Overview of Dark Sector physics

Maxim Pospelov

U of Minnesota and FTPI

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Feebly-Interacting Particles:
FIPs 2020 Workshop Report

P. Agrawal[1], M. Bauer[2,a], J. Beacham[3,a], A. Berlin[4], A. Boyarsky[5], S. Cebrian[6], X. Cid-Vidal[7], D. d'Enterria[3], A. De Roeck[3,a], M. Drewes[8], B. Echenard[9], M. Giannotti[10], G. F. Giudice[3,a], S. Gninenko[11], S. Gori[12,13], E. Goudzovski[14], J. Heeck[15], P. Hernandez[16,a], M. Hostert[24,25,38], I. Irastorza[6,a], A. Izmaylov[11], J. Jaeckel[17,a], F. Kahlhoefer[18], S. Knapen[3,b], G. Krnjaic[19,20,a], G. Lanfranchi[21,a,b,*], J. Monroe[22,a], V. Martinez-Outschoorn[23], J. Lopez-Pavon[16], S. Pascoli[2,39,a], M. Pospelov[24,25], D. Redigolo[3,26,b], A. Ringwald[27], O. Ruchayskiy[28] J. Ruderman[4,27,a], H. Russell[3], J. Salfeld-Nebgen[29], P. Schuster[30,a], M. Shaposhnikov[31,a], L. Shchutska[31], J. Shelton[32,a], Y. Soreq[33] Y. Stadnik[34], J. Swallow[14], K. Tobioka[35,36], Y.-D. Tsai[23,37]

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Plan for 3 lectures

1. Introduction. The need for new physics. Types of particle dark matter. Portals to new Physics. Phenomenology of particle dark matter in broad strokes.

2. Laboratory searches of dark matter and mediator particles. Beam experiments (colliders, beam dumps, intensity frontier). Direct detection efforts underground. Blind spots for direct detection.

3. Cosmological and astrophysical probes of dark sectors.

Evidence for New Physics

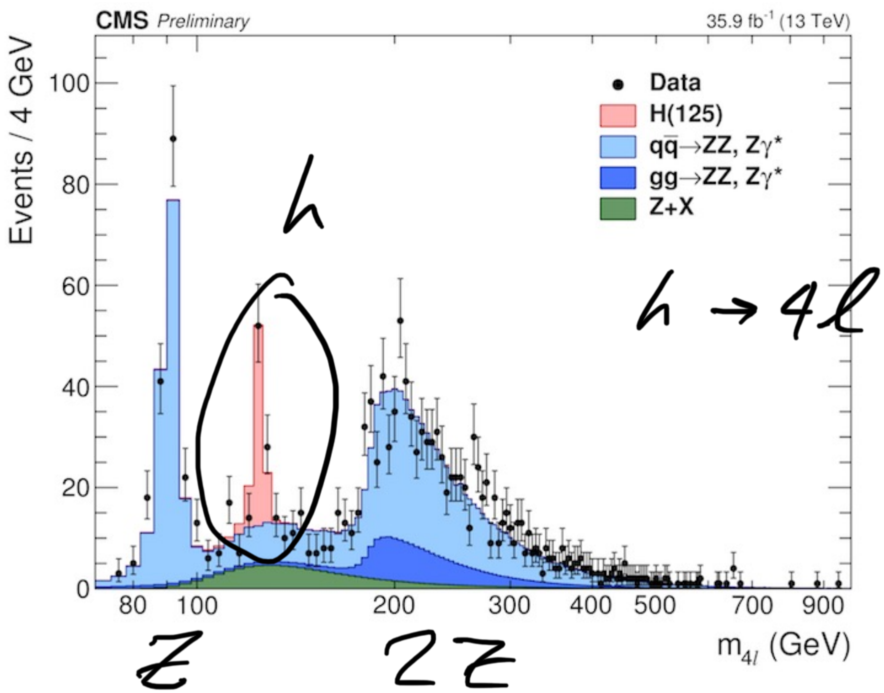
- Standard Model based on SU(3)*SU(2)*U(1) interactions is a well-established paradigm

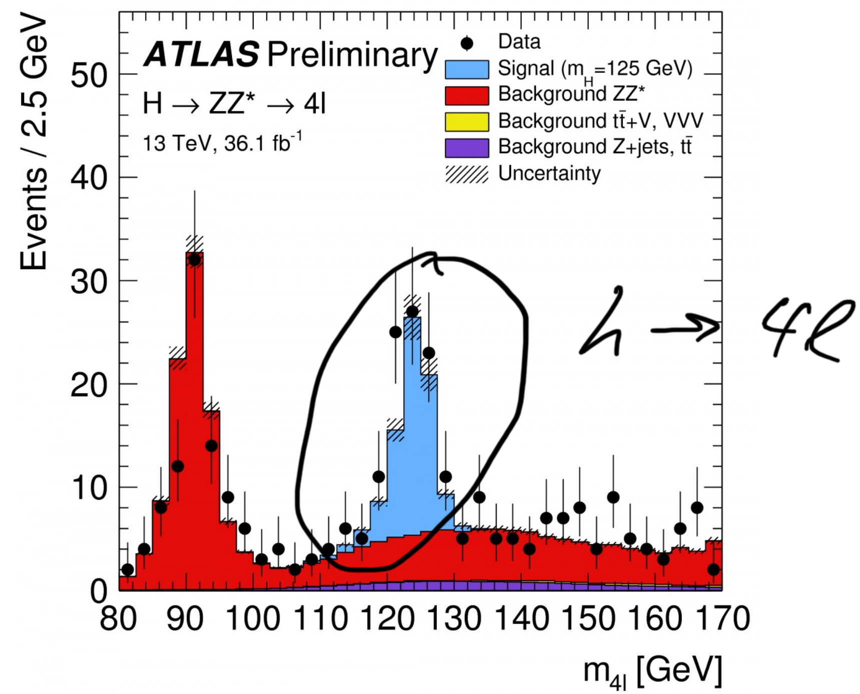
- Evidence for "New Physics" – interactions and particles and fields beyond the SM field content – comes from the neutrino physics and cosmology

- These are enormous subjects to cover in 3 lectures – but a lot of reference literature exists.

Higgs boson discovery

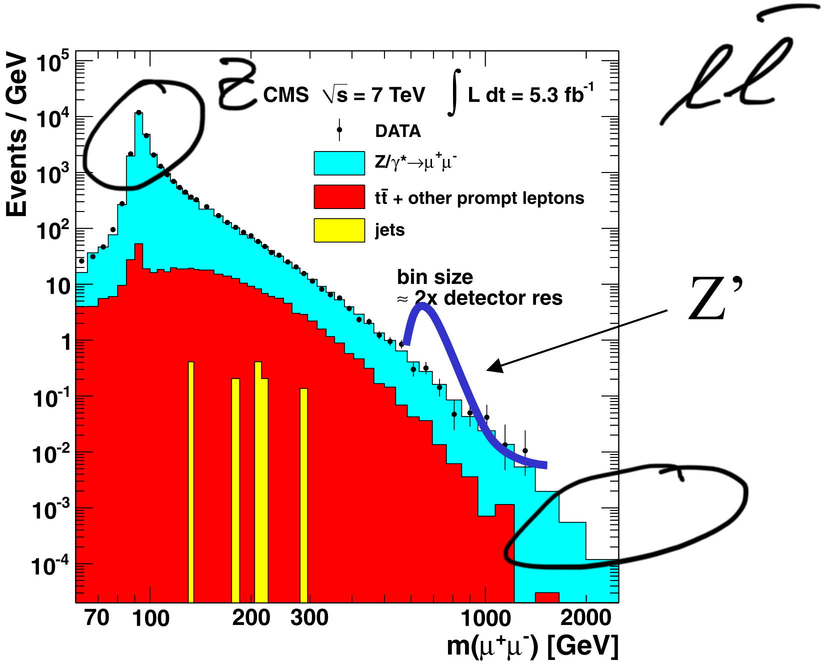
New particle and new type of fundamental force:

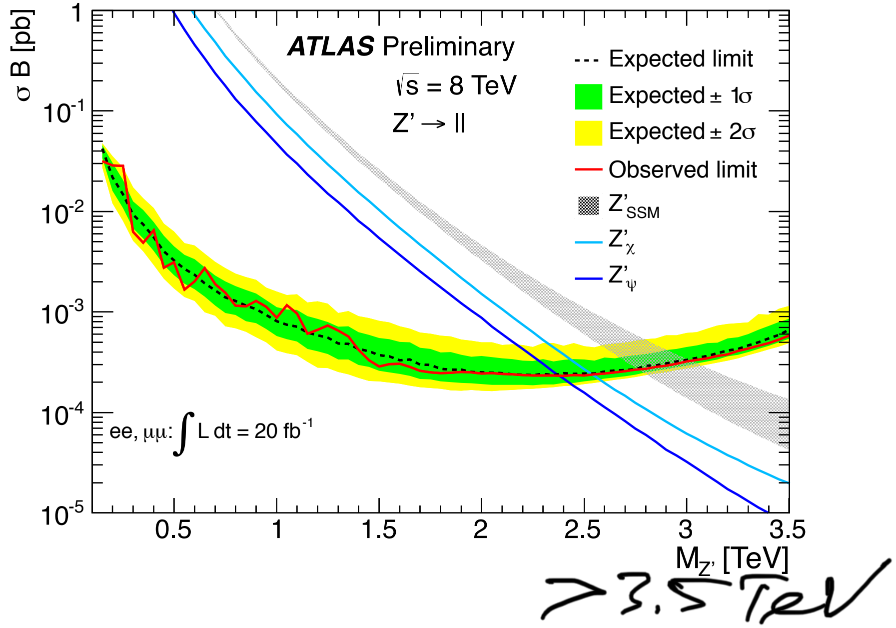


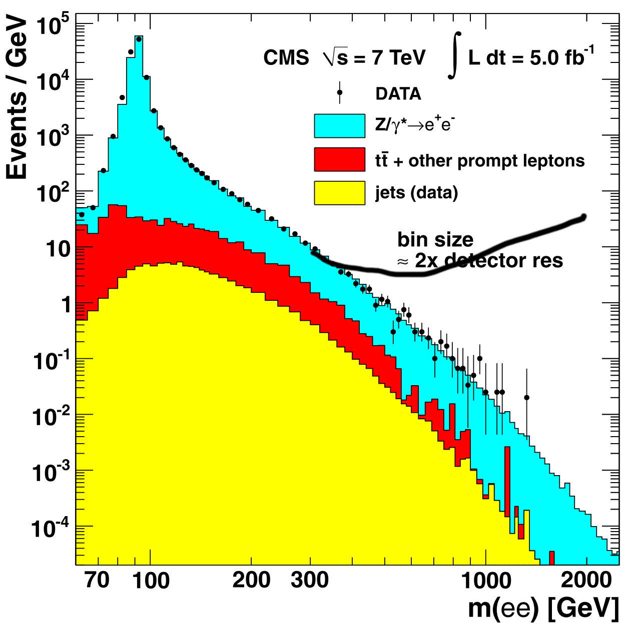


1. A new 0^+ resonance is observed at the LHC. ~50 years after prediction
2. Its properties are fully consistent with the properties of the Standard Model Higgs boson. Mass = 125 GeV (to 0.25%).
3. The discovery is remarkable because the prediction of the Higgs boson was based on theoretical consistency (and minimality!)

No New Physics at high energy thus far (?!)



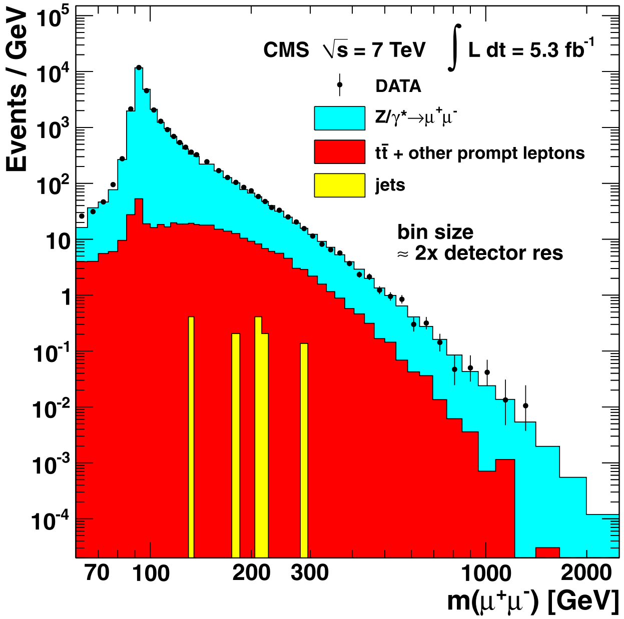


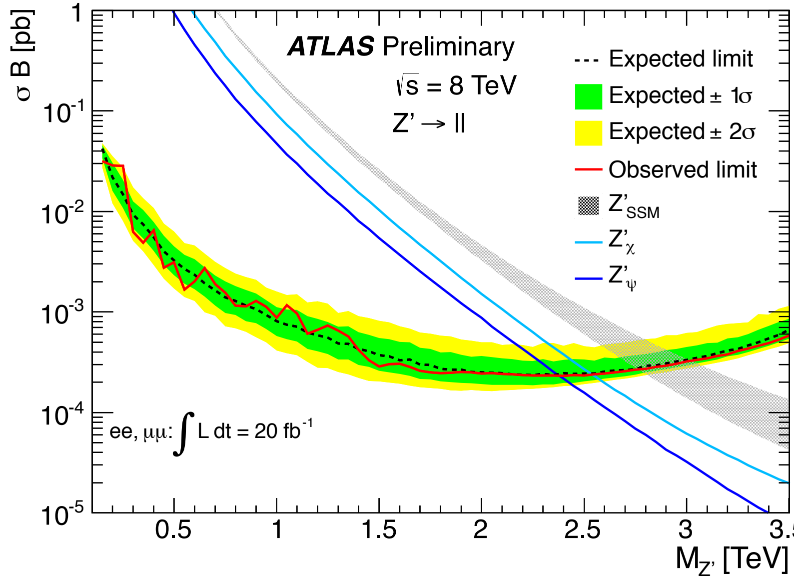


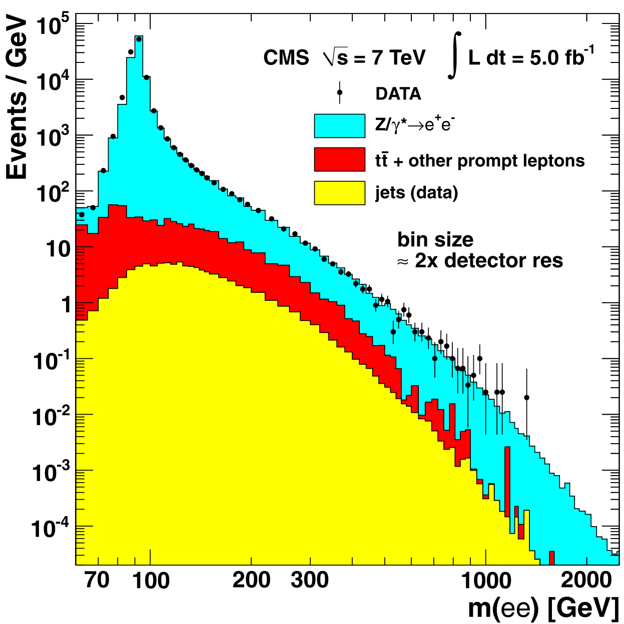
No hints for any kind of new physics. Strong constraints on SUSY, extra dimensions, technicolor resonances, etc.

Constraints on new *Z'* bosons push new gauge groups into multi-TeV territory.

No New Physics at high energy thus far (?!)







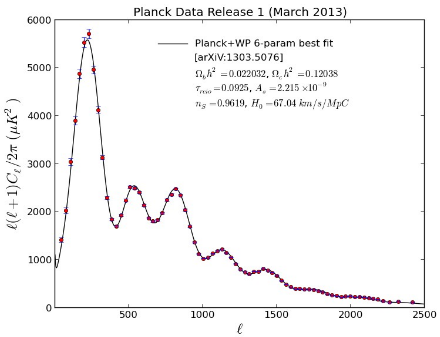
No hints for any kind of new physics. Strong constraints on SUSY, extra dimensions, technicolor resonances, etc.

Constraints on new *Z'* bosons push new gauge groups into multi-TeV territory.

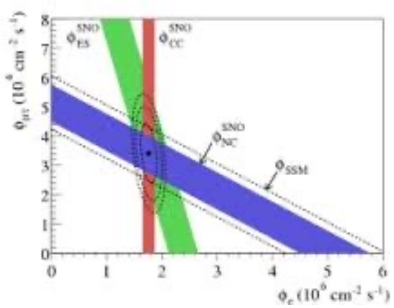
Are our basic assumptions wrong? Where else to look? What to do?

Clues for new physics

1. *Precision cosmology:* 6 parameter model (Λ-CDM) correctly describes statistics of 10^6 CMB patches. Existence of dark matter and dark energy. Strong evidence for inflation.

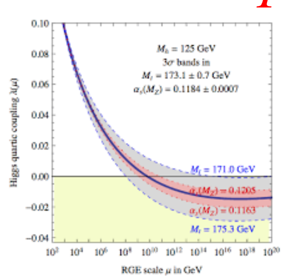


2. *Neutrino masses and mixing:* Give us a clue [perhaps] that there are new matter fields beyond SM. Some of them are not charged under SM.



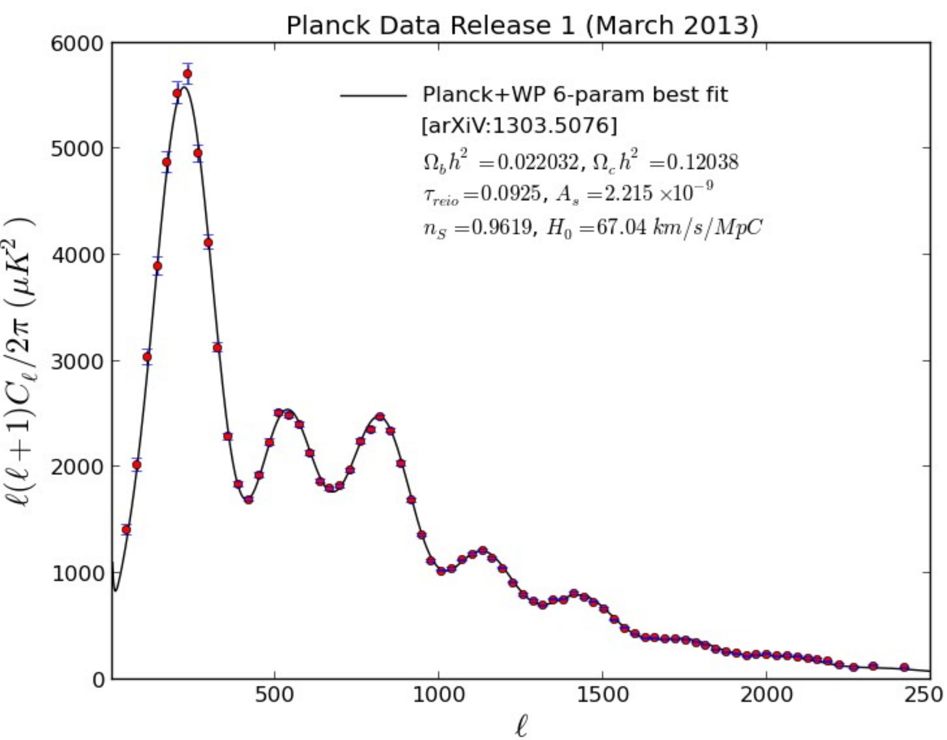
New physics

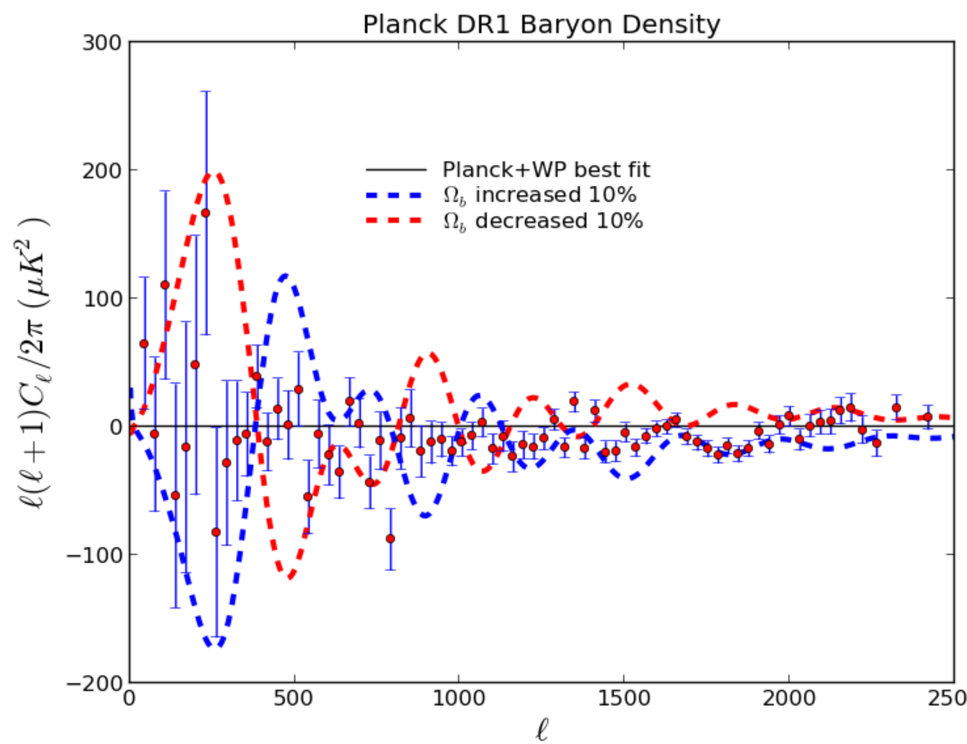
3. *Theoretical puzzles:* Strong CP problem, vacuum stability, hints on unification, smallness of m_h relative to highest scales (GUT, M_{Planck})



4. *"Anomalous results":* muon g-2, "proton radius puzzle", "cosmological lithium problem", small scale CDM problems…

Data from first Planck release in 2013





Parameter	Value (68%)
$\Omega_b h^2$	0.02207±0.00027
$\Omega_c h^2$	0.1198±0.0026 (is it high?)
$100\theta_*$ (acoustic scale at recombination)	1.04148±0.00062 (~ 500 parts per million accuracy)
τ	0.091±0.014 (WMAP seeded)
$\ln(10^{10}A_s)$	3.090±0.025
n_s	0.9585±0.0070 (<1 at > 5 σ)

Parameter	Value (95%)
Ω_K	-0.0005±0.006
$\Sigma\ m_\nu$ (eV)	<0.23
N_{eff}	3.30±0.54
Y_P	0.267±0.040

SM Lagrangian as an EFT

New orthodoxy: Standard Model Lagrangian includes all terms of canonical dimension 4 and less, consistent with three generations of quarks and leptons and the SU(3)*SU(2)*U(1) gauge structure at classical and quantum levels. strong weak EM

- Higgs is finally discovered. Alternatives (e.g. strong coupling at a TeV) are mostly dead/severely constrained.
- CP violation in the quark sector comes CKM
- Neutrinos contain intriguing clues (Masses and oscillations were not part of the 1967 Weinberg-Salam model).
- Problems: Strong CP problem, dark matter problem, neutrino mass problem, and more conceptual problems (gauge hierarchy).

SM as an Effective Field Theory

Typical BSM model-independent approach is to include all possible BSM operators + light new states explicitly.

$\mathcal{L}_{2020s} = - m_H^2\ (H^+_{SM} H_{SM}) +$ all dim 4 terms $(A_{SM},\ \psi_{SM},\ H_{SM}) +$

Neutrino mass operators (e.g. effective Dim=5) $\quad (LH)^2/\Lambda$

$+$(Wilson coeff. $/\Lambda^2$) $\times$ Dim 6 etc $(A_{SM},\ \psi_{SM},\ H_{SM})\ + \ldots$

all lowest dimension portals $(A_{SM},\ \psi_{SM},\ H,\ A_{DS},\ \psi_{DS},\ H_{DS}) \times$ portal couplings

$+$ dark sector interactions $(A_{DS},\ \psi_{DS},\ H_{DS})$

SM -- Standard Model

DS – Dark Sector – neutral under SM gauge fields

Neutral "portals" to the SM

Let us *classify* possible connections between Dark sector and SM

$H^+H\,(\lambda S^2 + A\,S)$ Higgs-singlet scalar interactions (scalar portal)

$B_{\mu\nu}V_{\mu\nu}$ "Kinetic mixing" with additional U(1)' group

(becomes a specific example of $J_\mu^i A_\mu$ extension)

LHN neutrino Yukawa coupling, N – RH neutrino

$J_\mu^i A_\mu$ requires gauge invariance and anomaly cancellation

It is very likely that the observed neutrino masses indicate that Nature may have used the *LHN* portal…

Dim>4

$J_\mu^A\ \partial_\mu a/f$ axionic portal

……….

$$\mathcal{L}_{\rm mediation} = \sum_{k,l,n}^{k+l=n+4} \frac{\mathcal{O}^{(k)}_{\rm med}\mathcal{O}^{(l)}_{\rm SM}}{\Lambda^n},$$

How to look for New Physics ?

1. High energy colliders.

$$\frac{1}{\Lambda^2}(\bar{e}e)(\bar{q}q) \to \sigma \propto \frac{E^2}{\Lambda^4} \to \Lambda > 10\,\mathrm{TeV}$$

2. Precision measurements, especially when a symmetry is broken

$$\frac{1}{\Lambda_{\mathrm{CP}}^2}(\bar{e}i\gamma_5 e)(\bar{q}q) \to \mathrm{EDM}, \frac{1}{\Lambda_{\mathrm{CP}}^2} < 10^{-10}G_F \to \Lambda_{\mathrm{CP}} > 10^7\,\mathrm{GeV}$$

3. Intensity frontier experiments where abnormal to SM appearance of FIPs (or sometimes disappearance, e.g. NA64) can be searched.

$$pp \to \pi, K, B \to HNL + X \to HNL \text{ decay to SM}$$

4. DM searches: *Atom + DM* → *visible energy*

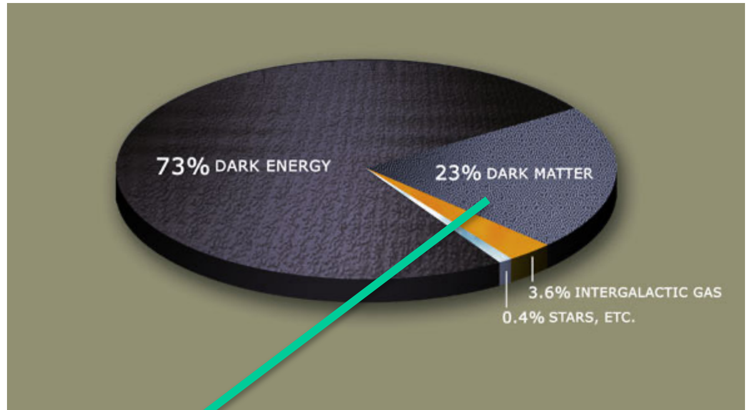
   DM + DM → *visible energy*

All these methods are employed to look for Dark Sector, and associated particles, such as Dark Matter and mediators.

These are the most relevant dark matter questions!

"Get in touch with DM" – story of *direct detection of DM*.

Why linking dark matter to particle physics is not easy



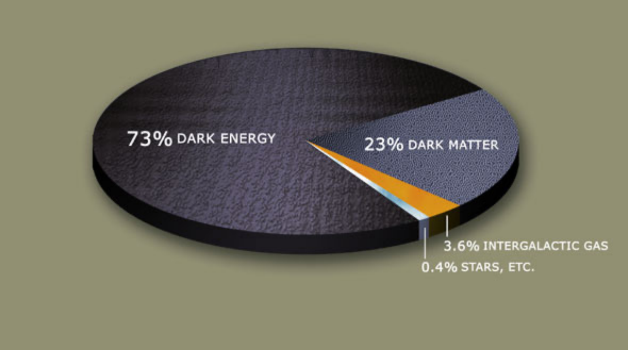
Av. Density ~ 0.3 GeV/cc – not a lot

$L_{min} \sim 10^{21}$ cm

$L_{exp} \sim$ few * 10^2 cm

We need to extrapolate 19 orders of magnitude!

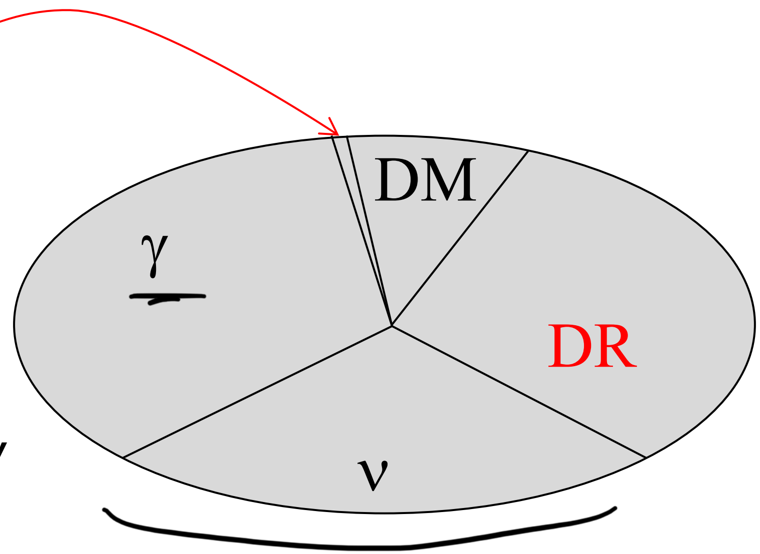
Theory is the first step!



Mass and number density of DM particles is unknown

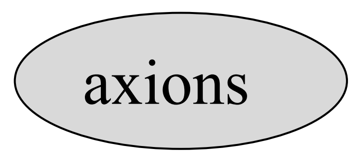
Atoms

In Energy chart they are 4%. In number density chart ~ 5 $\times 10^{-10}$ relative to γ



We have no idea about DM number densities. (WIMPs ~ 10^{-8} cm^{-3}; axions ~ 10^{9} cm^{-3}. Dark Radiation, Dark Forces – Who knows!).

Number density chart for axionic universe:



Lack of precise knowledge about nature of dark matter leaves a lot of room for existence of dark radiation, and dark forces – dark sector in general.

Lesson from precision cosmology:

1. Universe was relatively *simple* at T ~ 0.3 eV.
2. The dark matter was already "*in place*" at the time of the matter-radiation equality, when the potential wells created by DM started to grow. We see statistical evidence of H and He falling (and rebounding) from the DM gravitational wells. The amount of He and D is consistent with primordial nucleosynthesis
3. DM is not "made of ordinary atoms" – and there is 6 times more of it than of ordinary H and He. $\Omega_{\text{dark matter}} / \Omega_{\text{baryons}} = 5.4$

4. **What is it?** These are *not* known neutrinos: they would have to weigh ~ 50 eV (excluded), and would have a hard time making smaller scale structure (too hot to cluster on small scales). Simplicity of the early Universe, makes many of us suspect that the DM might be in the form of unknown (= e.g. beyond-SM) particles.

DM classification

At some early cosmological epoch of hot Universe, with temperature T >> DM mass, the abundance of these particles relative to a species of SM (e.g. photons) was

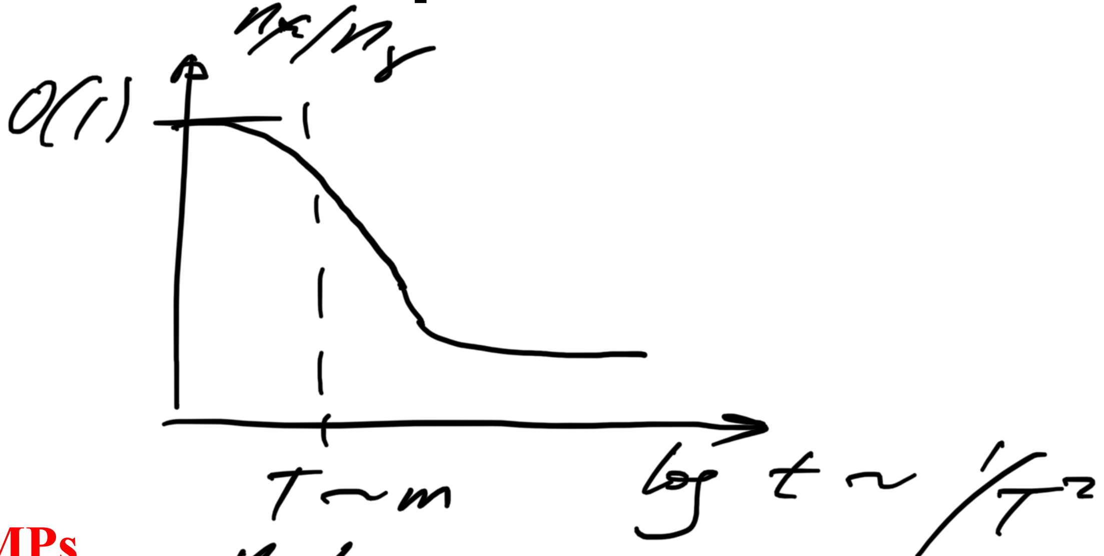
Normal: Sizable interaction rates ensure thermal equilibrium, $N_{DM}/N_\gamma = 1$. Stability of particles on the scale $t_{Universe}$ is required. *Freeze-out* calculation gives the required annihilation cross section for DM --> SM of order ~ 1 pbn, which points towards weak scale. These are **WIMPs**. Asymmetric DM is also in this category.

Very small: Very tiny interaction rates (e.g. 10^{-10} couplings from WIMPs). Never in thermal equilibrium. Populated by thermal leakage of SM fields with sub-Hubble rate (*freeze-in*) or by decays of parent WIMPs. [Gravitinos, sterile neutrinos, and other "feeble" creatures – call them **superweakly interacting MPs**]

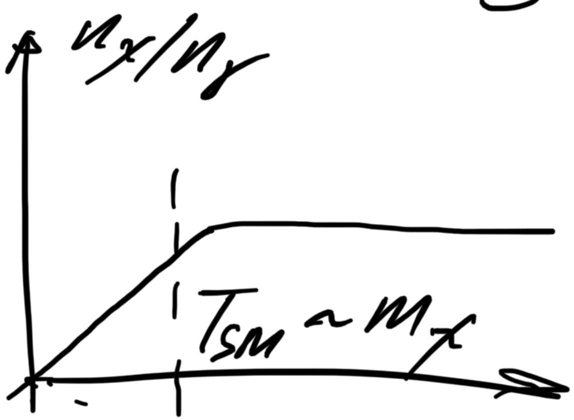
Huge: Almost non-interacting light, m< eV, particles with huge occupation numbers of lowest momentum states, e.g. $N_{DM}/N_\gamma \sim 10^{10}$. "Super-cool DM". Must be bosonic. Axions, or other very light scalar fields – call them **super-cold DM**.

Parametric dependence of the abundance

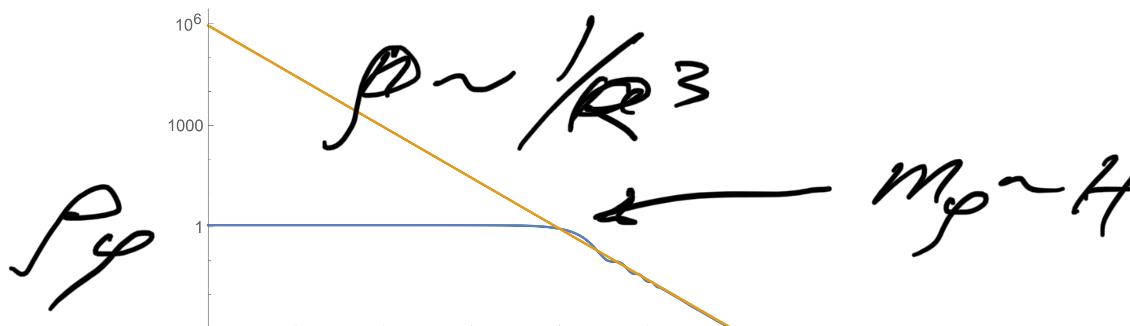
WIMPS



Super-WIMPs



Bosonic condensate DM

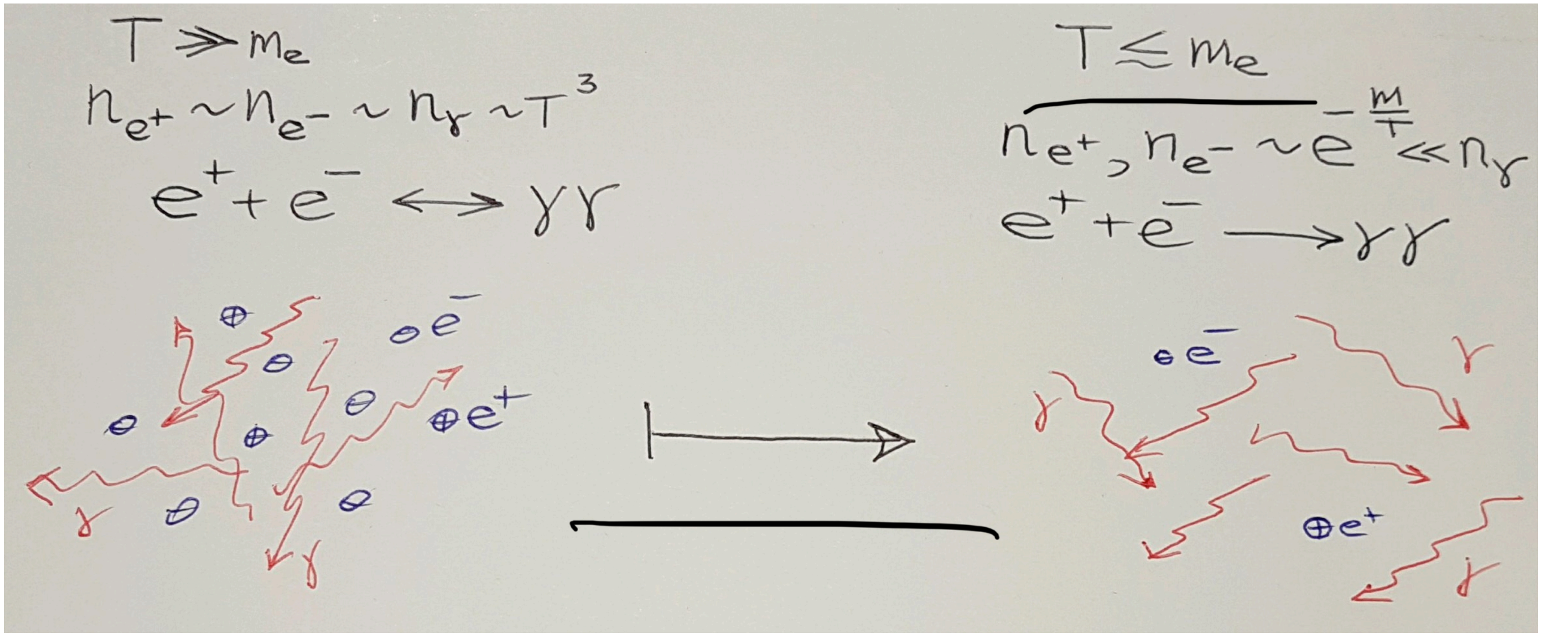


Weakly interacting massive particles

More technical definition: *required abundance is achieved via self-annihilation into the SM states.*

Annihilation in the early Universe

Let us follow the history of stable SM particles, e.g. electrons.



At temperatures T~ MeV and above (k=c=hbar=1 from now on), electrons and positrons are as abundant as photons. As T becomes smaller than m_e, the annihilation depletes charged particles, whose abundance becomes Boltzmann-suppressed. Process ends as you ran out of positrons.

WIMPs : "right abundance" as long as $< \sigma\,(v/c) > = 10^{-36}$ cm^2.

Cosmic Expansion

Einstein's → Freidmann's equation:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G_N T_{\mu\nu}$$

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G_N \rho$$

$$\frac{\ddot{R}}{R} = -\frac{8\pi}{3} G_N (\rho + 3p)$$

$$\dot{\rho} = -3H(\rho + p)$$

$$R(t)^3 = R_0^3 \frac{\Omega_m}{\Omega_\Lambda} \left[\sinh\left(\frac{3}{2}\Omega_\Lambda^{1/2} H_0 t\right)\right]^2$$

$\rho_\Lambda = R^0 \text{const}$

$\rho_m \sim \frac{\text{const}}{R^3}$

$\rho_{rad} \sim \frac{\text{const}}{R^4}$

$\frac{\dot{R}}{R} \sim \text{const} \frac{1}{R^2}$; $R \sim \sqrt{t}$

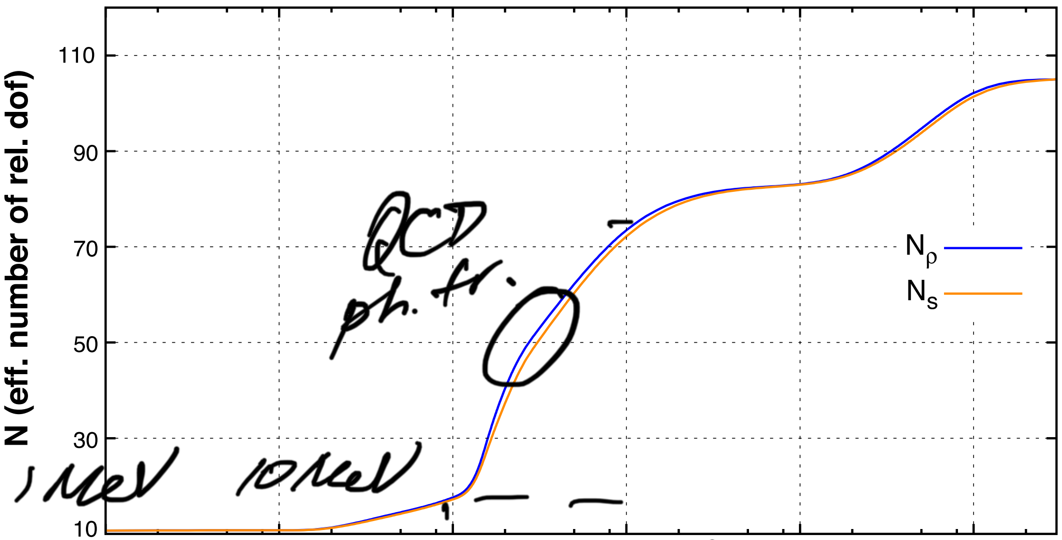
$H = \frac{\dot{R}}{R} = \frac{1}{2t}$

FIPs can contribute to the r.h.s. of these equations

Hot Universe

$$\rho = \left(\sum_{\rm B} g_{\rm B} + \frac{7}{8}\sum_{\rm F} g_{\rm F}\right)\frac{\pi^2}{30}T^4 \equiv \frac{\pi^2}{30}N(T)\,T^4$$

$$H(t) = \frac{1}{2t}$$



$$t = \left(\frac{90}{32\pi^3 G_{\rm N} N(T)}\right)^{1/2} T^{-2}\,.$$

$$H \simeq 1.6\sqrt{g_*}\,\frac{T^2}{M_{pl}}$$

$$M_{pl} \simeq 1.2\times 10^{19}\,\text{GeV}$$

$$\frac{t}{178\text{sec}} = \left(\frac{10^9\text{K}}{T}\right)^2$$

$$n_\gamma \simeq 0.24\,T^3$$

Equilibrium distribution

$$n_{F(B)}(T) = g_{F(B)}\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{\exp[\sqrt{p^2+m^2_{F(B)}}/T]\pm 1}$$

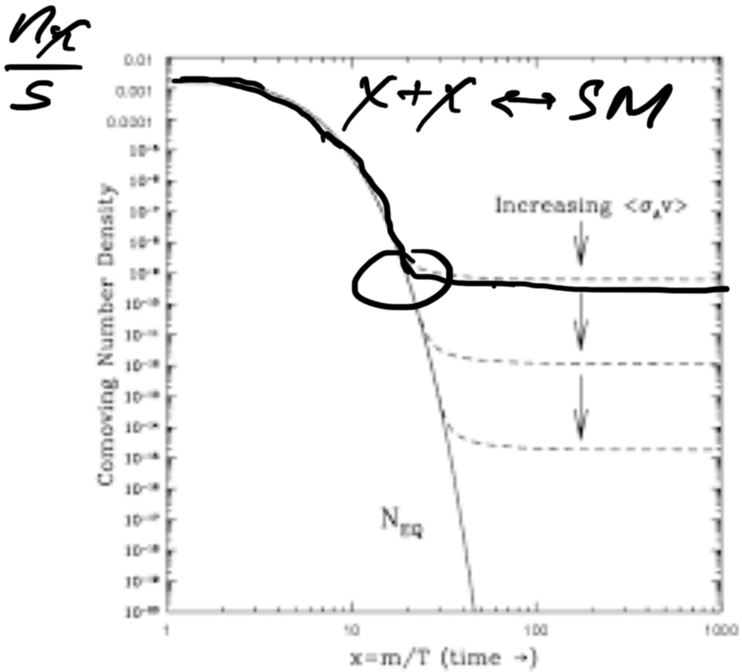
$$T \gg m_{F(B)} \to n_{F(B)} \sim n_\gamma;\quad T \ll m_{F(B)} \to n_{F(B)} \sim (mT)^{3/2}\exp(-m/T)$$

Weakly interacting massive particles

In case of electrons and positrons (when the particle asymmetry = 0), the end point is $n_e/n_{gamma} \sim 10^{-17}$. It is easy to see that this is a consequence of a large annihilation cross section ($\sim \alpha^2/m_e^2$).

We need a particle "X" with smaller annihilation cross section,

X + X → SM states.



Honest solution of Boltzmann equation gives a remarkably simple result. $\Omega_X = \Omega_{DM}$, observed if the annihilation rate is

$$\langle \sigma_{ann} v \rangle \approx 1\text{pbn} \times c$$

$10^{-36}\,\text{cm}^2 = \alpha^2/\Lambda^2$ → $\Lambda = 140$ GeV. **Λ ~ weak scale!!!** First implementations by (Lee, Weinberg; Dolgov, Zeldovich,….)

freeze-out formula: sketch of derivation

$\Gamma_{ann} \sim n_{\bar{\chi}} \cdot \langle \sigma v \rangle_{\chi\bar{\chi}}$ H

$>$ → equilibrium

$<$ → rate dropping out of eq.

$$n_\chi^{eq} \langle \sigma v \rangle \simeq H$$

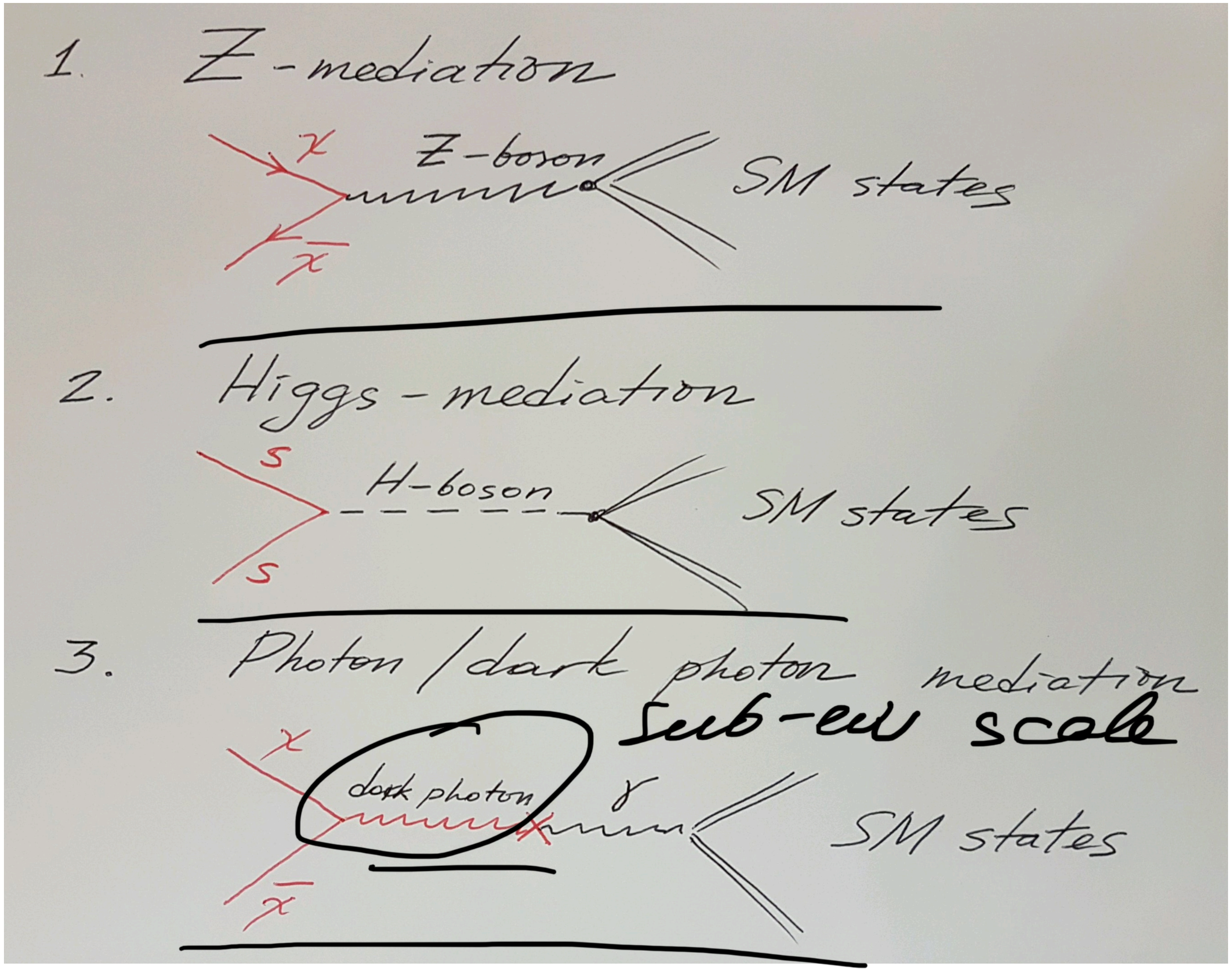
$$T_{freeze.out}/M \sim 1/20$$

After freeze out

$\frac{n_\chi}{n_\gamma} \sim const$ $\frac{\rho_\chi}{\rho_\gamma} \simeq \frac{M}{T}$

$\left(\frac{\rho_\chi}{\rho_\gamma}\right) \sim \left(\frac{\rho_{DM}}{\rho_\gamma}\right)_{obs}$ if $\langle \sigma v \rangle \sim 1/\text{pbn} \times C$

Examples of DM-SM mediation



Very economical extensions of the SM.

DM particles themselves + may be extra mediator force. Can be very predictive.

If dark matter annihilation is mediated by weak scale particles, the mass of dark matter is confined to ~ 10 –to-10000 GeV (Lee, Weinberg)

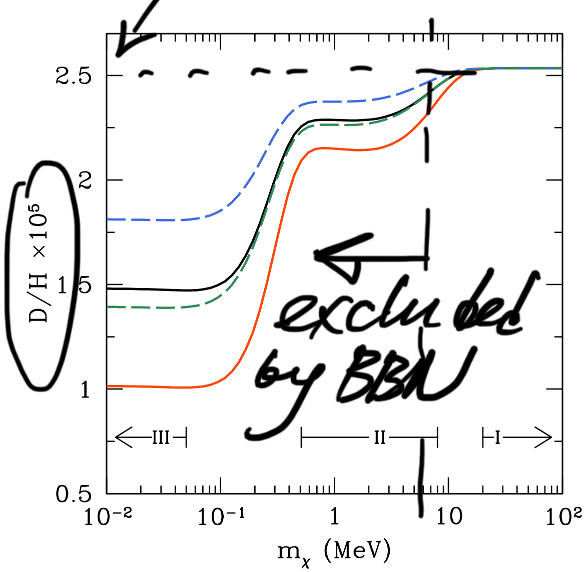
Lee-Weinberg window, light DM, BBN constraint

$\langle \sigma v \rangle \sim 10^{-36}\,\mathrm{cm}^2$ $\qquad$ $\langle \sigma v \rangle \sim \frac{\text{coupling}}{m_\chi^2}$

few GeV $\lesssim m_\chi \lesssim 10$ TeV

$\langle \sigma v \rangle \sim G_F^2 m_\chi^2 \Rightarrow$ few GeV m_χ

observed.



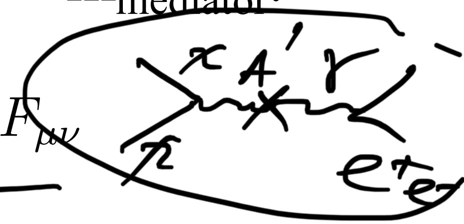
WIMP masses below ~ few GeV require sub-EW mediators.

WIMP masses below a few MeV are inconsistent with BBN

Examples of sub-GeV WIMPs

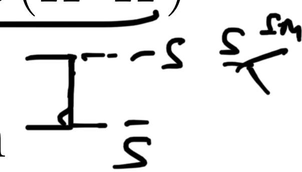
- Scalar dark matter talking to the SM via a "dark photon" (variants: L_{mu}-L_{tau} etc gauge bosons). With $2m_{DM} < m_{mediator}$:

$$\mathcal{L} = |D_\mu \chi|^2 - m_\chi^2 |\chi|^2 - \frac{1}{4} V_{\mu\nu}^2 + \frac{1}{2} m_V^2 V_\mu^2 - \frac{\epsilon}{2} V_{\mu\nu} F_{\mu\nu}$$



- Fermionic dark matter talking to the SM via a "dark scalar" that mixes with the Higgs. With $m_{DM} > m_{mediator}$:

$$\mathcal{L} = \overline{\chi}(i\partial_\mu \gamma_\mu - m_\chi)\chi + \lambda \overline{\chi}\chi S + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2} m_S^2 S^2 - AS(H^\dagger H)$$



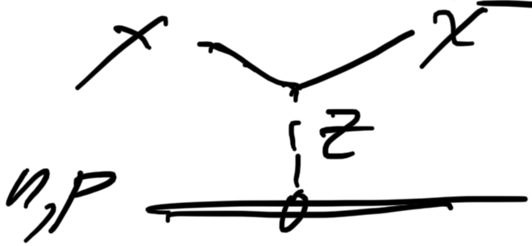
After EW symmetry breaking *S* ("dark Higgs") mixes with physical *h*, and can be light and weakly coupled provided that coupling A is small.

Take away point: *with lots of investment in searching for DM with masses > GeV, models with sub-GeV DM can be a blind spot.*

Theoretical predictions for $\sigma_{DM\text{-}N}$

- Unlike annihilation of WIMP DM (whose inferred cross section is quite model independent), the scattering cross section $\boldsymbol{\sigma_{DM\text{-}N}}$ does depend on the model.

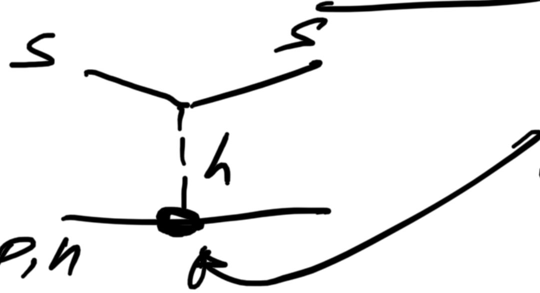
$\sigma_{DM\text{-}Nucleon}$ (Z-mediated) $\sim (1/8\pi)\, m_p^2 (G_F)^2 \sim (10^{-39}\text{-}10^{-38})$ cm^2 range.



σ

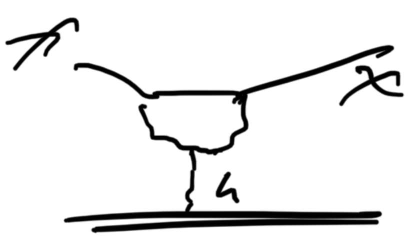
$g_{Znn} \sim \frac{g_1}{2}\left(-\frac{1}{2}\right)$

$\sigma_{DM\text{-}Nucleon}$ (Higgs-mediated) $\sim (10^{-4}\text{-}10^{-5}) \times \sigma_{DM\text{-}Nucleon}$ (Z-mediated)



$g_{hnn} \sim \frac{m_N (\sim 1/5)}{v} \sim \frac{1\,\text{GeV}}{246\,\text{GeV}}\, \frac{1}{5} \sim 10^{-3}$

$\sigma_{DM\text{-}Nucleon}$ (EW loop) $\sim 10^{-9} \times \sigma_{DM\text{-}Nucleon}$ (Z-mediated)

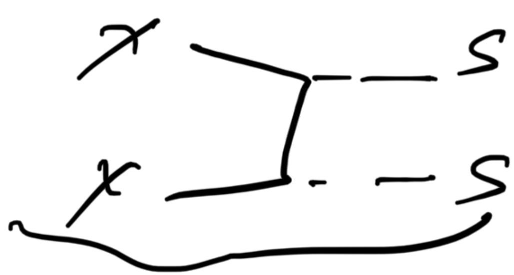


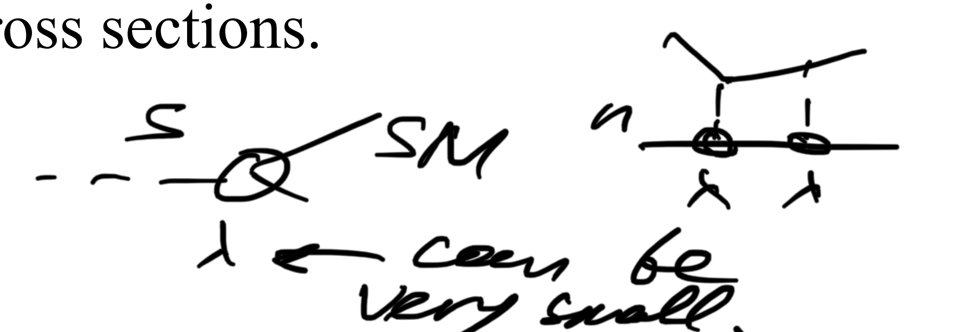
Scattering is *very* dependent on DM type

- Spin-dependent cross sections on nuclei with are ~ $1/A^2$ ~ 10^{-4} times smaller than spin-independent due to a coherence factor.
- Going Dirac → Majorana can greatly (~ 10^{-5}) suppress the rates.

$$\bar{\chi}\gamma_\mu\chi = 0$$
$$\bar{\chi}\gamma_\mu\gamma_5\chi \simeq \frac{\vec{\sigma}_\chi\cdot\vec{p}_\chi}{m} \qquad \frac{p_\chi}{m}\sim 10^{-3}c$$

- For some models there is no tree-level exchange between a nucleon and a DM particle. Loop level typically brings another $(\alpha_W/\pi)^2 \sim 10^{-4}$ suppression in the cross section.
- Secluded WIMPs (2 DM → 2 mediators followed by mediator decay to SM) can have terribly small cross sections.





Implication of the successful stream of Xe-based DM experiments

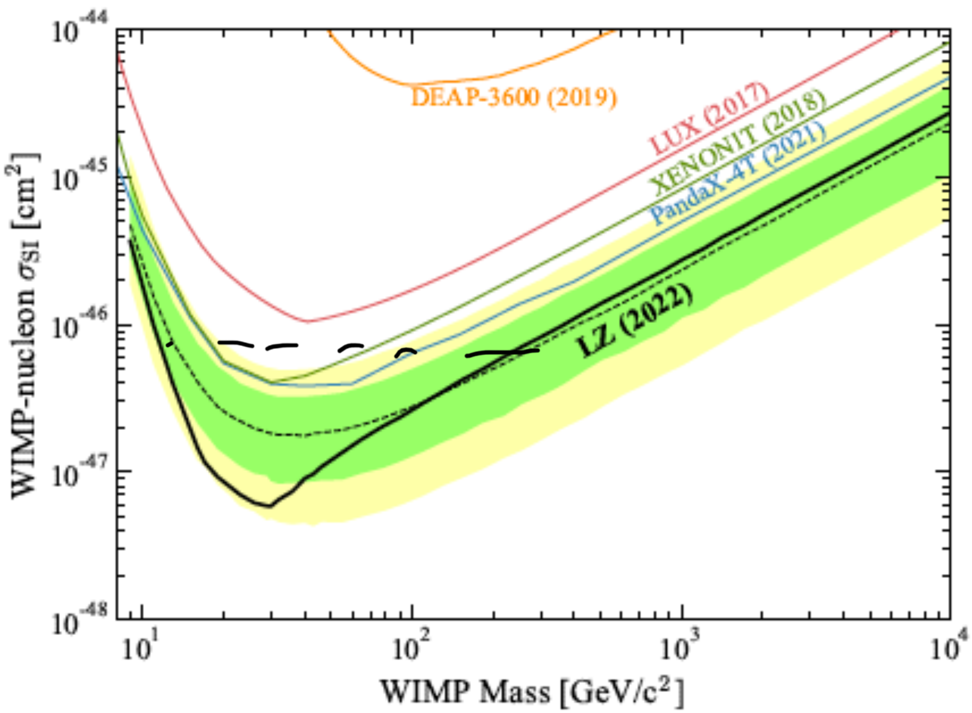
- Series of successful experiments: Xenon-100,1T,NT; LUX, LZ; PandaX's have pushed the limits on the nucleon cross section for weak-scale mediated Dark Matter.

- While Z-exchange based models (a-la Lee and Weinberg) has long been ruled out, new constraints put significant pressure on Higgs-mediated models, pushing them into multi-TeV territory. Loop-induced Higgs/W-box models (e.g. SUSY Higgsino-like) will "soon" be probed.

- Large mass and self-shielding properties also allow for the breakthrough sensitivities for the electron recoil ($E_{recoil} > 200$ eV), providing strong constraints on light DM, and on exotic particle emission from the Sun.

Interpreting recent LZ results for the Higgs-mediated scalar DM model

arXiv:2207.03764v1



- The best sensitivity at $m_{DM} \sim$ 30 GeV drops below 10^{-47}cm^2 benchmark

In the scaling regime, $m_{DM} > m_{Xe}$, the limit on the DM-nucleon cross section is $\sigma < 2.5\ 10^{-46}$ cm^2 (m_{DM}/TeV)

This has strong implications for particle physics models of WIMP DM.

Simplest DM model → $\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu S)^2 - m_0^2 S^2 + \lambda S^2 (H^\dagger H)$

Interpreting recent LZ results for the Higgs-mediated scalar DM model

Combining together a prior on the dark matter annihilation cross section,

$$\langle \sigma_{ann} v \rangle = \frac{\lambda^2}{4\pi m_S^2} \simeq 10^{-36} \mathrm{cm}^2 \times c$$

with the expression for the Higgs-boson-mediated nucleon-DM scattering cross section

$$\sigma_{pS} = \frac{\lambda^2}{\pi^2 m_S^2} \frac{m_p^2 \, (200\,\mathrm{MeV})^2}{m_h^4}$$

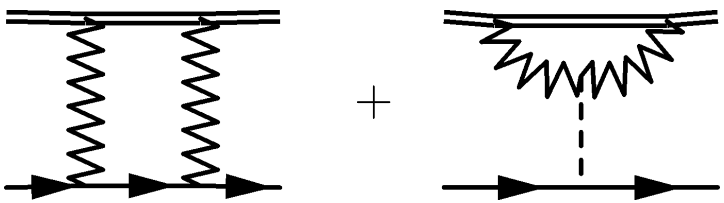
and using LZ limit $\sigma_{pS} < 2.5\ 10^{-46}\ \mathrm{cm}^2\ (m_S/\mathrm{TeV})$ we obtain the limit

$$m_S \gtrsim 1\,\mathrm{TeV}$$

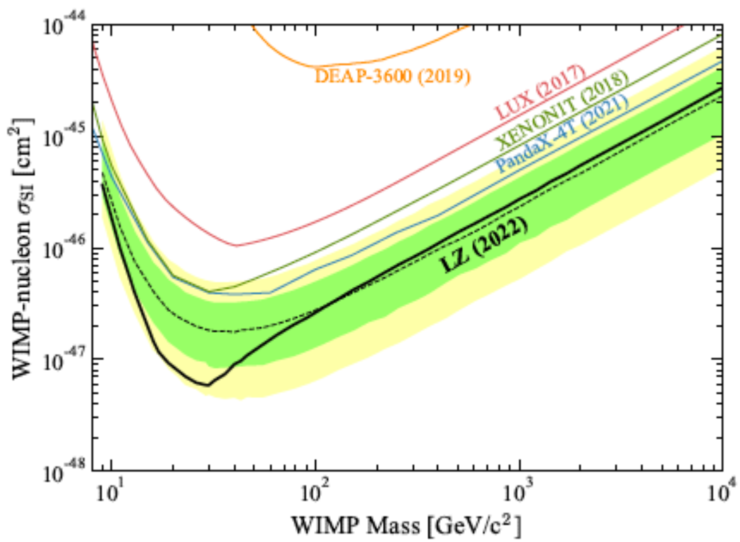
It implies that the coupling constant λ becomes moderately large, $\lambda > 0.15$, making it larger than the Higgs self-interaction coupling. *Subsequent experimental improvements may completely rule out this minimal type of models.*

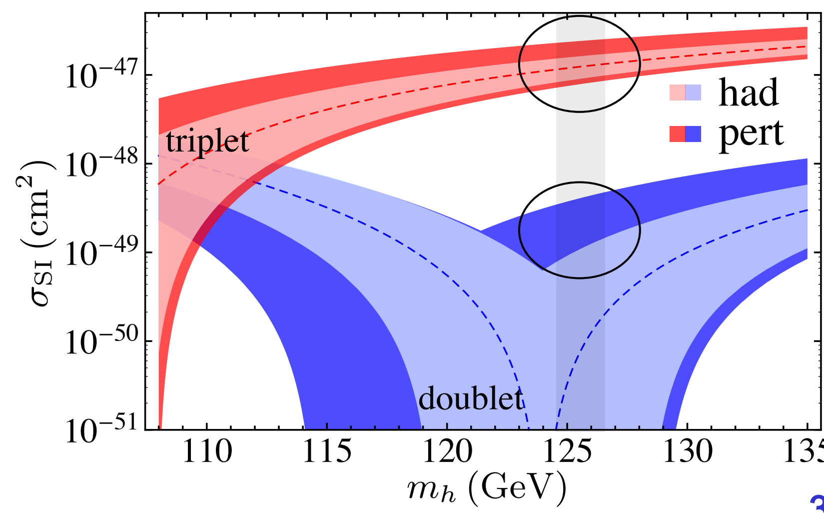
Next frontier – loop-mediated EW interaction

Models of heavy particles that have EW interactions but do not have a direct coupling to the Z-boson (e.g. due to small mass splitting) will interact via EW loops



W-box, and loop-induced Higgs exchange





From Hill, Solon, 2013

End of lecture 1

Freeze-in (i.e. superweakly interacting DM)

Initial abundance is negligible. Thermal production is small at all times $\Gamma_{SM \to DM}/H(T \sim m) << 1$.

Freeze-in dark matter

- Tiniest couplings needed, so that $\Gamma_{DM}/H(T \sim m) << 1$.
- Tiny couplings means that lifetime can be >> τ_{Universe}, and stability is not an issue. Both SM → χ and SM → $\chi\,\chi$ may be acceptable.
- Masses below MeV are Okay – no constraints from the BBN, typically

Sterile neutrinos

dark photons

gravitinos

Freeze-in example

Superweakly interacting massive particles. An example.

$$\mathcal{L} = \mathcal{L}_{\psi,A} + \mathcal{L}_{\chi,A'} - \frac{\epsilon}{2} F_{\mu\nu} F'_{\mu\nu} + \frac{1}{2} m^2_{A'} (A'_\mu)^2.$$

$$\mathcal{L}_{\psi,A} = -\frac{1}{4} F^2_{\mu\nu} + \bar{\psi}[\gamma_\mu (i\partial_\mu - eA_\mu) - m_\psi]\psi$$

$$\mathcal{L}_{\chi,A'} = -\frac{1}{4} (F'_{\mu\nu})^2 + \bar{\chi}[\gamma_\mu (i\partial_\mu - g' A'_\mu) - m_\chi]\chi,$$

Let us take for simplicity, $m_{\text{dark photon}} \rightarrow 0$, and $m_e < m_{DM} < m_\mu$ and consider electron + positron → DM.

$$\Gamma = \sum_{spin} |M|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \times \frac{f_1 f_2 d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^{12} 2^4 E_1 E_2 E_3 E_4}$$

After a long and tedious but otherwise trivial calculation we get,

$$d\Gamma = f_1 f_2 \frac{2^4 \pi^2}{2^8 \pi^6} E_1 E_2 dE_1 dE_2 ds \times \tilde{\Gamma}$$

$$= \frac{\alpha^2 f_1 f_2}{2\pi^3} dE_1 dE_2 ds \frac{2m^2 + s}{3s} \sqrt{1 - \frac{4m^2}{s}}$$

where $\alpha = \alpha_{\text{eff}} = \alpha_{\text{EM}} * \varepsilon$.

This is number of particles emitted per volume per time

Freeze-in example

Continued

Approximating $f_i \simeq \exp(-E_i/T)$, we get

$$\Gamma = \frac{\alpha^2 T}{3 \times 2\pi^3} \int_{4m^2} ds \times \sqrt{s} K_1(\sqrt{s}/T) \left(1 - \frac{4m^2}{s}\right)^{1/2} \frac{2m^2 + s}{s}$$

where *s* is the usual Mandelstam parameter.

$$\frac{n_{\chi+\bar{\chi}}}{n_{e^-}} = 2 \times \int_0^\infty \frac{dT}{TH} \times \frac{\Gamma}{n_{e^-}}.$$

$$\frac{n_{\chi+\bar{\chi}}}{n_{e^-}} = 2 \times \frac{\mathcal{C} m^4}{H(T=m) n_{e^-}(T=m)} \times \int_0^\infty I(x) dx,$$

$$\mathcal{C} = \epsilon^2 \frac{\alpha^2}{3 \times 2\pi^3}$$

Finally, the function *I(m/T)* that enters here is given by

$$x \equiv m/T \qquad I(m/T) = \int_{2x}^\infty 2y^2 dy \times K_1(y) \left(1 - \frac{4x^2}{y^2}\right)^{1/2} \frac{2x^2 + y^2}{y^2},$$

Freeze-in example

Continued

Numerically, we get

$$\frac{n_{\chi+\bar{\chi}}}{s} = 2 \times \frac{\mathcal{C} m^4}{s(T=m)H(T=m)} \times 4.16, \qquad \mathcal{C} = \epsilon^2 \frac{\alpha^2}{3 \times 2\pi^3}$$

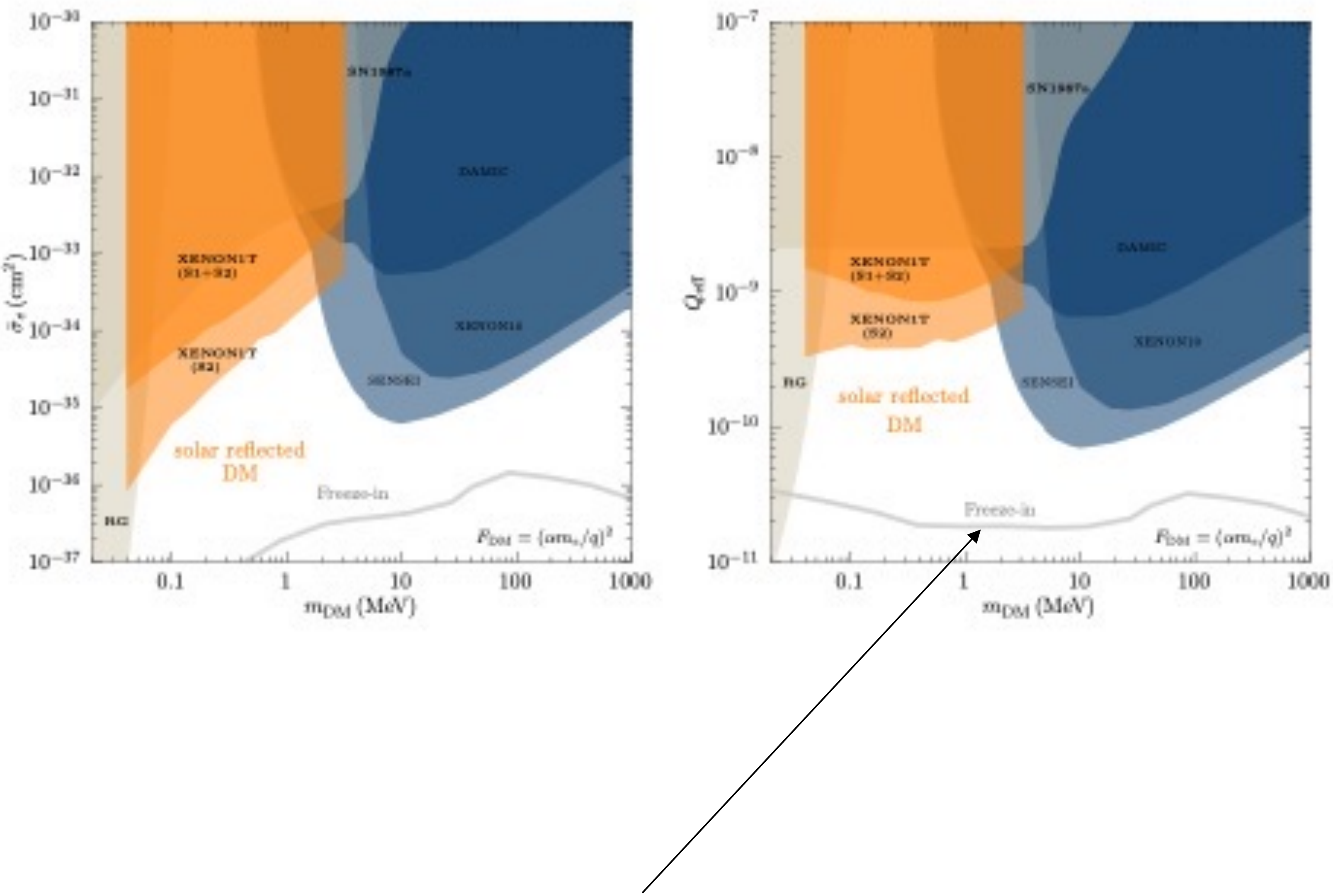
We need to adjust ε to get the correct abundance. Observed abundance is given by

$$s(T) = \frac{2\pi^2}{45} g_*(T) T^3; \quad g(T_*) = 2 + \frac{7}{8}(2 \times 2 + 3 \times 2) = \frac{43}{4},$$

$$\frac{n_{\chi+\bar{\chi}}}{s} = \frac{n_{\chi+\bar{\chi}}}{n_b}\frac{n_b}{s} = \frac{m_p}{m_\chi}\frac{\rho_{DM}}{\rho_b}\frac{n_b}{s} \simeq 4.3 \times 10^{-8} \times \frac{10\text{MeV}}{m_\chi},$$

Equating this, we get m-independent answer for a required value of ε :

$$\epsilon \simeq 1.96 \times 10^{-11}.$$



- We got a consistent number with existing literature.

Conclusions

Important points about WIMPs:

- abundance + BBN forces WIMPs into few MeV – 10 TeV windows, while requiring 1pbn × c annihilation cross section.
- ~5 GeV and up is constrained directly, most precisely by a suite of dual Xe TPC experiments. DM signal is very model-dependent. WIMPs are not in trouble.
- Models *with light mediators* can have WIMPs much lighter that Lee-Weinberg benchmark. *This is interesting experimentally*.

Important points about super-WIMPs (freeze-in DM):

- Mass can be even in a wider range. Couplings to SM is even smaller.
- Small couplings can mean suppression of decay rates. Quasi-stability often follows from here.
- Given a model, it is easy to calculate required coupling, often ~ 10^{-11}