UNIVERSITÀ DI PISA

INFN Istituto Nazionale di Fisica Nucleare
SEZIONE DI PISA

Strong CP and Modular Invariance

Arsenii Titov

Dipartimento di Fisica "Enrico Fermi", Università di Pisa, Italy
INFN, Sezione di Pisa, Italy

F. Feruglio, A. Strumia, AT, *JHEP* **07** (2023) 027 [2305.08908]
F. Feruglio, M. Parriciatu, A. Strumia, AT, 2406.01689

INVISIBLES24

Bologna, Italy
4 July 2024

Università di Pisa

INFN Istituto Nazionale di Fisica Nucleare
SEZIONE DI PISA

Strong CP [problem]
Strong CP [no problem]
Strong CP [violation]
Strong CP [conservation]

INVISIBLES24

Bologna, Italy
4 July 2024

Università di Pisa

INFN Istituto Nazionale di Fisica Nucleare
SEZIONE DI PISA

Strong CP [problem]
Strong CP [no problem]
Strong CP [violation]
Strong CP [conservation]

A Arsenii Titov

Which word is missing in the following title: "Strong CP and modular invariance"?

ChatGPT

The missing word is "violation". So the complete title is: "Strong CP violation and modular invariance".

INVISIBLES24

Bologna, Italy
4 July 2024

Outline

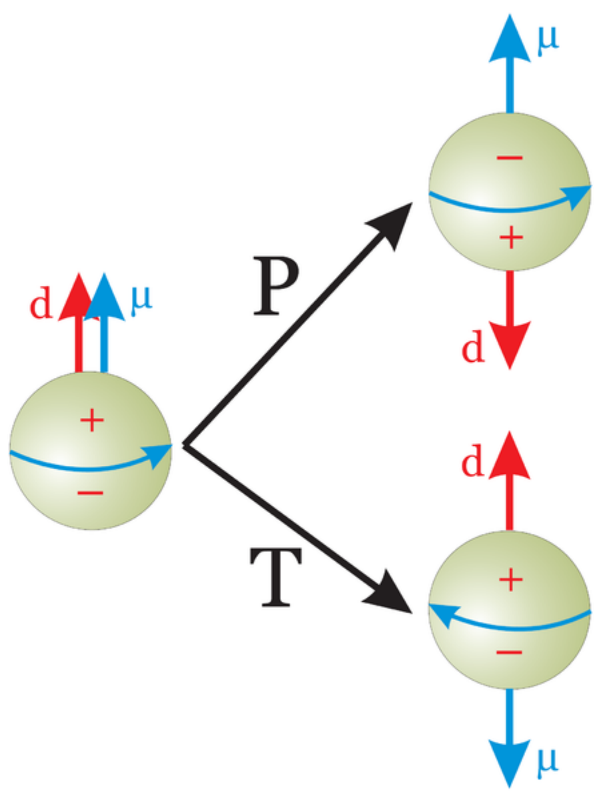
1. Strong CP problem
2. Existing solutions
3. Modular invariance and global supersymmetry
4. Modular anomalies and their cancellation
5. Models with SM quarks only
6. Models with heavy vector-like quarks
7. Corrections to $\bar{\theta} = 0$
8. Conclusions

The strong CP problem

$$\mathscr{L}_{\text{QCD}} = \overline{q}\left(i\not{D} - M_q\right) q - \frac{1}{4g_3^2} G^a_{\mu\nu} G^{a,\mu\nu} + \frac{\theta_{\text{QCD}}}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

$\boxed{\bar{\theta} = \theta_{\text{QCD}} + \arg\det M_q}$ CPV parameter

Neutron EDM d



$d = 2.4 \times 10^{-16}\ \bar{\theta}\ e \cdot$ cm Pospelov, Ritz, hep-ph/9908508v4

$|d| \leq 1.8 \times 10^{-26}\ e \cdot$ cm (90% C.L.) Abel et al., 2001.11966

$\boxed{|\bar{\theta}| \lesssim 10^{-10}}$

Why so small???

… and the CPV phase in the CKM matrix $\delta_{\text{CKM}} \approx 1.2$

Solution 1: the Axion

Promote $\bar{\theta}$ to a dynamical scalar field a, the axion, which washes out CP violation in QCD

$$\mathscr{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{32\pi^2}\frac{a}{f_a}G\tilde{G} + \ldots$$

$$\bar{\theta} = \frac{\langle a \rangle}{f_a} \qquad \text{with} \qquad \langle a \rangle = 0$$

New global $\mathrm{U}(1)_{\mathrm{PQ}}$

Peccei, Quinn, PRL **38** (1977) 1440; PRD **16** (1977) 1791

- spontaneously broken => the axion is a NGB
- anomalous under QCD $(\partial_\mu J^\mu_{\mathrm{PQ}} \propto G\tilde{G})$ => the axion is a pNGB

Quality problem

- Corrections of order $(f_a/M_{\mathrm{Pl}})^{\#}$ from higher-dimensional operators
- $\mathrm{U}(1)_{\mathrm{PQ}}$ should be an accidental symmetry in a complete model

Solution 2: CP (P) is symmetry of UV

- CP (P) is a symmetry of the UV
- It is broken spontaneously in such a way that $\bar{\theta} = 0$ and $\delta_{\text{CKM}} = \mathcal{O}(1)$

Nelson—Barr models

Nelson, PLB **136** (1984) 387; Barr, PRL **53** (1984) 329

New heavy vector-like quarks Q and scalars η with CPV complex VEVs $\langle \eta \rangle$

$$(q_R\ Q_R)\, M_q \begin{pmatrix} q_L \\ Q_L \end{pmatrix} = (q_R\ Q_R) \begin{pmatrix} y\mathrm{v}_H & y'\langle \eta \rangle \\ 0 & \mu \end{pmatrix} \begin{pmatrix} q_L \\ Q_L \end{pmatrix}$$

- CP is a symmetry => $\theta_{\text{QCD}} = 0$ and the couplings (y, y', μ) are real
- $\det M_q = y\mathrm{v}_H \mu$ is real (and positive) => $\arg \det M_q = 0$
- Effective light quark mass matrix depends on multiple $\langle \eta \rangle$ => $\delta_{\text{CKM}} \neq 0$

Additional matter, tuning, loop corrections…

Dine, Draper, 1506.05433

… or no problem to start with?

arXiv > hep-th > arXiv:2001.07152

Search...

Help

High Energy Physics – Theory

[Submitted on 20 Jan 2020 (v1), last revised 28 Sep 2021 (this version, v5)]

Absence of CP violation in the strong interactions

Wen-Yuan Ai, Juan S. Cruz, Bjorn Garbrecht, Carlos Tamarit

We derive correlation functions for massive fermions with a complex mass in the presence of a general vacuum angle. For this purpose, we first build the Green's functions in the one-instanton background and then sum over the configurations of background instantons. The quantization of topological sectors follows for saddle points of finite Euclidean action in an infinite spacetime volume and the fluctuations about these. For the resulting correlation functions, we therefore take the infinite-volume limit before summing over topological sectors. In contrast to the opposite order of limits, the chiral phases from the mass terms and from the instanton effects then are aligned so that, in absence of additional phases, these do not give rise to observables violating charge-parity symmetry. This result is confirmed when constraining the correlations at coincident points by using the index theorem instead of instanton calculus.

arXiv > hep-ph > arXiv:2404.16026

Search...

Help

High Energy Physics – Phenomenology

[Submitted on 24 Apr 2024]

CP conservation in the strong interactions

Wen-Yuan Ai, Bjorn Garbrecht, Carlos Tamarit

We discuss matters related to the point that topological quantization in the strong interaction is a consequence of an infinite spacetime volume. Because of the ensuing order of limits, i.e. infinite volume prior to summing over topological sectors, CP is conserved. Here, we show that this reasoning is consistent with the construction of the path integral from steepest-descent contours. We reply to some objections that aim to support the case for CP violation in the strong interactions that are based on the role of the CP-odd theta-parameter in three-form effective theories, the correct sampling of all configurations in the dilute instanton gas approximation and the volume dependence of the partition function. We also show that the chiral effective field theory derived from taking the volume to infinity first is in no contradiction with analyses based on partially conserved axial currents.

… or no problem to start with?

arXiv > hep-ph > arXiv:2403.13508

Search...

Help

High Energy Physics – Phenomenology

[Submitted on 20 Mar 2024]

Absence of strong CP violation

Gerrit Schierholz

Quantum Chromodynamics admits a CP-violating contribution to the action, the θ term, which is expected to give rise to a nonvanishing electric dipole moment of the neutron. Despite intensive search, no CP violations have been found in the strong interaction. This puzzle is referred to as the strong CP problem. There is evidence that CP is conserved in the confining theory, to the extent that color charges are totally screened for $\theta > 0$ at large distances. It is not immediately obvious that this implies a vanishing dipole moment though. With this Letter I will close the gap. It is shown that in the infinite volume hadron correlation functions decouple from the topological charge, expressed in terms of the zero modes. The reason is that hadrons have a limited range of interaction, while the density of zero modes vanishes with the inverse root of the volume, thus reducing the probability of finding a zero mode in the vicinity to zero. This implies, beyond doubt, that CP is conserved in the strong interaction.

arXiv > hep-th > arXiv:2404.19400

Search...

Help

High Energy Physics – Theory

[Submitted on 30 Apr 2024 (v1), last revised 10 Jun 2024 (this version, v2)]

The strong CP problem revisited and solved by the gauge group topology

F. Strocchi

We exploit the non-perturbative result that the θ angle which defines the vacuum structure is not a c-number free parameter, as suggested by the instanton semi-classical approximation, but instead one of the points of the spectrum of the central operator $\tilde{\theta}$ which describes the gauge group topology. Hence, the value of such an angle should not be \textit{a priori} fixed, but rather be determined, as any quantum operator, by the infinite volume limit of the functional integral, where the fermionic mass term uniquely fixes the phase, with $< \tilde{\theta} >= \theta_M$, the mass angle. Such an equality is stable under radiative corrections performed before the infinite volume limit of the functional integral.
The mechanisms is carefully controlled in the massive Schwinger model with attention to the infrared problems, to the volume effects induced by boundary terms and with a careful discussion of the infinite volume limit of the functional integral. The extension to the QCD case relies on the choice of modified APS boundary conditions which do not break chiral symmetry, allowing for the crucial role of the fermionic mass term in uniquely determining the phase, with a solution of the strong CP problem.

Our solution: CP + modular invariance

1. CP is a symmetry => $\theta_{\mathrm{QCD}} = 0$ (and real Lagrangian couplings)

2. Modular invariance/anomaly cancellation => $\arg\det M_q = 0$

3. CP is broken spontaneously by the VEV of a single complex scalar field, the modulus τ => $\delta_{\mathrm{CKM}} = \mathcal{O}(1)$

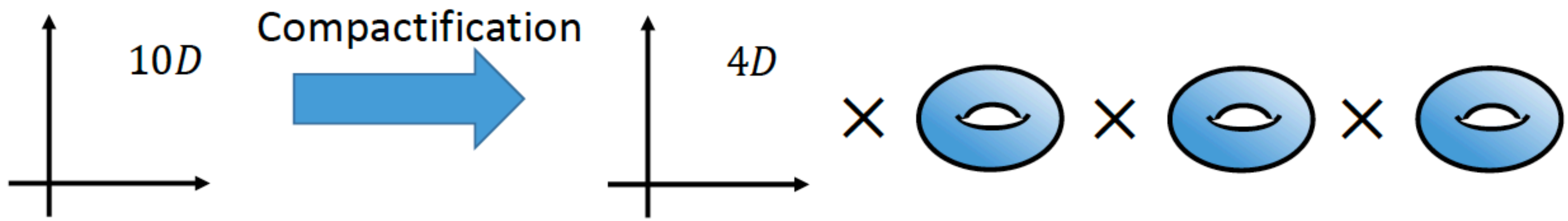
4. Quark mass hierarchies and mixing angles are reproduced by $\mathcal{O}(1)$ parameters

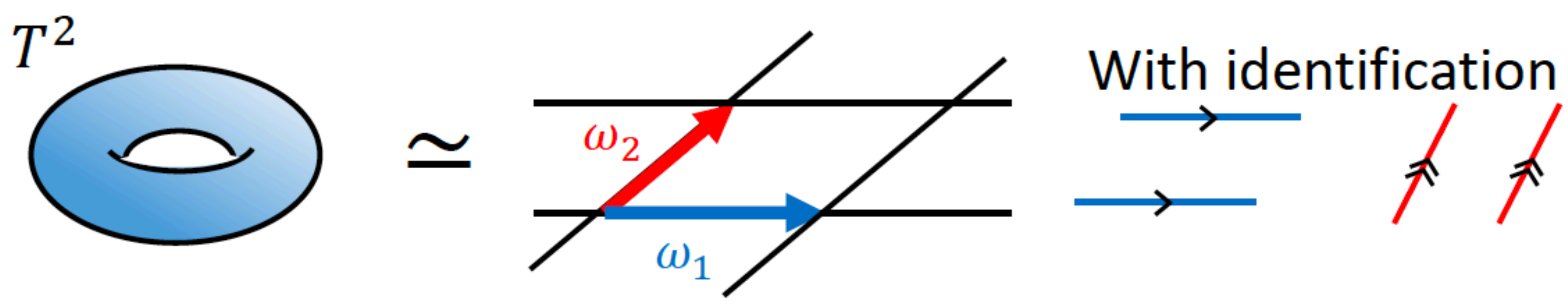
5. Corrections to $\bar{\theta} = 0$ are small under certain assumptions on SUSY breaking

Modular invariance

String theory requires extra dimensions

Images: Takuya H. Tatsuishi





$$\tau = \frac{\omega_2}{\omega_1}, \quad \mathrm{Im}\,(\tau) > 0$$

modulus

Lattice left invariant by modular transformations

$$\tau \to \gamma\tau \equiv \frac{a\tau + b}{c\tau + d} \qquad a, b, c, d \in \mathbb{Z} \qquad ad - bc = 1$$

These transformations form an infinite discrete group

Modular group

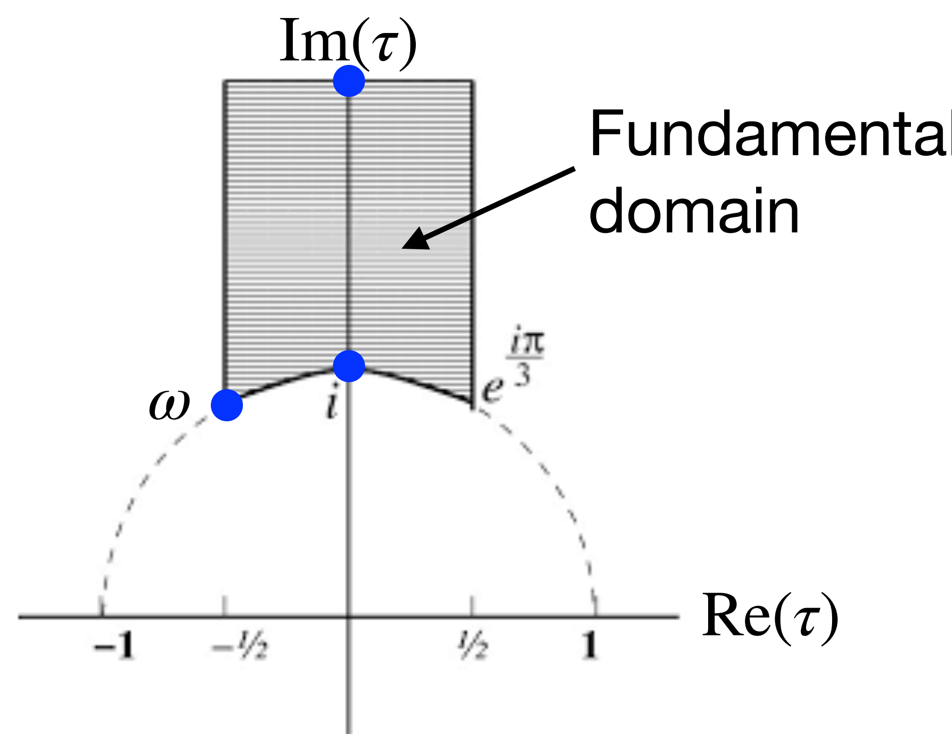
Homogeneous modular group

$$\Gamma = \langle S,\, T \mid S^4 = (ST)^3 = I \rangle \cong \mathrm{SL}(2,\mathbb{Z})$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\tau \xrightarrow{S} -\frac{1}{\tau} \qquad \tau \xrightarrow{T} \tau + 1$$

duality — discrete shift symmetry



Special points

- $\tau = i: \quad i \xrightarrow{S} -\frac{1}{i} = i \quad \Rightarrow \quad Z_4^S$
- $\tau = \omega \equiv e^{\frac{2\pi i}{3}}: \quad \omega \xrightarrow{ST} -\frac{1}{\omega+1} = \omega \quad \Rightarrow \quad Z_3^{ST} \times Z_2^{S^2}$
- $\tau = i\infty: \quad i\infty \xrightarrow{T} i\infty + 1 = i\infty \quad \Rightarrow \quad Z^T \times Z_2^{S^2}$

Modular forms

Holomorphic functions on $\mathscr{H} = \{\tau \in \mathbb{C} : \ \mathrm{Im}(\tau) > 0\}$ transforming under Γ as

$$\boxed{f(\gamma\tau) = (c\tau + d)^k f(\tau)\,, \qquad \gamma \in \Gamma}$$

k is weight, a non-negative even integer

Normalised Eisenstein series

$$E_k(\tau) = \frac{1}{2\zeta(k)} \sum_{(m,n)\neq(0,0)} \frac{1}{(m+n\tau)^k}$$

Each modular form of weight k can be written as a polynomial in E_4 and E_6

$$f(\tau) = \sum_{a,b\geq 0} c_{ab}\, E_4^a(\tau)\, E_6^b(\tau) \qquad \text{with} \qquad 4a + 6b = k$$

Modular weight k	0	2	4	6	8	10	12	14
Modular forms	1	–	E_4	E_6	$E_8 = E_4^2$	$E_{10} = E_4 E_6$	E_4^3, E_6^2	$E_{14} = E_4^2 E_6$

Modular-invariant SUSY theories

$\mathcal{N} = 1$ global SUSY action

$$\mathscr{L} = \int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\; K\left(\tau, e^{2V}\Phi, \tau^\dagger, \Phi^\dagger\right) + \left[\int \mathrm{d}^2\theta\; W(\tau, \Phi) + \frac{1}{16}\int \mathrm{d}^2\theta\, f(\tau)\, \mathscr{G}\,\mathscr{G} + \text{h.c.}\right]$$

Kähler potential K
(kinetic terms,
gauge interactions)

Superpotential W
(Yukawa interactions)

Gauge kinetic function f

$$f_3 = \frac{1}{g_3^2} - i\frac{\theta_{\mathrm{QCD}}}{8\pi^2}$$

Under modular transformations $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$

$$\begin{cases} \tau \to \dfrac{a\tau + b}{c\tau + d} & \tau \text{ is promoted to a (dimensionless) superfield} \\ \Phi \to \left(c\tau + d\right)^{-k_\Phi}\Phi & \text{matter supermultiplets} \\ V \to V & \text{vector supermultiplets} \end{cases}$$

Modular symmetry acts non-linearly

Ferrara et al., PLB **225** (1989) 363; PLB **233** (1989) 147; Feruglio, 1706.08749

Modular-invariant SUSY theories

$\mathcal{N} = 1$ global SUSY action

$$\mathscr{L} = \int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\; K\left(\tau, e^{2V}\Phi, \tau^\dagger, \Phi^\dagger\right) + \left[\int \mathrm{d}^2\theta\; W(\tau, \Phi) + \frac{1}{16}\int \mathrm{d}^2\theta\; f(\tau)\, \mathscr{G}\mathscr{G} + \text{h.c.}\right]$$

Kähler potential K
(kinetic terms,
gauge interactions)

Superpotential W
(Yukawa interactions)

Gauge kinetic function f

$$f_3 = \frac{1}{g_3^2} - i\frac{\theta_{\mathrm{QCD}}}{8\pi^2}$$

Modular invariance of the action requires

$$\begin{cases} K(\tau, \Phi, \tau^\dagger, \Phi^\dagger) \to K(\tau, \Phi, \tau^\dagger, \Phi^\dagger) + f_K(\tau, \Phi) + \overline{f_K}\left(\tau^\dagger, \Phi^\dagger\right) \\ W(\tau, \Phi) \to W(\tau, \Phi) \\ f(\tau) \to f(\tau) \end{cases}$$

Ferrara et al., PLB **225** (1989) 363; PLB **233** (1989) 147; Feruglio, 1706.08749

Modular-invariant SUSY theories

Minimal Kähler potential

$$K = -h^2 \log(-i\tau + i\tau^\dagger) + \sum_\Phi \frac{\Phi^\dagger e^{2V}\Phi}{(-i\tau + i\tau^\dagger)^{k_\Phi}}$$

Superpotential

$$W = Y^u_{ij}(\tau)\, u_{Ri}\, Q_j\, H_u + Y^d_{ij}(\tau)\, d_{Ri}\, Q_j\, H_d \qquad \left(u_R \equiv u^c\,, \;\; d_R \equiv d^c\right)$$

τ-dependent Yukawa couplings

$$Y^q_{ij}(\tau) \to (c\tau + d)^{k^q_{ij}}\, Y^q_{ij}(\tau) \qquad \text{with} \qquad k^q_{ij} = k_{q_{Ri}} + k_{Q_j} + k_{H_q}$$

are modular forms!

$$\boxed{Y^q_{ij}(\tau) = c^q_{ij}\, F_{k^q_{ij}}(\tau) \qquad \text{with} \qquad c^q_{ij} \in \mathbb{R} \qquad \text{because of CP}}$$

Gauge kinetic function

$$f = \frac{1}{g_3^2} \qquad\qquad \theta_{\text{QCD}} = 0 \;\; \text{because of CP}$$

Modular invariance and CP

Fields

$$\tau \xrightarrow{\mathrm{CP}} -\tau^\dagger \qquad \text{and} \qquad \Phi \xrightarrow{\mathrm{CP}} \Phi^\dagger$$

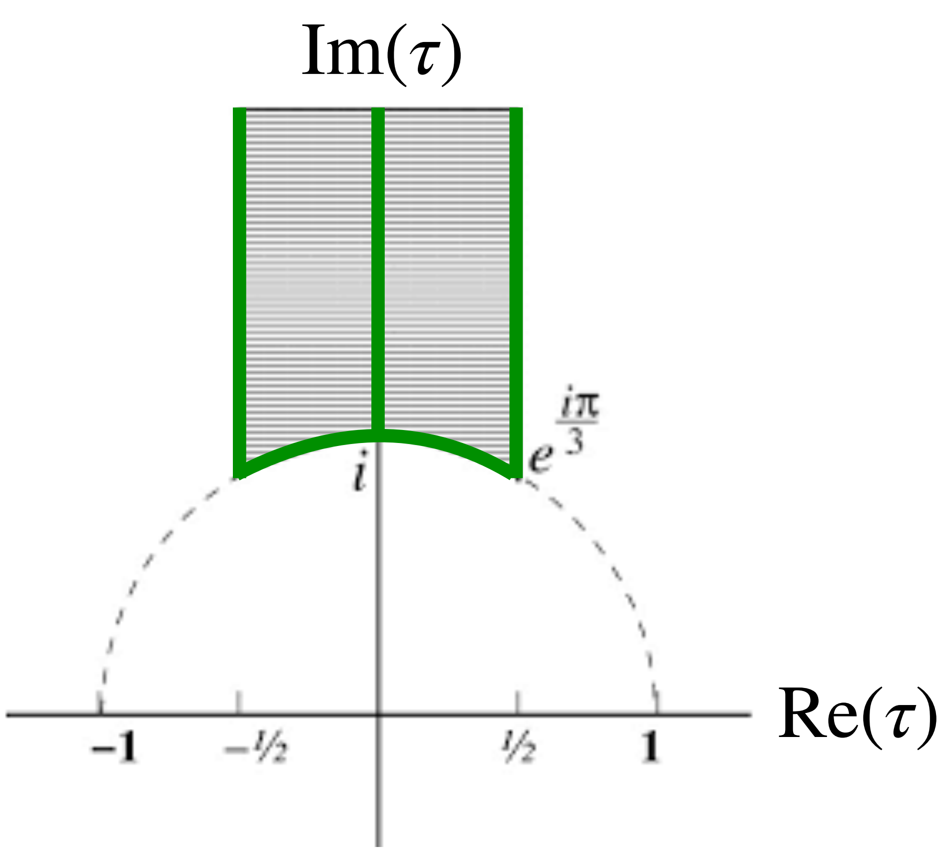
Modular forms

$$F(\tau) \xrightarrow{\mathrm{CP}} F(-\tau^*) = F(\tau)^*$$

CP-conserving values of τ

$\tau \xrightarrow{\mathrm{CP}} \boxed{-\tau^* = \gamma\tau}$ (goes to itself up to γ)

1. $\tau = iy \xrightarrow{\mathrm{CP}} iy$
2. $\tau = -\dfrac{1}{2} + iy \xrightarrow{\mathrm{CP}} \dfrac{1}{2} + iy = T\tau$
3. $\tau = e^{i\varphi} \xrightarrow{\mathrm{CP}} -e^{-i\varphi} = S\tau$



Novichkov, Penedo, Petcov, **AT**, 1905.11970; Baur, Nilles, Trautner, Vaudrevange, 1901.03251

Determinant of quark mass matrix

$$M_u = \mathrm{v}_u Y^u \qquad M_d = \mathrm{v}_d Y^d$$

$$\det M_q = \det M_u \det M_d \propto \det Y^u \det Y^d$$

$$Y^q(\tau) = \begin{pmatrix} F_{k^q_{11}} & F_{k^q_{12}} & F_{k^q_{13}} \\ F_{k^q_{21}} & F_{k^q_{22}} & F_{k^q_{23}} \\ F_{k^q_{31}} & F_{k^q_{32}} & F_{k^q_{33}} \end{pmatrix} \quad \Rightarrow$$

$\det Y^q(\tau)$ is a modular form of weight $k^q_{\det}$

$$k^q_{\det} = k^q_{11} + k^q_{22} + k^q_{33} = \ldots = \sum_{i=1}^{3} \left(k_{q_{Ri}} + k_{Q_i} \right) + 3k_{H_q}$$

And $\det Y^u(\tau) \det Y^d(\tau)$ is a modular form of weight $k_{\det}$

$$k_{\det} = k^u_{\det} + k^d_{\det} = \sum_{i=1}^{3} \left(2k_{Q_i} + k_{u_{Ri}} + k_{d_{Ri}} \right) + 3\left(k_{H_u} + k_{H_d} \right)$$

$$k_{\det} = 0 \quad \Rightarrow \quad \det Y^u(\tau) \det Y^d(\tau) = \text{(real) constant}$$

Matter fields and canonical normalisation

Gauge quantum numbers

	Q	u_R	d_R	L	e_R	H_u	H_d
$\mathrm{SU}(3)_C$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{SU}(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$
$\mathrm{U}(1)_Y$	$\frac{1}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	1	$\frac{1}{2}$	$-\frac{1}{2}$

Canonical normalisation

$$K \supset \frac{\Phi^\dagger \Phi}{(-i\tau + i\tau^\dagger)^{k_\Phi}} = \Phi^\dagger_{\text{can}} \Phi_{\text{can}} \qquad \Phi_{\text{can}} = \{\phi_{\text{can}}, \psi_{\text{can}}\}$$

$$\psi_{\text{can}} \to \left(\frac{c\tau + d}{c\tau^\dagger + d}\right)^{-\frac{k_\Phi}{2}} \psi_{\text{can}} = e^{-ik_\Phi \alpha(\tau)} \psi_{\text{can}} \qquad \alpha(\tau) = \arg(c\tau + d)$$

Modular symmetry acts on canonically normalised fields
as a τ-dependent phase rotation (with $\tau = \tau(x)$)

Cancellation of modular anomalies

Conditions for modular-gauge anomaly cancellation

$$\mathrm{SU}(3)_C: \quad A \equiv \sum_{i=1}^{3}\left(2k_{Q_i} + k_{u_{Ri}} + k_{d_{Ri}}\right) = 0$$

$$\mathrm{SU}(2)_L: \quad \sum_{i=1}^{3}\left(3k_{Q_i} + k_{L_i}\right) + k_{H_u} + k_{H_d} = 0$$

$$\mathrm{U}(1)_Y: \quad \sum_{i=1}^{3}\left(k_{Q_i} + 8k_{u_{Ri}} + 2k_{d_{Ri}} + 3k_{L_i} + 6k_{e_{Ri}}\right) + 3\left(k_{H_u} + k_{H_d}\right) = 0$$

Simplest solution

$$\boxed{k_Q = k_{u_R} = k_{d_R} = k_L = k_{e_R} = (-k, 0, k) \qquad \text{and} \qquad k_{H_u} + k_{H_d} = 0}$$

Cancellation of modular-QCD anomaly along with $k_{H_u} + k_{H_d} = 0$ implies

$$k_{\det} = \sum_{i=1}^{3}\left(2k_{Q_i} + k_{u_{Ri}} + k_{d_{Ri}}\right) + 3\left(k_{H_u} + k_{H_d}\right) = 0$$

Simplest example: quarks

Simplest non-trivial example giving $k_{\det} = 0$ and $A = 0$

$$k_Q = k_{u_R} = k_{d_R} = (-6, 0, 6) \quad \text{and} \quad k_{H_u} = k_{H_d} = 0$$

Yukawa matrices

$$Y^q = \begin{pmatrix} 0 & 0 & c^q_{13} \\ 0 & c^q_{22} & c^q_{23} E_6 \\ c^q_{31} & c^q_{32} E_6 & c^q_{33} E_4^3 + c'^q_{33} E_6^2 \end{pmatrix} \quad \Rightarrow \quad Y^q|_{\text{can}} = \begin{pmatrix} 0 & 0 & c^q_{13} \\ 0 & c^q_{22} & c^q_{23}\,(2\mathrm{Im}\tau)^3\,E_6 \\ c^q_{31} & c^q_{32}\,(2\mathrm{Im}\tau)^3\,E_6 & (2\mathrm{Im}\tau)^6\left[c^q_{33} E_4^3 + c'^q_{33} E_6^2\right] \end{pmatrix}$$

$$\det Y^q|_{\text{can}} = -\,c^q_{13} c^q_{22} c^q_{31} \in \mathbb{R}$$

Fixing $\tau = 1/8 + i$ and $\tan\beta = 10$

$$c^u_{ij} \approx 10^{-3} \begin{pmatrix} 0 & 0 & 1.56 \\ 0 & -1.86 & 0.87 \\ 1.29 & 4.14 & 3.51,\ 1.40 \end{pmatrix} \qquad c^d_{ij} \approx 10^{-3} \begin{pmatrix} 0 & 0 & 1.55 \\ 0 & -2.59 & 4.59 \\ 0.378 & 0.710 & 0.734,\ 1.76 \end{pmatrix}$$

reproduce the quark masses, mixing angles and δ_{CKM} at the GUT scale

Simplest example: leptons

$k_L = k_{e_R} = (-6, 0, 6)$

Weinberg operator $\mathscr{C}^\nu_{ij}(L_iH_u)(L_jH_u)$ for neutrino masses

Charged lepton Yukawa matrix and coefficient of the Weinberg operator

$$Y^e = \begin{pmatrix} 0 & 0 & c^e_{13} \\ 0 & c^e_{22} & c^e_{23}E_6 \\ c^e_{31} & c^e_{32}E_6 & c^e_{33}E_4^3 + c'^e_{33}E_6^2 \end{pmatrix} \qquad \mathscr{C}^\nu = \begin{pmatrix} 0 & 0 & c^\nu_{13} \\ 0 & c^\nu_{22} & c^\nu_{23}E_6 \\ c^\nu_{31} & c^\nu_{32}E_6 & c^\nu_{33}E_4^3 + c'^\nu_{33}E_6^2 \end{pmatrix}$$

Fixing $\tau = 1/8 + i$ and $\tan\beta = 10$

$$c^e_{ij} = 10^{-3}\begin{pmatrix} 0 & 0 & 1.29 \\ 0 & 5.95 & 0.35 \\ -2.56 & 1.47 & 1.01,\ 1.32 \end{pmatrix} \qquad c^\nu_{ij} = \frac{1}{10^{16}\ \text{GeV}}\begin{pmatrix} 0 & 0 & 3.4 \\ 0 & 7.1 & 1.2 \\ 3.4 & 1.2 & 0.19,\ 0.95 \end{pmatrix}$$

reproduce the lepton masses and mixings, including δ_{PMNS}

Models with larger modular weights

Yukawa matrices	Modular weights			Alternative bigger weights		
$Y_{u,d}$	$(u_L, d_L)_{1,2,3}$	$u_{R1,2,3}$	$d_{R1,2,3}$	$(u_L, d_L)_{1,2,3}$	$u_{R1,2,3}$	$d_{R1,2,3}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_6 \\ 1 & E_6 & E_4^3 + E_6^2 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -4 \\ 2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ -2 \\ 4 \end{pmatrix}$	$\begin{pmatrix} -8 \\ -2 \\ 4 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^2 \\ 1 & E_4 & E_4^3 + E_6^2 \end{pmatrix}$	$\begin{pmatrix} -6 \\ -2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -4 \\ -4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^2 \\ 1 & E_4^2 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4 E_6 \\ 1 & E_6 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ -2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 2 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^3 + E_6^2 \\ 1 & E_4 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ -4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 8 \end{pmatrix}$

Heavy quarks and singularities

- Heavy quarks are not needed for the mechanism to work, but assume they exist

$$k_q = (-6,\ -2, 0,\ +2,\ +6) \quad \text{and} \quad k_{H_u} = k_{H_d} = 0$$

Light chiral quarks Heavy vector-like quarks

- In the full theory $f_{\mathrm{UV}} \in \mathbb{R}$ and $\det M_{\mathrm{all}} \in \mathbb{R}$ => $\bar{\theta}_{\mathrm{UV}} = 0$

$$M_{\mathrm{all}} = \begin{pmatrix} M_{LL} & M_{LH} \\ M_{HL} & M_{HH} \end{pmatrix}$$

$$M_{\mathrm{light}} \approx M_{LL} - M_{LH} M_{HH}^{-1} M_{HL} \qquad M_{\mathrm{heavy}} \approx M_{HH}$$

Singularities where $\det M_{\mathrm{heavy}}(\tau) = 0$

(breakdown of EFT)

$$\det M_{\mathrm{all}} = \det M_{\mathrm{light}} \det M_{\mathrm{heavy}}$$

$$\det M_{\mathrm{light}} \to (c\tau + d)^{k_{\mathrm{light}}} \det M_{\mathrm{light}} \qquad \det M_{\mathrm{heavy}} \to (c\tau + d)^{k_{\mathrm{heavy}}} \det M_{\mathrm{heavy}}$$

$$k_{\mathrm{all}} = k_{\mathrm{light}} + k_{\mathrm{heavy}} = 0$$

EFT of light quarks

In the EFT of light quarks

$$\bar{\theta}_{\text{IR}} = \theta_{\text{QCD}} + \arg\det M_{\text{light}} = -8\pi^2\, \text{Im} f_{\text{IR}} + \arg\det M_{\text{light}}$$

The EFT has anomalous field content with $k_q = (-6, -2, 0)$

Anomaly is cancelled by a new contribution to the gauge kinetic function arising from the integration over the heavy quarks

$$f_{\text{IR}} = f_{\text{UV}} - \frac{1}{8\pi^2} \log\det M_{\text{heavy}}$$

Thus

$$\bar{\theta}_{\text{IR}} = \arg\det M_{\text{heavy}} + \arg\det M_{\text{light}} = \arg\det M_{\text{all}} = 0$$

Model with VLQs based on $\Gamma_2 \cong S_3$

Feruglio, Parriciatu, Strumia, **AT**, 2406.01689

	SM quarks			Extra vector-like quarks			
	Q	D^c	U^c	D'^c	D'	U'^c	U'
$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$	$2_{1/6}$	$1_{1/3}$	$1_{-2/3}$	$1_{1/3}$	$1_{-1/3}$	$1_{-2/3}$	$1_{2/3}$
Flavor symmetry Γ_2	$\mathbf{2} \oplus \mathbf{1}_0$	$\mathbf{2} \oplus \mathbf{1}_1$	$\mathbf{2} \oplus \mathbf{1}_0$	$\mathbf{2} \oplus \mathbf{1}_0$	$\mathbf{2} \oplus \mathbf{1}_1$	$\mathbf{2} \oplus \mathbf{1}_0$	$\mathbf{2} \oplus \mathbf{1}_0$
Modular weights k_Φ	-2	-2	-2	$+2$	$+2$	$+2$	$+2$

$$W_{\rm UV} \supset Q^T m^d D^c + Q^T n^d D'^c + D'^T N^d D^c + D'^T M^d D'^c$$

$$\mathscr{M}_d = \begin{pmatrix} m^d & n^d \\ N^d & M^d \end{pmatrix} \qquad m^d = v_d Y^d \qquad n^d = v_d Y'^d$$

$$m^d = 0_{3\times 3} \qquad n^d = n_d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha_d \end{pmatrix} \qquad N^d = N_d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \beta_d \end{pmatrix} \qquad M^d = M_d \begin{pmatrix} -Z_1^{(4)} + \gamma_{d1} Z_3^{(4)} & Z_2^{(4)} & \gamma_{d2} Z_1^{(4)} \\ Z_2^{(4)} & Z_1^{(4)} + \gamma_{d1} Z_3^{(4)} & \gamma_{d2} Z_2^{(4)} \\ \gamma_{d3} Z_2^{(4)} & -\gamma_{d3} Z_1^{(4)} & 0 \end{pmatrix}$$

$(Z_1^{(4)}(\tau), Z_2^{(4)}(\tau))^T \sim \mathbf{2}$ $\qquad Z_3^{(4)}(\tau) \sim \mathbf{1}_0$ $\qquad$ level $N = 2$ modular forms of weight $k = 4$

$$\det \mathscr{M}_d = \det \left[m^d - n^d \left(M^d\right)^{-1} N_d \right] \det M^d = -\det \left[n_d N_d \right] = -n_d^3 N_d^3 \alpha_d \beta_d \in \mathbb{R}$$

Corrections to $\bar{\theta} = 0$

M_{Pl}

$\Lambda_{\mathrm{flavour/CP}}$

Λ_{SUSY}

m_{SUSY}

v

Corrections to $\bar{\theta} = 0$

M_{Pl}

SUSY unbroken

- Modular invariance determines completely (up to real couplings) the functional dependence $W(\tau)$
- It is not the case for K, but $\bar{\theta}$ is insensitive to K
- No-renormalisation theorems Ellis, Ferrara, Nanopoulos, PLB **114** (1982) 231

$\Lambda_{\mathrm{flavour/CP}}$

Λ_{SUSY}

m_{SUSY}

v

Corrections to $\bar{\theta} = 0$

M_{Pl}

SUSY unbroken

- Modular invariance determines completely (up to real couplings) the functional dependence $W(\tau)$

$\Lambda_{\text{flavour/CP}}$

- It is not the case for K, but $\bar{\theta}$ is insensitive to K
- No-renormalisation theorems Ellis, Ferrara, Nanopoulos, PLB **114** (1982) 231

Λ_{SUSY}

SUSY breaking corrections

- In general, can be large

m_{SUSY}

- Small if $\Lambda_{\text{flavour/CP}} \gg \Lambda_{\mathrm{SUSY}}$ (as e.g. in gauge mediation) and soft breaking terms respect the flavour structure of the SM

v

$$\bar{\theta} \lesssim \frac{M_t^4 M_b^4 M_c^2 M_s^2}{\mathrm{v}^{12}} J_{\mathrm{CP}} \tan^6 \beta \sim 10^{-28} \tan^6 \beta$$

Corrections to $\bar{\theta} = 0$

$M_{\rm Pl}$

$\Lambda_{\rm flavour/CP}$

$\Lambda_{\rm SUSY}$

$m_{\rm SUSY}$

$\rm v$

SUSY unbroken

- Modular invariance determines completely (up to real couplings) the functional dependence $W(\tau)$
- It is not the case for K, but $\bar{\theta}$ is insensitive to K
- No-renormalisation theorems Ellis, Ferrara, Nanopoulos, PLB **114** (1982) 231

SUSY breaking corrections

- In general, can be large
- Small if $\Lambda_{\rm flavour/CP} \gg \Lambda_{\rm SUSY}$ (as e.g. in gauge mediation) and soft breaking terms respect the flavour structure of the SM

$$\bar{\theta} \lesssim \frac{M_t^4 M_b^4 M_c^2 M_s^2}{\rm v^{12}} J_{\rm CP} \tan^6\beta \sim 10^{-28} \tan^6\beta$$

SM corrections are negligible

- $\bar{\theta} \sim 10^{-18}$ at four loops Khriplovich, PLB **173** (1986) 193 Ellis, Gaillard, NPB **150** (1979) 141

Conclusions

- Modular invariance is inherent to toroidal compactifications in string theory
- It can be consistently implemented in a supersymmetric QFT
- The VEV of the modulus τ is the only source of spontaneous CP violation

$$\bar{\theta} = \theta_{\text{QCD}} + \arg\det M_q$$

- $\theta_{\text{QCD}} = 0$ because the UV theory is CP-conserving
- $\arg\det M_q = 0$ because of anomaly-free modular symmetry
- Corrections to $\bar{\theta} = 0$ are small under certain assumptions on SUSY breaking

Back-up slides

Phenomenology and cosmology

- Couplings to matter are suppressed by $1/h$ ($1/M_{\rm Pl}$ in SUGRA)
- No couplings to gauge bosons in the exact SUSY limit
- $m_\tau \gtrsim 10$ TeV not to spoil BBN
- Fermionic component of τ could be LSP and maybe DM
- Scalar potential $V(\tau) = V(-\tau^*)$ => CP-conjugated minima
  (domain walls are inflated away if CP breaking occurs before inflation)

Modular group

Homogeneous modular group

$$\Gamma = \left\langle S,\, T \mid S^4 = (ST)^3 = I \right\rangle \cong \mathrm{SL}(2,\mathbb{Z})$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\tau \xrightarrow{S} -\frac{1}{\tau} \qquad \tau \xrightarrow{T} \tau + 1$$

duality

discrete shift symmetry

Inhomogeneous modular group

$$\overline{\Gamma} = \left\langle S,\, T \mid S^2 = (ST)^3 = I \right\rangle \cong \mathrm{PSL}(2,\mathbb{Z}) = \mathrm{SL}(2,\mathbb{Z})/\{I, -I\}$$

In other words, $\mathrm{SL}(2,\mathbb{Z})$ matrices γ and $-\gamma$ are identified

$$\tau \xrightarrow{\gamma} \gamma\tau = \frac{a\tau + b}{c\tau + d} \qquad \tau \xrightarrow{-\gamma} (-\gamma)\,\tau = \frac{-a\tau - b}{-c\tau - d} = \gamma\tau$$

Modular forms

Holomorphic functions on $\mathscr{H} = \{\tau \in \mathbb{C} : \ \mathrm{Im}(\tau) > 0\}$ transforming under Γ as

$$\boxed{f\left(\gamma\tau\right) = \left(c\tau + d\right)^k f(\tau)\,, \qquad \gamma \in \Gamma}$$

k is weight, a non-negative even integer

$$\gamma = -I \quad \Rightarrow \quad f(\tau) = (-1)^k f(\tau) \quad \Rightarrow \quad k \text{ is even}$$

Modular forms are periodic and admit q-expansions

$$\gamma = T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \Rightarrow \quad f(\tau + 1) = f(\tau) \quad \Rightarrow \quad f(\tau) = \sum_{n=0}^{\infty} a_n q^n\,, \quad q = e^{2\pi i \tau}$$

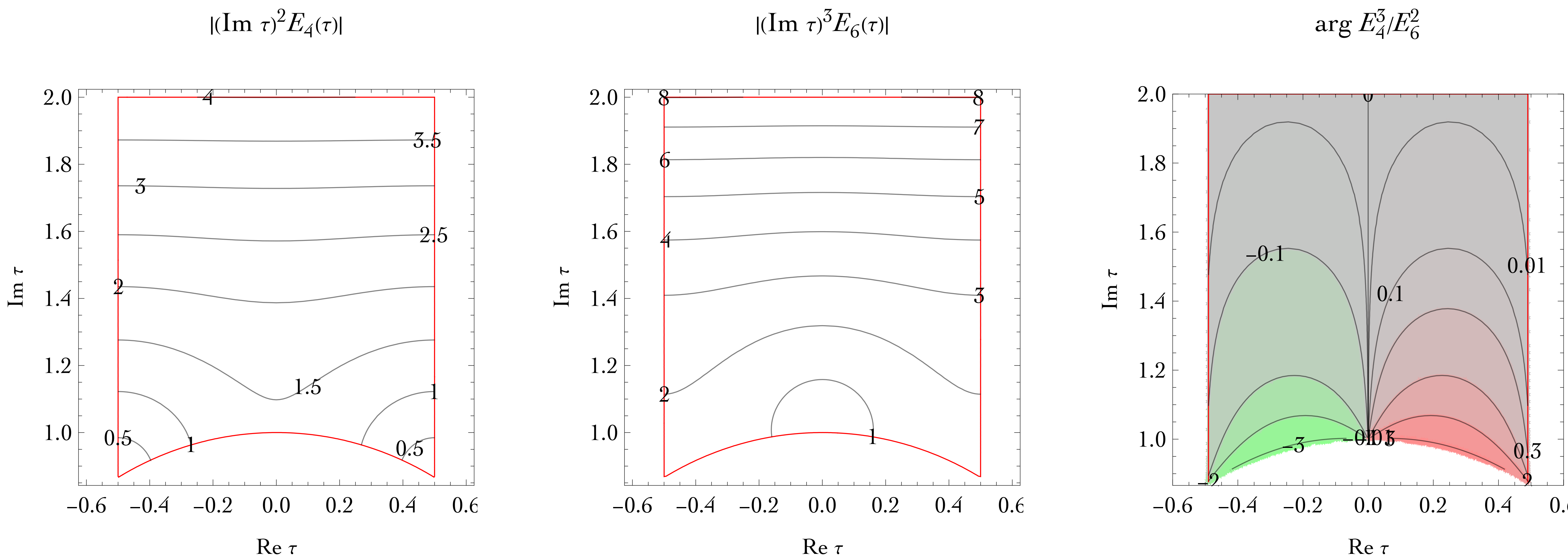
Modular forms of weight k form a linear space $\mathscr{M}_k$ of finite dimension

$$\dim \mathscr{M}_k = \begin{cases} 0 & \text{if } k \text{ is negative or odd} \\ \lfloor k/12 \rfloor & \text{if } k = 2 \pmod{12} \\ \lfloor k/12 \rfloor + 1 & \text{if } k \neq 2 \pmod{12} \end{cases}$$

E4 and E6

$$E_4(\tau) = 1 + 240 \sum_{n=1}^{\infty} \frac{n^3 q^n}{1 - q^n} = 1 + 240q + 2160q^2 + 6720q^3 + 17520q^4 + \mathcal{O}(q^5)$$

$$E_6(\tau) = 1 - 540 \sum_{n=1}^{\infty} \frac{n^5 q^n}{1 - q^n} = 1 - 504q - 16632q^2 - 122976q^3 - 532728q^4 + \mathcal{O}(q^5)$$



More on modular-gauge anomalies

$\Phi \to \Phi' = (c\tau + d)^{-k_\Phi} \Phi$

Jacobian J: $\mathscr{D}\Phi' = J\,\mathscr{D}\Phi$ Arkani-Hamed, Murayama, hep-th/9707133

$$\log J = -\frac{i}{64\pi^2} \int d^4x\, d^2\theta \left[\sum_\Phi T(\Phi)\, k_\Phi \right] W^a W^a \log(c\tau + d)$$

$T(\Phi)$ is the Dynkin index of the rep of Φ: $\mathrm{tr}\left(t_a t_b\right) = T(\Phi)\, \delta_{ab}$

$$\boxed{\sum_\Phi T(\Phi)\, k_\Phi = 0}$$

$$\mathrm{SU}(3)_C: \quad \sum_i \left(2k_{Q_i} + k_{u_{Ri}} + k_{d_{Ri}}\right) = 0$$

$$\mathrm{SU}(2)_L: \quad \sum_i \left(3k_{Q_i} + k_{L_i}\right) + k_{H_u} + k_{H_d} = 0$$

$$\mathrm{U}(1)_Y: \quad \sum_i \left(k_{Q_i} + 8k_{u_{Ri}} + 2k_{d_{Ri}} + 3k_{L_i} + 6k_{e_{Ri}}\right) + 3\left(k_{H_u} + k_{H_d}\right) = 0$$

Modular invariance and SUGRA

$\mathcal{N} = 1$ SUGRA action depends on

$$G = \frac{K}{M_{\text{Pl}}^2} + \log \left| \frac{W}{M_{\text{Pl}}^3} \right|^2$$

For G to be invariant, both K and W have to transform

$$K \to K + M_{\text{Pl}}^2 \left(F + F^\dagger\right) \qquad \text{and} \qquad W \to e^{-F} W$$

In the case of modular transformations

$$F = \frac{h^2}{M_{\text{Pl}}^2} \log(c\tau + d)$$

$$\boxed{W \to \left(c\tau + d\right)^{-k_W} W \qquad \text{with} \qquad k_W = \frac{h^2}{M_{\text{Pl}}^2} > 0}$$

The superpotential is a modular function, having singularities at some values of τ

$k_W \to 0$ rigid SUSY limit

Modular invariance and SUGRA

$$W = Y^u_{ij}(\tau)\, u_{Ri}\, Q_j\, H_u + Y^d_{ij}(\tau)\, d_{Ri}\, Q_j\, H_d$$

$$Y^q_{ij}(\tau) \to (c\tau + d)^{k^q_{ij}}\, Y^q_{ij}(\tau) \qquad \text{with} \qquad k^q_{ij} = k_{q_{Ri}} + k_{Q_j} + k_{H_q} - k_W$$

Furthermore, the Kähler transformation must be accompanied by a U(1) rotation

$$\psi \to e^{\frac{F - F^\dagger}{4}}\,\psi \qquad \qquad \lambda \to e^{-\frac{F - F^\dagger}{4}}\,\lambda$$

how gaugino enters the game

$$\psi_{\text{can}} \to \left(\frac{c\tau + d}{c\tau^\dagger + d}\right)^{\frac{k_W}{4} - \frac{k_\Phi}{2}} \psi_{\text{can}} \qquad \qquad \lambda \to \left(\frac{c\tau + d}{c\tau^\dagger + d}\right)^{-\frac{k_W}{4}} \lambda$$

Modular-QCD anomaly modifies as

$$A = \sum_{i=1}^{3} \left(2k_{Qi} + k_{u_{Ri}} + k_{d_{Ri}} - 2k_W\right) + Ck_W$$

$C = 3$ is quadratic Casimir of $\mathbf{8}$ of $\mathrm{SU}(3)_C$

Gluino mass

$$\bar{\theta} = \theta_{\text{QCD}} + \arg\det M_q + C \arg M_3$$

Assume $k_{\text{det}} = 0$ and the quark contribution to A vanishes. Then

$$\bar{\theta} = \theta_{\text{QCD}} + C \arg M_3$$

Gluino mass requires SUSY breaking

$$M_3 = \frac{g_3^2}{2} e^{K/2M_{\text{Pl}}^2} K^{i\bar{j}} D_{\bar{j}} W^\dagger f_i$$

Assuming $D_\tau W = 0$ and no additional phases from SUSY breaking

$$\arg M_3 = -\arg W$$

$$W = \ldots + \frac{c_0 M_{\text{Pl}}^3}{\eta(\tau)^{2k_W}} \qquad \text{and} \qquad f = \ldots + \frac{C k_W}{4\pi^2} \log \eta(\tau)$$

$$\bar{\theta} = -8\pi^2 \text{Im} f - C \arg W = 0$$

More on modular invariance in SUGRA

$$\det M_q \to \left(\frac{c\tau + d}{c\tau^\dagger + d}\right)^{\frac{k_{\det}}{2}} \det M_q \qquad k_{\det} = \sum_{i=1}^{3}\left(2k_{Q_i} + k_{u_{Ri}} + k_{d_{Ri}} - 2k_W\right) + 3\left(k_{H_u} + k_{H_d}\right)$$

$$M_3 \to \left(\frac{c\tau + d}{c\tau^\dagger + d}\right)^{\frac{k_W}{2}} M_3$$ (gluino mass arises only if SUSY is broken)

Finite modular groups

Infinite normal subgroups of $SL(2,\mathbb{Z})$, $N = 2\,,\, 3\,,\, 4\,, \ldots$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})\,, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

Principal congruence subgroups of the modular group

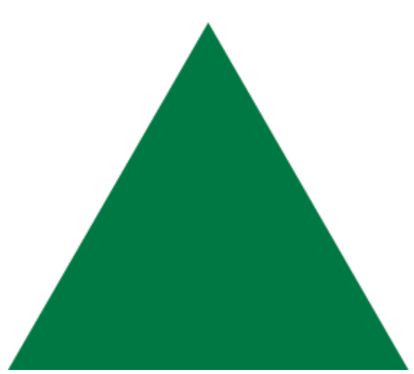
$$\overline{\Gamma}(2) \equiv \Gamma(2)/\{I, -I\} \qquad \overline{\Gamma}(N) \equiv \Gamma(N)\,, \quad N > 2$$

Finite modular groups

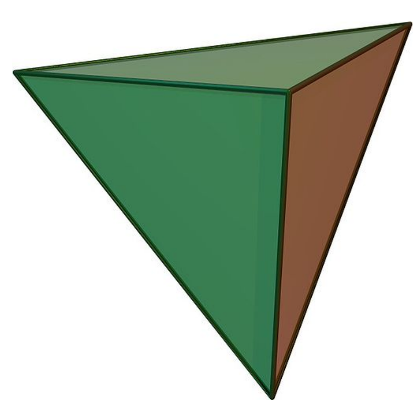
$$\Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N)$$

$$\Gamma_N = \left\langle S,\, T \mid S^2 = (ST)^3 = T^N = I \right\rangle\,, \quad N = 2\,,\, 3\,,\, 4\,,\, 5$$

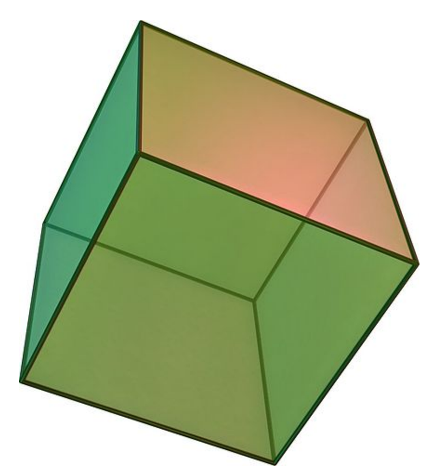
$\Gamma_2 \cong S_3$



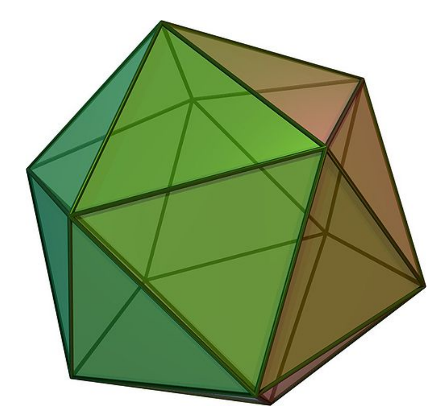
$\Gamma_3 \cong A_4$



$\Gamma_4 \cong S_4$



$\Gamma_5 \cong A_5$



Theories based on finite modular groups

$\mathcal{N}=1$ rigid SUSY matter action

$$\mathcal{S}=\int \mathrm{d}^4x\,\mathrm{d}^2\theta\,\mathrm{d}^2\overline{\theta}\;K(\tau,\overline{\tau},\psi,\overline{\psi})+\int \mathrm{d}^4x\,\mathrm{d}^2\theta\;W(\tau,\psi)+\int \mathrm{d}^4x\,\mathrm{d}^2\overline{\theta}\;\overline{W}(\overline{\tau},\overline{\psi})$$

Ferrara, Lust, Shapere, Theisen, PLB **225** (1989) 363

Ferrara, Lust, Theisen, PLB **233** (1989) 147

$$\begin{cases}\tau\to\dfrac{a\tau+b}{c\tau+d}\\ \psi_i\to\left(c\tau+d\right)^{-k_i}\rho_i(\tilde{\gamma})\,\psi_i\end{cases}\Rightarrow\begin{cases}W(\tau,\psi)\to W(\tau,\psi)\\ K(\tau,\overline{\tau},\psi,\overline{\psi})\to K(\tau,\overline{\tau},\psi,\overline{\psi})+f_K(\tau,\psi)+\overline{f_K}(\overline{\tau},\overline{\psi})\end{cases}$$

unitary representation of Γ_N

Feruglio, 1706.08749

$$W(\tau,\psi)=\sum_n\sum_{\{i_1,\ldots,i_n\}}\sum_s g_{i_1\ldots i_n,s}\left(Y_{i_1\ldots i_n,s}(\tau)\,\psi_{i_1}\ldots\psi_{i_n}\right)_{\mathbf{1},s}$$

$$\boxed{Y(\tau)\xrightarrow{\gamma}(c\tau+d)^{k_Y}\rho_Y(\tilde{\gamma})\,Y(\tau)}$$

$$k_Y=k_{i_1}+\ldots+k_{i_n}$$

$$\rho_Y\otimes\rho_{i_1}\otimes\ldots\otimes\rho_{i_n}\supset\mathbf{1}$$

Yukawa couplings are modular forms!

Models with finite modular symmetries

$$\begin{cases} \Phi \to \left(c\tau + d\right)^{-k_\Phi} \rho_\Phi(\gamma)\, \Phi \\ Y \to \left(c\tau + d\right)^{k_Y} \rho_Y(\gamma)\, Y \end{cases}$$

Feruglio, 1706.08749

ρ is a unitary representation of the finite modular group $\Gamma_N = \Gamma/\Gamma(N)$

$$\Gamma_2 \cong S_3 \qquad \Gamma_3 \cong A_4' = T' \qquad \Gamma_4 \cong S_4' \qquad \Gamma_5 \cong A_5'$$

Invariance of the superpotential $W \sim Y(\tau)\, \Phi_I \Phi_J \Phi_K$ under Γ and Γ_N requires

$$\begin{cases} k_Y = k_I + k_J + k_K \\ \rho_Y \otimes \rho_I \otimes \rho_J \otimes \rho_K \supset \mathbf{1} \end{cases}$$

Models with finite modular symmetries

$$\begin{cases} \Phi \to (c\tau + d)^{-k_\Phi} \rho_\Phi(\gamma)\, \Phi \\ Y \to (c\tau + d)^{k_Y} \rho_Y(\gamma)\, Y \end{cases}$$

Feruglio, 1706.08749

ρ is a unitary representation of the finite modular group $\Gamma_N = \Gamma / \Gamma(N)$

$$\Gamma_2 \cong S_3 \qquad \Gamma_3 \cong A_4' = T' \qquad \Gamma_4 \cong S_4' \qquad \Gamma_5 \cong A_5'$$

Invariance of the superpotential $W \sim Y(\tau)\, \Phi_I \Phi_J \Phi_K$ under Γ and Γ_N requires

$$\begin{cases} k_Y = k_I + k_J + k_K \\ \rho_Y \otimes \rho_I \otimes \rho_J \otimes \rho_K \supset \mathbf{1} \end{cases}$$

Penedo, Petcov, 2404.08032

Multi-dim irreps $\rho \sim \mathbf{r}\,, \quad \mathbf{r} > 1$ for SM quarks do not allow to simultaneously

- realise the proposed mechanism for $\bar{\theta} = 0$
- successfully describe quark masses and mixings

For the mechanism to work, SM quarks must furnish 1D irreps $\rho \sim \mathbf{1},\ \mathbf{1}',\ \mathbf{1}'', \ldots$

Models with finite modular symmetries

Minimal models (6 Lagrangian parameters per sector) Penedo, Petcov, 2404.08032

$$Y^q = \begin{pmatrix} c^q_{11} & 0 & c^q_{13} F^{(k')}_{\mathbf{1}^*} + c'^q_{13} F'^{(k')}_{\mathbf{1}^*} \\ 0 & c^q_{22} & c^q_{23} F^{(k)}_{\mathbf{1}^*} \\ 0 & 0 & c^q_{33} \end{pmatrix}$$

(up to weak basis transformations)

Modular weights are large

String compactifications
lead to smaller weights

(For models based on modular A_4
see also Petcov, Tanimoto, 2404.00858)

	Minimal models (I and II)
All Γ'_N	$(k, k') = (10, 12),\ (12, 14),\ (14, 16)$
S_3 only	$(10', 12),\ (10, 18'),\ (10', 18),\ (12, 12'),\ (12, 14'),\ (12, 16'),\ (12', 16),\ (12, 20'),\ (12', 20),\ (14', 16),\ (14, 18'),\ (14', 18),\ (14, 22'),\ (14', 22),\ (16, 16'),\ (16', 18),\ (16', 18'),\ (16, 20'),\ (16', 20),\ (18, 20'),\ (18', 20'),\ (20, 20'),\ (20', 22),\ (20', 22')$
A'_4 only	$(8', 12),\ (8', 18),\ (10', 12),\ (10, 16'),\ (10', 16),\ (10, 20''),\ (10', 20),\ (12, 12'),\ (12, 12''),\ (12, 14'),\ (12, 14''),\ (12, 16''),\ (12', 16),\ (12'', 16'),\ (12, 18'),\ (12, 18''),\ (12', 18),\ (12'', 18),\ (12, 22''),\ (12', 22),\ (12'', 22'),\ (14, 16'),\ (14', 16),\ (14', 16'),\ (14'', 16),\ (14'', 16'),\ (14', 18),\ (14, 20'),\ (14, 20''),\ (14', 20),\ (14', 20''),\ (14'', 20),\ (14'', 20'),\ (14, 24''),\ (14'', 24'),\ (16, 16''),\ (16', 16''),\ (16, 18'),\ (16, 18''),\ (16', 18'),\ (16', 18''),\ (16'', 18),\ (16'', 20'),\ (16, 22''),\ (16', 22''),\ (16'', 22),\ (16'', 22'),\ (16'', 26'),\ (18, 18'),\ (18, 18''),\ (18', 20),\ (18', 20'),\ (18', 20''),\ (18'', 20),\ (18'', 20'),\ (18'', 20''),\ (18, 22''),\ (18', 22),\ (18'', 22'),\ (18', 24''),\ (18'', 24'),\ (20, 22''),\ (20', 22''),\ (20'', 22''),\ (22, 22''),\ (22', 22''),\ (22'', 24'),\ (22'', 24''),\ (22'', 26')$

General conditions for $\bar{\theta} = 0$ and $\delta_{\text{CKM}} \neq 0$

$$\mathscr{L}_{\text{Yuk}} = U_i^c \, Y_{ij}^u \, Q_j \, H_u + D_i^c \, Y_{ij}^d \, Q_j \, H_d$$

1. $Y_{ij}^q = \sum_{\alpha_1 \dots \alpha_N} c^q_{ij,\alpha_1 \dots \alpha_N} z_1^{\alpha_1} \dots z_N^{\alpha_N}$ with z_a being 'scalars' with complex VEVs
   $\alpha_a \geq 0$ (no singularities)

2. $Y_{ij}^q \left(\lambda^{k_1} z_1, \dots, \lambda^{k_N} z_N \right) = \lambda^{d_{ij}^q} \, Y_{ij}^q \left(z_1, \dots, z_N \right) \quad \forall \lambda \in \mathbb{C}$ with $k_a \geq 0$

3. $\det \left[Y_{ij}^q \left(\lambda^{k_1} z_1, \dots, \lambda^{k_N} z_N \right) \right] = \lambda^{d_q} \det \left[Y_{ij}^q (z_1, \dots, z_N) \right]$ with $d_q \equiv \sum_i d_{ii}^q$

4. $d \equiv d_u + d_d = 0$

Feruglio, Parriciatu, Strumia, **AT**, 2406.01689

General conditions for $\bar{\theta} = 0$ and $\delta_{\mathrm{CKM}} \neq 0$

$$\mathscr{L}_{\mathrm{Yuk}} = U_i^c \, Y_{ij}^u \, Q_j \, H_u + D_i^c \, Y_{ij}^d \, Q_j \, H_d$$

$$d_{ij}^u = k_{U_i^c} + k_{Q_j} + k_{H_u} \qquad\qquad d_{ij}^d = k_{D_i^c} + k_{Q_j} + k_{H_d}$$

$$\det\left[Y_{ij}^u\left(\lambda^{k_1} z_1, \ldots, \lambda^{k_N} z_N\right)\right] = \det\left[\lambda^{d_{ij}^u} \, Y_{ij}^u(z_1, \ldots, z_N)\right] = \lambda^{d_u} \det\left[Y_{ij}^u(z_1, \ldots, z_N)\right]$$

$$d_u = \sum_{i=1}^{3}\left(k_{U_i^c} + k_{Q_i}\right) + 3k_{H_u} \qquad\qquad d_d = \sum_{i=1}^{3}\left(k_{D_i^c} + k_{Q_i}\right) + 3k_{H_d}$$

$$\boxed{d = \sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c}\right) + 3\left(k_{H_u} + k_{H_d}\right) = 0}$$

Feruglio, Parriciatu, Strumia, **AT**, 2406.01689

Patterns of Yukawa's for $\bar{\theta} = 0$ and $\delta_{\rm CKM} \neq 0$

$$Y = \begin{pmatrix} Y_{11}^{(0)} & Y_{12}^{(p_1-p_2)} & Y_{13}^{(p_1-p_3)} \\ Y_{21}^{(p_2-p_1)} & Y_{22}^{(0)} & Y_{23}^{(p_2-p_3)} \\ Y_{31}^{(p_3-p_1)} & Y_{32}^{(p_3-p_2)} & Y_{33}^{(0)} \end{pmatrix} \quad \text{with} \quad \det Y = \text{const}$$

- $p_1 = p_2 = p_3 \quad \Rightarrow \quad Y = \begin{pmatrix} Y_{11}^{(0)} & Y_{12}^{(0)} & Y_{13}^{(0)} \\ Y_{21}^{(0)} & Y_{22}^{(0)} & Y_{23}^{(0)} \\ Y_{31}^{(0)} & Y_{32}^{(0)} & Y_{33}^{(0)} \end{pmatrix} = \text{const}$
- $p_1 = p_2 \neq p_3 \quad \Rightarrow \quad Y = \begin{pmatrix} Y_{11}^{(0)} & Y_{12}^{(0)} & Y_{13}^{(p_1-p_3)}(z) \\ Y_{21}^{(0)} & Y_{22}^{(0)} & Y_{23}^{(p_1-p_3)}(z) \\ 0 & 0 & Y_{33}^{(0)} \end{pmatrix}$
- $p_1 \neq p_2 \neq p_3 \quad \Rightarrow \quad Y = \begin{pmatrix} Y_{11}^{(0)} & Y_{12}^{(p_1-p_2)}(z) & Y_{13}^{(p_1-p_3)}(z) \\ 0 & Y_{22}^{(0)} & Y_{23}^{(p_2-p_3)}(z) \\ 0 & 0 & Y_{33}^{(0)} \end{pmatrix}$

Feruglio, Parriciatu, Strumia, **AT**, 2406.01689 (see also Penedo, Petcov, 2404.08032)

QFT realisation with SUSY

Y depend on z_a and not on $\bar{z}_a$ => SUSY

$$W(\Phi) = U_i^c\, Y_{ij}^u(Z)\, Q_j\, H_u + D_i^c\, Y_{ij}^d(Z)\, Q_j\, H_d$$

$Z = \{Z_1, \ldots, Z_N\}$ dimensionless gauge-invariant chiral superfields

$M = \{U_i^c, D_i^c, E_i^c, Q_i, L_i, H_{u,d}\}$ matter and Higgs chiral superfields

$\Phi = \{M, Z\}$

The general conditions 1–4 are realised assuming invariance under

$$Q_i \to \Lambda^{-k_{Q_i}} Q_i \qquad U_i^c \to \Lambda^{-k_{U_i^c}} U_i^c \qquad D_i^c \to \Lambda^{-k_{D_i^c}} D_i^c \qquad Z_a \to \Lambda^{k_{Z_a}} Z_a$$

$$\boxed{\begin{aligned} Y_{ij}^u\left(\Lambda^{k_{Z_1}} Z_1, \ldots, \Lambda^{k_{Z_N}} Z_N\right) &= \Lambda^{k_{U_i^c}+k_{Q_j}+k_{H_u}}\, Y_{ij}^u\left(Z_1 \ldots, Z_N\right) \\ Y_{ij}^d\left(\Lambda^{k_{Z_1}} Z_1, \ldots, \Lambda^{k_{Z_N}} Z_N\right) &= \Lambda^{k_{D_i^c}+k_{Q_j}+k_{H_d}}\, Y_{ij}^d\left(Z_1, \ldots, Z_N\right) \end{aligned}}$$

Quark masses and mixings

At the GUT scale of 2×10^{16} GeV,
assuming MSSM with $\tan\beta = 10$ and SUSY breaking scale of 10 TeV

m_u/m_c	$(1.93 \pm 0.60) \times 10^{-3}$
m_c/m_t	$(2.82 \pm 0.12) \times 10^{-3}$
m_d/m_s	$(5.05 \pm 0.62) \times 10^{-2}$
m_s/m_b	$(1.82 \pm 0.10) \times 10^{-2}$
$\sin^2\theta_{12}$	$(5.08 \pm 0.03) \times 10^{-2}$
$\sin^2\theta_{13}$	$(1.22 \pm 0.09) \times 10^{-5}$
$\sin^2\theta_{23}$	$(1.61 \pm 0.05) \times 10^{-3}$
δ/π	0.385 ± 0.017

$$m_t = 87.46 \text{ GeV}$$
$$m_b = 0.9682 \text{ GeV}$$

Antusch, Maurer, 1306.6879
Yao, Lu, Ding, 2012.13390

Lepton masses and mixings

NuFIT 5.2 (2022)

with SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.4$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
$\sin^2\theta_{23}$	$0.451^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.569^{+0.016}_{-0.021}$	$0.412 \to 0.613$
$\theta_{23}/^\circ$	$42.2^{+1.1}_{-0.9}$	$39.7 \to 51.0$	$49.0^{+1.0}_{-1.2}$	$39.9 \to 51.5$
$\sin^2\theta_{13}$	$0.02225^{+0.00056}_{-0.00059}$	$0.02052 \to 0.02398$	$0.02223^{+0.00058}_{-0.00058}$	$0.02048 \to 0.02416$
$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.94$
$\delta_{\mathrm{CP}}/^\circ$	232^{+36}_{-26}	$144 \to 350$	276^{+22}_{-29}	$194 \to 344$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.507^{+0.026}_{-0.027}$	$+2.427 \to +2.590$	$-2.486^{+0.025}_{-0.028}$	$-2.570 \to -2.406$

Esteban et al., 2007.14792 and www.nu-fit.org

$$m_e/m_\mu = 0.0048 \pm 0.0002$$

$$m_\mu/m_\tau = 0.0565 \pm 0.0045$$