



GW signatures of domain walls

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Outline

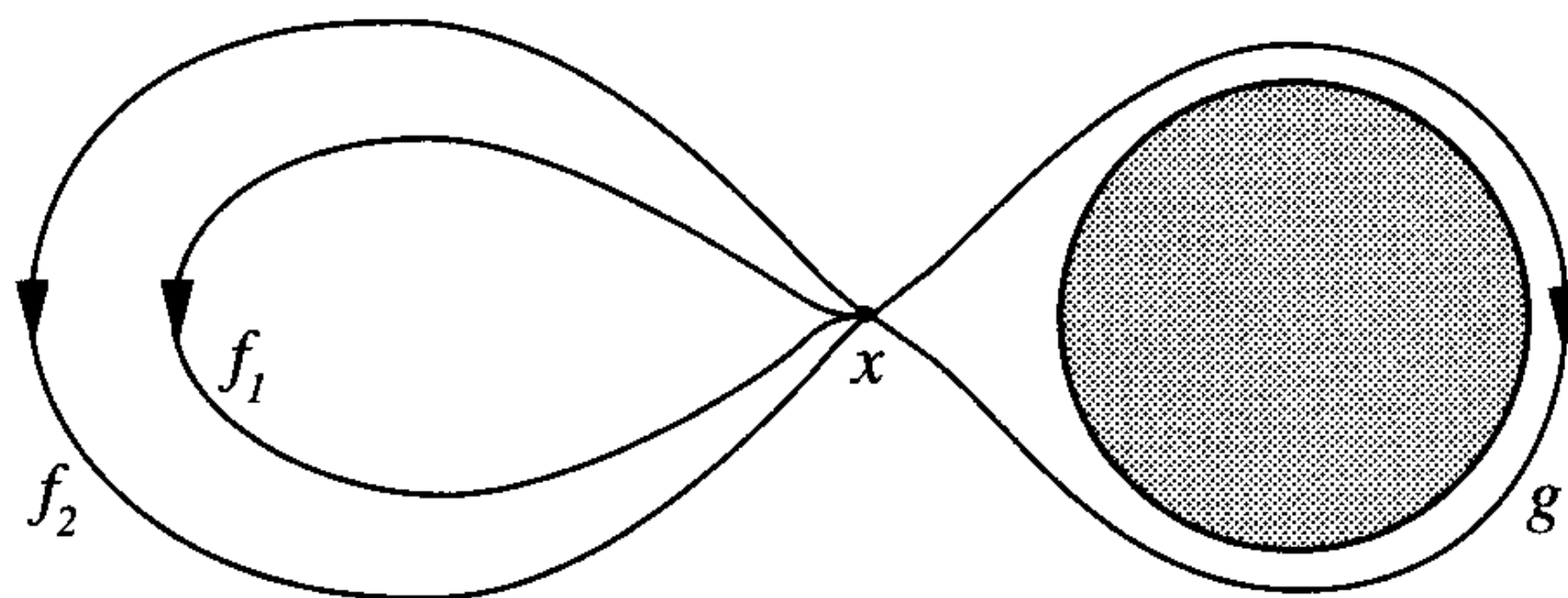
- Defects in field theory
- Formation in the early Universe
- Domain walls as seeds for bubble nucleation
- Domain walls as GW sources:
 - Impact of particle friction
 - Improved understanding of the scaling regime

Defects in field theory

- Topological defects are non-trivial (space dependent) solutions of the EOM.
- Their classification is based on the topological properties of the vacuum manifold \mathcal{M} :

Symmetry group G broken to subgroup $H \Rightarrow \mathcal{M} = G/H$ (coset).

- The type of defects that are supported depends on the non trivial homotopy group of \mathcal{M}



Space equivalent to $\mathcal{M} = S^1$ with
non trivial $\pi_1(S^1) = \mathbb{Z}$

Fig. from Vilenkin & Shellard

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Strings

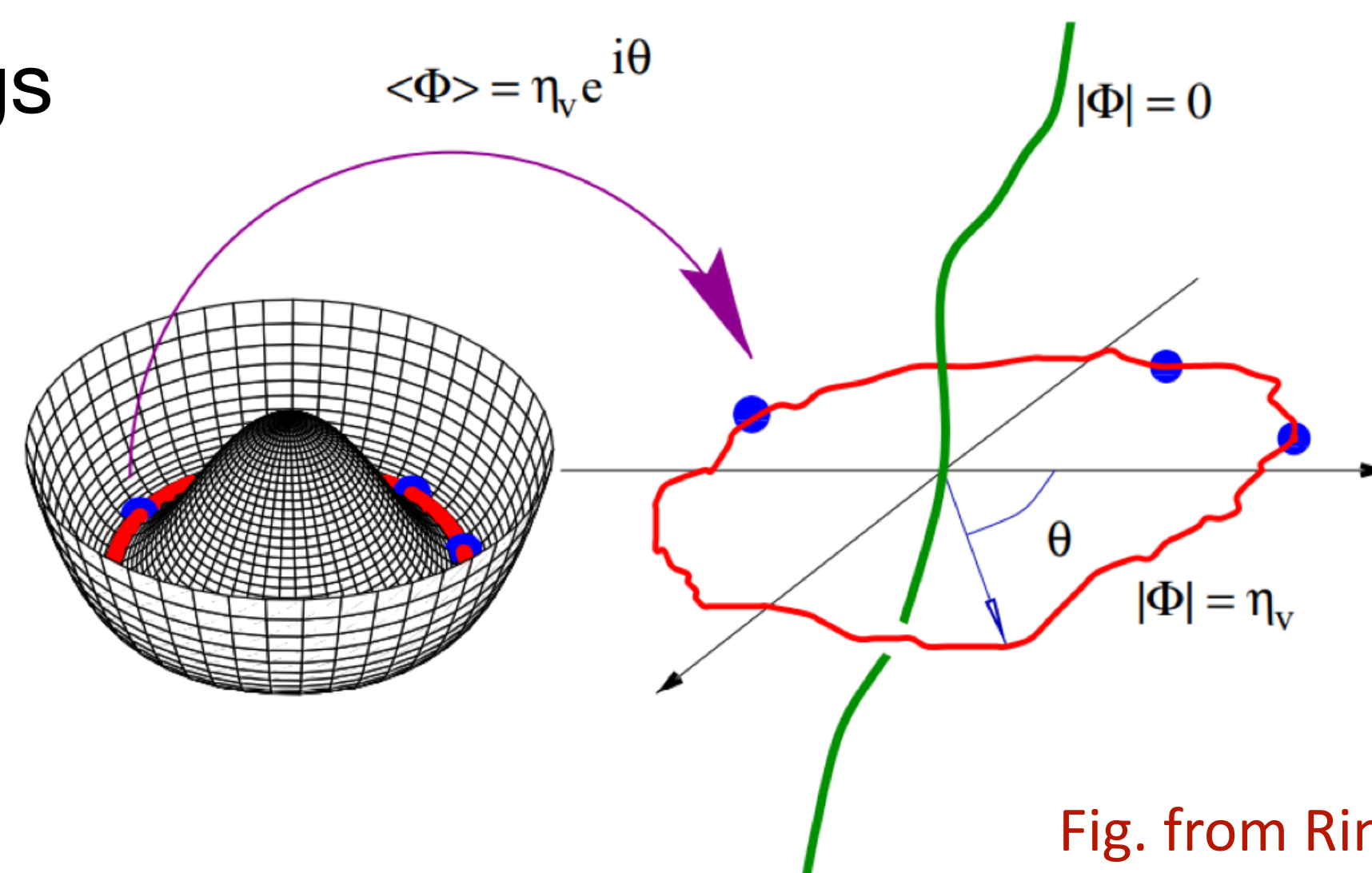


Fig. from Ringeval 2010

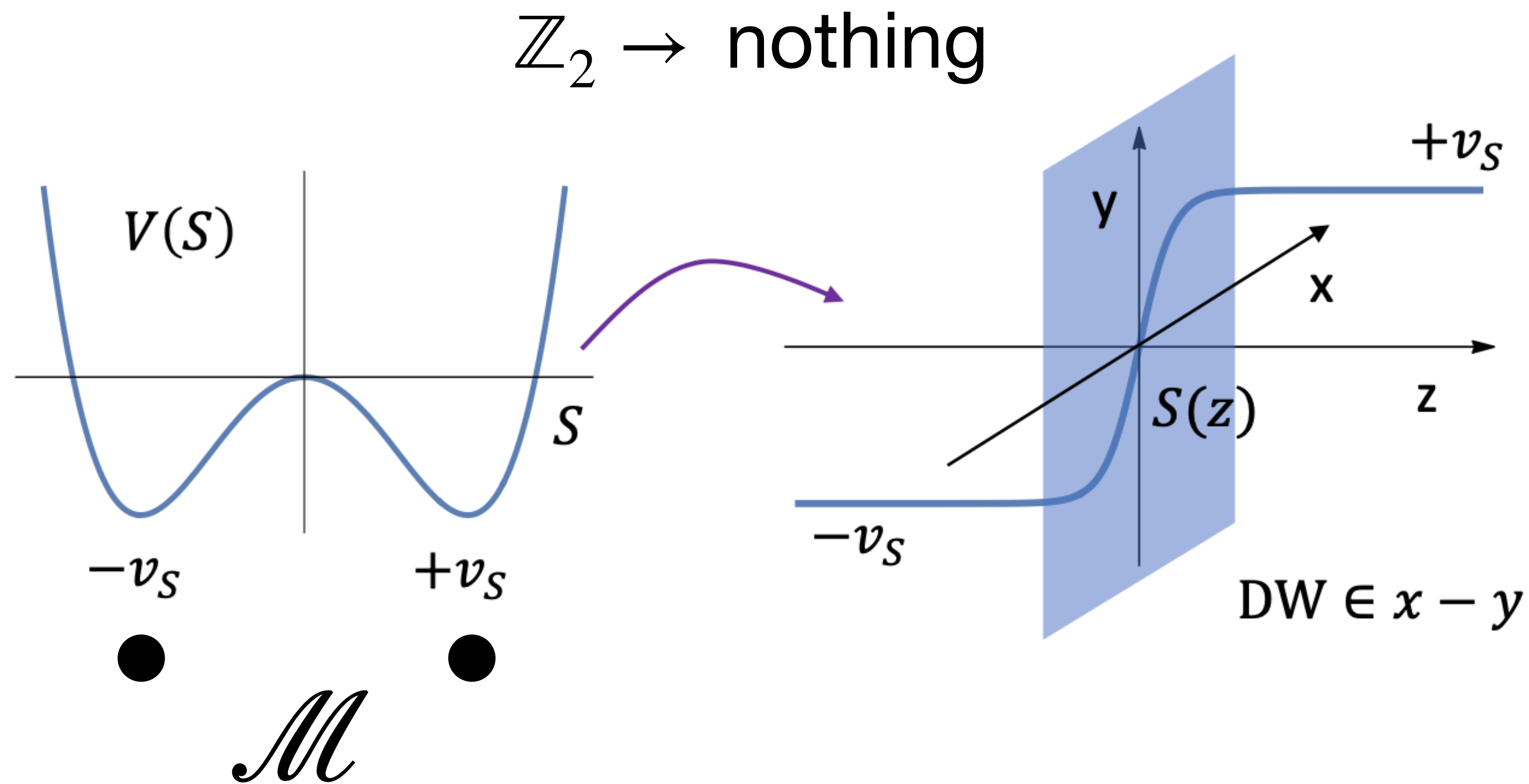
Defects in field theory

- Domain walls correspond to disconnected vacuum manifolds: breaking of discrete symmetries



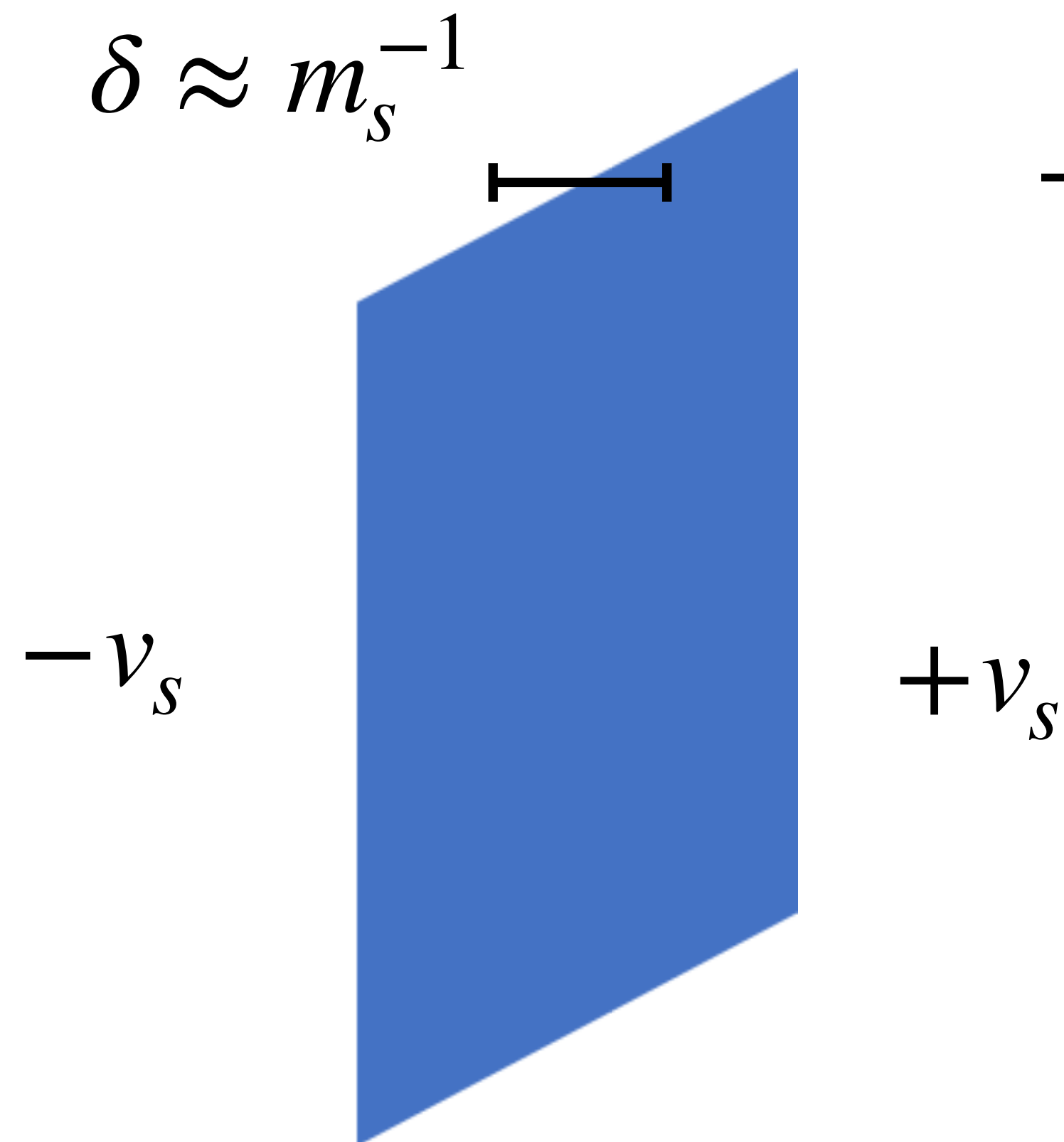
Defects in field theory

- Domain walls correspond to disconnected vacuum manifolds: breaking of discrete symmetries



Defects in field theory

- A domain wall solution is characterized by the tension σ of the wall and its width δ



- Domain wall mass per unit surface:

$$\sigma \approx m_s v_s^2$$

Formation of the network

- Domain walls form according to the Kibble mechanism:
 - Fluctuations of scalar field around T_c have finite correlation length $\xi(T) < d_H$
 - Uncorrelated patches will generally select different points of vacuum manifold \mathcal{M}

Zeldovich et al. 1975
Kibble 1976

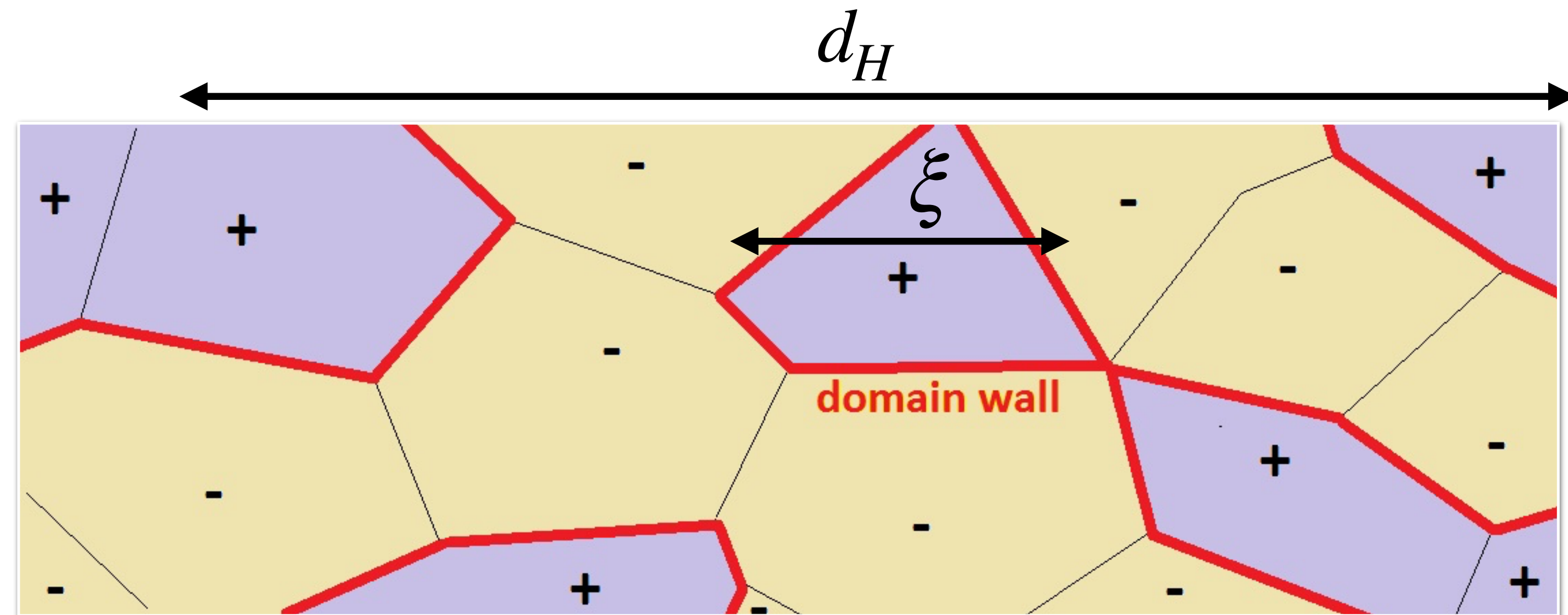
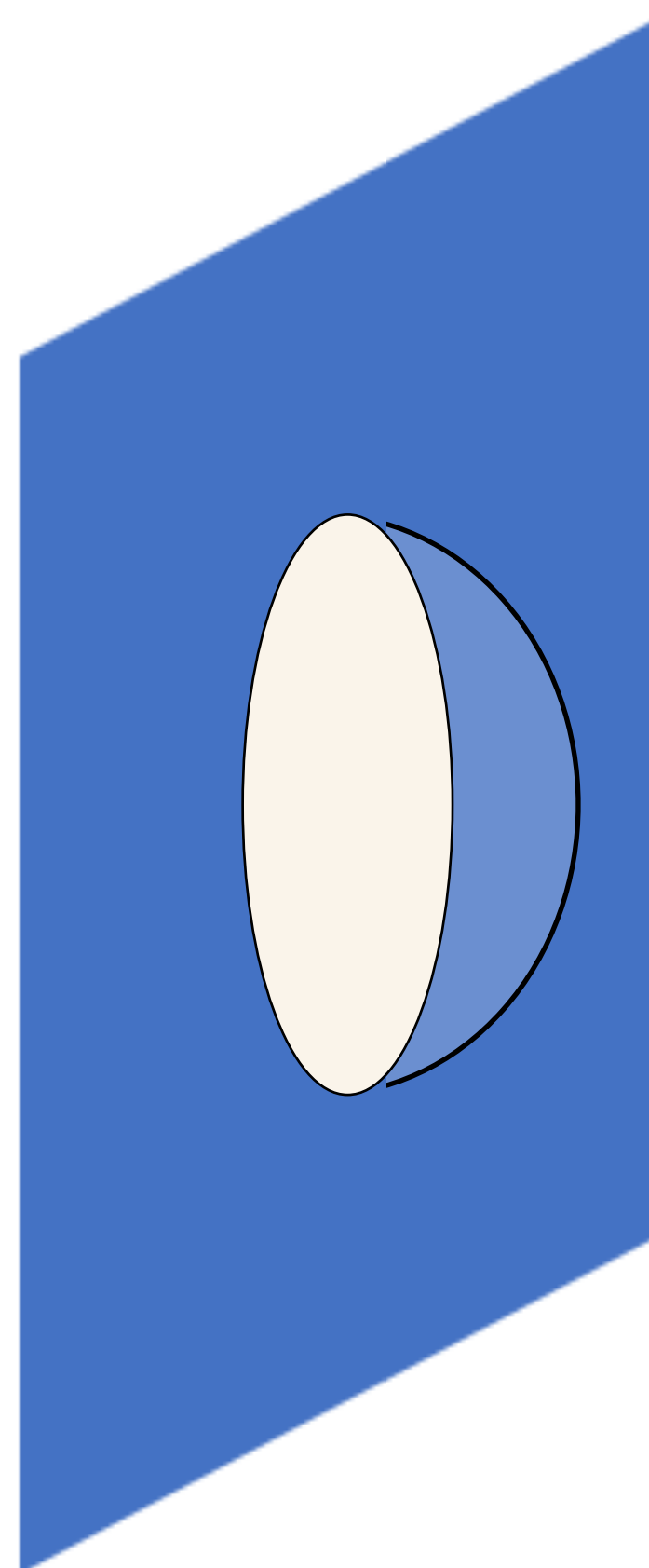


Fig. From MIT edu

Walls as impurities

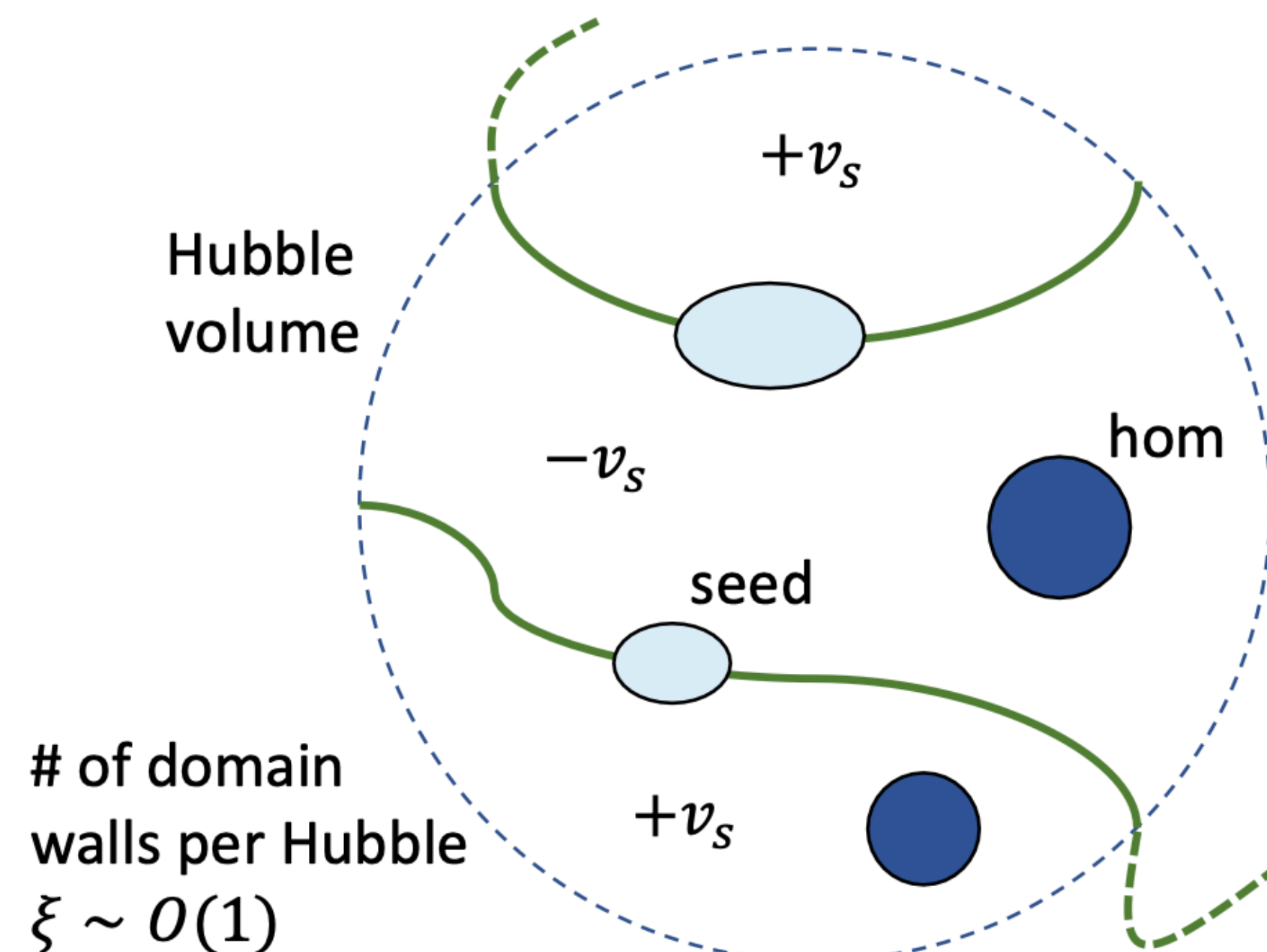
- The presence of domain walls at the time of a first order phase transition can induce exponentially enhanced nucleation on the surface



Seeded critical bubble

SB, Mariotti [2203.16450], PRL

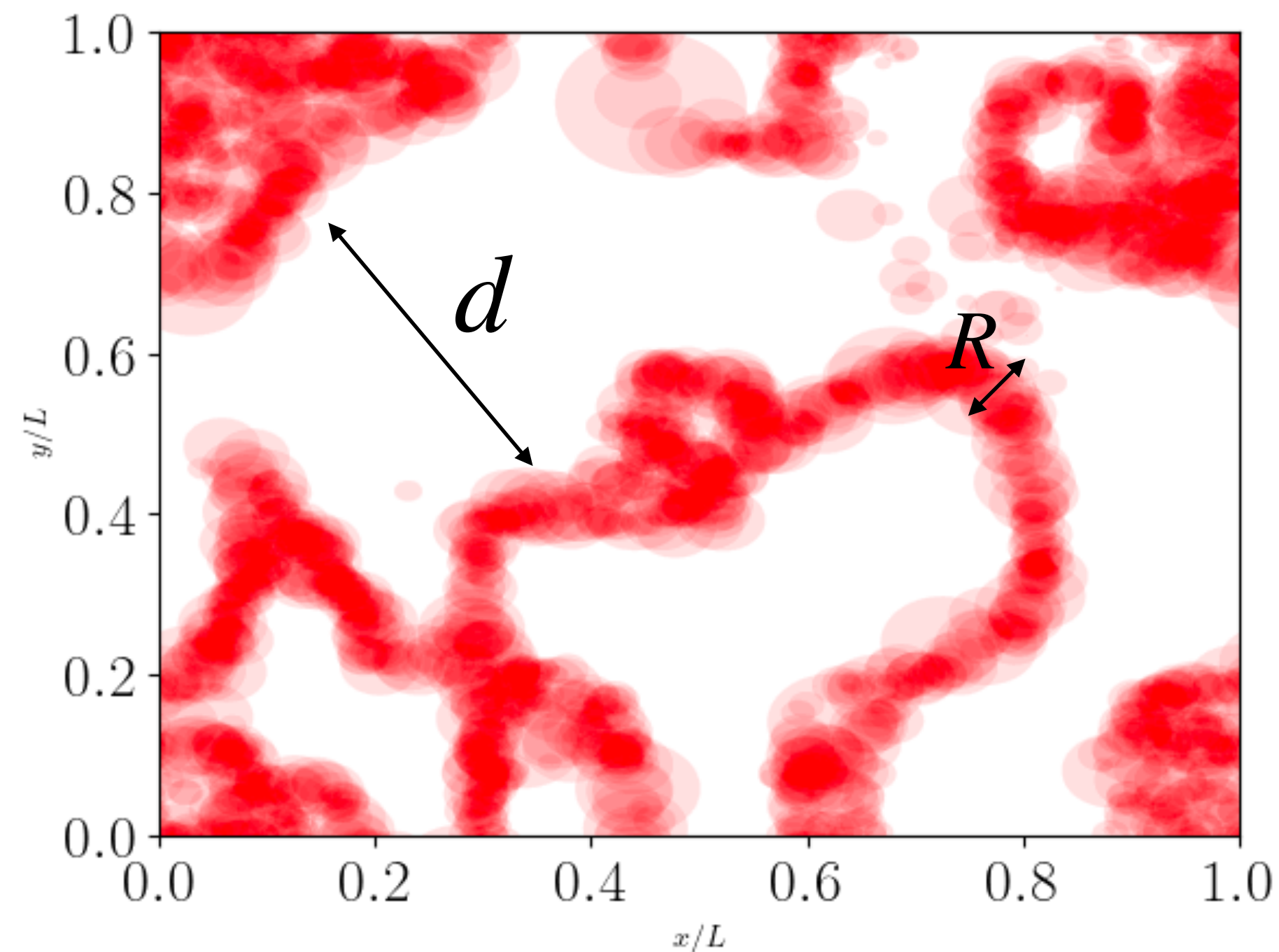
Agrawal, **SB**, Mariotti, Nee [2312.06749], JHEP



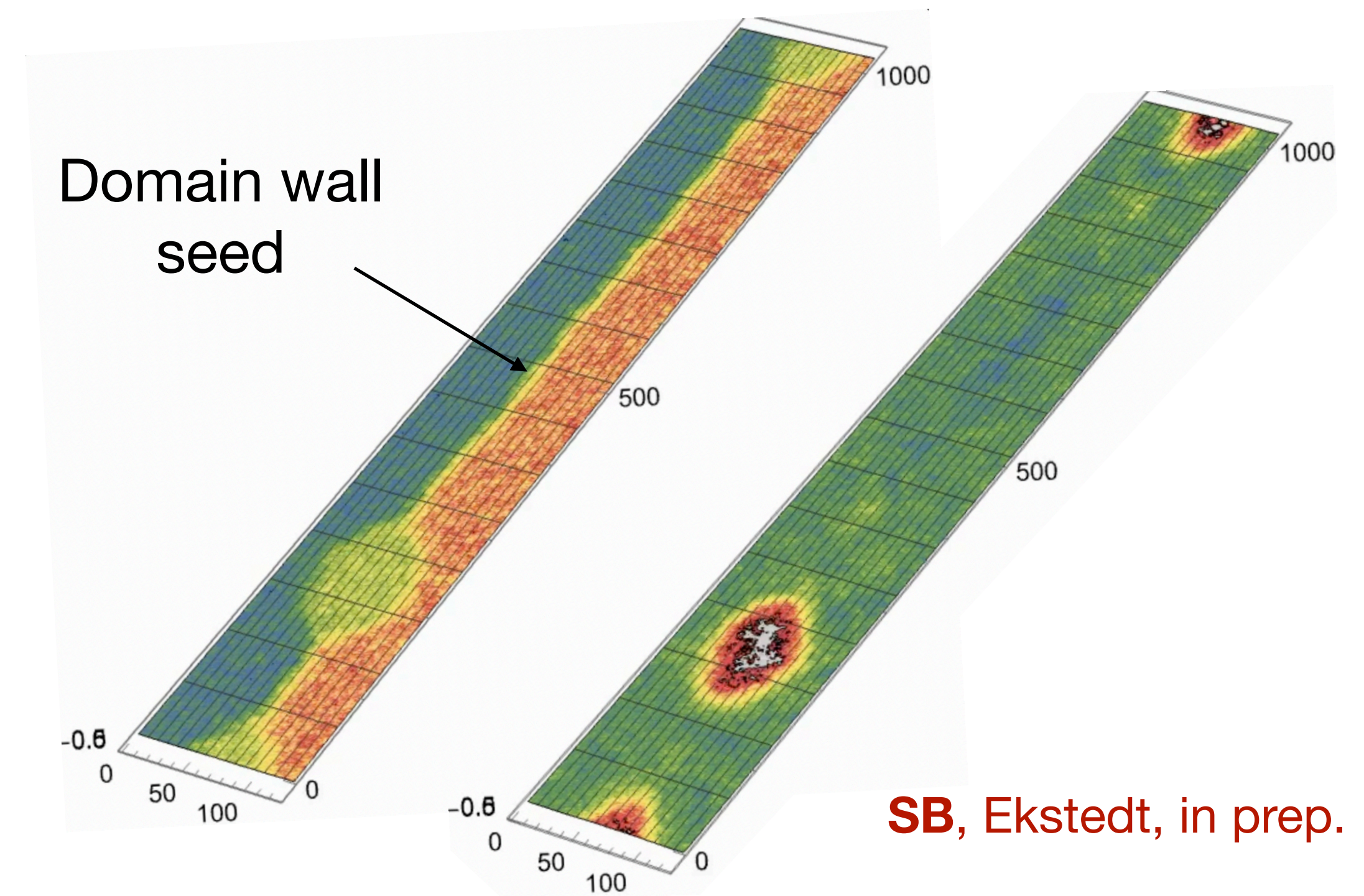
Same catalyzing effect can occur from axion strings: **SB**, Mariotti, [2405.08060]

Walls as impurities

- This scenario can be simulated from the hydrodynamical point of view, as well as real time seeded nucleation (Langevin approach)



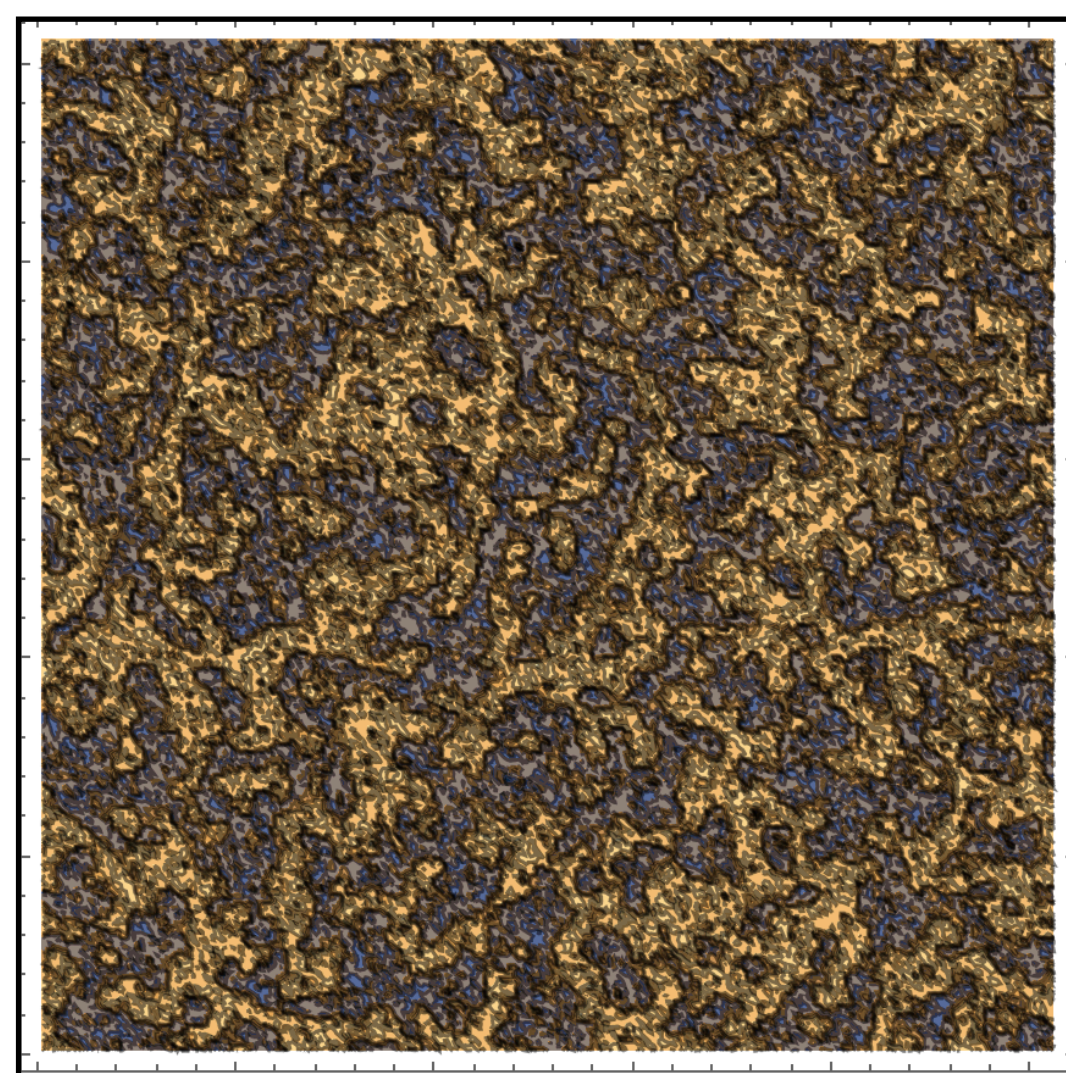
SB, Jinno, Konstandin, Rubira, Stomberg,
JCAP [2302.06952]

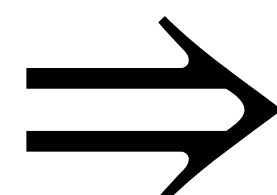


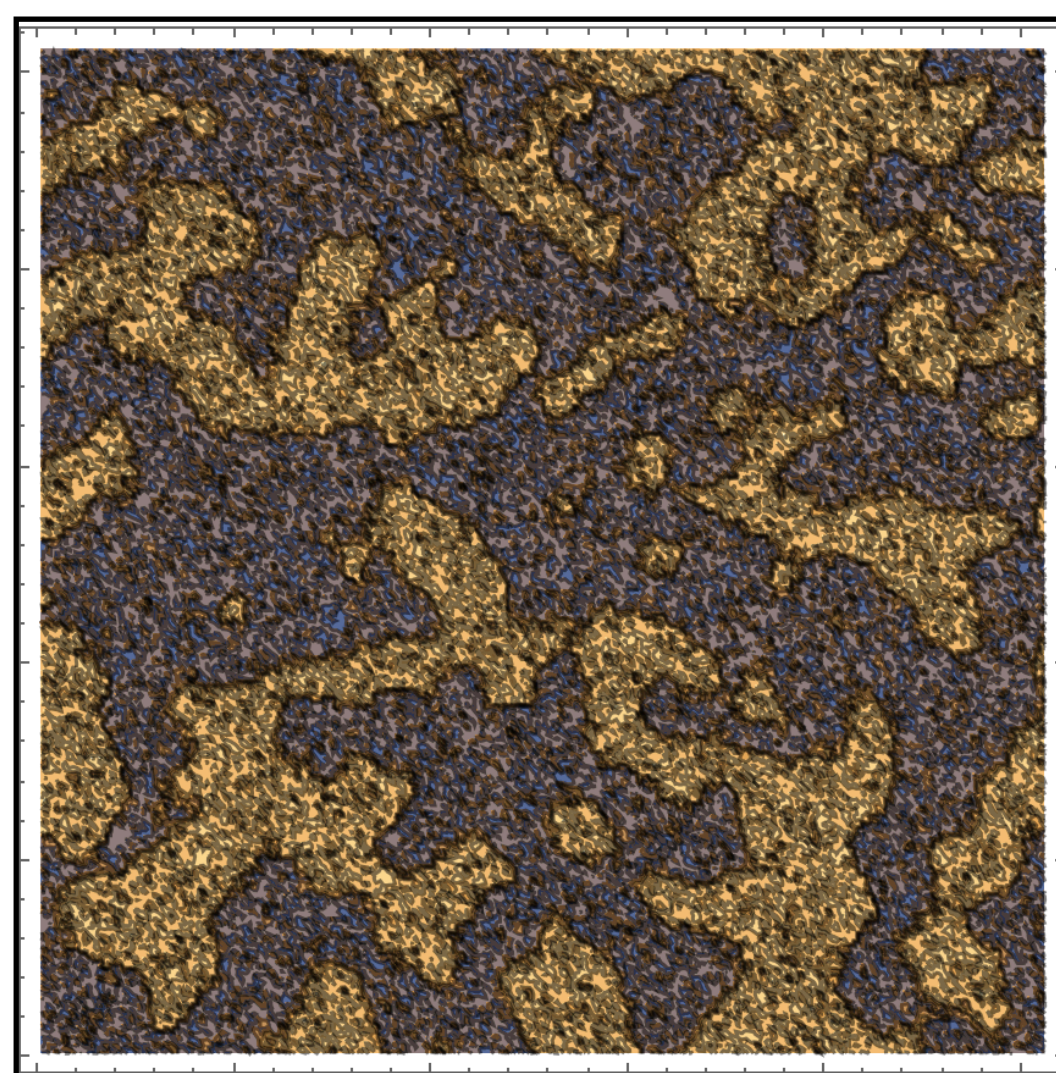
SB, Ekstedt, in prep.

Scaling regime

- After formation, the network reaches a dynamical attractor solution known as the scaling regime, with $\mathcal{O}(1)$ domain walls per Hubble volume at any time
- In scaling, the energy density of the network grows compared to the critical density
- This “domain wall problem” can be solved by a small energy or population bias that eventually annihilates the network




 time

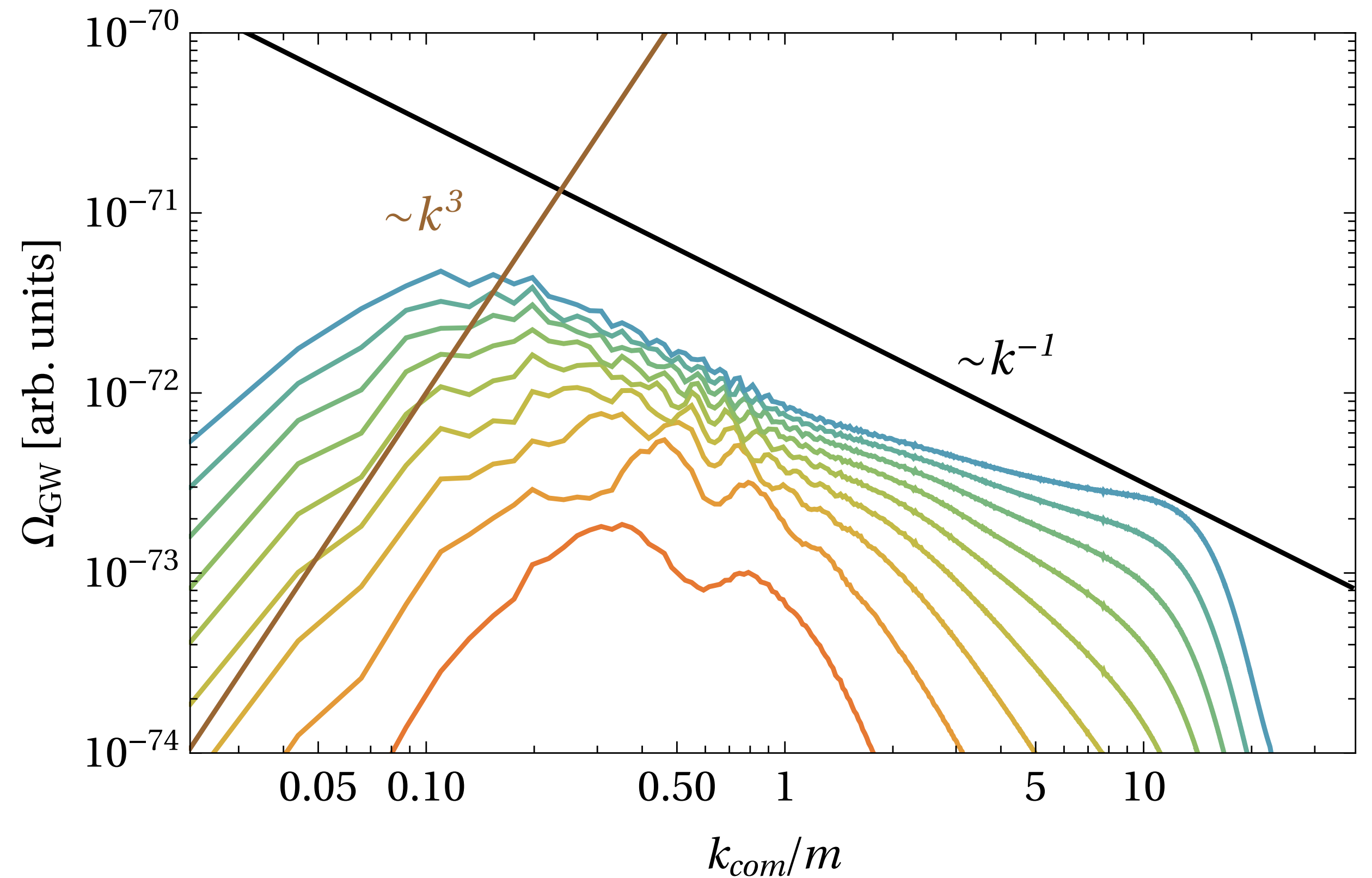


$$\frac{\rho_{\text{DW}}}{\rho_c} \sim G \sigma t$$

GWs from the scaling regime

- GWs are radiate by the domain walls during:
 - the scaling the regime (long-lasting source, dominated by later times)
 - the final phase of collapse and annihilation
- Assuming scaling ends at $T = T_*$:

$$\Omega_{\text{peak}} \sim 10^{-6} \alpha_*^2, \quad f_{\text{peak}} \sim H(T_*)$$



Friction domination

- Domain wall motion from the Nambu-Goto action

$$S = -\sigma \int d^3\zeta \sqrt{\gamma}$$

- Parameterize a possible friction force by

$$F^\nu = \frac{\sigma}{l_f} (u^\nu - x_{,a}^\nu \gamma^{ab} x_{,b}^\mu g_{\mu\sigma} u^\sigma)$$

- Equation of motion for the surface:

$$\ddot{x} + \left(3 \frac{\dot{a}}{a^2} + \frac{1}{l_f} \right) (1 - \dot{x}^2) a \dot{x} = \text{Curvature}$$

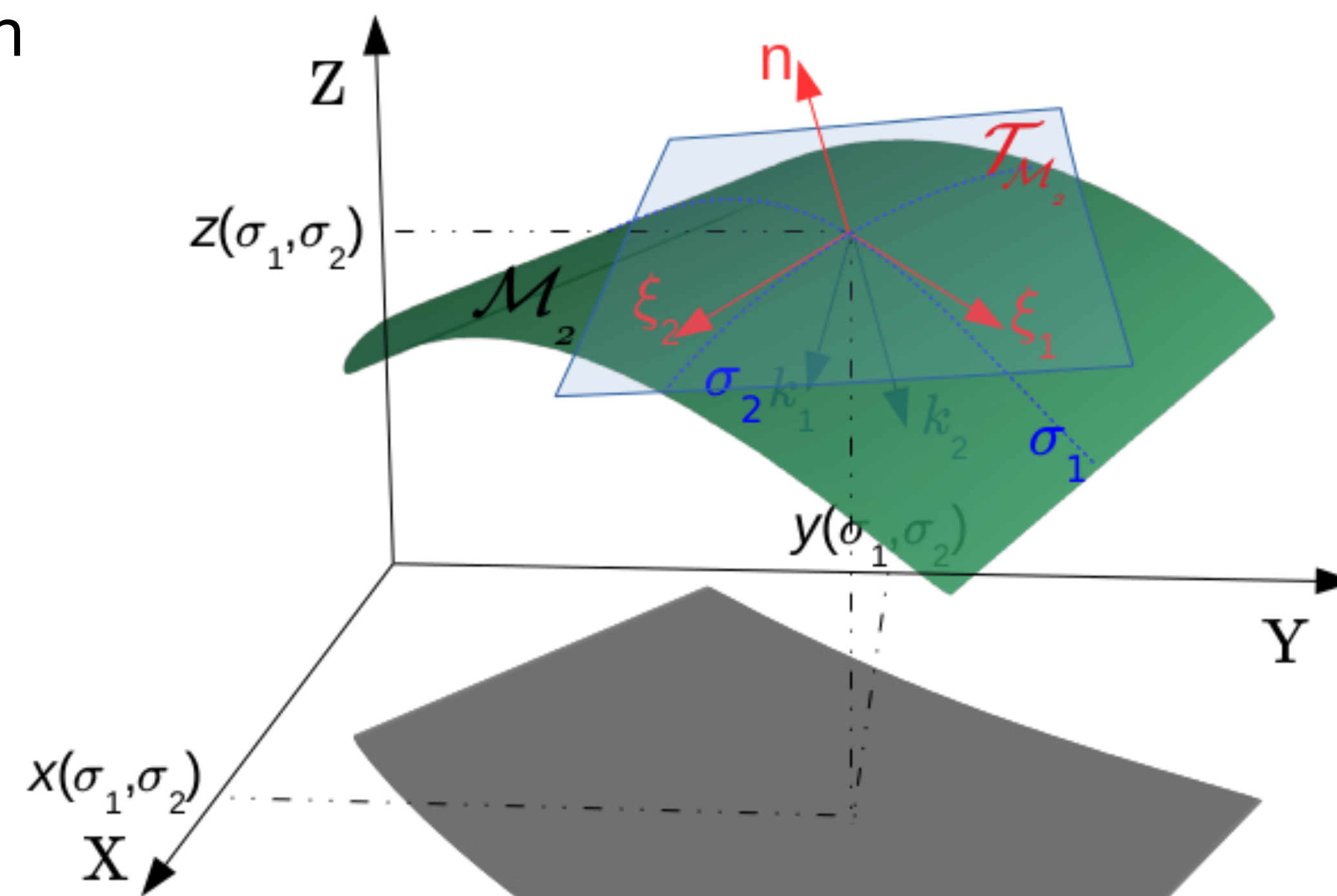


Fig. from Martins, Rybak, Avgoustidis, Shellard [1602.01322]

Friction domination

- Define a total damping length given by:

$$\frac{1}{l_d} = 3H + \frac{1}{l_f}$$

Scaling regime Friction domination

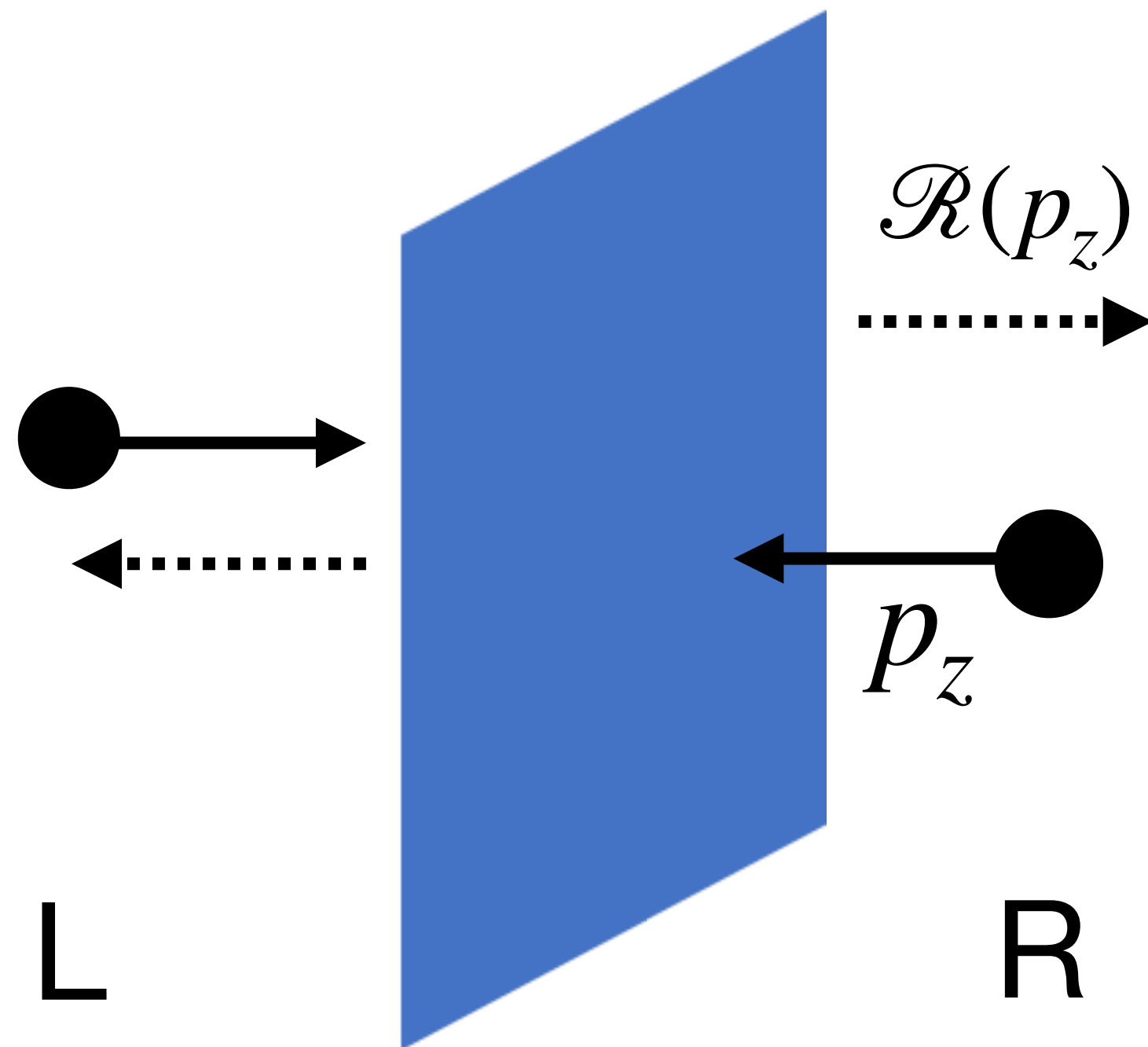
Friction domination

- The friction length can be evaluated given a particle physics model

$$\Delta P = P_R - P_L = 2 \int \frac{d^2 p}{(2\pi)^3} \int_0^\infty dp_z [f(-v) - f(v)] \frac{p_z^2}{E} \mathcal{R}(p)$$

Right and Left particle distribution in the wall frame

Reflection coefficient



$$\frac{1}{l_f} = \frac{\Delta P}{\sigma \gamma(v) v} \simeq \frac{\Delta P}{\sigma v}$$

Friction domination

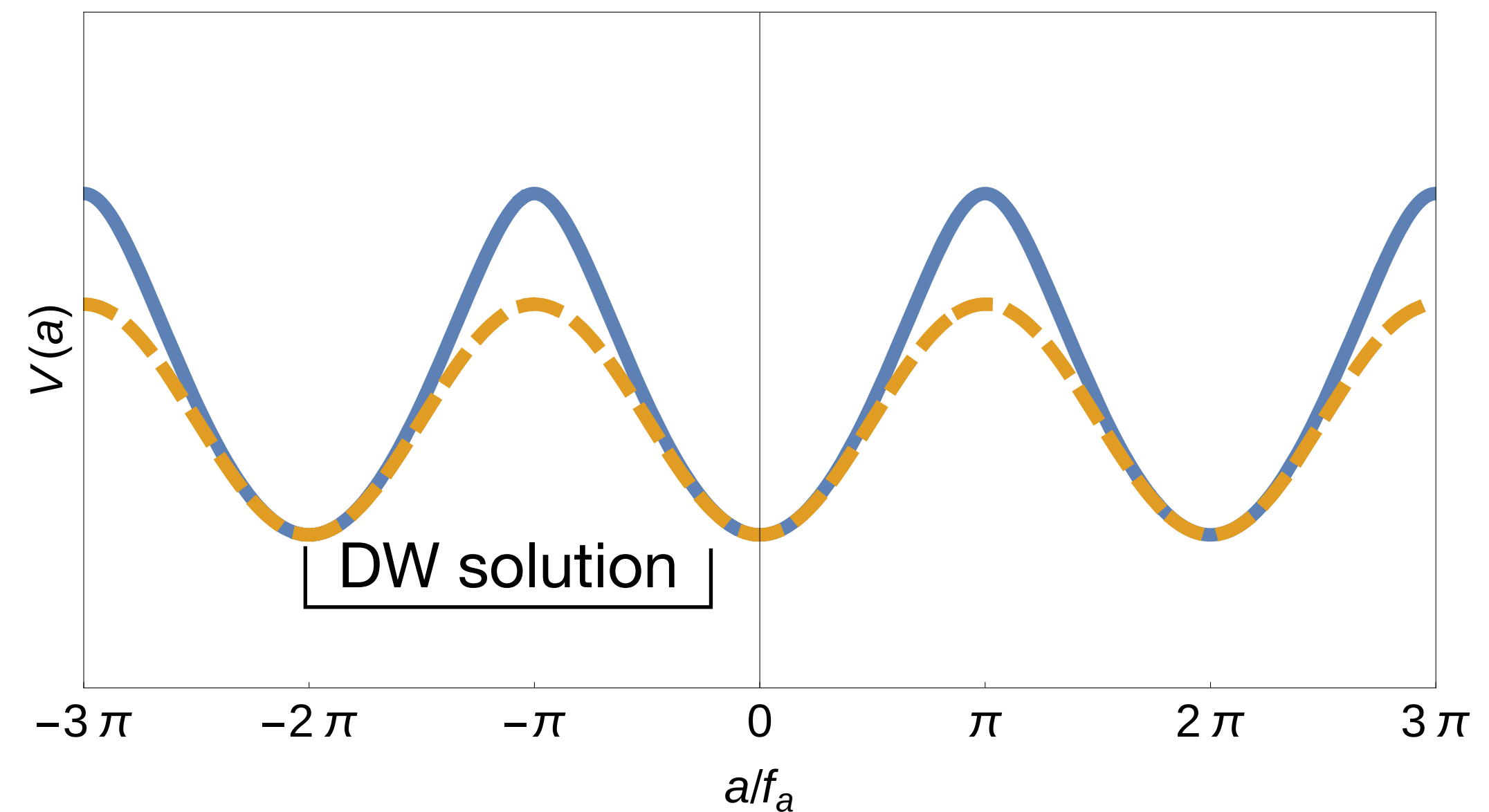
- Consider the case of fermions derivatively coupled to an axion-like-particle (ALP)

$$\mathcal{L}_{a\psi} = \frac{\kappa}{2N_{\text{DW}}f_a} \partial_\mu a \bar{\psi} \gamma^\mu \gamma_5 \psi + \bar{\psi} (i\not{\partial} - m_f) \psi$$

- Scattering off ALP domain walls:

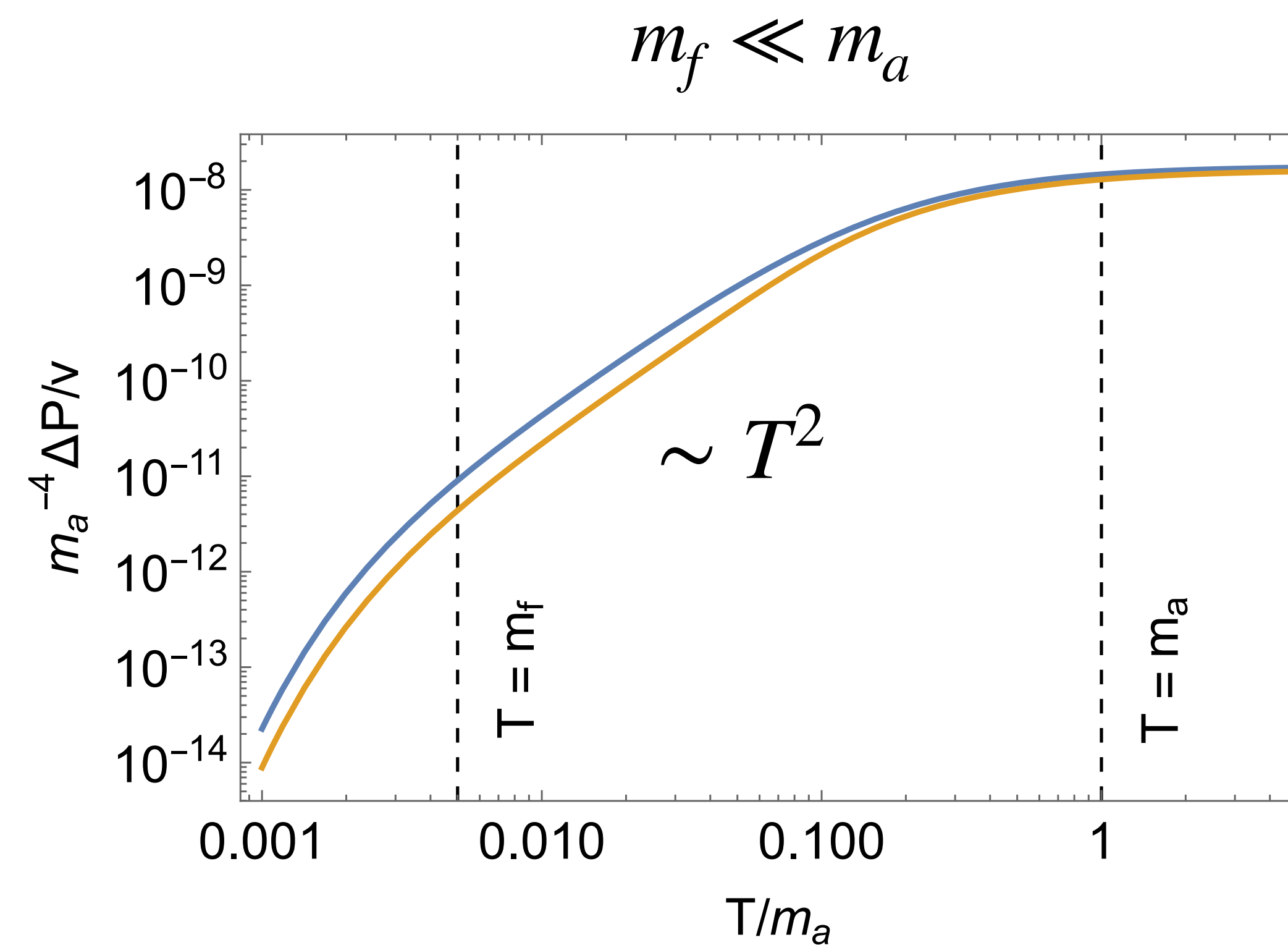
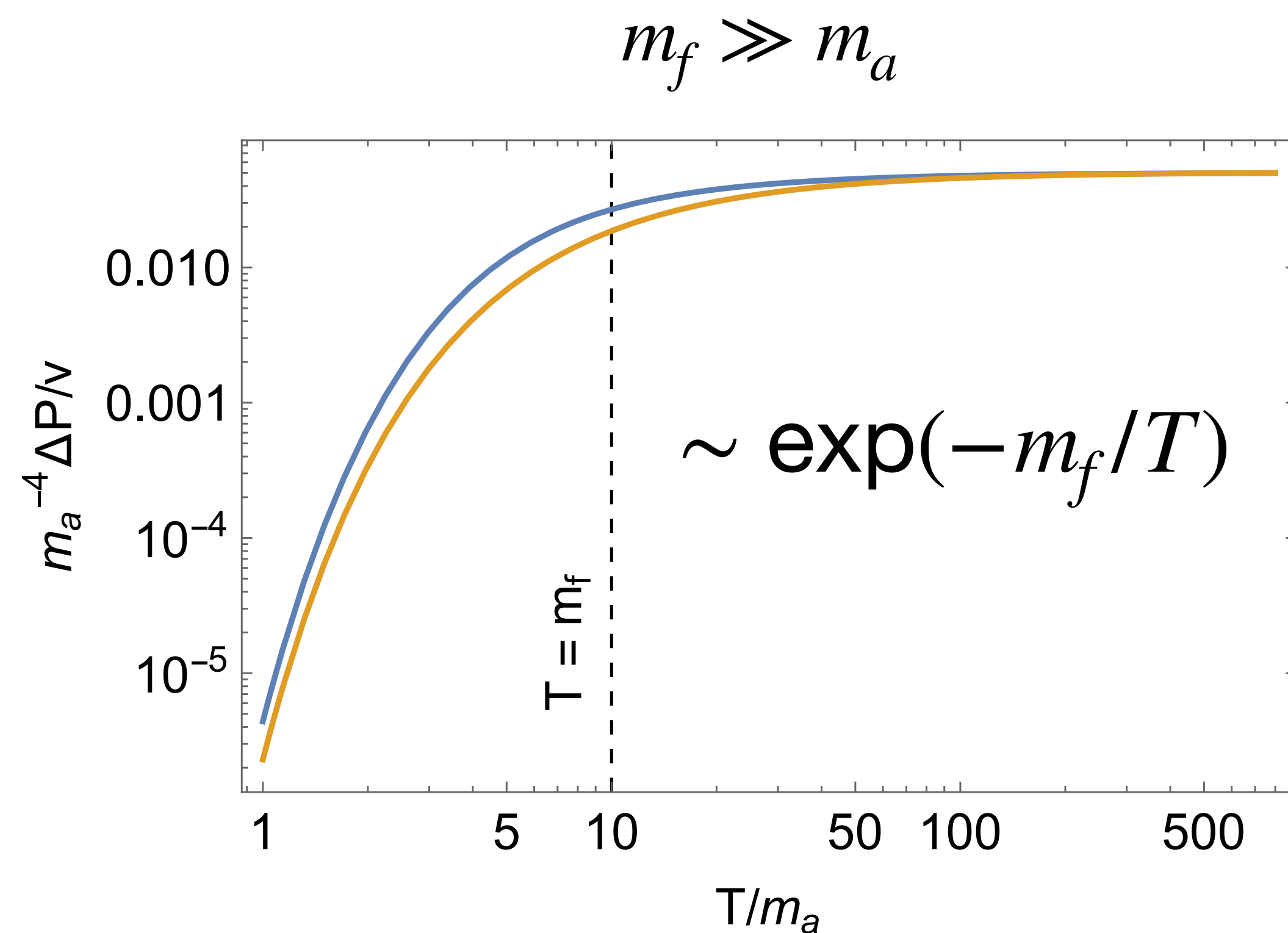
$$V(a) = \Lambda^4 \left[1 - \cos \left(\frac{aN_{\text{DW}}}{v_a} \right) \right]$$

$$a(z) = 4f_a \arctan (e^{m_a z})$$



Friction domination

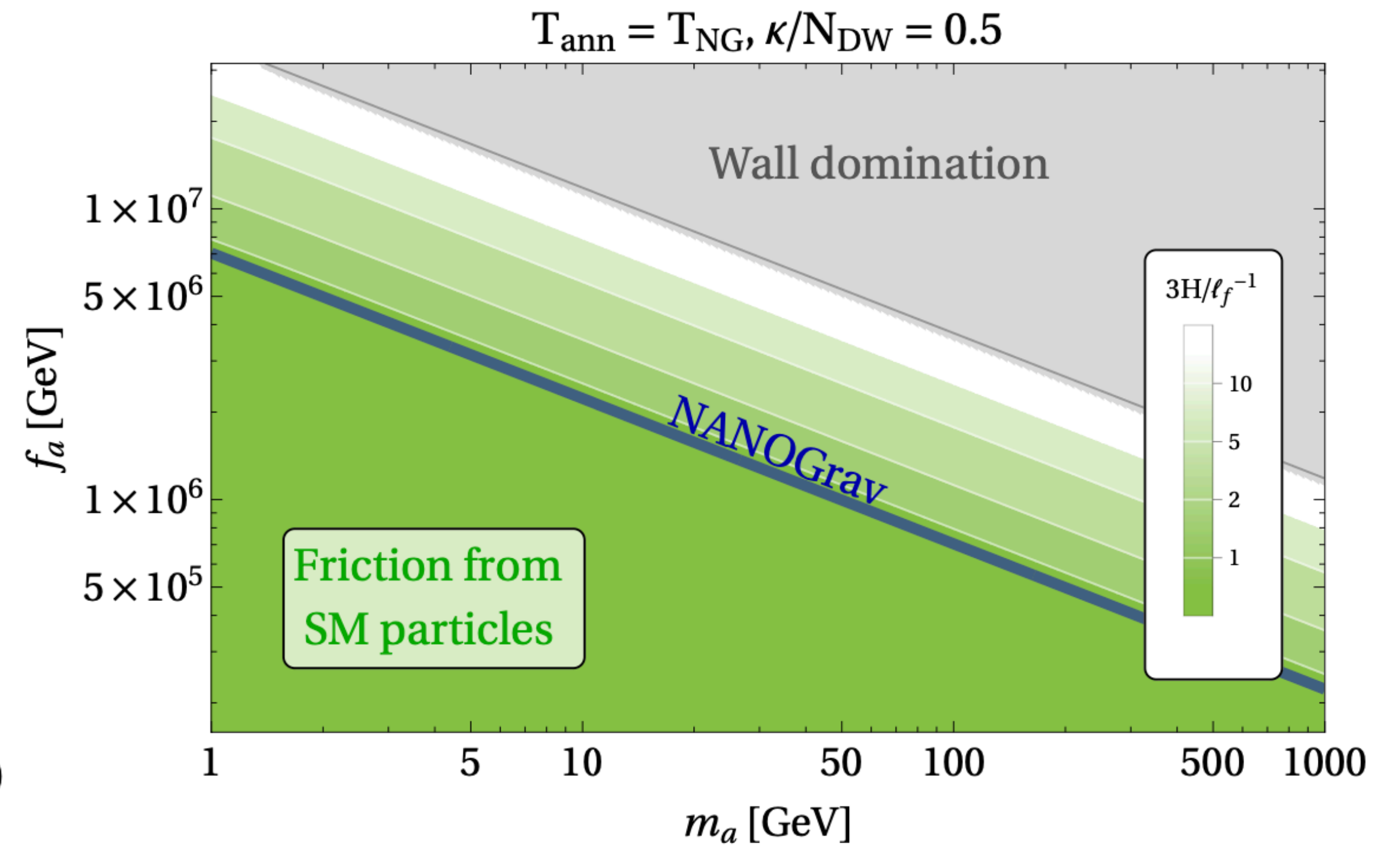
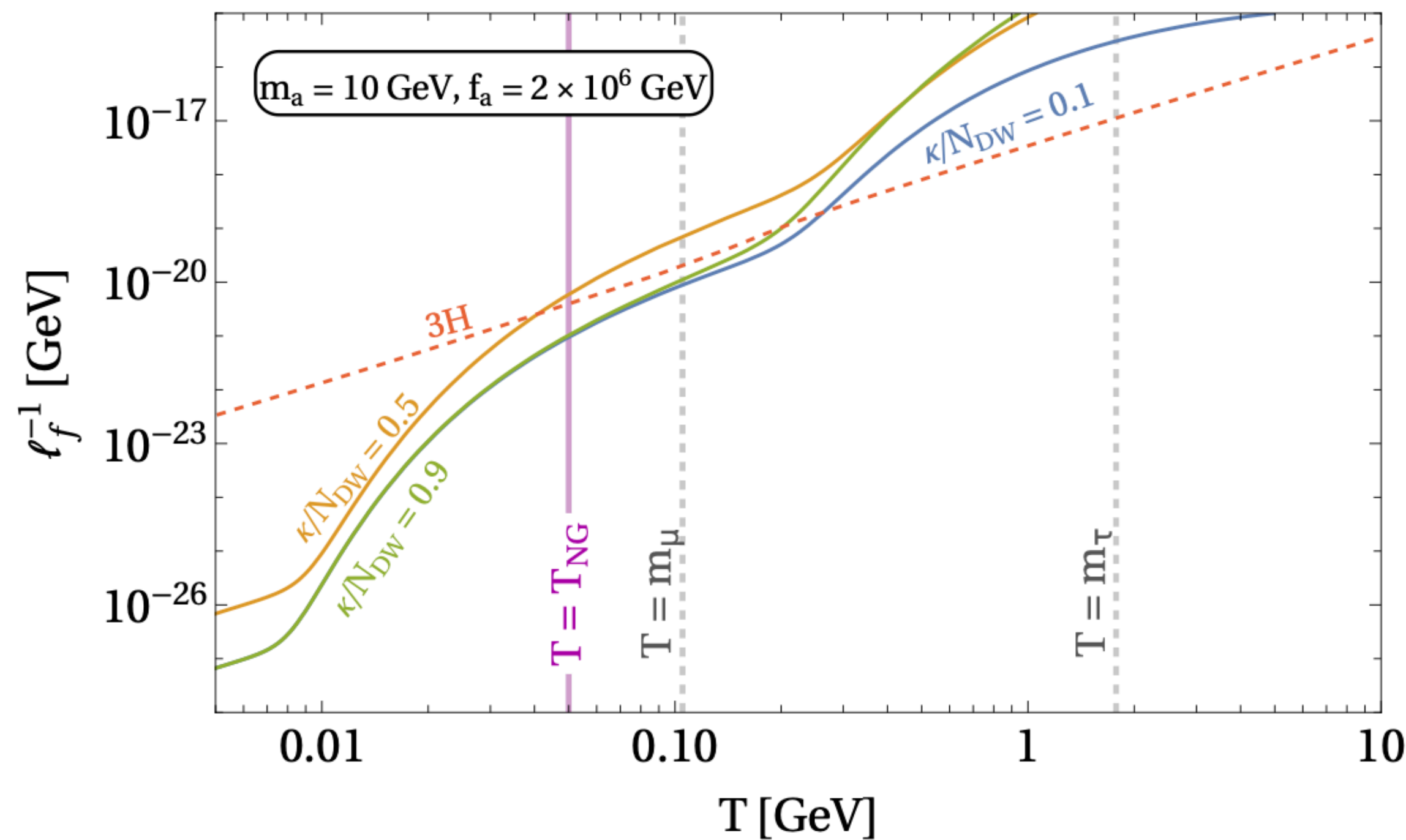
- Determine the reflection coefficient for the fermion scattering via Dirac equation



Friction domination

- ALPs coupled to **SM leptons** with strength $\sim 1/f_a$

SB, Mariotti, Rase, Sevrin, Turbang, [2210.14246], JCAP

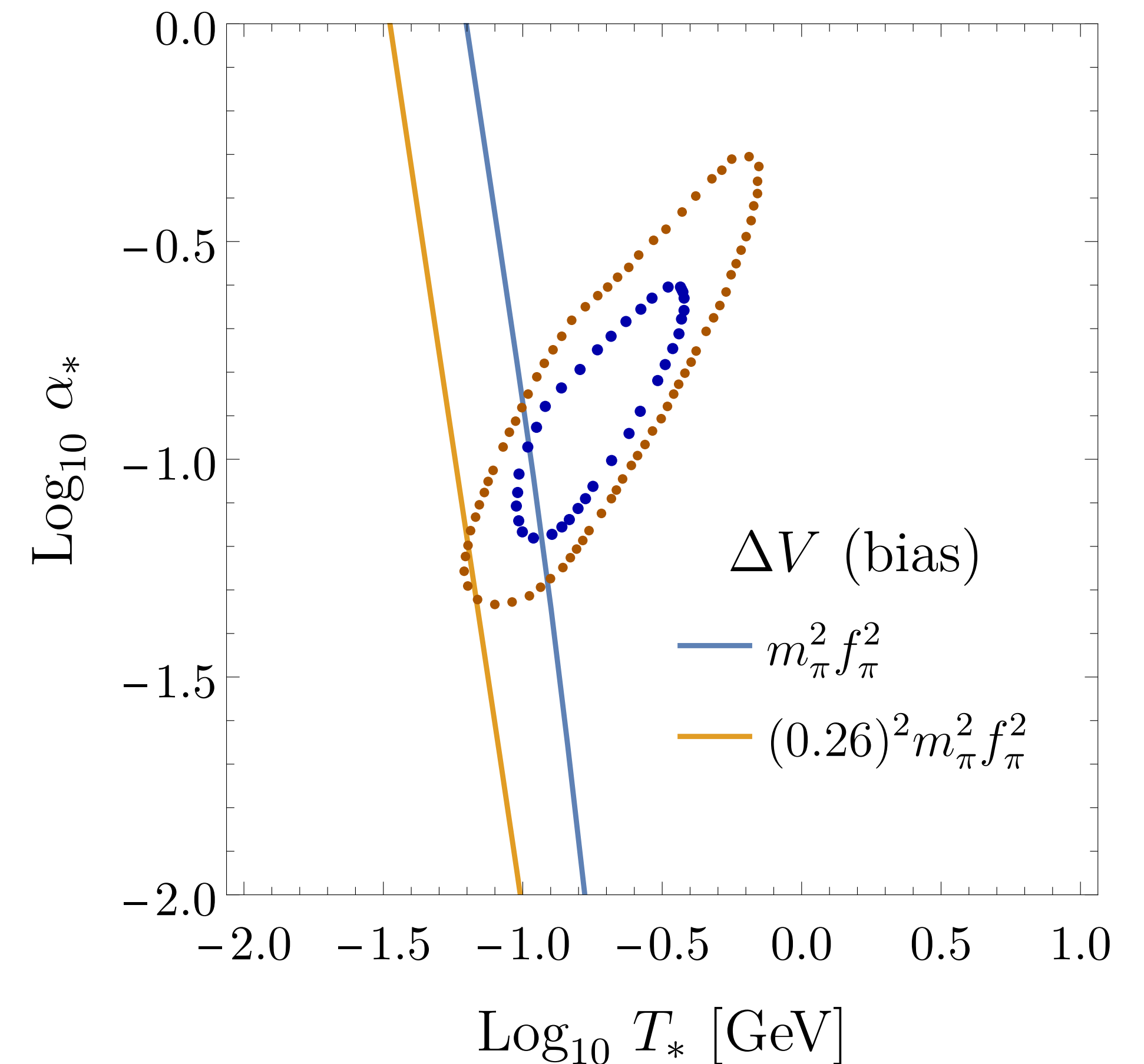


Friction domination

- ALP domain walls destroyed by QCD bias: natural annihilation at $T_* \sim 100 \text{ MeV}$
 - Introduce a coupling of the ALP to the gluons:

$$\mathcal{L}_a \supset \frac{\alpha_s}{4\pi} \frac{N_c}{v} G\tilde{G}$$

- This will generate a potential which will act as a bias for the pre-existing DW network



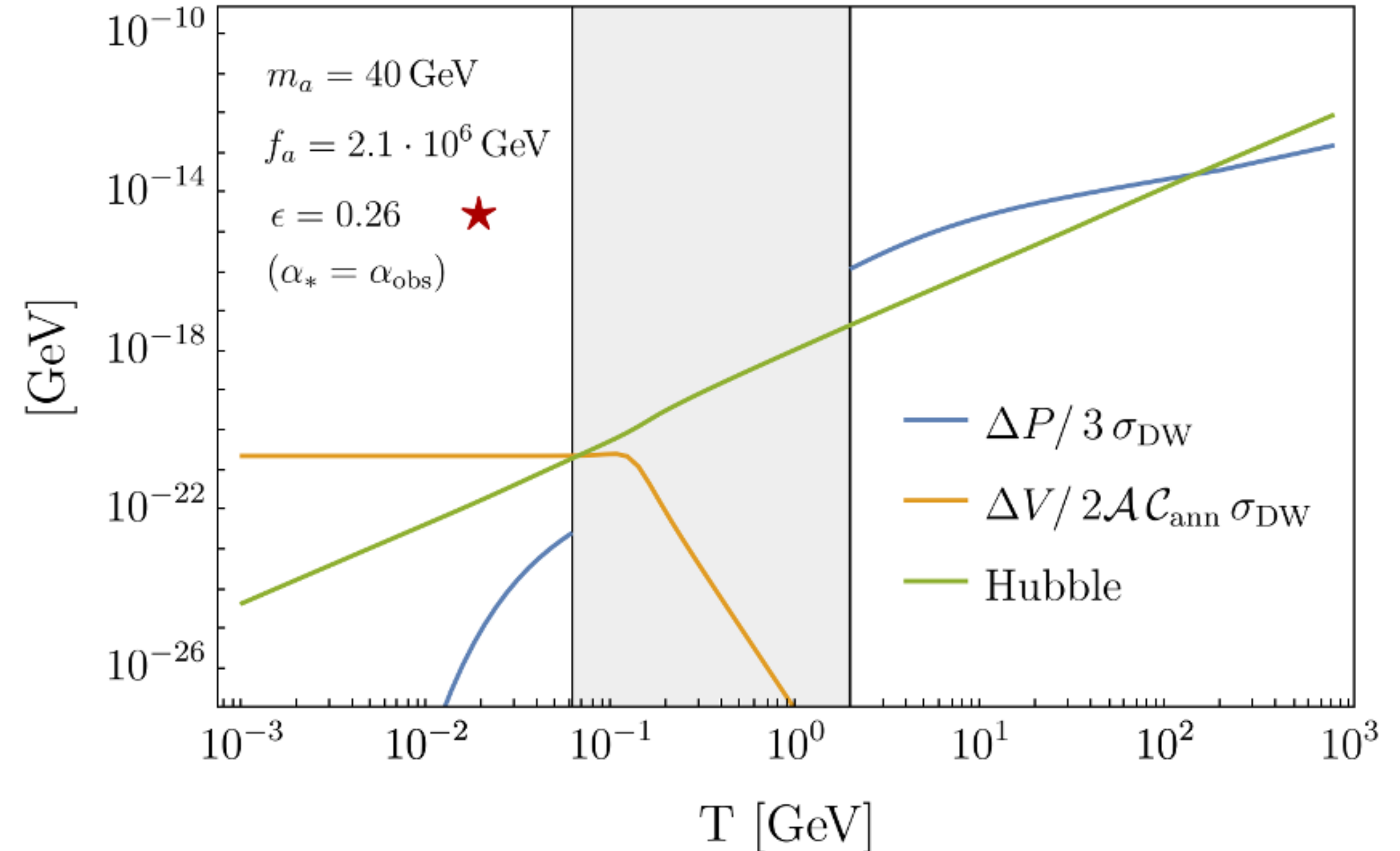
Friction domination

- Does this coupling with the QCD sector also induce friction from the ALP domain walls around the time of collapse?

▸ Crude approximations:

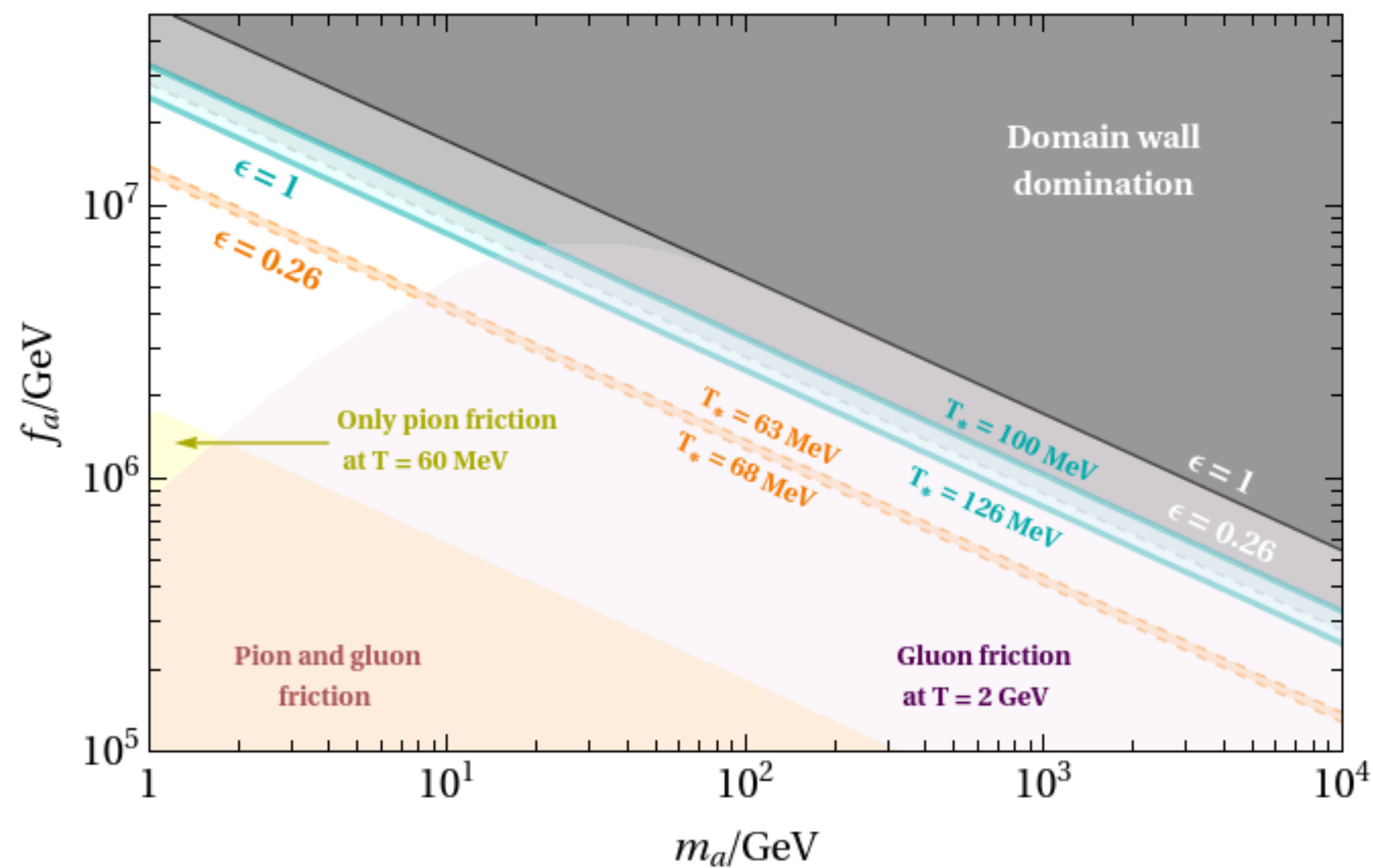
- Gluon scattering for $T > 2 \text{ GeV}$
- Pion scattering for $T < 60 \text{ MeV}$

SB, Mariotti, Rase, Sevrin,
[2302.06952], JCAP



Friction domination

- Summary plot for the NANOGrav 15yr parameter space of ALP domain walls:



Friction domination

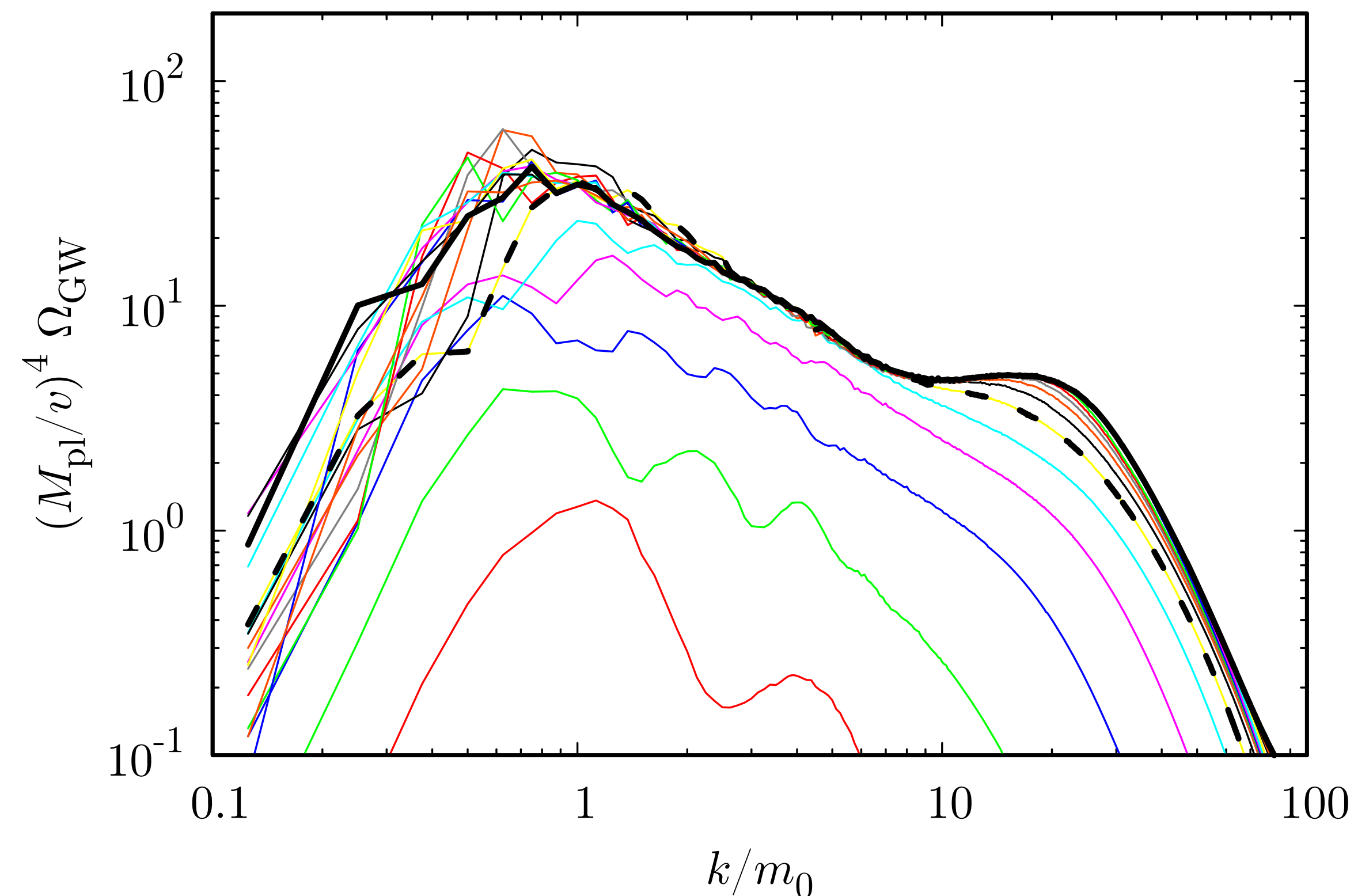
- Friction can be relevant also for the late dynamics of the domain wall network
- The corresponding implications for the GW emission have been studied within the velocity-one-scale model implying a suppressed amplitude, no actual numerical simulation so far
- When friction is relevant, possible new contribution to the GWs from the plasma?

GWs from the network collapse

- So far GW spectrum as given by the last moment of scaling prior to collapse
- Additional contribution from the actual phase of annihilation is expected
- This contribution can enhance the GW peak by 1-2 orders of magnitude

Kitajima, Lee, Murai, Takahashi, Yin,
PLB [2306.17146]

Ferreira, Notari, Pujolàs, Rompineve,
JCAP [2401.14331]

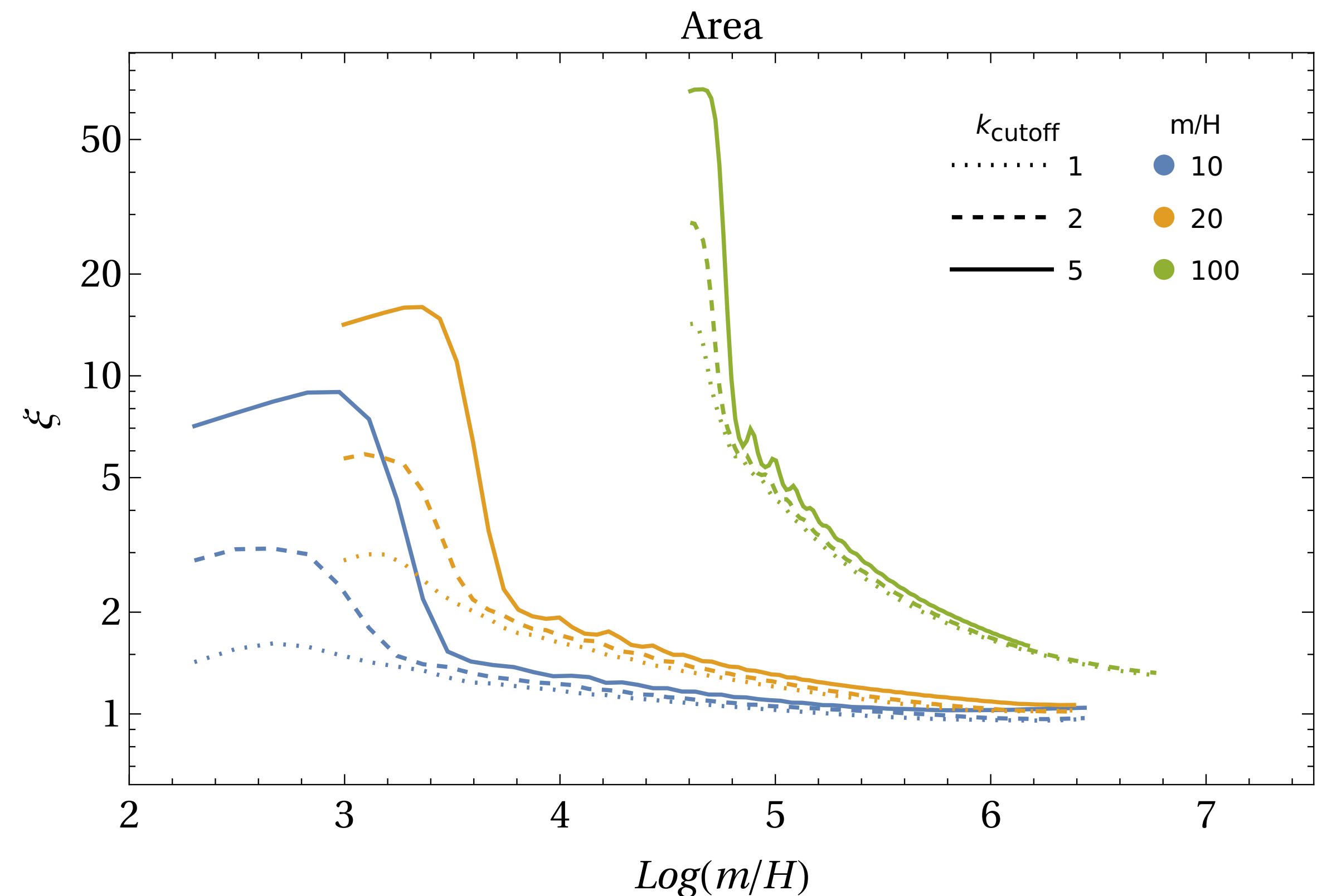


Scaling regime

- Simulations of domain wall networks support the scaling regime, but no systematic study of initial conditions (many simulations start with less than one wall per Hubble volume)

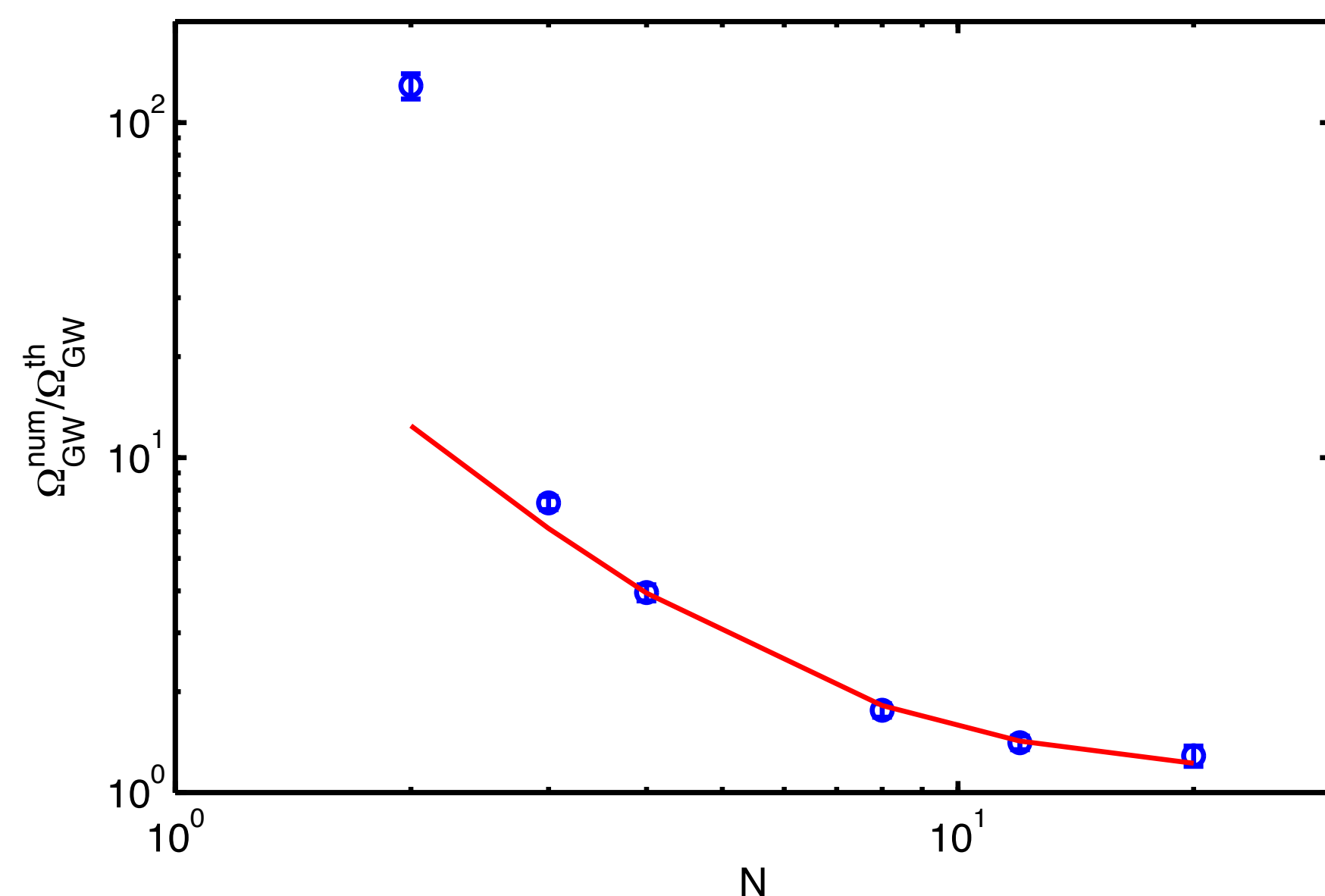
- Systematic study in terms of initial fluctuations and $m_s/H(T_i)$
- Infer the time evolution of ξ during the approach to scaling

Fig. From **SB**, Mariotti, Rase, Vanvlasselaer, in prep.

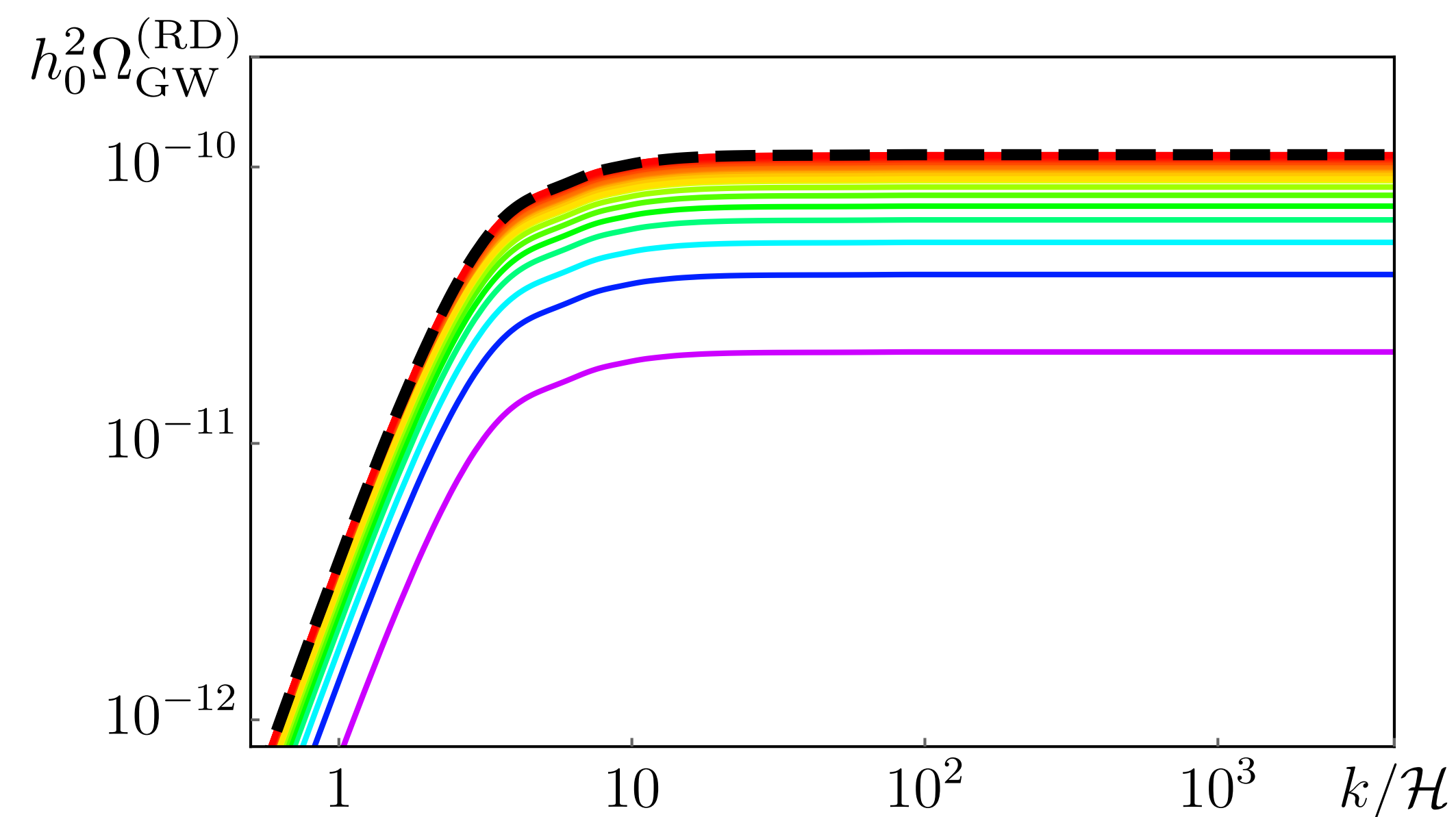


Scaling properties of the UETC

- A scaling network is characterized by a single scale set by the horizon size
- This argument has been used to prove the flat spectrum from scaling defects during radiation domination, and compared to explicit results from models with $O(N) \rightarrow O(N-1)$



Figueroa, Hindmarsh, Urrestilla, PRL [1212.5458]



Figueroa, Hindmarsh, Lizarraga, Urrestilla, PRD [2007.03337]

Scaling properties of the UETC

- The argument is based on dimensional analysis on the UETC of the stress-energy tensor:

$$\langle \Pi_{ij}^{\text{TT}}(\mathbf{k}, t) \Pi_{ij}^{\text{TT}*}(\mathbf{k}', t') \rangle = (2\pi)^3 \Pi^2(k, t, t') \delta_D(\mathbf{k} - \mathbf{k}')$$

- If the source is scaling Π^2 can only depend on $x_i = \kappa t_i$ up to trivial factors,

$$\Pi^2(k, t_1, t_2) = \frac{4v^4}{\sqrt{t_1 t_2}} C^T(x_1, x_2)$$

- The GW spectrum then reads:

$$\frac{d\rho_{\text{GW}}}{d \log k}(x, t) = \Omega_{\text{rad}} \frac{4}{\pi} \frac{M_P^2 H_0^2}{a(t)^4} \left(\frac{v}{M_P} \right)^4 \boxed{F^T(x)} = \mathbf{const.}$$

Scaling properties of the UETC

- Apply the same procedure to domain walls and study the scaling property of the UETC
- A similar (naive) ansatz for domain walls does not work:

$$\Pi^2(k, \tau_1, \tau_2) = \sigma^2(\tau_1\tau_2)^{1/2}C(x_1, x_2), \quad x = k\tau$$

- Need to include powers of the scale factor in the ansatz to account for the scaling in the DW network energy density

See also [2406.17053]

Scaling properties of the UETC

- Extract the scaling properties of the (equal time) Π^2 from the simulation, possibly determine the k^{-1} spectrum

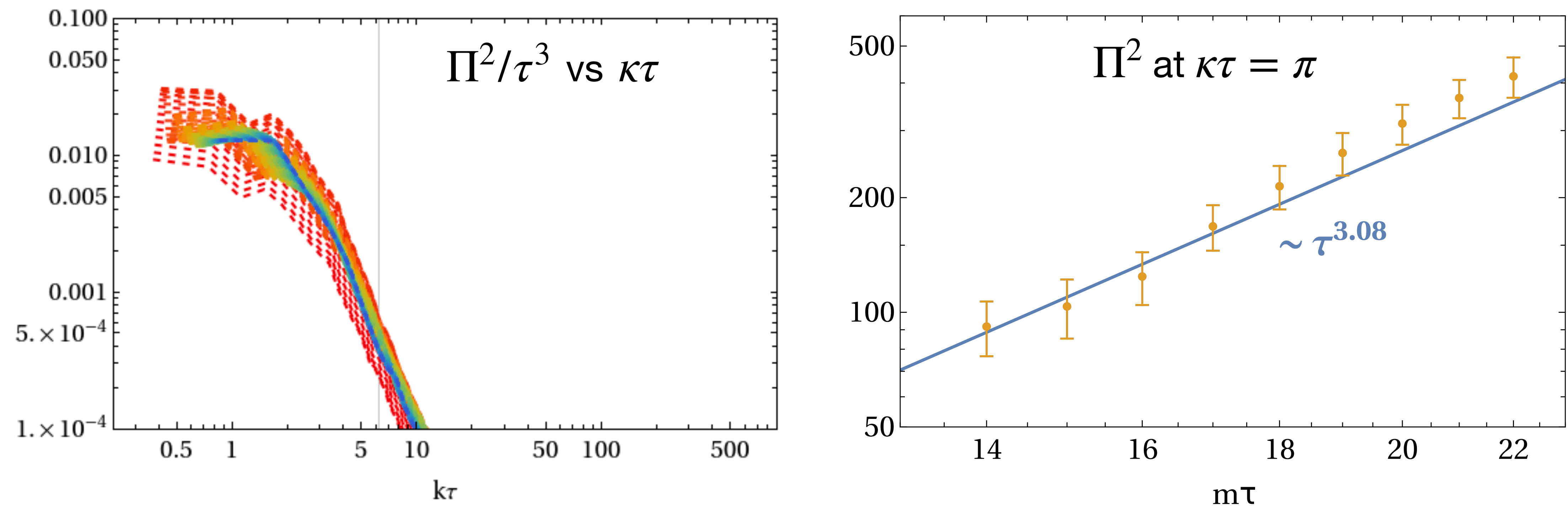


Fig. From **SB**, Mariotti, Rase, Vanvlasselaer, in prep.

Summary

- Domain walls can be themselves a powerful source of gravitational waves, and can also act as seeds for catalyzed bubble nucleation
- Friction with the plasma can be relevant at late times. This regime has not been studied with simulations, no clear indication for the GW spectrum so far
- Still room to improve the understanding of the scaling regime: systematic study of the approach to scaling and semi-analytical studies of the UETC.

Thank you!