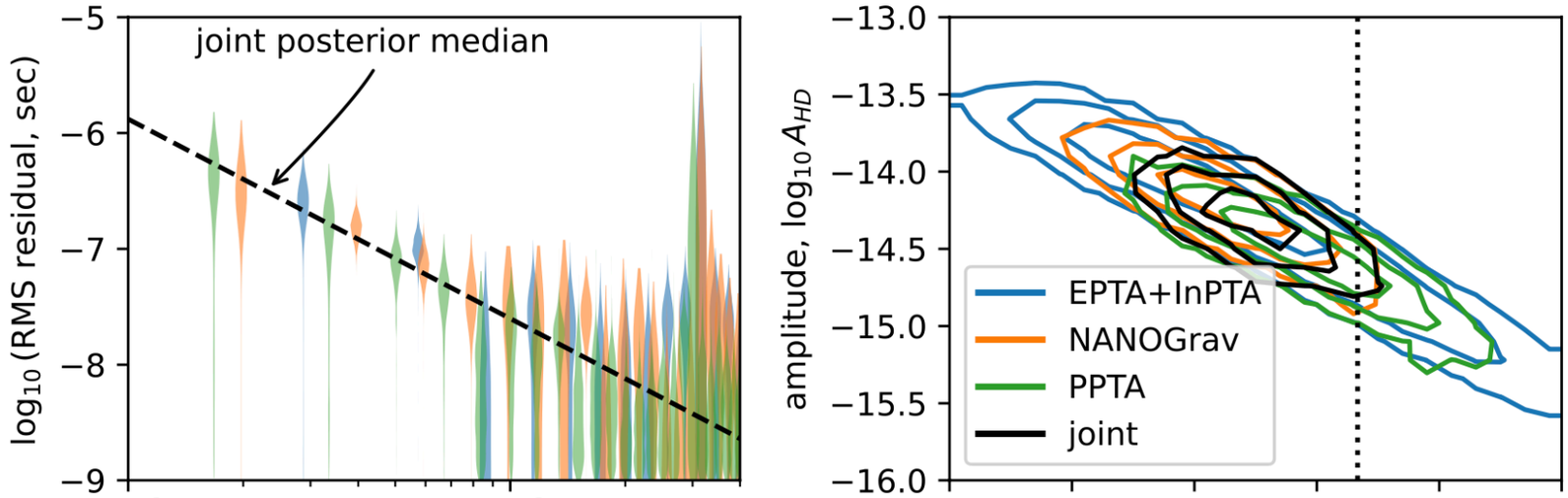


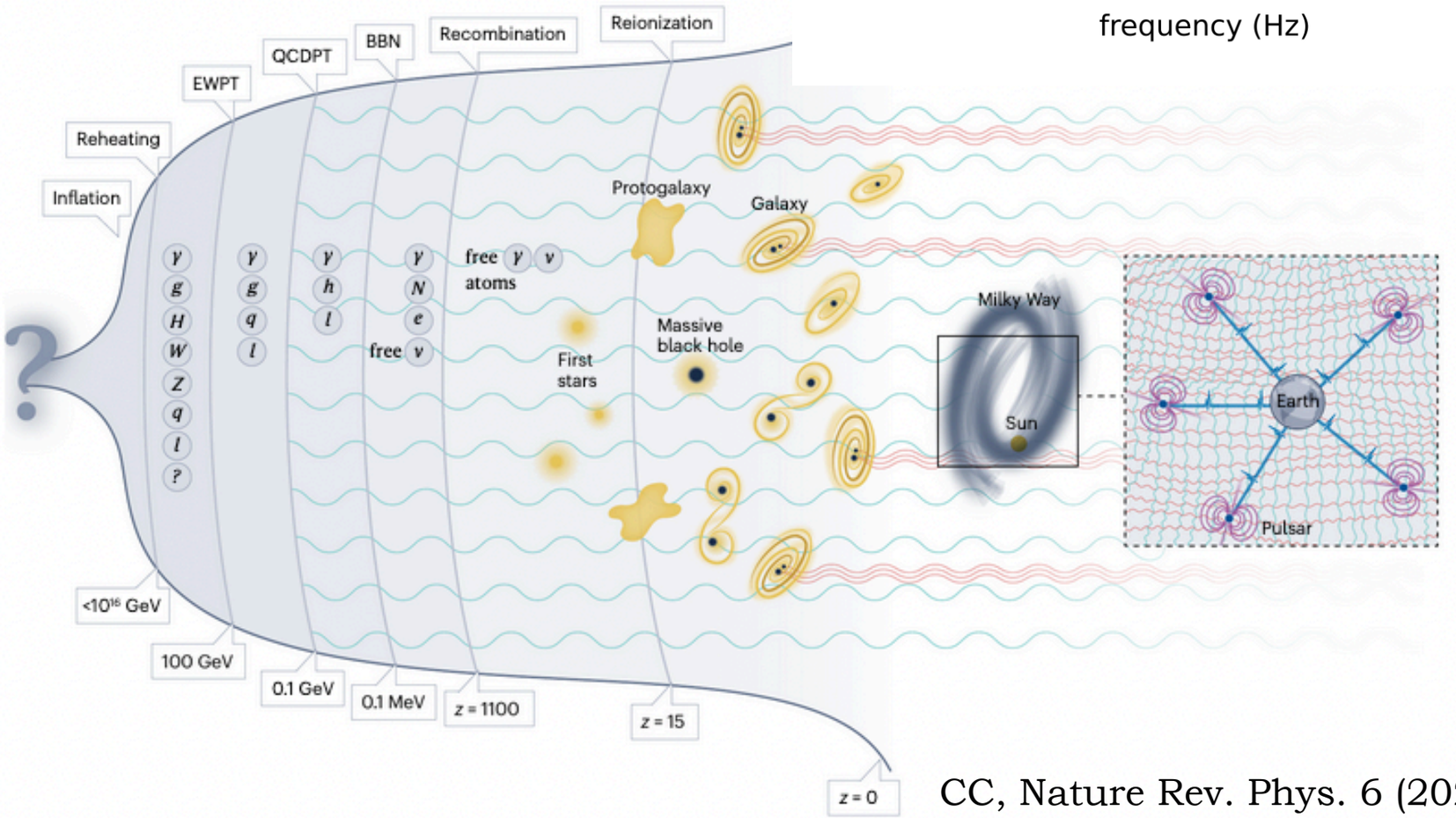
G. Agazie et al, arXiv:2306.16213

Pulsar timing arrays

- The slopes are shallower than 13/3 (but maybe the model isn't fully adapted…)
- The amplitude is consistent with the one from a SMBHBs SGWB
- All datasets are consistent within 1σ as shown by IPTA

IPTA Collaboration,
arXiv:2309.00693





But the signal could also be of primordial origin…

CC, Nature Rev. Phys. 6 (2024), 5291-293