Inflation and Dark Energy

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1. Inflation - including some neat additions
2. A bit more inflation and intro to Dark Energy
3. More Dark Energy and a few variants

Invisibles School, Bologna — June 24th - June 28th 2024

Before we start - given the week we are in:
If you don’t mention this

Denmark
outplay
England

Credit: OptaAnalyst

I won’t mention this

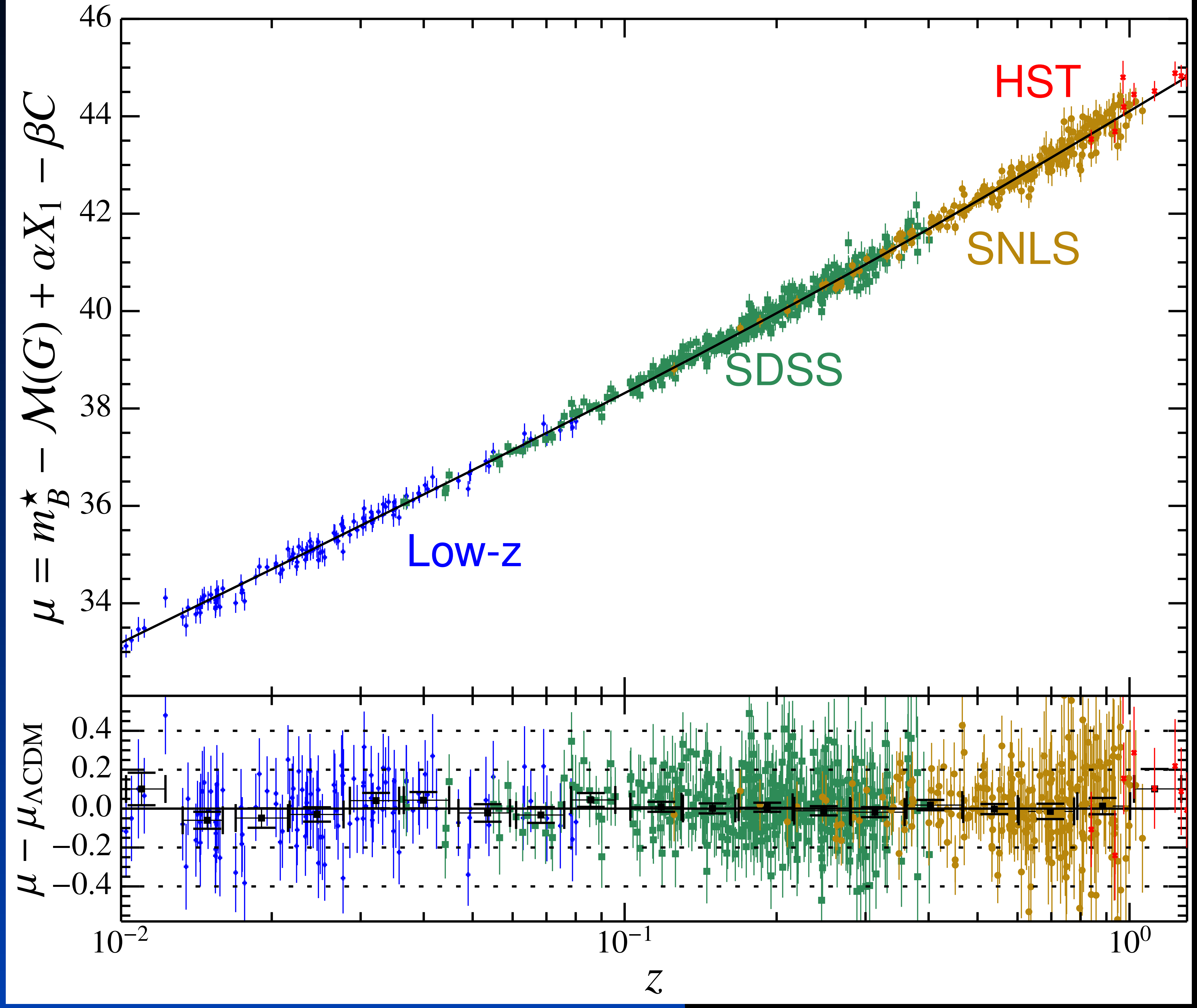
Spain outplay Italy

Credit: Carmen Jaspersen/Reuters

Agreed ?

The Big Bang – (1sec → today)

The cosmological principle -- isotropy and homogeneity on large scales



- **The expansion of the Universe**

$v = H_0 d$

$H_0 = 73.04 \pm 1.04$ km s^{-1} Mpc^{-1}

(Riess et al, 2022)

$H_0 = 67.4 \pm 0.5$ km s^{-1} Mpc^{-1}

(Planck 2018)

Is there a local v global tension ?

Betoule et al 2014

$$\text{Redshift } 1 + z = \frac{a_0}{a} \qquad H = \frac{\dot{a}}{a}$$

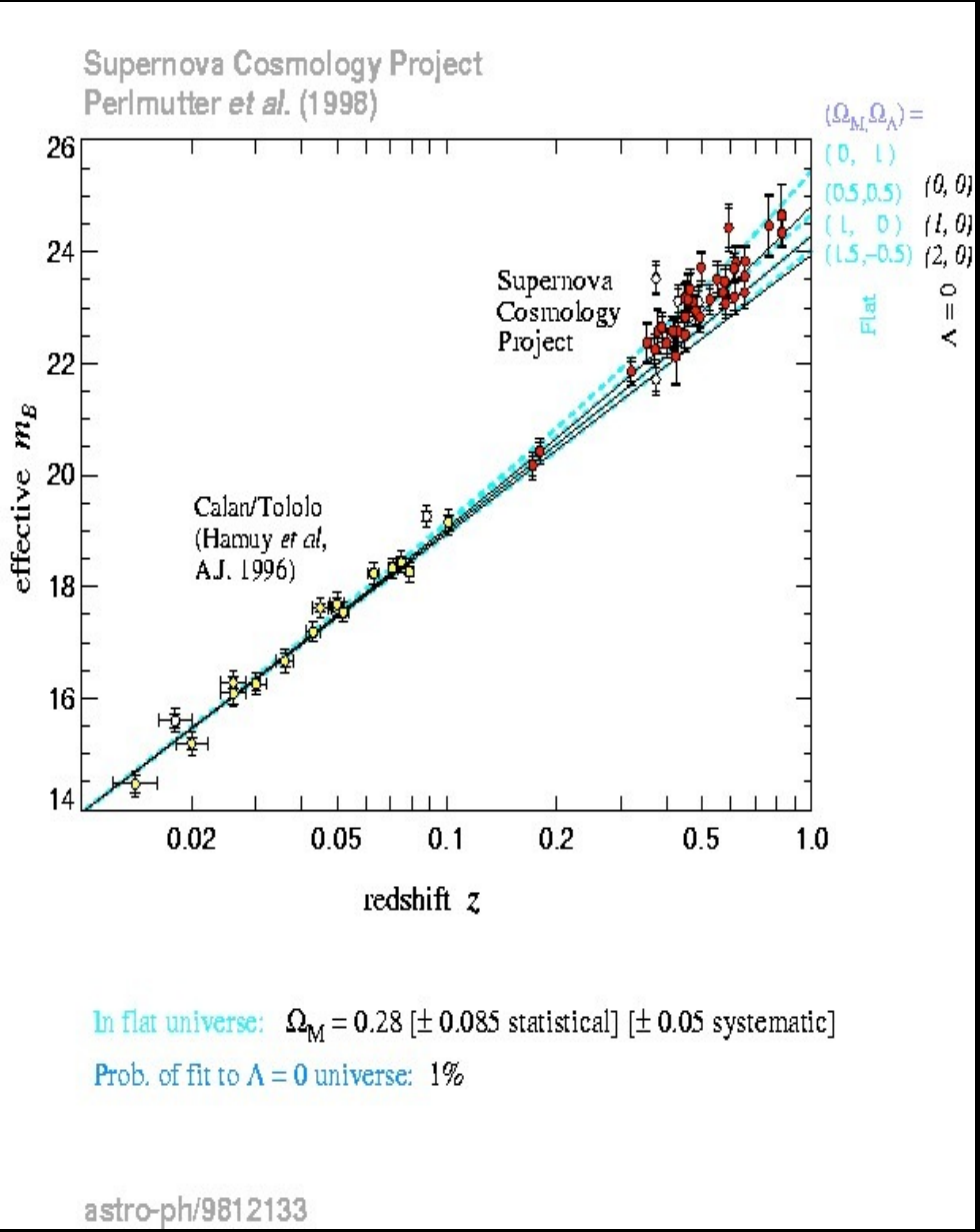
In fact the universe is accelerating !

Observations of distant supernova in galaxies indicate that the rate of expansion is increasing !

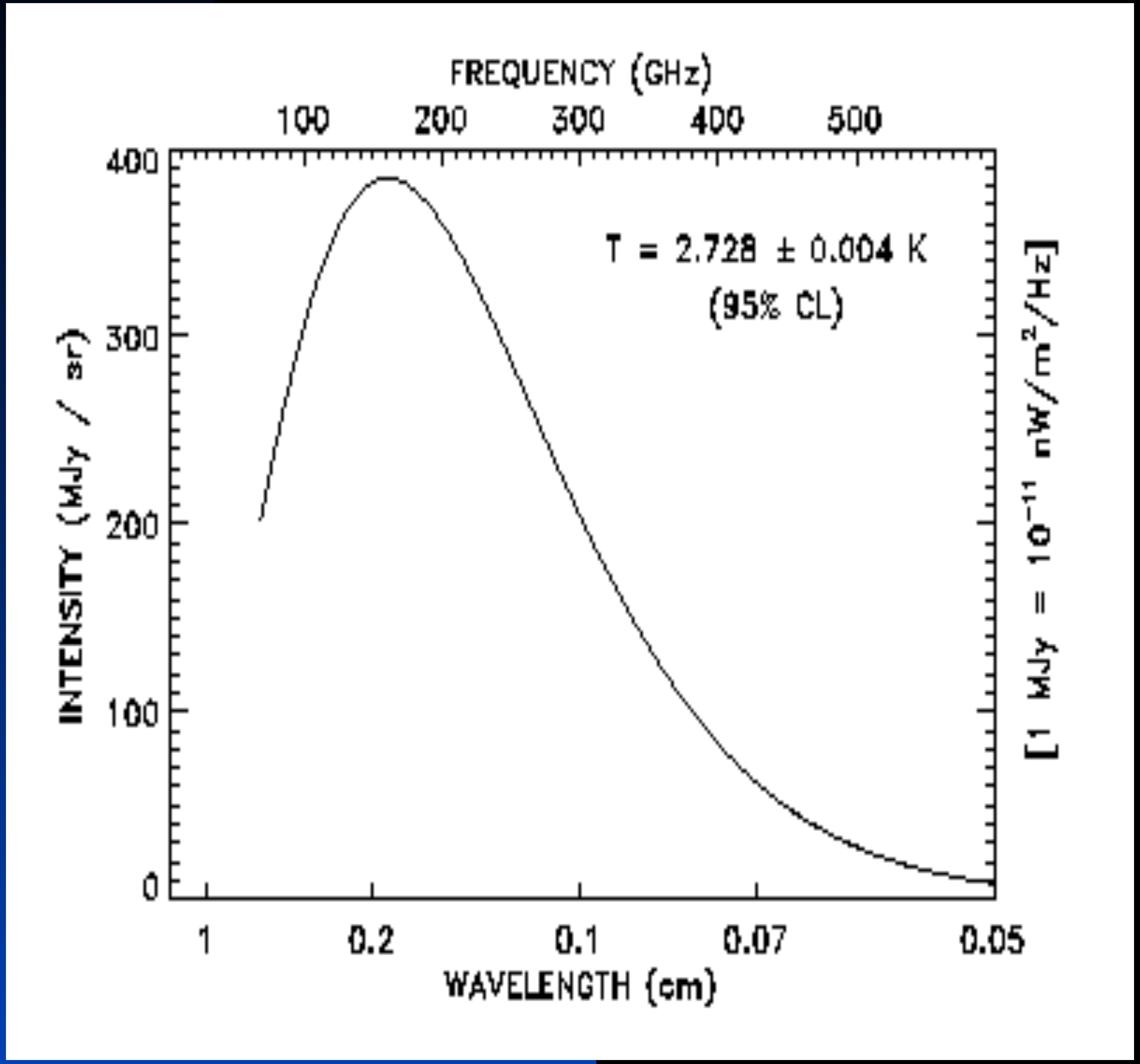
Huge issue in cosmology -- what is the fuel driving this acceleration?

We call it Dark Energy -- emphasises our ignorance!

Makes up 70% of the energy content of the Universe



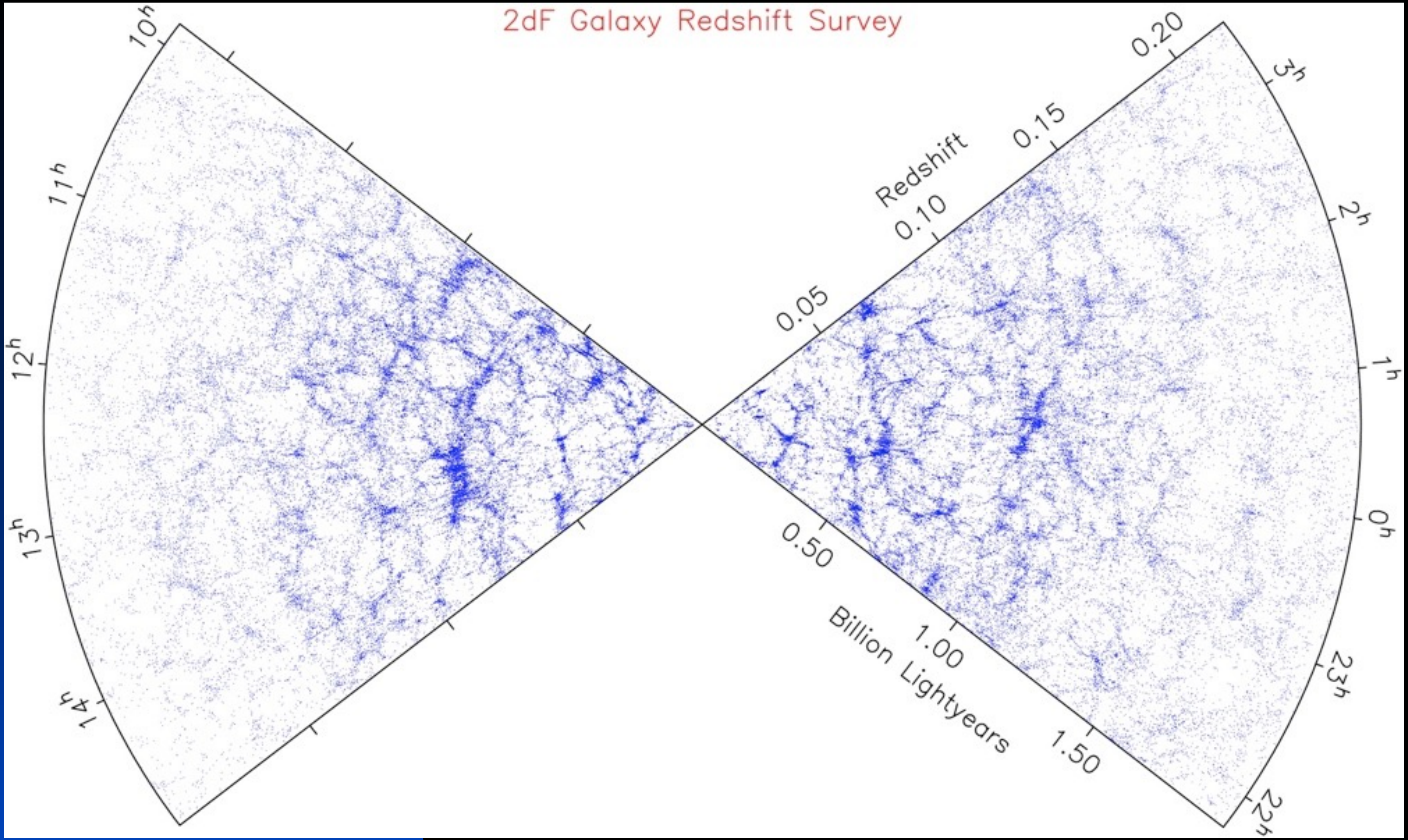
The Big Bang – (1sec → today)



Test 2

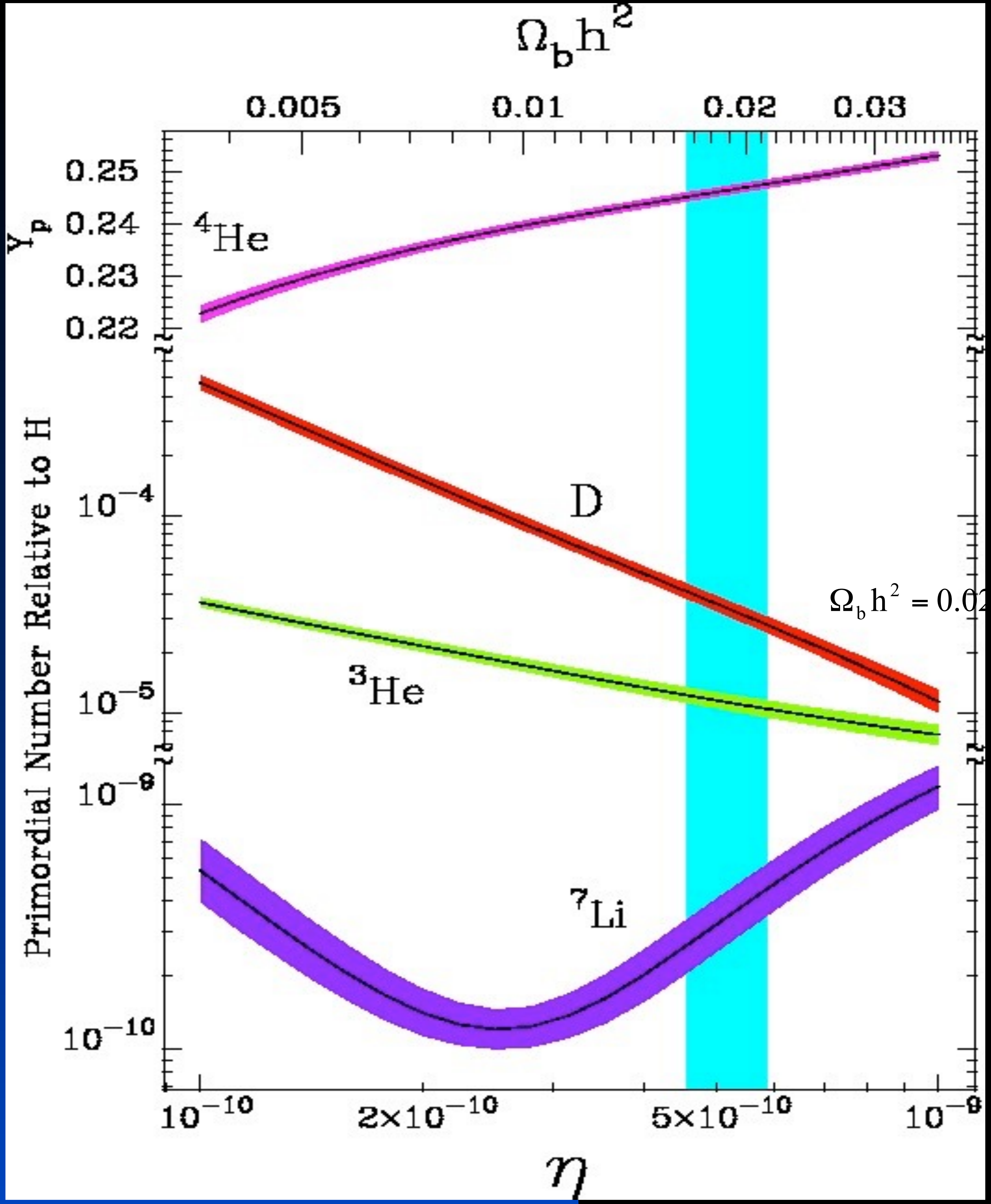
- **The existence and spectrum of the CMBR**
- **T_0=2.728 ± 0.004 K**
- Evidence of isotropy -- detected by COBE to such incredible precision in 1992
- Nobel prize for John Mather 2006

2dF Galaxy Redshift Survey



Homogeneous on large scales?

The Big Bang – (1sec → today)



$\Omega_b h^2$ = 0.02

Test 3

- **The abundance of light elements in the Universe.**
- **Most of the visible matter just hydrogen and helium.**

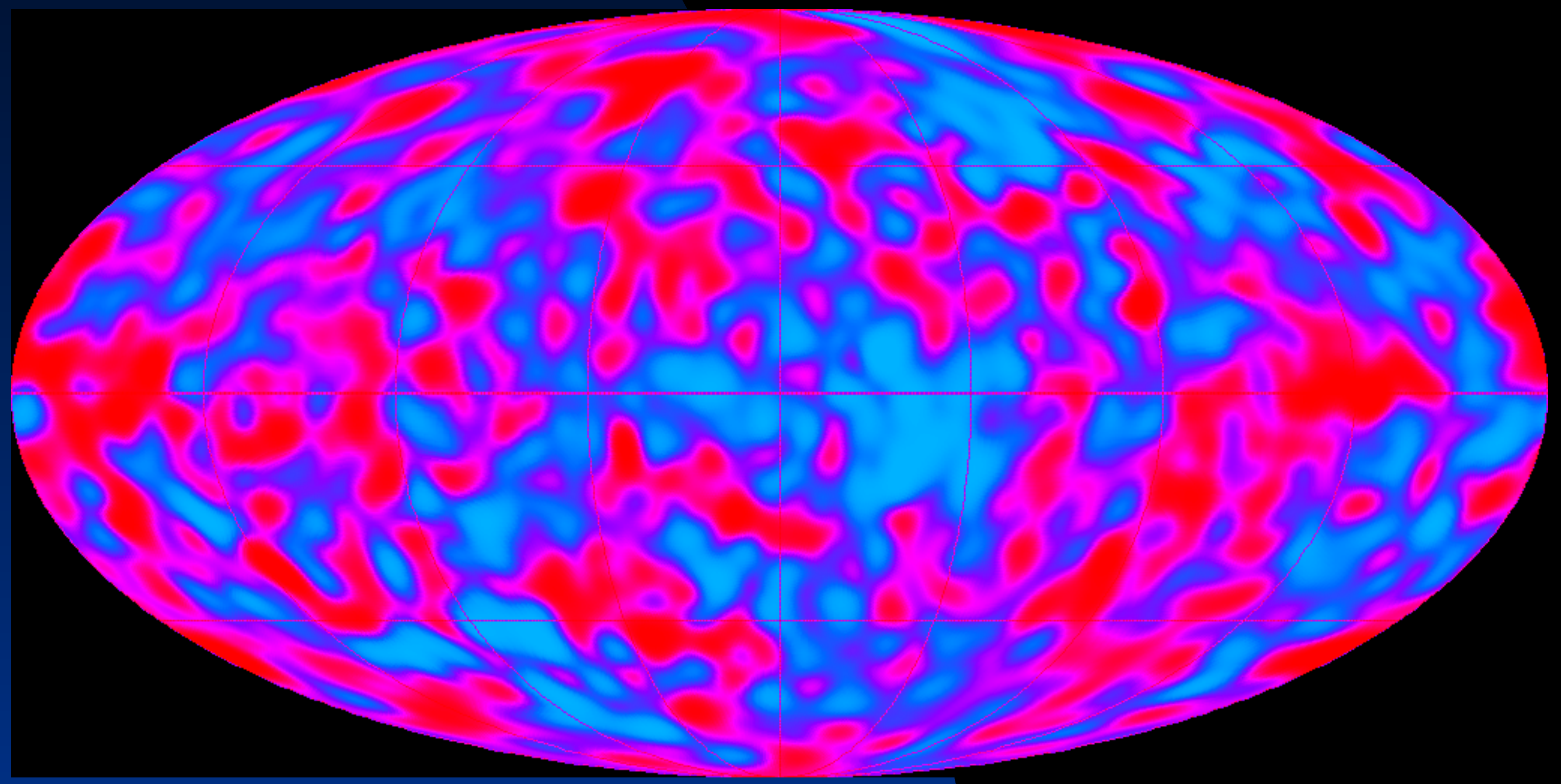
(baryons) -- $\Omega_b h^2 = 0.02242 \pm 0.00014$

Planck 2018

The Big Bang – (1sec → today)

Test 4

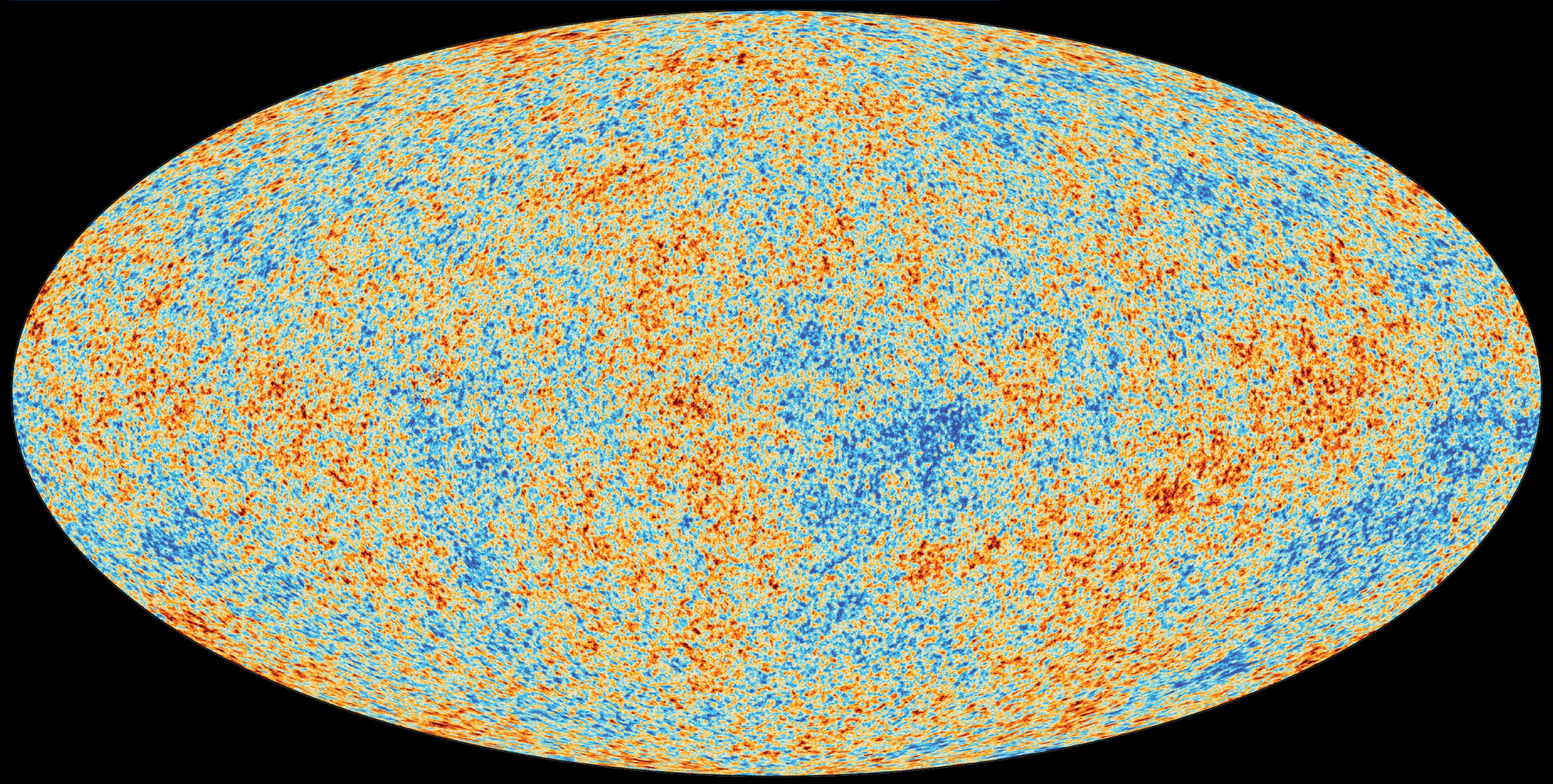
- Given the irregularities seen in the CMBR, the development of structure can be explained through gravitational collapse.



COBE - 1992, 2006

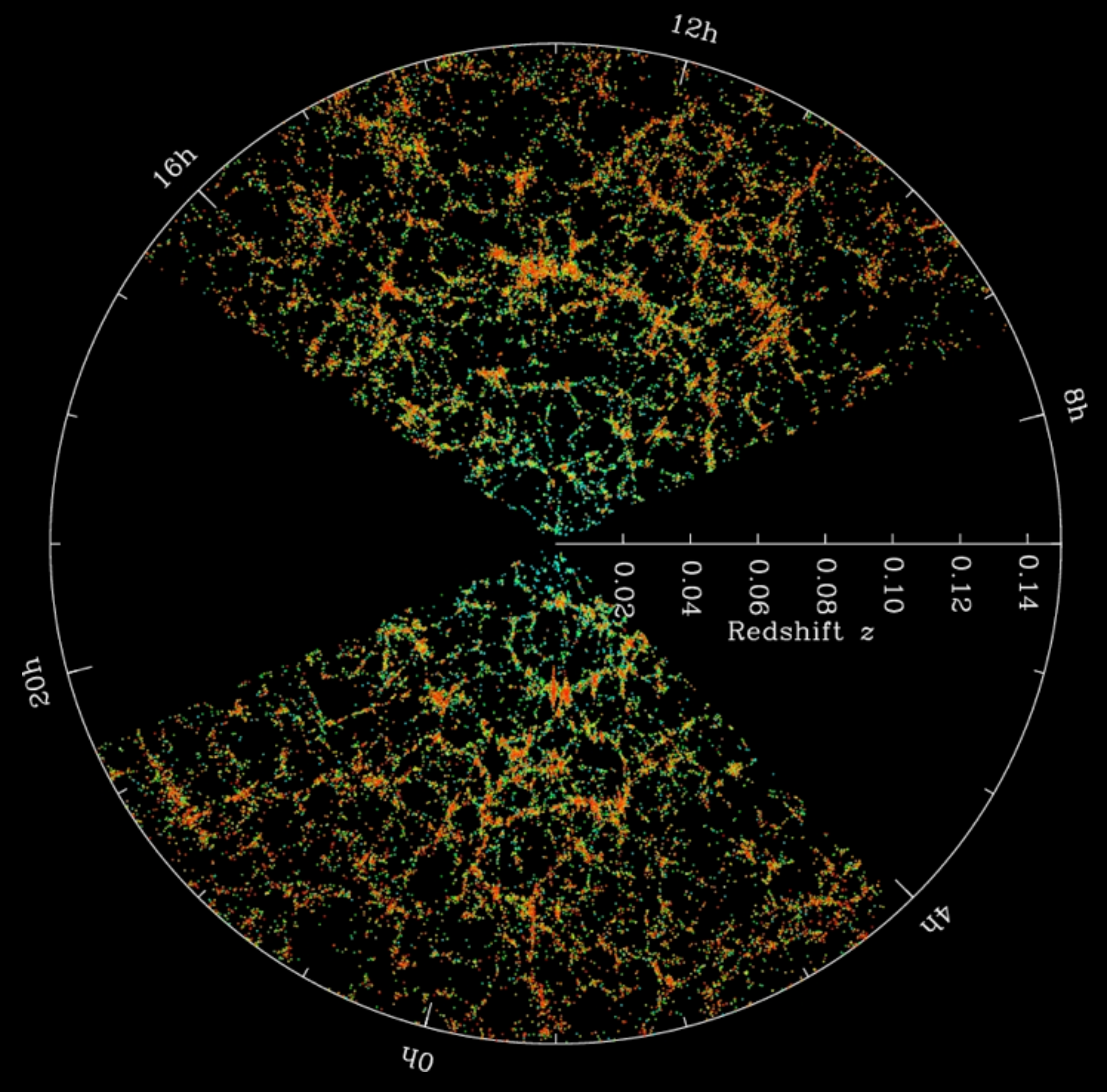
Nobel prize for

George Smoot



PLANCK-2018

SDSS



The key equations

Einstein GR: $$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Geometry   Matter   Cosm const - could be matter or geometry

Relates curvature of spacetime to the matter distribution and its dynamics.

Require metric tensor $g_{\mu\nu}$ from which all curvatures derived indep of matter:

Invariant separation of two spacetime points (μ,ν=0,1,2,3): $$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

Einstein tensor $G_{\mu\nu}$ -- function of $g_{\mu\nu}$ and its derivatives.
Energy momentum tensor $T_{\mu\nu}$ -- function of matter fields present.
For most cosmological substances can use perfect fluid representation for which we write

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu}$$

U^μ : fluid four vel = (1,0,0,0) - because comoving in the cosmological rest frame.
(ρ,p) : energy density and pressure of fluid in its rest frame

$$T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$$

Reminder of curvatures

Christoffel symbols:

$$\Gamma^{\mu}_{\nu\sigma} = \frac{1}{2} g^{\mu\lambda} (g_{\sigma\lambda,\nu} + g_{\nu\lambda,\sigma} - g_{\sigma\nu,\lambda})$$

Riemann's curvature tensor:

$$R^{\mu}_{\nu\sigma\gamma} = \Gamma^{\mu}_{\nu\gamma,\sigma} - \Gamma^{\mu}_{\nu\sigma,\gamma} + \Gamma^{\mu}_{\alpha\sigma}\Gamma^{\alpha}_{\gamma\nu} - \Gamma^{\mu}_{\alpha\gamma}\Gamma^{\alpha}_{\sigma\nu}$$

Ricci tensor:

$$R_{\mu\nu} = R^{\sigma}_{\mu\nu\sigma}$$

Ricci scalar:

$$R = R^{\mu}_{\mu}$$

Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

Not needed here

Cosmology - isotropic and homogeneous FRW metric

Copernican Principle: We are in no special place. Since universe appears isotropic around us, this implies the universe is isotropic about every point. Such a universe is also homogeneous.

Line element $$ds^2 = -dt^2 + a^2(t)dx^2$$

$$dx^2 = \frac{1}{1-kr^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

t -- proper time measured by comoving (i.e. const spatial coord) observer.
a(t) -- scale factor: k- curvature of spatial sections: k=0 (flat universe), k=-1 (hyperbolic universe), k=+1 (spherical universe)

Aside for those familiar with this stuff -- not chosen a normalisation such that a_0=1. We are not free to do that and simultaneously choose |k|=1. Can do so in the k=0 flat case.

Note: Crucial but not globally accepted — see for instance Watkins et al. results on large Bulk flows over 150-200 Mpc scales: eprint:2302.02028

Intro Conformal time : τ(t) $$\tau(t) \equiv \int^t \frac{dt'}{a(t')}$$

Implies useful simplification : $$ds^2 = a^2(\tau)(-d\tau^2 + dx^2)$$

Hubble parameter :
(often called Hubble constant) $$H(t) \equiv \frac{\dot{a}}{a}$$

Hubble parameter relates velocity of recession of distant galaxies from us to their separation from us

$$v = H(t)r$$

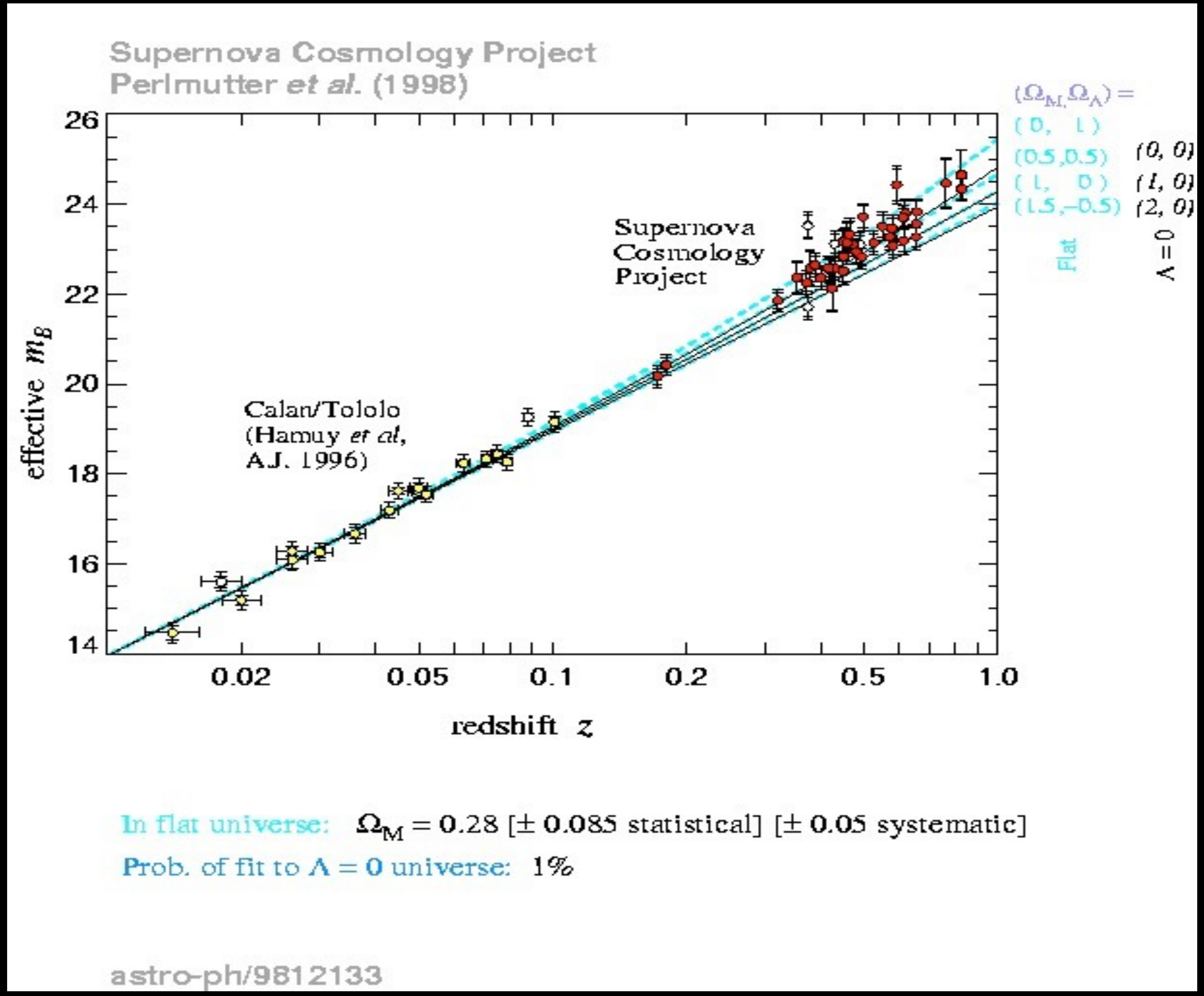
$$d = ax$$

$$\dot{d} = \dot{a}x + a\dot{x}$$

$$\dot{d} = Hd + a\dot{x}$$

$$\dot{d} = v + a\dot{x}$$

Hubble flow   peculiar velocity



$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$ applied to cosmology

Friedmann - the key bgd equation:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

a(t) depends on matter, $\rho(t)=\Sigma_i\rho_i$ -- sum of all matter contributions, rad, dust, scalar fields ...

Energy density $\rho(t)$: Pressure $p(t)$

Related through : $p = w\rho$

Eqn of state parameters: w=1/3 – Rad dom: w=0 – Mat dom: w=-1– Vac dom

Eqns (Λ=0):

Friedmann + Fluid energy conservation

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

$$\nabla^\mu T_{\mu\nu} = 0$$

Combine Friedmann and fluid equation to obtain Acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3}G(\rho + 3p) --- Accn$$

$$If\ \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

Inflation condition -- more later

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

$$\rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} \quad ; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

$$\mathrm{RD} \quad : \quad w = \frac{1}{3} : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-4} \quad ; \ a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{1}{2}}$$

$$\mathrm{MD} \quad : \quad w = 0 : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3} \quad ; \ a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$$

$$\mathrm{VD} \quad : \quad w = -1 : \rho(t) = \rho_0 \ ; \ a(t) \propto e^{Ht}$$

A neat equation

$$\rho_c(t) \equiv \frac{3H^2}{8\pi G} \quad ; \quad \Omega(t) \equiv \frac{\rho}{\rho_c}$$

$$\Omega > 1 \leftrightarrow k = +1$$

$$\Omega = 1 \leftrightarrow k = 0$$

$$\Omega < 1 \leftrightarrow k = -1$$

Friedmann eqn

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

Ω_m - baryons, dark matter, neutrinos, electrons, radiation

…

Ω_Λ - dark energy ; Ω_k - spatial curvature

$$\rho_c(t_0) \equiv 1.88h^2 * 10^{-29}\, \mathrm{gcm}^{-3}$$

Critical density

Bounds on H(z) -- Planck 2018 - (+BAO+lensing+lowE)

$$\mathbf{H^2(z) = H_0^2\left(\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{de}\exp\left(3\int_0^z \frac{1+w(z')}{1+z'}dz'\right)\right)}$$

(Expansion rate) -- H_0=67.66 ± 0.42 km/s/Mpc

(radiation) -- Ω_r = (8.5 ± 0.3) x 10^{-5} - (WMAP)

(baryons) -- $\Omega_b h^2$= 0.02242 ± 0.00014

(dark matter) -- $\Omega_c h^2$= 0.11933 ± 0.00091 —-(matter) - Ω_m = 0.3111 ± 0.0056

(curvature) -- Ω_k =0.0007 ± 0.0019

(dark energy) -- Ω_{de} = 0.6889 ± 0.0056 -- Implying univ accelerating today

(de eqn of state) -- 1+w = 0.028 ± 0.032 -- looks like a cosm const.

If allow variation of form : $w(z) = w_0 + w_a\, z/(1+z)$ then
w_0=-0.957 ± 0.08 and w_a = -0.29 ± 0.31 (68% CL) — (Planck 2018+SNe+BAO)

Important because distance measurements often rely on assumptions made about the background cosmology.

Recent developments — DESI (2024) - arXiv:2404.03002

$$w(z) = w_0 + w_a\, z/(1+z)$$

model/dataset	Ω_m	H_0 [$\mathrm{km\,s^{-1}\,Mpc^{-1}}$]	$10^3\Omega_K$	w or w_0	w_a
***w*CDM**					
DESI	0.293 ± 0.015	—	—	$-0.99^{+0.15}_{-0.13}$	—
DESI+BBN+θ_*	0.295 ± 0.014	$68.6^{+1.8}_{-2.1}$	—	$-1.002^{+0.091}_{-0.080}$	—
DESI+CMB	0.281 ± 0.013	$71.3^{+1.5}_{-1.8}$	—	$-1.122^{+0.062}_{-0.054}$	—
DESI+CMB+Panth.	0.3095 ± 0.0069	67.74 ± 0.71	—	-0.997 ± 0.025	—
DESI+CMB+Union3	0.3095 ± 0.0083	67.76 ± 0.90	—	-0.997 ± 0.032	—
DESI+CMB+DESY5	0.3169 ± 0.0065	66.92 ± 0.64	—	-0.967 ± 0.024	—
w_0w_aCDM					
DESI	$0.344^{+0.047}_{-0.026}$	—	—	$-0.55^{+0.39}_{-0.21}$	< -1.32
DESI+BBN+θ_*	$0.338^{+0.039}_{-0.029}$	$65.0^{+2.3}_{-3.6}$	—	$-0.53^{+0.42}_{-0.22}$	< -1.08
DESI+CMB	$0.344^{+0.032}_{-0.027}$	$64.7^{+2.2}_{-3.3}$	—	$-0.45^{+0.34}_{-0.21}$	$-1.79^{+0.48}_{-1.0}$
DESI+CMB+Panth.	0.3085 ± 0.0068	68.03 ± 0.72	—	-0.827 ± 0.063	$-0.75^{+0.29}_{-0.25}$
DESI+CMB+Union3	0.3230 ± 0.0095	66.53 ± 0.94	—	-0.65 ± 0.10	$-1.27^{+0.40}_{-0.34}$
DESI+CMB+DESY5	0.3160 ± 0.0065	67.24 ± 0.66	—	-0.727 ± 0.067	$-1.05^{+0.31}_{-0.27}$

This move towards phantom dark energy has generated a great deal of debate about the use of priors.

How old are we?

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2}$$

$$where\ \rho = \rho_m + \rho_r + \rho_\Lambda$$

$$t = \int \frac{da}{\dot{a}} = \int \frac{da}{aH}$$

$$t_0 = H_0^{-1}\int_0^1 \frac{x\,dx}{\left[\Omega_{m0}x + \Omega_{r0} + \Omega_{\Lambda 0}x^4 + (1-\Omega_0)x^2\right]^{1/2}}$$

$$where\ \Omega_0 = \Omega_{m0} + \Omega_{r0} + \Omega_{\Lambda 0}$$

$$Today: H_0^{-1} = 9.8\times 10^9\ h^{-1}\ years; h = 0.7$$

H_0^{-1} – –Hubble time

Useful estimate for age of universe

Ω_{m0}	Ω_{r0}	$\Omega_{\Lambda 0}$	t_0
1	0	0	9.4 Gyr
0.3	10^{-5}	0.7	13.4 Gyr
Open			
0.2	10^{-5}	0.2	12.4 Gyr
0.2	10^{-5}	0.6	13.96 Gyr
Closed			
0.3	10^{-5}	0.8	13.96 Gyr
0.4	10^{-5}	0.9	13.6 Gyr

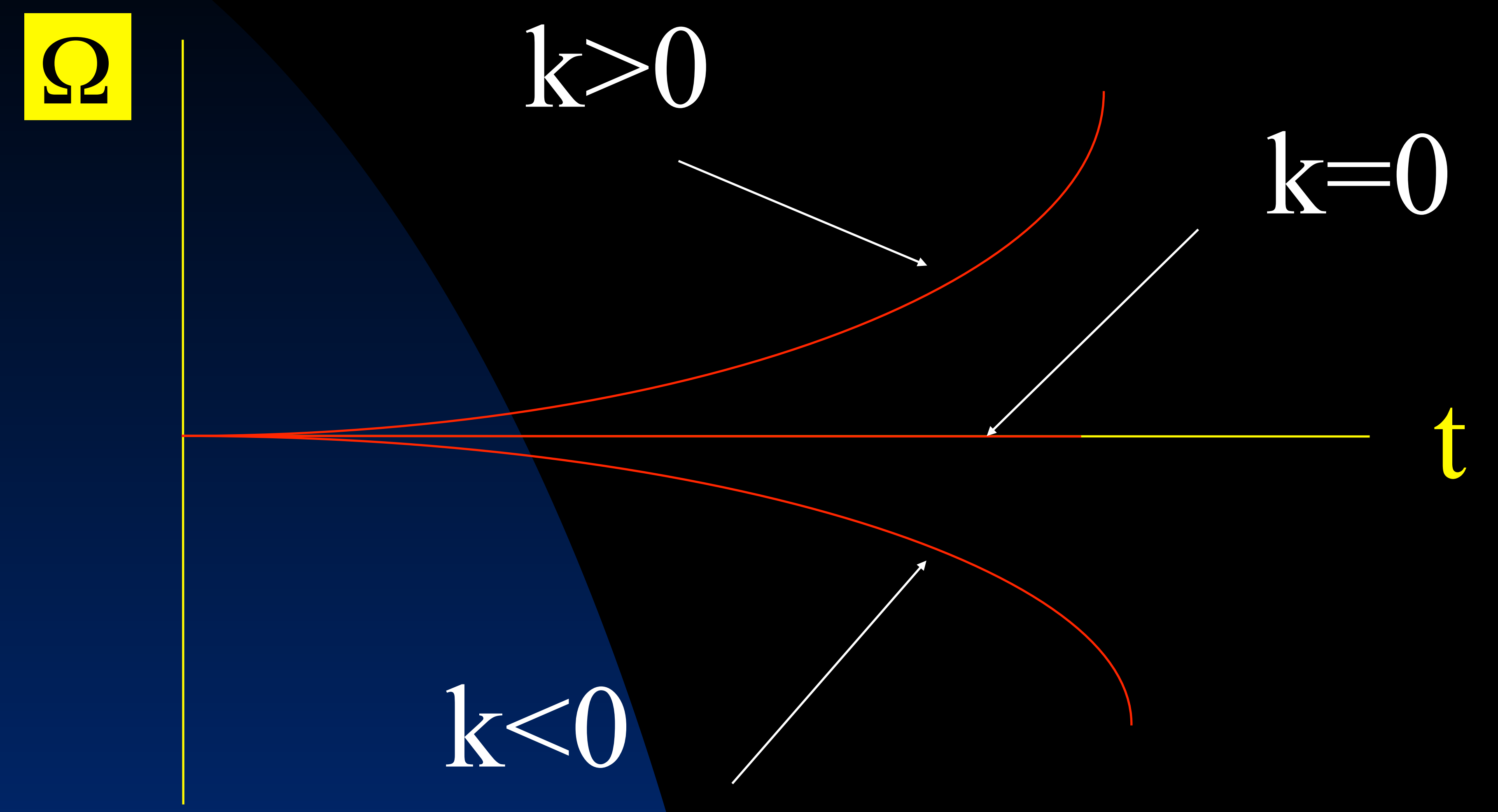
History of the Universe

10^{18} GeV	10^{-43} sec	10^{32} K	QG/String epoch (?)
			Inflation begins (?)
10^{3} GeV	10^{-10} sec	10^{15} K	Electroweak tran
1 GeV	10^{-4} sec	10^{12} K	Quark-Hadron tran
1 MeV	1 sec	10^{10} K	Nucleosynthesis
1 eV	10^{4} years	10^{4} K	Matter-rad equality
	10^{5} years	3.10^{3} K	Decoupling → microwave bgd.
10^{-3} eV	10^{10} years	3K	Present epoch with DE

The Big Bang – issues.

- Flatness problem – observed almost spatially flat cosmology requires fine tuning of initial conditions.
- Horizon problem -- isotropic distribution of CMB over whole sky appears to involve regions that were not in causal contact when CMB produced. How come it is so smooth?
- Monopole problem - where are all the massive defects which should be produced during GUT scale phase transitions.
- Relative abundance of matter – does not predict ratio baryons: radiation: dark matter.
- Origin of the Universe – simply assumes expanding initial conditions.
- Origin of structure in the Universe from initial conditions homogeneous and isotropic.
- The cosmological constant problem.

Flatness problem



Today: $\Omega < 1.1$

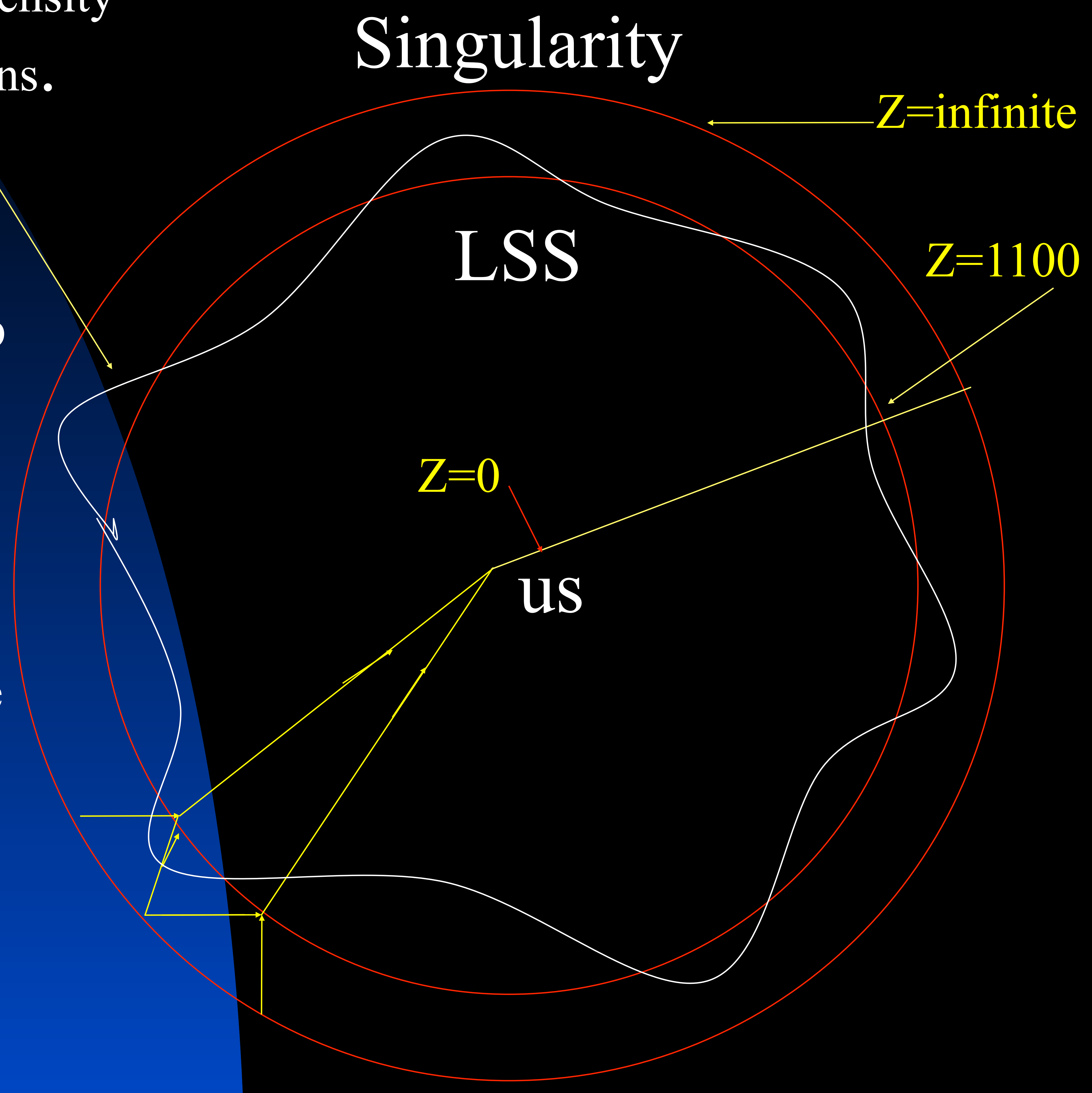
Why?

$$\Omega^{-1} = 1 - \frac{3k}{\kappa^2 \rho a^2} \propto \begin{cases} a & \text{mat} \\ a^2 & \text{rad} \\ a^{-2} & \Lambda \end{cases}$$

$$|\Omega(1s) - 1| = O(10^{-16})$$

Horizon problem

Primordial density fluctuations.



CMB photons emitted from opp sides of sky are in thermal equilibrium at same temp – but no time for them to interact before photons were emitted because of finite horizon size.

CMBR last interacted at 1+Z = 1100

300,000 yrs after big bang

Hubble radius was 2 degrees, 200 Mpc

LSS thickness – 15Mpc

Any region separated by > 2 deg – causally separated at decoupling.

Monopole problem

Monopoles are generic prediction of GUT type models.

They are massive stable objects, like domain walls and cosmic strings and many moduli fields.

They scale like cold dark matter, so in the early universe would rapidly come to dominate the energy density.

Must find a mechanism to dilute them or avoid forming them.

Some of the big questions in cosmology today

a) What is dark matter? -- 25% of the energy density

b) What is dark energy? -- 70% of the energy density. Does dark energy interact with other stuff in the universe?

c) Is dark energy really a new energy form or does the accelerating universe signal a modification of our theory of gravity?

d) What is the origin of the density perturbations, giving rise to structures?

e) Where is the cosmological gravitational wave background?

f) Are the fluctuations described by Gaussian statistics? If there are deviations from Gaussianity, where do they come from?

g) How many dimensions are there? Why do we observe only three spatial dimensions?

h) Was there really a big bang (i.e. a spacetime singularity)? If not, what was there before?

Enter Inflation

A period of accelerated expansion in the early Universe

Small smooth and coherent patch of Universe size less than (1/H) grows to size greater than the comoving volume that becomes entire observable Universe today.

Explains the homogeneity and spatial flatness of the Universe

and also explains why no massive relic particles predicted in say GUT theories

Leading way to explain observed inhomogeneities in the Universe

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3} G\,(\rho + 3p) - - - Accn$$

$$If\ \ \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

What is Inflation?

Any epoch of the Universe's evolution during which the comoving Hubble length is decreasing. It corresponds to any epoch during which the Universe has accelerated expansion.

$$\frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0 \leftrightarrow \ddot{a} > 0$$

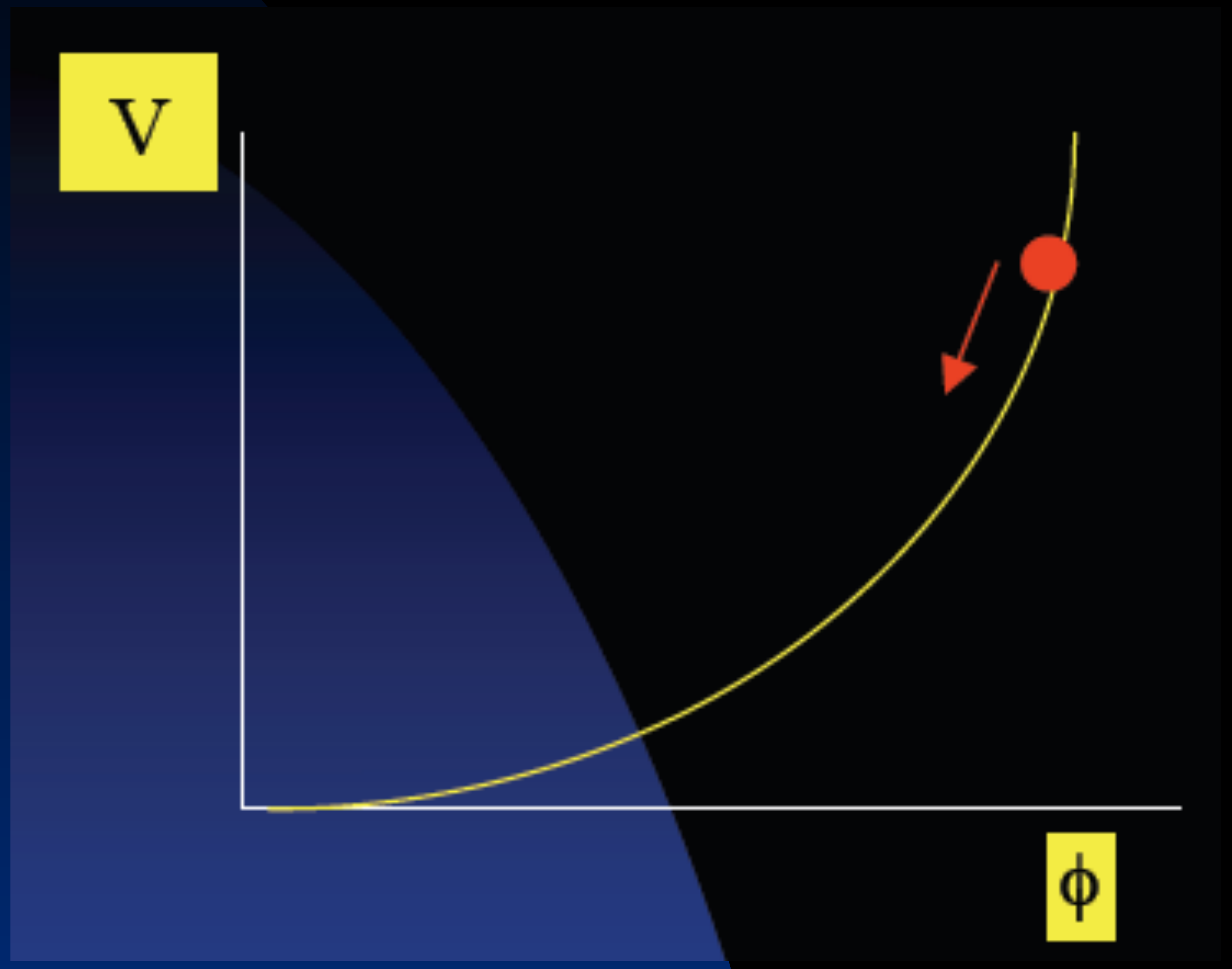
$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3}G(\rho + 3p) --- Accn$$

$$If\ \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

For inflation require material with negative pressure. Not many examples. One is a scalar field!

Intro fundamental scalar field -- like Higgs

If Universe is dominated by the potential of the field, it will accelerate!

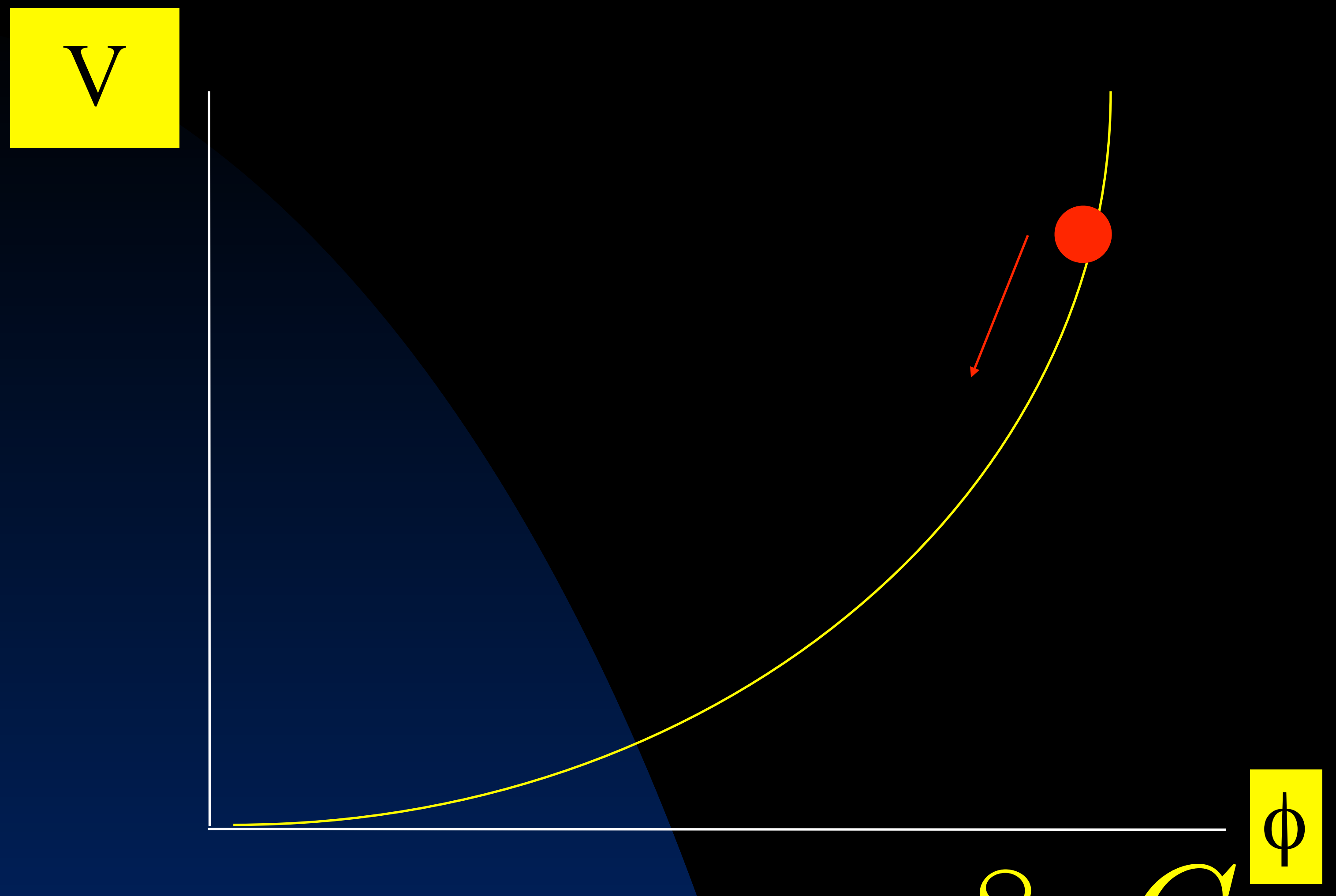


$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

We aim to constrain potential from observations.

During inflation as field slowly rolls down its potential, it undergoes quantum fluctuations which are imprinted in the Universe. Also leads to gravitational wave production.

Examples of inflation



Simplest case – homogeneous single scalar field

$$\rho_\phi = V(\phi) + \frac{\dot{\phi}^2}{2} \quad ; \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$$

EoM $$H^2 = \frac{8\pi G}{3}\rho_\phi \quad ; \quad \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Inflation $\ddot{a} > 0 \leftrightarrow (\rho + 3p) < 0 \leftrightarrow \dot{\phi}^2 \ll V(\phi)$ Slow roll approx

$$\Rightarrow \quad H^2 = \frac{8\pi G}{3}V(\phi) \quad ; \quad 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Also: $$\dot{H} = -4\pi G\dot{\phi}^2,$$

So, define a quantity which specifies how fast H changes during inflation

Prediction -- potential determines important quantities

Slow roll parameters [Liddle & Lyth 1992]

$$\epsilon = \frac{1}{16\pi G}\left[\frac{V'(\phi)}{V(\phi)}\right]^2$$

$$\eta = \frac{1}{8\pi G}\left[\frac{V''(\phi)}{V(\phi)}\right]$$

Slow roll inflation occurs when both of these slow roll conditions are << 1

End of inflation corresponds to ε=1

How much does the universe expand? Given by number of e-folds

$$N \equiv \ln\left(\frac{a(t_{\rm end})}{a(t_i)}\right) = \int_{t_i}^{t_e} H dt \simeq -\int_{\phi_i}^{\phi_e} \frac{V}{V'} d\phi$$

Last expression is true in the slow roll limit (for single field inflation).

Number of e-folds required

Solve say the Flatness problem:

Assume inflation until tend = 10^{-34} sec

Assume immediate radn dom until today, $t_0 = 10^{17}$ sec

Assume $|\Omega(t_0) - 1| \leq 0.01$

Now $|\Omega - 1| = \frac{k}{a^2 H^2}$; RD : $|\Omega - 1| \propto t$

$$|\Omega(10^{-34}\,s) - 1| \leq 0.01 * 10^{-34} * 10^{-17} \leq 10^{-54}$$

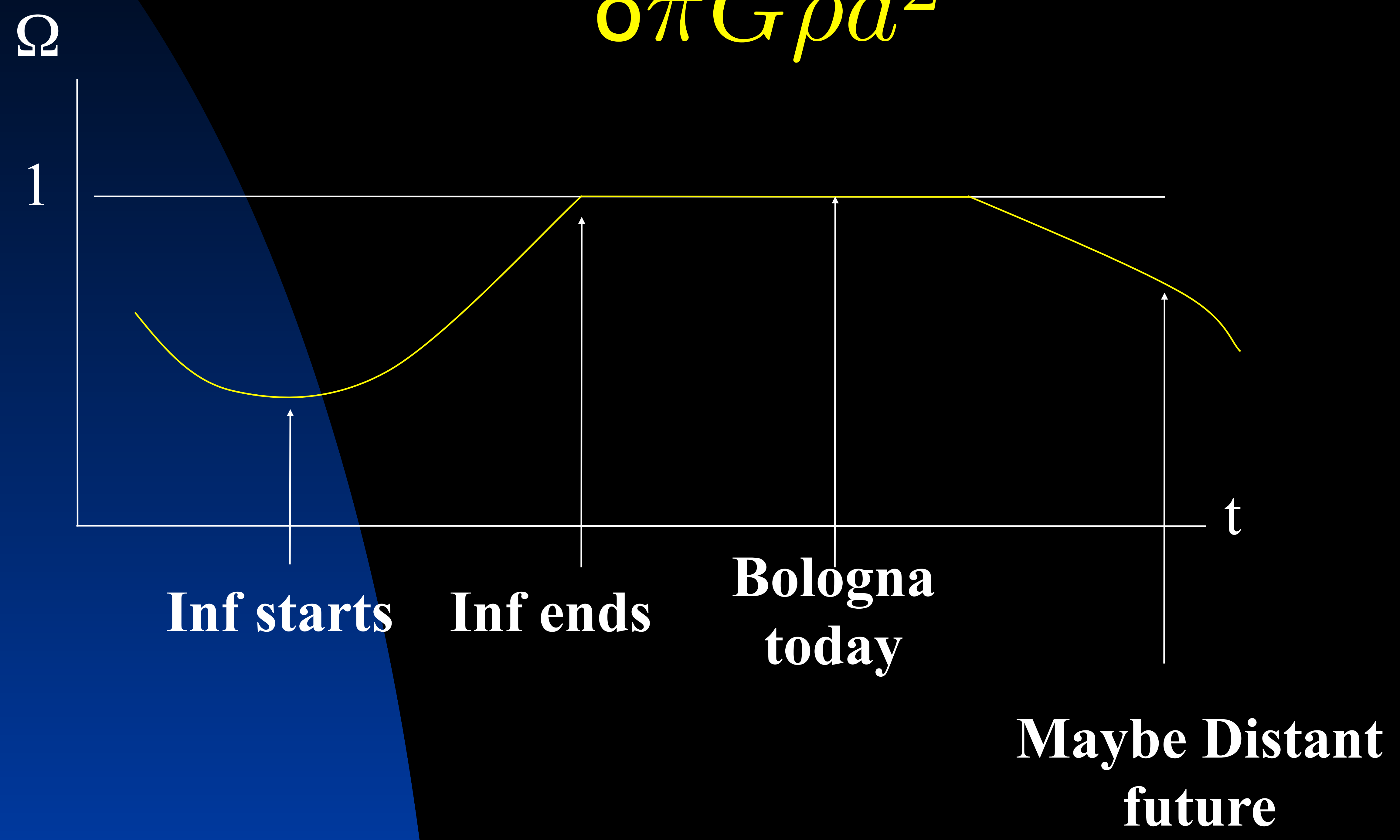
Inf $|\Omega - 1| \propto 1/a^2$ ⟹ $\frac{|\Omega_{end} - 1|}{|\Omega_{ini} - 1|} = a_i^2 / a_e^2 = 10^{-54}$

⟹ $N = \ln\left[\frac{a_{tend}}{a_{tini}}\right] \approx 62$

Solving the big bang problems

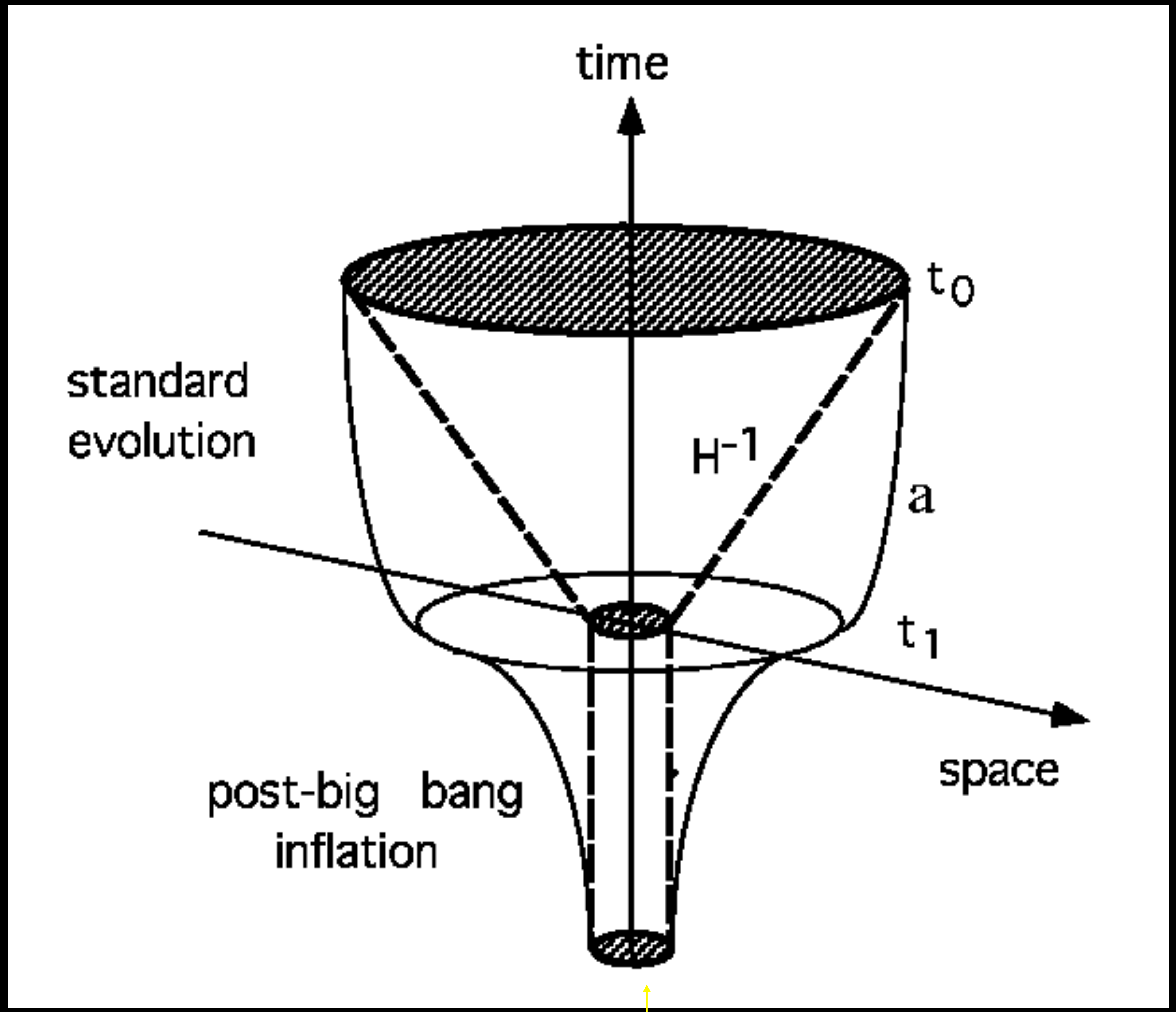
1. Flatness

$$\Omega^{-1} - 1 = -\frac{3k}{8\pi G\rho a^2} \propto a^{-2} \longrightarrow \exp(-2Ht)$$



2. Horizon problem:

Physical: H^{-1} const during inflation. Small initial patch can inflate. How likely is that? Question of initial conditions.



Initial causally connected region

3. Monopole problem: $\rho_{mon} \propto a^{-3} \rightarrow 0$ rapidly during inflation

Everything infact diluted away except for the inflaton field itself.

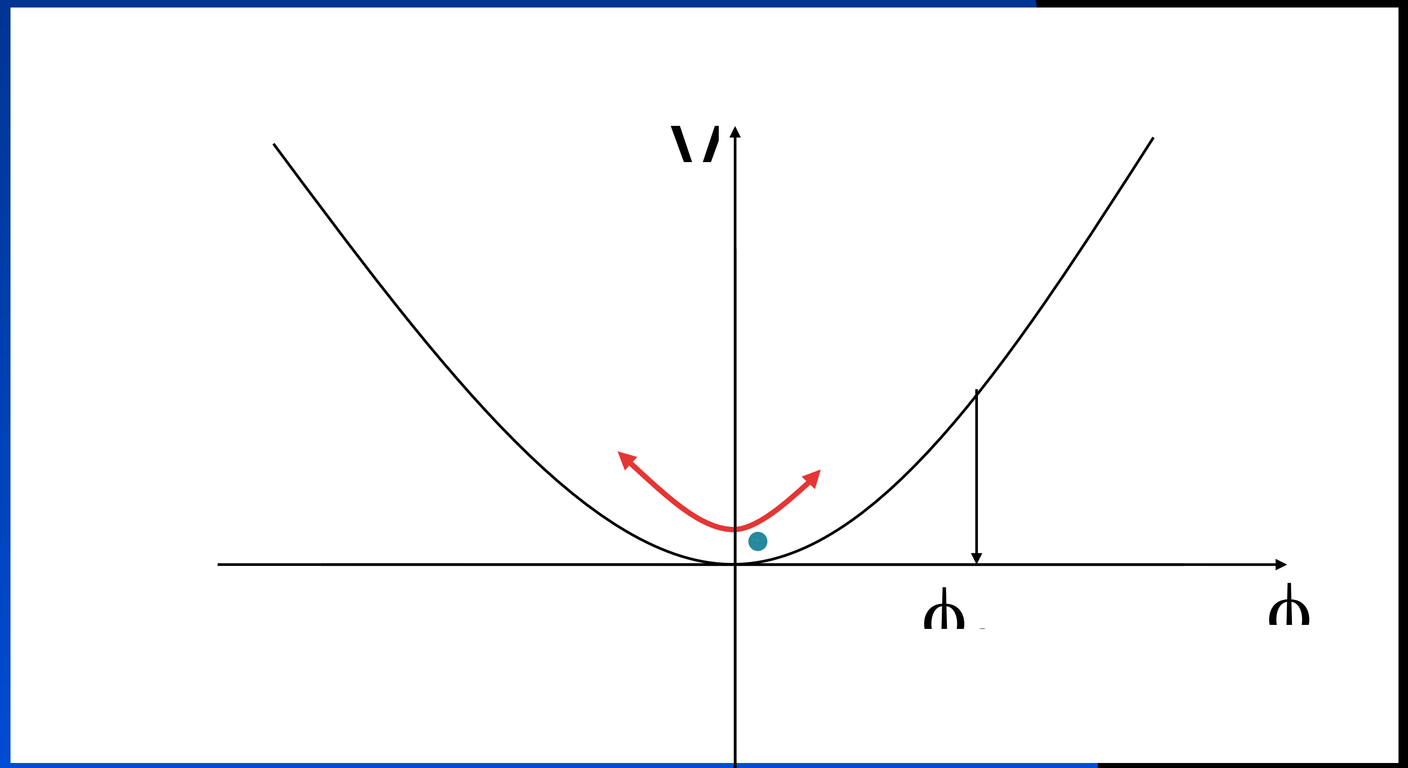
$T \propto a^{-1} \rightarrow 0$ rapidly during inflation

Hence need to reheat the universe at end of inflation

End of inflation

- Eventually SRA breaks down, as inflaton rolls to minima of its potential.

Experimental test of slow roll approximation – Aspen 2002



- Leaves a cold empty Universe apart from inflaton.
- Inflation has to end and the energy density of the inflaton field decays into particles. This is reheating and happens as the field oscillates around the minimum of the potential

End of inflation.

•Inflaton is coupled to other matter fields and as it rolls down to the minima it produces particles –perturbatively or through parametric resonance where the field produces many particles in a few oscillations.

•Dramatic consequences. Universe reheats, can restore previously broken symmetries, create defects again, lead to Higgs windings and sphaleron effects, generation of baryon asymmetry at ewk scale at end of a period of inflation.

•Important constraints: e.g.: gravitino production means : $T_{rh} < 10^9$ GeV -- often a problem!

More on this soon

The origins of perturbations -- the most important aspect of inflation

Idea: Inflaton field is subject to perturbations (quantum and thermal fluctuations). Those are stretched to superhorizon scales, where they become classical. They induce metric perturbations which in turn later become the first perturbations to seed the structures in the universe.

Also predict a cosmological gravitational wave background.

During inf $\phi(\underline{x}, t) = \phi_0(t) + \delta\phi(\underline{x}, t)$ ← Quantum fluc

Fourier modes: $\delta\phi(\underline{x}, t) = \sum_k \delta\phi_k(t)\, e^{ikx}$ → Generates fluc in matter and metric

$\delta_H^2(k)$ Scalar pertn – spectra of gaussian adiabatic density pertns generated by flucns in the scalar field and spacetime metric. Responsible for structure formation.

$h_{\mu\nu}$ $A_G(k)$ Tensor pertn in metric– gravitational waves.

Key features

During inflation comoving Hubble length (1/aH) decreases.

So, a given comoving scale can start inside (1/aH), be affected by causal physics, then later leave (1/aH) with the pertns generated being imprinted.

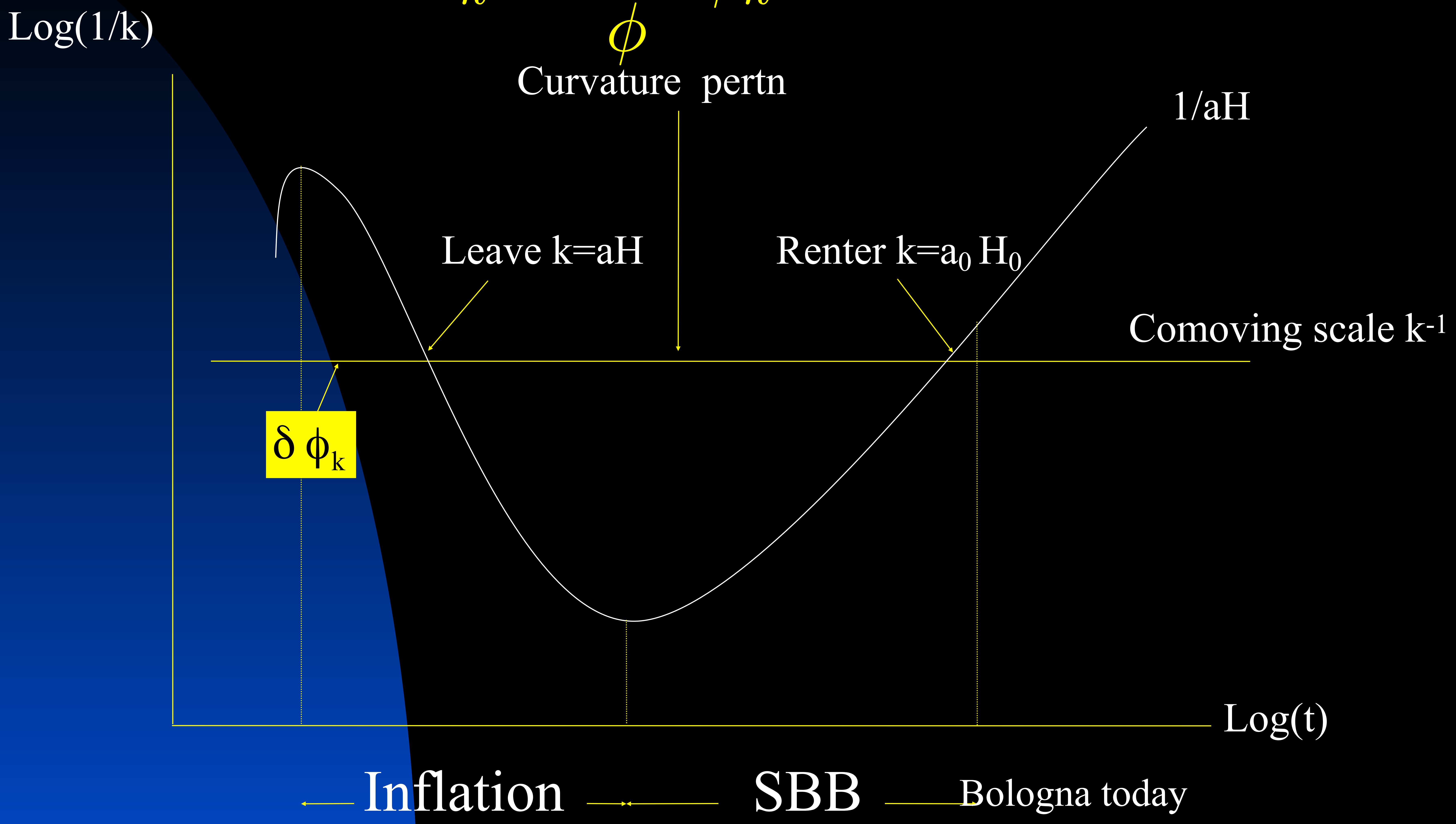
Quantum flucns in inflaton arise from uncertainty principle.

Pertns are created on wide range of scales and generated causally.

Size of irregularities depend on energy scale at which inflation occurs.

Pertn created causally, stretched by expansion.

$$\mathcal{R}_k = \frac{H}{\dot{\phi}} \delta\phi_k \simeq \text{const}$$



The power spectra

Focus on statistical measures of clustering.

Inflation predicts the amp of waves of a given k which obey gaussian statistics, the amplitude of each wave is chosen independently and randomly from its gaussian distribution. It predicts how the amplitude varies with scale — the power spectrum

Good approx -- power spectra as being power-laws with scale.

Density pertn

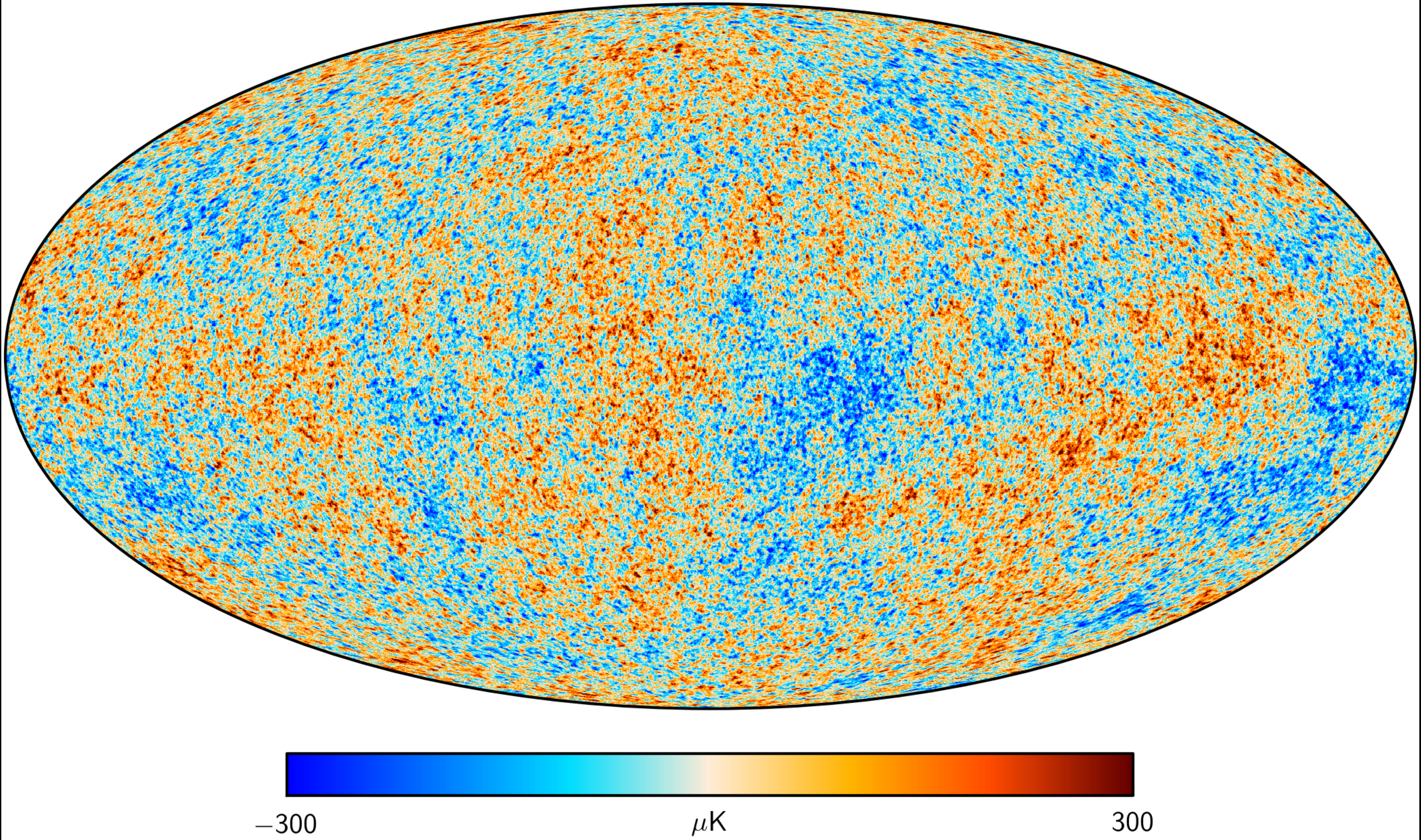
$$\delta_H^2(k) = \delta_H^2(k_0)\left[\frac{k}{k_0}\right]^{n-1}$$

Grav waves

$$A_G^2(k) = A_G^2(k_0)\left[\frac{k}{k_0}\right]^{n_G}$$

Four parameters

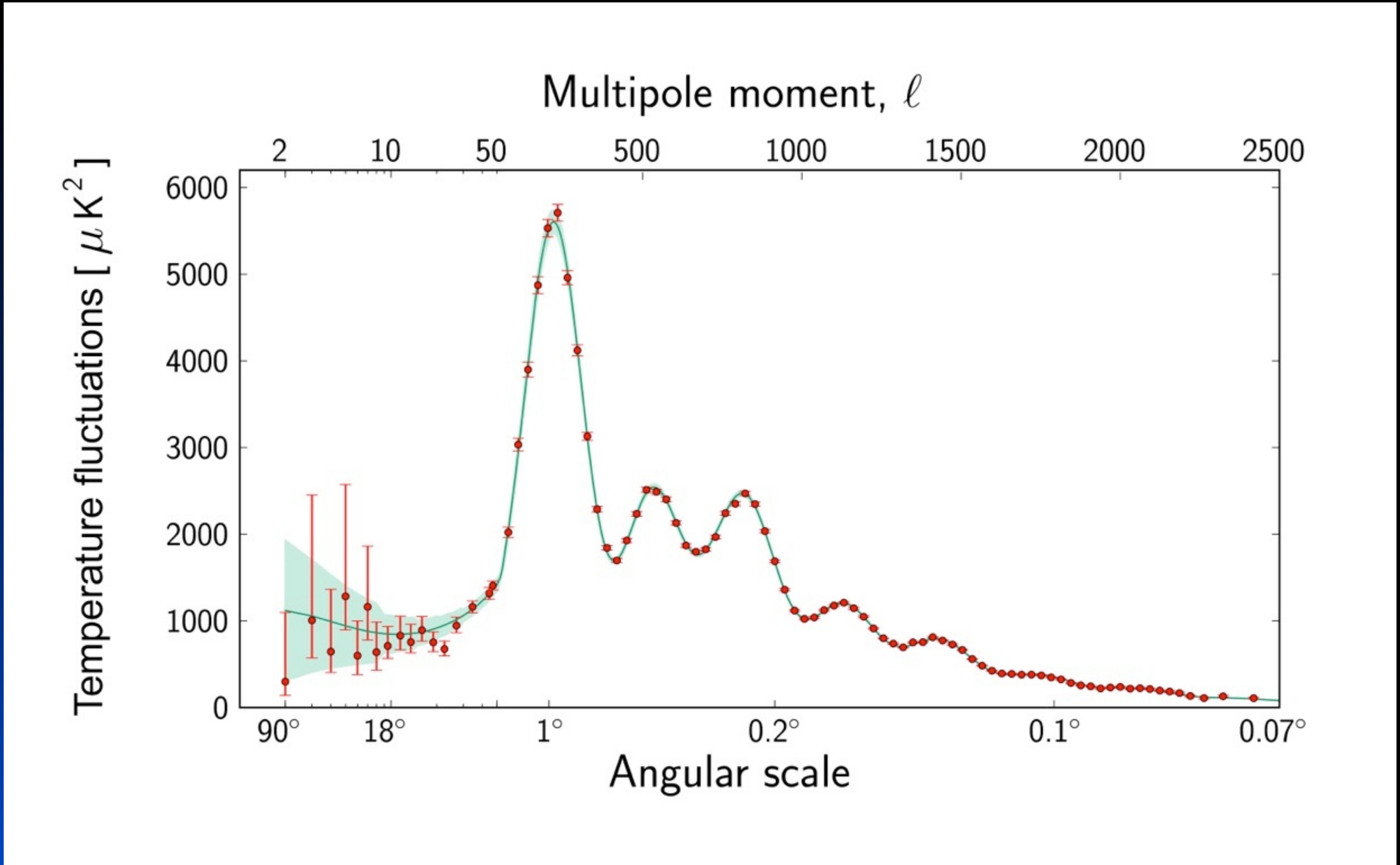
Planck- wow - minuscule temperature changes across the observable universe !



•Improvement over WMAP: ang resolution (x2.5), sensitvity (x10), freq coverage [9 bands (30-857 GHz) v 5 bands (23-90 GHz)]

Planck 2018 - ESA

Power spectrum - LCDM fit



Inflation seems to fit the data really well - the dots are the data, the solid line is the theory — although there are a few issues

Some formulae

Power spectra $P_\phi(k) = \frac{k^3}{2\pi^2}\left\langle |\delta\phi_k|^2 \right\rangle$

Vacuum soln $\left\langle |\delta\phi_k|^2 \right\rangle = \frac{H^2}{2k^3}$ → $P_\phi(k) = \left|\frac{H}{2\pi}\right|^2 \Bigg|_{k=aH(Exit)}$

Amp of density pertn $$\delta_H^2(k) = \frac{4}{25}\left(\frac{H}{\dot\phi}\right)^2\left(\frac{H}{2\pi}\right)^2_{k=aH}$$

SRA $\delta_H(k) \propto \kappa^{3/2}\left.\frac{V^{3/2}}{|V'|}\right]_{k=aH}$

WMAP: 60 efolds before tend → $\delta_H(k) \approx 1.91 * 10^{-5}$

→ $V^{1/4}/\varepsilon \leq 10^{16}$ GeV -- Lyth

In other words the properties of the inflationary potential are constrained by the CMB

Tensor pertns : amp of grav waves.

$$A_G(k) \propto \kappa^2 V^{1/2} \Big]_{k=aH}$$

Note: Amp of perts depends on form of potential. Tensor pertns gives info directly on potential but difficult to detect.

Observational consequences.

Precision CMBR expts like WMAP and Planck → probing spectra.

Standard approx – power law.

$$\delta_H^2(k) \propto k^{n-1} \; ; \; A_G^2(k) \propto k^{n_G}$$

$$n - 1 = \frac{d \ln \delta_H^2}{d \ln k} \; ; \; n_G = \frac{d \ln A_G^2}{d \ln k}$$

Power law ok, only a limited range of scales are observable.

For range 1Mpc → 10^4 Mpc : $\Delta \ln k \approx 9$

Crucial eqn $\frac{d \ln k}{d \phi} = \kappa \frac{V}{V'}$ ⟹ $n = 1 - 6\varepsilon + 2\eta \; ; \; n_G = -2\varepsilon$

n=1 ; n_G=0 – Harrison Zeldovich

CMBR → Measure relative importance of density pertns and grav waves.

$$R = \frac{C_2^{GW}}{C_2^{S}} \approx 4\pi\varepsilon$$

$$\text{where } \frac{\Delta T}{T} = \sum a_{lm} Y_m^l(\theta,\varphi),\; C_l = \left\langle |a_{lm}|^2 \right\rangle$$

C_l -- radiation angular power spectrum.

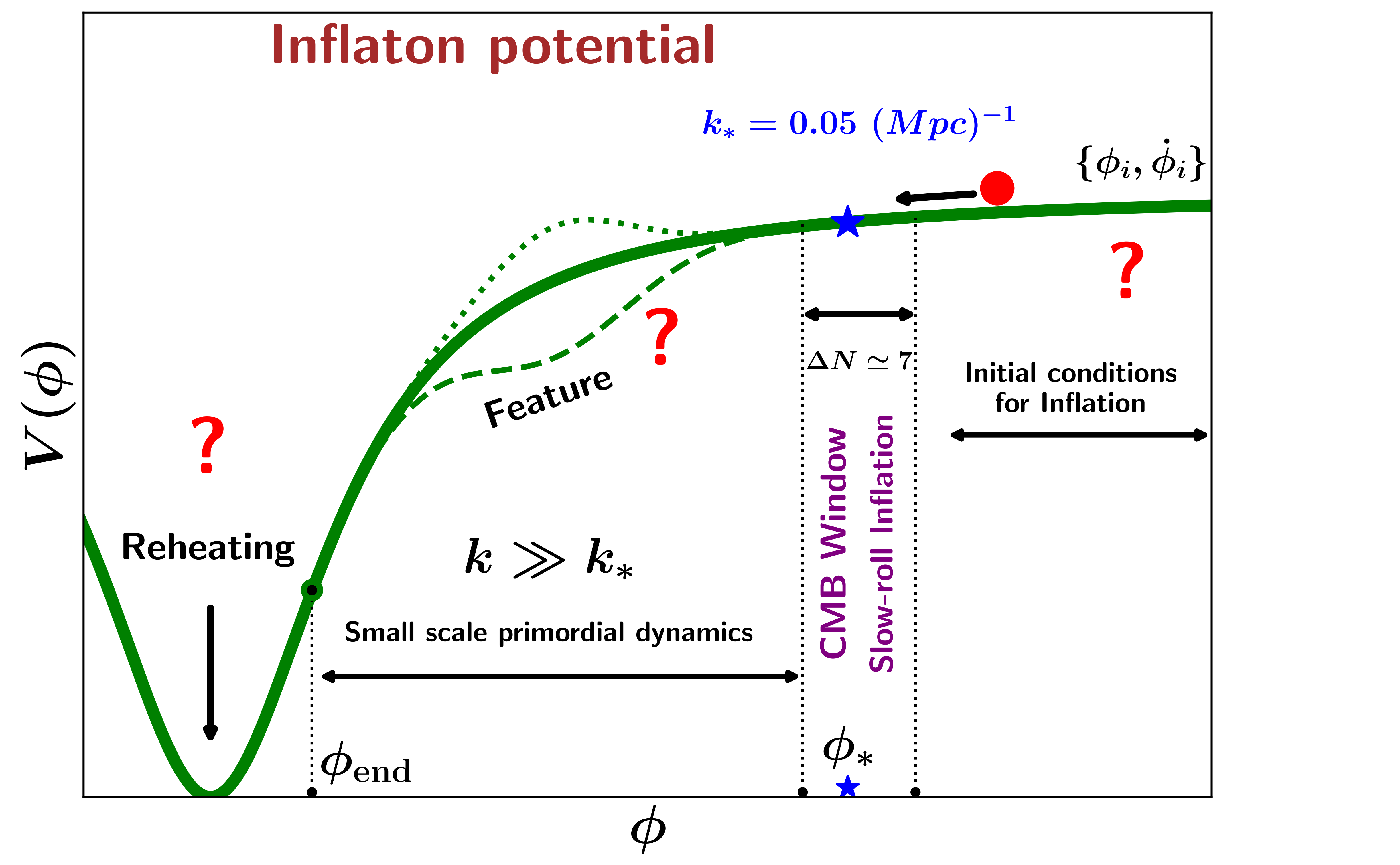
A unique test of inflation

$$R = -2\,\pi\,n_G$$

Indep of choice of inf model, relies on slow roll and power law approx. Unfortunately n_G too small for detection !

This is where the Bicep2 excitement was !

Inflation - brief recap



$$S[g_{\mu\nu},\phi] = \int d^4x\, \sqrt{-g}\, \left[\frac{m_p^2}{2} R - \frac{1}{2}\partial_\mu\phi\,\partial_\nu\phi\, g^{\mu\nu} - V(\phi)\right]$$

Einstein's equations assuming scalar field dominates the energy density

$$H^2 \equiv \frac{1}{3m_p^2}\rho_\phi = \frac{1}{3m_p^2}\left[\frac{1}{2}\dot\phi^2 + V(\phi)\right]$$

$$\dot H \equiv \frac{\ddot a}{a} - H^2 = -\frac{1}{2m_p^2}\dot\phi^2\,,$$

$$\ddot\phi + 3H\dot\phi + V_{,\phi}(\phi) = 0\,.$$

Credit: Swagat Mishra

Inflation can occur when potential dominated $\dot\phi^2 < V(\phi)$

When $\dot\phi^2 \ll V(\phi)$ with nearly flat potential dominating we obtain nearly exponential expansion at the background level

$$a \sim e^{Ht}$$

Inflation - produces the initial seeds for structure to grow through Quantum Fluctuations

Action for gravity plus inflaton

$$S[g_{\mu\nu},\phi] = \int \mathrm{d}^4x \sqrt{-g}\left(\frac{m_p^2}{2}R - \frac{1}{2}\partial_\mu\phi\,\partial_\nu\phi\,g^{\mu\nu} - V(\phi) + \ldots\right)$$

Metric including fluctuations

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\left(e^{2\Psi(t,\vec{x})}\,\delta_{ij} + h_{ij}(t,\vec{x})\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$

When mass of inflaton is small compared to Hubble rate : m<<H

Comoving curvature perturbation exists -
will become density and temperature fluctuations

$$-\zeta(t,\vec{x}) = \Psi + \frac{H}{\dot{\phi}}\frac{\delta\phi}{m_p}$$

Tensor perturbations which will become relic gravitational waves

$$h_{ij}(t,\vec{x})$$

Inflation - allows us to predict the form of the fluctuations for a given model

In particular during slow roll inflation, where the potential is flat enough and dominates the energy density

We have

$$\dot{\phi}^2 \ll V(\phi) \text{ and } \ddot{\phi} \ll V'(\phi)$$

We quantify the power spectrum and deviations from scale invariance in terms of slow roll parameters

$$\Rightarrow \boxed{\epsilon_H,\ |\eta_H| \ll 1} \quad \text{where} \quad \boxed{\epsilon_H = \frac{\dot{\phi}^2}{2m_p^2 H^2}, \quad \eta_H = \frac{-\ddot{\phi}}{H\dot{\phi}}}$$

The Power Spectrum for scalar and tensor fluctuations on large scales

$$\mathcal{P}_\zeta = \frac{1}{8\pi^2}\left(\frac{H}{m_p}\right)^2 \frac{1}{\epsilon_H} = A_S\left(\frac{k}{k_*}\right)^{n_S-1} \qquad \mathcal{P}_\mathcal{T} = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2 = A_\mathcal{T}\left(\frac{k}{k_*}\right)^{n_T}$$

Slow roll predictions:

$$n_S-1 = -4\epsilon_H + 2\eta_H, \qquad n_\tau = -2\epsilon_H, \qquad r \equiv \frac{A_\tau}{A_S} = 16\epsilon_{H*}$$

CMB observations:

$$A_s = 2.1 \times 10^{-9}$$

BICEP/Keck 2024:

$$A_\mathcal{T} \leq 3.6\%\, A_S$$

Scalar Spectral index: Red tilt

$n_S-1 \approx -0.033$

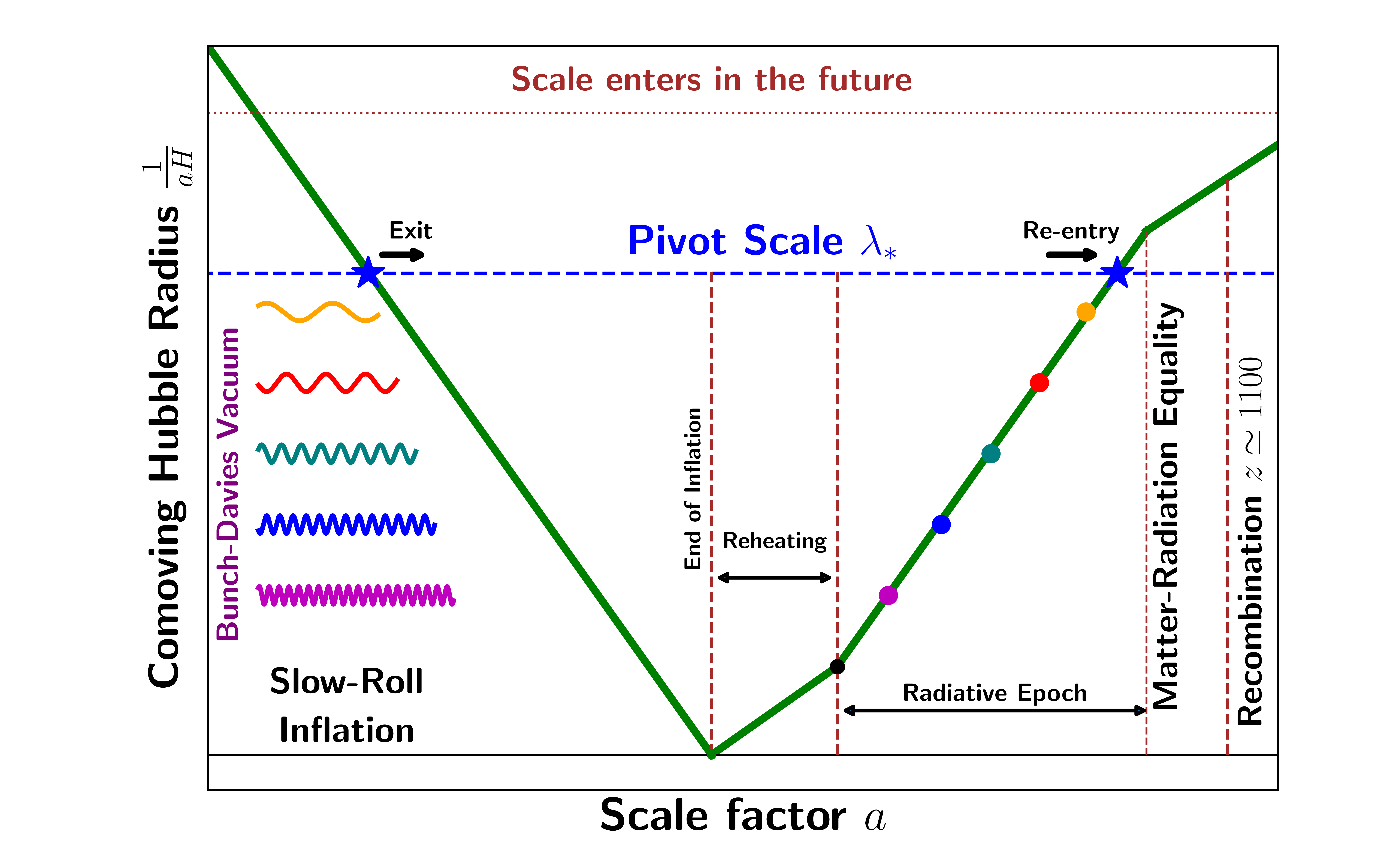
Tensor spectra index:

$|n_\tau| \lesssim 0.0045$

Prediction is nearly scale invariant and are very small on large scales

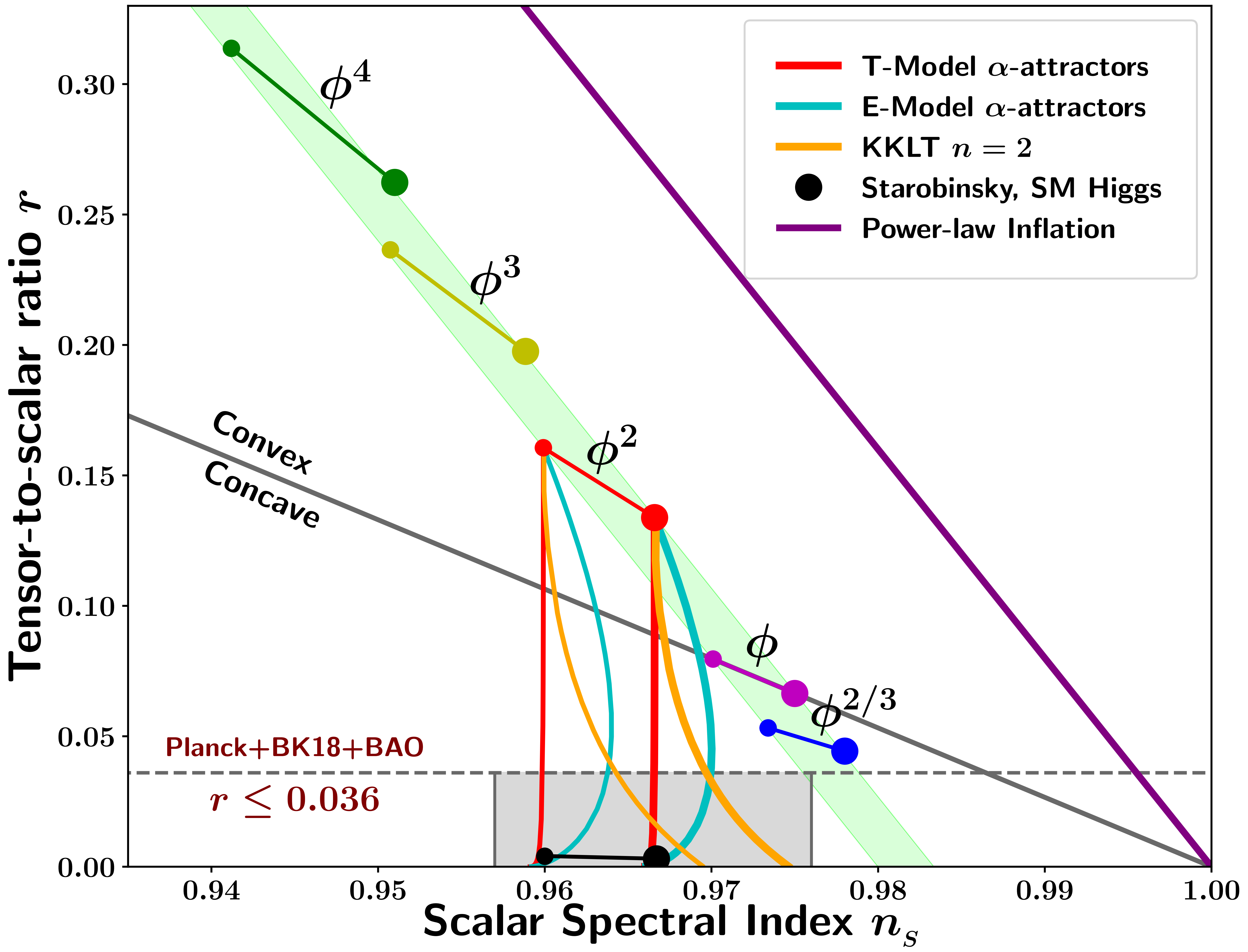
Implies $\epsilon_H < 0.002$ and $\eta_H > 0.01$ — we have a new hierarchy emerging - has implications for $V(\phi)$!

Pictorially may help



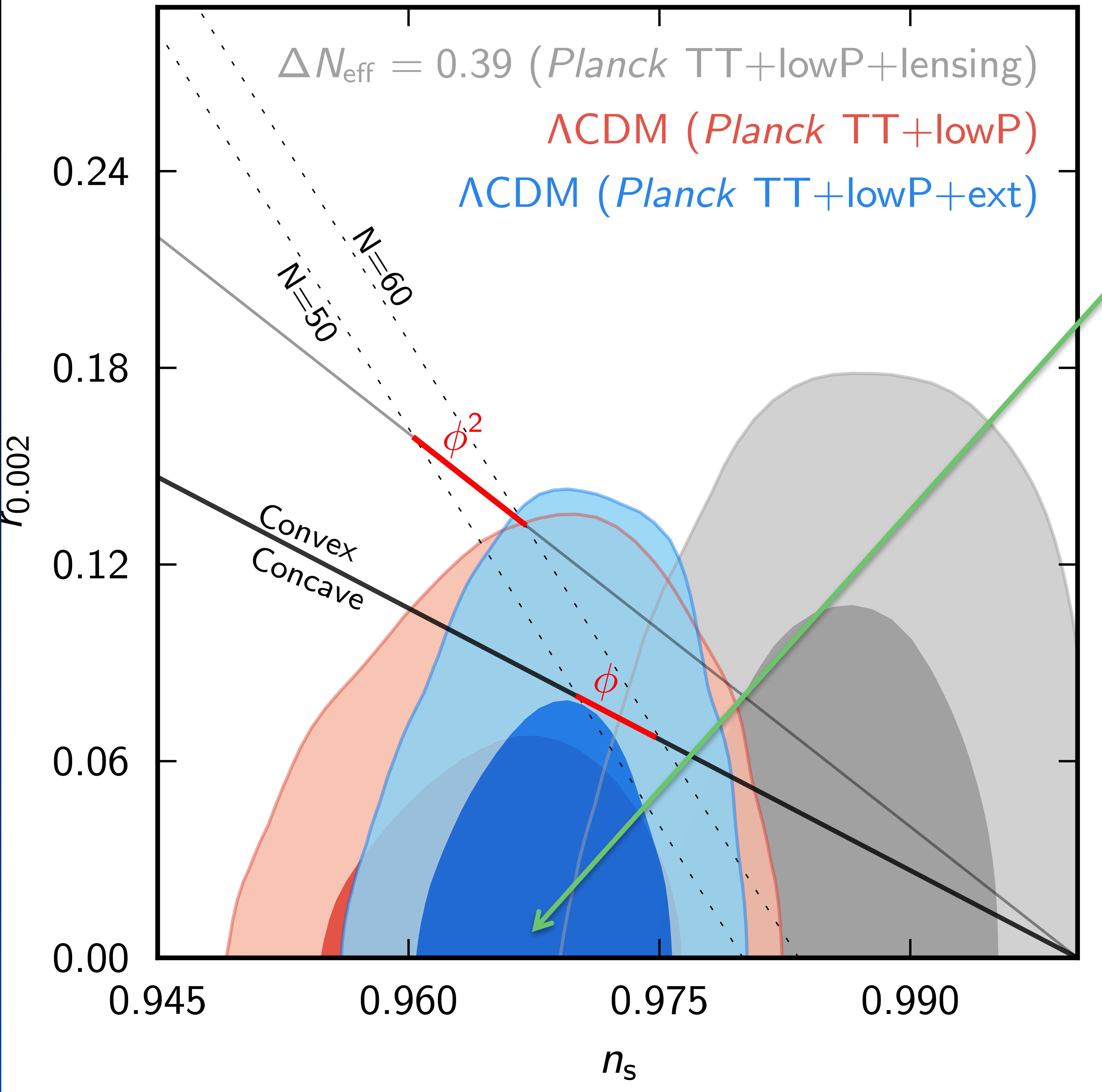
Credit: Swagat Mishra

The main way we constrain models of inflation from observation



Credit: Swagat Mishra

Real progress - compare with Planck collaboration 2014 - preliminary



preliminary

Starobinsky (R^2) inflation

$n_s \approx 1 - 2/N \approx 0.967$

$r \approx 12/N^2 \approx 0.0033$

$dn_s/d\ln k \approx -2/N^2 \approx -0.0006$

…… but, there is plenty of room at the top

(and to the side!)

Credit: Adam Moss 2013

Lecture 2

Inflation model building today -- big industry

Inflation and PBHs

Inflation and Reheating

Inflation and Cosmic Superstrings

First of all an extra slide from the soft skills course

Keep going to the final whistle blows ! Congratulations Italy !

Inflation model building today -- big industry

Multi-field inflation

Inflation in string theory and braneworlds

Inflation in extensions of the standard model

Cosmic strings formed at the end of inflation

The idea is clear though:

Use a combination of data (CMB, LSS, SN, BAO ...) to try and constrain models of the early universe through to models explaining the nature of dark energy today.

Loads of models — 300 models analysed with CMB and BAO data, 40% disfavoured, 20% favoured according to Jeffrey's scale of Bayesian evidence [Martin et al 2024]

Over 7,700 papers written with title Inflation in title !

Some examples keep it simple to get the idea – Chaotic Inflation

$$V(\phi) = \frac{m^2\phi^2}{2}$$ with $$\varepsilon = \frac{1}{2\kappa^2}\left[\frac{V'}{V}\right]^2 \quad ; \quad \eta = \frac{1}{\kappa^2}\left[\frac{V''}{V}\right]$$

Find: $$\varepsilon = \frac{2}{\kappa^2\phi^2} = \eta$$

SRA: $$H^2 = \frac{8\pi G}{3}V(\phi) \quad ; \quad 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Inf soln: $$\phi(t) = \phi_i - \frac{\sqrt{2}mt}{\sqrt{3}\kappa} \quad ;$$

$$a(t) = a_i \exp\left[\frac{\kappa m}{\sqrt{6}}\left(\phi_i t - \frac{mt^2}{\sqrt{6}\kappa}\right)\right]$$

End of inflation: $\varepsilon = 1 \Rightarrow \phi_e = \frac{\sqrt{2}}{\kappa}$

Num of e-folds: $N(\phi) = -\kappa^2 \int_{\phi}^{\phi_e} \frac{V}{V'} d\phi = \frac{\kappa^2 \phi^2}{4} - \frac{1}{2}$

N=60: $\phi_{60} \approx \frac{16}{\kappa} > \phi_e$

Scale just entering Hubble radius today, COBE/WMAP/Planck scale

Amp of den pertn: $\delta_H(k) = \frac{\kappa^3}{\sqrt{75\pi}} \frac{V^{3/2}}{|V'|}\Bigg]_{k=aH}$

Take to be 60 efolds before end of inflation.

Find: $\delta_H(k) = 12m\sqrt{G} \quad \text{where} \quad \kappa^2 \equiv 8\pi G$

Amp of grav waves: $A_G(k) = \sqrt{\frac{32}{75}} \, G V^{1/2} \Big]_{k=aH}$ 60 efolds before end of inflation.

Find: $A_G(k) \approx 1.4 m \sqrt{G}$

Normalise to Planck: $\delta_H(k) \approx 1.91 * 10^{-5}$

Find: $m = 2 * 10^{13}$ GeV **Constraint on inflaton mass!**

Spectral indices $n = 1 - 6\varepsilon + 2\eta \, ; \, n_G = -2\varepsilon$ Slow roll

Use values 60 e-folds before end of inflation.

$n = 0.97; \, n_G = -0.016$ Close to scale inv - totally ruled out !

2. Models of Inflation—variety is the spice of life.

(where is the inflaton in particle physics?)

(Lyth and Riotto, Phys. Rep. 314, 1, (1998), Lyth and Liddle (2009), Martin et al (2024))

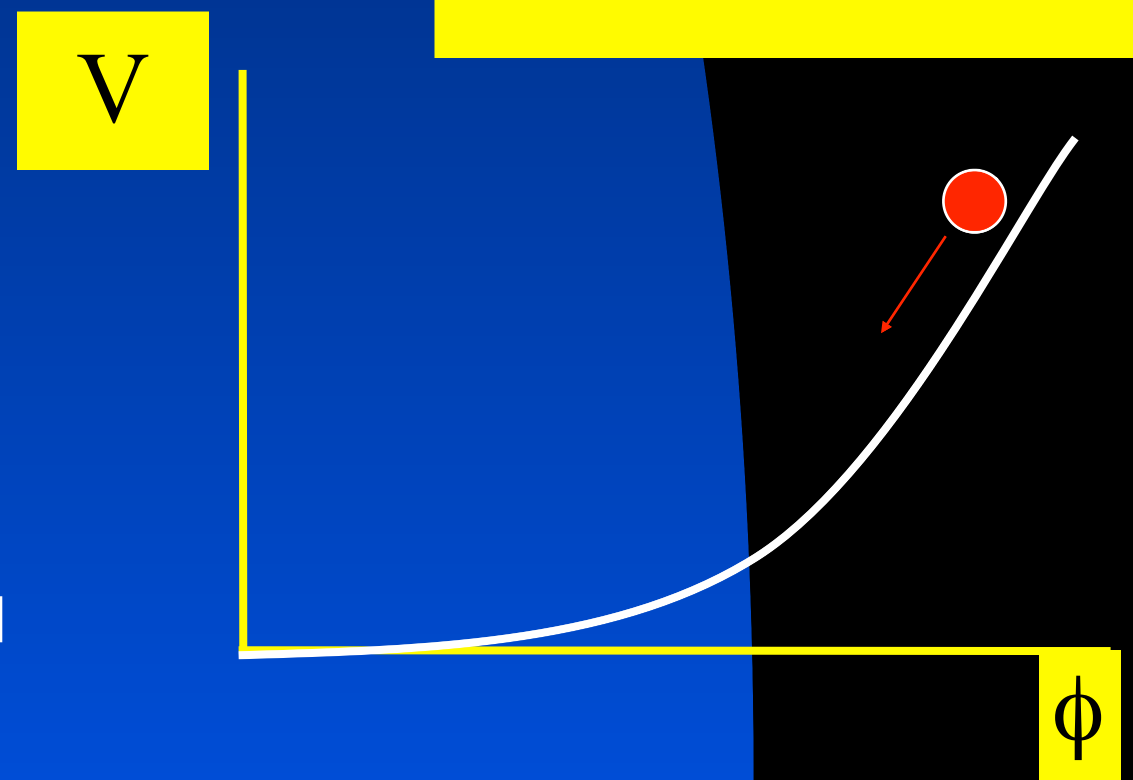
Field theory: $V(\phi) = V_0 + \frac{1}{2} m^2 \phi^2 + M\phi^3 + \lambda\phi^4 + \sum_{d=5}^{\infty} \lambda_d M_P^{4-d} \phi^d$

Quantum corrections give coefficients proportional to $\ln(\phi)$
and an additional term proportional to $\ln(\phi)$

1. Chaotic inflation .

$V(\phi) \propto \phi^p;\ \phi >> M_P;\ n - 1 = -(2+p)/2N;$

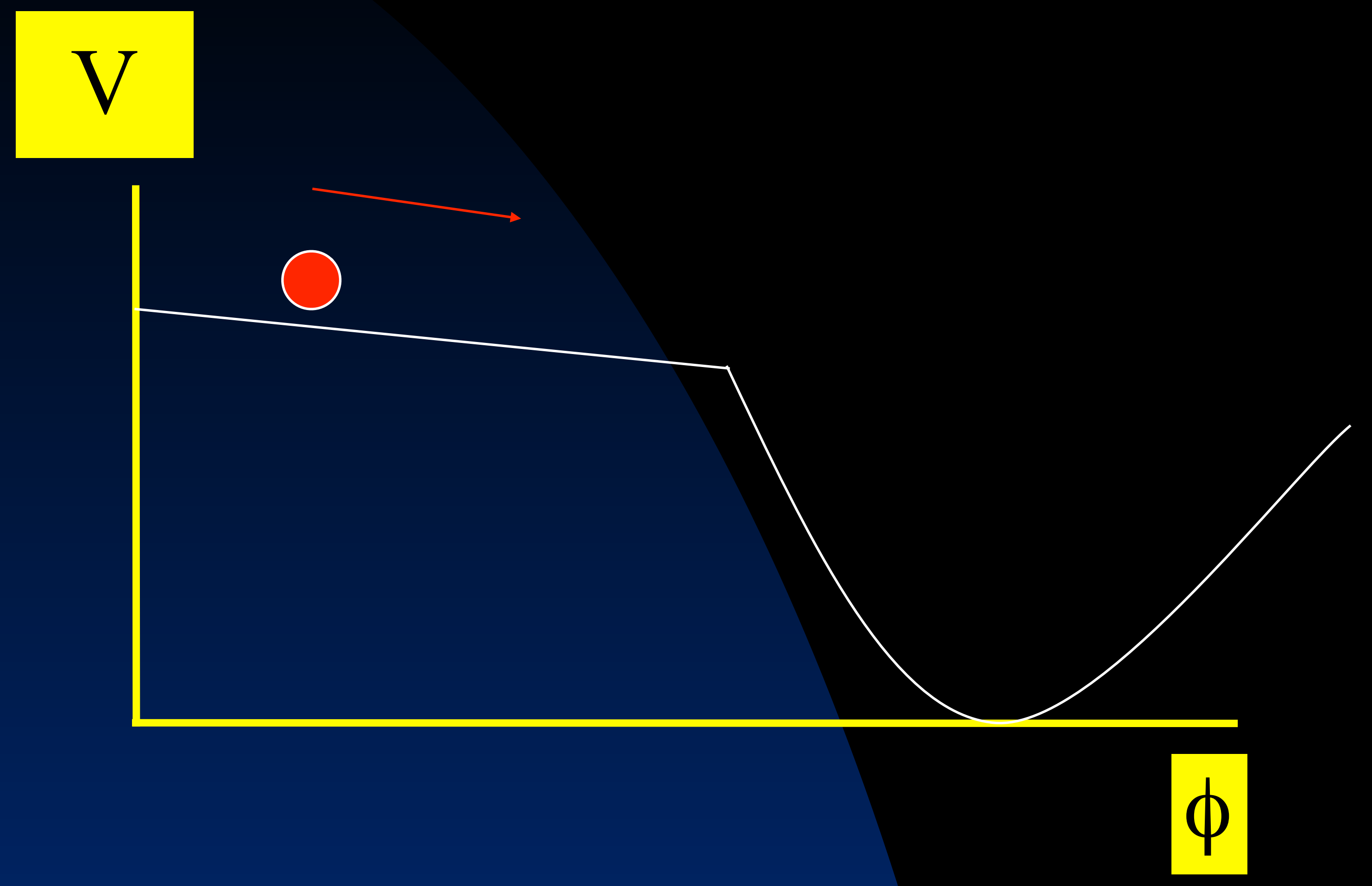
$R = -2\pi n_G = 3.1p/N \Rightarrow$ sig grav waves.



Inflates only for $\phi >> M_P$. Problem.
Why only one term? All other models inflate at $\phi < M_P$ and give negligible grav. waves.

2. New inflation

$$V(\phi) = V_0 - c\phi^p + \ldots;\quad p \geq 3;\quad n-1 = -\frac{2(p-1)}{(p-2)N}$$



$$V(\phi) = V_0 - \frac{1}{2}m^2\phi^2 + \ldots; \Rightarrow n-1 = -\frac{2M_P^2 m^2}{V_0}$$

$p = 2:$ modular, natural, quadratic inflation

3. Power-law inflation

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\phi}{m_P}\right);\quad p > 1;\quad n-1 = -\frac{2}{p}$$

1. Very useful because have exact solutions without recourse to slow roll. Similarly perturbation eqns can be solved exactly.

2. No natural end to inflation

4. Natural inflation

$$V(\phi) = V_0\left(1 + \cos\frac{\phi}{f}\right);$$

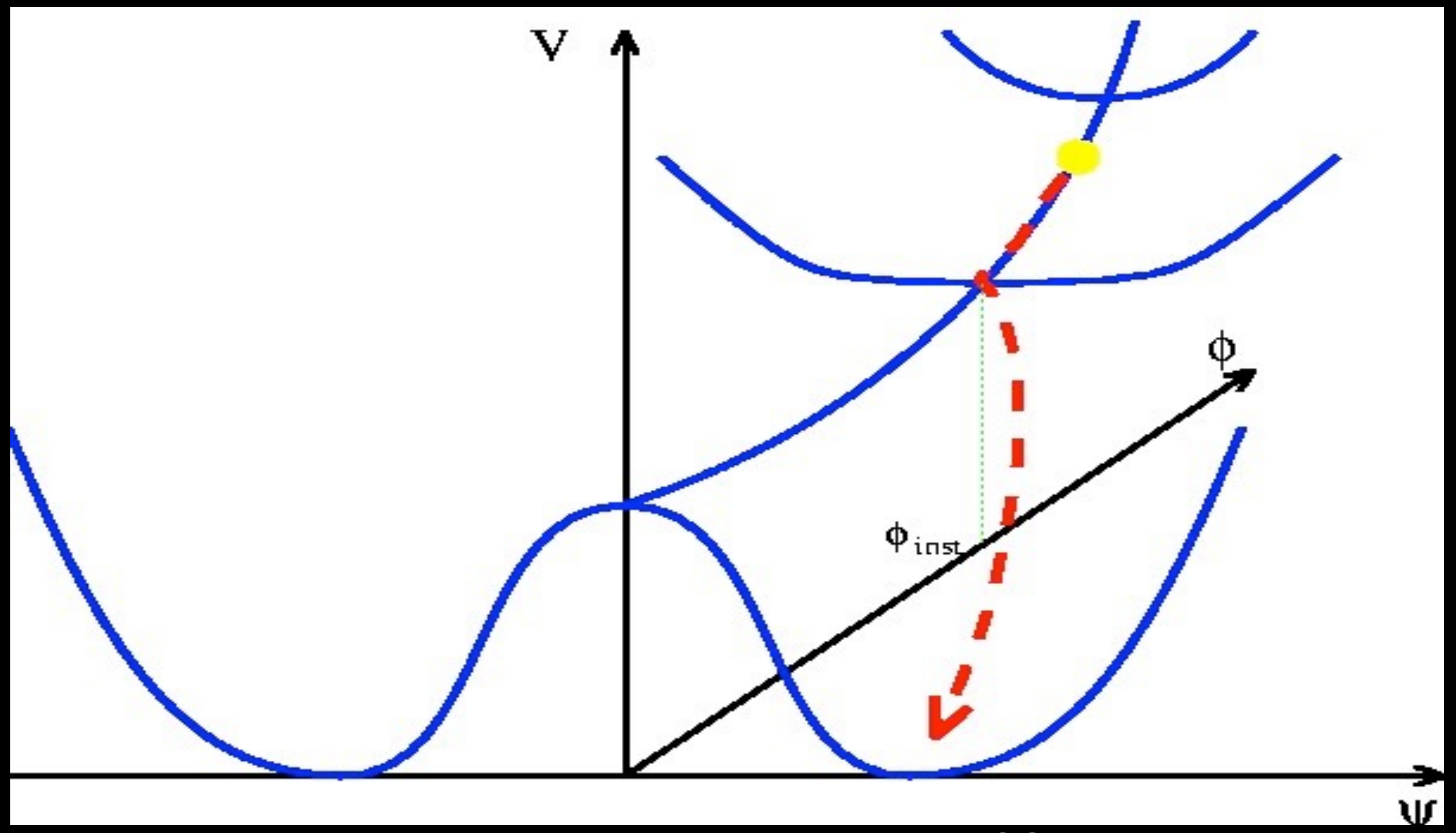
$$n - 1 < 0; \quad R - \text{negligible} - -\text{like New Inflation}$$

5. Hybrid inflation

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2;$$

$$n - 1 = \frac{2M_P^2 m^2}{V_0}$$

2 fields, inf ends when V_0 destabilised by 2[nd] non-inflaton field ψ

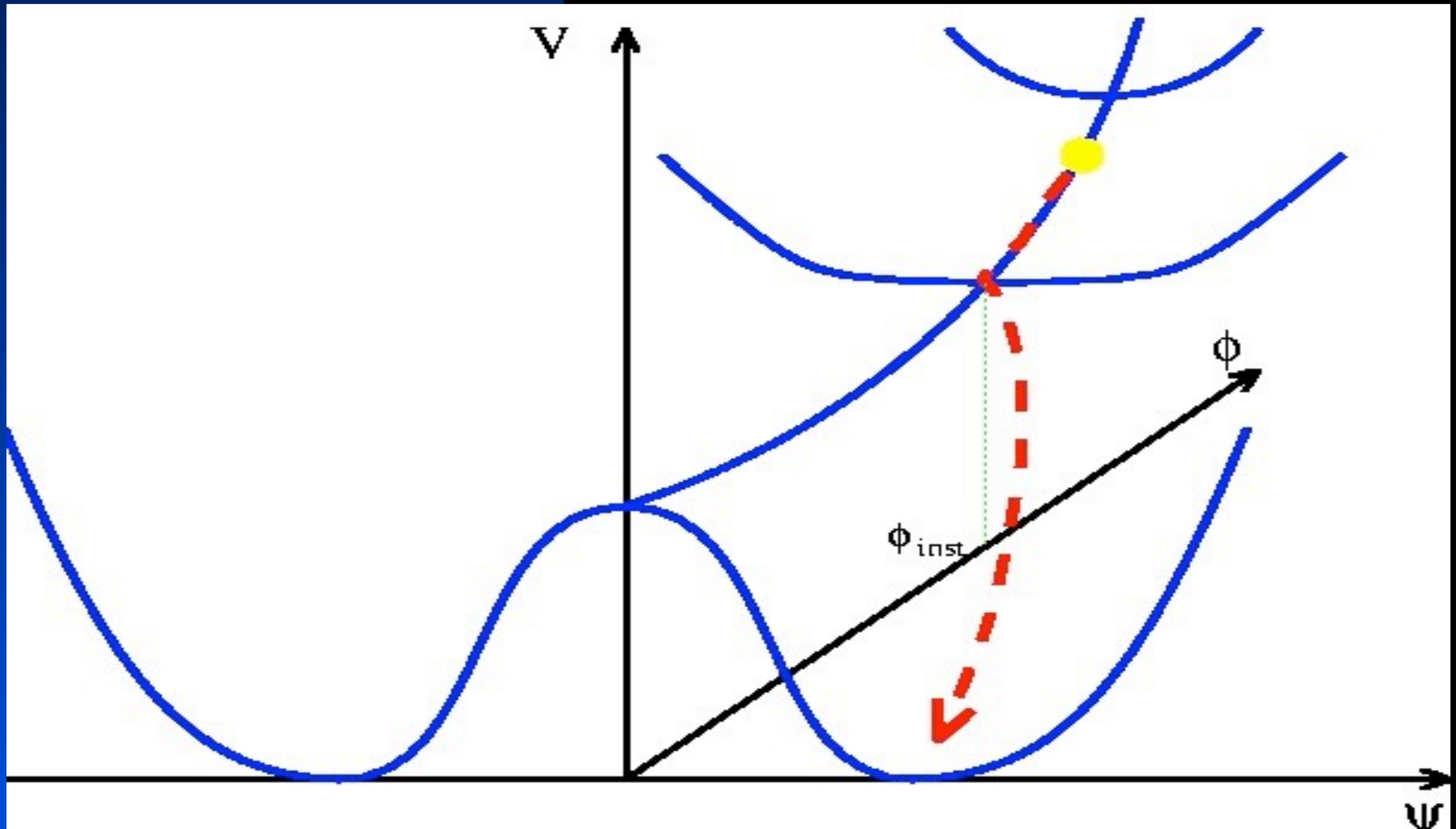


Two field inflation – more general

$$V(\phi,\psi) = \frac{1}{2} m_\phi^{\,2}\phi^2 + \frac{1}{2} g^2\phi^2|\chi|^2 + \frac{1}{4}\lambda\left(|\chi|^2 - \frac{m^2}{\lambda}\right)^2$$

Found in SUSY models.

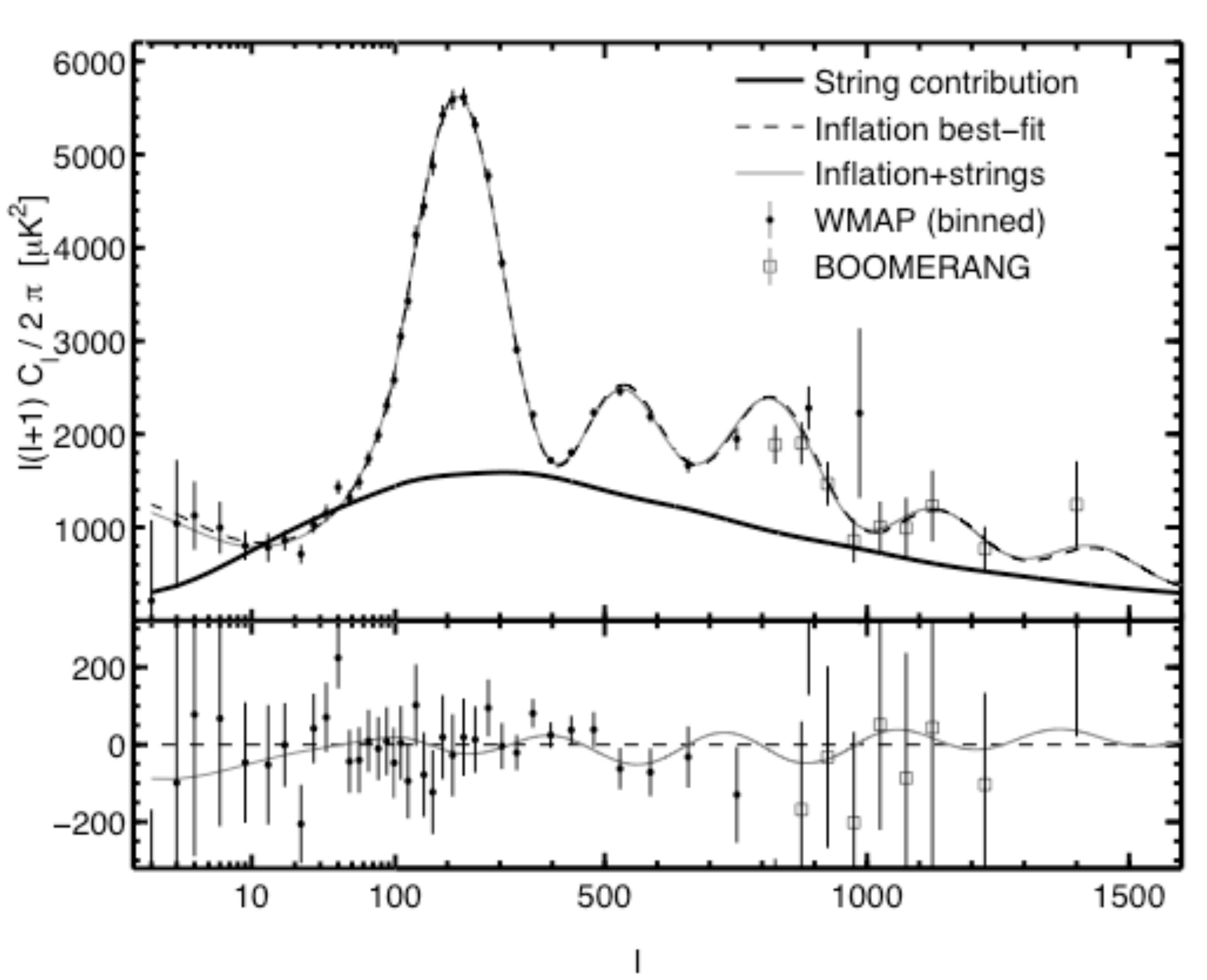
Better chance of success, plus lots of additional features, inc defect formation, ewk baryogenesis.



Inflation ends by triggering phase transition in second field.

Example of Brane inflation

Cosmic strings - may not do the full job but they can still contribute



Hybrid Inflation type models

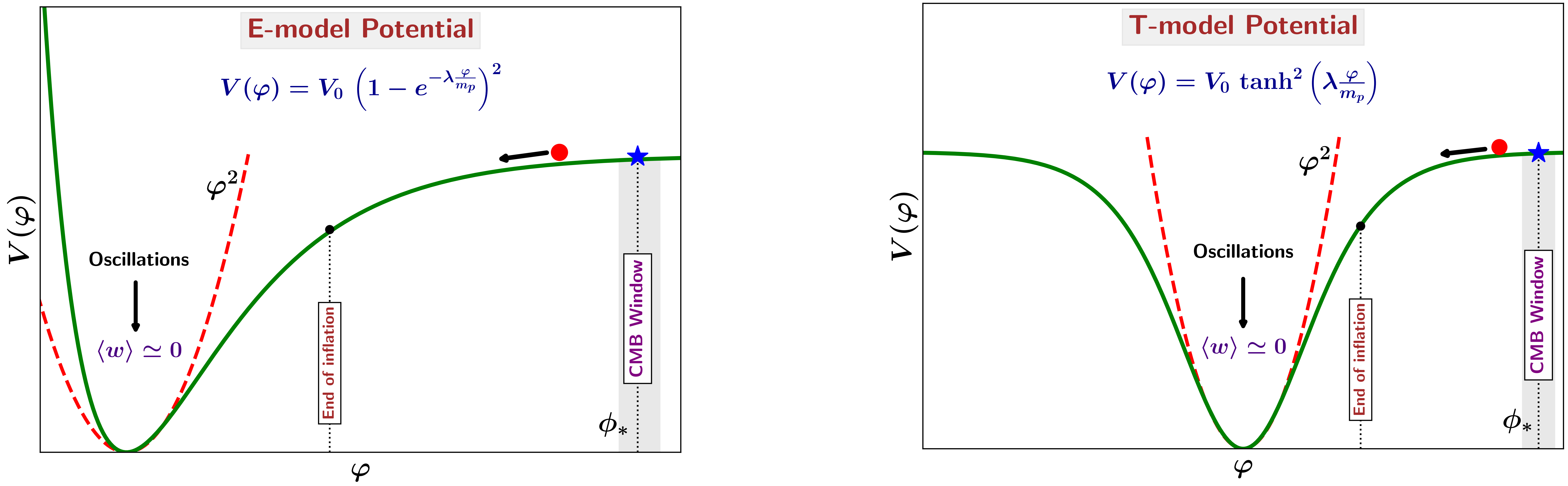
String contribution < 10% implies $G\mu < 2 \times 10^{-7}$.

Hindmarsh et al, 2019.

Alpha attractor E and T models of inflation

(Kallosh and Linde 2013)

$$V(\phi) = \frac{1}{2}m^2\phi^2 - |U(\phi)| \quad (\text{E-Model} \ \& \ \text{T-Model})$$



These fit more naturally with the recent Planck bounds on n and r.

Inflation in string theory -- non trivial

The η problem in Supergravity -- N=1 SUGR Lagrangian:

$$\mathcal{L} = -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} + V_F,$$

with

$$V_F = e^{K/M_p^2}\left[K^{\varphi\bar{\varphi}}D_\varphi W\overline{D_\varphi W} - \frac{3}{M_p^2}|W|^2\right]$$

and

$$D_\varphi = \partial_\varphi W + \frac{1}{M_p^2}\partial_\varphi K$$

$$K(\varphi,\bar{\varphi}) = K_0 + K_{\varphi\bar{\varphi}}\varphi\bar{\varphi} + \ldots$$

Expand K about φ=0

$$\begin{aligned}\mathcal{L} &\approx -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} - V_0\left(1 + K_{\varphi\bar{\varphi}}|_{\varphi=0}\frac{\varphi\bar{\varphi}}{M_p^2} + \ldots\right)\\ &= -\partial\phi\partial\bar{\phi} - V_0\left(1 + \frac{\phi\bar{\phi}}{M_p^2} + \ldots\right),\end{aligned}$$

Canonically norm fields ϕ

Have model indep terms which lead to contribution to slow roll parameter η of order unity

$$\Delta\eta = M_p^2\frac{\Delta V''}{V_0} = 1.$$

So, need to cancel this generic term possibly through additional model dependent terms.

Ex 1: Warped D3-brane D3-antibrane inflation where model dependent corrections to V can cancel model indep contributions

[Kachru et al (03) -- KLMMT].

Find:

$$V(\phi) = V_0(\phi) + \beta H^2 \phi^2$$

β relates to the coupling of warped throat to compact CY space. Can be fine tuned to avoid η problem

Ex 2: DBI inflation -- simple -- it isn't slow roll as the two branes approach each other so no η problem

Ex 3: Kahler Moduli Inflation [Conlon & Quevedo 05]

Inflaton is one of Kahler moduli in Type IIB flux compactification. Inflation proceeds by reducing the F-term energy. No η problem because of presence of a symmetry, an almost no-scale property of the Kahler potential.

$$V_{inf} = V_0 - \frac{4\tau_n W_0 a_n A_n e^{-a_n \tau_n}}{\mathcal{V}^2},$$

Inflaton moduli: τ_n

$$V_{inf} = V_0 - \frac{4\tau_n W_0 a_n A_n e^{-a_n\tau_n}}{\mathcal{V}^2},$$

Find:

$$\begin{aligned} 0.960 &< n < 0.967, \\ -0.0006 &< \frac{dn}{d\ln k} < -0.0008, \\ 0 &< |r| < 10^{-10}, \end{aligned}$$

with large volume modulus

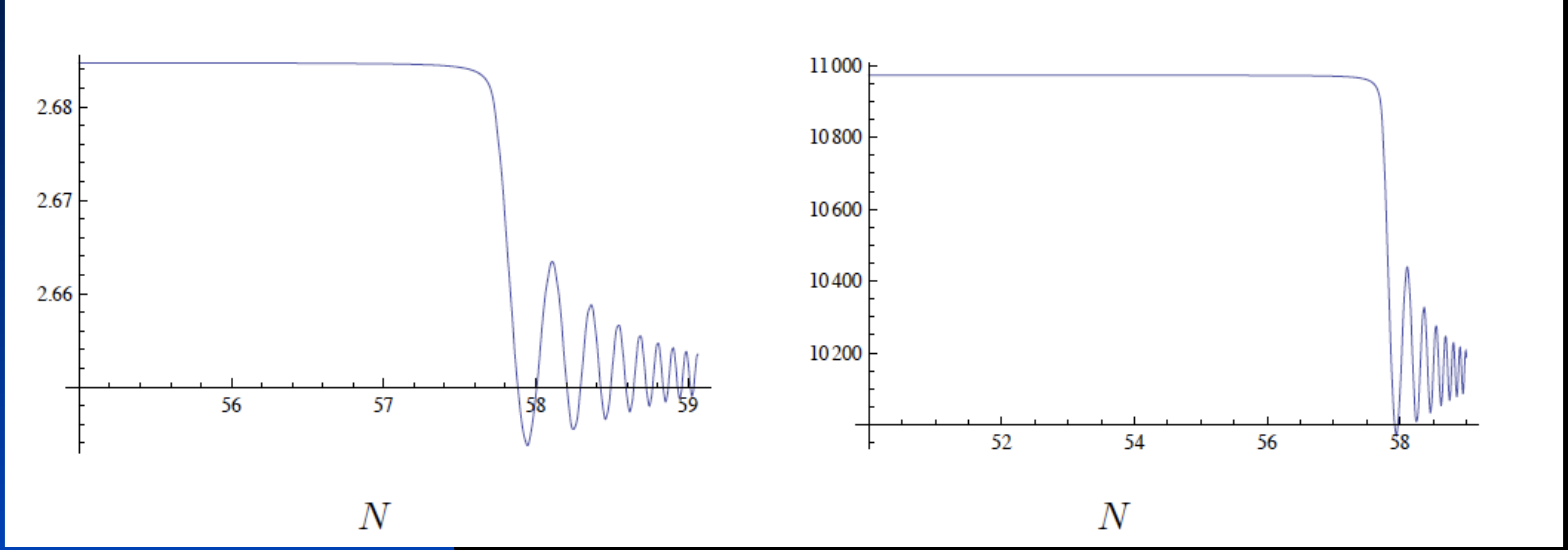
$$10^5 l_s^6 \leq \mathcal{V} \leq 10^7 l_s^6,$$

and

$$\begin{aligned} \eta &\approx -\frac{1}{N_e}, \\ \epsilon &< 10^{-12}, \end{aligned}$$

for $N_e \approx$50-60 efolds with low energy scale

$$V_{inf} \sim 10^{13}\text{GeV}.$$



Inflaton [Blanco-Pillado et al 09] **Volume modulus**

Can include curvaton as second evolving moduli -- Burgess et al 2010

Key inflationary parameters:

n: Planck and WMAP have detected it — it's not unity.

r: Tensor-to-scalar ratio : considered as a smoking gun for inflation but also produced by defects and some inflation models produce very little.

dn/dln k : Running of the spectral index, usually very small -- probably too small for detection.

f_{NL}: Measure of cosmic non-gaussianity. Still consistent with zero. Lots of current interest.

$G\mu$: string tension in Hybrid models where defects produced at end of period of inflation.

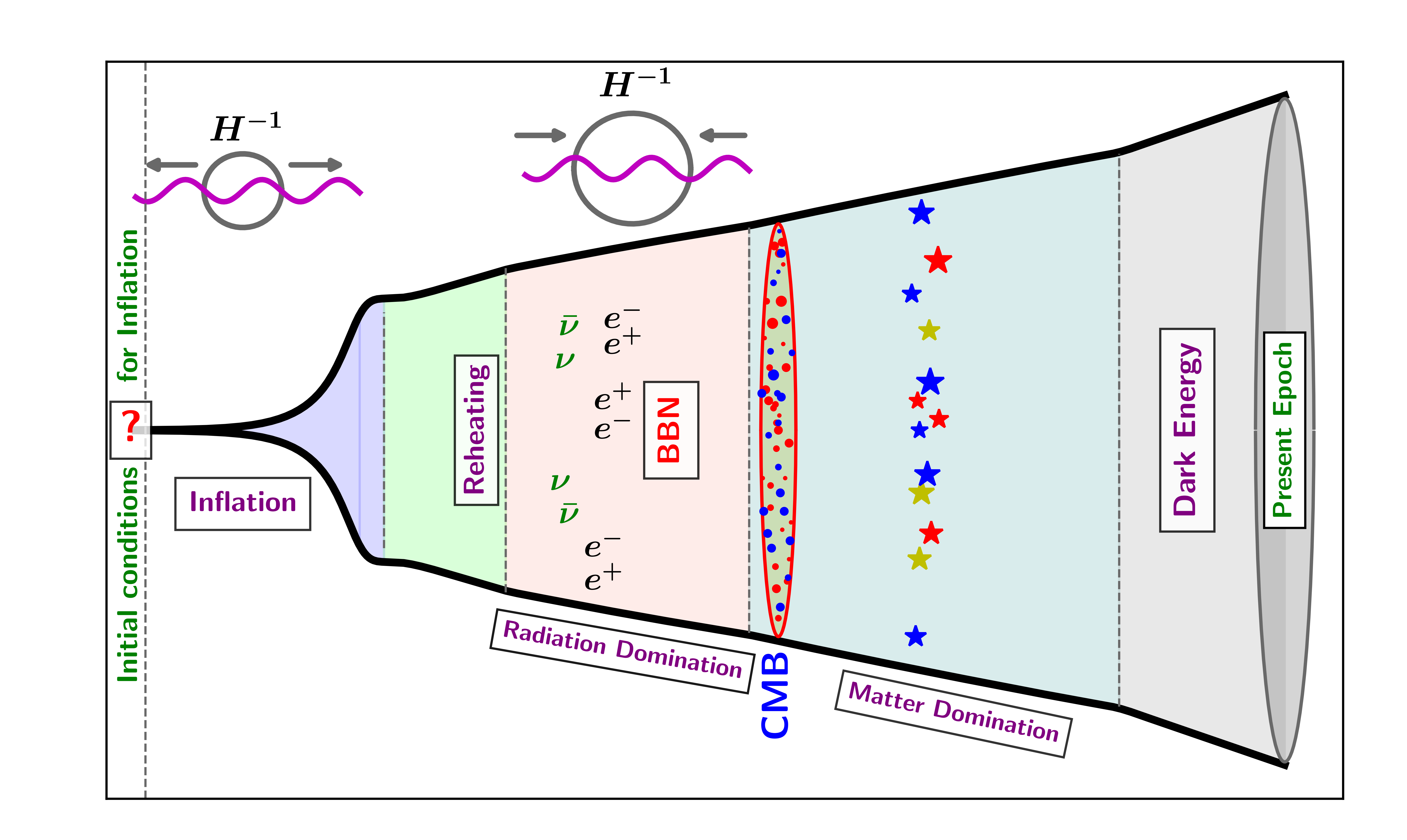
Also additional perturbation generation mechanisms (e.g. Curvaton)

Perturbations not from inflaton but from extra field and then couple through to curvature perturbation

Reheating the Universe after inflation has finished

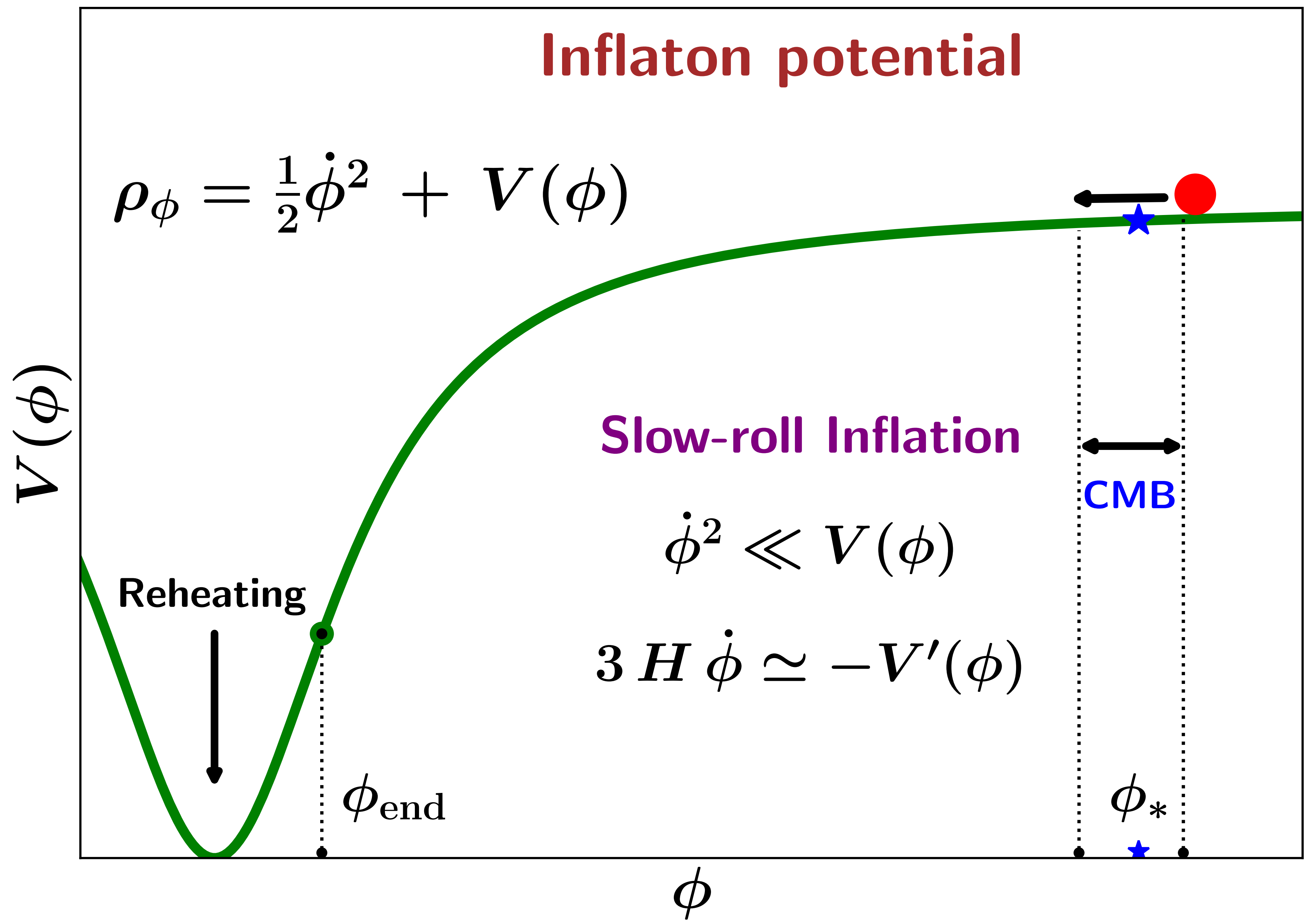
Inflation is the ultimate vacuum cleaner, it clears out pretty much everything, particles get diluted, radiation gets red shifted, we end inflation with a cold, empty large universe, not quite what we experience today.

We need to reheat the universe - we convert the remaining energy stored in the inflaton field into primordial particles through their interactions. We could consider this the beginning of the Hot Big Bang



Credit: Swagat Mishra

Reheating occurs as slow roll inflation finishes and inflaton oscillates about the potential minima



Inflaton oscillates about pot of form

$$V(\phi) \simeq V_0 \left(\frac{\phi}{m_p}\right)^{2n} \; ; \; n > 0$$

Experiences time averaged eos

$$\langle w_\phi \rangle = \frac{n-1}{n+1}$$

[Turner (1984)]

With oscillation period

$$T_{\rm osc} = \sqrt{\frac{8\pi m_p^2}{V_0}} \frac{\Gamma\left(1+\frac{1}{2n}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{2n}\right)} \left(\frac{\phi_0}{m_p}\right)^{1-n}$$

Since in this regime $m^2 \equiv V_{,\phi\phi} \gg H^2$ Inflaton experiences coherent oscillations about the min

For n=1, quadratic pot about the min $V(\phi) \propto \phi^2$ we see $\langle w_\phi \rangle = 0 \;\Rightarrow\; \rho_\phi \propto a^{-3}$

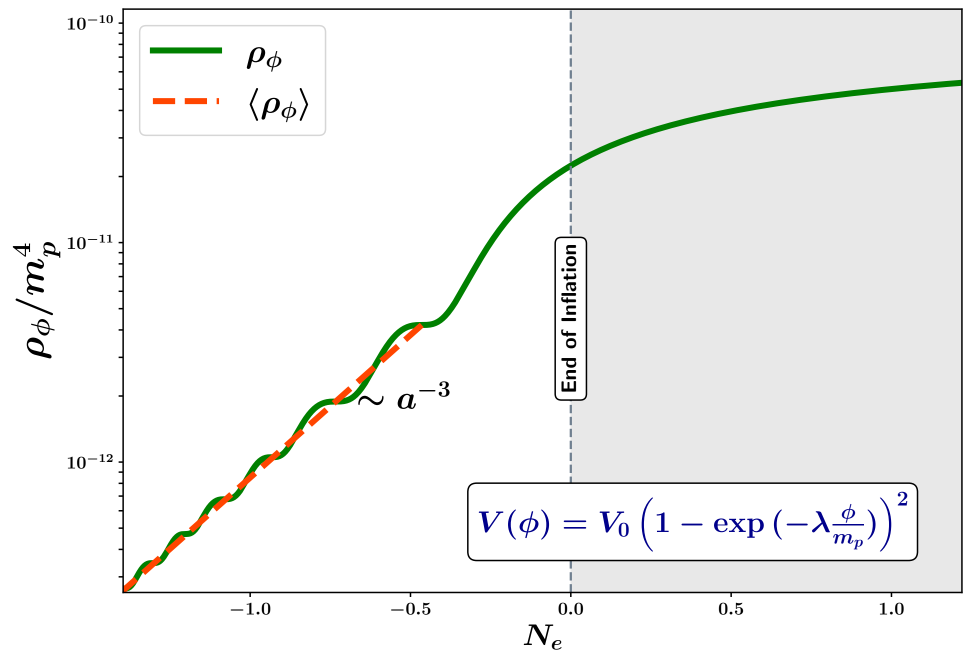
Inflaton behaves as matter and $T_{\rm osc}$ indep of ϕ_0 !

Since m >> H it follows $T_{\rm osc} << H^{-1} = 3\,t/2$

— under the adiabatic approximation

$$\phi(t) = \phi_0(t)\,\cos(mt)$$

With $\phi_0(t) \propto \left(\frac{m_p}{m}\right) \frac{1}{t}$



Credit: Swagat Mishra

Reheating from perturbative decay of inflaton

[Abbott et al, Albrecht et al, Linde, Dolgov 1980's]

Inflaton particles of mass m acting as CDM decay into fermions and bosons

$$\Gamma_{\phi\to\bar\psi\psi} = \frac{h^2\, m}{8\pi}\;;\quad \Gamma_{\phi\phi\to\chi\chi} = \frac{g^2\phi_0^2}{8\pi\, m}$$

Decay acts phenomenologically like additional friction term:

$$\ddot\phi + (3H + \boldsymbol{\Gamma})\,\dot\phi + V_{,\phi} = 0$$

Initially H >> Γ, reheating completes when H ~ Γ with reheat temperature T_{re}

$$\Gamma \simeq H = \sqrt{\frac{1}{3m_p^2}\,\rho(T_{\rm re})} \Rightarrow \Gamma = \sqrt{\frac{1}{3m_p^2}\,\frac{\pi^2}{30}\,g_*(T_{\rm re})\,T_{\rm re}^4}$$

Or:

$$T_{\rm re} \simeq \left(\frac{\pi^2 g_*}{90}\right)^{-1/4}\left(\Gamma\, M_p\right)^{1/2} = 2.4\times 10^{18}\times\left(\frac{\pi^2 g_*}{90}\right)^{-1/4}\left(\frac{\Gamma}{m_p}\right)^{1/2}\ {\rm GeV}$$

Decay to fermions with $h \lesssim 10^{-3}$ and $m = 10^{-5}\, m_p$: $\Gamma_{\phi\to\bar\psi\psi} = \frac{h^2\, m}{8\pi} \leq 4\times 10^{-13}\, m_p$ Hence low reheat temp $T_{\rm re} \leq 10^{12}\ {\rm GeV}$

Bosonic decay, ϕϕ—>χχ, recall $\phi_0 \propto 1/t$, hence $\Gamma_{\phi\phi\to\chi\chi} \propto 1/t^2$ But $H \propto 1/t \;\Rightarrow\; \Gamma_{\phi\phi\to\chi\chi} \ll H$

In that case reheating is incomplete and we have a coherent oscillating inflaton condensate

Reheating from non-perturbative decay of inflaton

[Kofman et al (1994), Shtanov et al (1995)]

Occurs when bosonic couplings high enough $g^2 \gtrsim 10^{-8}$

Particle production taking place in presence of oscillating inflaton condensate - via parametric resonance — collective phenomena

Occurs quickly efficiently and non-thermal

Not applicable to fermionic decay (Pauli exclusion)

Dynamics divided into three distinct phases:

1. Preheating (linear parametric resonance)

2. Backreaction (quenching of resonant particle production)

3. Scattering and thermalisation (perturbative decay, turbulence)

Inflaton φ decays to massless field χ

$$S[\varphi,\chi] = -\int d^4x\,\sqrt{-g}\left[\frac{1}{2}\,\partial_\mu\varphi\partial^\nu\varphi + V(\varphi) + \frac{1}{2}\,\partial_\mu\chi\partial^\nu\chi + \mathcal{I}(\varphi,\chi)\right]$$

Interaction:

$$\mathcal{I}(\phi,\,\chi) = \tfrac{1}{2}g^2\varphi^2\chi^2$$

Field equations:

$$\ddot{\varphi} - \frac{\nabla^2}{a^2}\varphi + 3H\dot{\varphi} + V_{,\varphi} + \mathcal{I}_{,\varphi} = 0$$

$$\ddot{\chi} - \frac{\nabla^2}{a^2}\chi + 3H\dot{\chi} + \mathcal{I}_{,\chi} = 0$$

Friedmann equation:

$$H^2 = \frac{1}{3\,m_p^2}\left[\frac{1}{2}\,\dot{\varphi}^2 + \frac{1}{2}\,\frac{\vec{\nabla}\varphi}{a}.\frac{\vec{\nabla}\varphi}{a} + V(\varphi) + \frac{1}{2}\,\dot{\chi}^2 + \frac{1}{2}\,\frac{\vec{\nabla}\chi}{a}.\frac{\vec{\nabla}\chi}{a} + \mathcal{I}(\varphi,\,\chi)\right]$$

Preheating in the linear regime :

$$\varphi(t,\vec{x}) = \phi(t) + \delta\varphi(t,\vec{x}) \to 0$$

$$\chi(t,\vec{x}) = \bar{\chi}(t) \to 0 + \delta\chi(t,\vec{x})$$

χ is in its vacuum state

At end of inflation :

$$\rho_\phi \gg \rho_\chi,\ \rho_{\delta\varphi}$$

condensate dominated

Evolution equations simplify

Stage 1: Preheating in the linear regime - parametric oscillator

Fourier modes:
$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left[\frac{k^2}{a^2} + g^2\,\phi(t)^2\right]\chi_k = 0$$

Ignoring the expansion (adiabatic regime) and recall for V(ϕ) ≈ 1/2 m² ϕ² we have
$$\phi(t) \simeq \phi_0(t)\,\cos(mt)$$

Obtain the Mathieu Equation:
$$\frac{\mathrm{d}^2\chi_k}{\mathrm{d}T^2} + \left[A_k - 2q\,\cos(2T)\right]\chi_k = 0$$

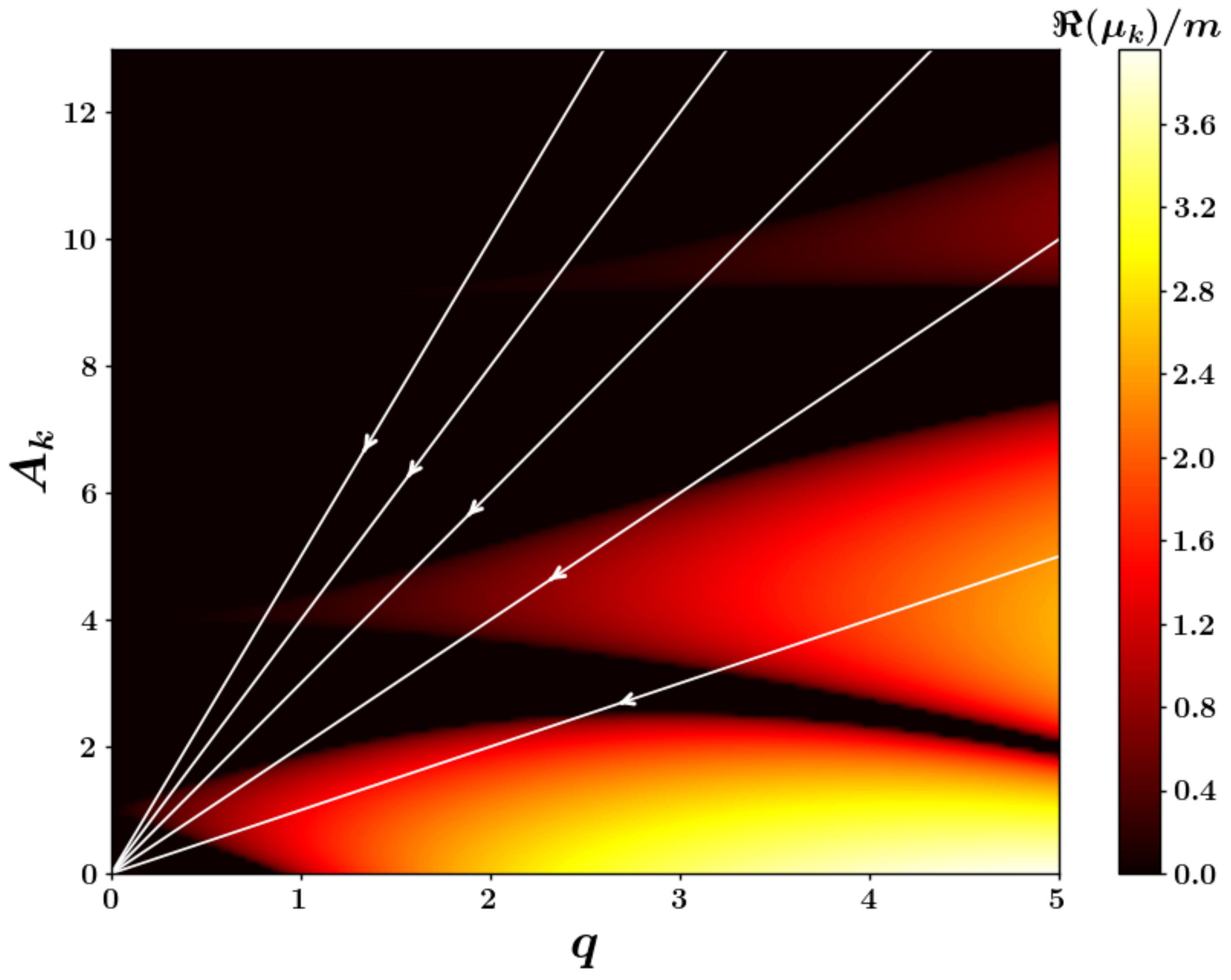
With $T = mt - \frac{\pi}{2}$ and resonance parameters: $q = \frac{g^2}{4}\left(\frac{\phi_0}{m}\right)^2 ; \quad A_k = \left(\frac{k}{m}\right)^2 + 2q$

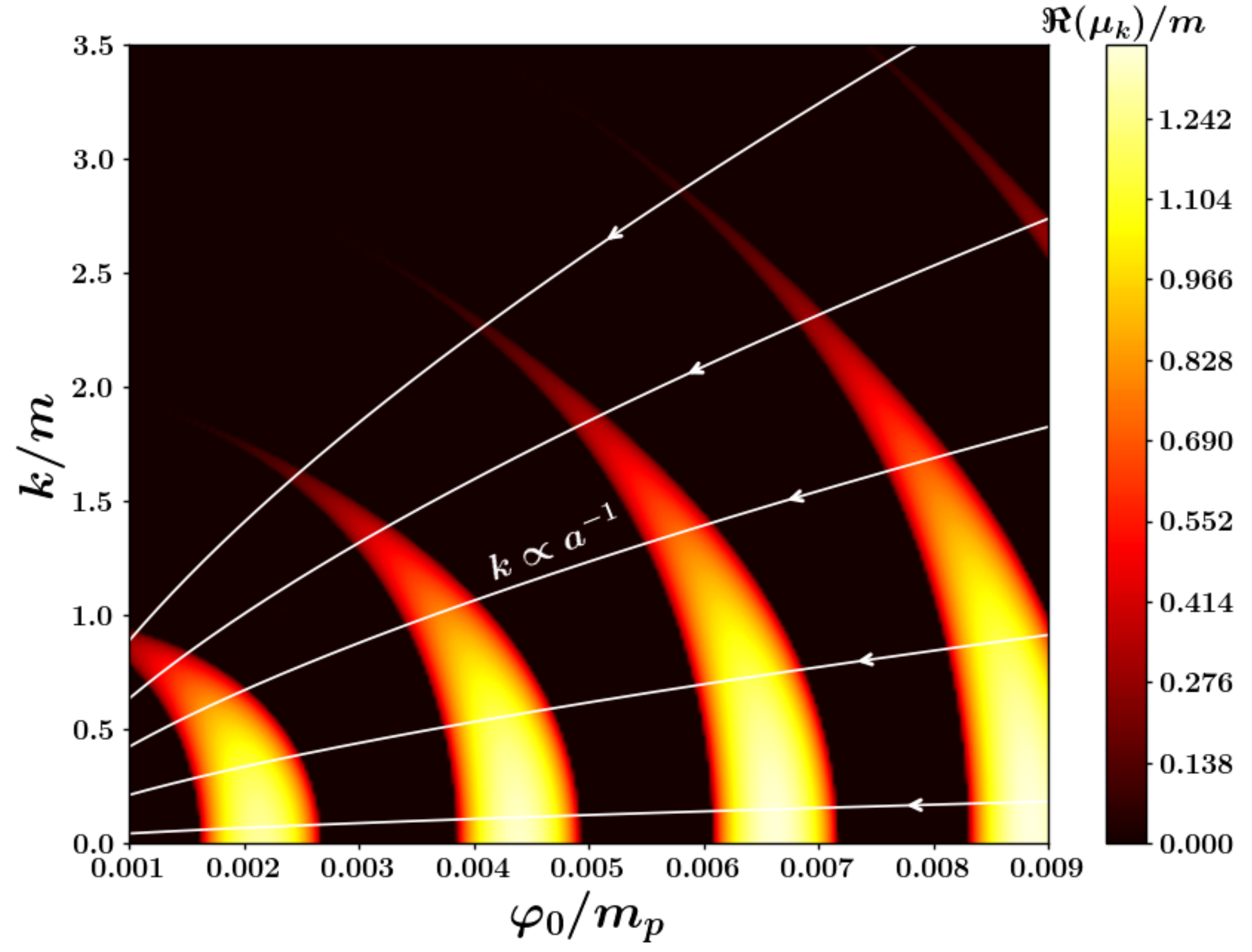
Write as
$$\frac{\mathrm{d}^2\chi_k}{\mathrm{d}T^2} + \Omega_\chi^2(k,T)\,\chi_k = 0 ; \qquad \Omega_\chi^2 = A_k - 2q\,\cos(2T)$$

Solutions from Floquet Theory
$$\chi_k(T) = \mathcal{M}_k^{(+)}(T)\,e^{\mu_k T} + \mathcal{M}_k^{(-)}(T)\,e^{-\mu_k T}$$

Exponential growing solutions for $\mathrm{Re}(\mu_k) \neq 0$.

Exponential growing solutions for $\mathrm{Re}(\mu_k) \neq 0$.





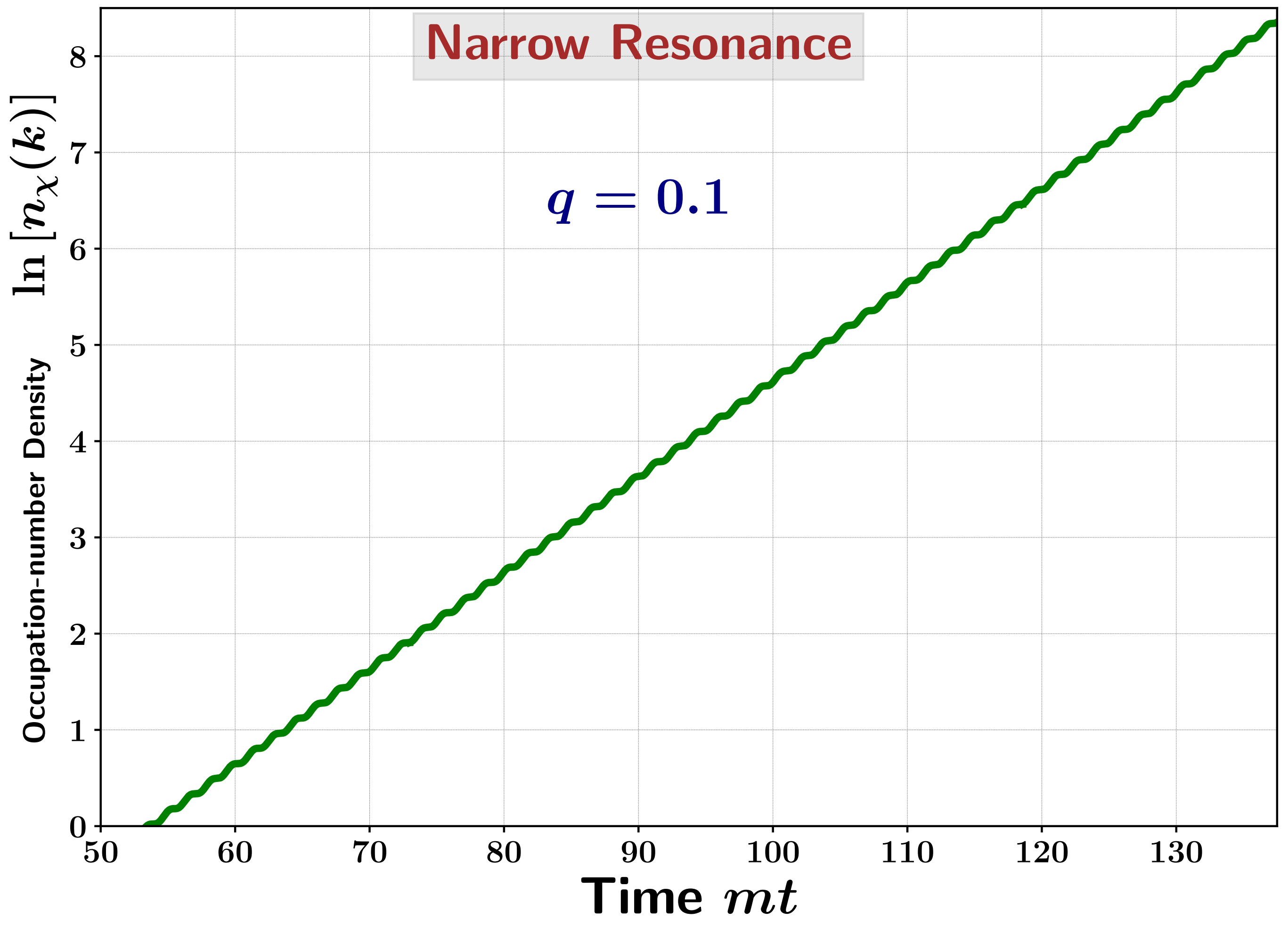
Narrow resonance

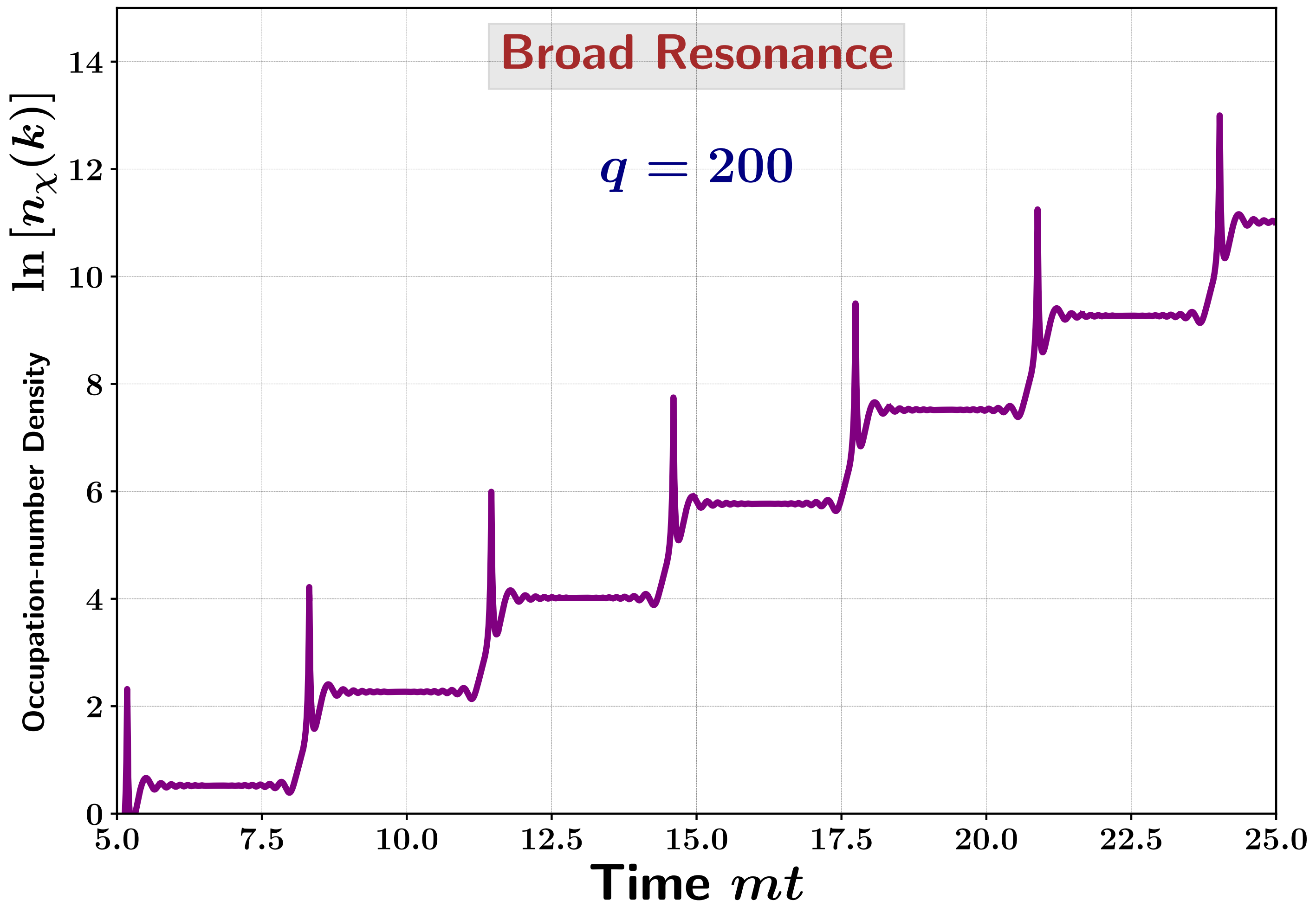
$$q = \frac{g^2}{4}\left(\frac{\phi_0}{m}\right)^2 \ll 1$$

Broad resonance

$$q = \frac{g^2}{4}\left(\frac{\phi_0}{m}\right)^2 \geq 1$$

$$n_\chi(k) \equiv \frac{\mathcal{E}_\chi(k)}{\Omega_\chi(k)} = \frac{1}{2\Omega_\chi}\left[\left|\frac{\mathrm{d}\chi_k}{\mathrm{dT}}\right|^2 + \Omega_\chi^2\,|\chi_k|^2\right] \propto e^{2\mu_k T}$$



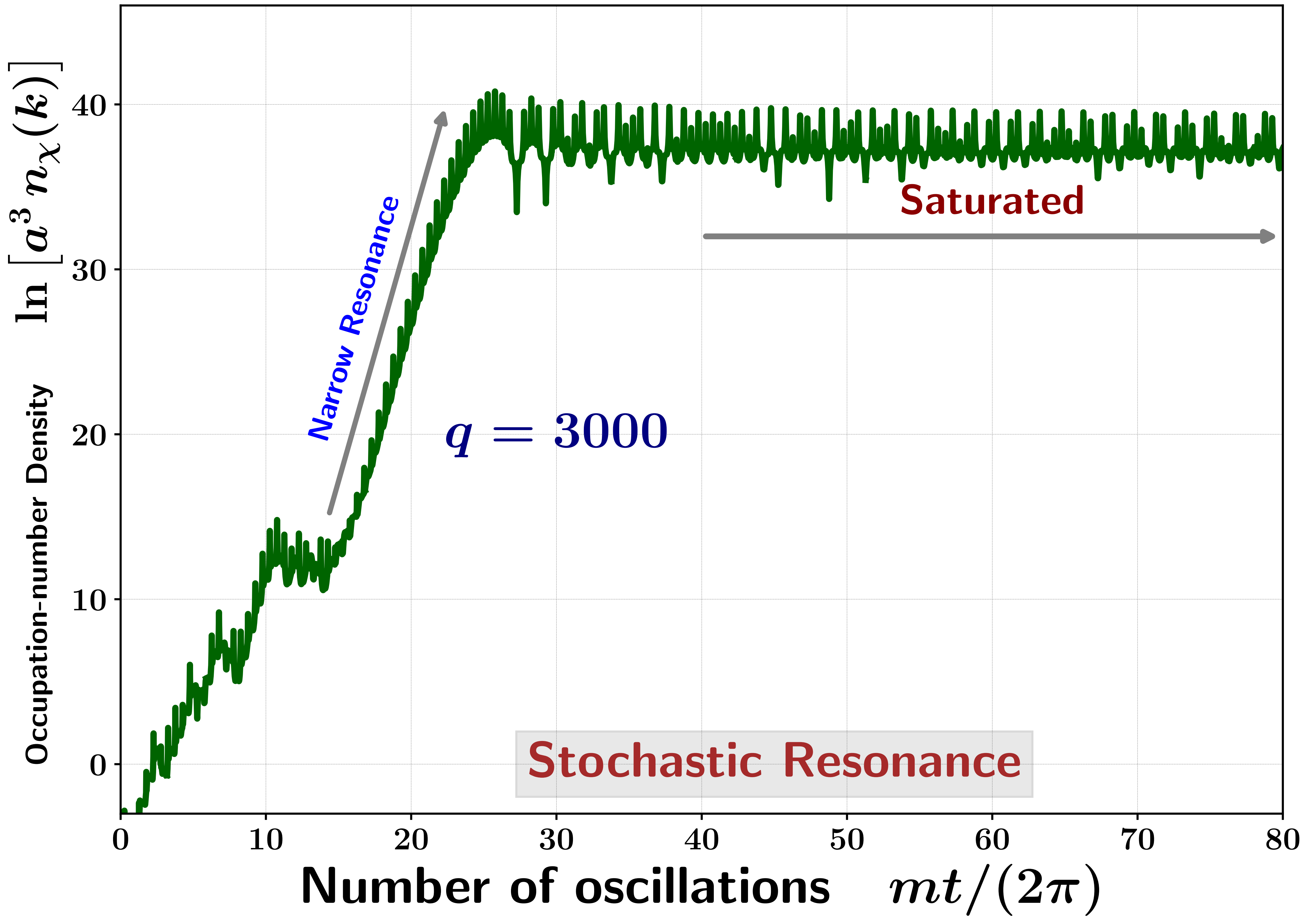


Occupation number density

Stage 2: Backreaction and quenching - shutting off the rapid particle production

System moves from Broad to Narrow residence as $\phi_0(t) \propto \left(\frac{m_p}{m}\right) \frac{1}{t}$

Particle production via resonance is quenched due to redshifting of q(t) and k/a(t), and the backreaction of χ(x,t) on ϕ(t)



Credit: Swagat Mishra

Stage 3: Perturbative decays take over

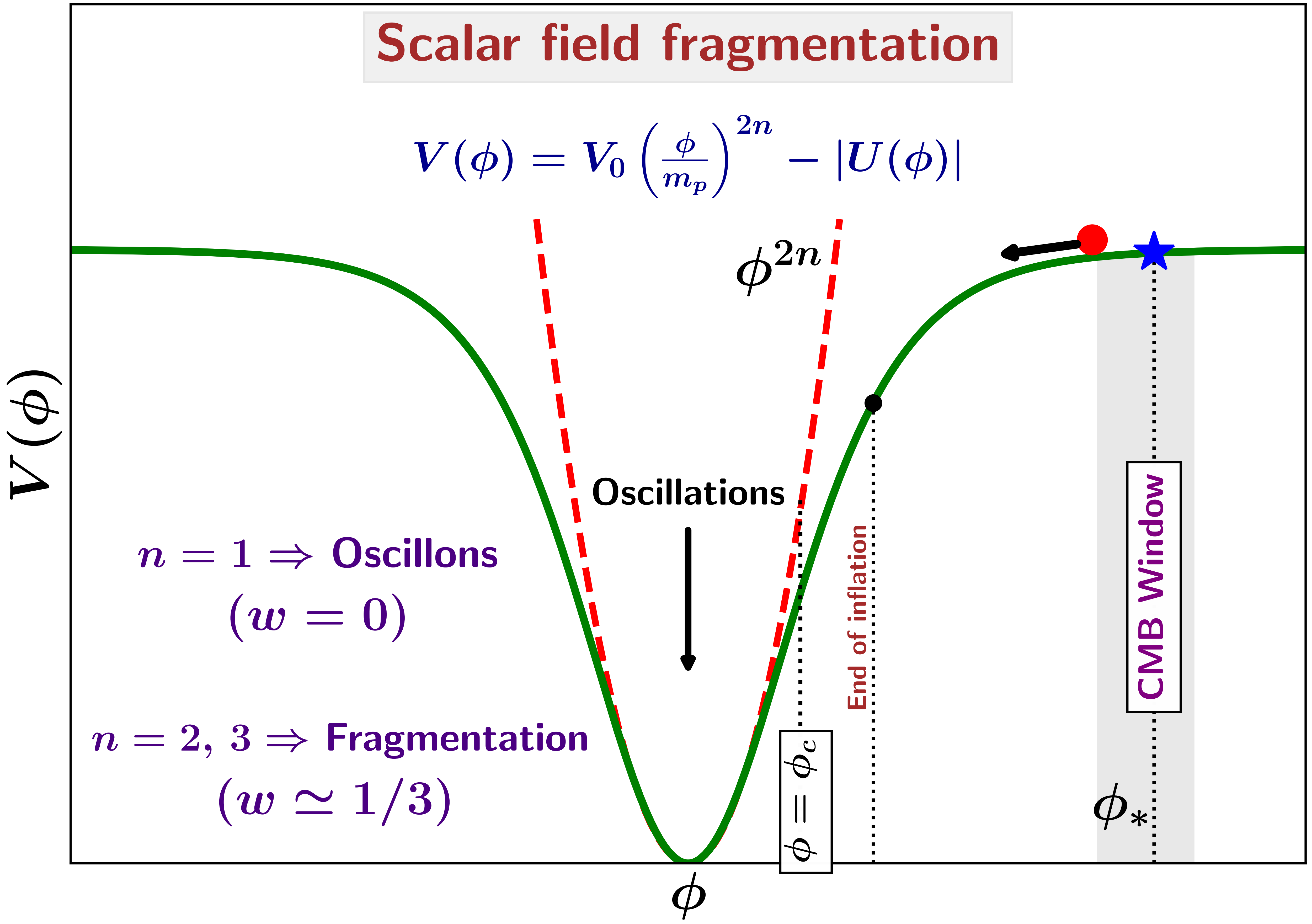
$$\varphi \longrightarrow \bar{\psi}\psi\,; \quad \varphi\varphi \longrightarrow \chi\chi$$

New possible feature not included so far arises from asymptotically flat potentials - motivated from CMB observations

$$V(\phi) = V_0 \left(\frac{\phi}{m_p}\right)^{2n} - |U(\phi)|$$

They have attractive self interactions allowing for the formations of long lived non-topological solitons like oscillons — provide a new route to reheating

[Amin et al 2010]



Credit: Swagat Mishra

Oscillons : a type of soliton, self supported localised long lived due to non-linear interactions
[Bogolyubsky & Makhankov 1978, Gleiser 1993, EJC et al 1995]

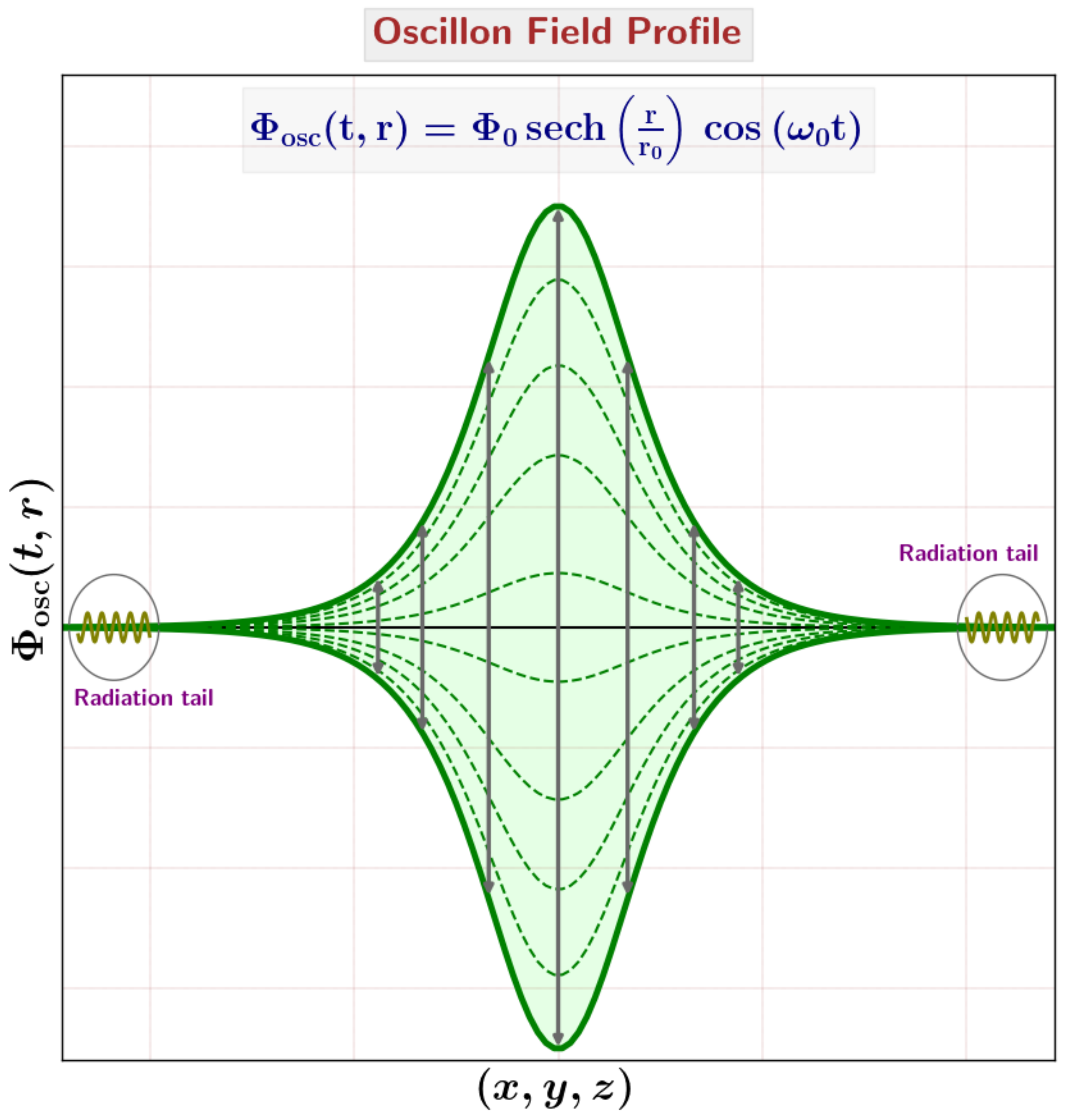
Can obtain semi-analytic solutions from small amplitude oscillations: $V(\varphi) \approx \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4} \mu \varphi^4 + \frac{g}{6} \lambda \varphi^6$

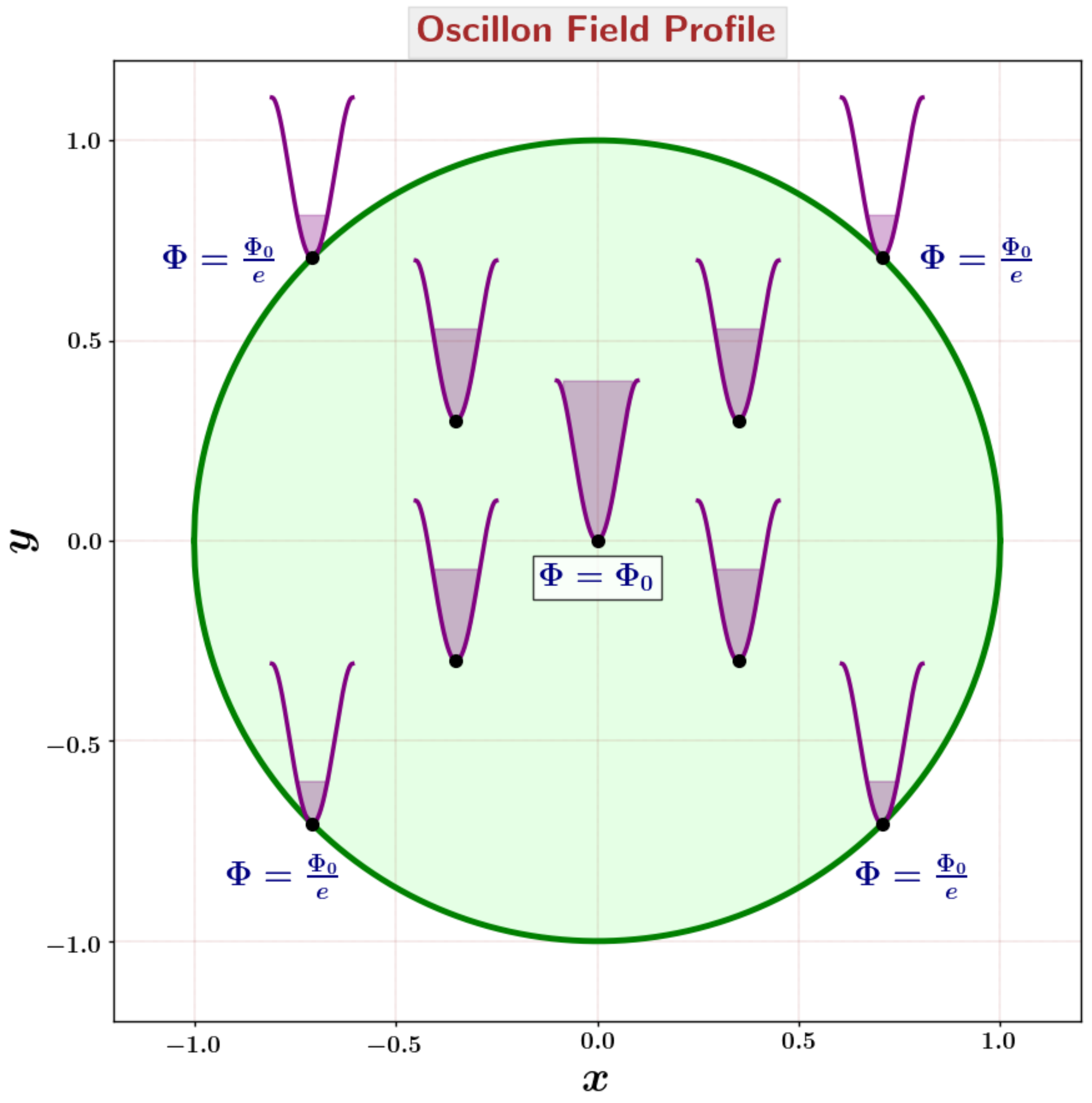
$$\boldsymbol{\varphi_{\rm osc}(t,r) \approx \Phi(r) \cos(\omega_0 t) + \ldots} \; ; \quad \omega_0 = m\sqrt{1 - \frac{\lambda^2 \alpha^2}{m^2}}$$

With core profile: $$\boldsymbol{\Phi(r) \approx \Phi_0 \,\mathrm{sech}\left(\frac{r}{r_0}\right)} \; ; \; \alpha^2 = \frac{g\Phi_0^2}{8\lambda}\left[3 - \frac{5}{3} g \Phi_0^2\right] \; ; \; r_0 = \frac{1}{\Phi_0}\sqrt{\frac{6}{\lambda} - \frac{10}{3\lambda} g\Phi_0^2}$$ [Amin et al, Mahbub and Mishra]

$$\textbf{Oscillon field} \quad \boldsymbol{\Phi(t,r) = \phi_0 \,\mathrm{sech}\left(\frac{r}{r_0}\right) \cos(\omega_0 t)}$$

Can survive for of order 10^9 oscillations depending on the interactions. [Zhang et al 2020]





Credit: Swagat Mishra

Oscillons : Can they form dynamically starting from natural conditions at the end of inflation ? [Lozanov & Amin 2017, Shafi et al 2024]

In the linear regime - we see behaviour similar to that already discussed. Self resonance . But then inflaton fragmentation kicks in

Linear regime, Fourier modes:

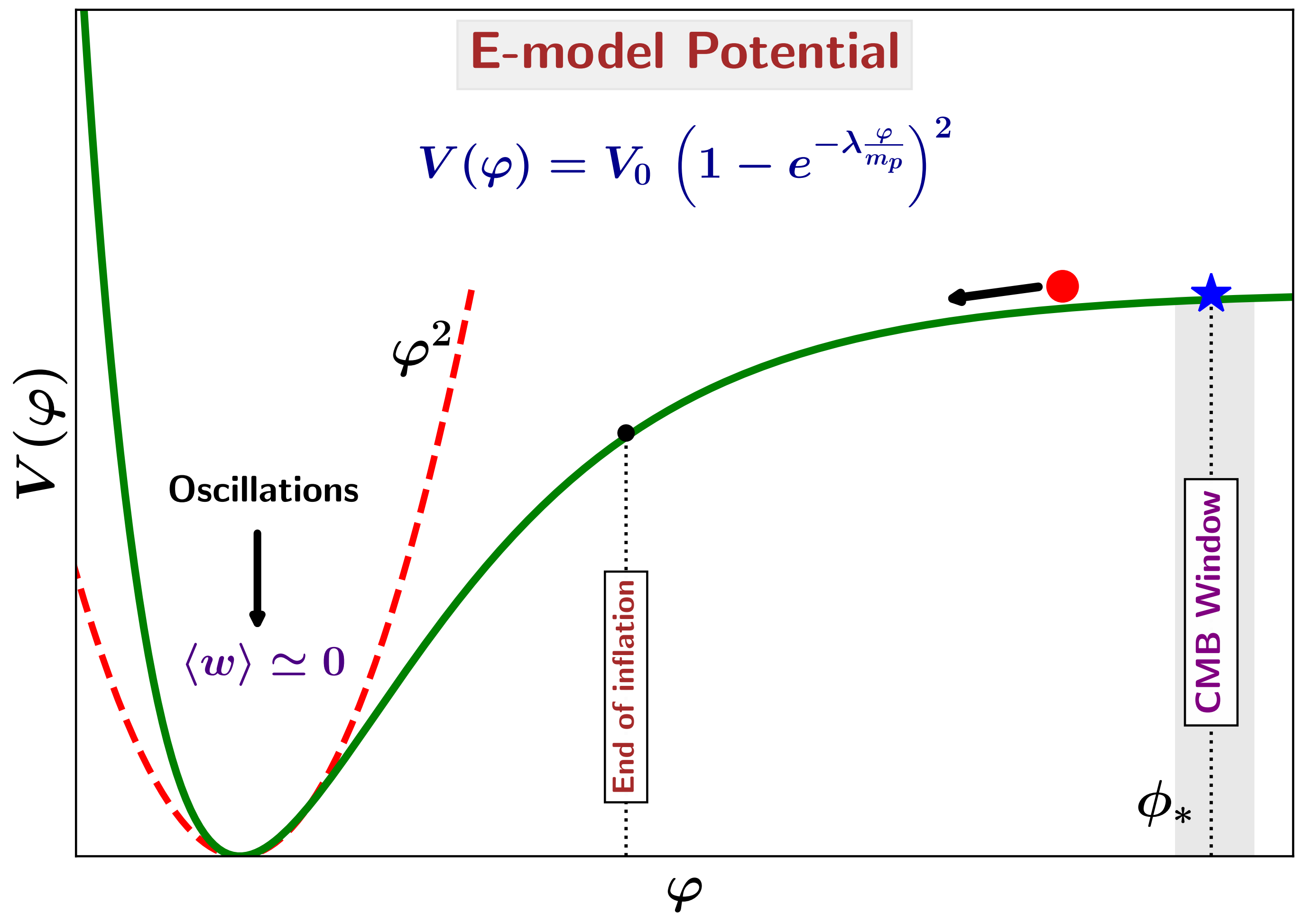
$$\delta\ddot{\varphi}_k + 3H\delta\dot{\varphi}_k + \left[\frac{k^2}{a^2} + V_{,\phi\phi}(\phi)\right]\delta\varphi_k = 0$$

Leads to exp growth of inflaton fluctuations with band structure similar to Mathieu resonance for quadratic case: $\delta\varphi_k(t) \propto e^{\mu_k m t}$

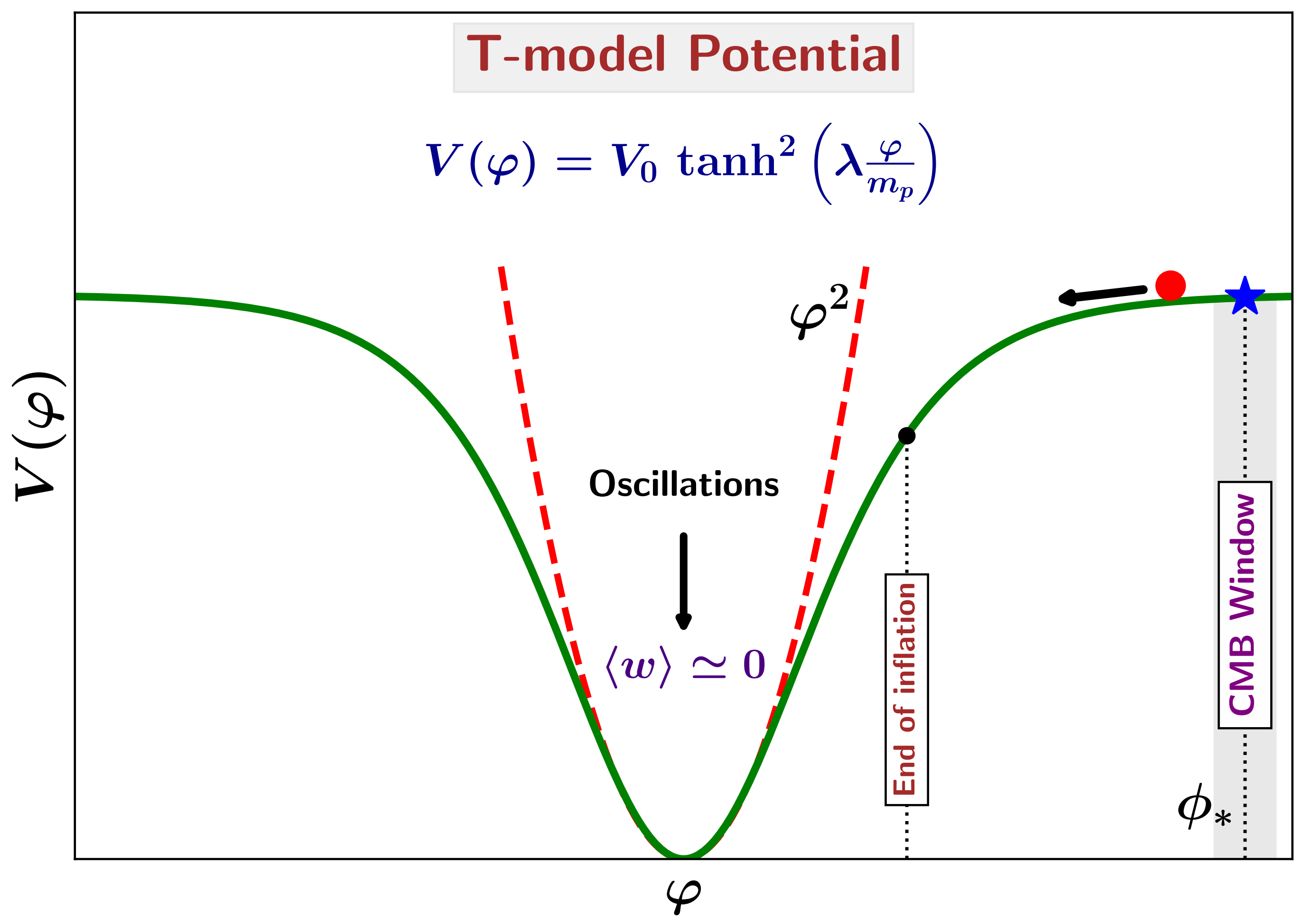
Potentials being considered - recall we need Asymptotically flat potentials

$$V(\phi) = \frac{1}{2}m^2\phi^2 - |U(\phi)| \quad (\textbf{E-Model} \;\&\; \textbf{T-Model})$$

Shafi et al- e-Print:2406-00108



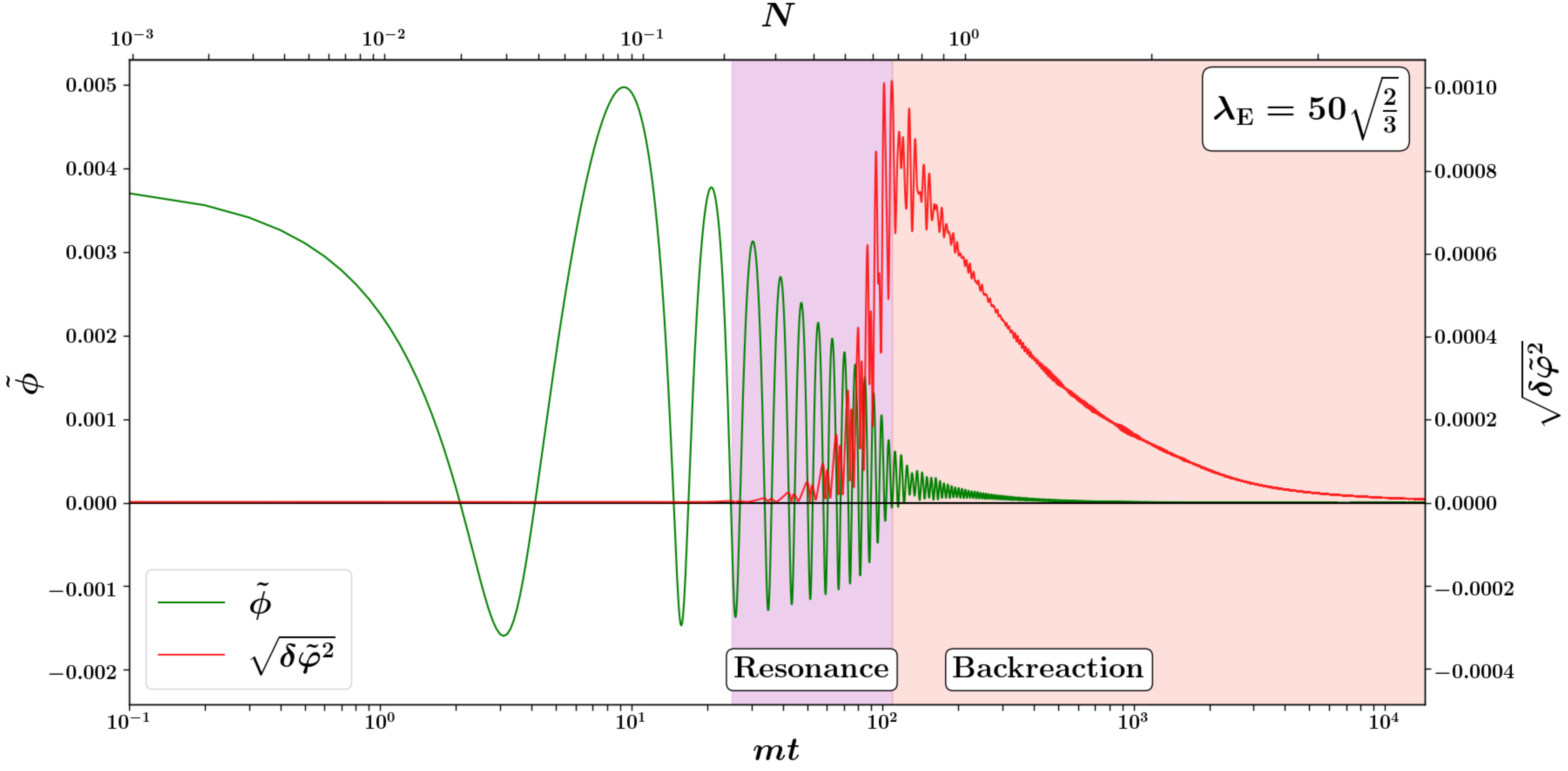
Asymmetric



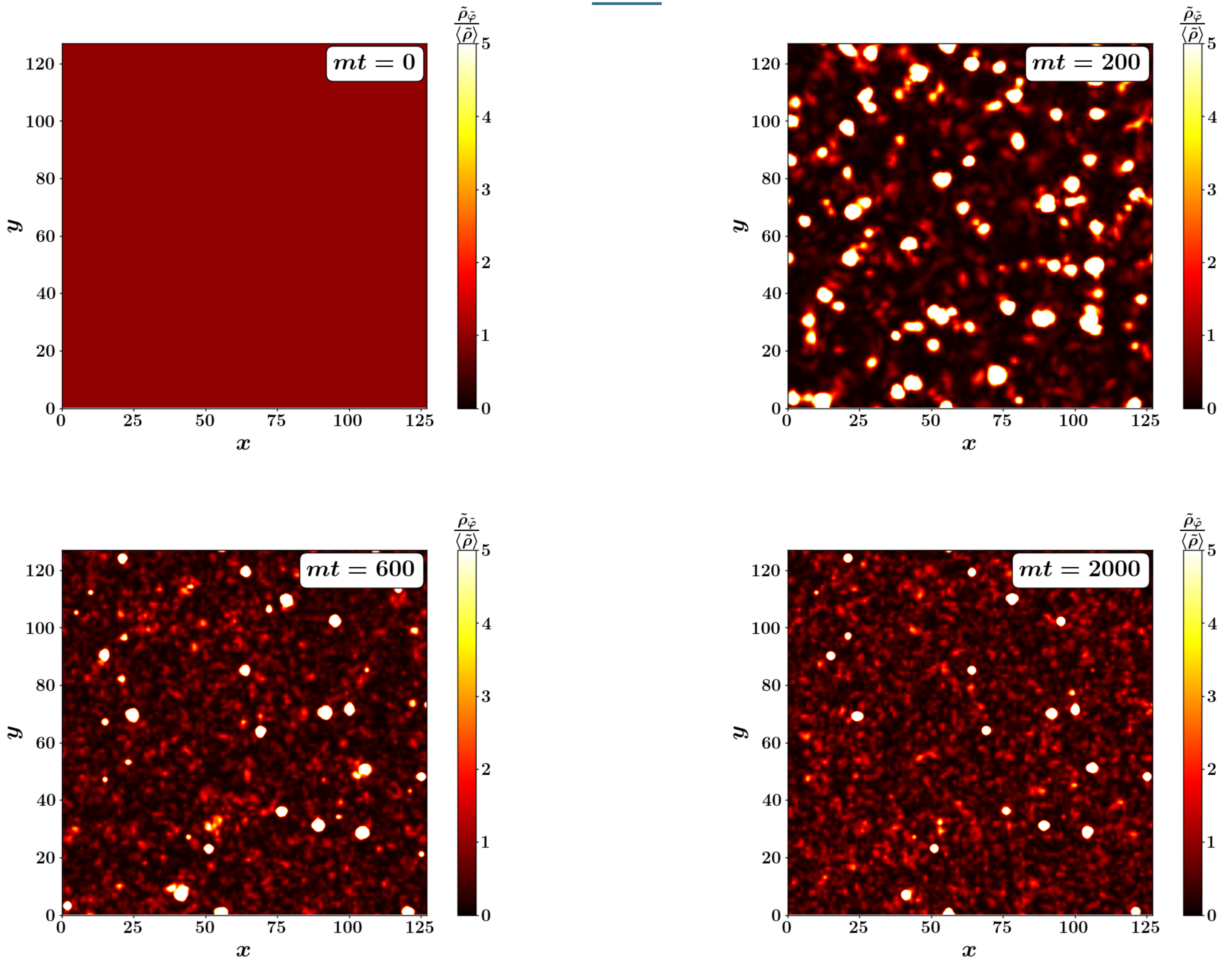
Symmetric

Oscillons : full non -linear evolution using CosmoLattice [Shafi et al 2024] - no external coupling

$$V(\varphi) = V_0 \left[1 - e^{-\lambda_E \frac{\varphi}{m_P}}\right]^2$$



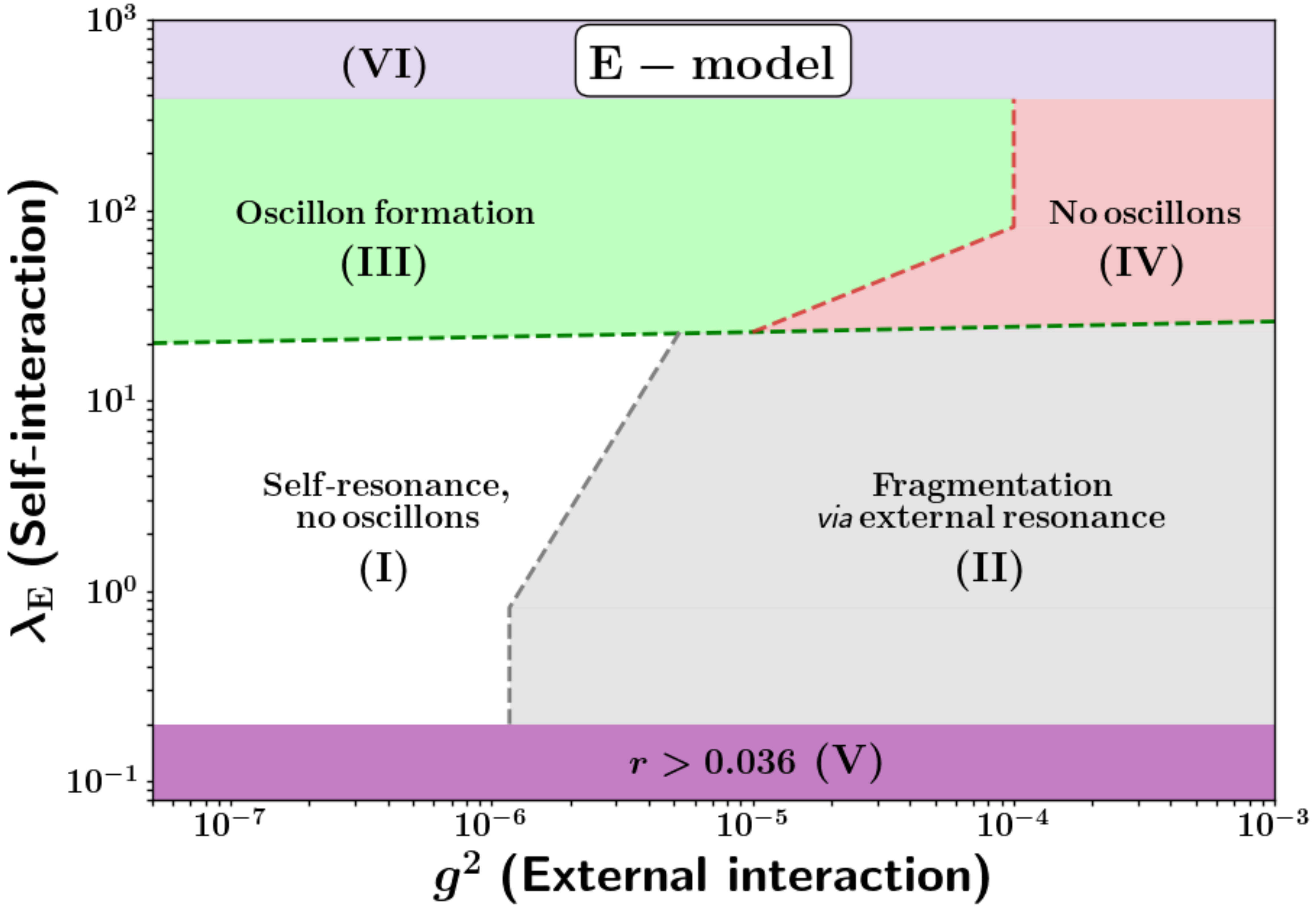
Oscillons formation - Asymmetric potential [Shafi et al 2024]

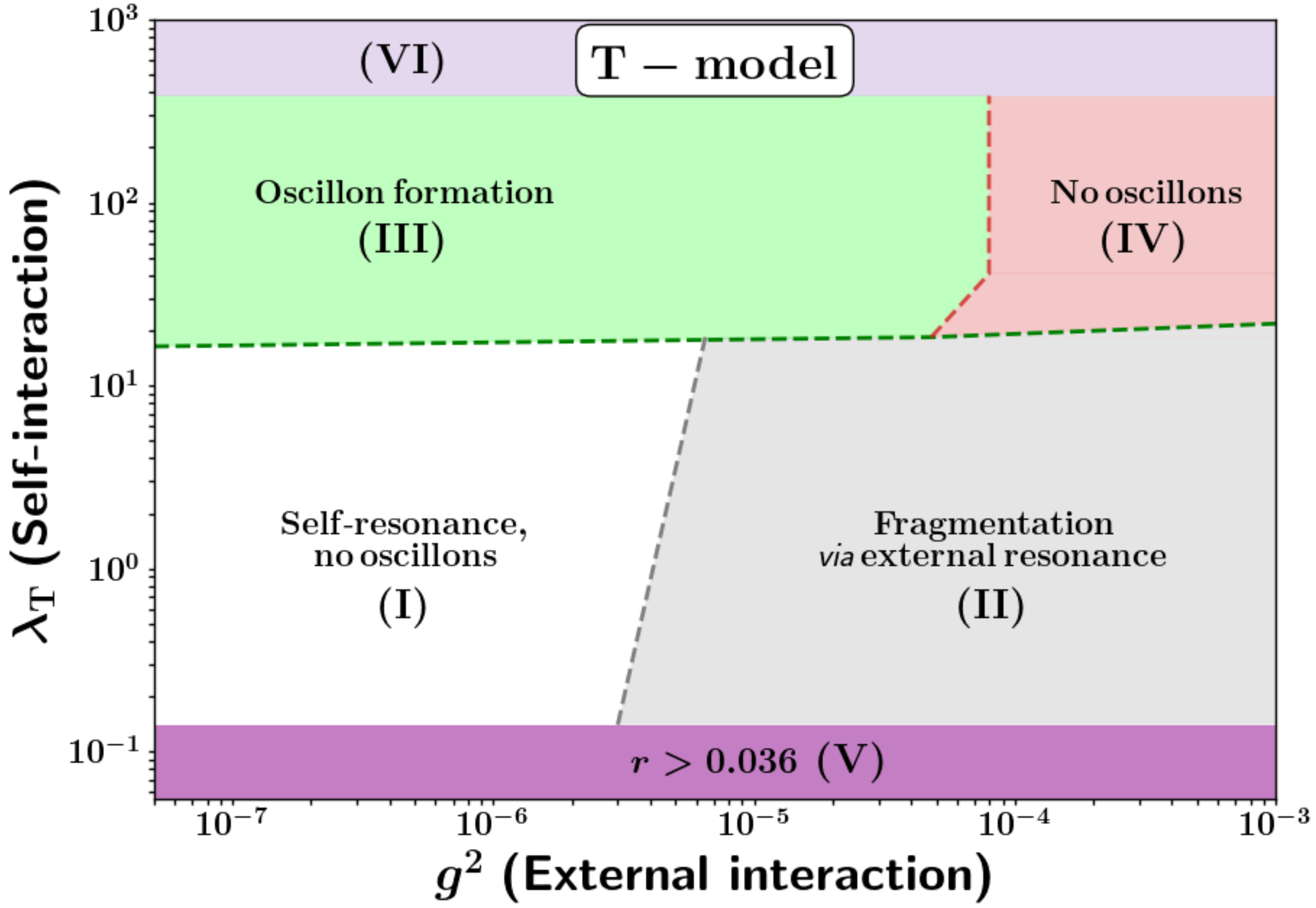


Oscillons in the presence of external coupling ? [Shafi et al 2024]

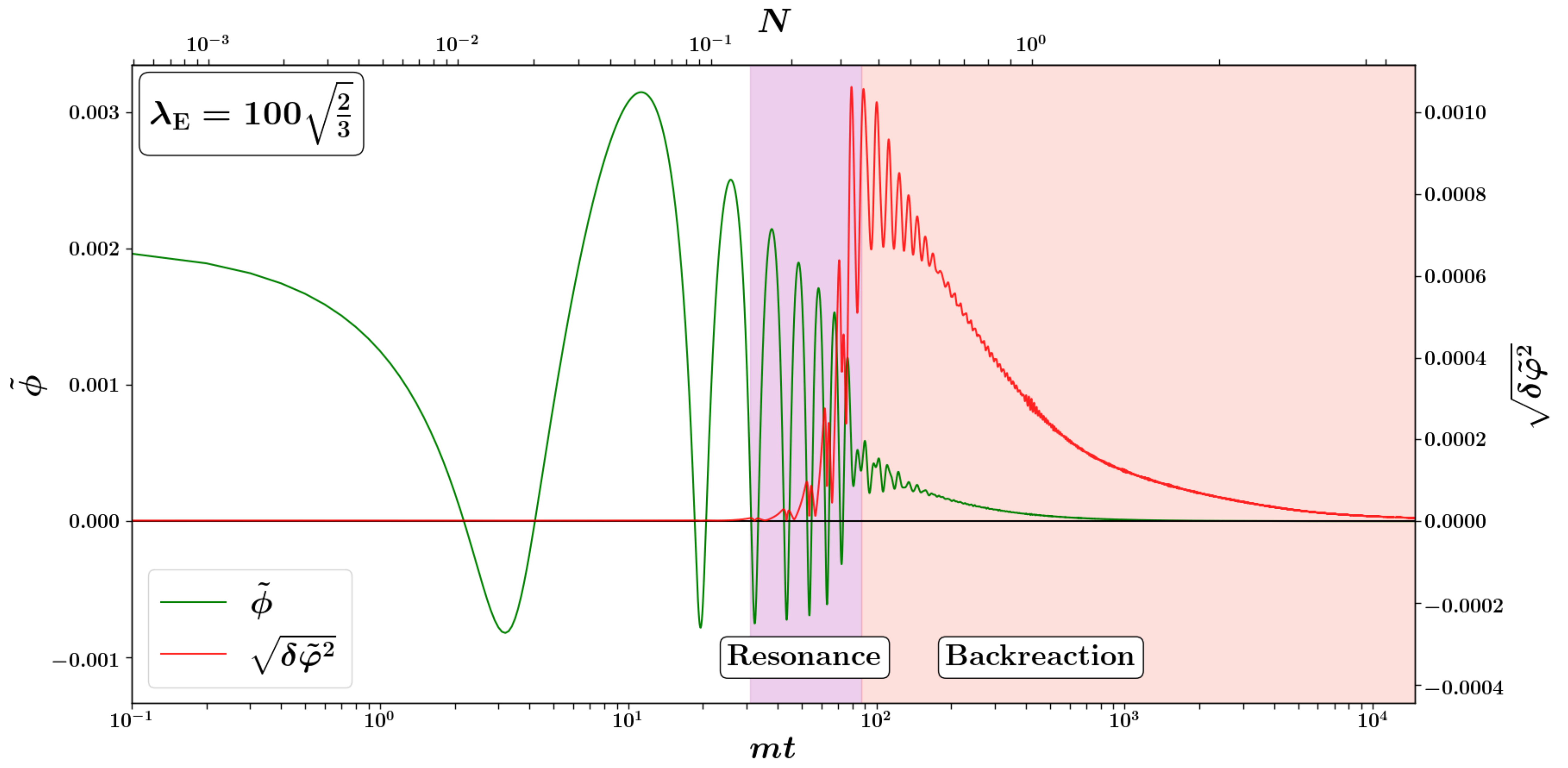
$$V(\varphi) = V_0 \left[1 - e^{-\lambda_E \frac{\varphi}{m_p}}\right]^2$$

$$V(\varphi) = V_0 \tanh^2\left(\lambda_T \frac{\varphi}{m_p}\right)$$

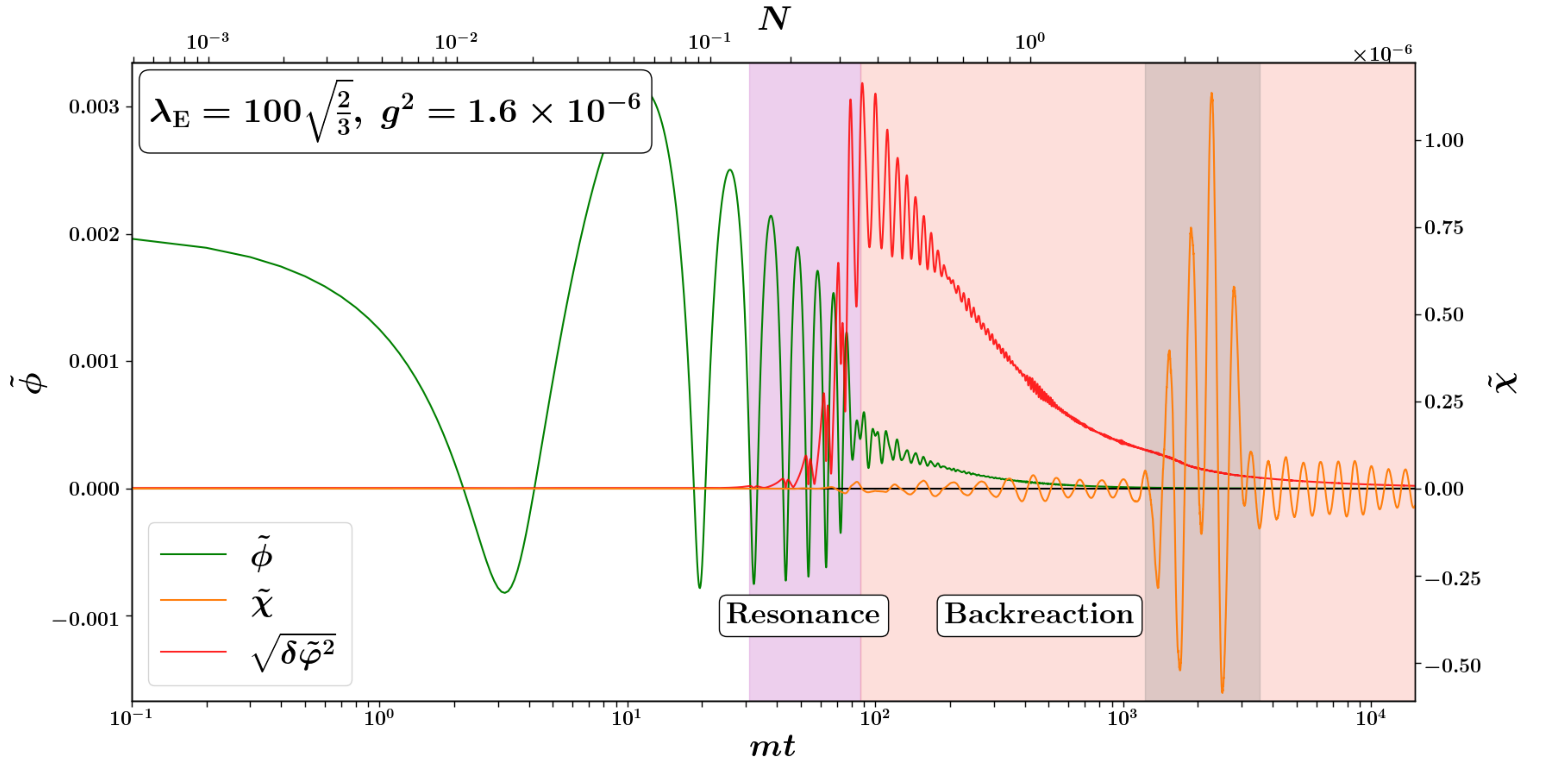




Oscillon formation and decay
with no external coupling

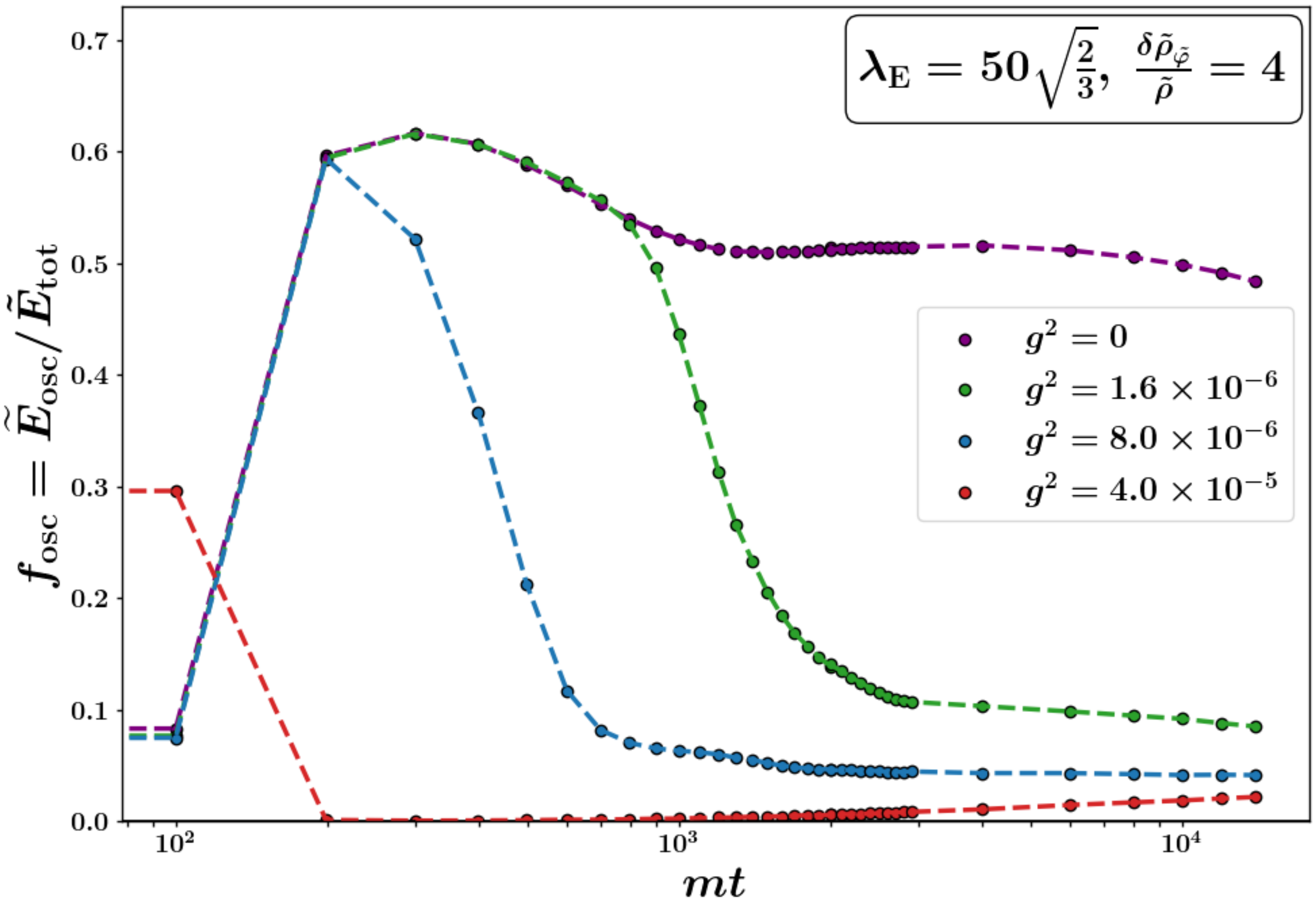


Oscillon formation and decay
with external coupling



[Shafi et al 2024]

Oscillon fractional abundance [Mahub and Mishra 2023]

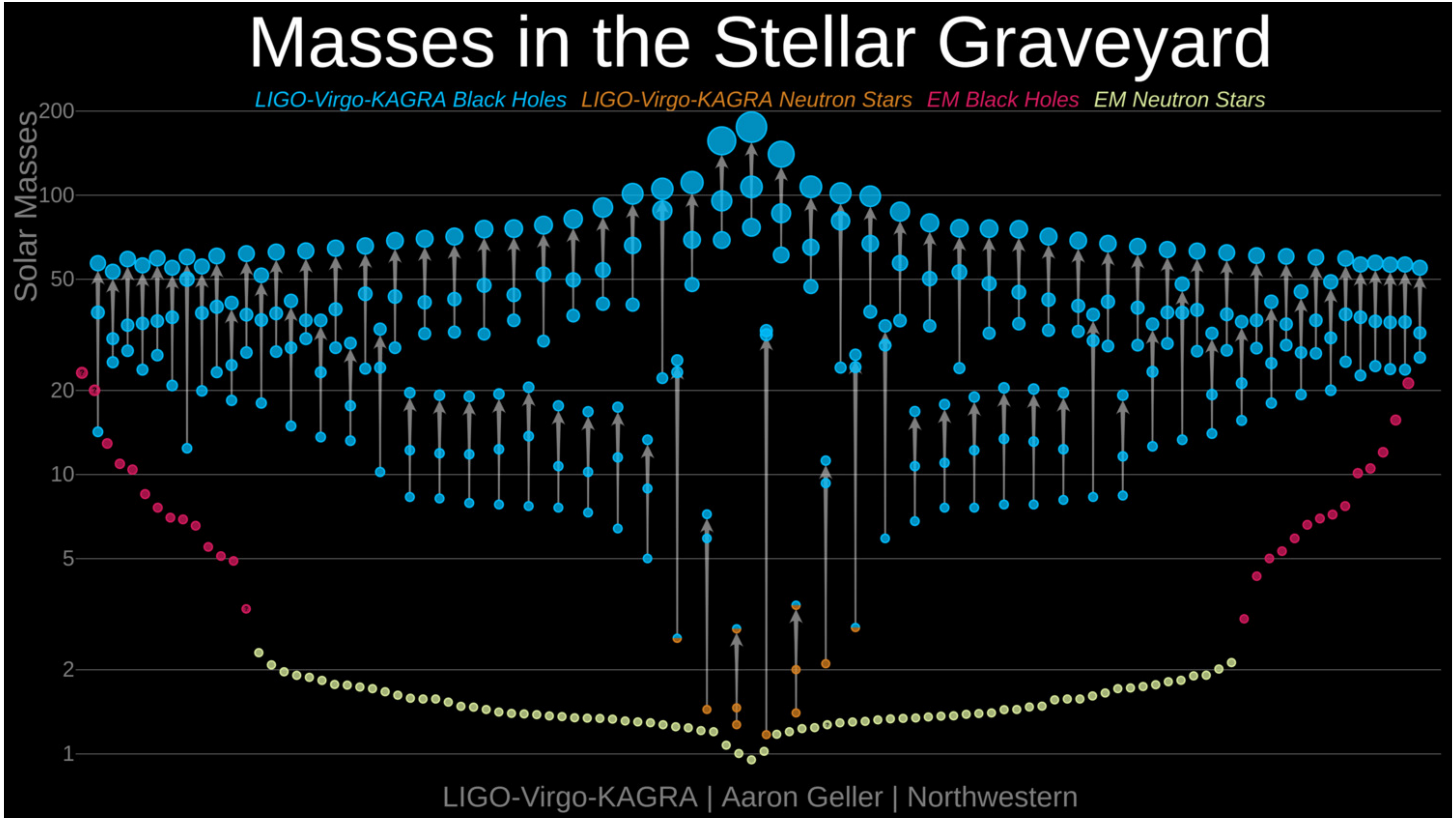


We find in the absence of external couplings oscillons form for both types of potentials and for generic initial conditions at the end of inflation

Inflation and Primordial Black Holes

$100 M_\odot$

$10 M_\odot$



Any of these primordial in origin ?

Credit: LIGO-Virgo-KAGRA consortium

Since LIGO's amazing direct detection of coalescing BH binaries, PBHs have had a resurgence of interest.

For reviews and future directions see Green & Kavanagh [arXiv: 2007.10722], Carr & Kuhnel [arXiv:2006.028380, Bird et al [arXiv:2203.08967]

Form from over densities in early Universe - before nucleosynthesis - non-baryonic [Zel'dovich & Novikov; Hawking]

They evaporate (Hawking radiation), lifetime longer than age of Universe for $M > 10^{15}$g — can make them a DM candidate [Hawking, Chapline]

Maybe some of the BHs in the binaries detected by LIGO-VIRGO are primordial [Bird et al, Clesse & Garcia-Bellido, Sasaki et al]

Formation

Favoured - collapse of large density perturbations (shortly after horizon entry) during radiation domination

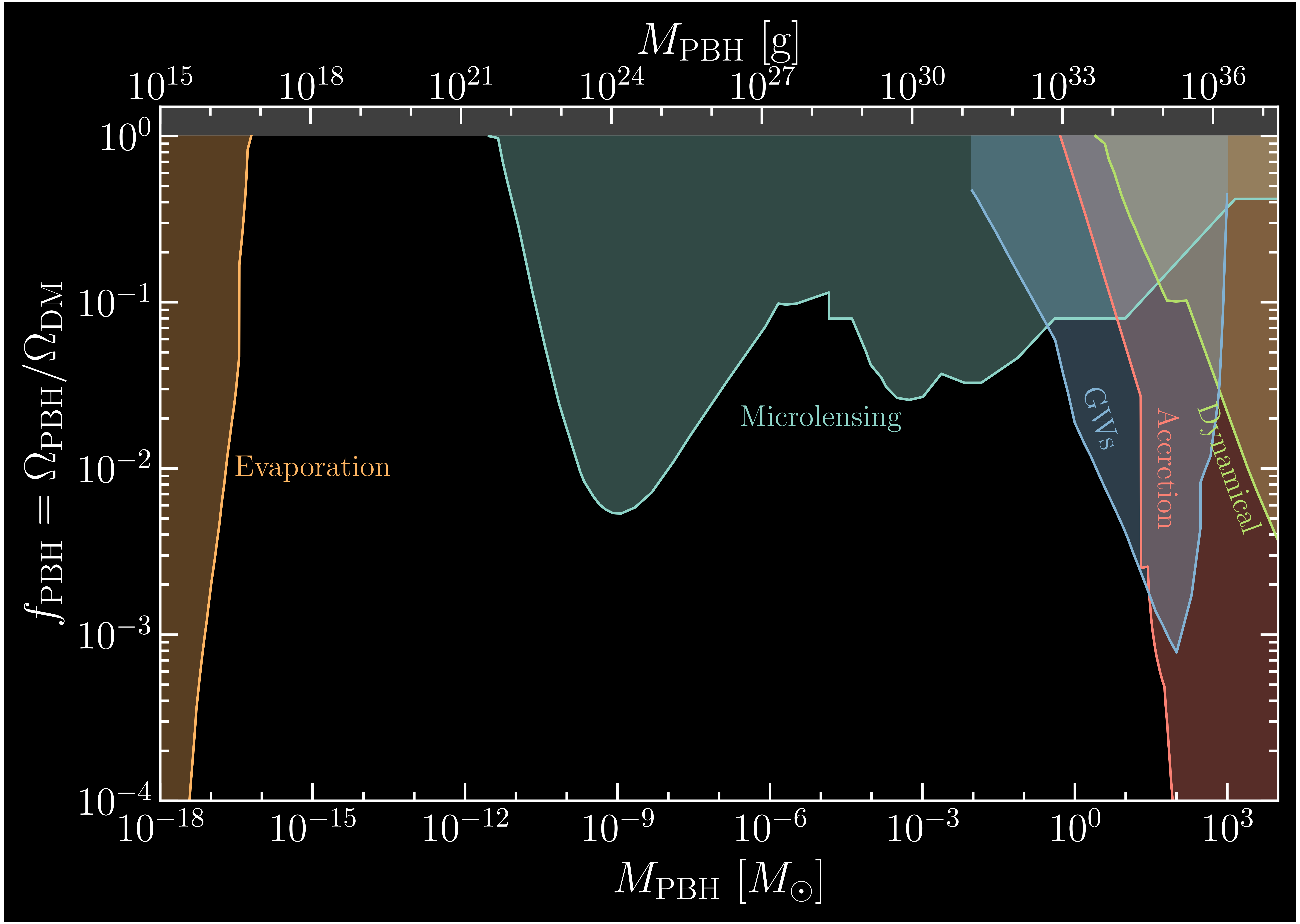
Also collapse of cosmic string loops [Hawking, Polnarev & Zemboricz], bubble collisions [Hawking, Moss & Stewart], fragmenting inflation condensates [Cotner & Kusenko]

Threshold for PBH formation [Carr] : $\delta \gtrsim \delta_c \sim w = p/\rho = 1/3$. — density contrast at horizon crossing, depends on shape of perturbation which depends on primordial power spectrum

PBH mass roughly equal to horizon mass

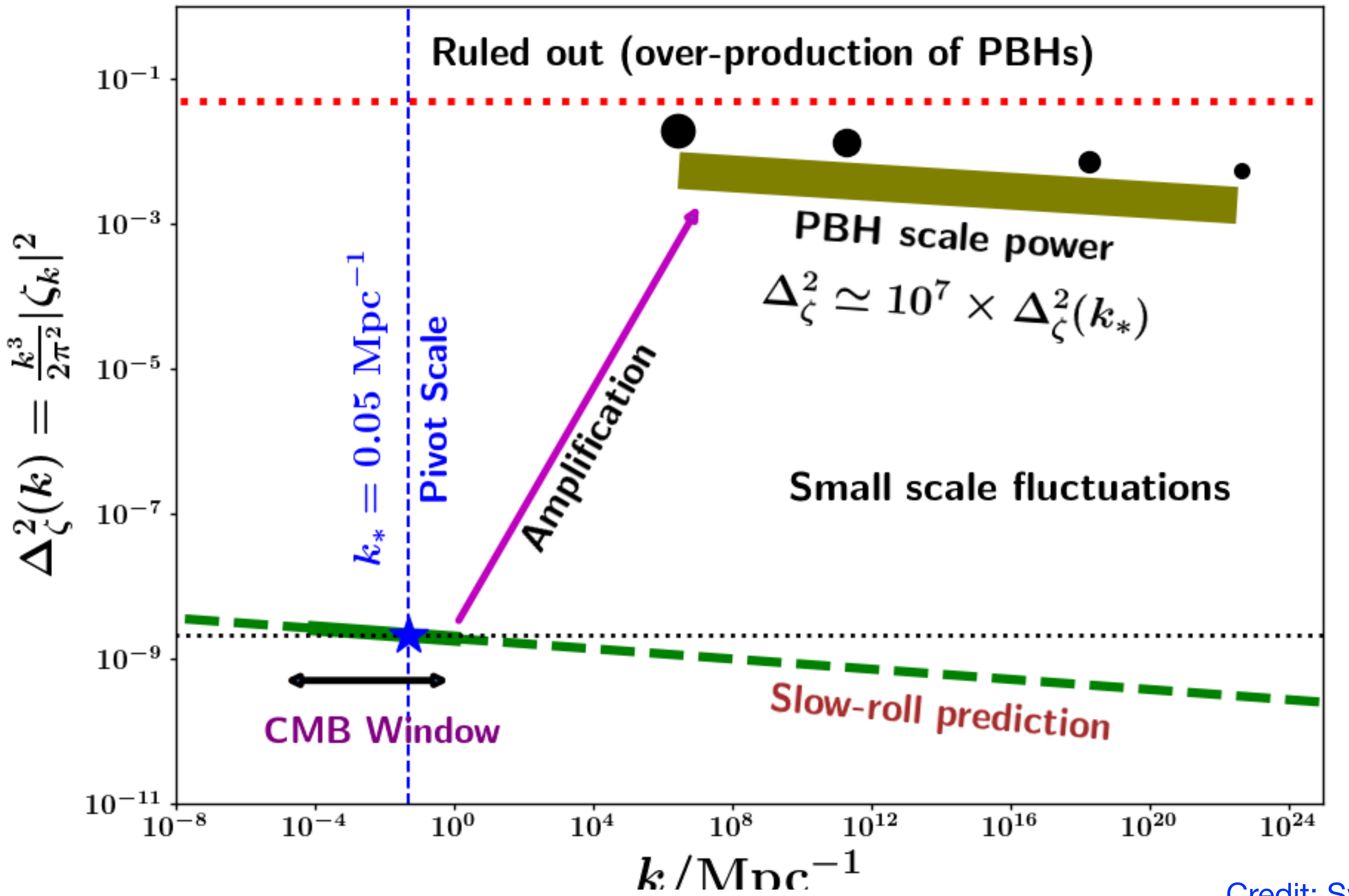
$$M_{\rm PBH} \sim 10^{15} g \left(\frac{t}{10^{-23}}\right) \sim M_{\rm sun} \left(\frac{t}{10^{-6} s}\right)$$

Present day bounds on PBHs as DM



Green and Kavanagh 2020

Required amplification for interesting PBH scenarios



Credit: Swagat Mishra

Primordial Black Holes are really really cool !

[Hawking 1971, Carr, Hawking 1974, Hawking 1974, Page 1975]

- Formed very early - typically within the first few seconds of the Hot Big Bang phase !
- We can use them to probe very small early Universe physics.
- Hawking told us, they have a temperature, and they evaporate as well as accrete.

- Hawking radiation - hard to detect.

$$\boxed{T_H = \frac{\hbar c^3}{8\pi G K_B M_{\rm BH}} = 6.19 \times 10^{-8} \left(\frac{M_\odot}{M_{\rm BH}}\right) \ K}$$

- Evaporation rate: $\frac{dm_{\rm BH}}{dt} = -\frac{g_\star}{3}\frac{m_{\rm Pl}^4}{m_{\rm BH}^2}$ ---> mass (t): $m_{\rm BH}^3 = m_0^2 - g_\star m_{\rm Pl}^4 t$ ---> lifetime: $\tau = \frac{m_0^3}{g_\star m_{\rm Pl}^4}$

- Initial mass of PBH evaporating today — about that of a mountain $M_c \simeq \left(\frac{t_0}{13.8 \ {\rm Gyr}}\right)^{\frac{1}{3}} \ 10^{15} \ {\rm gm}$

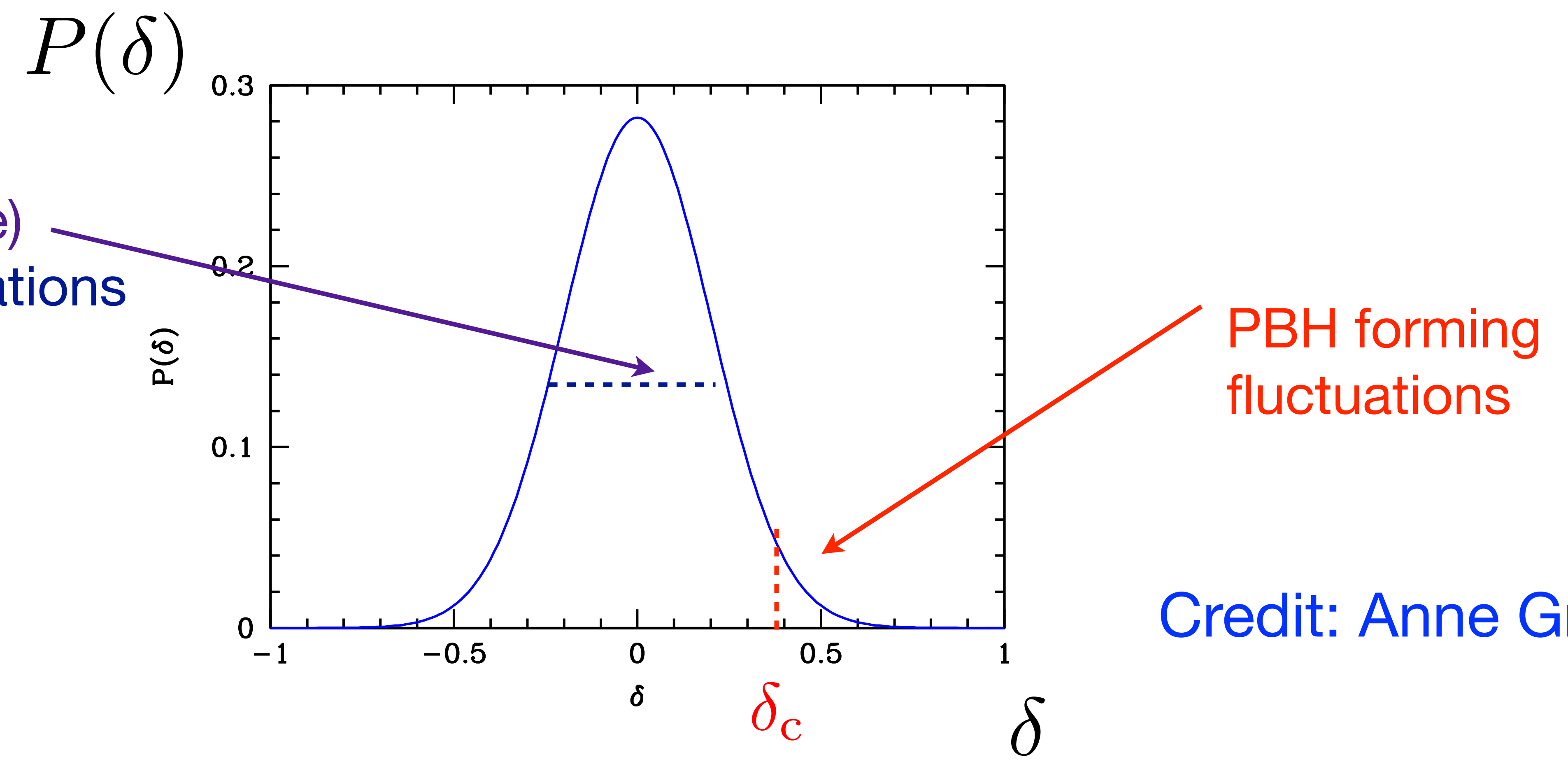
- Mass at formation $M_{\rm PBH} \simeq M_{\rm H} = 6 \times 10^4 \left(\frac{t}{1{\rm sec}}\right) M_{\rm sun}$ PBHs evaporating today formed around 10^{-23} sec into HBB phase

Initial PBH mass fraction (fraction of universe in regions dense enough to form PBHs)

$$\beta(M) \sim \int_{\delta_c}^{\infty} P(\delta(M_H)) d\delta(M_H)$$

For Gaussian probability distribution :

$$\beta(M) = \text{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma(M_H)}\right)$$



Credit: Anne Green

but in fact β must be small, hence $\sigma \ll \delta_c$ and $\beta(M) \sim \sigma(M_H) \exp\left(-\frac{\delta_c^2}{2\sigma^2(M_H)}\right)$

But PBH are matter, so in radiation their contribution to the energy density budget grows

$$\frac{\rho_{PBH}}{\rho_{rad}} \propto \frac{a^{-3}}{a^{-4}} \propto a$$

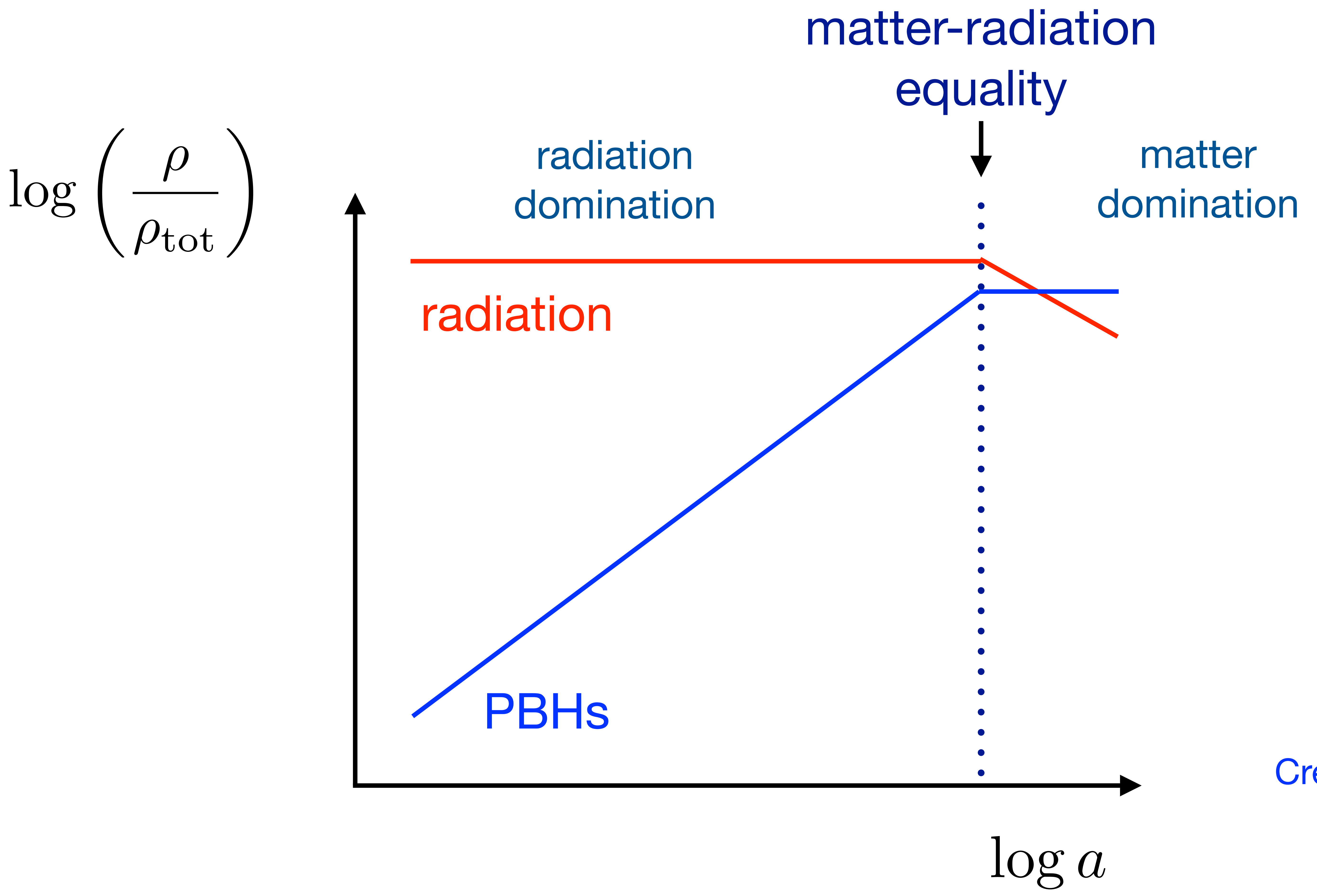
Relation between PBH initial mass function β and fraction of DM in form of PBHs, f:

$$\beta(M) \sim 10^{-9} f \left(\frac{M}{M_{sun}}\right)^{1/2}$$

So β must be small but non-negligible

But PBH are matter, so in radiation their contribution to the energy density budget grows

$$\frac{\rho_{\rm PBH}}{\rho_{\rm rad}} \propto \frac{a^{-3}}{a^{-4}} \propto a$$



Credit: Anne Green

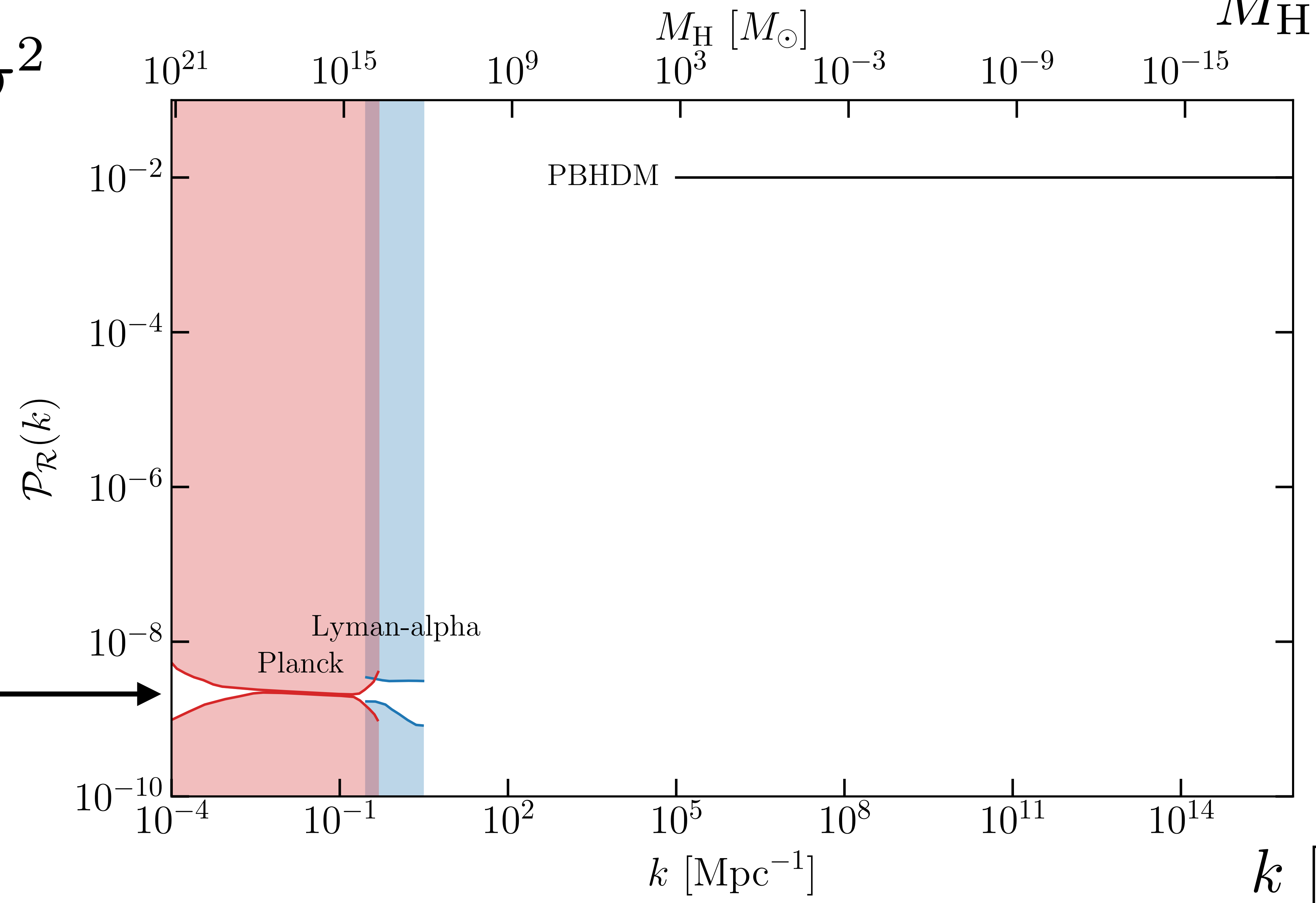
But on CMB we know primordial perturbations have amplitude $\sigma(M_H) \sim 10^{-5} \Rightarrow \beta(M) \sim \mathrm{erfc}(10^5) \sim \exp(-10^{10})$

Totally negligible if initial perturbations were close to scale invariant.

To form an interesting number of PBHs the primordial perturbations must be significantly larger ($\sigma^2(M_H)$~0.01) on small scales than on cosmological scales.

$M_H\ [M_\odot]$ ~ PBH mass

$\mathcal{P}_\mathcal{R}(k) \sim \sigma^2$



amplitude required for $f_{\mathrm{PBH}} \approx 1$

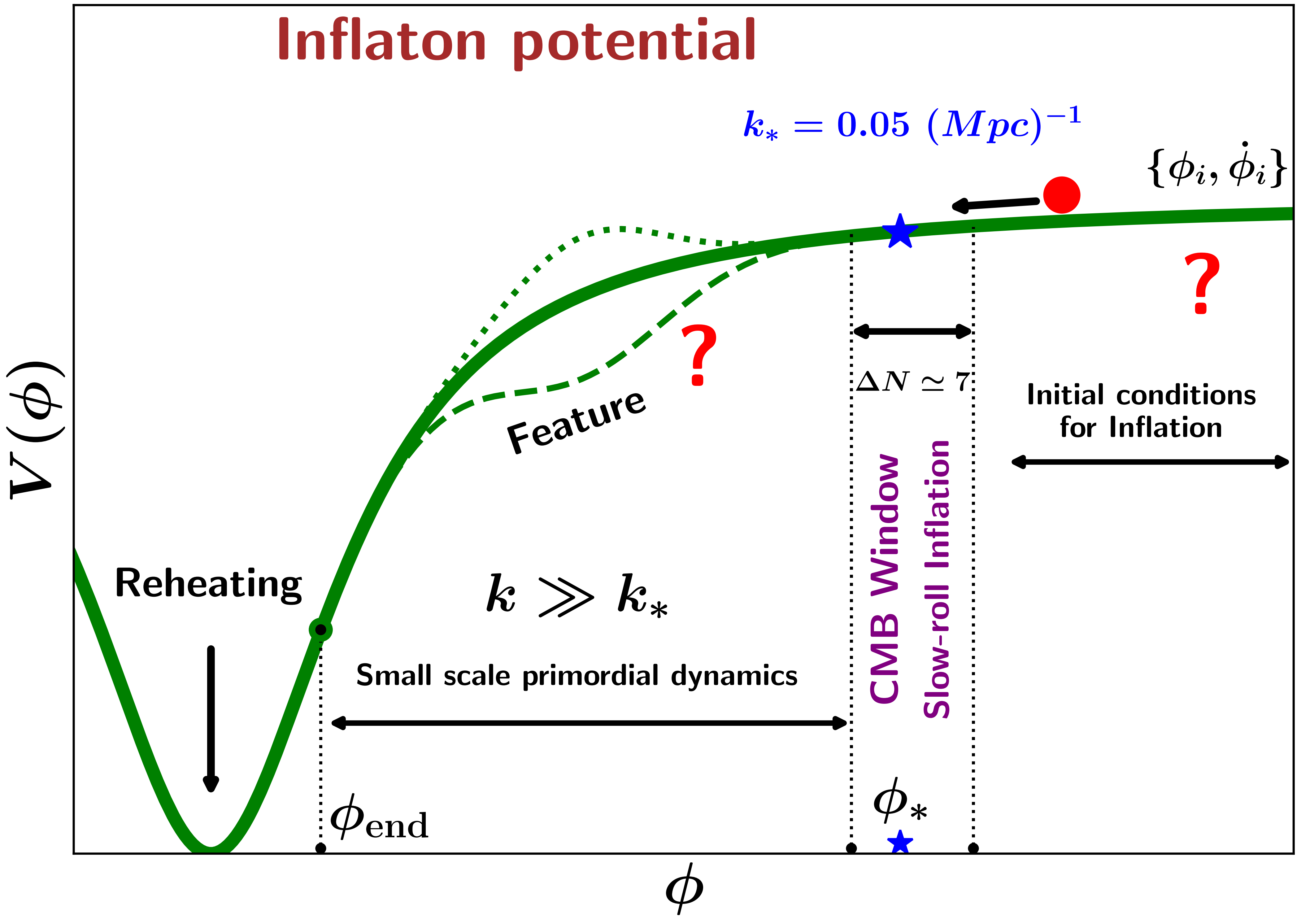
measured amplitude on cosmological scales

$k\ [\mathrm{Mpc}^{-1}]$

One approach — introduce non-gaussianity. PBHs form from rare large density fluctuations arising during inflation, change the shape of the tail of the probability distribution —> can significantly affect the PBH distribution

Credit: Anne Green

Inflation - brief recap



$$S[g_{\mu\nu},\phi] = \int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{m_p^2}{2}R - \frac{1}{2}\partial_\mu\phi\,\partial_\nu\phi\,g^{\mu\nu} - V(\phi)\right]$$

$$H^2 \equiv \frac{1}{3m_p^2}\rho_\phi = \frac{1}{3m_p^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\dot{H} \equiv \frac{\ddot{a}}{a} - H^2 = -\frac{1}{2m_p^2}\dot{\phi}^2\,,$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0\,.$$

$$\epsilon_H = -\frac{\dot{H}}{H^2} = \frac{1}{2m_p^2}\frac{\dot{\phi}^2}{H^2}\,,$$

$$\eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}} = \epsilon_H + \frac{1}{2\epsilon_H}\frac{\mathrm{d}\epsilon_H}{\mathrm{d}N}\,,$$

Slow roll parameters

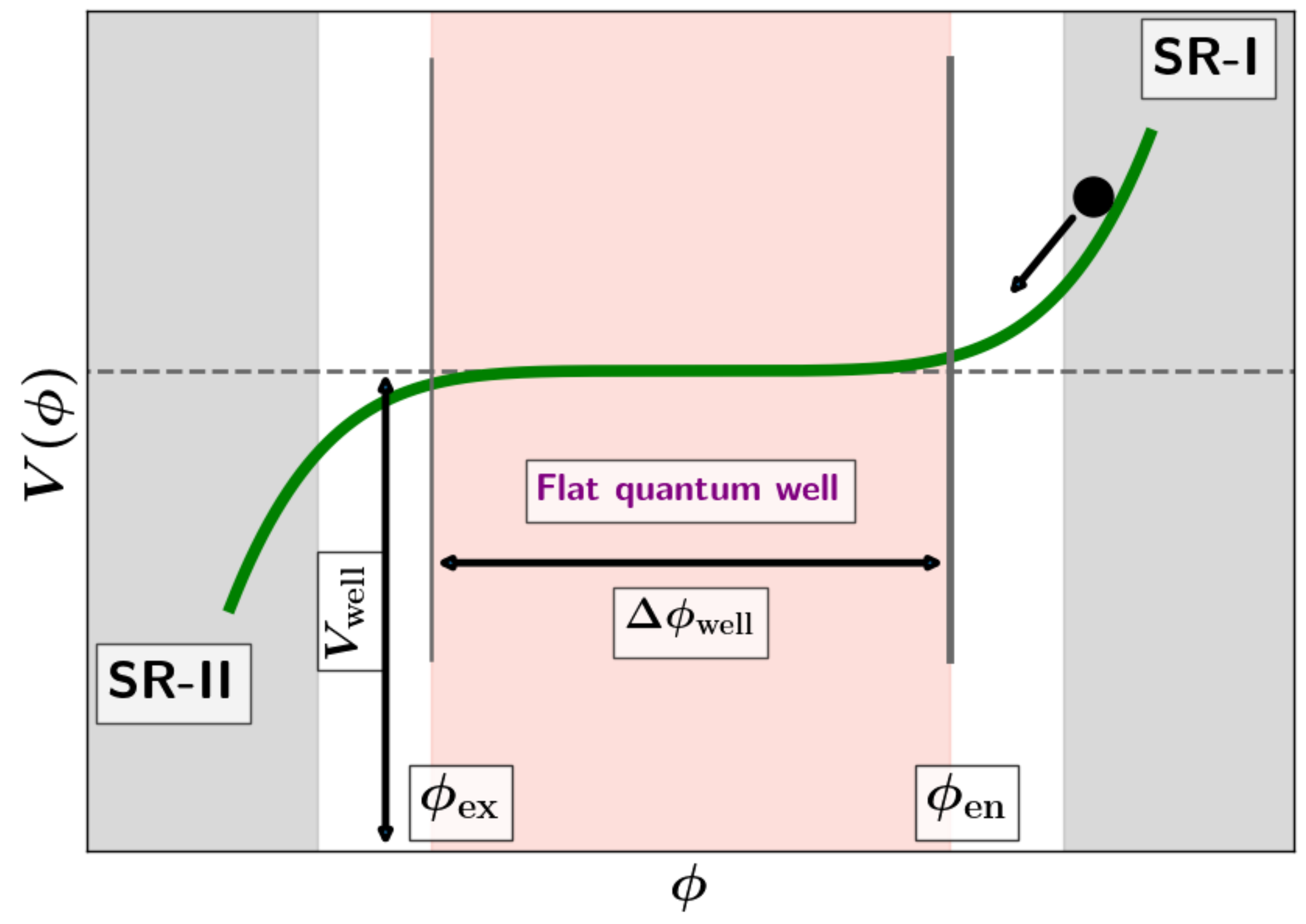
- **Quasi-de Sitter** inflation corresponds to the condition $\epsilon_H \ll 1$.
- **Slow-roll** inflation corresponds to both $\epsilon_H,\ \eta_H \ll 1$.

Introducing features into the inflaton potential - to generate the PBH abundance

Inflaton potential featuring **an approximate inflection point** or **a local bump/dip** at **low scales** slows down the inflaton leading to appreciable enhancement of scalar power-spectrum

$$P_\zeta = \frac{1}{8\pi^2}\left(\frac{H}{m_p}\right)^2 \frac{1}{\epsilon_H} \qquad \epsilon_H = \frac{1}{2m_p^2}\frac{\dot{\phi}^2}{H^2}$$

.



PBH formation requires enhancement of the inflationary power spectrum by a factor of 10^7 within less than 40 e-folds of expansion, the quantity Δ ln ε /Δ N, hence $|\eta_H|$ must grow to be of order unity, so violate the second slow roll condition. A flat plateau like region in the potential can allow this.

Ultra Slow roll inflation [Kinney (2005), Inoue and Yokoyama (2002)]

At intermediate field values, inflaton enters a transient period of USR. Since V'(ϕ)~0,

$$\ddot{\phi} + 3H\dot{\phi} = 0 \Rightarrow -\ddot{\phi}/H\dot{\phi} = +3, \quad \text{hence} \quad \eta_H = +3 \quad \text{(during USR)}$$

Inflaton speed drops exponentially with number of e-folds : $\dot{\phi} = \dot{\phi}_{\rm en}\, e^{-3\,H\,(t-t_{\rm en})} \propto e^{-3\,N}$

Critical entry velocity to just get across the plateau

$$\dot{\phi}_{\rm cr} = -3\,H\,\Delta\phi_{\rm well}, \qquad \pi_{\rm cr} = -3\,\Delta\phi_{\rm well}. \qquad \pi = \frac{{\rm d}\phi}{{\rm d}N} = \frac{\dot{\phi}}{H}$$

Quantum dynamics — stochastic inflation formalism - non - perturbative approach to calc the full primordial PDF [Starobinsky 1982]

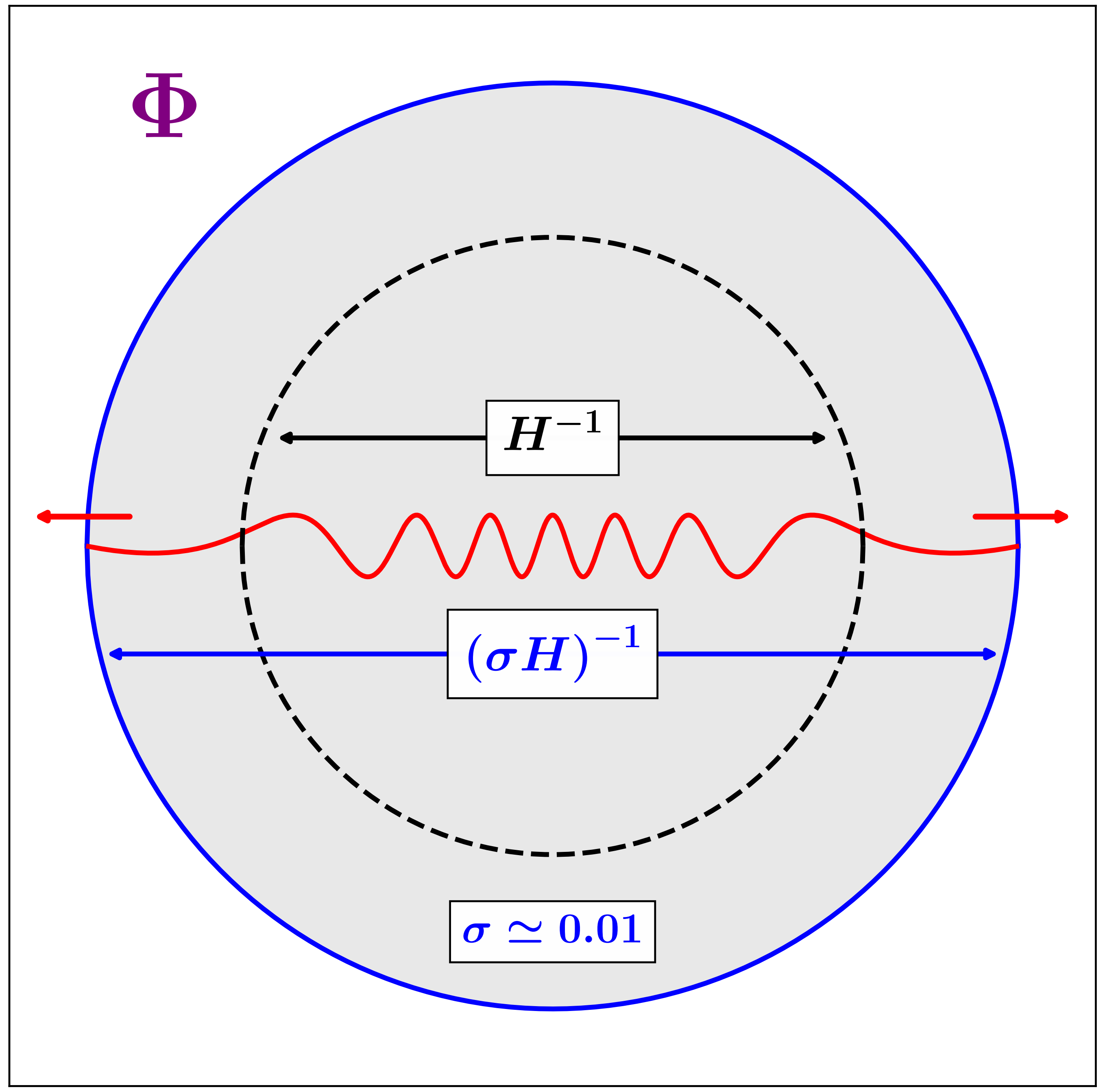
Effective long wavelength IR treatment of inflation, inflaton field is coarse grained over super Hubble scales k ≲ σaH, with const $\sigma \ll 1$.

Hubble exiting smaller scale UV modes are constantly converted into IR modes due to accelerated expansion.

Coarse grained inflaton field follows a Langevin-type-stochastic differential equation with stochastic noise terms sourced by the smaller scale UV modes, on top of classical drift terms sourced by V'(ϕ).

Split the Heisenberg operators of the inflaton $\hat{\phi}(N, \vec{x})$ and its conjugate momentum $\hat{\pi}_\phi = \mathrm{d}\hat{\phi}/\mathrm{d}N$ into the corresponding IR $\{\hat{\Phi}, \hat{\Pi}\}$ and UV $\{\hat{\varphi}, \hat{\pi}\}$ parts:

$$\hat{\phi} = \hat{\Phi} + \hat{\varphi} \quad , \quad \hat{\pi}_\phi = \hat{\Pi} + \hat{\pi}$$



Credit: Swagat Mishra

where the UV fields are defined as

$$\hat{\varphi}(N, \vec{x}) = \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^{\frac{3}{2}}} W\left(\frac{k}{\sigma a H}\right) \left[\phi_k(N)\, \hat{a}_{\vec{k}}\, e^{-i\vec{k}.\vec{x}} + \phi_k^*(N)\, \hat{a}_{\vec{k}}^\dagger\, e^{i\vec{k}.\vec{x}}\right]$$

$$\hat{\pi}(N, \vec{x}) = \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^{\frac{3}{2}}} W\left(\frac{k}{\sigma a H}\right) \left[\pi_k(N)\, \hat{a}_{\vec{k}}\, e^{-i\vec{k}.\vec{x}} + \pi_k^*(N)\, \hat{a}_{\vec{k}}^\dagger\, e^{i\vec{k}.\vec{x}}\right]$$

$W(k/\sigma a H)$ is the 'window function' Selects out modes with momentum k>σaH

With Swagat Mishra and Anne Green - e-Print:2303.17375 - JCAP 2023

Hamiltonian equations for coarse grained (IR) fields are Langevin equation

$$\frac{\mathrm{d}\hat{\Phi}}{\mathrm{d}N} = \hat{\Pi} + \hat{\xi}_\phi(N) \ ,$$

$$\frac{\mathrm{d}\hat{\Pi}}{\mathrm{d}N} = -\left(3-\epsilon_H\right)\hat{\Pi} - \frac{V_{,\phi}(\hat{\Phi})}{H^2} + \hat{\xi}_\pi(N) \ ,$$

where the field and momentum noise operators $\hat{\xi}_\phi(N)$ and $\hat{\xi}_\pi(N)$ are given by

$$\hat{\xi}_\phi(N) = -\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^{\frac{3}{2}}} \frac{d}{\mathrm{d}N} W\left(\frac{k}{\sigma a H}\right) \left[\phi_k(N)\,\hat{a}_{\vec{k}}\, e^{-i\vec{k}.\vec{x}} + \phi_k^*(N)\,\hat{a}_{\vec{k}}^\dagger\, e^{i\vec{k}.\vec{x}}\right]$$

$$\hat{\xi}_\pi(N) = -\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^{\frac{3}{2}}} \frac{d}{\mathrm{d}N} W\left(\frac{k}{\sigma a H}\right) \left[\pi_k(N)\,\hat{a}_{\vec{k}}\, e^{-i\vec{k}.\vec{x}} + \pi_k^*(N)\,\hat{a}_{\vec{k}}^\dagger\, e^{i\vec{k}.\vec{x}}\right]$$

Assume Window function with sharp IR/UV cut-off

$$W\left(\frac{k}{\sigma a H}\right) = \Theta\left(\frac{k}{\sigma a H} - 1\right) .$$

- Physically, the noise terms $\hat{\xi}_\phi$ and $\hat{\xi}_\pi$ in the Langevin equations are sourced by the constant outflow of UV modes into the IR modes
- As UV mode exits the cut-off scale $k = \sigma a H$ to become part of the IR field on super-Hubble scales, IR field receives a '*quantum kick*' with typical amplitude $\sim \sqrt{\langle 0|\hat{\xi}(N)\hat{\xi}(N')|0\rangle}$, where $|0\rangle$ is usually taken to be the Bunch-Davies vacuum.
- Given that $\sigma \ll 1$, this happens on ultra super-Hubble scales, where the UV modes must have already become classical fluctuations..

With $\xi_i = \{\xi_\phi, \xi_\pi\}$, equal-space noise correlators (auto-correlators) are

$$\langle \xi_i(N)\, \xi_j(N') \rangle = \Sigma_{ij}(N)\, \delta_D(N - N'),$$

where the noise correlation matrix Σ_{ij} is

$$\Sigma_{ij}(N) = (1 - \epsilon_H)\, \frac{k^3}{2\pi^2}\, \phi_{i_k}(N) \phi^*_{j_k}(N) \Big|_{k=\sigma a H} .$$

The noise correlation matrix is important !

Equivalent Fokker-Planck equation - time evolution of the PDF of $\{\Phi, \Pi\}$, subject to appropriate bcds.

$$\frac{\partial}{\partial \mathcal{N}} P_{\Phi_i}(\mathcal{N}) = \left[D_i \frac{\partial}{\partial \Phi_i} + \frac{1}{2} \Sigma_{ij} \frac{\partial^2}{\partial \Phi_i \partial \Phi_j} \right] P_{\Phi_i}(\mathcal{N})$$

where

$$D_i = \left\{ \Pi, -(3 - \epsilon_H)\, \Pi - \frac{V_{,\phi}(\Phi)}{H^2} \right\}$$

1. Absorbing boundary at $\phi^{(\mathrm{A})}$

$$P_{\Phi=\phi^{(\mathrm{A})},\Pi}(\mathcal{N}) = \delta_D(\mathcal{N}) ,$$

Closer to ϕ at end of inflation

2. Reflecting boundary at $\phi^{(\mathrm{R})}$

$$\frac{\partial}{\partial \Phi} P_{\Phi=\phi^{(\mathrm{R})},\Pi}(\mathcal{N}) = 0 .$$

Closer to ϕ at cmb scale

Characteristic function: $\chi_{\mathcal{N}}(q;\Phi_i)$, given by Fourier transform of the PDF $P_{\Phi_i}(\mathcal{N})$

$$\chi_{\mathcal{N}}(q;\Phi_i) \equiv \langle e^{i\,q\,\mathcal{N}} \rangle = \int_{-\infty}^{\infty} e^{i\,q\,\mathcal{N}} P_{\Phi_i}(\mathcal{N})\,\mathrm{d}\mathcal{N}\,,$$

CF then satisfies

$$\left[D_i \frac{\partial}{\partial \Phi_i} + \frac{1}{2}\Sigma_{ij}\frac{\partial^2}{\partial\Phi_i\partial\Phi_j} + iq \right] \chi_{\mathcal{N}}(q;\Phi_i) = 0\,,$$

with bcs

$$\chi_{\mathcal{N}}(q;\phi^{(\mathrm{A})},\Pi) = 1\,, \quad \frac{\partial}{\partial\Phi}\chi_{\mathcal{N}}(q;\phi^{(\mathrm{R})},\Pi) = 0\ .$$

Usual approach: assume noise matrix elements Σ_{ij} are of the de Sitter-type:

$$\Sigma_{\phi\phi} = (H/2\pi)^2\,, \qquad \Sigma_{\phi\pi},\,\Sigma_{\pi\pi} \simeq 0\,.$$

Quantum diffusion across a flat segment of the inflaton potential [Pattsion et al 2021]. Intro

$$f = \frac{\Phi - \phi_{\mathrm{ex}}}{\Delta\phi_{\mathrm{well}}}\,, \quad y = \frac{\Pi}{\pi_{\mathrm{cr}}}\,, \quad \mu^2 \simeq \frac{\Delta\phi_{\mathrm{well}}^2}{m_p^2}\frac{1}{v_{\mathrm{well}}}\,, \quad v_{\mathrm{well}} = V_{\mathrm{well}}/m_p^4\,,$$

f is the fraction of the flat well which remains to be traversed; y is the momentum relative to the critical momentum, V_{well} is the height of the flat quantum well.

Free stochastic diffusion : $\pi_{\rm en} \ll \pi_{\rm cr} \Rightarrow y_{\rm en} \ll 1 \longrightarrow$ the classical drift term can be ignored [Ezquiaga et al [2020], Pattison et al [2021]]

$$P_f(\mathcal{N}) = \sum_{n=0}^{\infty} A_n \sin\left[(2n+1)\,\frac{\pi}{2}\,f\right] \mathrm{e}^{-\Lambda_n \mathcal{N}} \,,$$

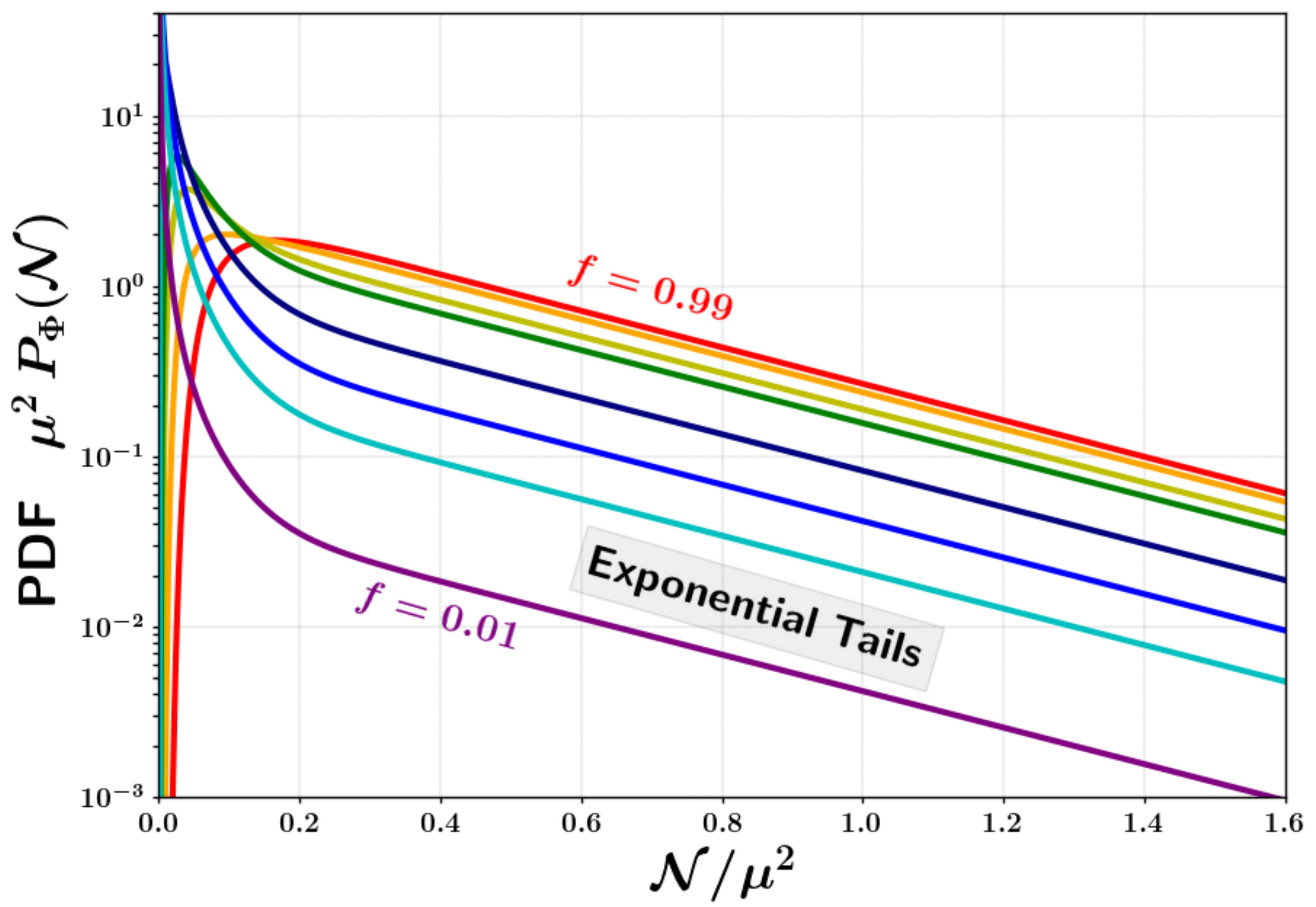
where

$$A_n = (2n+1)\frac{\pi}{\mu^2}\,, \qquad \Lambda_n = (2n+1)^2\frac{\pi^2}{4}\frac{1}{\mu^2}\,,$$

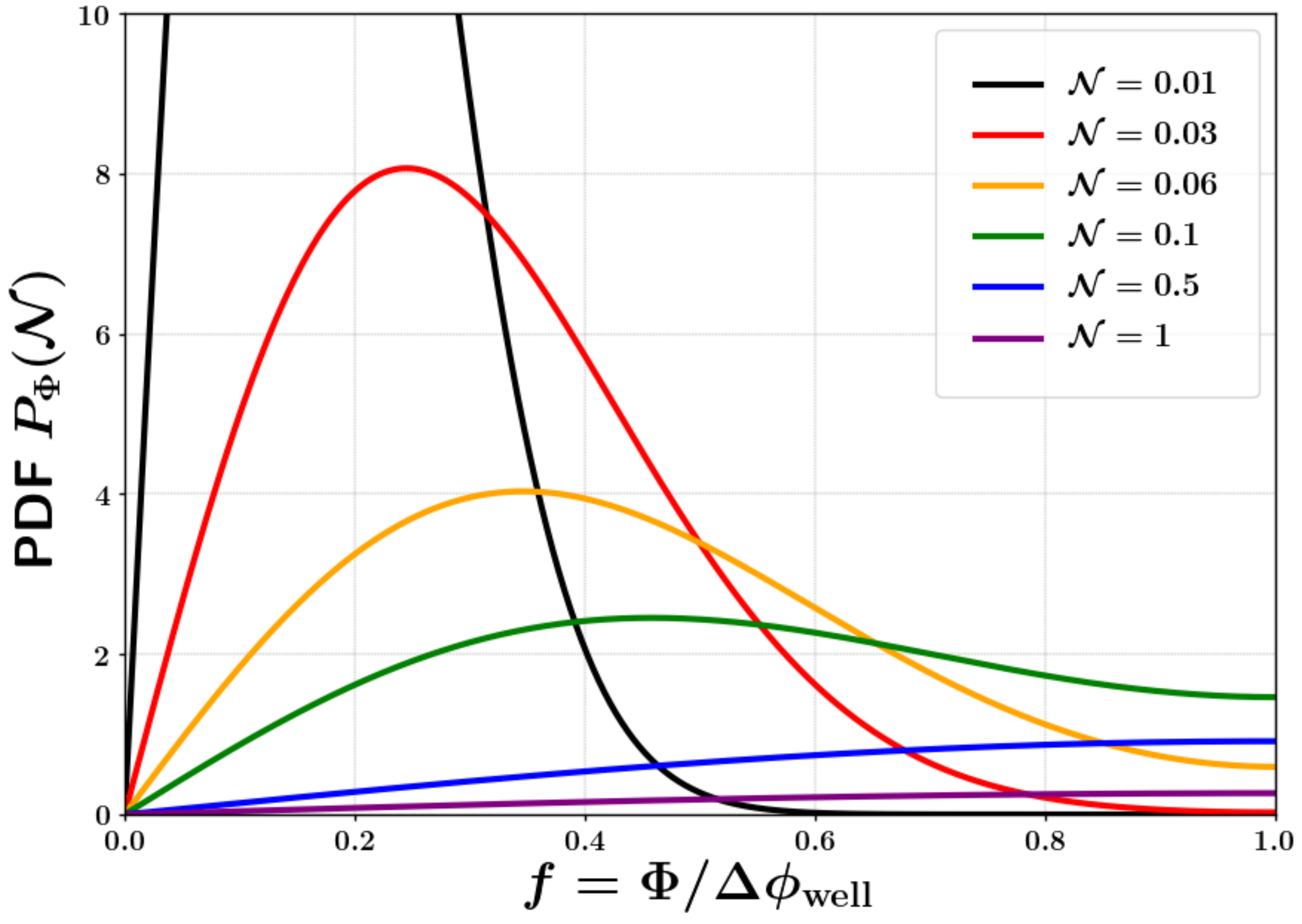
For $\mathcal{N} \gg 1$, PDF has an exponential tail

$$P_\Phi(\mathcal{N}) \simeq A_0\, e^{-\Lambda_0 \mathcal{N}}\,.$$

Changed shape of PDF from Gaussian in the tail



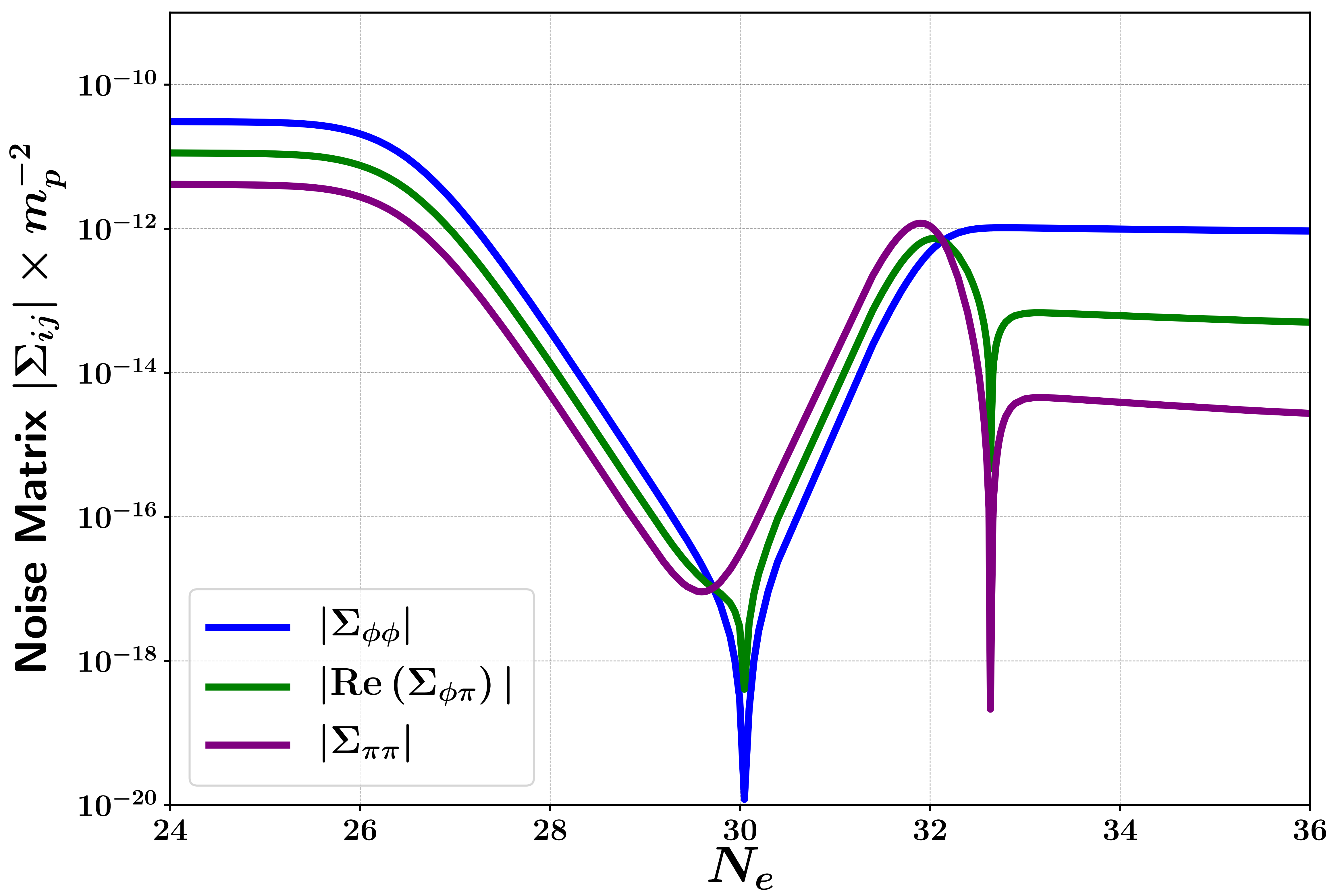
(a)

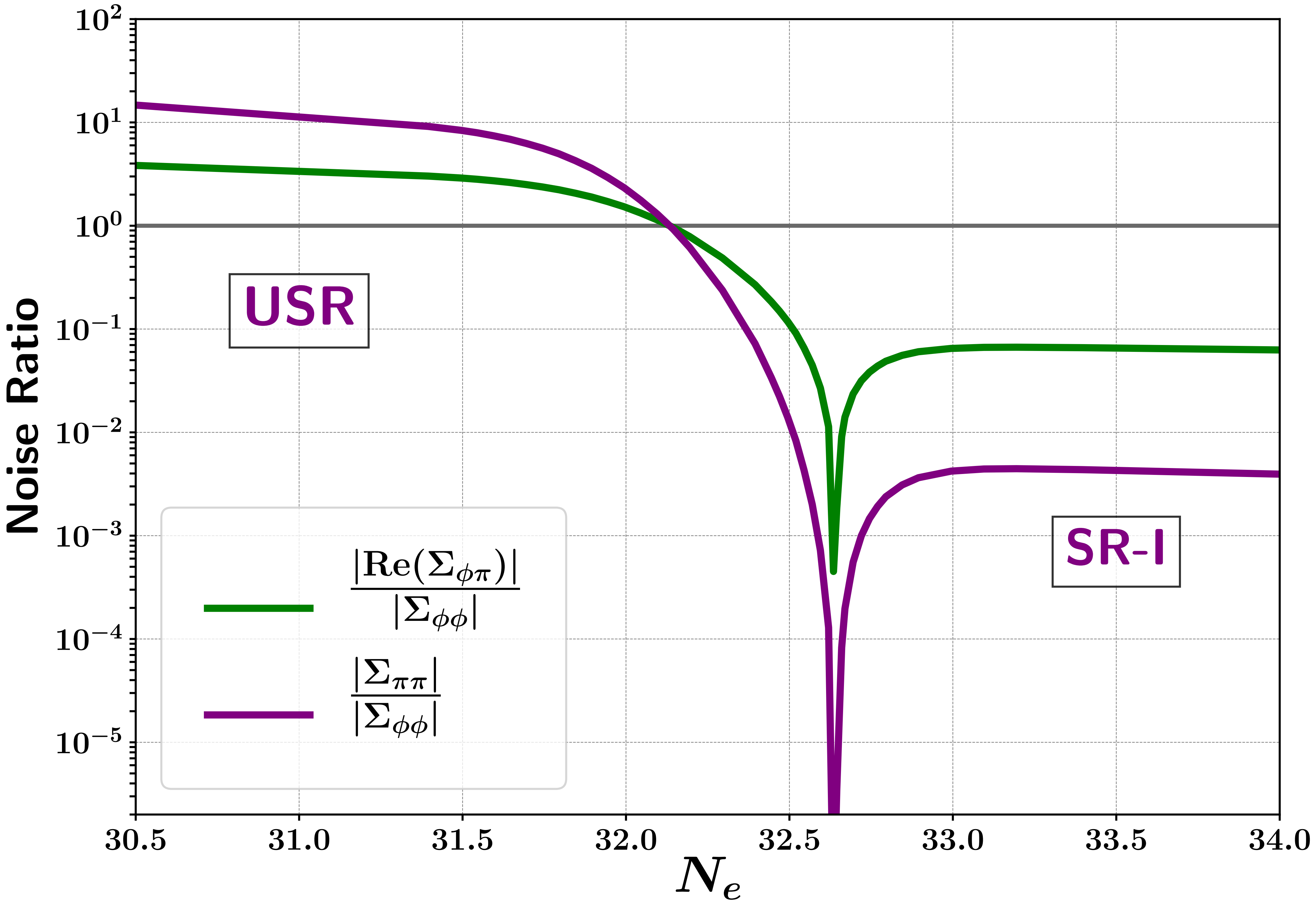


(b)

In reality — noise terms are more interesting !

Numerical noise matrix elements, Σ_{ij} - note the switching of dominant terms during USR





Noise ratios in the SR-1 to USR region

With Swagat Mishra and Anne Green - e-Print:2303.17375 - JCAP 2024

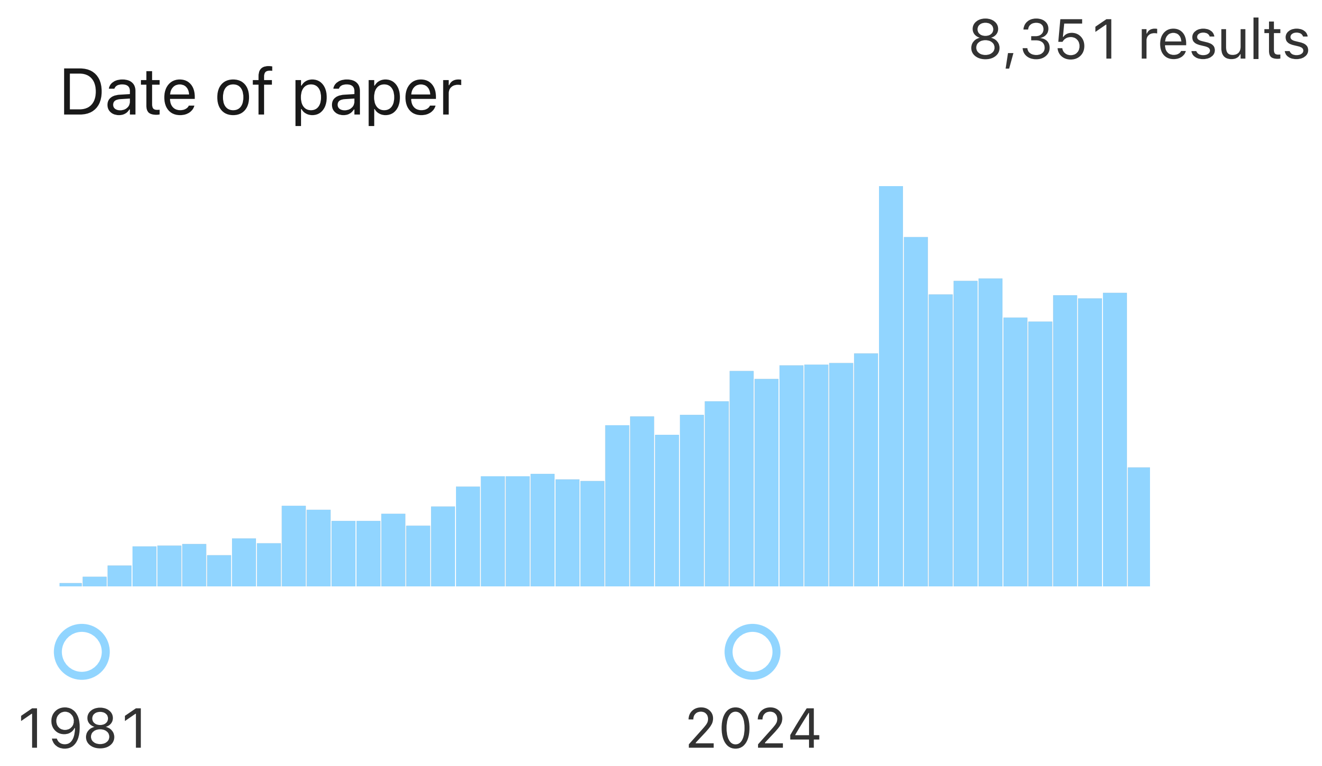
Fixed $M = 0.5\, m_p$, and bump parameters to be $A = 1.87 \times 10^{-3}$, $\tilde{\sigma} = 1.993 \times 10^{-2}$ and $\phi_0 = 2.005\, m_p$.

Gives amplification of the scalar power-spectrum, $\mathcal{P}_\zeta$, by a factor of 10^7 relative to its value on CMB scales.

Where's the inflaton ?

Loads of models — 300 models analysed with CMB and BAO data, 40% disfavoured, 20% favoured according to Jeffrey's scale of Bayesian evidence [Martin et al 2024]

Over 7,700 papers written with title Inflation in title .



To date, no accepted origin of the inflaton field. It should ideally be a fundamental field arising out of an underlying theory of particle physics like string theory - for a review see [Cicoli et al 2023].

But there appear to be issues there obtaining de Sitter solutions - it has led in part to the Swampland conjecture.

Of course inflation isn't de Sitter, but it looks like its not far from it with the Hubble parameter H slowly evolving during inflation.

One nice approach is due to Conlon and Quevdeo [2006] - Kahler Moduli Inflation.

It has some interesting features that exist between the end of inflation and reheating which we will look at briefly [Apers et al 2024]

Kahler Moduli Inflation [Conlon and Quevedo 2006]

Large volume scenario within a class of Type IIB flux compactifications on a Calibi-Yau orientifold]

Internal volume of CY:

$$\mathcal{V} = \frac{\alpha}{2\sqrt{2}} \left[(T_1 + \bar{T}_1)^{\frac{3}{2}} - \sum_{i=2}^{n} \lambda_i (T_i + \bar{T}_i)^{\frac{3}{2}} \right] = \alpha \left(\tau_1^{3/2} - \sum_{i=2}^{n} \lambda_i \tau_i^{3/2} \right)$$

Complex Kahler moduli $T_i = \tau_I + I\,\theta_i$

τ_I - volume of internal four cycles in CY

θ_i - axonic partners

Full scalar potential for moduli fields - don't look too closely !

$$\begin{aligned} V = & \sum_{\substack{i,j=2 \\ i<j}}^{n} \frac{A_i A_j \cos(a_i\theta_i - a_j\theta_j)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} e^{-(a_i\tau_i + a_j\tau_j)} \left(32(2\mathcal{V}+\xi)(a_i\tau_i + a_j\tau_j + 2a_ia_j\tau_i\tau_j) + 24\xi\right) \\ & + \frac{12W_0^2\xi}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} + \sum_{i=2}^{n} \left[\frac{12e^{-2a_i\tau_i}\xi A_i^2}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2} + \frac{16(a_iA_i)^2\sqrt{\tau_i}e^{-2a_i\tau_i}}{3\alpha\lambda_i(2\mathcal{V}+\xi)} \right. \\ & \left. + \frac{32e^{-2a_i\tau_i}a_iA_i^2\tau_i(1+a_i\tau_i)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)} + \frac{8W_0A_ie^{-a_i\tau_i}\cos(a_i\theta_i)}{(4\mathcal{V}-\xi)(2\mathcal{V}+\xi)} \left(\frac{3\xi}{2\mathcal{V}+\xi} + 4a_i\tau_i \right) \right] + V_{uplift}\,. \end{aligned}$$

Idea: displace just one moduli from its minimum, keeping the others fixed and show consistent slow roll inflation can be obtained with that moduli evolving back to its minima

Displace τ_2 with parameter $\rho \ll 1$ where

$$\rho \equiv \frac{\lambda_2}{a_2^{3/2}} : \sum_{i=2}^{n} \frac{\lambda_i}{a_i^{3/2}}$$

$$V_{LARGE} = \frac{BW_0^2}{\mathcal{V}^3} - \frac{4W_0 a_2 A_2 \tau_2 e^{-a_2\tau_2}}{\mathcal{V}^2}$$

Intriguing results obtained for 50-60 efolds:

$$\eta \simeq -\frac{1}{N_e}, \qquad \epsilon < 10^{-12},$$

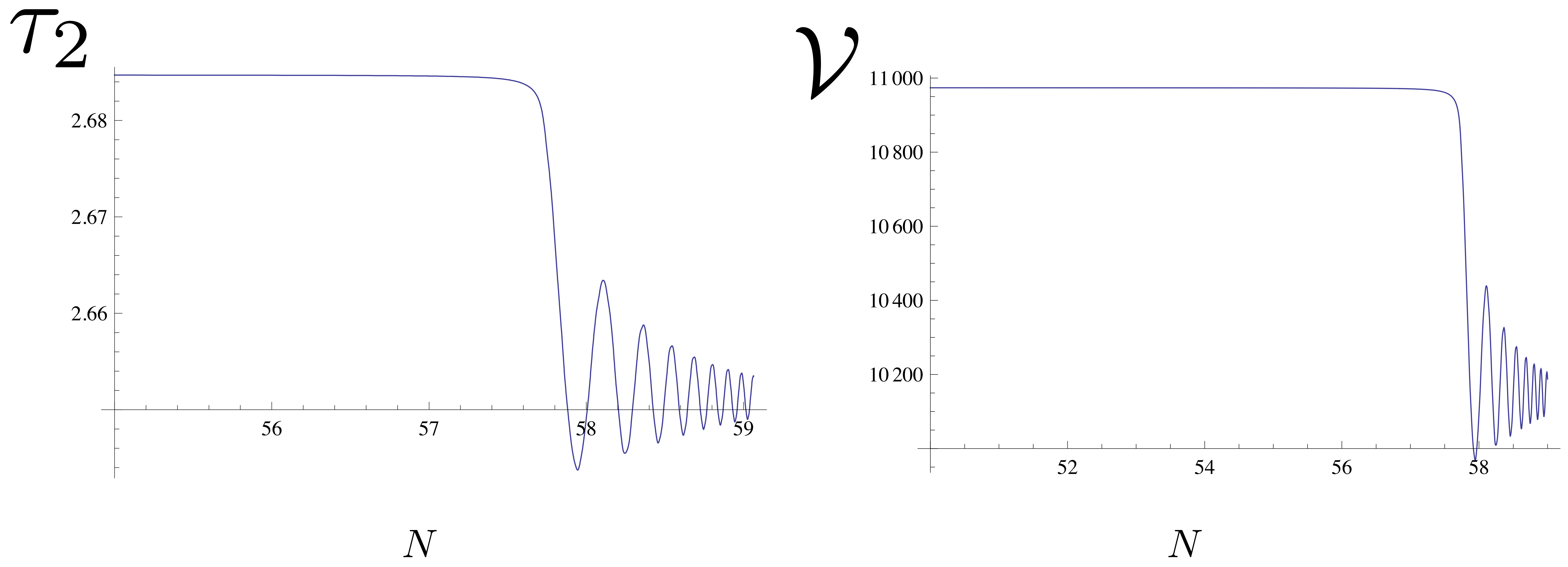
$$0.960 < n_s < 0.967, \qquad 0 < |r| < 10^{-10}$$

$$10^5 l_s^6 \leq \mathcal{V} \leq 10^7 l_s^6 .$$

Numerically solve the full equations. The question is what happens if we allow the moduli to evolve so that they all have to find their minima. Do we find the kind of evolution that Conlon and Quevedo assumed in their analytic model ?

[Blanco-Pillado et al 2009]

$$\tau_1^f = 2555.95, \quad \tau_2^f = 4.7752, \quad \tau_3^f = 2.6512, \quad \mathcal{V}^f = 10143.94363$$



Some more string inspired inflation models — [Cicoli et al 2023]

String model	n_s	r
Fibre Inflation	0.967	0.007
Blow-up Inflation	0.961	10^{-10}
Poly-instanton Inflation	0.958	10^{-5}
Aligned Natural Inflation	0.960	0.098
N-Flation	0.960	0.13
Axion Monodromy	0.971	0.083
D7 Fluxbrane Inflation	0.981	5×10^{-6}
Wilson line Inflation	0.971	10^{-8}
D3-$\overline{\text{D3}}$ Inflation	0.968	10^{-7}
Inflection Point Inflation	0.923	10^{-6}
D3-D7 Inflation	0.981	10^{-6}
Racetrack Inflation	0.942	10^{-8}
Volume Inflation	0.965	10^{-9}
DBI Inflation	0.923	10^{-7}

As you can see there are many - some close to being or already ruled out !

After inflation ! [Apers et al 2024]

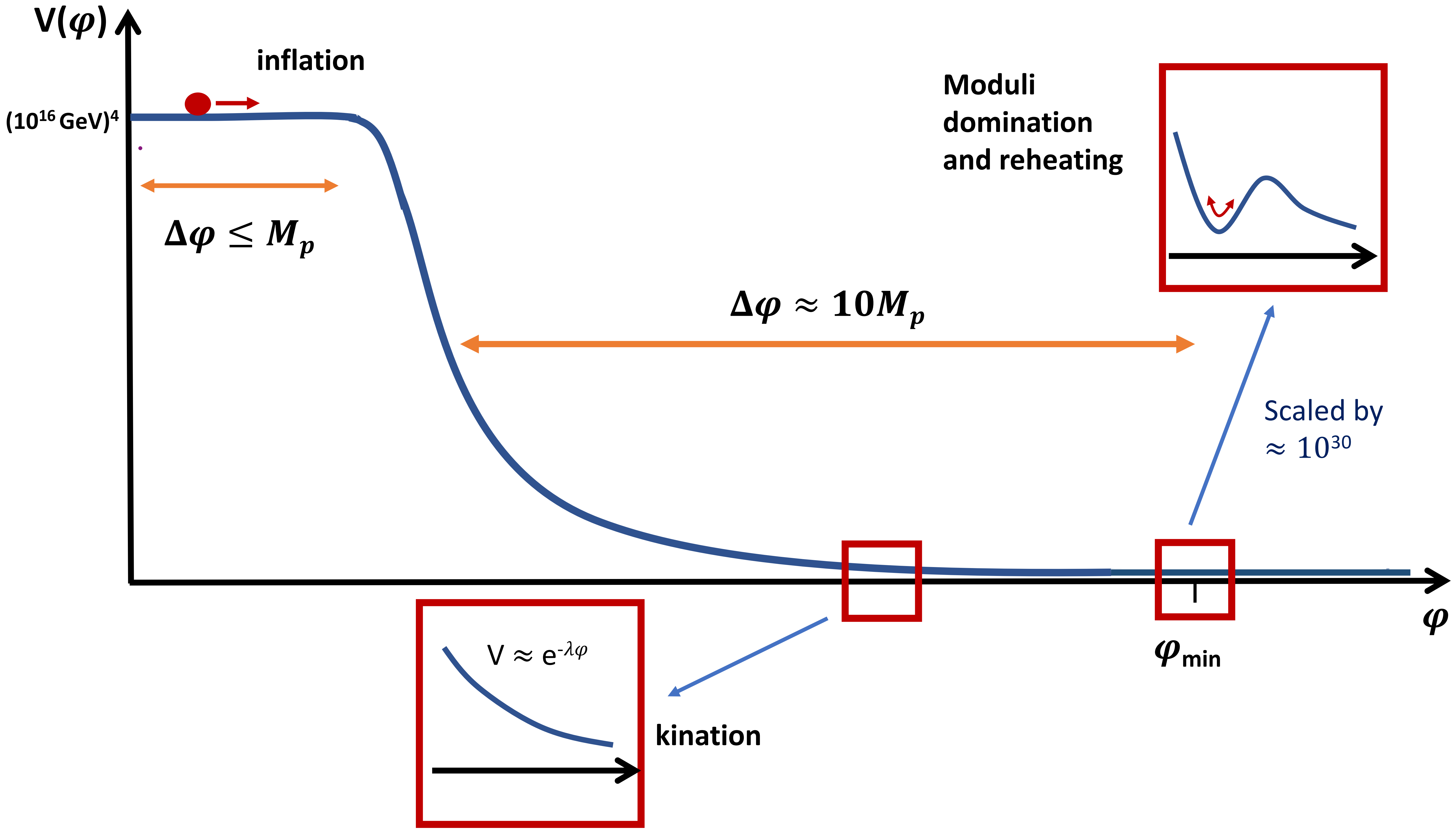
The bit between the end of inflation and the thermal HBB - some 30 orders of magnitude in time.

Potentially new stringy features could emerge which would modify the standard picture.

For example, large field displacements between end of inflation and final vacuum - under control !

No necessary relationship between inflaton field and field responsible for reheating. In fact in D3-anti D3 brane case, inflaton disappears.

Long Kination and moduli dominated epoch leading to moduli driven reheating



[Cicoli et al 2023]

During Kination - potential term subdominant:

$$\ddot{\Phi} + 3H\dot{\Phi} = 0, \qquad 3H^2 M_P^2 = \frac{\dot{\Phi}^2}{2}$$

where : $\frac{\Phi}{M_P} = \sqrt{\frac{2}{3}} \ln \mathcal{V}$

Kinating field satisfies : $\Phi(t) = \Phi(t_0) + \sqrt{\frac{2}{3}} M_P \ln\left(\frac{t}{t_0}\right)$ with : $a(t) \sim t^{1/3}$

Travels roughly one Planck distance in one Hubble time

Example of Kiination with : $V(\Phi) = V_0 e^{-\lambda\Phi/M_P}$ with : $\lambda > \sqrt{6}$

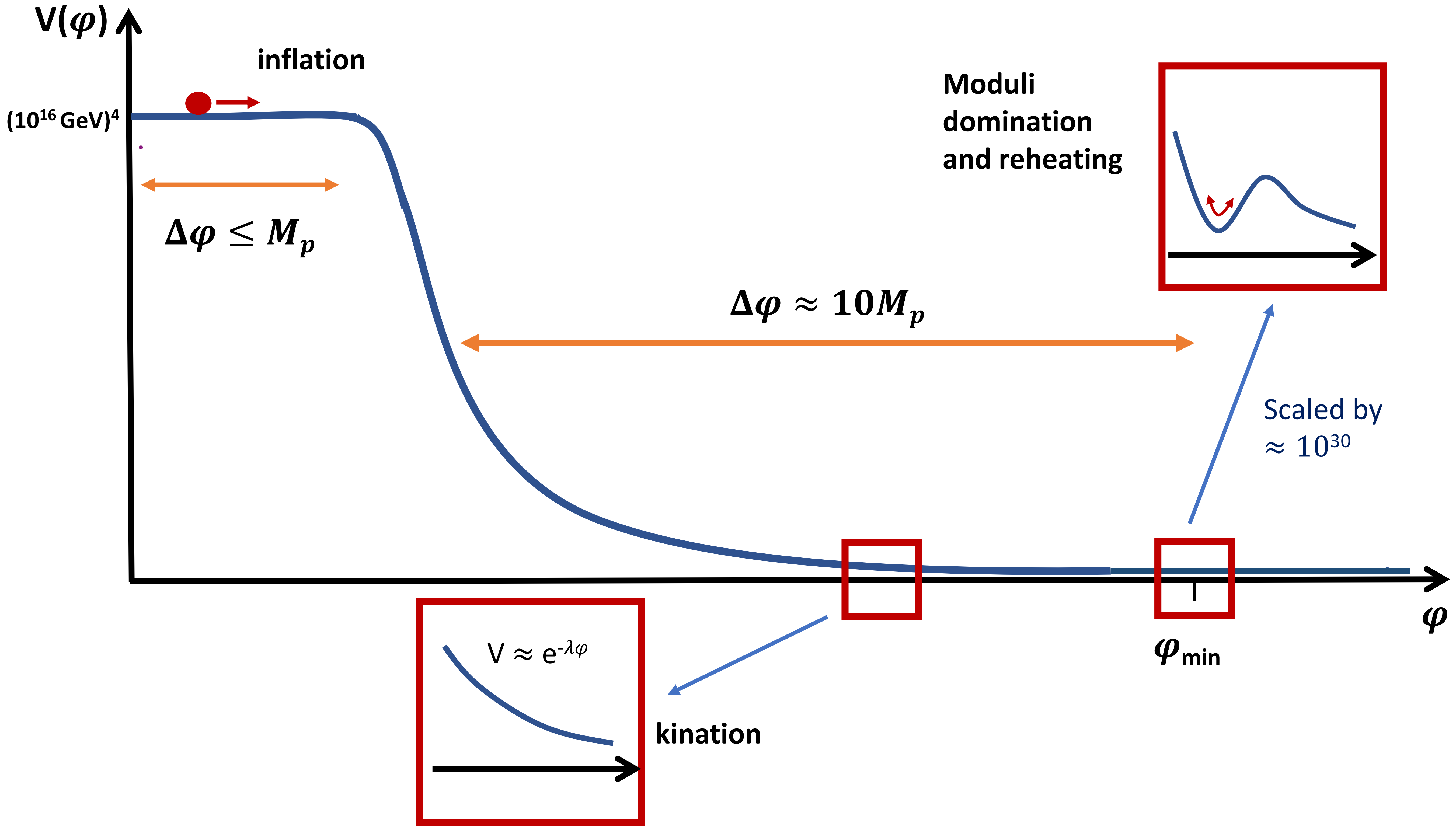
But as V decreases during Kination and as: $\rho_{kin} \sim \frac{1}{a(t)^6},$

Eventually any residual radiation or matter becomes dominant and enter Tracker regime where the radiation and ϕ track each other

Tracker field satisfies : $\Phi(t) = \Phi(t_0) + \frac{2M_P}{\lambda} \ln\left(\frac{t}{t_0}\right)$ with : $a(t) \sim t^{1/2}$

Again travels roughly one Planck distance in one Hubble time

Guides the field into the min of the moduli potential where reheating can occur



Time varying standard model parameters because determined by evolving moduli fields !

Gauge couplings, Yukawa couplings and axion decay constants - could be different from today.

Perturbations in the field grow during Kination and into the tracker regime before the moduli are stabilised and reheating occurs - potential for new exciting pre BBN physics ! [Apers et al 2024]

Cosmic string tensions will evolve in time, and a new network formation process could emerge from the formation of loops -

[with Sanchez Gonzalez, Conlon and Hardy 2024]

$m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}$ with $G\mu \sim m_s^2$ hence $G\mu \sim t^{-1}$

Will concentrate on one important element of this use of Tracker behaviour - the overshoot problem [Brustein and Steinhardt 93] !

The barrier that has to eventually trap the moduli field can be 20 or more orders of magnitude smaller than the energy scale during inflation. The field should simply shoot straight past and decompactify spacetime !

In cosmology as in many areas of physics we often deal with systems that are inherently described through a series of coupled non-linear differential equations.

By determining the late time behaviour of some combination of the variables, we often see that they may approach some form of attractor solution.

From the stability of these attractor solutions we can learn about the system.

Moreover the phase plane description of the system is often highly intuitive enabling easy analysis and understanding of the system.

Examples inc the relative energy densities in scalar fields compared to the bgd rad and matter densities, as well as the relative energy density in cosmic strings.

Wetterich (88) Peebles and Ratra (88),

EJC, Liddle and Wands

Enter Tracker solutions:

Scalar field: $$\phi : \rho_\phi = \frac{\dot\phi^2}{2} + V(\phi); \ p_\phi = \frac{\dot\phi^2}{2} - V(\phi)$$

EoM:
$$\dot H = -\frac{\kappa^2}{2}(\dot\phi^2 + \gamma\rho_b)$$
$$\dot\rho_b = -3\gamma\rho_b$$
$$\ddot\phi = -3H\dot\phi - \frac{dV}{d\phi}$$

+ constraint:
$$H^2 = \frac{\kappa^2}{3}(\rho_\phi + \rho_b)$$

Intro new variables x and y:

$$x = \frac{\kappa\dot\phi}{\sqrt{6}H}; \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}; \quad \lambda \equiv \frac{-1}{\kappa V}\frac{dV}{d\phi}; \quad \Gamma - 1 \equiv \frac{d}{d\phi}\left(\frac{1}{\kappa\lambda}\right)$$

Eff eqn of state: $\gamma_\phi = \frac{2\dot{\phi}^2}{2V+\dot{\phi}^2}$; $\Omega_\phi = \frac{\kappa^2\rho_\phi}{3H^2} = x^2 + y^2$

Friedmann eqns and fluid eqns become:

$$x' = -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x[2x^2 + \gamma(1 - x^2 - y^2)]$$
$$y' = -\lambda\sqrt{\frac{3}{2}}xy + \frac{3}{2}y[2x^2 + \gamma(1 - x^2 - y^2)]$$
$$\lambda' = -\sqrt{6}\lambda^2(\Gamma - 1)$$
$$\frac{\kappa^2\rho_{\rm b}}{3H^2} + x^2 + y^2 = 1$$

where $x' \equiv \frac{d}{d(\ln a)}$

Note: $0 \le \gamma_\phi \le 2$; $0 \le \Omega_\phi \le 1$

Scaling solutions: (x`=y`=0)

No:	x_c	y_c	Existance	Stability	Ω_ϕ	γ_ϕ
1	0	0	$\forall\lambda,\gamma$	SP: $0<\gamma$ SN: $\gamma=0$	0	Undefined
2a	1	0	$\forall\lambda,\gamma$	UN: $\lambda<\sqrt{6}$ SP: $\lambda>\sqrt{6}$	1	2
2b	-1	0	$\forall\lambda,\gamma$	UN: $\lambda>-\sqrt{6}$ SP: $\lambda<-\sqrt{6}$	1	2
3	$\frac{\lambda}{\sqrt{6}}$	$\left(1-\frac{\lambda^2}{6}\right)^{1/2}$	$\lambda^2\le 6$	SP: $3\gamma<\lambda^2<6$ SN: $\lambda^2<3\gamma$	1	$\frac{\lambda^2}{3}$
4	$\left(\frac{3}{2}\right)^{1/2}\frac{\gamma}{\lambda}$	$\left[\frac{3(2-\gamma)\gamma}{2\lambda^2}\right]^{1/2}$	$\lambda^2\ge 3\gamma$	SN: $3\gamma<\lambda^2<\frac{24\gamma^2}{9\gamma-2}$ SS: $\lambda^2>\frac{24\gamma^2}{9\gamma-2}$	$\frac{3\gamma}{\lambda^2}$	γ

$$V = V_0 e^{-\lambda\kappa\phi}$$

Late time attractor is scalar field dominated

$\lambda^2 \le 6$

Field mimics background fluid.

$\lambda^2 \ge 3\gamma$

EJC, Liddle and Wands

$$V = V_0 e^{-\lambda \kappa \phi}$$

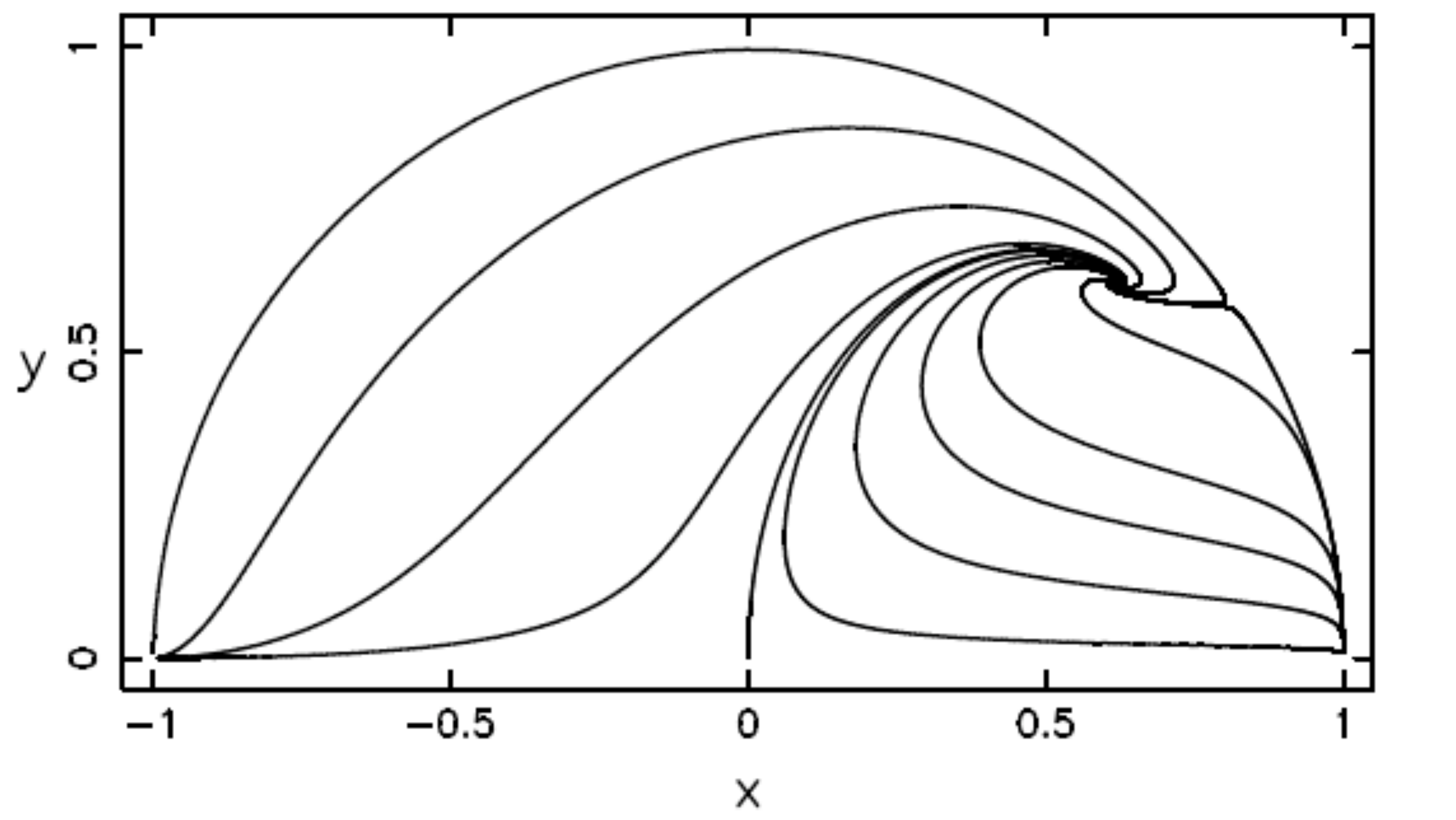


FIG. 3. The phase plane for $\gamma = 1$, $\lambda = 2$. The scalar field dominated solution is a saddle point at $x = \sqrt{2/3}$, $y = \sqrt{1/3}$, and the late-time attractor is the scaling solution with $x = y = \sqrt{3/8}$.

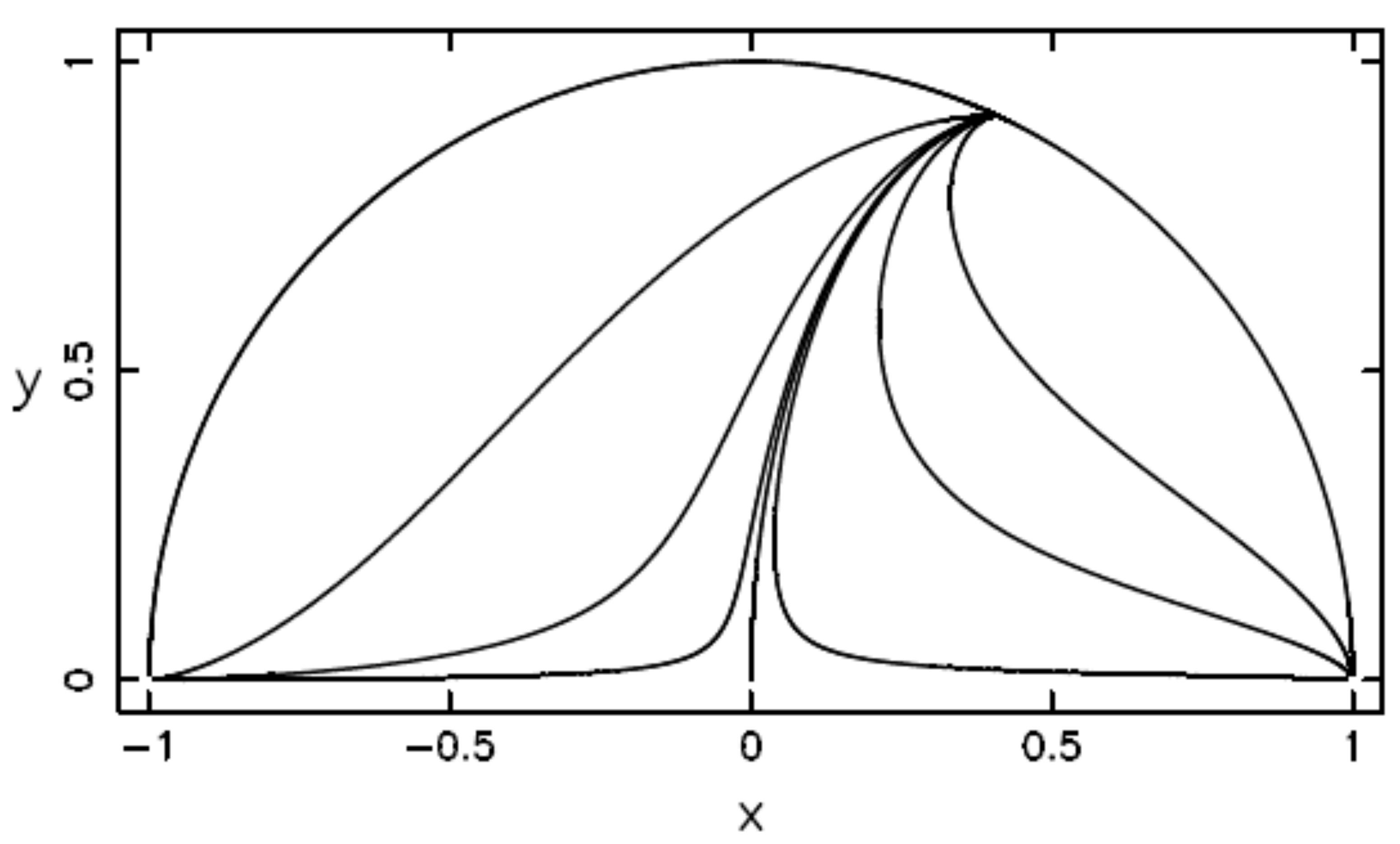


FIG. 2. The phase plane for $\gamma = 1$, $\lambda = 1$. The late-time attractor is the scalar field dominated solution with $x = \sqrt{1/6}$, $y = \sqrt{5/6}$.

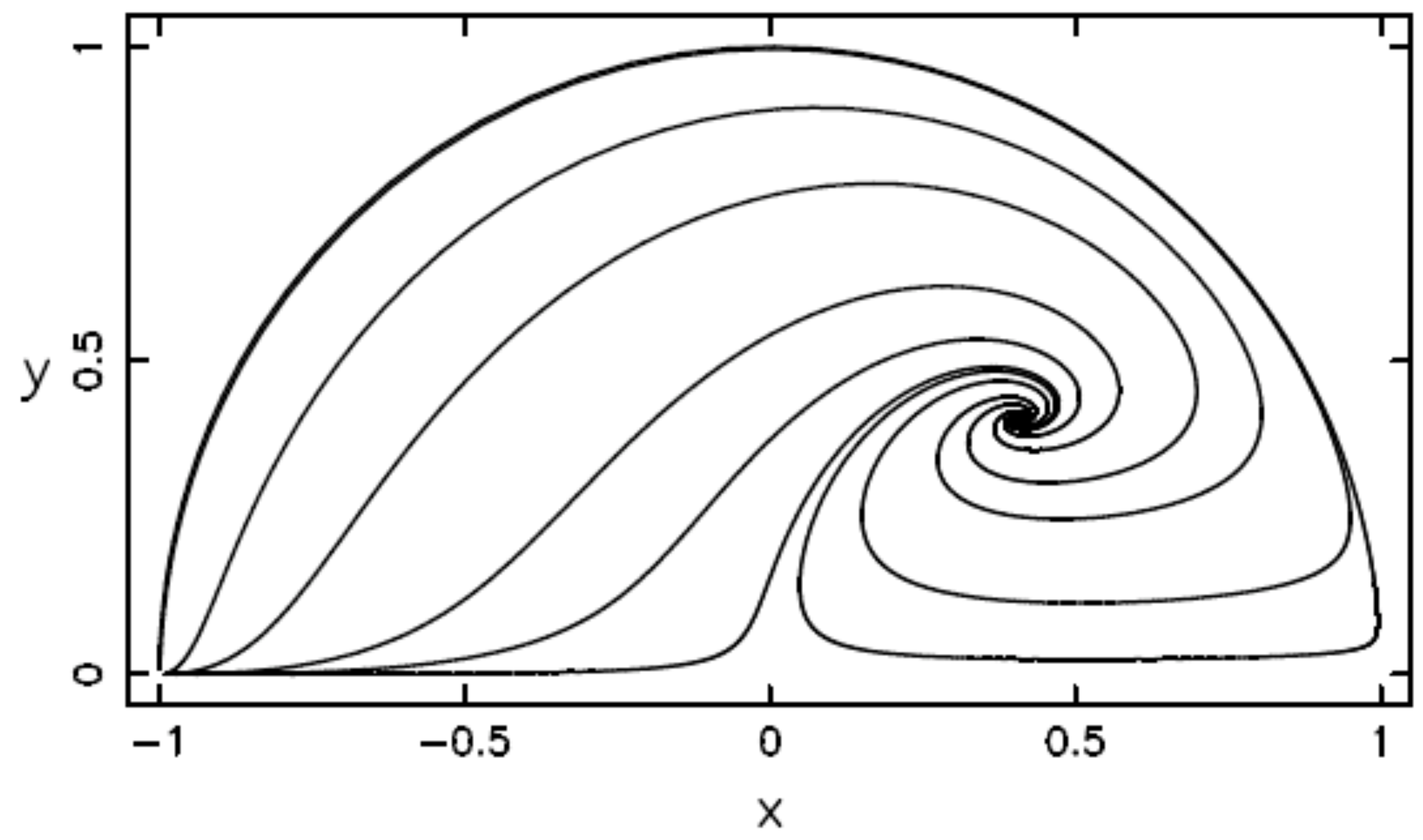
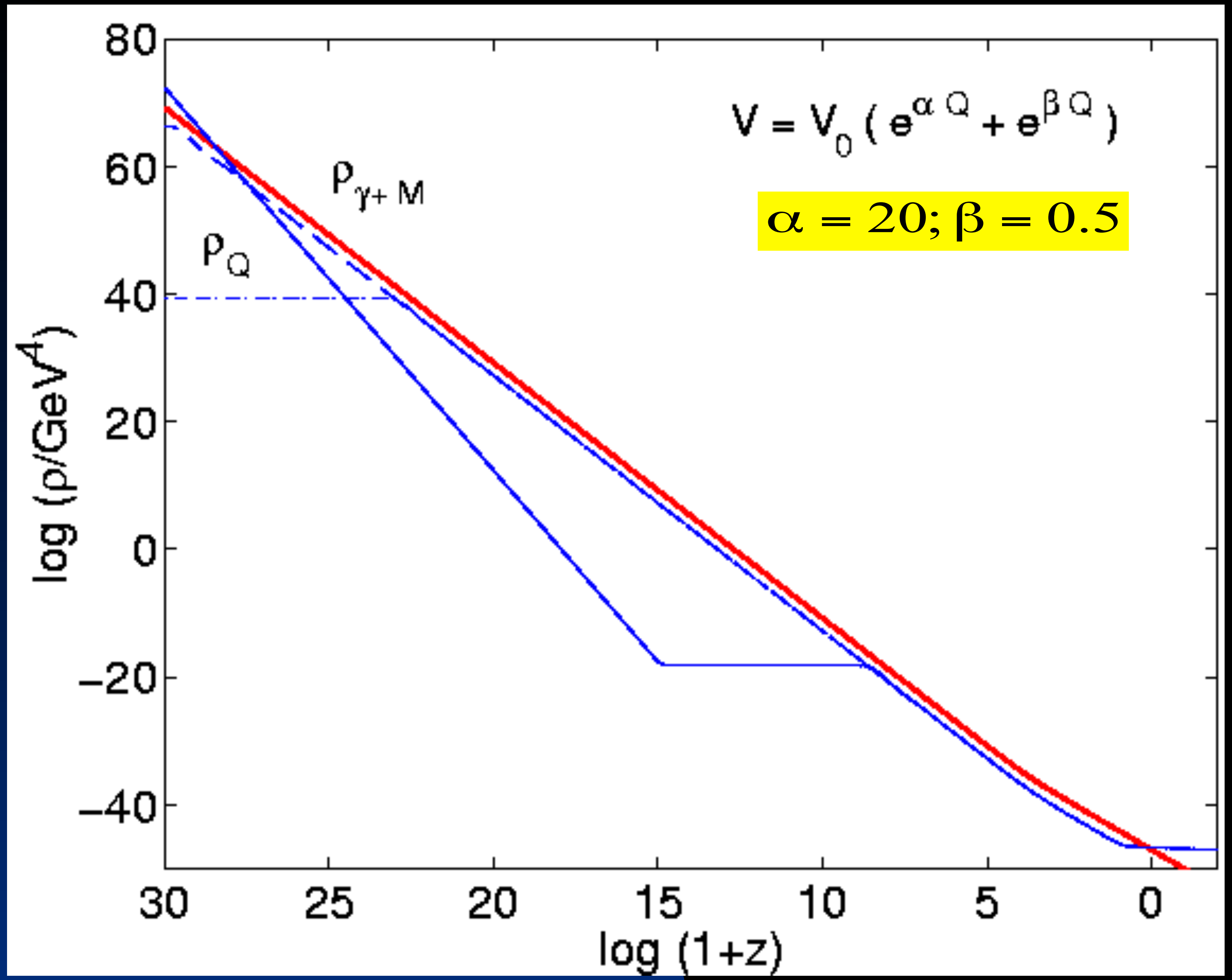
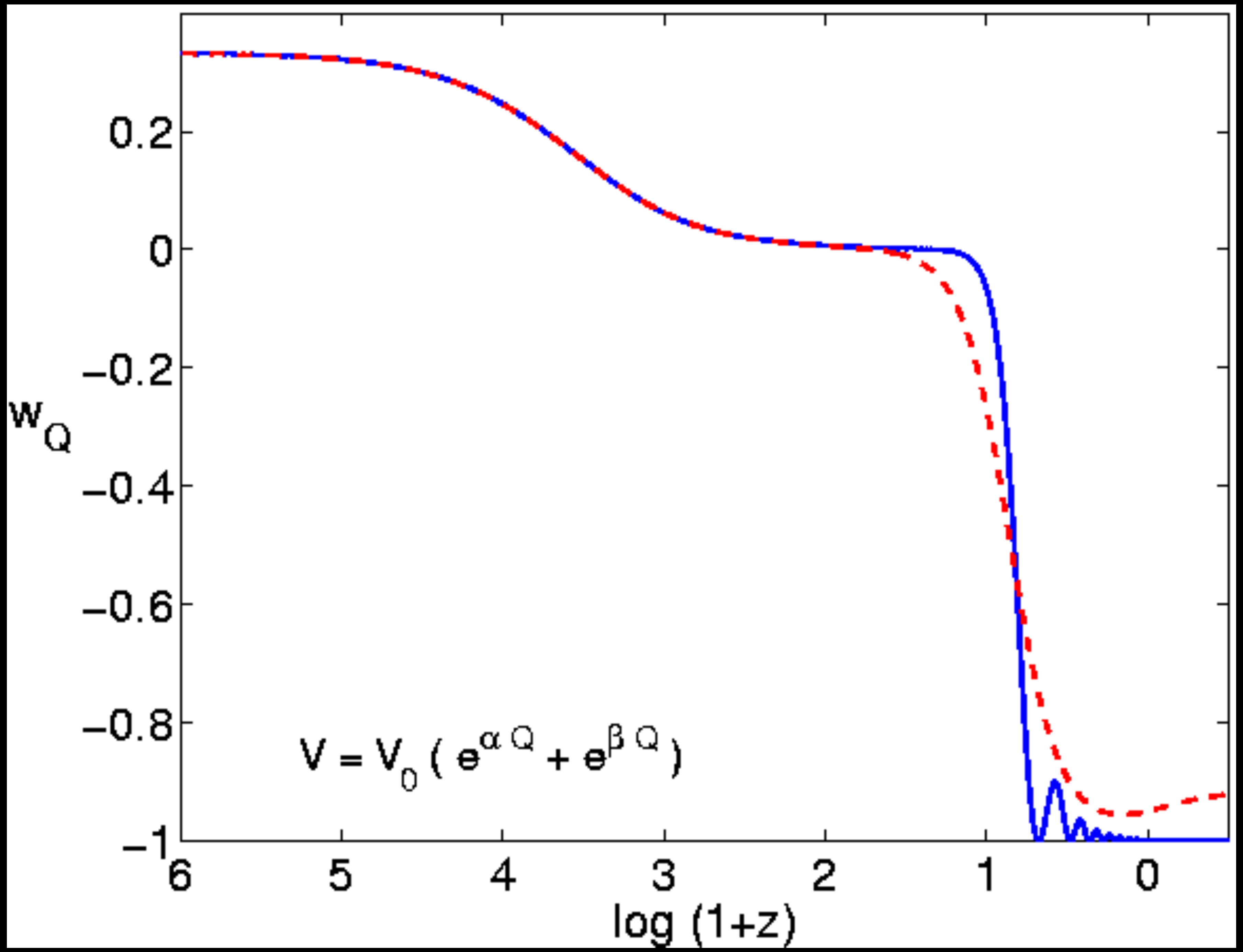


FIG. 4. The phase plane for $\gamma = 1$, $\lambda = 3$. The late-time attractor is the scaling solution with $x = y = \sqrt{1/6}$.

1. Scaling solutions in Dark Energy - Quintessence





Scaling for wide range of i.c.

Fine tuning:

$$V_0 \approx \rho_\phi \approx 10^{-47}\ \mathrm{GeV}^4 \approx (10^{-3}\ \mathrm{eV})^4$$

Mass:

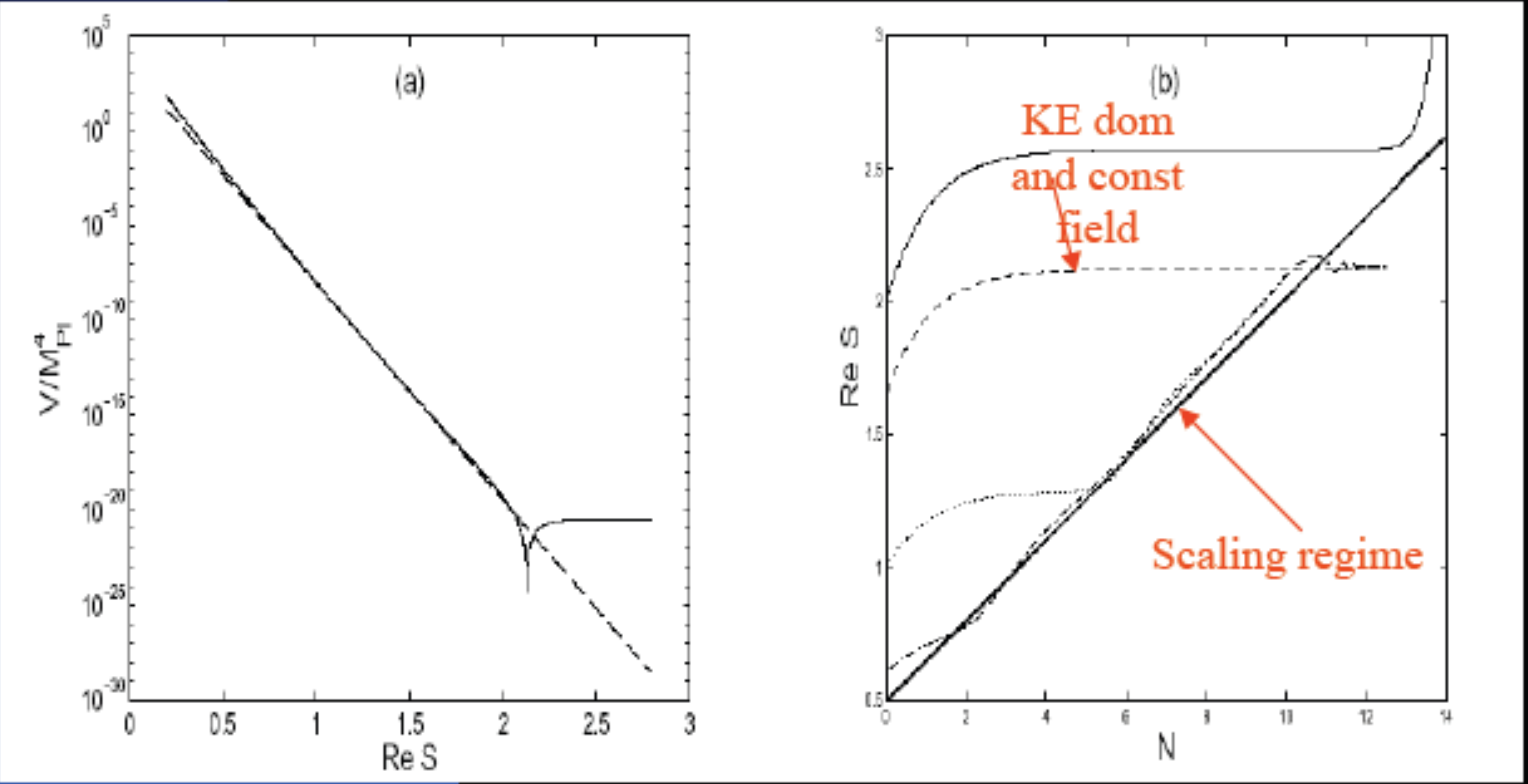
$$m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \mathrm{eV}$$

Fifth force !

2. Useful way of stabilising moduli in string cosmology. Sources provide extra friction when potentials steep.

Barreiro, de Carlos and EC : hep/th-9805005

Brustein, Alwis and Martins : hep-th/0408160



Two condensate model with $V \sim e^{-a\mathrm{Re}S}$ as approach minima

Barreiro et al : hep-th/0506045

3. Stabilising volume moduli ($\sigma=\sigma_r+\sigma_i$) in KKLT [Kachru et al 2003]

$$\ddot{\sigma_r} + 3H\dot{\sigma_r} - \frac{1}{\sigma_r}(\dot{\sigma_r}^2 - \dot{\sigma_i}^2) + \frac{2\sigma_r^2}{3}\partial_{\sigma_r}V = 0$$

$$\ddot{\sigma_i} + 3H\dot{\sigma_i} - \frac{2}{\sigma_r}\dot{\sigma_r}\dot{\sigma_i} + \frac{2\sigma_r^2}{3}\partial_{\sigma_i}V = 0$$

$$\dot{\rho_b} + 3H\gamma\rho_b = 0$$

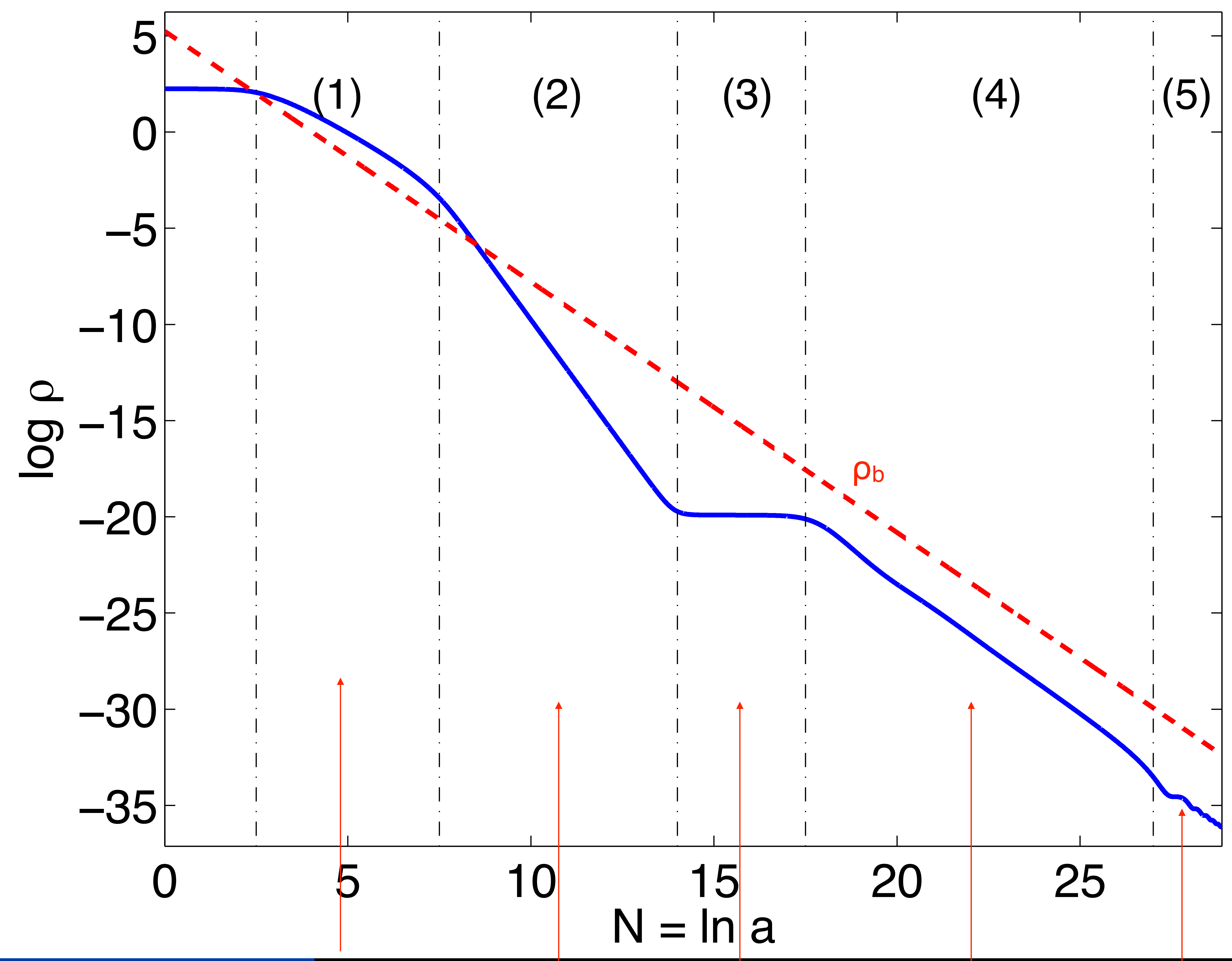
$$3H^2 = \frac{3}{4\sigma_r^2}(\dot{\sigma_r}^2 + \dot{\sigma_i}^2) + V + \rho_b$$

$$V = \frac{\alpha A e^{-\alpha\sigma_r}}{2\sigma_r^2}\left[A\left(1+\frac{\alpha\sigma_r}{3}\right)e^{-\alpha\sigma_r} + W_0\cos(\alpha\sigma_i)\right] + \frac{C}{\sigma_r^3}.$$

[including contribution from D term to uplift the potential to de Sitter]

[for discussion on validity of D term addition see also Burgess et al 2003; Achucarro et al 2006]

Evolution of energy density of $\phi \propto \ln\sigma_r$ in KKLT and Kallosh Linde type potentials



Flat potential: Scalar field dominated

Steeper pot Kinetic field dominated

Field frozen in pot

Scaling or tracking regime

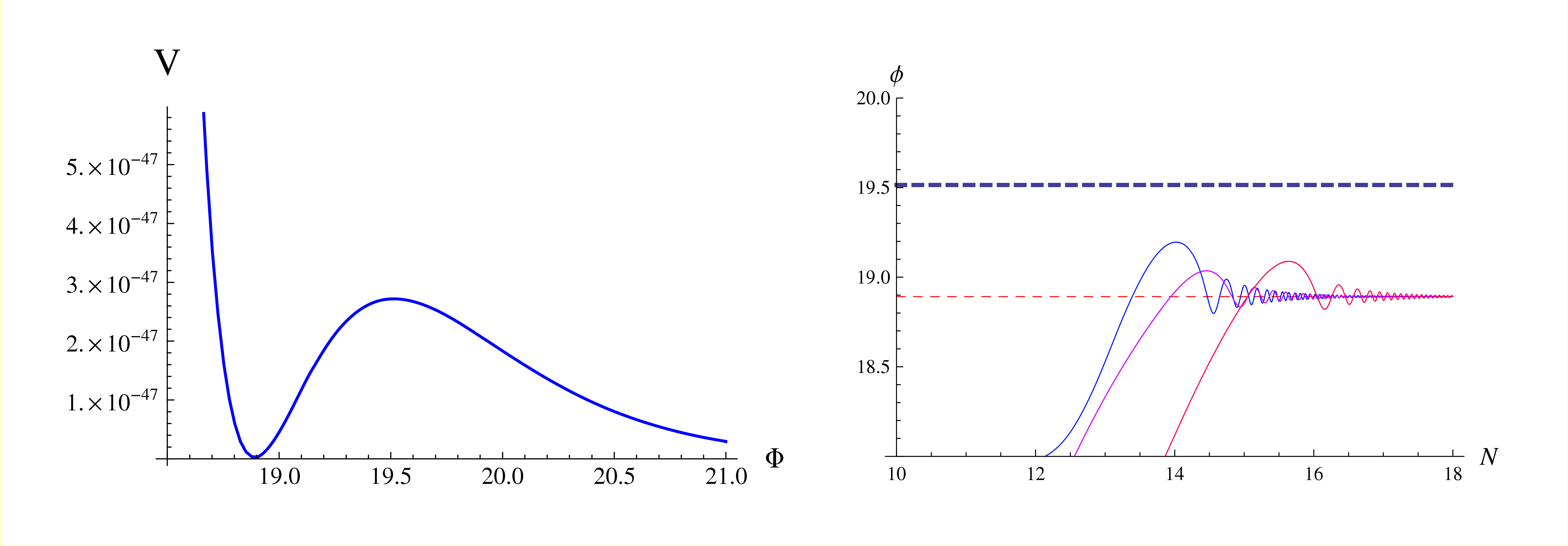
Added friction from scaling regime slows field down and stabilises it in min of potential

[Brustein et al 2004; Barreiro et al 2005]

4. Large volume modulus inflation - high scale inflation & low scale SUSY co-existing [Conlon et al 2008]

$$V = V_0\left((1-\epsilon\,\Phi^{3/2})e^{-\sqrt{27/2}\Phi} + C\,e^{-10\Phi/\sqrt{6}} + De^{-11\Phi/\sqrt{6}} + \delta e^{-\sqrt{6}\Phi}\right)$$

Toy example - but general features



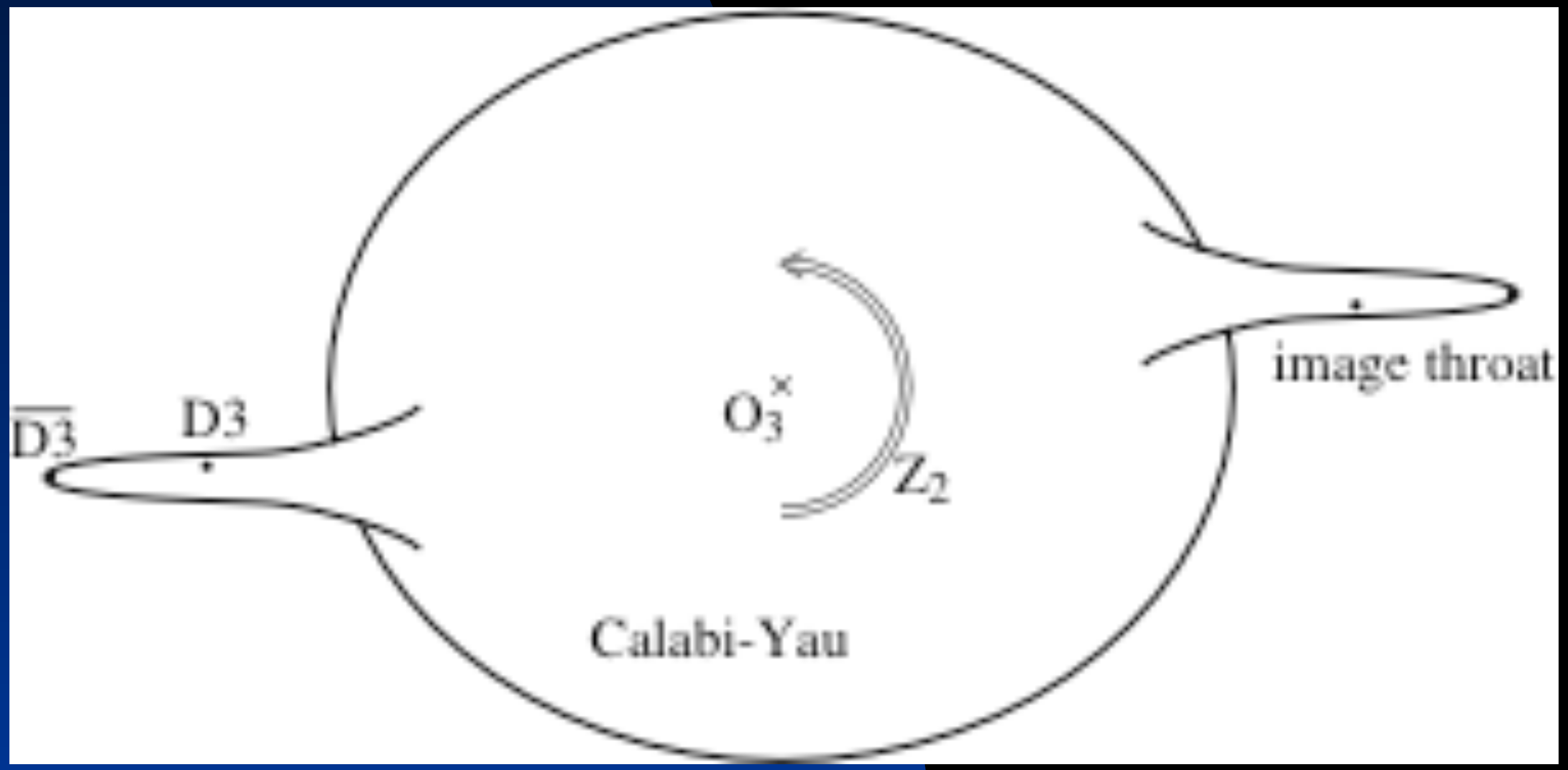
Steep potential after inflation would normally have runaway solutions but presence of radiation leads to additional Hubble Friction which leads to attractor behaviour and field settles in its minimum.

Strings in KLMT © model -- an example.

[Kachru, Kallosh, Linde, Maldacena, McAllister & Trivedi 03]

IIB string theory on CY manifold, orientifolded by Z_2 sym with isolated fixed points, become O3 planes. Warped metric:

$$ds^2 = e^{2A(x_\perp)}\eta_{\mu\nu}dx^\mu dx^\nu + ds_\perp^2 \ .$$



Inflaton: sep of D3 and anti D3 in throat.

Annihilation in region of large grav redshift,

$$min\{e^{A(x_\perp)}\} = e^{A_0} \ll 1$$

Redshift in throat important. Inflation scale and string tension, as measured by a 10 dim inertial observer, are set by string physics -- close to the four-dimensional Planck scale. Corresponding energy scales as measured by a 4 dim obs are suppressed by a factor of e^{A_0}

Strings surviving inflation:

D-brane-antibrane inflation leads to formation of D1 branes in non-compact space [**Dvali & Tye; Burgess et al; Majumdar & Davis; Jones, Sarangi &Tye; Stoica & Tye**]

Form strings, not domain walls or monopoles.

$$10^{-11} \leq G\mu \leq 10^{-6}$$

In general for cosmic strings to be cosmologically interesting today we require that they are not too massive (from CMB constraints), are produced after inflation (or survive inflation) and are stable enough to survive until today [**Dvali and Vilenkin (2004); EJC,Myers and Polchinski (2004), Conlon et al (2024)**].

What sort of strings? Expect strings in non-compact dimensions where reheating will occur: F1-brane (fundamental IIB string) and D1 brane localised in throat. [Jones,Stoica & Tye, Dvali & Vilenkin]

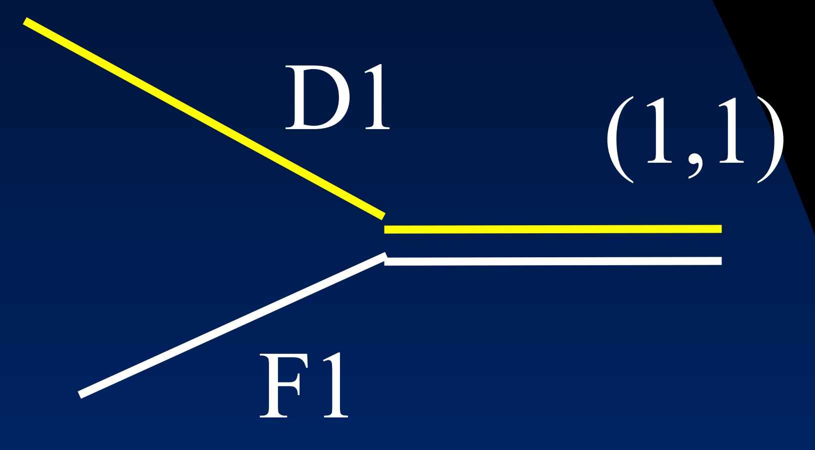
D1 branes - defects in tachyon field describing D3-anti D3 annihilation, so produced by Kibble mechanism.

Strings created at end of inflation at bottom of inflationary throat. Remain there because of deep pot well. Eff 4d tensions depend on warping and 10d tension $\bar{\mu}$

$$\mu = e^{2A(x_\perp)}\bar{\mu}$$

F1-branes and D1-branes --> also (p,q) strings for relatively prime integers p and q. [Harvey & Strominger; Schwarz]

Interpreted as bound states of p F1-branes and q D1-branes [Polchinski;Witten]



Tension in 10d theory:

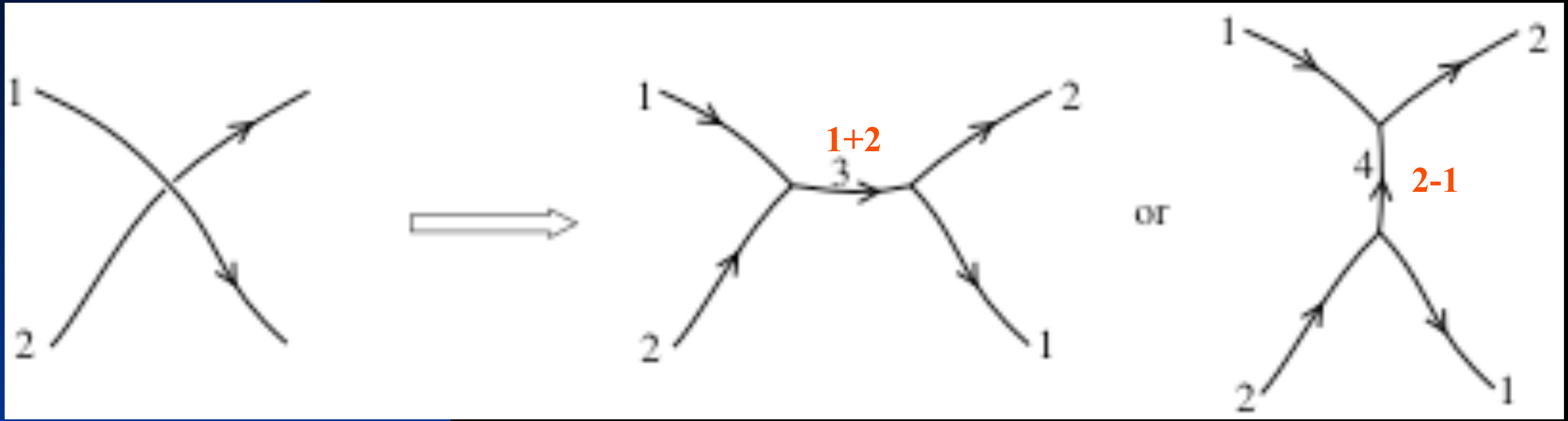
$$\mu_i \equiv \mu_{(p_i,q_i)} = \frac{\mu_F}{g_s}\sqrt{p_i^2 g_s^2 + q_i^2}$$

Distinguishing cosmic superstrings

1. Intercommuting probability for gauged strings P~1 always ! In other words when two pieces of string cross each other, they reconnect. Not the case for superstrings -- model dependent probability [Jackson et al 04].
2. Existence of new `defects' D-strings allows for existence of new hybrid networks of F and D strings which could have different scaling properties, and distinct observational effects.

(p,q) string networks -- exciting prospect.

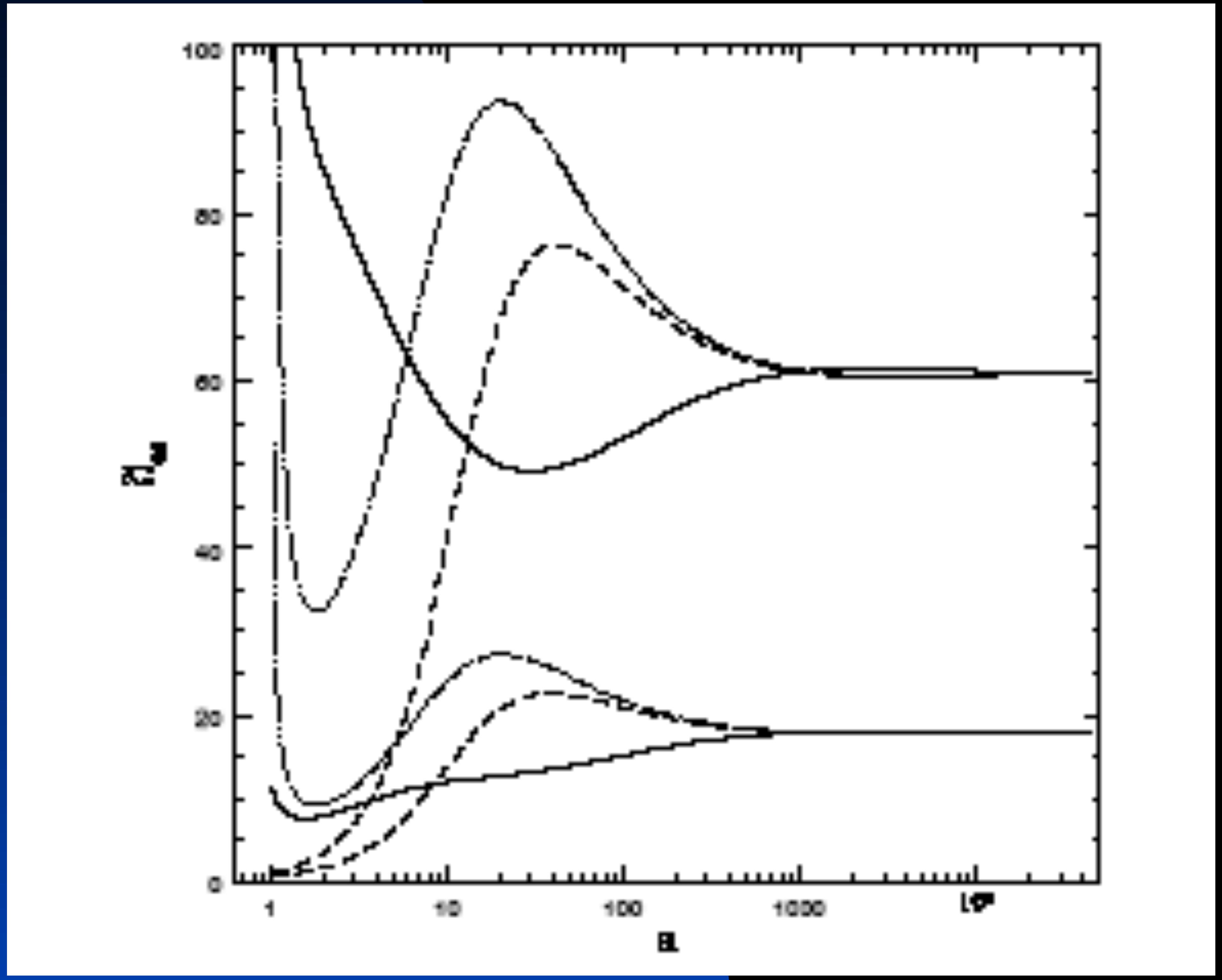
Two strings of different type cross, can not intercommute in general -- produce pair of trilinear vertices connected by segment of string.



What happens to such a network in an expanding background? Does it scale or freeze out in a local minimum of its PE [Sen]?Then it could lead to a frustrated network scaling as w=-1/3

Including multi-tension cosmic superstrings

[Tye et al 05, Avgoustidis and Shellard 07, Urrestilla and Vilenkin 07, Avgoustidis and EJC 10, Rybak et al 18].



Density of (p,q) cosmic strings.

Density of D1 strings.

Scaling achieved indep of initial conditions, and indep of details of interactions.

Modelling a network —single one-scale model: (Kibble + many...)

Infinite string density $\rho = \frac{\mu}{L^2}$

$$\dot{\rho} = -2\frac{\dot{a}}{a}\rho - \frac{\rho}{L}$$

Expansion — Loss to loops

Correlation length $L(t) = \xi(t)t, \quad a(t) \sim t^{\beta}$ Scale factor

$$\frac{\dot{\xi}}{\xi} = \frac{1}{2t}\left(2(\beta - 1) + \frac{1}{\xi}\right)$$

Scaling solution $\xi = [2(1-\beta)]^{-1}.$

Need this to understand the behaviour with the CMB.

Velocity dependent model: (Shellard and Martin)

$$\dot{\rho} = -2\frac{\dot{a}}{a}(1+v^2)\rho - \frac{\tilde{c}\,v\rho}{L},$$

RMS vel of segments

$$\dot{v} = (1-v^2)\left(\frac{k}{L} - 2\frac{\dot{a}}{a}v\right)$$

Curvature type term encoding small scale structure

$$k = \frac{2\sqrt{2}}{\pi}\left(\frac{1-8v^6}{1+8v^6}\right)$$

$$\xi^2 = \frac{k(k+\tilde{c})}{4\beta(1-\beta)}, \qquad v^2 = \frac{k(1-\beta)}{\beta(k+\tilde{c})}$$

Both correlation length and velocity scale

Multi tension string network:(Avgoustidis & Shellard 08, Avgoustidis & EJC 10, Rybak et al 18)

$$\dot{\rho}_i = -2\frac{\dot{a}}{a}(1+v_i^2)\rho_i - \frac{c_i v_i \rho_i}{L_i} - \sum_{a,k} \frac{d^k_{ia}\bar{v}_{ia}\mu_i \ell^k_{ia}(t)}{L_a^2 L_i^2} + \sum_{b,\, a\leq b} \frac{d^i_{ab}\bar{v}_{ab}\mu_i \ell^i_{ab}(t)}{L_a^2 L_b^2}$$

Expansion

Loop of \`i' string

Segment of \`i' collides with \`a' to form segment \`k' -- removes energy

Segment of \`i' forms from collision of \`a' and \`b' -- adds energy

$$\dot{v}_i = (1-v_i^2)\left[\frac{k_i}{L_i} - 2\frac{\dot{a}}{a}v_i + \sum_{b,\, a\leq b} b^i_{ab}\frac{\bar{v}_{ab}}{v_i}\frac{(\mu_a+\mu_b-\mu_i)}{\mu_i}\frac{\ell^i_{ab}(t)L_i^2}{L_a^2 L_b^2}\right]$$

$$v_{ab} = \sqrt{v_a^2 + v_b^2}$$

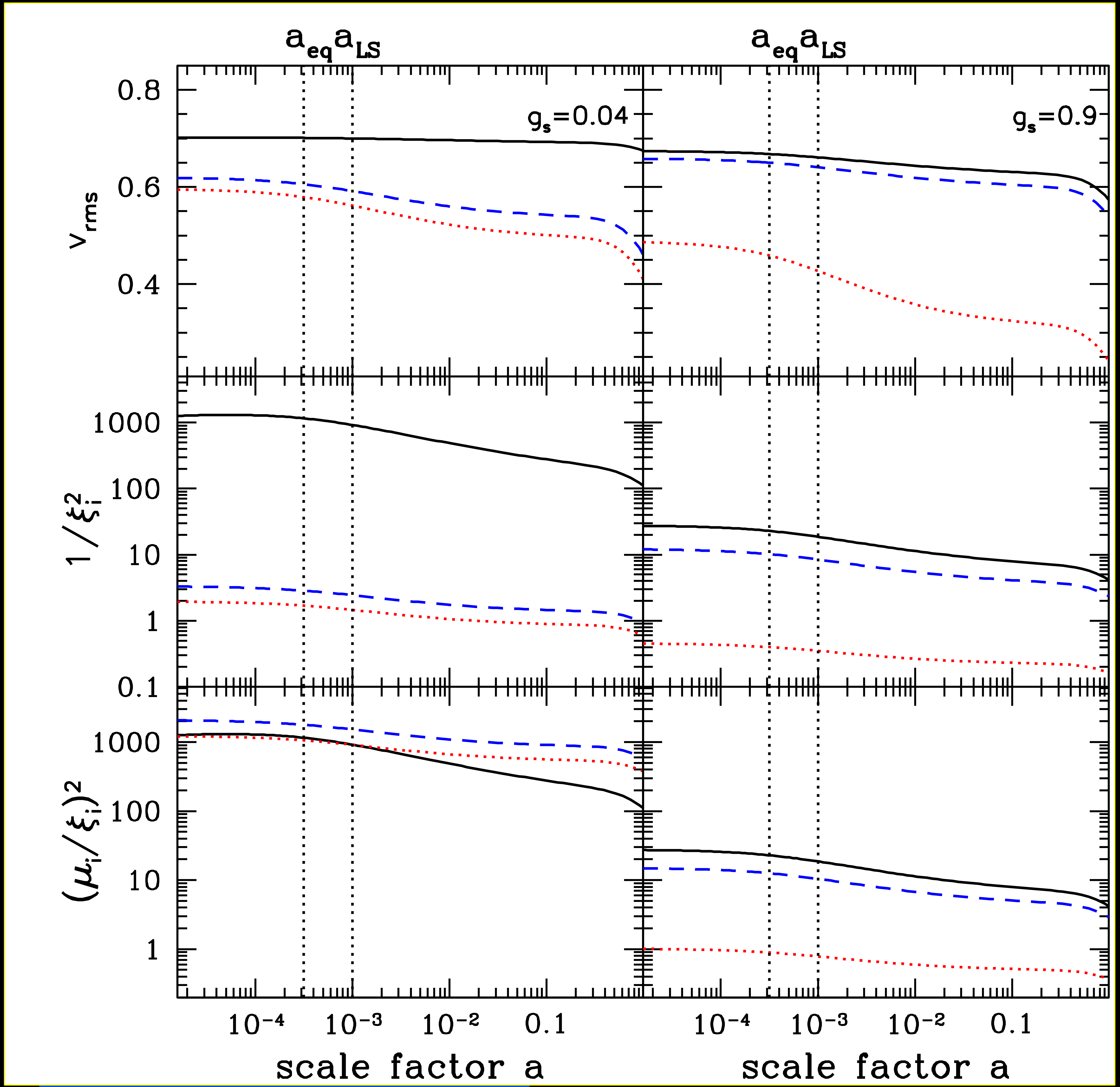
$$\mu_i \equiv \mu_{(p_i,q_i)} = \frac{\mu_F}{g_s}\sqrt{p_i^2 g_s^2 + q_i^2}$$

$$\rho_i = \frac{\mu_i}{L_i^2}$$

\`k' segment length $\ell^k_{ij} = \dfrac{L_i L_j}{L_i + L_j}$

d^k_{ia} incorporate the probabilities of intercommuting and the kinetic constraints. They have a strong dependence on the string coupling g_s

$$\{(p,q)_i\} = \{(1,0),(0,1),(1,1),(1,2),(2,1),(1,3),(3,1)\}\ , \quad (i = 1,...,7)$$



Avgoustidis et al (PRL 2011)

Example - 7 types of (p,q) string. Only first three lightest shown - scaling rapidly reached in rad and matter.

Densities of rest suppressed.

Black -- (1,0) -- Most populous
Blue dash -- (0,1)
Red dot dash -- (1,1)

Deviation from scaling at end as move into Λ domination.

Note lighter F strings dominate number density whilst heavier and less numerous D strings dominate power spectrum for at smaller g_s, where as they are comparable at large g_s ~1

General Network Behaviour

- Scaling for all string types

  (though we keep the first 7 lightest strings)

- Only 3 lightest components

  (F, D, FD strings)

- Hierarchy in number densities

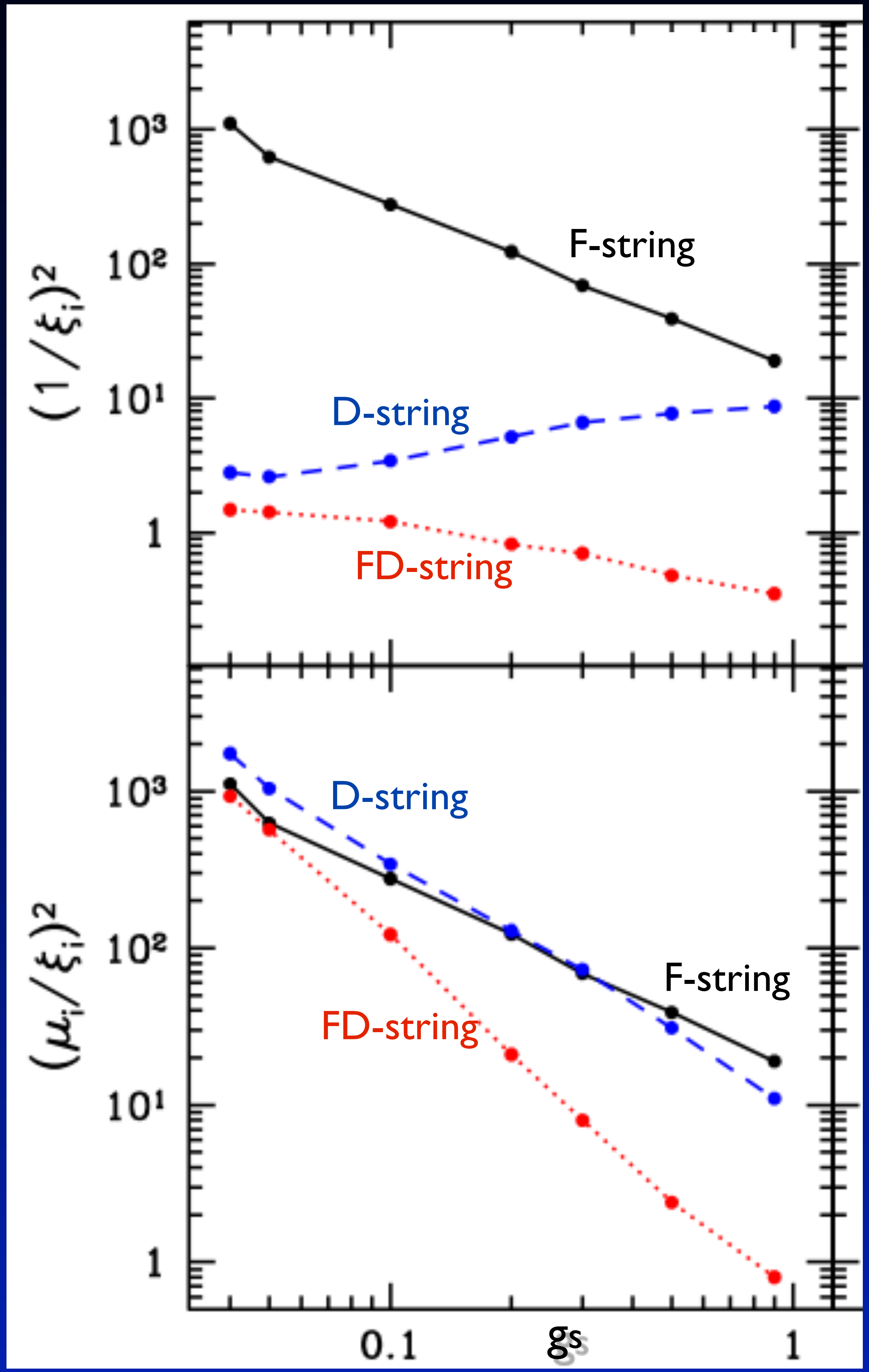
  $N_F > N_D > N_{FD}$

- Hierarchy in tensions

  $\mu_{FD} > \mu_D > \mu_F$

- Number density vs "CMB" density

  Competition depending on g_s



Strings and the CMB

Modified CMBACT (Pogosian) to allow for multi-tension strings.
Shapes of string induced CMB spectra mainly obtained form large scale properties of string such as correlation length and rms velocity given from the earlier evolution eqns.
Normalisation of spectrum depends on:

$$C_l^{strings} \propto \sum_{i=1}^{N} \left(\frac{G\mu_i}{\xi_i} \right)^2$$

i.e. on tension and correlation lengths of each string

Since strings can not source more than 10% of total CMB anisotropy, we use that to determine the fundamental F string tension which is otherwise a free parameter. So μ_F chosen to be such that:

$$f_s = C^{TT}_{strings} / C^{TT}_{total} = 0.1$$

where

$$C^{TT} \equiv \sum_{\ell=2}^{2000} (2\ell + 1) C_\ell^{TT}$$

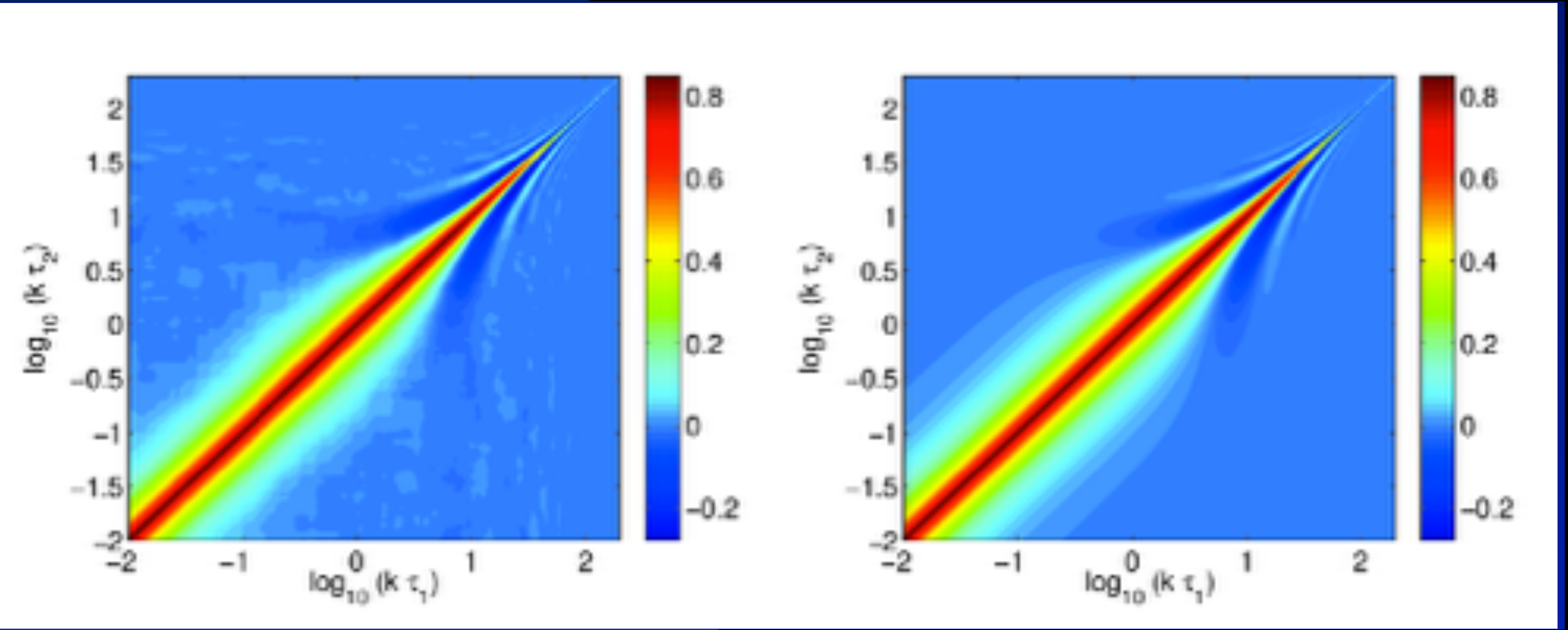
Strings and the CMB

Strings are active, incoherent sources —> require UETC:

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi\, \Theta(k,\tau_1)\Theta(k,\tau_2)$$

Model network as made of unconnected string segments with lengths and velocities given by VOS model

Compute integrals analytically [Avgoustidis et al 2012]

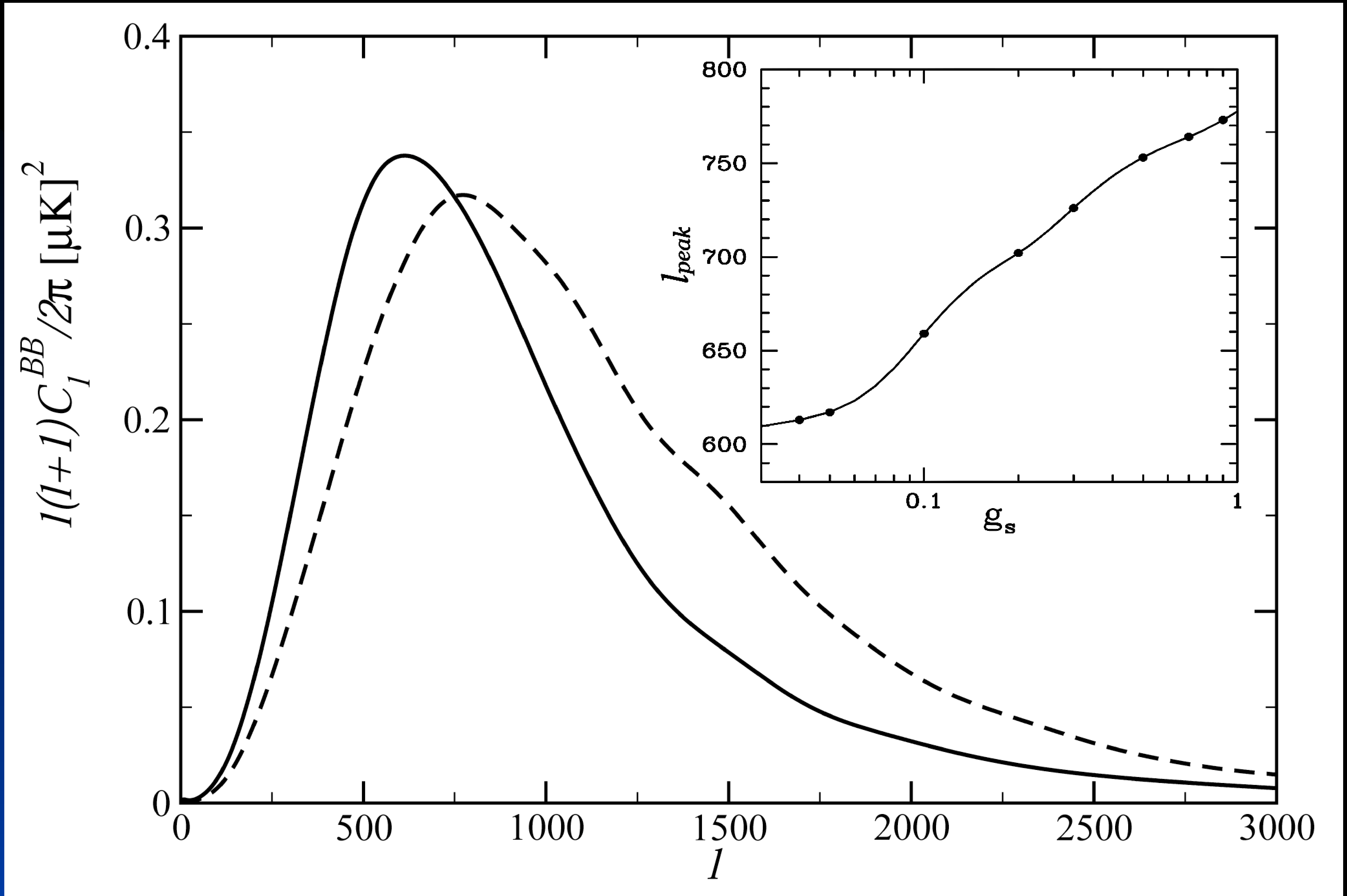


USM - 8 hours

Analytic - 20 secs

Can get Cl's in a few minutes: MCMC analysis including network parameters now possible [Charnock et al 2016]

B-mode Power Spectrum due to strings



B-mode power spectra for g_s = 0.04 (solid) and g_s = 0.9 (dash) normalised so that strings contribute 10% of the total CMB anisotropy.

Inset figure -- the position of the peak as a function of string coupling. Note the shift of the peak to lower l values as the string coupling is reduced.

Possible to discriminate them in future experiments like QUIET and Polarbear.

Results - cosmic strings:

$$G\mu < 1.1 \times 10^{-7} - \text{ Planck2015 TT}$$
$$G\mu < 9.6 \times 10^{-8} - \text{ Planck2015 TT } + \text{ Pol } + \text{ lowP}$$
$$G\mu < 8.9 \times 10^{-8} - \text{ Planck2015 TT } + \text{ Pol } + \text{ lowP } + \text{ BKPlanck}$$

No constraints on c_r and α, slight preference for higher values of c_r

and lower values of α

Results - cosmic superstrings:

$$G\mu_F < 2.8 \times 10^{-8} - \text{ Planck2015 TT } + \text{ lowP}$$

when marginalised over c_s, g_s and w

Currently looking at three point correlation function for evidence of non-gaussianity and B mode polarisation effects - initial results show signal is extremely small and in fact analytically tensor bi-spectrum vanishes.

Recent nanoGrav results consistent with network of cosmic superstrings - exciting for the time being.

Conclusions

Single field Inflation has become the standard paradigm for primordial density fluctuations.

Tight constraints are emerging on the slow roll parameters — possible two scales emerging

Reheating the Universe is an area that has received relatively little attention.

Possible role of non-topological solitons like oscillons in models with asymptotically flat potentials - a new observational route

They could lead to PBHs formed In the early universe - - require modification from standard slow roll inflation

An accurate calculation of the full PDF of the perturbations is required to calculate their abundance.

Where is the inflaton in string theory? Have looked at a particular example and seen the possible importance of the kinating period between the end of inflation and the onset of reheating - some 30 orders of magnitude in time, when lots could happen !

Have looked at cosmic superstrings which could form at the end of a period of string driven inflation !

Aspects not mentioned include:

Multi field inflation

Non-Gaussianity constraints

The link if any between inflation in the early and late universe !

What if inflation never happened ?

Extra slides

Conclusions cont…

Have not discussed many elements of PBH physics:

Role in Information paradox [Hawking 1971,1974]
Role as a catalysis of Ewk phase transition [Gregory et al 2014]
Possible role of PBH Planck mass relics in dark matter constraints [Zeldovich 1984, MacGibbon 1987]
Alternative formation mechanisms such as collapsing cosmic string loops
or from bubble collisions. [Hawking Moss & Stewart 1982]
Baryogenesis scenarios from PBH evaporations [Zeldovich & Starobinski 1976]
PBHs decay by evaporation - interesting attractor solution where PBHs in equilibrium with radiation in both radiation dominated and matter dominated universe - might lead to interesting new features. [Barrow et al 1992]

For objects that as far as we know have never been detetced, PBHs offer staggering constraints on cosmological models.

Analytic treatment of instantaneous transition - works really nicely

Ansatz - motivated by numerical results

$$\eta_H(\tau) = \eta_1 + (\eta_2 - \eta_1)\ \Theta(\tau - \tau_1)$$

Assume piecewise constant η_H - makes instantaneous (yet finite) transition η_1->η_2 at time $\tau = \tau_1$

Obtain

$$\nu^2 - \frac{1}{4} \equiv \frac{z''}{z}\,\tau^2 = \mathcal{A}\,\tau\,\delta_D(\tau-\tau_1) + \nu_1^2 - \frac{1}{4} + \left(\nu_2^2 - \nu_1^2\right)\ \Theta(\tau - \tau_1)\,,$$

where

$$\mathcal{A} = \eta_2 - \eta_1\,, \quad \nu_{1,2}^2 - \frac{1}{4} = 2 - 3\,\eta_{1,2} + \eta_{1,2}^2\,.$$

Note the delta function - gives the rapid dip

Nosie matrix elements

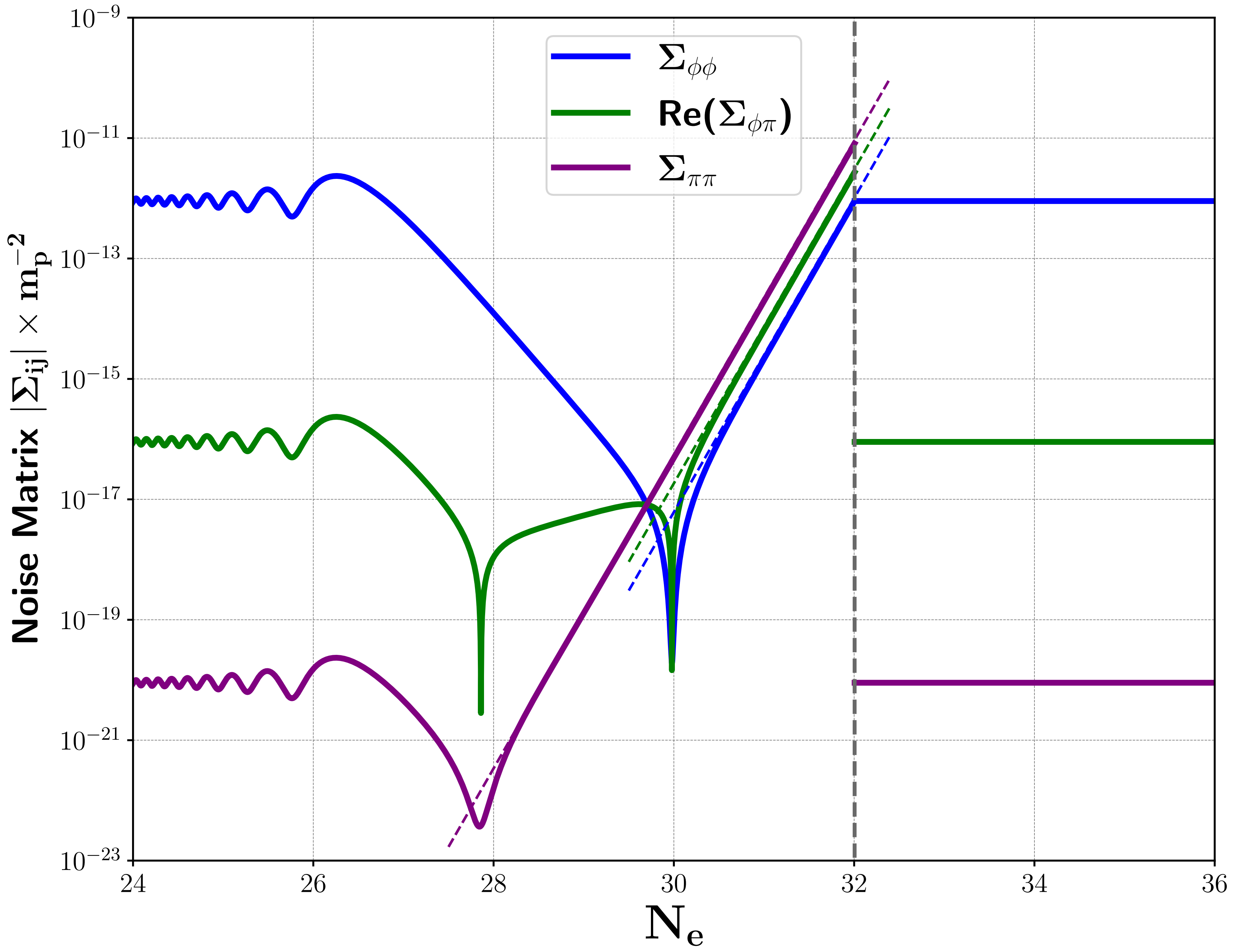
$$\Sigma_{\phi\phi} = \left(\frac{H}{2\pi}\right)^2 T^2 \left|\sqrt{2k}\,v_k(T)\right|^2\Big|_{T=\sigma}\,,$$

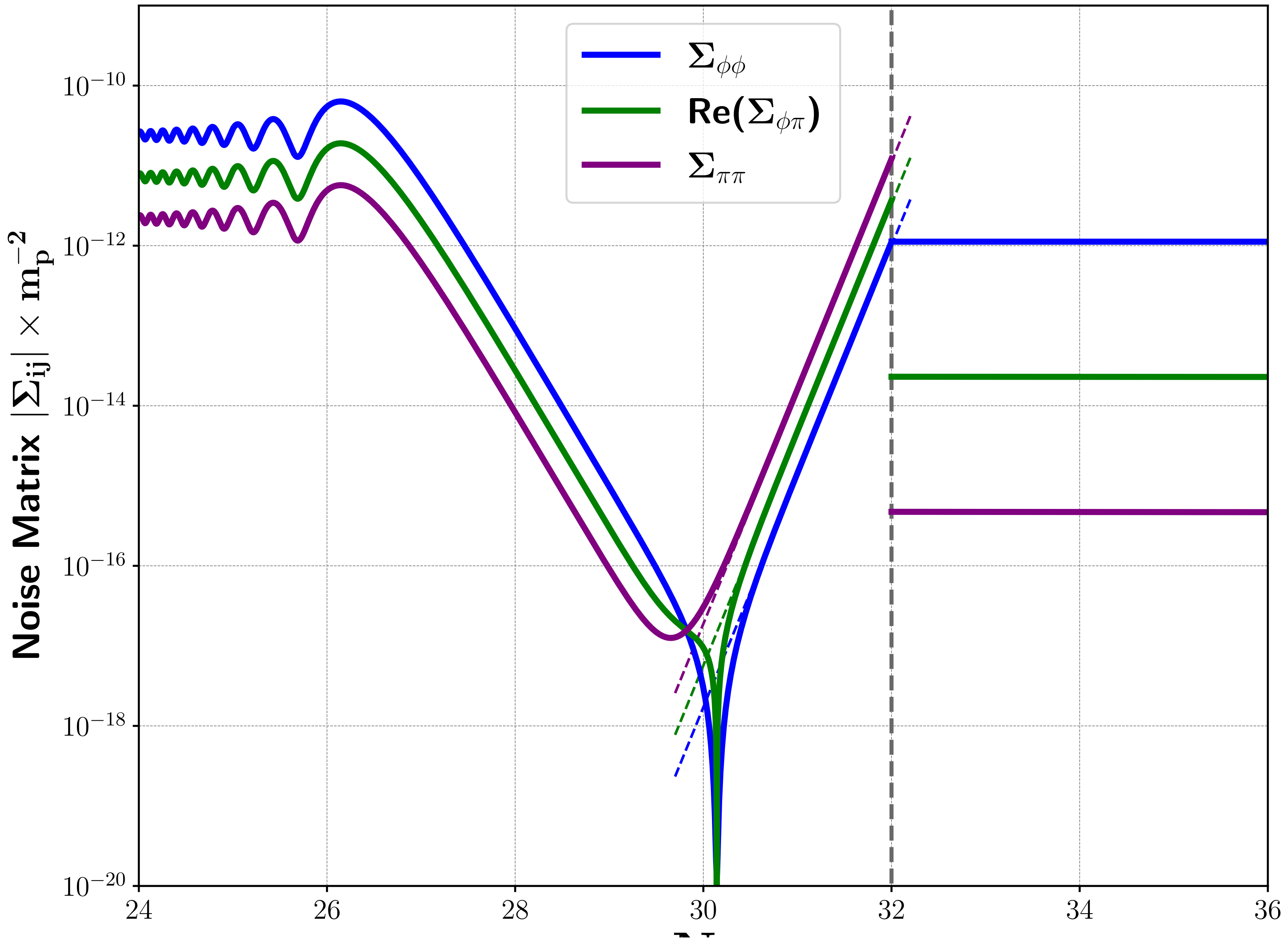
$$\mathrm{Re}\,(\Sigma_{\pi\phi}) = -\left(\frac{H}{2\pi}\right)^2 T^2 \mathrm{Re}\left(\sqrt{2k}v_k^*(T)\left[T\frac{\mathrm{d}}{\mathrm{d}T}\left(\sqrt{2k}v_k(T)\right) + \sqrt{2k}v_k(T)\right]\right)\Big|_{T=\sigma}$$

$$\Sigma_{\pi\pi} = \left(\frac{H}{2\pi}\right)^2 T^2 \left|T\,\frac{\mathrm{d}}{\mathrm{d}T}\left(\sqrt{2k}\,v_k(T)\right) + \sqrt{2k}\,v_k(T)\right|^2\Big|_{T=\sigma}\,,$$

de Sitter case: $\nu_1 = \nu_2 = 3/2$

Instantaneous transition - from SRI: $\nu_1 = 1.52$ to USR with $\nu_2 = 1.8$





Features of analytic solution

Pre transition epoch $T \geq T_1$ with $\nu = \nu_1$

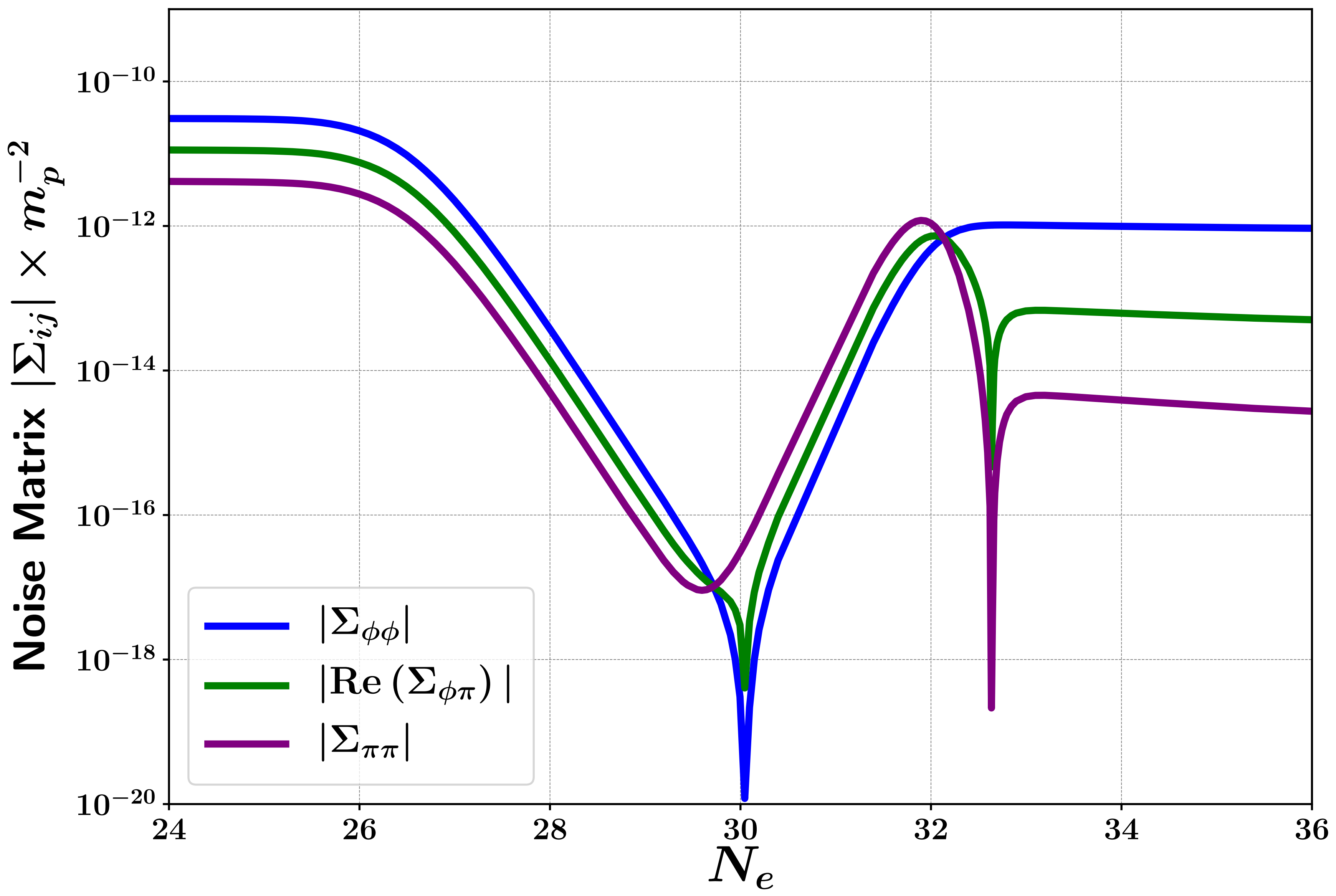
$$\Sigma_{\phi\phi} : |\mathrm{Re}(\Sigma_{\phi\pi})| : \Sigma_{\pi\pi} \to 1 : \left(\nu_1 - \frac{3}{2}\right) : \left(\nu_1 - \frac{3}{2}\right)^2$$

Immediately after transition epoch $\Sigma_{ij} \propto e^{2\mathcal{A}N_e}$ ($\mathcal{A} \equiv \eta_2 - \eta_1 = 3.32$) and

$$\Sigma_{\phi\phi} : |\mathrm{Re}(\Sigma_{\phi\pi})| : \Sigma_{\pi\pi} \to 1 : \mathcal{A} : \mathcal{A}^2$$

Sufficiently late times, $T \ll T_1$, same as above but with $\nu = \nu_2$,

$$\Sigma_{\phi\phi} : |\mathrm{Re}(\Sigma_{\phi\pi})| : \Sigma_{\pi\pi} \to 1 : \left(\nu_2 - \frac{3}{2}\right) : \left(\nu_2 - \frac{3}{2}\right)^2$$



Full numerical solution

• But when power spectrum sufficiently amplified for an interesting abundance of PBHs, $\pi_{\rm en} \simeq \pi_{\rm cr} \Rightarrow y_{\rm en} \simeq 1$.

• Then, both classical drift and stochastic diffusion become important (at least initially during the entry into the USR segment).

• Furthermore, the de Sitter approximations for the noise matrix elements might breakdown during the transition into the USR phase [Ahmadi et al 2022] .

• Consequently, it becomes important to estimate the noise matrix elements more accurately.

Case 1: Noise matrix elements in stochastic inflation with featureless potential – slow roll case

Evolution of modes $\{\phi_k, \pi_k\}$ given via Mukhanov-Sasaki equation which in terms of conformal time τ is

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0 \ ,$$

where

$$z = a m_p \sqrt{2\epsilon_H} \ ,$$
$$\frac{z''}{z} = (aH)^2 \left[2 + 2\epsilon_H - 3\eta_H + 2\epsilon_H^2 + \eta_H^2 - 3\epsilon_H\eta_H - \frac{1}{aH}\,\eta_H'\right]$$

and in the spatially flat gauge: $$\phi_k = \frac{v_k}{a} \ , \quad \pi_k = \frac{\mathrm{d}}{\mathrm{d}N}\left(\frac{v_k}{a}\right)$$

Early times, all mode sub horizon -> impose Bunch Davies i.c

$$\lim_{k\tau \longrightarrow -\infty} v_k(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}$$

Intro new time variable:

$$T = -k\tau = \frac{k}{aH}$$

MS-eqn becomes :

$$\frac{\mathrm{d}^2 v_k}{\mathrm{d}T^2} + \left(1 - \frac{\nu^2 - \frac{1}{4}}{T^2}\right) v_k = 0,$$

$$\nu^2 = \frac{1}{(aH)^2} \frac{z''}{z} + \frac{1}{4}.$$

For slow-roll inflation, $\nu^2 \geq 9/4$ at early times and increases monotonically towards the end of inflation.

Case of Pure dS limit, both $\epsilon_H, \eta_H = 0$, leading to $z''/z = 2a^2H^2$ and $\nu^2 = 9/4$.

Obtain mode solution:

$$v_k(T) = \frac{1}{\sqrt{2k}}\left(1 + \frac{i}{T}\right)e^{iT}$$

And exact noise matrix elements, (recall evaluated at $k = \sigma a H$, hence when $T = \sigma$)

$$\Sigma_{\phi\phi} = (1+\sigma^2)\left(\frac{H}{2\pi}\right)^2$$

$$\mathrm{Re}(\Sigma_{\phi\pi}) = -\sigma^2\left(\frac{H}{2\pi}\right)^2$$

$$\Sigma_{\pi\pi} = \sigma^4\left(\frac{H}{2\pi}\right)^2$$

For $\sigma = 0.01$ say have $\Sigma_{\phi\phi} : \Sigma_{\phi\pi} : \Sigma_{\pi\pi} = 1 : 10^{-4} : 10^{-8}$ - which is why $\Sigma_{\phi\pi}$ and $\Sigma_{\pi\pi}$ usually ignored.

Case of slow roll inflation where $\epsilon_H, \eta_H \ll 1$, the slow-roll parameters **but** do not exactly vanish.

For realistic SR potentials, ν is roughly equal to $3/2$ and evolves slowly and monotonically. We obtain

$$v_k(T) = e^{i(\nu+\frac{1}{2})\frac{\pi}{2}} \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{2k}} \sqrt{T}\, H_\nu^{(1)}(T)\,,$$

And on superhorizon scales:

$$\Sigma_{\phi\phi} = 2^{2(\nu-\frac{3}{2})} \left[\frac{\Gamma(\nu)}{\Gamma(3/2)}\right]^2 \left(\frac{H}{2\pi}\right)^2 T^{2(-\nu+\frac{3}{2})}\,,$$

$$\mathrm{Re}\,(\Sigma_{\phi\pi}) = -2^{2(\nu-\frac{3}{2})} \left[\frac{\Gamma(\nu)}{\Gamma(3/2)}\right]^2 \left(\frac{H}{2\pi}\right)^2 \left(-\nu+\frac{3}{2}\right) T^{2(-\nu+\frac{3}{2})}\,,$$

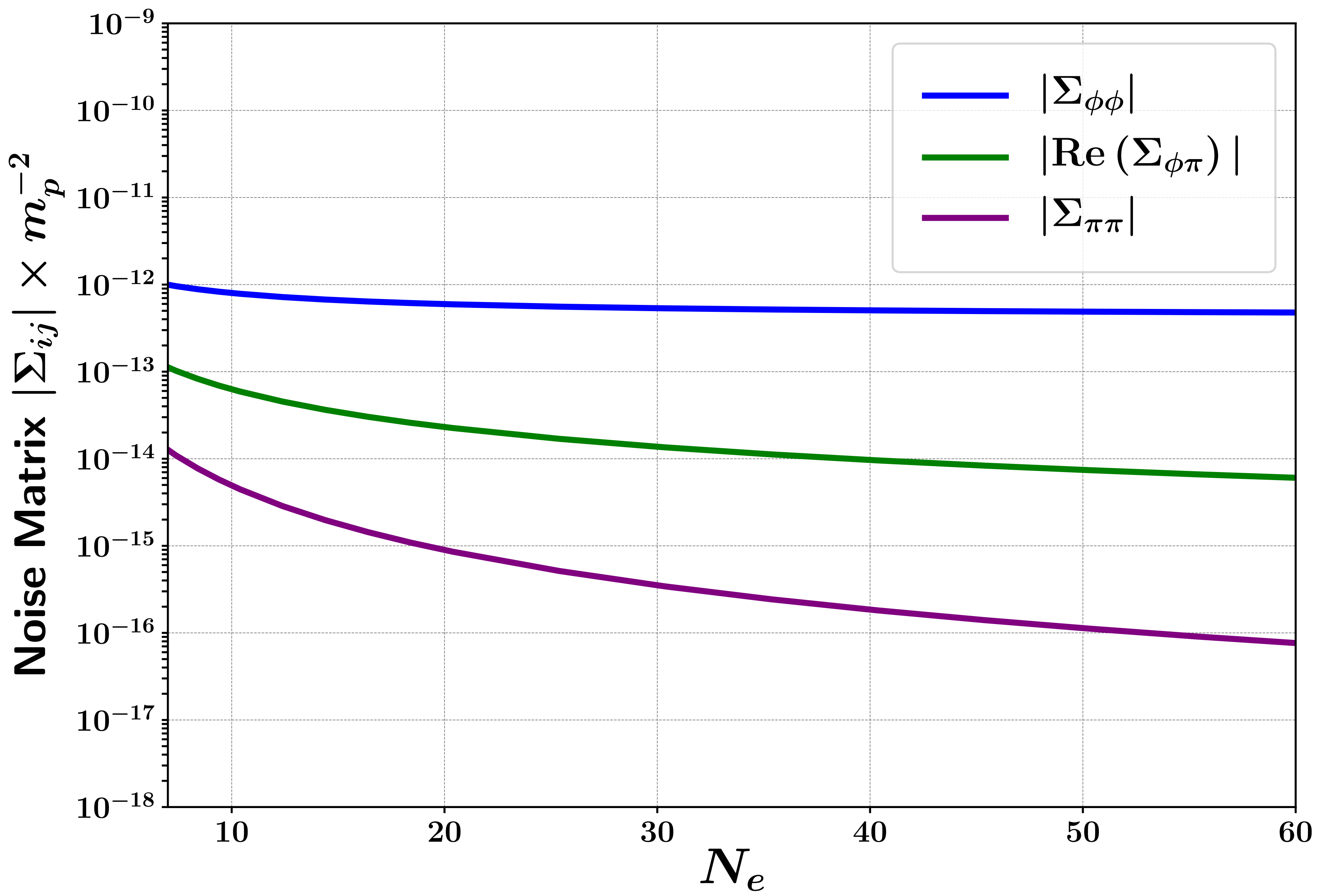
$$\Sigma_{\pi\pi} = 2^{2(\nu-\frac{3}{2})} \left[\frac{\Gamma(\nu)}{\Gamma(3/2)}\right]^2 \left(\frac{H}{2\pi}\right)^2 \left(-\nu+\frac{3}{2}\right)^2 T^{2(-\nu+\frac{3}{2})}\,.$$

Note, the hierarchy of noise terms no longer necessarily present

For D-brane KKLT type potential

$$V(\phi) = V_0 \frac{\phi^2}{M^2+\phi^2}$$

we find for large N_e, $\Sigma_{\phi\phi} : |\mathrm{Re}(\Sigma_{\phi\pi})| : \Sigma_{\pi\pi} = 1 : 10^{-2} : 10^{-4}$ unlike de Sitter case.



Case of potentials with a slow-roll violating feature, like USR with $\epsilon_H \ll 1$, while $\eta_H \geq 1$

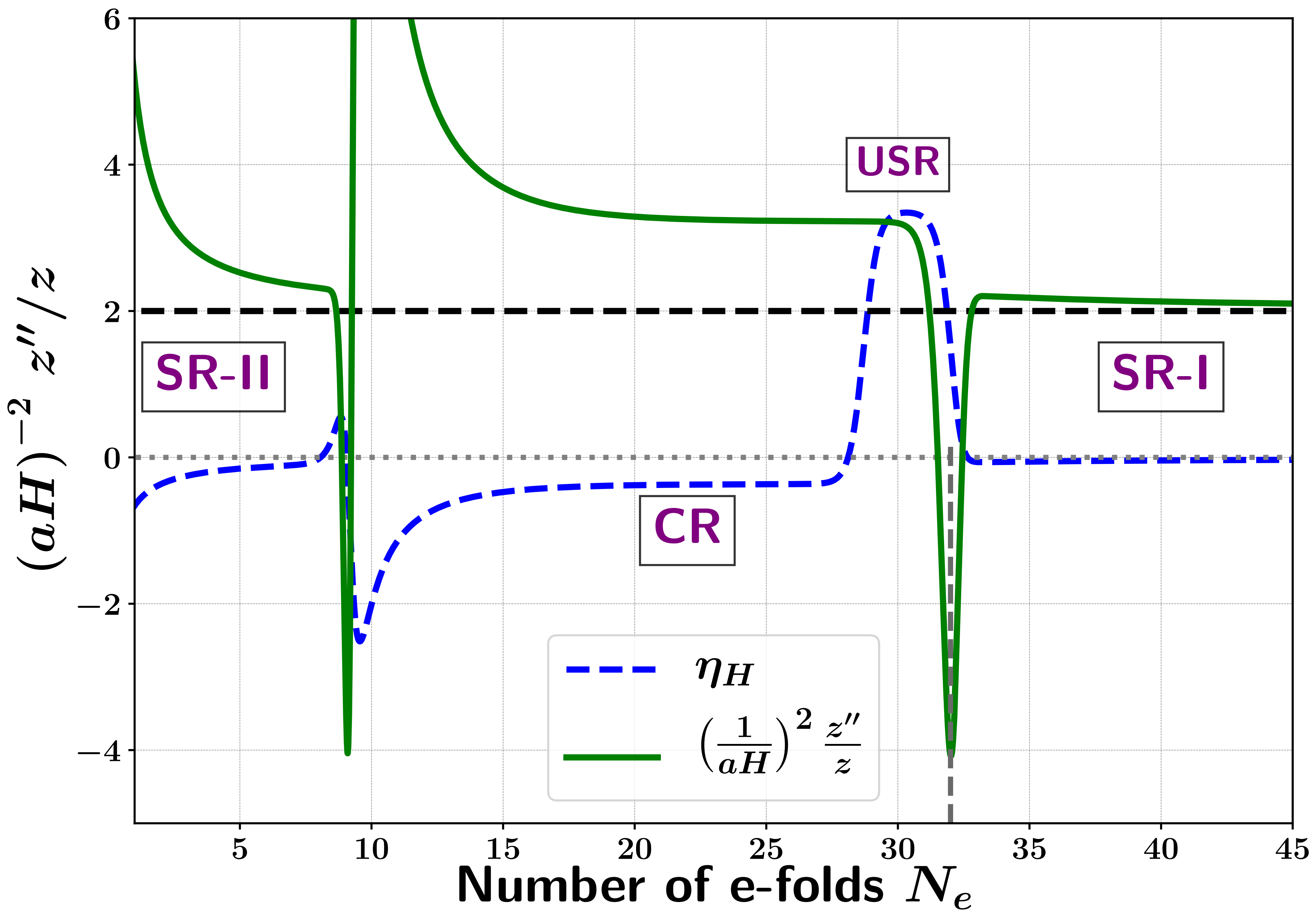
Dynamics undergoes number of phases driven by η_H. We now have :

$$\frac{1}{(aH)^2}\,\frac{z''}{z} \simeq 2 - 3\,\eta_H + \eta_H^2 + \tau\,\frac{\mathrm{d}\eta_H}{\mathrm{d}\tau}$$

Specific example, a modified KKLT potential with an additional tiny Gaussian bump-like feature [Mishra et al 2019]:

$$V_{\mathrm{b}}(\phi) = V_0\,\frac{\phi^2}{M^2+\phi^2}\,\left[1 + A\,\exp\left(-\frac{1}{2}\,\frac{(\phi-\phi_0)^2}{\tilde{\sigma}^2}\right)\right],$$

where A, $\tilde{\sigma}$ and ϕ_0 represent the height, width and position of the bump respectively.

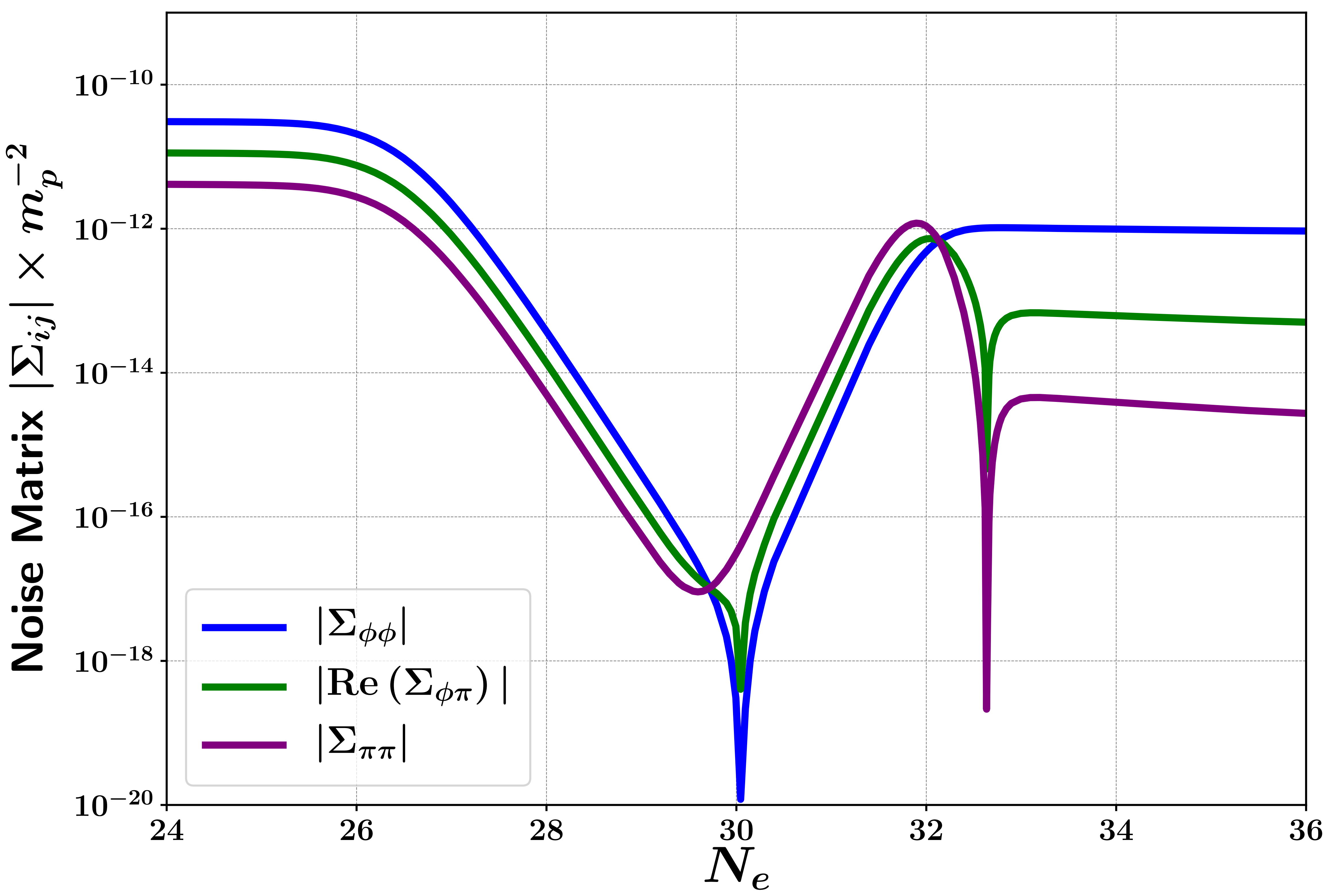


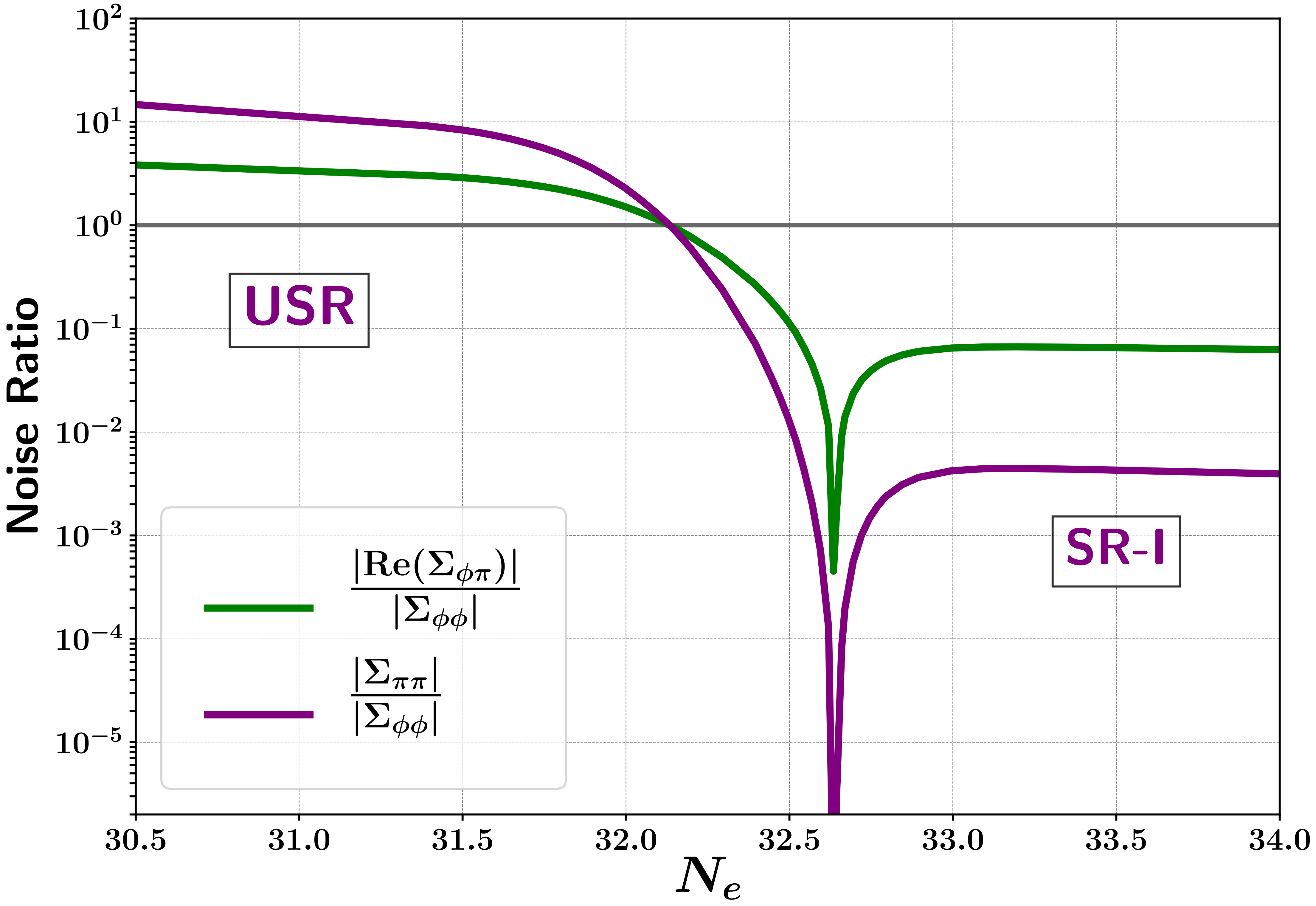
Fixed $M = 0.5\,m_p$, and bump parameters to be $A = 1.87 \times 10^{-3}$, $\tilde{\sigma} = 1.993 \times 10^{-2}$ and $\phi_0 = 2.005\,m_p$.

Gives amplification of the scalar power-spectrum, $\mathcal{P}_\zeta$, by a factor of 10^7 relative to its value on CMB scales.

In reality — noise terms are more interesting !

Numerical noise matrix elements, Σ_{ij} - note the switching of dominant terms during USR





Noise ratios in the SR-1 to USR region

With Swagat Mishra and Anne Green - e-Print:2303.17375 - JCAP 2024

Fixed $M = 0.5\,m_p$, and bump parameters to be $A = 1.87 \times 10^{-3}$, $\tilde{\sigma} = 1.993 \times 10^{-2}$ and $\phi_0 = 2.005\,m_p$.

Gives amplification of the scalar power-spectrum, $\mathcal{P}_\zeta$, by a factor of 10^7 relative to its value on CMB scales.

Outstanding steps to calculate the PBH mass fraction

Have calculated the stochastic noise matrix elements Σ_{ij}, for a sharp transition from SR to USR

Aim is to determine the PDF of the number of e-folds, $P_{\Phi,\Pi}(\mathcal{N})$, by solving the adjoint Fokker-Planck eqn

$$\frac{\partial}{\partial \mathcal{N}} P_{\Phi_i}(\mathcal{N}) = \left[D_i \frac{\partial}{\partial \Phi_i} + \frac{1}{2} \Sigma_{ij} \frac{\partial^2}{\partial \Phi_i \partial \Phi_j} \right] P_{\Phi_i}(\mathcal{N}) .$$

$$P_{\Phi,\Pi}(\mathcal{N}) = \sum_{n=0}^{\infty} B_n(\Phi,\Pi)\, e^{-\Lambda_n \mathcal{N}}$$

$B_n(\Phi,\Pi)$ to be determined from b.c. and expressions for Σ_{ij}

Then calculate the mass fraction of PBHs $\beta_{\rm PBH}$.

$$\beta(\Phi,\Pi) \equiv \int_{\zeta_c}^{\infty} P(\zeta_{\rm cg})\, \mathrm{d}\zeta_{\rm cg} = \int_{\zeta_c + \langle \mathcal{N}(\Phi,\Pi) \rangle}^{\infty} P_{\Phi,\Pi}(\mathcal{N})\, \mathrm{d}\mathcal{N}$$

$$\beta(\Phi,\Pi) = \sum_{n=0}^{\infty} \frac{B_n(\Phi,\Pi)}{\Lambda_n} \exp\left[-\Lambda_n \left[\zeta_c + \sum_{m=0}^{\infty} \frac{B_m(\Phi,\Pi)}{\Lambda_m^2} \right] \right]$$

Of course this might not be possible !

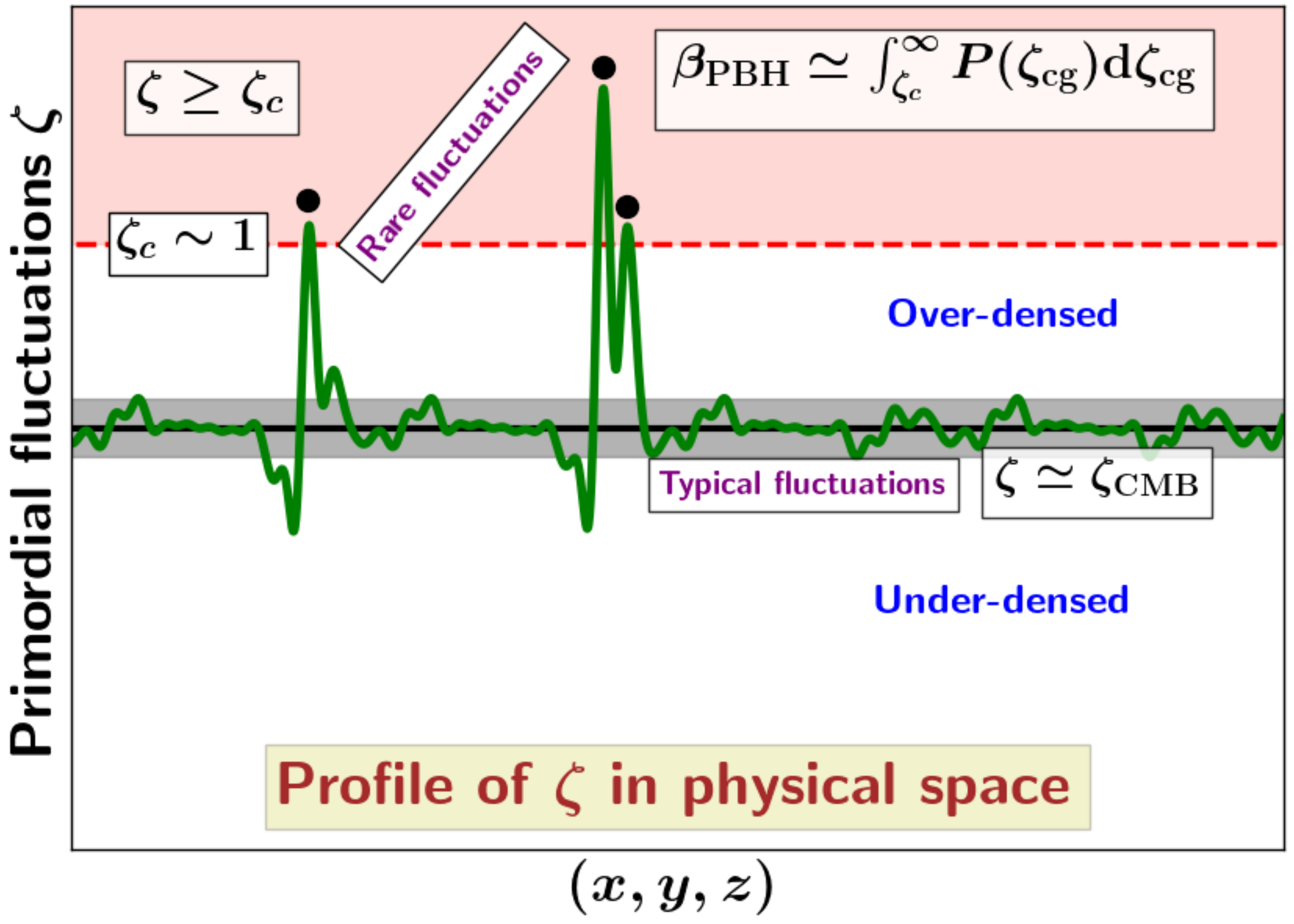
$$\beta^{\rm G}(\Phi,\Pi) \simeq \frac{\sigma_{\rm cg}}{\sqrt{2\pi}\zeta_c} \exp\left[-\frac{\zeta_c^2}{2\sigma_{\rm cg}^2} \right]$$

Compare with Gaussian PDF for typical fluctuations in the perturbative approach, $\beta^{\rm G}(\Phi,\Pi)$

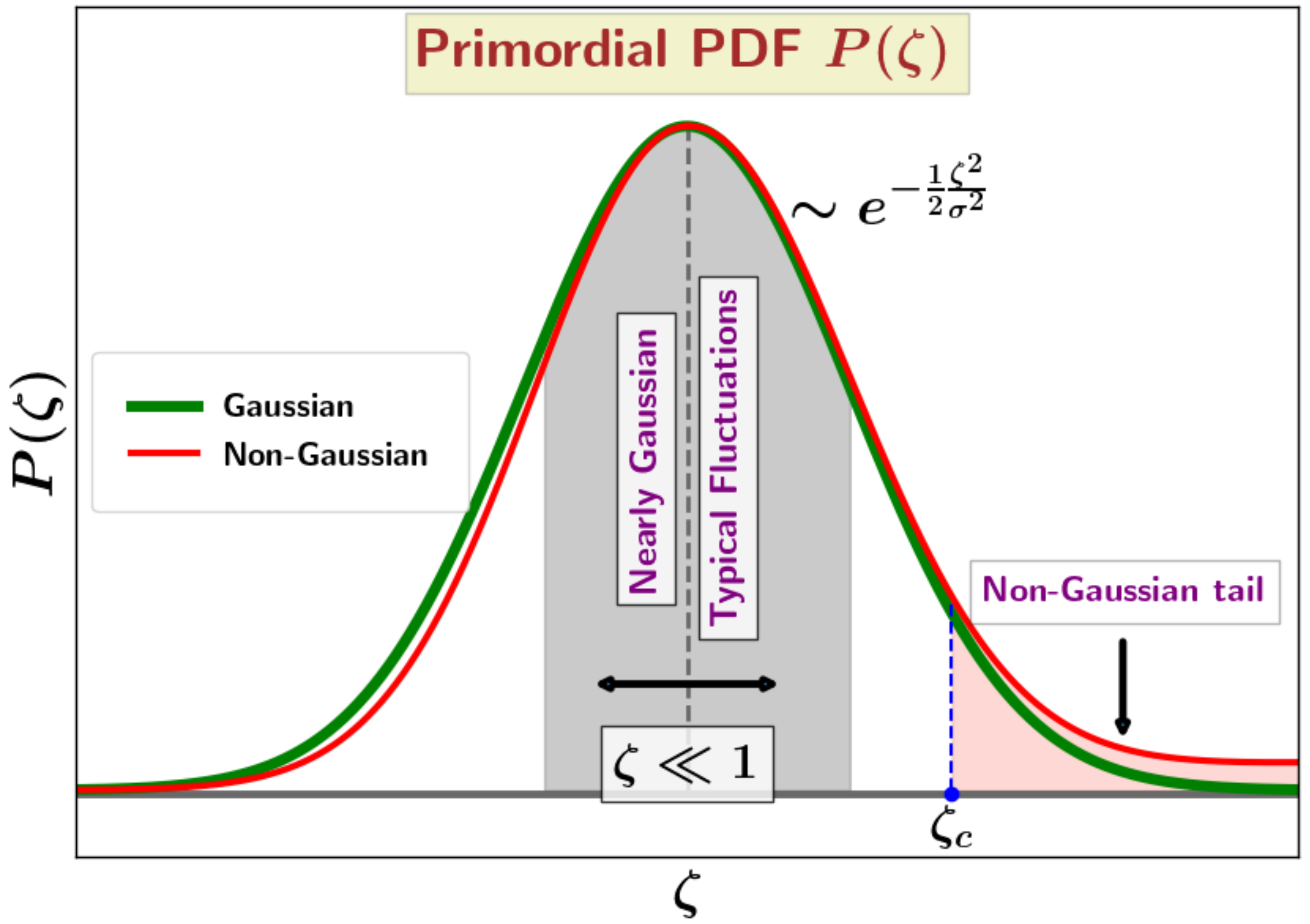
$$\sigma_{\rm cg}^2(\Phi,\Pi) = \int_{k(\Phi,\Pi)}^{k_e} \frac{\mathrm{d}k}{k} \mathcal{P}_\zeta(k) \ .$$

Why might a non-gaussian PDF of Primordial Fluctuations help with creating PBHs ?

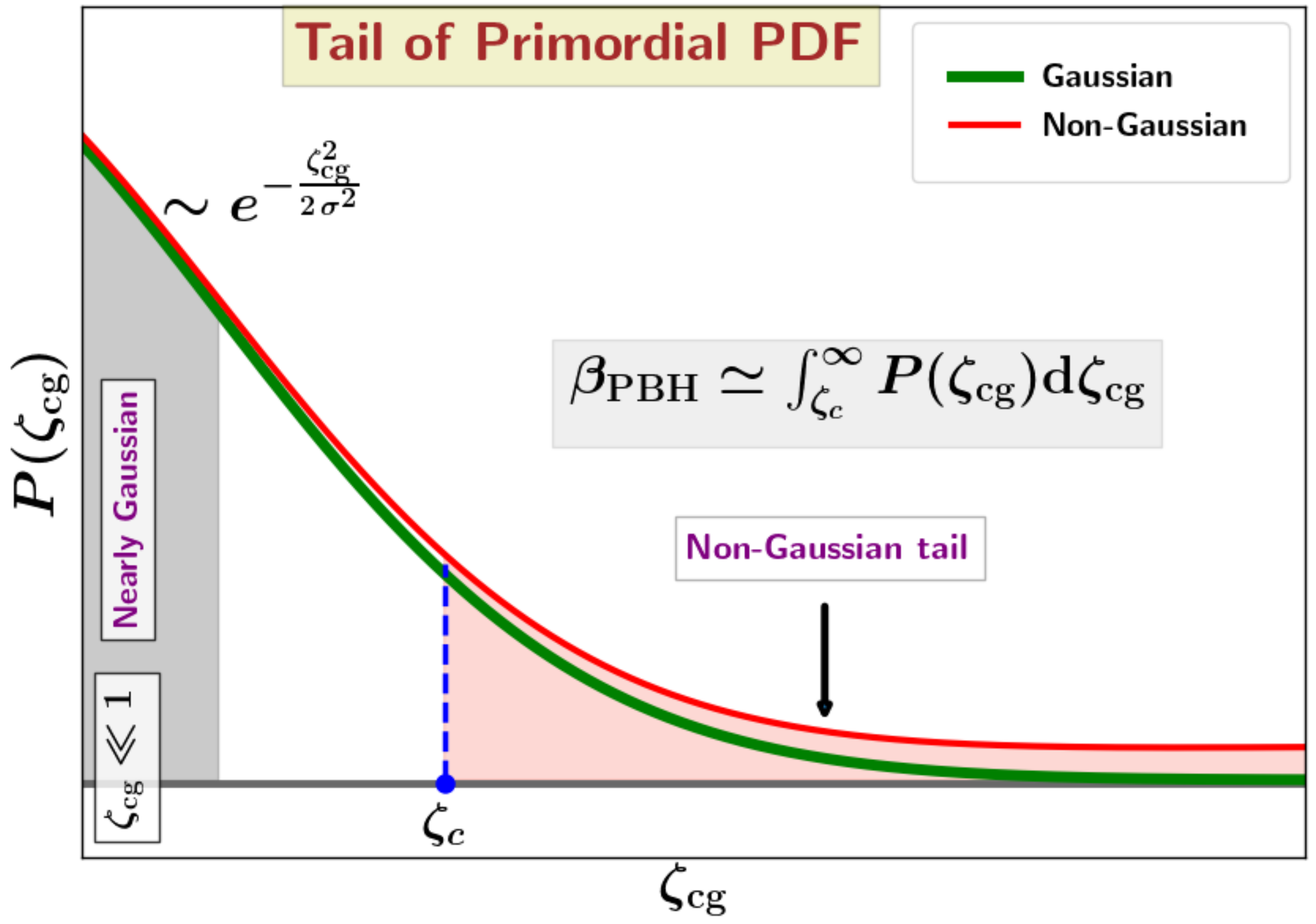
We expect PBHs to form from rare peaks in the fluctuations in the density contrast



For small fluctuations we expect the PDF to be Gaussian

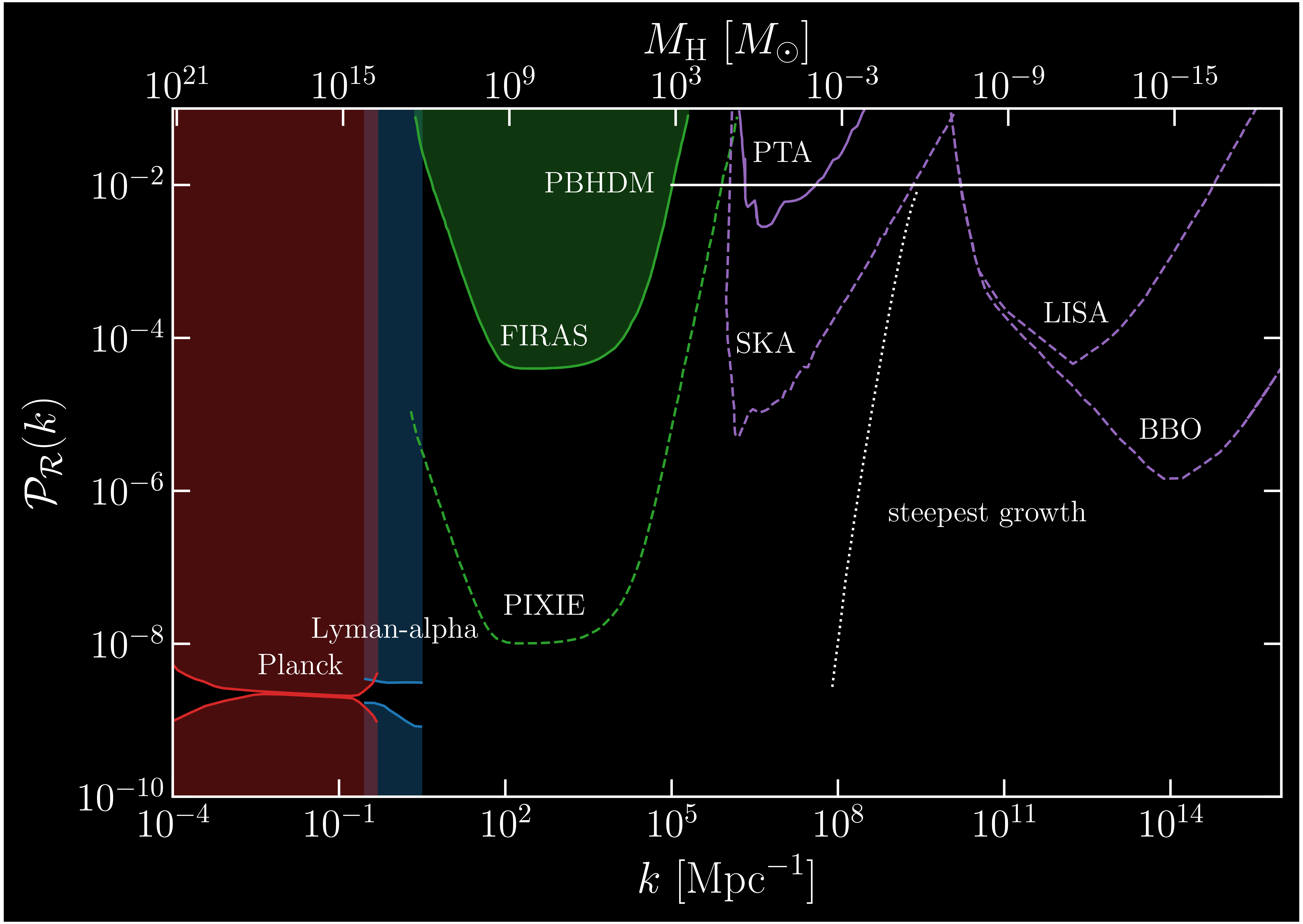


But deviations from Gaussian for large fluctuations could increase the PDF enhancing the likelihood of forming PBHs



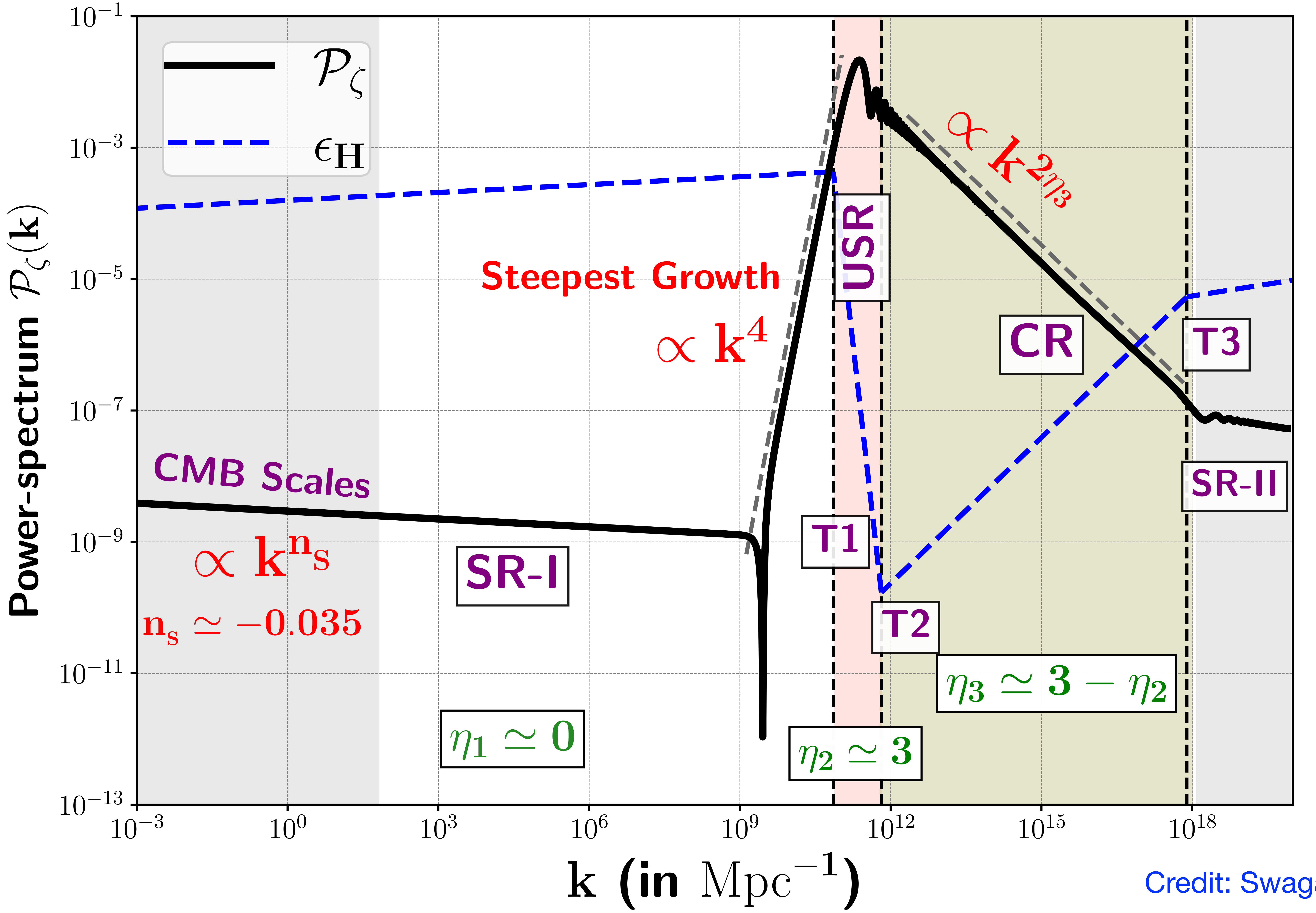
Credit: Swagat Mishra

Observational Constraints on Power Spectrum - very little on small scales



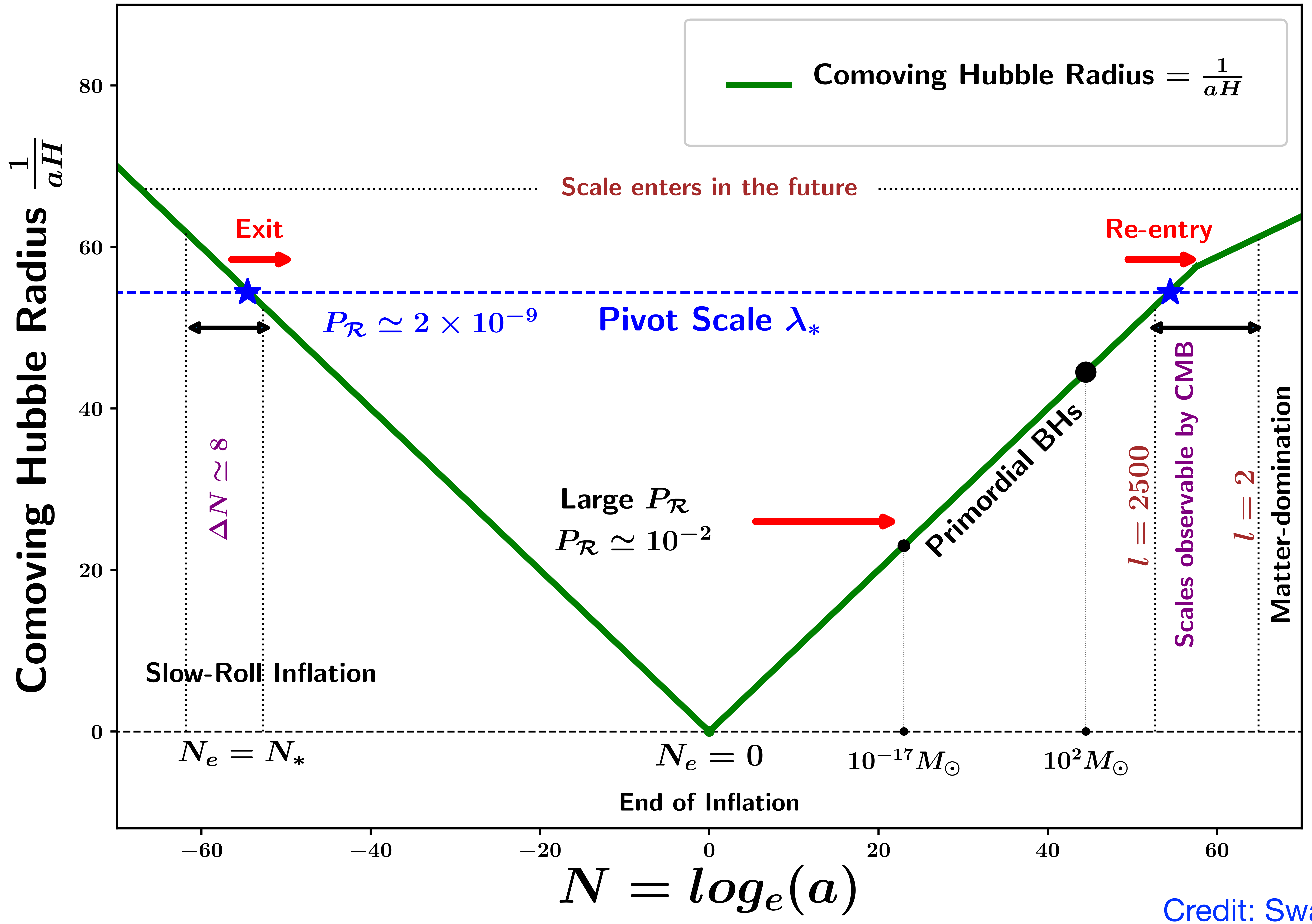
Carr et al 2020; Green and Kavanagh 2020

In terms of a power spectrum generated from inflation we require



Credit: Swagat Mishra

PBH size fluctuations re-enter on different scales

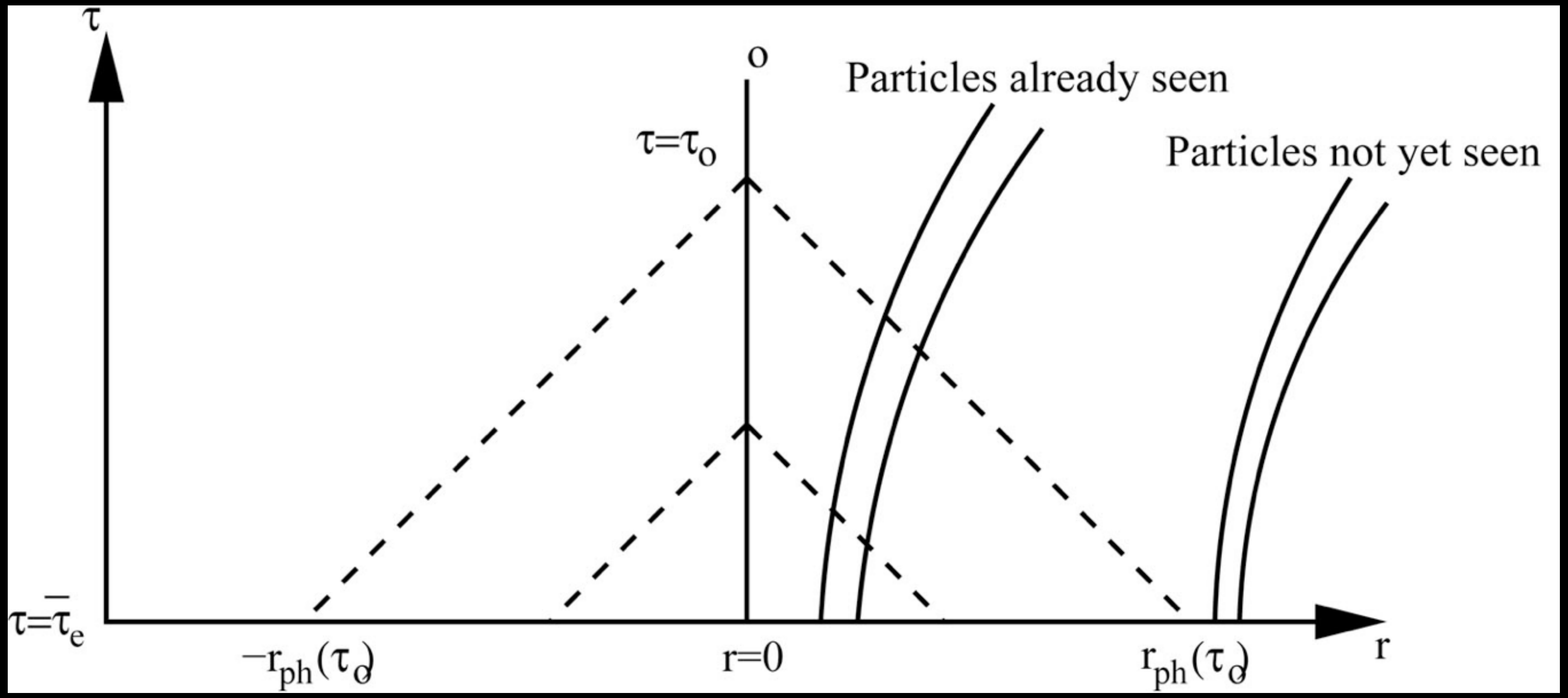


Credit: Swagat Mishra

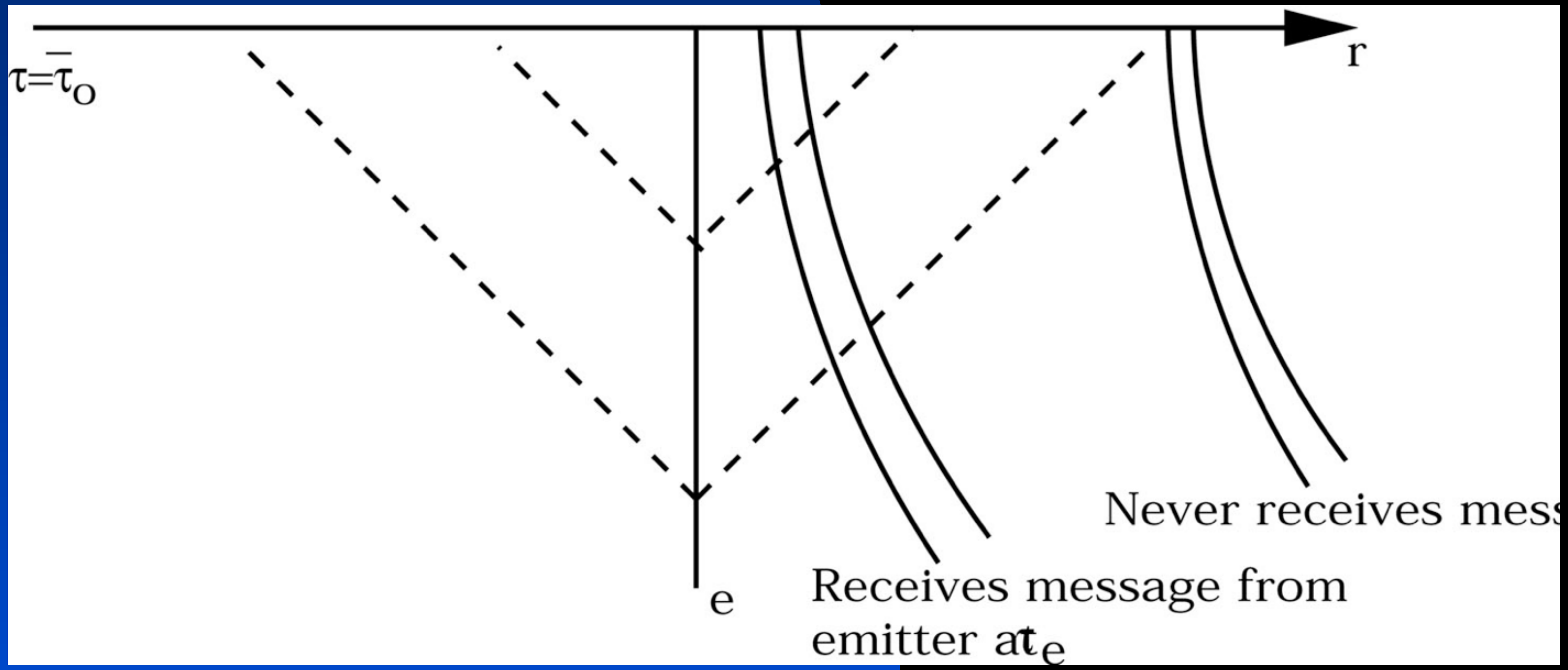
Horizons -- crucial concept in cosmology

a) *Particle horizon:* is the proper distance at time t that light could have travelled since the big bang (i.e. at which *a=0*). It is given by

$$d_p(t) = a(t) \int_0^t \frac{dt'}{a(t')}$$



b) *Event horizon: is* the proper distance at time t that light will be able to travel in the future:



$$d_{\rm EH}(t) = a(t) \int_t^\infty \frac{dt'}{a(t')}$$

Trodden and Carroll 03