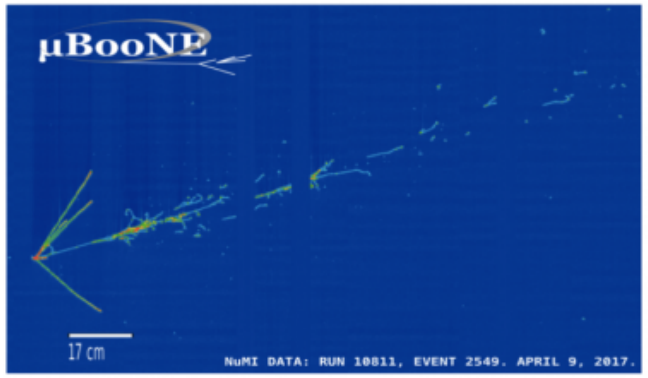
More Axion Stars from Strings

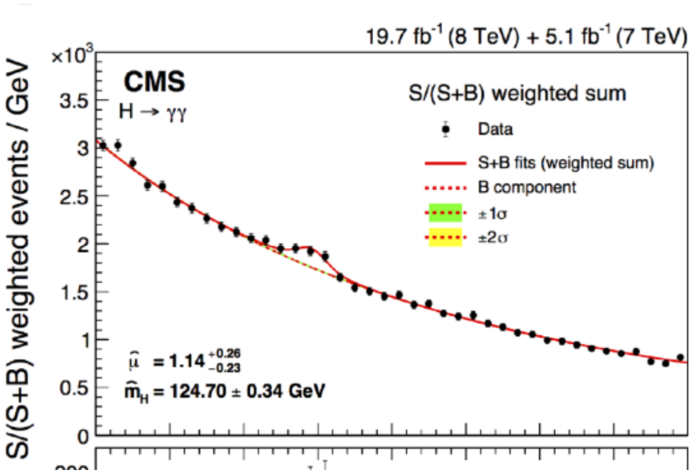
Edward Hardy

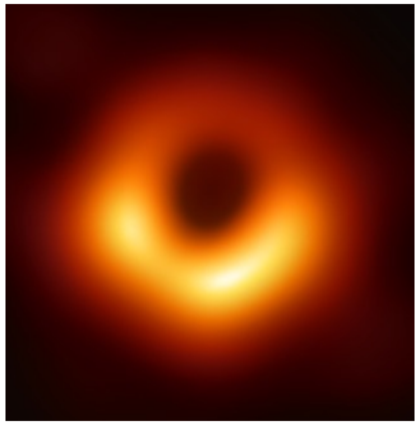
UNIVERSITY OF OXFORD

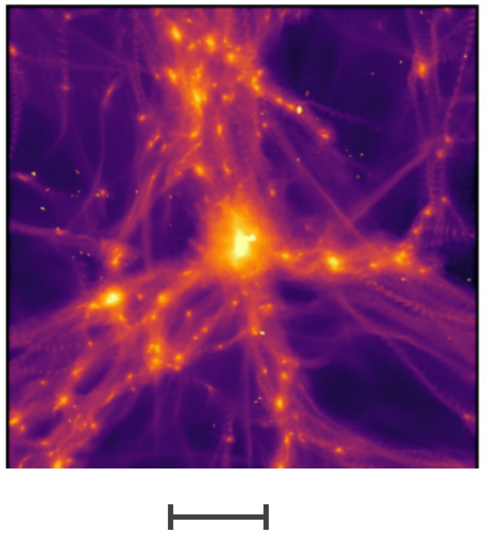
2405.19389 Gorghetto, EH, Villadoro

Dark Matter



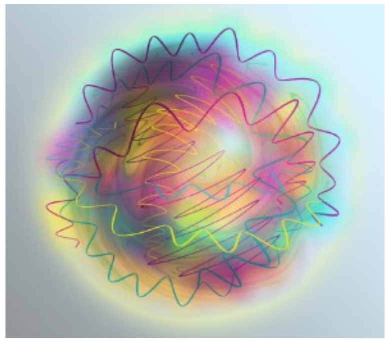






$\lambda_{\text{d.B.}}$

$$\frac{n_B - \bar{n}_B}{n_\gamma} \simeq 10^{-10}$$

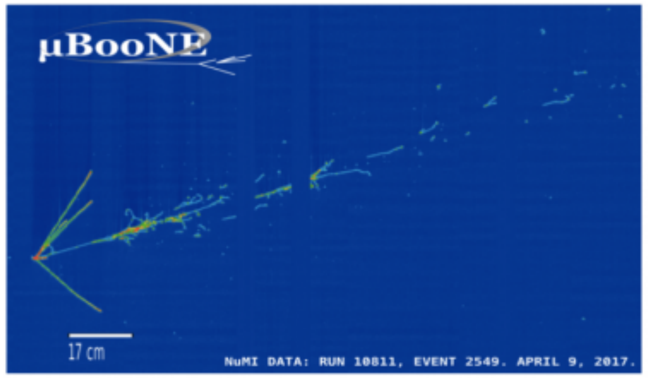


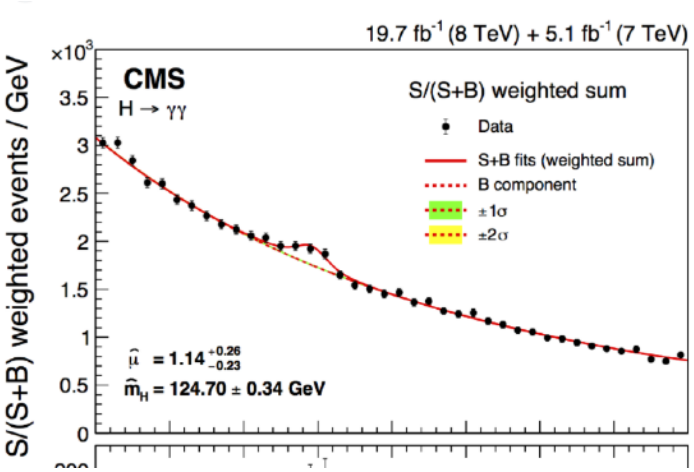
$$\Lambda_{\text{UV}}\ e^{-\frac{1}{2b_0\ \alpha(\Lambda_{\text{UV}})}}$$

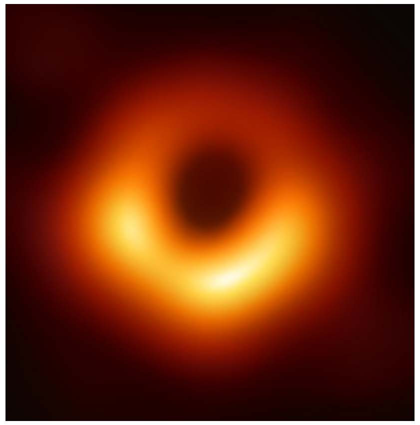
Dark Matter

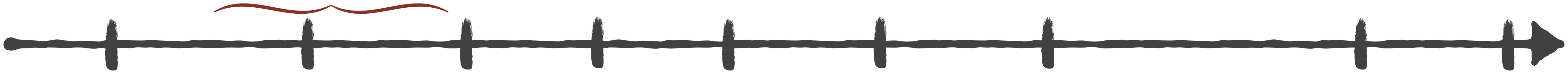
QCD

axion

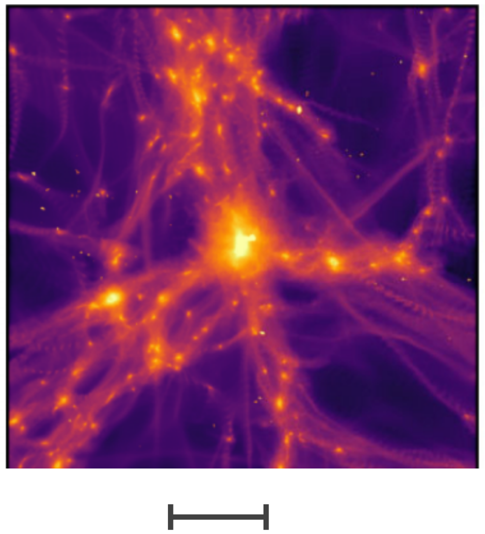








$10^{-20}\mathrm{eV}$ $10^{-5}\mathrm{eV}$ eV keV MeV GeV TeV $10^{16}\mathrm{GeV}$ $10^{-14}M_\odot$



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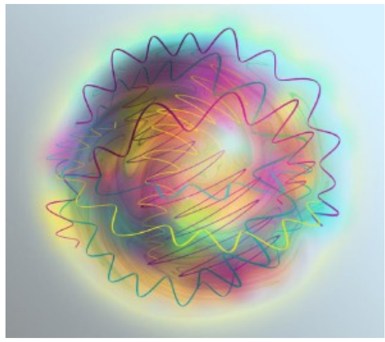


$$\Lambda_{\mathrm{UV}}\; e^{-\frac{1}{2b_0\,\alpha(\Lambda_{\mathrm{UV}})}}$$

The QCD axion

$$\mathcal{L} \supset \theta_0 \frac{\alpha_S}{8\pi} G\tilde{G}$$

$$\theta' = \theta_0 + \arg\left(\mathrm{Det} M_q\right) \lesssim 10^{-10}$$

Strong CP problem

The QCD axion

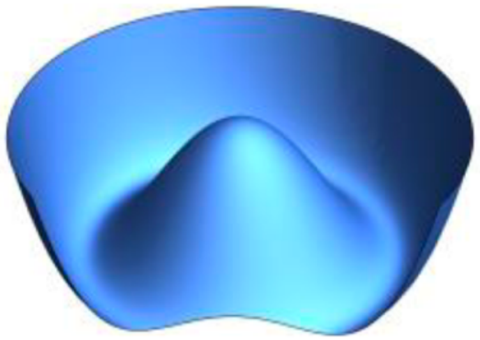
$$\mathcal{L} \supset \theta_0 \frac{\alpha_S}{8\pi} G\tilde{G} \qquad \theta' = \theta_0 + \arg\left(\mathrm{Det} M_q\right) \lesssim 10^{-10}$$

Strong CP problem

- Axion a, shift symmetry $a \to a + c$
- Candidate axions generic in high energy theories

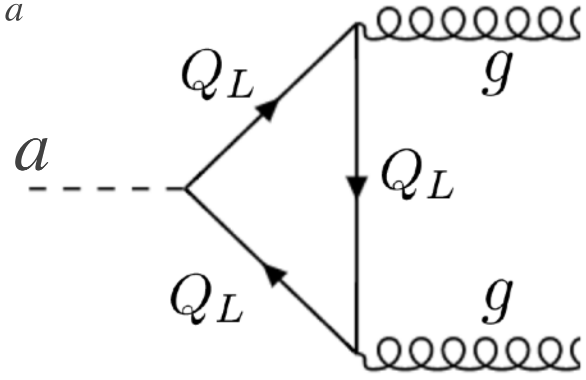
$$\boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

$$\mathscr{L} = (\partial\phi)^2 - \frac{m_r^2}{2f_a^2}\left(|\phi|^2 - \frac{f_a^2}{2}\right)^2$$



$$\phi = \frac{f_a + r}{\sqrt{2}} e^{ia/f_a}$$

$$\theta \equiv a/f_a$$

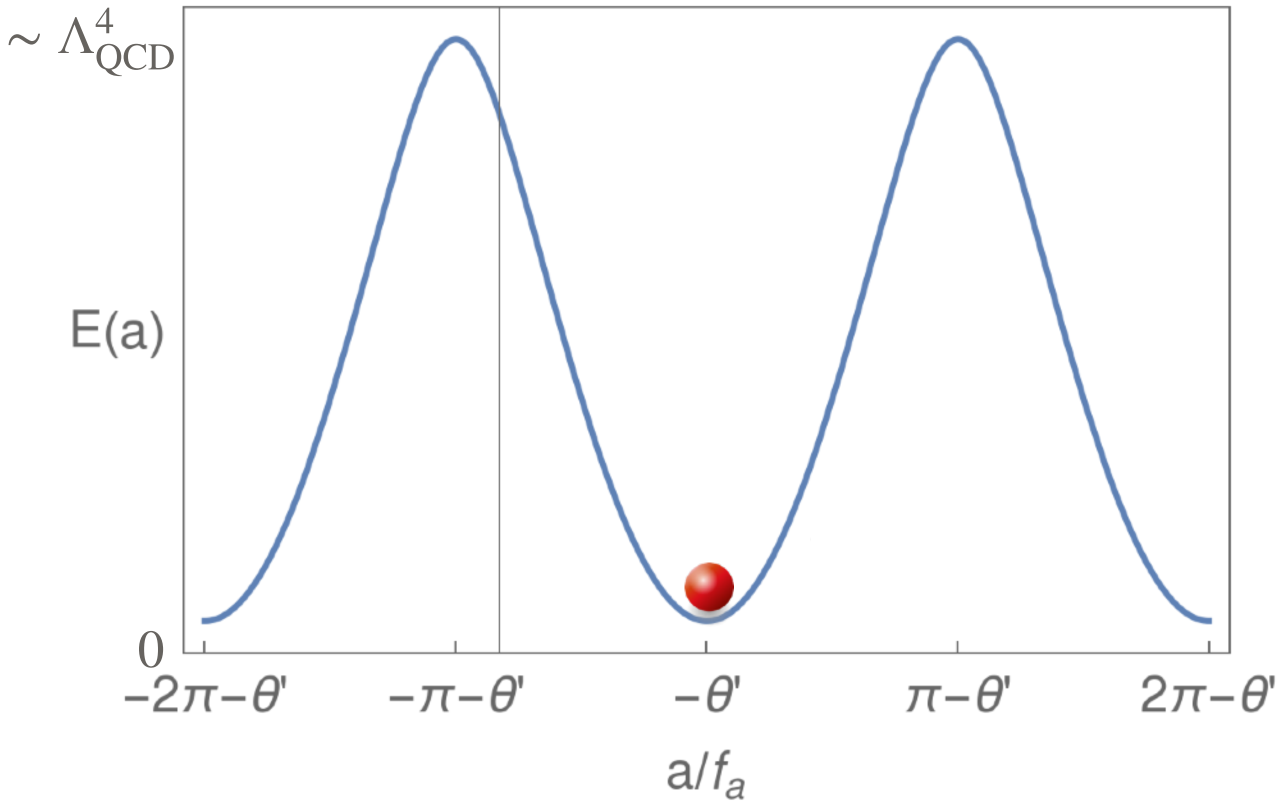


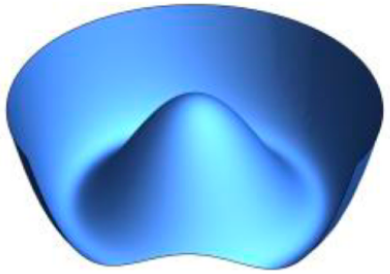
The QCD axion

$$\boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

$$E\left(a\right) \geq E\left(a = -\theta'\right)$$

$$\theta_{\rm tot} = \langle a \rangle + \theta' = 0$$





[Vafa, Witten]

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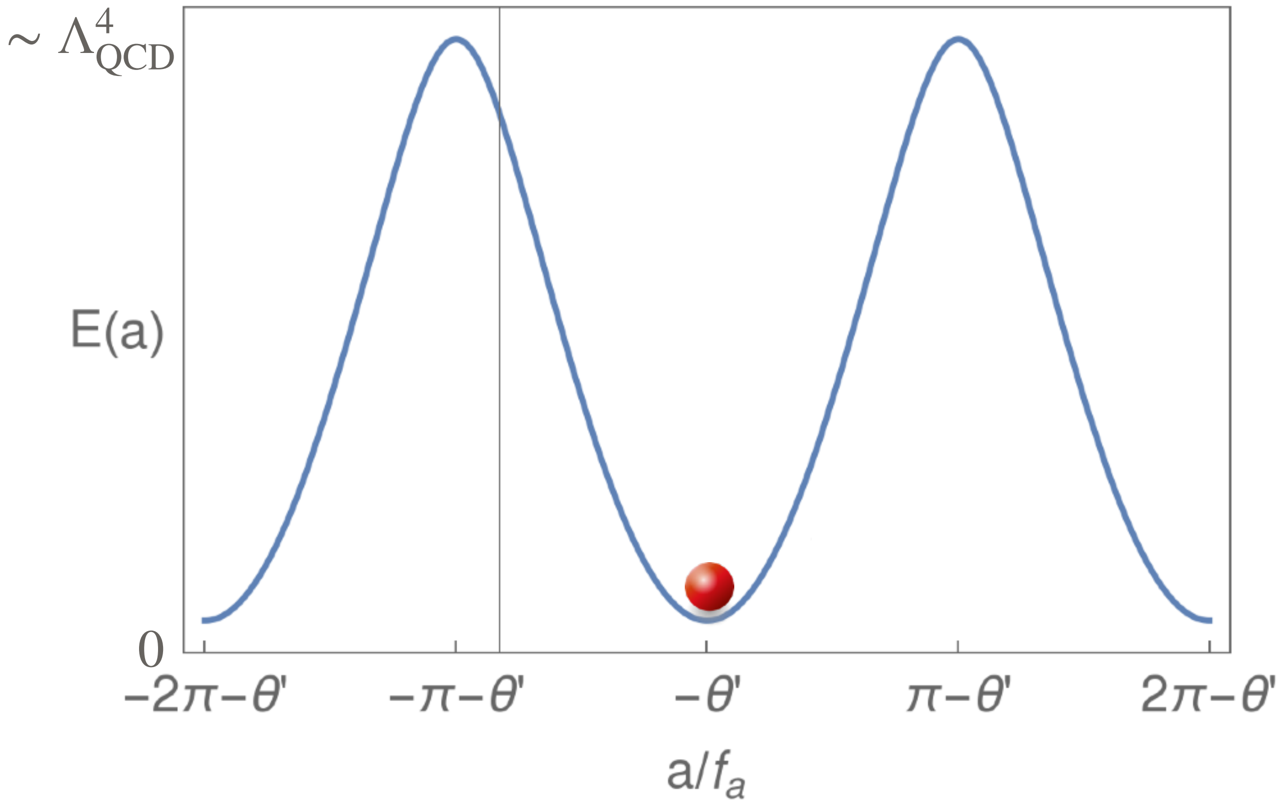
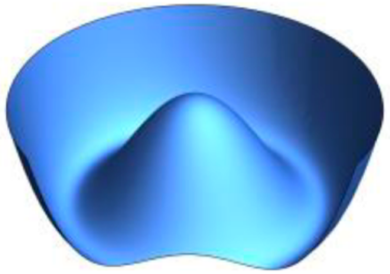




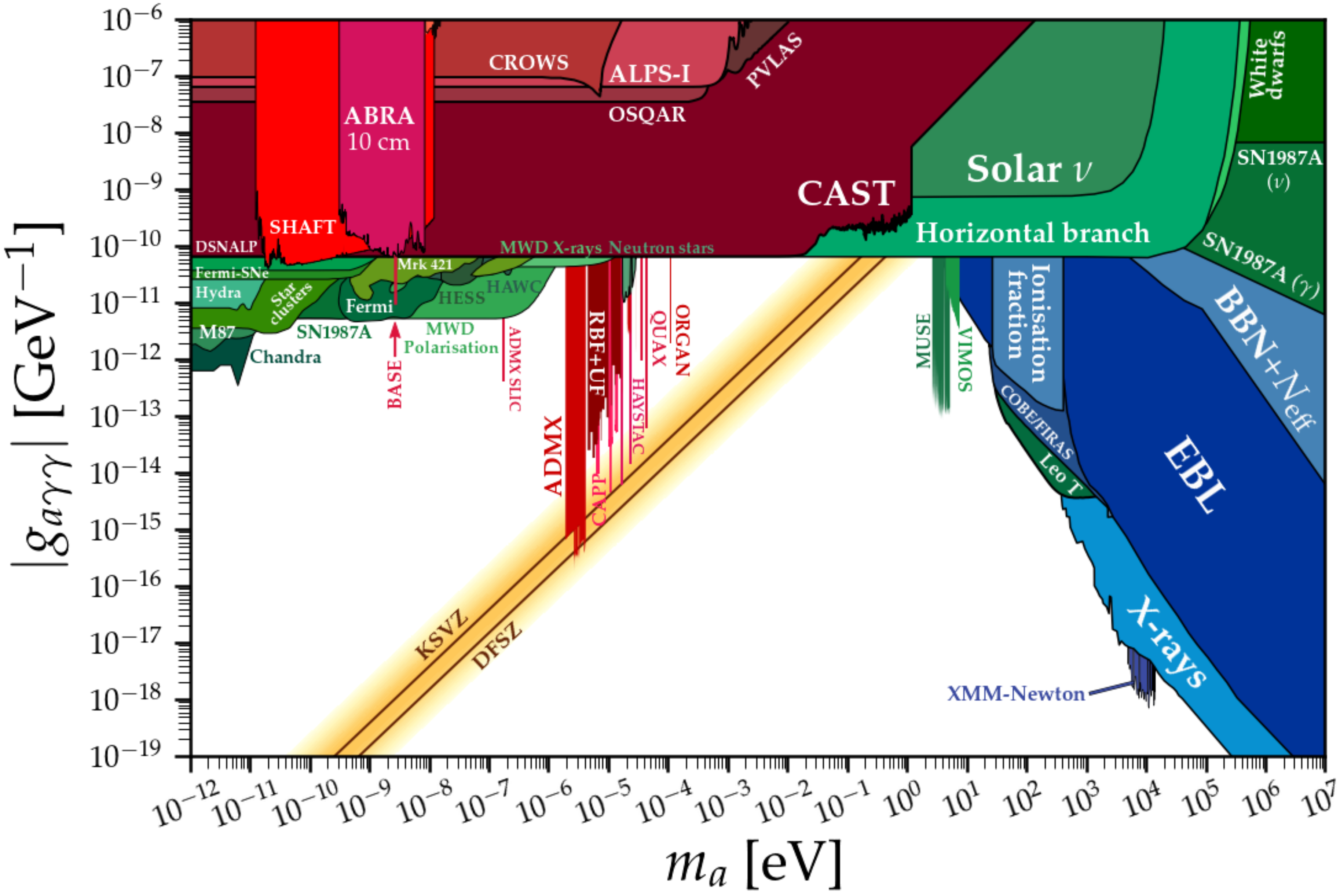
[Vafa, Witten]

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2} + \ldots$$

$$\boxed{m_a = 5.70(6)(4)\ \mu\text{eV}\left(\frac{10^{12}\text{GeV}}{f_a}\right)}$$

Searches

$$\mathscr{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a \; F_{\mu\nu} \tilde{F}^{\mu\nu}$$



https://cajohare.github.io/AxionLimits

Searches

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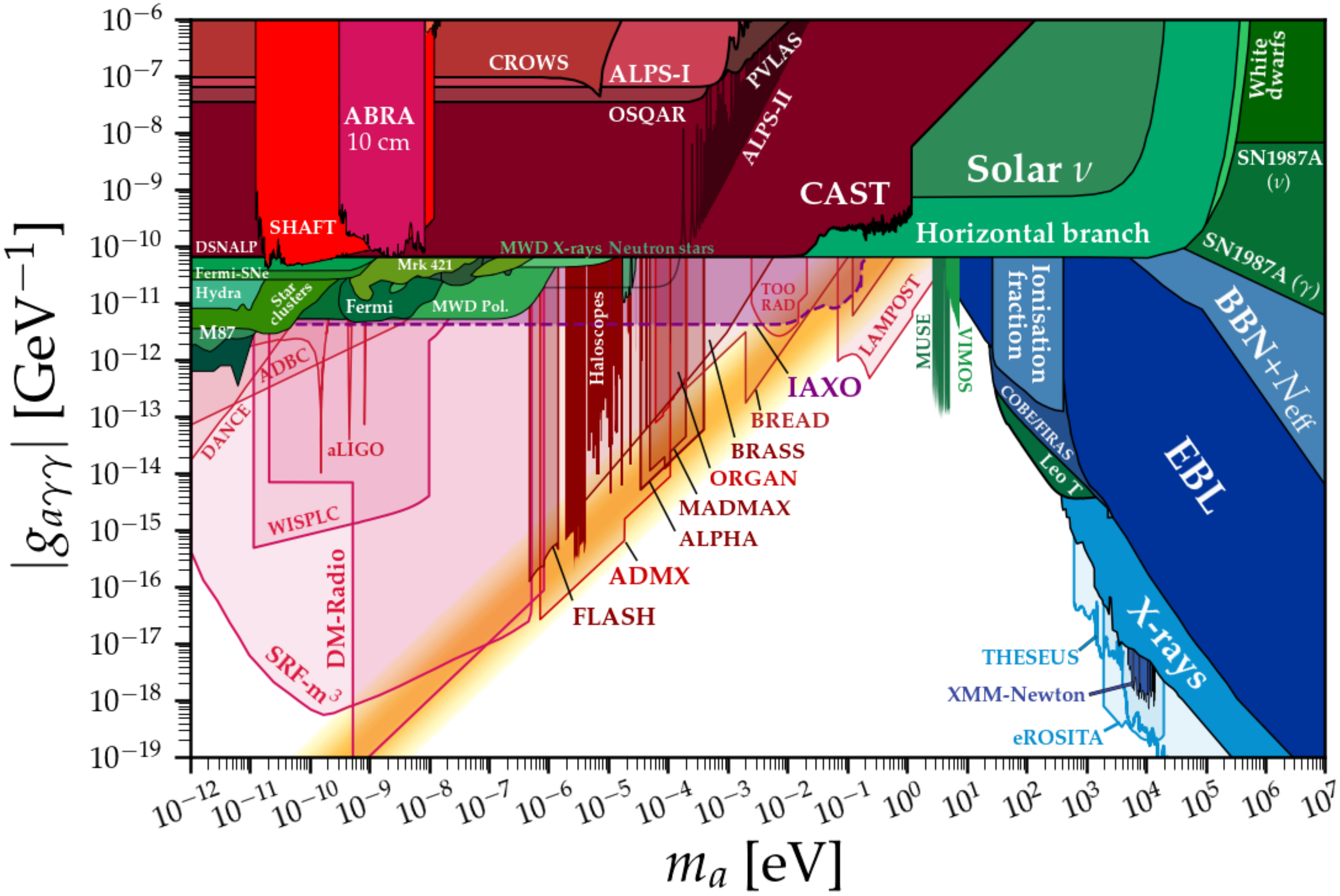


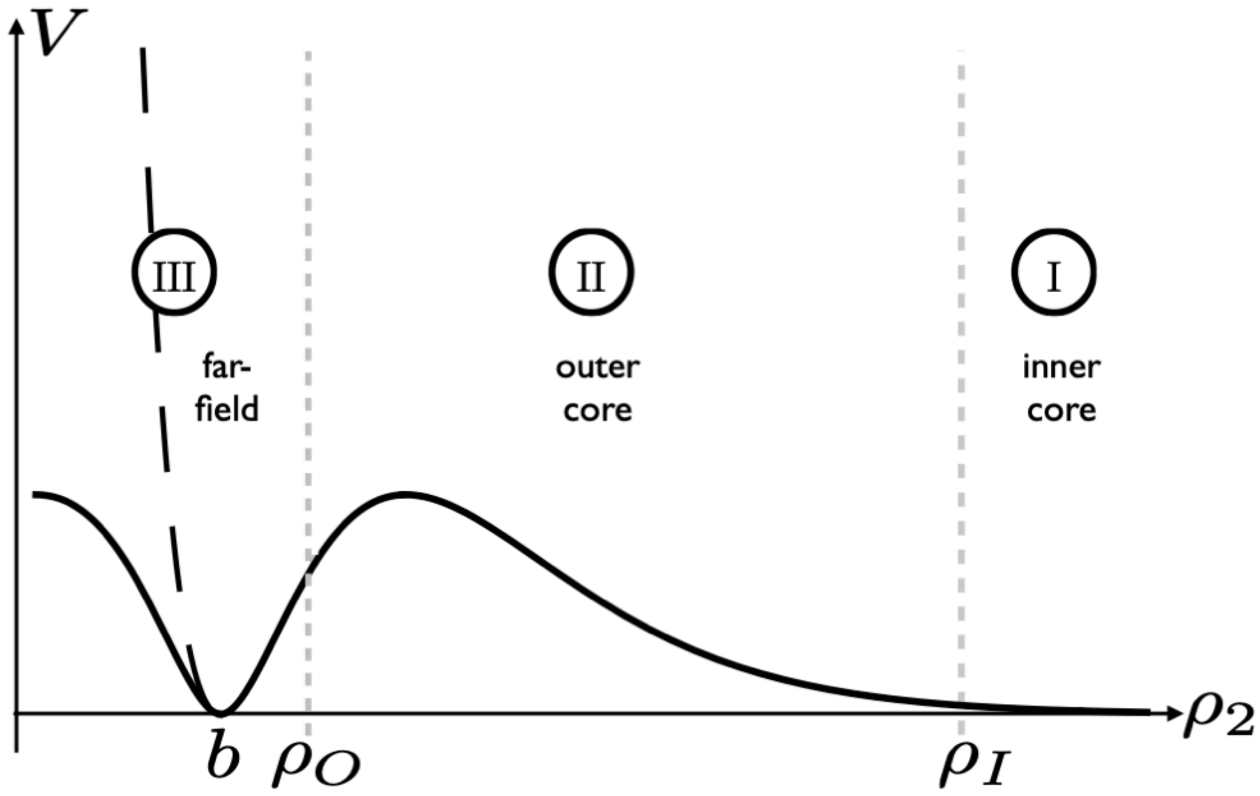
https://cajohare.github.io/AxionLimits

Caution

The dynamics of simple field theory axions, with $N_W = 1$

Possibly important differences for string theory axions, e.g.

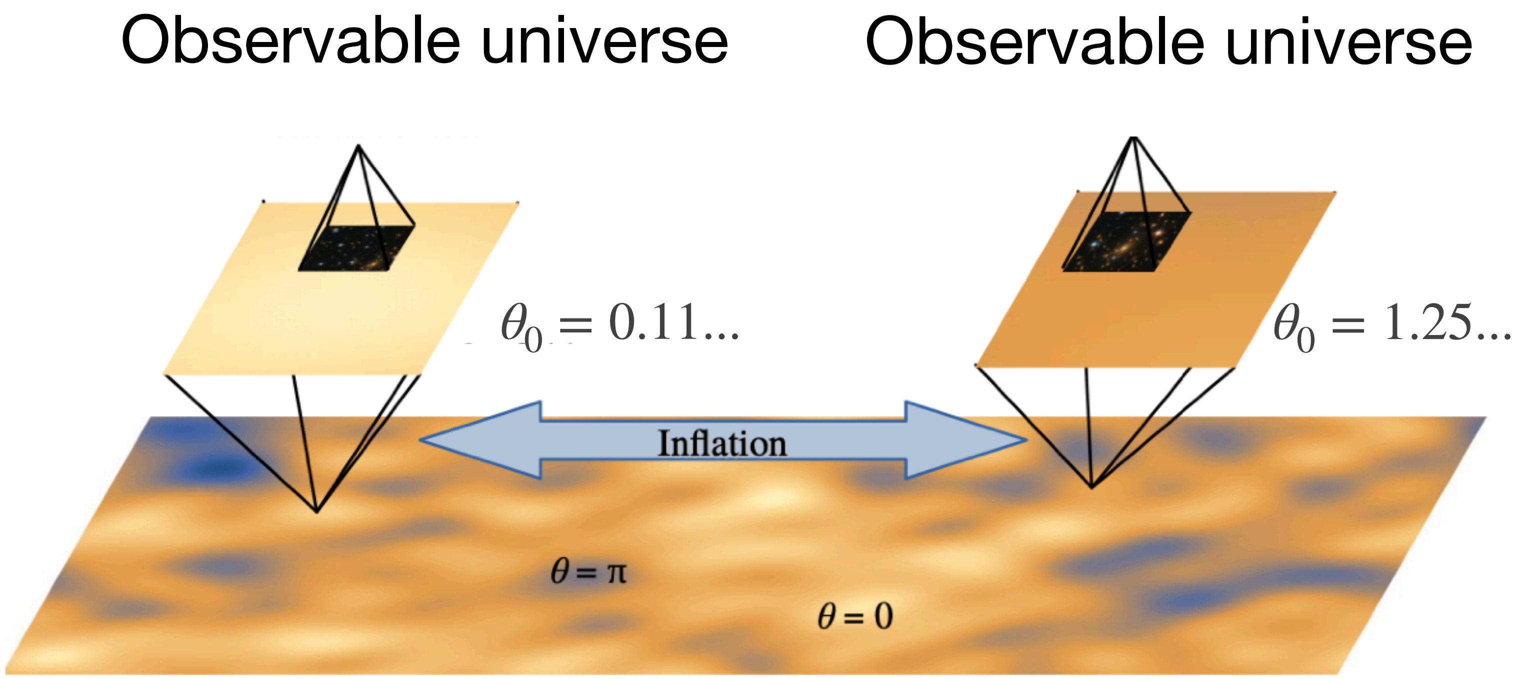
- *production of strings*
- *core structure*
- *KK modes*
- *cosmological history*



[March-Russell, Tillim]

Initial conditions

Pre-inflationary



Observable universe

Observable universe

$\theta_0 = 0.11...$

$\theta_0 = 1.25...$

Inflation

$\theta = \pi$

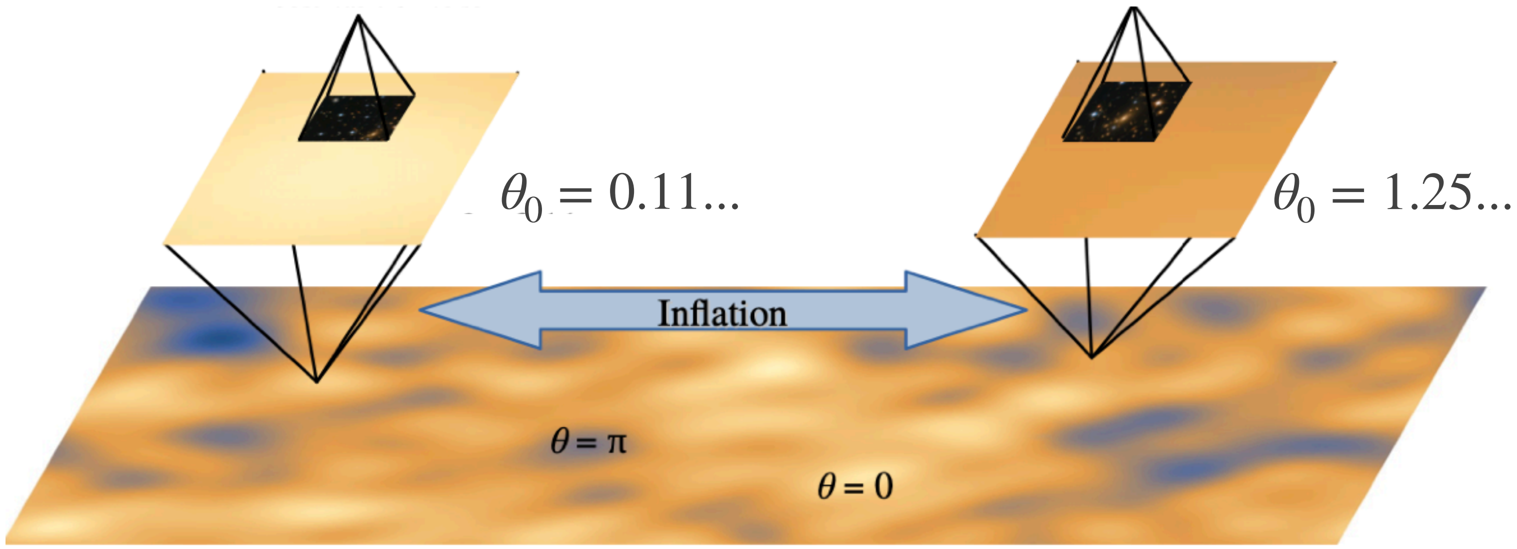
$\theta = 0$

Initial conditions

Pre-inflationary

Observable universe

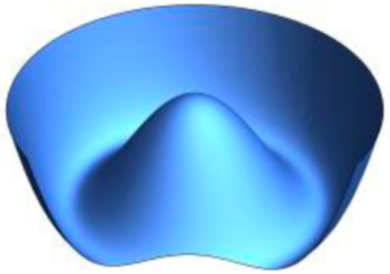
Observable universe

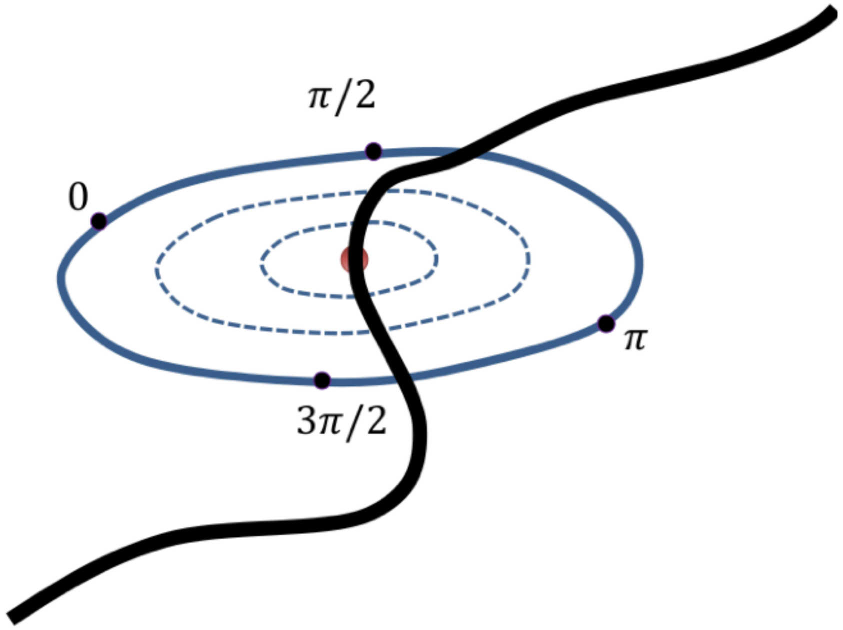


Post-inflationary

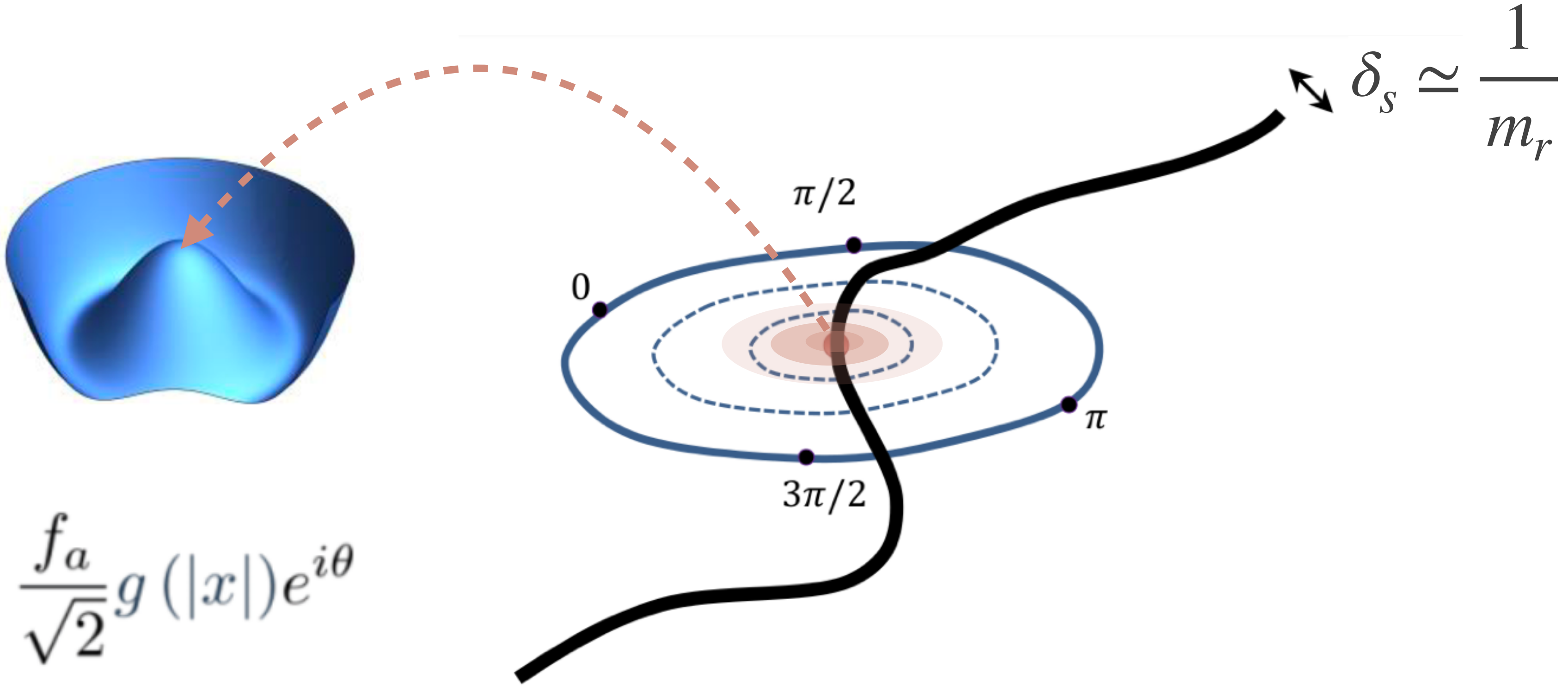
Observable universe

Topological strings





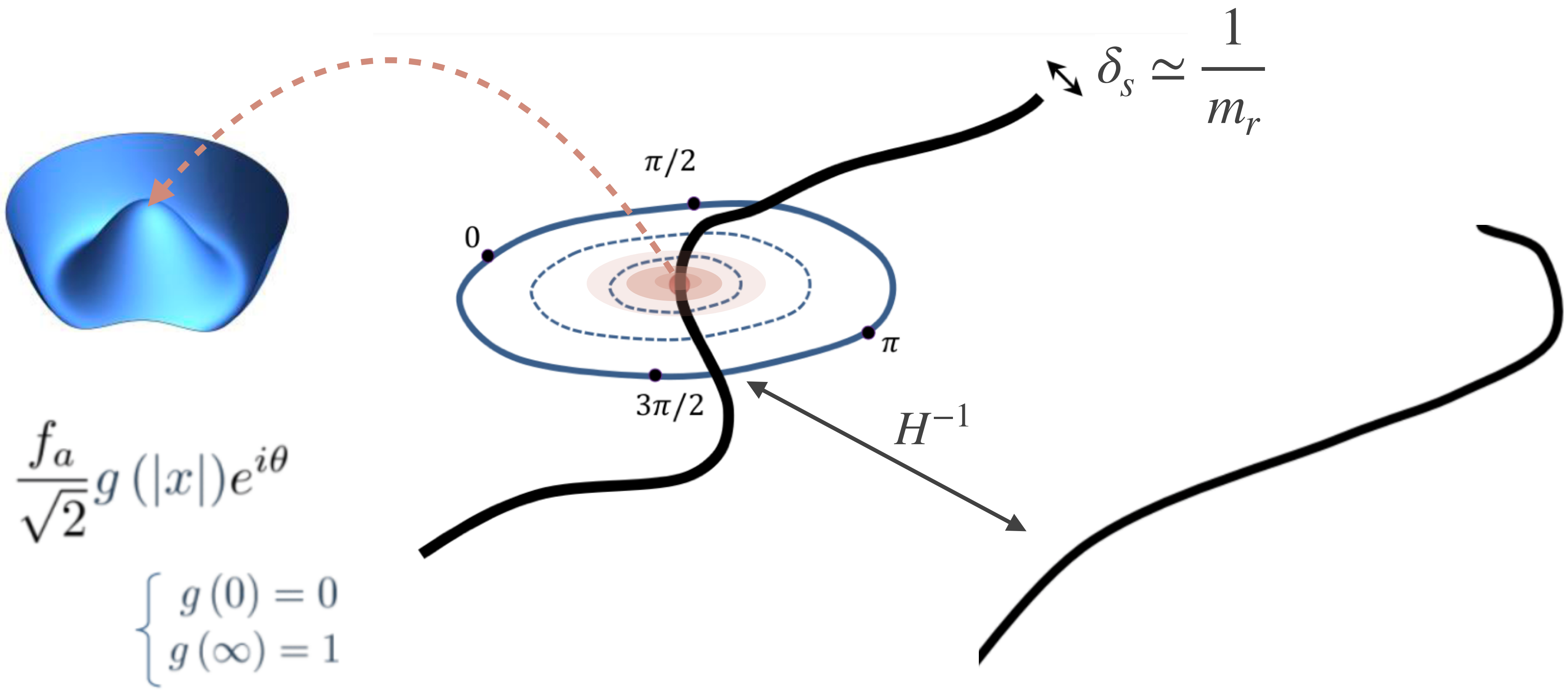
Topological strings



$$\phi = \frac{f_a}{\sqrt{2}} g\left(|x|\right) e^{i\theta}$$

$$\begin{cases} g\left(0\right) = 0 \\ g\left(\infty\right) = 1 \end{cases}$$

Topological strings



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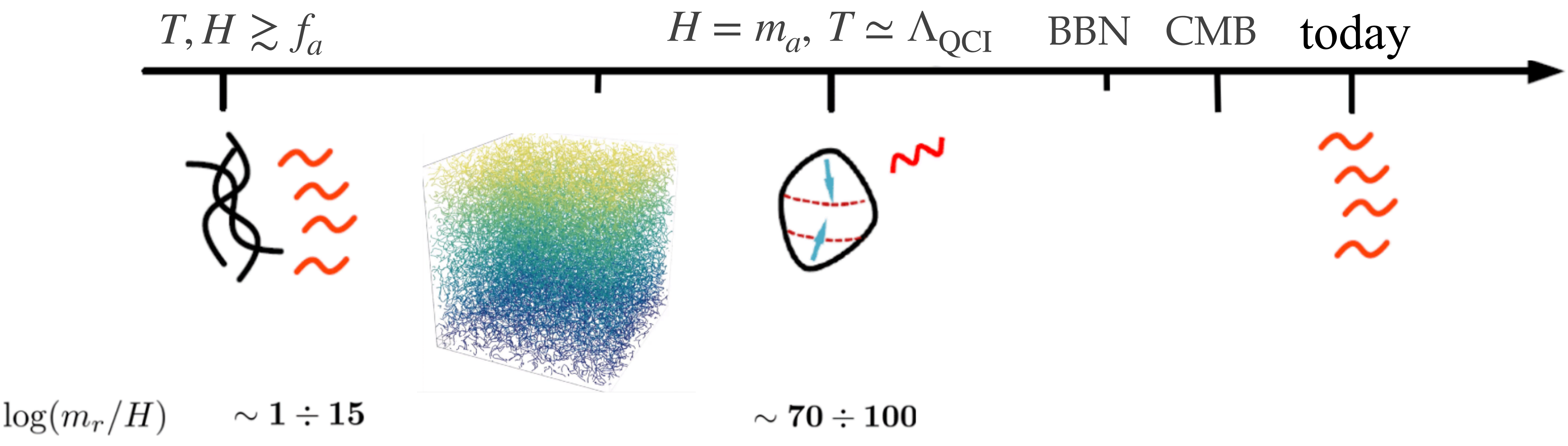
Full evolution



$T, H \gtrsim f_a$

$H = m_a,\ T \simeq \Lambda_{\rm QCI}$

BBN

CMB

today

$\log(m_r/H)$

$\sim \mathbf{1 \div 15}$

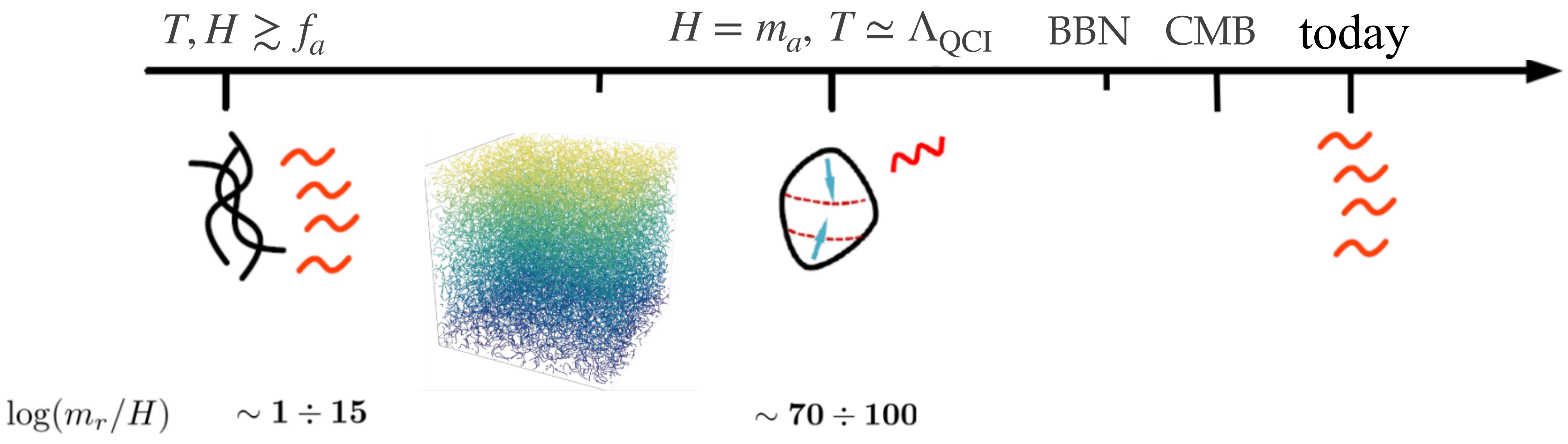
$\sim \mathbf{70 \div 100}$

Strings form

Domain walls form

Relic axions

Full evolution



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Strings form

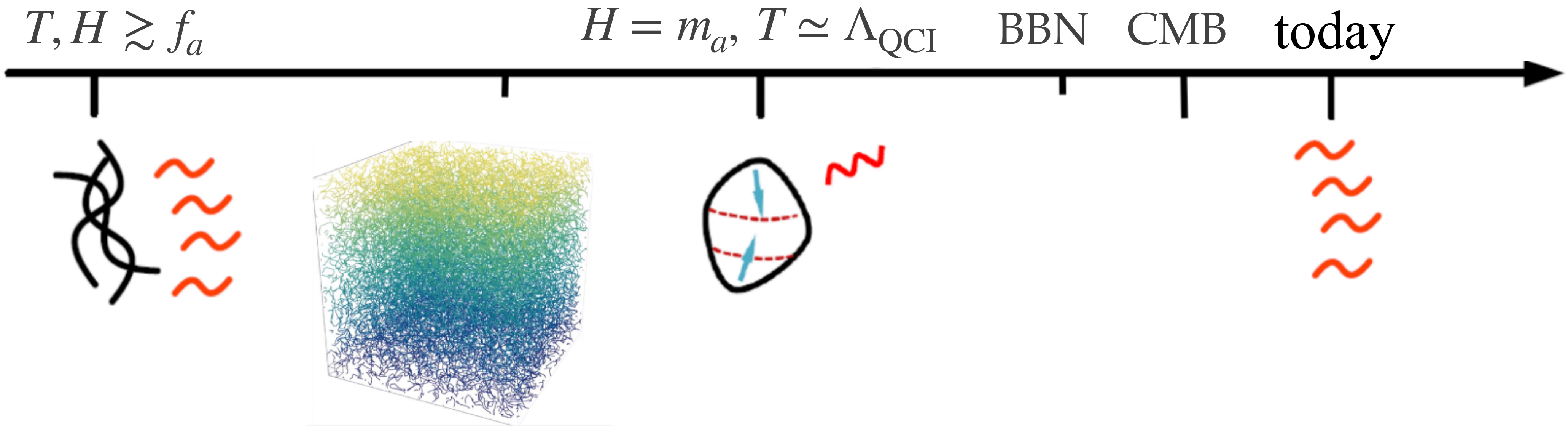
Domain walls form

Relic axions

Dynamics:

- *nonlinear*
- *large scale separation*

Full evolution



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BBN

CMB

today

$\log(m_r/H)$ $\sim \mathbf{1 \div 15}$ $\sim \mathbf{70 \div 100}$

Strings form

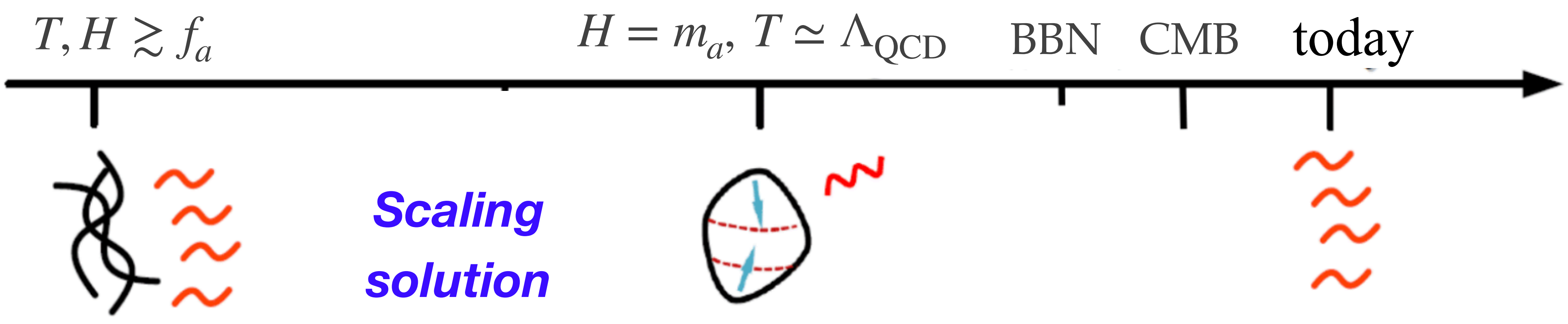
Domain walls form

Relic axions

Simulate →

Extrapolate

Full evolution



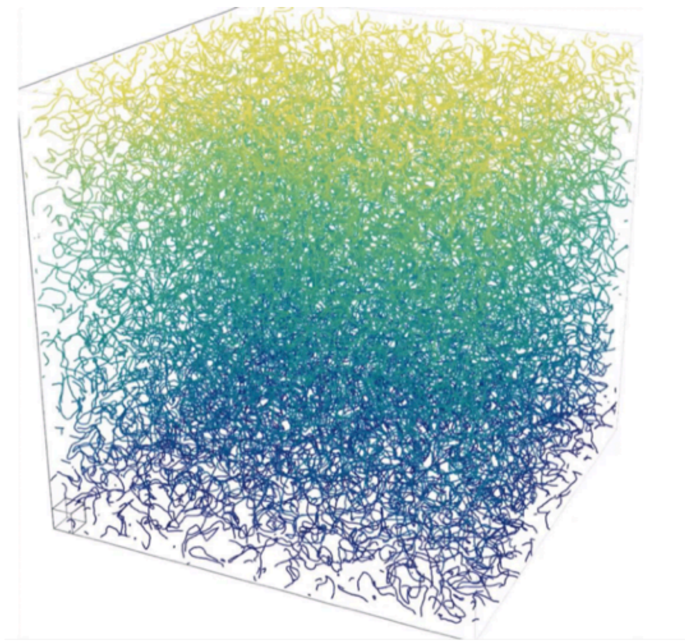
$T, H \gtrsim f_a$ $H = m_a,\ T \simeq \Lambda_{\mathrm{QCD}}$ BBN CMB today

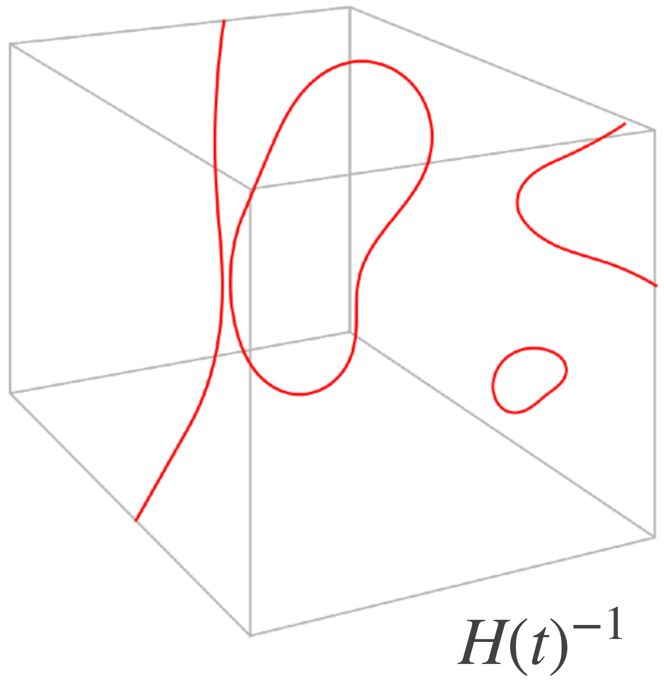
Scaling solution

Strings form

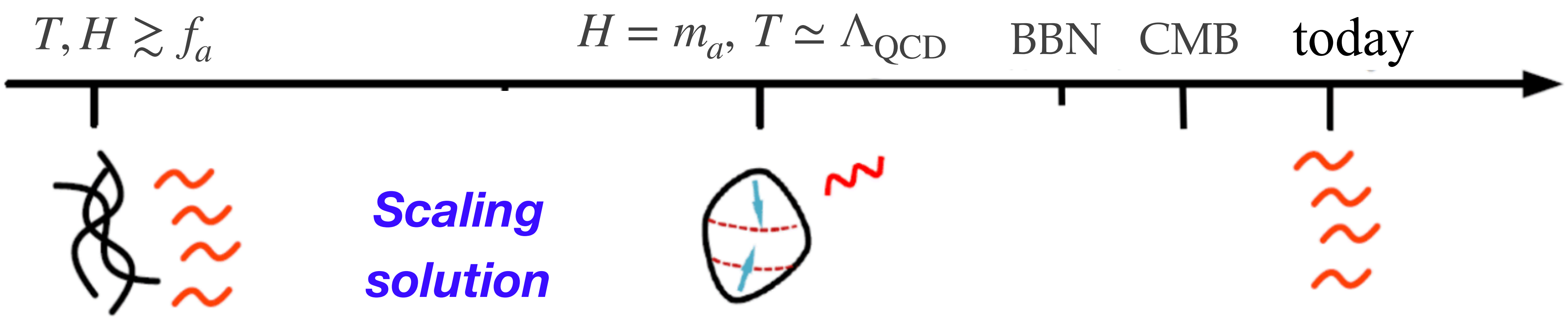
Strings destroyed

Relic axions





Full evolution



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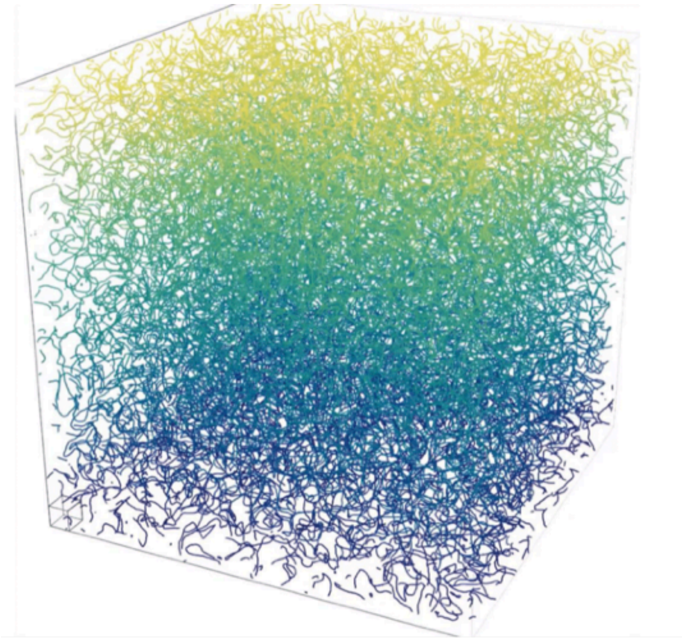
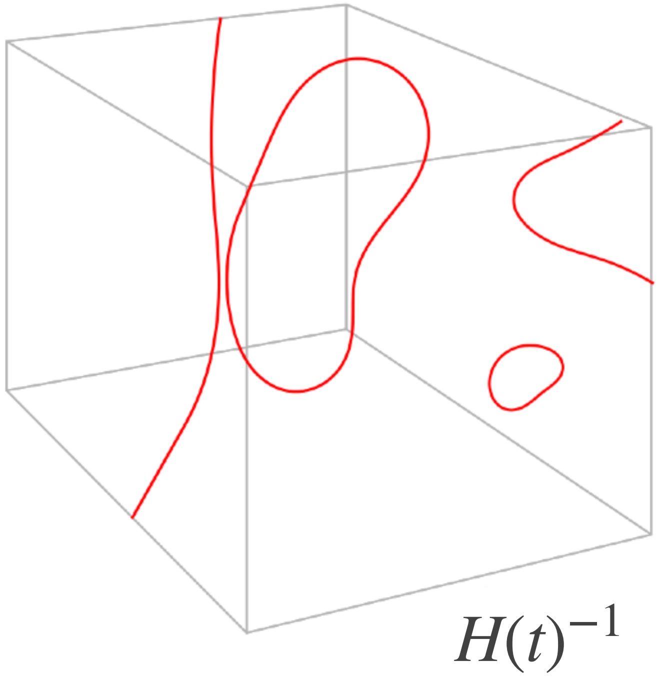


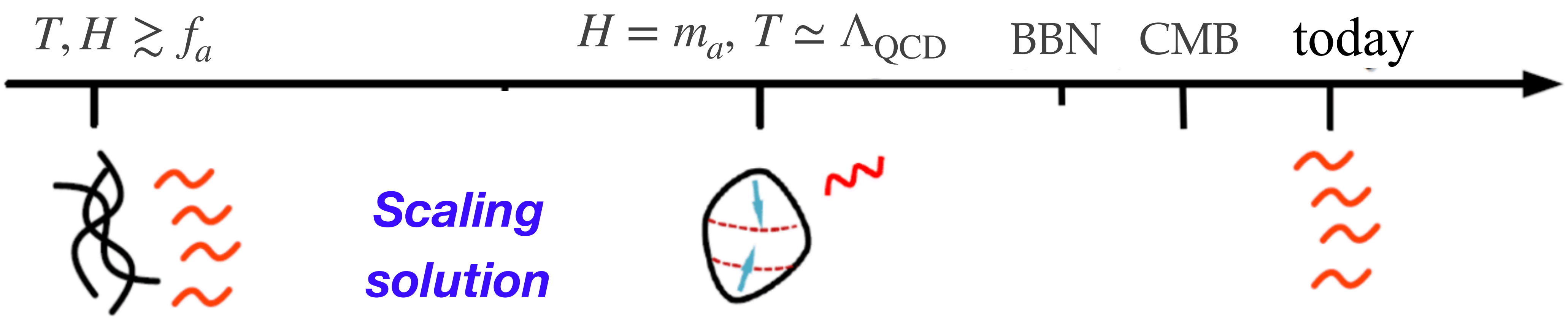


$$\Omega_{DM} \simeq 0.23 \implies$$

$$f_a \lesssim 10^{10}\ \text{GeV}$$

$$m_a \gtrsim 0.5\ \text{meV}$$

Full evolution



$T, H \gtrsim f_a$

$H = m_a,\ T \simeq \Lambda_{\rm QCD}$

BBN

CMB

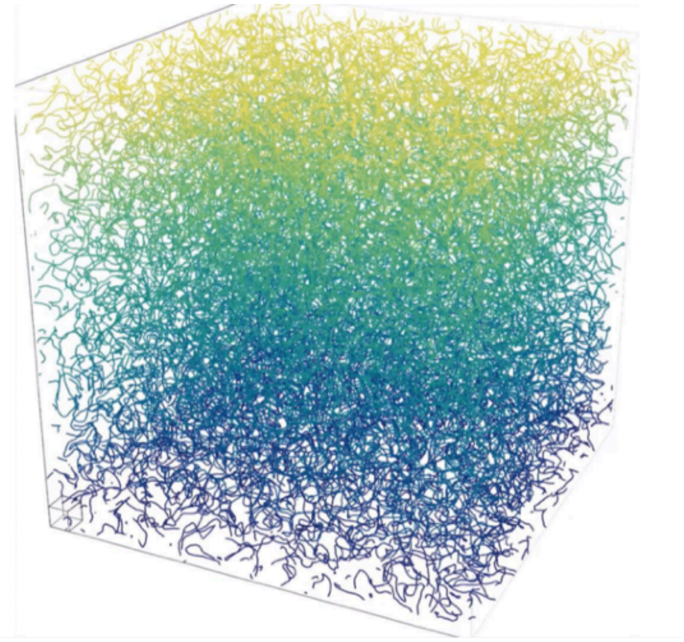
today

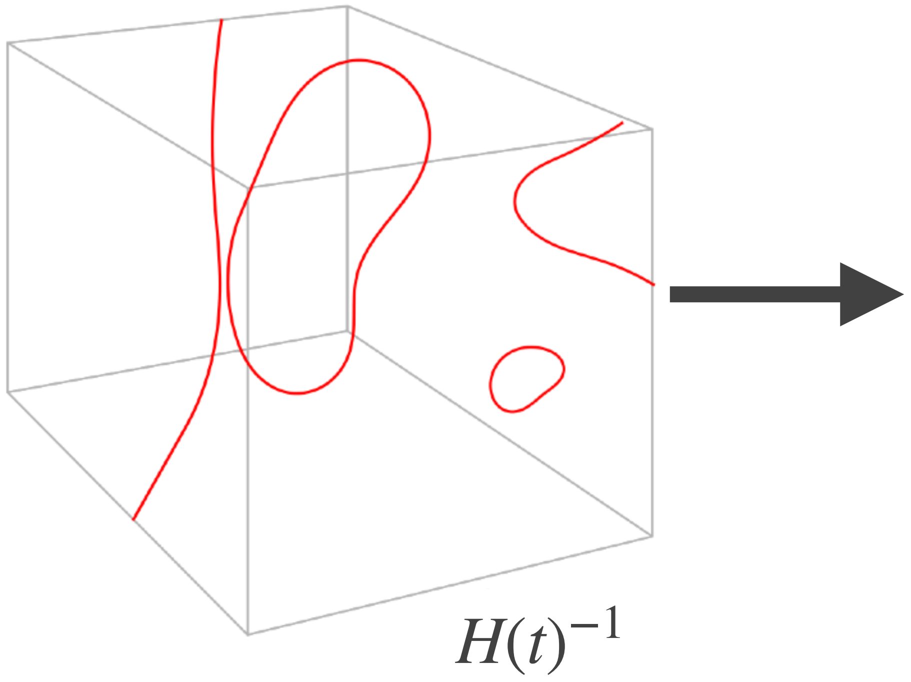
Scaling solution

Strings form

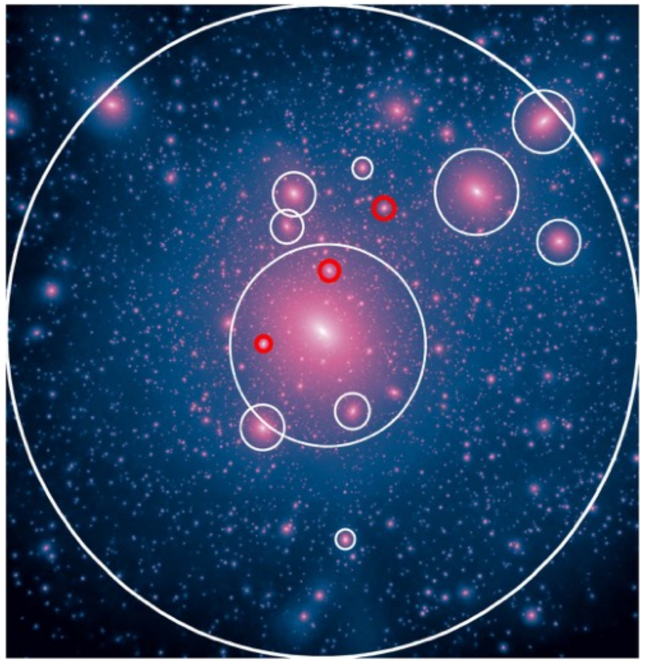
Strings destroyed

Relic axions





Axions with spatial fluctuations



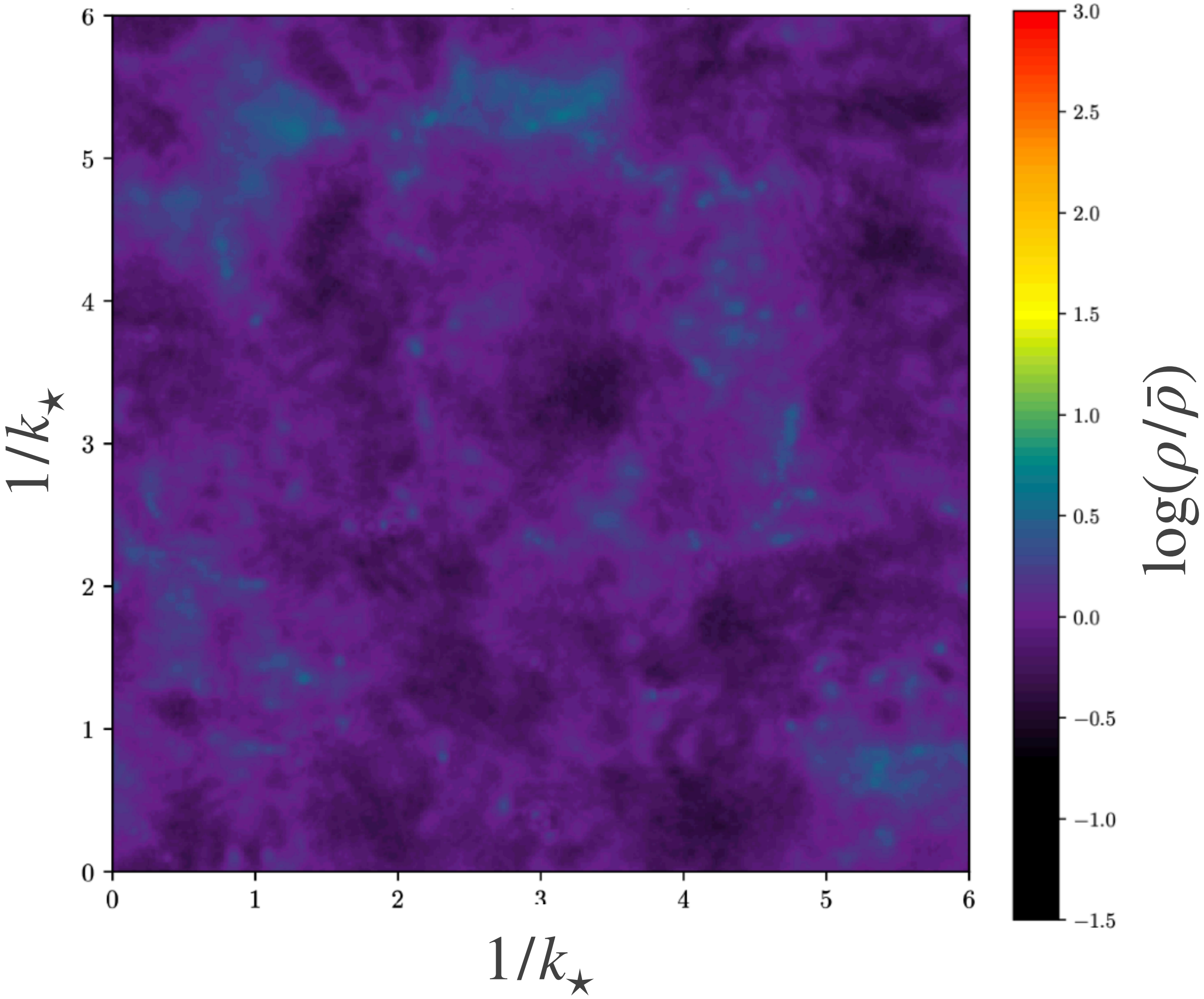
[Eggemeier et al]

Dark matter substructure

“Axion miniclusters”

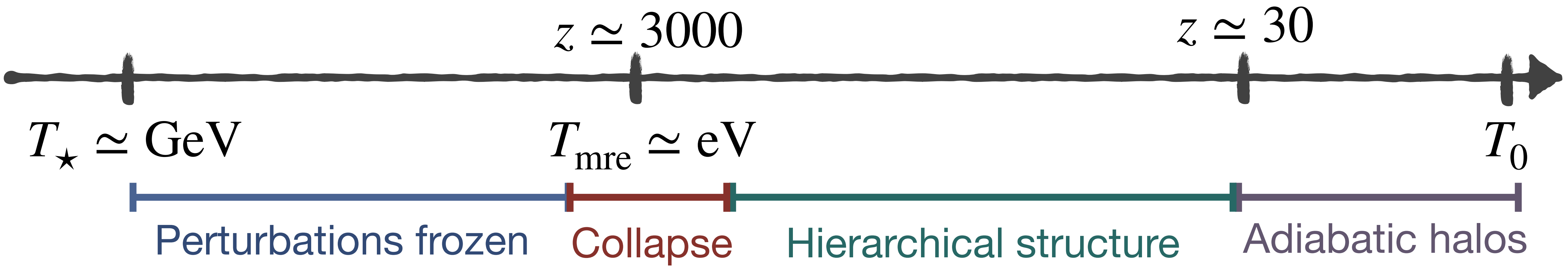
Initial perturbations

Order one fluctuations on co-moving scales $\simeq H_\star$ when $H = m_a(T)$



[Eggemeier et al]

Standard picture



$z \simeq 3000$

$z \simeq 30$

$T_\star \simeq \mathrm{GeV}$

$T_{\mathrm{mre}} \simeq \mathrm{eV}$

T_0

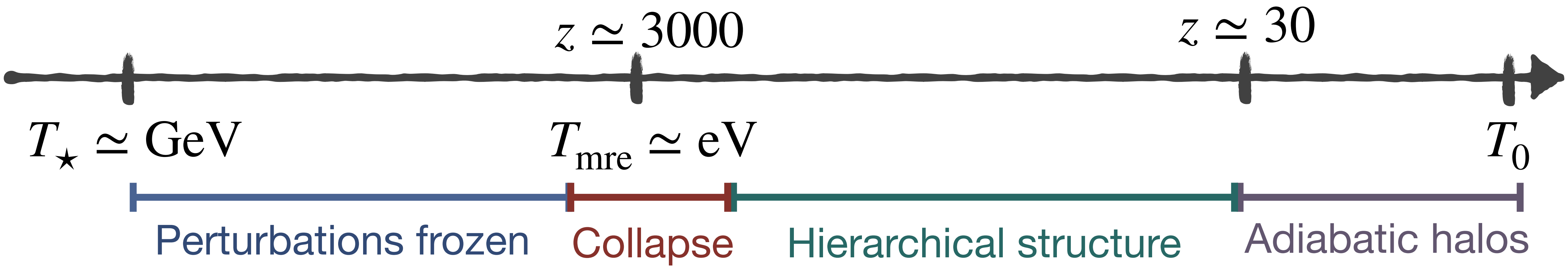
Perturbations frozen

Collapse

Hierarchical structure

Adiabatic halos

Standard picture



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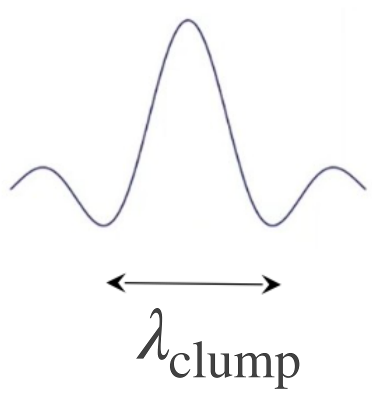
Wave effects at matter radiation equality

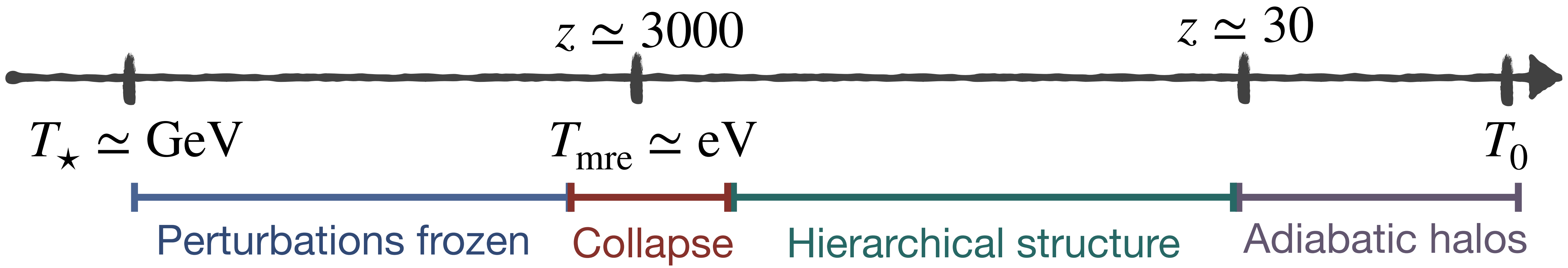


$$\lambda_{\mathrm{dB}} = \frac{1}{m_a v} = \frac{1}{m_a (GM/\lambda_{\mathrm{clump}})^{1/2}} = \frac{1}{\lambda_{\mathrm{clump}} (4\pi G \rho m_a^2)^{1/2}}$$

"Quantum" Jeans scale:

$$\boxed{\lambda_J \simeq (G\rho m_a^2)^{1/4}} \qquad k_J/R = (16\pi G \rho m_a^2)^{-1/4}$$

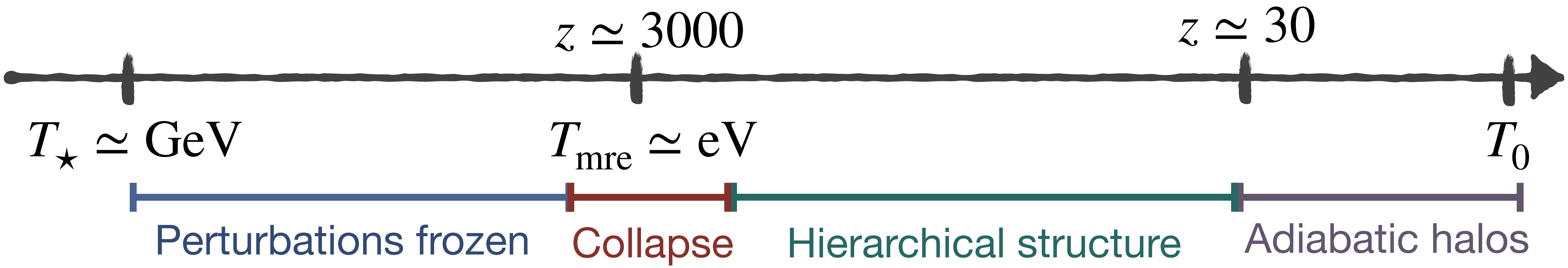
Standard picture



ALP:

$$\left.\frac{k_p}{k_J}\right|_{\rm MRE} = \frac{k_{p\star} a_\star / a_{\rm MRE}}{(16\pi G \rho_{\rm MRE} m^2)^{1/4}} \simeq \frac{k_{p\star}}{H_\star}$$

Standard picture



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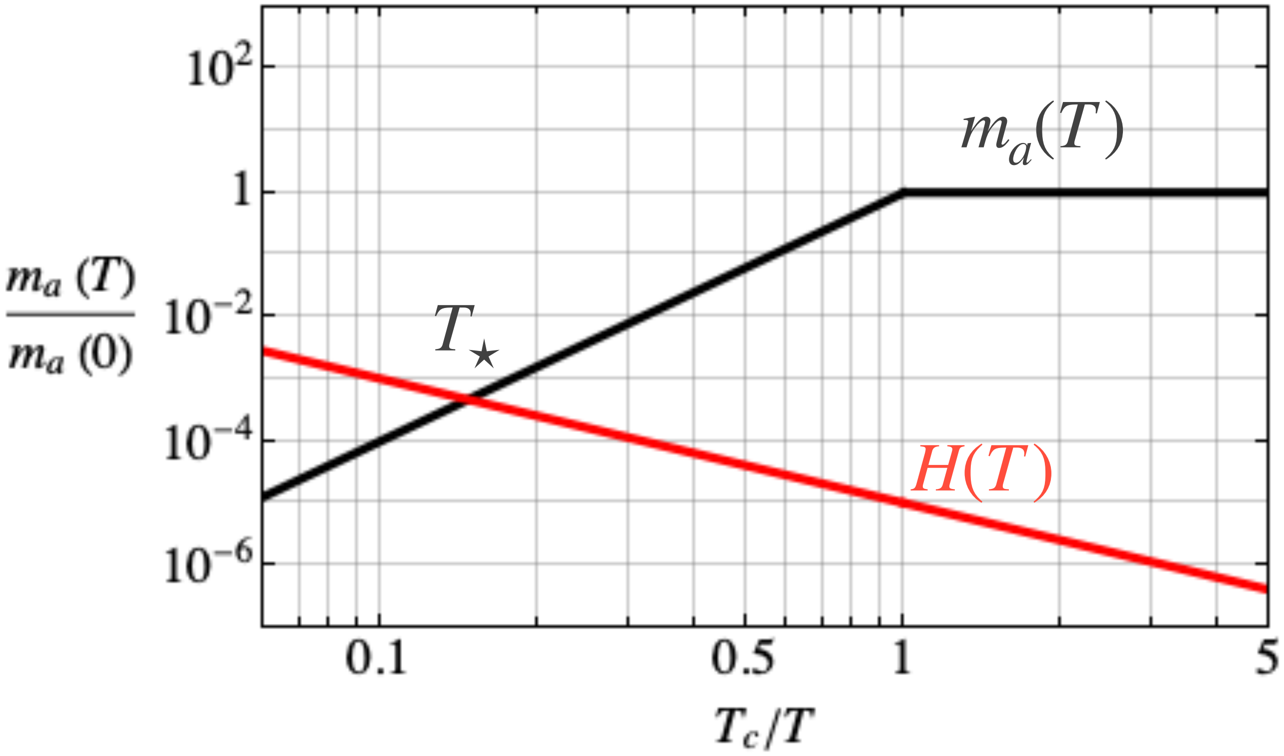
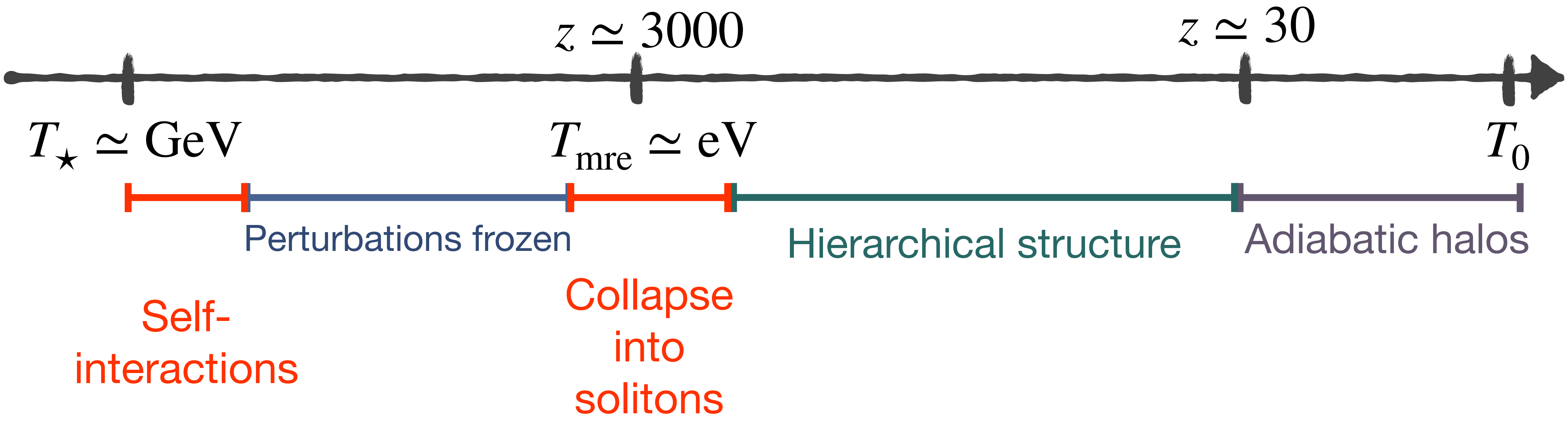
QCD axion:

$$\left.\frac{k_p}{k_J}\right|_{\rm MRE} = \frac{k_{p\star}}{6^{1/4}H_\star}\left(\frac{m_\star}{m}\right)^{1/2}$$

$$\sim 10^{-3}\,\frac{k_{p\star}}{H_\star}$$



New aspects



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T_0

Perturbations frozen

Hierarchical structure

Adiabatic halos

Self-interactions

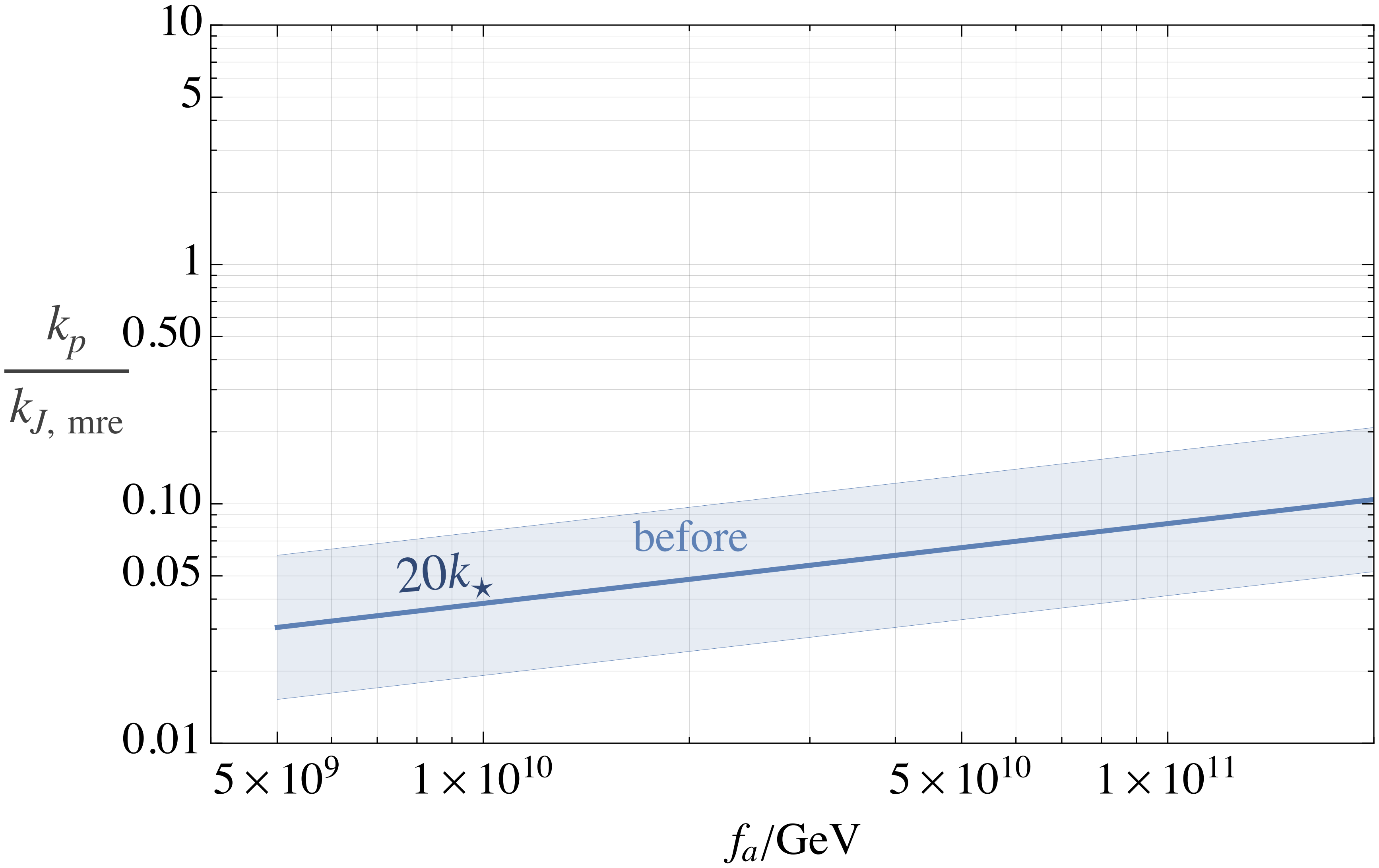
Collapse into solitons

QCD axion:

$$\left.\frac{k_p}{k_J}\right|_{\mathrm{MRE}} = \xcancel{\frac{k_{p\star}}{6^{1/4} H_\star}\left(\frac{m_\star}{m}\right)^{1/2} \sim 10^{-3}\,\frac{k_{p\star}}{H_\star}}$$

Self-interactions

$$\frac{k_\star}{k_{J,\mathrm{eq}}} \simeq 0.002 \left(\frac{f_a}{10^{10}\ \mathrm{GeV}} \right)^{1/3}$$



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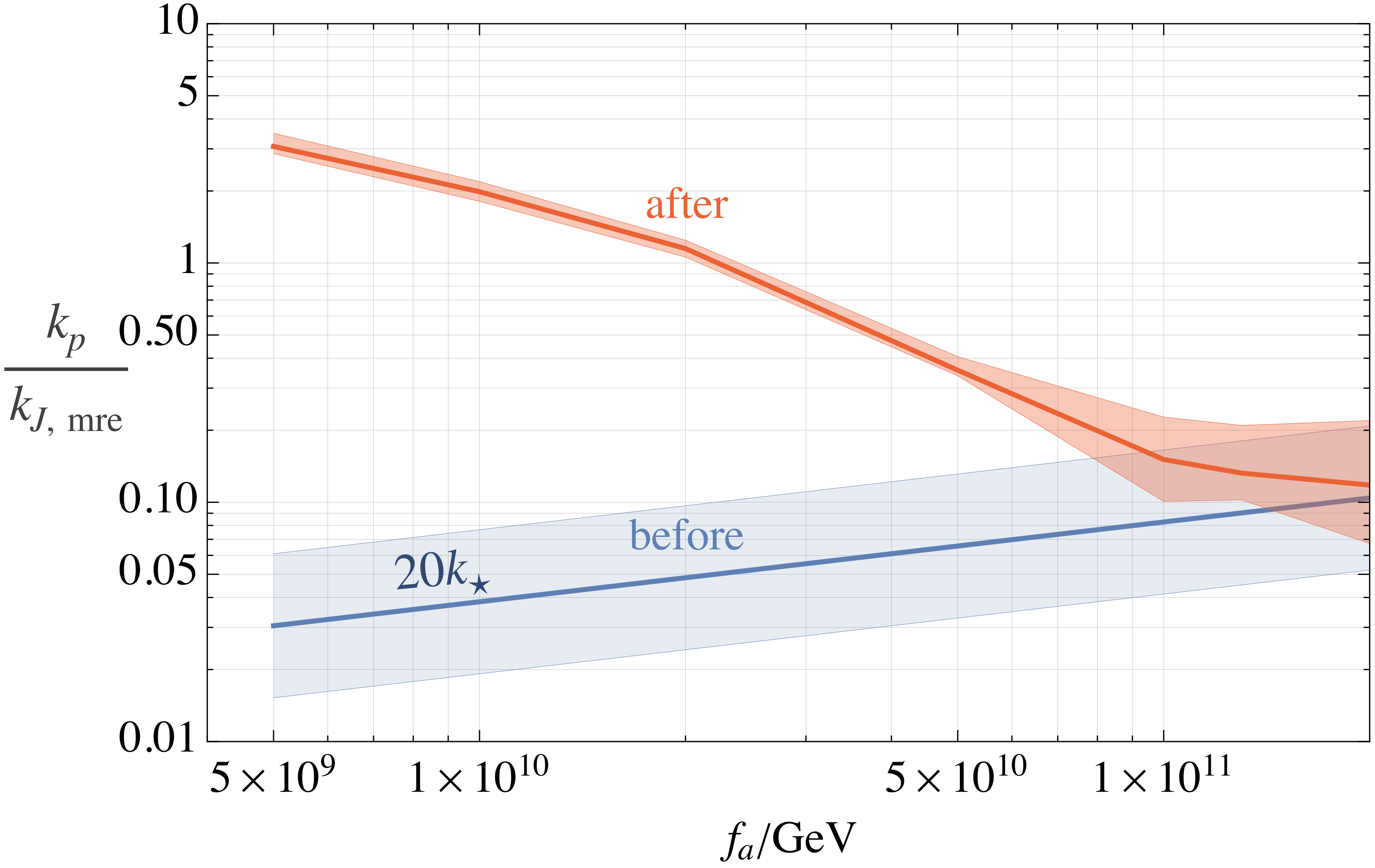

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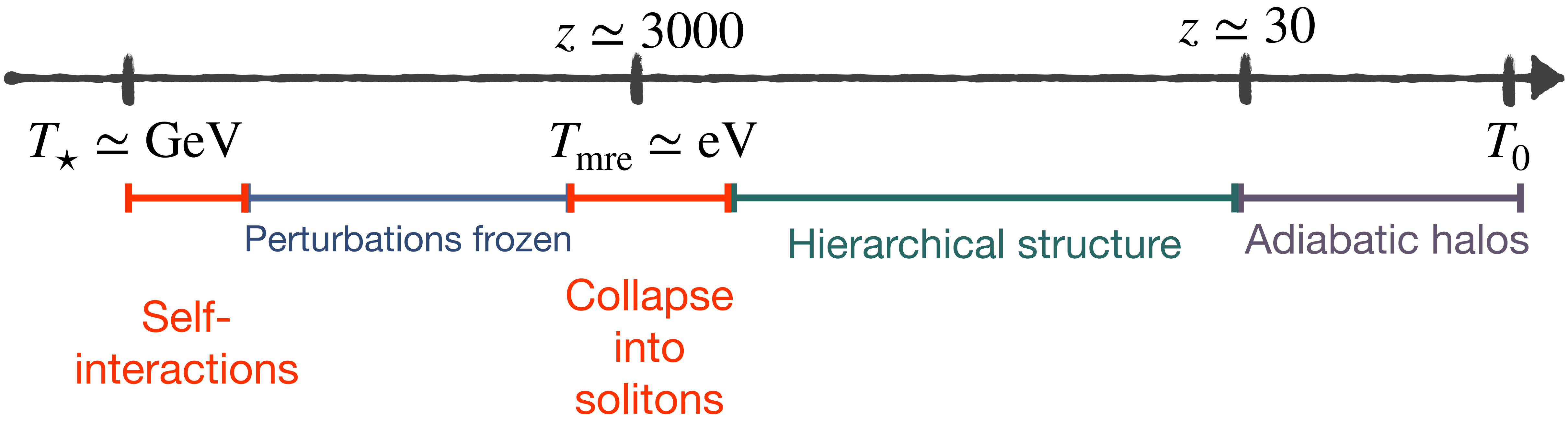
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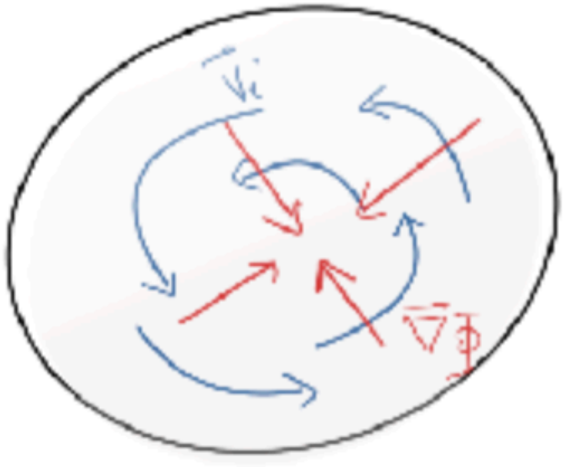
Halos vs solitons

Halos

$\Phi_Q \simeq 0$

→ gravitational potential balanced by velocity term



Angular momentum “supports” the gravitational potential

Halos vs solitons

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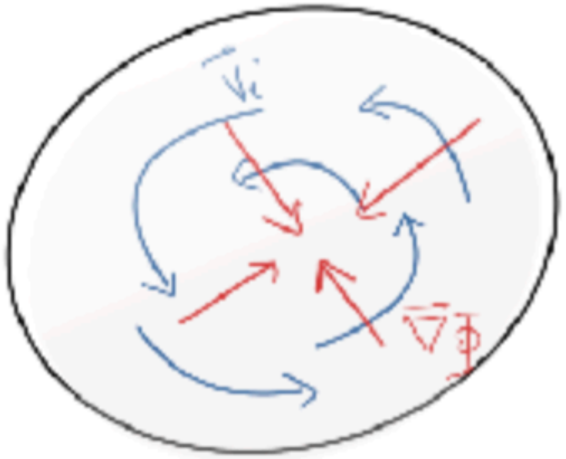

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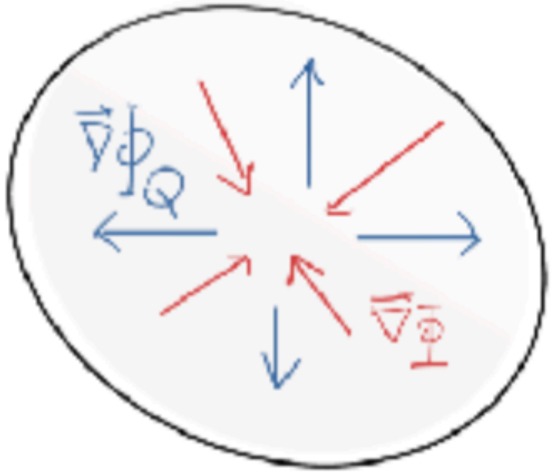
$\Phi_Q = -\Phi \qquad \vec{v} = 0$

→ gravitational potential balanced by quantum pressure *“Axion star”*



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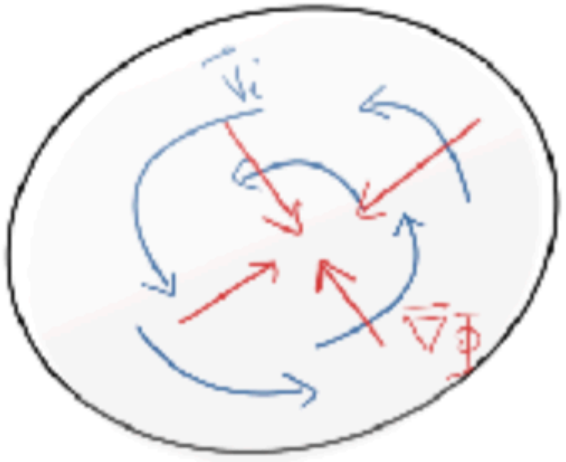

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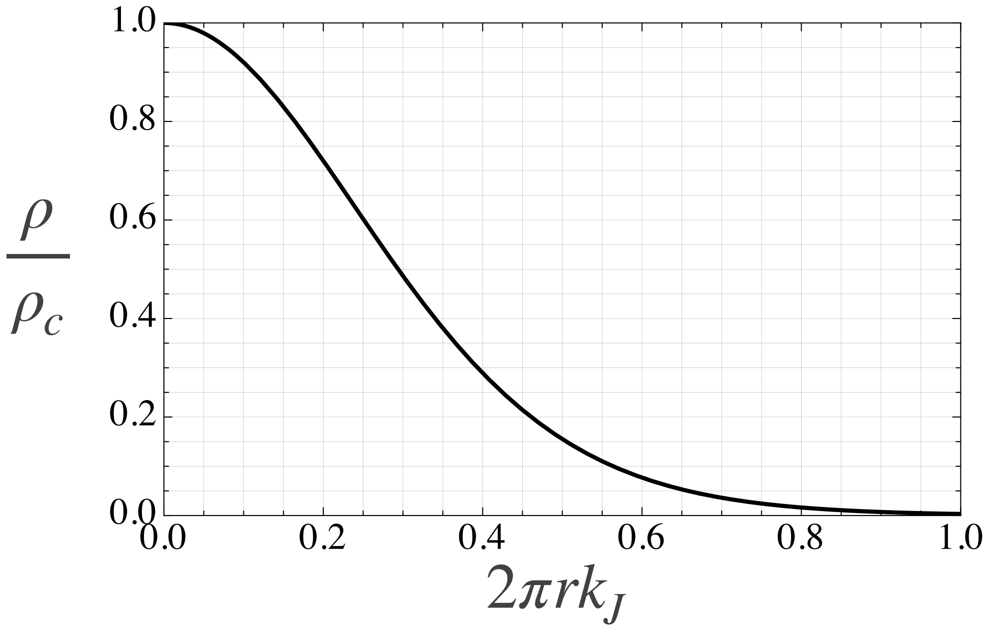

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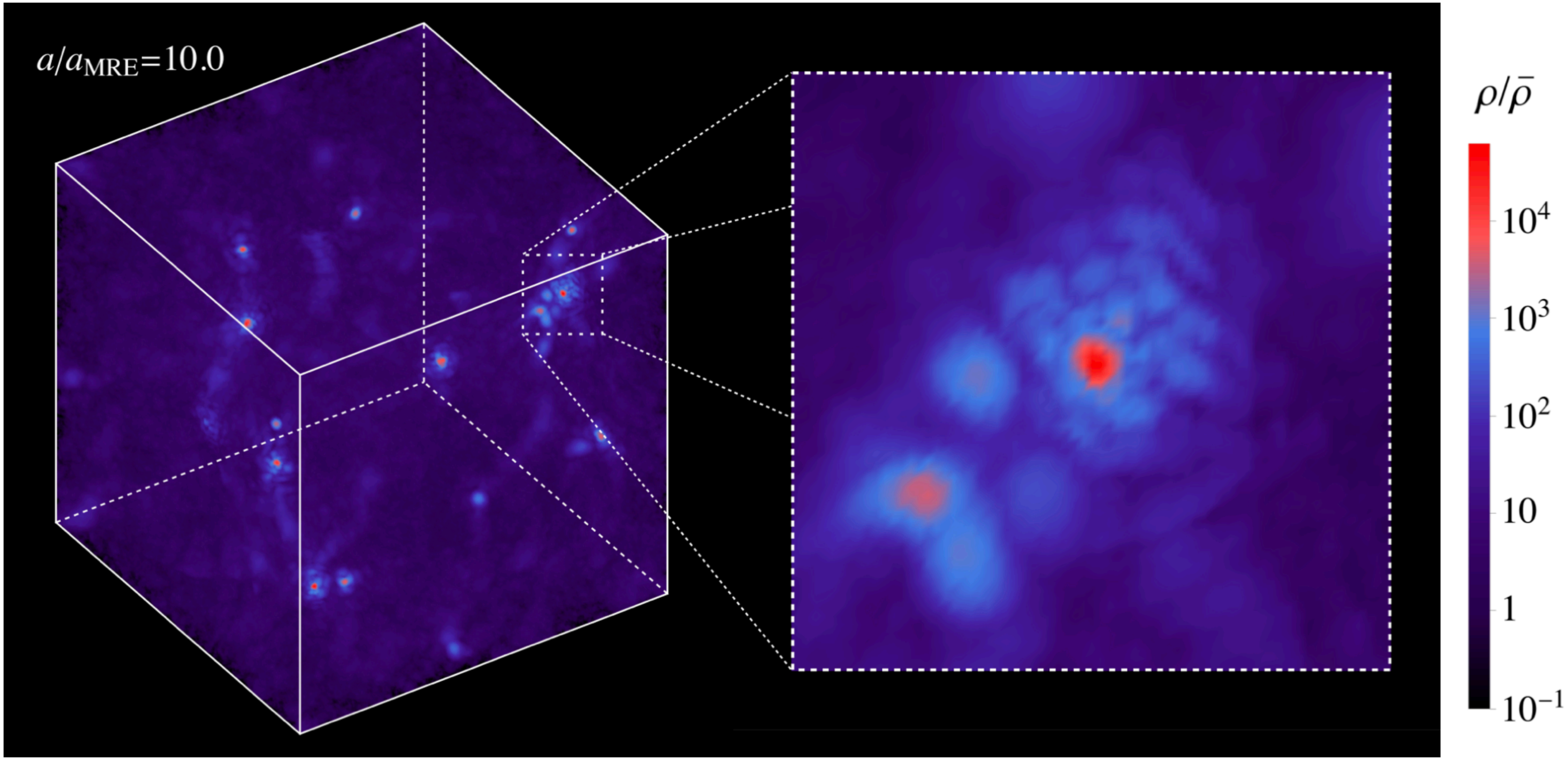
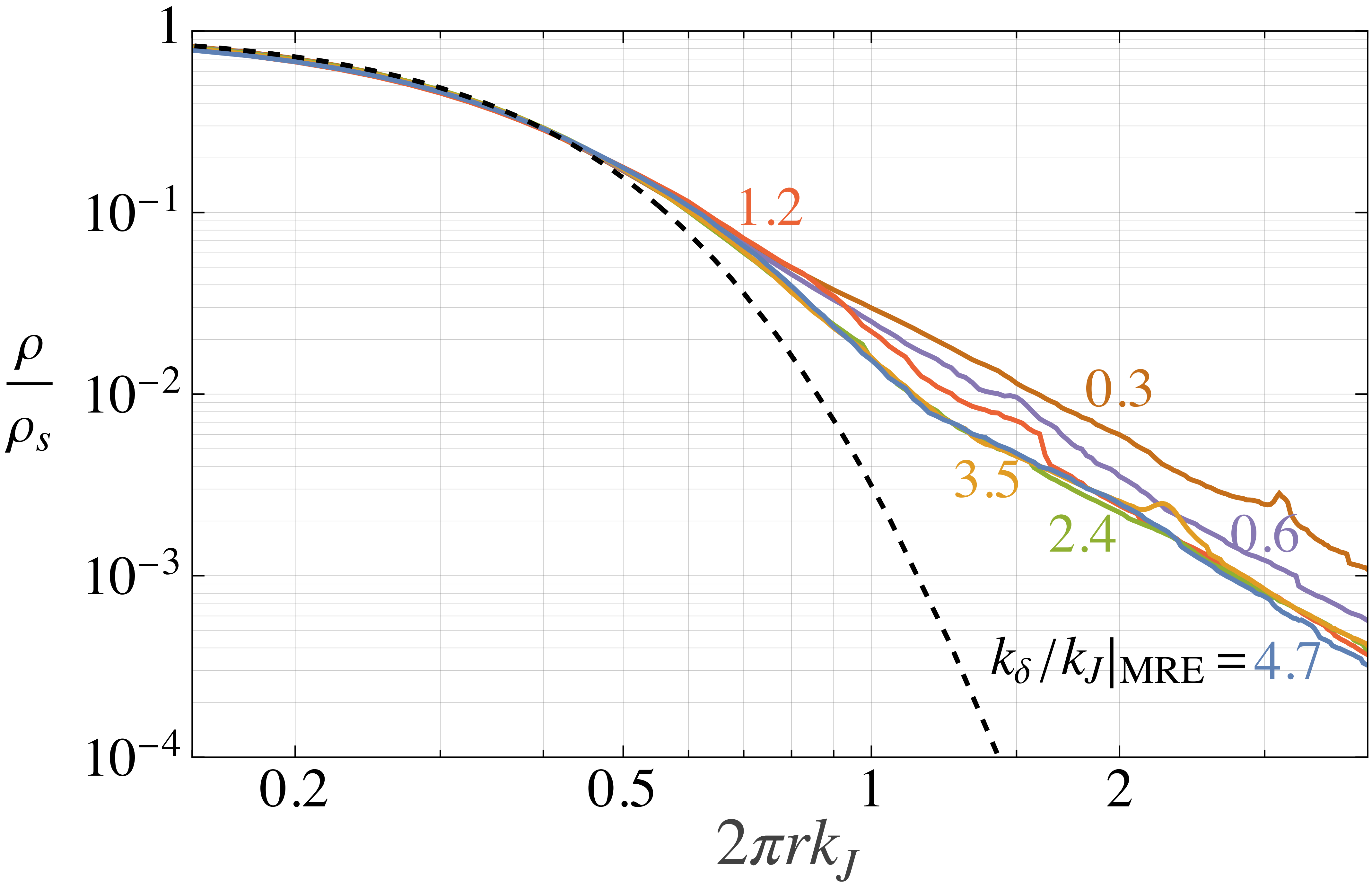
Simulations



Properties of the substructure



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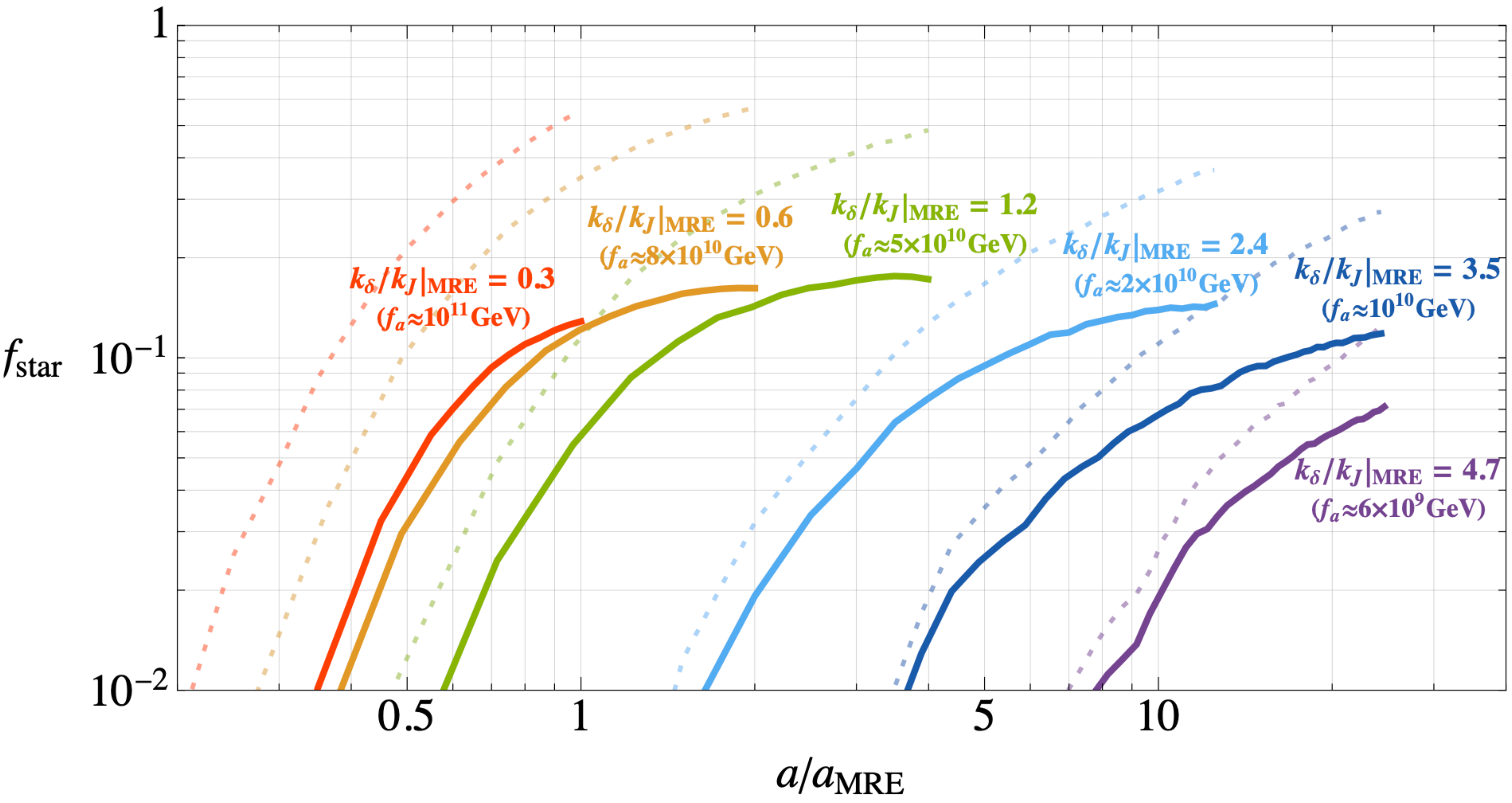

Properties of the substructure

$$\bar{M}_s \approx 2 \cdot 10^{-19}\, M_\odot \left(\frac{f_a}{10^{10}\ \mathrm{GeV}}\right)^{\frac{5}{2}}$$

$$R_{0.1} \simeq 4.2 \times 10^6\ \mathrm{km} \left(\frac{f_a}{10^{10}\ \mathrm{GeV}}\right)^2 \left(\frac{10^{-19} M_\odot}{M_s}\right)$$

$$\bar{\rho}_s \approx 0.1\ \mathrm{eV}^4 \left(\frac{f_a}{10^{10}\ \mathrm{GeV}}\right)^4$$

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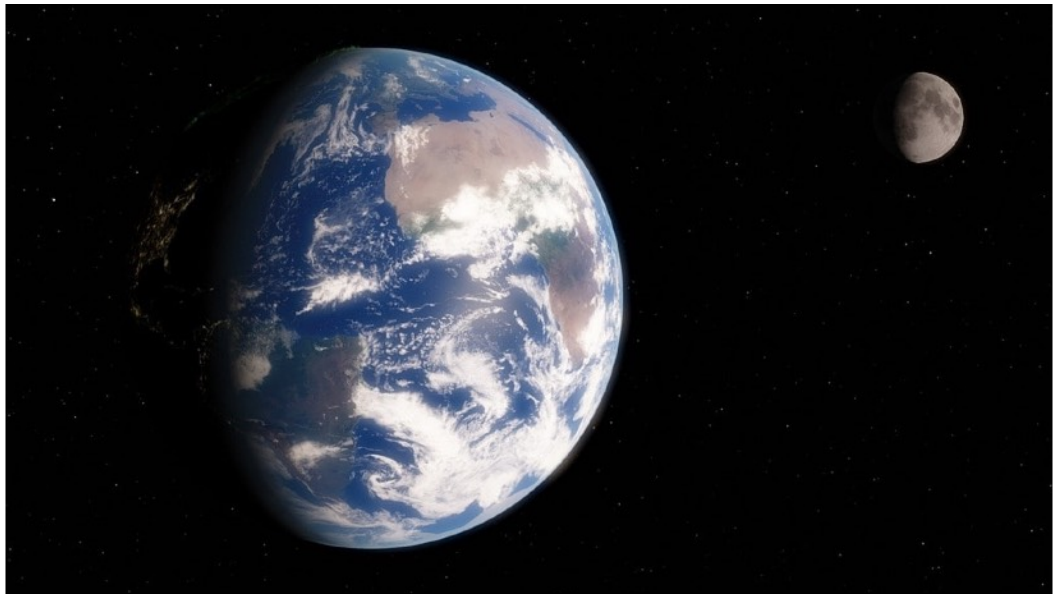


Properties of the substructure

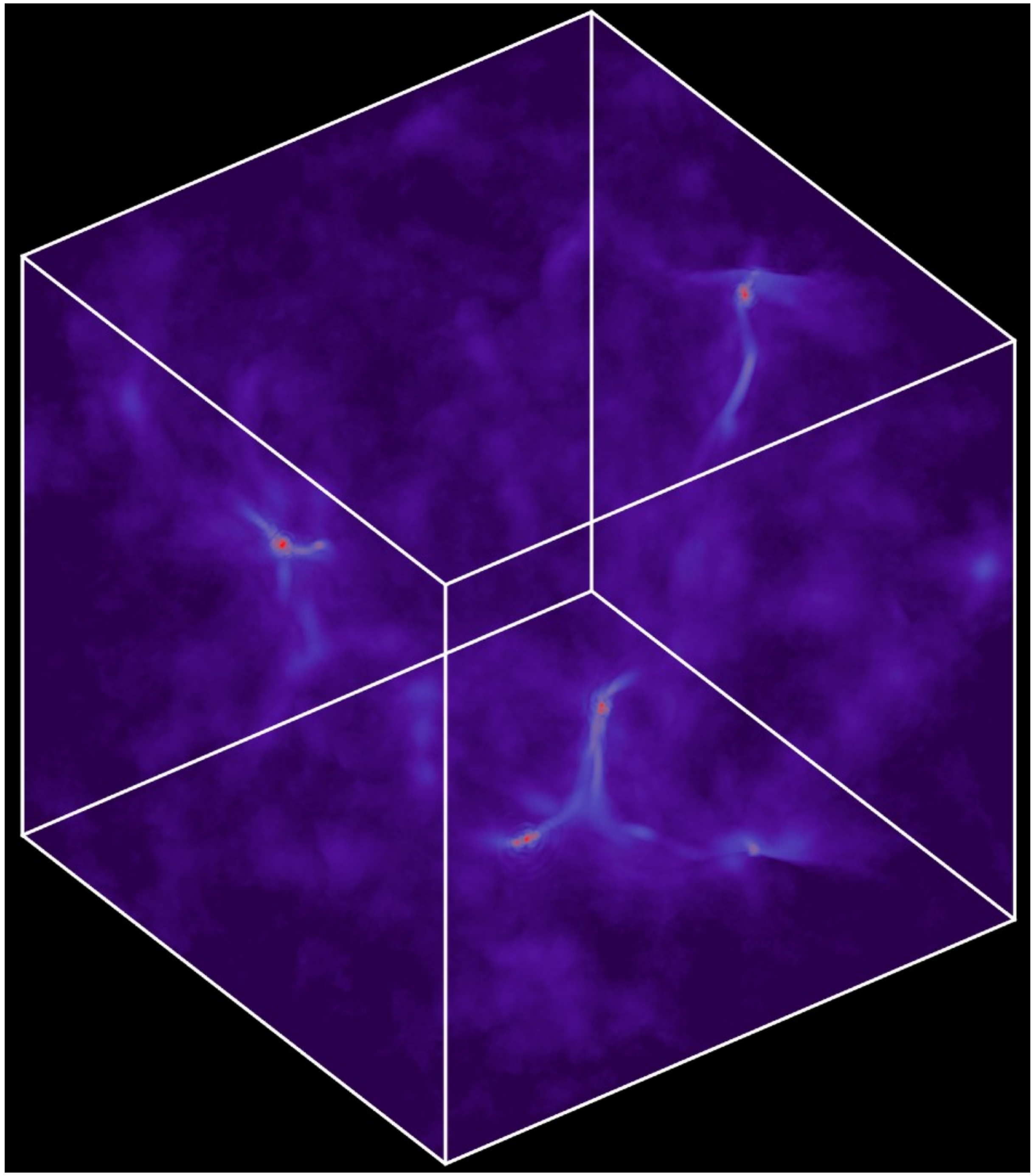
$$\tau_\oplus = 5\ \text{yrs} \left(\frac{R_{0.1}}{R}\right)^2 \left(\frac{0.1}{f_{\text{star}}}\right) \left(\frac{\bar{M}_s}{10^{-19} M_\odot}\right)^3 \left(\frac{10^{10}\ \text{GeV}}{f_a}\right)^4$$

$$\Delta t \simeq \frac{2R_{0.1}}{v_r}\sqrt{1-\frac{R^2}{R_{0.1}^2}} = 8\ \text{hrs} \left(\frac{10^{-19} M_\odot}{\bar{M}_s}\right) \left(\frac{f_a}{10^{10}\ \text{GeV}}\right)^2 \sqrt{1-\frac{R^2}{R_{0.1}^2}}$$

Factor $\gtrsim 10^6$ enhancement compared to background DM density

Summary

- Previously neglected self-interactions at $T \simeq \Lambda_{\rm QCD}$ move energy to the UV
- Fluctuations on scales $k_p \simeq k_{J,\rm MRE}$
- Structures that form around MRE are solitonic “axion stars”
- 20% of DM axions bound



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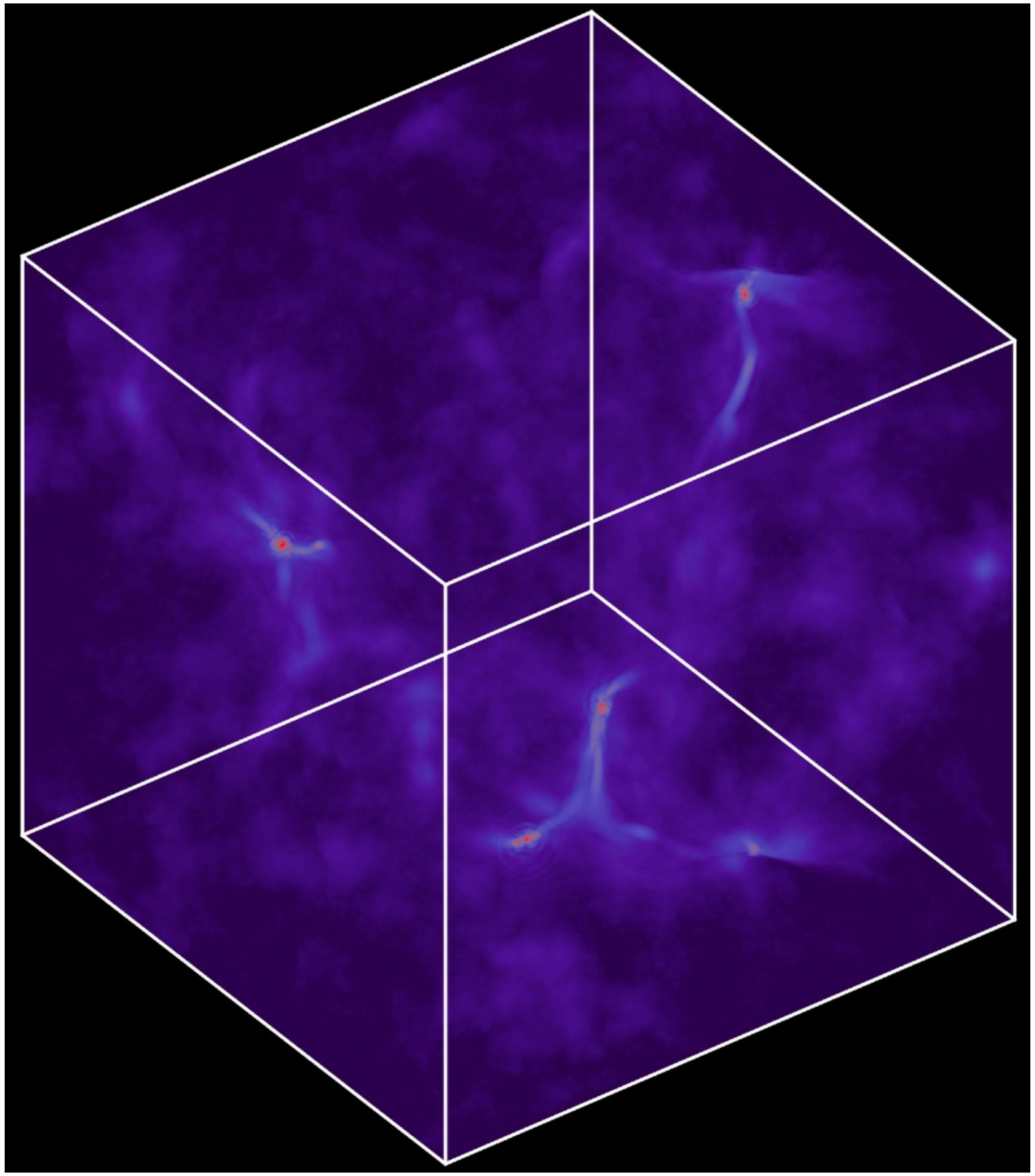


Schrödinger Poisson & QP

$$\left(i\partial_t + \frac{\nabla^2}{2m_a} - m_a\Phi\right)\psi = 0$$

$$\nabla^2\Phi = \frac{4\pi G}{R}\left(|\psi|^2 - \left\langle|\psi|^2\right\rangle\right)$$

Magdelung Transformation

$$\psi = \sqrt{\rho}e^{i\theta}$$

$$\vec{v} = \frac{1}{m_a}\nabla\theta$$

$$\longleftrightarrow$$

Continuity: $\partial_t\rho_i + 3H\rho_i + R^{-1}\nabla\cdot\left(\rho\vec{v}\right) = 0$

Euler: $\partial_t\vec{v} + H\vec{v} + R^{-1}\left(\vec{v}\cdot\nabla\right)\vec{v} = -R^{-1}\left(\nabla\Phi + \nabla\Phi_Q\right)$

Perfect fluid with
"quantum pressure": $\Phi_Q \equiv -\dfrac{\hbar^2}{2R^2m_a^2}\dfrac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}$

Quantum pressure
negligible if $\nabla\Phi \gg \nabla\Phi_Q$

$$4\pi GR^2\rho \gg \frac{1}{2R^2m_a^2}\nabla^2\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \sim \frac{1}{4R^2m_a^2}k^4$$

$$\frac{k_J}{R} = \left(16\pi G\rho m_a^2\right)^{1/4}$$

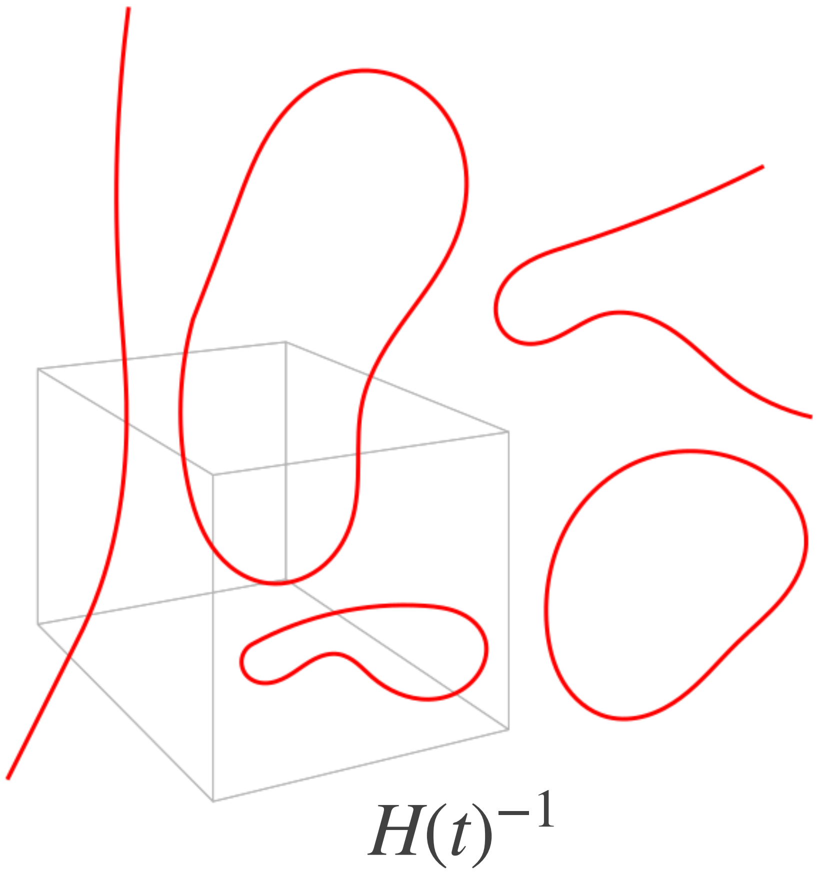
$k \ll k_J$ Overdensities dominated by Φ

→ grow and collapse

$k \gg k_J$ Overdensities dominated by Φ_Q

→ prevented from collapsing and oscillate

Scaling



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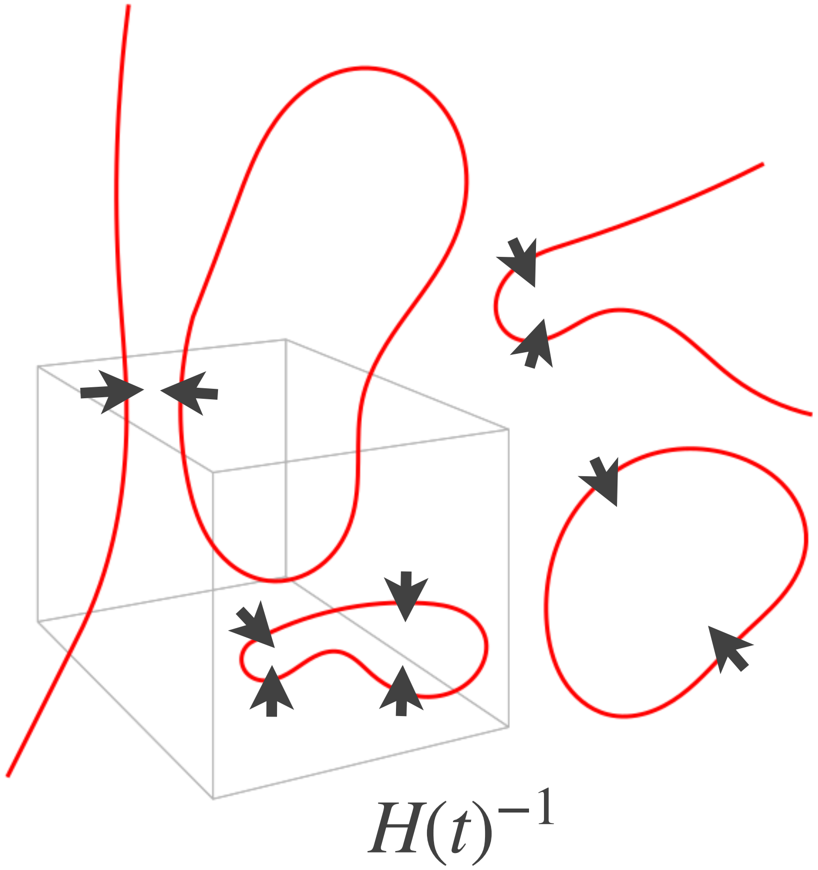

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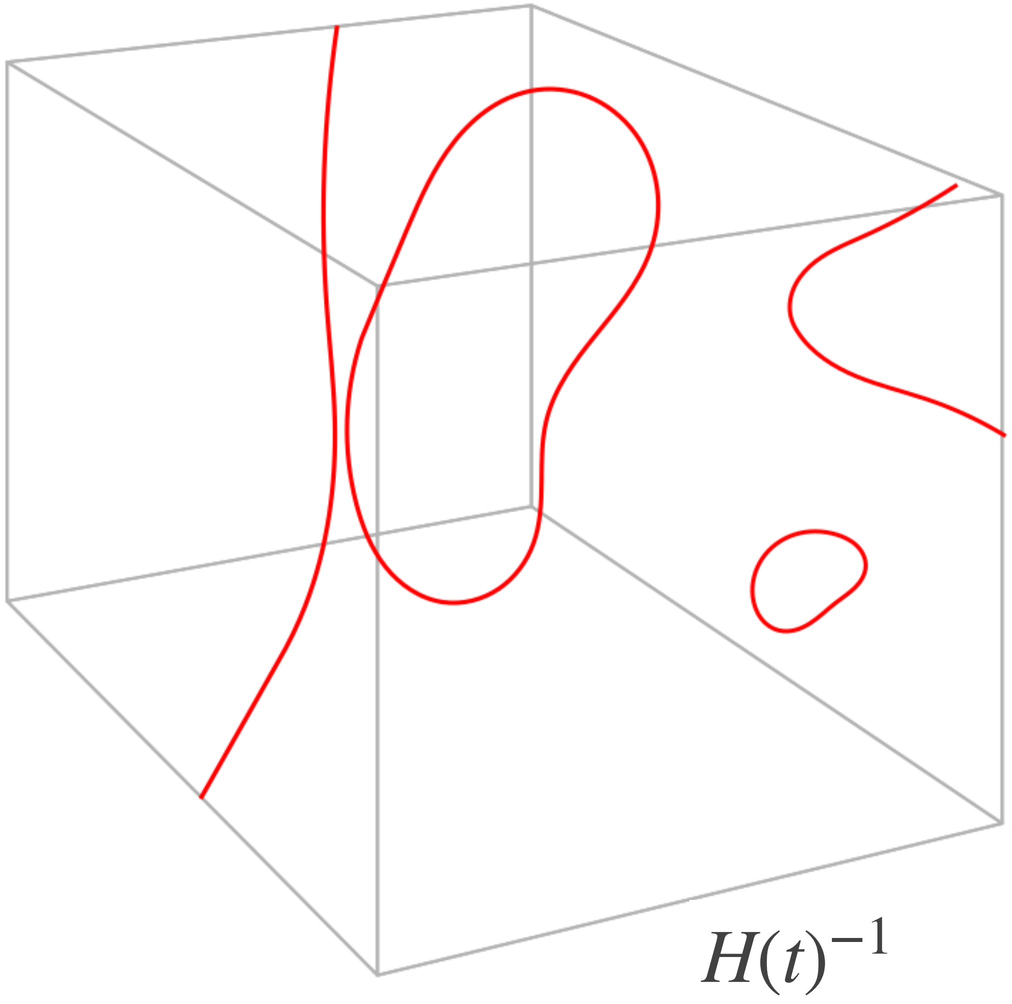

$\xi(t) =$ Length of string in Hubble lengths per Hubble volume

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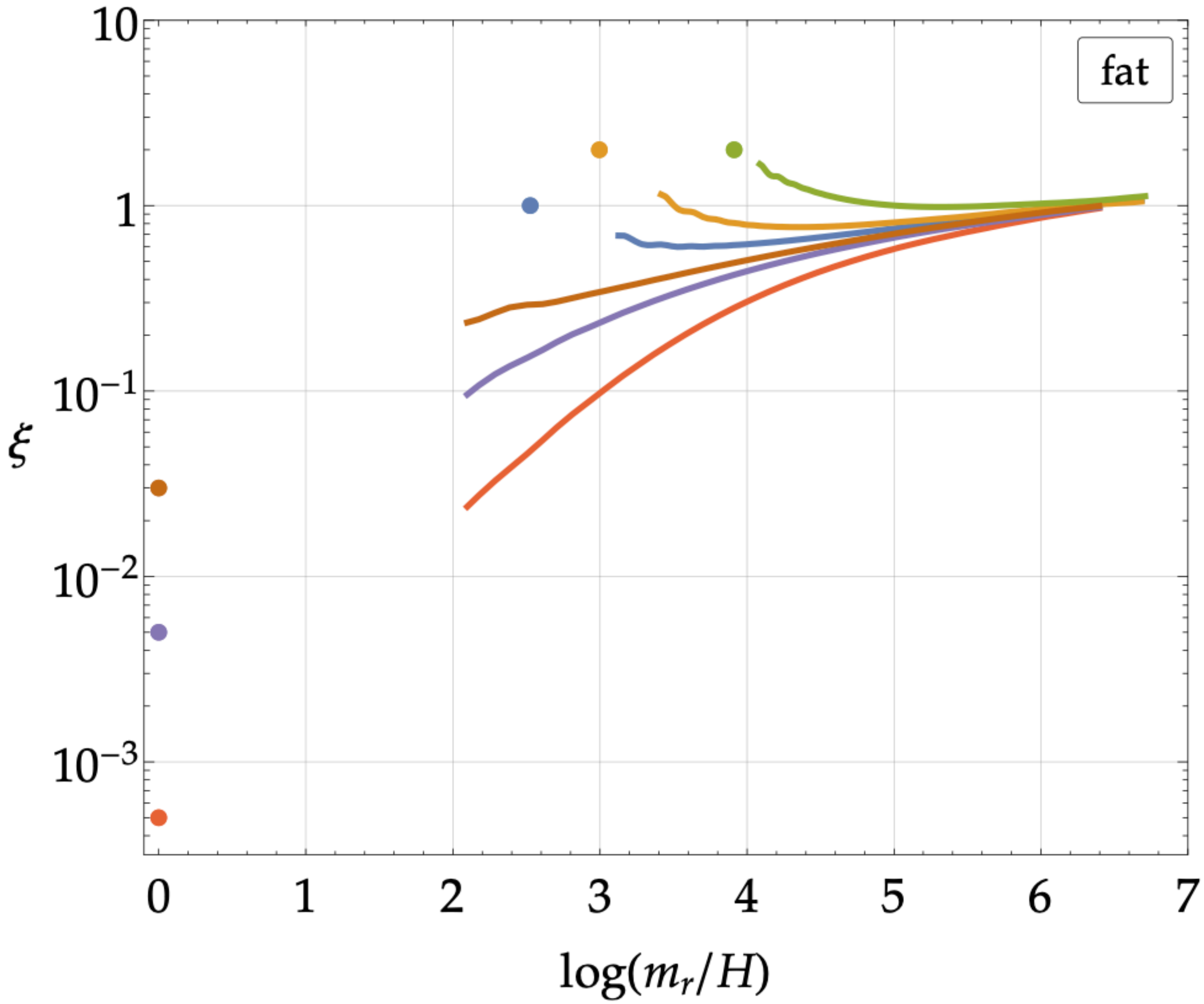

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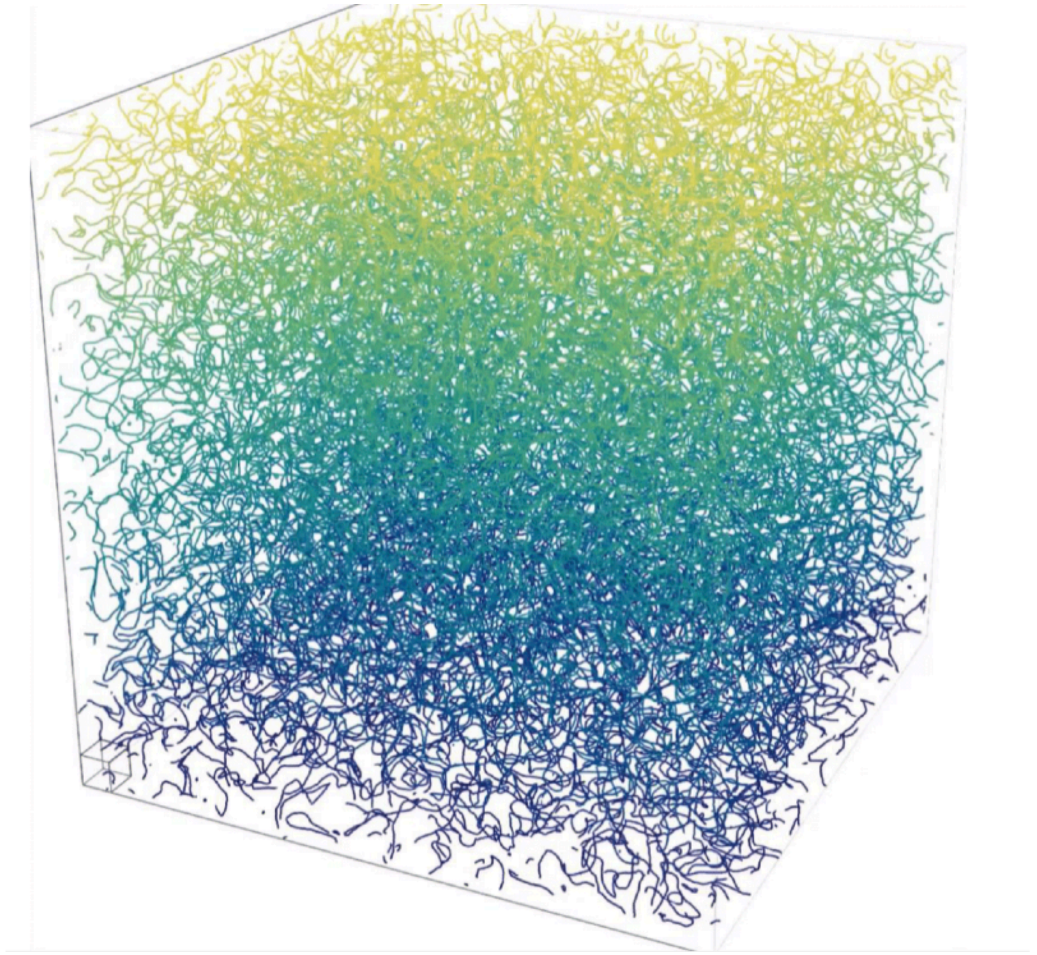
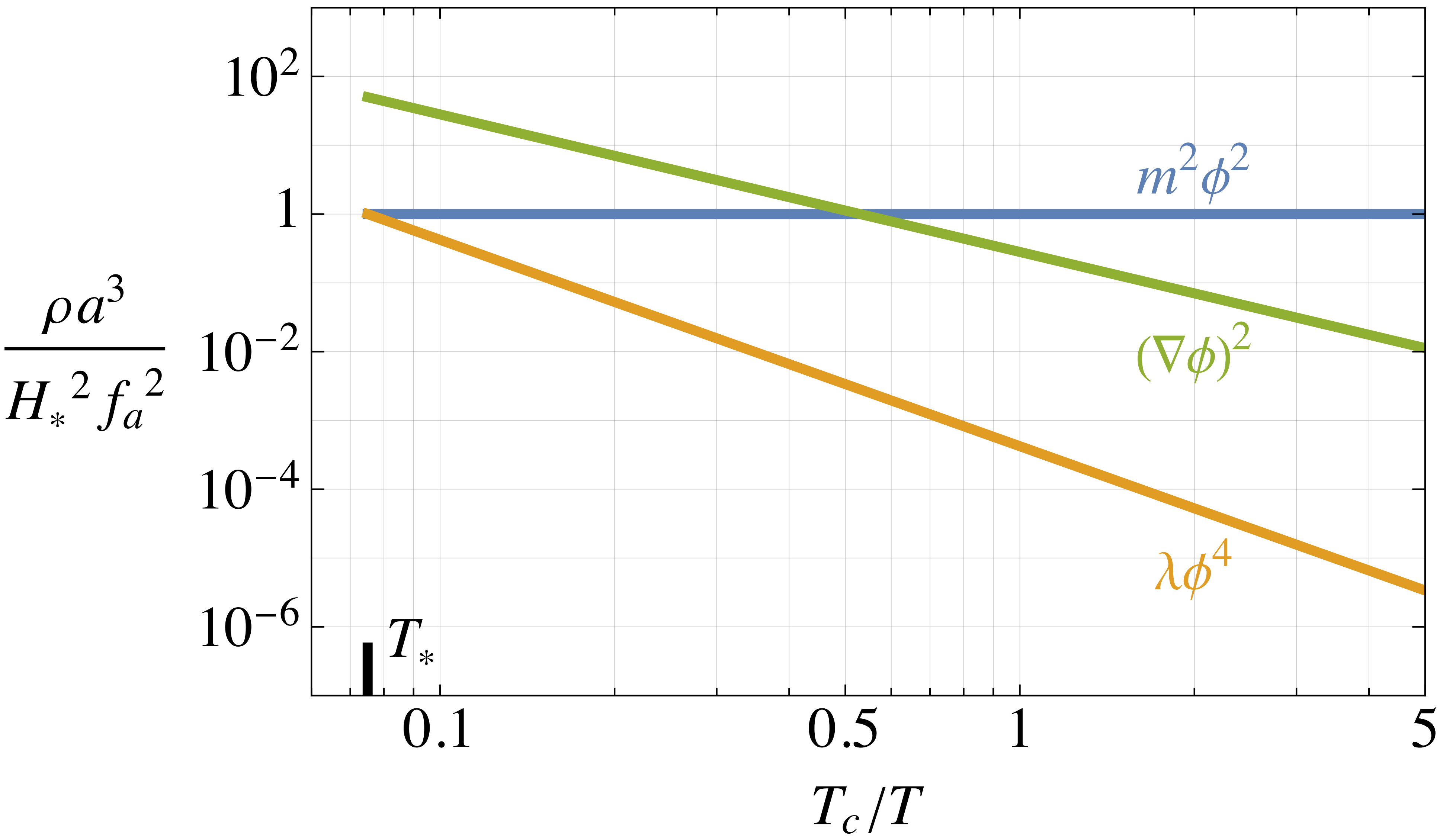


Self-interactions

ALP:



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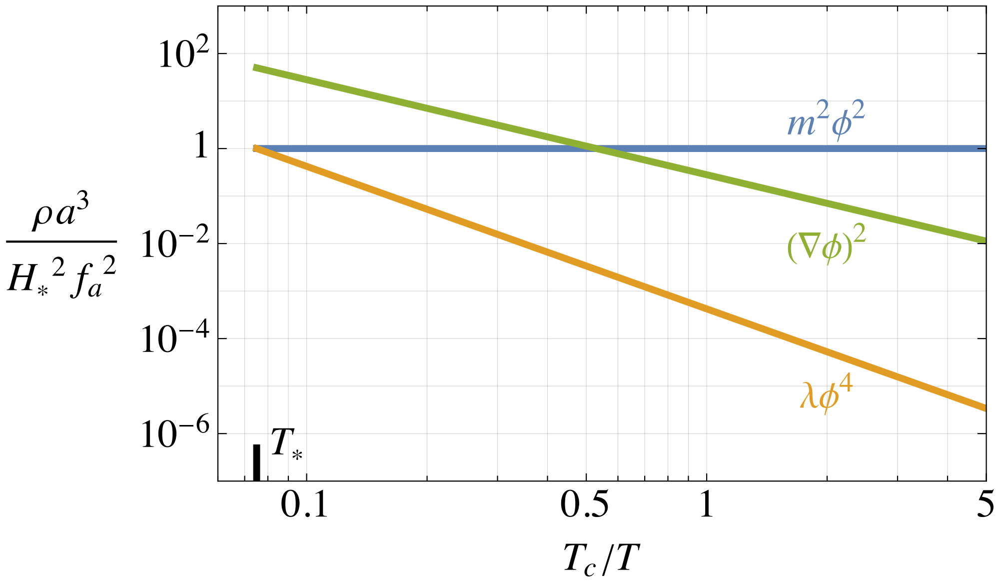
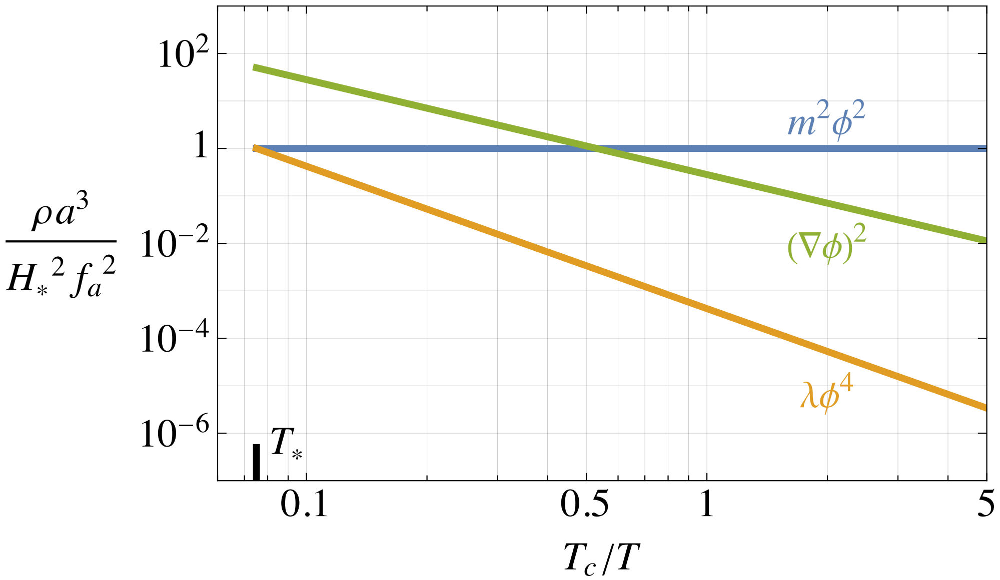


QCD axion:

Self-interactions

ALP:



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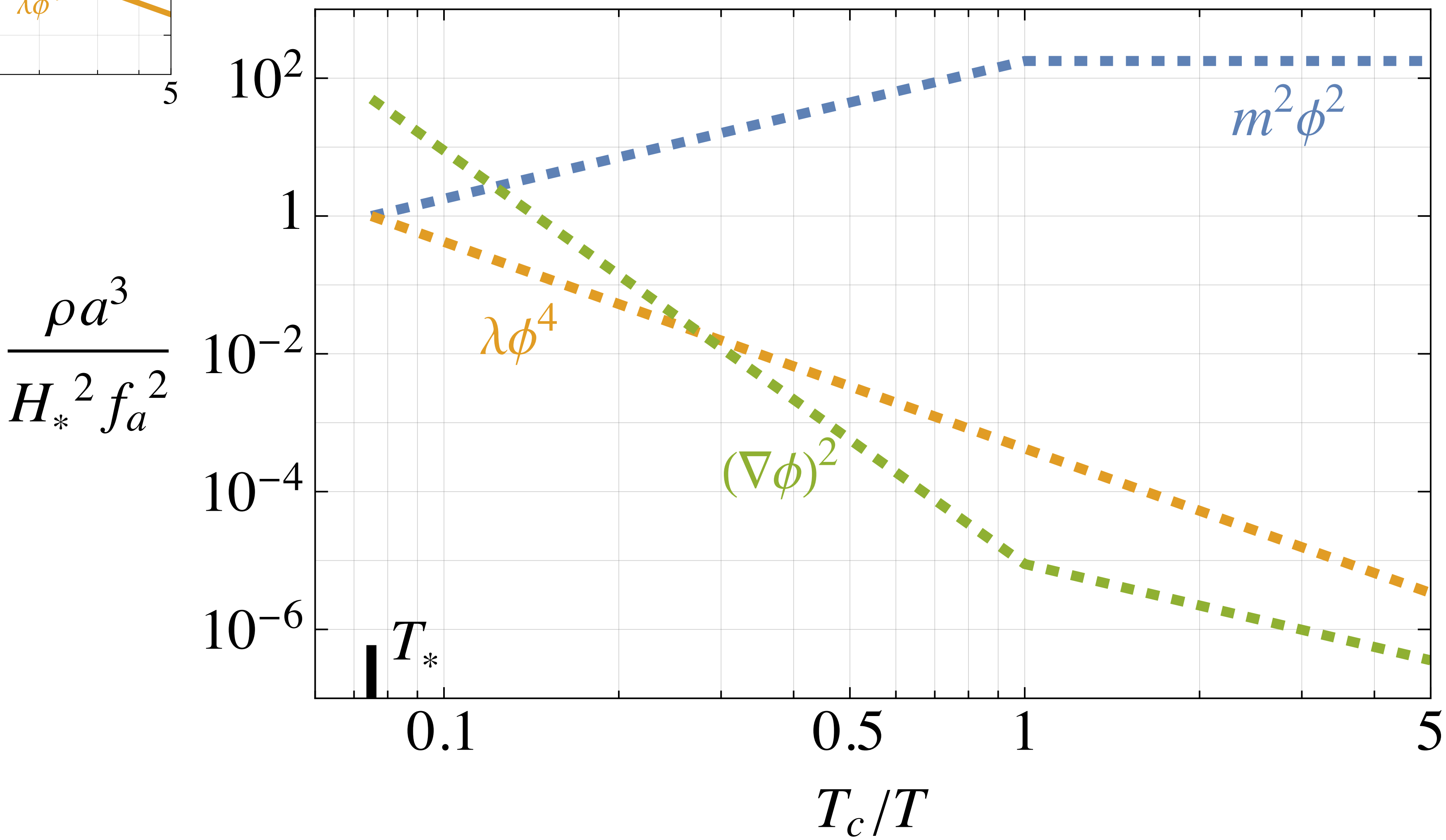

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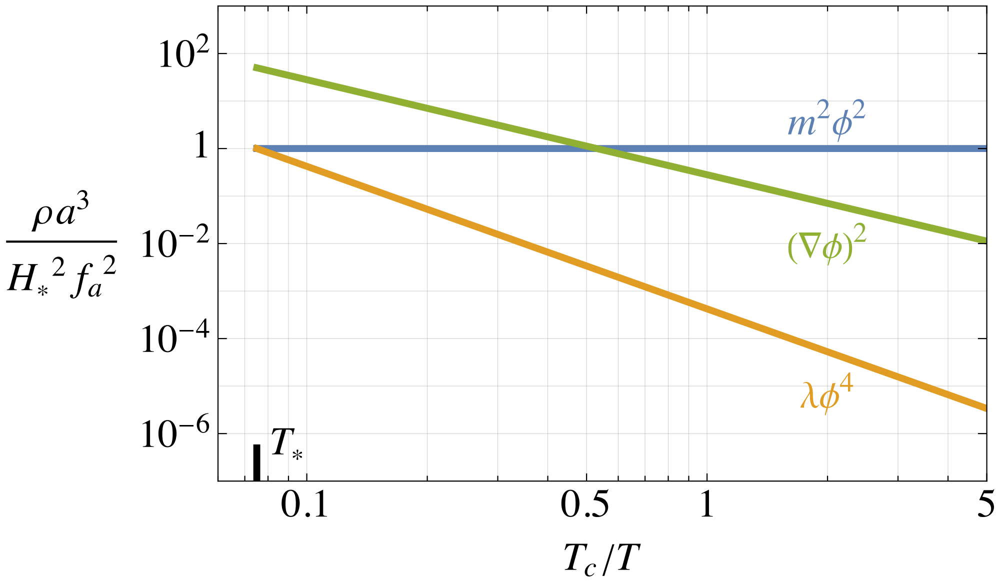

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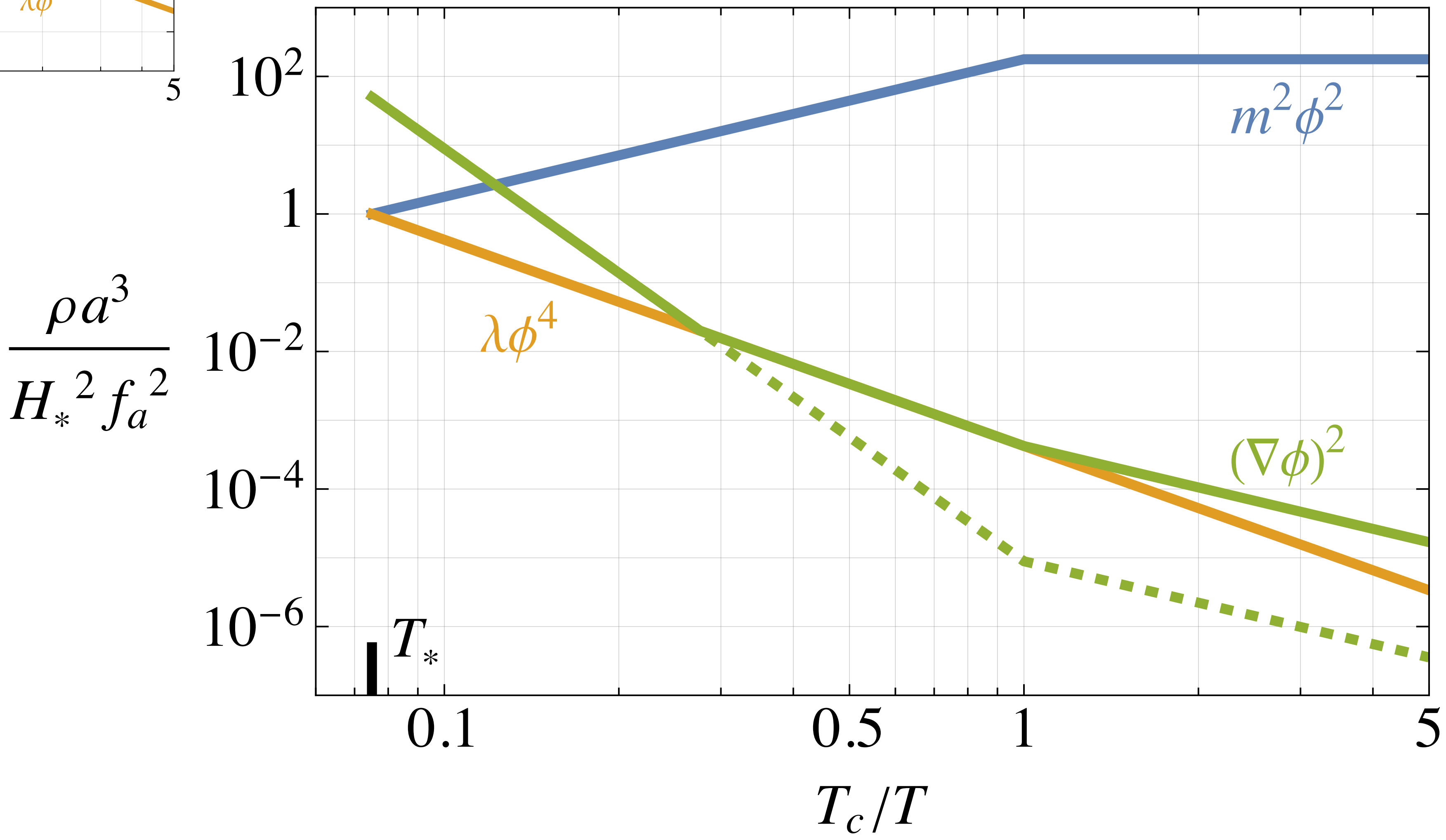

Self-interactions

$$\mathscr{L} = \frac{1}{2}(\partial a)^2 - V(a), \quad \text{with} \quad V(a) = \frac{1}{2}m_a^2 a^2 - \frac{\lambda}{4!}a^4 \ , \qquad \lambda \simeq \frac{m_a^2}{f_a^2} \ll 1$$

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$$a = \frac{1}{\sqrt{2m_a^2 R^3}}\left(\psi e^{-im_a t} + c.c.\right)$$

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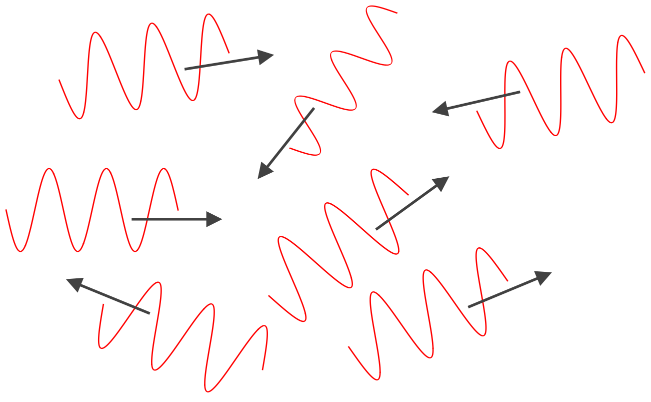

The perturbative regime $k^2 \gtrsim k_c^2$

$$t_{\rm rel} = 64 \frac{m_a^5 k_p^2}{\lambda^2 \rho^2}$$

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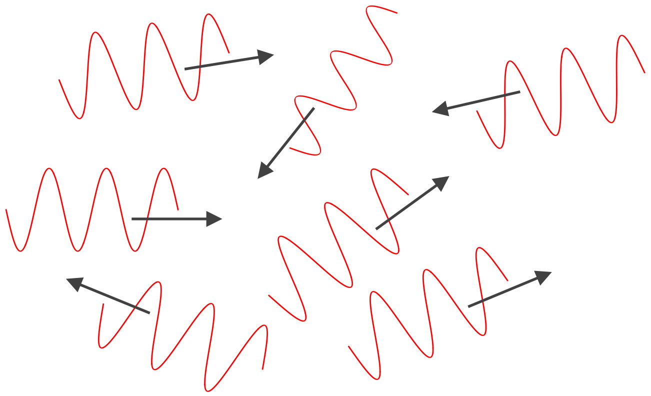

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The non-perturbative regime $k^2 \lesssim k_c^2$?

$$t_c \equiv \frac{8m_a^3}{\lambda\bar{\rho}_a}$$

Flat space

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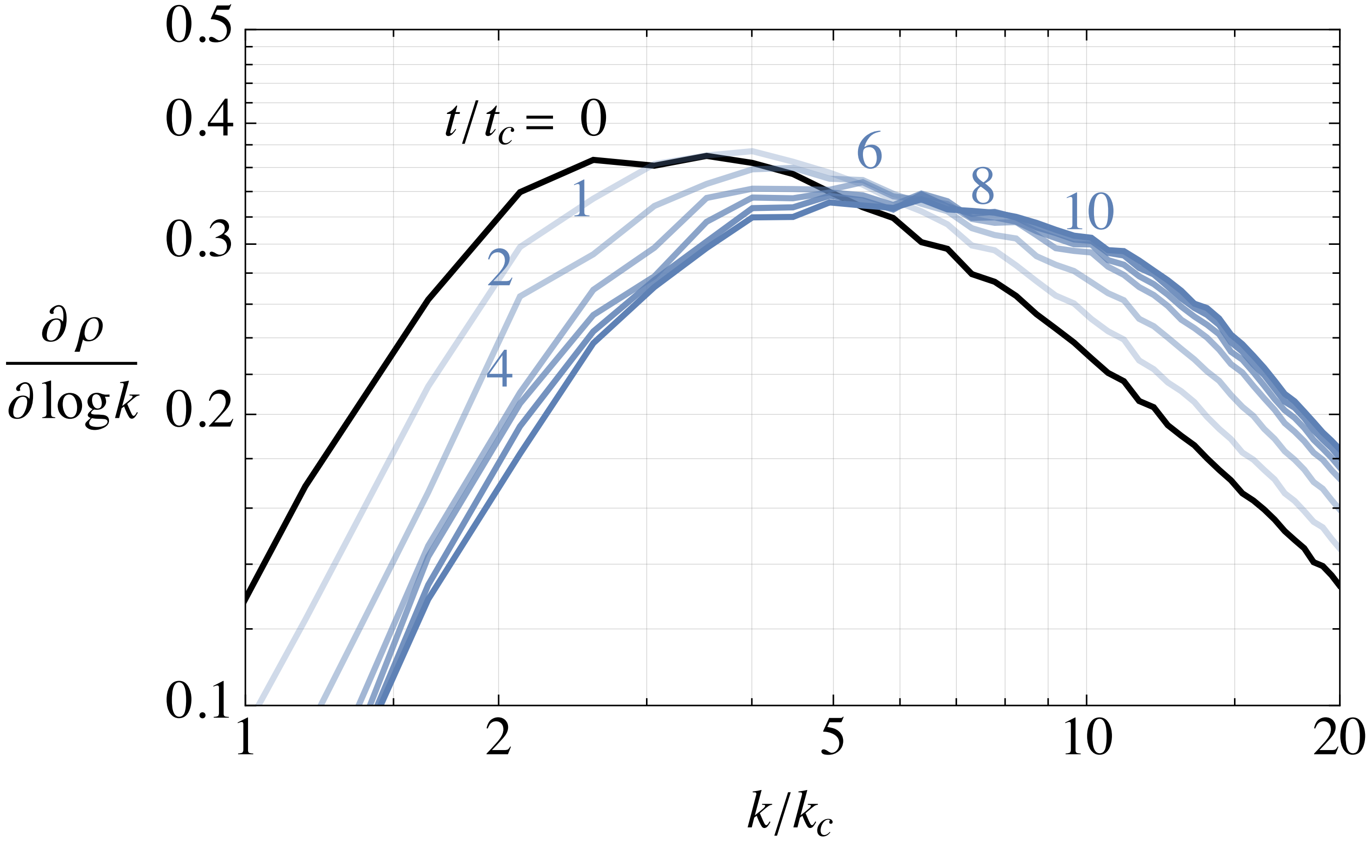
Perturbative regime $k^2 \lesssim k_c^2$



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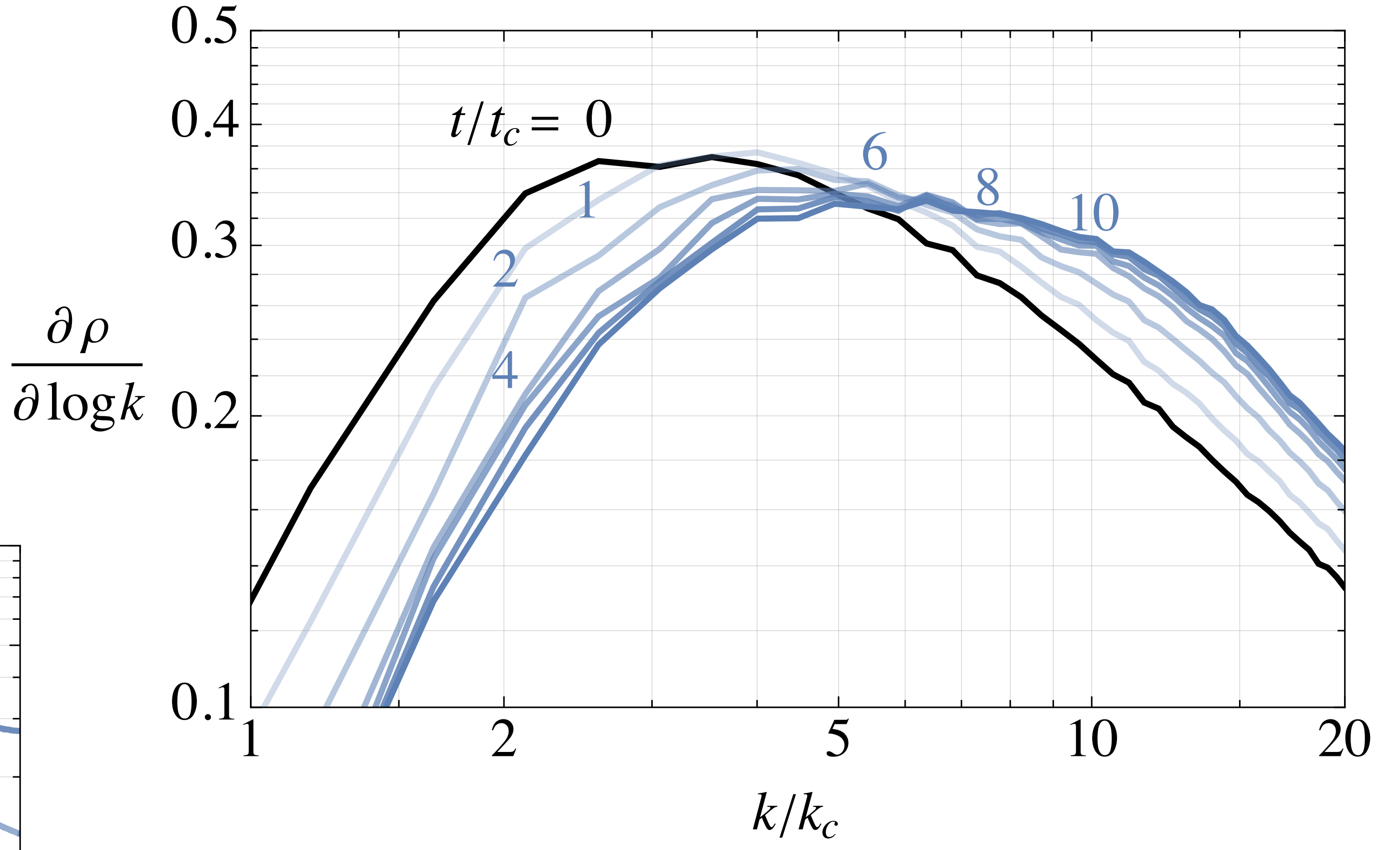

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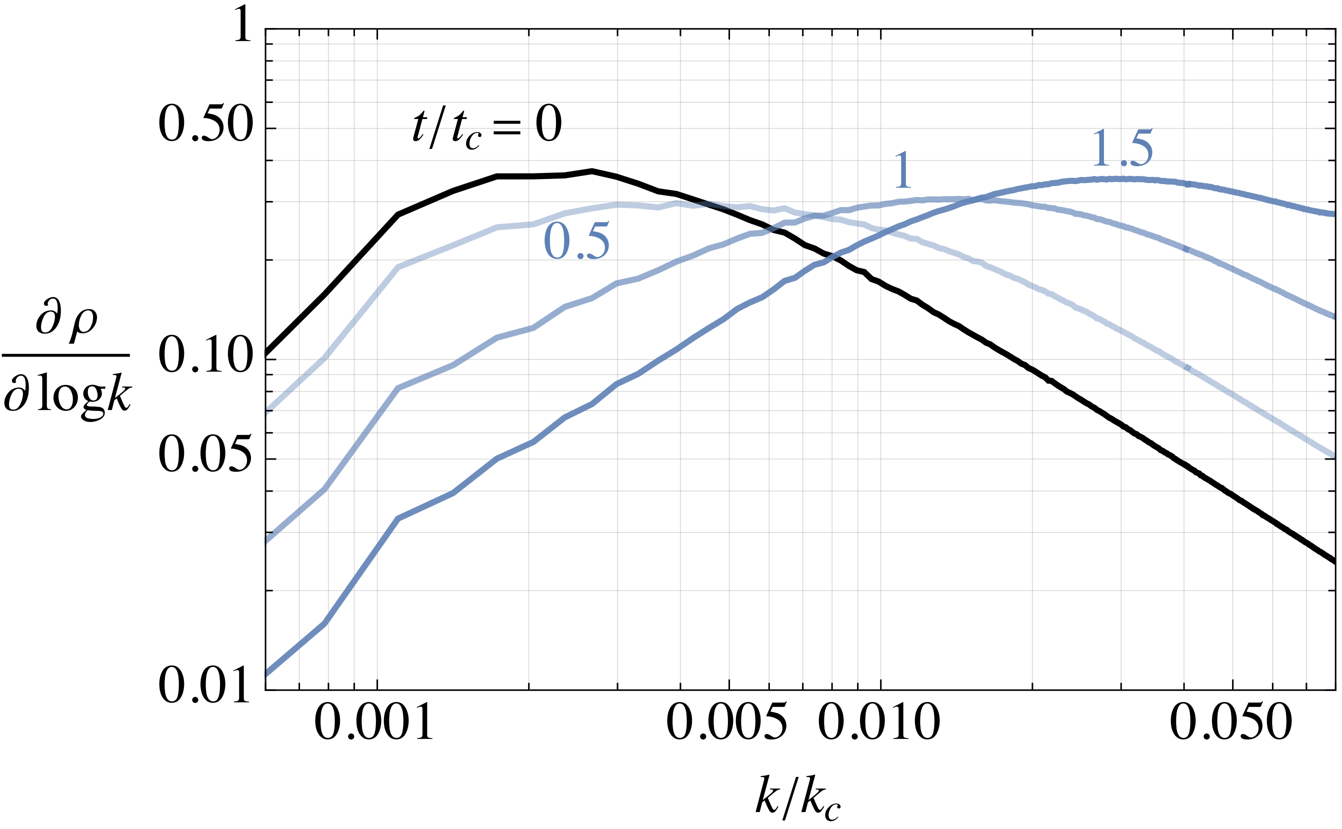
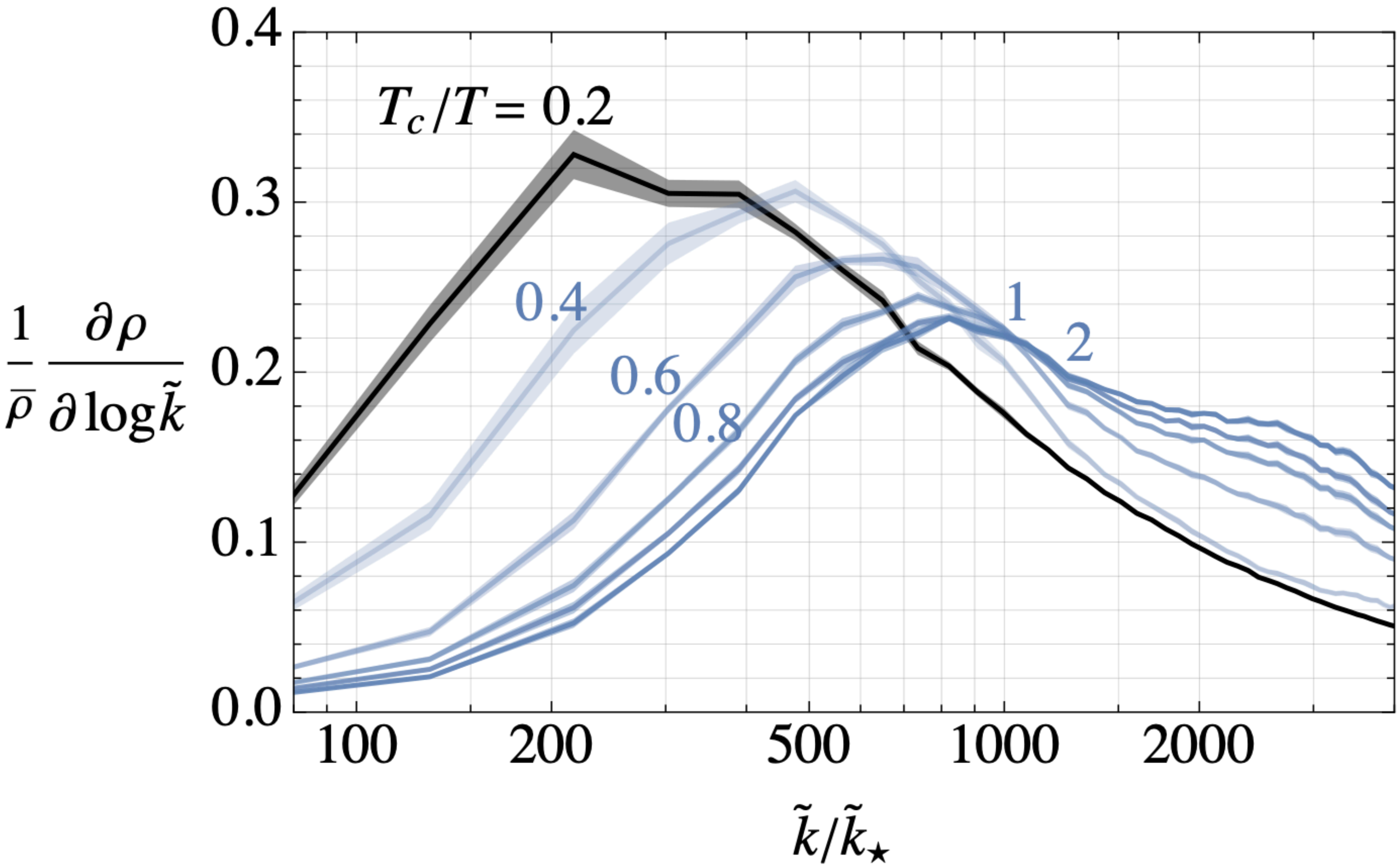


$$t_{\rm rel} \gg t_c$$

Results from simulations

$f_a = 10^{10}$ GeV



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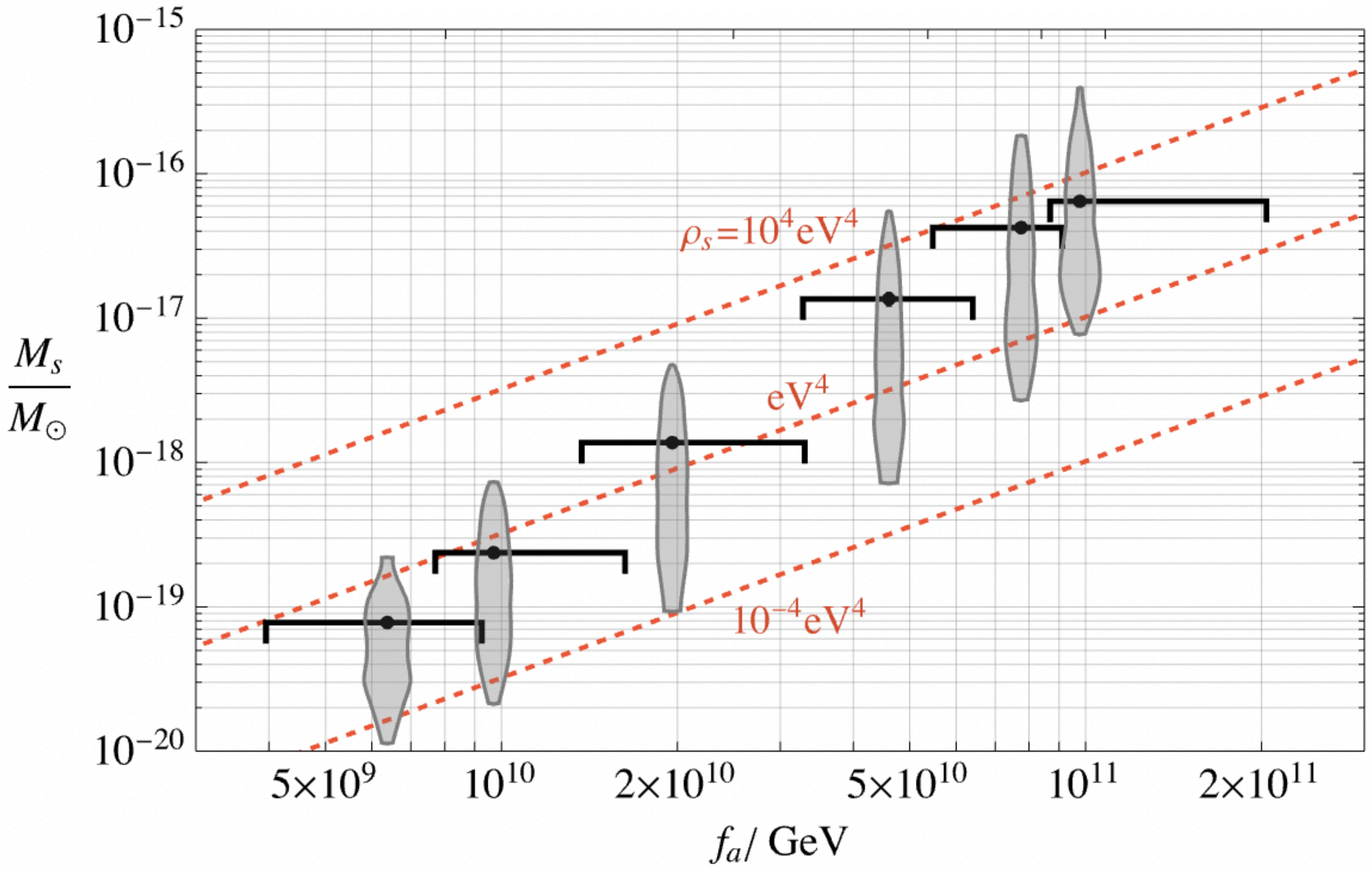

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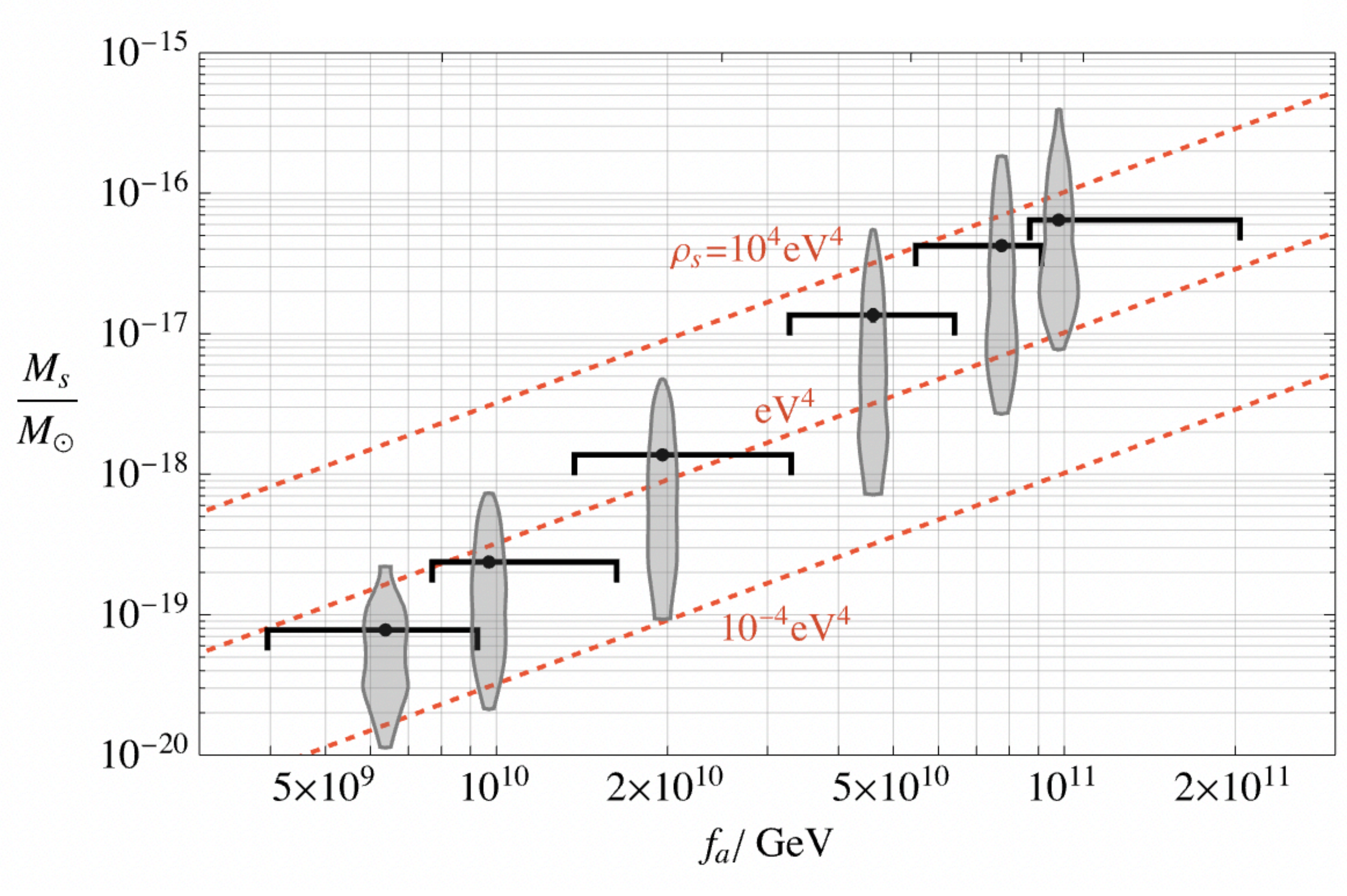

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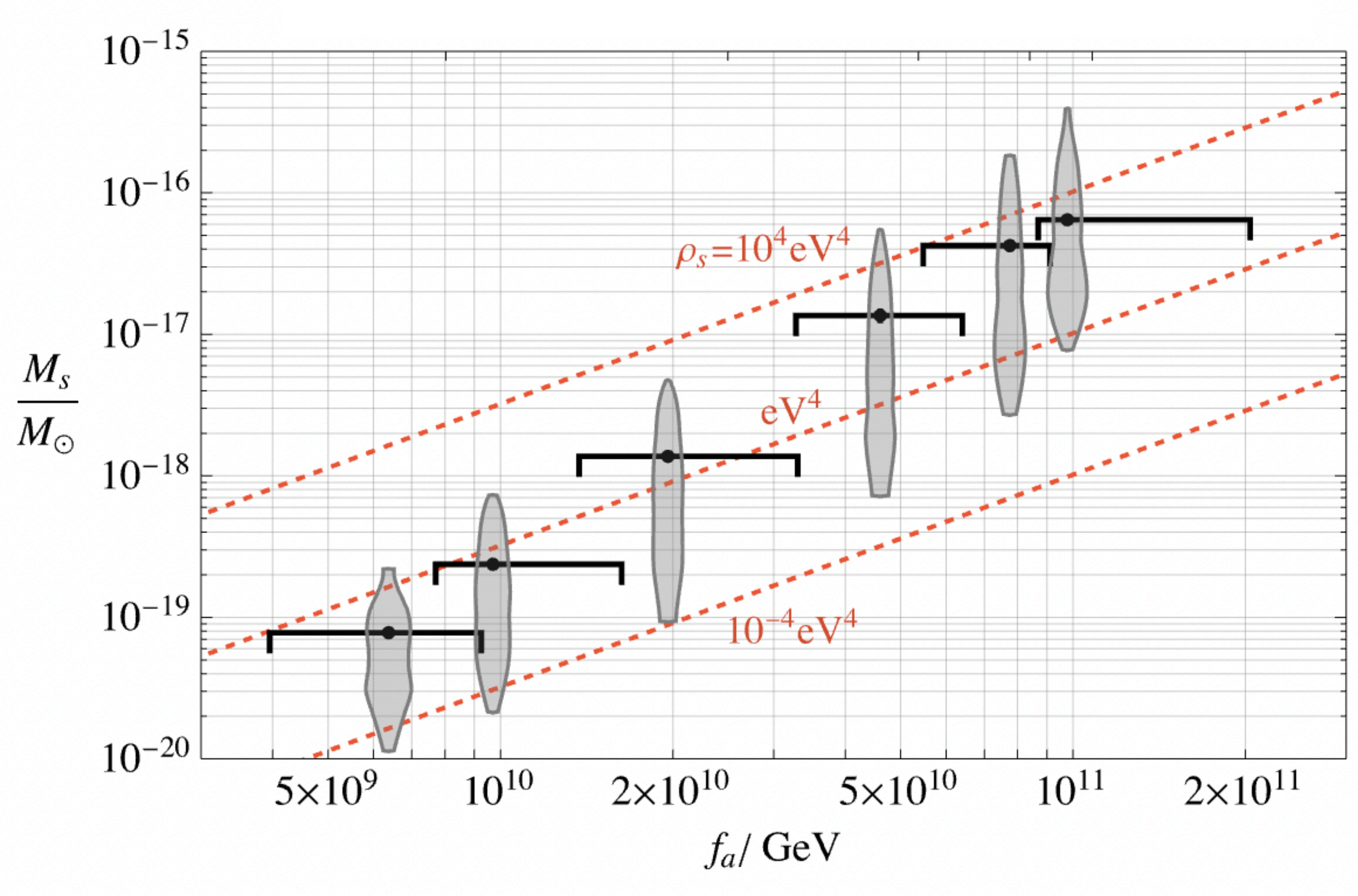

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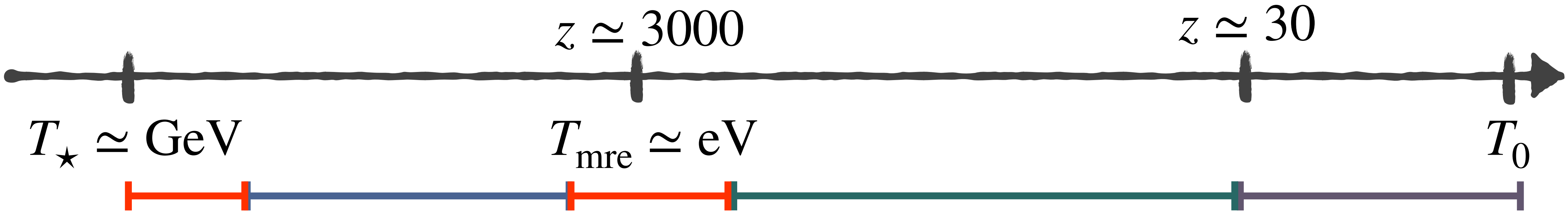
Subsequent evolution?



$z \simeq 3000$

$z \simeq 30$

$T_\star \simeq \text{GeV}$

$T_{\text{mre}} \simeq \text{eV}$

T_0

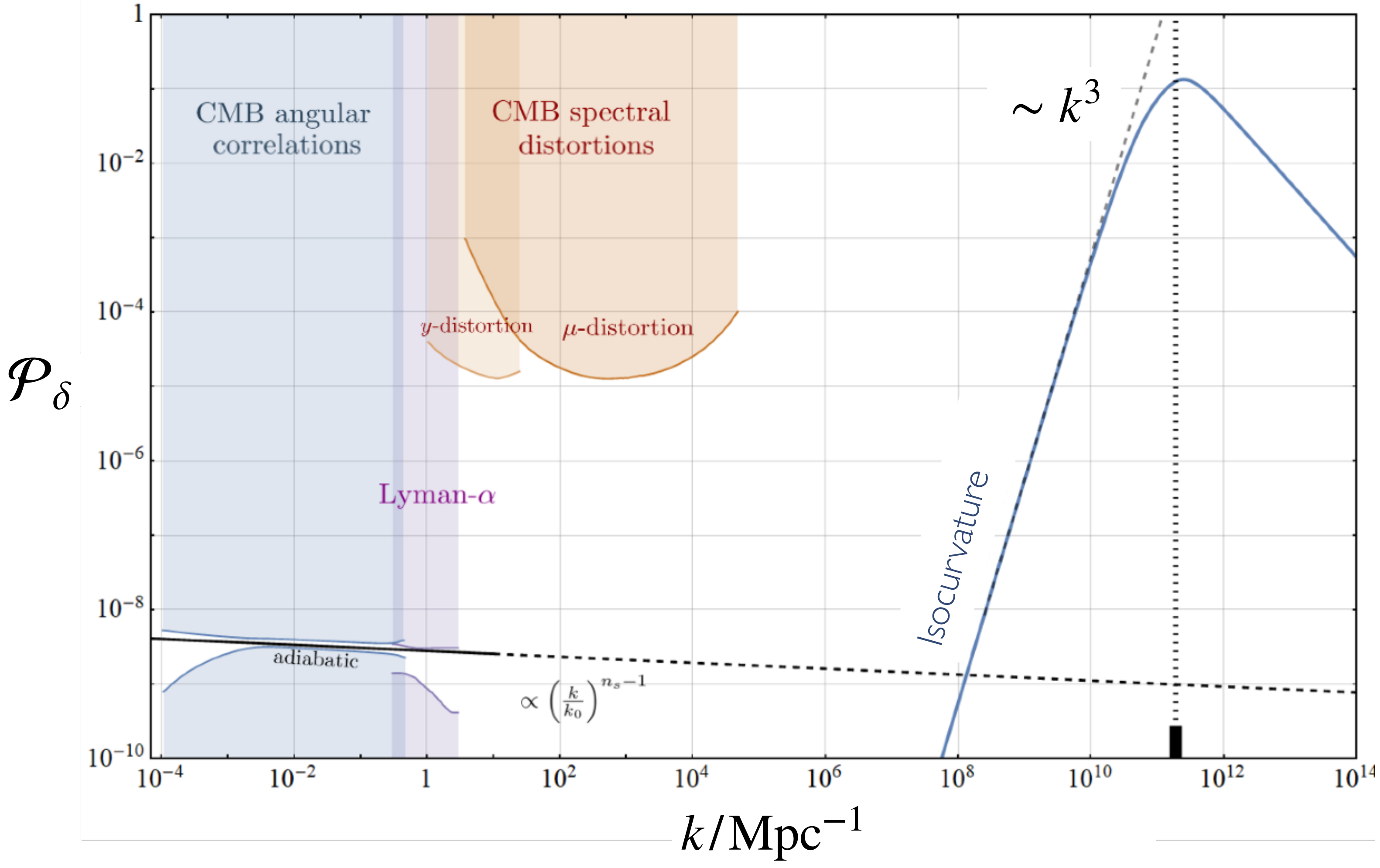
Self-interactions

Perturbations frozen

Collapse into solitons

Hierarchical structure

Adiabatic halos



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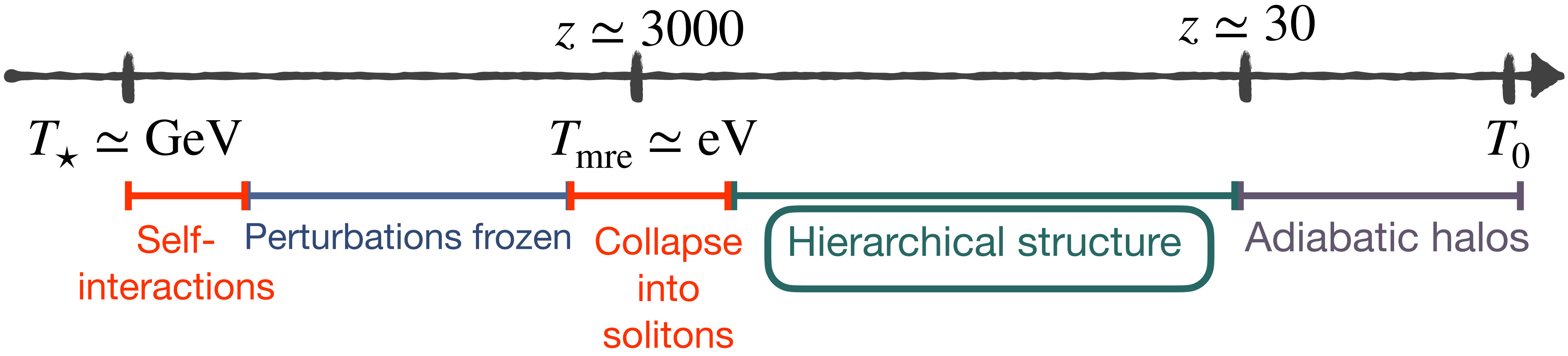

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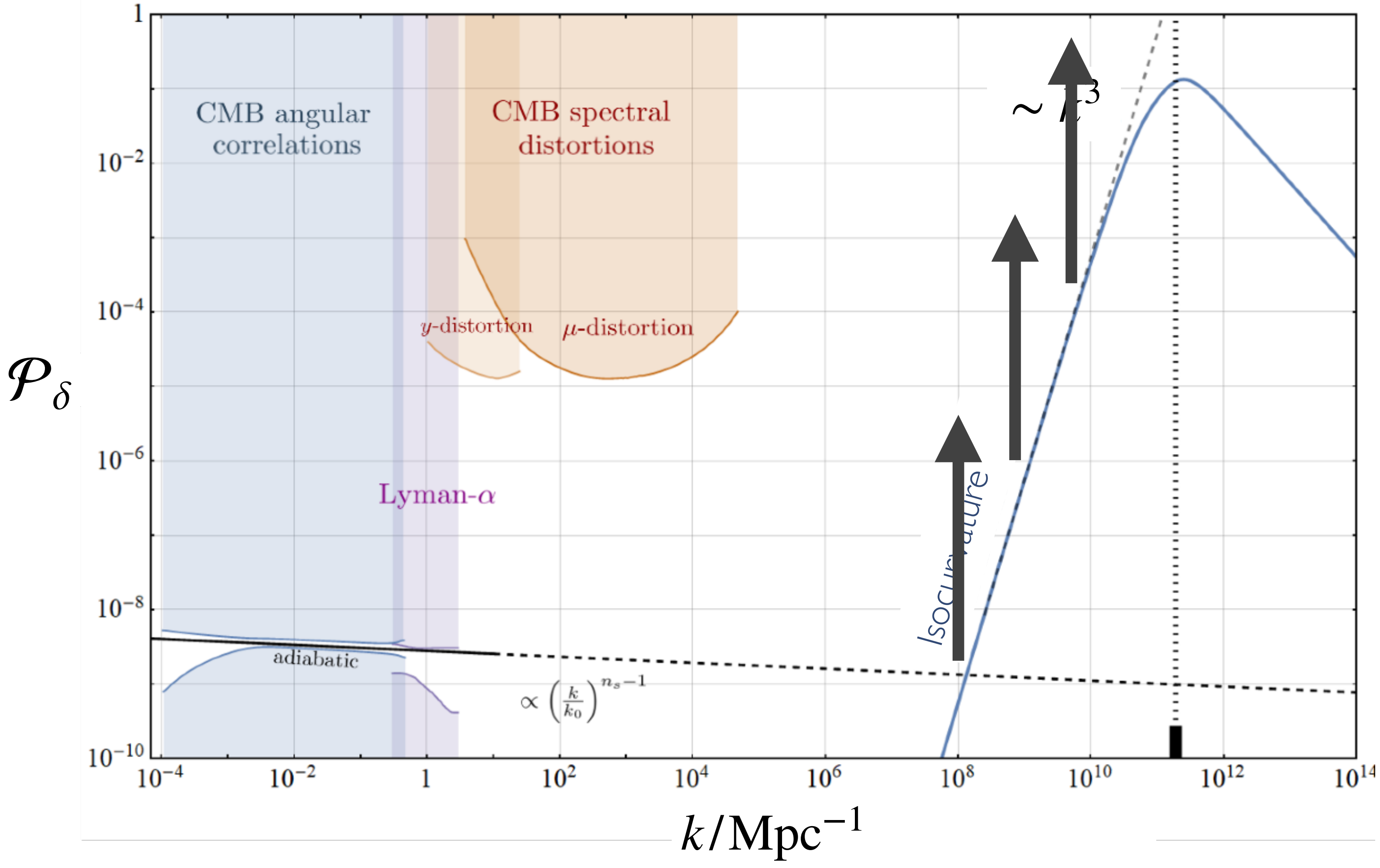

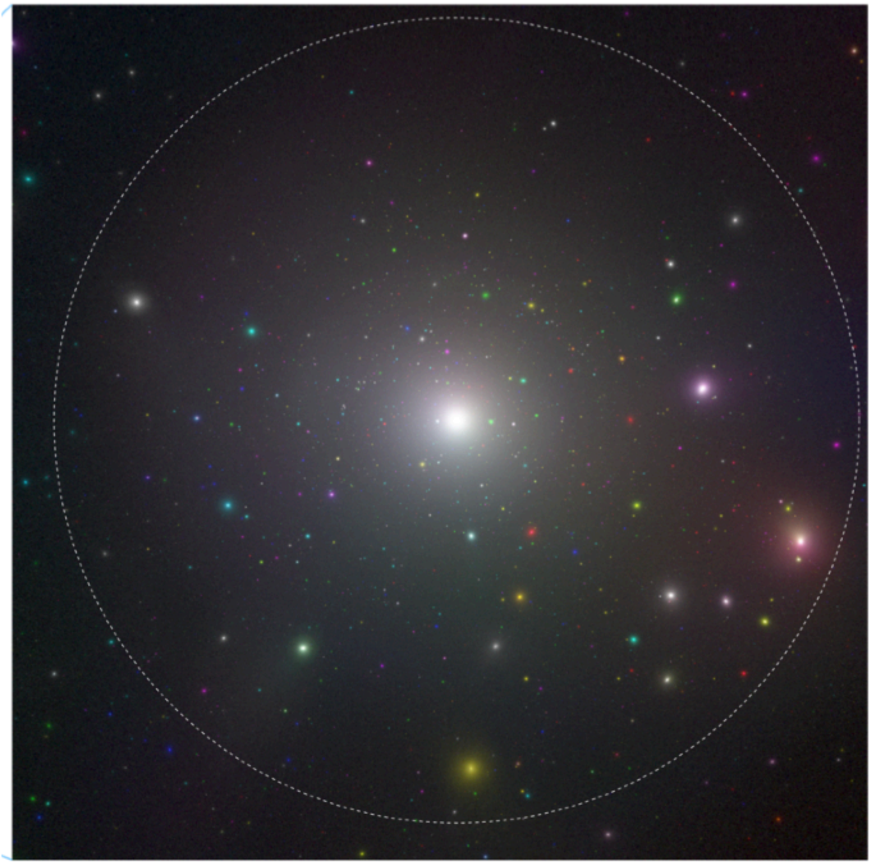

[Eggemeier et al]

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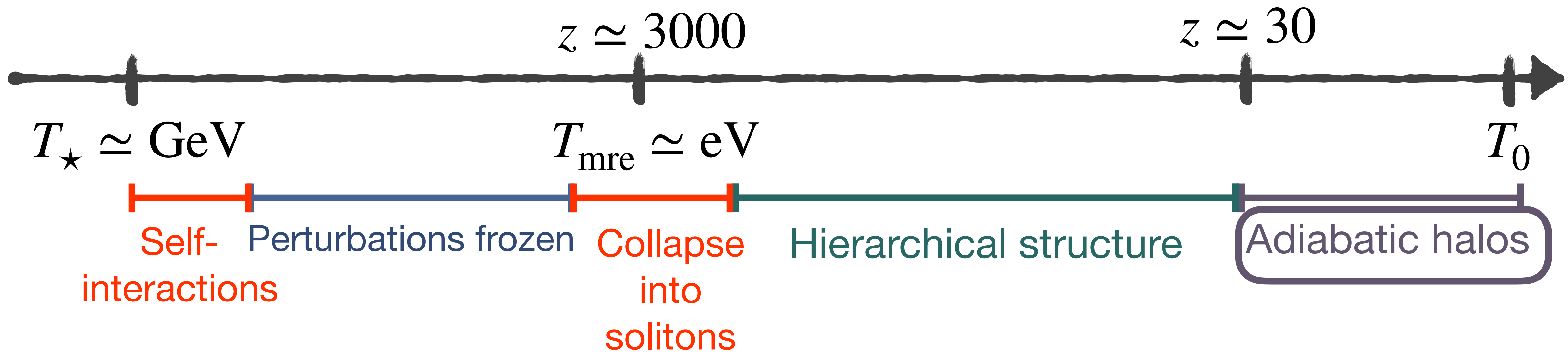

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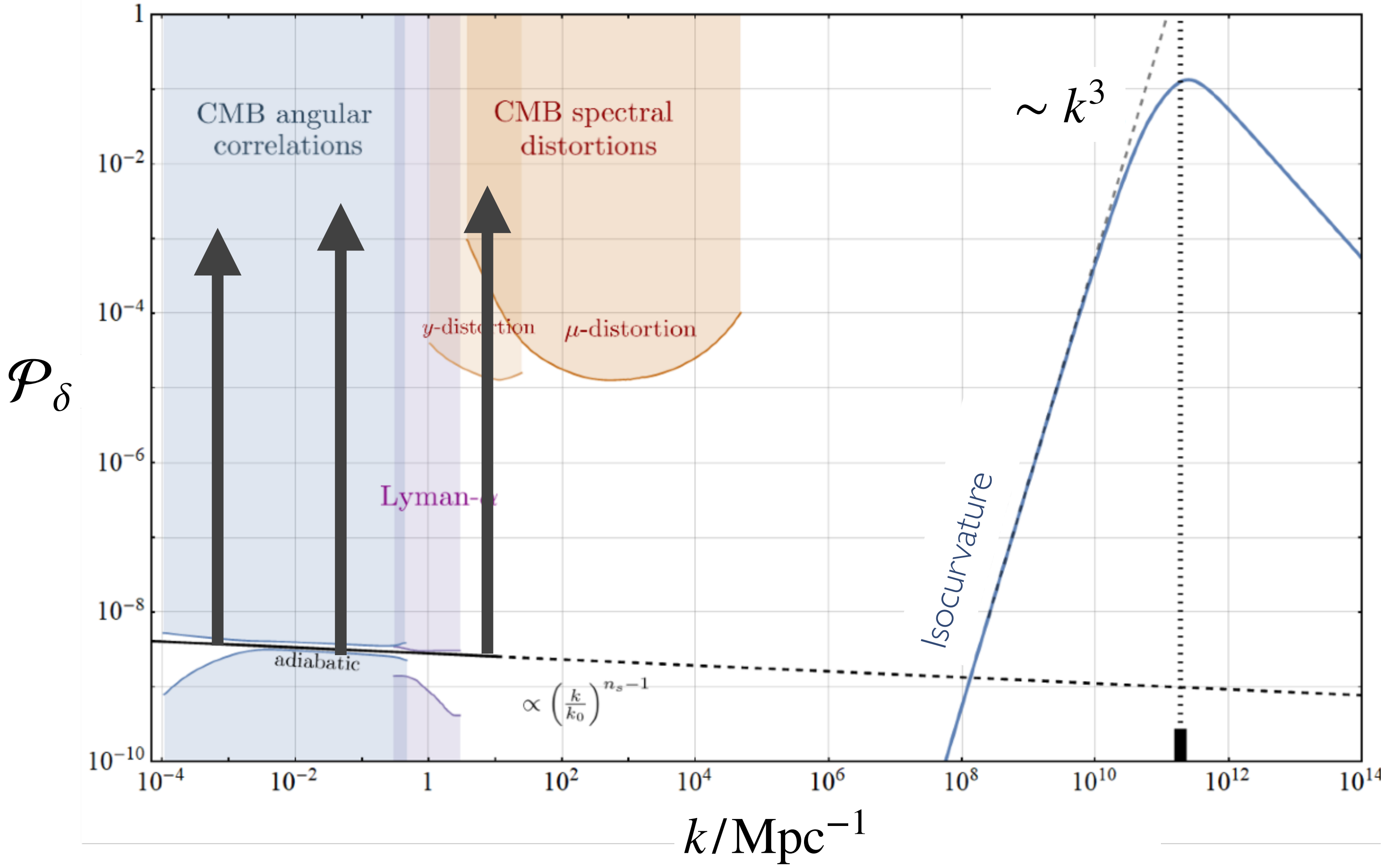
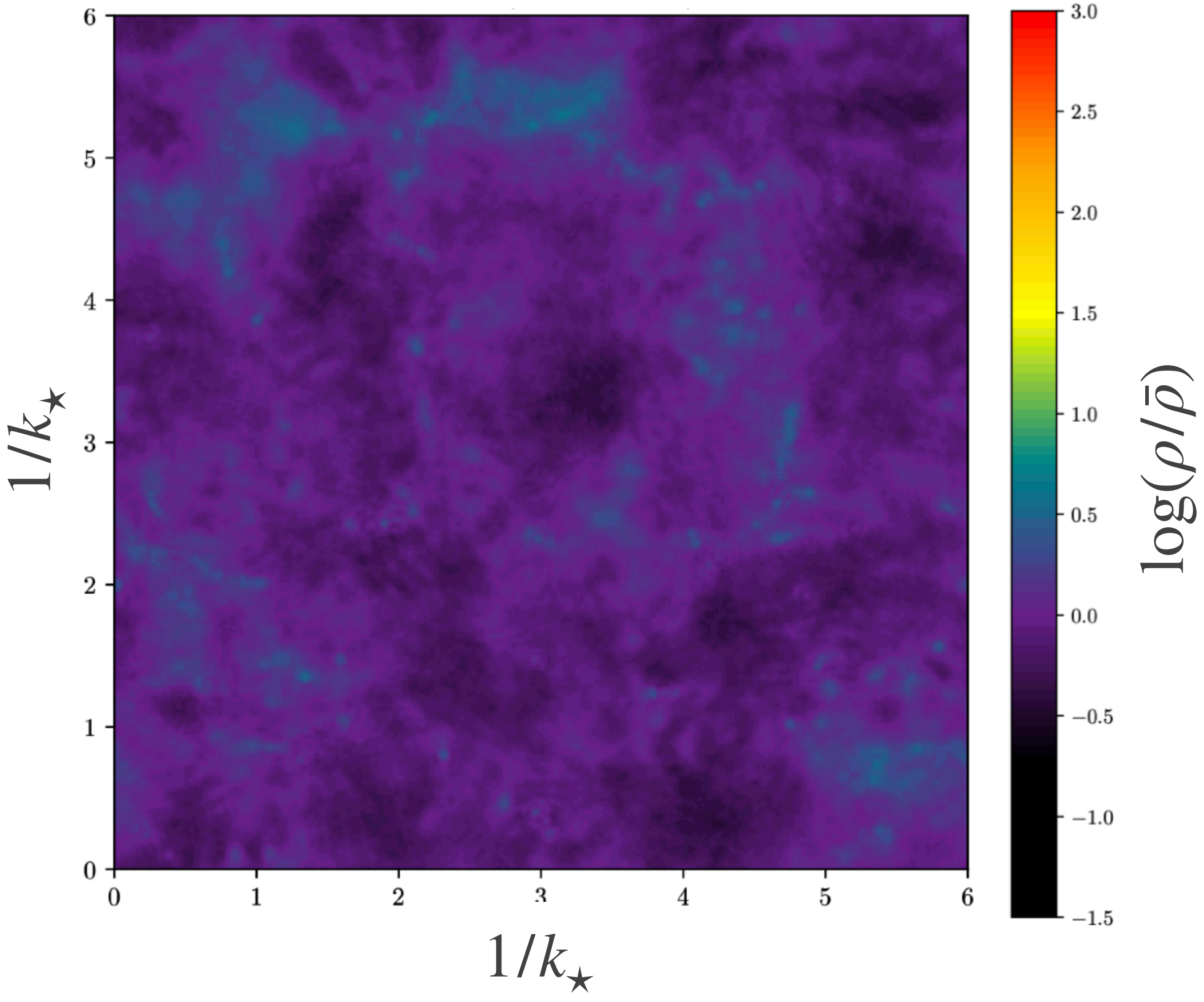


Initial perturbations

Order one fluctuations on co-moving scales $\simeq H_\star$ when $H = m_a(T)$



[Eggemeier et al]

$$\delta(x) \equiv \frac{\boxed{\rho(x)} - \bar{\rho}}{\bar{\rho}}$$

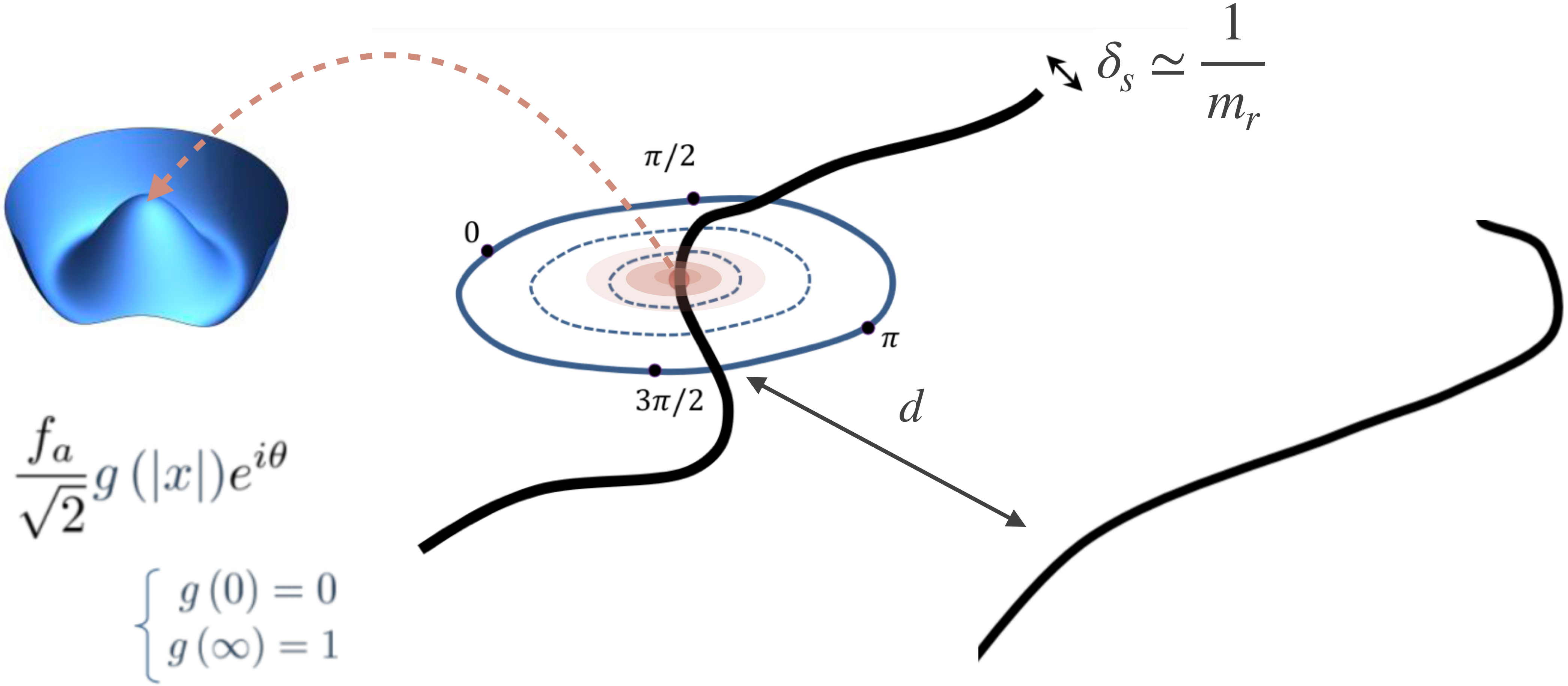
Dark matter density

Density power spectrum

$$\langle \tilde{\delta}^*(\vec{k})\tilde{\delta}(\vec{k}')\rangle \equiv \frac{2\pi^2}{k^3}\delta^3(\vec{k}-\vec{k}')\boxed{\mathcal{P}_\delta(|\vec{k}|)}$$

$$\mathcal{P}_\delta(k) \sim \frac{\partial\rho}{\partial \log k}$$

Topological strings



$$\phi = \frac{f_a}{\sqrt{2}} g\left(|x|\right) e^{i\theta}$$

$$\begin{cases} g\left(0\right) = 0 \\ g\left(\infty\right) = 1 \end{cases}$$

$$\mu = \frac{E}{L} \sim \underbrace{\boxed{\pi f_a^2}}_{\text{Core}} \underbrace{\boxed{\log \frac{d}{m_r^{-1}}}}_{\text{Gradient}}$$

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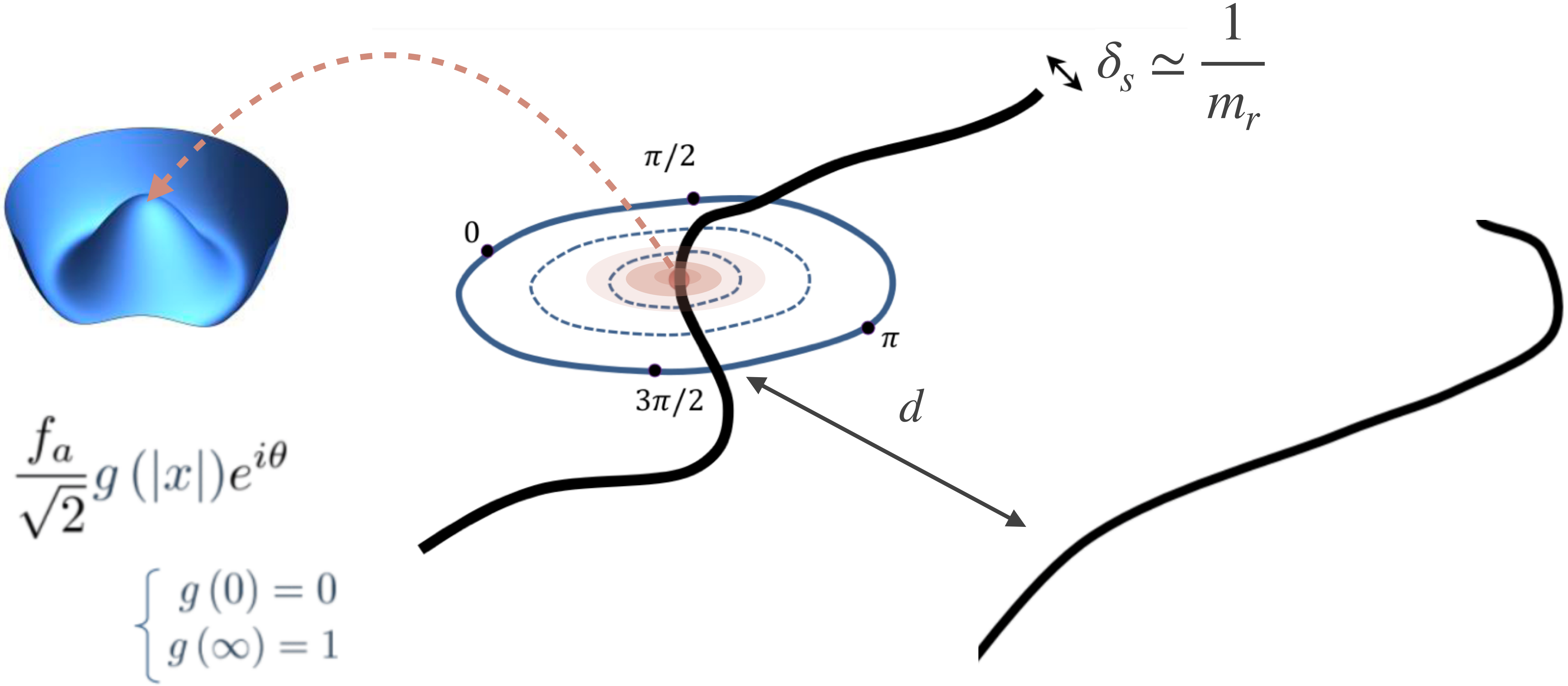

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$H \sim T^2/M_{\text{Pl}}$

Grows logarithmically

with time