### **Inflation and Dark Energy**

#### **Ed Copeland -- Nottingham University**

1 Invisibles School, Bologna — June 24th - June 28th 2024

- 1. Inflation including some neat additions
- 2. A bit more inflation and intro to Dark Energy
- 3. More Dark Energy and a few variants



### **Before we start - given the week we are in: If you don't mention this**

Denmark outplay England

Credit: OptaAnalyst





Credit: Carmen Jaspersen/Reuters





#### **I won't mention this**





#### Spain outplay Italy

4



The cosmological principle -- isotropy and homogeneity on large scales

• **The expansion of the Universe v=H0d H0=73.04±1.04 km s-1 Mpc-1 (Riess et al, 2022) H0=67.4±0.5 km s-1 Mpc-1 (Planck 2018)**

Is there a local v global tension **?**

*H* =  $\dot{a}$ *a*

tance modulus redshift redshift redshift redshift redshift relation of the best-fit  $\mathbb{R}^n$ **Betoule et al 2014** Redshift  $1 + z =$ line. *Bottom:* Residuals from the best-fit ⇤CDM cosmology as *a*0 *a*



a function of redshift. The weighted average of the weighted average of the residuals in  $\mathcal{C}$ 





In flat universe:  $\Omega_M = 0.28$  [ $\pm$  0.085 statistical] [ $\pm$  0.05 systematic] Prob. of fit to  $A = 0$  universe: 1%

astro-ph/9812133

### **In fact the universe is accelerating !**

Observations of distant supernova in galaxies indicate that the rate of expansion is increasing !

> We call it Dark Energy -emphasises our ignorance!

Huge issue in cosmology -- what is the fuel driving this acceleration?

Makes up 70% of the energy content of the Universe



# **The Big Bang – (1sec today)**

#### **Test 2**

• **The existence and spectrum of the CMBR** 

 $\bullet$   $T_0=2.728 \pm 0.004$  K

• Evidence of isotropy - detected by COBE to such incredible precision in 1992

• Nobel prize for John Mather 2006

#### 2dF Galaxy Redshift Survey



#### Homogeneous on large scales?

8



# **The Big Bang – (1sec today)**

#### **Test 3**

- **The abundance of light elements in the Universe.**
- **Most of the visible matter just hydrogen and helium.**

 $(baryons) -  $\Omega_b h^{2}= 0.02242 \pm 0.00014$  8 2018$ 

 $10^{-9}$ 

#### • **Given the irregularities seen in the CMBR, the development of structure can be explained through gravitational collapse.**



### **Test 4 The Big Bang – (1sec today)**



*Tµ* = diag(*, p, p, p*) 10 U<sup>µ</sup>: fluid four vel =  $(1,0,0,0)$  - because comoving in the cosmological rest frame.

$$
ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu
$$

Geometry Matter Cosm const - could be matter or geometry

# **The key equations**

Einstein GR:

Invariant separation of two spacetime points  $(\mu, \nu=0,1,2,3)$ :

Relates curvature of spacetime to the matter distribution and its dynamics.

Require metric tensor  $g_{\mu\nu}$  from which all curvatures derived indep of matter:

Einstein tensor  $G_{\mu\nu}$  -- function of  $g_{\mu\nu}$  and its derivatives. Energy momentum tensor  $T_{\mu\nu}$  -- function of matter fields present. we write

For most cosmological substances can use perfect fluid representation for which

#### $T_{\mu\nu} = (\rho + p)U_{\mu}U_{\nu} + pg_{\mu\nu}$

(ρ,p) : energy density and pressure of fluid in its rest frame

 $G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$ 

=

**1** 

2

 $g^{\mu\lambda}(g_{\sigma\lambda,\nu}+g_{\nu\lambda,\sigma}-g_{\sigma\nu,\lambda})$  $= \Gamma^{\mu}_{\nu\gamma,\sigma} - \Gamma^{\mu}_{\nu\sigma,\gamma} + \Gamma^{\mu}_{\alpha\sigma} \Gamma^{\alpha}_{\gamma\nu} - \Gamma^{\mu}_{\alpha\gamma} \Gamma^{\alpha}_{\sigma\nu}$ 

> $R_{\mu\nu} = R_{\mu\nu}^{\sigma}$  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}$ 2  $g_{\mu\nu}R$  $R = R^{\mu}_{\mu}$ *µ*

*R<sup>µ</sup>*  $\nu \sigma \gamma$ Riemann's curvature tensort

 $\Gamma^{\mu}_{\nu}$ 

。<br>レσ

# **Reminder of curvatures**

Christoffel symbols:

Ricci tensor:

Ricci scalar:

Einstein tensor:

Not needed here

$$
ds^2 = -dt^2 + a^2(t)dx^2
$$

$$
\frac{1}{1-kr^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)
$$



#### **Cosmology - isotropic and homogeneous FRW metric**

Copernican Principle: We are in no special place. Since universe appears isotropic around us, this implies the universe is isotropic about every point. Such a universe is also homogeneous.

Line element

#### $dx^2 =$ **1**

t -- proper time measured by comoving (i.e. const spatial coord) observer. a(t) -- scale factor: k- curvature of spatial sections: k=0 (flat universe), k=-1 (hyperbolic universe), k=+1 (spherical universe)

Aside for those familiar with this stuff -- not chosen a normalisation such that  $a_0=1$ . We are not free to do that and simultaneously choose  $|k|=1$ . Can do so in the  $k=0$  flat case.

12 Note: Crucial but not globally accepted — see for instance Watkins et al. results on large Bulk flows over 150-200 Mpc scales: eprint:2302.02028

#### *<sup>t</sup> dt a*(*t* )  $ds^2 = a^2(\tau)(-d\tau^2 + dx^2)$ *H*(*t*) *a*  $\mathbf{\dot{a}}$ *a*



astro-ph/9812133

 $\tau(t) \equiv$ Intro Conformal time : τ(t)

Implies useful simplification :

 $\nu = H(t)r$  $d = ax$  $d = \dot{a}x + a\dot{x}$  $\dot{d} = Hd + a\dot{x}$  $d = v + ax$ Hubble flow peculiar velocity

Hubble parameter : (often called Hubble constant) Hubble parameter relates velocity of recession of distant galaxies from us to their separation from us

$$
\nabla^{\mu}T_{\mu\nu}=0
$$



 $H^2$  ≡

 $H^2$ 

 $\dot{\rho}$  +

$$
\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}
$$

a(t) depends on matter,  $\rho(t)=\sum_i \rho_i$  -- sum of all matter contributions, rad, dust, scalar fields...

- Energy density  $\rho(t)$ : Pressure  $p(t)$ 
	- $Related through : p = w\rho$
- Eqn of state parameters: w=1/3 Rad dom: w=0 Mat dom: w=-1– Vac dom

**Eqns (**Λ**=0):** 

**Friedmann + Fluid energy conservation**

#### $G_{\mu\nu} = 8\pi GT_{\mu\nu} - \Lambda g_{\mu\nu}$  applied to cosmology

$$
\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2}
$$

$$
3(\rho + p)\frac{\dot{a}}{a} = 0
$$

$$
\rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} \quad ; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}
$$

RD : 
$$
w = \frac{1}{3} : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-4}
$$
 ;  $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{1}{2}}$   
MD :  $w = 0 : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3}$  ;  $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$ 

RD : 
$$
w = \frac{1}{3}
$$
 :  $\rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-4}$  ;  $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{1}{2}}$   
MD :  $w = 0$  :  $\rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3}$  ;  $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$ 

 $V\Box$  :  $w=-1:\rho(t)=\rho_0$  ;  $a(t)\propto e^H$ 

If  $\rho + 3p < 0 \Rightarrow \ddot{a} > 0$ 

# **Combine Friedmann and fluid equation to obtain Acceleration equation:**

$$
\frac{\ddot{a}}{a} = -\frac{8\pi}{3}G(\rho + 3p) - - Accn
$$

$$
H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2}
$$

$$
\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0
$$

Inflation condition -- more later

# $\Omega > 1 \leftrightarrow k = +1$  $\Omega=1 \leftrightarrow k=0$  $\Omega < 1 \leftrightarrow k = -1$ **A neat equation**







Ωm - baryons, dark matter, neutrinos, electrons, radiation

...

 $\Omega_{\Lambda}$  - dark energy;  $\Omega_{k}$  - spatial curvature

 $\rho_c(t_0) = 1.88h^2 * 10^{-29} gcm^{-3}$ 

17 Bounds on  $H(z)$  -- Planck 2018 - (+BAO+lensing+lowE) (Expansion rate)  $- H_0 = 67.66 \pm 0.42$  km/s/Mpc  $rac{1}{2}$  (radiation) --  $\Omega_r = (8.5 \pm 0.3) \times 10^{-5}$  - (WMAP)  $(baryons) - \Omega_b h^2 = 0.02242 \pm 0.00014$ (dark matter) --  $\Omega_c h^{2} = 0.11933 \pm 0.00091$  —-(matter) -  $\Omega_m = 0.3111 \pm 0.0056$  $(curvature) -  $\Omega_k = 0.0007 \pm 0.0019$$ (dark energy)  $- \Omega_{de} = 0.6889 \pm 0.0056 -$  Implying univ accelerating today (de eqn of state)  $-1+w = 0.028 \pm 0.032 - 1$ ooks like a cosm const. If allow variation of form :  $w(z) = w_0 + w_a z/(1+z)$  then  $w_0$ =-0.957  $\pm$  0.08 and  $w_a$  = -0.29  $\pm$  0.31 (68% CL) — (Planck 2018+SNe+BAO) Important because distance measurements often rely on assumptions made about the  $\mathbf{H}^2(\mathbf{z}) = \mathbf{H_0^2}$  $\sqrt{2}$  $\Omega_{\rm r}(1+z)^4 + \Omega_{\rm m}(1+z)^3 + \Omega_{\rm k}(1+z)^2 + \Omega_{\rm de}$  exp  $\sqrt{2}$ 3  $\int_0^1$ 0  $1 + w(z')$  $1 + z'$ dz ⇥⇥

background cosmology.

#### $w(z) = w_0 + w_a z/(1+z)$  $W(z) = W_0 + W_2 z/(1 + z)$ DESI+BBN+✓⇤ <sup>0</sup>*.*<sup>296</sup> *<sup>±</sup>* <sup>0</sup>*.*014 68*.*<sup>52</sup> *<sup>±</sup>* <sup>0</sup>*.*69 0*.*3+4*.*<sup>8</sup> 5*.*<sup>4</sup> — —

#### Recent developments — DESI (2024) - arXiv:2404.03002



 $06/23/2008$  whoutom dark aparazz has generated a great daal of dahata ahal<sup>8</sup>t the  $1$ DESI+CMB+Union3 0*.*3095 *±* 0*.*0083 67*.*76 *±* 0*.*90 — 0*.*997 *±* 0*.*032 — 8/23/2008 nhanto DESI 0*.*<sup>313</sup> *<sup>±</sup>* <sup>0</sup>*.*049 — 87+100 <sup>85</sup> 0*.*70+0*.*<sup>49</sup> 0*.*<sup>25</sup> *<sup>&</sup>lt;* 1*.*<sup>21</sup> This move towards phantom dark energy has generated a great deal of debate about the use of priors.



### **How old are we?**

$$
H^{2} = \frac{\dot{a}^{2}}{a^{2}} = \frac{8\pi}{3}G\rho - \frac{k}{a^{2}}
$$
\n
$$
t_{0} = H_{0}^{-1}\int_{0}^{1} \frac{x dx}{\left[\Omega_{m}x + \Omega_{r0} + \Omega_{A0}x^{4} + (1 - \Omega_{0})x^{2}\right]^{1/2}}
$$
\nwhere  $\rho = \rho_{m} + \rho_{r} + \rho_{A}$   
\nwhere  $\Omega_{0} = \Omega_{m0} + \Omega_{r0} + \Omega_{A0}$   
\n $t = \int \frac{da}{\dot{a}} = \int \frac{da}{aH}$   
\n $Today: H_{0}^{-1} = 9.8 \times 10^{9} h^{-1} \text{ years; } h = 0.7$   
\n $\Omega_{m0} = \Omega_{r0} = \Omega_{A0} - t_{0}$   
\n $\Omega_{r0} = \Omega_{A0} - t_{0}$   
\n $0.3 = 10^{-5} - 0.7 = 13.4 \text{ Gyr}$   
\nUseful estimate for age of  
\n $0.2 = 10^{-5} - 0.2 = 12.4 \text{ Gyr}$   
\n $0.2 = 10^{-5} - 0.6 = 13.96 \text{ Gyr}$   
\n $0.3 = 10^{-5} - 0.8 = 13.96 \text{ Gyr}$   
\n $0.4 = 10^{-5} - 0.9 = 13.6 \text{ Gyr}$ 

# **History of the Universe**



# **The Big Bang – issues.**

- Flatness problem observed almost spatially flat cosmology requires fine tuning of initial conditions.
- Horizon problem -- isotropic distribution of CMB over whole sky appears to involve regions that were not in causal contact when CMB produced. How come it is so smooth?
- Monopole problem where are all the massive defects which should be produced during GUT scale phase transitions.
- Relative abundance of matter does not predict ratio baryons: radiation: dark matter.
- Origin of the Universe simply assumes expanding initial conditions.
- Origin of structure in the Universe from initial conditions homogeneous and isotropic.
- The cosmological constant problem.

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# **Horizon problem**

Primordial density fluctuations.

CMB photons emitted from opp sides of sky are in thermal equilibrium at same temp – but no time for them to interact before photons were emitted because of finite horizon size.

> <u>08/11/20region separated by > 2 deg – causally separated at deco20</u> Anny *region separated by > 2 deg – causally separated at decoupling.*



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# **Monopole problem**

- Monopoles are generic prediction of GUT type models.
- They are massive stable objects, like domain walls and cosmic strings and many moduli fields.
	- They scale like cold dark matter, so in the early universe would rapidly come to dominate the energy density.
	- Must find a mechanism to dilute them or avoid forming them.

#### **Some of the big questions in cosmology today**

- a) What is dark matter? -- 25% of the energy density
- b) What is dark energy? -- 70% of the energy density. Does dark energy interact with other stuff in the universe?
- c) Is dark energy really a new energy form or does the accelerating universe signal a modification of our theory of gravity?
- d) What is the origin of the density perturbations, giving rise to structures?
- e) Where is the cosmological gravitational wave background?
- f) Are the fluctuations described by Gaussian statistics? If there are deviations from Gaussianity, where do they come from?
- g) How many dimensions are there? Why do we observe only three spatial dimensions?
- 08/11/2011 25 h) Was there really a big bang (i.e. a spacetime singularity)? If not, what was there before?

#### Enter Inflation

- A period of accelerated expansion in the early Universe
- Small smooth and coherent patch of Universe size less than (1/H) grows to size greater than the comoving volume that becomes entire observable Universe today.
	- Explains the homogeneity and spatial flatness of the Universe
	- and also explains why no massive relic particles predicted in say GUT theories
		- Leading way to explain observed inhomogeneities in the Universe

$$
\frac{\ddot{a}}{a} = -\frac{8\pi}{3}G(\rho + 3p) - - Accn
$$





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# **What is Inflation?**

Any epoch of the Universe's evolution during which the comoving Hubble length is decreasing. It corresponds to any epoch during which the Universe has accelerated expansion.

 $\ddot{a}$ *a*  $=-\frac{8\pi}{2}$ 3

# at<br>1

For inflation require material with negative pressure. Not many examples. One is a scalar field!

$$
\frac{d}{dt}\left(\frac{H^{-1}}{a}\right)
$$





- Intro fundamental scalar field -- like Higgs
- If Universe is dominated by the potential of the field, it will accelerate!

$$
\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)
$$
  

$$
p = \frac{1}{2}\dot{\phi}^2 - V(\phi)
$$

- We aim to constrain potential from observations.
- During inflation as field slowly rolls down its potential, it undergoes quantum fluctuations which are imprinted in the Universe. Also leads to gravitational wave production.

$$
\text{EnM} \quad H^2 = \frac{8\pi G}{3} \rho_{\phi} \quad ; \quad \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0
$$
\n
$$
\text{Inflation } \ddot{a} > 0 \leftrightarrow (\rho + 3p) < 0 \leftrightarrow \dot{\phi}^2 \ll V(\phi) \quad \text{Slow roll}
$$
\n
$$
H^2 = \frac{8\pi G}{3} V(\phi) \quad ; \quad 3H\dot{\phi} + \frac{dV}{d\phi} = 0
$$

 $= -4\pi G\phi$  $\dot{b}^2$ 

So, define a quantity which specifies how fast H changes during inflation



# **Examples of inflation**

single scalar field

$$
\rho_{\phi} = V(\phi) + \frac{\phi^2}{2} \quad ; \quad p_{\phi} = \frac{\phi^2}{2} - V(\phi)
$$

 $N \equiv \ln \left( \frac{a(t_{\rm end})}{a(t)} \right)$  $a(t_i)$ ⇥ =

$$
\int_{t_i}^{t_e} H dt \simeq - \int_{\phi_i}^{\phi_e} \frac{V}{V'} d\phi
$$



Slow roll inflation occurs when both of these slow roll conditions are << 1

Prediction -- potential determines important quantities Slow roll parameters [Liddle & Lyth 1992]

End of inflation corresponds to  $\varepsilon$ =1 How much does the universe expand? Given by number of e-folds

Last expression is true in the slow roll limit (for single field inflation).

Assume

\n
$$
|\Omega(t_0) - 1| \leq 0.01
$$
\nNow

\n
$$
|\Omega - 1| = \frac{k}{a^2 H^2}; \quad RD : |\Omega - 1| \propto t
$$
\n
$$
|\Omega(10^{-34} s) - 1| \leq 0.01 * 10^{-34} * 10^{-17} \leq 10^{-54}
$$
\nfor

\n
$$
|\Omega - 1| \propto \frac{1}{a^2} \implies \frac{|\Omega_{\text{end}} - 1|}{|\Omega_{\text{ini}} - 1|} = \frac{a_i^2}{a_e^2} = 10^{-54}
$$
\nor

\n
$$
N = \ln \left[ \frac{a_{\text{tend}}}{a_{\text{tini}}} \right] \approx 62
$$

- Solve say the Flatness problem:
- Assume inflation until tend  $= 10^{-34}$  sec
- Assume immediate radn dom until today,  $t_0 = 10^{17}$  sec



# **Number of e-folds required**



# $\frac{\partial n}{\partial \pi G \rho a^2} \propto a^{-2} \longrightarrow \exp(-2Ht)$

**Maybe Distant future** 

2. Horizon problem:

Physical: H-1 const during inflation. Small initial patch can inflate. How likely is that? Question of initial conditions.

 $\rho_{\rm mon} \propto a^{-3} \rightarrow 0$  $08/11/2011$   $1.33$ 3. Monopole problem:  $\rho_{\rm mon} \propto a^{-3} \rightarrow 0$  rapidly during inflation Everything infact diluted away except for the inflaton field itself. Hence need to reheat the universe at end of inflation  $T \propto a^{-1} \rightarrow 0$ rapidly during inflation



Initial causally connected region

# **End of inflation**



• Eventually SRA breaks down, as inflaton rolls to minima of its potential.

• Leaves a cold empty Universe apart from inflaton.

d a d minimum of the potential  $\frac{1}{34}$ Inflation has to end and the energy density of the inflaton field decays into particles. This is reheating and happens as the field oscillates around the

Experimental test of slow roll approximation – Aspen 2002



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# **End of inflation.**

•Inflaton is coupled to other matter fields and as it rolls down to the minima it produces particles –perturbatively or through parametric resonance where the field produces many particles in a few oscillations.

•Dramatic consequences. Universe reheats, can restore previously broken symmetries, create defects again, lead to Higgs windings and sphaleron effects, generation of baryon asymmetry at ewk scale at end of a period of inflation.

•Important constraints: e.g.: gravitino production means :  $T_{rh}$  < 109 GeV  $$ often a problem!

### More on this soon

### **The origins of perturbations -- the most important aspect of inflation**

Idea: Inflaton field is subject to perturbations (quantum and thermal fluctuations). Those are stretched to superhorizon scales, where they become classical. They induce metric perturbations which in turn later become the first perturbations to seed the structures in the universe.

Also predict a cosmological gravitational wave background.



Tensor pertn in metric– gravitational waves.
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## **Key features** During inflation comoving Hubble length (1/aH) decreases.

- -
- -

So, a given comoving scale can start inside (1/aH), be affected by causal physics, then later leave (1/aH) with the pertns generated being imprinted.

Quantum flucns in inflaton arise from uncertainty principle.

Pertns are created on wide range of scales and generated causally.

Size of irregularities depend on energy scale at which inflation occurs.



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$$
\delta_H^2(k) = \delta_H^2(k_0) \left[ \frac{k}{k_0} \right]^{n-1}
$$

$$
A_G^2(k) = A_G^2(k_0) \left[ \frac{k}{k_0} \right]^{n_G}
$$

# **The power spectra**

Good approx -- power spectra as being power-laws with scale.

Four parameters

Focus on statistical measures of clustering.

Inflation predicts the amp of waves of a given k which obey gaussian statistics, the amplitude of each wave is chosen independently and randomly from its gaussian distribution. It predicts how the amplitude varies with scale — the power spectrum

Density pertn

Grav waves

# Planck- wow - minuscule temperature changes across the observable universe !



300  $\mu$ K •Improvement over WMAP: ang resolution (x2.5), sensitvity (x10), freq coverage 40 [9 bands (30-857 GHz) v 5 bands (23-90 GHz)] Planck 2018 - ESA

# **Power spectrum - LCDM fit**





### 41 Inflation seems to fit the data really well - the dots are the data, the solid line is the theory — although there are a few issues

$$
\frac{1^2}{k^3}
$$
  $P_{\phi}(k) = \left|\frac{H}{2\pi}\right|^2_{k=aH(Exit)}$   

$$
\frac{2}{H}(k) = \frac{4}{2\pi} \left(\frac{H}{i}\right)^2 \left(\frac{H}{2\pi}\right)^2
$$

$$
\left(\frac{h}{25}\right) = \frac{25}{2\pi} \left(\frac{2\pi}{k} - aH\right)
$$



# **Some formulae**

$$
\left\langle \left\vert \delta\,\varphi_{k}\right\vert ^{2}\right\rangle
$$









Vacuum soln

before tend

$$
\delta_{\rm H} \text{ (k)} \approx 1.91*10^{-5}
$$

In other words the properties of the inflationary potential are constrained by the CMB

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$$
\left|\mathbf{A}_{G}(\mathbf{k}) \propto \kappa^{2} \left| \mathbf{V}^{1/2} \right| \right|_{\mathbf{k} = \mathbf{a} \mathbf{H}}
$$

Tensor pertns: amp of grav waves.

> Note: Amp of perts depends on form of potential. Tensor pertns gives info directly on potential but difficult to detect.

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# **Observational consequences.**

Precision CMBR expts like WMAP and Planck  $\rightarrow$  probing spectra.

Standard approx – power law.

For range  $1 \text{Mpc} \rightarrow 10^4$  $\frac{d \ln k}{d \phi} = \kappa \frac{V}{V'}$ Crucial eqn

 $\delta_H^2(k) \propto k^{n-1}$ ;  $A_G^2(k) \propto k^{n}$  $n-1 = \frac{d \ln \delta_H^2}{d \ln k}$ ;  $n_G = \frac{d \ln A_G^2}{d \ln k}$ 

## n=1; n<sub>G</sub>=0 – Harrison Zeldovich

Power law ok, only a limited range of scales are observable.

$$
Mpc: \Delta ln k \approx 9
$$

$$
n = 1 - 6\epsilon + 2\eta; n_G = -2\epsilon
$$

## $CMBR \rightarrow Measure$  relative importance of density pertns and grav waves.

$$
R = \frac{C_2^{GW}}{C_2^S} \approx 4\pi\epsilon
$$
  
where  $\frac{\Delta T}{T} = \sum a_{lm} Y_m^1(\theta, \varphi), C_1 = \langle |a_{lm}|^2 \rangle$ 

- A unique test of inflat
	-

Indep of choice of inf model, relies on slow roll and power law approx. Unfortunately  $n_G$  too small for detection !

This is where the Bicep2 excitement was !

*Cl* -- radiation angular power spectrum.

$$
r = -2 \pi n_{\rm G}
$$





*,* (2.2)

*,* (2.3)

slow-roll parameters ✏*<sup>H</sup>* and ⌘*H*, defined by

$$
a\ \cdot
$$

### Inflation - brief recan potential *V* () which is minimally coupled to gravity. The system is described by the action **R** is the Ricci scalar and  $P$  is the metric tensor. Specializing to the spatial specializing to the spatial spatial spatial spatial specializing to the spatial spatial spatial spatial spatial spatial spatial spatial spat Inflation - brief recap Source of Inflation: A Scalar Field of Inflation: A Scalar Field of Inflation: A Scalar Field of Inflation: A<br>Source of Inflation: A Scalar Field of Inflation: A Scalar Field of Inflation: A Scalar Field of Inflation: A And Einstein's equations imply And Einstein's equations imply

**P R** is the Ricci scalar and *Ricci scalar field dominates the energy* Einstein's equations assuming scalar field dominates the energy density = i<mark>nstein's eq</mark>u 3  $\frac{1}{\sqrt{2}}$ ⇢ + 3 *p* 

$$
\vec{H}^{2} \equiv \frac{1}{3m_{p}^{2}} \rho_{\phi} = \frac{1}{3m_{p}^{2}} \left[ \frac{1}{2} \dot{\phi}^{2} + V(\phi) \right]
$$
\n
$$
\dot{H} \equiv \frac{\ddot{a}}{a} - H^{2} = -\frac{1}{2m_{p}^{2}} \dot{\phi}^{2},
$$
\n
$$
\ddot{\phi} + 3 H \dot{\phi} + V_{,\phi}(\phi) = 0.
$$

**oocur when potential dominated**

\n
$$
\boxed{\dot{\phi}^2 < V(\phi)}
$$

$$
S[g_{\mu\nu},\phi]=\int\, \mathrm{d}^4x\,\sqrt{-g}\,\left[\frac{m_p^2}{2}\,R-\frac{1}{2}\,\partial_\mu\phi\,\partial_\nu\phi\,g^{\mu\nu}-V(\phi)\right]
$$



$$
a\sim e^{Ht}
$$

*,* (2.3)

slow-roll parameters ✏*<sup>H</sup>* and ⌘*H*, defined by

*H*<sup>2</sup>



 $\frac{1}{2}$  and  $\frac{1}{2}$  allegation of the background level early flat potentia **0** ominating **v**  $\dot{\phi}^2 \ll V(\phi) \;$  with nearly flat potential dominating we obtain nearly exponential ex with nearly flat potential dominating we obtain nearly exponential expansion at the background level otential domi <u>rating we obtain n</u>

3

### Credit: Swagat Mishra

### Inflation can occur when potential dominated **ri**ation can occur when potential dominated

### Inflation - produces the initial seeds for structure to grow through Quantum Fluctuations System = Gravity (*gµ*⌫) + Scalar Field () Iflation - produces the initial seeds for structure to grow through *m*<sup>2</sup>

Tensor perturbations which will become relic gravitational waves Gravitation will be seemed relief are Tensor perturbations which will become relic gravitational waves  $h_{ij}(t,\bar{x})$ 

02/09/2010 47  $\frac{02}{09/2010}$ 

*p*

Action for gravity plus inflaton<br>*Action* for gravity plus inflaton Action for gravity plus inflat

$$
\text{for} \quad S[g_{\mu\nu}, \phi] = \int \, \mathrm{d}^4 x \, \sqrt{-g} \, \left( \frac{m_p^2}{2} \, R - \frac{1}{2} \, \partial_\mu \phi \, \partial_\nu \phi \, g^{\mu\nu} - V(\phi) + \ldots \right)
$$

**Metric including fluctuations** 
$$
ds^{2} = -dt^{2} + a^{2}(t) \left[ \left( e^{2\Psi(t,\vec{x})} \delta_{ij} + h_{ij}(t,\vec{x}) \right) dx^{i} dx^{j} \right]
$$

Particularly two light fields (*m* ⌧ *H*) are guaranteed to exist – Particularly two light fields (*m* ⌧ *H*) are guaranteed to exist – When mass of inflaton is small compared to Hubble rate: m<<H a mass of inflaton is small compared to Hubble rate : m< <sup>2</sup> + *a*2(*t*) h ⇣*e*<sup>2</sup> (*t,*<sup>~</sup> *<sup>x</sup>*) *ij* + *hij* (*t,* ~ *x*)

Comoving curvature perturbation exists

\nwill become density and temperature fluctuations

\nTensor perturbations which will become relic gravitational waves

\n
$$
h_{ij}(t, \vec{x}) = \Psi + \frac{H}{\dot{\phi}} \frac{\delta \phi}{m_p}
$$

Swagat Saurav Mishra, CAPT, Nottingham PBHs and Tail of PDF

### will become density and temperature fluctuations  $\left\vert \begin{array}{ccc} \text{S}^{(0)}, \text{w}) & \text{I} & \text{i} \ & \text{\o} & \text{m}_p \end{array} \right.$ Comoving curvature perturbation exists oving curvature perturbation exists -  $\vert -\zeta(t,\bar{x})\vert$ will become density and temperature fluctuations **Fig. 1**

⌘

### d*x<sup>i</sup>* d*x<sup>j</sup>*

i

## Inflation - allows us to predict the form of the fluctuations for a given model

## In particular during slow roll inflation, where the potential is flat enough and dom

We have

⇡2

*a*

*n<sup>S</sup>* ' 0*.*033 *, |n<sup>T</sup> |* 0*.*0045

Physical wavelength *<sup>p</sup>* = CMB observations: *IMB observation* 

Scalar Spectral index: Red tilt ral<br>It

In particular during slow roll inflation, where the potential is flat enough and dominates the energy density ) ✏*H, |*⌘*H|* ⌧ 1 where ✏*<sup>H</sup>* =

Physical wavelength *<sup>p</sup>* =

*n<sup>S</sup>* ' 0*.*033 *, |n<sup>T</sup> |* 0*.*0045

The Power Spectrum for scalar and tensor fluctuati The Power Spectrum for scalar and tensor fluctuations on large scales The Power-Spectrum for scalar and tensor From CMB observations, During slow-roll (SR) inflation, ˙<sup>2</sup> ⌧ *<sup>V</sup>* () and ¨ ⌧ *<sup>V</sup>* <sup>0</sup>



✓ *H*

We quantify the power spectrum and deviations from scale invariance in terms of  $\Rightarrow$   $\epsilon_H$ <br>slow roll parameters slow roll parameters deviations from scale invariance in terms of ) ✏*H, |*⌘*H|* ⌧ 1 where ✏*<sup>H</sup>* =

> diction<br>— ⇡2 Slow roll predictions: ◆<sup>2</sup> 1 ✓ *k* ◆*n<sup>S</sup> P*⇣ =

> > *k*

ru<br>Drediction is nearly scale invariant and are very sm ale invariant and are very small on large scales Prediction is nearly scale invariant and are n Swagat Saurav Mishra, CAPT, Nottingham PBHs and Tail of PDF Prediction is nearly scale invariant and are very small on large scales *n<sup>S</sup>* ' 0*.*033 *, |n<sup>T</sup> |* 0*.*0045 *k n<sup>S</sup>* ' 0*.*033 *, |n<sup>T</sup> |* 0*.*0045

*n<sup>S</sup>* ' 0*.*033 *, |n<sup>T</sup> |* 0*.*0045

02/09/2010 48 **1<0.00<sup>2</sup>/3hd nu > 0.01 — we have a new hierarchy emerging - has imp<sup>48</sup>** Swagat Saurav Mishra, CAPT, Nottingham PBHs and Tail of PDF **EXAMPLES CHAP.** (slightly red-tilted) and the slightly red-tilted (slightly red-tilted) and the slightly red-CMB pivot scale *<sup>k</sup>*⇤ = 0*.*05 Mpc<sup>1</sup> (slightly red-tilted) Implies  $\epsilon$ <sub>H</sub><0.00<sup>2</sup> and η $\mu$  > 0.01 — we have a new hierarchy emerging - has implications for V(φ) !

We have  
\n
$$
\dot{\phi}^2 \ll V(\phi)
$$
 and  $\ddot{\phi} \ll V'(\phi)$   
\ntify the power spectrum and  
\n $\Rightarrow \boxed{\epsilon_H, |\eta_H| \ll 1}$  where  $\boxed{\epsilon_H = \frac{\dot{\phi}^2}{2m_p^2H^2}, \eta_H = \frac{-\ddot{\phi}}{H\dot{\phi}}}$   
\n $\Rightarrow$  The Power Spectrum for scalar and tensor fluctuations on large scales

$$
\mathcal{P}_{\zeta} = \frac{1}{8\pi^2} \left(\frac{H}{m_p}\right)^2 \frac{1}{\epsilon_H} = A_S \left(\frac{k}{k_*}\right)^{\eta_S - 1} \qquad \qquad \mathcal{P}_{\mathcal{T}} = \frac{2}{\pi^2} \left(\frac{H}{m_p}\right)^{\eta_S - 1}
$$

$$
A_s = 2.1 \times 10^{-9}
$$

Physical wavelength *<sup>p</sup>* =  $\text{ctral index:} \qquad n_{\text{S}}\text{-}1\approx \text{-}0.033$  $\mathsf{N}\mathsf{S}$ - I $\approx$  -0.033 Physical wavelength *<sup>p</sup>* = 2*m*<sup>2</sup>

$$
n_S-1 = -4\epsilon_H + 2\eta_H, \qquad n_\tau = -2\epsilon_H, \qquad r \equiv
$$

*k*⇤



$$
\mathcal{P}_{\mathcal{T}} = \frac{2}{\pi^2} \left( \frac{H}{m_p} \right)^2 = A_{\mathcal{T}} \left( \frac{k}{k_*} \right)^{n_T}
$$

$$
n_{\tau} = -2\epsilon_H, \qquad r \equiv \frac{A_{\tau}}{A_S} = 16\epsilon_{H*}
$$

$$
A_{\mathcal{T}} \leq 3.6\% A_S \qquad \text{BICEP/Keck 2024:}
$$

 $A\mathcal{T} \geq 0.0$  /0  $AS$ 

n<sub>S</sub>-1≈ -0.033 **Ignetra index:** In<sub>τ</sub> | ≲ 0.0045  $n_{\tau}$  |  $\leq 0.0045$  $\ln l - \Omega \Omega A F$ 

From CMB observations,

Primordial Power-spectra at least at large scales –

*A<sup>T</sup>* 3*.*6% *A<sup>S</sup>*

*P<sup>T</sup>* =

⇡2

*m<sup>p</sup>*

*k*⇤

Spectral indices

Scale factor *a*

## Pictorially may help





### The main way we constrain models of inflation from observation

Credit: Swagat Mishra



### Starobinsky (R<sup>2</sup>) inflation

 $n_s \approx 1-2/N \approx 0.967$  $r \approx 12/N^2 \approx 0.0033$ dn<sub>s</sub>/dlnk  $\approx -2/N^2 \approx -0.0006$ 

...... but, there is plenty of room at the top

(and to the side!)

## Real progress - compare with Planck collaboration 2014 - preliminary

Credit: Adam Moss 2013

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## **Lecture 2**

Inflation model building today -- big industry



Inflation and PBHs

Inflation and Reheating

Inflation and Cosmic Superstrings

### First of all an extra slide from the soft skills course



- Inflation model building today -- big industry
	- Multi-field inflation
	- Inflation in string theory and braneworlds
- Inflation in extensions of the standard model
- Cosmic strings formed at the end of inflation
	- The idea is clear though:
- Use a combination of data (CMB, LSS, SN, BAO ...) to try and constrain models of the early universe through to models explaining the nature of dark energy today.
	- Loads of models 300 models analysed with CMB and BAO data, 40% disfavoured, 20% favoured according to Jeffrey's scale of Bayesian evidence [Martin et al 2024]
		- Over 7,700 papers written with title Inflation in title ! 54

## **Some examples keep it simple to get the idea – Chaotic Inflation**



$$
\frac{1}{2\kappa^2} \left[ \frac{V'}{V} \right]^2 \quad ; \quad \eta = \frac{1}{\kappa^2} \left[ \frac{V''}{V} \right]
$$

$$
\frac{\pi G}{3}V(\phi) \quad ; \quad 3H\dot{\phi} + \frac{dV}{d\phi} = 0
$$

$$
\frac{\sqrt{2mt}}{\sqrt{3}\kappa}
$$
;  

$$
xp\left[\frac{\kappa m}{\sqrt{6}}\left(\phi_{i}t-\frac{mt^{2}}{\sqrt{6}\kappa}\right)\right]
$$





# today, COBE/WMAP/Planck scale

Take to be 60 efolds before



Amp of grav<br>waves:  $A_G(k) = \sqrt{\frac{32}{75}} \text{ GV}^{\frac{1}{2}}\Big|_{k=aH}$ 

## 60 efolds before end of inflation.



## Find:  $m = 2 * 10^{13}$  GeV **Constraint on inflaton mass!**

## indices  $n = 1 - 6\varepsilon + 2\eta$ ;  $n_G = -2\varepsilon$  Slow roll

## Find:

 $A_G(k) \approx 1.4 m\sqrt{G}$ 

Normalise to Planck:  $\delta_{\rm H}$  (k)  $\approx 1.91*10^{-5}$ 



 $\frac{11 - 0.91}{11}$ ,  $\frac{11}{11}$ ,  $\frac{11}{11}$ ,  $\frac{11}{11}$ ,  $\frac{11}{11}$ ,  $\frac{11}{11}$ ,  $\frac{11}{11}$  totally ruled out ! Close to scale inv -

# Spectral



## Use values 60 e-folds before end of inflation.



## **2. Models of Inflation—variety is the spice of life. (where is the inflaton in particle physics?)**

Quantum corrections give coefficients proportional to  $ln(\phi)$ and an additional term proportional to  $\ln(\phi)$ 

08/11/2011 58 Inflates only for φ>>MP . Problem. Why only one term? All other models inflate at  $\phi$ <M<sub>p</sub> and give negligible grav. waves.

**1. Chaotic inflation .**

**(Lyth and Riotto, Phys. Rep. 314, 1, (1998), Lyth and Liddle (2009), Martin et al (2024))**

Field theory:  $V(\phi) = V_0 + \frac{1}{2}m^2$ 

V

$$
(\phi^2 + M\phi^3 + \lambda\phi^4 + \sum_{d=5}^{\infty} \lambda_d M_P^{4-d} \phi^d)
$$

$$
V(\phi) \propto \phi^p; \phi \gg M_p; n-1 = -(2+p)/2N;
$$

 $R = -2\pi n_G = \frac{3.1p}{N}$   $\Rightarrow$  sig grav waves.



$$
-c\phi^p + \dots; \ p \ge 3; \ n - 1 = -\frac{2(p-1)}{(p-2)N}
$$

$$
= V_0 - \frac{1}{2} m^2 \phi^2 + ...; \Rightarrow n - 1 = -\frac{2M_p^2 m^2}{V_0}
$$

 $p = 2: \mod$ ular, natural, quadratic inflation

$$
\left(-\sqrt{\frac{16\pi}{p} \frac{\phi}{m_p} \frac{\theta}{\ddot{p}}}, p > 1; n - 1 = -\frac{2}{p}\right)
$$

- **1. Very useful because have exact solutions without recourse to slow roll. Similarly perturbation eqns can be solved exactly.** 
	- **2. No natural end to inflation**

**5. Hybrid inflation**

$$
V(\phi) = V_0 + \frac{1}{2}
$$
  
n - 1 = 
$$
\frac{2M_P^2 m^2}{2}
$$

08/11/2011 60 **2 fields, inf ends when V0 destabilised by 2nd non-inflaton field** ψ

 $1 + \cos \frac{\phi}{f}$ 

## - negligible - - like New Inflation



**4. Natural inflation**

$$
V(\phi) = V_0 \left(1 + \frac{1}{n-1} \right)
$$



## **Two field inflation – more general**

$$
V(\phi, \psi) = \frac{1}{2} m_{\phi}^{2} \phi^{2} + \frac{1}{2} g^{2} \phi^{2} |\chi|^{2} + \frac{1}{4} \lambda \left( |\chi|^{2} - \frac{m^{2}}{\lambda} \right)^{2}
$$

- Found in SUSY models.
- Better chance of success, plus lots of additional features, inc defect formation, ewk baryogenesis.

Inflation ends by triggering phase transition in second field.

Example of Brane inflation

## Cosmic strings - may not do the full job but they can still contribute



## 06/23/2008 62 String contribution < 10% implies *G*µ < 2 x10−<sup>7</sup> . Hybrid Inflation type models Hindmarsh et al, 2019.

## Alpha attractor E and T models of inflation (Kallosh and Linde 2013)

$$
V(\phi) = \frac{1}{2}m^2\phi^2 - |l|
$$



### 06/23/2008 63 \* Thoeo fit more noturelly with the reedship or Swagat Swagat Swagat Saurav Mishra, Capt, Nottingham Reheating and Oscillons and Oscillons and Oscillons and O **These fit more naturally with the recent Planck bounds on n and r.**

## $U(\phi)$  | (E-Model & T-Model)





## *g***<sub>n</sub> f** *nn**n**nsupergravity -- <b>N=1 SUGR Lagrangian:* **Inflation in string theory -- non trivial**

$$
\mathcal{L} = -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} + V_F, \quad \text{with}
$$

and 
$$
D_{\varphi} = \partial_{\varphi} W + \frac{1}{M_p^2} \partial_{\varphi} K
$$

$$
\begin{array}{rcl}\n\mathcal{L} & \approx & -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} - V_0 \left( 1 + K_{\varphi\bar{\varphi}}|_{\varphi=0} \frac{\varphi\bar{\varphi}}{M_p^2} + \dots \right) \\
& = & -\partial\phi\partial\bar{\phi} - V_0 \left( 1 + \frac{\phi\bar{\phi}}{M_p^2} + \dots \right),\n\end{array}
$$

### **Expand K about φ=0**

**Canonically norm fields ϕ**

 $\Delta q = 44\pi$   $\tau_{\ell}$   $\tau_{\ell}$  theorohoditional model dependents **So, need to cancel this generic term possibly through additional model dependent terms.**

## **Have model indep terms which lead to contribution to slow roll parameter η of order unity**

$$
V_F = e^{K/M_p^2} \left[ K^{\varphi\bar{\varphi}} D_{\varphi} W \overline{D_{\varphi} W} - \frac{3}{M_p^2} |W|^2 \right]
$$

$$
K(\varphi,\bar{\varphi})=K_0+K_{\varphi\bar{\varphi}}\varphi\bar{\varphi}+\ldots
$$

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**Ex 1: Warped D3-brane D3-antibrane inflation where model dependent corrections to V can cancel model indep contributions [Kachru et al (03) -- KLMMT].** 

 $V(\phi) = V_0(\phi) + \beta H^2 \phi^2$ 

*B* relates to the coupling of warped **throat to compact CY space. Can be fine tuned to avoid η problem** 



**Ex 2: DBI inflation -- simple -- it isn't slow roll as the two branes approach each other so no η problem** 

**Ex 3: Kahler Moduli Inflation [Conlon & Quevedo 05]**

**Inflaton is one of Kahler moduli in Type IIB flux compactification. Inflation proceeds by reducing the F-term energy. No η problem because of presence of a symmetry, an almost no-scale property of the Kahler potential.** 

$$
V_{inf}=V_0-\frac{4\tau_n W_0 a_n A_n}{\mathcal{V}^2}
$$

**Inflaton moduli: τn** 



 $V_{inf} = V_0 - \frac{4\tau_n W_0 a_n A_n e^{-a_n \tau_n}}{V^2}$ 

**volume modulus** 

 $10^5 l_s^6 \leq \mathcal{V} \leq 10^7 l_s^6$ 

**Inflaton [Blanco-Pillado et al 09] Volume modulus**  Can include curvaton as second evolving moduli -- Burgess et al 2010

**with low energy scale**

 $V_{inf} \sim 10^{13}$ GeV.



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**Key inflationary parameters:** 

- n: Planck and WMAP have detected it it's not unity.
- r: Tensor-to-scalar ratio : considered as a smoking gun for inflation but also produced by defects and some inflation models produce very little.
- dn/dln k : Running of the spectral index, usually very small -- probably too small for detection.
- f<sub>NL</sub>: Measure of cosmic non-gaussianity. Still consistent with zero. Lots of current interest.
- Gµ: string tension in Hybrid models where defects produced at end of period of inflation.
- Also additional perturbation generation mechanisms (e.g. Curvaton) Perturbations not from inflaton but from extra field and then couple through to curvature perturbation



ultimate vacuum cleaner, it clears out pretty much everything, pa<br>with a cold, empty large universe, not quite v Inflation is the ultimate vacuum cleaner, it clears out pretty much everything, particles get diluted, radiation gets red shifted, we end inflation with a cold, empty large universe, not quite what we experience today.

We need to reheat the universe - we convert the remaining energy stored in the inflaton field into primordial particles through their interactions. We could consider this the beginning of the Hot Big Bang

### Reheating the Universe after inflation has finished

Credit: Swagat Mishra





### Reheating occurs as slow roll inflation finishes and inflaton oscillates about the potential minima Inflaton EoM: ¨ + 3*H*˙ + *V ,* = 0 hes and inflaton oscillates ow roll inflation finishes and inflaton oscillates about the potential minim



$$
V(\phi) \simeq V_0 \left(\frac{\phi}{m_p}\right)^{2n} \bigg|; \ n > 0
$$



### ! Inflaton decay into fermions and bosons *via* Feynman processes **Cay of inflaton** [Abbott et al, Albrecht et al, Linde, Dolgov 1980's] al, Albrecht et al, Linde, Dolgov 1980's] <u>reference de caracter de la persona de caracter de caracter de contracte de la personalidad de la personalidad<br>Contracteur de la personalidad de </u> <mark>bbott et</mark> a <sup>1</sup>, *i* worden be di,

! ¯ =

### ) Relatively low reheating temperature ! For bosonic decay ! , since <sup>2</sup> / <sup>1</sup>*/t*<sup>2</sup> ) ! / <sup>1</sup>*/t*<sup>2</sup> In that case reheating is incomplete and we have a coherent oscillating inflaton condensate

Inflaton particles of mass m acting as CDM decay into fermions and bosons Reheating *via* Perturbative decay

Decay acts phenomenologically like additional friction term:  $\left[\phi + (3H + \Gamma)\,\phi + V\right]$  *M<sup>p</sup>*  $\frac{1}{2}$ *y* like additional fri

Initially  $H >> F$ , reheating completes when  $H \sim F$  with reheat temperature  $T_{\text{re}}$ **25** when H ∼ Γ w .<br>itl n rehe Initially H >> F, reheating completes when H ~ F with reheat tempera

 $\frac{9^{*}}{0}$  $\overline{\phantom{0}}$  $b_p$  )  $~{\rm GeV} \Big\vert$  $\mathcal{S}(10^{-19}m_{\odot}$  Hence low reheat temp  $T_{\odot}< 10^{12}$  (  $\text{GeV}$  $\sqrt{\pi}$  $\frac{1}{2}$  $\frac{1}{2}$ *p* \*\*Abbott, Farhi, Wise; \*\*Albrecht, Steinhardt, Turner; \*\*Dolgov, Linde (1982)  $\mathcal{S}$  $S = 2.4 \times 10^{10} \times \left( \frac{1}{2.8} \right)$  (  $\left( \frac{1}{2.8} \right)$  (  $\left( \frac{1}{2.8} \right)$  (  $\left( \frac{1}{2.8} \right)$  ) (  $\left( \frac{1}{2.8$  $T_{\rm re} \simeq$  $\left(\frac{\pi^2 g_*}{90}\right)^{-1/4}$  $\sqrt{2}$  $\Gamma$   $M_p$  $\sqrt{1/2}$  $= 2.4 \times 10^{18} \times$ 90  $\left( \right)$ *m<sup>p</sup>*  $\sqrt{1/2}$ GeV 10<sup>-5</sup> m<sub>p</sub>:  $\Gamma_{\rm \star}$  ,  $\bar{J}_{\rm \star}$   $\rm =$   $\frac{h^2\, m}{\sim} < 4\times 10^{-13}\, m_{\rm \star}$  . Hence low rehe  $\rightarrow_{\chi\chi}\propto 1/$ *h*<sup>2</sup> *m* But  $H \propto 1$ Or:  $T_{\text{re}} \simeq \left(\frac{m}{90}\right) \quad (\Gamma M_p)^{\frac{1}{2}} = 2.4 \times 10^{18} \times \left(\frac{m}{90}\right) \quad (\frac{1}{m_p})^{\frac{1}{2}}$ Decay to fermions with h≲10<sup>-3</sup> and m= 10<sup>-5</sup> m<sub>p</sub>:  $\ \Gamma_{\phi\to\bar{\psi}\psi}=$  $h^2$   $m$  $8\pi$  $\leq 4 \times 10^{-13} \, m_p$  Hence low reheat temp  $T_{\rm re} \leq 10^{12} \,\, \mathrm{GeV}$ leading to an upper  $\Gamma_{\phi\phi\to\chi\chi}\propto 1/t^2$  and the set of the set of  $I$  $\begin{bmatrix} 1 & \sqrt{16} & \$ Bosonic decay, φφ—>χχ, recall φ $_0$  ∝ 1/t, hence  $\;\;\Gamma_{\phi\phi\to\chi\chi}\propto 1/t^2$  But  $\;\;\;H\propto 1/t\;\;\Rightarrow\;\;\Gamma_{\phi\phi\to\chi\chi}\ll H$ Hence low reheat temp  $\overline{\mathbf{S}}$  $\sqrt{\pi^2 g}$ leading to an upper bound *<sup>T</sup>*re <sup>10</sup><sup>12</sup> GeV.  $D$  europe temperaturely with the temperature of  $D$  entropy  $D$  $\textbf{Bosonic decay, } \boldsymbol{\phi} \boldsymbol{\phi} \boldsymbol{-\!\!>}{\color{black} \times}$ yx, recall  $\boldsymbol{\phi}_0$   $_{\!\!\infty}$  1/t, hence  $\quad \Gamma_{\boldsymbol{\phi} \boldsymbol{\phi} \boldsymbol{\rightarrow} \chi \chi} \propto 1/t^2$ But  $\sqrt{P}$  $\leq 4 \times 10^{-15} \, m_p$  Hence low reheat temp  $T_{\rm re} \leq 10^{12} \; {\rm GeV}$ But  $H \propto 1/t \Rightarrow \Gamma_{\phi\phi \to \chi\chi} \ll H$ 





### Reheating from perturbative decay of inflaton [Abbott et al, Albrecht et al, Linde, Dolgov 1980's] ✓⇡<sup>2</sup>*g*⇤

d bosons 
$$
\Gamma_{\phi \to \bar{\psi}\psi} = \frac{h^2 m}{8\pi}; \quad \Gamma_{\phi\phi \to \chi\chi} = \frac{g^2 \phi_0^2}{8\pi m}
$$

nonenologically like additional friction term:

\n
$$
\begin{aligned}\n\ddot{\phi} + (3H + \Gamma) \, \dot{\phi} + V_{,\phi} &= 0 \\
\end{aligned}
$$

$$
\text{ompletes when }\mathsf{H}\text{ $\sim$ $\Gamma$ with reheat temperature $\mathsf{T}_{\text{re}}$} \quad \Gamma\simeq H=\sqrt{\frac{1}{3m_p^2}\,\rho(T_{\text{re}})}\Rightarrow\Gamma=\sqrt{\frac{1}{3m_p^2}\,\frac{\pi^2}{30}\,g_*(T_{\text{re}})\,T_{\text{re}}^4}
$$
\n
$$
\left[T_{\text{re}}\simeq\left(\frac{\pi^2g_*}{90}\right)^{-1/4}\left(\Gamma\,M_p\right)^{1/2}=2.4\times10^{18}\times\left(\frac{\pi^2g_*}{90}\right)^{-1/4}\,\left(\frac{\Gamma}{m_p}\right)^{1/2}\,\text{GeV}\right]
$$

### Reheating from non-perturbative decay of inflaton [Kofman et al (1994), Shtanov et al (1995)]

- Occurs when bosonic couplings high enough  $g^2 \ge 10^{-8}$
- Particle production taking place in presence of oscillating inflaton condensate via parametric resonance — collective phenomena
	- Occurs quickly efficiently and non-thermal
	- Not applicable to fermionic decay (Pauli exclusion)
		- Dynamics divided into three distinct phases:
		- 1. Preheating (linear parametric resonance)
	- 2. Backreaction (quenching of resonant particle production)
	- 3. Scattering and thermalisation (perturbative decay, turbulence)

Inflaton  $\varphi$  decays to massless field  $\chi$  $\blacksquare$  and action  $\varphi$  of action  $\varphi$  of action  $\varphi$  of action  $\varphi$  of action  $\varphi$ described by the action of the action of<br>The action of the action of

$$
\boxed{S[\varphi,\chi]=-\int\mathrm{d}^4x\,\sqrt{-g}\left[\frac{1}{2}\,\partial_{\mu}\varphi\partial^{\nu}\varphi+V(\varphi)+\frac{1}{2}\,\partial_{\mu}\chi\partial^{\nu}\chi+\mathcal{I}(\varphi,\chi)\right]}
$$

Interaction:

 $\lambda$ 



 $\sum$ 

 $\ddot{\chi} - \frac{\nabla^2}{a^2}$ 

 $2 \frac{9}{10}$ 

 $\nabla^2$ 

 $\overline{\nabla^2}$ 

 $\cdot \frac{\nabla^2}{\partial \theta} \varphi$ 

 $\ddot{\varphi}-\frac{\nabla^2}{a^2}$ 

$$
\ddot{\varphi} - \frac{\nabla^2}{a^2} \varphi + 3H\dot{\varphi} + V_{,\varphi} + \mathcal{I}_{,\varphi} = 0
$$

$$
\frac{\nabla^2}{a^2}\chi + 3H\dot{\chi} + \mathcal{I}_{,\chi} = 0
$$

 $\Omega$  $\frac{1}{2}$  +  $\frac{1}{2}$  $\overline{\Omega}$  $\sqrt{1}$ *a*  $\vec{x}$ <sup>+ 0</sup> 2 ˙ 2 *a . a* + *I*('*,* )  $\mathbf{v}(t,\vec{x}) = \nabla(t)\hat{\mathbf{v}} + \delta \mathbf{v}(t,\vec{x}) + \mathbf{X}$  is in its va  $\mathcal{S}_{\mathcal{S}}$  and  $\mathcal{S}_{\mathcal{S$  $\vec{x}$  =  $\phi(t) + \delta \varphi(t, \vec{x})$ <sup>-0</sup>  $\bar{f}(t,\vec{x})$  =  $\bar{\chi}(t) + \delta \chi(t,\vec{x})$  x is in its vacuum state

At end of inflation :  $\rho_{\phi} \gg \rho_{\chi},\, \rho_{\delta\varphi}$  condensate dominated

er are regions of the linear regime of the linear regime of the linear region of the linear regions of the linear regions of the linear region of the linear region of the linear regions of the linear regions of the linear

$$
\chi)=\tfrac{1}{2}g^2\varphi^2\chi^2
$$

=



$$
\lambda \qquad a^2 \qquad \lambda + 911\lambda + 2, \chi = 0
$$
\nFriedmann equation:

\n
$$
H^2 = \frac{1}{3m_p^2} \left[ \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} \frac{\vec{\nabla} \varphi}{a} \cdot \frac{\vec{\nabla} \varphi}{a} + V(\varphi) + \frac{1}{2} \dot{\chi}^2 + \frac{1}{2} \frac{\vec{\nabla} \chi}{a} \cdot \frac{\vec{\nabla} \chi}{a} + \mathcal{I}(\varphi, \chi) \right]
$$

Preheating in the linear regime :

Field equations:

in the linear regime : 
$$
\varphi(t, \vec{x}) = \phi(t) +
$$

$$
\chi(t, \vec{x}) = \bar{\chi}(t) +
$$

At end of inflation : 
$$
\rho_{\phi} \gg \rho_{\chi}, \rho_{\delta\varphi}
$$
 condensate dominated
# Equation of dynamics of the Fourier modes

 $\ddot{\chi}_k + 3H\dot{\chi}_k +$ Fourier modes:  $\ddot{\chi}_k$  $\ddot{\mathbf{v}}_k + 3H\dot{\mathbf{v}}_k + \left|\frac{k^2}{k}\right|$  $\frac{1}{2}a+3H\dot{\chi}_{k} + \Big\vert$ 

 $\frac{1}{2}$  Ignoring the expansion of space (adiabatic approximation) of  $\frac{1}{2}$ and (regime) candidates it (1)  $\frac{1}{2}$  is  $\frac{1}{2}$  in  $\frac{1}{2}$  in  $\frac{1}{2}$ Ignoring the expansion (adiabatic regime) and recall for  $V(\phi) \approx 1/2$  m<sup>2</sup>  $\phi$ <sup>2</sup> we have  $\int$  or  $\int$   $\frac{1}{2}$  cos (*x*)  $\int$  for  $\int$   $\frac{1}{2}$  (*A*)  $\int$   $\frac{1}{2}$  (*O*)  $\int$   $\frac{1}{2}$ 

> Obtain the Mathieu Equation: u Equai





Fourier modes: 
$$
\ddot{\chi}_k + 3H\dot{\chi}_k + \left[\frac{k^2}{a^2} + g^2 \phi(t)^2\right] \chi_k = 0
$$

$$
\frac{\mathrm{d}^2 \chi_k}{\mathrm{d} T^2} + \Big[A_k - 2q \, \cos \left( 2T \right) \Big] \chi_k = 0
$$

$$
ext{egime} \text{ and } \text{recall for } V(\varphi) \approx 1/2 \text{ m}^2 \varphi^2 \text{ we have } \left[ \phi(t) \simeq \phi_0(t) \cos(mt) \right]
$$

$$
= mt - \frac{\pi}{2} \qquad \text{and resonance parameters:} \qquad q = \frac{g^2}{4} \left(\frac{\phi_0}{m}\right)^2; \quad A_k = \left(\frac{k}{m}\right)^2 + 2q
$$
\n
$$
\frac{d^2 \chi_k}{dT^2} + \Omega_\chi^2(k, T) \chi_k = 0; \quad \boxed{\Omega_\chi^2 = A_k - 2q \cos(2T)}
$$



 $\frac{1}{\sqrt{2}}$ 

 $\sim$  SSM Lecture Notes (2024). The state of the state  $\sim$ 

Solutions from Floquet Theory  $\chi_k(T) = \mathcal{M}_k^{(+)}(T) \; e^{\mu_k T}$  .

 $\sqrt{(T)}$   $\sqrt{(T)}$   $\sqrt{(T)}$   $\sqrt{(T)}$   $\sqrt{(T)}$ let Theory  $\chi_k(T) = \mathcal{M}_k^{(+)}$ 

Stage 1: Preheating in the linear regime - parametric oscillator **ime - parametric**  *k*2  $\frac{1}{2}$  cillator *a*<br>*Pheating in the linear regime - parame* the linear regime - parametric oscillator

$$
\boxed{\chi_k(T) = \mathcal{M}_k^{(+)}(T) e^{\mu_k T} + \mathcal{M}_k^{(-)}(T) e^{-\mu_k T}}
$$

Exponential growing solutions for  $Re(\mu_k) \neq 0$ .

d<sup>2</sup>*<sup>k</sup>*

h

Write as

#### Narrow resonance The Broad production Broad production in  $\mathbf{B}$ Broad resonance







### Narrow resonance

$$
q=\frac{g^2}{4}\left(\frac{\phi_0}{m}\right)^2\geq 1
$$



#### Exponential growing solutions for  $\text{Re}(\mu_k) \neq 0$ .  $L$ Aponential grown ig Solutions for May  $\mu$   $\approx$  0.

 $n_\chi(k) \equiv$ 

 $\mathcal{E}_{\chi}(k)$ 







=

1

 $\lceil$   $\begin{array}{c} \end{array}$ 

 $\mathrm{d}\chi_k$ 

 $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\vert$ 

2

 $+ \Omega_\chi^2 \, |\chi_k|^2$ 

 $\overline{1}$ 

 $\propto e^{2\mu_k T}$ 

 $\mathrm{d}\mathrm{T}$ 

 $2\Omega_\chi$ 

 $\Omega_\chi(k)$ 

System moves from Broad to Narrow residence as & *T*osc is independent of 0. System moves from Broad to Narrow residence as  $\phi_0(t) \propto \left(\frac{m_p}{m}\right)$ !Resonance parameter *<sup>q</sup>* <sup>=</sup> *<sup>g</sup>*2<sup>2</sup>

> Stage 3: Perturbative decays take over  $\varphi \longrightarrow \bar{\psi}\psi \, ; \quad \varphi \varphi \longrightarrow \chi \chi$

Stage 2: Backreaction and quenching - shutting off the rapid particle production tion and quenching - shutting off the rapid particle production

Particle production via resonance is quenched due to redshifting of q(t) and k/a(t), and the backreaction of  $\chi(x,t)$  on  $\varphi(t)$ *k*<sup>2</sup>

\n
$$
\phi_0(t) \propto \left(\frac{m_p}{m}\right) \frac{1}{t}
$$
\n



New possible feature not included so far arises from asymptotically flat potentials - motivated from CMB observations <sup>=</sup> *e*<sup>2</sup> (*t,*<sup>~</sup> *<sup>x</sup>*) *ij* + *hij* (*t,* ~

$$
V(\phi)=V_0\left(\frac{\phi}{m_p}\right)^{2n}-|U(\phi)|
$$

They have attractive self interactions a the formations of long lived non-topological<br>Colitons like oscillons — provide a new route to They<br>the **They have attractive self interactions allowing for** have the self-interaction of the self-interaction of the self-interaction of the self-interaction of the self-<br>The self-interaction of the self-interaction of the self-interaction of the self-interaction of the self-intera solitons like oscillons — provide a new route to reheating

*V*

(

 $\boldsymbol{\phi}$ 



**)** 

\*\*Amin *et. al* & Lozanov *et. al* (2010-2020) Credit: Swagat Mishra



[Amin et al 2010]





Oscillons : a type of soliton, self supported localised long lived due to non-linear interactions For soliton, sen supported localised long lived due to hon-linear litteractions<br>[Bogolyubsky & Makhankov 1978, Gleiser 1993, EJC et al 1995]<br> $\lambda$  and  $q$  as  $\lambda$ ! Analytical results based on small-amplitude oscillations o<br>D 2 m supported localised form<br>Makhankov 1978, Gleiser

Can obtain semi-analytic solutions from small amplitude oscillations:  $V(\varphi) \approx$ 1 2  $m^2\varphi^2-\frac{\lambda}{4}\,\mu\,\varphi^4+\frac{g}{6}\,\lambda\,\varphi^6$  $\delta(r) \cos(\omega_0 t) + \cdots$   $\omega_0 - m_1 \left(1 - \frac{\lambda^2 \alpha^2}{r^2} \right)$  $g\Phi_0^2$   $\begin{bmatrix} r \end{bmatrix}$ ,  $a^2$   $g\Phi_0^2$   $\begin{bmatrix} 0 & 5 \end{bmatrix}$   $a^2$   $h^2$   $h^2$   $h^2$   $h^2$   $h^2$   $h^2$   $h^2$   $h^2$ <sup>1</sup> <sup>2</sup>↵<sup>2</sup> [Amin et al, Mahbub and Mishra] ! The core profile of an Oscillon *V* (') ⇡ <sup>2</sup> <sup>4</sup> *<sup>µ</sup>* '<sup>4</sup> <sup>+</sup> *<sup>g</sup>* <sup>6</sup> '<sup>6</sup>  $\blacksquare$  Suppose the formulation of  $\setminus \mathcal{P}$  $\boldsymbol{\varphi}_{\textbf{osc}}(t,r) \thickapprox \Phi(r) \cos{(\omega_0\,t)}\,+\, ... \, \vert; \quad \omega_0 = m$  $\overline{\phantom{a}}$  $1 - \frac{\lambda^2 \alpha^2}{m^2}$  $m^2$  $\left[\left(r_0\right)^{3}\right]$  $\left[\begin{array}{c}3^{9}^{10}\end{array}\right]$ ,  $10 - \Phi_0 \sqrt{\lambda}$   $3\lambda^{9}$ <sup>1</sup> 8 3 0 ; *r*<sup>0</sup> = <sup>0</sup> <sup>10</sup> With core profile: c solutions from small amplitude oscillations:  $V($ 'osc(*t, r*) ⇡ (*r*) cos (!<sup>0</sup> *t*) + *...* ; !<sup>0</sup> = *m* <sup>1</sup> <sup>2</sup>↵<sup>2</sup> *m*<sup>2</sup>  $\frac{1}{\sqrt{1-\frac{1$  $\Phi(r) \approx \Phi_0 \, \text{sech} \left( \frac{r}{r_c} \right)$ *r*0 ◆  $;\,\alpha^2$ =  $g\Phi_0^2$ 0  $8\lambda$  $3 - \frac{5}{3}$ 3  $g\Phi_0^2$  $\overline{\phantom{a}}$  $; r_0 =$ 1  $\Phi_0$  $\sqrt{6}$  $\frac{6}{\lambda} - \frac{10}{3\lambda}g\Phi_0^2$ 0 \*\*Rajaraman(1987), \*\*Gleiser *et. al*; \*\*Amin *et. al* ; \*\*Mahbub, SSM (2023)

$$
\frac{\varphi_{\rm osc}(t,r) \approx \Phi(r) \cos{(\omega_0)}}{\Phi(r) \approx \Phi_0 \,\text{sech}\left(\frac{r}{r_0}\right)}; \,\alpha^2 = \frac{g\Phi_0^2}{8\lambda}.
$$



**Oscillon field** 
$$
\Phi(t,r) = \phi_0 \operatorname{sech}\left(\frac{r}{r_0}\right) \cos(\omega_0 t)
$$

Credit: Swagat Mishra



Can survive for of order 109 oscillations depending on the interactions. [Zhang et al 2020]

Oscillons : Can they form dynamically starting from natural conditions at the end of inflation ? [Lozanov & Amin 2017, Shafi et al 2024]  $S = \frac{1}{2}$  self-resonance and inflaton fragmentation  $\frac{1}{2}$  . The self-resonance and  $\frac{1}{2}$ 

In the linear regime - we see behaviour similar to that already discussed. Self resonance. But then inflaton fragmentation kicks in In the linear regime, Fourier model functions satisfying the complete satisfying  $\mathcal{L}(\mathcal{A})$ na – wa saa hahavinur similar to that alraady discussed. Self resonance . Rut thar

Linear regime, Fourier modes:



Asymmetric

Leads to exp growth of inflaton fluctuations with band structure similar to Mathieu resonance for quadratic case:  $\ \delta\varphi_k(t)\propto e^{\mu_kmt}$ *Poince Carrie Rodan Worldog / Reyniptonodily natipotent* Band - recall we need Asymptotically flat potentials Potentials being considered - recall we need Asymptotically flat potentials ) Exponential growth of inflaton fluctuations '*k*(*t*) / *<sup>e</sup><sup>µ</sup>kmt* Potentials being considered - recall we need Asymptotically fiat potentials

 $\sqrt{1 - \frac{1}{2}}$  ( $\sqrt{1 - \frac{1}{2}}$ <sup>2</sup>*m*<sup>2</sup><sup>2</sup> *<sup>|</sup>U*()*<sup>|</sup>* (E-Model & T-Model) Shafi et al- e-Print:2406-00108  $\frac{1}{2}m^2\phi^2 - |U(\phi)| \quad \text{(E-Model & T-Model)}$ 

$$
\text{urier modes:} \quad \left[ \ddot{\delta \varphi}_k + 3H \dot{\delta \varphi}_k + \left[ \frac{k^2}{a^2} + V_{,\phi\phi}(\phi) \right] \delta \varphi_k = 0 \right]
$$

$$
V(\phi)=\tfrac{1}{2}m^2\phi^2-|
$$



Symmetric





Oscillons : full non -linear evolution using CosmoLattice [Shafi et al 2024] - no external coupling In particular we consider the E-model potential of the form



# Oscillons formation - Asymmetric potential [Shafi et al 2024]











$$
\boldsymbol{V}(\boldsymbol{\varphi})=V_0\,\tanh^2\left(\lambda_{\textrm{\tiny T}}\frac{\boldsymbol{\varphi}}{m_p}\right)
$$

# In particular we consider the E-model potential of the E-model potential of the form of th

$$
\boldsymbol{V}(\boldsymbol{\varphi}) = V_0 \, \left[ 1 - e^{-\lambda_{\rm E} \, \frac{\boldsymbol{\varphi}}{m_p}} \right]^2
$$



Oscillons in the presence of external coupling ? [Shafi et al 2024] Oscilions in the presence of external coupling: ponan et



### Oscillon formation and decay with no external coupling

### Oscillon formation and decay with external coupling



### [Shafi et al 2024]

### Oscillon fractional abundance [Mahub and Mishra 2023]



We find in the absence of external couplings oscillons form for both types of potentials and for generic initial conditions at the end of inflation





Any of these primordial in origin? Credit: LIGO-Virgo-KAGRA consortium





10*M*

# Inflation and Primordial Black Holes





Threshold for PBH formation [Carr] :  $\delta \gtrsim \delta_c$ ~w=p/ $\rho = 1/3$ .  $-$  density contrast at horizon crossing, depends on shape of perturbation which depends on

PBH mass roughly equal to horizon mass

- Since LIGO's amazing direct detection of coalescing BH binaries, PBHs have had a resurgence of interest.
- For reviews and future directions see Green & Kavanagh [arXiv: 2007.10722], Carr & Kuhnel [arXiv:2006.028380, Bird et al [arXiv:2203.08967]
	- Form from over densities in early Universe before nucleosynthesis non-baryonic [Zel'dovich & Novikov; Hawking]
- They evaporate (Hawking radiation), lifetime longer than age of Universe for M>1015g can make them a DM candidate [Hawking, Chapline]
	- Maybe some of the BHs in the binaries detected by LIGO-VIRGO are primordial [Bird et al, Clesse & Garcia-Bellido, Sasaki et al ]

$$
M_{\rm PBH} \sim 10^{15} g \left( \frac{t}{10^{-23}} \right) \sim M_{\rm sun} \left( \frac{t}{10^{-6} s} \right)
$$
06/23



### **Formation**

- Favoured collapse of large density perturbations (shortly after horizon entry) during radiation domination
- Also collapse of cosmic string loops [Hawking, Polnarev & Zemboricz], bubble collisions [Hawking, Moss & Stewart], fragmenting inflation condensates [Cotner & Kusenko]
	- primordial power spectrum



# Present day hounds on PRHs as DM Present day bounds on PBHs as DM

### Green and Kavanagh 2020

## Posuired emplification for interesting DDLL seeperies Required amplification for interesting PBH scenarios



# Primordial Black Holes are really really cool !

• Formed very early - typically within the first few seconds of the Hot Big Bang phase!<br>
19 apply 1 minutes and the Hot Big Bang Phase • Hawking told us, they have a temperature, and they evaporate as well as accrete.

 $\overline{1}$ 3  $10^{15}$  gm

 $M_{\rm sun}$ • Mass at formation  $M_{\rm PBH}\simeq M_{\rm H}=6\times10^4$  |  $\to$  |  $M_{\rm sun}$  PBHs evaporating today formed around 10-23 sec into HBB phase



$$
T_H = \frac{\hbar c^3}{8\pi G K_B M_{\rm BH}} = 6.19 \times 10^{-8} \left(\frac{M_{\odot}}{M_{\rm BH}}\right) K
$$

• Evaporation rate:

$$
\frac{dm_{\rm BH}}{dt} = -\frac{g_{\star}}{3} \frac{m_{\rm Pl}^4}{m_{\rm BH}^2} \text{---}\rangle \text{ mass (t): } m_{\rm BH}^3 = m_0^2 - g_{\star} m_{\rm Pl}^4 t \quad \text{---}\rangle \text{ lifetime:} \quad \tau = \frac{m_0^3}{g_{\star} m_{\rm Pl}^4}
$$

$$
M_{\rm PBH} \simeq M_{\rm H} = 6 \times 10^4 \left( \frac{t}{1 {\rm sec}} \right) \, \rm M
$$

• Initial mass of PBH evaporating today — about that of a mountain

 $\frac{1}{2}$   $M_c \simeq \left(\frac{\nu_0}{12.8 \text{ C K} \cdot \text{m}}\right)$   $10^{15} \text{ gm}$  $\left(\frac{t_0}{13.8 \text{ Gyr}}\right)$ 

[Hawking 1971, Carr, Hawking 1974, Hawking 1974, Page 1975]

- 
- We can use them to probe very small early Universe physics.
- whing to bail about you probol for you and they evan<br>wiking told us they have a temperature and they evan
- Hawking radiation hard to detect. Hawking radiation - hard to detect.

Initial PBH mass fraction (fraction of universe in regions dense enough to form PBHs)

$$
(M) \sim \int_{\delta_c}^{\infty} P(\delta(M_H)) d\delta(M_H)
$$



but in fact β must be small, hence  ${\tt \sigma} \ll {\tt \delta_c}\,$  and  $\;\;\beta(M) \sim \sigma(M_{\rm H}) \exp\Big($ 

But PBH are matter, so in radiation their contribution to the energy density budget grows Relation between PBH initial mass function β and fraction of DM in form of PBHs, f:

So  $\beta$  must be small but non-negligible

$$
\frac{\beta(M)}{M_{\rm H})} = \text{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma(M_H)}\right)
$$

### PBH forming fluctuations

$$
\left(-\frac{\delta_{\rm c}^2}{2\sigma^2(M_{\rm H})}\right)
$$

Credit: Anne Green

$$
\frac{\rho_{\rm PBH}}{\rho_{\rm rad}} \propto \frac{a^{-3}}{a^{-4}} \propto a
$$
  

$$
\beta(M) \sim 10^{-9} f \left(\frac{M}{M_{\rm sun}}\right)^{1/2}
$$

$$
\frac{\rho_{\rm PBH}}{\rho_{\rm rad}} \propto \frac{a^{-3}}{a^{-4}} \propto a
$$



### But PBH are matter, so in radiation their contribution to the energy density budget grows  $\rho_{\rm PBH}\sim a^{-3}$ grows with the second with the<br>Second with the second with the  $\rho_{\rm PBH}$  $\rho_{\rm rad}$  $\propto$  $\frac{a}{a-4} \propto a$



To form an interesting number of PBHs the primordial perturbations must be significantly larger ( $\sigma^2(M_H)$ ~0.01) on small scales than on cosmological scales.

But on CMB we know primordial perturbati
$$
\phi_{\rm s} = \phi_0(M_H) \sim 10^{-5} \Rightarrow \beta(M) \sim \text{erfc}(10^5) \sim \exp(-10^{10})
$$

Totally negligible if initias proturbations were closs to scale invariant.



Credit: Anne Green

One approach — introduce non-gaussianity. PBHs form from rare large density fluctuations arising during inflation, change the shape of the tail of the probability distribution —> can significantly affect the PBH distribution



slow-roll parameters ✏*<sup>H</sup>* and ⌘*H*, defined by

$$
S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[ \frac{m_p^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} - V(\phi) \right]
$$

$$
H^{2} \equiv \frac{1}{3m_{p}^{2}} \rho_{\phi} = \frac{1}{3m_{p}^{2}} \left[\frac{1}{2}\dot{\phi}^{2} + V(\phi)\right]
$$
\nInitial conditions

\n
$$
\ddot{H} \equiv \frac{\ddot{a}}{a} - H^{2} = -\frac{1}{2m_{p}^{2}} \dot{\phi}^{2},
$$
\ninitial conditions

\n
$$
\ddot{\phi} + 3 H \dot{\phi} + V_{,\phi}(\phi) = 0.
$$

d✏*<sup>H</sup>*

 $\text{flation corresponds to the condition } \epsilon_H \ll 1.$  $\text{lition } \epsilon_H \ll 1$ • Quasi-de Sitter inflation corresponds to the condition  $\epsilon_H \ll 1$ .

$$
\epsilon_H = -\frac{\dot{H}}{H^2} = \frac{1}{2m_p^2} \frac{\dot{\phi}^2}{H^2},
$$
  
\n
$$
\eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}} = \epsilon_H + \frac{1}{2\epsilon_H} \frac{d\epsilon_H}{dN},
$$
  
\nSlow roll parameters

*Slow roll* Slow roll parameters



- 
- Slow-roll inflation corresponds to both  $\epsilon_H$ ,  $\eta_H \ll 1$ .

Inflation - brief recap potential *V* () which is minimally coupled to gravity. The system is described by the action d*s*<sup>2</sup> 3*m*<sup>2</sup> **Inflation - brief recap** 

#### roducing features into the inflaton potent  $\frac{1}{2}$ become since the imater potential to generate the PDT abandance Introducing features into the inflaton potential - to generate the PBH abundance

Inflaton potential featuring an approximate inflection point or a local bump/dip at low scales slows down the inflaton leading to appreciable enhancement of scalar power-spectrum r a local bump/dip at low scales slows down the inflaton. Inflat 1 reatur:<br>**ɔ/dip** a t low scales slows down the inflaton

.

 $S_{\rm eff}$  Swagat Saurav Mishra, Capt, Nottingham  $P_{\rm eff}$  and  $\Gamma_{\rm eff}$ 

$$
P_{\zeta} = \frac{1}{8\pi^2} \left(\frac{H}{m_p}\right)^2 \frac{1}{\epsilon_H}
$$
  
\n $\epsilon_H = \frac{1}{2m_p^2} \frac{\dot{\phi}^2}{H^2}$   
\n $\frac{1}{2m_p^2}$   
\n $\frac{1}{2m_p^2$ 

Inflaton speed drops exponentially with number of e-folds :  $\dot{\phi} = \dot{\phi}_{\text{en}} e^{-3H(t-t_{\text{en}})} \propto e^{-3N}$ 

*P*⇣ = 1 8⇡<sup>2</sup> ✓ *H m<sup>p</sup>* ◆<sup>2</sup> 1 ✏*H* ✏*<sup>H</sup>* = 1 2*m*<sup>2</sup> *p* ˙2 *H*<sup>2</sup> Criteria for PBH from single field PBH formation requires enhancement of the inflationary power spectrum by a factor of 107 *<sup>N</sup>*USR() = <sup>1</sup> <sup>2</sup> Intermediate scale featuring a Ultra Slow roll inflation [Kinney (2005), Inoue and Yokoyama (2002)] (USR) (see Refs. [54, 61, 67, 80, 81]). Since *V* power for PBH formation. At intermediate field values, inflaton enters a transient period of USR. Since V'(ɸ)~0, *N*USR() can be used in the '*non-linear classical N formalism*' to determine the PDF of <sup>3</sup> Successful Reheating mechanism. *V*()end PBH ⇤ PBH Feature *k*⇤ = 0*.*05 (*Mpc*)<sup>1</sup> CMB Reheating Toy Model for PBH from Inflation potential can allow this. ¨ + 3*H*˙ = 0 ) ¨*/H*˙ = +3*,* hence ⌘*<sup>H</sup>* = +3 (during USR) Given that PBH formation requires the enhancement of the inflationary power spectrum by a factor of 10<sup>7</sup> within less than 40 e-folds of expansion (see *e.g.* Ref. [79]), the quantity ln ✏*H/N*, and hence *|*⌘*H|*, must grow to become of order unity, thereby violating the second slow-roll condition in Eq. (2.8). In the particular case of a flat plateau region in the potential at intermediate field values, the inflaton enters a transient period of ultra slow-roll *,* = 0, from the equation of motion Eq. (2.5), ¨ + 3*H*˙ = 0 ) ¨*/*(*<sup>H</sup>* ˙) = +3, leading to ⌘*<sup>H</sup>* = +3 (during USR)*.* (2.12) Figure 2: A zoomed-in version of Fig. 1 in order to schematically illustrate the intermediate flat quantum well feature (highlighted with pink shading) in the inflaton potential. The height and width of the flat segment are denoted by *V*well and well respectively. After exiting the first slow-roll phase (SR-I) near the CMB window, the inflaton enters the flat region at = en at intermediate field values. During this USR phase, the effects of quantum diffusion might The total number of e-folds of expansion during the USR period up to , where ex <sup>3</sup> log ⇣⇡en ⇡ ⌘ = <sup>3</sup> log ✓ ⇡en ⇡en 3 ( en) ⇡ = d d*N* = *<sup>H</sup> .* (2.15) From the above expressions, it is clear that the dynamics of inflation during USR is flat regime (flat quantum well)<sup>3</sup> is characterised by its width well <sup>=</sup> en ex, and height, *V*well. During this regime ˙ = ˙ en 3*H* ( en) *.* (2.14) The total number of e-folds of expansion during the USR period up to , where ex en is given by

within less than 40 e-folds of expansion, the quantity Δ ln ε /Δ N, hence ||ημ| must grow to 30 VIOIQLE LITE SCOOTIC SIOW TON Construction of the co be of order unity, so violate the second slow roll condition. A flat plateau like region in the

Inflaton speed drops exponentially with number of e-folds : 
$$
\dot{\phi} = \dot{\phi}_{en} e^{-3H(t-t_{en})} \propto e^{-3N}
$$

 $\mathbf{r}$ Critical entry velocity to just get across the plateau

$$
\dot{\phi}_{en} e^{-3H(t-t_{en})} \propto e^{-3N}
$$

 $\mathcal{A}$  as a consequence, the inflaton speed drops exponentially with the number of e-folds during du



become significant and hence one should use the stochastic inflation for stochastic inflation for maligned infla<br>Inflation for the stochastic inflation for maligned inflation for the stochastic inflation for the stochastic

the primordial PDF of  $\mu$  Later, the inflaton emerges from the inflaton emerges from the USR phase to another slow-roll  $\mu$ 

$$
\dot{\phi}_{\rm cr} = -3 H \Delta \phi_{\rm well}, \qquad \pi_{\rm cr} = -3 \Delta \phi_{\rm well}. \qquad \qquad \pi = \frac{d\phi}{dN} = \frac{\dot{\phi}}{H}
$$

#### ˆ  $\boldsymbol{\mathsf{h}}$ astic inflation formalism - <code>non</code> - <code>p</code>

d*N* bounded by the set of th **operators of the inflaton of the inflaton controller controller the interest of the interest of the interest o** ˆ r scale UV modes are constantly

ov modes are conclumly<br>
vin-type-stochastic differe evir .<br>ri

> J. ˆ

 $W\left(k/\sigma aH\right)$  is the 'window function'  $\qquad$  Selects out modes with momentum k> $\sigma$ aH With Swagat Mishra and Anne Green - e-Print:2303.17375 - JCAP 2023

Quantum dynamics - stochastic inflation formalism - non - perturbative approach to calc the full primordial PDF [Starobinsky 1982] Effective long wavelength IR treatment of inflation, inflaton field is coarse grained over super Hubble scales k  $\leq \sigma$ aH, with const  $\sigma \ll 1$ . Hubble exiting smaller scale UV modes are constantly converted into IR modes due to accelerated expansion. Coarse grained inflaton field follows a Langevin-type-stochastic differential equation with stochastic noise terms sourced by the smaller scale UV modes, on top of classical drift terms sourced by V'(φ). ˆ */*d*N* **rential equation with stochastic noise terms sore** Biong wardiongen in the Bathlond of immation, immatori hold to soarco granted over dapor i fabble soarco . ˆ */*d*N*  $\frac{1}{2}$  ,  $\frac{1}{2}$  and the strict of the sequence with stockation noise terms UV modes, on top of classical drift terms sourced by V'(φ). nanor soare of modes are constantly corrected into in modes add to accordiated expansion.

ˆ

where the UV fields are defined as

d*N* v modes, on top of classical of

Split the Heisenberg operators of the inflaton  $\hat{\phi}(N, \vec{x})$  and its conjugate momentum  $\hat{\pi}_{\phi} = d\hat{\phi}/dN$  into the corresponding IR  $\{\hat{\Phi}, \hat{\Pi}\}$  and UV  $\{\hat{\varphi}, \hat{\pi}\}$  parts: ˆ of the inflaton  $\hat{\phi}(N, \vec{x})$  and its conjugate momentum  $\hat{\pi}_{\phi} = d\hat{\phi}/dN$  into the  $\mathbf{n}$ *,* ⇧ nĮ  $\text{ponding IR } \{\Phi, \Pi\} \text{ and } \text{UV } \{\hat{\varphi}, \hat{\pi}\} | \text{M}$ the Heisenberg operators of the inflaton  $\hat{\phi}(N, \vec{x})$  and its conjugate momentum  $\hat{\pi}_{\phi} = d\hat{\phi}/dN$  into the

$$
\hat{\phi} = \hat{\Phi} + \hat{\varphi} \quad , \quad \hat{\pi}_{\phi} = \hat{\Pi} + \hat{\pi}
$$

$$
\hat{\varphi}(N,\vec{x}) = \int \frac{\mathrm{d}^3 \vec{k}}{(2\pi)^{\frac{3}{2}}} W\left(\frac{k}{\sigma a H}\right) \left[\phi_k(N)\,\hat{a}_{\vec{k}} e^{-i\vec{k}.\vec{x}} + \phi_k^*(N)\,\hat{a}_{\vec{k}}^\dagger e\right]
$$

$$
\hat{\pi}(N,\vec{x}) = \int \frac{\mathrm{d}^3 \vec{k}}{(2\pi)^{\frac{3}{2}}} W\left(\frac{k}{\sigma a H}\right) \left[\pi_k(N)\,\hat{a}_{\vec{k}} e^{-i\vec{k}.\vec{x}} + \pi_k^*(N)\,\hat{a}_{\vec{k}}^\dagger e\right]
$$



 $\overline{\phantom{a}}$ <sub>ទ</sub> wi ˆ Selects out modes with momentum k>oaH cr ˆ t:23  $2^{\prime}$ With Swagat Mishra and Anne Green - e-Print:2303.17375 - JCAP 2023

Credit: Swagat Mishra



✓ *k* ✓ *k* ◆  $\xi_{\pi}$  in the Langevin equations are sourced by the constant outflow of UV

*a*ngcvn.<br>*H* to be timed a same part of the IR field on super-Hubble scales, IR field receives a '*quantum kick*' with typical amplitude  $\sim \sqrt{\langle 0|\xi(N)\xi(N')|0\rangle}$ , where  $|0\rangle$  is usually taken to be to well-behavies-vacuum.<br>The physical space of physical space. See Refs. [71, 83, 84] for more discussion. In more discussion. In more • As UV mode exits the cut-off scale  $k = \sigma aH$  to become part of the IR field on super-Hubble scales, IR  $\overline{\phantom{a}}$  $\langle 0|$  $\overbrace{\mathcal{L}}^{\wedge}$  $\xi(N)$  $\overrightarrow{\mathcal{L}}$  $\xi(N')|0\rangle$ , where  $|0\rangle$  is usually taken to be • Given that  $\sigma \ll 1$ , this happens on ultra super-Hubble scales, where the UV modes must have already



where the omer ıt

- Physically, the noise terms  $\xi_{\phi}$  and  $\xi_{\pi}$  in the Langevin equations are sourced by the constant outflow of UV  $t_{\rm H}$  and come  $\sigma_{\rm H}$  in the physical space. See Refs.  $\sigma_{\rm H}$  $\overbrace{\mathcal{L}}^{\wedge}$  $\xi_\phi$  and  $\overrightarrow{\mathcal{L}}$ modes into the IR modes
- 

#### (3.2), the equations for the coarse-grained fields are [67] **Hamiltonian equations for coarse grained (IR) fields are Langevin equation** ions:<br>. n: **decised**<br> *<u>y</u>* ˆ ינ<br>ו ˆ

$$
\frac{d\hat{\Phi}}{dN} = \hat{\Pi} + \hat{\xi}_{\phi}(N) ,
$$

$$
\frac{d\hat{\Pi}}{dN} = -(3 - \epsilon_H) \hat{\Pi} - \frac{V_{,\phi}(\hat{\Phi})}{H^2} + \hat{\xi}_{\pi}(N) ,
$$
where the field and momentum noise operators  $\hat{\xi}_{\phi}(N)$  and  $\hat{\xi}_{\pi}(N)$  are given by  

$$
\hat{\xi}_{\phi}(N) = -\int \frac{d^3 \vec{k}}{(2\pi)^{\frac{3}{2}}} \frac{d}{dN} W\left(\frac{k}{\sigma aH}\right) \left[\phi_k(N)\hat{a}_{\vec{k}} e^{-i\vec{k}.\vec{x}} + \phi_k^*(N)\hat{a}_{\vec{k}}^{\dagger} e^{i\vec{k}.\vec{x}}\right]
$$

$$
\hat{\xi}_{\pi}(N) = -\int \frac{d^3 \vec{k}}{(2\pi)^{\frac{3}{2}}} \frac{d}{dN} W\left(\frac{k}{\sigma aH}\right) \left[\pi_k(N)\hat{a}_{\vec{k}} e^{-i\vec{k}.\vec{x}} + \pi_k^*(N)\hat{a}_{\vec{k}}^{\dagger} e^{i\vec{k}.\vec{x}}\right]
$$

Assume Window function with sharp IR

the Bunch-Davies vacuum.

become classical fluctuations..

 $With \; \mathbf{\xi}_i = \{\mathbf{\xi}_\phi, \mathbf{\xi}_\pi\},$  equal-space noise correlators (auto-correl

#### where the noise correlation matrix  $\Sigma_{ij}$  is h*N* (*i*)i =  $\overline{C}$



## The noise correlation matrix is i

Equivalent Fokker-Planck equation - time evolution of the  $\overline{\text{PDF}}$  of  $\{\Phi, \Pi\}$ , subject to appropriate bcds. time evolution of the  $\overline{PDF}$  of  $\{\Phi, \Pi\}$ , subject to appropriate bcds.

*.*

With 
$$
\xi_i = \{\xi_{\phi}, \xi_{\pi}\}
$$
, equal-space noise correlators (auto-correlators) are  

$$
\langle \xi_i(N) \xi_j(N') \rangle = \Sigma_{ij}(N) \delta_D(N - N')
$$
where the noise correlation matrix  $\Sigma_{ij}$  is

$$
\Sigma_{ij}(N) = (1 - \epsilon_H) \frac{k^3}{2\pi^2} \phi_{i_k}(N) \phi_{j_k}^*(N) \bigg|_{k = \sigma aH}.
$$

The noise correlation matrix is important !

 $\frac{1}{\partial \Phi}P_{\Phi=\phi^{\text{(R)}},\Pi}(\mathcal{N})=0$  *.* Closer to  $\,\varphi$  at cmb scale Closer to  $\phi$  at cmb scale  $(\Lambda) = 0$  $\overline{a}$ *r* to φ at cmb scale

$$
\frac{\partial}{\partial \Phi}P_{\Phi=\Phi}
$$

$$
\frac{\partial}{\partial N} P_{\Phi_i}(\mathcal{N}) = \left[ D_i \frac{\partial}{\partial \Phi_i} + \frac{1}{2} \Sigma_{ij} \frac{\partial^2}{\partial \Phi_i \partial \Phi_j} \right] P_{\Phi_i}(\mathcal{N}) \qquad \text{where} \qquad \qquad D_i = \left\{ \Pi, -(3 - \epsilon_H) \Pi - \frac{V_{,\phi}(\Phi)}{H^2} \right\}
$$

1. Absorbing boundary at  $\phi^{(A)}$ 

*<i>p* (*N*)  $\bar{X}$  (*N*)  $\Omega$  Pleasar to check  $P_{\Phi=\phi^{(\mathrm{A})},\Pi}(\mathcal{N})=\delta_D(\mathcal{N})\;, \quad \text{Closer to $\varphi$ at end of inflation}$  $I(\Lambda)$   $I(\Lambda)$  closes to the exploition  $-\phi = \phi^{(1)}$ , 1

end of inflation and a reflection and a reflection  $\mathcal{O}(\mathbf{R})$  $\omega$ . Items 2. Reflecting boundary at  $\phi^{(R)}$ 



 $\chi_{\mathcal{N}}(q; \Phi)$ 

$$
\Phi_i) \equiv \langle e^{i\,q\,\mathcal{N}} \rangle = \int_{-\infty}^{\infty} e^{i\,q\,\mathcal{N}} P_{\Phi_i}(\mathcal{N}) \,\mathrm{d}\mathcal{N},
$$

CF then satisfies



$$
\frac{1}{i} + \frac{1}{2} \Sigma_{ij} \frac{\partial^2}{\partial \Phi_i \partial \Phi_j} + iq \bigg] \chi_{\mathcal{N}}(q; \Phi_i) = 0,
$$

with bcs



Usual approach: assume noise matrix  $\epsilon$ 

Quantum diffusion across a flat segment of the inflaton potential [Pattsion et al 2021]. Intro

$$
_{\rm }(A)_{\rm ,\,II})=1\,,\quad \frac{\partial}{\partial\Phi}\,\chi_{\mathcal{N}}(q;\phi^{(\mathrm{R})},\mathrm{II})=0\,\,.
$$

*f* is the fraction of the flat well which remains to be traversed; *y* is the momentum relative to the critical momentum,  $V_{\text{well}}$  is the height of the flat quantum well.

Characteristic function:  $\chi_{\mathcal{N}}(q; \Phi_i)$ , given by Fourier transform of the PDF  $P_{\Phi_i}(\mathcal{N})$ 

x elements 
$$
\Sigma_{ij}
$$
 are of the de Sitter-type:  
\n $\Sigma_{\phi\phi} = (H/2\pi)^2$ ,  $\Sigma_{\phi\pi}$ ,  $\Sigma_{\pi\pi} \simeq 0$ .

$$
f = \frac{\Phi - \phi_{\rm ex}}{\Delta \phi_{\rm well}} \,, \quad y = \frac{\Pi}{\pi_{\rm cr}} \,, \quad \mu^2 \simeq \frac{\Delta \phi_{\rm well}^2}{m_p^2} \, \frac{1}{v_{\rm well}} \,, \quad v_{\rm well} = V_{\rm well}/m_p^4 \,,
$$

al  $[2020]$ , Pattison et al  $[2021]$ 

 $P_f(\mathcal{N}) = \sum$ *n*=0

where

$$
P_f(\mathcal{N}) = \sum_{n=0}^{\infty} A_n \sin \left[ (2n+1) \frac{\pi}{2} f \right] e^{-\Lambda_n \mathcal{N}},
$$
  
 
$$
A_n = (2n+1) \frac{\pi}{\mu^2}, \qquad \Lambda_n = (2n+1)^2 \frac{\pi^2}{4} \frac{1}{\mu^2},
$$

For  $\mathcal{N} \gg 1,$  PDF has an exponential tail

 $P_{\Phi}$  (



Free stochastic diffusion :  $\pi_{en} \ll \pi_{cr} \Rightarrow y_{en} \ll 1 \longrightarrow$  the classical drift term can be ignored [Ezquiaga et  $\pi$  (*x*)=0  $\pi$  (*x*)=0  $\pi$  /*the classical drift torm can be ignored* [F<sub>*z</sub>*</sub>

$$
(\mathcal{N}) \simeq A_0 e^{-\Lambda_0 \mathcal{N}}.
$$

*<sup>µ</sup>*<sup>2</sup> *,* ⇤*<sup>n</sup>* = (2*<sup>n</sup>* + 1)<sup>2</sup> ⇡<sup>2</sup>





potential Eq. (4.28), with = 0*.*01, leading to a realistic smooth transition from SR-I to USR. The plot shows that the plot shows the plot shows the behaviour of the USR regime. He will be used the USR regime. The USR regime of the USR regime of the USR regime.  $\alpha$ Fixed  $M = 0.5 m_p$ , and bump parameters to be  $A = 1.87 \times 10^{-3}$ , relative to the field noise, ⌃, in the USR epoch. With Swagat Mishra and Anne Green - e-Print:2303.17375 - JCAP 2024  $t = 1$  to  $\sigma = 10^{-3}$  and  $t = 1000$  to  $10^{-2}$  the momentum induced noise terms,  $t = 10^{-3}$  and  $t = 0.00$  momentum induced noise terms,  $t = 10^{-3}$  and  $t = 0.00$  momentum induced noise terms,  $t = 10^{-3}$  and  $t = 0.00$  momen Fixed  $M = 0.5 m_p$ , and bump parameters to be  $A = 1.87 \times 10^{-3}$ ,  $\tilde{\sigma} = 1.993 \times 10^{-2}$  and  $\phi_0 = 2.005 m_p$ . Gives amplification of the scalar power-spectrum,  $P_{\zeta}$ , by a factor of  $10^{7}$  relative to its value on CMB scales.

### In reality — noise terms are more interesting !

#### Numerical noise matrix elements,  $\Sigma_{ij}$  - note the switching of dominant terms during USR value the switching of dominant terms during JISR



# Where's the inflaton ?

Loads of models — 300 models analysed with CMB and BAO data, 40% disfavoured, 20% favoured according to Jeffrey's scale of Bayesian evidence [Martin et al 2024]

Over 7,700 papers written with title Inflation in title .

 $\overline{\phantom{a}}$ To date, no accepted origin of the inflaton field. It should ideally be a fundamental field arising out of an underlying theory of particle physics like string theory - for a review see [Cicoli et al 2023].

But there appear to be issues there obtaining de Sitter solutions - it has led in part to the Swampland conjecture.





Of course inflation isn't de Sitter, but it looks like its not far from it with the Hubble parameter H slowly evolving during inflation.

One nice approach is due to Conlon and Quevdeo [2006] - Kahler Moduli Inflation.

It has some interesting features that exist between the end of inflation and reheating which we will look at briefly [Apers et al 2024]

k<br>K1⊂ = 9α5/3λj = 9α5/<br>Andrew = 9α5/3λj =

Large volume scenario within a class of Type IIB flux compactifications on a Calibi-Yau orientifold] can be written in the form, we have th<br>, we have the form, we have the form, we have the form, we have the form of the form, we have the form of the Large volume scenario within a d

Internal volume of CY:  $\mathbf{r}$ 

ners. The parameters α and λ<sup>i</sup> are model dependent constants that can be computed once

 $\overline{\phantom{a}}$ 

Full scalar potential for moduli fields - don't look too closely !

$$
\mathcal{V} = \frac{\alpha}{2\sqrt{2}} \left[ (T_1 + \bar{T}_1)^{\frac{3}{2}} - \sum_{i=2}^n \lambda_i (T_i + \bar{T}_i)^{\frac{3}{2}} \right] = \alpha \left( \tau_1^{3/2} - \sum_{i=2}^n \lambda_i \tau_i^{3/2} \right)
$$

α (4V − ξ)<br>3αβ = ξημιουργίας<br>3αβ = ξημιουργίας

where Complex Kahler moduli T<sub>i</sub> = τ<sub>i</sub> + I θ<sub>i</sub>, with τi describing the volume of volume o  $\tau_{\text{I}}$  - volume of internal four cycles in  $\text{C}\gamma$ I - volume of internal four cycles in CY θ<sub>i</sub> - axonic partners Complex Kahler moduli T

$$
V = \sum_{\substack{i,j=2 \\ i  
+ 
$$
\frac{12W_0^2 \xi}{(4V - \xi)(2V + \xi)^2} + \sum_{i=2}^{n} \left[ \frac{12e^{-2a_i \tau_i} \xi A_i^2}{(4V - \xi)(2V + \xi)^2} + \frac{16(a_i A_i)^2 \sqrt{\tau_i} e^{-2a_i \tau_i}}{3\alpha \lambda_i (2V + \xi)} + \frac{32e^{-2a_i \tau_i} a_i A_i^2 \tau_i (1 + a_i \tau_i)}{(4V - \xi)(2V + \xi)} + \frac{8W_0 A_i e^{-a_i \tau_i} \cos(a_i \theta_i)}{(4V - \xi)(2V + \xi)} \left( \frac{3\xi}{2V + \xi} + 4a_i \tau_i \right) + V_{uplift}.
$$
$$

### Kahler Moduli Inflation [Conlon and Quevedo 2006] Following information [Conformation gacveas 2000] Moduli Inflation [Conlon and Quevedon

**Intriguing results** obtained for 50-60 efolds:

Idea: displace just one moduli from its minimum, keeping the others fixed and show consistent slow roll inflation can be obtained with that moduli evolving back to its minima on can be obtained with that moduli evolving back to its minima enough values of τ<sup>2</sup> so that V<sup>2</sup>e<sup>−</sup>a2τ<sup>2</sup> ≪ 1. Taking into account that we are in the slow roll 50-60) they obtained the results

> $\frac{1}{2}$  $\frac{1}{2}$

$$
\text{Displace } \tau_2 \text{ with parameter } \rho \ll 1 \text{ where}
$$
\n
$$
\rho \equiv \frac{\lambda_2}{a_2^{3/2}} : \sum_{i=2}^n \frac{\lambda_i}{a_i^{3/2}}
$$
\n
$$
V_{LARGE} = \frac{BW_0^2}{\mathcal{V}^3} - \frac{4W_0 a_2 A_2 \tau_2 e^{-a_2 \tau_2}}{\mathcal{V}^2}
$$
\n
$$
\eta \simeq -\frac{1}{N_c}, \qquad \epsilon < 10^{-12},
$$
\n
$$
\text{obtained for}
$$
\n
$$
0.960 < n_s < 0.967, \quad 0 < |r| < 10^{-10}
$$
\n
$$
10^5 l_s^6 \leq \mathcal{V} \leq 10^7 l_s^6,
$$



Numerically solve the full equations. The question is what happens if we allow the moduli to evolve so that they all have to find their minima. Do we find the kind of evolution that Conlon and Quevedo assumed in their analytic model ? [Blanco-Pillado et al 2009]  $\frac{1}{58}$  which  $\frac{56}{25}$  The  $\frac{57}{25}$   $\frac{1}{58}$   $\frac{1}{58}$   $\frac{1}{28}$   $\frac{1}{28}$   $\frac{59}{28}$ The global medicine of the pool 56 57 | 58 | V V <sup>v</sup> 59 id of evolution that Confort and Queveup assu

$$
\tau_1^f = 2555.95, \quad \tau_2^f = 4.7752, \quad \tau_3^f = 2.6512, \quad \mathcal{V}^f = 10143.94363
$$



Table 4: Comparison among the predictions for the scalar spectral index and the tensor-to-scalar ratio of the main models of string inflation,

# Some more string inspired inflation models — [Cicoli et al 2023]

# *String model*

Fibre Inflation

Blow-up Inflation

Poly-instanton Infl

Aligned Natural Inf

*N*-Flation

Axion Monodron

D7 Fluxbrane Inflation

Wilson line Inflat

D3-D3 Inflatio

Inflection Point Inf

D3-D7 Inflation

Racetrack Inflati

Volume Inflatio

DBI Inflation

As you can see there are many - some close to being or already ruled out !



- The bit between the end of inflation and the thermal HBB some 30 orders of magnitude in time.
- Potentially new stringy features could emerge which would modify the standard picture. While it is true that the second consider the standard picture is the standard picture from individuary from Stringy reducted could critery of writen would modify the Standard picture.
- For example, large field displacements between end of inflation and final vacuum under control!
- No necessary relationship between inflaton field and field responsible for reheating. In fact in D3-anti D3 brane case, inflaton disappears. may be preserved in some string theory models, the standard cosmology may be modified in (at least) three ways. inship between inflaton field and field responsible for reneating. In fact in D3-anti D3 brane ca
	- Long Kination and moduli dominated epoch leading to moduli driven reheating there being no necessary relationship between the inflaton field and the field responsible formulation of the f<br>I and Kination and moduli dominated enoch leading to moduli driven reheating. the expectation of a long moduli-dominated epoch in the universe culminating in moduli-driven reheating. These

After inflation ! [Apers et al 2024] more general treatment of the standard cosmology can consult e.g. [67], while an earlier discussion of aspects



### [Cicoli et al 2023]



then the field enters a kination epoch as the potential energy grows ever more sub-dominant. Eventually any residual radiation or matter becomes dominant and enter Tracker regime where the radiation and φ track each other nere the radiation and  $\varphi$  track each other

Tracker field satisfies :  $\Phi(t) = \Phi(t_0) + \frac{2 M_P}{\ln} \ln \left( \frac{t}{t_0} \right)$  with Tracker field satisfies :  $\Phi(t)=\Phi(t_0)+\frac{--1}{\sqrt{2}}\ln\left(\frac{1}{t}\right)$  with :  $a(t)\thicksim t^{1/2}$ ield s  $\overline{\phantom{a}}$ 

During Kination - potential term subdominant:	\n $\ddot{\Phi} + 3H\dot{\Phi} = 0, \qquad \text{where} : \qquad \frac{\Phi}{M_P} = \frac{\dot{\Phi}^2}{2}$ \n
---	--

Again travels foughly one marked relation ustar<br>Note: the above is not the consequence of the above is not the above is not the above in the above is not the above in the above is not the above in the above is not the abov is a *tracker* solution with fixed proportions of the energy density in potential, kinetic and fluid Again travels roughly one Planck distance in one Hubble time As mentioned above, during the kination epoch, the modulus field is (e↵ectively) massless, and from Eq. (2.4), we see that the computation is increasing the computation of the comp <sup>2</sup>*/*<sup>3</sup> and

any other fluid (in particular, radiation or matter) will grow in importance relative to the kinating scalar and eventually catch up. For exponential potential potential potentials, the endpoint of this evolution of this evolution of the endpoint of this evolution of this evolution of this evolution of this evolution o  $\sqrt{3}$  Guides the field into the min of the moduli poter During a tracker solution along an exponential potential *V* () = *V*0*e/M<sup>P</sup>* , the evolution ili potent **A**<br>
ial wher *<u>Premeating can occur</u>*  $\blacksquare$ Guides the field into the min of the moduli potential where reheating can occur traversing a significantly transPlanckian distance in field space during any extended tracker Guides the field into the min of the moduli potential where reheating can occur and overtake the kinetic energy. The kinetic energy is the kinetic energy of the kinetic energy is the kinetic<br>And the kinetic energy is the kinetic energy is the kinetic energy is the kinetic energy is the kinetic energy.

$$
\text{Kinating field satisfies:} \qquad \qquad \Phi(t) = \Phi(t_0) + \sqrt{\frac{2}{3}} M_P \ln\left(\frac{t}{t_0}\right)
$$
\n
$$
\text{Travels roughly one Plensk distance in one H}
$$

3 *t*0 **documber 10 Travels roughly one Planck distance in one Hubble time** is a *tracker* solution with fixed proportions of the energy density in potential, kinetic and fluid I ravels roughly one Planck distance in one Hu Travels roughly one Planck distance in one Hubble time (*t*<sub>0</sub>)<br>13/2 and so *m tayels roughly one Planck distance in one Hubble time* 

 $\sigma = \frac{1}{\sqrt{d}} \int d^4x \, dx$ Example of Kiination with : Example of Kiination with :  $V(\Phi) = V_0 e^{-\lambda \Phi/M_P}$  with :  $\lambda > \sqrt{6}$ Example of Kination with  $V(\Psi) = V_0 e^{-\lambda \frac{1}{2} m T}$  with  $\lambda > 0$ 

But as V decreases during Kination and as:  $\qquad \qquad \rho_{kin} \sim \frac{1}{\rho(f) 6},$ But as V decreases during Kination and as:  $\rho_{kin} \sim$ but as y decreases during Kination and as.  $\mu_{kin} \sim a(t)^6$ , But as V decreases during Kination and as:  $\rho_{kin} \sim \frac{1}{\sqrt{1-\rho}}$  .



Example of Kiination with: 
$$
V(\Phi) = V_0 e^{-\lambda \Phi/M_P}
$$
 with:  $\lambda > \sqrt{6}$ 

\nBut as V decreases during Kination and as:  $\rho_{kin} \sim \frac{1}{a(t)^6}$ ,

$$
\text{Tracker field satisfies:}\qquad \qquad \Phi(t) = \Phi(t_0) + \frac{2M_P}{\lambda} \ln\left(\frac{t}{t_0}\right) \qquad \text{ with:}\quad \mathsf{a(t)} \thicksim \mathsf{t}^{1/2}
$$

 $1/3$ Hubble times, whether in the early or late universe, the kinating field traverses mutiple Planckian However, as entergy distorted to the redshift of the state of the justified as the potential energy is comparable to the kinetic one (the end of inflation). In externalistime-limited; as  $\frac{1}{\sqrt{2}}$  and  $\frac{1}{\sqrt$ 

inflation (*V*barrier ' 1020*V*inf). new exciting pre BBN physics ! [Apers et al 2024] volume evolves as *V* / *t* during this epoch (see [14] and ing Kinduon and into the tracker regime before<br>All physics ITA

Cosmic string tensions will evolve in time, and a new network formation process could emerge from the f [15] for more detailed discussion).  $\overline{a}$ fundamental strings, with a tension  $\mathsf{f}\mathsf{with}$  and [with Sanchez Gonzalez, Conlon and Hardy 2024]



period of inflation at relative to the start of the start of the Hot Big Bang. We show here the start of the start of the start of the case of a kination  $\sigma$ 

is best understood, the canonical modulus  $\mathsf{C}$ anno canonical modulus  $\mathsf{V}$ uka aaug<br>a Gauge couplings, Yukawa couplings and axion dec decrease.<br>Second in the second which in the second second terminal in the second terminal in the second terminal in the

$$
G\mu\sim t^{-1}
$$

$$
m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}
$$
 with  $G\mu \sim m_s^2$  hence  $G\mu \sim t^{-1}$ 

- Time varving standard model parameters because determined by evolving moduli fie Time varying standard model parameters because determined by evolving moduli fields ! Time varying standard model parameters because determined by evolving moduli fields decreasing tension. The rate of power loss from a loop itying standard model parameters because determined by evolving modi  $\overline{\text{ws}}$  is a numerical factor that depends on the precise of  $\overline{\text{w}}$ 
	- Caugo couplings. Vulcawe couplings and avion docay sonstants sould be different from Gauge couplings, Yukawa couplings and axion decay constants - could be different from today. *Rmax*(*t*) = *Rmax,i ti .* (25) The most obvious string of the most obvious string theory condition. loday.<br>Simulations suggest and suggest .
- Perturbations in the field grow during Kination and into the tracker regime before the moduli are stabilised and reheating occurs potential for where  $\alpha$  is a numerical factor that depends on the precise of **eheating occurs - potential for** 
	- Eputh Sanchez Gonzalez, Conlon and Hardy 2024 1 **Oscillation 19 and so show that the lifetime of a loop of loop of a loop of a loo** Cosmic string tensions will evolve in time, and a new network formation process could emerge from the formation of loops mass *lsµ* is (using *µ* = *m*<sup>2</sup> Cosmic string tensions will evolve in time, and a new network formation process could emerge from the formation of loops -

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Will concentrate on one important element of this use of Tracker behaviour - the overshoot problem [Brustein and Steinhardt 93] !

The barrier that has to eventually trap the moduli field can be 20 or more orders of magnitude smaller than the energy scale during inflation. The field should simply shoot straight past and decompactify spacetime !

In cosmology as in many areas of physics we often deal with systems that are inherently described through a series of coupled non-linear differential equations.

By determining the late time behaviour of some combination of the variables, we often see that they may approach some form of attractor solution.

From the stability of these attractor solutions we can learn about the system.

Moreover the phase plane description of the system is often highly intuitive enabling easy analysis and understanding of the system.

Examples inc the relative energy densities in scalar fields compared to the bgd rad and matter densities, as well as the relative energy density in cosmic strings.
Enter Tracker solutions:

Wetterich (88) Peebles and Ratra (88),

EJC, Liddle and Wands

$$
\frac{\dot{b}^2}{2} + V(\phi); \ p_{\phi} = \frac{\dot{\phi}^2}{2} - V(\phi)
$$



 $\frac{\kappa\dot{\phi}}{\sqrt{6}H}$ ;  $y =$ 

 $x =$ 

 $\overline{\mathcal{K}}$ 

 $\frac{1}{\sqrt{2}}$ 

 $\sqrt{ }$ 

+ constraint:

$$
H^2 = \frac{\kappa^2}{3} (\rho_{\phi} + \rho_{\rm b})
$$

Intro new variables x and y:

$$
\frac{\sqrt{V}}{3H}, \lambda \equiv \frac{-1}{\kappa V} \frac{dV}{d\phi}, \Gamma - 1 \equiv \frac{d}{d\phi} \left(\frac{1}{\kappa \lambda}\right)
$$

$$
\frac{2\dot{\phi}^2}{2V+\dot{\phi}^2}, \quad \Omega_{\phi} = \frac{\kappa^2 \rho_{\phi}}{3H^2} = x^2 + y^2
$$

Friedmann eqns and fluid eqns become:



$$
+\lambda\sqrt{\frac{3}{2}y^2} + \frac{3}{2}x[2x^2 + \gamma(1 - x^2 - y^2)]
$$
  

$$
\frac{3}{2}xy + \frac{3}{2}y[2x^2 + \gamma(1 - x^2 - y^2)]
$$

## Field mimics background fluid.  $\lambda^2 \ge 3\gamma$



## Scaling solut



$$
tions: (x=y'=0)
$$

### EJC, Liddle and Wands

$$
V = V_0 e^{-\lambda \kappa \phi}
$$



The phase plane for  $\gamma = 1, \lambda = 2$ . The scalar FIG. 3. field dominated solution is a saddle point at  $x = \sqrt{2/3}$ ,  $y = \sqrt{1/3}$ , and the late-time attractor is the scaling solution with  $x = y = \sqrt{3/8}$ .





The phase plane for  $\gamma = 1, \lambda = 1$ . The FIG. 2. late-time attractor is the scalar field dominated solution with  $x = \sqrt{1/6}, y = \sqrt{5/6}.$ 



FIG. 4. The phase plane for  $\gamma = 1$ ,  $\lambda = 3$ . The late-time attractor is the scaling solution with  $x = y = \sqrt{1/6}$ .



$$
0^{-47} \text{ GeV}^4 \approx (10^{-3} \text{ eV})^4
$$

## 1. Scaling solutions in Dark Energy - Quintessence

## 2. Useful way of stabilising moduli in string cosmology. Sources provide extra friction when potentials steep.



- Barreiro, de Carlos and EC : hep/th-9805005
- Brustein, Alwis and Martins : hep-th/0408160



114 Two condensate model with V~e-aReS as approach minima Barreiro et al : hep-th/0506045

### 3. Stabilising volume moduli (σ=σr+σi) in KKLT **[Kachru et al 2003]** namics of the latter is given in terms of the scale factor  $\overline{\phantom{a}}$ model, along with the evolution equation for the backalusing volume moduli (o=d o moduli (<del>a q 1a)</del> in WW T water **2** σ <u>ρλέο ΙΖΥ</u> Ζ΄ Πρωτά

ρ<sup>b</sup> = ρb<sup>0</sup>/a<sup>3</sup><sup>γ</sup> . (7)

Kino da katika matang kata sa kata sa<br>Kino da kata sa kata s

Kino da katika matang kata

It is worth splitting the equations of motion for the

complex chiral superfields into those for their real and

<sup>I</sup> ) refers to the real (imaginary) part

Kino de la Regional de la Regional<br>Regional de la Regional de la Regio

Kina na matsayan na matsay<br>Matsayan na matsayan na ma

of the scalar fields and ∂j<sup>R</sup> (∂j<sup>I</sup> ) are used to denote the

derivative of the potential with respect to the real (imag-

 $\ddot{\sigma_r} + 3H\dot{\sigma_r} - \frac{1}{\sigma_s}$  $\sigma_r$  $\big(\dot{\sigma_r}^2 - \dot{\sigma_i}$  $\ddot{\sigma_{i}} + 3H\dot{\sigma_{i}} - \frac{2}{\sigma_{i}}$  $\overline{\sigma}_r$ σ ¨ <sup>i</sup> + 3Hσ ˙  $\frac{1}{\sigma}$   $\frac{1}{2}$  $\begin{array}{c} \mathbf{1} \end{array}$ 3  $\dot{\sigma}_i$  r

$$
\ddot{\sigma_r} + 3H\dot{\sigma_r} - \frac{1}{\sigma_r}(\dot{\sigma_r}^2 - \dot{\sigma_i}^2) + \frac{2\sigma_r^2}{3}\partial_{\sigma_r}V = 0
$$
  

$$
\ddot{\sigma_i} + 3H\dot{\sigma_i} - \frac{2}{\sigma_r}\dot{\sigma_r}\dot{\sigma_i} + \frac{2\sigma_r^2}{3}\partial_{\sigma_i}V = 0
$$
  

$$
\dot{\rho_b} + 3H\gamma\rho_b = 0
$$

$$
^{2}+\dot{\sigma_{i}}^{2})+V+\rho_{b}
$$





e<br>C

115 σ3 **Fincluding**  $\overline{\rm G}$  on validity of  $\overline{\rm D}$  term addition see also Burgess et al 2003; Achue  $\sigma$  and  $\sigma$  and  $\sigma$  and  $\sigma$  and  $\sigma$  and  $\sigma$  are not as an extremum in  $\sigma$ ud11<br>..  $\overline{1}$ contribution from D term to uplift the potential to de Sitter The possibility of  $\Gamma$ [including contribution from D term to uplift the potential to de Sitter] [for discussion on validity of D term addition see also Burgess et al 2003; Achucarro et al 2006]

tential has an extremum in  $\sigma$  and  $\sigma$ 

$$
V = \frac{\alpha A e^{-\alpha \sigma_r}}{2\sigma_r^2} \left[ A \left( 1 + \frac{\alpha \sigma_r}{3} \right) e^{-\alpha \sigma_r} + W_0 \cos(\alpha \sigma_i) \right]
$$

<sup>I</sup> ) refers to the real (imaginary) part

of the scalar fields and ∂<sup>j</sup><sup>R</sup> (∂<sup>j</sup><sup>I</sup> ) are used to denote the

derivative of the potential with respect to the real (imag-

ory with a stabilized volume modulus,  $\omega$  stabilized volume modulus,  $\omega$  put for  $\omega$ 

figure 1 we show a contour plot of this scalar potential,

In this section, the results are shown in terms of the initial

abundance Ω<sup>b</sup> which will allow us to compare them with

In general one can identify up to five regions in the

evolution of a scalar field with these types of potentials.

In figure 2 we show a typical evolution going through the

five regions although, of course, not all initial conditions

will give rise to an evolution that will go through all of



bered from 1 to 5, are described in the text. We have taken the text. We have taken the text. We have taken th

### Evolution of energy density of  $\phi \propto \ln \sigma_r$  in KKLT and Kallosh Linde type potentials

[Brustein et al 2004; Barreiro et al 2005]

### 1 Lerge velume modulus inflation high seels inflation & lew seels CIICV as avist 4. Large volume modulus inflation - high scale inflation & low scale SUSY co-existing [Conlon et al 2008] To illustrate this idea, we start by studying moduli evolution in the following toy



model describing a field  $\mathcal{A}$  with a field  $\mathcal{A}$  with a positive and  $\mathcal{A}$  with a potential  $\mathcal{A}$ 

$$
- C e^{-10\Phi/\sqrt{6}} + De^{-11\Phi/\sqrt{6}} + \delta e^{-\sqrt{6}\Phi}
$$



s but presence of radiation leads to additional Hubble Friction d field settles in its minimum. which ican to attracted control below and the souls better in the finith ay solutions but presence of radiation leads to additional Hubble Friction<br>Maviour and field settles in its minimum. Steep potential after inflation would normally have runaway solutions but presence of radiation leads to additional Hubble Friction which leads to attractor behaviour and field settles in its minimum.

Toy<br>example - but general features



inflation is dominated by the cubic term, inflation starting at the inflection point is eternal,

the spectral index *n*<sup>s</sup> ≈ 0*.*93, and the amplitude of perturbations of the metric produced

during inflation satisfies the COBE–WMAP normalization for

## Strings in KLMT <sup>©</sup> model -- an example.

[Kachru, Kallosh, Linde, Maldacena, McAllister & Trivedi 03]

 $ds^2 = e^{2A(x_\perp)} \eta_{\mu\nu} dx^\mu dx^\nu + ds^2$ .



IIB string theory on CY manifold, orientifolded by  $Z_2$  sym with isolated fixed points, become O3 planes. Warped metric:

> Inflaton: sep of D3 and anti D3 in throat.

 $03/16/2012$  dim obs are suppressed by a factor of Redshift in throat important. Inflation scale and string tension, as measured by a 10 dim inertial observer, are set by string physics -- close to the fourdimensional Planck scale. Corresponding energy scales as measured by a 4

Annihilation in region of large grav redshift,



- D-brane-antibrane inflation leads to formation of D1 branes in noncompact space [**Dvali & Tye; Burgess et al; Majumdar & Davis; Jones, Sarangi &Tye; Stoica & Tye**]
	- Form strings, not domain walls or monopoles.

In general for cosmic strings to be cosmologically interesting today we require that they are not too massive (from CMB constraints), are produced after inflation (or survive inflation) and are stable enough to survive until today [**Dvali and Vilenkin (2004); EJC,Myers and Polchinski (2004), Conlon et al (2024)**].

## Strings surviving inflation:

## $10^{-11} \leq G\mu \leq 10^{-6}$

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What sort of strings? Expect strings in non-compact dimensions where reheating will occur: F1-brane (fundamental IIB string) and D1 brane localised in throat. [Jones,Stoica & Tye, Dvali & Vilenkin]

Strings created at end of inflation at bottom of inflationary throat. Remain there because of deep pot well. Eff 4d tensions depend on warping and 10d tension  $\bar{\mu}$ 

D1 branes - defects in tachyon field describing D3-anti D3 annihilation, so produced by Kibble mechanism.

F1-branes and D1-branes --> also (p,q) strings for relatively prime integers p and q. [Harvey & Strominger; Schwarz]

Interpreted as bound states of p F1-branes and q D1-branes [Polchinski;Witten]

- -



Tension in 10d theory:

 $\mu_i = \mu_{(p_i)}$ 

$$
q_i) = \frac{\mu_F}{g_s} \sqrt{p_i^2 g_s^2 + q_i^2}
$$

## Distinguishing cosmic superstrings

1. Intercommuting probability for gauged strings P~1 always ! In other words when two pieces of string cross each other, they reconnect. Not the case for superstrings -- model dependent probability [Jackson et al 04].

2. Existence of new `defects' D-strings allows for existence of new hybrid networks of F and D strings which could have different scaling properties, and distinct observational effects.

## (p,q) string networks -- exciting prospect.

Two strings of different type cross, can not intercommute in general -- produce pair of trilinear vertices connected by segment of string.

What happens to such a network in an expanding background? Does it scale or freeze out in a local minimum of its PE [Sen]?Then it could lead to a frustrated network scaling as w=-1/3



Scaling achieved indep of initial conditions, and indep of details of Density of D1 interactions.

Density of  $(p,q)$ cosmic strings.

strings.

## Including multi-tension cosmic superstrings

[Tye et al 05, Avgoustidis and Shellard 07, Urrestilla and Vilenkin 07, Avgoustidis and EJC 10, Rybak et al 18].





*µ L*<sup>2</sup>  $\dot{a}$ *a*  $\rho-\frac{\rho}{L}$ *L* Modelling a network —single one-scale model: (Kibble + many...)  $L(t) = \xi(t)t, a(t) \sim t$  $\beta$  $2(\beta - 1) + \frac{1}{\zeta}$ ⇥ ⇥ Scale factor Expansion Loss to loops

*.*

Need this to understand the behaviour with the CMB.

*a*  $\mathbf{\dot{\Omega}}$ 

= 2

 $\dot{v}$ 

*a*

 $\xi^2$ = *k*(*k* + ˜*c*)  $4\beta(1-\beta)$ 

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### Velocity dependent model: (Shellard and Martin)

 $(1 + v^2)\rho - \frac{\tilde{c}v}{L}$ ˜ *L ,*  $\dot{v} = (1 - v^2)$  *k*  $L^2$  2  $\boldsymbol{\dot{a}}$ *a v* ⇥ 2  $\sqrt{2}$  $\boldsymbol{\pi}$ <sup>1</sup> <sup>8</sup>*v*<sup>6</sup> 1+8*v*<sup>6</sup> ⇥  $v^2$ =  $k(1 - \beta)$ (*k* + ˜*c*)

 $k$  =  $k$  = Curvature type term encoding small scale structure

 $\dot{\rho}$ 

Both correlation length and velocity scale

RMS vel of segments

Multi tension string network:(Avgoustidis & Shellard 08, Avgoustidis & EJC 10, Rybak et al 18)

 $\rho_i =$  $\overline{\mu}$ *i*  $L_i^2$ *i*  $\ell_{ij}^k =$  $L_i L_j$  $L_i + L_j$  $d_{ia}^k$ 06/23/2008 127  $\dot{v}_i = (1-v_i^2)$ Γ ⇤ *ki Li*  $-2$  $\dot{a}$ *a*  $v_i + \sum$  $b, a \leq b$  $b^i_{ab}$  $\bar{v}_{ab}$  $v_{\bm{i}}$  $(\mu_a + \mu_b - \mu_i)$  $\overline{\mu}$ *i*  $\frac{\ell_{ab}^i(t)}{L_i^2}$  $L_a^2L_b^2$ ⇥ ⌅  $\dot{\rho}_i = -2$  $\boldsymbol{\dot{a}}$ *a*  $(1 + v_i^2)$  $\frac{2}{i}$ ) $\rho_i - \frac{c_i v_i \rho_i}{I}$ *Li*  $a,k$  $d_{ia}^k \bar{v}_{ia} \mu_i \ell_{ia}^k(t)$  $L^2_a L^2_i$ +  $b, a \leq b$  $d^i_{ab}\bar{v}_{ab}\mu_i\ell^i_{ab}(t)$  $L_a^2L_b^2$  $v_{ab} =$  $\overline{\phantom{a}}$  $v_a^2 + v_b^2$  *pi*  $= \mu_{(p_i, q_i)} =$ *µ<sup>F</sup> gs*  $\overline{\phantom{a}}$  $p_i^2 g_s^2 + q_i^2$ Expansion Loop of `i' Loop of `i' collides with `a' to form segment `k' -- removes energy Segment of `i' forms from collision of `a' and `b' -- adds energy `k' segment length incorporate the probabilities of intercommuting and the kinetic constraints. They have a strong dependence on the string coupling gs

Black -- (1,0) -- Most populous Blue dash  $(0,1)$ Red dot dash  $(1,1)$ 

Example - 7 types of (p,q) string. Only first three lightest shown - scaling rapidly reached in rad and matter.

Deviation from scaling at end as move into Λ domination.

## $\{(p,q)_i\} = \{(1,0), (0,1), (1,1), (1,2), (2,1), (1,3), (3,1)\}, (i = 1, ..., 7)$



Note lighter F strings dominate number density whilst heavier and less numerous D strings dominate power spectrum for at smaller gs, where as they are comparable at large  $g_s \sim 1$ 

Densities of rest suppressed.

Avgoustidis et al (PRL 2011)

# General Network Behaviour

• Hierarchy in number densities  $N_F > N_D > N_{FD}$ 

• Scaling for all string types (though we keep the first 7 lightest strings)

• Hierarchy in tensions

 $\mu_{FD}$  >  $\mu_{D}$  >  $\mu_{F}$ 

• Only 3 lightest components (F, D, FD strings)

• Number density vs "CMB" density Competition depending on gs

F-string



$$
C_l^{strings} \propto \sum_{i=1}^N \left(\frac{G\mu_i}{\xi_i}\right)^2
$$

$$
C^{TT} \equiv \sum_{\ell=2}^{2000} (2\ell+1) C_{\ell}^{TT}
$$

### Strings and the CMB

Since strings can not source more than 10% of total CMB anisotropy, we use that to determine the fundamental F string tension which is otherwise a free parameter. So  $\mu_F$  chosen to be such that:

 $f_s = C_{strings}^{TT}/C_{total}^{TT} = 0.1$ 

Modified CMBACT (Pogosian) to allow for multi-tension strings. Shapes of string induced CMB spectra mainly obtained form large scale properties of string such as correlation length and rms velocity given from the earlier evolution eqns. Normalisation of spectrum depends on:

i.e. on tension and correlation lengths of each string

where

## Strings and the CMB Computing CMB signals from strings

Strings are active, incoherent sources — **require UETC**:



### USM - 8 hours USM - 8 hours Analytic - 20 secs

ASU-Tufts Workshop, 03/02/14 4/20

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$$
\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \,d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi \, \Theta(k,\tau_1)\Theta(k,\tau_2)
$$

Model network as made of unconnected string segments with lengths and<br>
velocities given by VOS model segments and versions and very vost model velocities given by VOS model velocities given by vUS model<br>Compute integrals analytically [Avgoustidis et al 2012

# (~20 seconds)

<u>اع</u>

## Compute integrals analytically [Avgoustidis et al 2012]

lower by all the string coupling is reduced. contribute 10% of the total CMB anisotropy. Possible to discriminate them in future experiments like QUIET and Polarbear.

B-mode power spectra for  $g_s = 0.04$  (solid) and  $g_s = 0.9$  (dash) normalised so that strings

- Inset figure -- the position of the peak as a function of string coupling. Note the shift of the peak to
	-

### B-mode Power Spectrum due to strings



 $G\mu < 8.9 \times 10^{-8}$  – Planck2015 TT + Pol + lowP + BKPlanck

- Results cosmic strings:
	- $G\mu < 1.1 \times 10^{-7} \text{Planck}2015$  TT
- $G\mu < 9.6 \times 10^{-8} \text{Planck}2015 \text{ TT } + \text{ Pol } + \text{ low}P$ 
	-
- No constraints on  $c_r$  and  $\alpha$ , slight preference for higher values of  $c_r$ 
	- and lower values of  $\alpha$
	- Results cosmic superstrings:
	- $G\mu_F < 2.8 \times 10^{-8} \text{Planck}2015 \text{ TT } + \text{ low}P$ 
		- when marginalised over c<sub>s</sub>, g<sub>s</sub> and w
- Currently looking at three point correlation function for evidence of nongaussianity and B mode polarisation effects - initial results show signal is extremely small and in fact analytically tensor bi-spectrum vanishes.
- Recent nanoGrav results consistent with network of cosmic superstrings exciting for the time being.



### **Conclusions**

- Single field Inflation has become the standard paradigm for primordial density fluctuations.
- Tight constraints are emerging on the slow roll parameters possible two scales emerging
	- Reheating the Universe is an area that has received relatively little attention.
- Possible role of non-topological solitons like oscillons in models with asymptotically flat potentials a new observational route
	- They could lead to PBHs formed In the early universe - require modification from standard slow roll inflation
		- An accurate calculation of the full PDF of the perturbations is required to calculate their abundance.
- Where is the inflaton in string theory? Have looked at a particular example and seen the possible importance of the kinating period between the end of inflation and the onset of reheating - some 30 orders of magnitude in time, when lots could happen !
	- Have looked at cosmic superstrings which could form at the end of a period of string driven inflation !
		- Aspects not mentioned include:
			- Multi field inflation
			- Non-Gaussianity constraints
		- The link if any between inflation in the early and late universe !
			- What if inflation never happened ?



**Extra slides**

### **Conclusions cont…**

Have not discussed many elements of PBH physics:

Role in Information paradox [Hawking 1971,1974] Role as a catalysis of Ewk phase transition [Gregory et al 2014] Possible role of PBH Planck mass relics in dark matter constraints [Zeldovich 1984, MacGibbon 1987] Alternative formation mechanisms such as collapsing cosmic string loops or from bubble collisions. [Hawking Moss & Stewart 1982] Baryogenesis scenarios from PBH evaporations [Zeldovich & Starobinski 1976] PBHs decay by evaporation - interesting attractor solution where PBHs in equilibrium with radiation in both radiation dominated and matter dominated universe - might lead to interesting new features. [Barrow et al 1992]

For objects that as far as we know have never been detetced, PBHs offer staggering constraints on cosmological models.



Then of instantaneous transition - works really merely  
\n
$$
\eta_H(\tau) = \eta_1 + (\eta_2 - \eta_1) \Theta(\tau - \tau_1)
$$

Assume piecewise constant  $\eta_H$  - makes instantaneous (yet finite) transition  $\eta_1$ -> $\eta_2$  at time  $\tau = \tau_1$ <u>stant n</u> า<sub>้</sub><br>ไห d kes inst stan **US** *z* k (**Qi**erentiability), (4.40)

Obtain  
\nwhere 
$$
\nu^2 - \frac{1}{4} \equiv \frac{z''}{z} \tau^2 = A \tau \delta_D (\tau - \tau_1) + \nu_1^2 - \frac{1}{4} + (\nu_2^2 - \nu_1^2) \Theta(\tau - \tau_1),
$$
\n
$$
\text{Note the delta function}
$$
\n
$$
A = \eta_2 - \eta_1, \quad \nu_{1,2}^2 - \frac{1}{4} = 2 - 3 \eta_{1,2} + \eta_{1,2}^2.
$$
\nNote the delta function

Note the delta function -

$$
\Sigma_{\phi\phi} = \left(\frac{H}{2\pi}\right)^2 T^2 \left|\sqrt{2k} v_k(T)\right|^2 \Big|_{T=\sigma},
$$
\n
$$
\text{Re}\left(\Sigma_{\pi\phi}\right) = -\left(\frac{H}{2\pi}\right)^2 T^2 \text{Re}\left(\sqrt{2k} v_k^*(T) \left[T\frac{\mathrm{d}}{\mathrm{d}T}\left(\sqrt{2k} v_k(T)\right) + \sqrt{2k} v_k(T)\right]\right) \Big|_{T=\sigma}
$$
\n
$$
\Sigma_{\pi\pi} = \left(\frac{H}{2\pi}\right)^2 T^2 \left|T\frac{\mathrm{d}}{\mathrm{d}T}\left(\sqrt{2k} v_k(T)\right) + \sqrt{2k} v_k(T)\right|^2 \Big|_{T=\sigma},
$$

Analytic treatment of instantaneous transition - works really nicely Analytic treatment of instantaneous transition - works really nicely

 $k$  ( $k$  ) are the mode functions before and after the mode functions before and after the transitions  $\alpha$ 

Nosie matrix elements

### $1yti$

Ansatz - motivated by numerical results



**1. The expression of analytic solution expressions for the pre-transition enoise matrix elements in the pre-transition expressions for the pre-transition expressions for the pre-transition enormalism in the pre-transition** 

Pre transition epoch  $T > T_1$  with  $\nu =$ Pre transition epoch  $T \geq T_1$  with  $\nu = \nu_1$  $\frac{1}{2}$  Equips U quidi Pro transition epoch  $T > T_1$  with u.



2 can be shown to satisfy the algebraic equation of the algebraic equations of the algebraic equations of the a<br>Equations of the algebraic equations of the algebraic equations of the algebraic equations of the algebraic eq Figure 6: The numerical literature of the modified Kingdom and modified Kingdom and modified Kingdom and modified K Full numerical solution



Immediately after transition epoch  $\Sigma_{ij} \propto e^{2\mathcal{A}N_e}$  ( $\mathcal{A} \equiv \eta_2 - \eta_1 = 3.32$ ) and 2. Immediately after the transition, ⌃*ij* / *<sup>e</sup>*2*AN<sup>e</sup>* , and ⌃ : *<sup>|</sup>*Re(⌃⇡)*<sup>|</sup>* : ⌃⇡⇡ ! 1 : *<sup>A</sup>* : *<sup>A</sup>*<sup>2</sup> *.* (4.73)

$$
\Sigma_{\phi\phi}:|\text{Re}(\Sigma_{\phi\pi})|:\Sigma_{\pi\pi}\to 1:\left(\nu_2-\frac{3}{2}\right):\left(\nu_2-\frac{3}{2}\right)^2
$$

### Instantaneous transition - from SRI:  $\nu_1 = 1.52$  to USR with  $\nu_2 = 1.8$

elements fall nearly-exponentially with ⌃*ij* ⇠ *<sup>e</sup>*2*AN<sup>e</sup>* . The ratio ⌃ : *<sup>|</sup>*Re(⌃⇡)*<sup>|</sup>* : ⌃⇡⇡ is

$$
\Sigma_{\phi\phi}:|\mathrm{Re}(\Sigma_{\phi\pi})|:\Sigma_{\pi\pi}\to 1:\left(\nu_1-\frac{3}{2}\right):\left(\nu_1-\frac{3}{2}\right)^2
$$

$$
\Sigma_{\phi\phi}: |\text{Re}(\Sigma_{\phi\pi})| : \Sigma_{\pi\pi} \to 1 : \mathcal{A} : \mathcal{A}^2
$$

 $\Omega_{1}$  ficiently late times,  $T \ll T$ , same as above by with  $y = y_{0}$  $\overline{\phantom{a}}$  $\frac{3}{2}$   $\frac{3}{2}$ ,  $\frac{3}{2}$ ,  $\frac{3}{2}$ ,  $\frac{4}{2}$ , but with  $\frac{3}{2}$ , but with  $\frac{2}{3}$ Sufficiently late times,  $T \ll T_1$ , same as above but with  $\nu = \nu_2$ ,

• But when power spectrum sufficiently amplified for an interesting abundance of PBHs,  $\pi_{\rm en}\simeq\pi_{\rm cr}\Rightarrow y_{\rm en}\simeq 1.$ In this section we calculate the expressions for the noise matrix elements ⌃*ij* , *i.e.* the correlators  $\frac{1}{2}$  spectrum sufficiently amplified for an interesting abundance of PRHs  $\pi$   $\sim \pi$   $\rightarrow u$   $\sim 1$ approximation to the slow-roll  $\frac{1}{2}$  and  $\frac{1}{2}$  for slow-roll and the slow-roll and the slow-roll and  $\frac{1}{2}$ 

• Then, both classical drift and stochastic diffusion become important (at least initially during the entry into the USR segment). noth classical drift and stochastic diffusion become important (at least initially during the s both classical drile and stochastic diffusion become important (at least initially during the pure de Sitter approximation to the substitution to the substitution obtained using the slow-roll approximation of the slow-roll approximation of the slow-roll approximation of the slow-roll approximation of the slow-roll approximation o The key equations that we use are the following: the following: the definition of the noise operators,  $\alpha$ classical drift and stochastic diffusion become important (at least initially during the entry noise correlations of Eq. (3.14), with the noise correlation matrix  $\hat{P}$  being  $\hat{P}$  being  $\hat{P}$ 

where

• Furthermore, the de Sitter approximations for the noise matrix elements might breakdown during the transition into the USR phase [Ahmadi et al 2022] .  $\bullet$  Consequently, it becomes important to estimate the noise matrix elements more accurately. The key equations that we use are the following: the definition of the noise operators, ermore, the <mark>de Sitter approximations for the noise matrix elements might breakdown</mark> during the crossing the coarseon into the USK phase [Animadi et al 2022]. It is important to note that these UV-noise mode functions are to be computed, not

Case 1: Noise matrix elements in stochastic inflation with featureless potential – slow roll case

ion of modes  $\{\phi_k, \pi_k\}$  given via Mukhanov-Sasaki equation which in terms of conformal time  $\begin{pmatrix} 1 & 10 \\ 0 & 10 \end{pmatrix}$  8  $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Evolution of modes  $\{\phi_k, \pi_k\}$  given via Mukhanov-Sasaki equation which in terms of conformal time  $\tau$  is *<sup>k</sup>* + *<sup>k</sup>*<sup>2</sup> *<sup>z</sup>*<sup>00</sup> *z v<sup>k</sup>* = 0 *,* (4.1)

 $v_k''$  +

$$
v''_k + \left(k^2 - \frac{z''}{z}\right)v_k = 0,
$$

where  
\n
$$
z = am_p\sqrt{2\epsilon_H},
$$
\n
$$
\frac{z''}{z} = (aH)^2 \left[ 2 + 2\epsilon_H - 3\eta_H + 2\epsilon_H^2 + \eta_H^2 - 3\epsilon_H\eta_H - \frac{1}{aH} \eta'_H \right]
$$
\nand in the spatially flat gauge: 
$$
\phi_k = \frac{v_k}{a}, \quad \pi_k = \frac{d}{dN} \left( \frac{v_k}{a} \right)
$$

and in the spatially flat gauge:

From Sec. 3.2, it is clear that in order to accurately compute the noise matrix elements

### Early times, all mode sub horizon -> impose Bunch Davies i.c lime de  $\alpha$  ly times, all mode sub horizon -> impose Bunch Davies i.c  $\qquad \qquad \lim_{L\to\infty} \, v_k(x)$  $\frac{1}{2}$

Intro new time variable:  $T^{\prime} = -k\tau = \frac{1}{aH}$ 

$$
\lim_{k\tau \longrightarrow -\infty} v_k(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}
$$

$$
T = -k\tau =
$$

During and the conformal time  $MS$  -eqn becomes: MS-eqn becomes : correspond to *T* and *T* and

Intro new time variable:

\n
$$
T = -k\tau = \frac{k}{aH}
$$

 $v_k = 0,$ 

$$
\frac{\mathrm{d}^2v_k}{\mathrm{d}T^2}+\left(1-\frac{\nu^2-\frac{1}{4}}{T^2}\right)v_k=0,
$$



For slow-roll  $\overline{1}$ v-ro<br>20 ation,  $\nu^2 \geq 9/4$  at early times and  $\n *V or de de or or*$ <sup>1</sup>/<sub>4</sub>  $\frac{1}{4}$  increases monotonically towards th For slow-roll inflation,  $\nu^2 \geq 9/4$  at early times and increases monotonically towards the end of inflation.

 $e^{-ik\tau}$ During time  $\mathsf{MS}\text{-}\mathsf{eqn}$  becomes the conformal time  $\mathsf{MS}\text{-}\mathsf{eqn}$  becomes the

$$
\nu^2 = \frac{1}{(aH)^2} \frac{z''}{z} + \frac{1}{4}
$$

Obtain mode solution:

And **exact noise matrix elements**, (recall evaluated at  $k = \sigma aH$ , hence when  $T = \sigma$ )

$$
v_k(T) = \frac{1}{\sqrt{2k}} \left( 1 + \frac{i}{T} \right) e^{iT}
$$



**Case of Pure dS limit**, both  $\epsilon_H$ ,  $\eta_H = 0$ , leading to  $z''/z = 2a^2H^2$  and  $\nu^2 = 9/4$ .

$$
\phi = (1 + \sigma^2) \left(\frac{H}{2\pi}\right)^2
$$

$$
\left(\Sigma_{\phi\pi}\right) = -\sigma^2 \left(\frac{H}{2\pi}\right)^2
$$

$$
\Sigma_{\pi\pi} = \sigma^4 \left(\frac{H}{2\pi}\right)^2
$$

For  $\sigma = 0.01$  say have  $\Sigma_{\phi\phi} : \Sigma_{\phi\pi} : \Sigma_{\pi\pi} = 1 : 10^{-4} : 10^{-8}$  - which is why  $\Sigma_{\phi\pi}$  and  $\Sigma_{\pi\pi}$  usually ignored.

### Case of slow roll inflation where  $\epsilon_H$ ,  $\eta_H \ll 1$ , the slow-roll parameters but do not exactly vanish.

For realistic SR potentials,  $\nu$  is roughly equal to 3/2 and evolves slowly and monotonically. We obtain

$$
v_k(T) = e^{i(\nu + \frac{1}{2})\frac{\pi}{2}} \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{2k}} \sqrt{T} H_{\nu}^{(1)}(T),
$$
  
\n
$$
\Sigma_{\phi\phi} = 2^{2(\nu - \frac{3}{2})} \left[ \frac{\Gamma(\nu)}{\Gamma(3/2)} \right]^2 \left( \frac{H}{2\pi} \right)^2 T^{2(-\nu + \frac{3}{2})},
$$
  
\n
$$
\text{Re}(\Sigma_{\phi\pi}) = -2^{2(\nu - \frac{3}{2})} \left[ \frac{\Gamma(\nu)}{\Gamma(3/2)} \right]^2 \left( \frac{H}{2\pi} \right)^2 \left( -\nu + \frac{3}{2} \right) T^{2(-\nu + \frac{3}{2})},
$$
 Note, the hier  
\nterms no long  
\n
$$
\Sigma_{\pi\pi} = 2^{2(\nu - \frac{3}{2})} \left[ \frac{\Gamma(\nu)}{\Gamma(3/2)} \right]^2 \left( \frac{H}{2\pi} \right)^2 \left( -\nu + \frac{3}{2} \right)^2 T^{2(-\nu + \frac{3}{2})}.
$$

And on superhorizon scales:

Note, the hierarchy of noise terms no longer necessarily present



For D-brane KKLT type potential

$$
V(\phi) = V_0 \frac{\phi^2}{M^2 + \phi^2}
$$

$$
\frac{1}{(aH)^2}\frac{z''}{z}
$$

$$
\frac{\gamma}{\tau} \simeq 2 - 3 \eta_H + \eta_H^2 + \tau \frac{\mathrm{d}\eta_H}{\mathrm{d}\tau}
$$

Specific example, <mark>a modified KKLT potential with an additional tiny Gaussian bump-like feature</mark> [Mishra et al 2019]: evolution of ⌘*<sup>H</sup>* and *z*00*/z* for this potential is shown in Fig. 5. We have fixed *M* = 0*.*5 *mp*,

$$
V_{\rm b}(\phi)=V_0\,\frac{\phi^2}{M^2+\phi^2}\,\left[1+A\,\exp\left(-\,\frac{1}{2}\,\frac{(\phi-\phi_0)^2}{\tilde{\sigma}^2}\right)\right]\,,
$$

where A,  $\tilde{\sigma}$  and  $\phi_0$  represent the height, width and position of the bump respectively.

Gives amplification of the scalar power-spectrum,  $\mathcal{P}_{\zeta}$ , by a factor of  $10^{7}$  relative to its value on CMB scales. for a de Sitter expansion (Eq. 4.8), namely ⌫ = 3*/*2. In the modified KKLT case, (1*/aH*)2*z*00*/z*

Case of potentials with a slow-roll violating feature, like USR with  $\epsilon_H \ll 1$ , while  $\eta_H \ge 1$ Dynamics undergoes number of phases driven by  $\eta_H$ . We now have :  $R = \frac{1}{2}$ phase which a slow-roll violating leature, like Opit while  $\epsilon_H$ 



potential Eq. (4.28), with = 0*.*01, leading to a realistic smooth transition from SR-I to USR. The plot shows that the plot shows the plot shows the behaviour of the USR regime. He will be used the USR regime. The USR regime of the USR regime of the USR regime.  $\alpha$ Fixed  $M = 0.5 m_p$ , and bump parameters to be  $A = 1.87 \times 10^{-3}$ , relative to the field noise, ⌃, in the USR epoch. With Swagat Mishra and Anne Green - e-Print:2303.17375 - JCAP 2024  $t = 1$  to  $\sigma = 10^{-3}$  and  $t = 1000$  to  $10^{-2}$  the momentum induced noise terms,  $t = 10^{-3}$  and  $t = 0.00$  momentum induced noise terms,  $t = 10^{-3}$  and  $t = 0.00$  momentum induced noise terms,  $t = 10^{-3}$  and  $t = 0.00$  momen Fixed  $M = 0.5 m_p$ , and bump parameters to be  $A = 1.87 \times 10^{-3}$ ,  $\tilde{\sigma} = 1.993 \times 10^{-2}$  and  $\phi_0 = 2.005 m_p$ . Gives amplification of the scalar power-spectrum,  $P_{\zeta}$ , by a factor of  $10^{7}$  relative to its value on CMB scales.

### In reality — noise terms are more interesting !

### Numerical noise matrix elements,  $\Sigma_{ij}$  - note the switching of dominant terms during USR value the switching of dominant terms during JISR


## Outstanding steps to calculate the PBH mass fraction

 $\overline{1}$ The adjoint FPE for the PDF *P<sup>i</sup>* (*N* ) corresponding to the general Langevin equation, Have calculated the stochastic noise matrix elements  $\Sigma_{ij}$ , for a sharp transition from SR to USR a sharp transition from SR to USR, using both analytical and numerical techniques. Our Have calculated the stochastic noise matrix elements  $\Sigma_{ij}$ , for a sharp transition from  $\zeta$ Have calculated the stochastic noise matrix elements  $\Sigma_{\pm}$  for a sharp transition from SR to US coarse grained curvature perturbation, *P*(⇣*cg*), above the threshold for PBH formation, ⇣*c*. *a* snarp trains

Aim is to determine the

Aim is to determine the PDF of the number of e-folds,  $P_{\Phi,\Pi}(\mathcal{N})$ , by solving the adjoint Fokker-Planck eqn Alm is to determine the PDF of the number of e-folds,  $P_{\Phi,\Pi}(\mathcal{N})$ , by solving the adjoint Fokker-Planck Aim is to determine the PDF of the number of e-folds,  $P_{\Phi,\Pi}(\mathcal{N})$ , by solving the adjoint Fokke *<sup>N</sup><sup>c</sup>* <sup>=</sup> ⇣*<sup>c</sup>* <sup>+</sup> <sup>h</sup>*<sup>N</sup>* (*,* ⇧)<sup>i</sup> <sup>=</sup> ⇣*<sup>c</sup>* <sup>+</sup><sup>X</sup> ⇤2 *n*

 $B_n(\Phi,\Pi)\,e^{-\Lambda_n\,{\cal N}}$ 

 $P_{\Phi,\Pi}(\mathcal{N}) = \sum_{n=0}^{\infty} B_n(\Phi,\Pi) e^{-\Lambda_n \mathcal{N}}$   $B_n(\Phi,\Pi)$  to be determined from b.c. and expressions for  $\Sigma_{ij}$  $\frac{1}{2}$  $B_n(\Phi, \Pi)$  to be determined from b.c. a

$$
\frac{\partial}{\partial N} P_{\Phi_i}(\mathcal{N}) = \left[ D_i \frac{\partial}{\partial \Phi_i} + \frac{1}{2} \Sigma_{ij} \frac{\partial^2}{\partial \Phi_i \partial \Phi_j} \right] P_{\Phi_i}(\mathcal{N}).
$$

$$
P_{\Phi,\Pi}(\mathcal{N}) = \sum_{n=0}^{\infty} B_n(\Phi,\Pi) e^{-\Lambda_n \mathcal{N}} \qquad B_n(\Phi,\Pi)
$$
 to be determined from b.c. and

 $\mathbf{m}$  conditions includes an absorbing boundary at smaller field values ( $\mathbf{m}$ ) conditions (A) closer to the total values end of the mass inflation of  $\epsilon$  points. Then calculate the mass fraction of PBHs  $\beta_{\rm PBH}$ . Then calculate the mass fraction of PBHs  $\beta_{\text{DDH}}$ . conditions and using the correct expressions for the noise matrix elements ⌃*ij* as determined and then calculate the mass fraction is given by the set of  $\mathbf{r}_{\text{max}}$  $\int$ Then ⇣*c* ate the mass ⇣*c*+h*N* (*,*⇧)i  $2Hs \beta_{PBH}$ . which leads to the final expression for the  $f$ inflation for the state of the s<br>The state of the st

$$
\beta(\Phi,\Pi) \equiv \int_{\zeta_c}^{\infty} P(\zeta_{cg}) d\zeta_{cg} = \int_{\zeta_c + \langle \mathcal{N}(\Phi,\Pi) \rangle}^{\infty} P_{\Phi,\Pi}(\mathcal{N}) d\mathcal{N}
$$

$$
\beta(\Phi,\Pi) = \sum_{n=0}^{\infty} \frac{B_n(\Phi,\Pi)}{\Lambda_n} \exp\left[-\Lambda_n \left[\zeta_c + \sum_{m=0}^{\infty} \frac{B_m(\Phi,\Pi)}{\Lambda_m^2}\right]\right]
$$
Of course this might not k  
gaussian PDF for typical fluctuations  
ive approach,  $\beta^{\rm G}(\Phi,\Pi)$ 
$$
\sigma_{cg}^2(\Phi,\Pi) = \int_{k(\Phi,\Pi)}^{k_e} \frac{dk}{k} \mathcal{P}_{\zeta}(k) .
$$

roacn Compare with Gaussian PDF for ty in the perturbative approach,  $\beta^{\mathrm{G}}(\Phi,\Pi)$ 

*.* (5.4)

Of course this might not be possible !



## Why might a non-gaussian PDF of Primordial Fluctuations help with creating PBHs ?

We expect PBHs to form from rare peaks in the fluctuations in the density contrast



For small fluctuations we expect the PDF to be Gaussian



 $(x,y,z)$ 



But deviations from Gaussian for large fluctuations could increase the PDF enhancing the likelihood of forming PBHs

# Small scale power spectrum is not constrained!



Observational Constraints on Power Spectrum - very little on small scales

#### Carr et al 2020; Green and Kavanagh 2020

## In terms of a power spectrum generated from inflation we require



### PBH size fluctuations re-enter on different scales



 $d_p(t) = a(t)$  *<sup>t</sup>* 0 *dt a*(*t* )

## **Horizons -- crucial concept in cosmology**

a) *Particle horizon:* is the proper distance at time t that light could have travelled since the big bang (i.e. at which *a=0*). It is given by





b) *Event horizon: is* the proper distance at time t that light will be able to travel in the future: