Hierarchies from landscape probability gradients and critical boundaries

> Oleksii Matsedonskyi TUM based on 2311.10139

> > Invisibles 2024

Gauge Hierarchy problem:

$$\delta m_h^2 \propto \Lambda^2 \leftarrow {any phi}$$
interac

e.g.
$$\frac{m_P^2}{m_h^2} \sim 10^{34}$$

Single vacuum* approaches:

$$\delta m_h^2 = 0 \Lambda^2 + \mathcal{O}(100 GeV)$$

supersymmetry
compositeness
extra dimensions

$$m_h^2 \subset (-\Lambda^2, \Lambda^2)$$

$$m_h^2 \subset (-\Lambda^2, \Lambda^2)$$



$$m_h^2 \subset (-\Lambda^2, \Lambda^2)$$



$$m_h^2 \subset (-\Lambda^2, \Lambda^2)$$





Landscape/dynamical approaches:

$$m_h^2 \subset (-\Lambda^2, \Lambda^2)$$

orobability

see also e.g.



- Hook, Marques-Tavares 1607.01786
- Fonseca, Morgante, Sato, Servant 1911.08473

$$m_h^2 \subset (-\Lambda^2, \Lambda^2)$$



Preview of the final mechanism

Landscapes for both mH and CC. Why?

Preview of the final mechanism

Landscapes for both mH and CC. Why?

$$\frac{m_P^4}{\Lambda_{cc}(obs)} \sim 10^{120}$$

- most straightforward approach to the smallness of CC is landscape + anthropics
- dynamics of the two landscapes generically interfere hence it is natural to consider them together









Probability measures

What are the probabilities to observe different vacua?



 $\chi \propto$ some fundamental parameter e.g. m_{H}^{2}

Probability measures

What are the probabilities to observe different vacua?

1. standard volume-weighted measure

A. D. Linde, Phys. Lett. B 175, 395 (1986).

- A. D. Linde, D. A. Linde, and A. Mezhlumian, Phys. Rev. D 49, 1783 (1994), gr-qc/9306035.
- A. D. Linde and A. Mezhlumian, Phys. Lett. B 307, 25 (1993), gr-qc/9304015.

2. local measures

- R. Bousso, Phys. Rev. Lett. 97, 191302 (2006), hep-th/0605263.
- L. Susskind (2007), 0710.1129.
- Y. Nomura, Astron. Rev. 7, 36 (2012), 1205.2675.

Probability to observe some type of vacuum (labeled e.g. by the Higgs mass)

overall volume of∞ this vacuum atsome proper time t

*Youngness paradox: assumed to be solved by a version of the stationary measure prescription

Probability gradients



Probability gradients



Probability gradients



$\dot{P}_i = -P_i \sum_{j \neq i} \Gamma_{i \to j} + \sum_{j \neq i} P_j \Gamma_{j \to i} + \frac{3H_i P_i}{2H_i P_i}$

Highest "parent" minimum

$$\dot{P}_0 \simeq 3H_0P_0$$

eternal 'stationary' inflation:

$$P_0 = C_0 e^{3H_0 t}$$





• Lower vacuum:

$$\dot{P}_1 \simeq 3H_1P_1 + P_0\Gamma_{0\to 1}$$

eternal 'stationary' inflation:

$$P_1 = C_1 e^{3H_0 t}$$

Volume-weighted measures **Probability gradients** $\Gamma_{0 \rightarrow 1}$ V $3H_0P_1 = \dot{P}_1 \simeq 3H_1P_1 + P_0\Gamma_{0\to 1}$ ()compensates "missing" χ expansion i = 1

$$\Rightarrow P_1 = C_1 e^{3H_0 t}$$
$$\Rightarrow C_1 = \frac{\Gamma_{0 \to 1}}{3(H_0 - H_1)} C_0$$

Volume-weighted measures **Probability gradients** $\Gamma_{0 \rightarrow 1}$ V $3H_0P_1 = \dot{P}_1 \simeq 3H_1P_1 + P_0\Gamma_{0\to 1}$ compensates "missing" χ expansion i = 1

$$\Rightarrow P_i = C_i e^{3H_0 t}$$
$$\Rightarrow C_i = \frac{\Gamma_{(i-1) \to i}}{3(H_0 - H_i)} C_{(i-1)}$$

Volume-weighted measures **Probability gradients** $\Gamma_{0 \rightarrow 1}$ V $3H_0P_1 = \dot{P}_1 \simeq 3H_1P_1 + P_0\Gamma_{0\to 1}$ ()compensates "missing" χ expansion i = 1

$$\Rightarrow P_i = C_i e^{3H_0 t}$$

$$\Rightarrow C_i = \left[\prod_{k=0}^i \frac{\Gamma_{(k-1) \to k}}{3(H_0 - H_k)}\right] C_0$$

Probability gradients

numerically:



HM tunneling (|m|<H):



Probability gradients

numerically:



HM tunneling (|m|<H):









We need to scan mH and introduce the boundaries

Higgs-VEV dependent critical boundary



 $V(\phi, h) \supset \mu_{\phi}^2 h^2 \cos(\phi/f) + M^2 h^2 \cos(\phi/F)$

Higgs-VEV dependent critical boundary



$$V(\phi, h) \supset \mu_{\phi}^{2}h^{2}\cos(\phi/f) + M^{2}h^{2}\cos(\phi/F)$$

$$\downarrow$$

$$m_{h}^{2} = M^{2}\cos(\phi/F) + \cdots$$

Higgs-VEV dependent critical boundary




Armadillo



Armadillo



mH and CC from gradients & boundaries



mH and CC from gradients & boundaries

Parameter space

 $m_{\phi} \simeq 10^{-20} eV \dots 1GeV$

Motivation

Extrapolation of black hole complementarity to inflationary space.

The physically meaningful description of the universe should be confined to a region of space accessible to some hypothetical observer.

R. Bousso, Phys. Rev. Lett. 97, 191302 (2006), hep-th/0605263.

L. Susskind (2007), 0710.1129.

Y. Nomura, Astron. Rev. 7, 36 (2012), 1205.2675.

What is P(vac)?

Time that a worldline spends (or a number of times it enters) in a given vacuum on its way to AdS

$$\dot{P}_i = -P_i \sum_{j \neq i} \Gamma_{i \to j} + \sum_{j \neq i} P_j \Gamma_{j \to i}$$

Linde, 0611043

Probability gradients



Probability gradients



1. Dominated by initial conditions

e.g. "quantum creation of the universe"

$$P(t=0) \propto \exp\left[-\frac{3}{8}\frac{m_P^4}{V(\chi)}\right] \propto \exp\left[\frac{8\pi^2}{3}\frac{V(\chi)}{H^4}\right]$$

A. D. Linde, Lett. Nuovo Cim. 39, 401 (1984).A. Vilenkin, Phys. Rev. D 30, 509 (1984).

Probability gradients



2. I.C. + Dynamics

$$P = \exp[\kappa t] P_{t=0}, \text{ with } \kappa_{ij} = \Gamma_{j \to i} - \delta_{ij} \sum_{k} \Gamma_{j \to k}$$

Probability gradients



2. I.C. + Dynamics

$$P_i \simeq \frac{1}{i!} (\kappa t)^i P_{t=0} \simeq \frac{1}{i!} (\Gamma t)^i$$

Probability gradients



3. Equilibrium independent of I.C. (if no sinks)

$$P_i \propto \exp\left[rac{3}{8}rac{m_P^4}{V(\chi_i)}
ight] \propto \exp\left[-rac{8\pi^2}{3}rac{V(\chi_i)}{H^4}
ight]$$

Probability gradients

3 regimes, end of slow-roll picks the time of sampling.

Regime 2 has probability defined by Γ similarly to the V-weighted case.

All the pheno associated with the relaxion. (although param. space is somewhat different)



other triggers discussed e.g. in Arkani-Hamed, D'Agnolo, Kim 2012.04652



Banerjee, OM, Kim, Perez 2004.02899



Graham, Kaplan, Rajendran 1504.07551

• These were only 'local' probes:

$$V(\phi, h) \simeq \frac{1}{2} V_{\phi}'' \phi^2 + V_{\phi h}'' \phi h + \dots$$

• These were only 'local' probes:

$$V(\phi, h) \simeq \frac{1}{2} V_{\phi}'' \phi^2 + V_{\phi h}'' \phi h + \dots$$

- Can one probe the global landscape structure? e.g. ϕ displacement by density effects:
 - talk by Javi Serra
 - Balkin, Serra, Springmann, Stelzl, Weiler 2106.11320
 - Hook, Huang 1904.00020

Conclusions

Dynamical solution for the Higgs mass in the presence of the CC landscape for two "orthogonal" measures.

Conclusions

Dynamical solution for the Higgs mass in the presence of the CC landscape for two "orthogonal" measures.

When I told Rocky Kolb that I was going to be talking about eternal inflation, he said, "That's OK, we can talk about physics later." A.Guth, 0002188

Predictions are uncertain, which doesn't mean that they are not physically significant.

Conclusions

Dynamical solution for the Higgs mass in the presence of the CC landscape for two "orthogonal" measures.

When I told Rocky Kolb that I was going to be talking about eternal inflation, he said, "That's OK, we can talk about physics later." A.Guth, 0002188

Predictions are uncertain, which doesn't mean that they are not physically significant.

Landscapes & anthropics \neq giving up on exp testablity: potential probes from astrophysics to colliders

Thank you!

back-up slides

"Youngness paradox"



eternal inflation driven by vacuum 1

"Youngness paradox"



"Stationary measure"



A. D. Linde, JCAP 06, 017 (2007), 0705.1160
A. D. Linde, V. Vanchurin, and S. Winitzki, JCAP 01, 031 (2009), 0812.0005

"Stationary measure"

gist: P are compared at the time of reaching stationarity



A. D. Linde, JCAP 06, 017 (2007), 0705.1160
A. D. Linde, V. Vanchurin, and S. Winitzki, JCAP 01, 031 (2009), 0812.0005

Stochastic approach

$$V = \Lambda + \frac{1}{2}m^2\phi^2 \quad \Rightarrow \quad P_{\nu} = \exp\left[-A\phi^2\right]\left\{\mathbf{c}_+ D_{\nu}\left[B\phi\right] + \mathbf{c}_- D_{\nu}\left[-B\phi\right]\right\}$$

$$\begin{split} A\phi^2 &= \frac{4\pi^2}{3} \frac{V(\phi) - V(0)}{H(0)^4}, \\ B\phi &= \left\{ 4\frac{4\pi^2}{3} \frac{|V(\phi) - V(0)|}{H(0)^4} \sqrt{1 - \frac{9}{\pi} \frac{H(0)^4}{m^2 m_P^2}} \right\}^{1/2} \operatorname{sign}[\phi] \\ \nu &= \frac{9(H(0)^2 - H_s^2) + m^2}{2|m^2|\sqrt{1 - \frac{9}{\pi} \frac{H(0)^4}{m^2 m_P^2}}} - \frac{1}{2}. \end{split}$$

asymptote:
$$D_{\nu}(x) \xrightarrow[x \to \infty]{} |x|^{\nu} e^{-x^2/4}$$

$$\xrightarrow[x \to -\infty]{} (-1)^{\nu} |x|^{\nu} e^{-x^2/4} + \frac{\sqrt{2\pi}}{\Gamma[-\nu]} |x|^{-\nu-1} e^{x^2/4}$$

Stochastic approach



65

mH and CC from gradients & boundaries

$$\begin{aligned} \mathsf{FPV:} \quad \dot{P}_{n_{\phi},n_{\chi}} &= \Gamma_{\downarrow\phi} P_{n_{\phi}-1,n_{\chi}} + \Gamma_{\downarrow\chi} P_{n_{\phi},n_{\chi}-1} + 3H_{n_{\phi},n_{\chi}} P_{n_{\phi},n_{\chi}} \\ \mathsf{factorization:} \quad P_{n_{\phi},n_{\chi}} &= \left[\prod_{i=1}^{n_{\phi}} \frac{\Gamma_{\downarrow\phi}}{3i\Delta H_{\phi}}\right] \left[\prod_{j=1}^{n_{\chi}} \frac{\Gamma_{\downarrow\chi}}{3j\Delta H_{\chi}}\right] C_{0} e^{3H_{s}t} \end{aligned}$$

anthropic line:

 $n_{\chi}|_{V^{(\text{today})}=0} = \frac{1}{2\pi} \frac{F_{\chi}}{f_{\chi}} \arccos\left(const - (M_{\phi}/M_{\chi})^4 \cos(2\pi n_{\phi}f_{\phi}/F_{\phi})\right) \simeq -\kappa n_{\phi} + const_{\chi}^2$

P on anthropic line:

$$P(\phi,\chi)|_{V=0} \propto \left(\frac{\Gamma_{\phi}}{3(n_{\phi}/e)\Delta H_{\phi}}\right)^{n_{\phi}} \left(\frac{\Gamma_{\chi}}{3(n_{\chi}/e)\Delta H_{\chi}}\right)^{n_{\chi}} \propto \left(\frac{e\Gamma_{\phi}}{3n_{\phi}\Delta H_{\phi}} \left(\frac{3n_{\chi}\Delta H_{\chi}}{e\Gamma_{\chi}}\right)^{\kappa}\right)^{n_{\phi}}$$

peaked at correct mh if:

$$\frac{e\Gamma_{\phi}}{3n_{\phi}\Delta H_{\phi}} \left(\frac{3n_{\chi}\Delta H_{\chi}}{e\Gamma_{\chi}}\right)^{\kappa} > 1 \quad \text{where} \qquad \kappa = \frac{N_{\chi}}{N_{\phi}} \frac{M_{\phi}^4}{M_{\chi}^4} \frac{\sin\phi_0}{\sin\chi_0}, \quad N_{\phi} = \frac{F_{\phi}}{f_{\phi}}, \quad N_{\chi} = \frac{F_{\chi}}{f_{\chi}}$$
66

Similar approaches

V-weighted

assuming non-eternal

M. Geller, Y. Hochberg, and E. Kuflik, Phys. Rev. Lett. **122**, 191802 (2019), 1809.07338. C. Cheung and P. Saraswat (2018), 1811.12390.

G. F. Giudice, M. McCullough, and T. You, JHEP 10, 093 (2021), 2105.08617.



Inflating to the weak scale

→ Inflating to the weak scale: Geller, Hochberg, Kuflik 1809.07338



→ for any $\{\phi, \chi\}$ giving a correct Higgs mass there will be another $\{\phi, \chi\}$ giving wrong Higgs mass and the same vacuum energy

Inflating to the weak scale



→ for any $\{\phi, \chi\}$ giving a correct Higgs mass there will be another $\{\phi, \chi\}$ giving wrong Higgs mass and the same vacuum energy

Stochastic approach



χ

Stochastic approach



$$\dot{P} = \frac{\partial}{\partial\phi} \left(\frac{H^{3(1-\beta)}}{8\pi^2} \frac{\partial}{\partial\phi} (H^{3\beta}P) \right) + \frac{\partial}{\partial\phi} \left(\frac{V'}{3H}P \right) + 3HP$$

 \Rightarrow analogous P distribution

Stochastic approach



$$V = \Lambda + \frac{1}{2}m^2\phi^2$$

general solution:

eigenmodes of $\nu \propto -H_s^2 + \dots$ Giudice,McCullough,You, 2105.08617

$$P_{\nu} = \exp\left[-A\phi^{2}\right] \left\{ \mathbf{c}_{+}D_{\nu}\left[B\phi\right] + \mathbf{c}_{-}D_{\nu}\left[-B\phi\right] \right\} \, \boldsymbol{e}^{3H_{s}t}$$


Matching



 $P_{\nu} = \exp\left[-A\phi^{2}\right] \left\{ \mathbf{c}_{+}D_{\nu}\left[B\phi\right] + \mathbf{c}_{-}D_{\nu}\left[-B\phi\right] \right\}$







CC solution?



$$= \Delta \Lambda_{cc\,\chi} \simeq M_{\chi}^4 / N_{\chi}$$

has to be within $\Lambda_{cc(obs.)} \simeq 10^{-47} {\rm GeV}^4 \quad (1)$

CC solution?



In addition, $P(\chi)$ prefers less tunnelings, hence higher Λ , close to the upper anthropic bound $\sim 10^3 \Lambda_{cc(obs.)}$ \Rightarrow one needs a sufficiently mild grad $P(\chi)$ (2)

CC solution?



In addition, $P(\chi)$ prefers less tunnelings, hence higher Λ , close to the upper anthropic bound ~ $10^3 \Lambda_{cc(obs.)}$ \Rightarrow one needs a sufficiently mild grad $P(\chi)$ (2)

We evade (1), (2) by assuming some additional finescanning sector.

Slow-roll inflation



We assume some slow-roll inflation in the background, responsible for eternal inflation at a scale *H_s*

Parameter space



Parameter space



Local measures

Probability gradients

3 regimes, end of slow-roll picks the time of sampling.

Regime 2 has no probability-vacuum energy degeneracy.

* Although the degeneracy can be broken e.g. by changing slope after inflation.

Local measures

Parameter space:



(Other params similar to volume-weighted)

Local measures

Parameter space:

Main bounds on Ne:

1) domain walls

2) requirement to erase V-dependent initial conditions

$$\frac{1}{n_{\phi}!} \left[\Gamma_{\phi\downarrow} t_R \right]^{n_{\phi}} > \exp\left[-\frac{8\pi^2}{3} \frac{V(0) - V(n_{\phi})}{H^4} \right]$$