Lecture 3
Approaches to understanding Dark Energy

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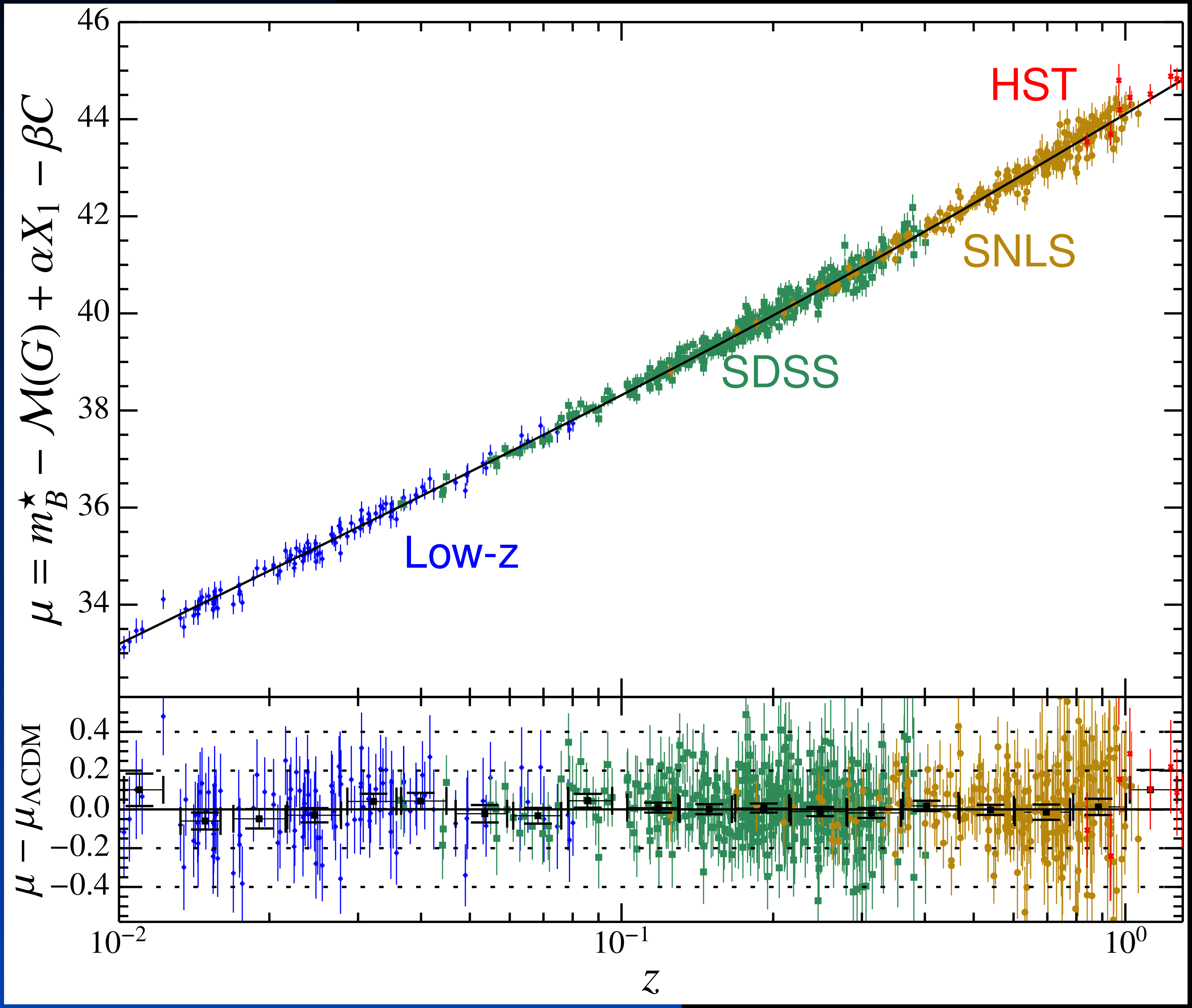
1. Brief recap of evolution of the universe: assumptions and evidence supporting them - pointing out issues where they may occur.
2. Approaches to Dark Energy and Modified Gravity.
3. Testing screening mechanisms in the laboratory.
4. Hubble tension and approaches to Early Dark Energy
5. Impact of GW discovery on late time cosmology.
6. Dark Energy and the String Swampland
7. Recent large z results if quasars can be standard candles

Invisibles School, Bologna — June 24th - June 28th 2024

The Big Bang – (1sec → today)

The cosmological principle -- isotropy and homogeneity on large scales



- **The expansion of the Universe $v=H_0d$**

H_0=73.04±1.04 km s^{-1} Mpc^{-1}

(Riess et al, 2022)

H_0=67.4±0.5 km s^{-1} Mpc^{-1}

(Planck 2018)

Is there a local v global tension ?

Betoule et al 2014

$$\text{Redshift } 1 + z = \frac{a_0}{a} \qquad H = \frac{\dot{a}}{a}$$

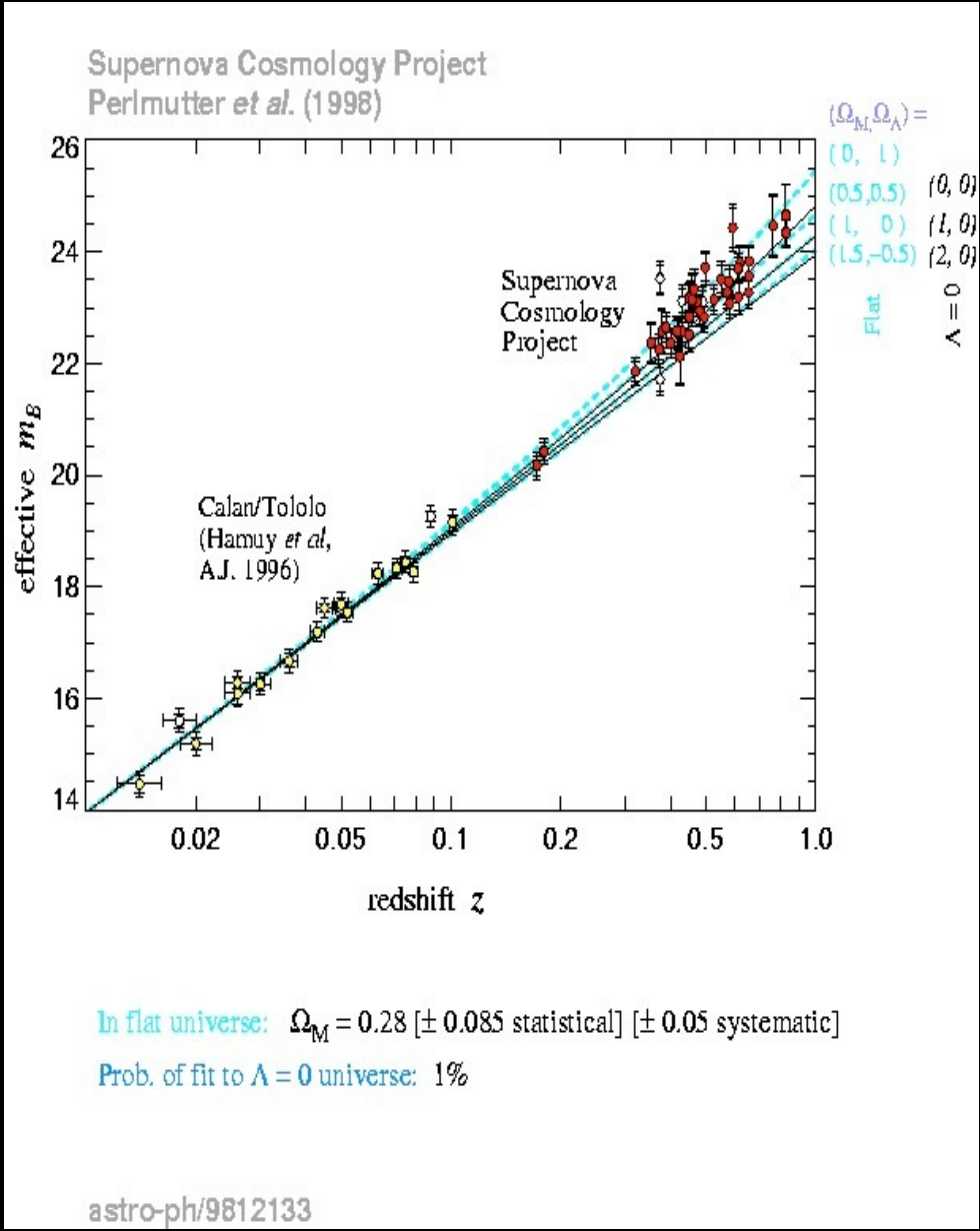
In fact the universe is accelerating !

Observations of distant supernova in galaxies indicate that the rate of expansion is increasing !

Huge issue in cosmology -- what is the fuel driving this acceleration?

We call it Dark Energy -- emphasises our ignorance!

Makes up 70% of the energy content of the Universe



$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$ applied to cosmology

Friedmann - the key bgd equation:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

a(t) depends on matter, $\rho(t)=\Sigma_i\rho_i$ -- sum of all matter contributions, rad, dust, scalar fields ...

$$\text{Energy density } \rho(t): \quad \text{Pressure } p(t)$$

$$\text{Related through}: p = w\rho$$

Eqn of state parameters: w=1/3 – Rad dom: w=0 – Mat dom: w=-1– Vac dom

Eqns (Λ=0):

Friedmann + Fluid energy conservation

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

$$\nabla^\mu T_{\mu\nu} = 0$$

A neat equation

$$\rho_c(t) \equiv \frac{3H^2}{8\pi G} \quad ; \quad \Omega(t) \equiv \frac{\rho}{\rho_c}$$

$$\Omega > 1 \leftrightarrow k = +1$$

$$\Omega = 1 \leftrightarrow k = 0$$

$$\Omega < 1 \leftrightarrow k = -1$$

Friedmann eqn

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

Ω_m - baryons, dark matter, neutrinos, electrons, radiation ...

Ω_Λ - dark energy ; Ω_k - spatial curvature

$$\rho_c(t_0) \equiv 1.88 h^2 * 10^{-29} \, \mathrm{g\,cm^{-3}}$$

Critical density

Bounds on H(z) -- Planck 2018 - (+BAO+lensing+lowE)

$$\mathbf{H^2(z) = H_0^2\left(\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{de}\exp\left(3\int_0^z \frac{1+w(z')}{1+z'}dz'\right)\right)}$$

(Expansion rate) -- H_0=67.66 ± 0.42 km/s/Mpc

(radiation) -- $\Omega_r = (8.5 \pm 0.3) \times 10^{-5}$ - (WMAP)

(baryons) -- $\Omega_b h^2 = 0.02242 \pm 0.00014$

(dark matter) -- $\Omega_c h^2 = 0.11933 \pm 0.00091$ —-(matter) - $\Omega_m = 0.3111 \pm 0.0056$

(curvature) -- $\Omega_k = 0.0007 \pm 0.0019$

(dark energy) -- $\Omega_{de} = 0.6889 \pm 0.0056$ -- Implying univ accelerating today

(de eqn of state) -- $1+w = 0.028 \pm 0.032$ -- looks like a cosm const.

If allow variation of form : $w(z) = w_0 + w_a\, z/(1+z)$ then
$w_0 = -0.957 \pm 0.08$ and $w_a = -0.29 \pm 0.31$ (68% CL) — (Planck 2018+SNe+BAO)

Important because distance measurements often rely on assumptions made about the background cosmology.

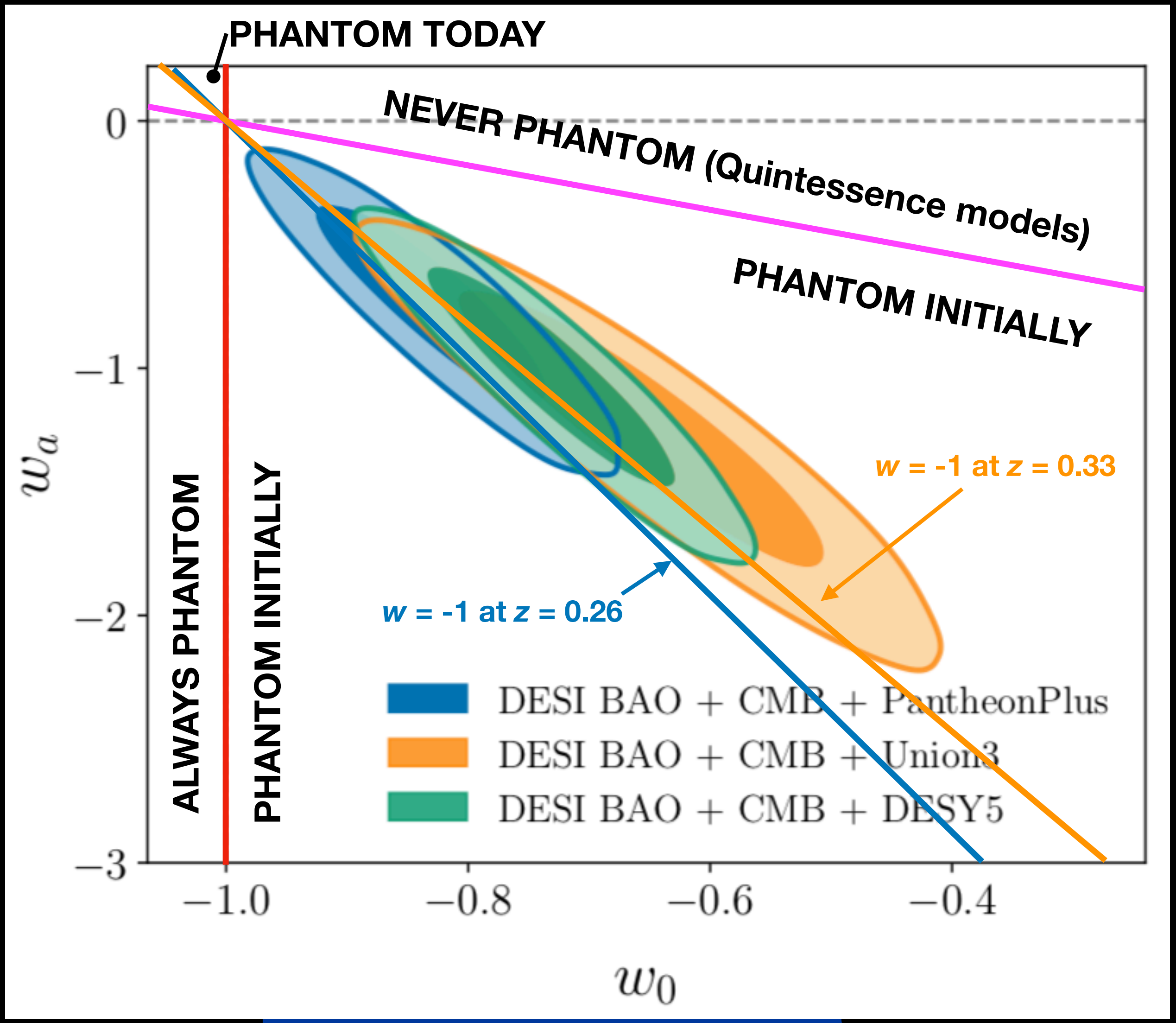
Recent developments — DESI (2024) - arXiv:2404.03002

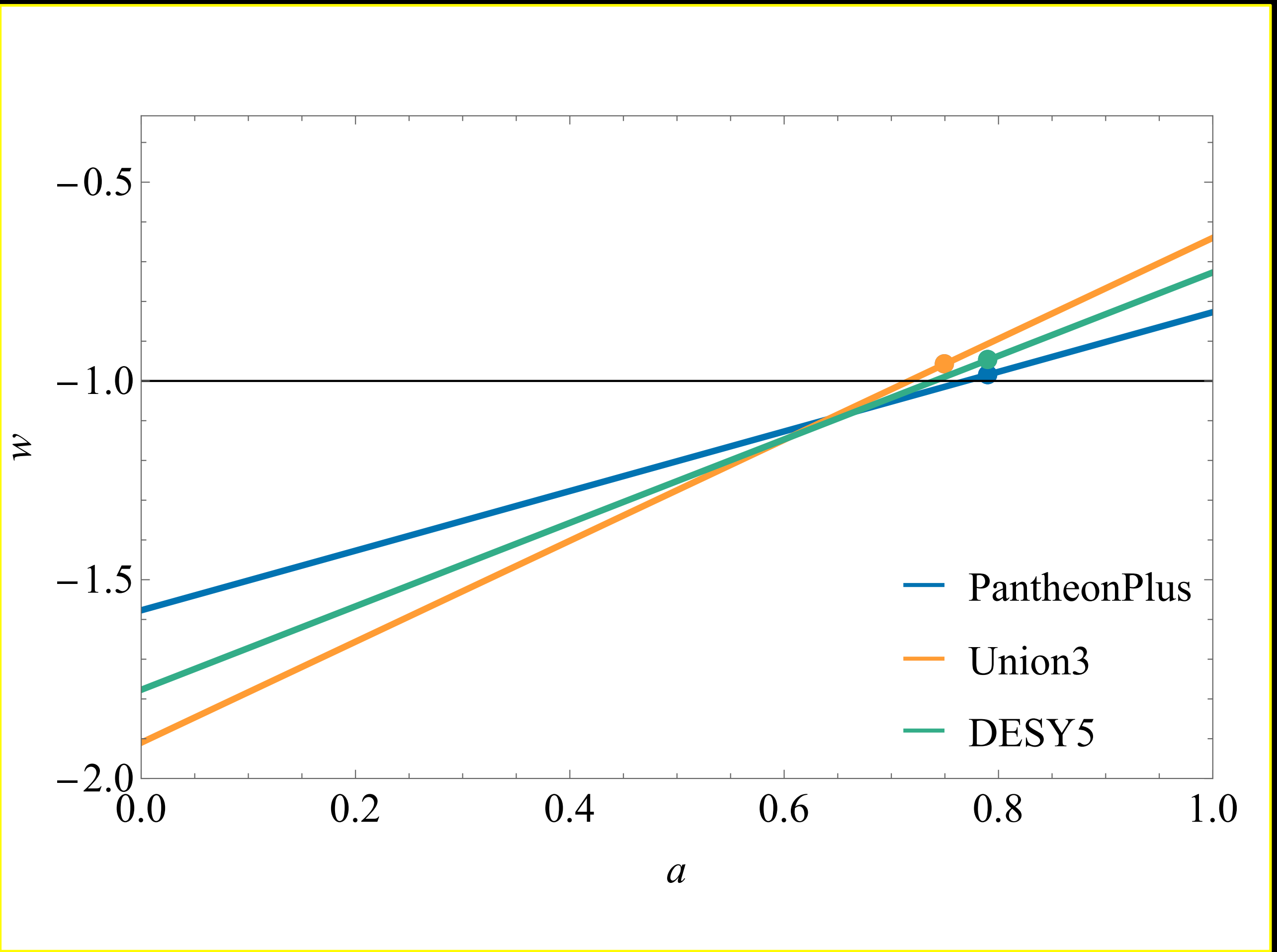
$$w(z) = w_0 + w_a\, z/(1+z)$$

model/dataset	Ω_m	H_0 $[\mathrm{km\,s^{-1}\,Mpc^{-1}}]$	$10^3\Omega_K$	w or w_0	w_a
***w*CDM**					
DESI	0.293 ± 0.015	—	—	$-0.99^{+0.15}_{-0.13}$	—
DESI+BBN+θ_*	0.295 ± 0.014	$68.6^{+1.8}_{-2.1}$	—	$-1.002^{+0.091}_{-0.080}$	—
DESI+CMB	0.281 ± 0.013	$71.3^{+1.5}_{-1.8}$	—	$-1.122^{+0.062}_{-0.054}$	—
DESI+CMB+Panth.	0.3095 ± 0.0069	67.74 ± 0.71	—	-0.997 ± 0.025	—
DESI+CMB+Union3	0.3095 ± 0.0083	67.76 ± 0.90	—	-0.997 ± 0.032	—
DESI+CMB+DESY5	0.3169 ± 0.0065	66.92 ± 0.64	—	-0.967 ± 0.024	—
w_0w_aCDM					
DESI	$0.344^{+0.047}_{-0.026}$	—	—	$-0.55^{+0.39}_{-0.21}$	< -1.32
DESI+BBN+θ_*	$0.338^{+0.039}_{-0.029}$	$65.0^{+2.3}_{-3.6}$	—	$-0.53^{+0.42}_{-0.22}$	< -1.08
DESI+CMB	$0.344^{+0.032}_{-0.027}$	$64.7^{+2.2}_{-3.3}$	—	$-0.45^{+0.34}_{-0.21}$	$-1.79^{+0.48}_{-1.0}$
DESI+CMB+Panth.	0.3085 ± 0.0068	68.03 ± 0.72	—	-0.827 ± 0.063	$-0.75^{+0.29}_{-0.25}$
DESI+CMB+Union3	0.3230 ± 0.0095	66.53 ± 0.94	—	-0.65 ± 0.10	$-1.27^{+0.40}_{-0.34}$
DESI+CMB+DESY5	0.3160 ± 0.0065	67.24 ± 0.66	—	-0.727 ± 0.067	$-1.05^{+0.31}_{-0.27}$

This move towards phantom dark energy ($w < -1$) has generated a great deal of debate about the use if priors.

$$w(z) = w_0 + w_a\, z/(1+z)$$



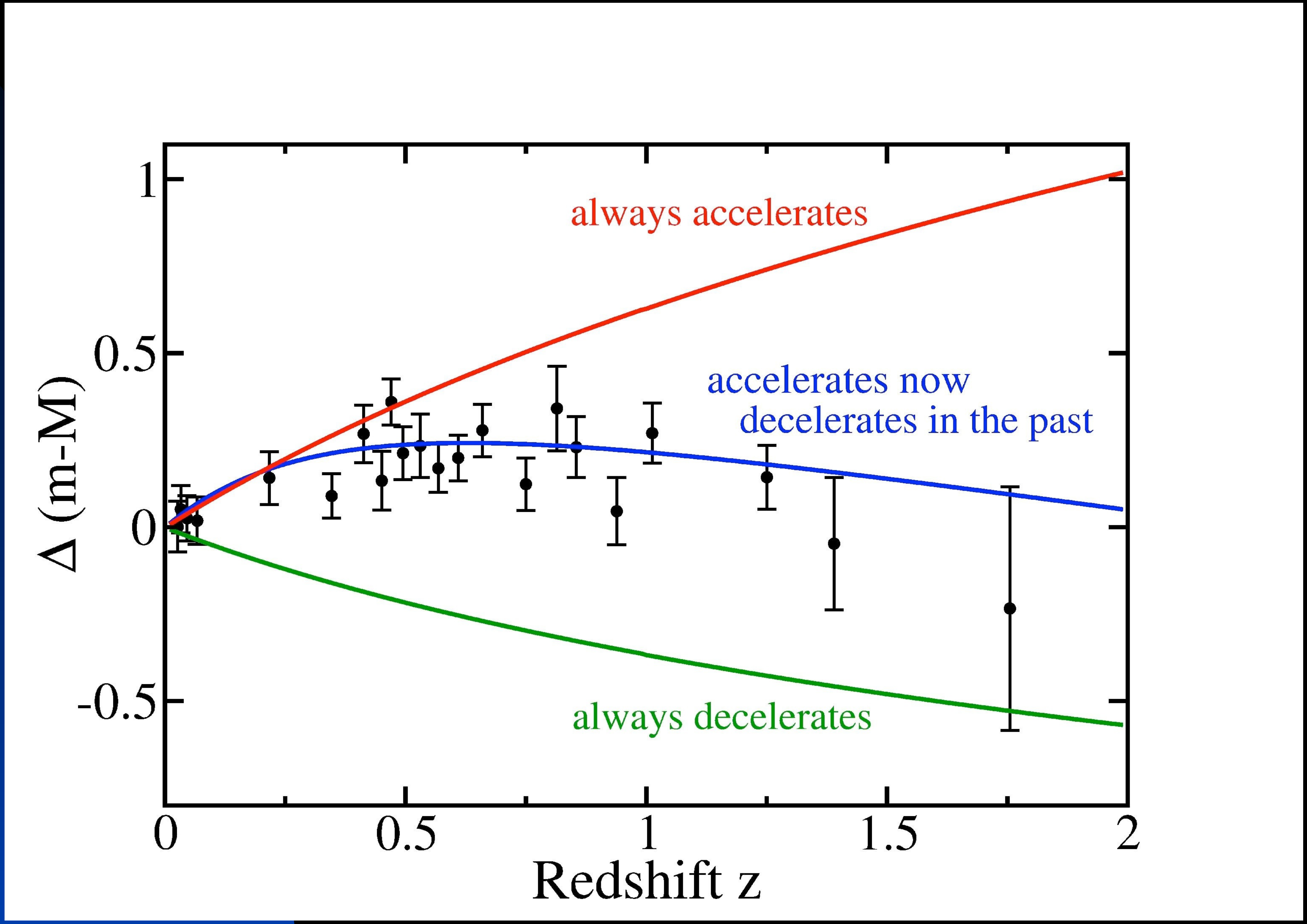


Cortes and Liddle 2024, using DESI 2024

Cortes and Liddle 2024

It looks like the phantom like dynamical dark energy in the past is driven by the data points taken close to today - see the pivot points. Yet, we know the energy density in dark energy drops rapidly in the past. Can we be so certain the slopes are really sending us in phantom regimes in the past ?

The acceleration has not been forever -- pinning down the turnover will provide a very useful piece of information.



Huterer 2010

Help address cosmic coincidence problem ! A region hopefully EUCLID will be able to probe in a few weeks

Evidence for Dark Energy?

$$3.\ \Omega_0 = \Omega_m + \Omega_\Lambda$$

Enter CMBR:

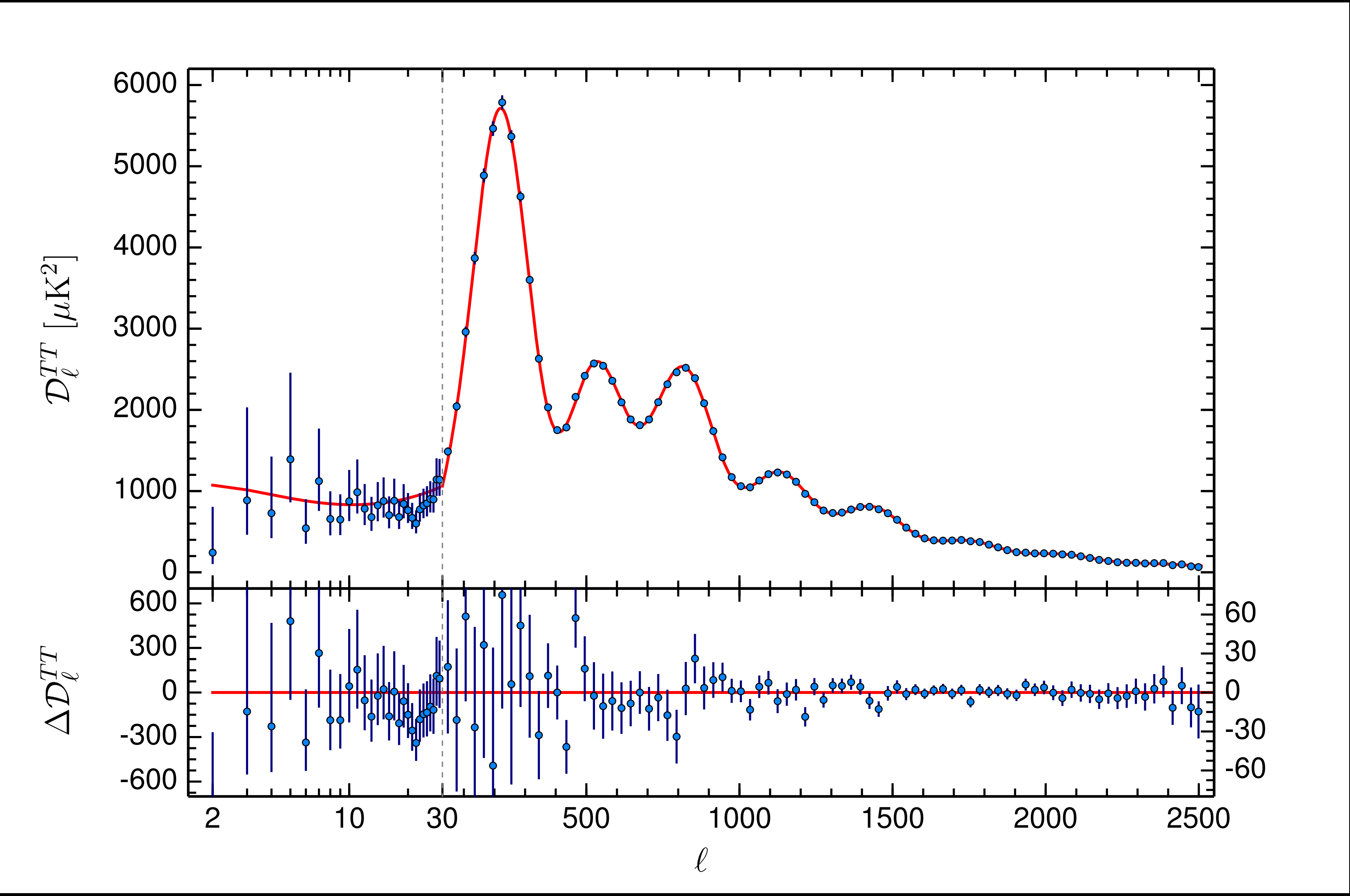
Provides clue. 1st angular peak in power spectrum.

$$l_{\text{peak}} \approx \frac{220}{\sqrt{\Omega_0}}$$

↓

$\Omega_k = 0.0007 \pm 0.0019$

Planck 2018



Planck TT spectrum (2015)

Different approaches to Dark Energy include amongst many:

A true cosmological constant -- but why this value - CCP ?

Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.

Modifications of Einstein gravity leading to acceleration today.

Anthropic arguments.

Perhaps GR but Universe is inhomogeneous.

Hiding the cosmological constant -- its there all the time but just doesn’t gravitate and something else is driving the acceleration.

Yet to be proposed ...

Brief reminder why the cosmological constant is regarded as a problem?

The CC gravitates in General Relativity:

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{16\pi G} - \rho_{\rm vac}\right)$$

$$G_{\mu\nu} = -8\pi G \rho_{\rm vac} g_{\mu\nu}$$

Now: $\rho_{\rm vac}^{\rm obs} \ll \rho_{\rm vac}^{\rm theory}$

Just as well because anything much bigger than we have and the universe would have looked a lot different to what it does look like. In fact structures would not have formed in it.

Estimate what the vacuum energy should be :

$$\rho_{\rm vac}^{\rm theory} \sim \rho_{\rm vac}^{\rm bare}$$

$$+$$

zero point energies of each particle

$$+$$

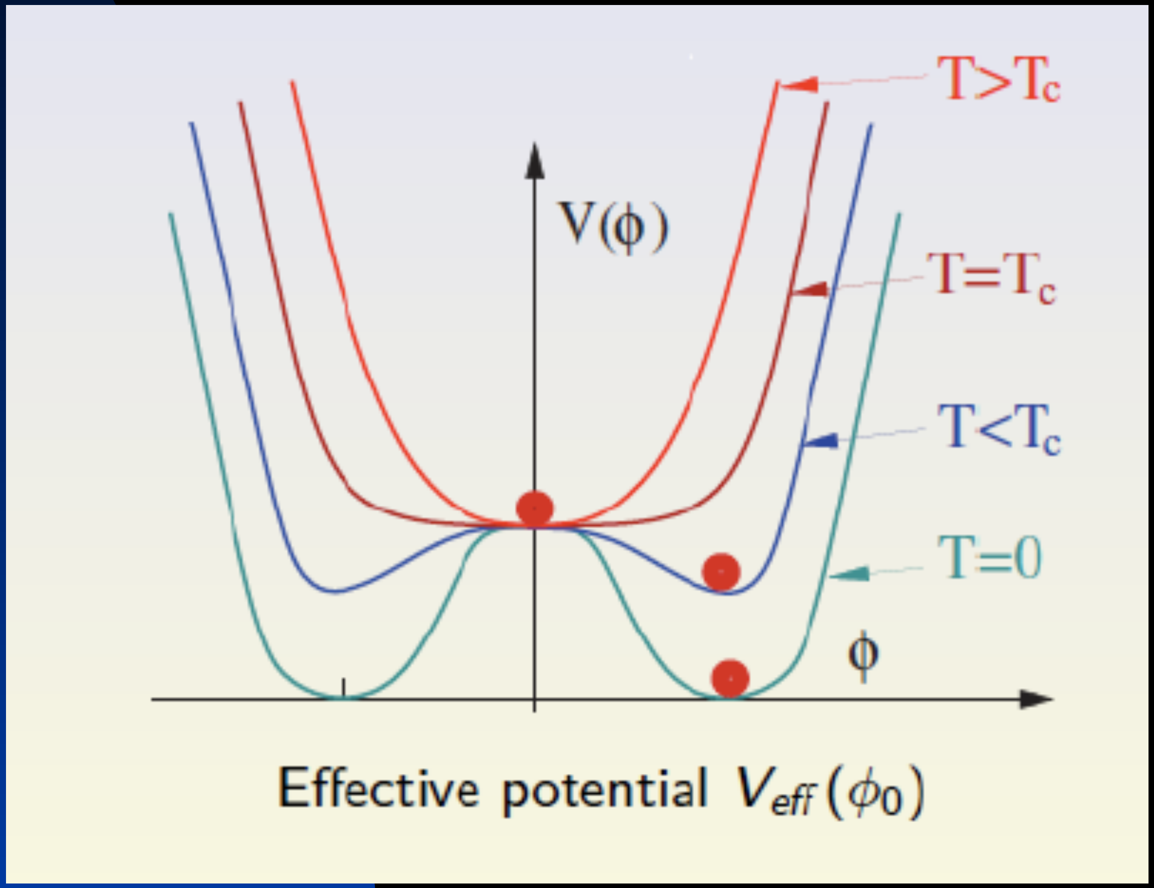
contributions from phase transitions in the early universe zero point energies of each particle

For many fields (i.e. leptons, quarks, gauge fields etc...):

$$< \rho > = \frac{1}{2} \sum_{\text{fields}} g_i \int_0^{\Lambda_i} \sqrt{k^2 + m^2} \frac{d^3 k}{(2\pi)^3} \simeq \sum_{\text{fields}} \frac{g_i \Lambda_i^4}{16\pi^2}$$

where g_i are the dof of the field (+ for bosons, - for fermions).

contributions from phase transitions in the early universe



$$\Delta V_{\rm ewk} \sim (200 \text{ GeV})^4$$

$$\Delta V_{\rm QCD} \sim (0.3 \text{ GeV})^4$$

Quantum Gravity cut-off — $(10^{18}\ \mathrm{GeV})^4$ — fine tuning to 120 decimal places

SUSY cut-off — $(\mathrm{TeV})^4$ — fine tuning to 60 decimal places

EWK phase transition — $(200\mathrm{GeV})^4$ — fine tuning to 56 decimal places

QCD phase transition — $(0.3\mathrm{GeV})^4$

Muon — $(100\mathrm{MeV})^4$

fine tuning to 44 decimal places

electron — $(1\ \mathrm{MeV})^4$ — fine tuning to 36 decimal places

$(\mathrm{meV})^4$ — Observed value of the effective cosmological constant today !

String - theory -- where are the realistic models?

`No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGR models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

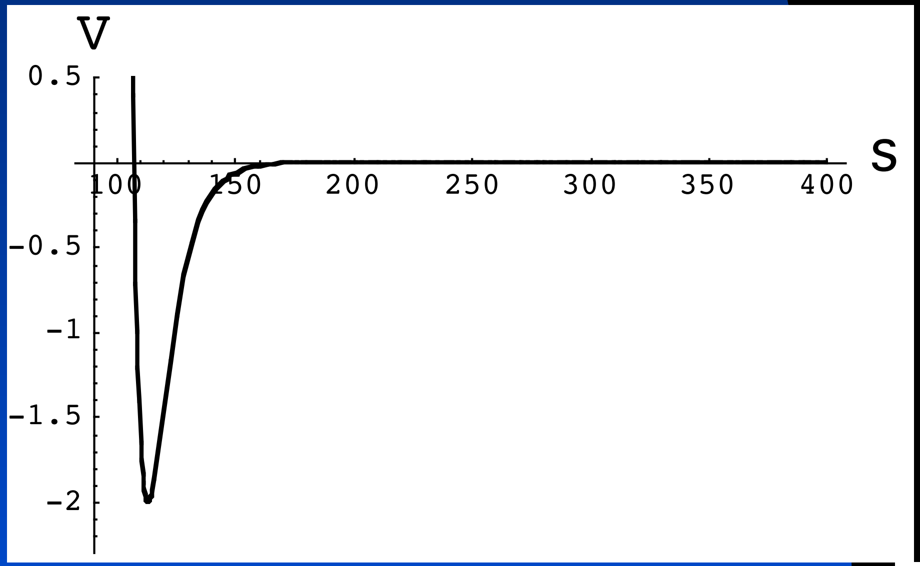
Avoid no-go theorem by relaxing conditions of the theorem.

1. Allow internal space to be time-dependent scalar fields (radion)

2. Brane world set up require uplifting terms to achieve de Sitter vacua hence accn

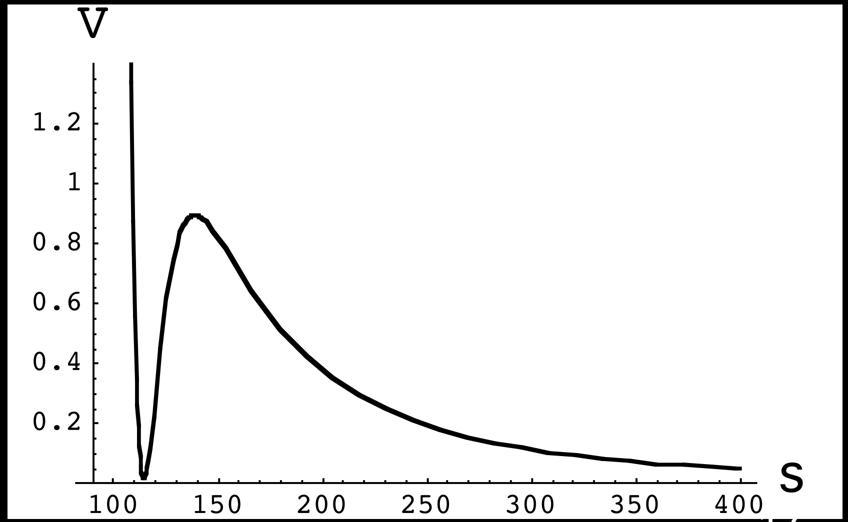
Example of stabilised scenario: Metastable de Sitter string vacua in TypeIIB string theory, based on stable highly warped IIB compactifications with NS and RR three-form fluxes. [Kachru, Kallosh, Linde and Trivedi 2003]

Metastable minima arises from adding positive energy of anti-D3 brane in warped Calabi-Yau space.



AdS minimum

$$V_{\mathrm{KKLT}} = V_{\mathrm{AdS}} + \frac{D}{\sigma^2}$$



Metastable dS minimum

The String Landscape approach

Type IIB String theory compactified from 10 dimensions to 4.

Internal dimensions stabilised by fluxes. Assumes natural AdS vacuum uplifted to de Sitter vacuum through additional fluxes !

Many many vacua ~ 10^{500} ! Typical separation ~ $10^{-500}\ \Lambda_{pl}$

Assume randomly distributed, tunnelling allowed between vacua --> separate universes .

Anthropic : Galaxies require vacua < $10^{-118}\ \Lambda_{pl}$ [Weinberg] Most likely to find values not equal to zero!

Landscape gives a realisation of the multiverse picture.

There isn't one true vacuum but many so that makes it almost impossible to find our vacuum in such a Universe which is really a multiverse.

So how can we hope to understand or predict why we have our particular particle content and couplings when there are so many choices in different parts of the universe, none of them special ?

SUSY large extra dimensions and Lambda - Burgess et al 2013, 2015

Soln to 6D Einstein-Maxwell-scalar with chiral gauged sugr.

In more than 4D, the 4D vac energy can curve the extra dimensions.

Proposal: Physics is 6D above 0.01eV scale with SUSY bulk. We live in 4D brane with 2 extra dim. 4D vac energy cancelled by Bulk contributions - quintessence like potential generated by Qu corrections leading to late time accn.

Sequestering Lambda - Kaloper and Padilla 2013-2016

IR soln to the problem - initial version adds a global term to Einstein action

Introduce global dynamical variables Λ, λ

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \Lambda - \lambda^4\mathcal{L}(\lambda^{-2}g^{\mu\nu}, \Psi)\right] + \sigma\left(\frac{\Lambda}{\lambda^4\mu^4}\right)$$

λ sets the hierarchy between matter scales and M_{pl}

$$\frac{m_{phys}}{M_{pl}} = \frac{\lambda m}{M_{pl}}$$

Padilla 2015

Eq of motion:

$$M_{pl}^2 G^{\mu}{}_{\nu} = \tau^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu}\langle \tau^{\alpha}{}_{\alpha}\rangle$$

$$T^{\mu}_{\nu} = -V_{vac}\delta^{\mu}_{\nu} + \tau^{\mu}_{\nu}$$

where: $$\Lambda = \frac{1}{4}\langle T^{\alpha}{}_{\alpha}\rangle, \qquad \langle Q\rangle = \frac{\int d^4x Q\sqrt{g}}{\int d^4x\sqrt{g}}$$ spacetime volume must be finite

Vacuum energy drops out at each and every loop order

Universe has finite spacetime volume

Ends in a crunch
w=-1 is transient
Ω_k>0

collapse triggered by dominating dark energy

Linear potential $V=m^3\varphi$

form protected by shift symmetry,
size of m^3 technically natural

Local version of sequestering can accommodate infinite universe [Kaloper et al 2015]

Self tuning - with the Fab Four

with Charmousis, Padilla and Saffin

PRL 108 (2012) 051101; PRD 85 (2012) 104040

In GR the vacuum energy gravitates, and the theoretical estimate suggests that it gravitates too much.

Basic idea is to use self tuning to prevent the vacuum energy gravitating at all.

The cosmological constant is there all the time but is being dealt with by the evolving scalar field.

Most general scalar-tensor theory with second order field equations:

[G.W. Horndeski, Int. Jour. Theor. Phys. 10 (1974) 363-384]

The action which leads to required self tuning solutions :

$$\begin{aligned}
\mathcal{L}_{john} &= \sqrt{-g}V_{john}(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\
\mathcal{L}_{paul} &= \sqrt{-g}V_{paul}(\phi)P^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi \\
\mathcal{L}_{george} &= \sqrt{-g}V_{george}(\phi)R \\
\mathcal{L}_{ringo} &= \sqrt{-g}V_{ringo}(\phi)\hat{G}
\end{aligned}$$

In other words it can be seen to reside in terms of the four arbitrary potential functions of ϕ coupled to the curvature terms.

Covers most scalar field related modified gravity models studied to date.

fab four cosmology

TABLE I: Examples of interesting cosmological behaviour for various fixed points with $\sigma = 0$.

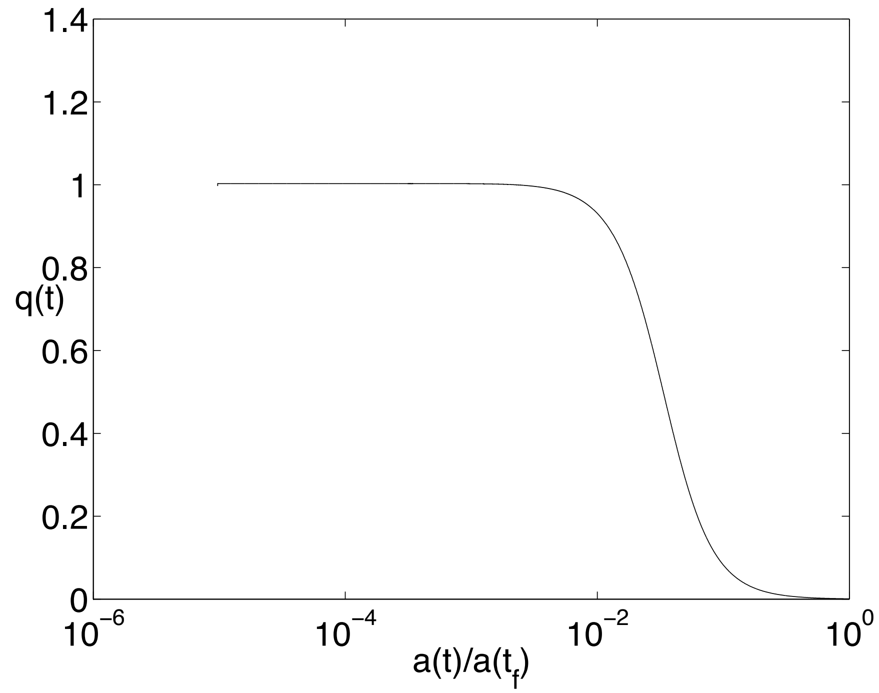
Case	cosmological behaviour	$V_j(\phi)$	$V_p(\phi)$	$V_g(\phi)$	$V_r(\phi)$
Stiff fluid	$H^2 \propto 1/a^6$	$c_1\phi^{\frac{4}{\alpha}-2}$	$c_2\phi^{\frac{6}{\alpha}-3}$	0	0
Radiation	$H^2 \propto 1/a^4$	$c_1\phi^{\frac{4}{\alpha}-2}$	0	$c_2\phi^{\frac{2}{\alpha}}$	$-\frac{\alpha^2}{8}c_1\phi^{\frac{4}{\alpha}}$
Curvature	$H^2 \propto 1/a^2$	0	0	0	$c_1\phi^{\frac{4}{\alpha}}$
Arbitrary	$H^2 \propto a^{2h}, \quad h \neq 0$	$c_1(1+h)\phi^{\frac{4}{\alpha}-2}$	0	0	$-\frac{\alpha^2}{16}h(3+h)c_1\phi^{\frac{4}{\alpha}}$

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

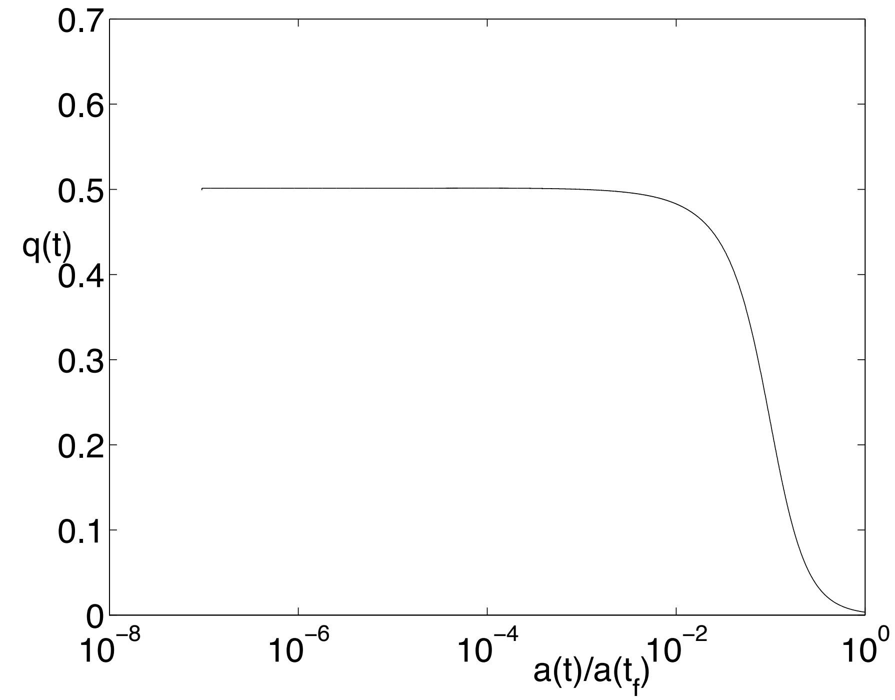
$$a \sim t^p \sim t^{-1/h}$$

$$q = -\frac{p(p-1)}{p^2} = -(1+h)$$

"radiation"



"matter"



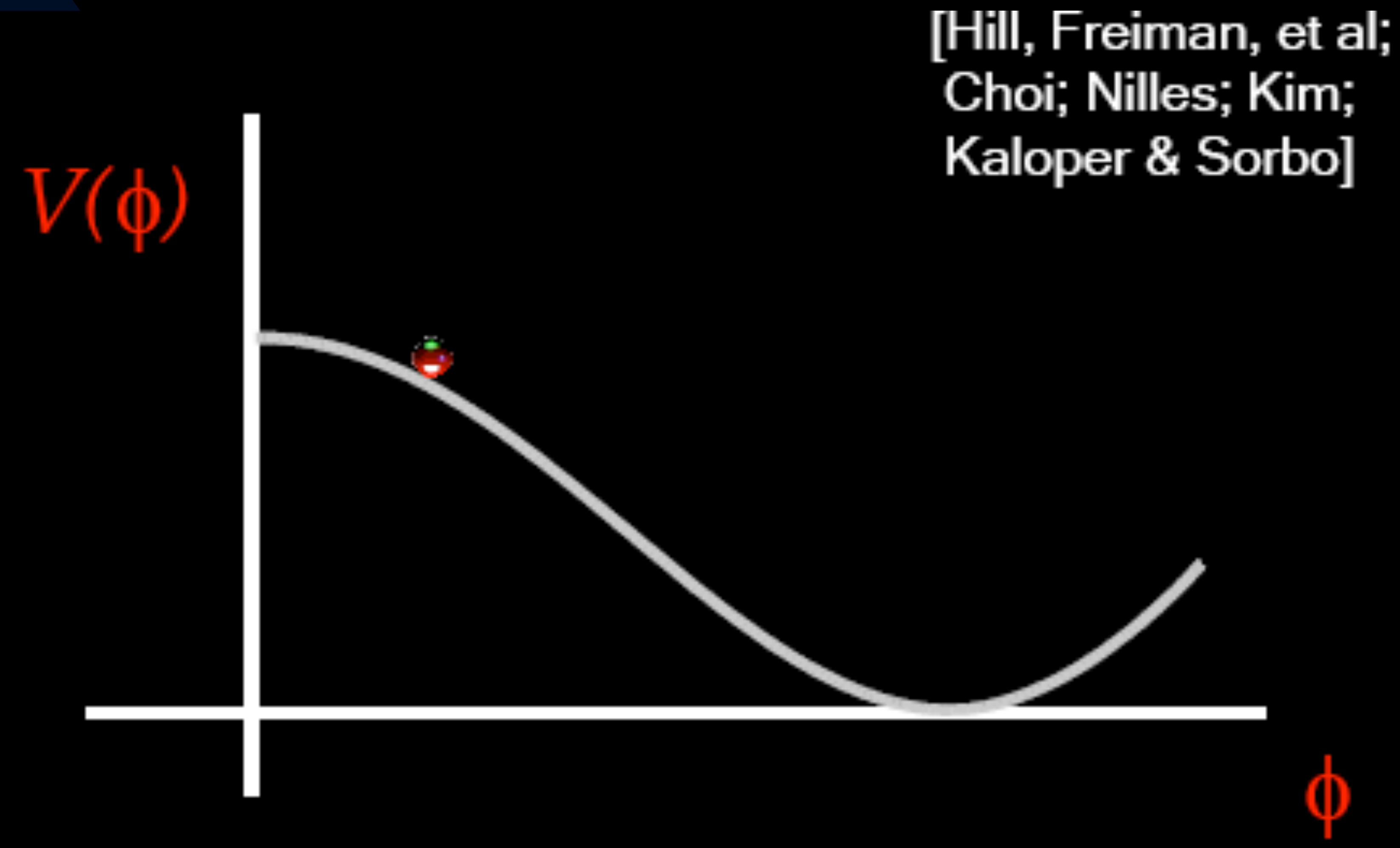
See also:

Appleby et al JCAP 1210 (2012) 060; Amendola et al PRD 87 (2013) 2, 023501; Martin-Moruno et al PRD 91 (2015) 8, 084029; Babichev et al arXiv:1507.05942 [gr-qc] ; Emond et al JCAP 05 (2019) 038

Particle physics inspired models of dark energy ?

Pseudo-Goldstone Bosons -- approx sym ϕ --> ϕ + const.

Leads to naturally small masses, naturally small couplings



Barbieri et al

See Yoga model of Burgess et al 2021 for new approach at solving the CCP via relaxation mechanism and obtaining dynamical DE

$$V(\phi) = \lambda^4(1 + \cos(\phi/F_a))$$

Axions could be useful for strong CP problem, dark matter and dark energy — ex. Quintessential Axion.

Axions could be useful for strong CP problem, dark matter and dark energy.

Strong CP problem intro axion : $m_a = \frac{\Lambda^2_{\rm QCD}}{F_a};\ F_a\ -\ \text{decay constant}$

PQ axion ruled out but invisible axion still allowed: $10^9\ \text{GeV} \leq F_a \leq 10^{12}\ \text{GeV}$

Sun stability CDM constraint

String theory has lots of antisymmetric tensor fields in 10d, hence many light axion candidates.

Can have $F_a \sim 10^{17}$-10^{18} GeV

Quintessential axion -- dark energy candidate [Kim & Nilles].

Requires $F_a \sim 10^{18}$ GeV which can give:

$$E_{\rm vac} = (10^{-3}\ \text{eV})^4 \to m_{\rm axion} \sim 10^{-33}\ \text{eV}$$

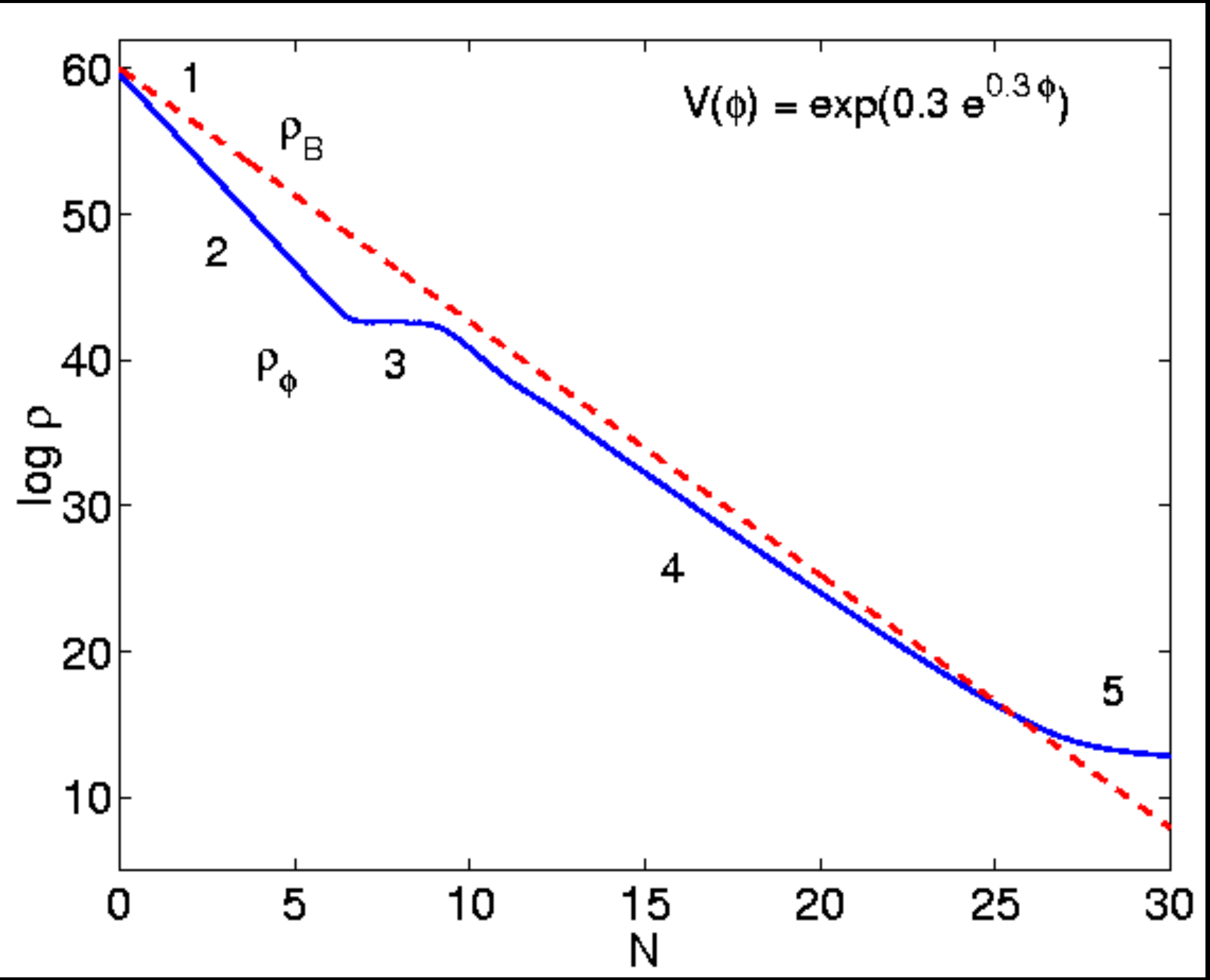
Because axion is pseudoscalar -- mass is protected, hence avoids fifth force constraints

Dynamical Dark Energy

Wetterich 1987,
Caldwell et al 1998

Slowly rolling scalar fields Quintessence

1. **PE → KE**
2. **KE dom scalar field energy den.**
3. **Const field.**
4. **Attractor solution: almost const ratio KE/ PE.**
5. **PE dom.**

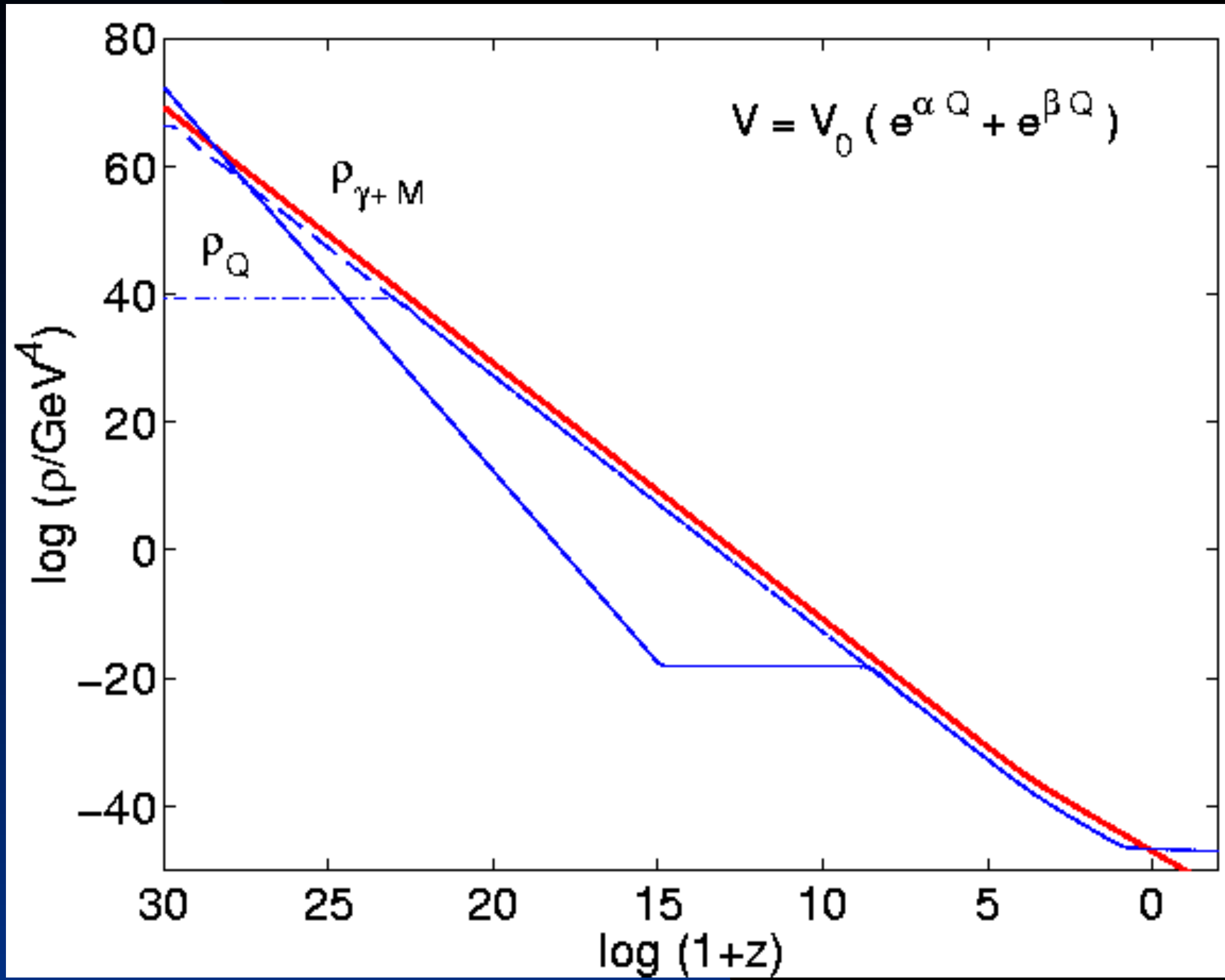


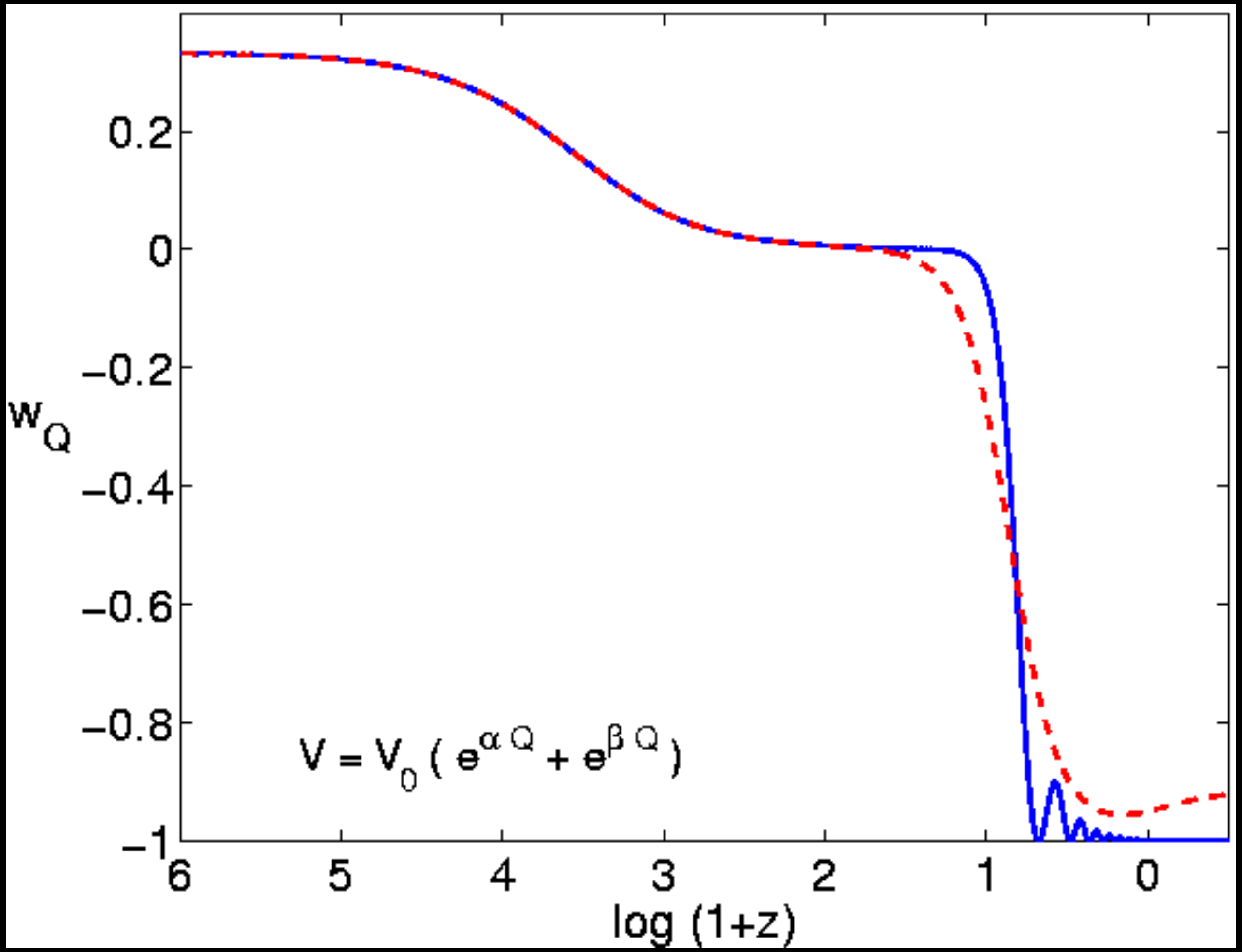
Nunes

Attractors make initial conditions less important

$$V(\phi) = V_1 + V_2$$
$$= V_{01} e^{-\kappa\lambda_1\phi} + V_{02} e^{-\kappa\lambda_2\phi}$$

Barreiro, EJC and Nunes 2000





$\alpha = 20; \beta = 0.5$

Scaling for wide range of i.c.

Fine tuning: $V_0 \approx \rho_\phi \approx 10^{-47}\ \mathrm{GeV}^4 \approx (10^{-3}\ \mathrm{eV})^4$

Mass: $m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \mathrm{eV}$

Generic issue Fifth force - require screening mechanism!

Tracker solutions :

In cosmology as in many areas of physics we often deal with systems that are inherently described through a series of coupled non-linear differential equations.

Such systems often can not be solved analytically, yet they can be analysed through determining the late time behaviour of some combination of the variables, where they may approach some form of attractor solution, attractors in variables that are not always the basic variables the underlying equations describe.

By determining the nature of these attractor solutions (their stability for example) one can learn a great deal about the system in general.

Moreover the phase plane description of the system is often highly intuitive enabling easy analysis and understanding of the system.

In cosmology this is particularly useful. The universe is very old, and the existence of scaling solutions where a quantity becomes constant enables one to find the regime where scaling occurs, and then simply rescale the quantities to obtain their values today -- thereby avoiding doing a simulation for 13.7 Billion years !

Examples include the relative energy densities in scalar fields compared to the background radiation and matter densities, as well as the relative energy density in cosmic strings.

In general such a phase plane analysis reduces the order of the differential equations being investigated by introducing new variables which are themselves derivatives of the original variables.

Example in cosmology :

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\rho - \frac{k}{a^2} + \Lambda$$ Friedmann eqn

$$\dot{\rho} + 3H(\rho + p) = 0$$ Fluid eqn.

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}(\rho + 3p) + \frac{\Lambda}{3}$$ Acceleration eqn

Note:

where $\kappa^2 \equiv 8\pi G$

$$\frac{\ddot{a}}{a} \geq 0 <-> = (\rho + 3p) \leq 0$$

Tracker solutions

Wetterich,

Peebles and Ratra,

EJC, Liddle and Wands

Scalar field: $\phi:\ \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)\ ;\ p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$

EoM:

$$\dot{H} = -\frac{\kappa^2}{2}(\dot{\phi}^2 + \gamma\rho_B)$$

$$\dot{\rho}_B = -3\gamma H \rho_B$$

$$\ddot{\phi} = -3H\dot{\phi} - \frac{dV}{d\phi}$$

+ constraint:

$$H^2 = \frac{\kappa^2}{3}(\rho_\phi + \rho_B)$$

Intro: $x = \frac{\kappa\dot{\phi}}{\sqrt{6}H}$ $y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}$ $\lambda \equiv \frac{-1}{\kappa V}\frac{dV}{d\phi}$ $\Gamma - 1 \equiv \frac{d}{d\phi}\left(\frac{1}{\kappa\lambda}\right)$

Eff eqn of state: $\gamma_\phi = \frac{\dot{\phi}^2}{V + \frac{\dot{\phi}^2}{2}} = \frac{2x^2}{x^2 + y^2}$ $\Omega_\phi = \frac{\kappa^2 \rho_\phi}{3H^2} = x^2 + y^2$

Friedmann eqns and fluid eqns become:

$$x' = -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right]$$

$$y' = -\lambda\sqrt{\frac{3}{2}}xy + \frac{3}{2}y\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right]$$

$$\lambda' = -\sqrt{6}\lambda^2(\Gamma - 1)$$

$$\frac{\kappa^2 \rho_\gamma}{3H^2} + x^2 + y^2 = 1$$

where $' \equiv d/d(\ln a)$

Note: $0 \le \gamma_\phi \le 2 : 0 \le \Omega_\phi \le 1$

Scaling solutions: (x`=y`=0)

No:	x_c	y_c	Existance	Stability	Ω_ϕ	γ_ϕ
1	0	0	$\forall\lambda,\gamma$	SP: $0<\gamma$ SN: $\gamma=0$	0	Undefined
2a	1	0	$\forall\lambda,\gamma$	UN: $\lambda<\sqrt{6}$ SP: $\lambda>\sqrt{6}$	1	2
2b	-1	0	$\forall\lambda,\gamma$	UN: $\lambda>-\sqrt{6}$ SP: $\lambda<-\sqrt{6}$	1	2
3	$\frac{\lambda}{\sqrt{6}}$	$\left(1-\frac{\lambda^2}{6}\right)^{1/2}$	$\lambda^2\le 6$	SP: $3\gamma<\lambda^2<6$ SN: $\lambda^2<3\gamma$	1	$\frac{\lambda^2}{3}$
4	$\left(\frac{3}{2}\right)^{1/2}\frac{\gamma}{\lambda}$	$\left[\frac{3(2-\gamma)\gamma}{2\lambda^2}\right]^{1/2}$	$\lambda^2\ge 3\gamma$	SN: $3\gamma<\lambda^2<\frac{24\gamma^2}{9\gamma-2}$ SS: $\lambda^2>\frac{24\gamma^2}{9\gamma-2}$	$\frac{3\gamma}{\lambda^2}$	γ

$$V = V_0 e^{-\lambda\kappa\phi}$$

Late time attractor is scalar field dominated

$\lambda^2 \le 6$

Field mimics background fluid.

$\lambda^2 \ge 3\gamma$

Nucleosynthesis bound → $\lambda^2 > 20$

EJC, Liddle and Wands

$$V = V_0 e^{-\lambda \kappa \phi}$$

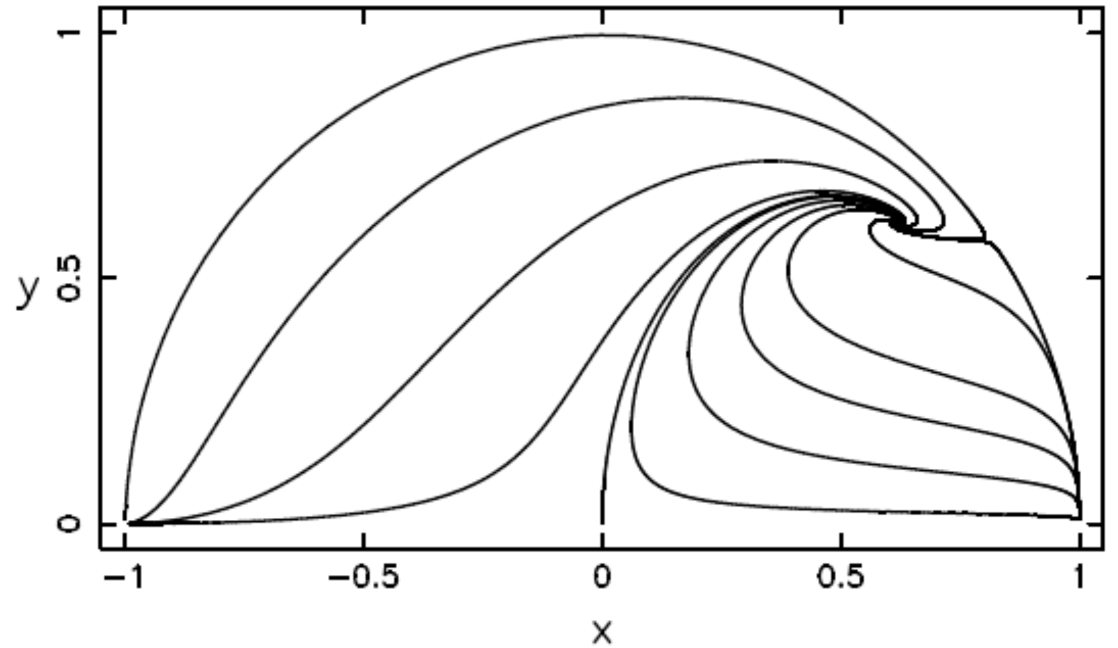


FIG. 3. The phase plane for $\gamma = 1$, $\lambda = 2$. The scalar field dominated solution is a saddle point at $x = \sqrt{2/3}$, $y = \sqrt{1/3}$, and the late-time attractor is the scaling solution with $x = y = \sqrt{3/8}$.

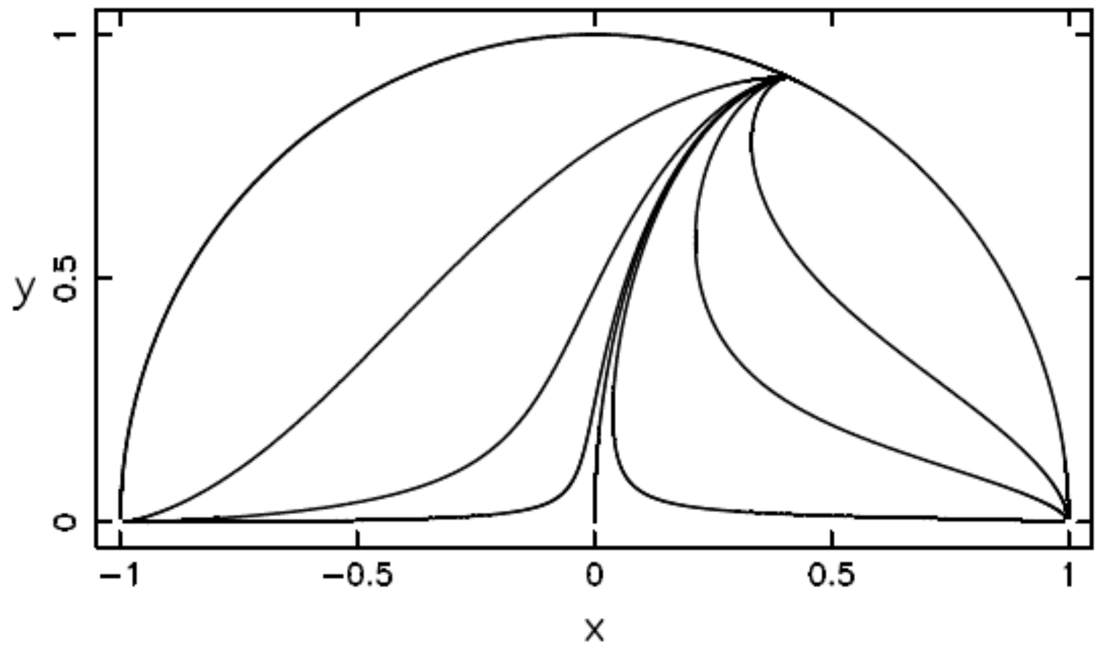


FIG. 2. The phase plane for $\gamma = 1$, $\lambda = 1$. The late-time attractor is the scalar field dominated solution with $x = \sqrt{1/6}$, $y = \sqrt{5/6}$.

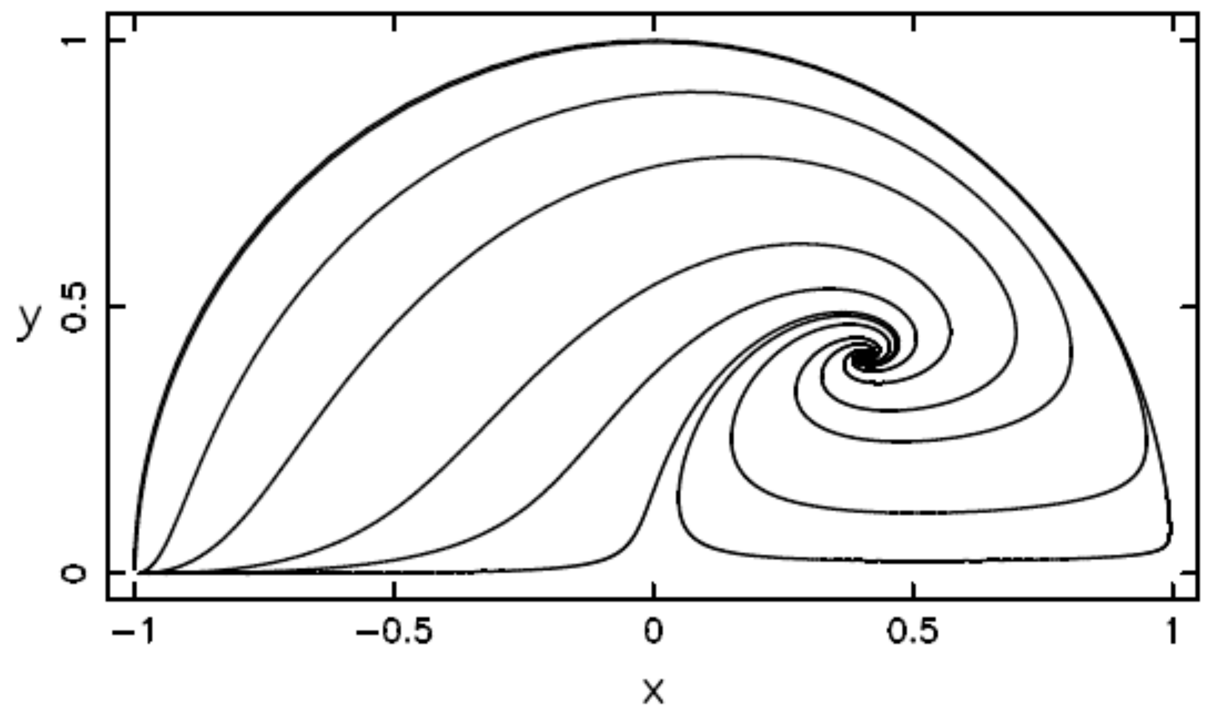


FIG. 4. The phase plane for $\gamma = 1$, $\lambda = 3$. The late-time attractor is the scaling solution with $x = y = \sqrt{1/6}$.

Stability criteria

Expand about critical points $x = x_c + u \,, \qquad y = y_c + v \,,$

Sub into evoln eqns

$$x' = -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right]$$
$$y' = -\lambda\sqrt{\frac{3}{2}}xy + \frac{3}{2}y\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right] \,,$$

Yields first order pertn eqns

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \mathcal{M}\begin{pmatrix} u \\ v \end{pmatrix}$$

General solution where $m_\pm$ are eigenvalues of M

$$u = u_+ \exp(m_+ N) + u_- \exp(m_- N)$$
$$v = v_+ \exp(m_+ N) + v_- \exp(m_- N)$$

Fluid-dominated solution:

$$m_- = -\frac{3(2-\gamma)}{2} \,, \qquad m_+ = \frac{3\gamma}{2} \,.$$

Kinetic-dominated solutions, $(x_c = \pm 1, \, y_c = 0)$:

$$m_- = \sqrt{\frac{3}{2}}\left(\sqrt{6} \mp \lambda\right) \,, \qquad m_+ = 3(2-\gamma)$$

Scalar field dominated solution:

$$m_- = \frac{\lambda^2 - 6}{2} \,, \qquad m_+ = \lambda^2 - 3\gamma$$

Scaling solution:

$$m_\pm = -\frac{3(2-\gamma)}{4}\left[1 \pm \sqrt{1 - \frac{8\gamma(\lambda^2 - 3\gamma)}{\lambda^2(2-\gamma)}}\right]$$

2. Applications in dark energy models

One approach to dark energy involves assuming the dark energy is dynamical, not due to an underlying cosmological constant. That is assumed to be zero from some as yet unknown symmetry argument and what we are left with is an evolving scalar field which came to dominate recently.

Depending on the underlying potential such a field can undergo a period of tracking where it mimics the background energy density before coming to dominate at late times.

All such models I am aware of require various degrees of fine tuning as we shall see

Coincidence problem – why now?

$$\frac{\ddot{a}}{a} \geq 0 <-> = (\rho + 3p) \leq 0$$

If:

$$\rho_x = \rho_x^0 \, a^{-3(1+w_x)}$$

Universe dom by Quintessence at:

$$z_x = \left(\frac{\Omega_x}{\Omega_m}\right)^{\frac{1}{3w_x}} - 1$$

$$\left(\frac{\Omega_x}{\Omega_m}\right) = \frac{7}{3} \rightarrow z_x = 0.5,\ 0.3 \text{ for } w_x = -\frac{2}{3},\ -1$$

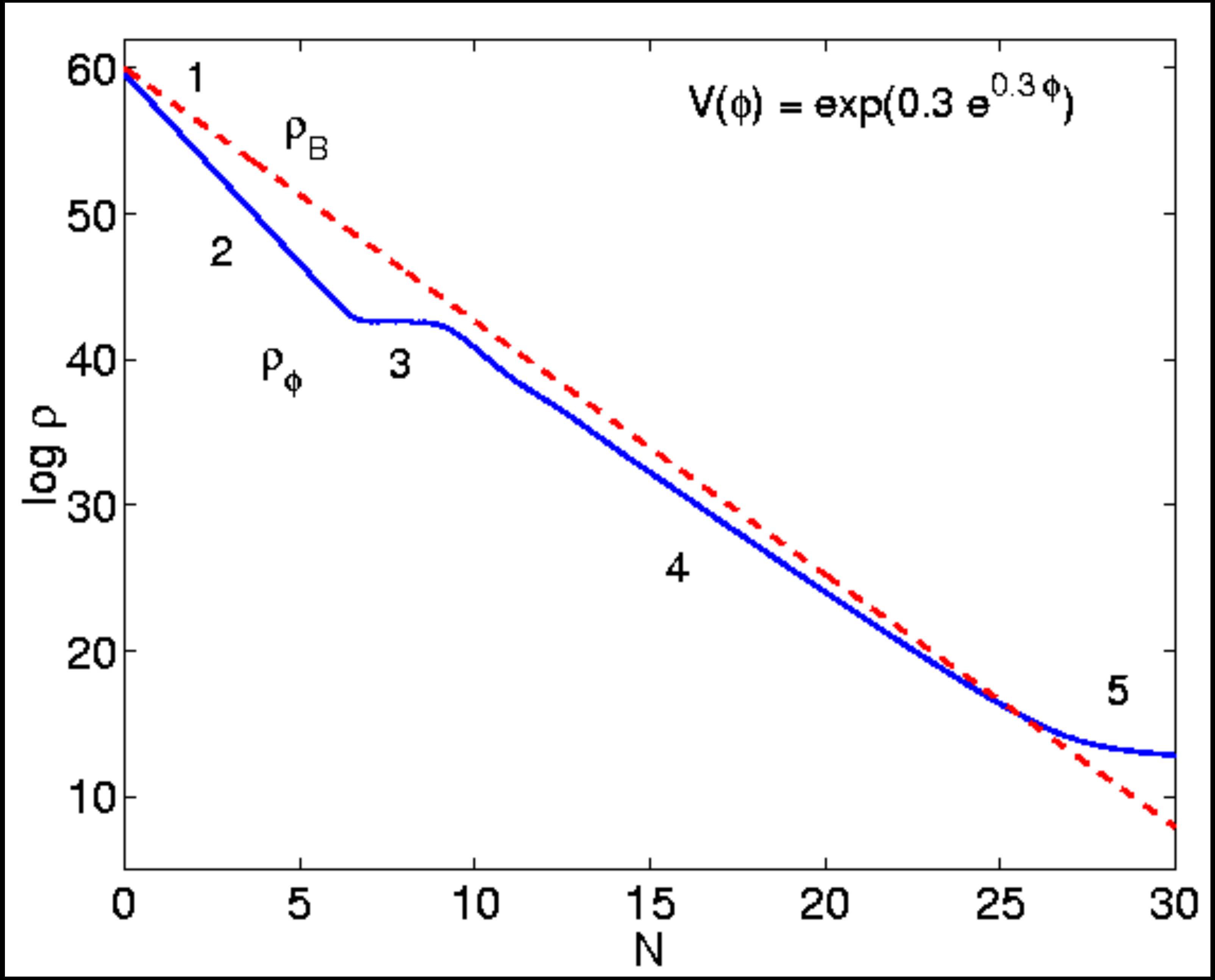
Univ accelerates at:

$$z_a = \left(-(1+3w_x)\frac{\Omega_x}{\Omega_m}\right)^{\frac{-1}{3w_x}} - 1$$

$$z_a = 0.7,\ 0.5 \text{ for } w_x = -\frac{2}{3},\ -1$$

Slowly rolling scalar fields
Quintessence - Generic behaviour

1. **PE → KE**
2. **KE dom scalar field energy den.**
3. **Const field.**
4. **Attractor solution: almost const ratio KE/ PE.**
5. **PE dom.**

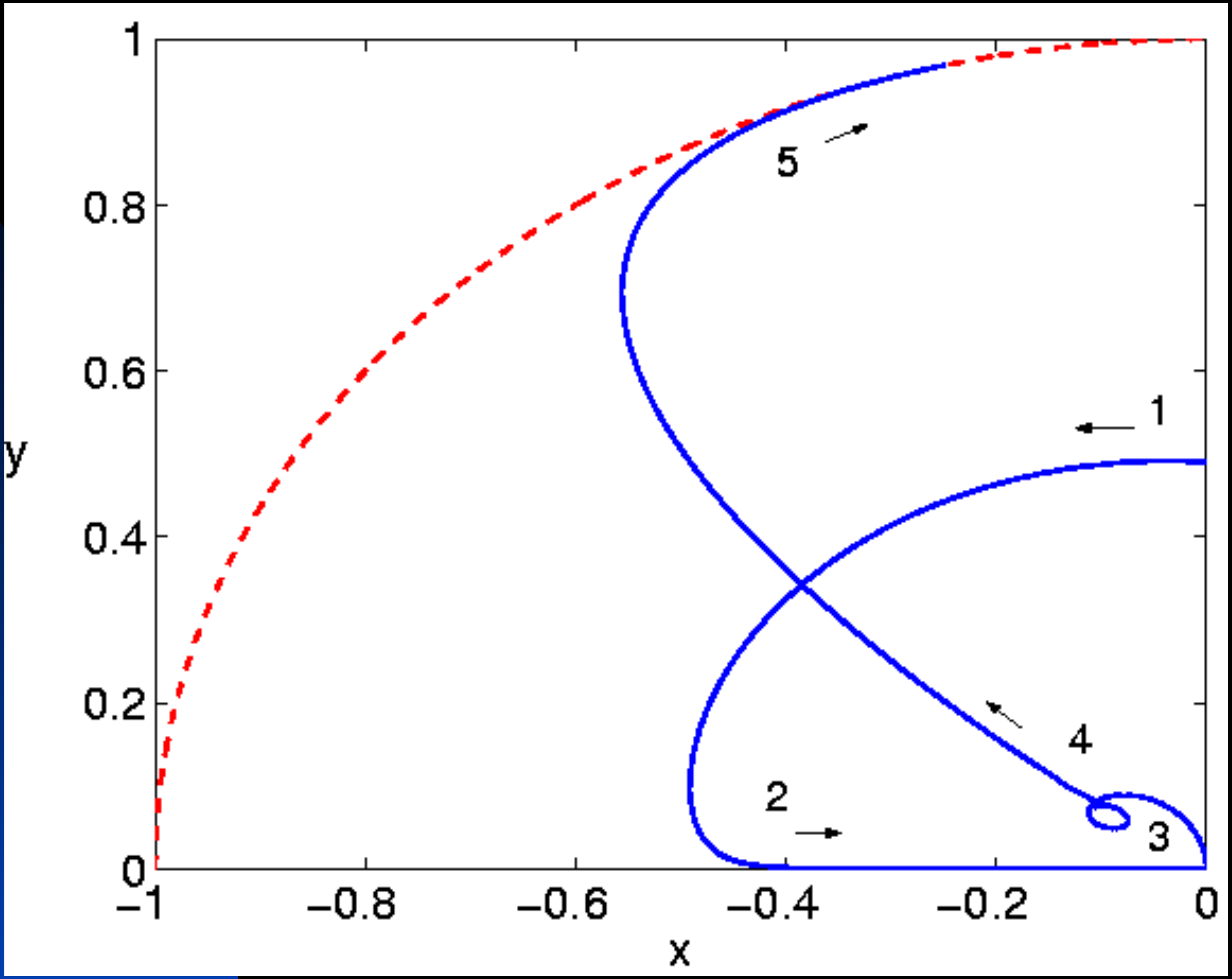


Nunes

Attractors make initial conditions less important

Phase Plane picture

Nunes



Typical example : Scaling solutions with exponential potentials. (EJC, Liddle and Wands)

$$V(\phi) = V_0 e^{-\kappa\lambda\phi}$$

Original Quintessence model

Peebles and Ratra;

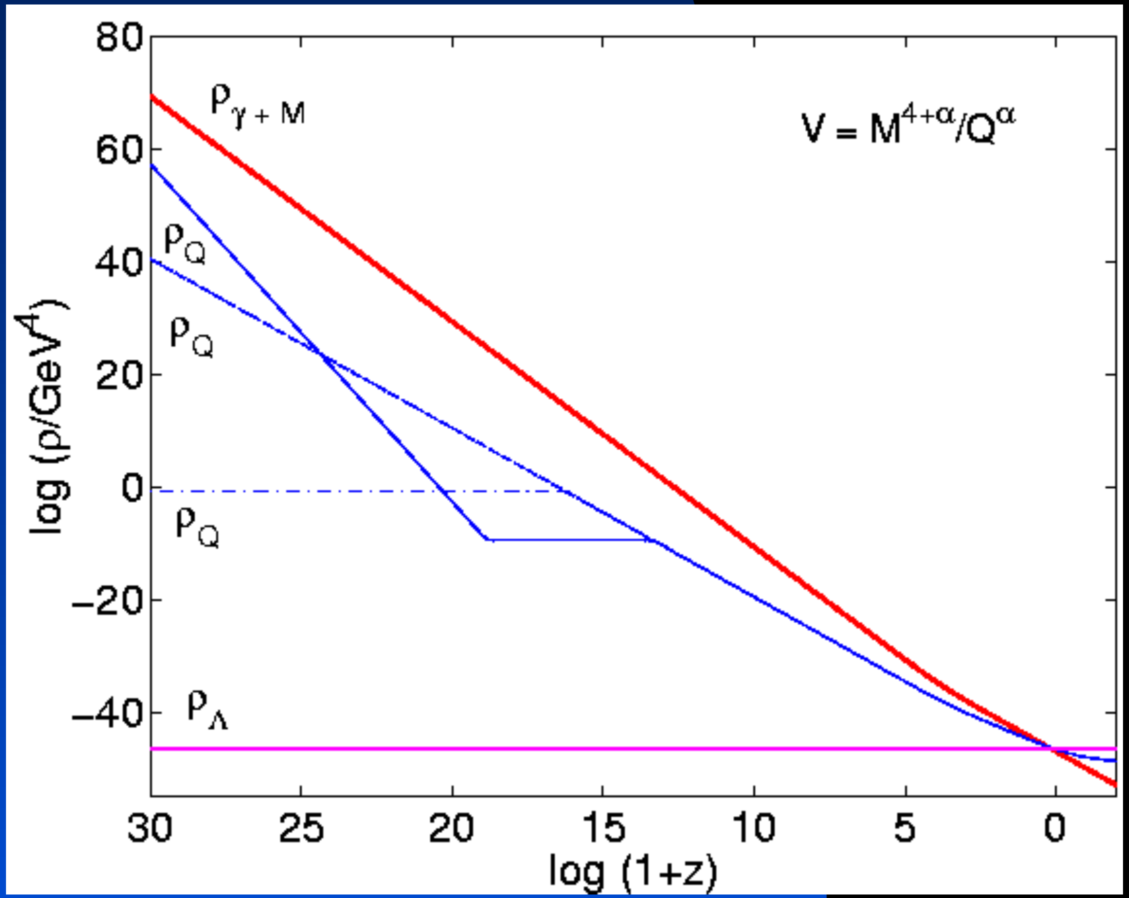
Zlatev, Wang and Steinhardt

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}}$$

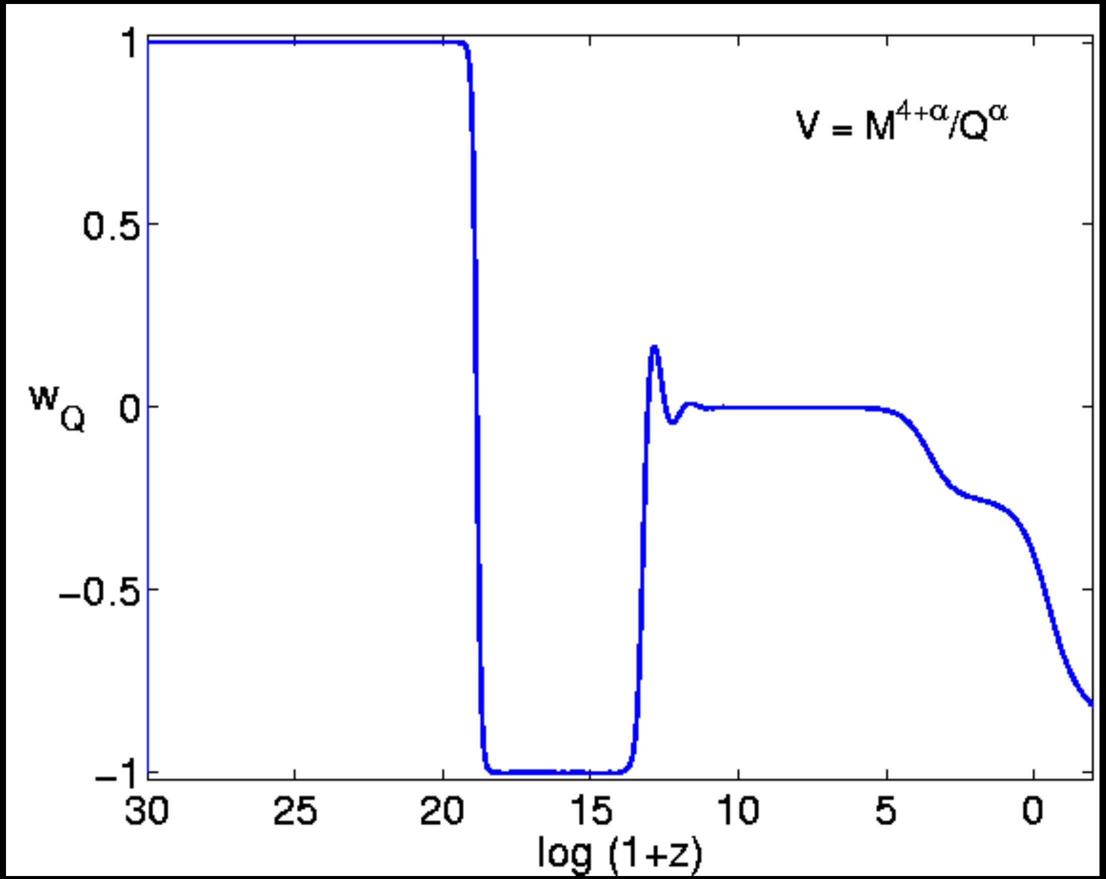
$$\lambda = \frac{-\alpha}{\kappa\phi}$$

$$\Gamma - 1 \equiv \frac{1}{\alpha}$$

Find: $$\phi = \phi_i \left(\frac{a}{a_i}\right)^{3(1+w_B)/(2+\alpha)}$$ and $$w_\phi = \frac{\alpha w_B - 2}{2+\alpha}$$



$\alpha = 6$



Fine Tuning in Quintessence

Need to match energy density in Quintessence field to current critical energy density.

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}}$$

$$\rho_{\Lambda} \le \frac{H_0^2}{\kappa^2} \approx 10^{-47}\ \mathrm{GeV}^4$$

Find: $$y_c^2 = \frac{\kappa^2 V}{3H^2} \propto \kappa^2 \phi^2$$

so: $$H^2 = \frac{V}{\phi^2} \propto \kappa^2 \rho_{\phi} \Rightarrow \phi_0 \approx M_{pl}$$

Hence: $$M = \left[\rho_{\phi}^{0}\, M_{pl}^{\alpha}\right]^{1/4+\alpha} \Rightarrow \alpha = 2; M = 1\, GeV$$

A few models

1. Inverse polynomial – found in SUSY QCD - Binetruy

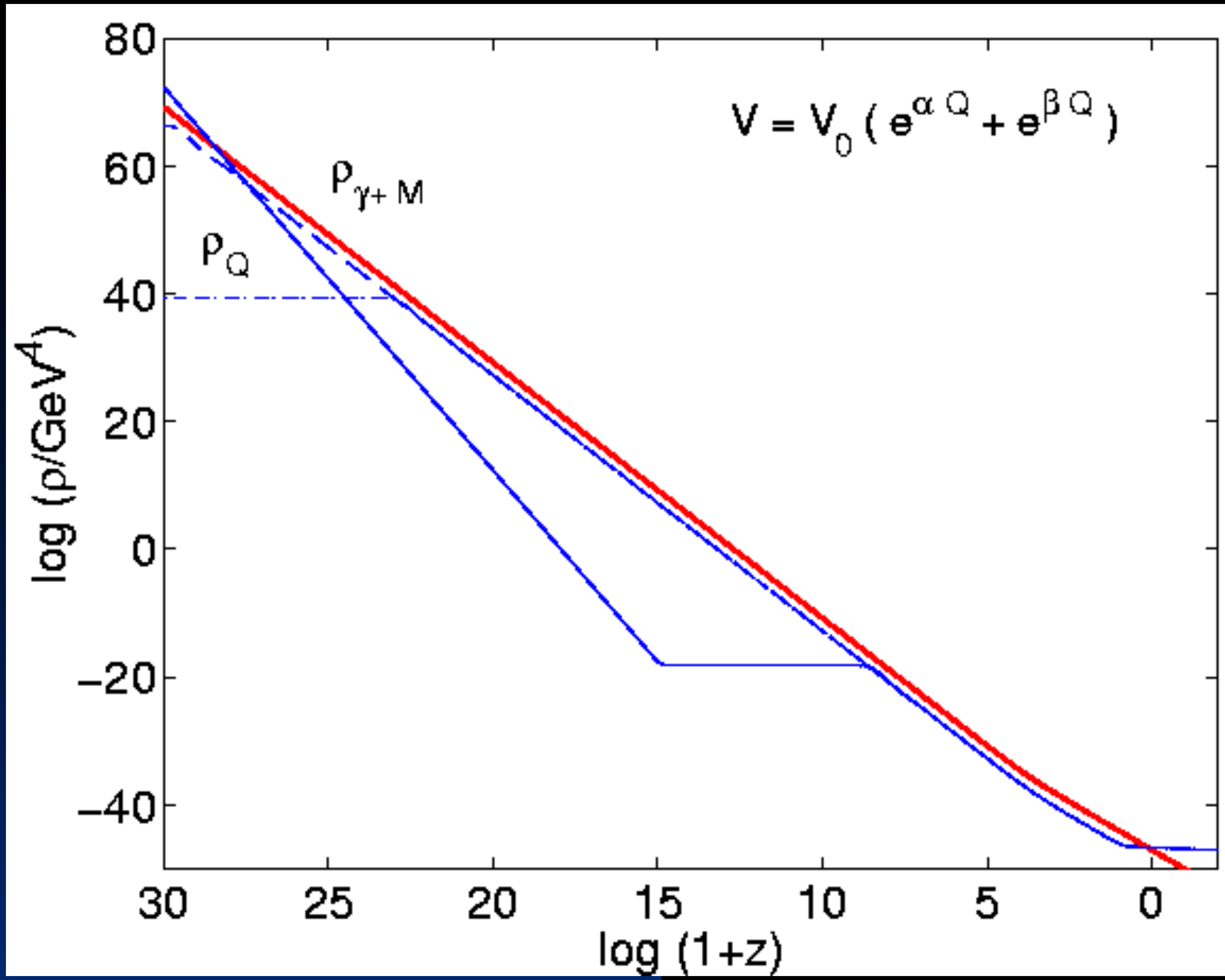
2. Multiple exponential potentials – SUGR and String compactification.

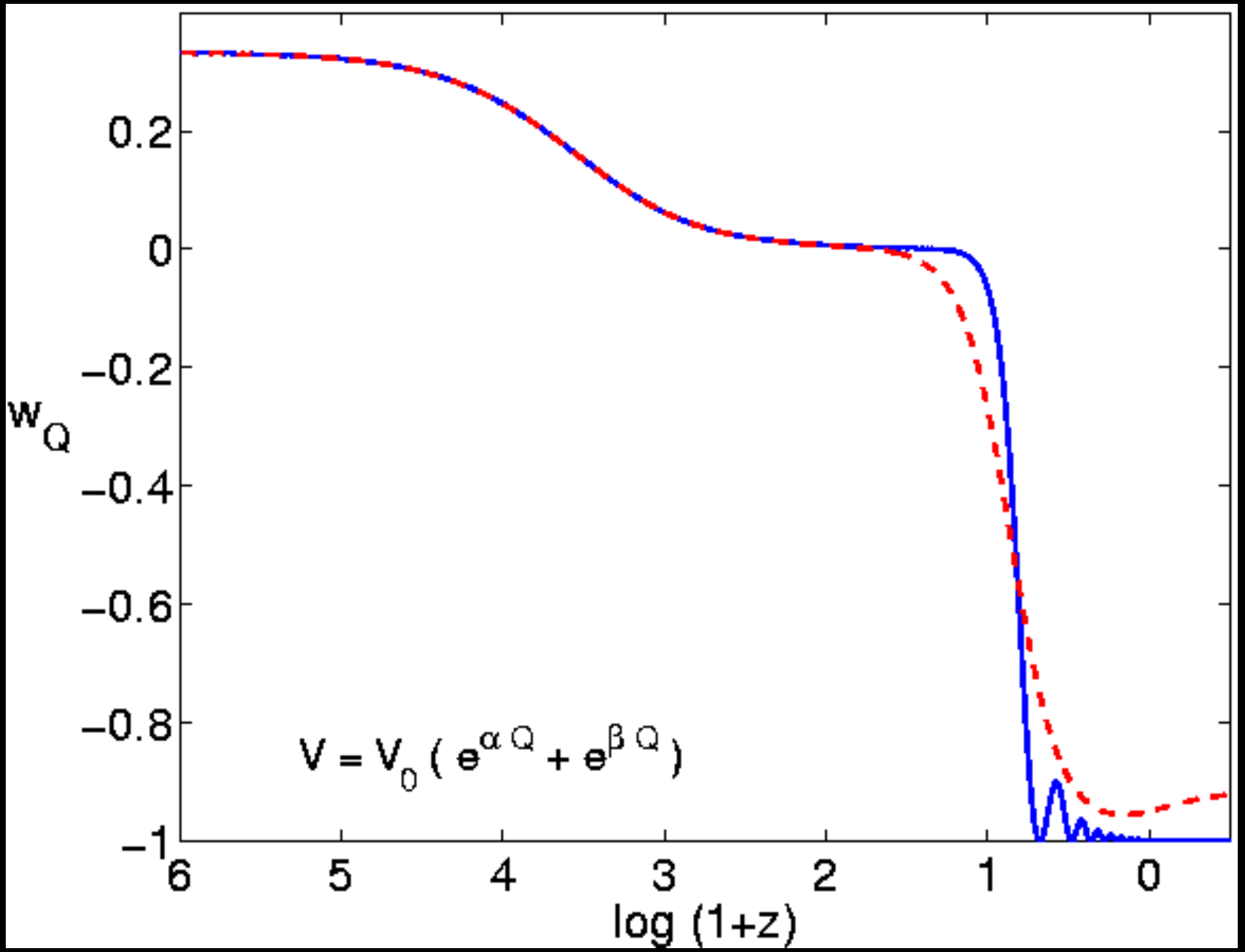
Barreiro, EC, Nunes

$$V(\phi) = V_1 + V_2$$
$$= V_{01} e^{-\kappa\lambda_1\phi} + V_{02} e^{-\kappa\lambda_2\phi}$$

Enters two scaling regimes depends on lambda, one tracking radiation and matter, second one dominating at end. Must ensure do not violate nucleosynthesis constraints.

$\alpha = 20;\ \beta = 0.5$





Scaling for wide range of i.c.

Fine tuning:

$$V_0 \approx \rho_\phi \approx 10^{-47}\ \mathrm{GeV}^4 \approx (10^{-3}\ \mathrm{eV})^4$$

Mass:

$$m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \mathrm{eV}$$

Fifth force !

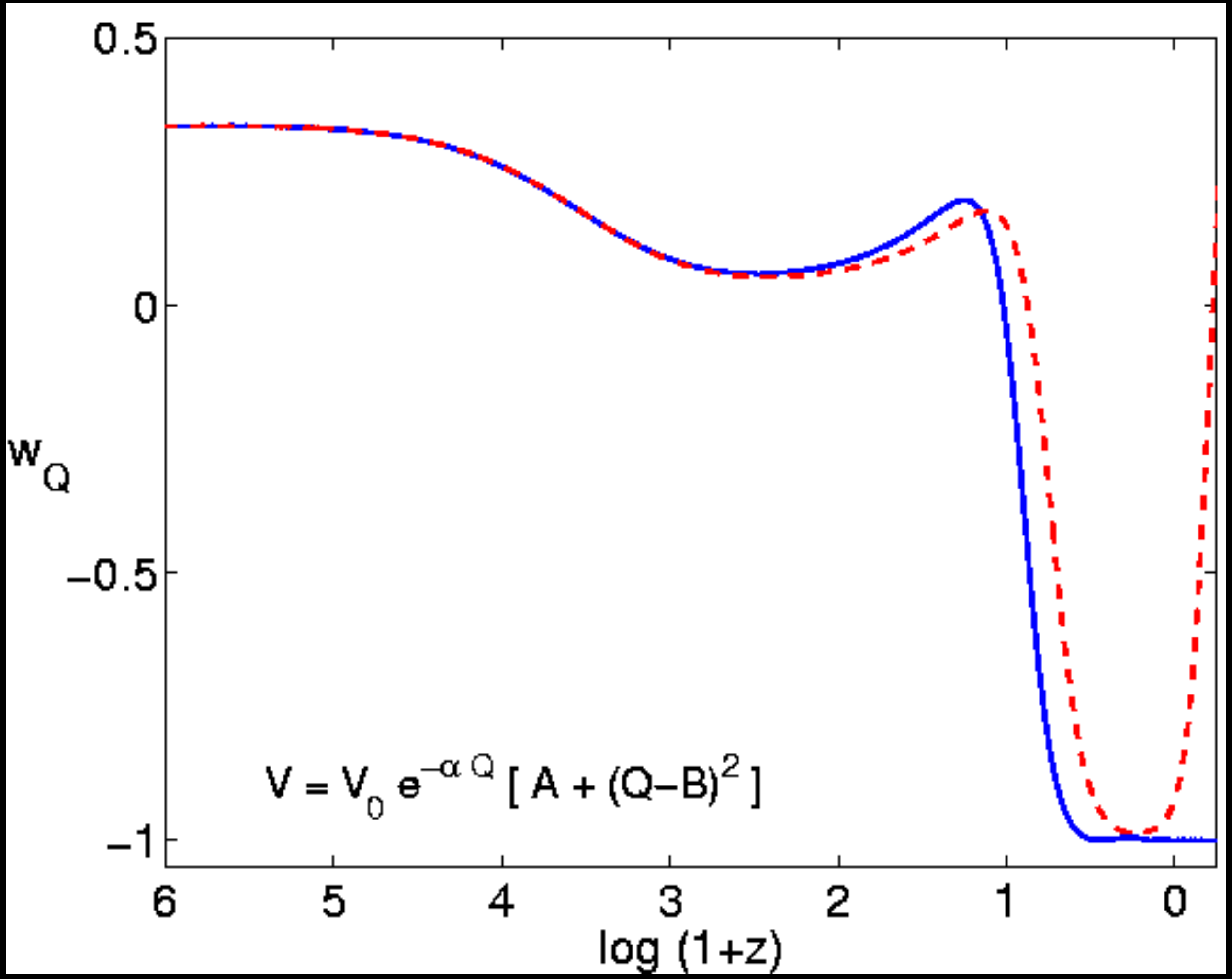
3. Albrecht-Skordis model – Albrecht and Skordis

$$V(\phi) = V_0 \, e^{-\alpha\kappa\phi}\left[A + (\kappa\phi - B)^2\right]$$

-- Brane models

Early times: exp dominates and scales as rad or matter.

Field gets trapped in local minima and univ accelerates



Fine tuned as in previous cases.

K-essence v Quintessence

K-essence -- scalar fields with non-canonical kinetic terms. Advantage over Quintessence through solving the coincidence model? -- Armendariz-Picon, Mukhanov, Steinhardt

Long period of perfect tracking, followed by domination of dark energy triggered by transition to matter domination -- an epoch during which structures can form.

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{16\pi G}R + K(\phi)\tilde{p}(X)\right]$$

$$K(\phi) > 0,\ X = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$$

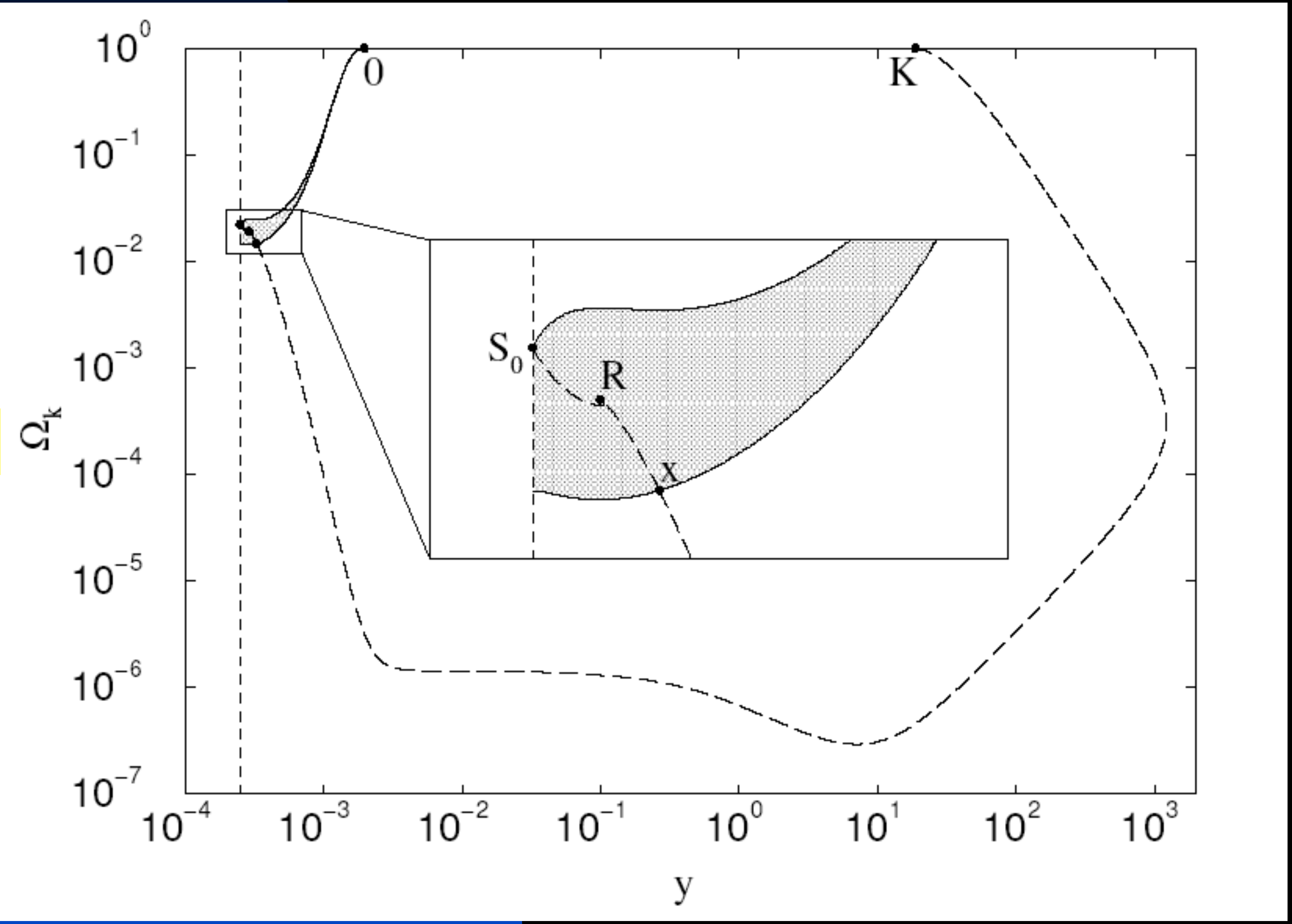
Eqn of state $w_k = \frac{\tilde{p}(X)}{\tilde{\varepsilon}(X)} = \frac{\tilde{p}(X)}{2X\tilde{p}'(X) - \tilde{p}(X)}$ can be < -1

However also requires similar level of find tuning as in Quintessence

Fine tuning in K-essence as well [Malquarti, EJC, Liddle]

Not so clear that K-essence solves the coincidence problem. The basin of attraction into the regime of tracker solutions is small compared to those where it immediately goes into K-essence domination.

Ω_k



$y = 1/\sqrt{X}$

Shaded region is basin of attraction for stable tracker solution at point R. All other trajectories go to K-essence dom at point K.

Based on K-essence model astro-ph/0004134, Armendariz-Picon et al.

Phantom Dark Energy - a way to get w<-1 — [Caldwell 2002]

Recall a canonical homogeneous scalar field

$$\rho = -T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \quad p = T^i_i = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Eqn of state

$$w_\phi = \frac{p}{\rho} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}\,.$$

Bounded -1 < w < 1 - Quintessence

Intro ghost field (negative KE)

$$w_\phi = \frac{p}{\rho} = \frac{\dot{\phi}^2 + 2V(\phi)}{\dot{\phi}^2 - 2V(\phi)}\,.$$

w < -1 if PE dominates

Curvature of the universe grows towards infinity within a finite time if dominated by a phantom field — leads to a Big Rip

UV Quantum instabilities - energy density unbounded from below, vacuum unstable against production of ghosts and normal (positive energy) fields. Even if the ghosts are decoupled from matter fields, they couple to gravitons which mediate vacuum decay: vacuum —> 2 ghosts + 2 photons

For those who like a few details - imagine a universe with matter and a phantom field.

Fluid eqn for phantom, energy density grows for w<-1

$$\dot{\rho} = -3H(\rho + p) \quad \Rightarrow \quad \frac{d\ln\rho}{d\ln a} = -3(1+w) \quad \Rightarrow \quad \rho \propto a^{-3(1+w)}$$

Dimensionless phantom, energy density

$$\Omega_X(a) \equiv \frac{\rho_X(a)}{\rho_{\rm crit}(a)} = \frac{\rho_X(a)}{\rho_{\rm m}(a) + \rho_X(a)} = \frac{\Omega_{X,0}a^{-3(1+w_X)}}{\Omega_{m,0}a^{-3} + \Omega_{X,0}a^{-3(1+w_X)}}$$

Hence

$$\Omega_X(a) = \left(1 + \frac{\Omega_{m,0}}{\Omega_{X,0}} a^{3w_X}\right)^{-1}$$

If $\Omega_{X,0}$=0.75, w_X=-2, have 99.9% in Phantom when a=2.6

Scale factor diverges in finite time

$$\Delta t = \frac{2H_\star^{-1}}{3|1+w_X|}$$

where $H_\star$ - value of H when DE first dominates

The problem of coupling DE and DM directly with scalars

[D'Amico, Hamil & Kaloper 2016]

Generate loop corrections to the DE mass.

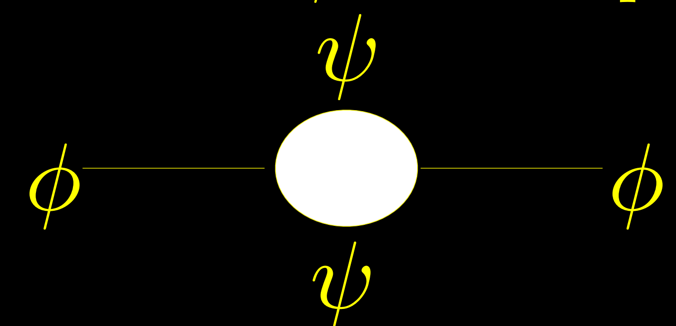
Consider Yukawa type coupling between DE scalar and DM fermion $g\phi\bar{\psi}\psi$

Now since it is DE: $m_\phi \simeq H \sim 10^{-33} eV$

Very light so long range attractive 5th force: $Pot: \Phi(r) \sim g^2/r$

Must be less than grav attraction of DM particles by say factor 10 $g < m_\psi/(10 m_{\rm pl})$

Loop correction to DE mass from DM



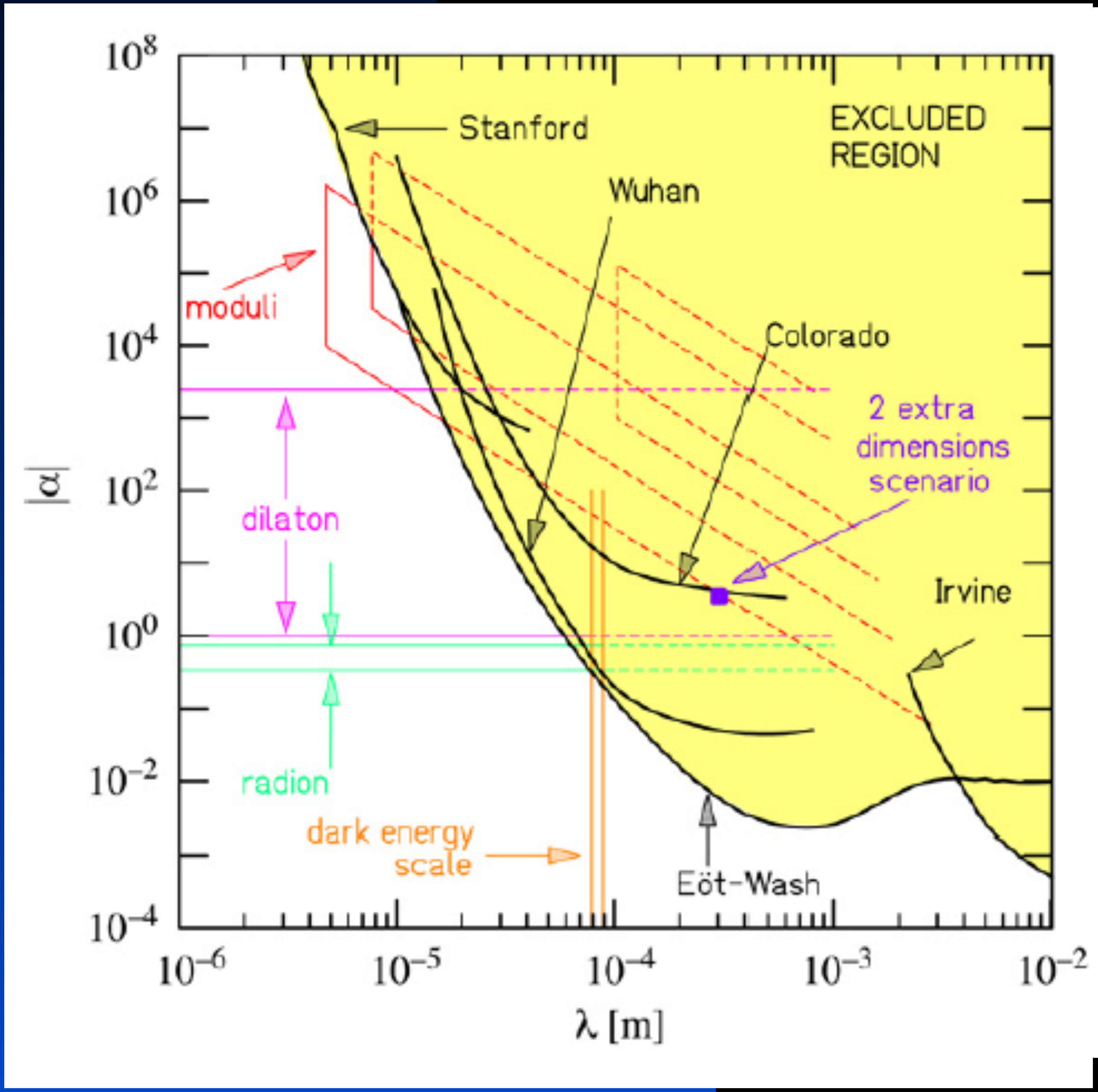
$$\delta m_\phi^2 \simeq g^2 m_\psi^2 < m_\psi^4/(10 m_{\rm pl})^2$$

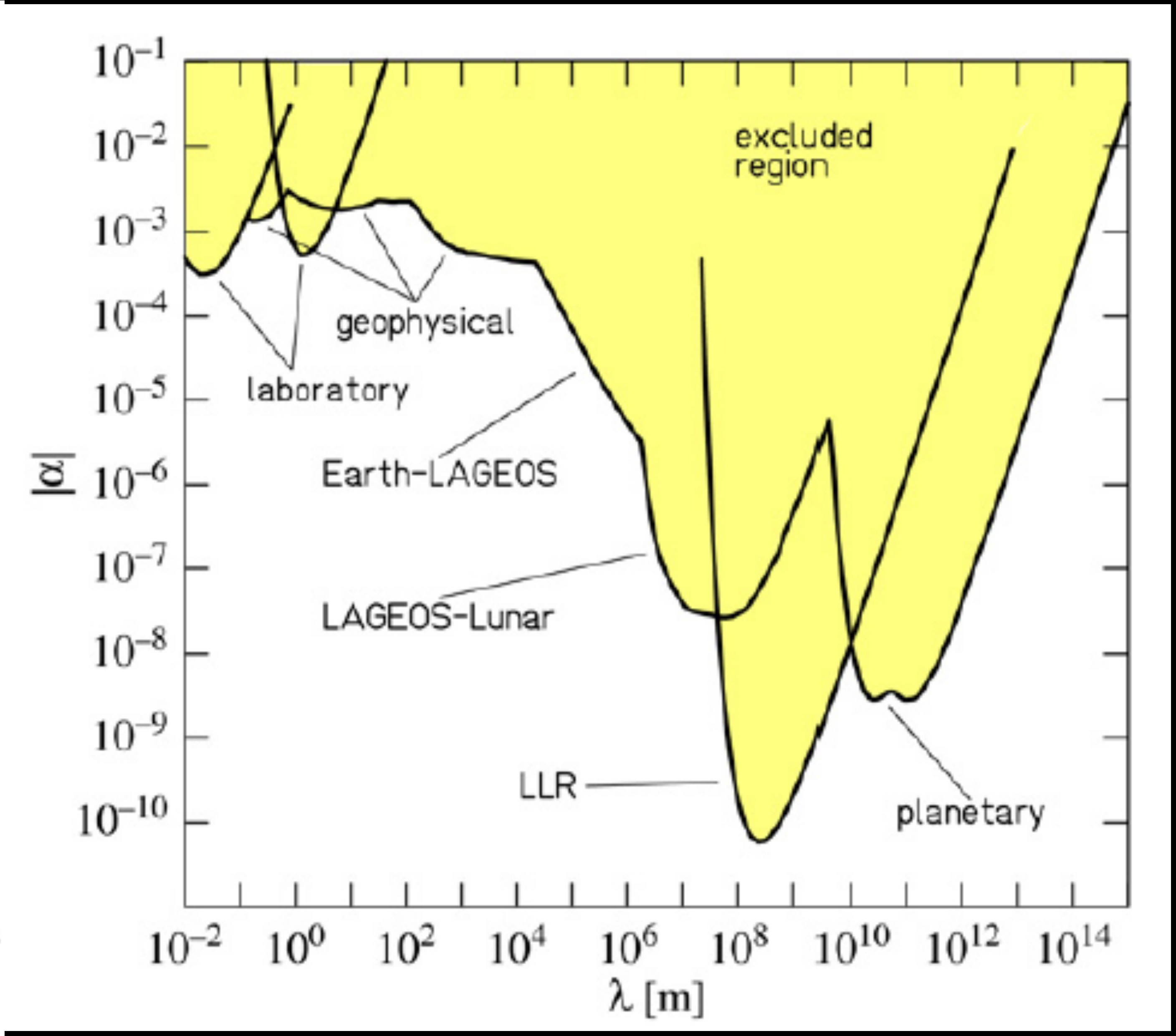
Require: $\delta m_\phi^2 < H_0^2$ implying : $m_\psi < 10^{-3} eV$

But then the required light DM isn't cold - or go for an axion with a protected mass or a different coupling between DM and DE

Quintessence tends to lead to existence of Yukawa Fifth Force - very tightly constrained.

$$F(r) = G\frac{m_1 m_2}{r^2}\left[1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right]$$





Adelberger 2009.

Screening mechanisms - a route to hide the fifth forces

1. Chameleon fields [Khoury and Weltman (2003) …]

Non-minimal coupling of scalar to matter in order to avoid fifth force type constraints on Quintessence models: the effective mass of the field depends on the local matter density, so it is massive in high density regions and light (m~H) in low density regions (cosmological scales).

2. K-essence [Armendariz-Picon et al …]

Scalar fields with non-canonical kinetic terms. Includes models with derivative self-couplings which become important in vicinity of massive sources. The strong coupling boosts the kinetic terms so after canonical normalisation the coupling of fluctuations to matter is weakened -- screening via Vainshtein mechanism

Similar fine tuning to Quintessence -- vital in brane-world modifications of gravity, massive gravity, degravitation models, DBI model, Galileon's, ...

3. Symmetron fields [Hinterbichler and Khoury 2010 …]

vev of scalar field depends on local mass density: vev large in low density regions and small in high density regions. Also coupling of scalar to matter is prop to vev, so couples with grav strength in low density regions but decoupled and screened in high density regions.

Chameleon bare non-linear potential

$$V_{\rm eff}(\phi) = V(\phi) + \left(1\frac{\phi}{M}\right)\rho\,.$$

with self-interaction strength Λ

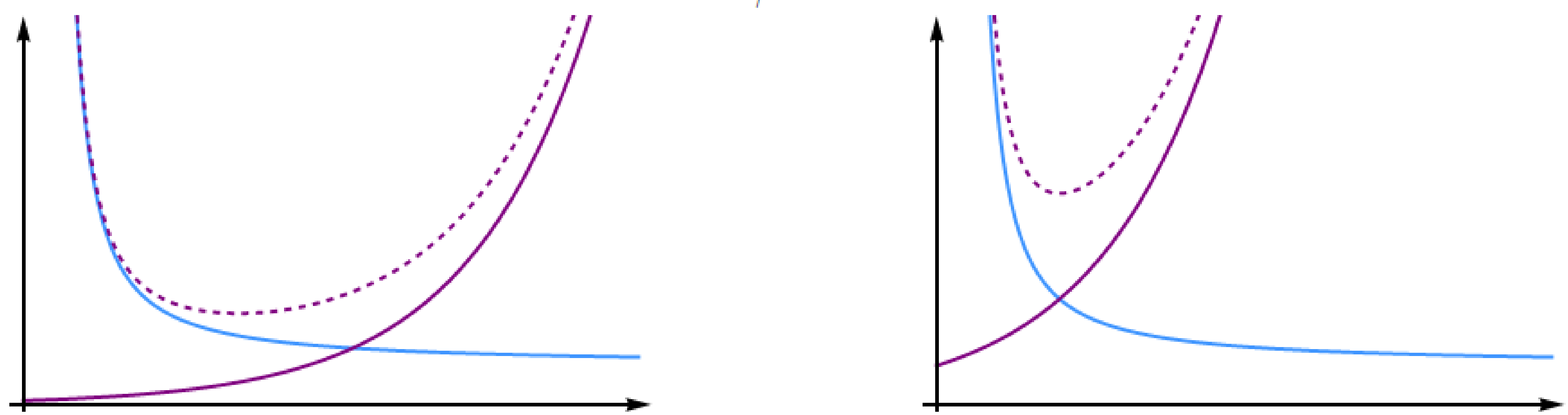
$$V(\phi) = \frac{\Lambda^5}{\phi}$$

[Khoury and Weltman, PRL 93 171104 (2004)]

The mass of the chameleon changes with the environment

Field is governed by an effective potential

$$V_{\rm eff} = \frac{\Lambda^5}{\phi} + \frac{\phi}{M}\rho$$



Low density High density

$$\phi_{\rm min}(\rho) \;=\; \left(\frac{\Lambda^5 M}{\rho}\right)^{1/2}\,,$$

$$m_{\rm min}(\rho) \;=\; \sqrt{2}\left(\frac{\rho^3}{\Lambda^5 M^3}\right)^{1/4}\,.$$

coupling constants

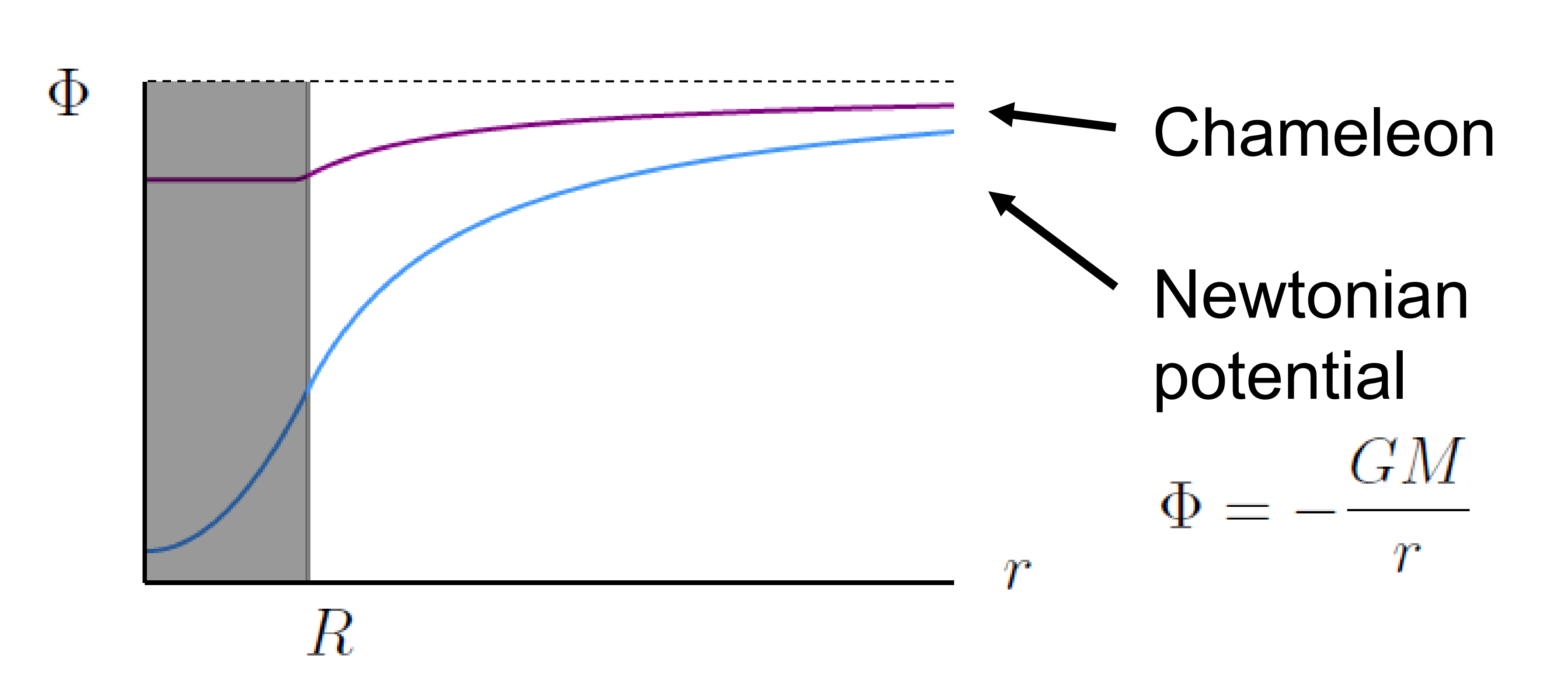
$$10^{-5}eV < \Lambda < 10^{-1}eV$$

$$10^{-14}M_p < M < M_p$$

How does this type of potential help with the fifth force constraints?

The fact it is density (or environment) dependent means that in less dense areas it is light (as required for dark energy) and in denser regions it is massive (as required by solar system tests).

The increased mass makes it harder for the Chameleon field to adjust its value, leads to the associated force being screened.



The Chameleon potential well around a massive object is shallower than for standard light scalar fields - hence the associated force is reduced.

The sources we consider for the chameleon field are const density and spherically symmetric.

$$\rho(r) = \rho_A \Theta(R_A - r) + \rho_{\rm bg}\Theta(r - R_A)\ ,$$

ρ_A, R_A – –density and radius of the source

ρ_{bg} – –density of bgd env surrounding the ball

There is a universal form for the scalar potential which comes from solving the eom in all the regimes and matching across boundaries - suitable for weakly and strongly perturbing objects:

$$\phi = \phi_{\rm bg} - \lambda_A \frac{1}{4\pi R_A}\frac{M_A}{M}\frac{R_A}{r}e^{-m_{\rm bg} r}$$

$$\lambda_A = \begin{cases} 1\ , & \rho_A R_A^2 < 3M\phi_{\rm bg} \\ 1 - \frac{S^3}{R_A^3} \approx 4\pi R_A \frac{M}{M_A}\phi_{\rm bg}\ , & \rho_A R_A^2 > 3M\phi_{\rm bg} \end{cases}$$

$$m_{\rm bg}^2 = d^2V_{\rm eff}/d\phi^2|_{\phi_{\rm bg}}$$

The parameter λ determines how responsive an object is to the chameleon field.

For small $m_{bg}r$ the ratio of the acceleration of a test particle due to chameleon and gravity is

$$\frac{a_\phi}{a_N} = \frac{\partial_r \phi}{M}\frac{r^2}{GM_A} = 3\lambda_A\left(\frac{M_P}{M}\right)^2$$

If λ =1 this could be a big effect ! On cosmological scales though λ << 1

And so we begin to think about measuring this effect in Laboratory experiments.

We see that the chameleon effects are not screened for `small' objects that do not probe the scalar non-linearities. This will be the case if λ =1 or:

$$\frac{1}{4\pi R_A}\frac{M_A}{M} \ll \phi_{\rm bg}$$

To achieve this we either require an expt with:

$$\phi_{\rm bg} \text{ is large } -- \text{ high quality vacuum}$$
$$\frac{M_A}{R_A} << 1 -- \text{ atoms}$$

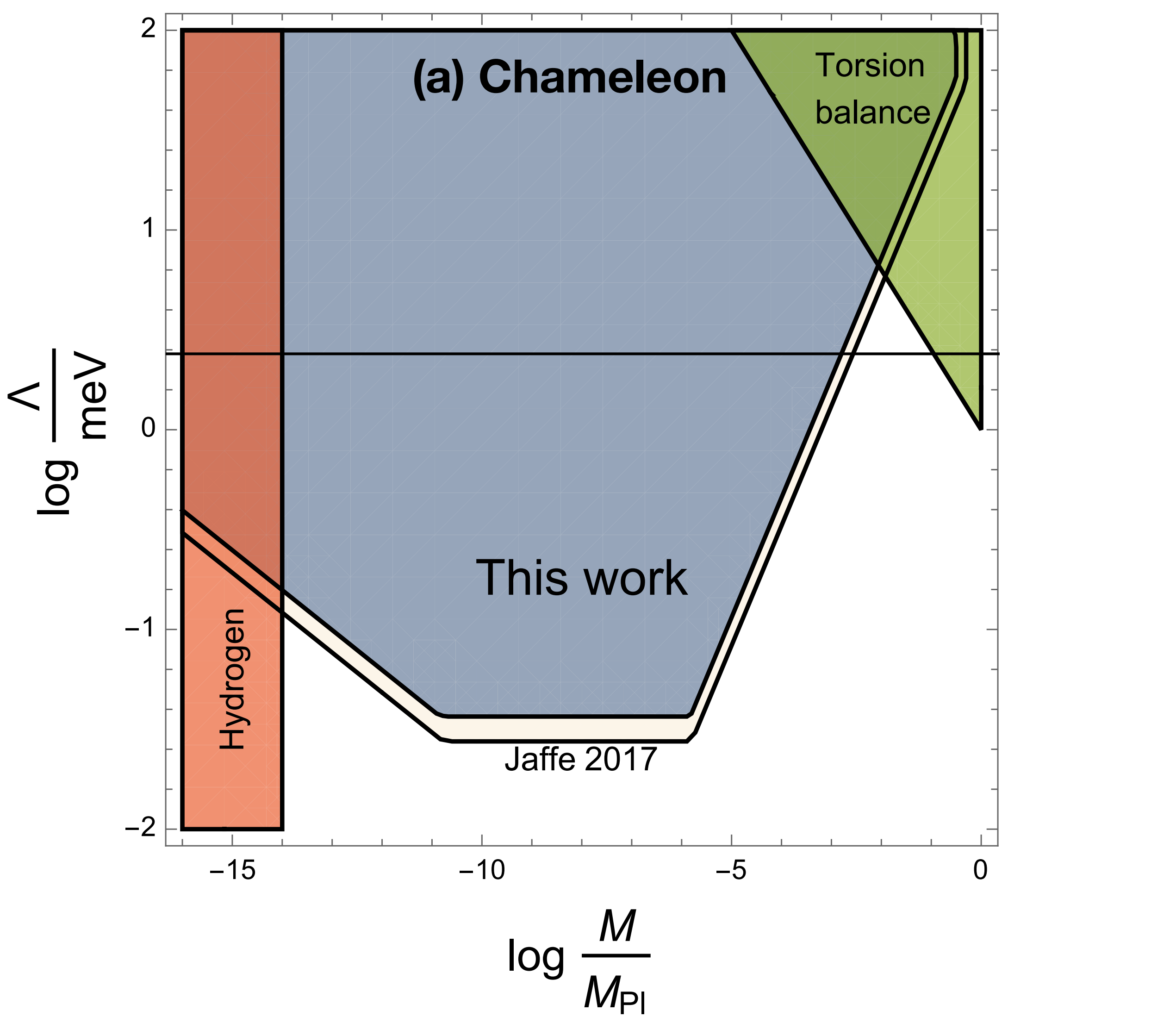
The idea is to use a vacuum chamber with walls thick enough so that the interior can be screened from external chameleon field fluctuations

Dark Energy Direct Detection Experiment [Burrage, EC, Hinds 2015,Hamilton et al 2015]

We normally associate DE with cosmological scales but here we use the lab !

Atom Interferometry - testing Chameleons Idea: Individual atoms in a high vacuum chamber are too small to screen the chameleon field and so are very sensitive to it - can detect it with high sensitivity. Can use atom interferometry to measure the chameleon force - or more likely constrain the parameters !

$$\nabla^2\phi = -\frac{\Lambda^2}{\phi^2} + \frac{\rho}{M}$$



$$F_r = \frac{GM_AM_B}{r^2}\left[1 + 2\lambda_A\lambda_B\left(\frac{M_P}{M}\right)^2\right]$$

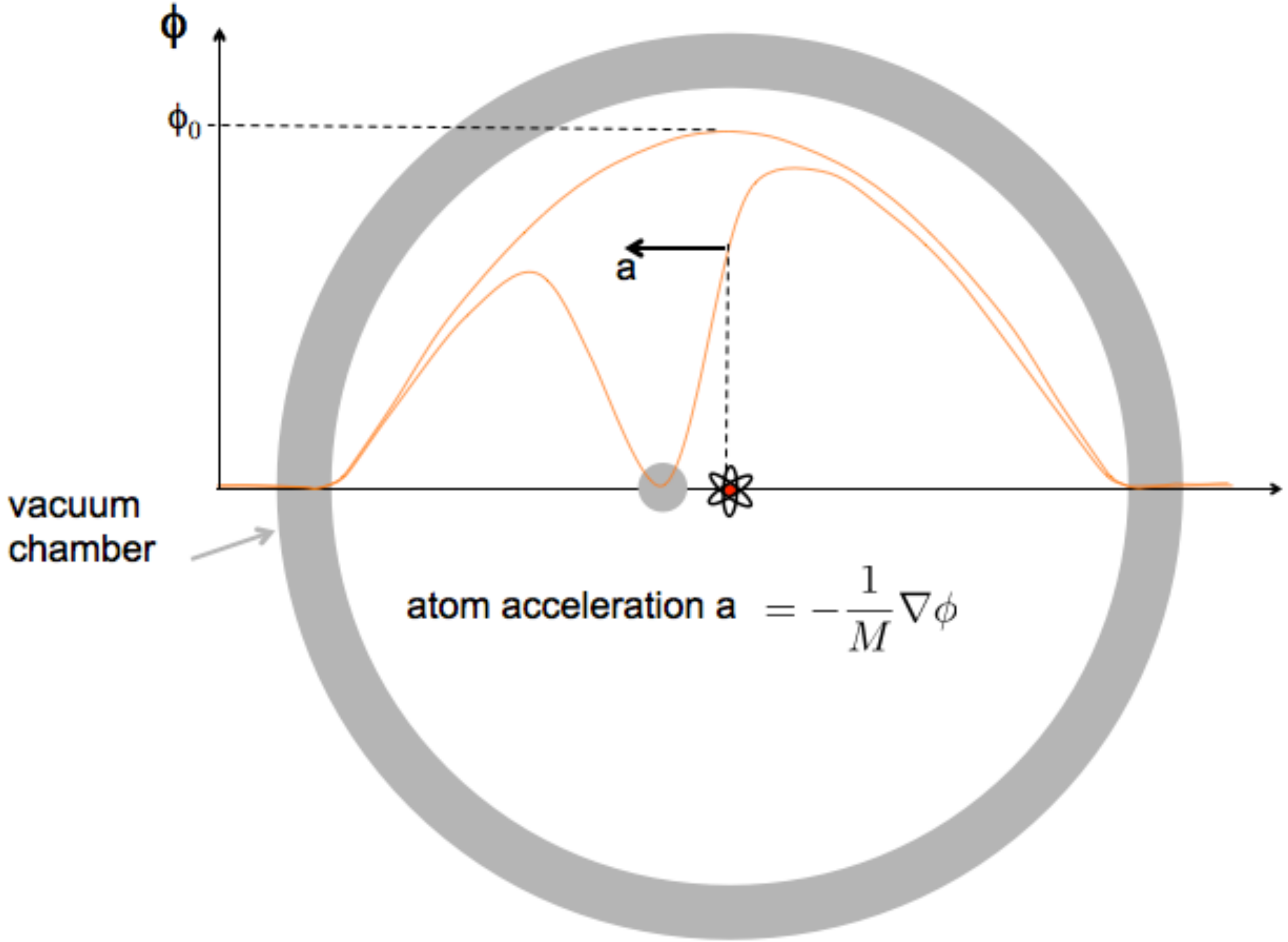
$$\lambda_i = 1 \text{ for } \rho_i R_i^2 < 3M\phi_{bg}$$

$$\lambda_i = \frac{3M\phi_{bg}}{\rho_i R_i^2} \text{ for } \rho_i R_i^2 > 3M\phi_{bg}$$

Sph source A and test object B near middle of chamber experience force between them - usually ƛ<<1 in cosmology but for atom ƛ=1 - reduced suppression

[Sabulsky et al 2019]

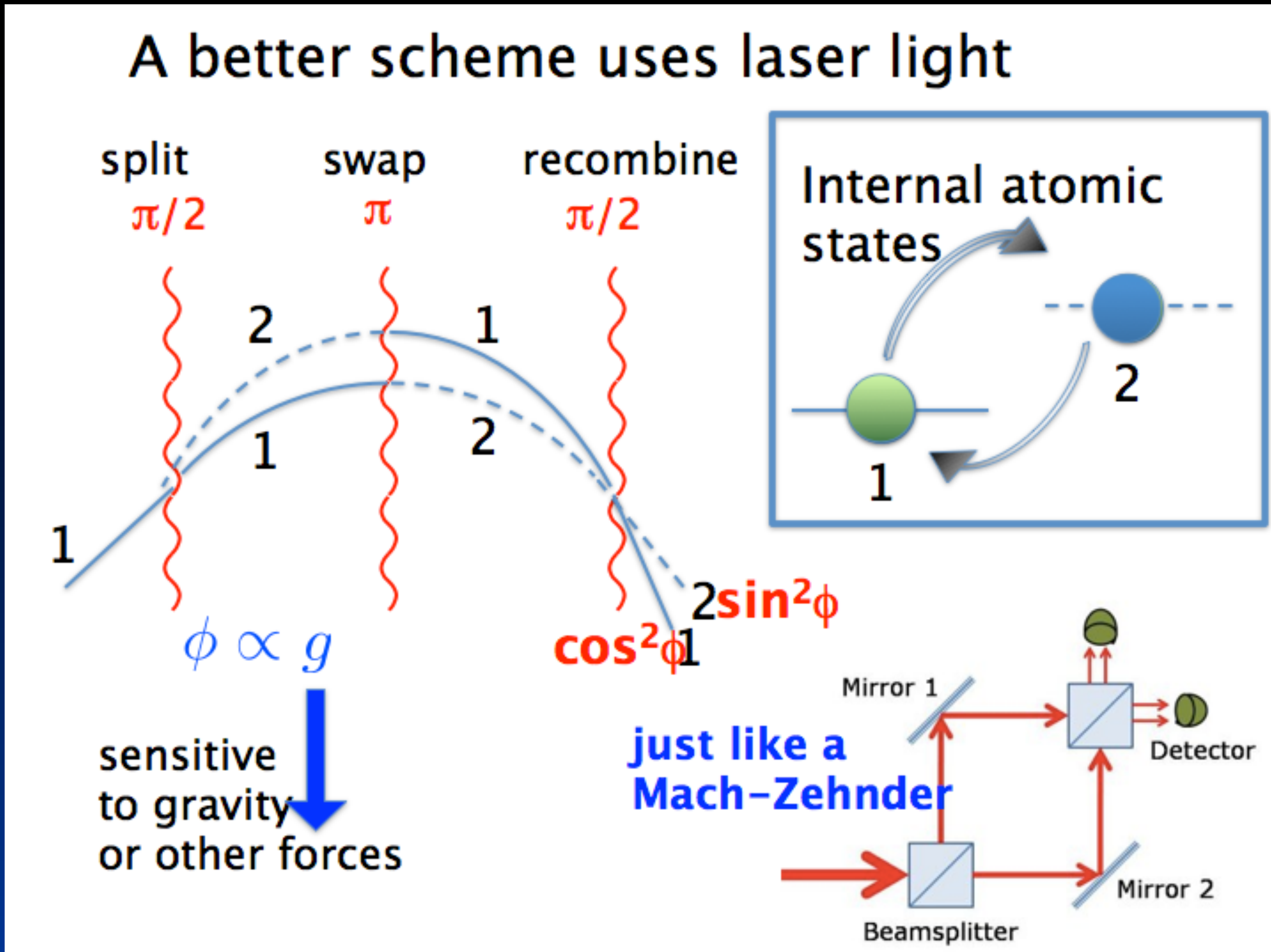
Measure ϕ in a high vacuum chamber



atom acceleration a $= -\frac{1}{M}\nabla\phi$

Ed Hinds

Use Atom Interferometry of atoms in free fall [Burrage, EC, Hinds 2015]



Ed Hinds

Raman interferometry uses a pair of counter-proagating laser beams, pulsed on three times, to split the atomic wave function, imprint a phase difference, and recombine the wave function.

The output signal of the interferometer is proportional to $\cos^2 \varphi$, with

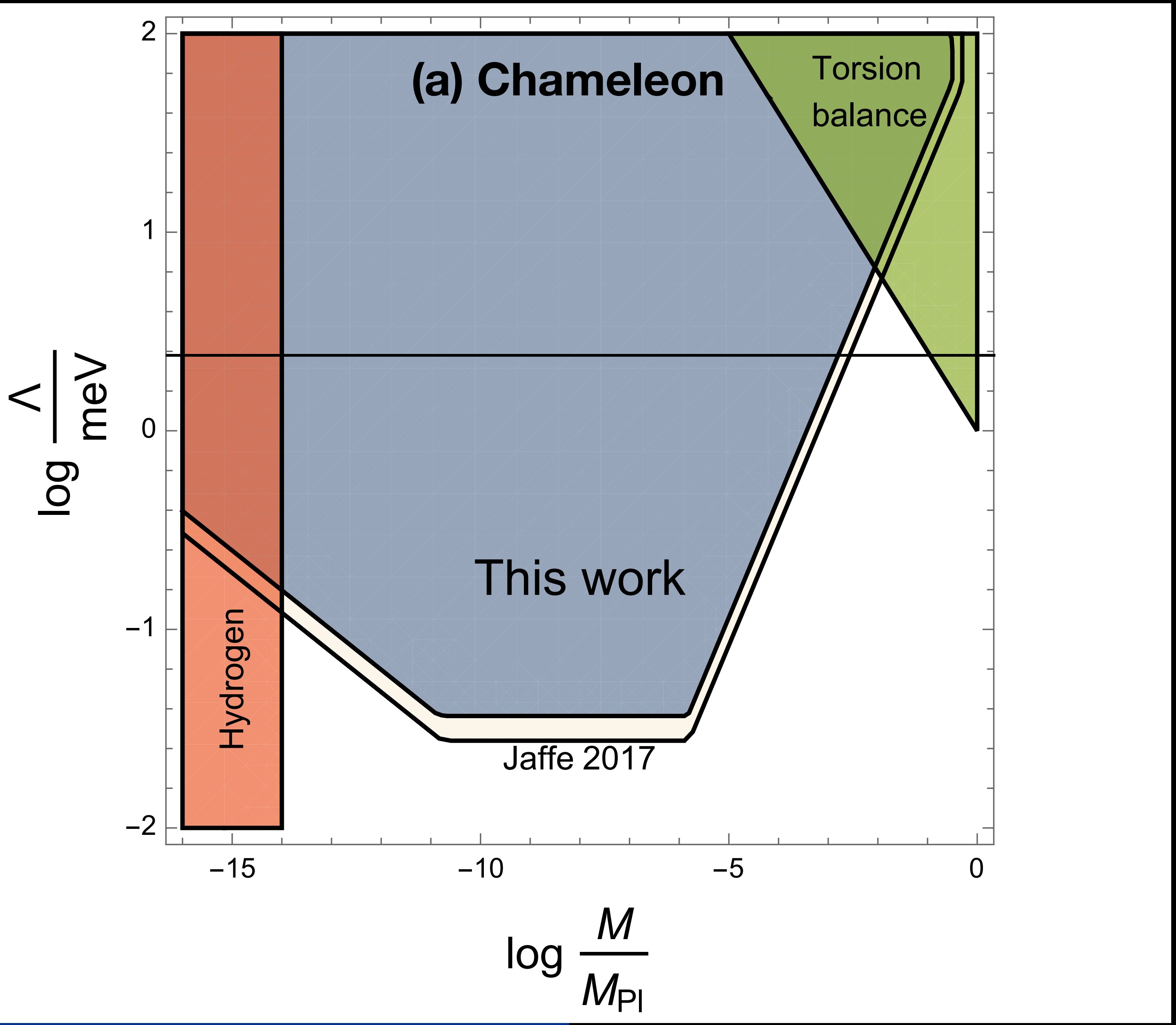
$$\varphi = (\underline{k}_1 - \underline{k}_2).\underline{a}T^2$$

$\underline{k}_{1,2}$ − −wavevectors of the 2 beams

T − −time interval between pulses

$\underline{a}$ − −acceleration of the atom

Sensitivity to acc'n of rubidium atoms due to sphere placed in Chamber radius 10cm, Pressure 10^{-10} Torr



[Sabulsky et al 2019]

$$V_{\rm eff}(\phi) = V(\phi) + \left(\frac{\phi}{M}\right)\rho$$

$$V(\phi) = \frac{\Lambda^5}{\phi}$$

Systematics:

Stark effect, Zeeman effect, Phase shifts due to scattered light, movement of beams - negligible at 10^{-6} g and controllable for 10^{-9} g

Accn due to chameleon force outside an Al sphere of radius R_A = 19mm and screening factor $\lambda_A \ll 1$.

Λ-M area above solid black line excluded by atom interferometry expt measuring 10^{-6} g - easy !

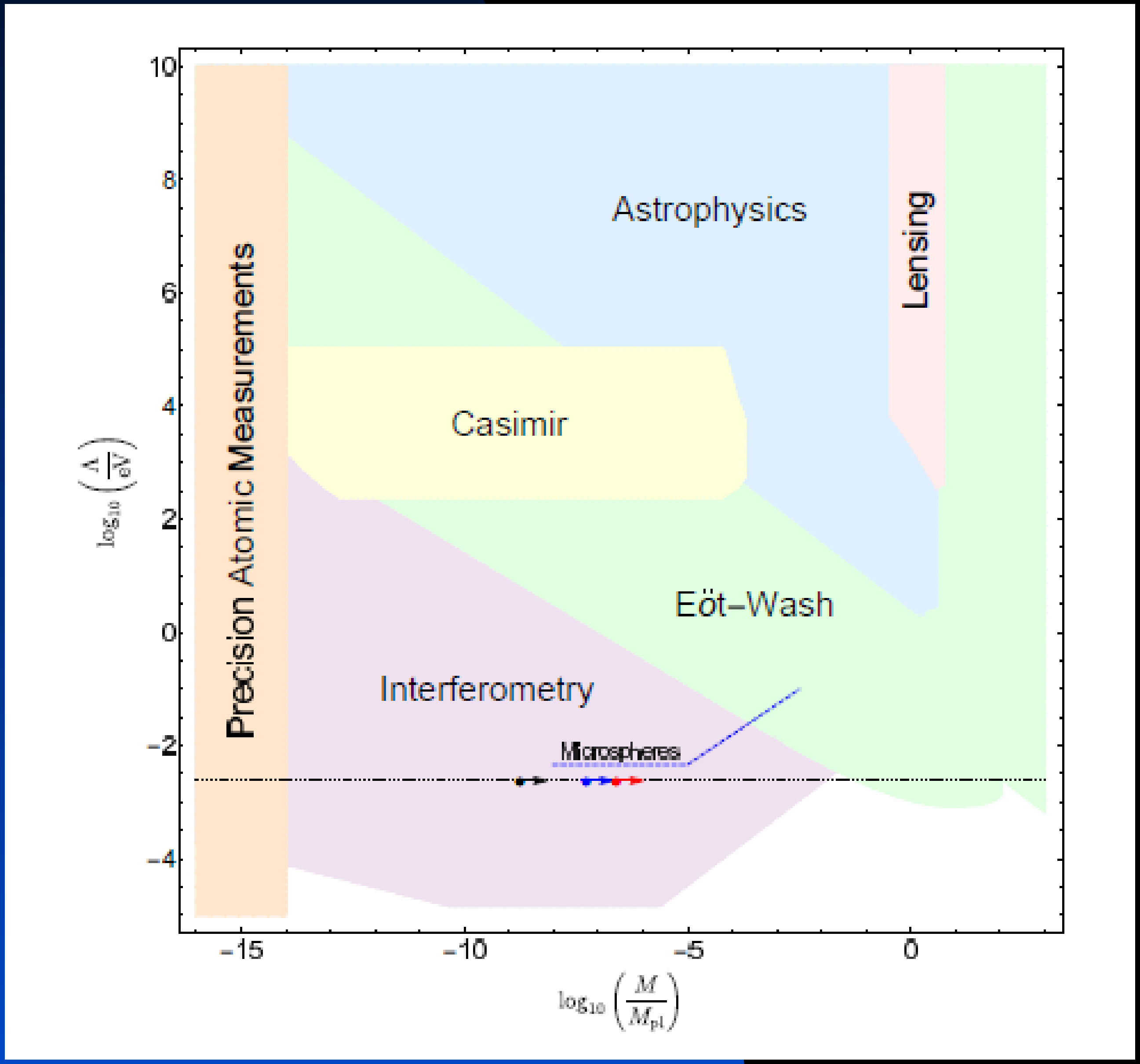
Our result indicates acceleration due to chameleon < 18 x10^{-9} g (90% CL) - can reach M_P !

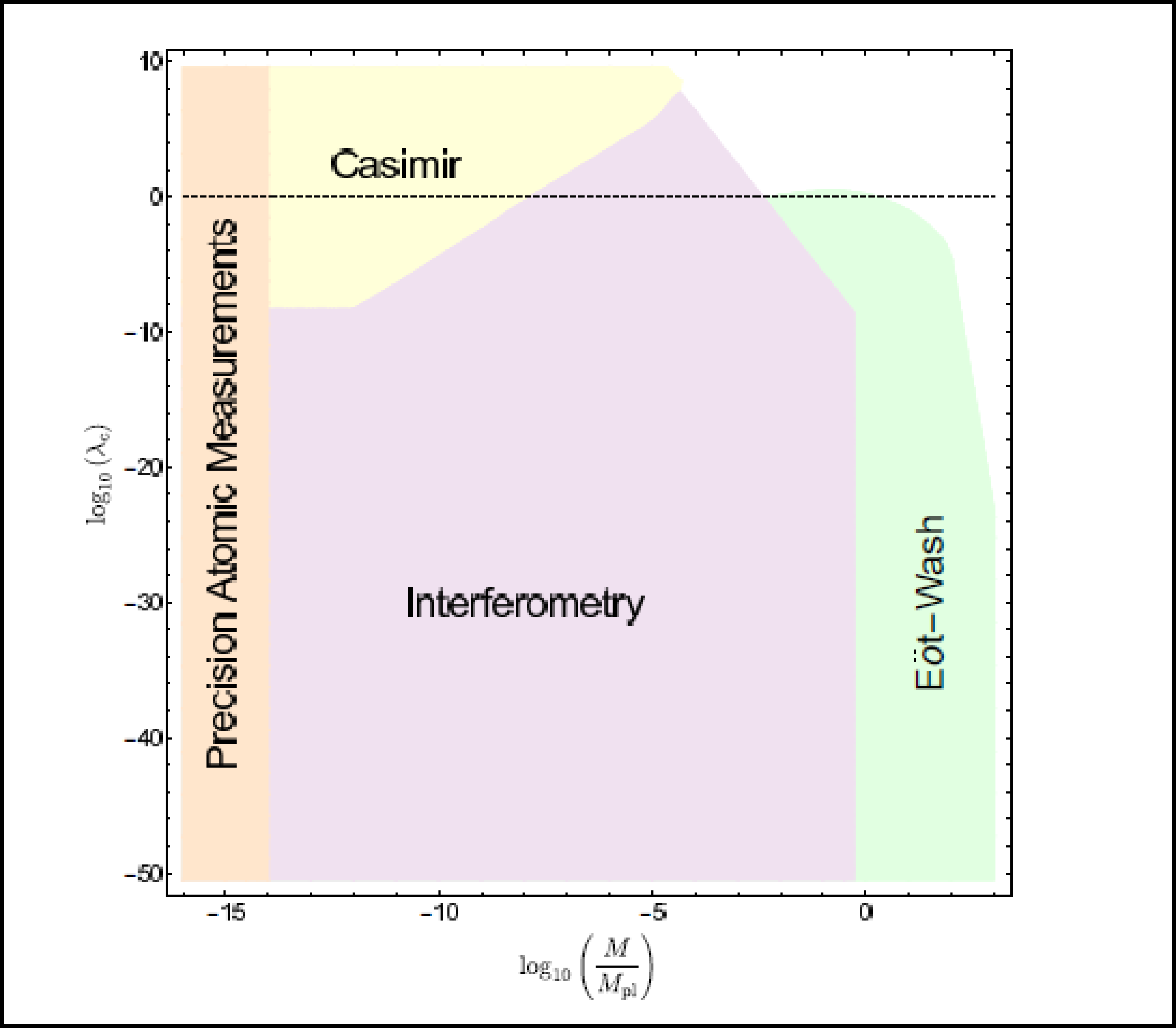
Combined chameleon constraints [Burrage & Sakstein 2017]

$$V_{\rm eff}(\phi) = V(\phi) + \left(\frac{\phi}{M}\right)\rho$$

$$V(\phi) = \frac{\Lambda^5}{\phi}$$

$$V(\phi) = \frac{\Lambda}{4}\phi^4$$





Screening mechanisms - Symmetron [Hinterbichler & Khoury 2010]

Model:

$$\tilde{V}(\varphi) \equiv V(\varphi) - \mathcal{L}_m[g] = -\frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4 - \mathcal{L}_m[g],$$

Scalar field conformally coupled to matter through Jordan frame metric $g_{\mu\nu}$ related to Einstein frame metric $\hat{g}_{\mu\nu}$:

$$g_{\mu\nu} = A^2(\varphi)\tilde{g}_{\mu\nu}$$

with

$$A(\varphi) = 1 + \frac{\varphi^2}{2M^2} + \mathcal{O}\left(\frac{\varphi^4}{M^4}\right),$$

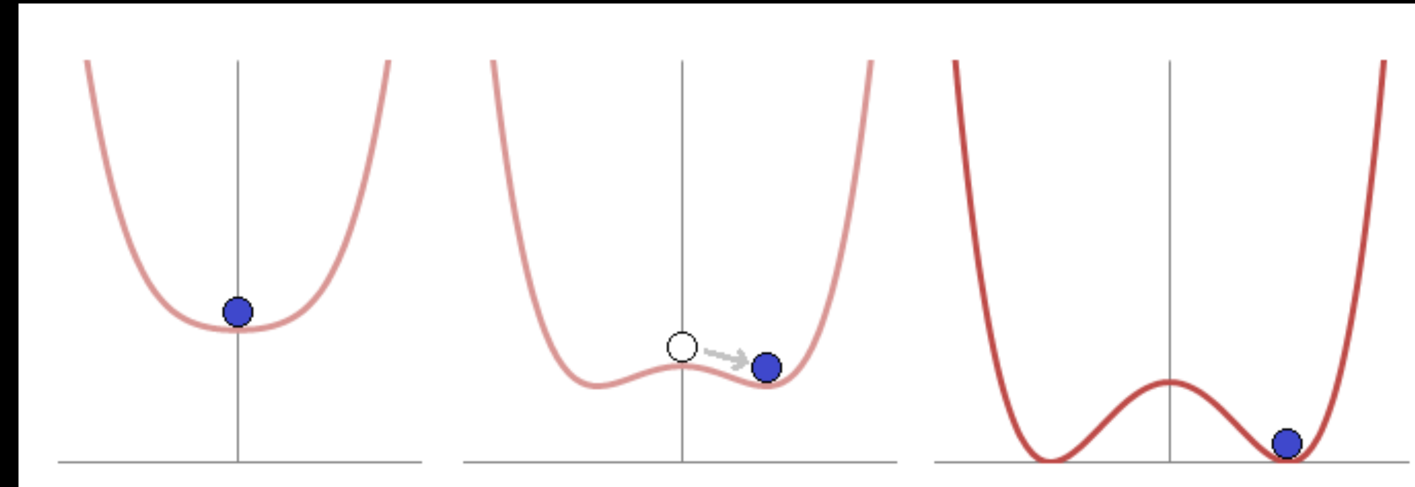
Coupling to matter leads to a fifth force which vanishes as $\varphi \to 0$

$$\vec{F}_{\rm sym} = \vec{\nabla}A(\varphi) = \frac{\varphi}{M^2}\vec{\nabla}\varphi.$$

Treating matter fields as a pressure less perfect fluid we obtain the classical Einstein frame potential

$$\tilde{V}(\varphi) = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\varphi^2 + \frac{1}{4}\lambda\varphi^4,$$

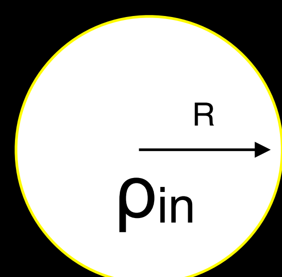
$$\tilde{V}(\varphi) = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\varphi^2 + \frac{1}{4}\lambda\varphi^4,$$



High density: $\rho/M^2 > \mu^2$:

Low density: $\rho/M^2 < \mu^2$:

Spherical source radius R:



ρ_{out}

with $\rho_{in}/M^2 > \mu^2$ and $\rho_{out}/M^2 < \mu^2$

Define:

$$m_{\rm in}^2 = \rho_{\rm in}/M^2 - \mu^2 > 0, \quad m_{\rm out}^2 = 2(\mu^2 - \rho_{\rm out}/M^2) > 0, \quad v \equiv m_{\rm out}/\sqrt{\lambda},$$

Assuming $m_{out}\, r \ll 1$ we find:

$$\varphi(r) = \frac{\pm v}{m_{\rm in} r} \begin{cases} \dfrac{\sinh m_{\rm in} r}{\cosh m_{\rm in} R}, & 0 < r < R \\ \left[\dfrac{\sinh m_{\rm in} R}{\cosh m_{\rm in} R} + m_{\rm in}(r - R)\right], & R < r. \end{cases}$$

Radiatively Stable Symmetron [Burrage, EC, Millington, PRL 2016]

Idea: rather than symmetry breaking at tree level in regions of low density, sym breaking arises radiatively in similar regions via CW mechanism.

Begin with scale invariant model minimally coupled to gravity in Jordan Frame

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(\phi)\mathcal{R} - \Lambda + \mathcal{L} + \mathcal{L}_m\right],$$

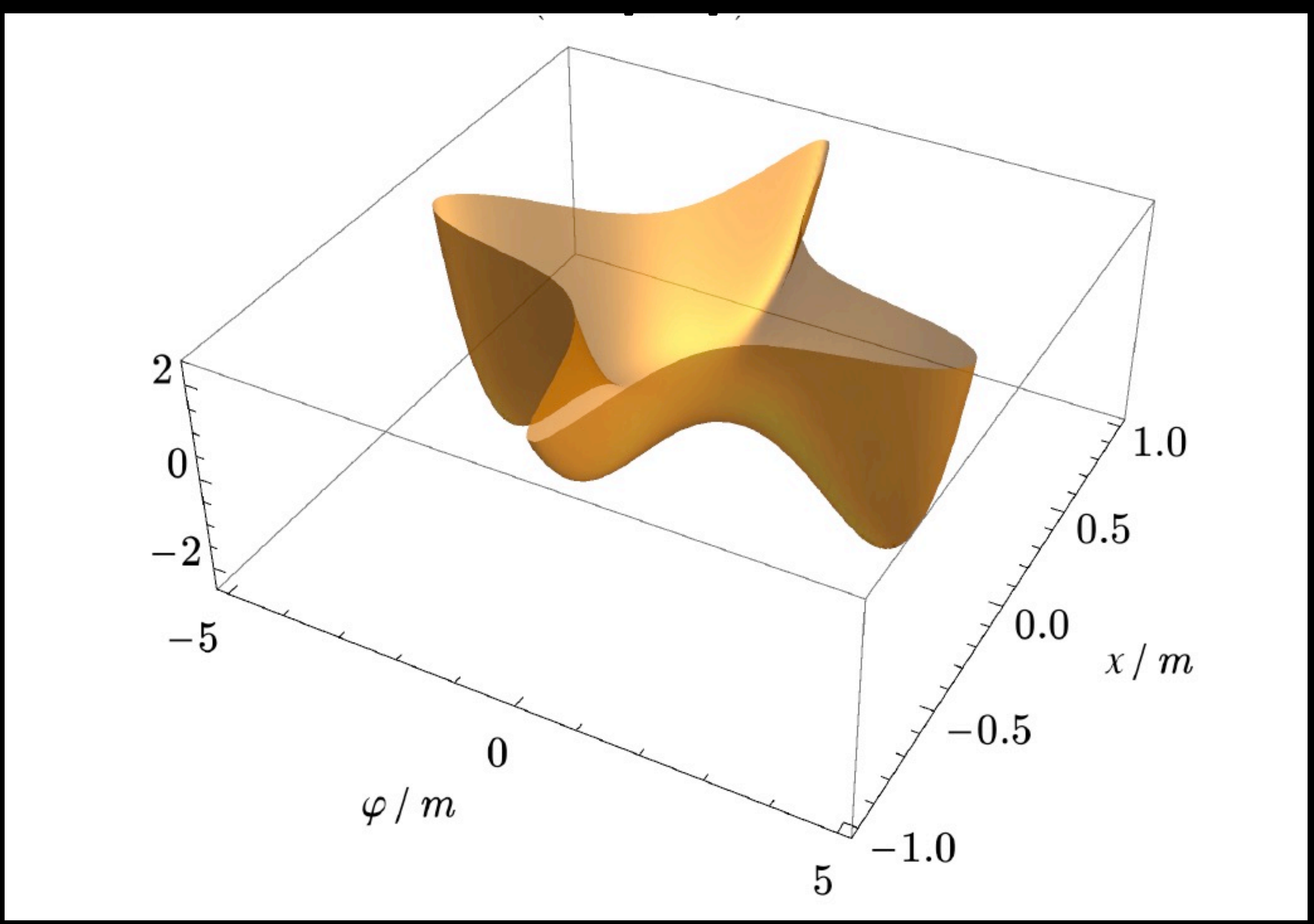
$$-\mathcal{L} = \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + \frac{1}{2}X_{,\mu}X^{,\mu} + \frac{\lambda}{4}\phi^2X^2 + \frac{\kappa}{4!}X^4,$$

$$F(\phi) = 1 + \frac{\phi^2}{M^2},$$

One Loop Effective Potential

Assuming: gravitational sector is a classical source so neglect all gravitational perturbations; neglect gradient effects so Mink bgd, constant field profiles in loop integrals; treat matter as p=0 perfect fluid

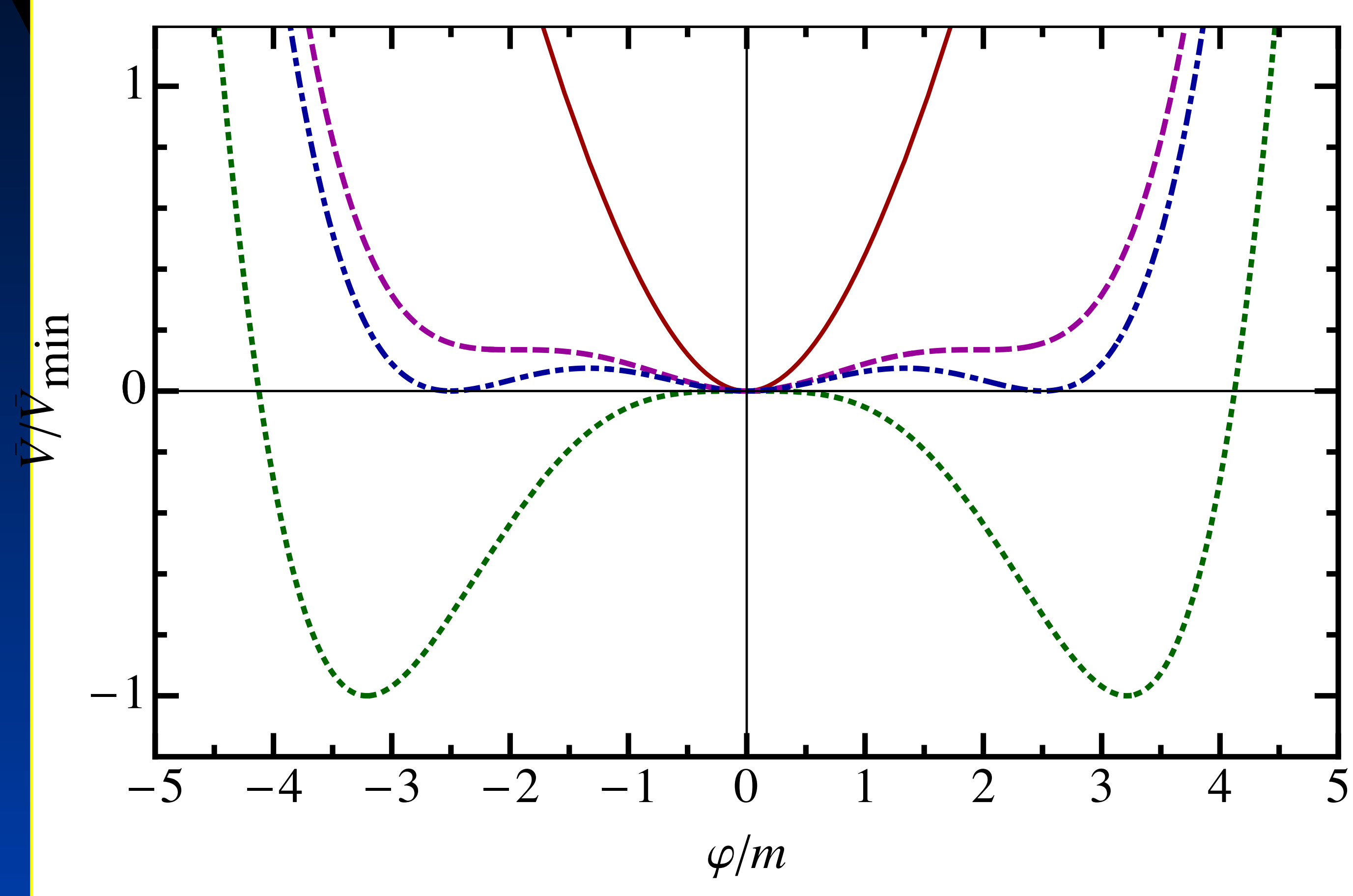
Global minimum along χ=0



[Garbrecht, Millington 2015, Burrage et al 2016]

Renormalised one loop potential for symmetron field when λ=κ

$$V(\varphi) = \frac{1}{2}F(\varphi)\mathcal{R} + \left(\frac{\lambda}{16\pi}\right)^2 \varphi^4 \left(\ln\frac{\varphi^2}{m^2} - \frac{17}{6}\right).$$



Fun dynamics - five roots, symmetry restored as density of matter increases. Potential low temperature first order phase transitions, bubbles and domain walls !

Radiatively Stable Symmetron [Burrage, EC, Millington, PRL 2016]
Renormalised one loop potential for symmetron field

Radiative screening mechanism

ρ

symmetry restored: one global minimum; fifth force screened.

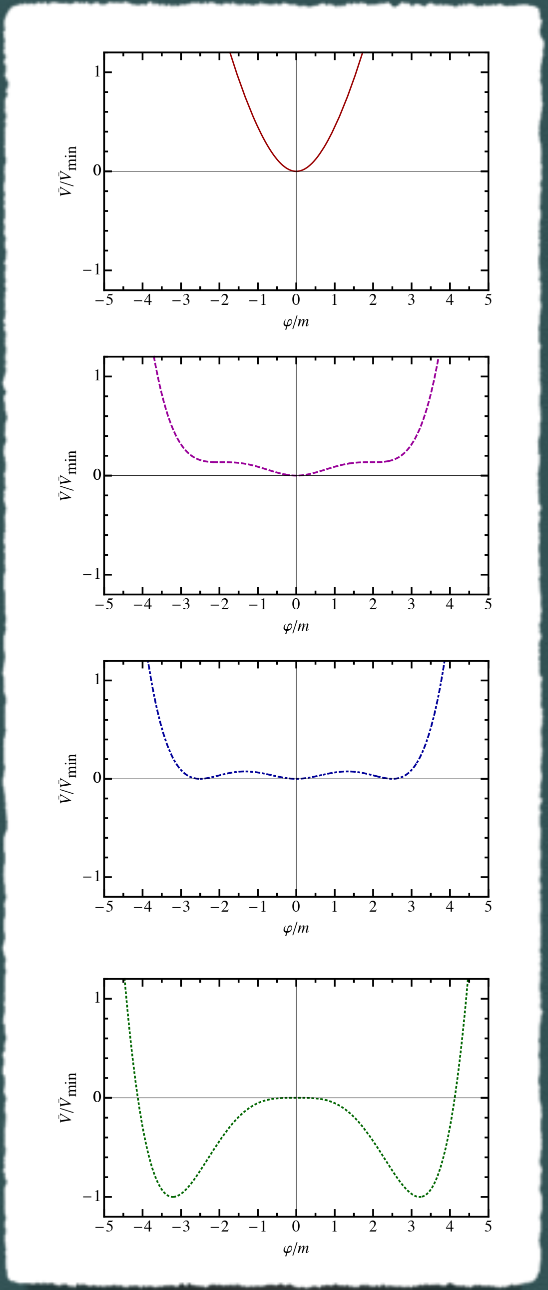
$\left(\frac{\lambda}{8\pi}\right)^2 e^{4/3}\, m^2 M^2$ **critical point**: one global minimum and two inflection points.

Tunneling to global symmetric minimum.

$\frac{1}{2}\left(\frac{\lambda}{8\pi}\right)^2 e^{11/6}\, m^2 M^2$ **degenerate point**: three degenerate global minima.

Tunneling to global symmetry-breaking minima.

symmetry broken: two global minima and a flat maximum.



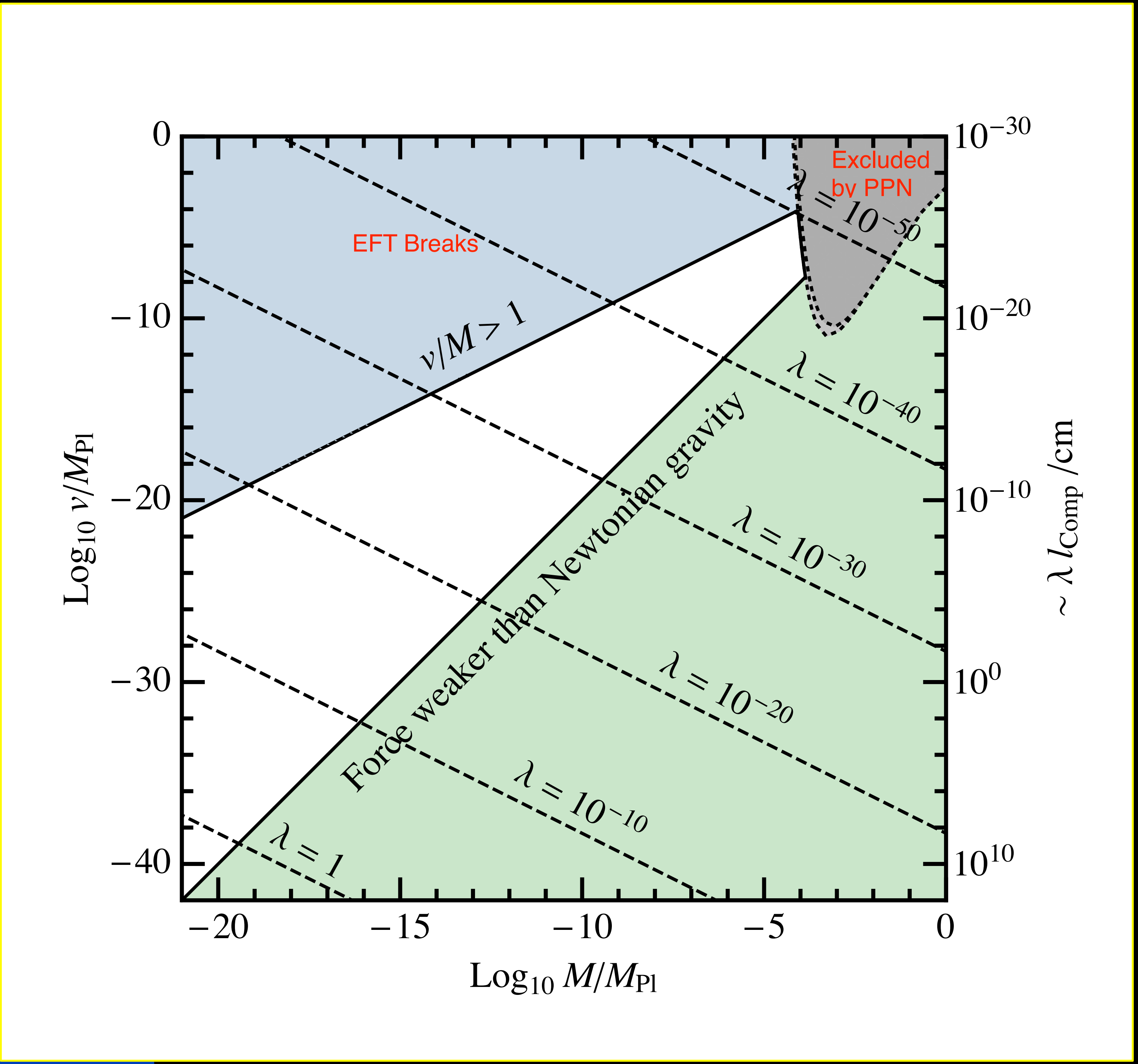
Fun dynamics - five roots, symmetry restored as density of matter increases.
Potential low temperature first order phase transitions, bubbles and domain walls !

Pete Millington

Constraints

Radiatively stable if: $\phi_{min}/M < 1$ $\lambda > (v_H/M_{Pl})^2$

Also satisfy Eöt-Wash and be in sym broken phase in current cosmological vacuum



Benchmark values : $\lambda \sim 10^{-18}$ $v \sim 10^3$ TeV $M \sim 10^{-5} M_{Pl}$

gives $l_{Comp} \sim$ 1cm — tabletop fifth force experiment scales.

Symmetrons & rotation curves - screening in galaxies [Burrage, EC & Millington 2017]

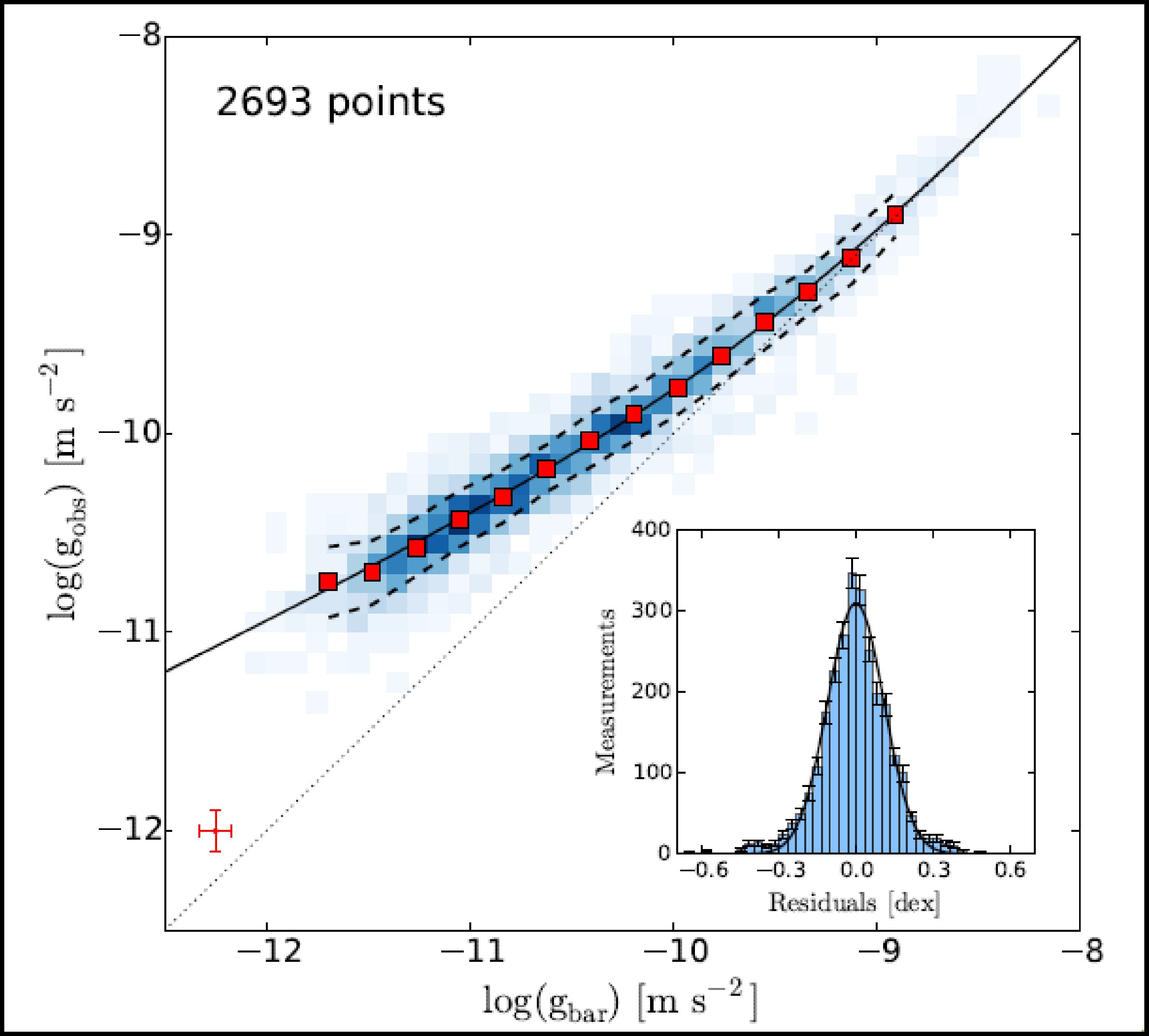
Radial acceleration relation from 153 galaxies (also known as mass discrepancy acceleration relation) [McGaugh et al PRL 2016]

$$g_{\rm obs(bar)}(r) = \frac{V^2_{\rm obs(bar)}(r)}{r} = \frac{GM_{\rm obs(bar)}(r)}{r^2}$$

Empirical fit:

$$g_{\rm obs} = \frac{g_{\rm bar}}{1 - e^{-\sqrt{g_{\rm bar}/g_{\dagger}}}}$$

where $g_{\dagger} = 1.20 \pm 0.02(\mathrm{rand}) \pm 0.24(\mathrm{sys}) \times 10^{-10}\ \mathrm{ms}^{-2}$.



Explanations include: MOND [Milgrom 2016], MOG [Moffat 2016], Emergent Gravity [Verlinde 2016], Dissipative DM [Keller & Waldsley 2016], Superfluid DM [Hodson et al 2016], some weird thing called ΛCDM [Ludlow et al PRL 2017] + us + others …

Symmetron explanation [Burrage, EC and Millington 2017]

$$g_{\rm obs} = \frac{g_{\rm bar}}{1 - e^{-\sqrt{g_{\rm bar}/g_\dagger}}}$$

$$g_{\rm obs(bar)}(r) = \frac{V^2_{\rm obs(bar)}(r)}{r} = \frac{GM_{\rm obs(bar)}(r)}{r^2}$$

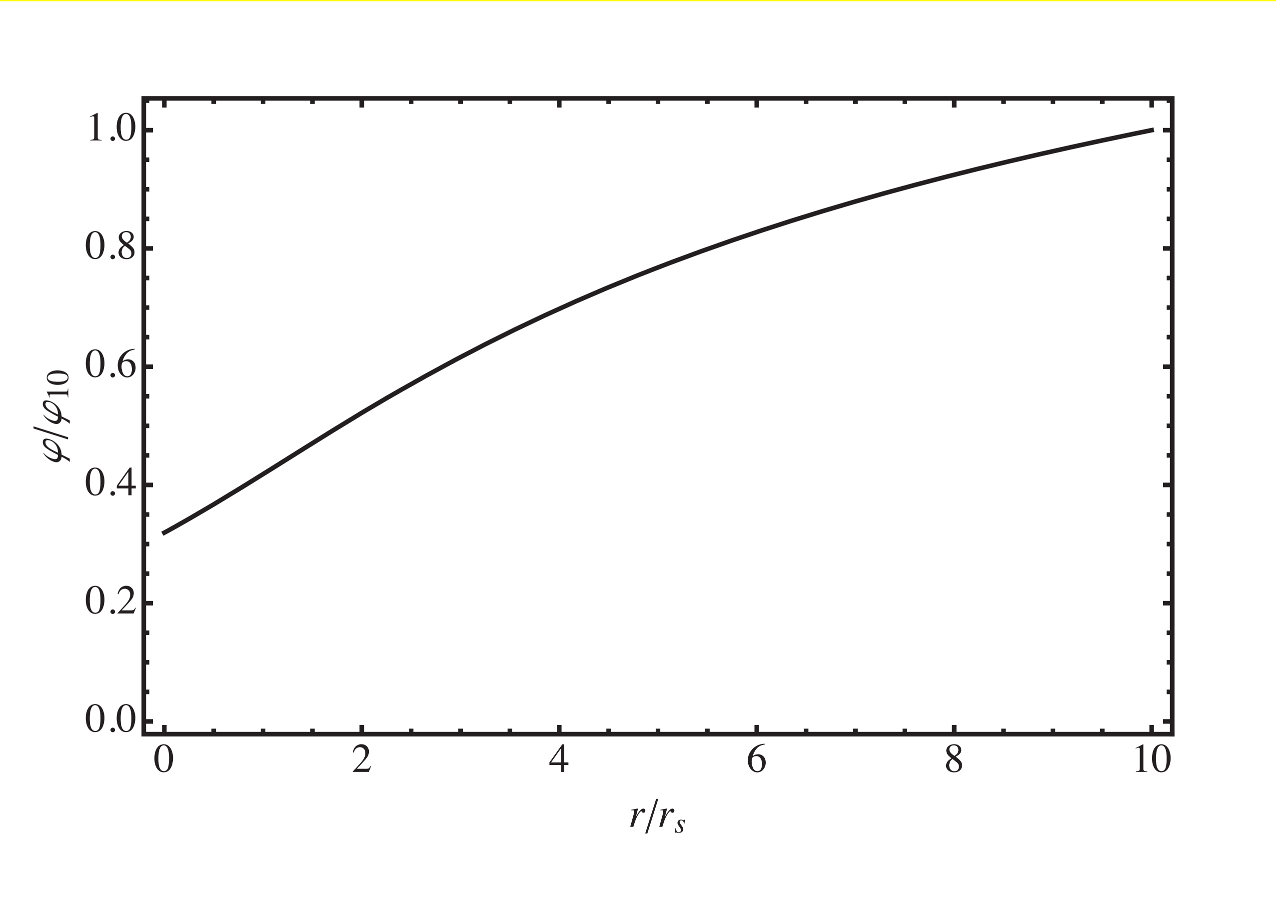
Rotation curve explained if symmetron profile satisfies:

$$g_{\rm sym}(r) = \frac{c^2}{2}\frac{d}{dr}\left(\frac{\varphi(r)}{M}\right)^2 = \frac{g_{\rm bar}(r)}{e^{\sqrt{g_{\rm bar}(r)/g_\dagger}} - 1}$$

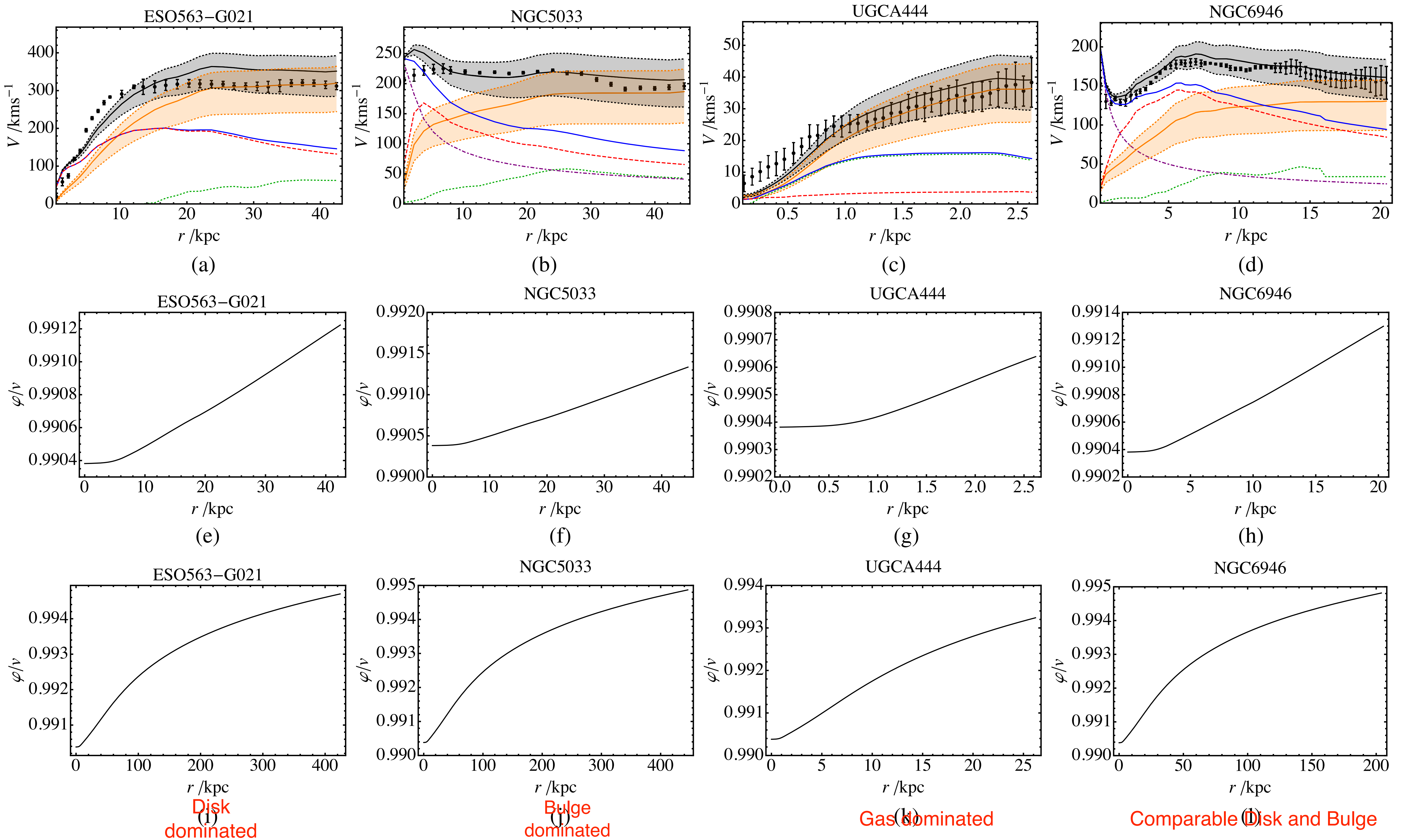
Assuming an exponential disc profile for the galaxy $\Sigma(r) = \Sigma_0 e^{-r/r_s}$ we obtain:

$$\mathcal{M}_{\rm bar}(r) = \mathcal{M}_0 \int_0^r \frac{\mathrm{d}r'}{r_s}\frac{r'}{r_s} e^{-r'/r_s}$$
$$= \mathcal{M}_0\left[1 - e^{-r/r_s}\left(1 + \frac{r}{r_s}\right)\right],$$

Hence the required symmetron profile to explain observed accn without dark matter



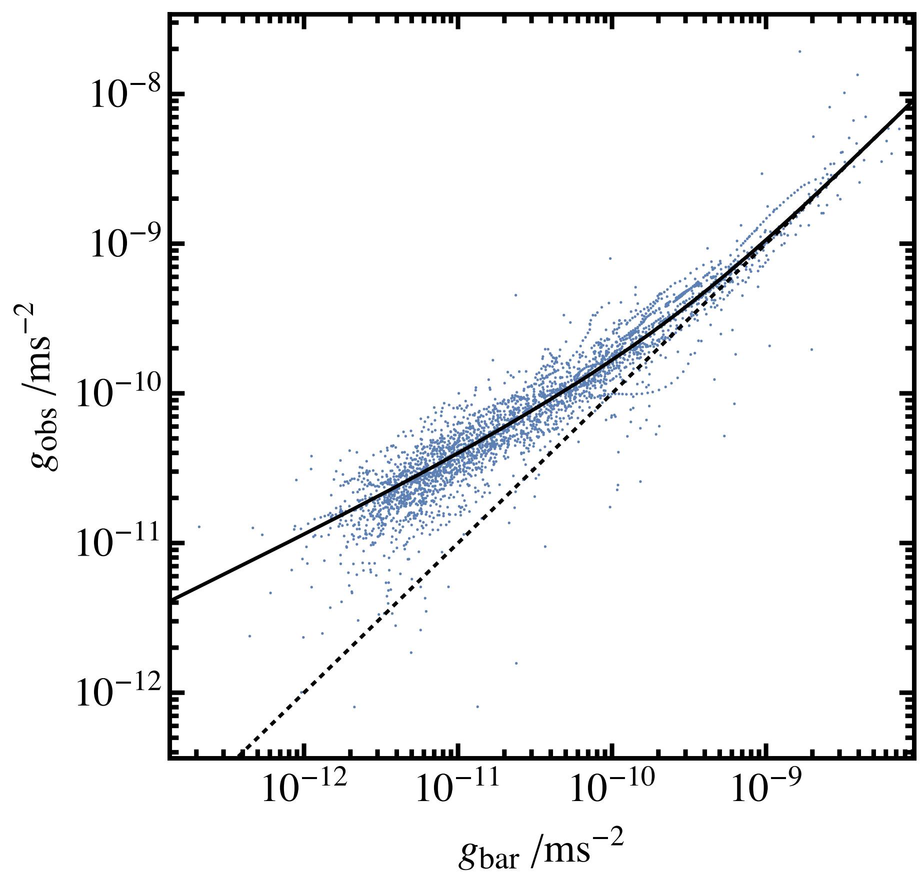
Real galaxies in the SPARC dataset [Burrage, EC & Millington 2017, SPARC, Lelli et al 2016]



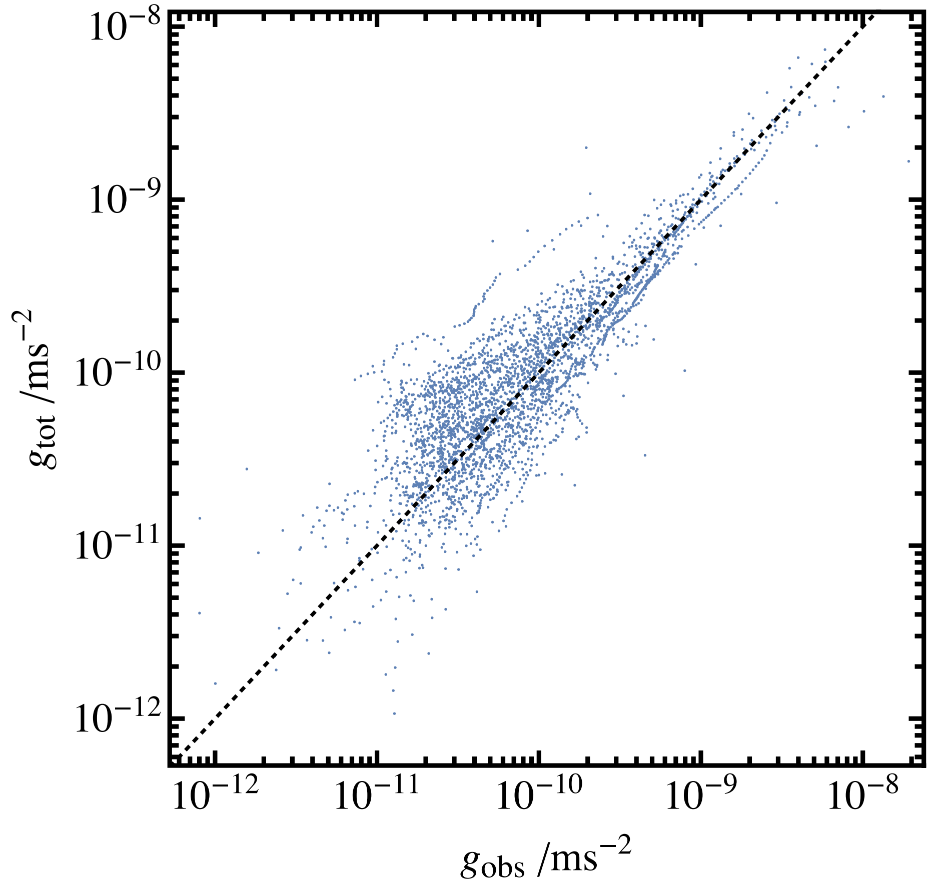
$$M = M_{\rm Pl}/10 \text{ and } \bar{\rho}_0 = 1\ M_\odot\,{\rm pc}^{-3},\ v/M = 1/150, \text{ and } \mu = 3\times10^{-39}\ {\rm GeV}:$$

Comparison with real data

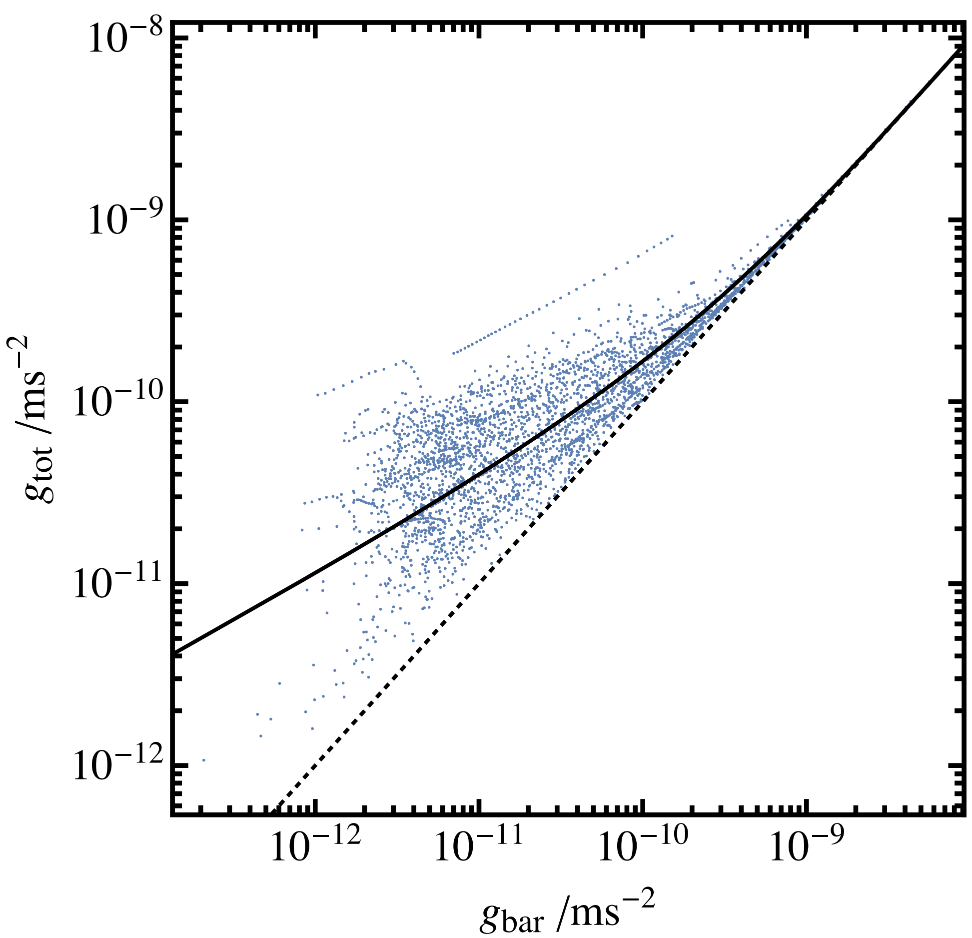
[Burrage, EC and Millington 2017]



(a) observed versus baryonic



(b) symmetron prediction versus observed



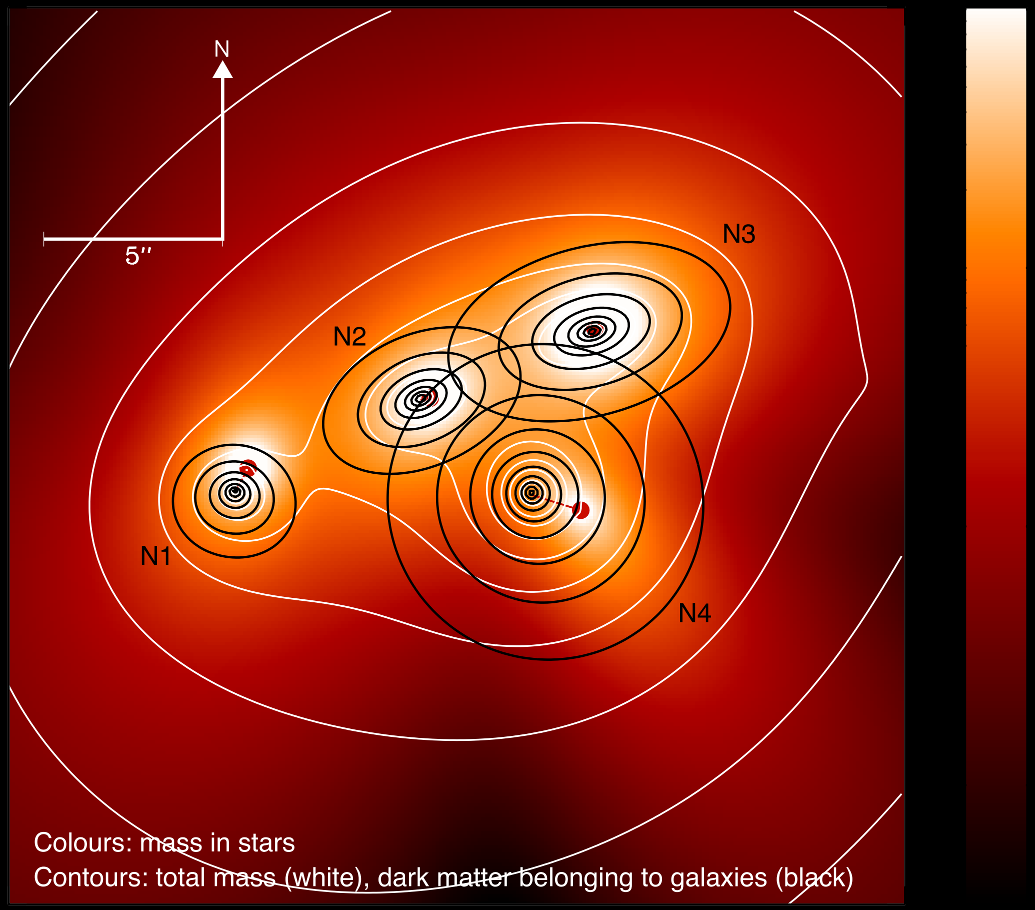
(c) symmetron prediction versus baryonic

Recent result — this radial acceleration relation (RAR) is the fundamental correlation governing the radial (in-disk) dynamics of late type galaxies. It can not be tightened - it sounds to me as if it is an important relation for any model to predict.

[R. Stiskalek and H. Desmond — arXiv:2305.19978017]

Other interesting aspects [Burrage, EC and Millington 2017]

`Kink-kink' interactions of the symmetron profiles, as well as the response of the symmetron field to the change in the gas distribution may produce an offset between the stellar and DM components in colliding systems such as observed in Abell 2827



[Taylor et al 2017]

Disk Stability - known that baryonic component alone insufficient to stabilise disks of galaxies to barlike modes, spherical DM halo fixes that.

Energy stored in symmetron field has similar stabilising effect. Requires constraint

$$\frac{\mu}{\mathrm{GeV}} \gtrsim \frac{2\times 10^{-41}}{\sqrt{\alpha n}}\left(\frac{v}{M_{\mathrm{Pl}}}\right)^{-1},$$

Modified Gravity models can couple to the standard model particles - we can use particle collisions to look for fifth forces [Brax et al (2016), Aaboud et al (2019), S.Sevillano Munoz et al (2022)]

Brans Dicke

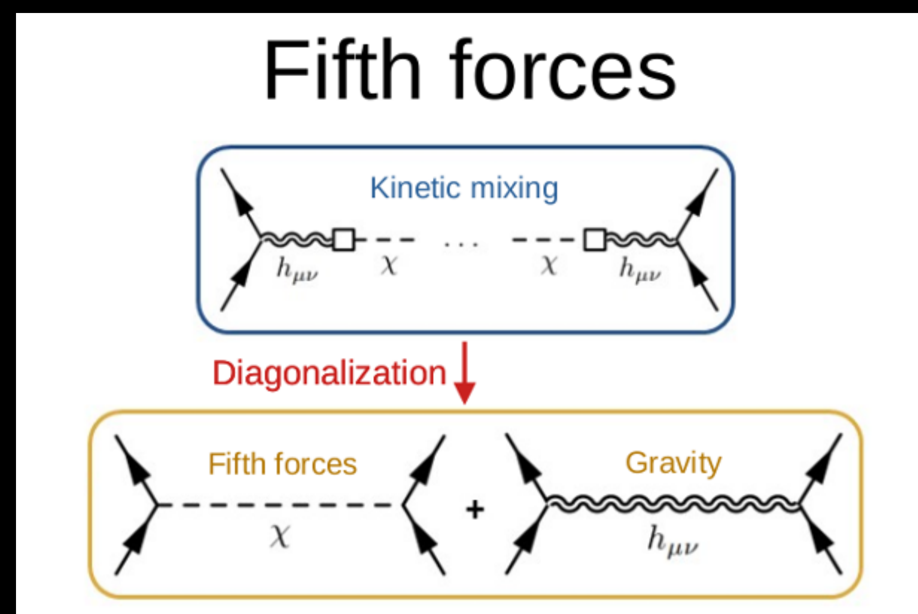
$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{F(X)}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu X\partial_\nu X - U(X) + \mathcal{L}_{\mathrm{m}}\{\psi_i, \phi_i, g_{\mu\nu}, \ldots\}\right]$$

Expand around Mink space

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \cdots$$
$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu},$$

Fifth forces leak into the system via a kinetic mixing with gravity

Modified theory of gravity
+
Standard Model



General Relativity
+
Beyond the Standard Model

[credit: Sergio Sevillano Munoz]

Once we have BSM description we can calculate from quantum corrections the scattering amplitudes. But they are long and tedious to do. They require: expanding of gravity, canonical normalisation, expanding around non-trivial vevs, obtaining the kinetic mixings to graviton and then mass mixings

In flat space, particle phenomenologists use FeynRules - Mathematica package that goes from a Lagrangian gives Feyn Rules and phenomenology.

What about Gen Rel plus BSM ?

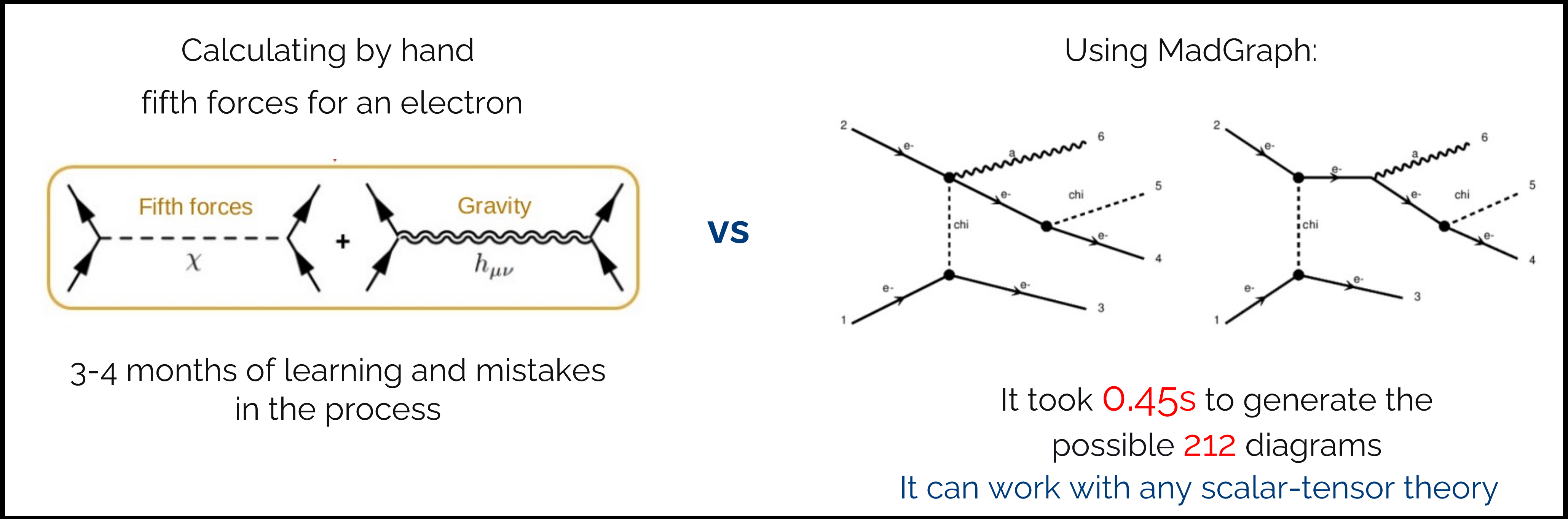
Enter FeynMG written primarily by Sergio Sevillano Munoz

A sub package of Feyn Rules

[S.Sevillano Munoz et al, arXiv:2211.14300]

Allows user to insert new scalar dof and any grav theory. Can then perform the necessary operations to calculation the BSM description.

Test scalar-tensor theories in colliders



[credit: Sergio Sevillano Munoz]

Modifying Gravity rather than looking for Dark Energy - non trivial

Any theory deviating from GR must do so at late times yet remain consistent with Solar System tests. Potential examples include:

•f(R), f(G) gravity -- coupled to higher curv terms, changes the dynamical eqns for the spacetime metric. Need chameleon mechanism [Starobinski 1980, Carroll et al 2003, Joyce et al 2015…]

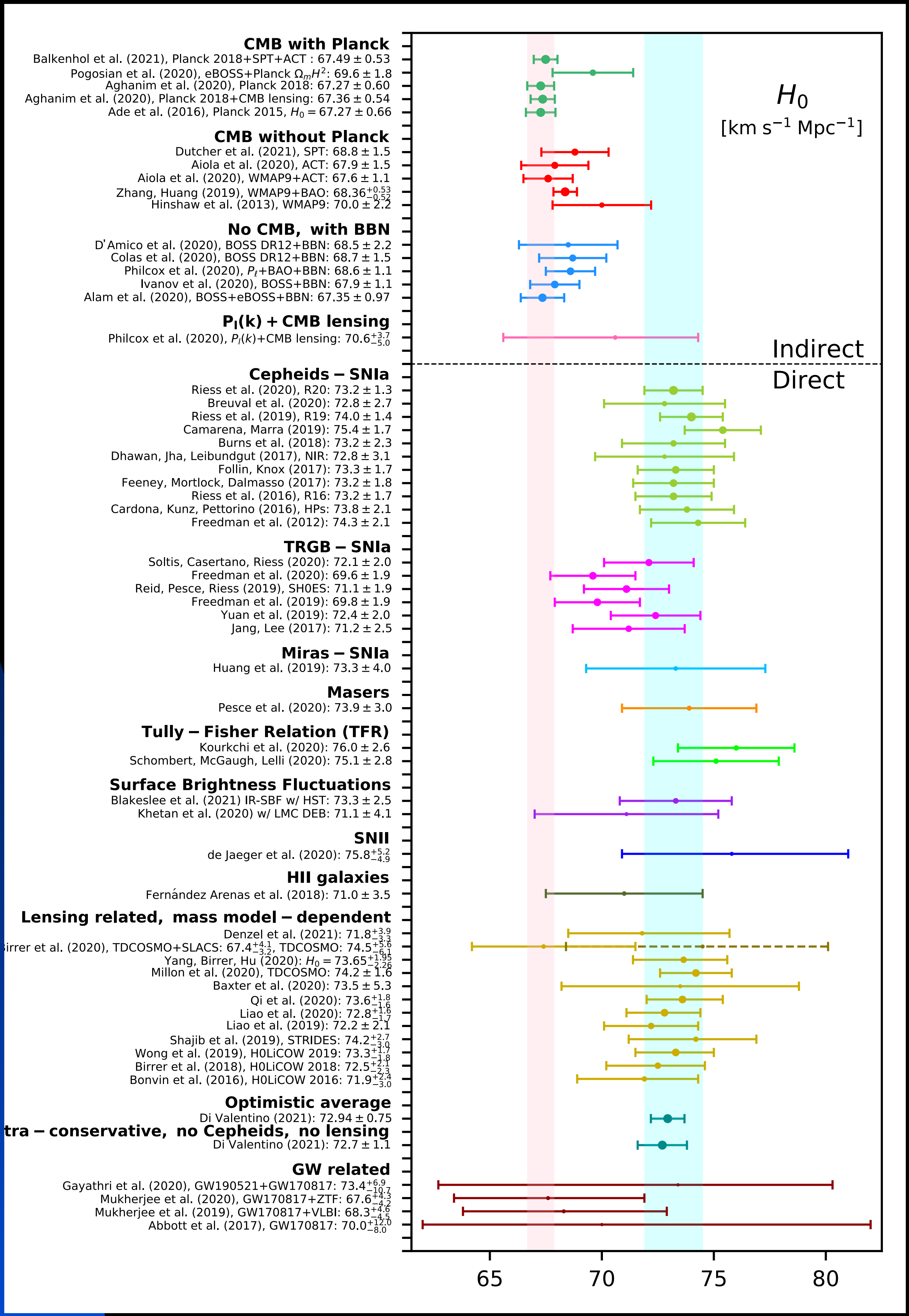
- Modified source gravity -- gravity depends on nonlinear function of the energy.
- Gravity based on the existence of extra dimensions -- DGP gravity

We live on a brane in an infinite extra dimension. Gravity is stronger in the bulk, and therefore wants to stick close to the brane -- looks locally four-dimensional.

Tightly constrained -- both from theory [ghosts] and observations

- Scalar-tensor theories including higher order scalar-tensor lagrangians -- examples include Galileon models
- Massive gravity theories dRGT [de Rham et al 2011…]

Return to Hubble tension - local v global - Early Dark Energy



[Di Valentino et al 2019]

Lots of approaches being taken to determine H_0

H_0=67.4±0.5 km s^{-1} Mpc^{-1} (Planck) v H_0=73.2±1.3 km s^{-1} Mpc^{-1} (SHOES)

Assuming the tension is a sign of new physics - many theoretical approaches.

Most of them make use of the standard ruler imprinted in the cmb maps - the Sound Horizon - the distance sound waves could propagate in a plasma from t=0 to t=1100.

Measure the angular size on the cmb, so have a distance and redshift to cmb.

One approach - use new physics early on to reduce the physical size of the sound horizon, hence decrease the distance we infer to the cmb (rem we measure the angular separation) - implying the H_0 we infer increases !

$$r_s^* = \int_{z_*}^{\infty} \frac{dz}{H(z)} c_s(z) \quad \rightarrow D_A \sim \frac{r_s^*}{\theta_s^*} \quad \rightarrow H_0$$

Recall $D_A \sim 1/H_0$

So the idea, have new physics early on, alter the energy density, change H(z). Concentrate here on EDE but also possible to have late time modifications to resolve the tension [Zhao et al, Nature Ast 2017; Wang et al, Astro J. Lett 2018]

The particle cosmologists tool of choice — a (pseudo) scalar field - ϕ

ϕ initially frozen on its potential c/o Hubble friction - like DE with w=-1

As H~m, rolls down potential and oscillates.

Need late time w>0, so EDE energy density decays faster than matter.

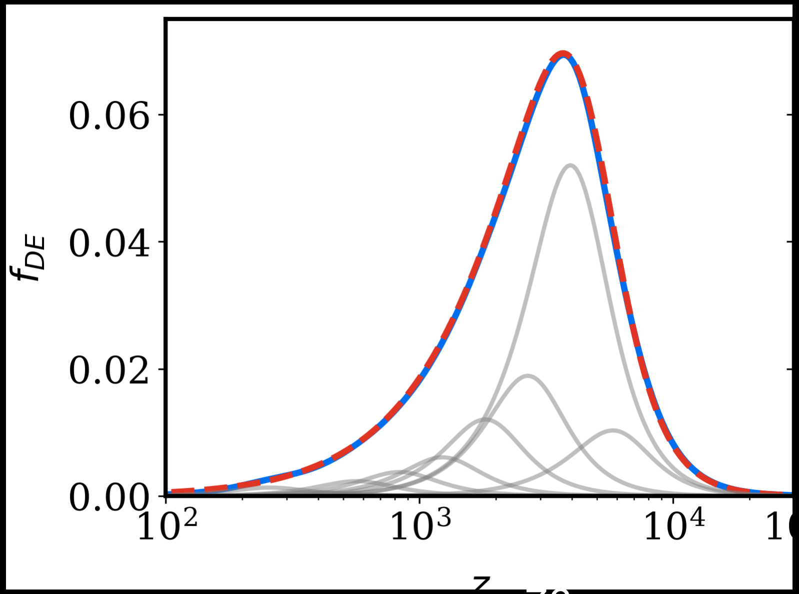
Three EDE examples:

axion EDE [Poulin et al, PRL 2019]

$$V(\phi) = m^2 f^2 (1 - \cos(\phi/f))^n, \quad m \sim 10^{-27} eV, \;\; f \sim 10^{26} eV, \;\; n = 3$$

Near minimum - eos - $w_\phi = \dfrac{n-1}{n+1} = \dfrac{1}{2} > 0$

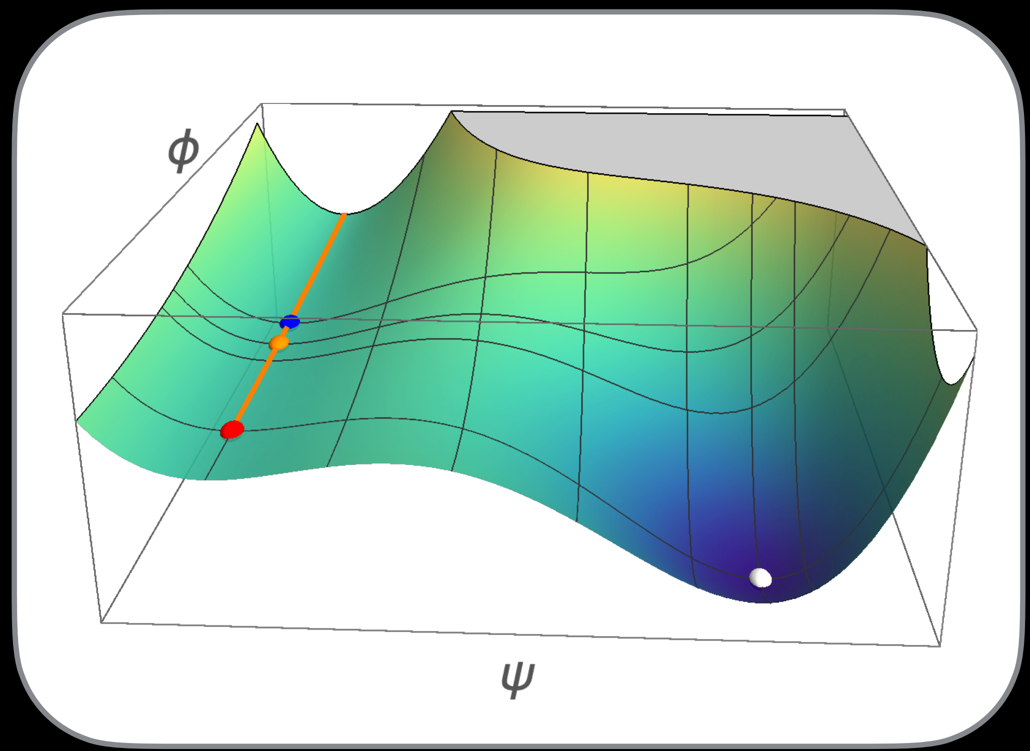
Note occurs around matter radiation equality



[Moss et al, 2021]

New EDE — driven by a first order phase transition [Niedermann and Sloth, PRD 2021]

$V(\psi,\phi) = \frac{\lambda}{4}\psi^4 + \frac{1}{2}\beta M^2\psi^2 - \frac{1}{3}\alpha M\psi^3 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\tilde{\lambda}\phi^2\psi^2,$ ψ is tunneling field, ϕ trigger field



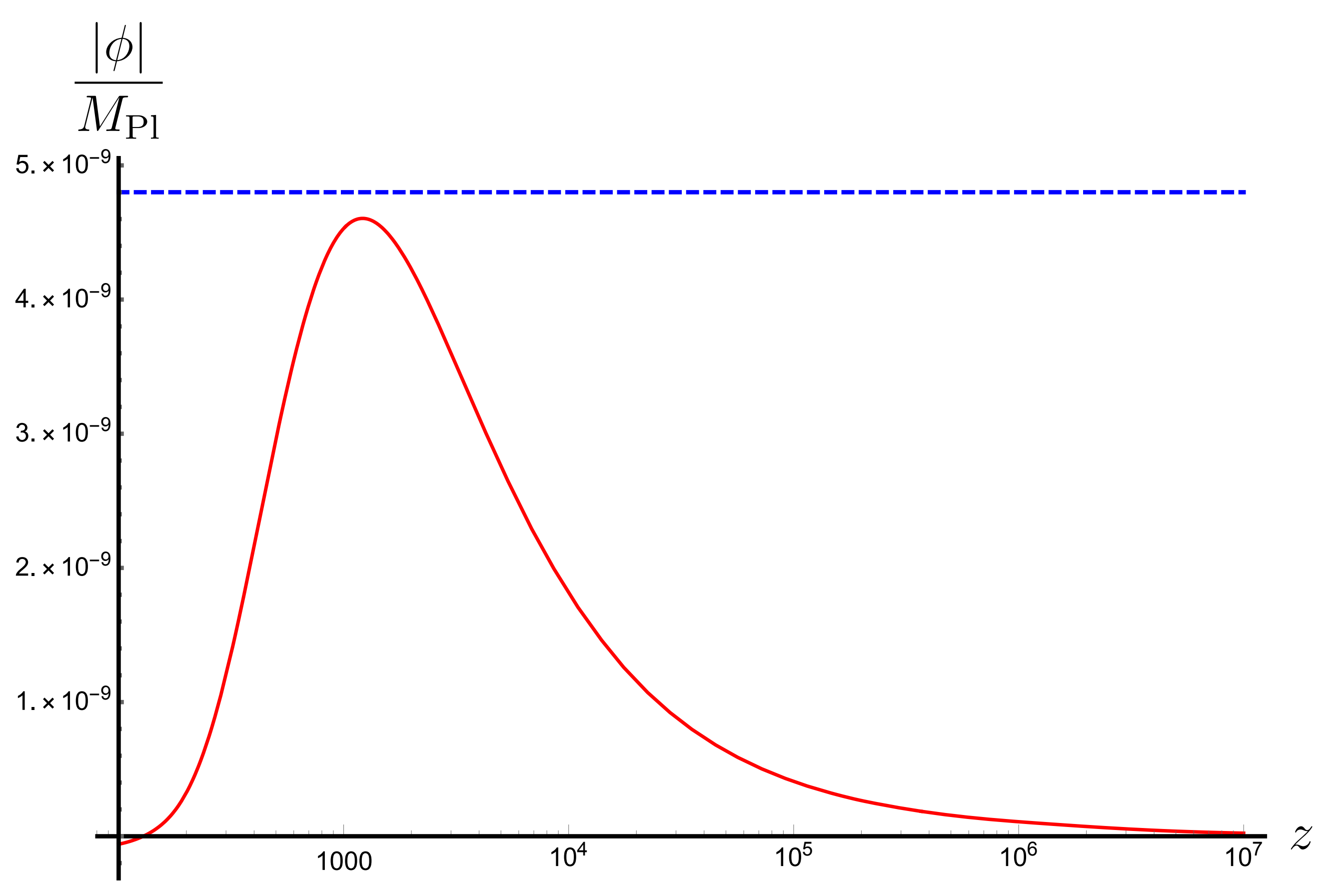
False vacuum decay of ψ from cosm const source to decaying field with const eos w>0 around eV scale.

$H_0 = 71.4 \pm 1.0 kms^{-1}Mpc^{-1}$, with decay at $z_* = 4920^{+620}_{-730}$ and with $f_{NEDE} = 0.126^{+0.03}_{-=.03}$

Massive neutrino driven EDE — [Sakstein and Trodden, PRL 2020, for earlier related work see Amendola et al 2008]

Idea: If EDE field ϕ is coupled to neutrinos with strength β, it receives a large injection of energy around the time that neutrinos become non-relativistic, which is when their temp ~ their mass, just before matter-rad equality.

Nice feature - neutrino decoupling provides trigger for EDE by displacing ϕ from min of it's potential $V(\phi) = \lambda\phi^4/4$.



$m_\nu = 0.5eV$, $\beta = 4 \times 10^{-4}$, $\lambda = 10^{-75}$

For approaches resolving the Hubble tension using impact of screened fifth forces on the distance ladder see [Desmond et al, PRD 2019, Baker et al, Rev Mod Phys 2021]

More general approach to DE - spike model

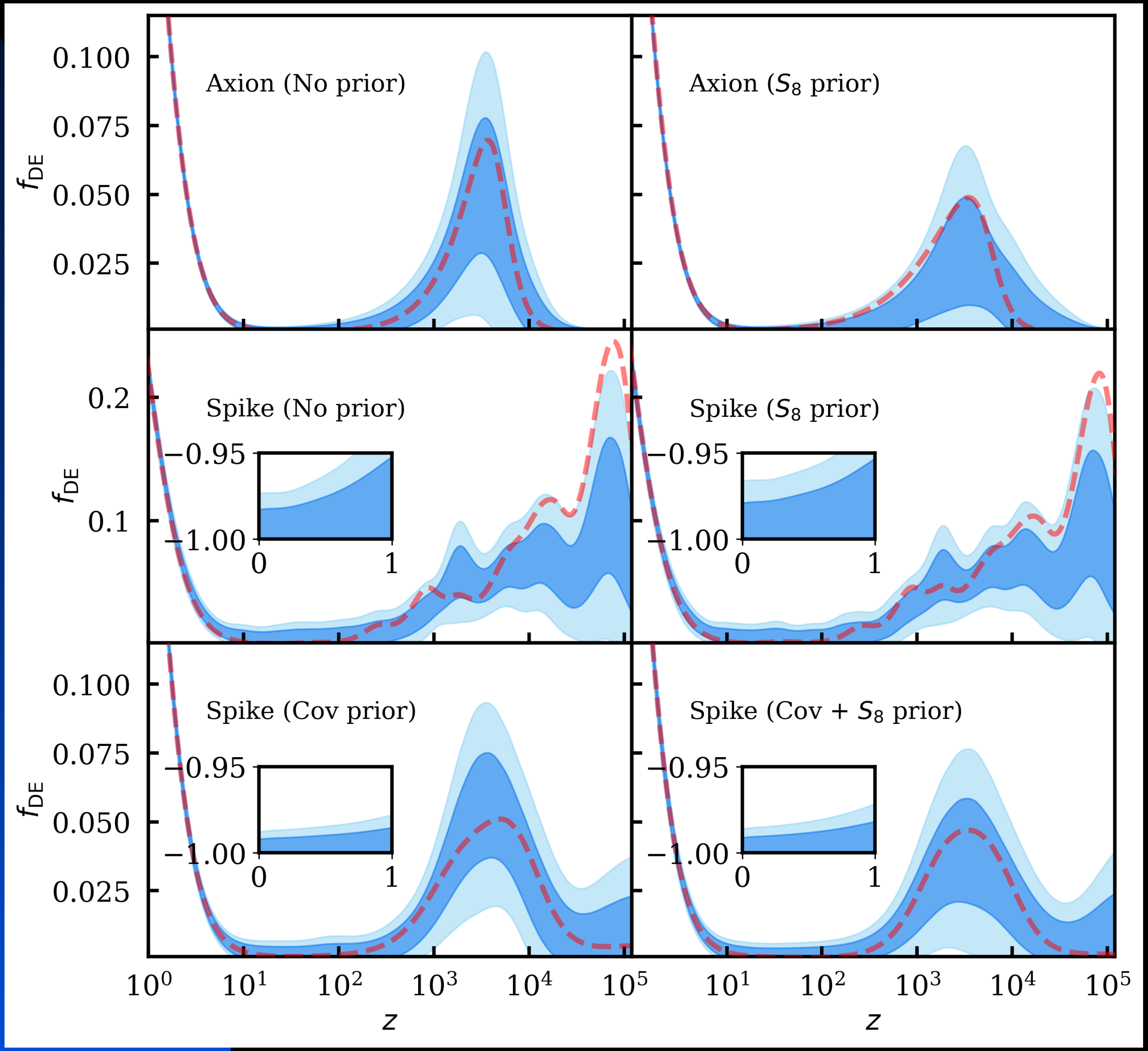
[Moss, EJC, Bamford and Clarke 2021 - for similar approach see also Lin et al 2019 and Hojjati et al 2013]

Model DE by perfect fluid with series of bins in energy density, with eos $-1 \leq w \leq 1$. Combine with cmb, BAO and local H_0 data obtain improvement over ΛCDM with DE contributing significantly between $z \sim 10^4 - 10^5$ and $c_s^2 \sim 1/3$.

$\Delta\chi^2 = -10.8$

$\Delta\chi^2 = -34.4$

$\Delta\chi^2 = -14.0$



inc DES S_8 prior

$S_8 = 0.776 \pm 0.017$

A few details

Parameter	ΛCDM	Axion Fluid	Spike	Spike (+ Covariance Prior)
H_0	68.48 ± 0.32 (68.44)	$70.03^{+0.81}_{-1.1}$ (70.95)	$72.25^{+0.93}_{-1.2}$ (73.59)	$70.9^{+1.0}_{-1.3}$ (71.29)
Ω_m	0.3001 ± 0.0041 (0.3006)	$0.2975^{+0.0044}_{-0.0049}$ (0.2950)	$0.3027^{+0.0062}_{-0.0055}$ (0.2978)	0.2948 ± 0.0054 (0.2952)
n_s	0.9729 ± 0.0030 (0.9728)	$0.9810^{+0.0060}_{-0.0073}$ (0.9834)	0.9703 ± 0.0083 (0.9636)	$0.9805^{+0.0081}_{-0.0063}$ (0.9833)
c_s^2	-	-	$0.334^{+0.021}_{-0.039}$ (0.3125)	$0.401^{+0.10}_{-0.090}$ (0.4153)
w_n	-	$0.475^{+0.087}_{-0.18}$ (0.3523)	-	-
z_c	-	10240^{+2000}_{-8000} (5460)	-	-
$f_\mathrm{EDE}(z_\mathrm{c})$	-	$0.0272^{+0.0097}_{-0.021}$ (0.03609)	-	-
S_8	0.8075 ± 0.0077 (0.8073)	0.814 ± 0.010 (0.8133)	0.8182 ± 0.0099 (0.8183)	$0.812^{+0.011}_{-0.0094}$ (0.8151)
χ^2_H0	15.5	4.7 (**-10.8**)	0.1 (**-15.4**)	3.7 (**-11.8**)
χ^2_Planck	1017.0	1020.0 (3.0)	1009.2 (**-7.8**)	1018.3 (1.3)
χ^2_ACT	240.7	235.3 (**-5.4**)	225.3 (**-15.4**)	234.4 (**-6.3**)
χ^2_S8	3.4	4.8 (1.4)	6.2 (2.8)	5.3 (1.9)
χ^2_data	2316.7	2305.9 (**-10.8**)	2281.4 (**-35.4**)	2302.8 (**-14.0**)
χ^2_prior	0.0	0.0	0.0	3.8
$\Delta \ln E$	-	-	-	5.0

The high z behaviour of EDE changes the radiation driving envelope that modifies the high *l* CMB power spectrum, potentially alleviating the tension between Planck and ACT data -see [Hill et al 2021]

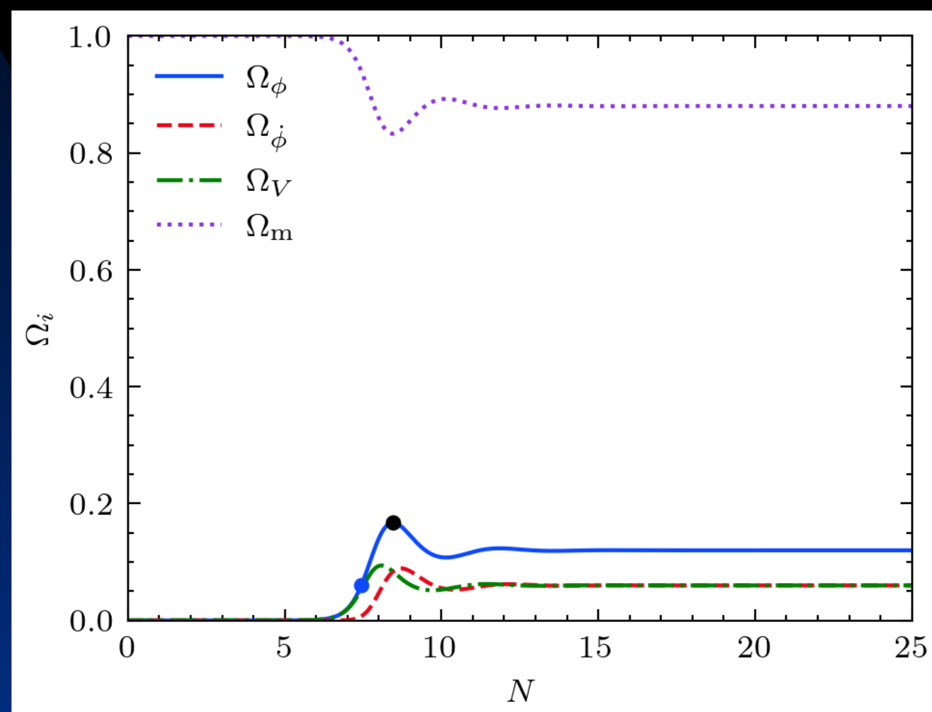
Note - none of these models really address the S_8 tension - cmb v lss

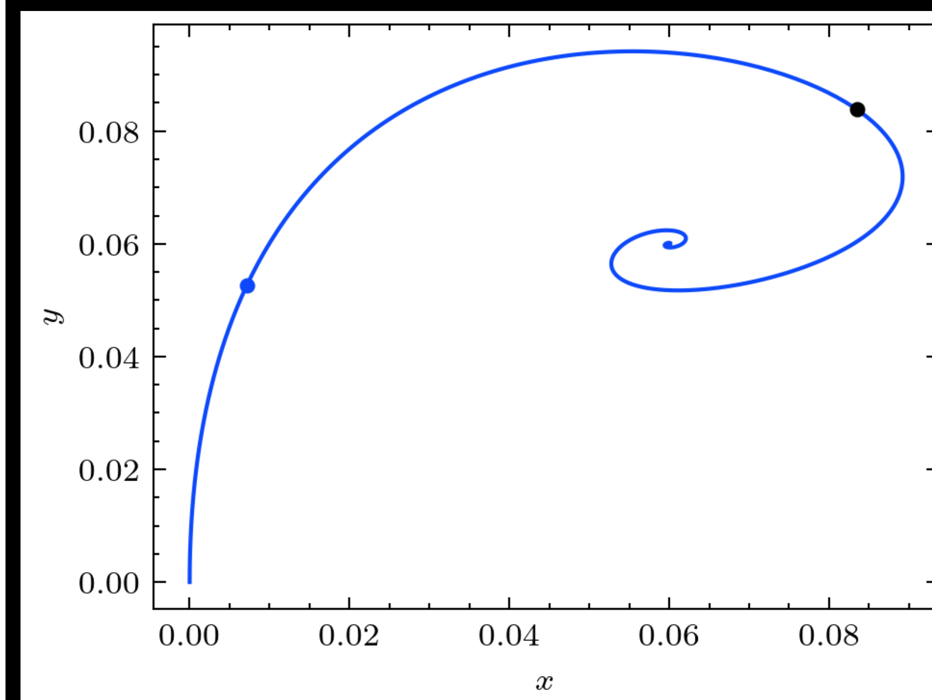
Once the 33 spike parameters inc, find moderate Bayesian evidence for EDE [following the approach developed in [Crittendon et al, JCAP 2012; Zhao et al, PRL 2012]]

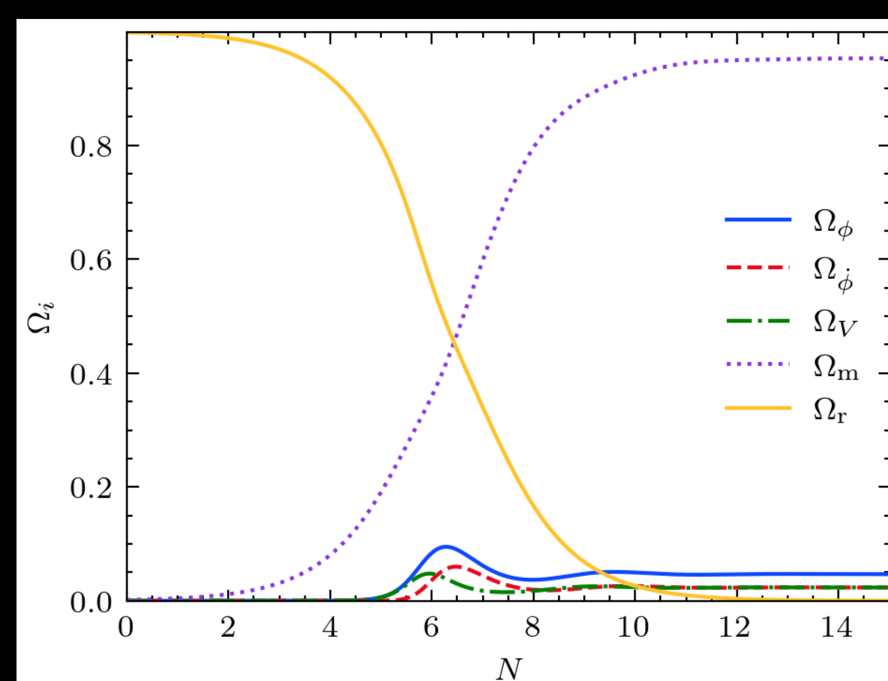
A nice feature of scaling solutions - they tend to generate bumps in their energy density as they approach their attractor solutions

$$H^2 = \frac{\kappa^2}{3}\left(\rho_r + \rho_m + \rho_{cc} + \frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \qquad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H} \quad : \quad \Omega_\phi = \frac{\kappa^2\rho_\phi}{3H^2} = x^2 + y^2 \quad : V(\phi) = V_0 \exp(-\kappa\lambda\phi)$$







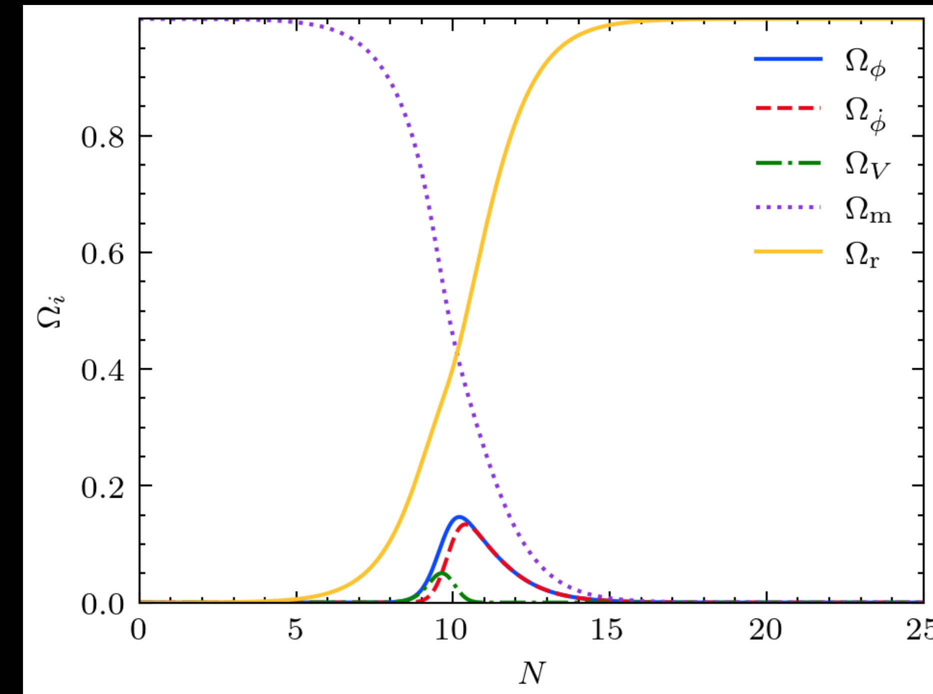
Quintessence peak around matter-radiation equality

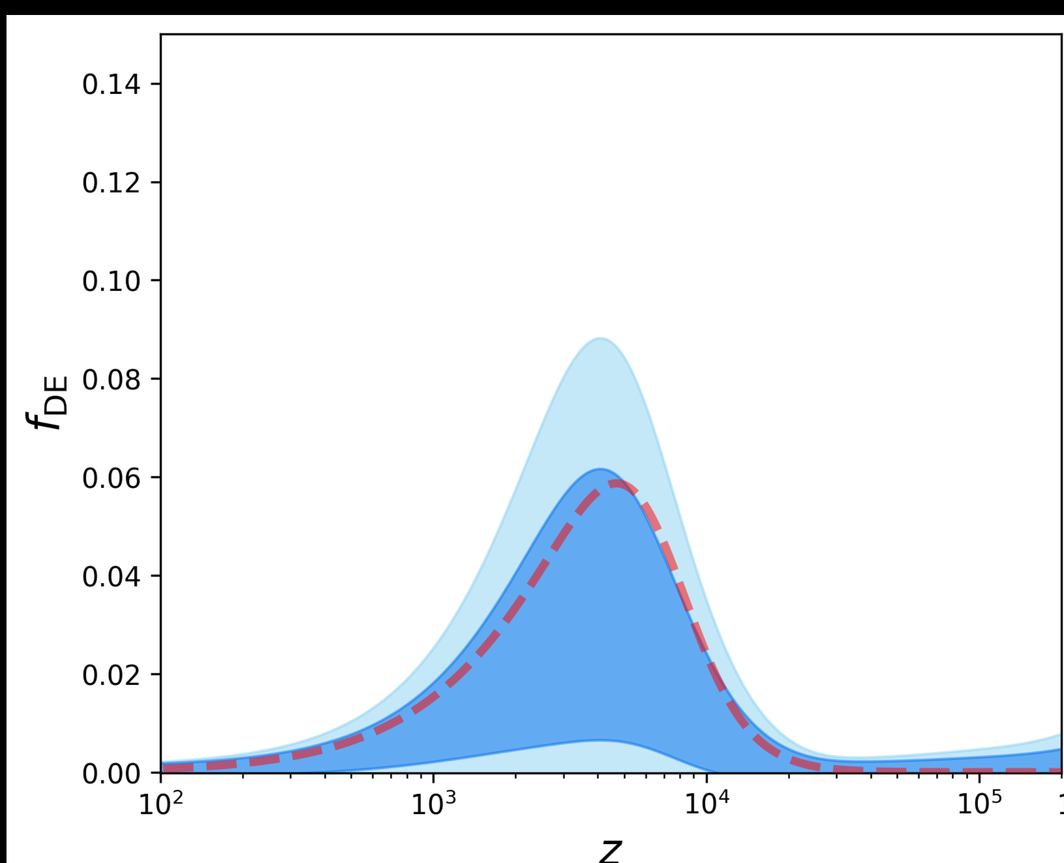
[EJC, A. Moss, S. Sevillano Muńoz, J.D. White 2023]

Also for K-essence type behaviour, as long as there is an attractor it wants to go to.

$$\mathcal{L} = \frac{X(\dot\phi)^n}{M^{4(n-1)}} - V(\phi) \quad : \quad X(\dot\phi) \equiv \tfrac{1}{2}\dot\phi^2 \quad : \quad V(\phi) = V_0 \exp\left(-\kappa\lambda\phi\right)$$

n=2

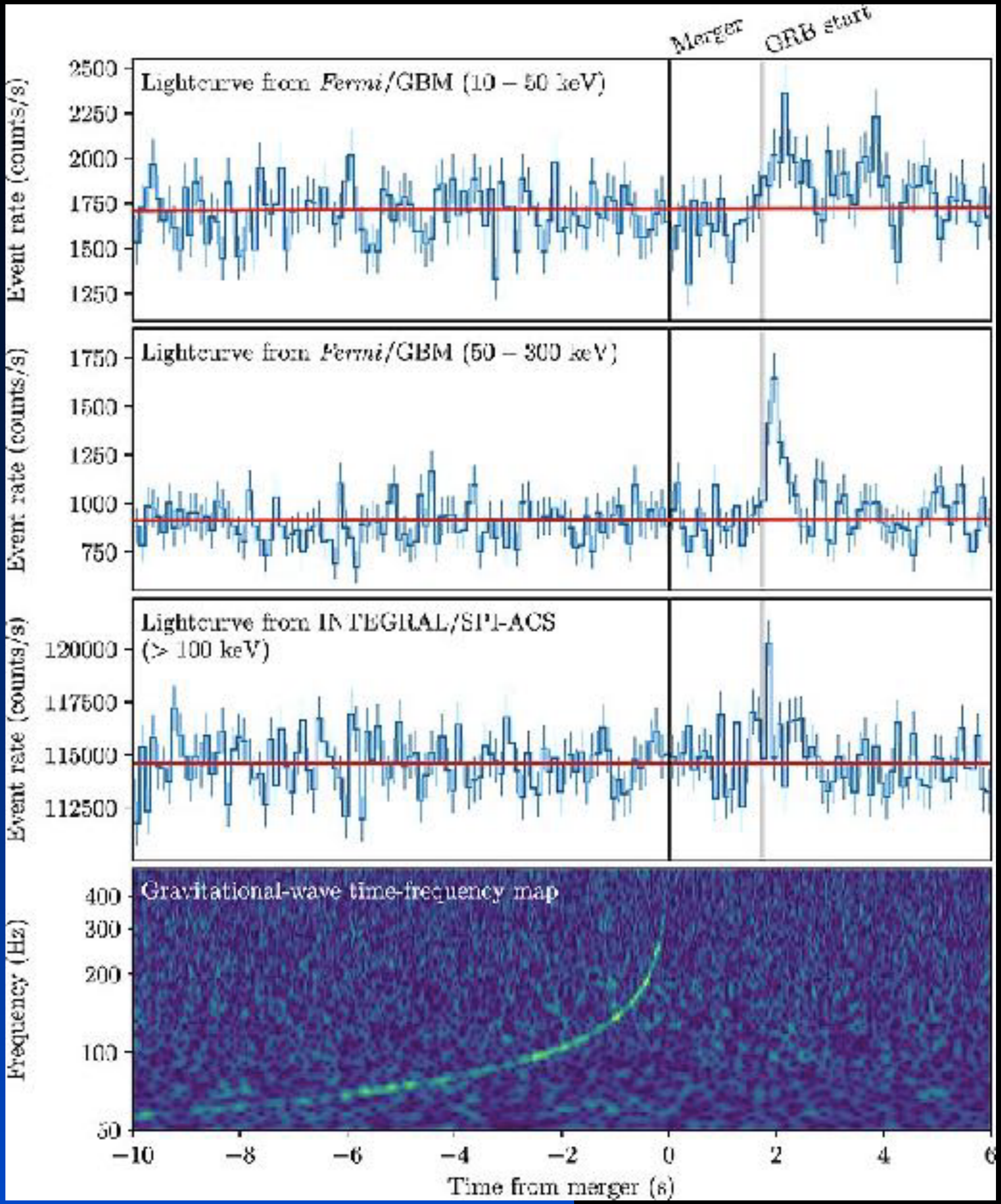




Parameter	ΛCDM	K-essence	Fang
H_0	68.16 ± 0.34 (68.17)	69.6 ± 1.1 (70.45)	$68.15^{+0.40}_{-0.35}$ (68.15)
$\Omega_{\rm b}h^2$	$0.02247^{+0.00011}_{-0.000094}$ (0.02248)	$0.02248^{+0.00014}_{-0.00016}$ (0.02251)	0.02247 ± 0.00012 (0.02246)
$\Omega_{\rm c}h^2$	0.11829 ± 0.00077 (0.1183)	$0.1237^{+0.0039}_{-0.0044}$ (0.1278)	0.11830 ± 0.00084 (0.1183)
$n_{\rm s}$	0.9715 ± 0.0030 (0.9715)	0.9804 ± 0.0078 (0.9873)	0.9716 ± 0.0032 (0.9720)
$\log(10^{10}A_{\rm s})$	$3.056^{+0.012}_{-0.013}$ (3.052)	$3.064^{+0.013}_{-0.017}$ (3.058)	3.057 ± 0.014 (3.056)
$\tau_{\rm reio}$	$0.0586^{+0.0060}_{-0.0068}$ (0.05654)	$0.0574^{+0.0064}_{-0.0083}$ (0.05122)	0.0586 ± 0.0071 (0.05881)
$r_d h$	100.50 ± 0.60 (100.5)	100.71 ± 0.70 (100.5)	100.47 ± 0.64 (100.5)
S_8	0.8181 ± 0.0091 (0.8161)	$0.829^{+0.013}_{-0.014}$ (0.8378)	0.8183 ± 0.0095 (0.8182)
$\chi^2_{\rm H0}$	17.0	6.3 (**-10.7**)	17.1 (0.1)
$\chi^2_{\rm Planck}$	1014.7	1017.1 (2.4)	1015.1 (0.4)
$\chi^2_{\rm ACT}$	240.4	234.4 (**-6.0**)	240.3 (-0.2)
$\chi^2_{\rm data}$	2312.2	2297.9 (**-14.3**)	2312.5 (0.3)

MCMC fit : constraints on Quintessence from sound speed and K-essence from rate at which energy density drops

The impact of the simultaneous detection of GWs and GRBs on Modified Gravity models !



Credit: LIGO-VIRGO Collaboration.

GW 170817 and GRB 170817A

speed of GW waves

$$c_T^2 = 1 + \alpha_T$$

$$\Delta t \simeq 1.7s$$

$$\rightarrow |\alpha_T| \leq 10^{-15}$$

Implication for scalar-tensor theories - [Horndeski (1974), Deffayet et al 2011]

Lagrangian couples field and curvature terms: $\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i$

$$\mathcal{L}_2 = K \qquad \mathcal{L}_3 = -G_3 \Box\phi$$

$$\mathcal{L}_4 = G_4 R + G_{4,X}\left[(\Box\phi)^2 - \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi\right]$$

$$\mathcal{L}_5 = G_5 G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,X}\left[(\nabla\phi)^3 - 3\nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi\Box\phi + 2\nabla^\nu\nabla_\mu\phi\nabla^\alpha\nabla_\nu\phi\nabla^\mu\nabla_\alpha\phi\right]$$

where $G_i = G_i(\phi, X)$ and $X = -\nabla^\mu\phi\nabla_\mu\phi/2$

Linearise theory and map to alpha parameter :

$$M_*^2 \alpha_T = 2X \left[2G_{4,X} - 2G_{5,\phi} - (\ddot{\phi} - H\dot{\phi})G_{5,X}\right]$$

$$M_*^2 = 2(G_4 - 2XG_{4,X} + XG_{5,\phi} - H\dot{\phi}XG_{5,X})$$

Recall:

$$|\alpha_T| \leq 10^{-15}$$

Many authors assumed the following saying they held barring fine-tuned cancellation:

$$G_{4,X} = G_{5,\phi} = G_{5,X} = 0$$

This of course satisfies the bound meaning any model that satisfies those conditions (such as GR, f(R), Quintessence) is perfectly viable.

Creminelli & Vernizzi (2017), Baker et al (2017), Sakstein & Jain (2017), Ezquiaga & Zumalacárregui (2017)

Crucially though it does not imply that models that do not satisfy the assumptions are ruled out !

Copeland et al, PRL (2019)

Ex: Fab Four - self tuning solutions with a large Cosmological Constant:

$$G_X^{(2)} = V^{(J)} - 2V_\phi^{(P)} X + 4V_{\phi\phi}^{(R)} \left(1 - \ln|8\pi G X|\right)$$

$$G_\phi^{(3)} = \frac{1}{2} V_\phi^{(P)} X + \frac{2}{3} V_{\phi\phi}^{(R)} \ln|8\pi G X|$$

$$G_X^{(3)} = \frac{1}{2} V^{(P)} + \frac{2}{3} V_\phi^{(R)} \frac{1}{X}$$

Four arbitrary potentials- John, Paul, Ringo, George

$$|\alpha_T| \leq 10^{-15}$$

$$\left[\frac{3}{2} V^{(P)} X + 2V_\phi^{(R)}\right] \left(\ddot{\phi} - H\dot{\phi}\right) = -V^{(J)} X - V_\phi^{(P)} X^2 - 4V_{\phi\phi}^{(R)} X$$

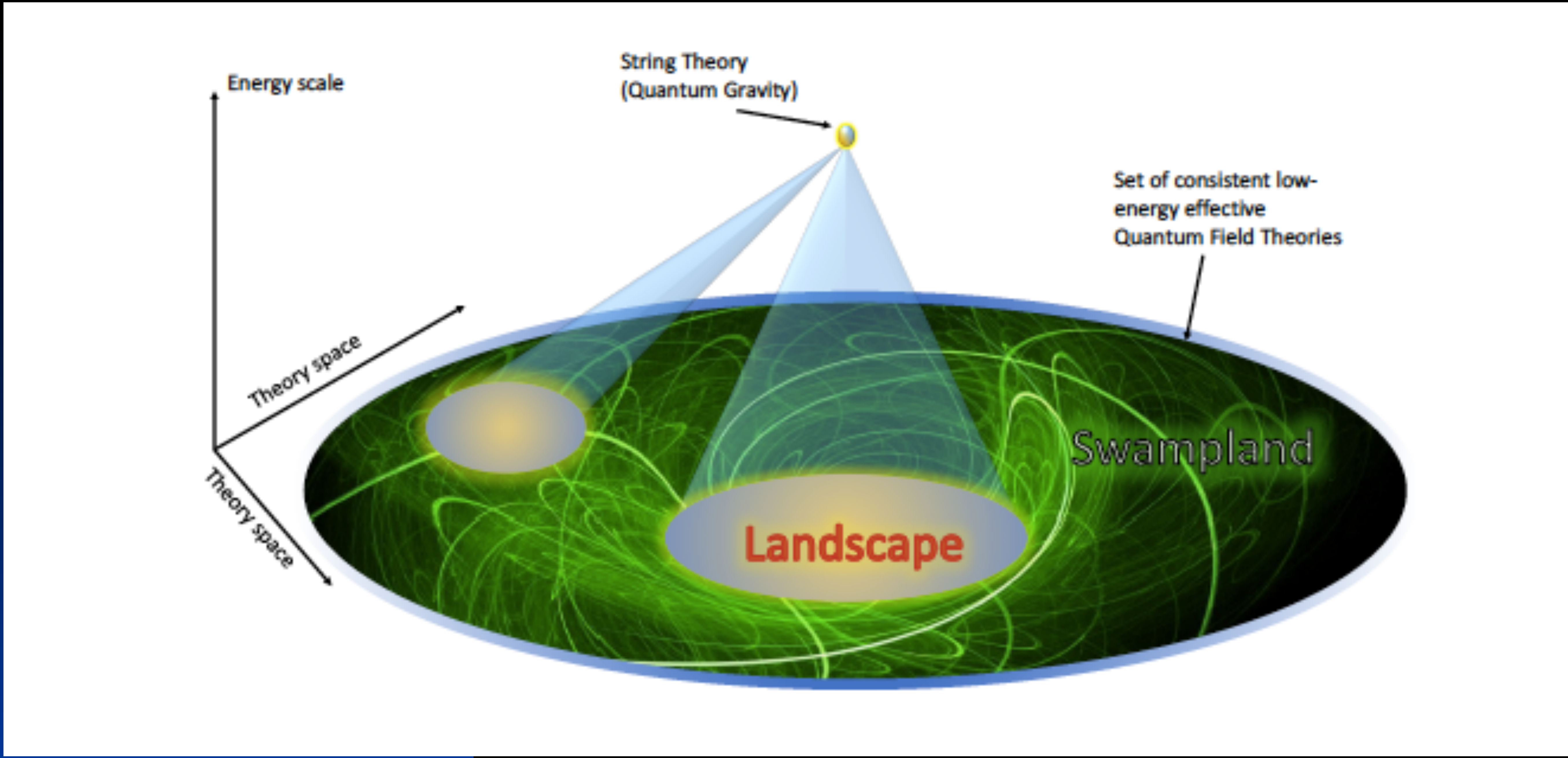
Cosmological Solutions : [EJC,Padilla, Saffin and Skordis 2018]

Case	behaviour	$V^{(J)}$	$V^{(P)}$	$V^{(G)}$	$V^{(R)}$
Stiff	$H^2 = H_0^2/a^6$	$c_1\phi^{4/\alpha-2}$	$c_2\phi^{6/\alpha-3}$	0	0
Radiation	$H^2 = H_0^2/a^4$	$c_1\phi^{4/\alpha-2}$	0	$c_2\phi^{2/\alpha}$	$-\frac{\alpha^2}{8}c_1\phi^{4/\alpha}$
Curvature	$H^2 = H_0^2/a^2$	0	0	0	$c_1\phi^{4/\alpha}$
Arbitrary $w \neq -1$	$H^2 = H_0^2 a^{-3(1+w)}$	$-\frac{1}{2}c_1(1+3w)\phi^{4/\alpha-2}$	0	0	$\frac{9\alpha^2(1-w^2)}{64}c_1\phi^{4/\alpha}$
Matter-I	$H^2 = H_0^2 a^{-3}$	$c_1\phi^{n+4}$	$c_2\phi^{n+6}$	0	$\frac{2n-3}{16(2n+7)(n+6)}c_1\phi^{n+6}$
Matter-II	$H^2 = H_0^2 a^{-3}$	$c_1\phi^{n+4}$	0	$c_2\phi^{n+3}$	$-\frac{(n+3)(2n+5)}{8(2n+7)(n+6)}c_1\phi^{n+6}$
Matter-III	$H^2 = H_0^2 a^{-3}$	$-\frac{1}{2}c_1\phi^4$	0	0	$\frac{1}{16}c_1\phi^6$
Matter-IV	$H^2 = H_0^2 a^{-3}$	$-45\sqrt{2}\phi^5$	$-\frac{75067}{225}\frac{1}{M^2}\phi^7$	$-M^2\phi^4$	$\frac{143}{168}\sqrt{2}\phi^7$

Table 1: Table of solutions from Copeland-Padilla-Saffin

All of these solutions except Stiff fluid satisfy the GW bound and in doing so determine either the coefficient alpha or n in the potentials.

Dark Energy and the String Swampland [Agrawal et. al. 2018]



[Credit: E. Palti 2018]

String Swampland [Vafa 2005]

The class of theories that appear perfectly acceptable as low energy QFT but can not be in the Landscape of string theories at high energies.

Dark Energy and the String Swampland [Agrawal et. al. 2018]

They make use of 2 main criteria:

1. The Swampland Distance Conjecture. Range traversed by a scalar field in field space is bounded by

$$\frac{|\Delta\phi|}{M_{\rm Pl}} < \Delta < O(1)$$

If go large distance D in field space, a tower of light modes appear with mass scale

$$m \sim M_{\rm Pl}\exp(-\alpha D), \quad \alpha \sim O(1)$$

which invalidates the effective action being used.

2. There is a lower bound on $\dfrac{|\nabla_\phi V(\phi)|}{V(\phi)} > c \sim O(1), \quad \text{when} \quad V > 0$

motivated by difficulty in obtaining reliable deS vacua, and string constructions of scalar potentials.

The constants are not well constrained yet. But if constraint 2 is accepted (which it isn't yet by many), it would clearly rule out ΛCDM as the source of the current acceleration.

Quintessence type models work well though with model independent constraints of $c < 0.6$, $c < 3.5\,\Delta$.

$$V(\phi) = V_1 e^{\lambda_1 \phi / M_{\rm Pl}} + V_2 e^{\lambda_2 \phi / M_{\rm Pl}}$$

[Barreiro, EC, Nunes 2000]

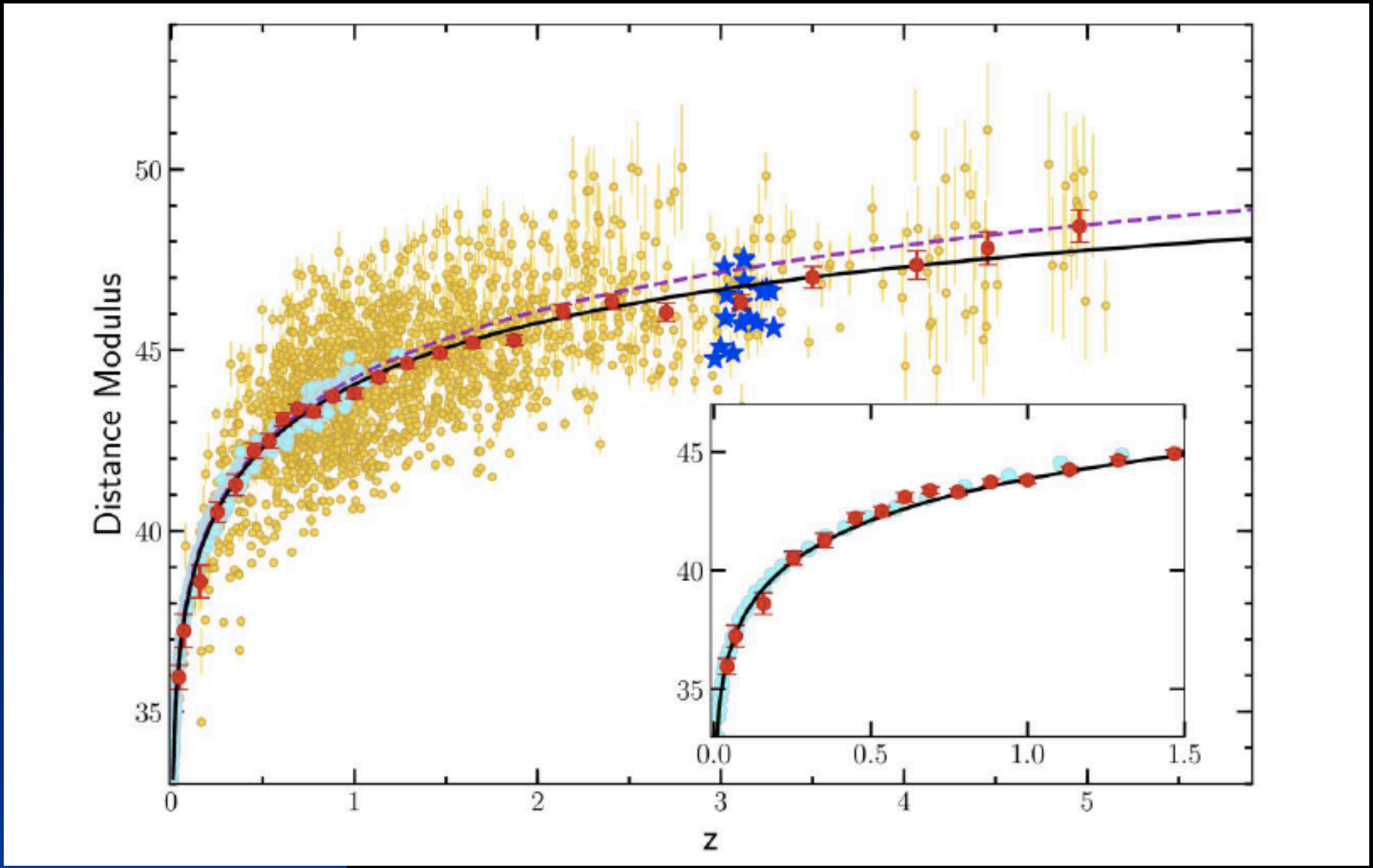
$$\lambda_1 \gg \sqrt{3}, \quad \lambda_2 = c = 0.6$$

For a range of initial conditions, evolves so that it initially scales with the background matter density and then at late times comes to dominate whilst satisfying criteria 1 and 2. In fact they find:

$$\Delta \geq \frac{1}{3} c \, \Omega_\phi^0$$

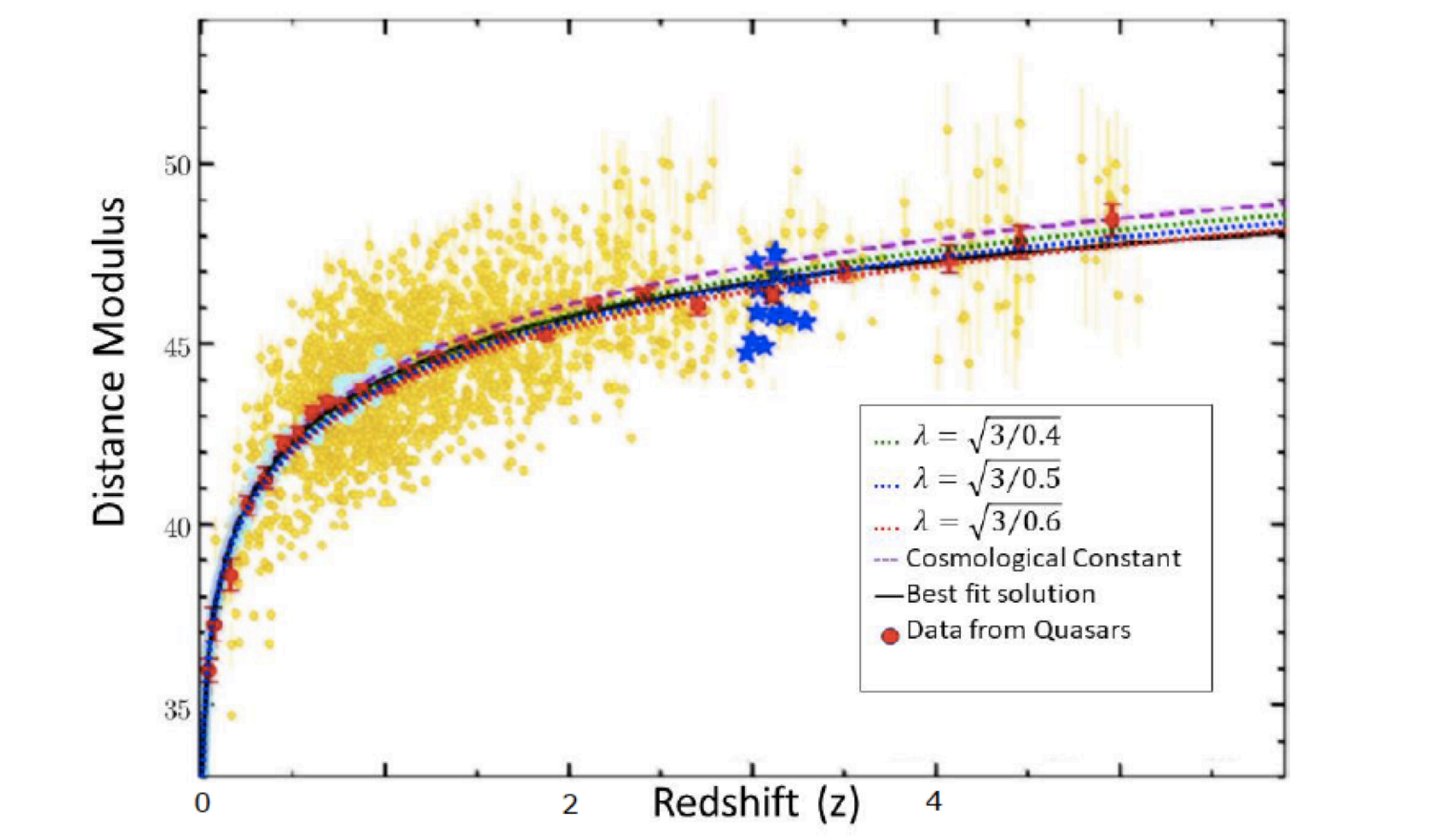
Early days but might lead to genuine new constraints on the nature of dark energy - still somewhat unclear how robust the bound is.

Quasars as Standard Candles ? [Risaliti & Lusso. Nat. Astron. 2019]



Developed a technique they argue allows quasars to be treated as std candles. Here of order 1600 quasars (yellow,blue) out to z~5. Inset is comparison to SN (cyan) showing good agreement to z~1.4 with dashed magenta line is ΛCDM with $\Omega_M \sim 0.31 \pm .05$ - extrapolated out to z~5.

Evolving Dark Energy ?



Ex: $V(\phi) = V_1 \exp(\sqrt{2}\phi/2) + V_2 \exp(\lambda\phi), \quad \sqrt{5} < \lambda < \sqrt{7.5}$

Early days - key is are quasars standard candles !

No time to discuss:

Interacting dark energy and dark matter - [Amendola 2000, Farrar and Peebles 2006, … Kase and Tsujikawa 2020]

Novel cosmologies which avoid a CC (but don't resolve the CCP) - acceleration from entropic forces - GREA [Garcia-Bellido and Espinosa-Portales 2021].

Questioning evidence for dark energy - [Nielsen et al 2016, Colin et al 2019]

And much more ….

Conclusions

1. Quintessence type approaches to the nature of dark energy and the current acceleration of the Universe provides alternative to Landscape.
2. Need to screen this which leads to models such as axions, Higgs-dilatons, chameleons, non-canonical kinetic terms etc.. -- many of these have their own issues.
3. Atoms are small enough that the chameleon or symmetron field can't react to it quickly enough and they remain unscreened in high vacuum.
4. Emergence of GW and multi-messenger astronomy opens up a new direction to constrain and rule out modified gravity models, but we need to be careful how we do it. [see Baker et al Rev Mod Phys 2021]
5. Is the Hubble tension telling us something about dark energy or MG? Time will tell - maybe LIGO will tell us over the coming years !
6. Is the Swampland telling us something about dark energy?
7. How can we go locally beyond SN1a ? Quasars ?

Extra slides if time permits

Testing models - consider coupled dark energy-dark matter.

Have seen provides a nice way to explain coincidence problem.

What is most general phenomenological model we can construct?

Three distinct classes of mixed models with couplings intro at the level of the action [Pourtsidou, Skordis , EC 2013, 2015]

- Consider Dark Energy (DE) coupled to Cold Dark Matter (c) [e.g. Kodama & Sasaki '84, Ma & Bertschinger '95]

- $T^{(\mathrm{c})}$ and $T^{(\mathrm{DE})}$ are not separately conserved:

$$\nabla_\mu T^{(\mathrm{c})\mu}{}_\nu = -\nabla_\mu T^{(\mathrm{DE})\mu}{}_\nu = J_\nu \neq 0$$

- Various forms of coupling have been considered. Examples:

$$J_\nu \propto \rho_c \nabla_\nu \phi \qquad \text{[Amendola '00]}$$
$$J_\nu \propto \rho_c\, u_\nu^{(\mathrm{c})} \qquad \text{[Valiviita et al '08]}$$

- FRW background with $\bar{J}_\nu = (\bar{J}_0, \bar{J}_i)$ and linear perturbations $(\delta J_0, \delta J_i)$. Note that $\bar{J}_i = 0$ because of isotropy. The CDM energy density equation becomes

$$\dot{\bar{\rho}}_\mathrm{c} + 3\mathcal{H}\bar{\rho}_\mathrm{c} = -\bar{J}_0$$

With thanks to my collaborator Alkistis Pourtsidou for lending me some of the following slides.

Using the fluid pull back formalism we consider the fluid/particle number density n.

- The action for GR and a fluid is

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}f(n)$$

- $f(n)$ is (in principle) an arbitrary function, whose form determines the equation of state and speed of sound of the fluid

- For pressureless matter (CDM) $f(n) \propto n$

- Stress-energy tensor is given by

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}$$

- Can match ρ, P to the fluid function $f(n)$ as

$$\Rightarrow \rho = f, \quad P = n\frac{df}{dn} - f$$

- We want to construct a model where the fluid with number density n (e.g. CDM) is explicitly coupled to a DE field ϕ

- Invariants: $Y = \frac{1}{2}(\nabla_\mu\phi)^2$, $Z = u^\mu\nabla_\mu\phi$

- Our general Lagrangian has the form

$$L = L(n, Y, Z, \phi)$$

- Example: Usual quintessence has

$$L = Y + V(\phi) + f(n)$$

Type 1 models. $L(n, Y, Z, \phi) = F(Y, \phi) + f(n, \phi)$ ex: $f(n) = g(n)e^{\alpha(\phi)}$

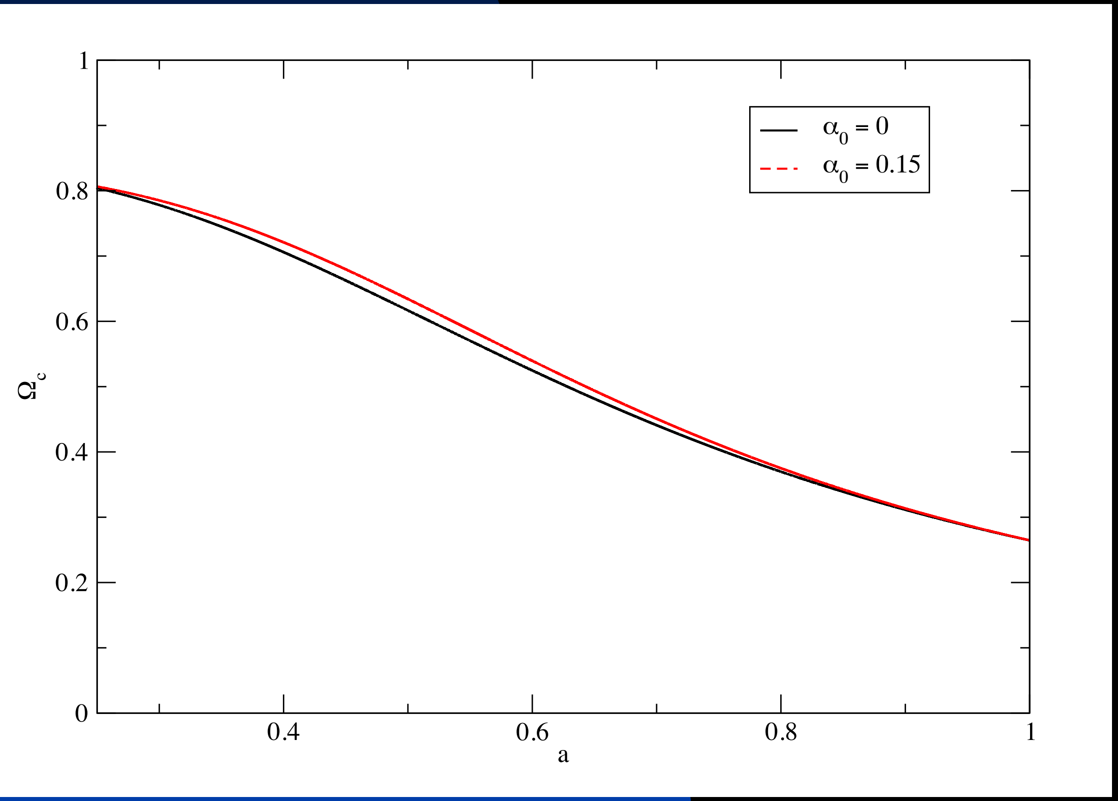
Could be K-essence scalar field coupled to matter, or Quintessence if F=Y+V(ϕ)

Coupling current $J_\mu = -\rho \frac{d\alpha(\phi)}{d\phi} \nabla_\mu \phi$ [generalized Amendola model]

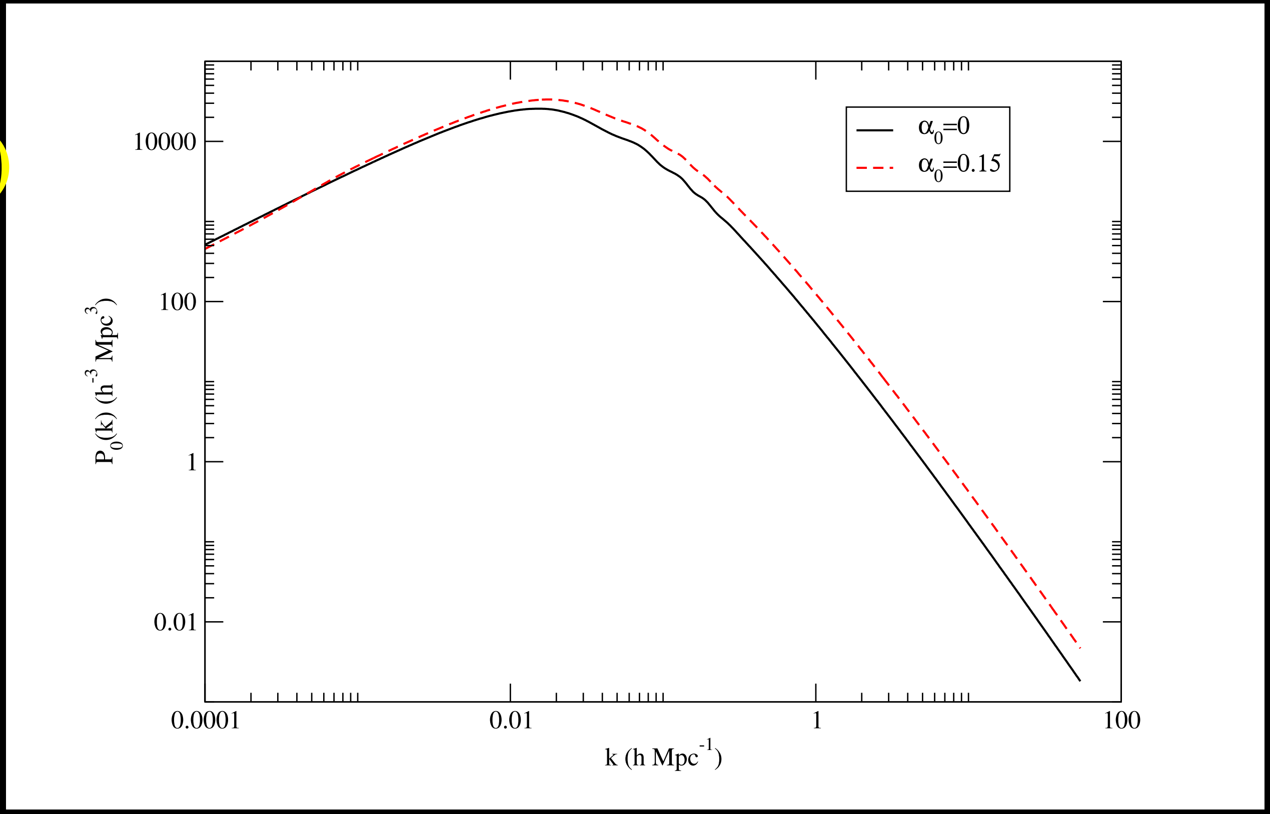
Choose $\alpha(\phi) = \alpha_0 \phi$ with α_0 const and study observational signatures in CMB and matter power spectra (modified CAMB code).

Note the evolution of CDM density: $\bar{\rho}_c = \bar{\rho}_{c,0} a^{-3} e^{\alpha(\phi)}$

Ω_C



P(k)

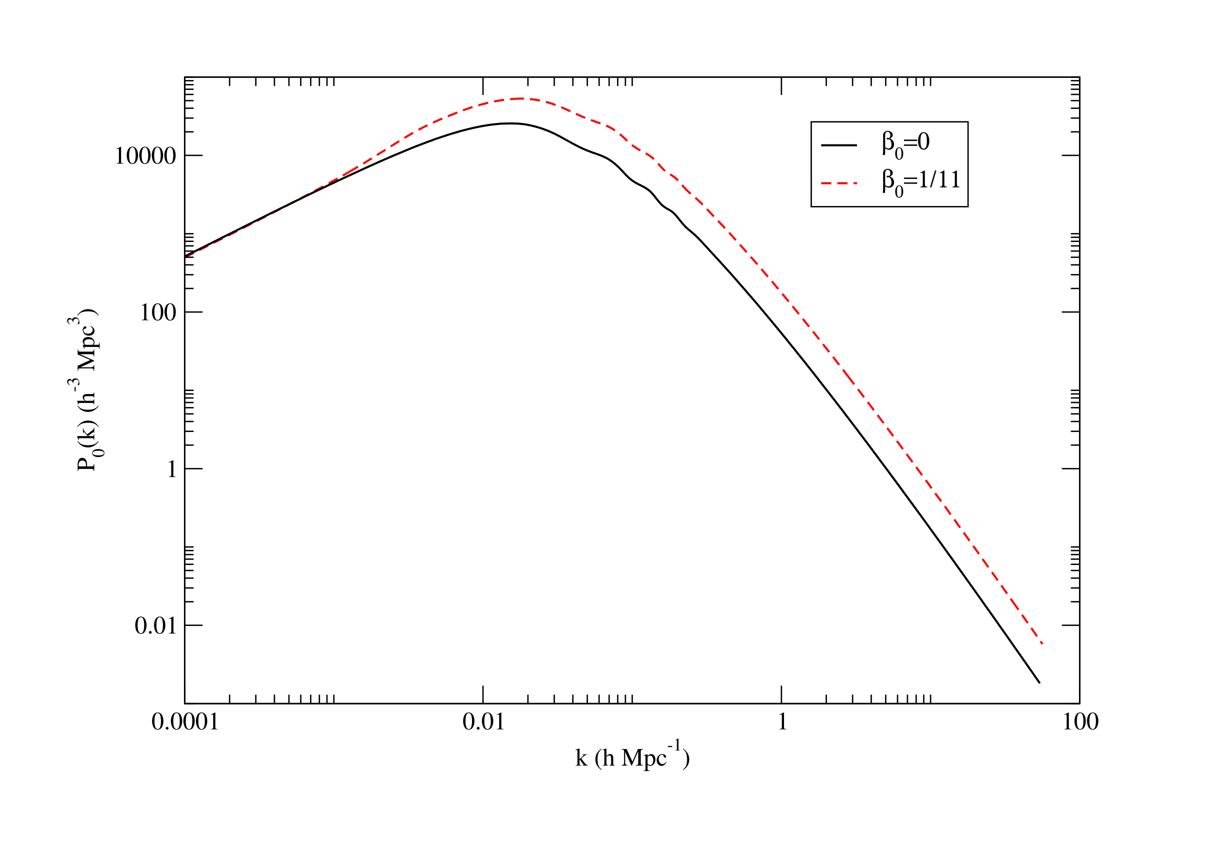


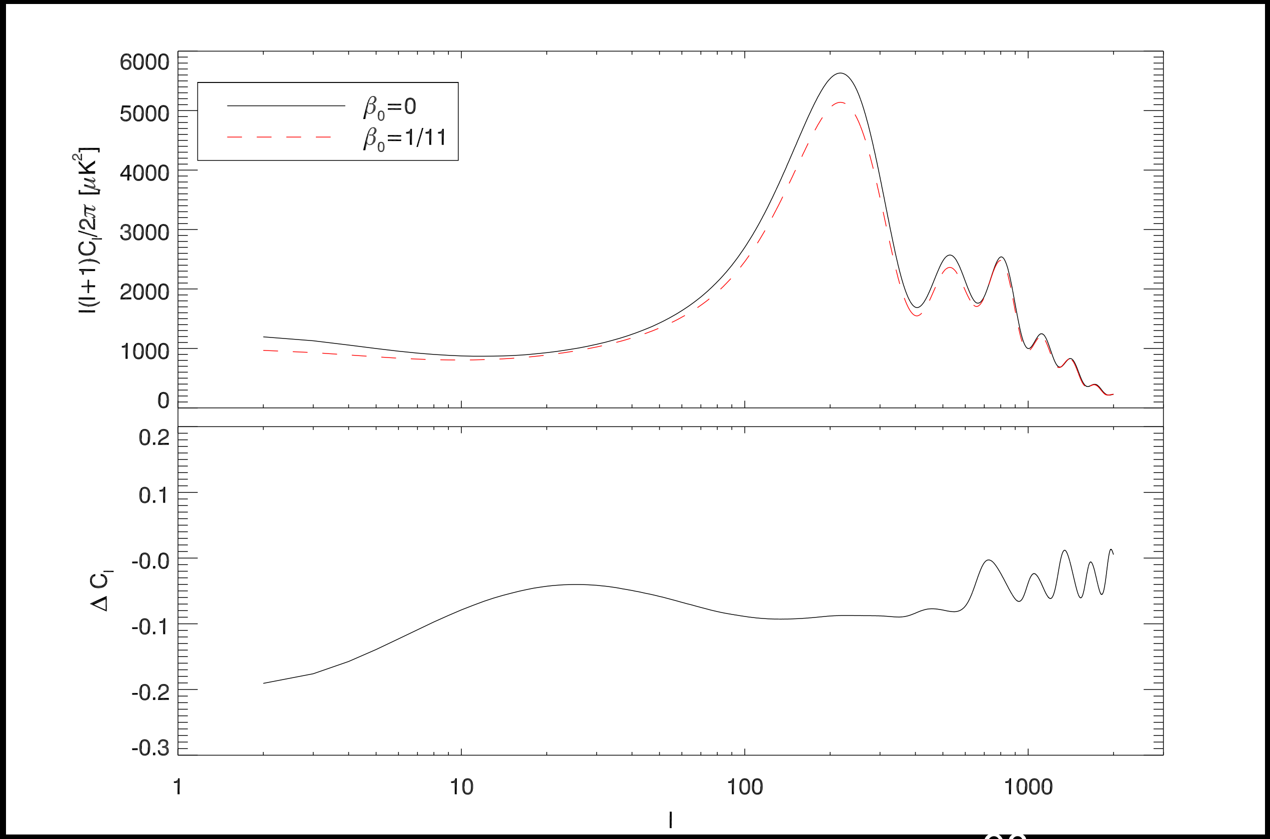
More DM at early times, equality earlier - only small scale pertns have time to enter horizon and grow during radiation dom - growth enhanced, small scale power increases, larger sigma8

Type 2 models. $L(n, Y, Z, \phi) = F(Y, \phi) + f(n, Z)$

ex: $\bar{\rho}_c = \bar{\rho}_{c,0} a^{-3} \bar{Z}^{\frac{\beta_0}{1-\beta_0}}$

Since $\bar{Z} = -\dot{\bar{\phi}}/a$, $\bar{\rho}_c$ depends on the time derivative $\dot{\bar{\phi}}$ instead of $\bar{\phi}$ itself which is a notable difference from the Type-1 case.





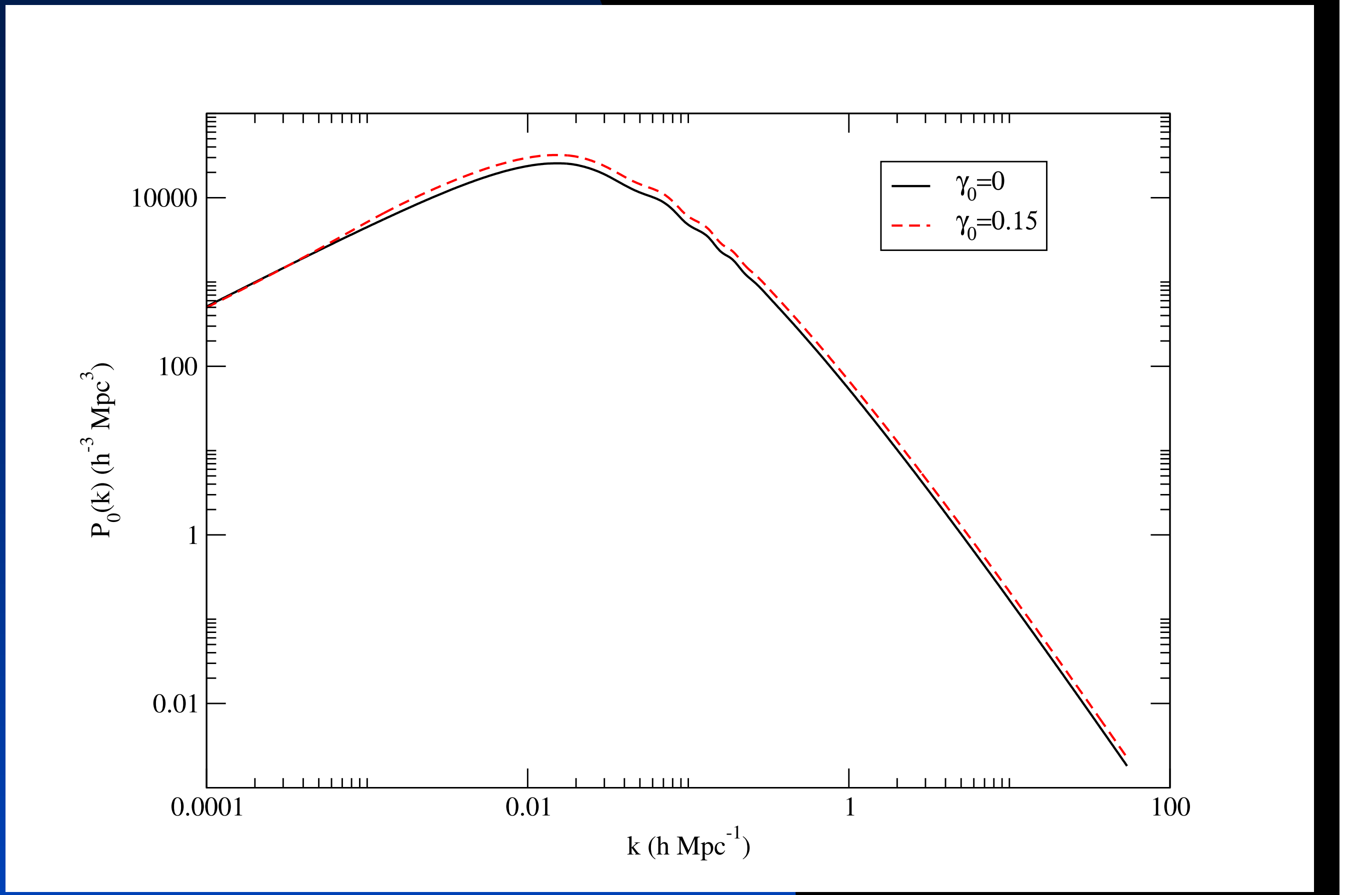
Type 3 models. $L(n, Y, Z, \phi) = F(Y, Z, \phi) + f(n)$ ex: $F = Y + V(\phi) + \gamma(Z)$ $\gamma(Z) = \gamma_0 Z^2$

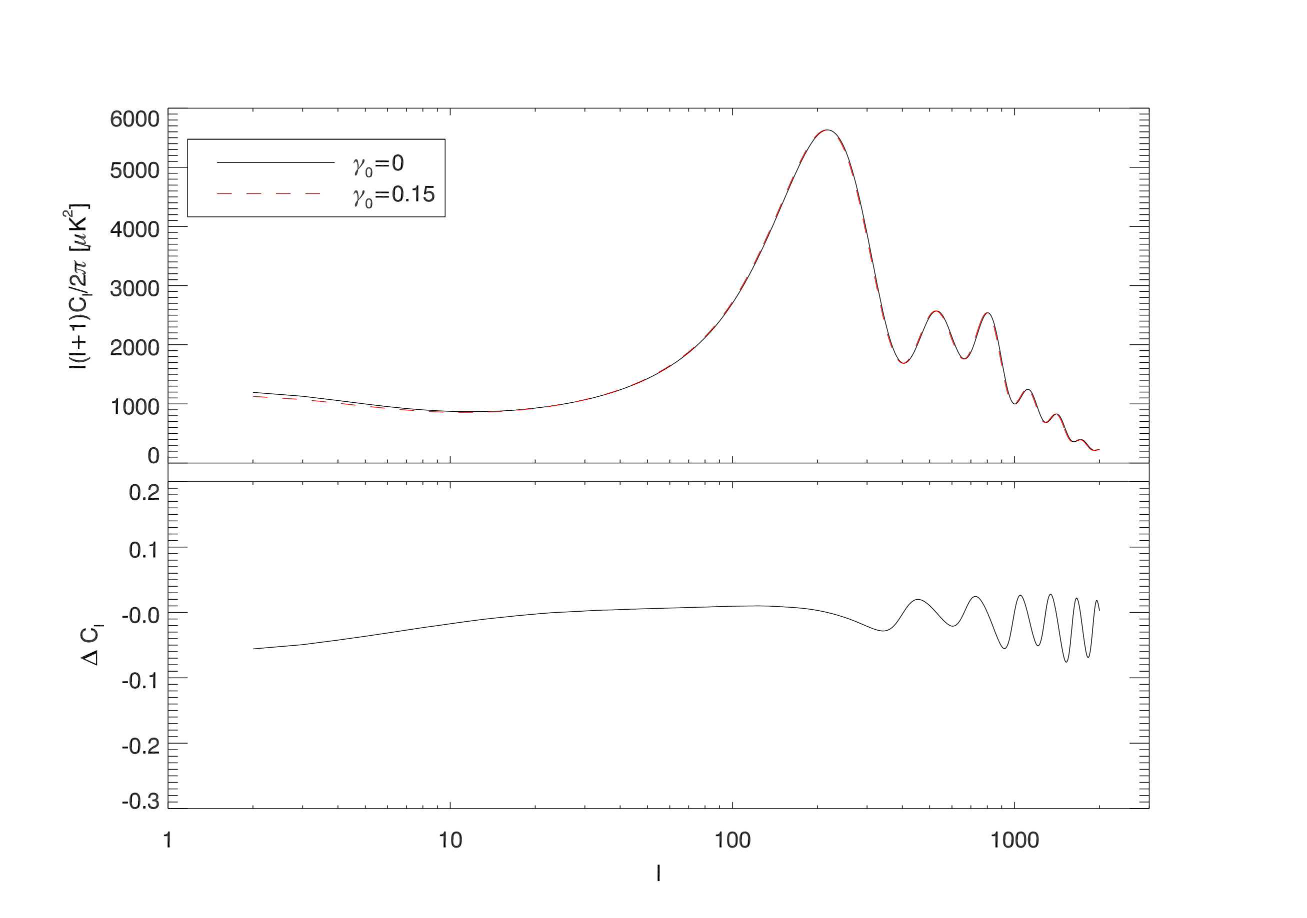
Type 3 models have $\bar{J}_0 = 0$ They involve pure momentum transfer

no coupling at the background field equations!

$$\dot{\bar{\rho}}_c + 3\mathcal{H}\bar{\rho}_c = 0$$

Furthermore, the energy-conservation equation remains uncoupled even at the linear level, i.e. $\delta \equiv \delta\rho/\bar{\rho}$ obeys uncoupled equation.





Parameterise these mixed models - extend the PPF formalism [Hu 08, Skordis 08]

[Skordis, Pourtsidou, EC 2015]

Parameterising these mixed models - extend the PPF formalism [Hu 08, Skordis 08]

$$G_{\mu\nu} = T^{(\text{known})}_{\mu\nu} + U_{\mu\nu}$$

Tensor $U_{\mu\nu}$ contains the unknown fields/modifications, i.e. effective dark energy. Can depend on additional fields, metric etc. Example $f(R)$ gravity with $f_R = \frac{df}{dR}$:

$$U_{\mu\nu} = \nabla_\mu \nabla_\nu f_R - f_R R_{\mu\nu} + \left(\frac{1}{2} f - \nabla^2 f_R\right) g_{\mu\nu}$$

Assuming that there are no interactions between the two sectors, use $\nabla_\mu G^\mu{}_\nu = 0$ and $\nabla_\mu T^\mu{}_\nu = 0$ to get

$$\nabla_\mu U^\mu{}_\nu = 0$$

$\rightarrow$ Field equations for the modifications.

[Skordis, Pourtsidou, EC 2015]

- Consider Dark Energy (DE) coupled to Cold Dark Matter (c).
- $\nabla_\mu G^\mu{}_\nu = 0$ still true, but $T = T^{(\text{c})}$ and $U = T^{(\text{DE})}$ not separately conserved

$$\nabla_\mu T^{(\text{c})\mu}{}_\nu = -\nabla_\mu T^{(\text{DE})\mu}{}_\nu = J_\nu$$

- Split $J_\nu = \bar{J}_\nu + \delta J_\nu$ (note $\delta J_i = \nabla_i S$).
- FRW background with $\bar{J}_0$, $\bar{J}_i = 0$
- We want to parameterise δJ_0 and S in terms of metric and fluid variables.

- δJ_0 and S are written in terms of the DM and DE fluid variables, the metric variables and their derivatives.

- Notation: $\delta = \delta\rho/\bar{\rho}$

$$\delta J_0 = -6A_1\dot{\hat{\Phi}} - 6A_2(\dot{\hat{\Phi}} + \mathcal{H}\hat{\Psi}) + A_3\delta_{\rm DE} + A_4\delta_c + A_5\theta_{\rm DE} + A_6\theta_c + \bar{J}_0\Psi,$$

$$S = -6B_1\dot{\hat{\Phi}} - 6B_2(\dot{\hat{\Phi}} + \mathcal{H}\hat{\Psi}) + B_3\delta_{\rm DE} + B_4\delta_c + B_5\theta_{\rm DE} + B_6\theta_c,$$

- We have 12 free functions. Different models have different sets of non-zero (A_i, B_i).

- $\bar{J}_0 = \Gamma\bar{\rho}_c$ [Valiviita et al]. This model has

$$\delta J_0 = \bar{J}_0(\delta_c + \Psi); \quad S = \bar{J}_0\theta_c$$

$\Rightarrow$ The only non-zero coefficients are:

$$A_4 = B_6 = \bar{J}_0$$

- $\bar{J}_0 = -\beta\bar{\rho}_c\dot{\bar{\phi}}$ [Amendola]

$\Rightarrow$ The non-zero coefficients are:

$$A_3 = \frac{\bar{J}_0}{1 + w_\phi}, \quad A_4 = \bar{J}_0, \quad A_5 = \beta\bar{\rho}_c a^2\frac{dV}{d\phi}$$

$$B_5 = \bar{J}_0$$

Model/Coefficients	Q	A_1	A_2	A_3	A_4	A_5	A_6	B_1	B_2	B_3	B_4	B_5	B_6
Coupled Quintessence	$-\beta_A\bar{\rho}_c\dot{\bar{\phi}}$	-	-	$\frac{Q}{1+w}$	Q	$\beta_A\bar{\rho}_c a^2 V_\phi$	-	-	-	-	-	Q	-
$J_\mu \propto u_\mu$	$a\Gamma_{int}\bar{\rho}_c$	-	-	-	Q	-	-	-	-	-	-	-	Q
elastic scattering	-	-	-	-	-	-	-	-	-	-	-	$-\rho_{DE}(1+w)an_D\sigma_D$	$-B_5$
Type-1	$-\bar{\rho}_c\alpha_\phi\dot{\bar{\phi}}$	-	-	$\frac{Qc_s^2}{1+w}$	Q	$Q\left[\frac{\alpha_{\phi\phi}}{\alpha_\phi} - \frac{c_s^2\dot{\bar{\phi}}\bar{K}_\phi}{(1+w)\bar{K}}\right]$	-	-	-	-	-	Q	-
Type-2	$\frac{\bar{Z}\beta_Z\bar{\rho}_c}{1+\bar{Z}\beta}\dot{\bar{Z}}$	-	A_2	A_3	A_4	A_5	-	-	-	-	-	Q	-
Type-3	-	-	-	-	-	-	-	-	-	B_3	-	B_5	$-B_5 + \frac{3\mathcal{H}\bar{Z}F_Z c_s^2}{1-\frac{\bar{Z}\bar{F}_Z}{\bar{\rho}_c}}$

TABLE II: Specific models and their PPF coefficients. The coupled Quintessence model is a subcase of Type 1 with $\alpha_\phi = \beta_A$. The elastic scattering model is in fact distinct from Type-3 (see text at the end of section III D). For the coefficients A_2, A_3, A_4 and A_5 in the case of Type-2 see (70). For the coefficients B_3 and A_5 in the case of Type-3 see (86). For the remaining functions the reader is referred to each specific example in the text.

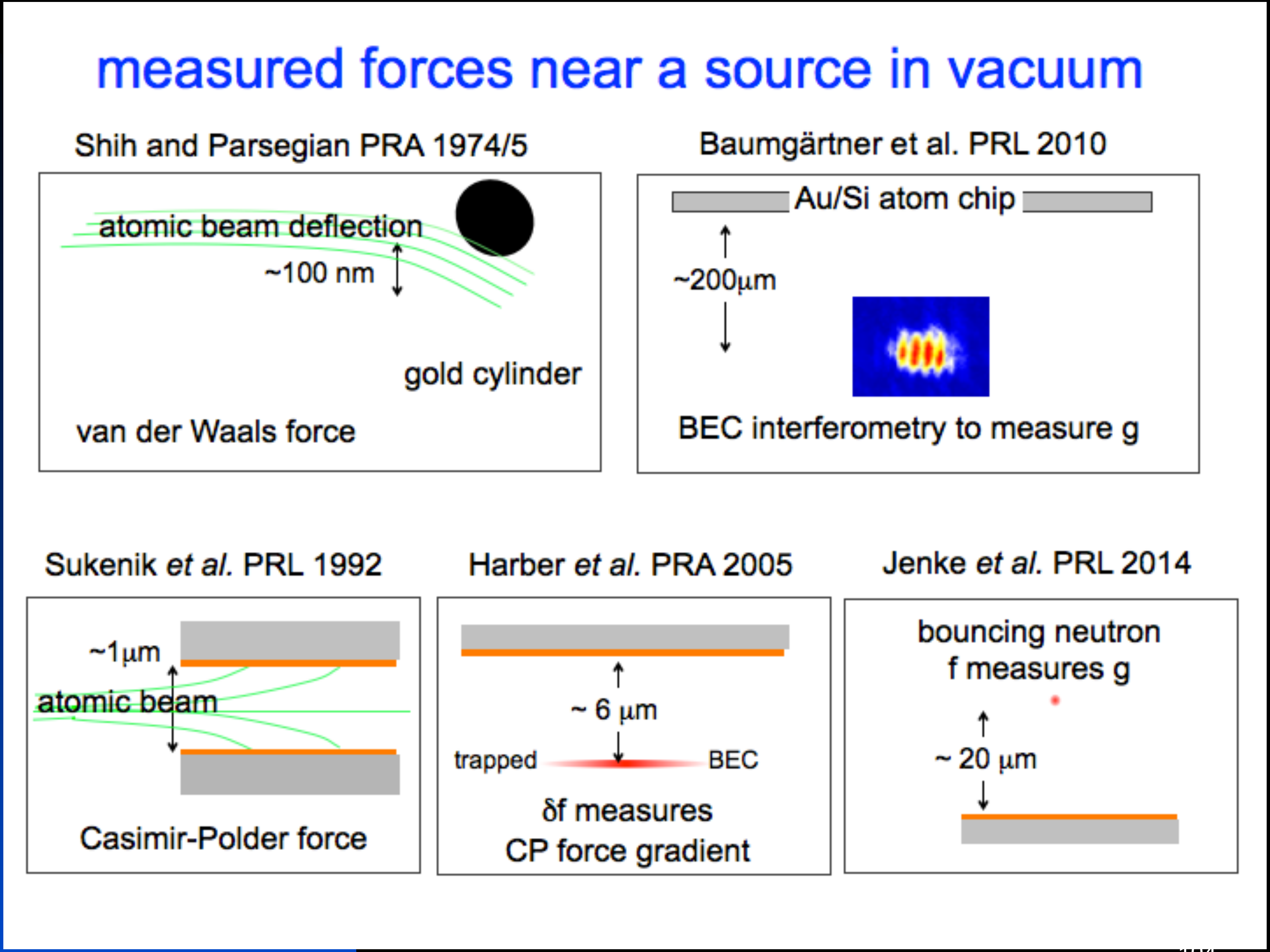
Basic assumptions: Bgd cosmology is FRW soln, field eqns are at most 2nd order in time derivatives and are gauge invariant.

Once you know the field eqns PPF parameterisation is useful tool for phenomenological model building.

Interesting that in all the models we looked at A_1, A_6, B_1, B_2, B_4 are all zero. What models are there where they are non-zero?

See also very nice related work in Amendola, Barreiro and Nunes 2014 [Assisted coupled quintessence]; Amendola et al 2013 [Observables and unobservables in DE cosmologies]

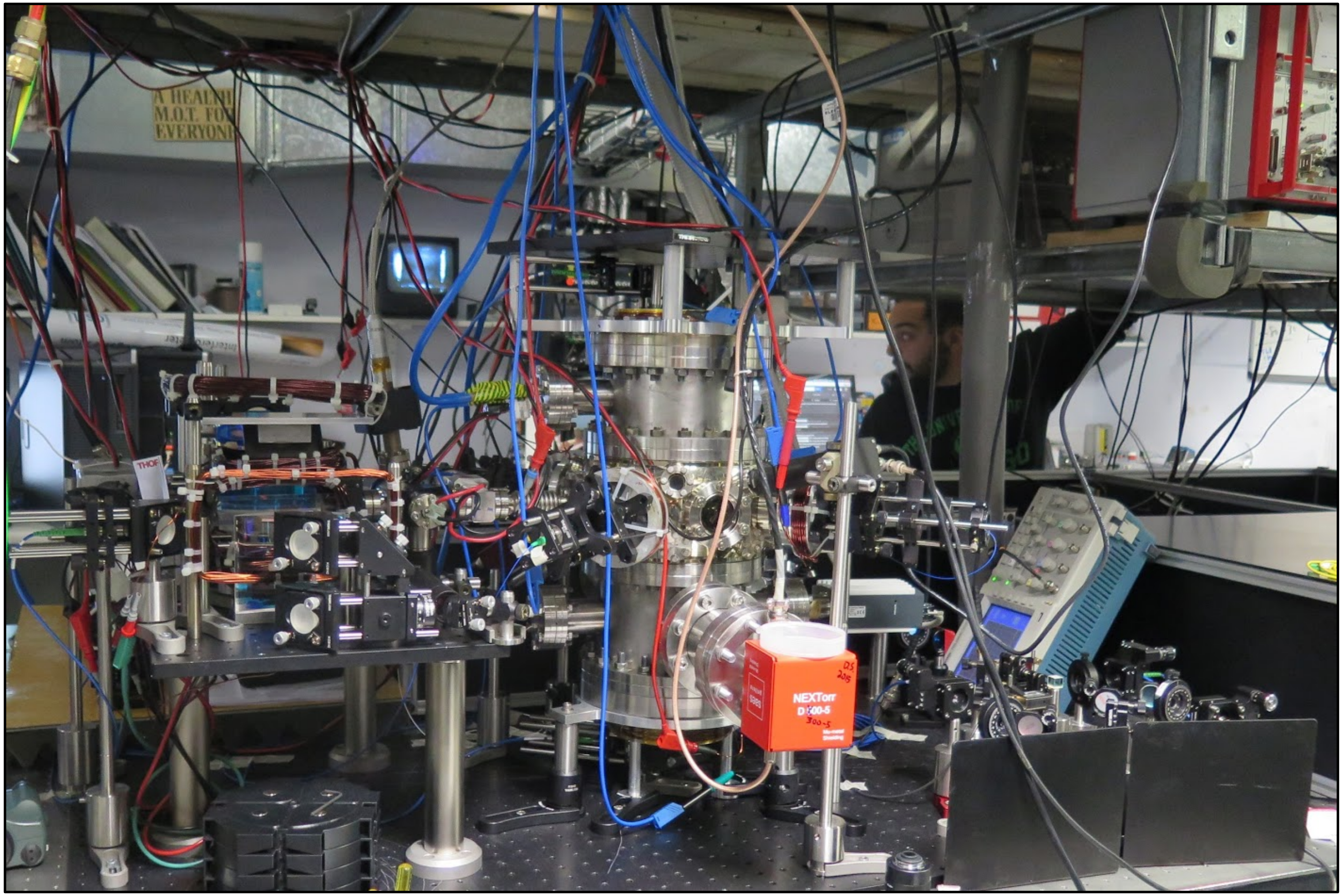
We can constrain the chameleon with any measurement of interactions between atoms and macroscopic objects/surfaces in high vacuum environments



Ed Hinds

Chameleon experiment constructed at Imperial College

Centre for Cold Matter (Ed Hinds group)

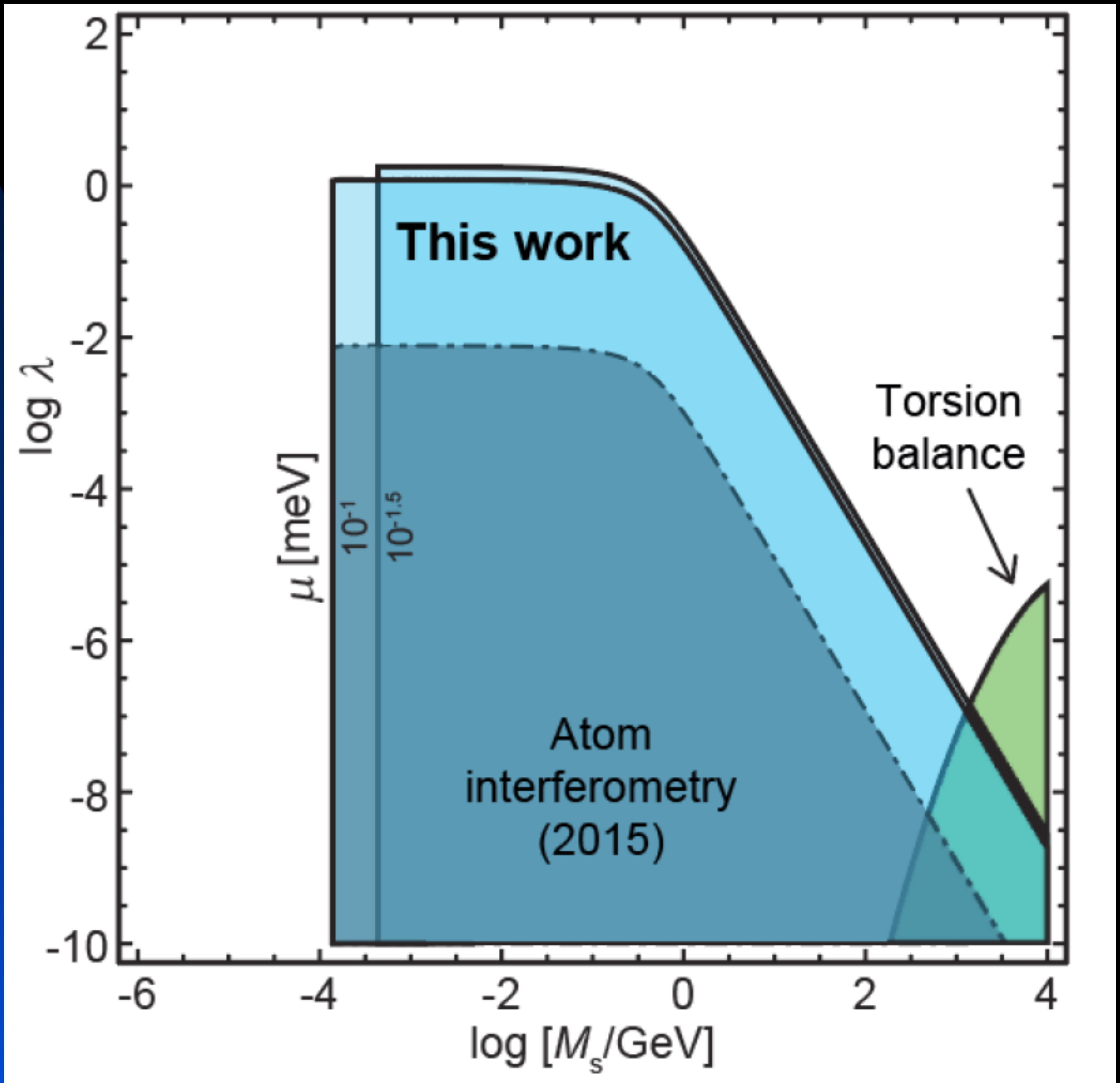


Experiment rotated by 90 degrees from the Berkeley experiment - no sensitivity to Earth's gravity

[Dylan Sabulsky, Indranil Dutta and Ed Hinds]

Symmetron constraints [Jaffe et al 2016; Burrage et al 2106, Brax & Davis 2016]

$$V_{\rm eff}(\phi) = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\phi^2 + \frac{\lambda}{4}\phi^4$$

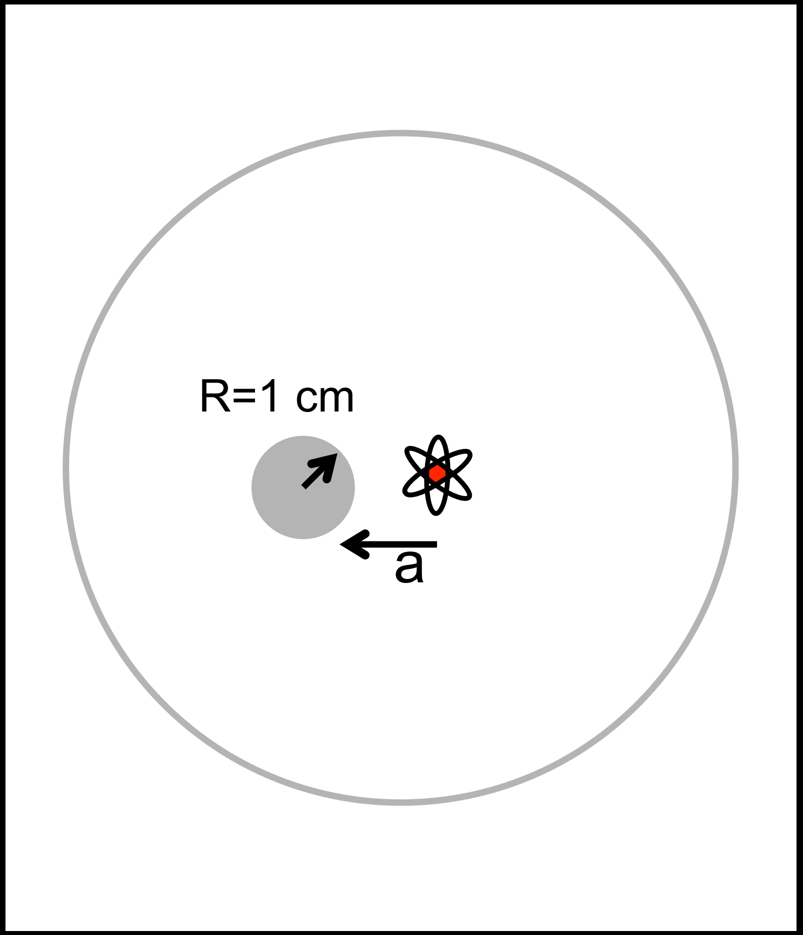


Consider now a source object A and test object B (atom) near the middle of the chamber. The force between uniform spheres a distance r apart, due to the combined effect of gravity and the chameleon field is :

$$F_r = \frac{G M_A M_B}{r^2}\left[1 + 2\lambda_A \lambda_B \left(\frac{M_P}{M}\right)^2\right]$$

where

$$\lambda_i = \begin{cases} 1 & \rho_i R_i^2 < 3M\phi_{\mathrm{bg}} \\ \frac{3M\phi_{\mathrm{bg}}}{\rho_i R_i^2} & \rho_i R_i^2 > 3M\phi_{\mathrm{bg}} \end{cases}$$



Fifth force experiments to date tend to have $\lambda_A \ll 1$ and $\lambda_B \ll 1$ because the objects are large and dense and ϕ_{bg} is small in the high terrestrial bgd density. Resulting double suppression of the force is so strong, expt bounds are not very stringent.

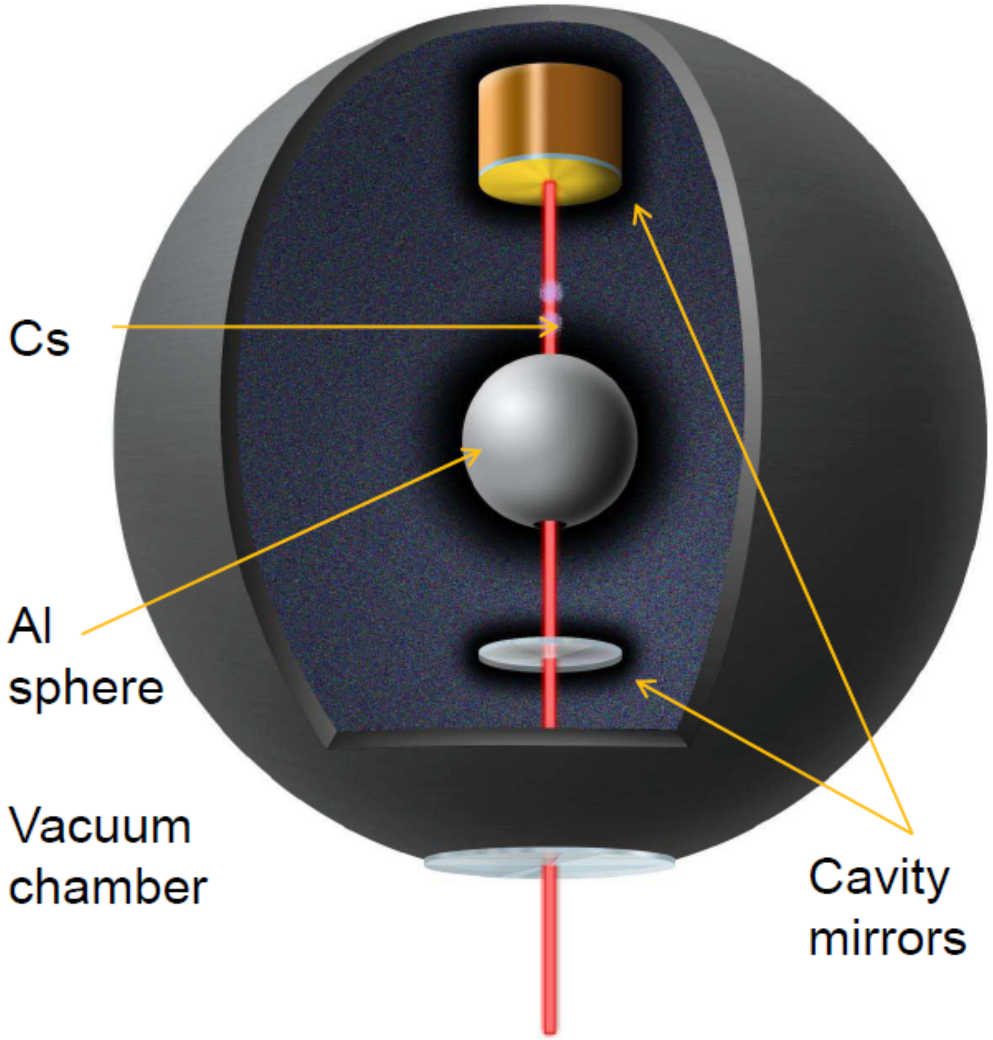
However, can achieve $\lambda_B = 1$ by using an atom in high vacuum where $\rho_B R^2_B << M \phi_{bg}$

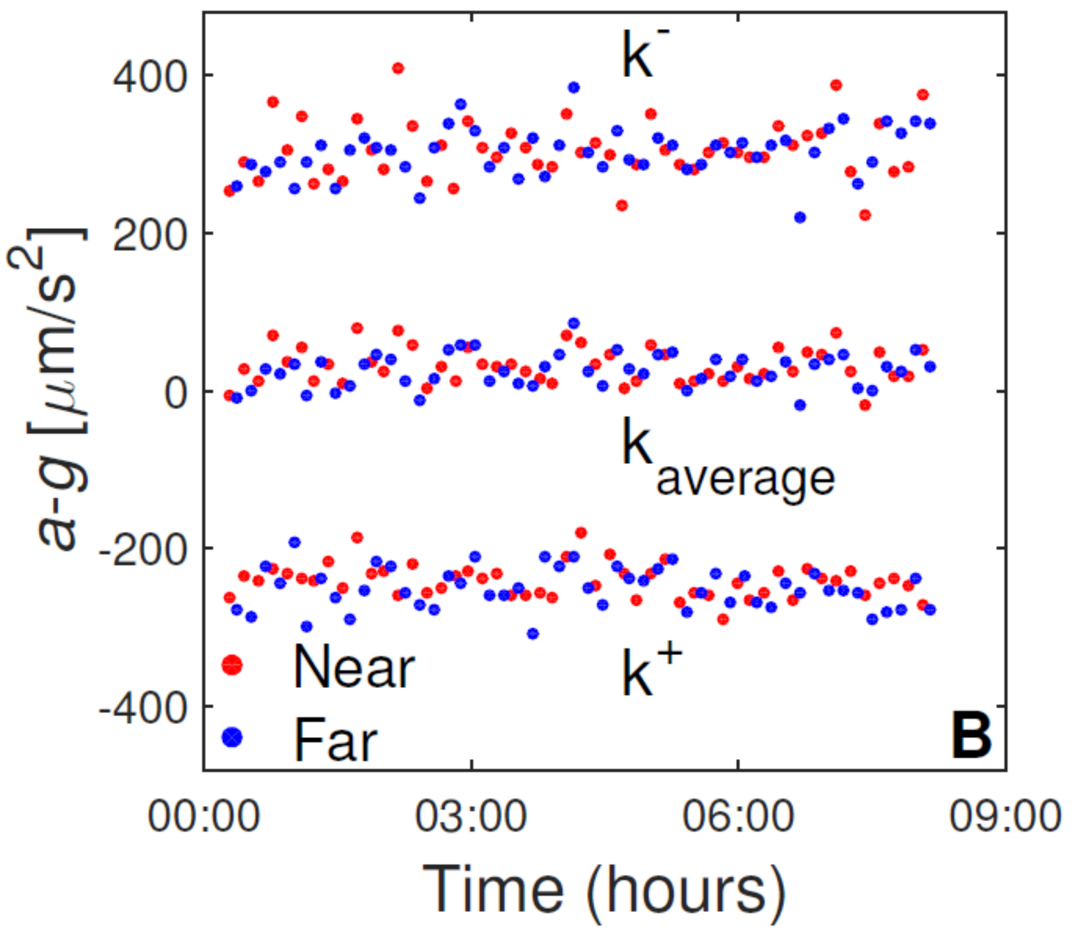
Then the acceleration towards a macroscopic test mass is only singly suppressed and atom interferometry can easily detect it.

The experiment was performed in Berkeley within a few months of the proposal

Berkley Experiment

Using an existing set up with an optical cavity
The cavity provides power enhancement, spatial filtering, and a precise beam geometry

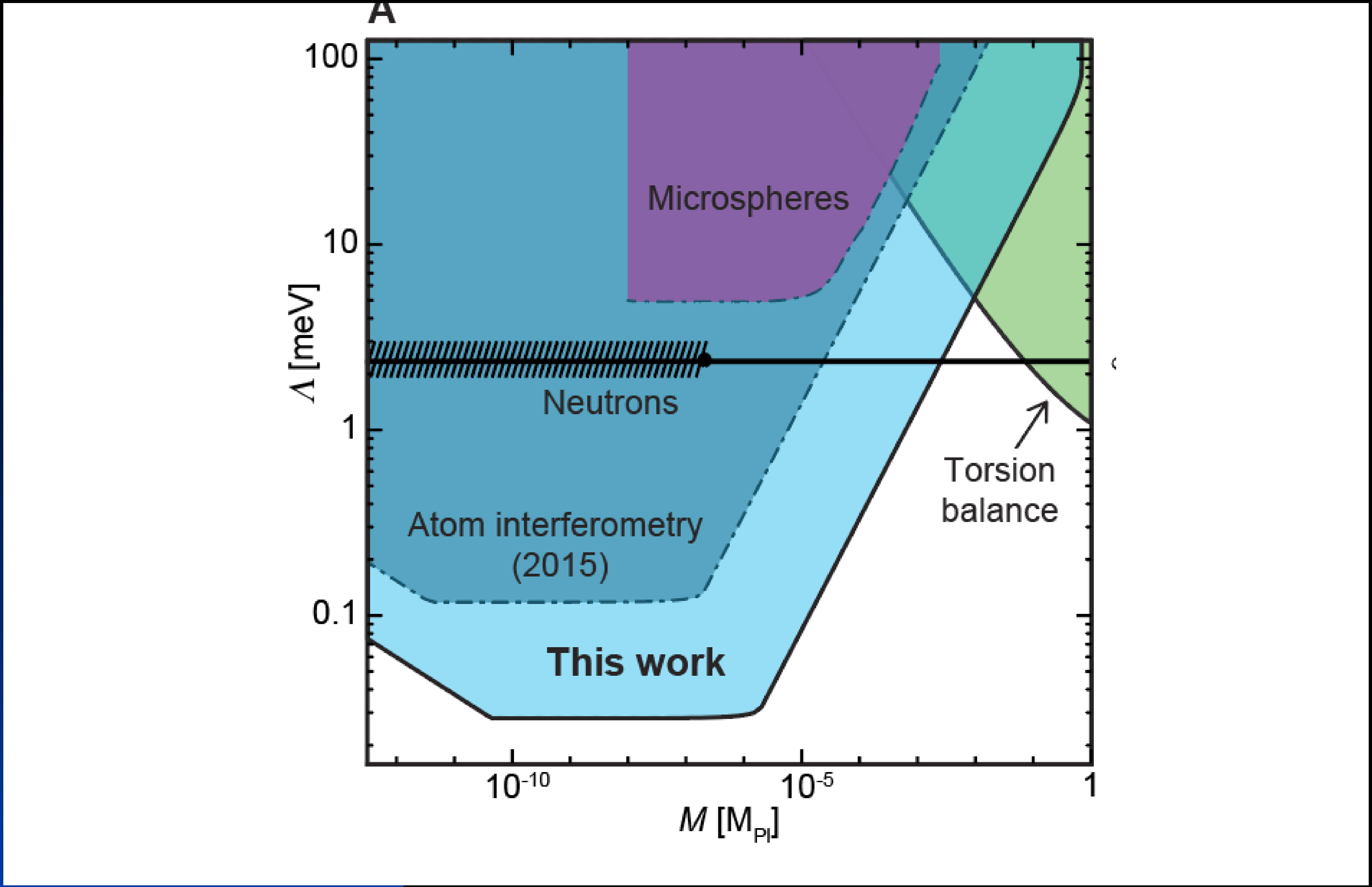




Hamilton et al. (2015)

Slide thanks to Clare Burrage

Berkeley Experiment



Hamilton et al 2015, Jaffe et al 2016 - already increased limits on Chameleons by over two orders of magnitude.

Ex: Fab Four - self tuning solutions with a large Cosmological Constant:

$$G_X^{(2)} = V^{(J)} - 2V_\phi^{(P)} X + 4V_{\phi\phi}^{(R)} \left(1 - \ln|8\pi G X|\right)$$

$$G_\phi^{(3)} = \frac{1}{2} V_\phi^{(P)} X + \frac{2}{3} V_{\phi\phi}^{(R)} \ln|8\pi G X|$$

$$G_X^{(3)} = \frac{1}{2} V^{(P)} + \frac{2}{3} V_\phi^{(R)} \frac{1}{X}$$

Four arbitrary potentials- John, Paul, Ringo, George

$$|\alpha_T| \leq 10^{-15}$$

$$\left[\frac{3}{2} V^{(P)} X + 2V_\phi^{(R)}\right] \left(\ddot{\phi} - H\dot{\phi}\right) = -V^{(J)} X - V_\phi^{(P)} X^2 - 4V_{\phi\phi}^{(R)} X$$

Cosmological Solutions : [EJC,Padilla, Saffin and Skordis 2018]

Case	behaviour	$V^{(J)}$	$V^{(P)}$	$V^{(G)}$	$V^{(R)}$
Stiff	$H^2 = H_0^2/a^6$	$c_1\phi^{4/\alpha-2}$	$c_2\phi^{6/\alpha-3}$	0	0
Radiation	$H^2 = H_0^2/a^4$	$c_1\phi^{4/\alpha-2}$	0	$c_2\phi^{2/\alpha}$	$-\frac{\alpha^2}{8}c_1\phi^{4/\alpha}$
Curvature	$H^2 = H_0^2/a^2$	0	0	0	$c_1\phi^{4/\alpha}$
Arbitrary $w \neq -1$	$H^2 = H_0^2 a^{-3(1+w)}$	$-\frac{1}{2}c_1(1+3w)\phi^{4/\alpha-2}$	0	0	$\frac{9\alpha^2(1-w^2)}{64}c_1\phi^{4/\alpha}$
Matter-I	$H^2 = H_0^2 a^{-3}$	$c_1\phi^{n+4}$	$c_2\phi^{n+6}$	0	$\frac{2n-3}{16(2n+7)(n+6)}c_1\phi^{n+6}$
Matter-II	$H^2 = H_0^2 a^{-3}$	$c_1\phi^{n+4}$	0	$c_2\phi^{n+3}$	$-\frac{(n+3)(2n+5)}{8(2n+7)(n+6)}c_1\phi^{n+6}$
Matter-III	$H^2 = H_0^2 a^{-3}$	$-\frac{1}{2}c_1\phi^4$	0	0	$\frac{1}{16}c_1\phi^6$
Matter-IV	$H^2 = H_0^2 a^{-3}$	$-45\sqrt{2}\phi^5$	$-\frac{75067}{225}\frac{1}{M^2}\phi^7$	$-M^2\phi^4$	$\frac{143}{168}\sqrt{2}\phi^7$

Table 1: Table of solutions from Copeland-Padilla-Saffin

All of these solutions except Stiff fluid satisfy the GW bound and in doing so determine either the coefficient alpha or n in the potentials.