

Beyond the SM at Finite Density



Javi Serra



Instituto de
Física
Teórica
UAM-CSIC



CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

in collaboration w/ R. Balkin, K. Springmann, S. Stelzl, and A. Weiler

Light Scalars Beyond the SM at Finite Density



Javi Serra



Instituto de
Física
Teórica
UAM-CSIC

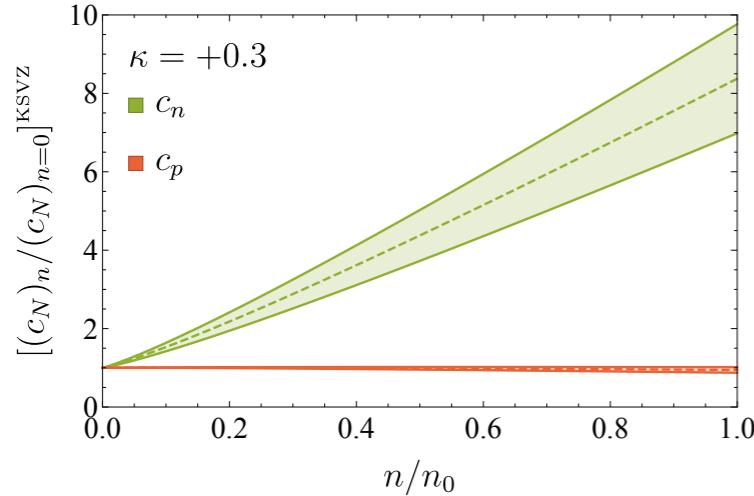


in collaboration w/ R. Balkin, K. Springmann, S. Stelzl, and A. Weiler

Program to study the physics of well-motivated scalar fields at finite density.

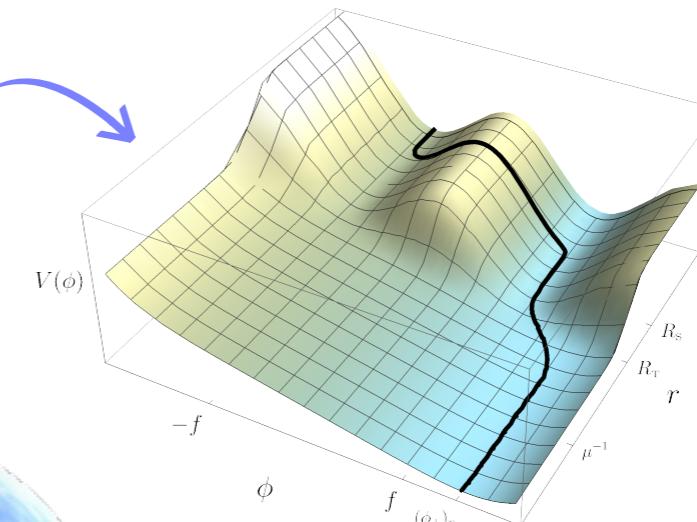
QCD Axion

arXiv:2003.04903



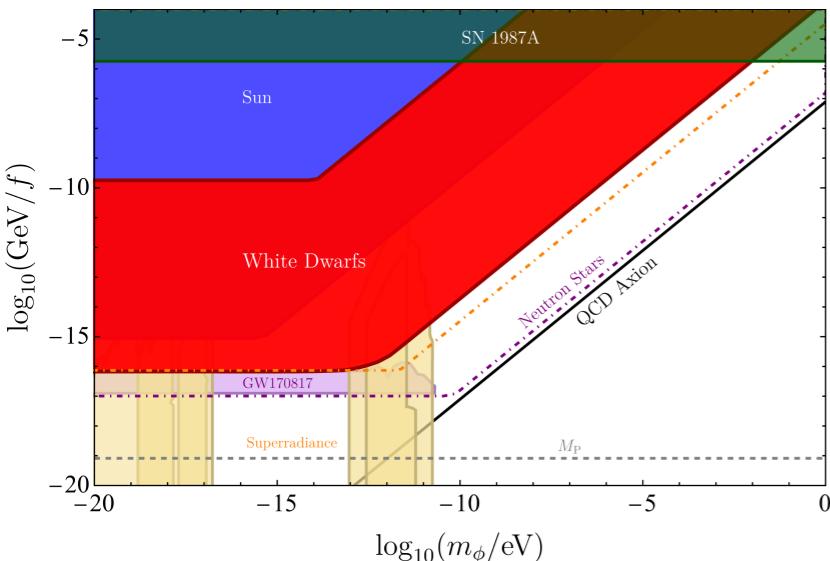
Phase Transitions

arXiv:2105.13354



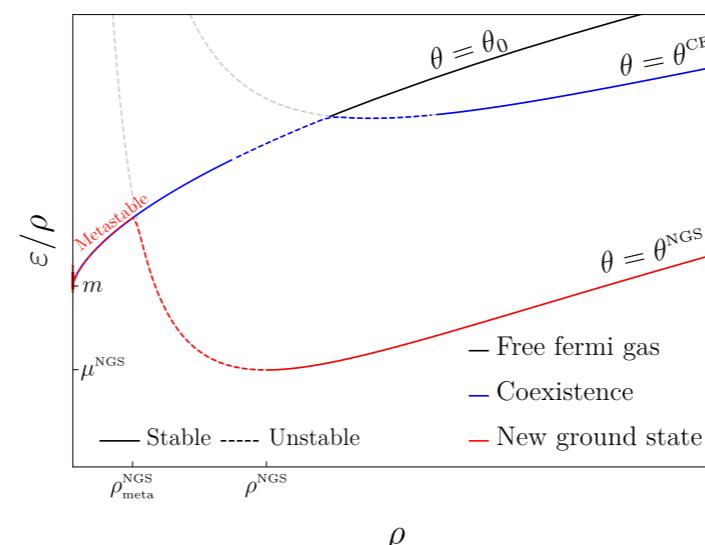
Lighter QCD Axion

arXiv:2211.02661



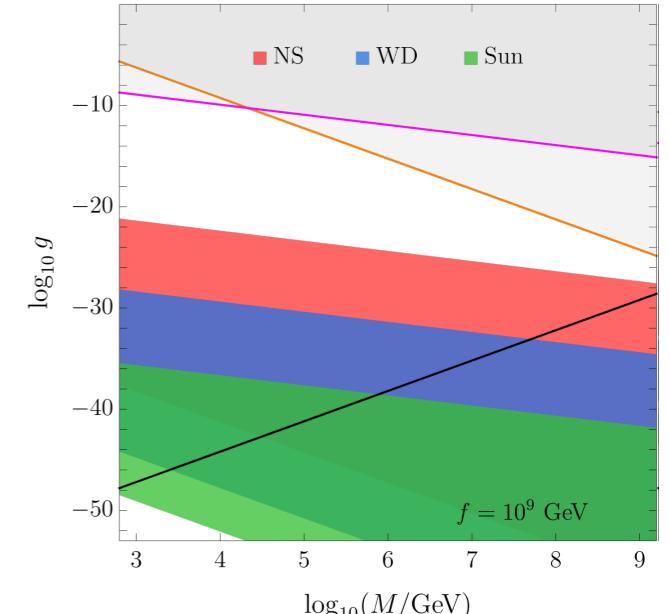
New Ground State

arXiv:2307.14418



Relaxion

arXiv:2106.11320



Motivations

Strong-CP Problem

Violation of NDA expectations in the QCD sector.

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Theory

$$\theta = O(1)$$

Experiment
(nEDM)

$$\theta \lesssim 10^{-10}$$

$$\delta_{\text{CKM}} = O(1)$$

The puzzling absence of CP violation in the interactions of hadrons.

see e.g. E. Hardy and C. Tamarit vs A. Ramos talks

Strong-CP Problem

Violation of NDA expectations in the QCD sector.

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Theory

$$\theta = O(1)$$

Experiment
(nEDM)

$$\theta \lesssim 10^{-10}$$

$$\delta_{\text{CKM}} = O(1)$$

The puzzling absence of CP violation in the interactions of hadrons.

see e.g. E. Hardy and C. Tamarit vs A. Ramos talks



QCD Axion

Nambu-Goldstone boson of QCD-anomalous $U(1)_{\text{PQ}}$ symmetry

$$\mathcal{L}_{\theta+a} = \frac{1}{2}(\partial_\mu a)^2 + \left(\theta + \frac{a}{f_a} \right) \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

$\xrightarrow{\quad a \quad}$



QCD infrared dynamics makes axion solve strong CP-problem.

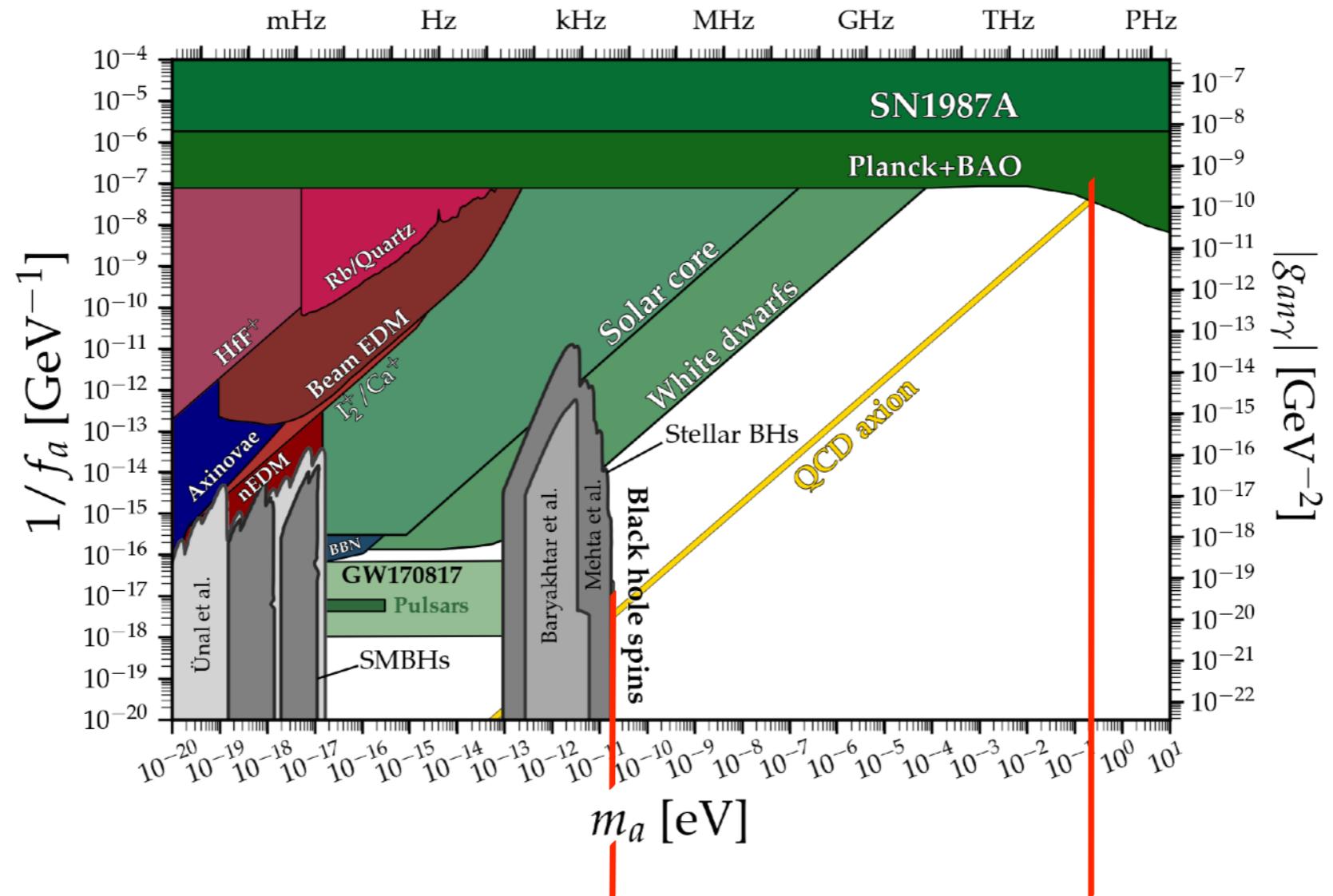
$$V(a) \sim -m_\pi^2 f_\pi^2 \cos \frac{a}{f_a}$$

$$\langle a \rangle = 0 \quad m_a \sim m_\pi f_\pi / f_a$$

$$m_\pi f_\pi \approx 130 \text{ MeV}$$

QCD Axion

Current status of QCD-axion parameter space.



Superradiance

Baryakhtar et al. '20

Mehta et al. '20

...

$$10^{-11} \lesssim m_a/\text{eV} \lesssim 10^{-1}$$

$$10^7 \lesssim f_a/\text{GeV} \lesssim 10^{17}$$

Hot DM component

Caloni et al. '22

rev. Caputo, Raffelt '24

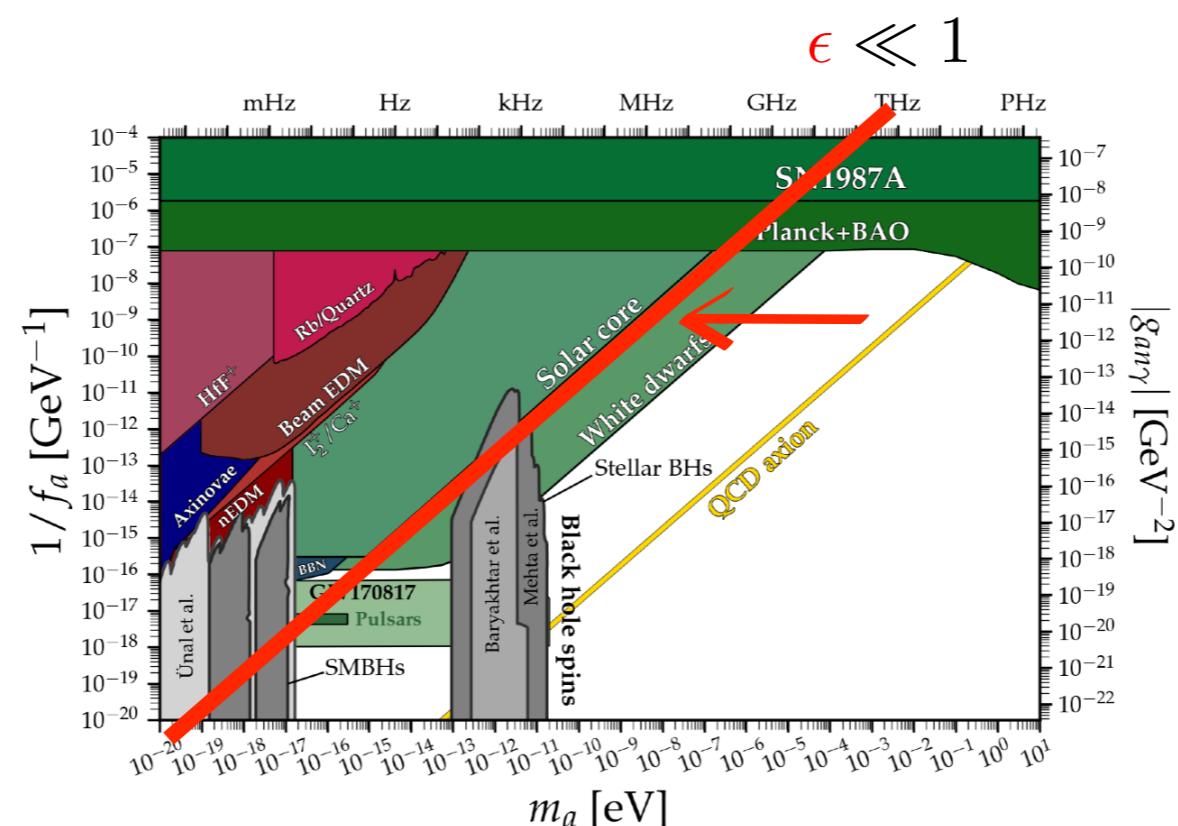
...

Light QCD Axion

Lighter version of the canonical QCD axion.

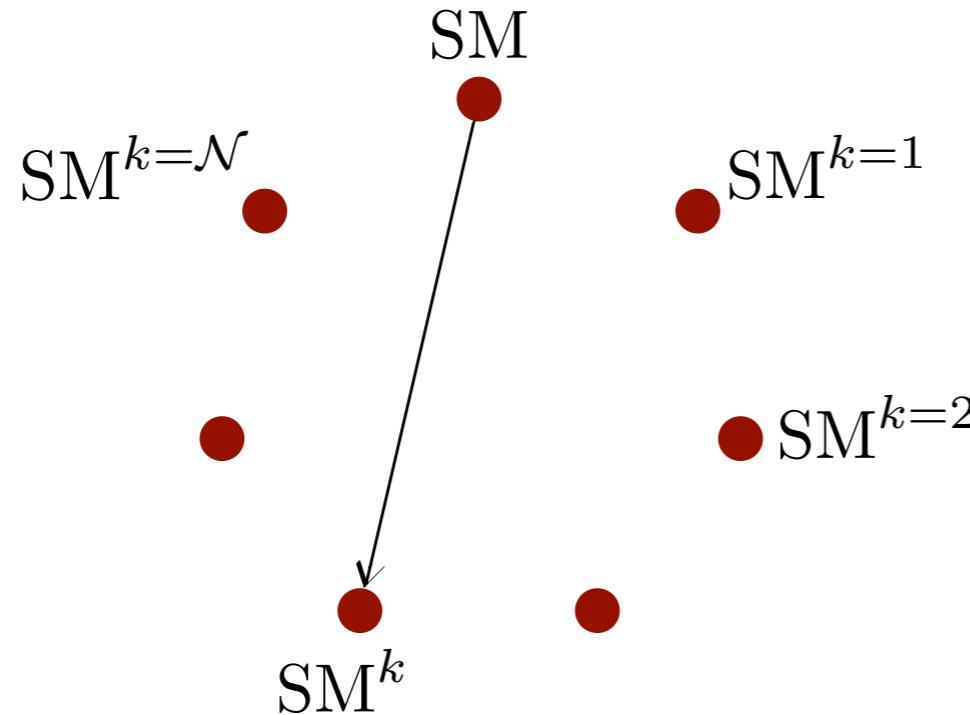
$$V_\epsilon(a) = \epsilon V(a) = -\epsilon m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{a}{2f_a}}$$

$$\langle a \rangle = 0 \quad m_a^2 = \epsilon \frac{m_\pi^2 f_\pi^2}{f_a^2} \frac{m_u m_d}{(m_u + m_d)^2}$$



Z_N QCD Axion

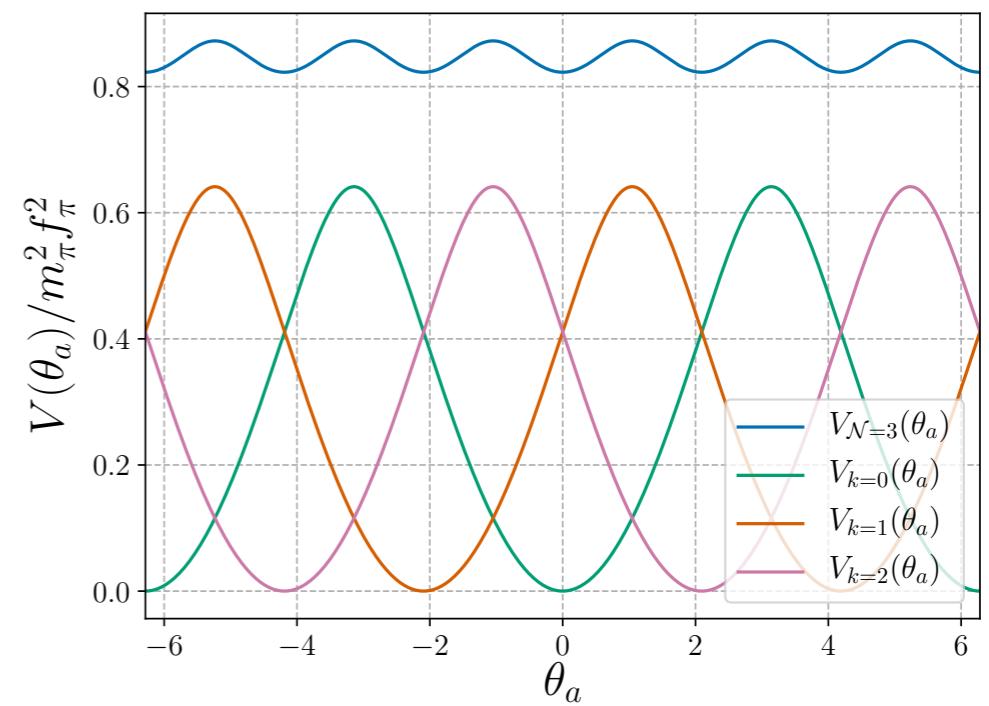
Lighter QCD axion based from discrete Z_N (N odd) symmetry.



$$V_N(a) \simeq -\frac{m_\pi^2 f_\pi^2}{N^2} \cos N \frac{a}{f_a}$$

$$\langle a \rangle = \frac{\pi k}{N} \quad m_a^2 = \frac{m_\pi^2 f_\pi^2}{f_a^2} \sqrt{\frac{1-z}{\pi(1+z)}} N^{3/2} z^N$$

$$a \rightarrow a + \frac{2\pi k}{N} f_a$$



Cosmological Constant / Electroweak Hierarchy Problems

Violation of NDA expectations in the gravitational / Higgs sectors

$$\mathcal{L}_{d<4} = \sqrt{-g} (\Lambda_{\text{CC}} + m_H^2 |H|^2)$$

Theory

$$\frac{\Lambda_{\text{CC}}}{M_{\text{Pl}}^4} = O(1) \quad \frac{m_H^2}{M_{\text{Pl}}^2} = O(1)$$

Experiment
(LHC, Universe)

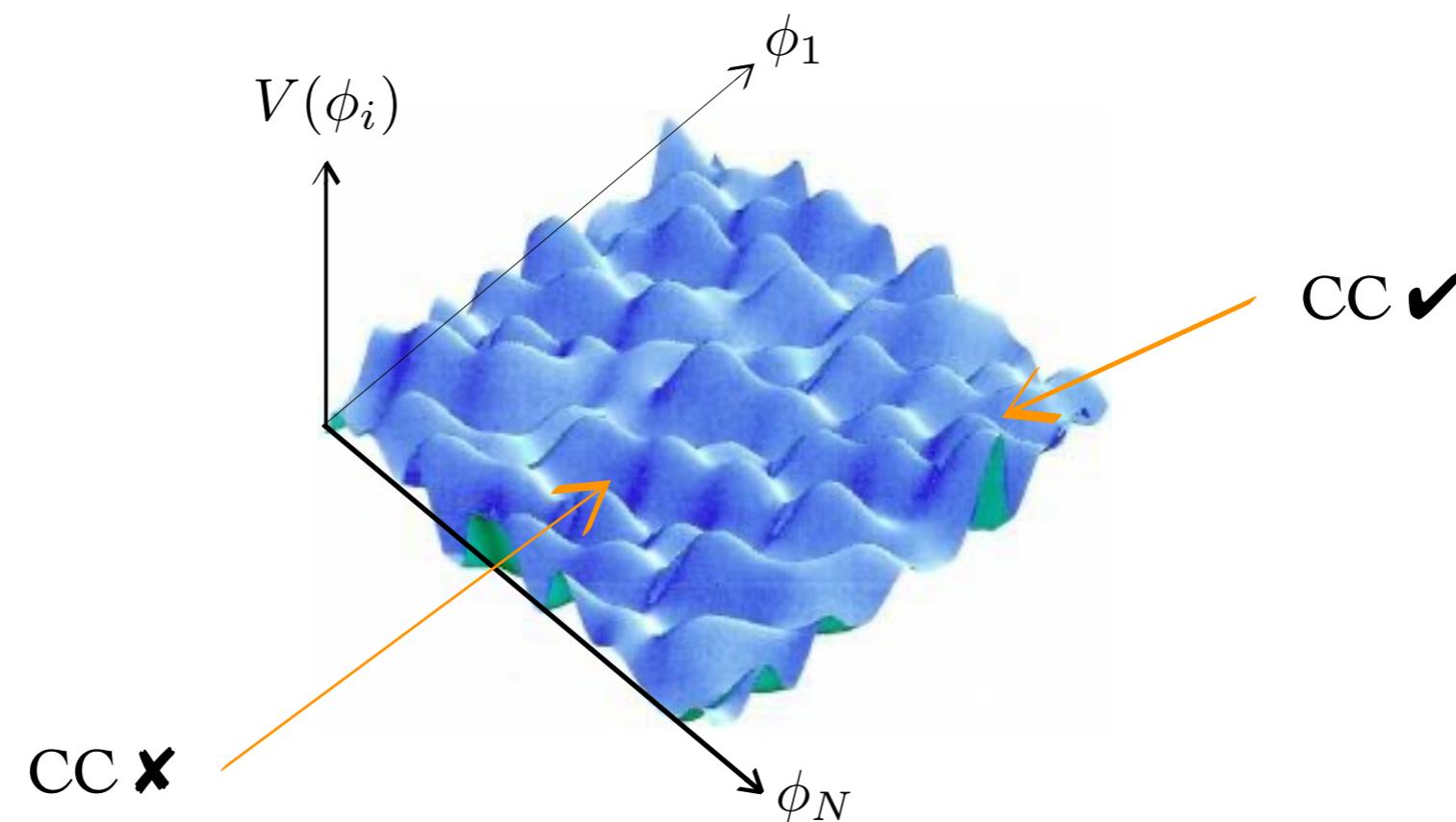
$$\frac{\Lambda_{\text{CC}}}{M_{\text{Pl}}^4} \sim 10^{-120} \quad \frac{m_H^2}{M_{\text{Pl}}^2} \sim 10^{-30}$$

The perplexing existence of a big universe w/ big things in it.



Landscapes

Low-energy dimensionful parameters are not fundamentally calculable by finely scanned.

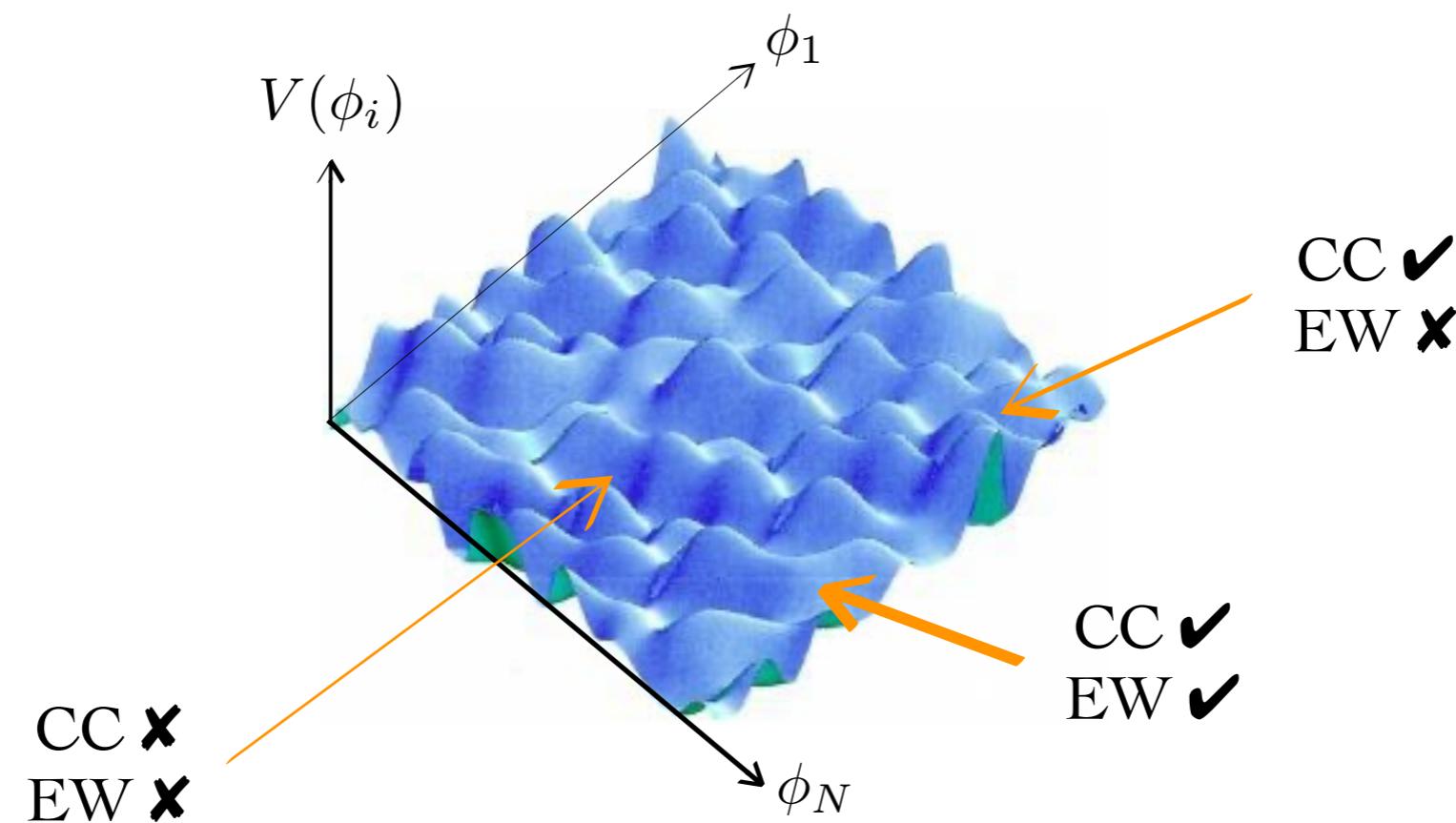


Arguably the best explanation for the tiny size of the cosmological constant.

[along w/ selection mechanism of our vacuum among the multiverse of vacua]

Landscapes

Low-energy dimensionful parameters are not fundamentally calculable by finely scanned.



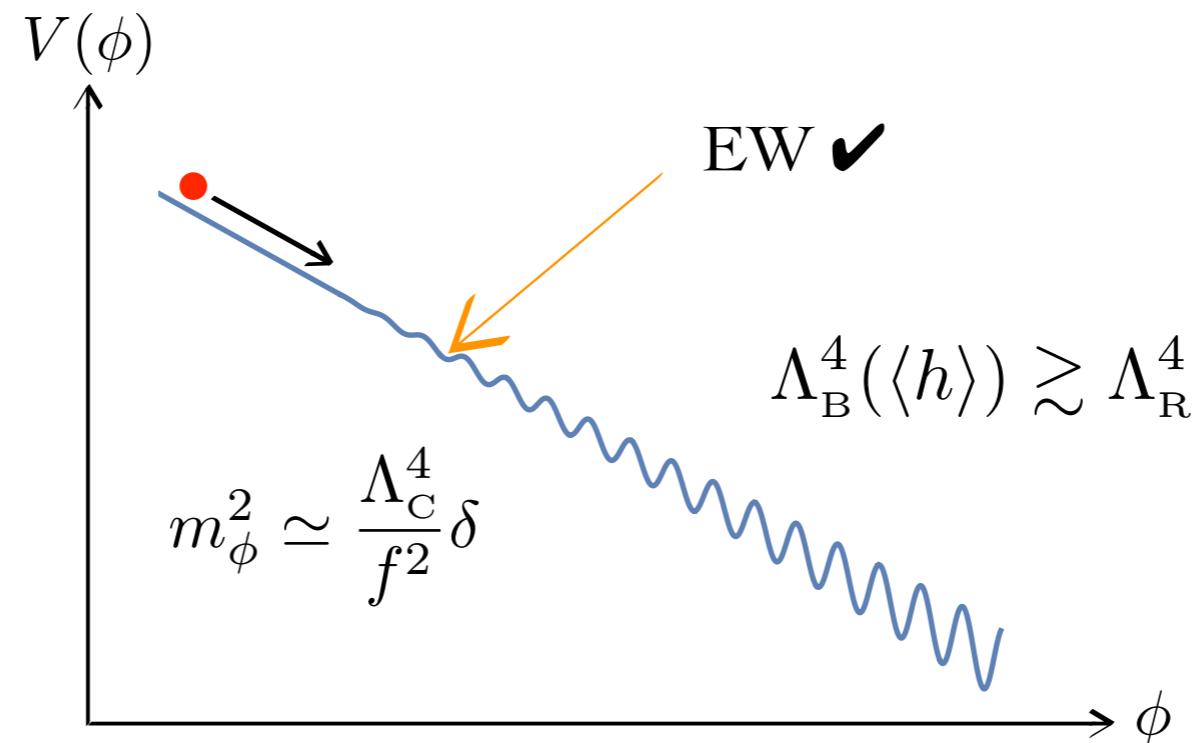
see e.g. O. Matsedonskyi talk

Alternative to standard symmetry approaches: supersymmetry/compositeness.

[along w/ selection mechanism of our vacuum among the multiverse of vacua]

Relaxion

Dynamical selection of small electroweak scale via weakly-coupled light axion-like scalar.



$$V(\phi) = -\Lambda_R^4 \frac{\phi}{f} + \underline{\Lambda_B^4(h)} \cos \frac{\phi}{f}$$

Non-vanishing Higgs VEV triggers QCD-like* barrier that stops relaxion's evolution.

* QCD-axion: $\Lambda_B^4(h) \sim m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \frac{h}{v}$

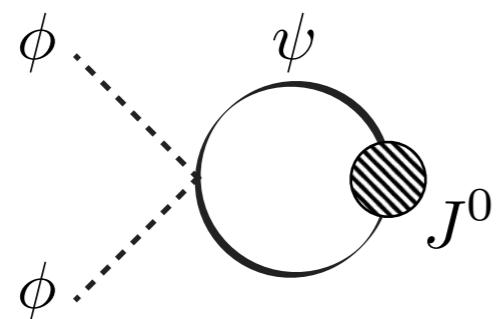


Finite Density

Finite Density Effects

Properties of **scalar** coupled to conserved charge change in **system** w/ non-vanishing density.

$$\mathcal{L} \supset \phi^2 \bar{\psi} \psi - V(\phi)$$



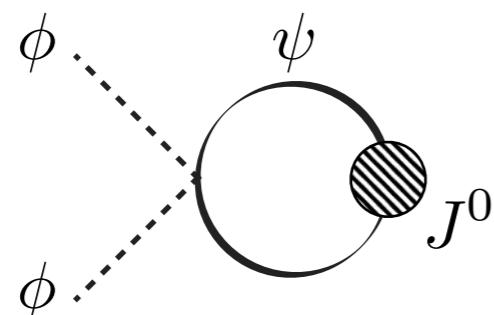
$$n = \langle J^0 \rangle \xrightarrow{\text{NR}} \simeq \langle \bar{\psi} \psi \rangle$$

$$\Delta V(\phi) = n\phi^2$$

Finite Density Effects

Properties of **scalar** coupled to conserved charge change in **system** w/ non-vanishing density.

$$\mathcal{L} \supset \phi^2 \bar{\psi} \psi - V(\phi)$$



$$n = \langle J^0 \rangle \xrightarrow{\text{NR}} \langle \bar{\psi} \psi \rangle \quad \Delta V(\phi) = n\phi^2$$

Free Fermi gas:

$$n = 2 \int^{k_F} \frac{d^3 k}{(2\pi)^3} = \frac{k_F^3}{3\pi^2} \quad k_F(\mu) = \sqrt{\mu^2 - m^2} \Theta(\mu - m)$$

Formally, fermion loops evaluated w/ finite density ($T = 0$) propagator:

$$iG(p) = (\not{p} + m) \left[\frac{i}{p^2 - m^2 + i\epsilon} - 2\pi\delta(p^2 - m^2)\Theta(p^0)\Theta(k_F - |\vec{p}|) \right]$$

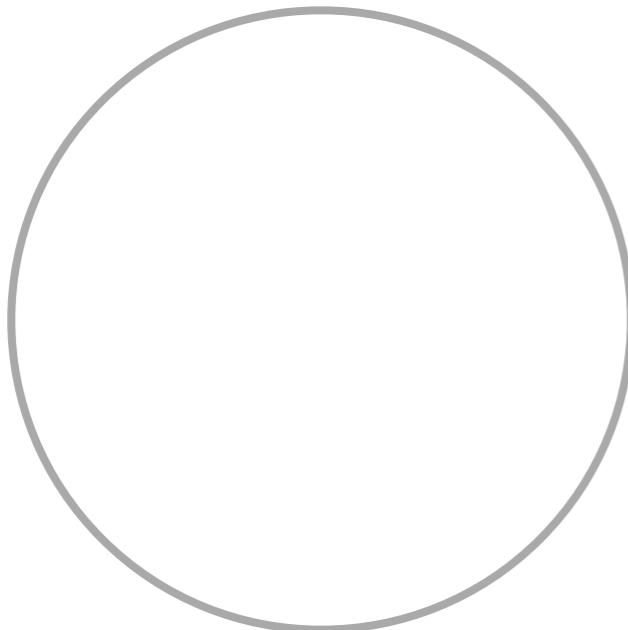
vacuum	medium (Fermi sea)
--------	--------------------

Finite Density Effects

Properties of **scalar** coupled to conserved charge change in **system** w/ non-vanishing density.

$$\mathcal{L} \supset \phi^2 \bar{\psi} \psi - V(\phi)$$

Vacuum

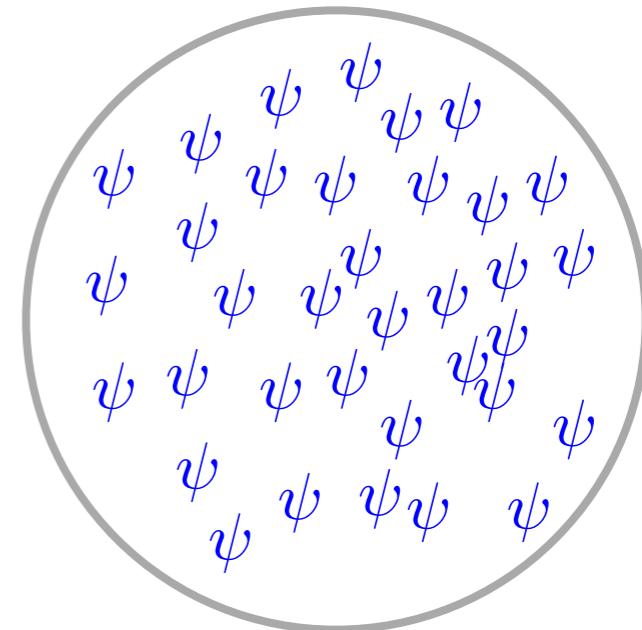


$$V(\phi)$$

$$n = \langle J^0 \rangle \simeq \langle \bar{\psi} \psi \rangle$$



Medium

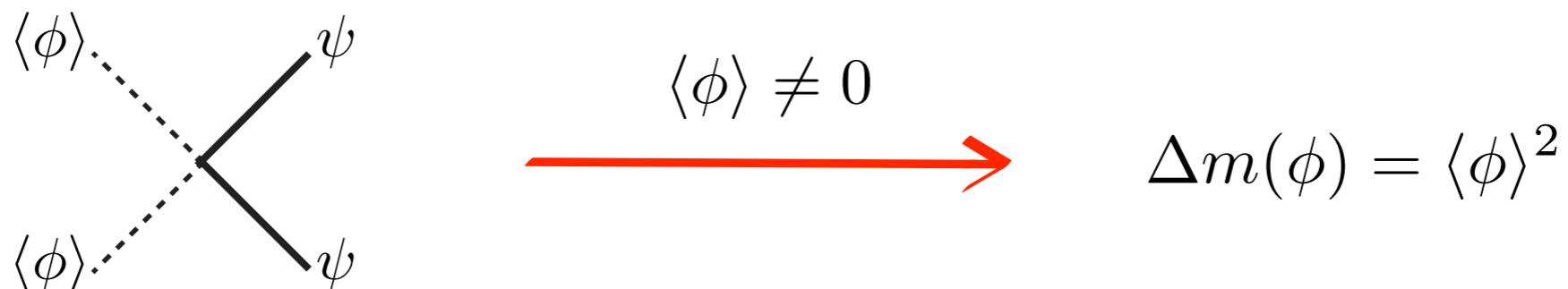


$$V(\phi) + n\phi^2$$

Finite Density Effects

Properties of **system** coupled to **scalar** change if scalarization take place.

$$\mathcal{L} \supset \phi^2 \bar{\psi} \psi - m \bar{\psi} \psi$$



Scalar-dependent grand-canonical potential:

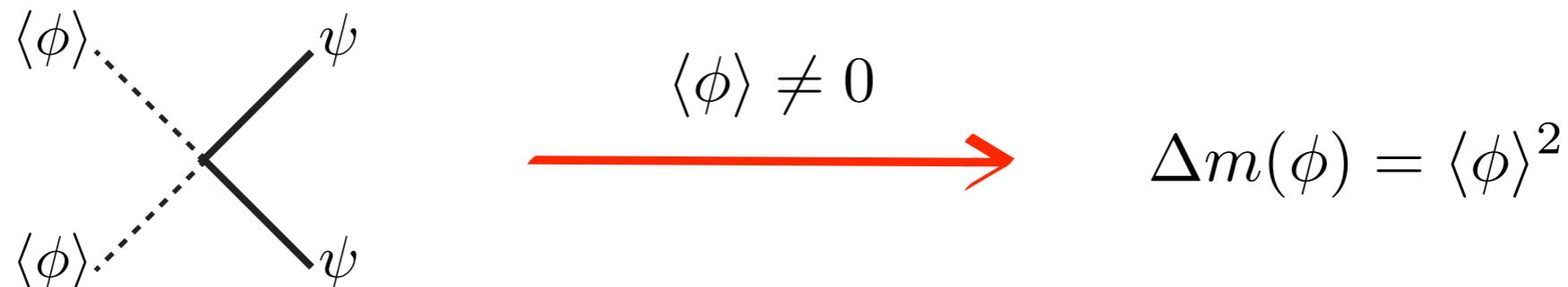
$$\Omega = \Omega(\mu, \phi)$$

$$\left. \frac{\partial \Omega}{\partial \mu} \right|_{\phi} = n \quad \left. \frac{\partial \Omega}{\partial \phi} \right|_{\mu} = 0$$

Finite Density Effects

Properties of **system** coupled to **scalar** change if scalarization take place.

$$\mathcal{L} \supset \phi^2 \bar{\psi} \psi - m \bar{\psi} \psi$$



Scalar-dependent grand-canonical potential:

$$\Omega = \Omega(\mu, \phi)$$

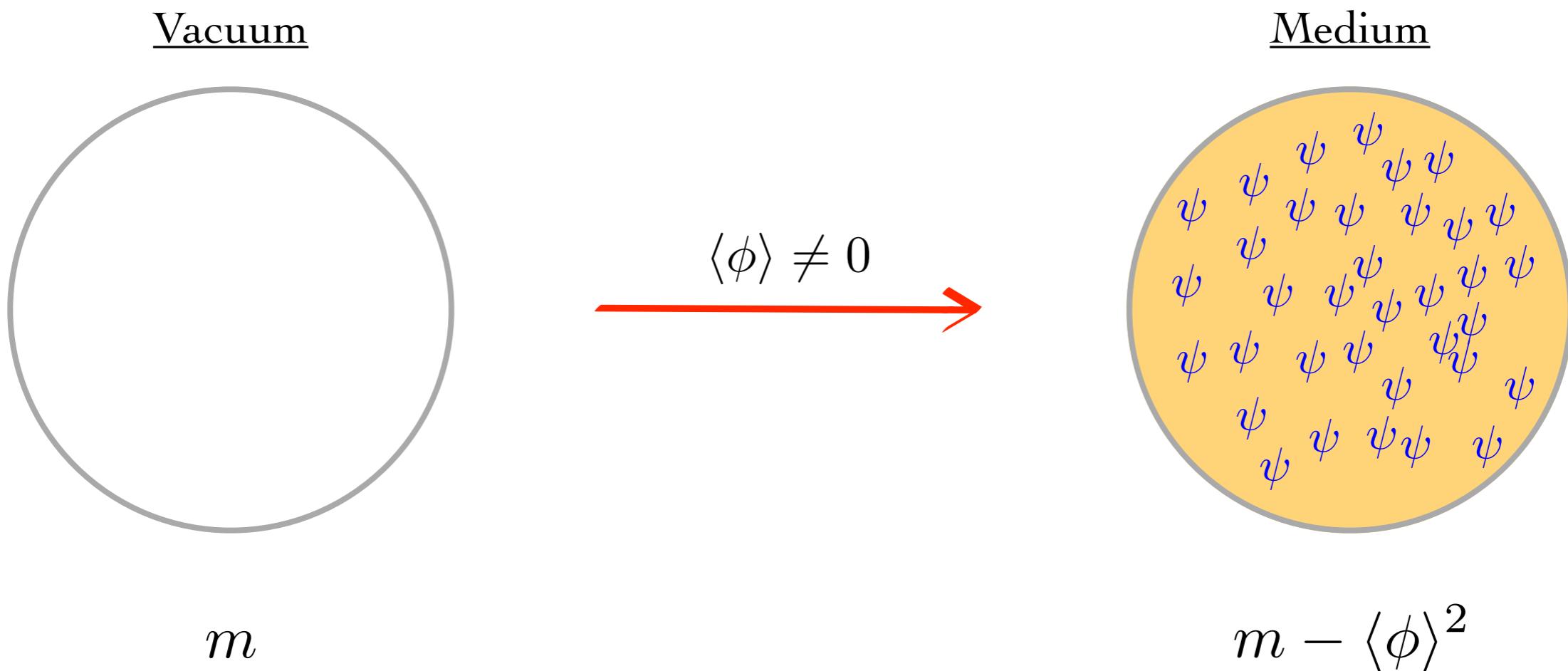
$$\left. \frac{\partial \Omega}{\partial \mu} \right|_{\phi} = n \quad \left. \frac{\partial \Omega}{\partial \phi} \right|_{\mu} = 0$$

Free Fermi gas: $\Omega(\mu, \phi) = \Omega_{\psi}(\mu, m(\phi)) + V(\phi)$ $\xrightarrow[\text{NR}]{\partial_{\phi}|_{\mu}}$ $\frac{\partial V}{\partial \phi} + n \frac{\partial m}{\partial \phi} = 0$

Finite Density Effects

Properties of **system** coupled to **scalar** change if scalarization take place.

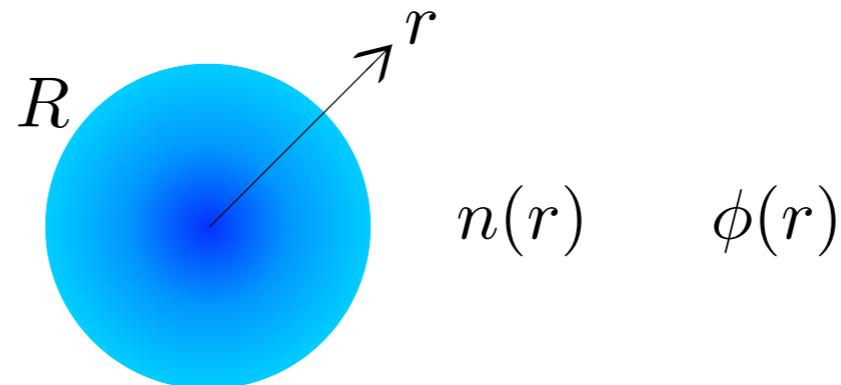
$$\mathcal{L} \supset \phi^2 \bar{\psi} \psi - m \bar{\psi} \psi$$



$$m - \langle \phi \rangle^2$$

Stars

Compact stellar objects: finite size, spherically symmetric, dense systems.



Tolman-Oppenheimer-Volkoff + scalar equations:

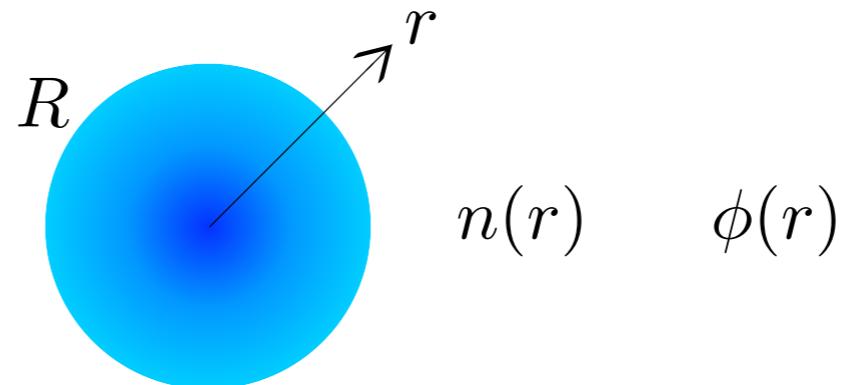
$$M' = 4\pi r^2 \varepsilon \left[1 + O\left(\frac{\phi'^2}{\epsilon}\right) \right]$$

$$p' = -\frac{GM}{r^2} \varepsilon \left[1 + O\left(\frac{GM}{r}, \frac{p}{\varepsilon}, \frac{\phi'^2}{\epsilon}\right) \right] - \phi' \frac{\partial \Omega}{\partial \phi}$$

$$\left(\phi'' + \frac{2}{r} \phi' \right) \left[1 + O\left(\frac{GM}{r}\right) \right] = \frac{\partial \Omega}{\partial \phi}$$

Stars

Compact stellar objects: finite size, spherically symmetric, dense systems.



Tolman-Oppenheimer-Volkoff + scalar equations:

$$M' = 4\pi r^2 \varepsilon \left[1 + O(\frac{\phi'^2}{\epsilon}) \right]$$

$$p' = -\frac{GM}{r^2} \varepsilon \left[1 + O(\frac{GM}{r}, \frac{p}{\varepsilon}, \frac{\phi'^2}{\epsilon}) \right] - \cancel{\phi'} \frac{\partial \Omega}{\partial \phi}$$

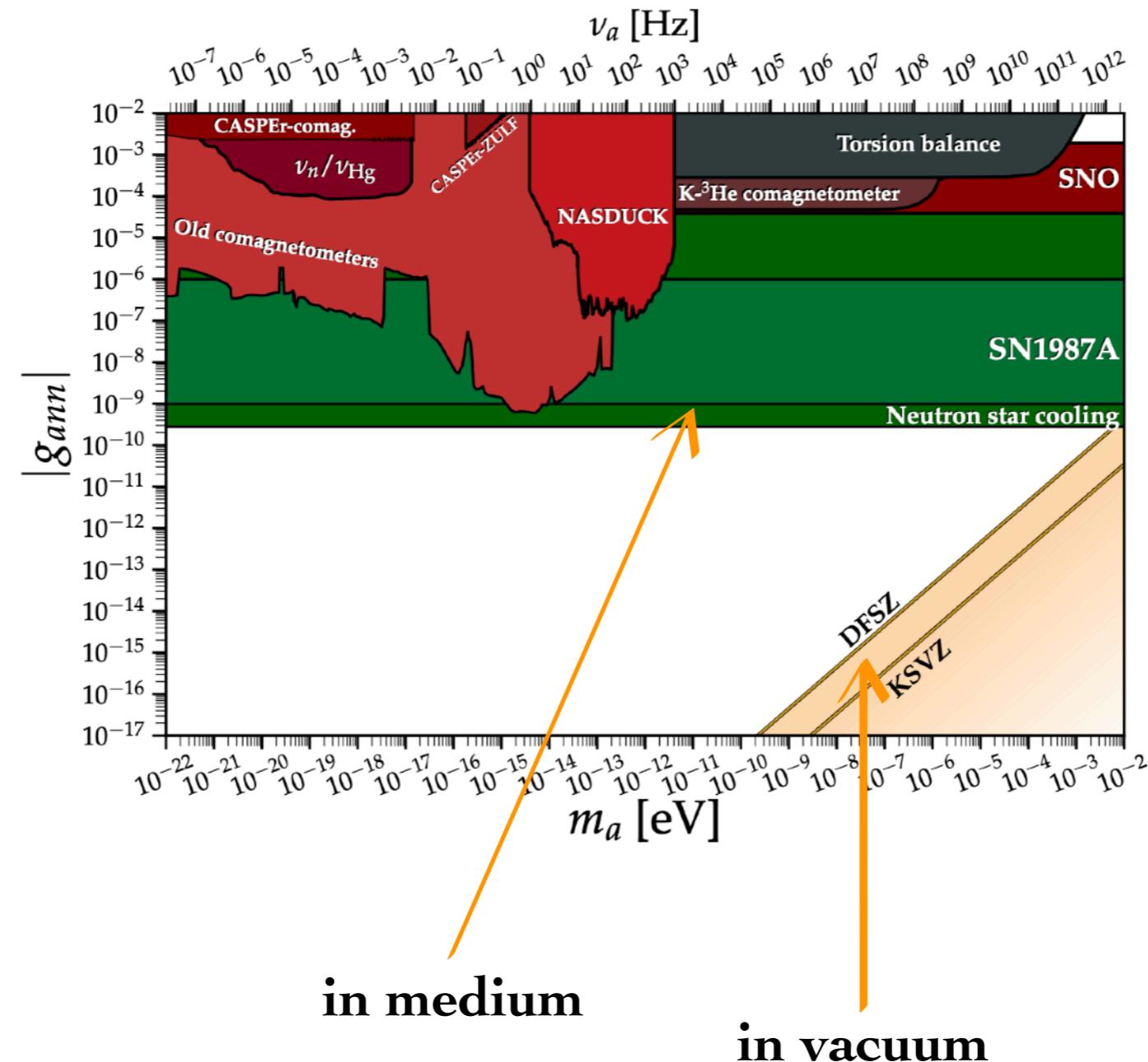
$$\cancel{(\phi'' + \frac{2}{r}\phi')} \left[1 + O(\frac{GM}{r}) \right] = \frac{\partial \Omega}{\partial \phi}$$

* negligible gradient energy limit: $\frac{1}{\bar{m}_\phi(n)} \ll R$

QCD Axion

Stellar Emission: Bounds

Supernova and Neutron star cooling bounds not systematic in finite-density effects.

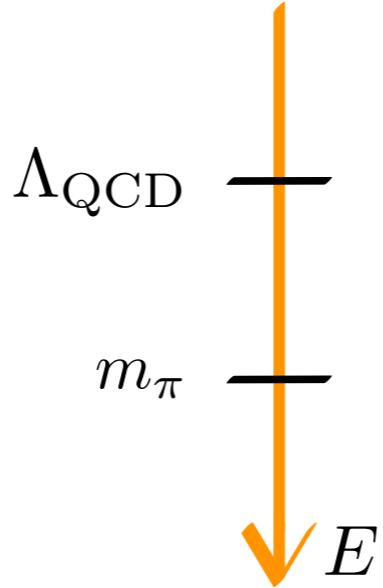


QCD Axion Couplings: in Vacuum

Model-independent low-energy couplings to protons/neutrons from axion-pion mixing.

$$\mathcal{L}_a \supset \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{\partial_\mu a}{2f_a} \sum_q c_q^0 \bar{q} \gamma^\mu \gamma_5 q$$

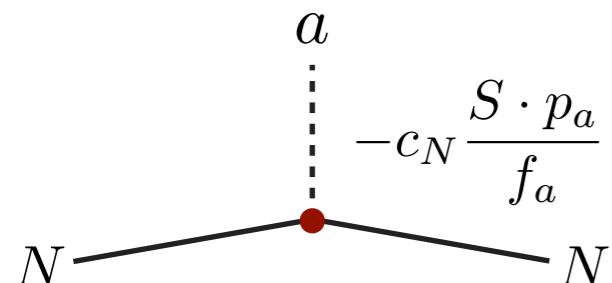
model independent model dependent



KSVZ axion: $c_q^0 \simeq 0$

Axion derivative coupling to non-relativistic neutrons/protons below pion mass.

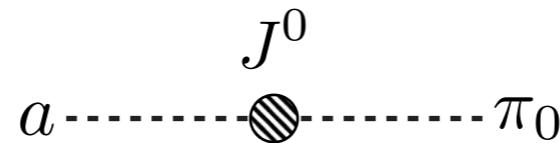
$$\frac{\partial_\mu a}{f_a} \bar{N} S^\mu (g_A c_{u-d} \sigma_3 + g_0 c_{u+d} \mathbb{I}) N \quad N = \binom{p}{n}$$



$$c_N^{\text{KSVZ}} = \begin{pmatrix} -0.42(3) \\ +0.02(3) \end{pmatrix} \leftarrow \text{accidental cancellation}$$

QCD Axion Couplings: in Medium

- Axion-meson mixing angles: $c_{u\pm d}$



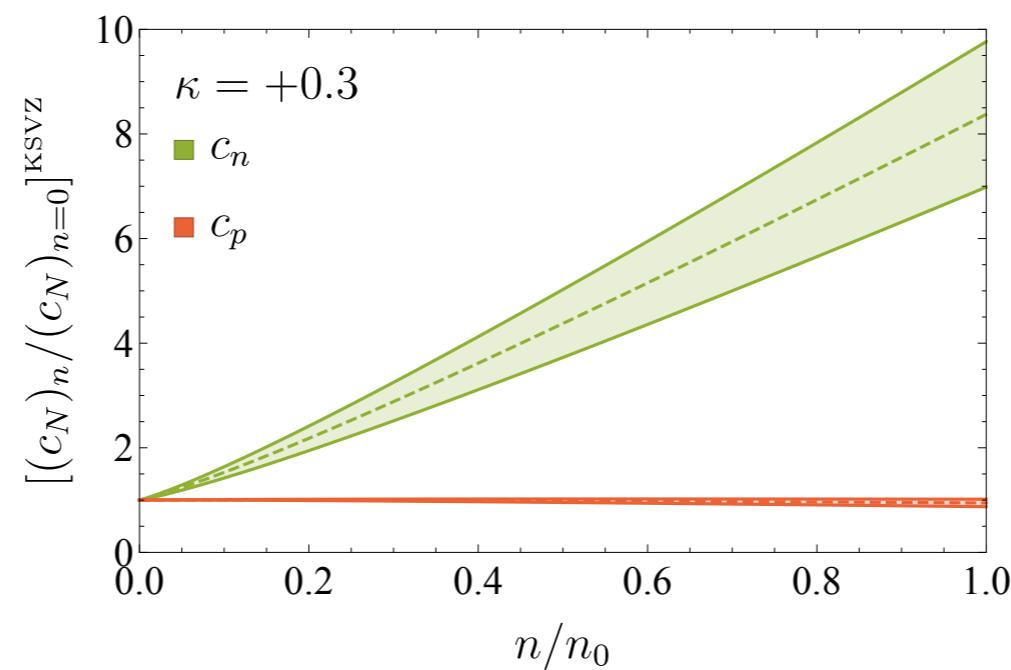
- Nuclear matrix elements: g_A, g_0



$$k_F \simeq 260 \text{ MeV} (n/n_0)^{1/3}$$

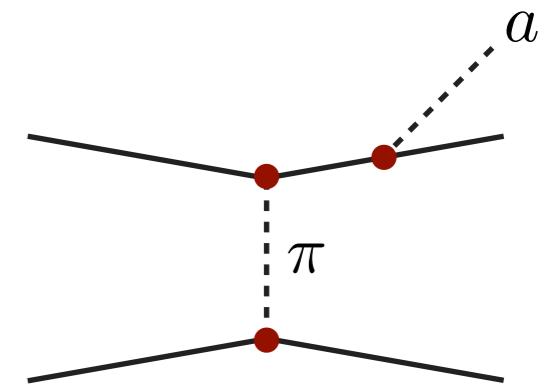
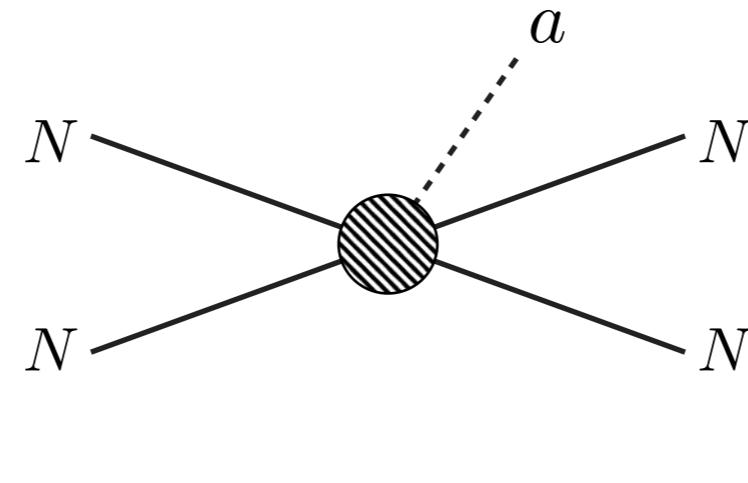
$$k_F > m_\pi$$

$$(k_F/\Lambda_\chi)^2 \simeq 15\% (n/n_0)^{2/3}$$

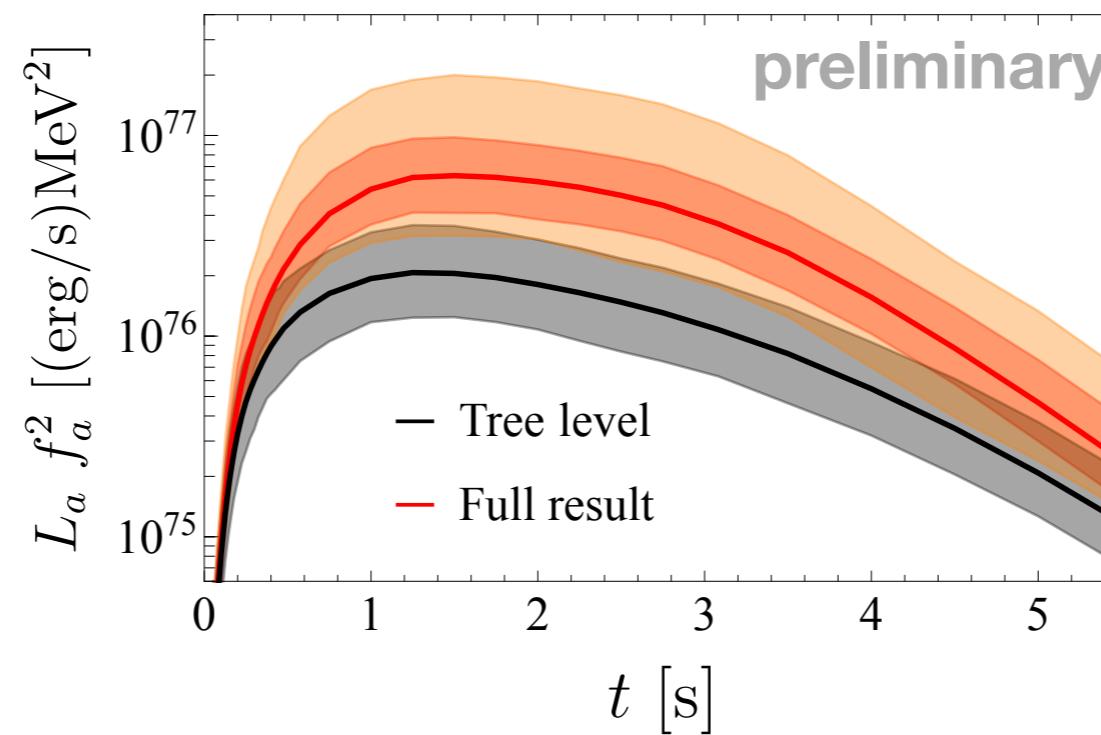


QCD Axion Emission: in Medium

Axion emissivity in Supernova and Neutron star cooling, systematically.



Raffelt, Seckel '88



SN bound meaningful (th. error), NS cooling bound $O(1)$ uncertain; similar for DFSZ/KSVZ.

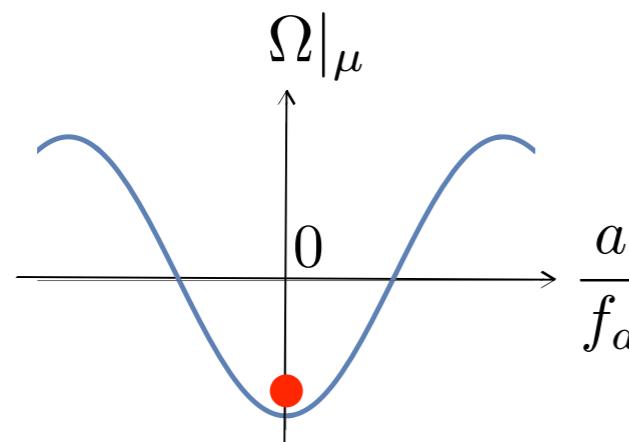
Lighter QCD Axion

Axionalization

Lighter QCD axion develops non-trivial profile in dense enough systems.

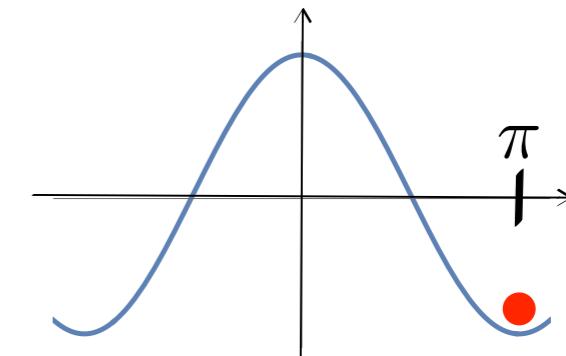
$$V_\epsilon(a) \simeq -\epsilon m_\pi^2 f_\pi^2 \left(\cos \frac{a}{f_a} - 1 \right)$$

$$m_N(a) - m_N \simeq +g m_N \left(\cos \frac{a}{f_a} - 1 \right) \quad g \simeq \sigma_N/m_N$$



$$\frac{\partial \Omega}{\partial \phi} \Big|_\mu = 0$$

$$n > \epsilon \frac{m_\pi^2 f_\pi^2}{\sigma_N} \frac{n_c}{n}$$



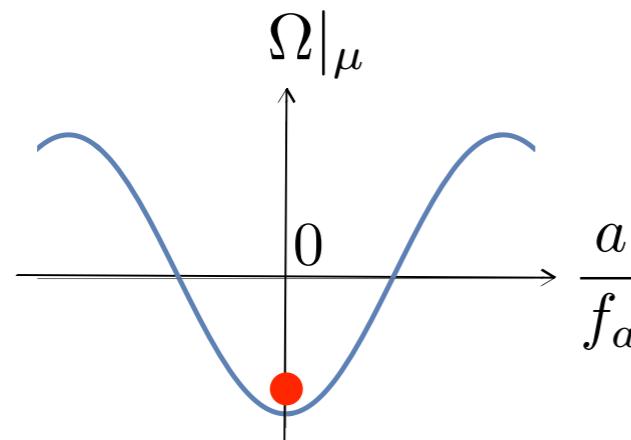
$$\Delta m_N \simeq -32 \text{ MeV} \left(\frac{\sigma_N}{50 \text{ MeV}} \right)$$

Axionalization

Lighter QCD axion develops non-trivial profile in dense enough systems.

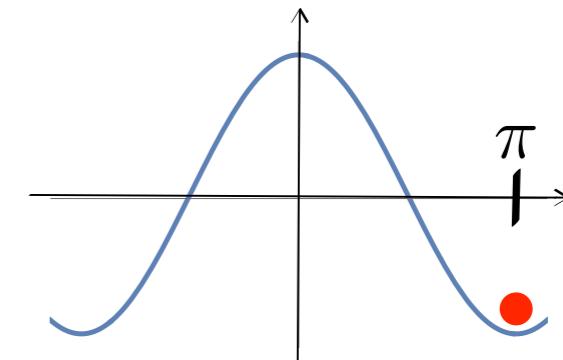
$$V_\epsilon(a) \simeq -\epsilon m_\pi^2 f_\pi^2 \left(\cos \frac{a}{f_a} - 1 \right)$$

$$m_N(a) - m_N \simeq +g m_N \left(\cos \frac{a}{f_a} - 1 \right) \quad g \simeq \sigma_N / m_N$$

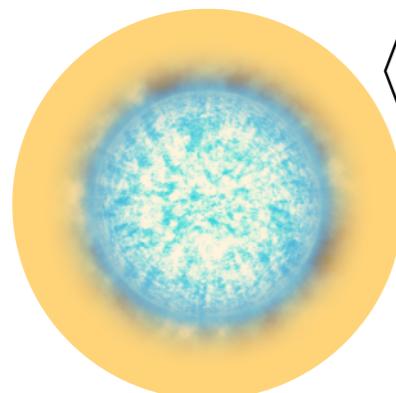


$$\frac{\partial \Omega}{\partial \phi} \Big|_\mu = 0$$

$$n > \epsilon \frac{m_\pi^2 f_\pi^2}{\sigma_N} \frac{n_c}{n}$$



$$\Delta m_N \simeq -32 \text{ MeV} \left(\frac{\sigma_N}{50 \text{ MeV}} \right)$$

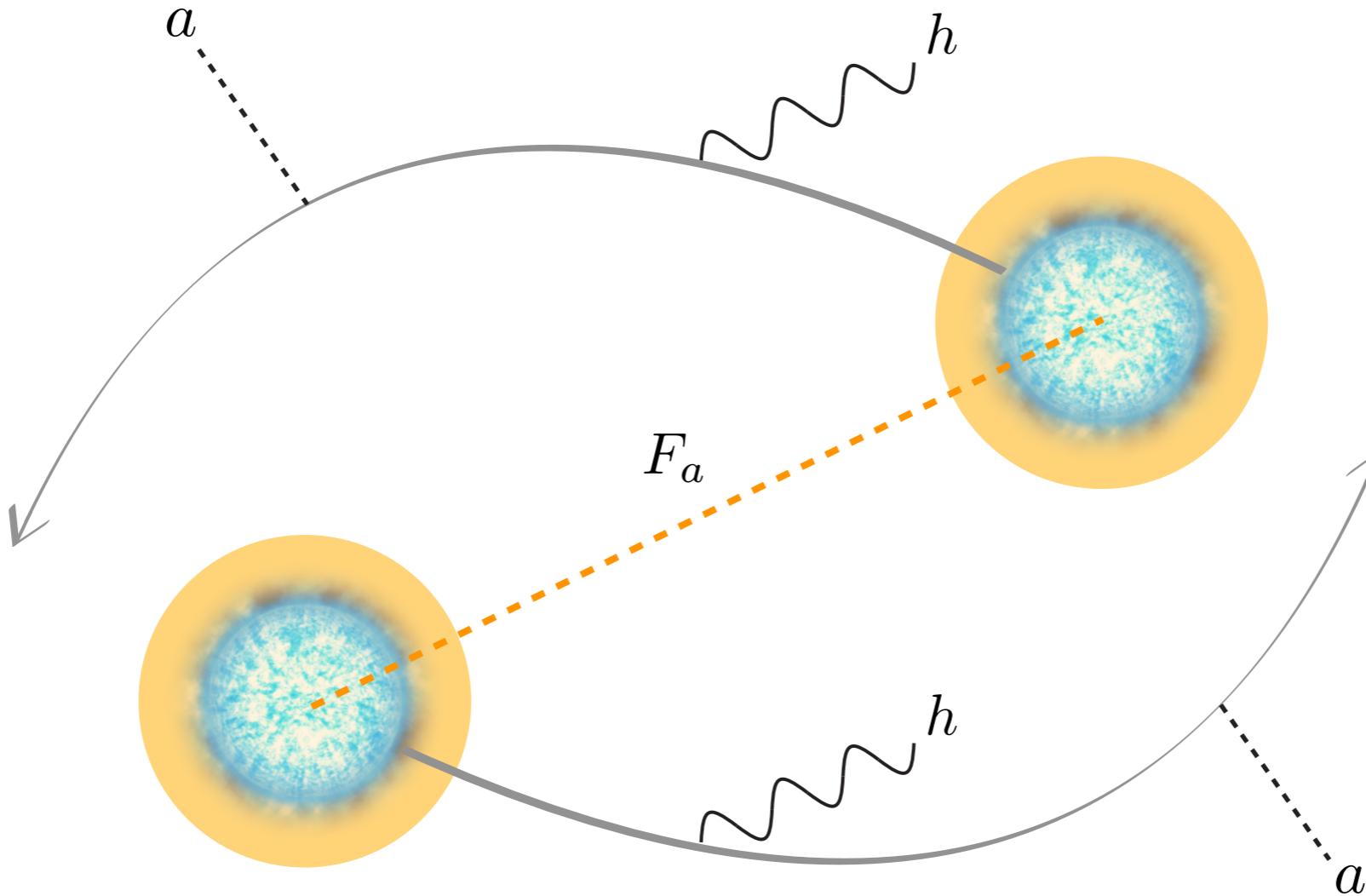


$$\langle a \rangle = \pi f_a$$

$$R \gtrsim \frac{1}{\bar{m}_\phi(n)} = 10^4 \text{ km} \left(\frac{f}{10^{16} \text{ GeV}} \right) \left(\frac{10^{-4} \text{ MeV}^3}{n_{\text{WD}}} \right)^{1/2}$$

Neutron Star Mergers

Lighter QCD axion mediates long-range force between axionalized neutron stars.



Merger dynamics affected for Planck-scale decay constants.

New Ground State

Lighter QCD axion leads to new scalarized ground state of (free) matter.

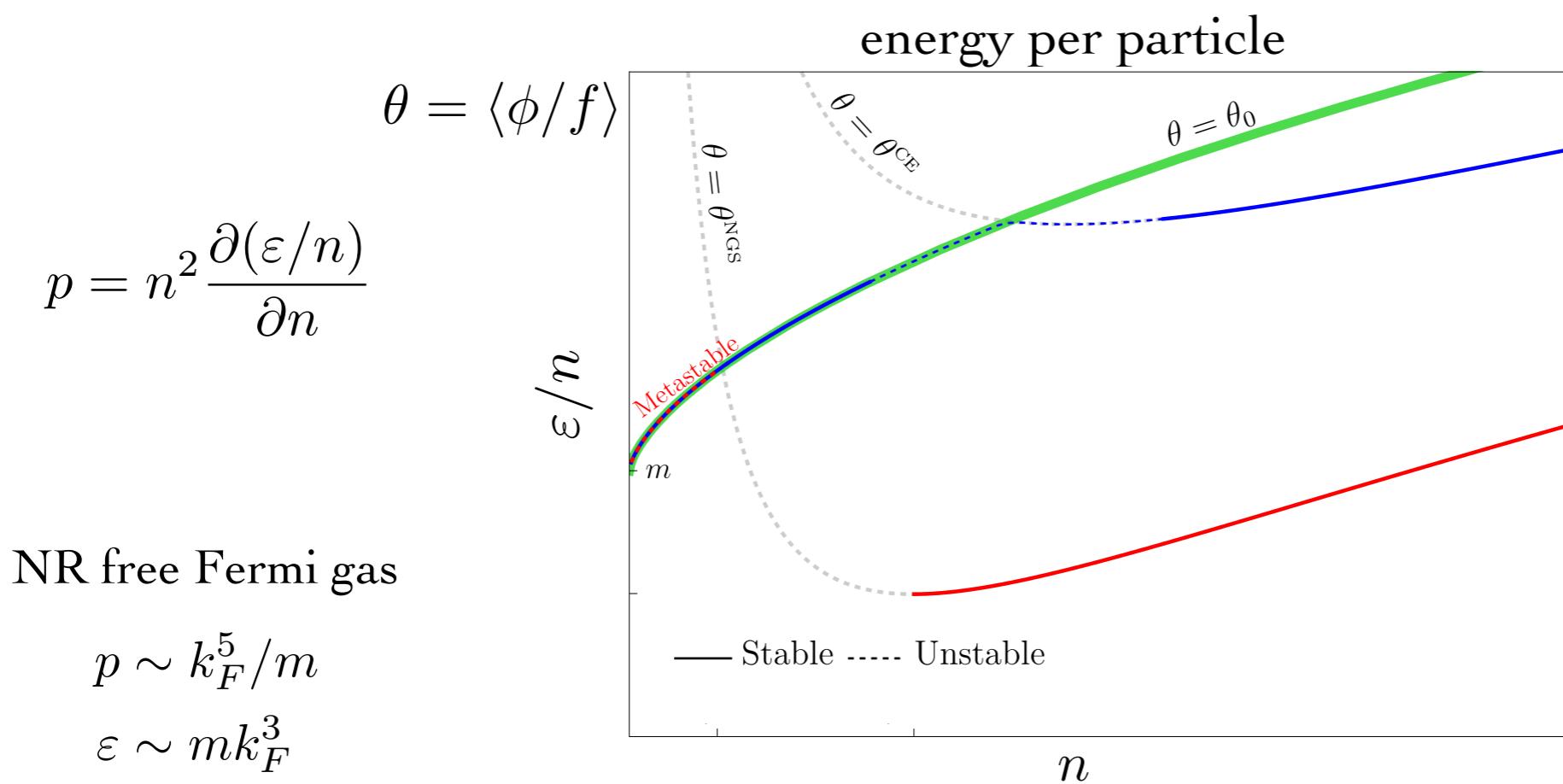
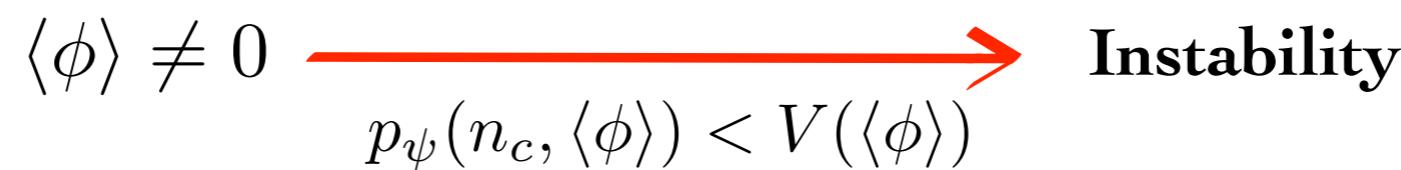
$$\begin{aligned}\varepsilon(n, \phi) &= \varepsilon_\psi(n, \phi) + V(\phi) \\ p(n, \phi) &= p_\psi(n, \phi) - V(\phi)\end{aligned}\quad \psi = n, e + p, e + p + n$$

$$\begin{array}{c} \langle \phi \rangle \neq 0 \\[1ex] p_\psi(n_c, \langle \phi \rangle) < V(\langle \phi \rangle) \end{array} \xrightarrow{\hspace{10em}} \textbf{Instability}$$

New Ground State

Lighter QCD axion leads to new scalarized ground state of (free) matter.

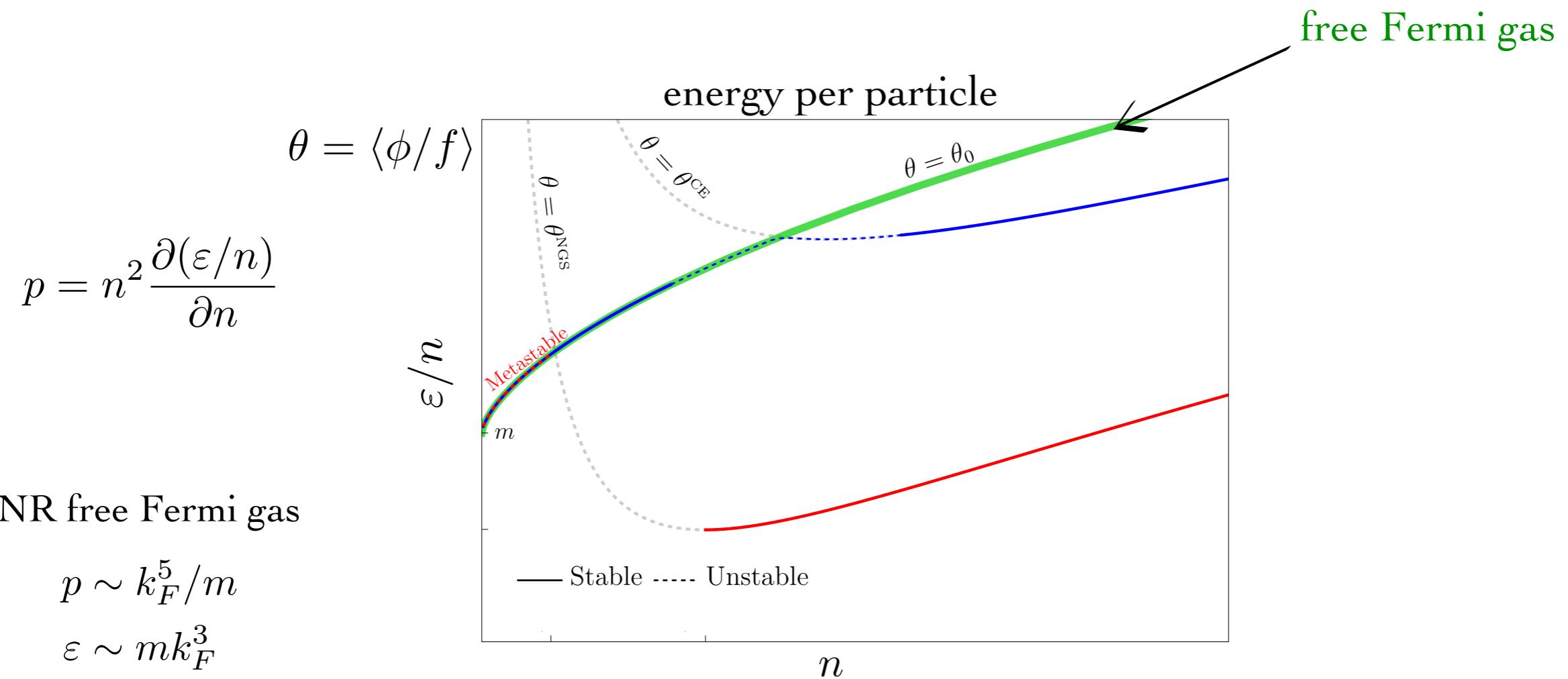
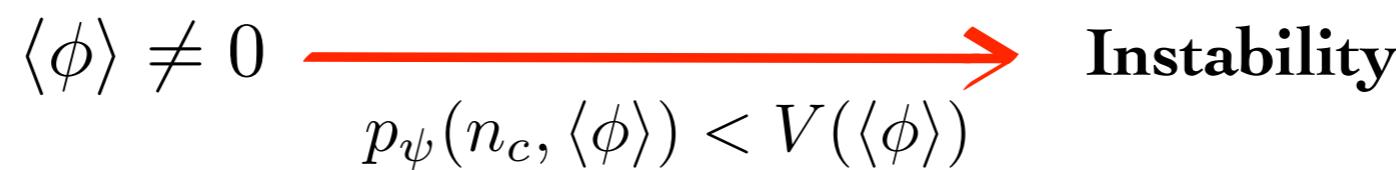
$$\begin{aligned}\varepsilon(n, \phi) &= \varepsilon_\psi(n, \phi) + V(\phi) \\ p(n, \phi) &= p_\psi(n, \phi) - V(\phi)\end{aligned}\quad \psi = n, e + p, e + p + n$$



New Ground State

Lighter QCD axion leads to new scalarized ground state of (free) matter.

$$\begin{aligned}\varepsilon(n, \phi) &= \varepsilon_\psi(n, \phi) + V(\phi) \\ p(n, \phi) &= p_\psi(n, \phi) - V(\phi)\end{aligned}\quad \psi = n, e + p, e + p + n$$

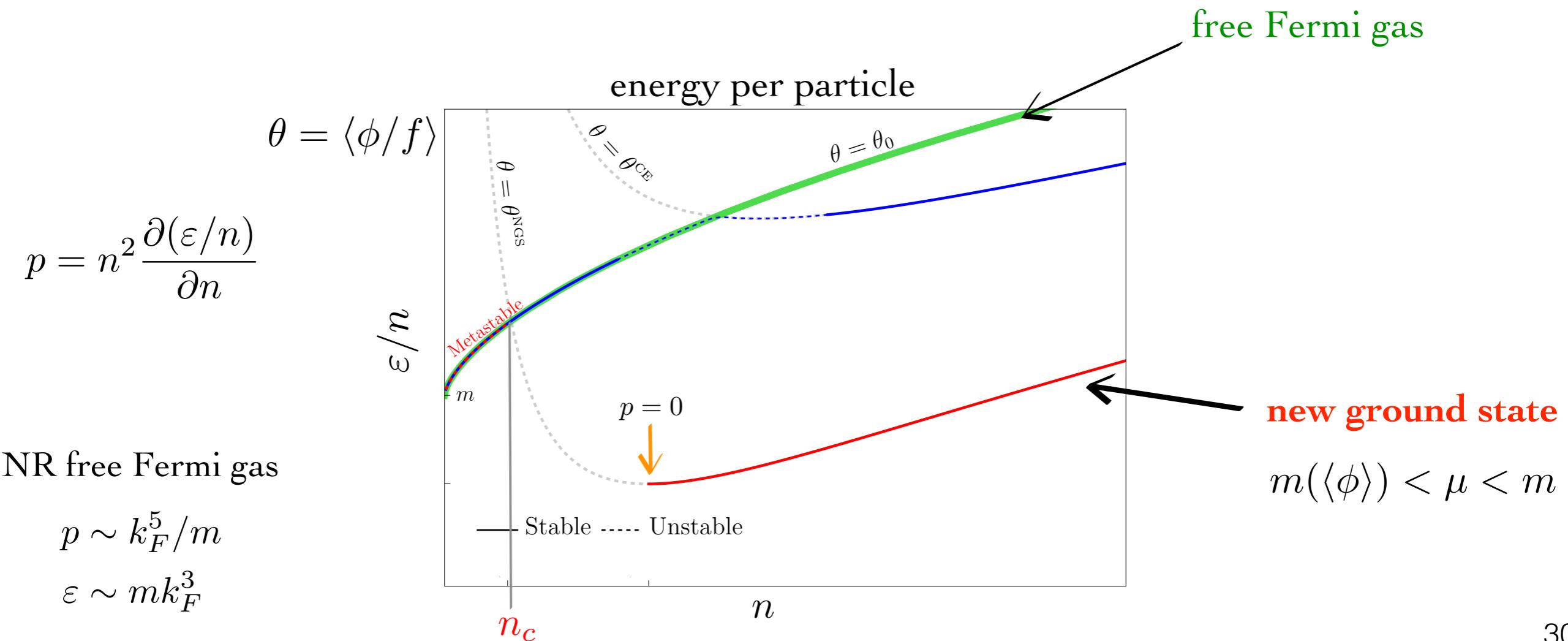


New Ground State

Lighter QCD axion leads to new scalarized ground state of (free) matter.

$$\begin{aligned}\varepsilon(n, \phi) &= \varepsilon_\psi(n, \phi) + V(\phi) \\ p(n, \phi) &= p_\psi(n, \phi) - V(\phi)\end{aligned}\quad \psi = n, e + p, e + p + n$$

$\langle \phi \rangle \neq 0$ \longrightarrow **Instability**
 $p_\psi(n_c, \langle \phi \rangle) < V(\langle \phi \rangle)$



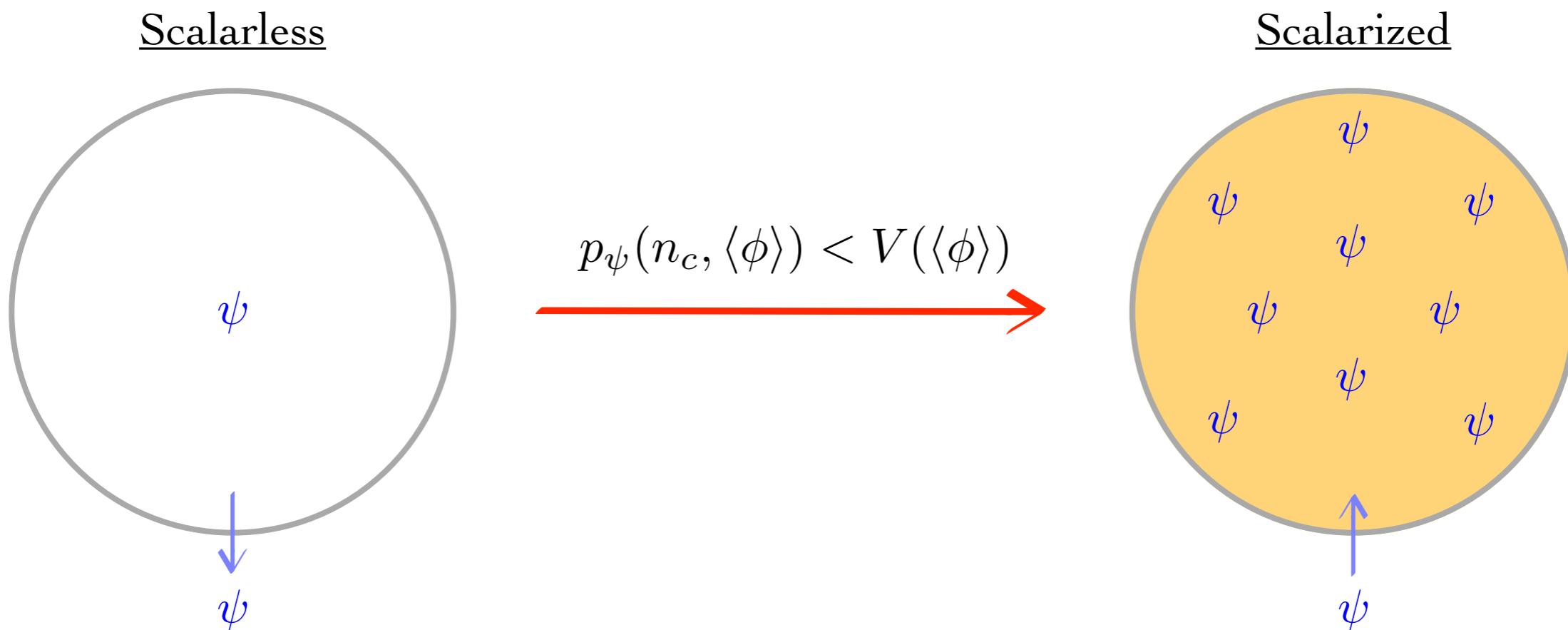
New Ground State

Lighter QCD axion leads to new scalarized ground state of (free) matter.

$$\varepsilon(n, \phi) = \varepsilon_\psi(n, \phi) + V(\phi)$$

$$p(n, \phi) = p_\psi(n, \phi) - V(\phi)$$

$$\psi = n, e + p, e + p + n$$



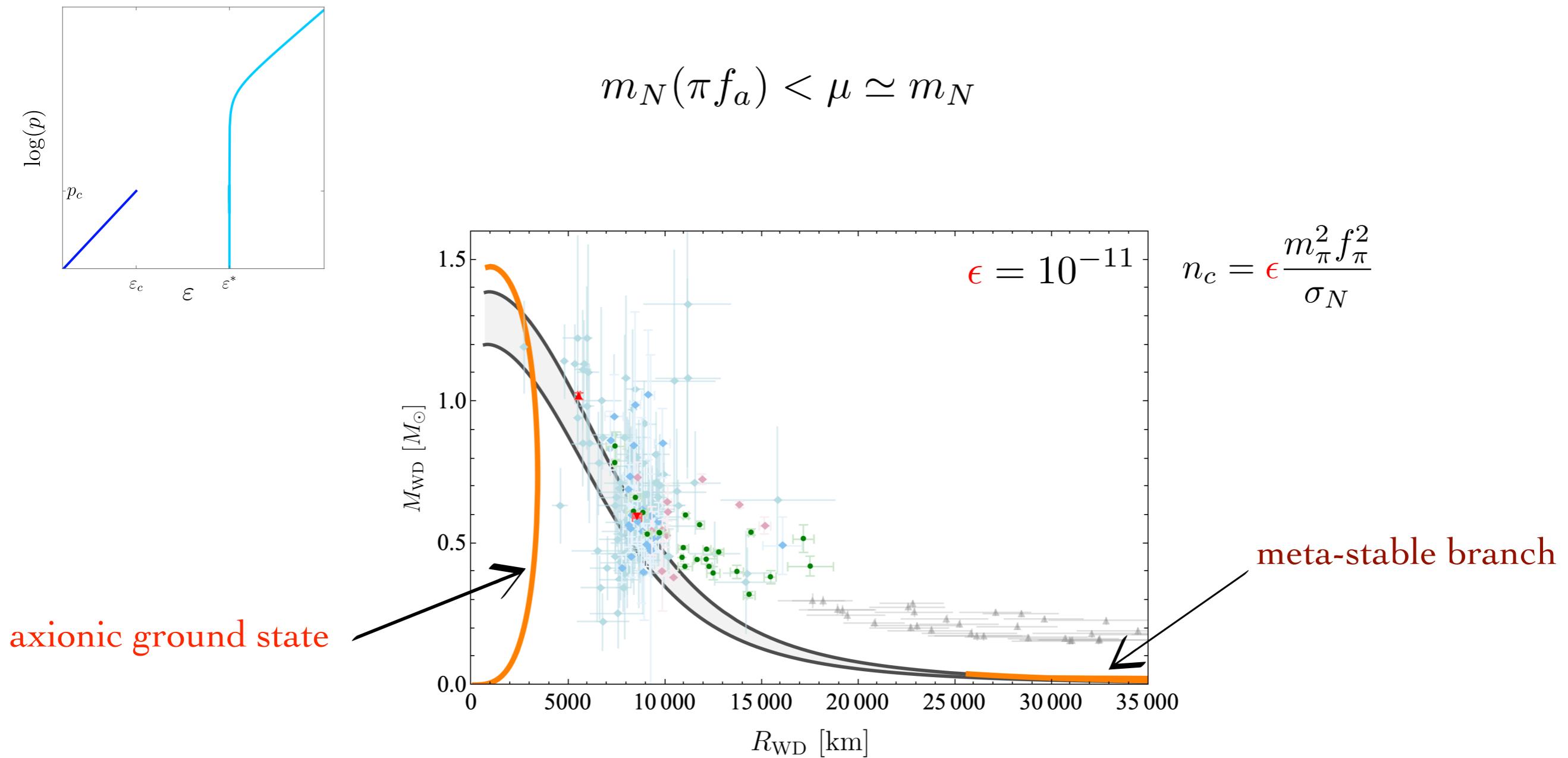
White-Dwarf Mass-Radius Gap

Discontinuity in Equation of State leads to a gap in M - R plane of WDs.

$$p \simeq p_e$$

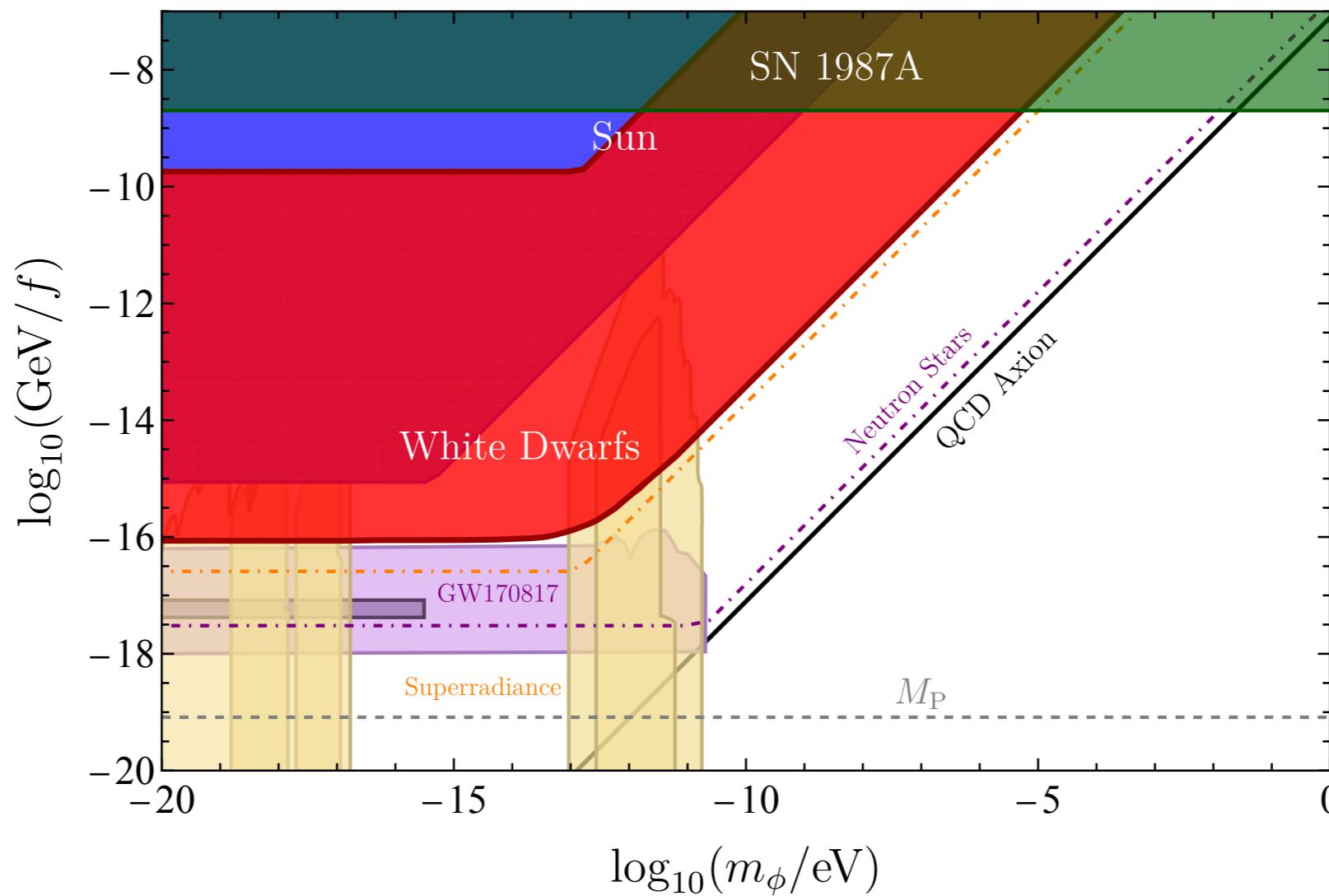
$$\varepsilon \simeq 2m_N n$$

$$n = n_e = n_p$$



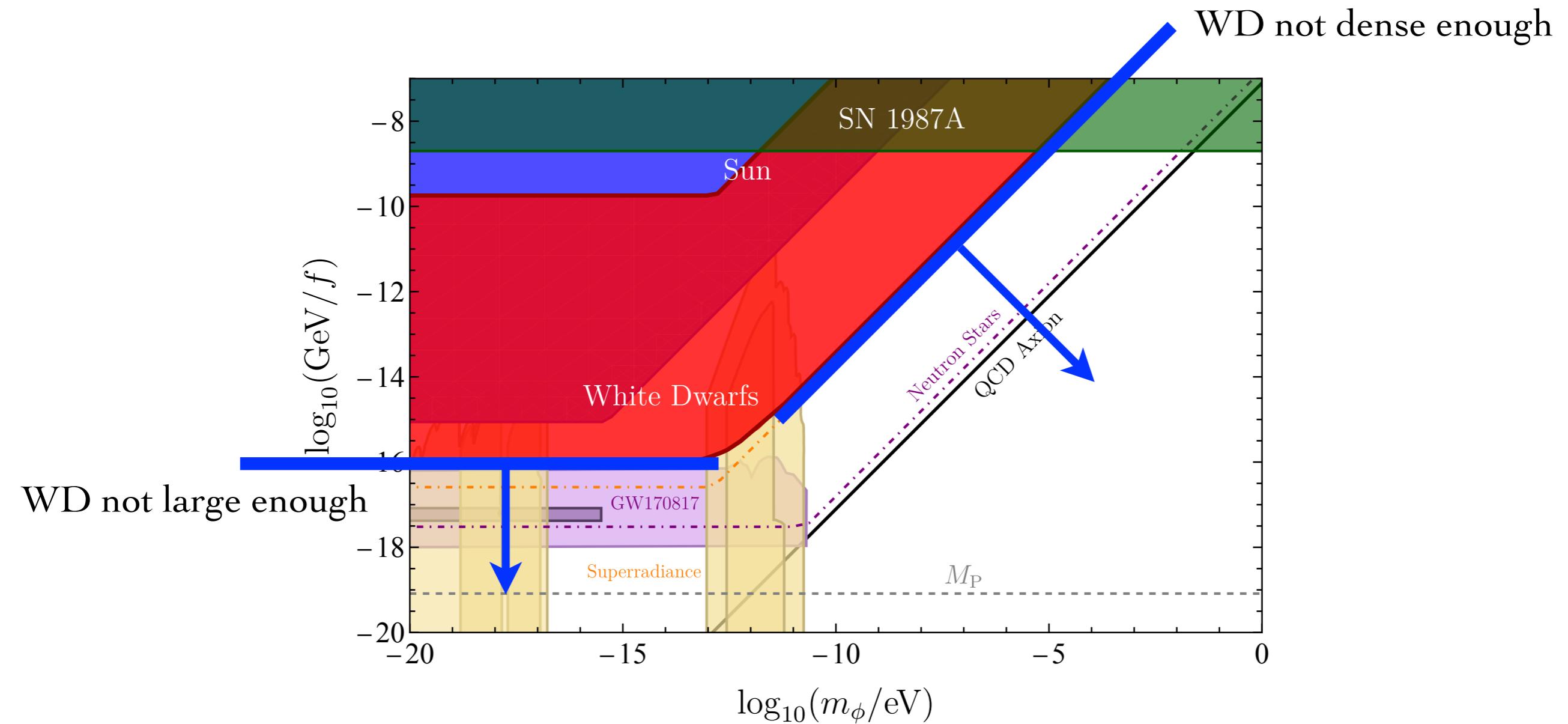
Experimental Bounds

Strong bounds on lighter QCD axion from absence of observed gap.



Experimental Bounds

Strong bounds on lighter QCD axion from absence of observed gap.



[Almost identical bounds hold for the Z_N realization.]

QCD Scalars

Heavier (and larger) Neutron Stars

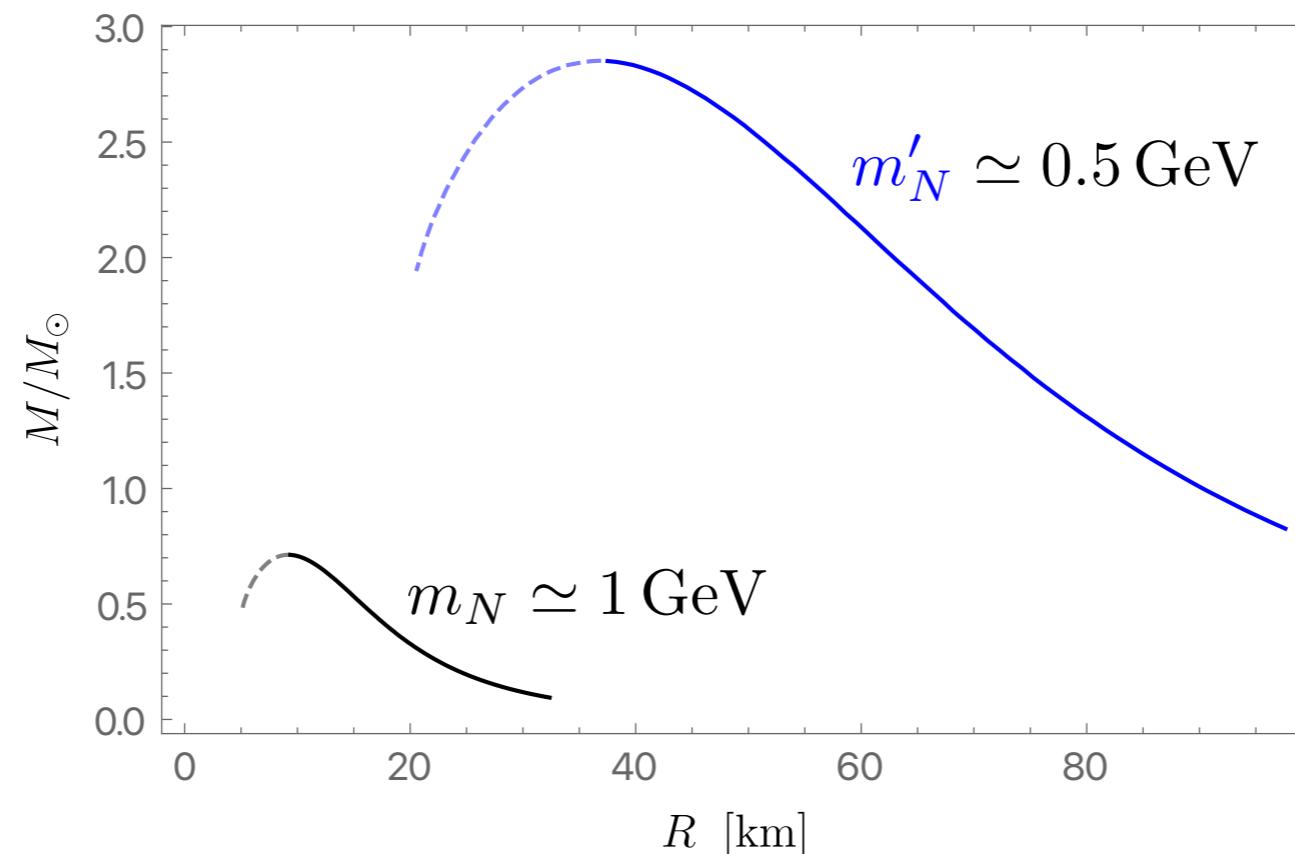
Mass (and radius) of NS made of a free Fermi gas neutrons increases w/ its mass.

$$\frac{M_{\max}(m_N)}{M_{\max}(m'_N)} \simeq \left(\frac{m'_N}{m_N} \right)^2$$

$$m(r) \sim m_N n_N r^3$$

$$\varepsilon_N \sim m_N n_N \sim m_N^4$$

$$\varepsilon_g \sim m(r) m_N n_N / M_{\text{Pl}}^2 r$$



Active QCD-coupled scalars realize this.

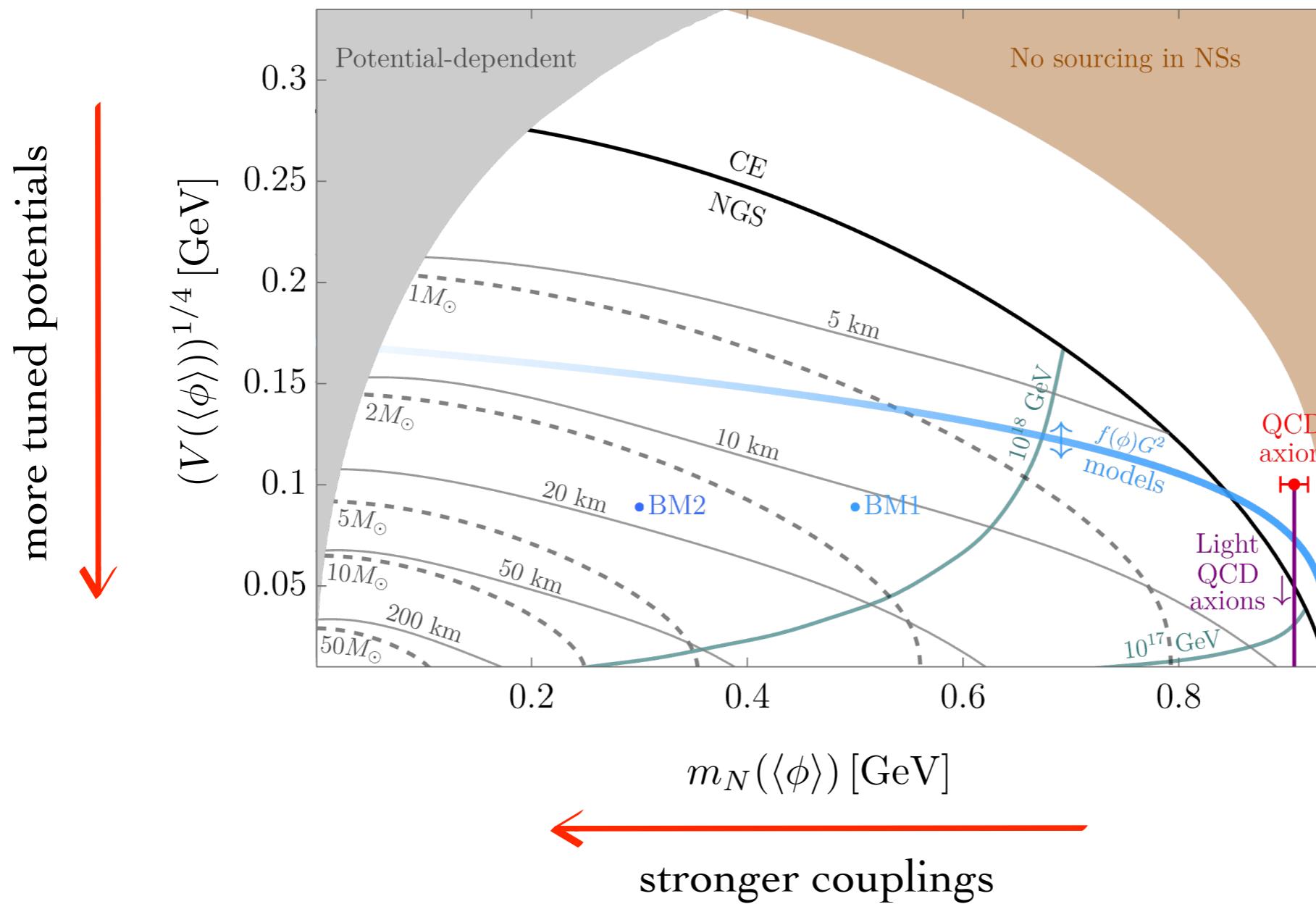
$$m'_N \equiv m_N(\langle \phi \rangle)$$

Scalarized Stellar Landscape

Mass and radius of (free-)neutron stars expand a much larger range than in the “SM”.

$$\varepsilon(n, \phi) = \varepsilon_N(n, m_N(\phi)) + V(\phi)$$

$$p(n, \phi) = p_N(n, m_N(\phi)) - V(\phi)$$



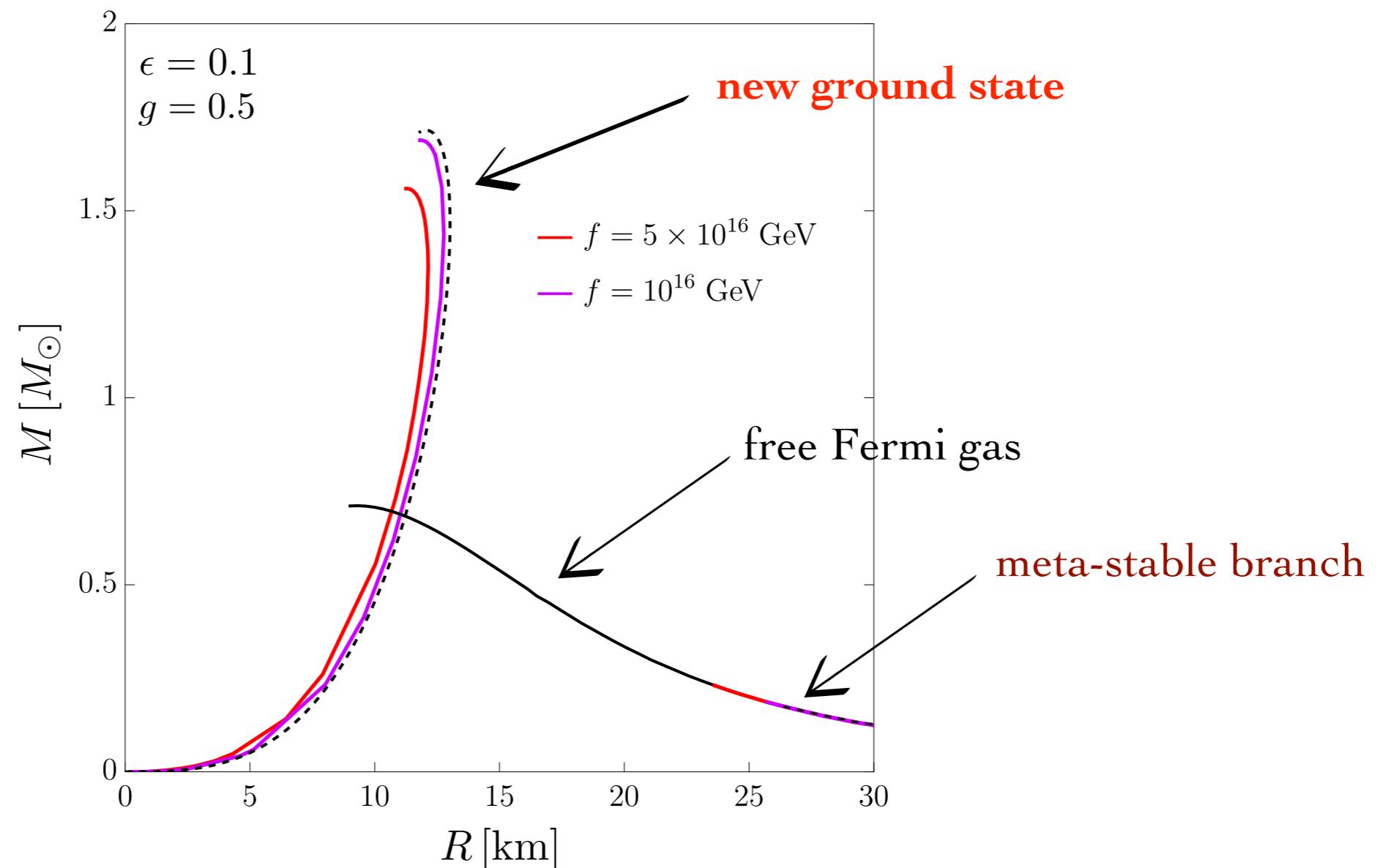
$$\frac{M_{\max}}{M'_{\max}} \approx \left(\frac{m'_N}{m_N} \right)^2$$

$$\frac{M_{\max}}{M'_{\max}} \approx \left(\frac{V(\langle \phi \rangle')}{V(\langle \phi \rangle)} \right)^{1/2}$$

Heavy Neutron Stars from ALPs

$$V(\phi) = -\epsilon m_\pi^2 f_\pi^2 \left(\cos \frac{\phi}{f} - 1 \right)$$

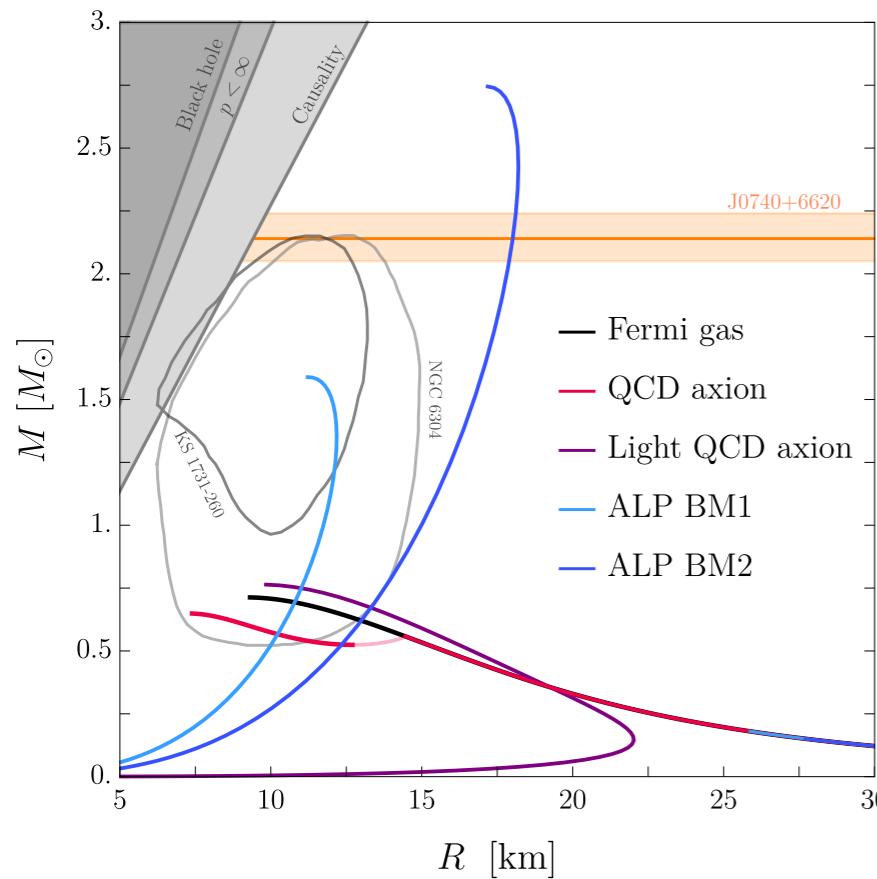
$$m_N(\phi) - m_N = +g m_N \left(\cos \frac{\phi}{f} - 1 \right)$$



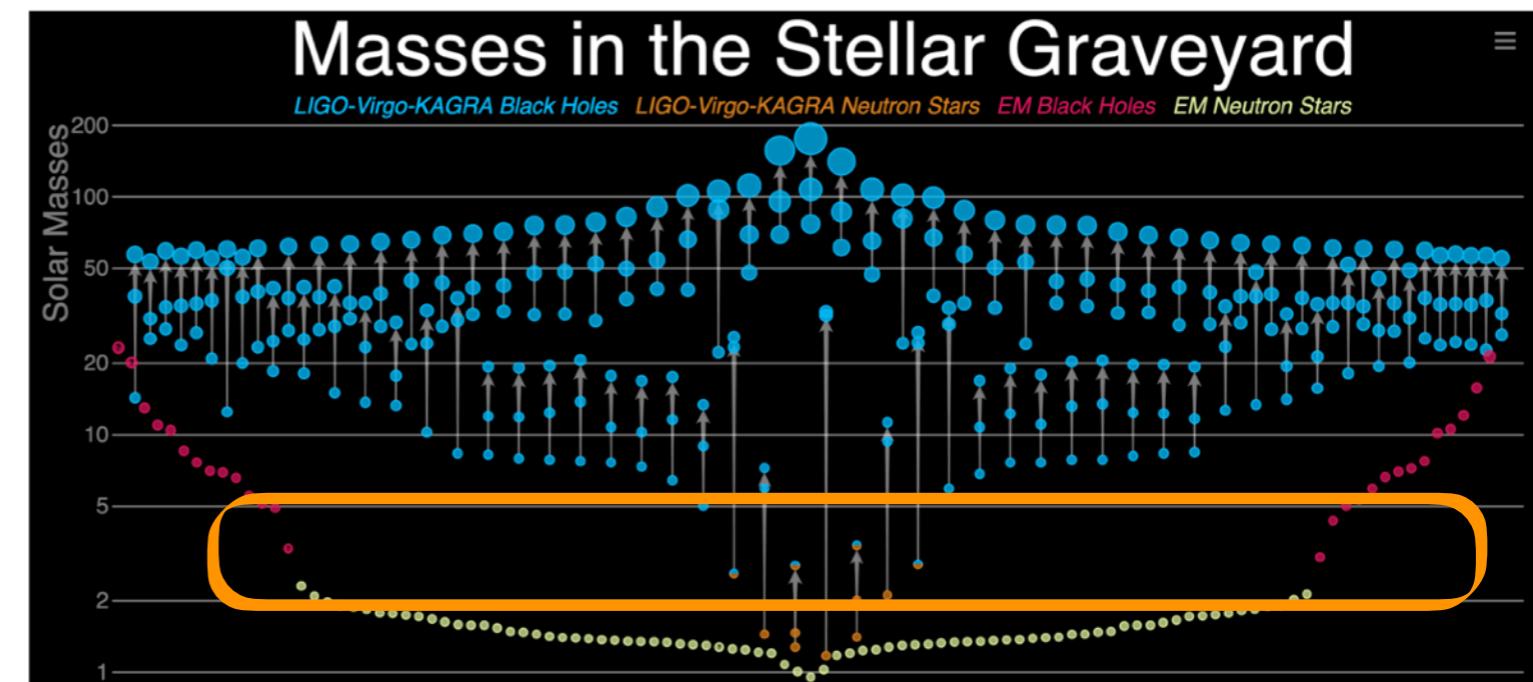
Experimental Observations

Simplified yet physically transparent analysis provides new benchmarks to compare w/ data.

X-ray obs.



Gravitationa wave obs.



Much more data to come in the future from NICER and LIGO-Vigro-KAGRA.

[M_{\max} causal bounds do not generically hold for scalarized NSs.]



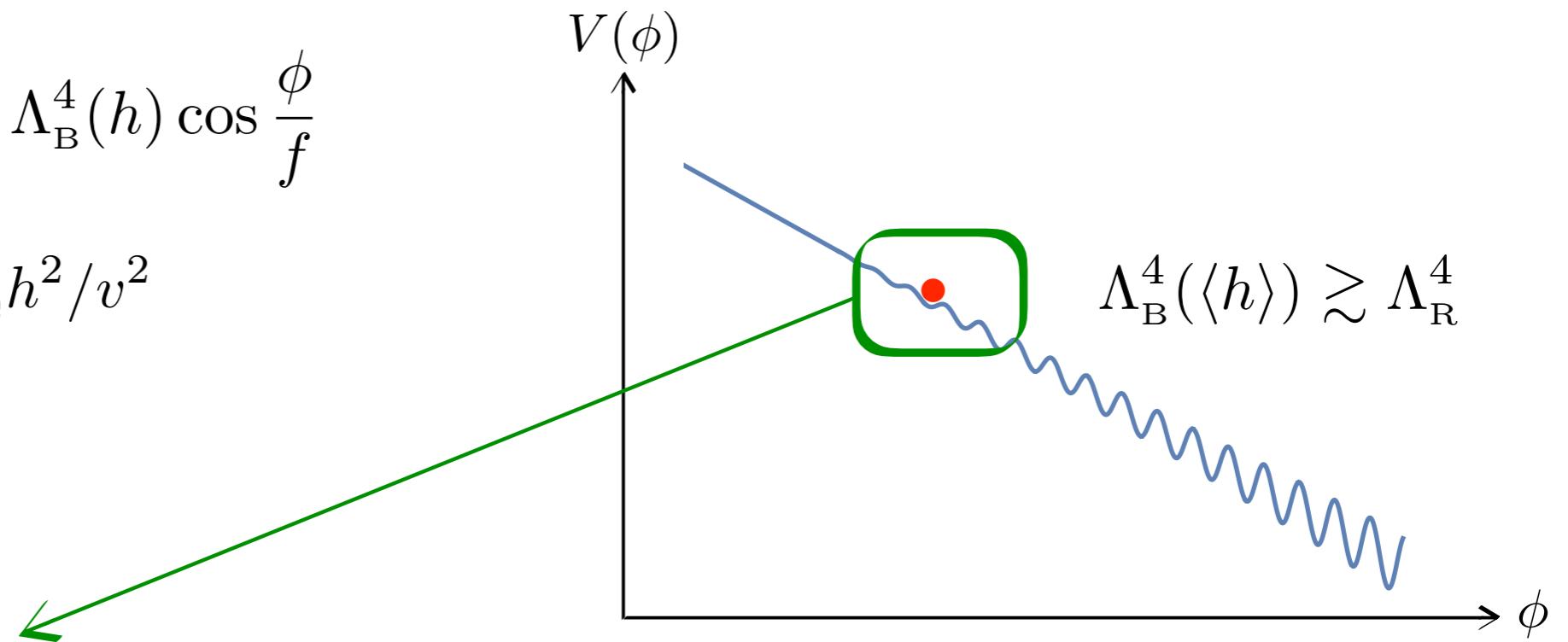
Relaxion

Relaxion at Finite Density

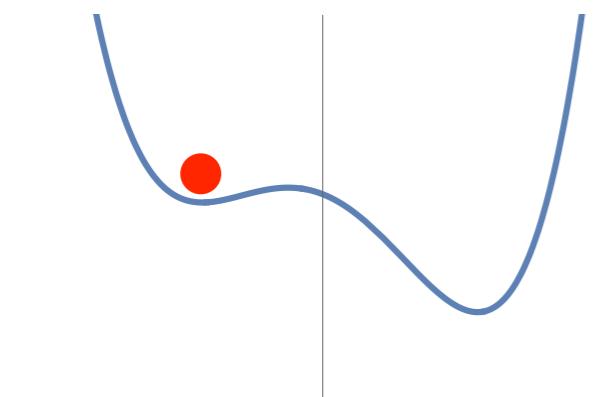
Vacua of (non-QCD) relaxion change in medium, due to Higgs dependence.

$$V(\phi) = -\Lambda_{\text{R}}^4 \frac{\phi}{f} + \Lambda_{\text{B}}^4(h) \cos \frac{\phi}{f}$$

$$\Lambda_{\text{B}}^4(h) = \Lambda_{\text{C}}^4 h^2 / v^2$$



$n = 0$

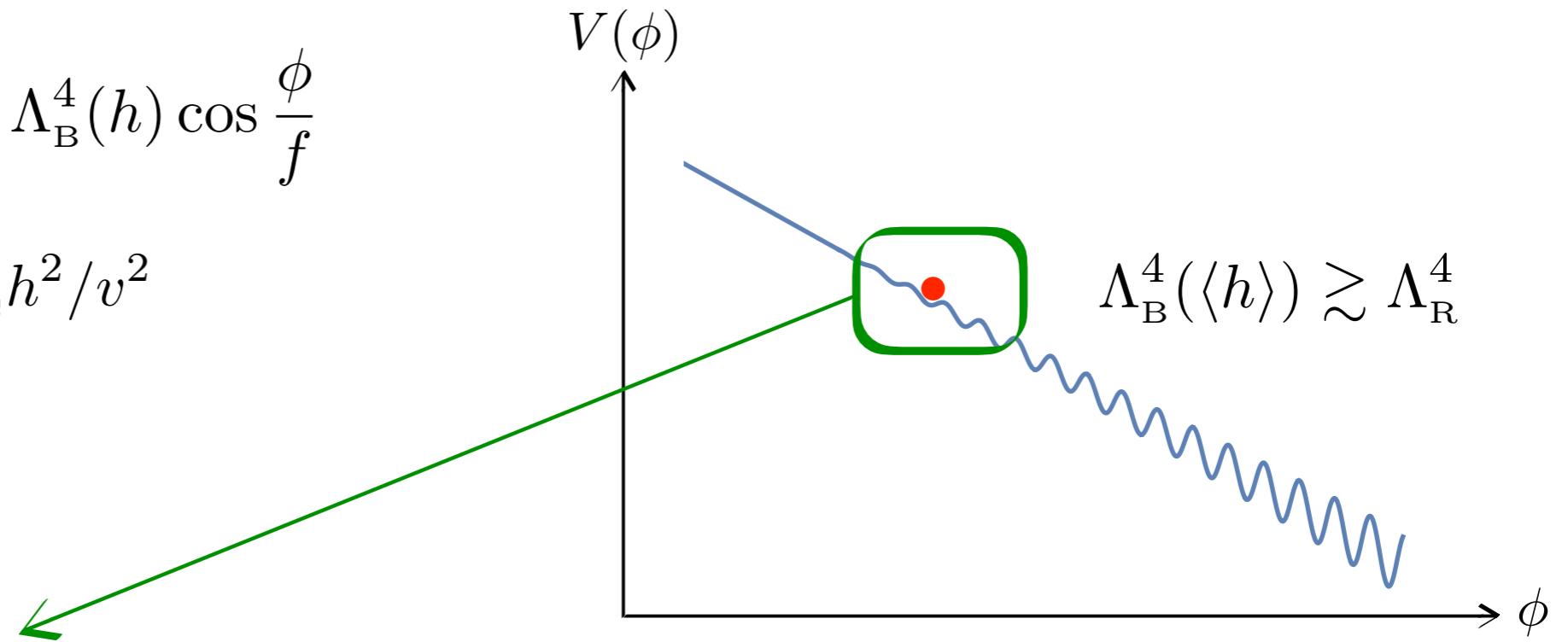


Relaxion at Finite Density

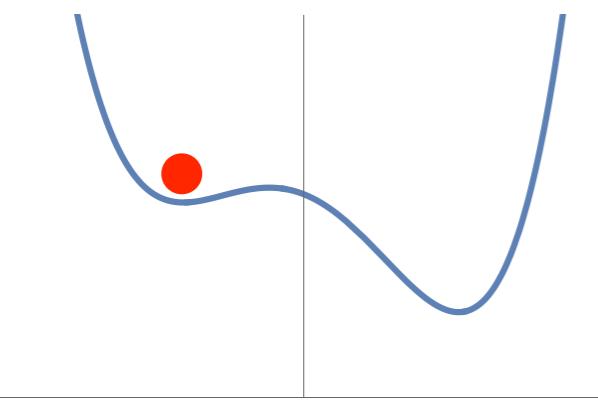
Vacua of (non-QCD) relaxion change in medium, due to Higgs dependence.

$$V(\phi) = -\Lambda_{\text{R}}^4 \frac{\phi}{f} + \Lambda_{\text{B}}^4(h) \cos \frac{\phi}{f}$$

$$\Lambda_{\text{B}}^4(h) = \Lambda_{\text{C}}^4 h^2 / v^2$$



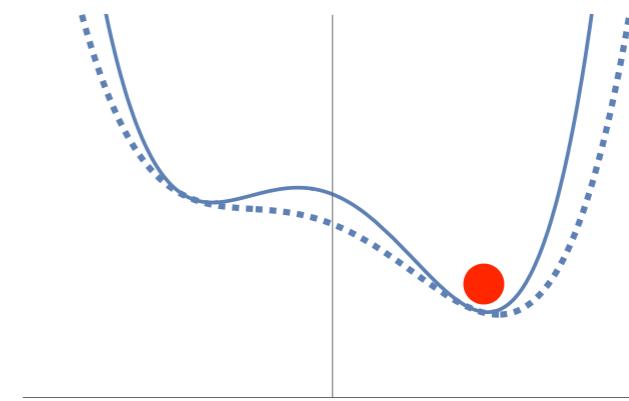
$n = 0$



$$\delta h^2 \sim \frac{\sigma_N}{v} \frac{n}{\lambda_H v^3}$$

$$\Lambda_{\text{B}}^4(n_c) \equiv \Lambda_{\text{R}}^4$$

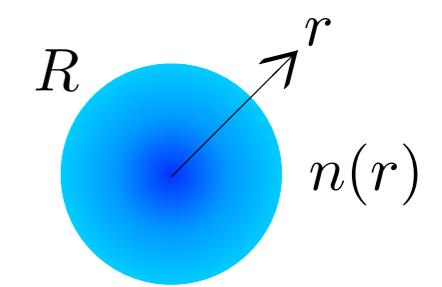
$n > n_c$



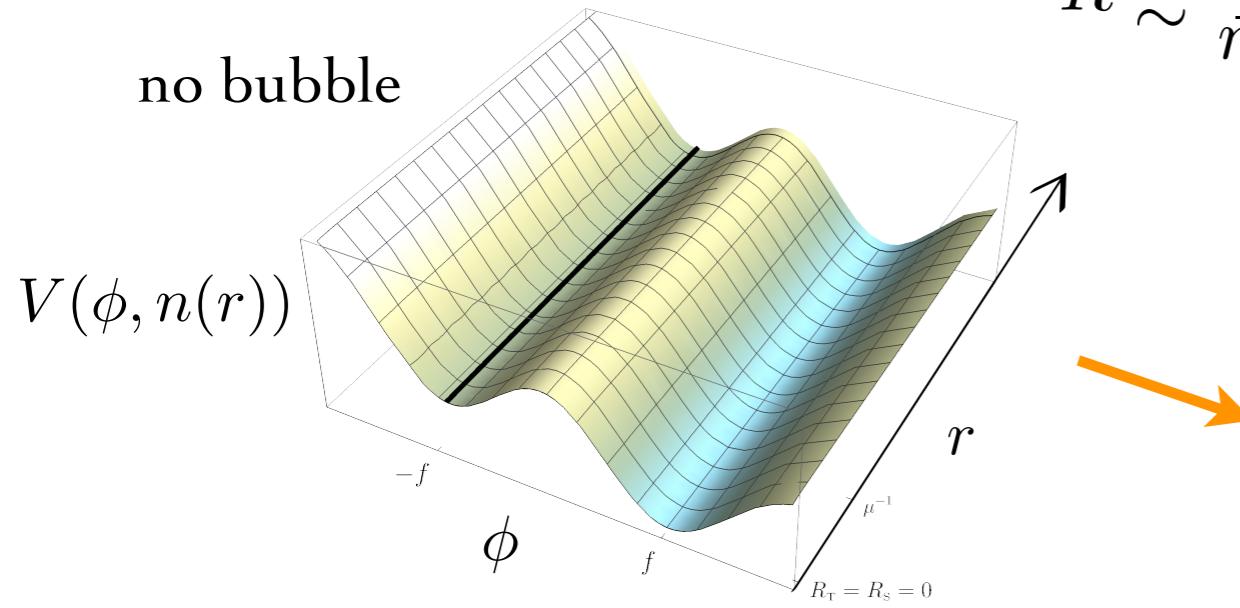
Classical transition between vacua allowed above critical density.

Bubble Formation and Expansion

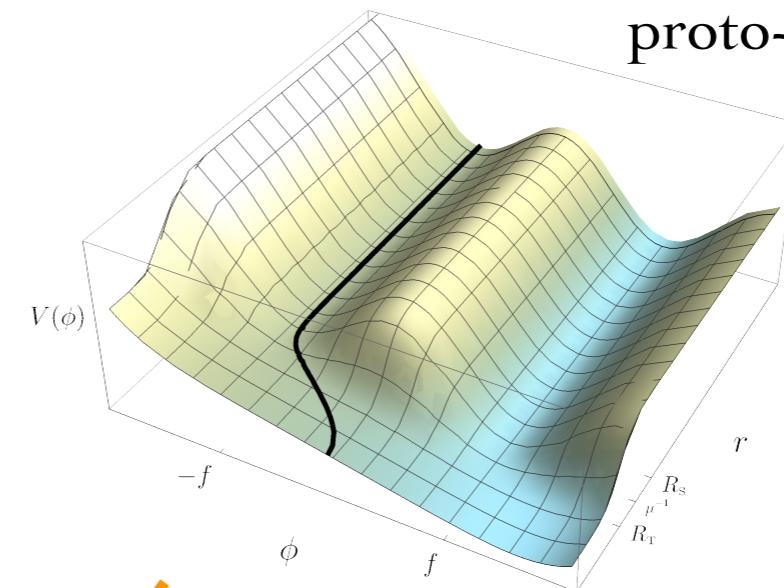
$$R \gtrsim \frac{1}{\bar{m}_\phi(n)} \simeq \frac{f}{\Lambda_R^2}$$



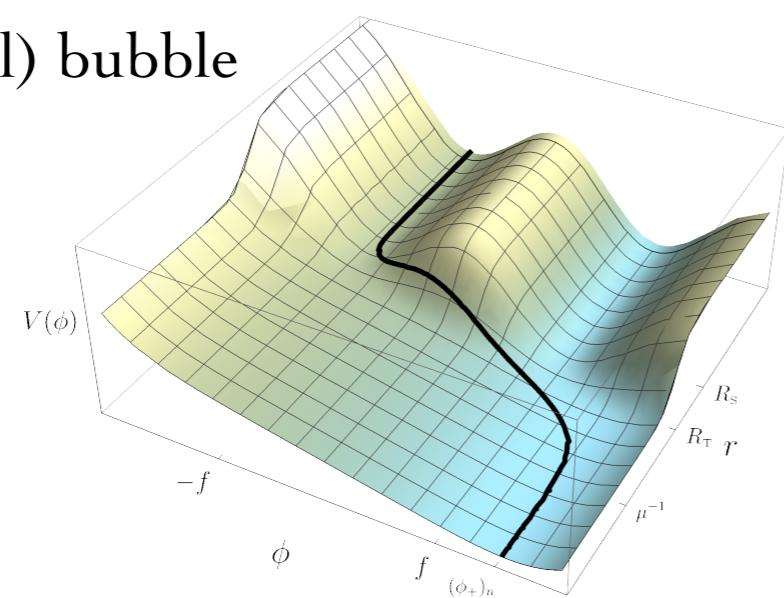
no bubble



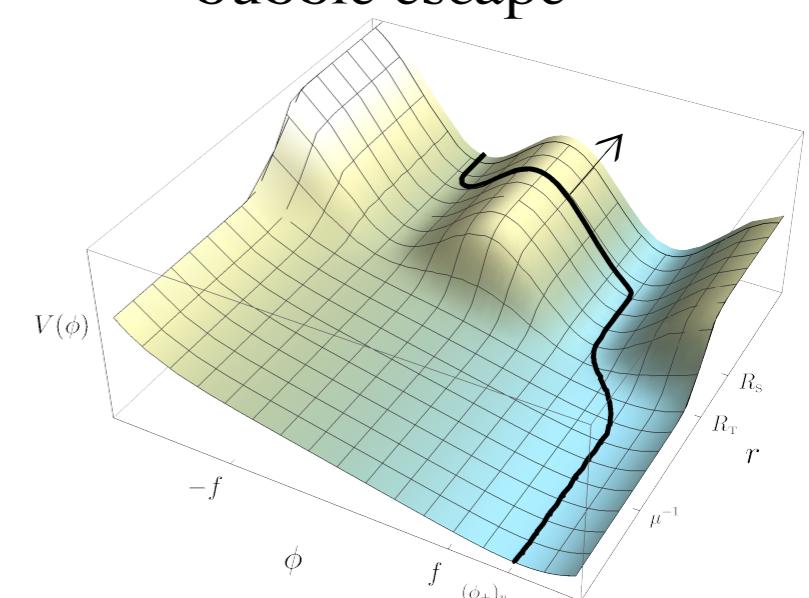
proto-bubble



(full) bubble

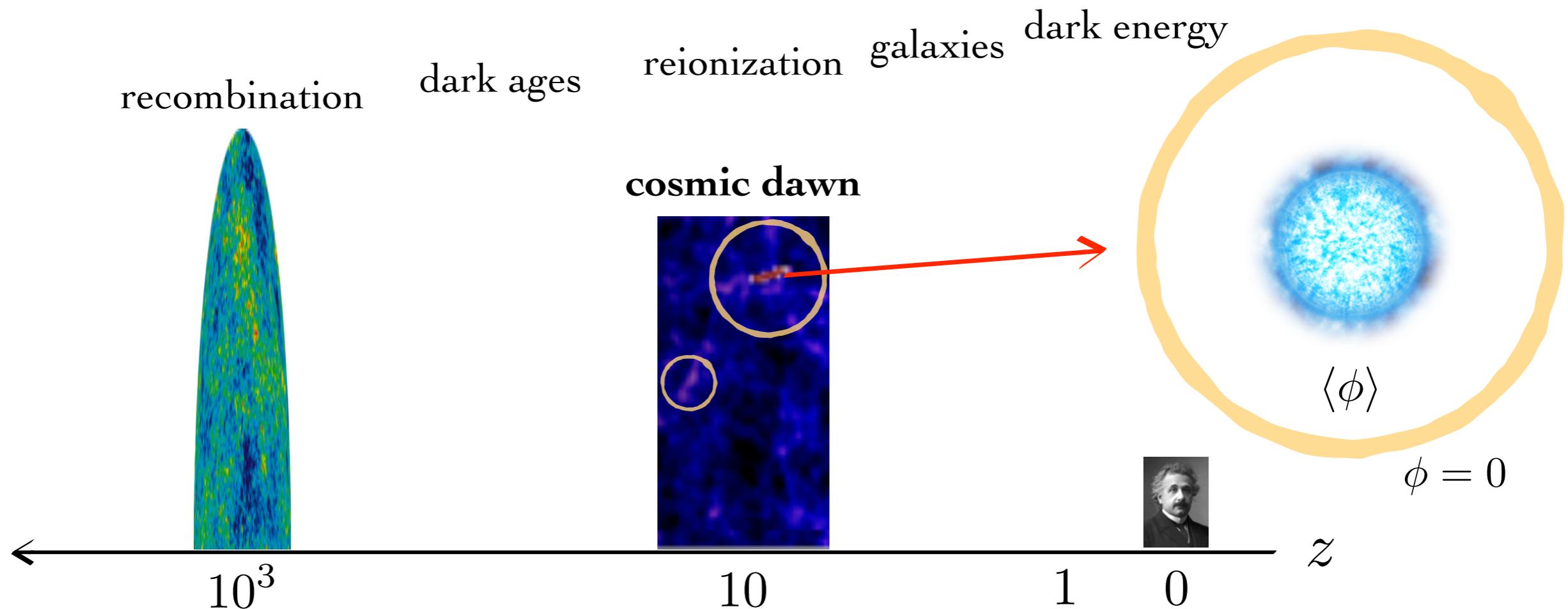


bubble escape

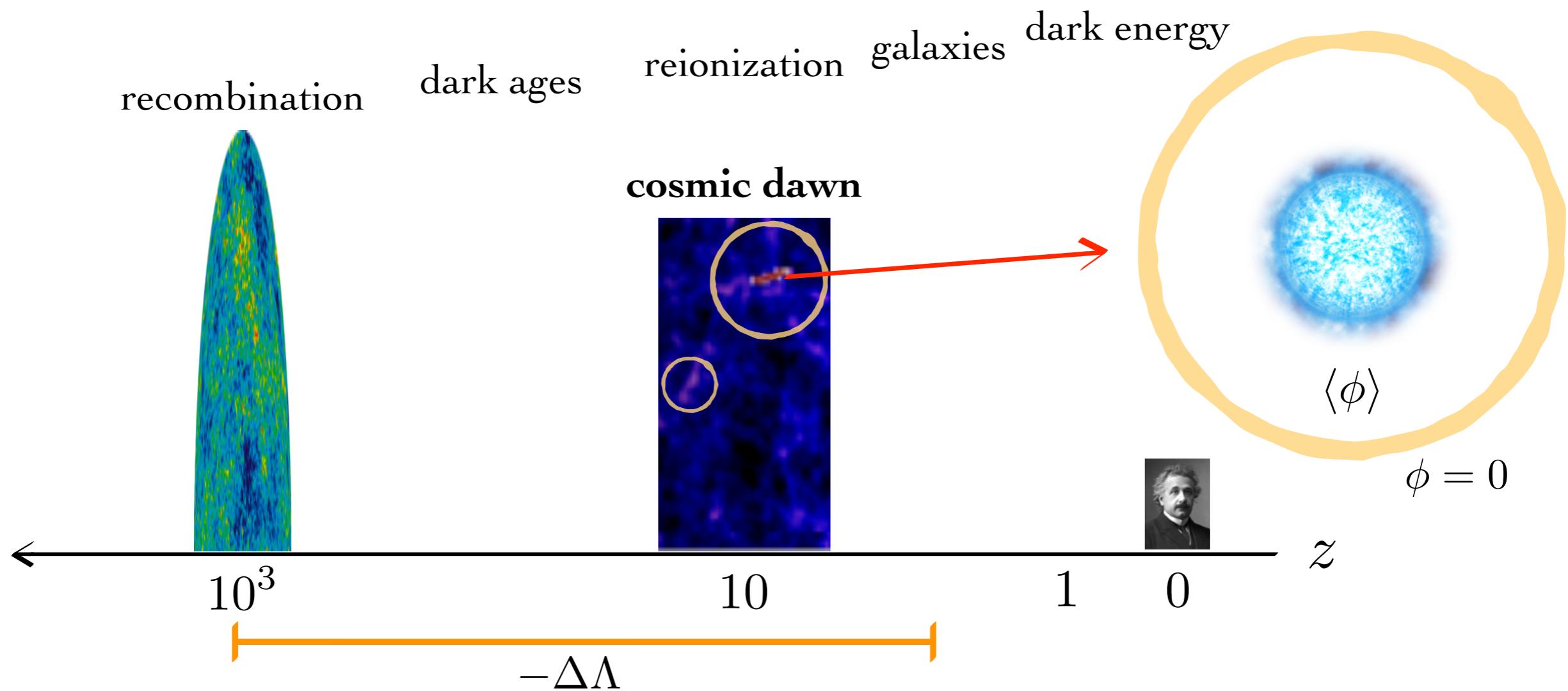


[Verified w/ numerical solutions of scalar EoM.]

Phase Transition at Cosmic Dawn



Phase Transition at Cosmic Dawn



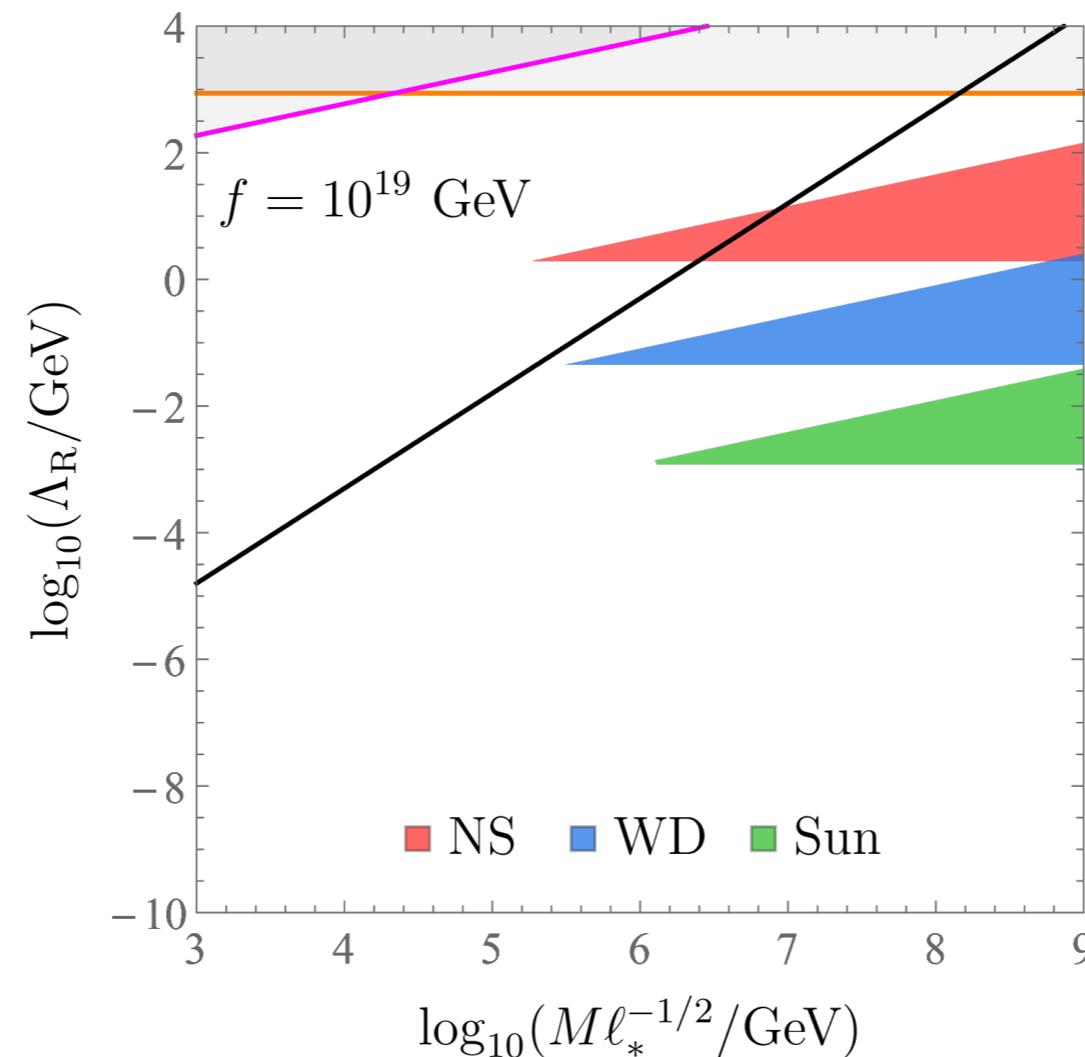
If our universe (within horizon) completely transitions to lower-energy minimum:

$$-\Delta\Lambda \sim \Lambda_R^4 \gtrsim \left(\frac{f}{R}\right)^2 \approx \Lambda_0 \times 10^{15} \left(\frac{f}{10 \text{ TeV}}\right)^2 \left(\frac{10 \text{ km}}{R}\right)^2$$

For dense enough small stars, change in dark energy (CC) way too large.

Experimental Bounds

Large fraction of relaxion parameter space would have led to too large change in CC.



$$-\Delta\Lambda \lesssim \Lambda_0 \times 10^2$$

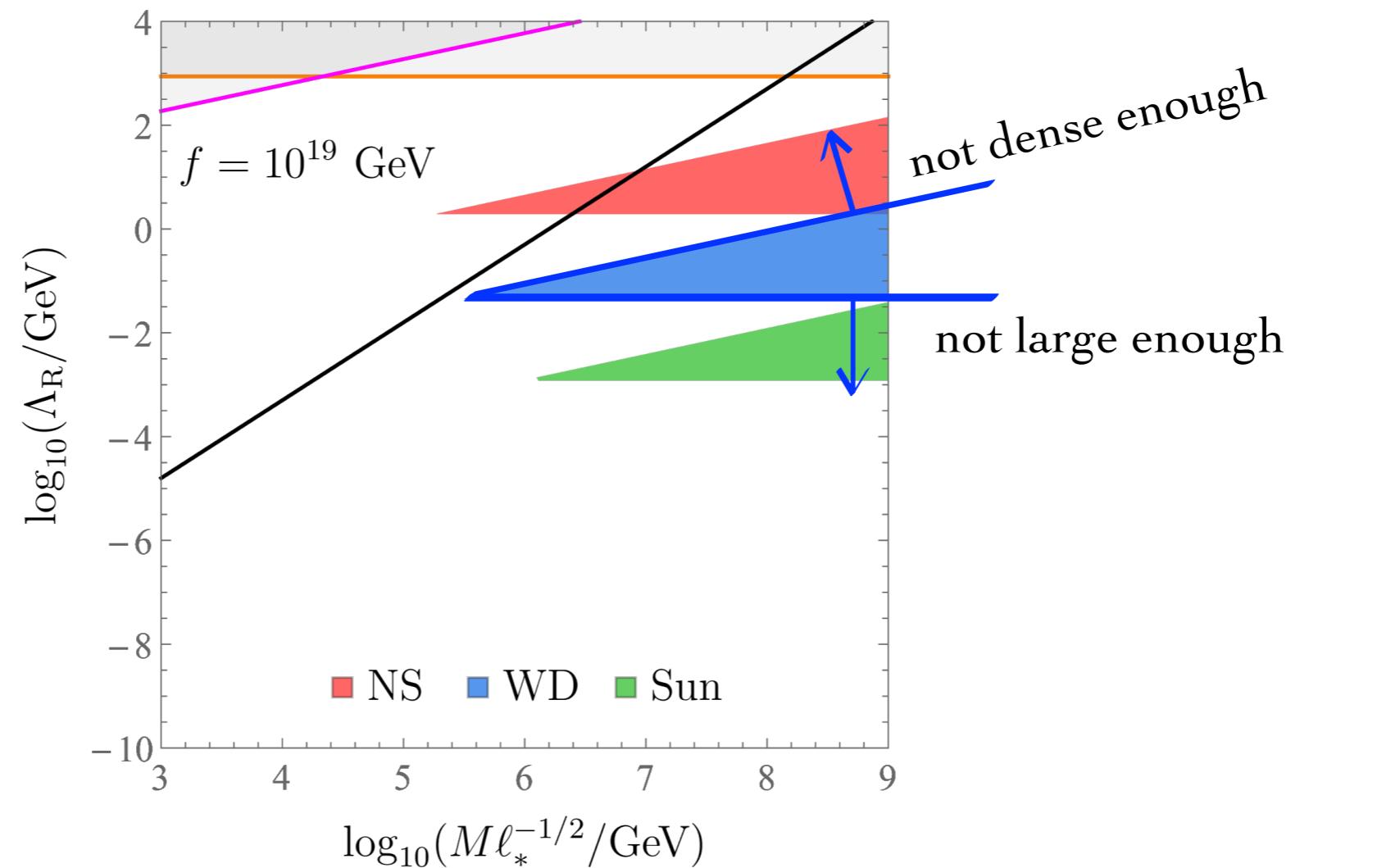
Karwall, Kamionkowski '16

M = cutoff

ℓ_* = position minimum

Experimental Bounds

Large fraction of relaxion parameter space would have led to too large change in CC.



$$-\Delta\Lambda \lesssim \Lambda_0 \times 10^2$$

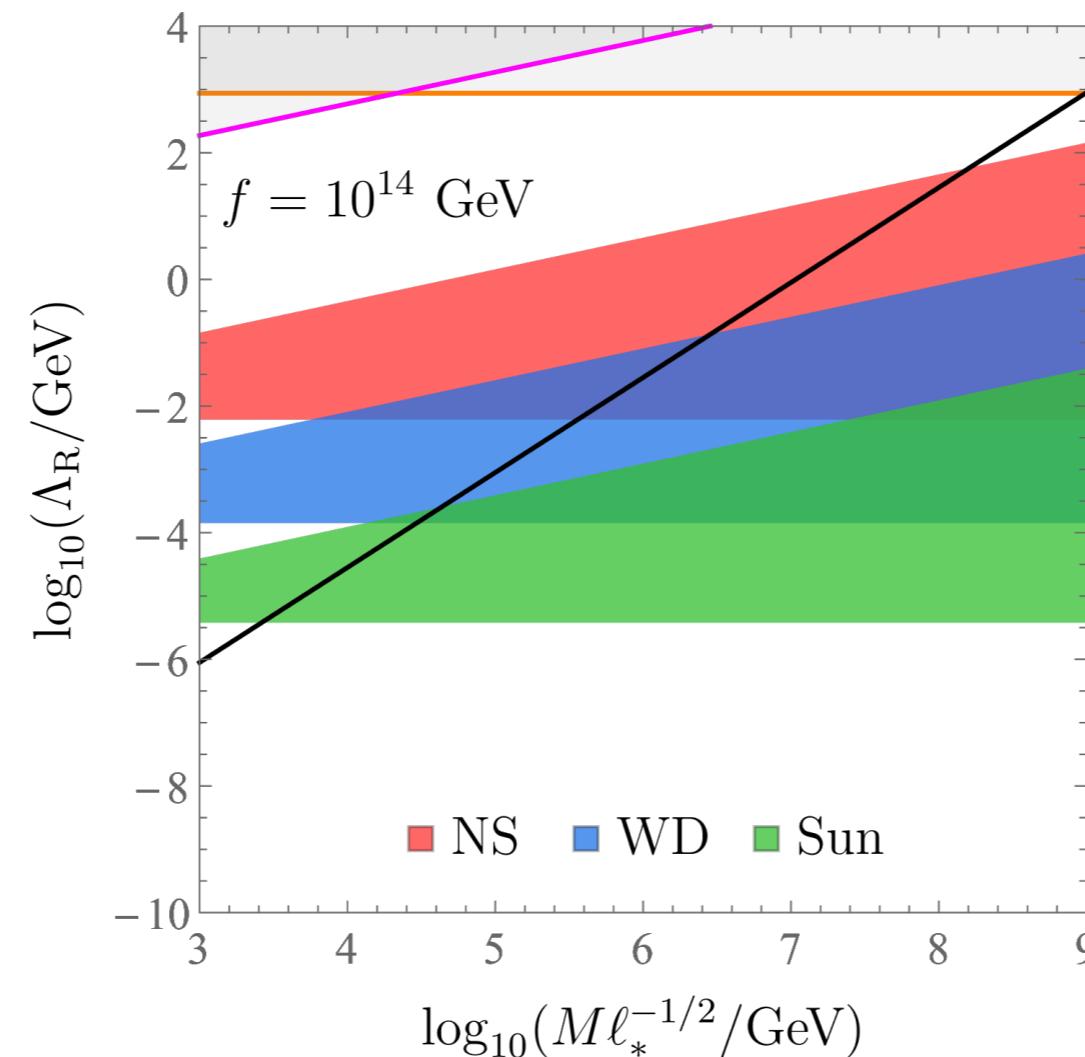
Karwall, Kamionkowski '16

M = cutoff

ℓ_* = position minimum

Experimental Bounds

Large fraction of relaxion parameter space would have led to too large change in CC.



$$-\Delta\Lambda \lesssim \Lambda_0 \times 10^2$$

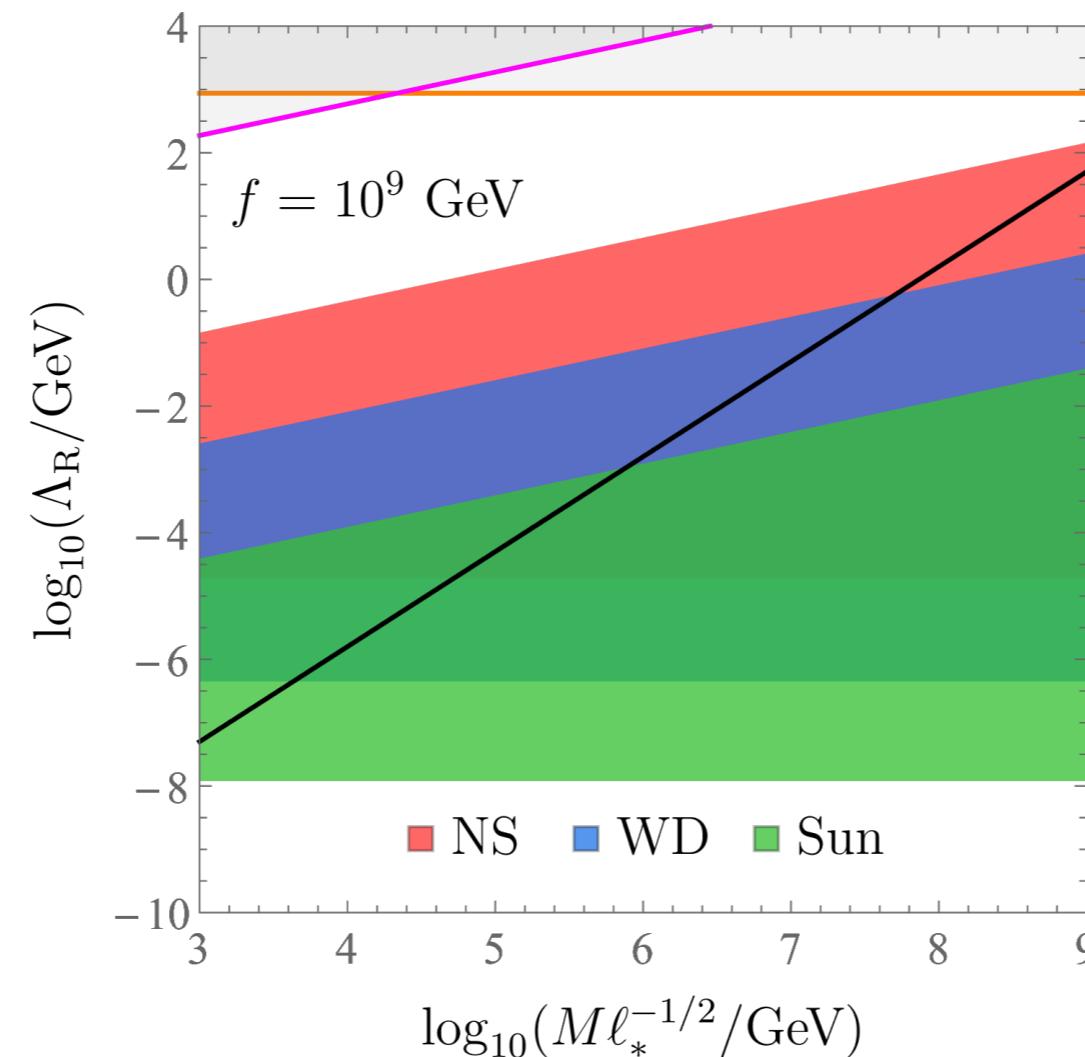
Karwall, Kamionkowski '16

M = cutoff

ℓ_* = position minimum

Experimental Bounds

Large fraction of relaxion parameter space would have led to too large change in CC.



$$-\Delta\Lambda \lesssim \Lambda_0 \times 10^2$$

Karwall, Kamionkowski '16

M = cutoff

ℓ_* = position minimum

Conclusions

Rich interplay between light, weakly-coupled BSM physics and finite density systems:

- QCD-axion emission in supernovae and neutron stars.
- Lighter QCD-axion bounds from $M-R$ relation of white dwarfs.
- Heavy neutron stars in new ground state from QCD-coupled scalars.
- Relaxion bounds from absence of phase transitions at cosmic dawn.
- ...

To probe solutions to the main naturalness problems of the SM.

Thank you.

Finite Density Field Theory

Grand-canonical potential density:

$$\hat{\Omega} = \mathcal{H} - \mu J^0$$

Chemical potential as temporal component of gauge field associated w/ conserved charge:

$$\partial_0 \rightarrow \partial_0 + i\mu q$$

Ensemble averages w/ density matrix (path) integration:

$$\langle \mathcal{O} \rangle = \frac{\text{Tr } \mathcal{O} \hat{\rho}}{\text{Tr } \hat{\rho}} \quad \hat{\rho} = e^{-\frac{1}{T}(H - \mu Q)} \quad Z(T, \mu, \mathcal{V}) = \text{Tr } \hat{\rho}$$



$$\Omega(T, \mu) = -\frac{T \ln Z}{\mathcal{V}} = \varepsilon - \mu n = -p$$

Light Scalars

Scalar fields of interest are light in contrast to the star's density or temperature.

$$m_\phi \ll n^{1/3} = 1/\ell$$

coherently coupled

$$n \gtrsim \frac{\partial V/\partial\phi}{-\partial m/\partial\phi} \sim \frac{m_\phi^2 f^2}{g_\psi m}$$

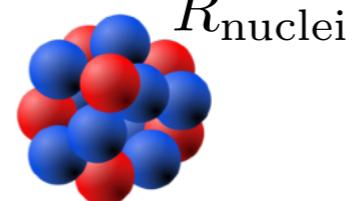
density-sensitive V

$$m_\phi \lesssim T$$

stellar emission

[Energy gradients too large to be source by single nuclei.]

$$m_\phi \ll \frac{1}{R_{\text{nuclei}}}$$



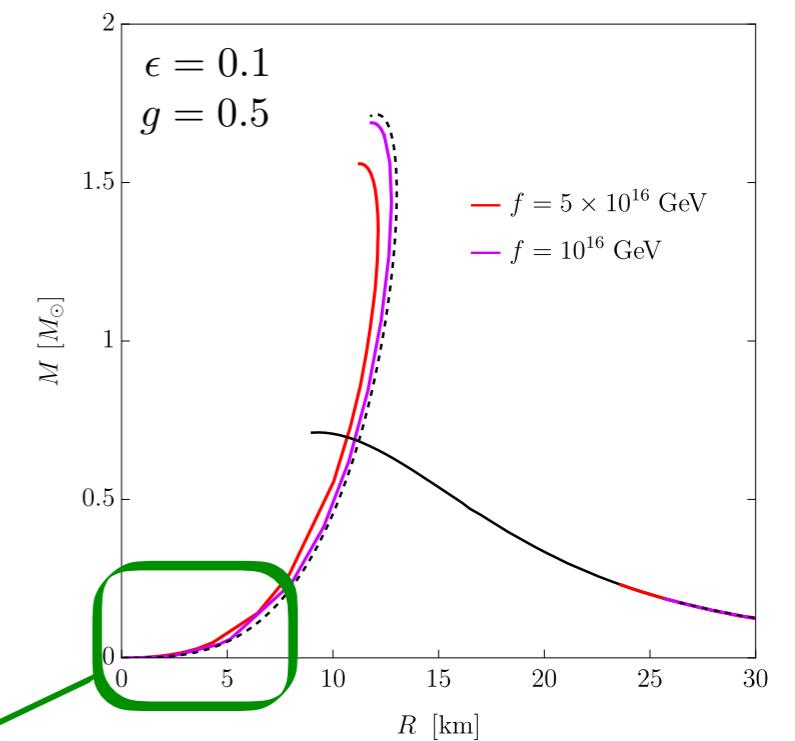
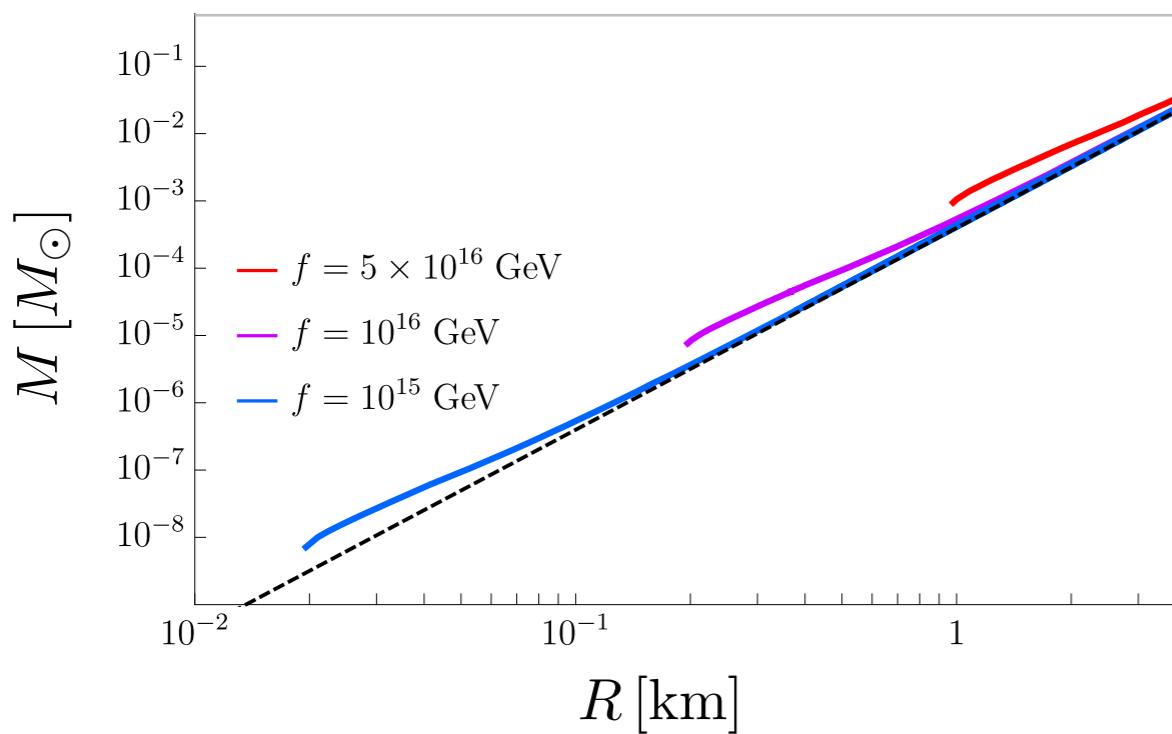
Self-Bound Objects

NGS implies stable objects held together by gradient pressure (i.e. no gravity).

$$\frac{\partial \Omega}{\partial \phi} = \frac{\partial V}{\partial \phi} + n \frac{\partial m_N}{\partial \phi}$$

$$p' = -\phi' \frac{\partial \Omega}{\partial \phi}$$

$$\phi'' + \frac{2}{r}\phi' = \frac{\partial \Omega}{\partial \phi}$$

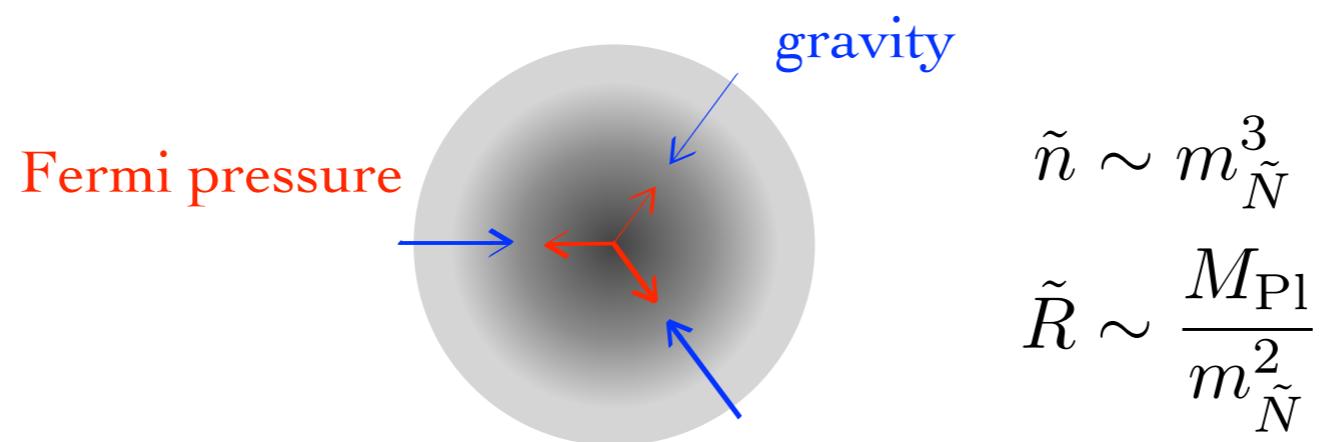


$$R_{\text{SBO}}^{\min} \sim \frac{1}{m_\phi}$$

Dark Compact Objects

Dark Stars

Classical vacuum transitions seeded by dark-baryon stars.



$$\tilde{n} \sim m_{\tilde{N}}^3$$

$$\tilde{R} \sim \frac{M_{\text{Pl}}}{m_{\tilde{N}}^2}$$

$$\frac{\Lambda_B^4(\tilde{n})}{\Lambda_B^4} \simeq 1 - \frac{\sigma_{\tilde{N}} \tilde{n}}{\Lambda_C^4}$$

dense enough: $\Lambda_B^4(\tilde{n}) < \Lambda_R^4$

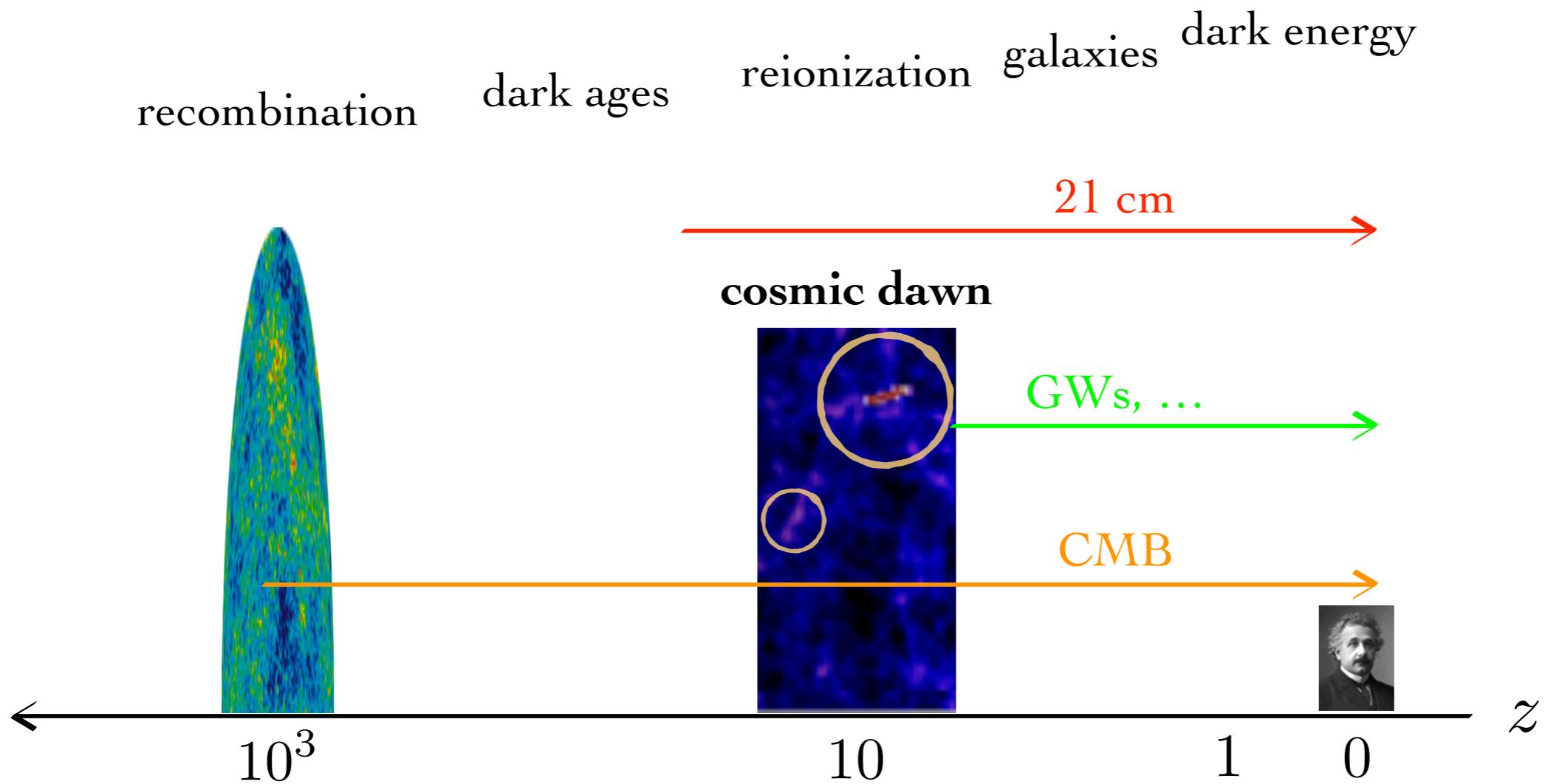
large enough: $m_{\tilde{N}} \lesssim \Lambda_C \sqrt{\frac{M_{\text{Pl}}}{f}}$

↓

$$-\Delta\Lambda \gtrsim m_{\tilde{N}}^4 \left(\frac{f}{M_{\text{Pl}}} \right)^2 \approx \Lambda_0 \times 10^{-2} \left(\frac{m_{\tilde{N}}}{10 \text{ keV}} \right)^4 \left(\frac{f}{10 \text{ TeV}} \right)^2$$

Dark stars could seed phase transitions a priori consistent w/ change in dark energy.

Experimental Probes



Many potential probes of a phase transition at cosmic dawn.