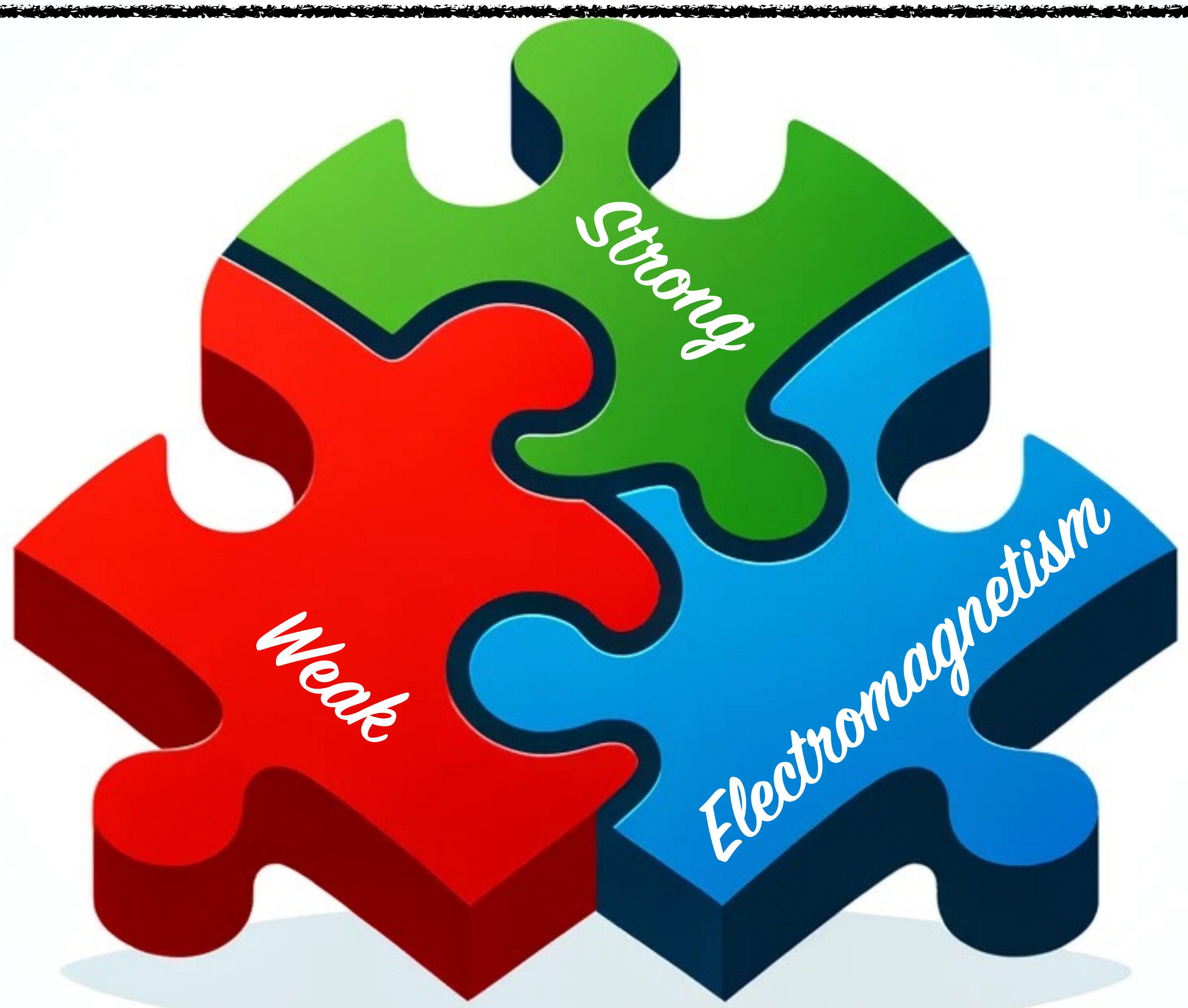
Proton Decay and Gravitational Waves as Complementary Tests of Grand Unification



Jessica Turner

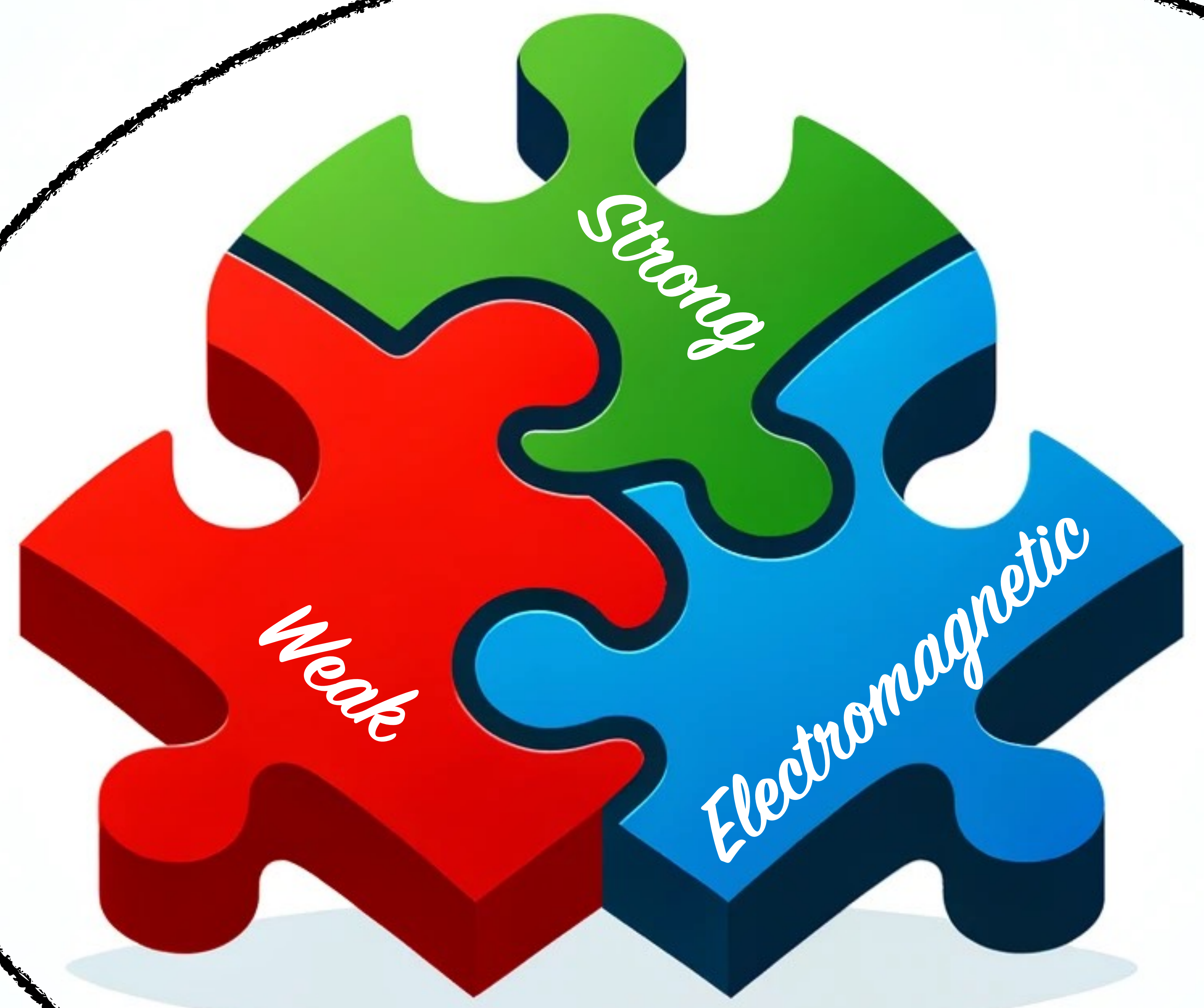
Institute for Particle Physics Phenomenology, Durham University

Durham University

INVISIBLES24

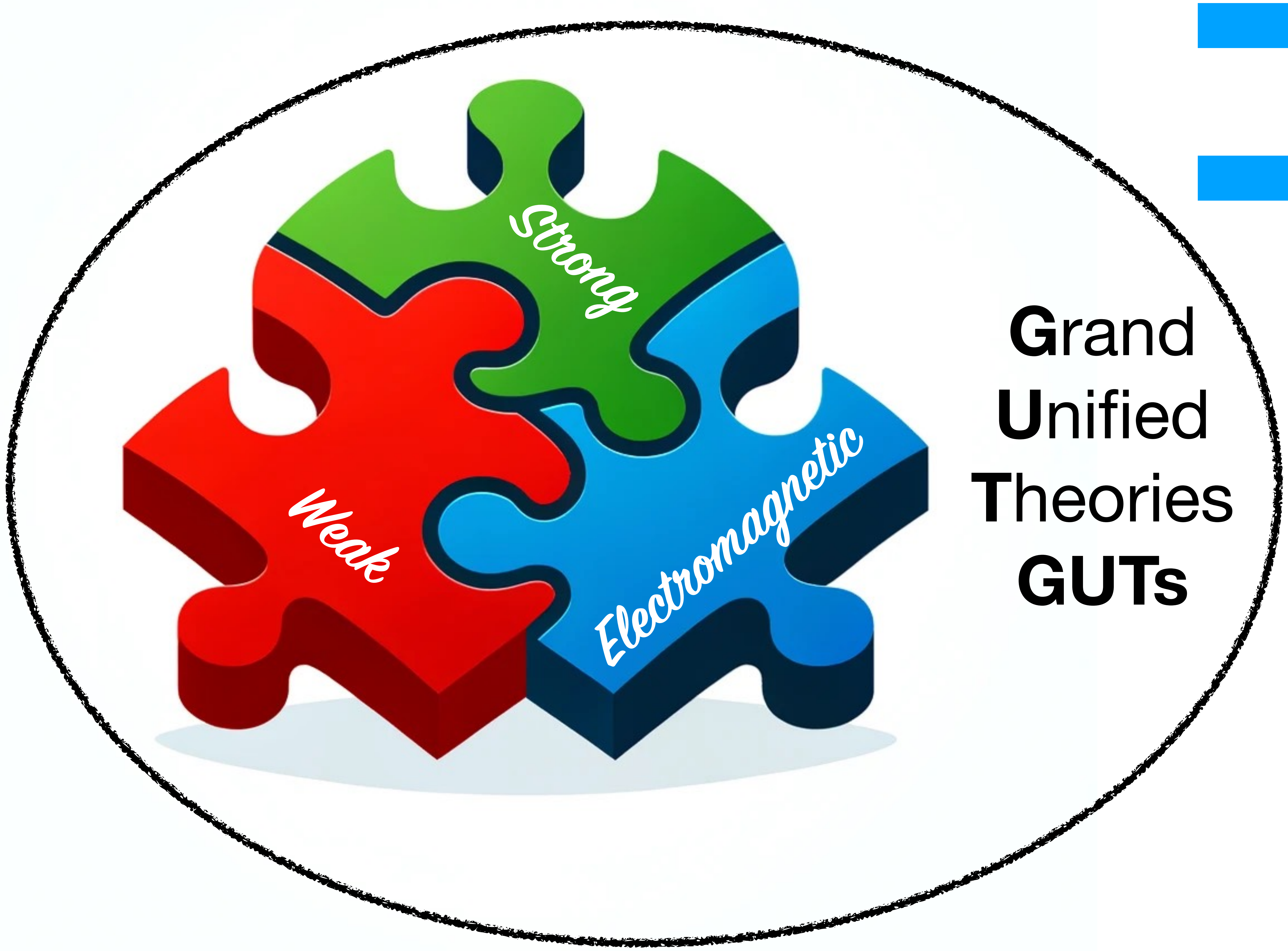
IP3

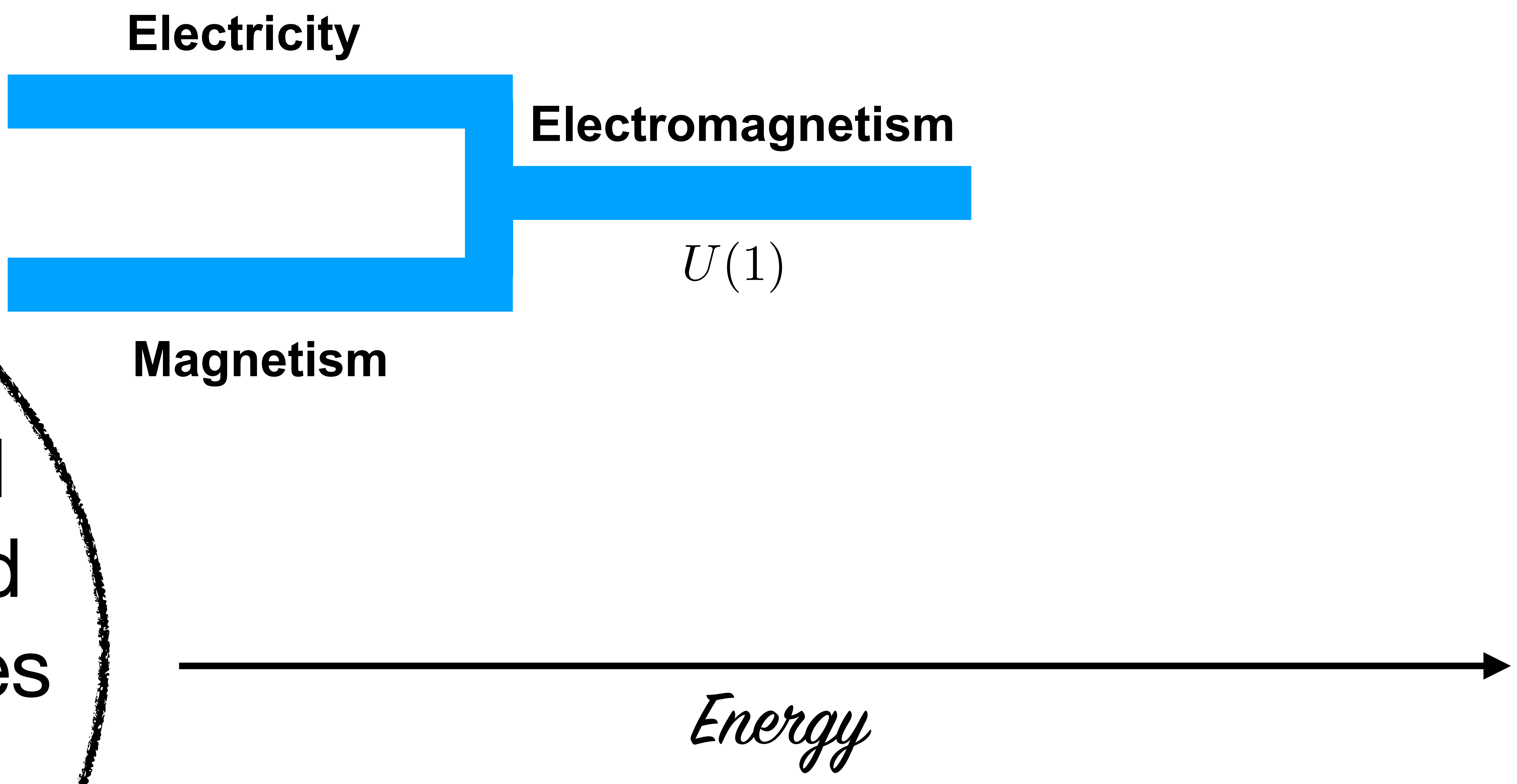
Motivation for Grand Unification



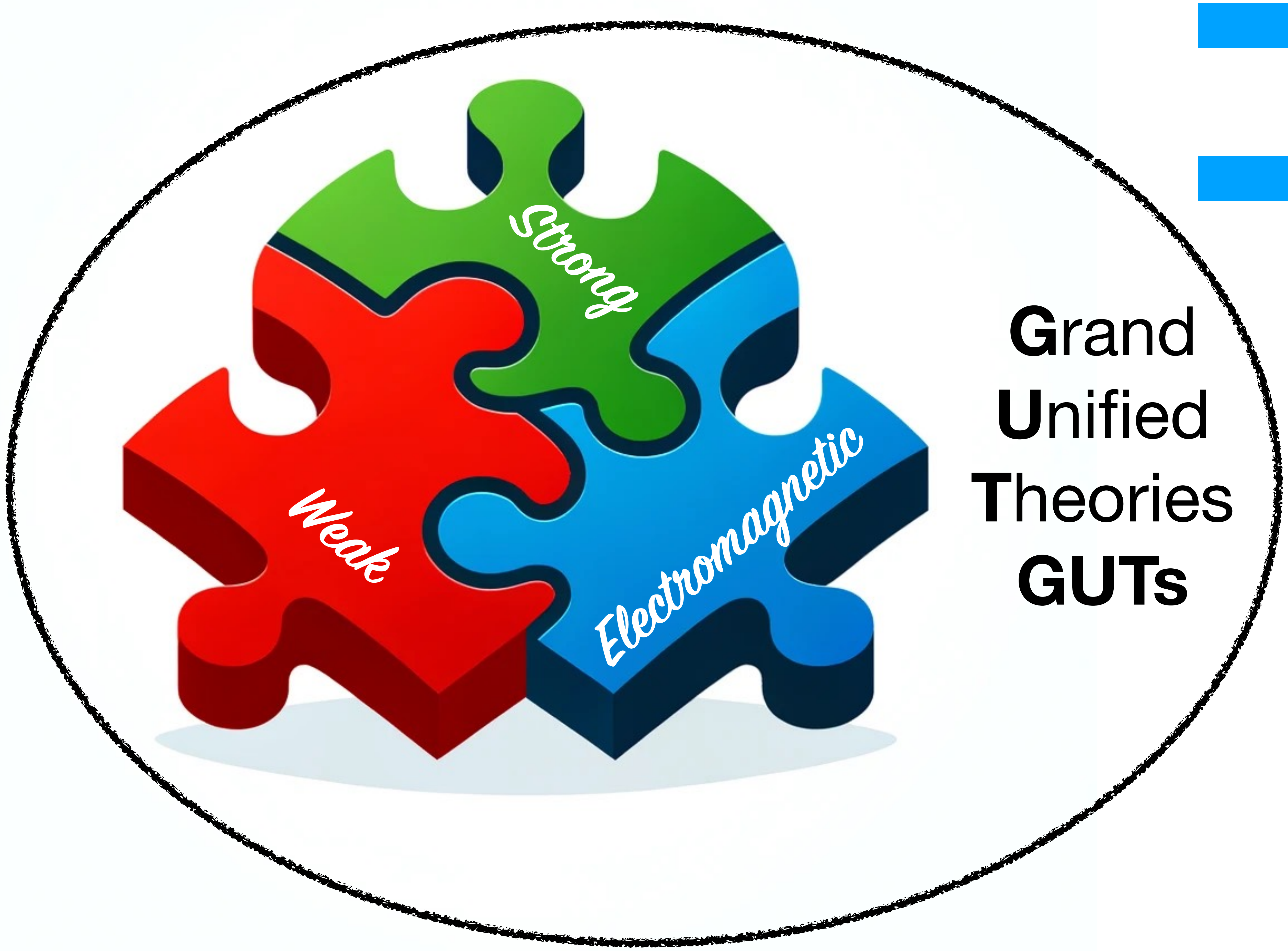
Grand
Unified
Theories
GUTs

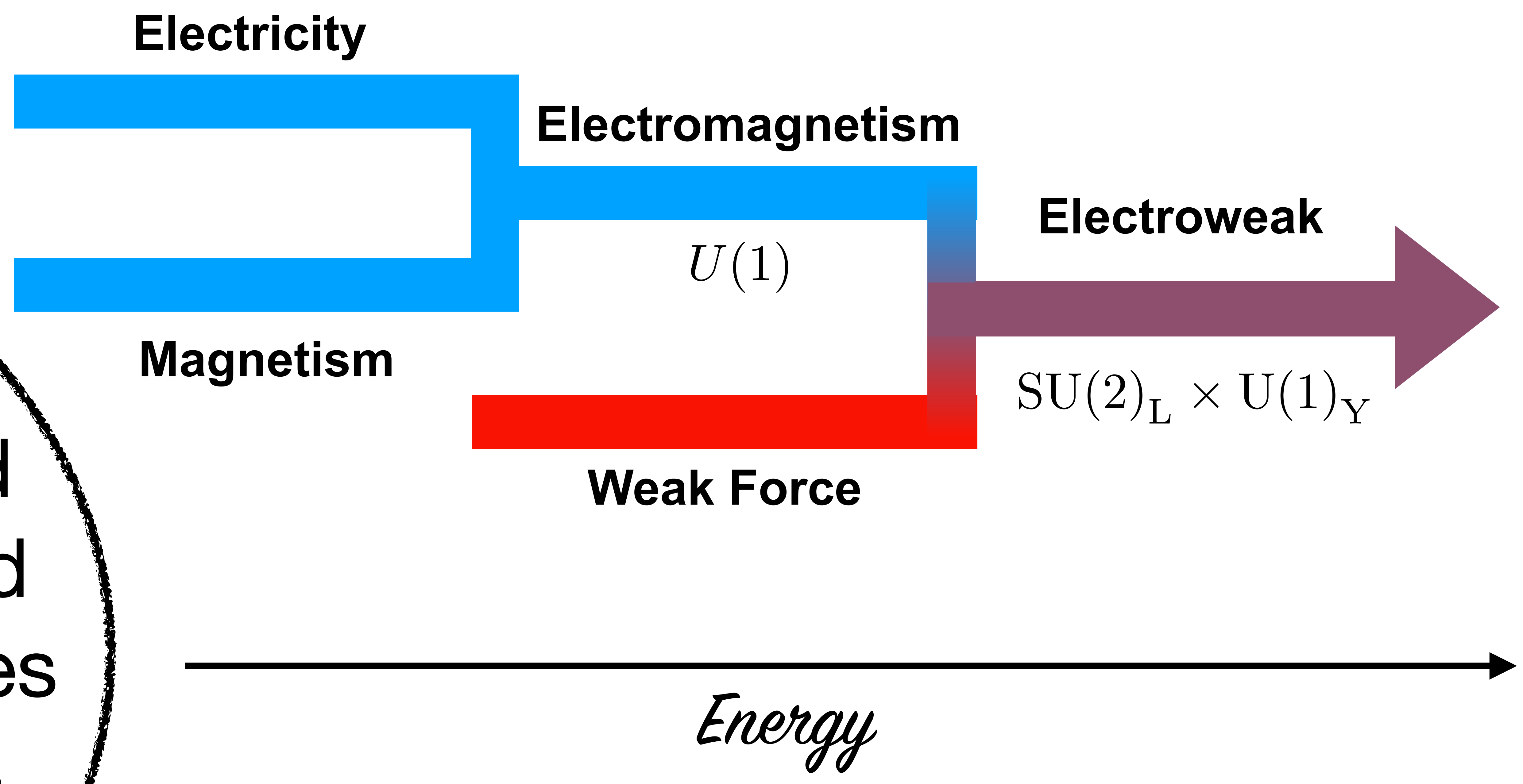
Motivation for Grand Unification



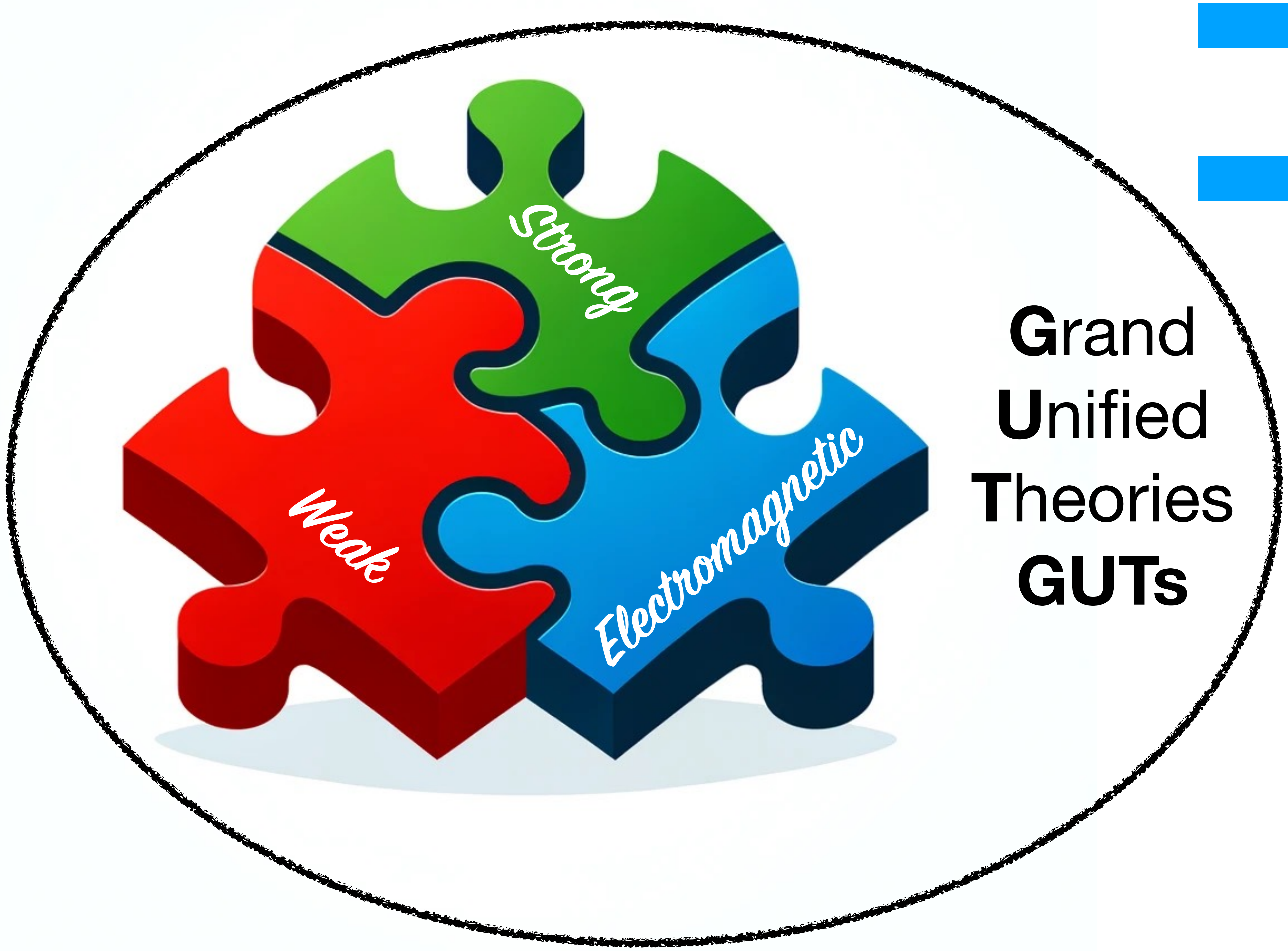


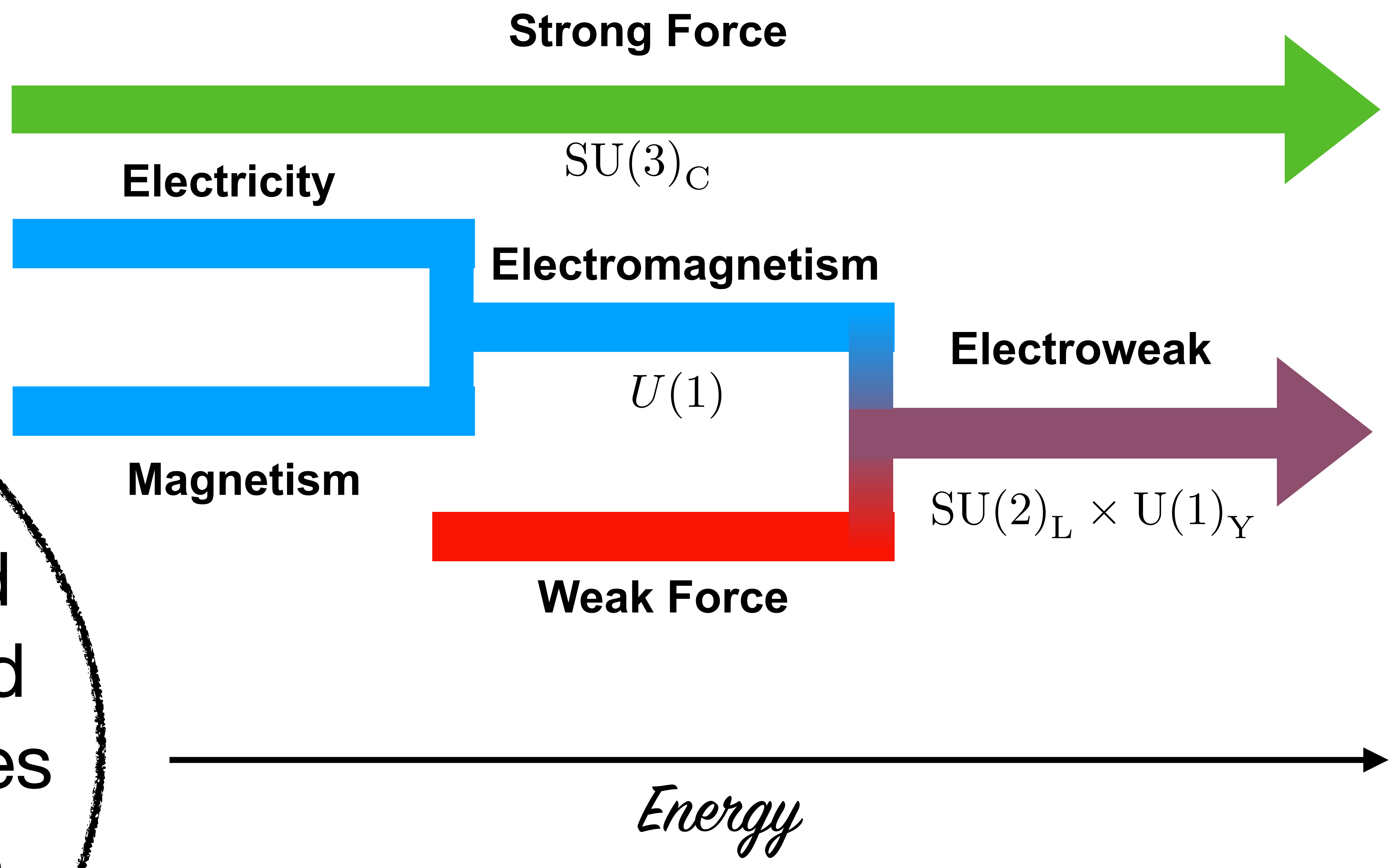
Motivation for Grand Unification



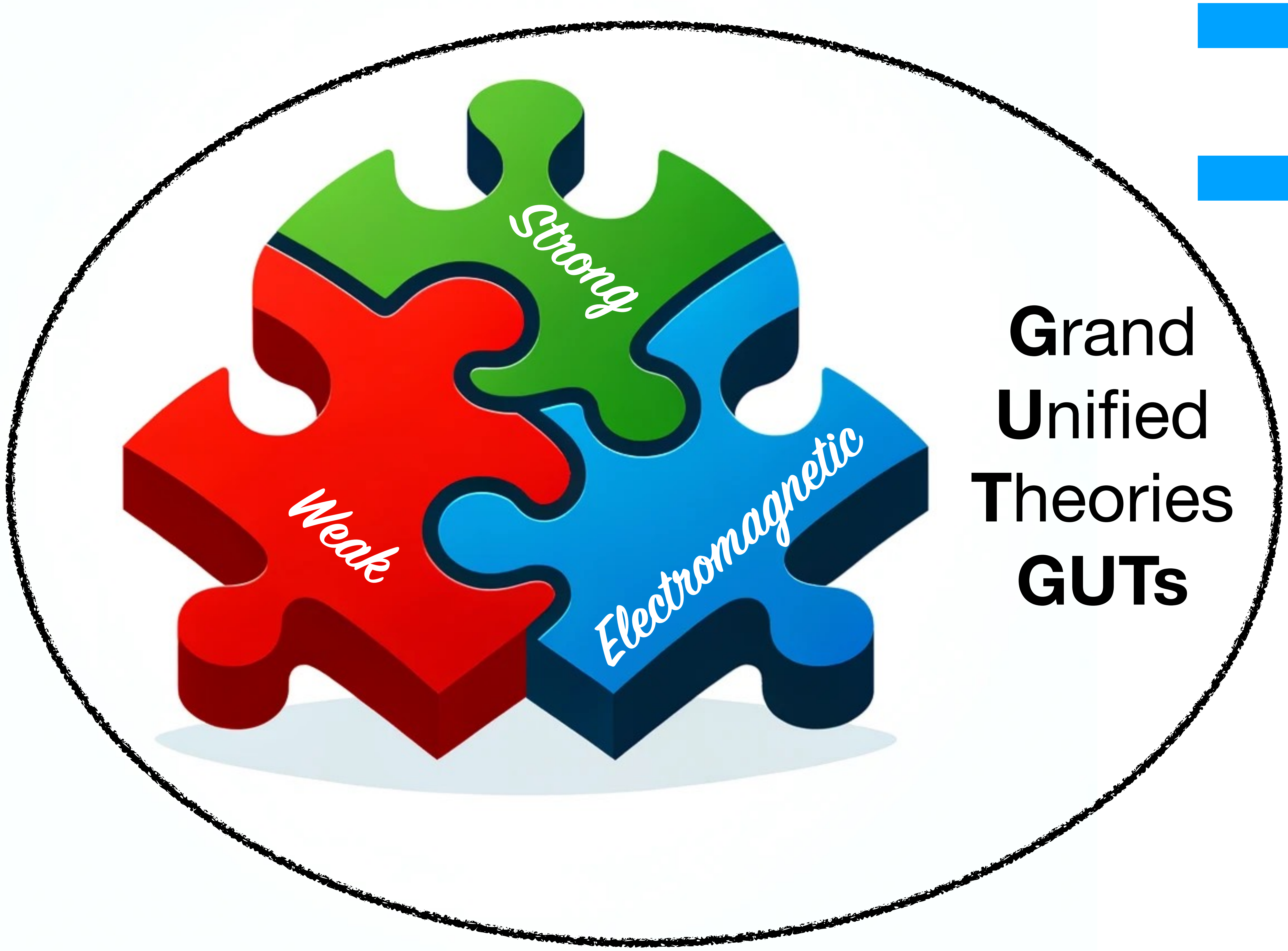


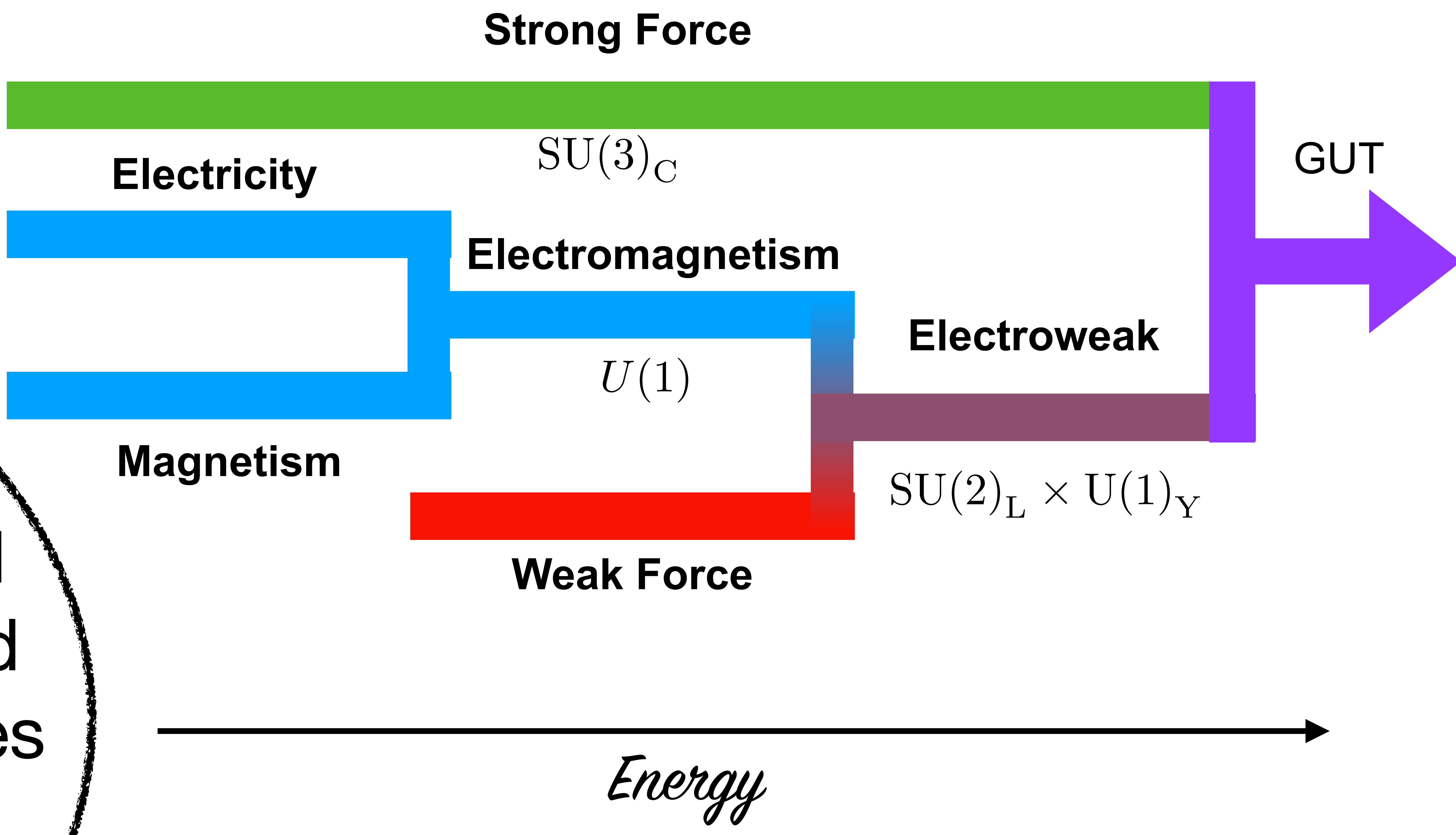
Motivation for Grand Unification



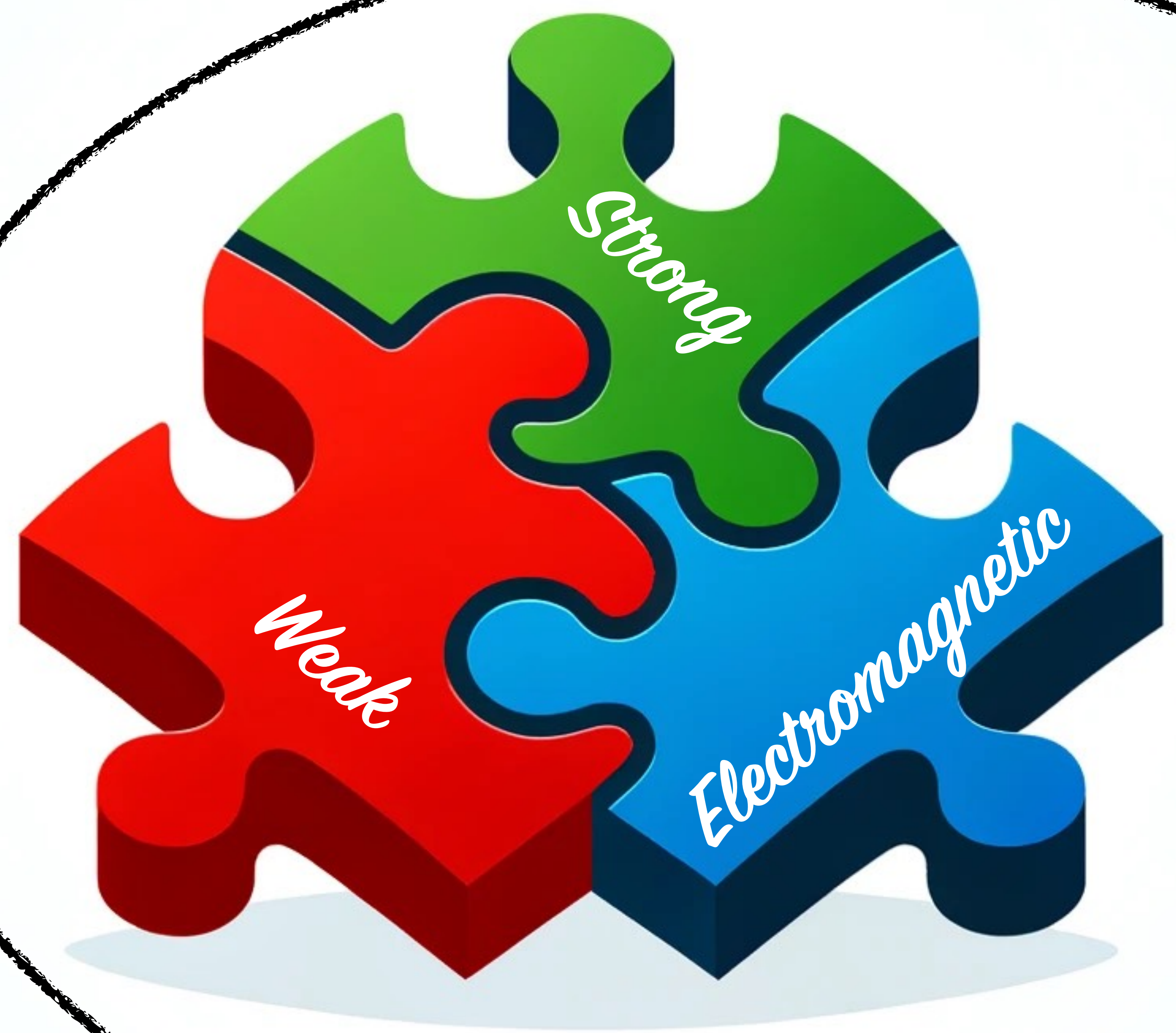


Motivation for Grand Unification

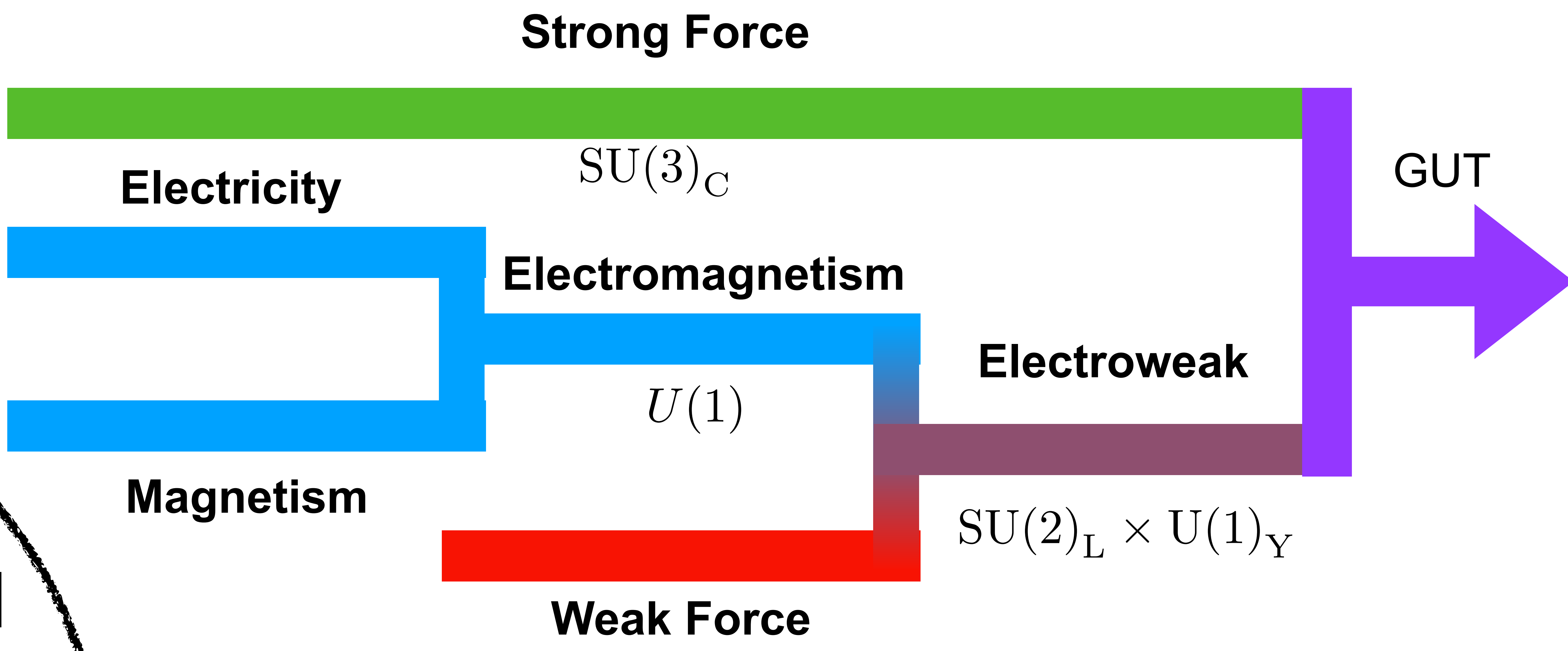




Motivation for Grand Unification



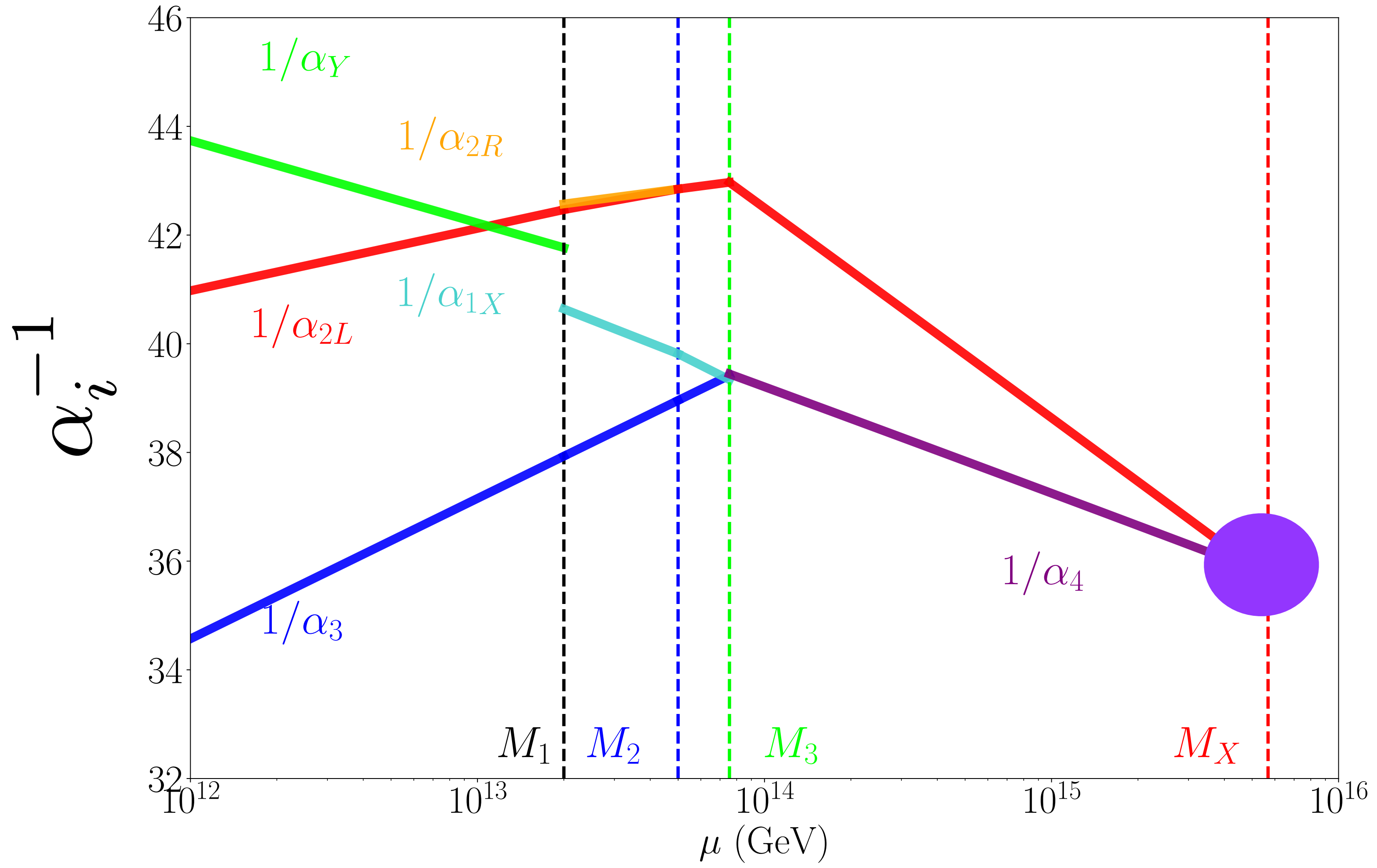
Grand
Unified
Theories
GUTs



Forces are unified

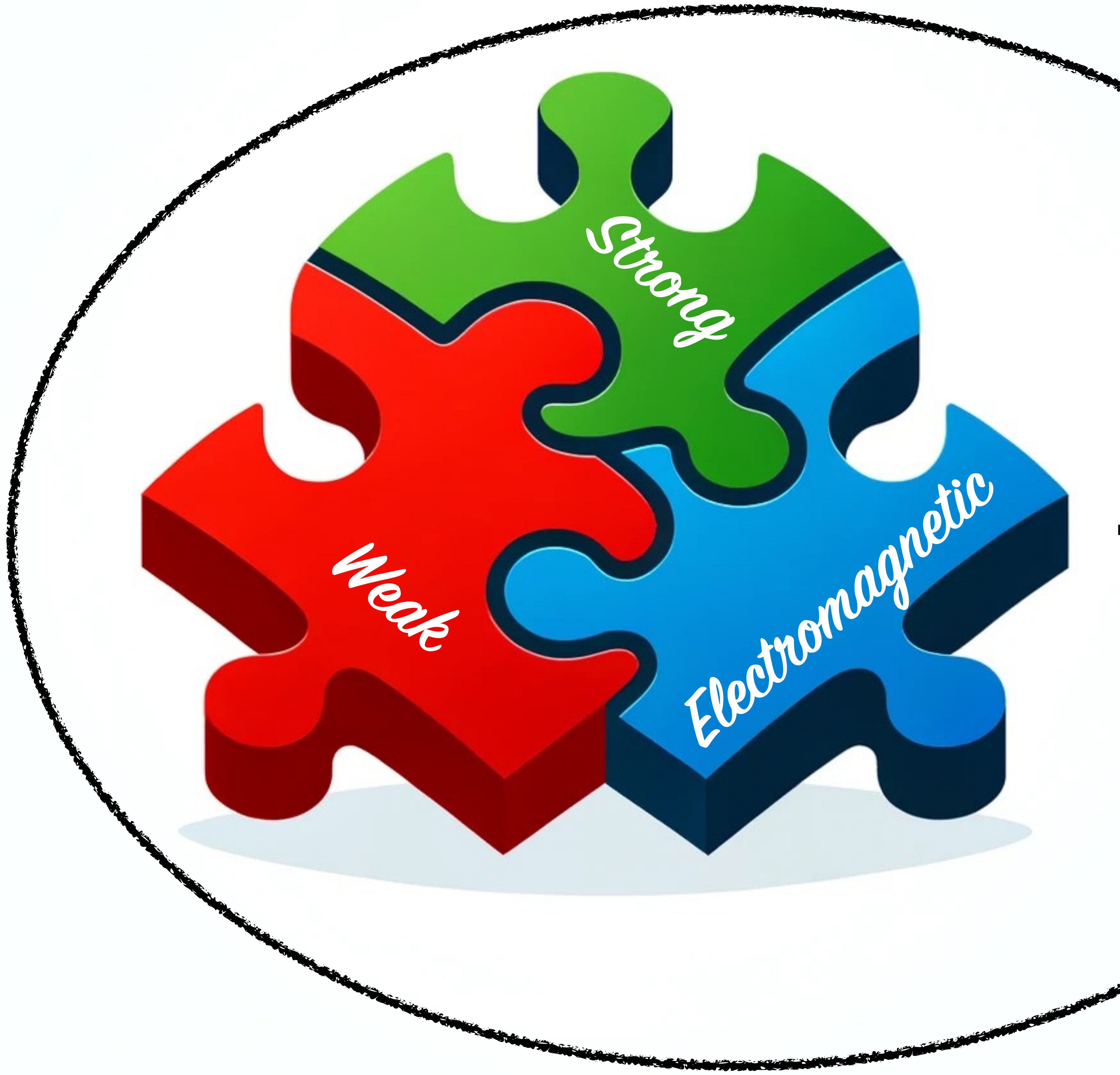
$$G_{\mathrm{GUT}} \supset G_{\mathrm{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

g_X $\quad g_3 \quad g_2 \quad g_1$

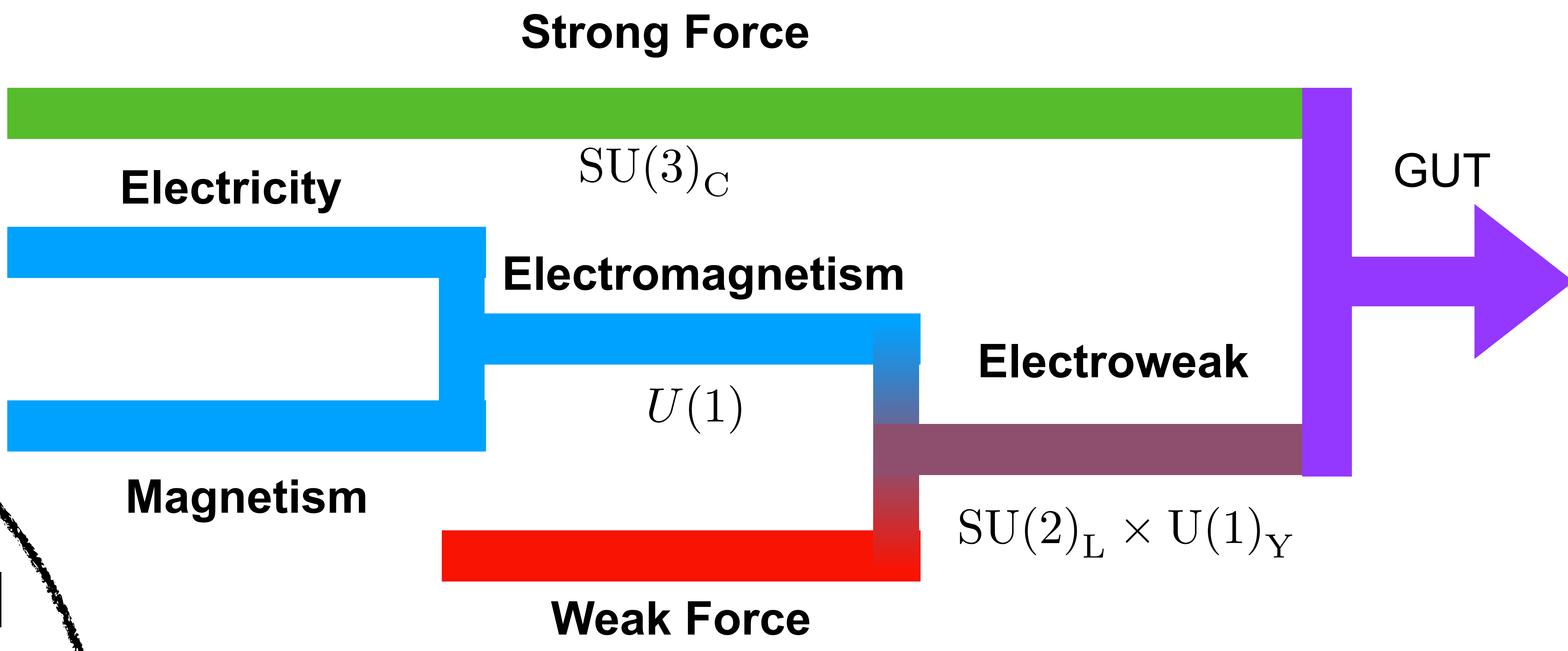


GUT scale
M_X

Motivation for Grand Unification



Grand Unified Theories GUTs

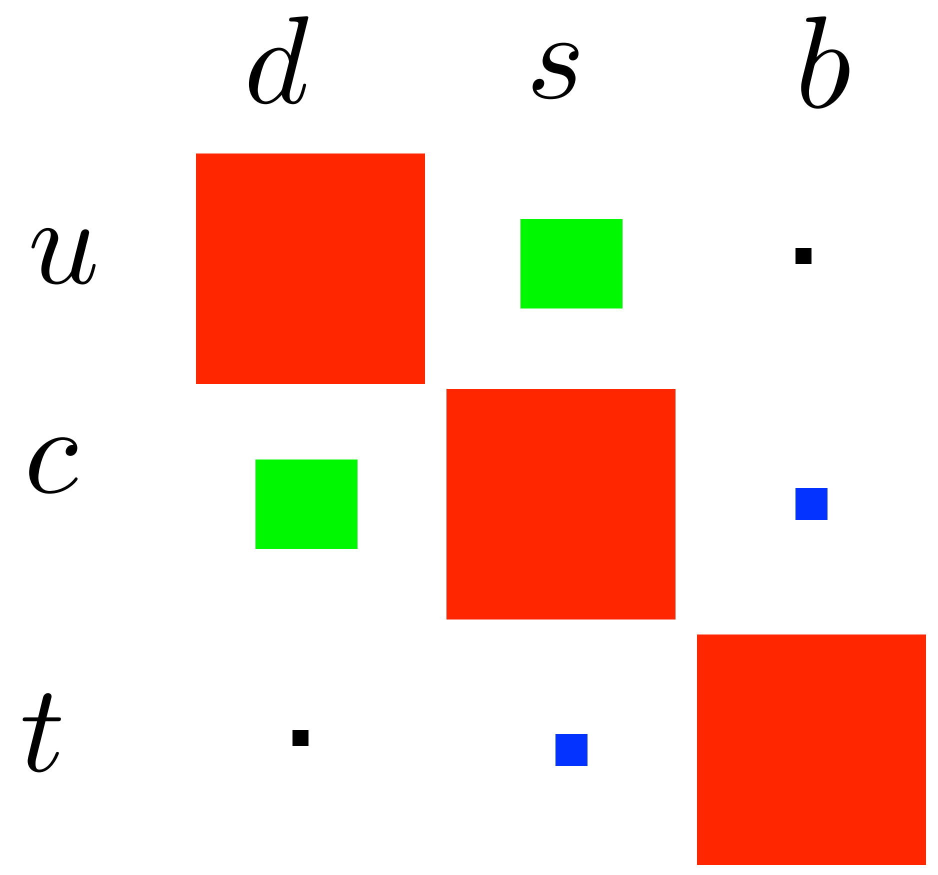


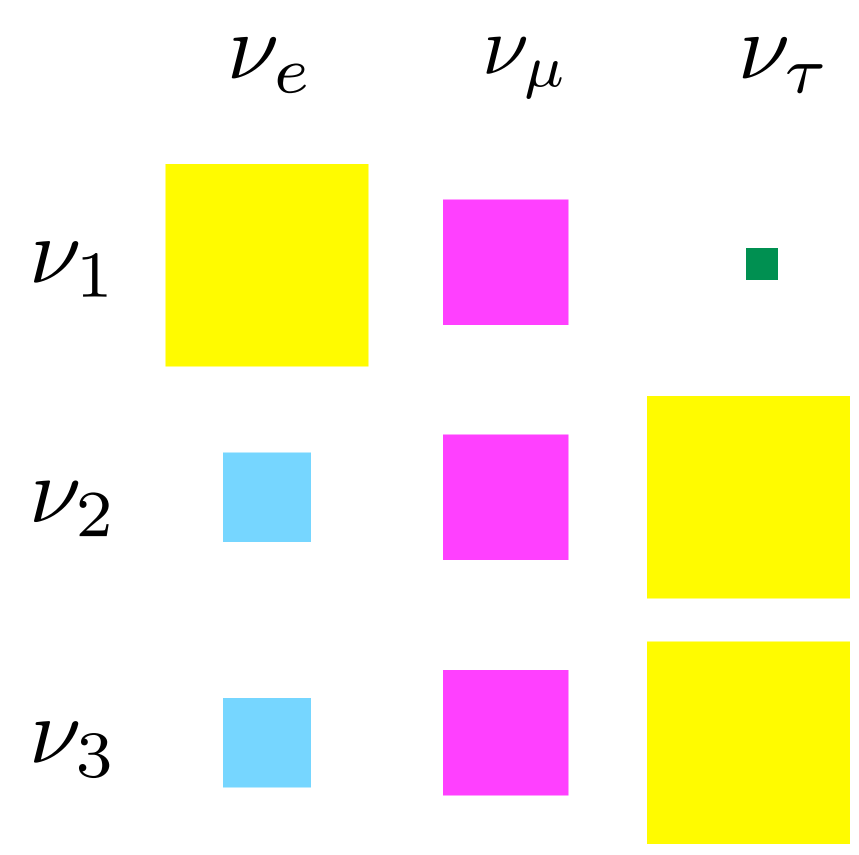
Matter is unified

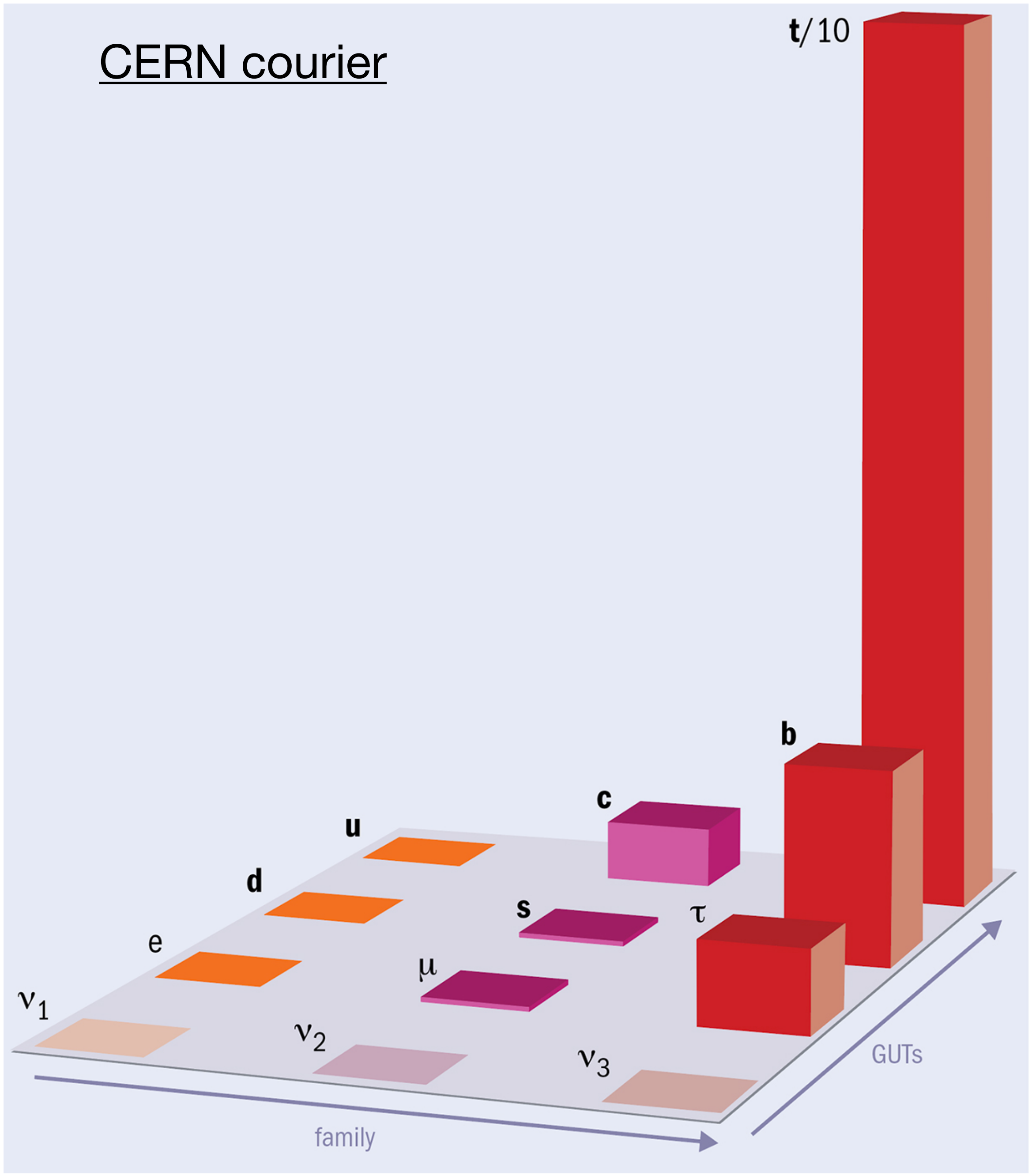
$$\bar{\mathbf{5}} : \begin{pmatrix} d_r \\ d_g \\ d_b \\ e \\ \nu_e \end{pmatrix}$$

Motivation for Grand Unification

Flavour Puzzle

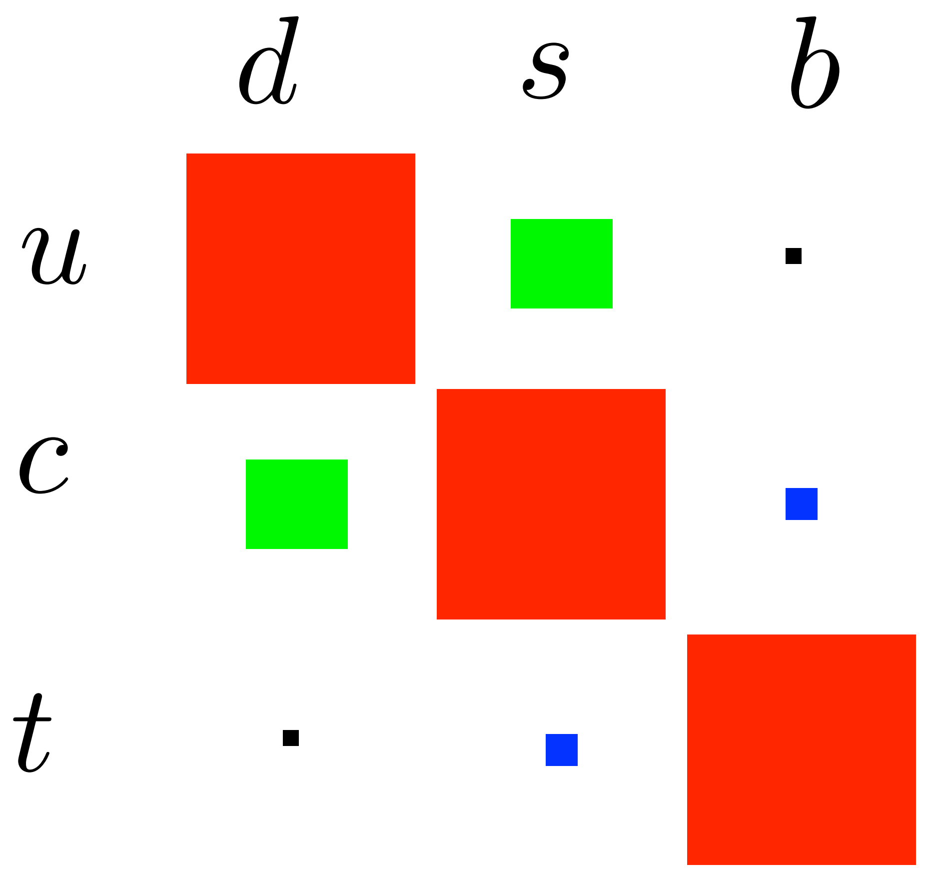


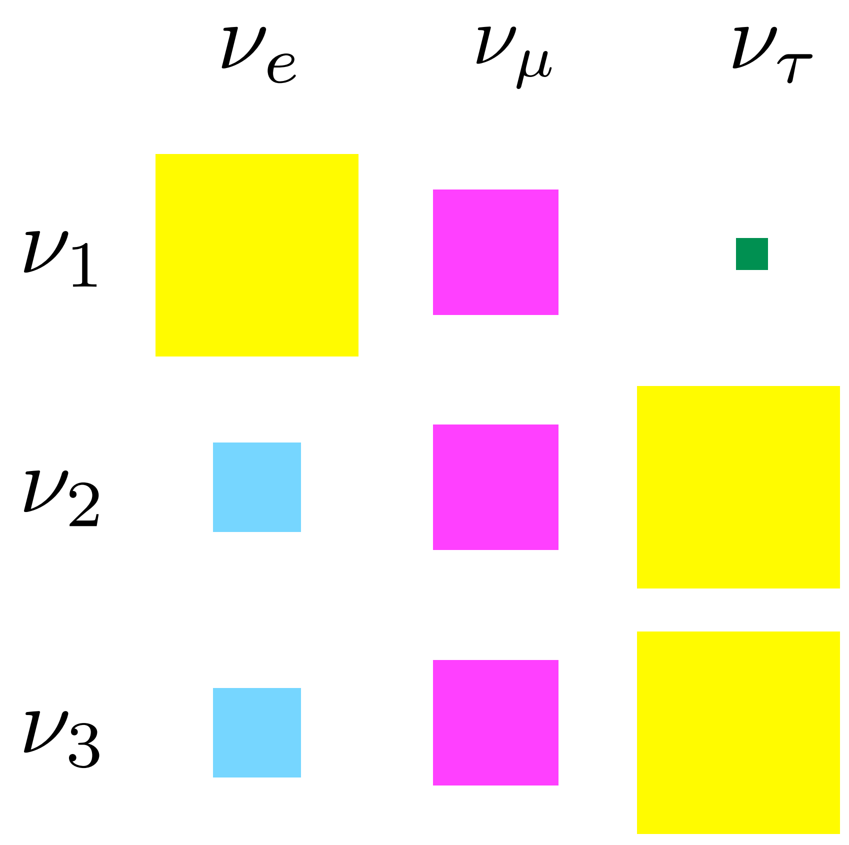


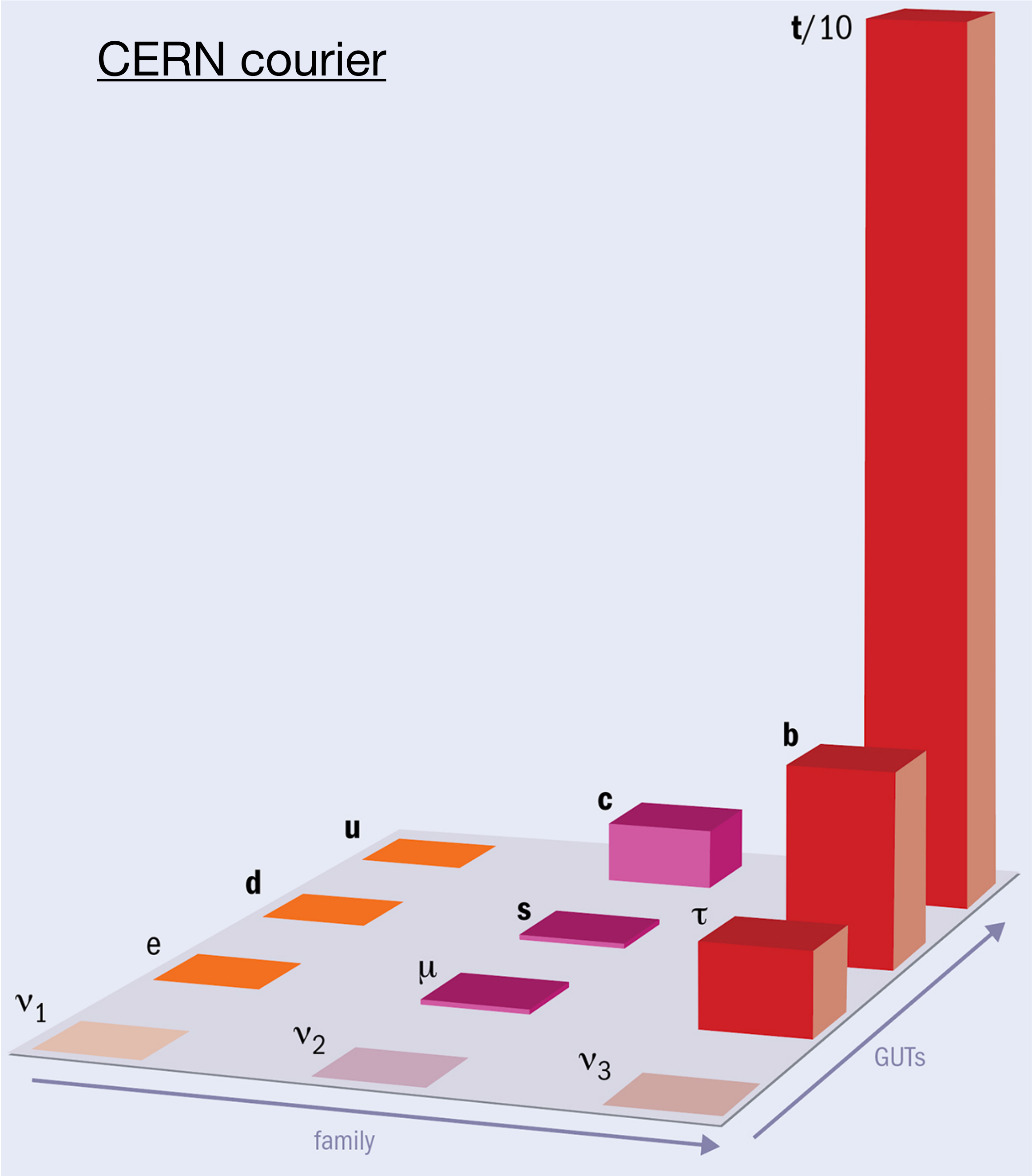


Motivation for Grand Unification

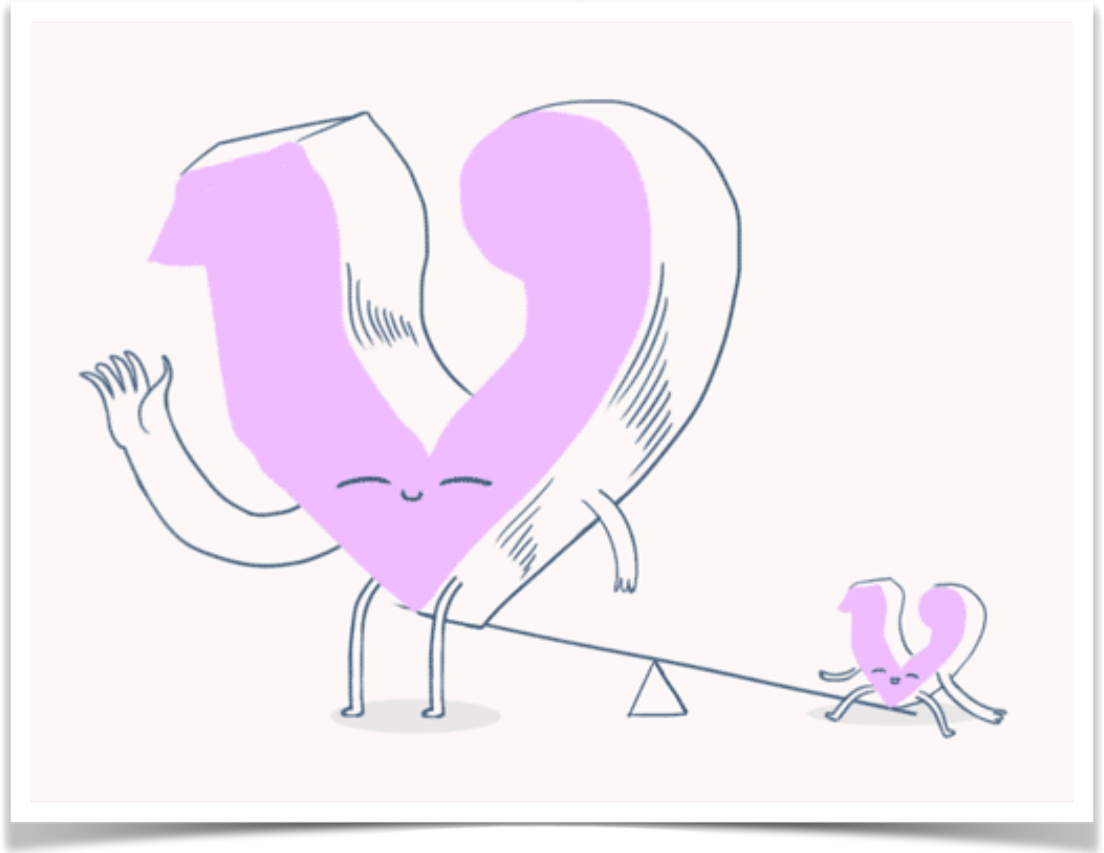
Flavour Puzzle





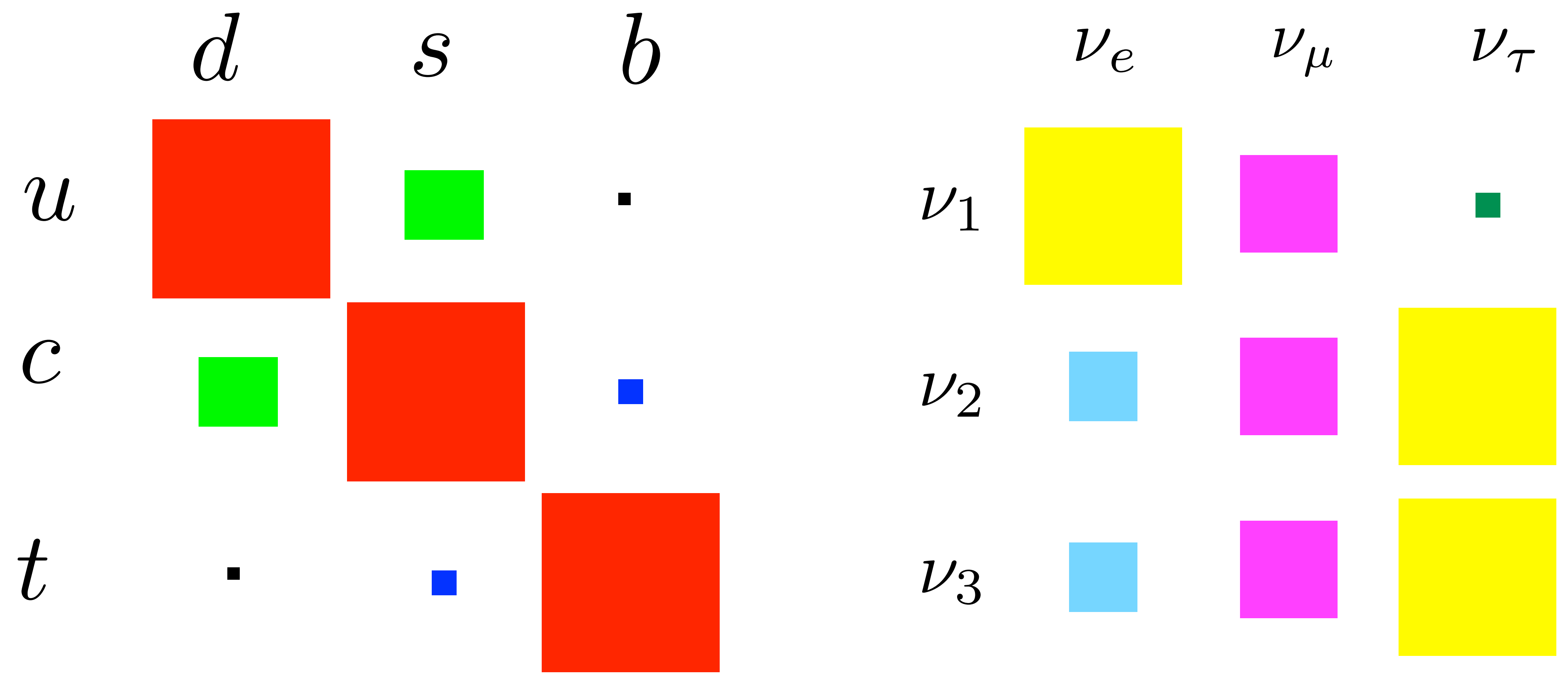


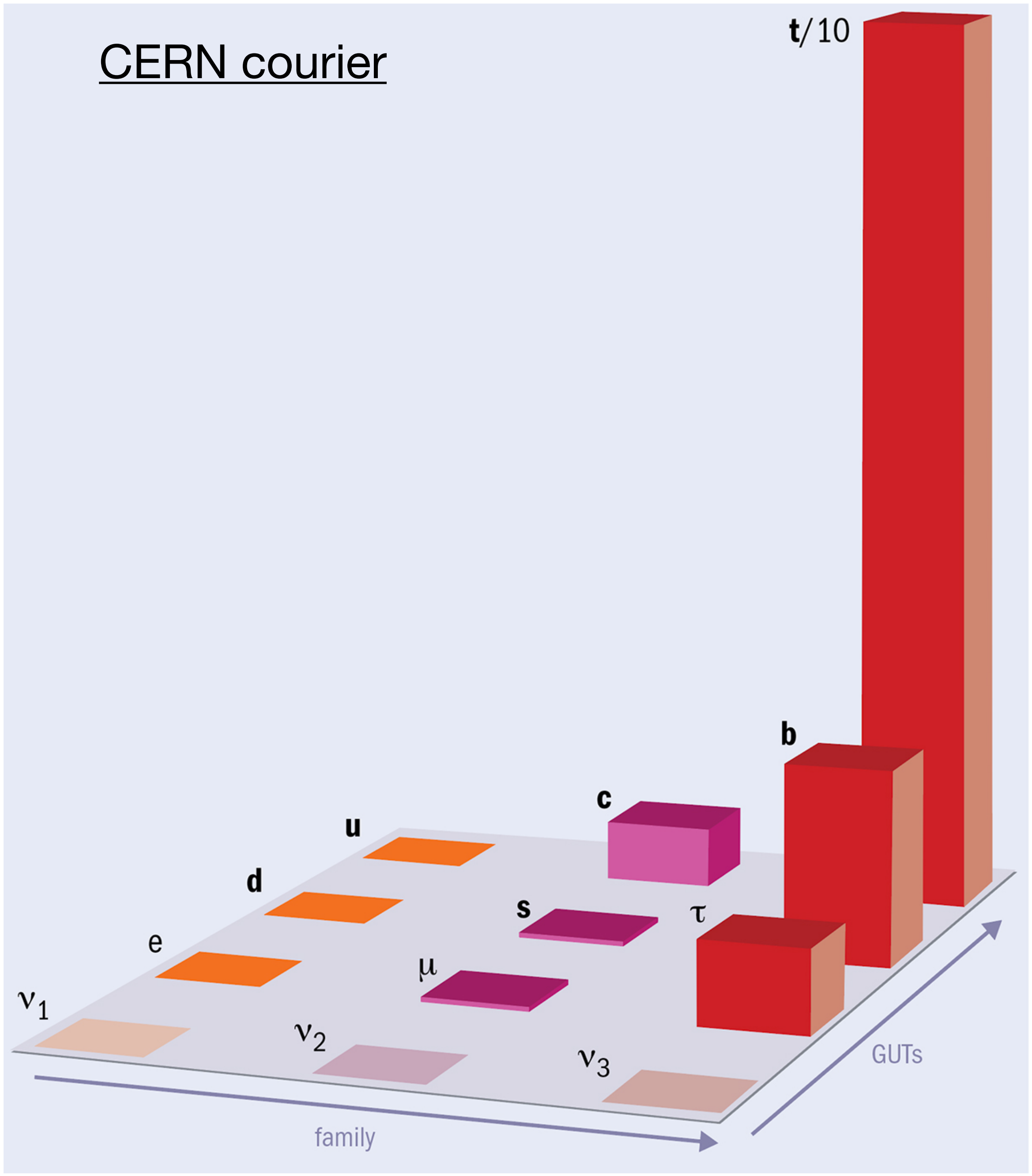
Neutrino Masses



Motivation for Grand Unification

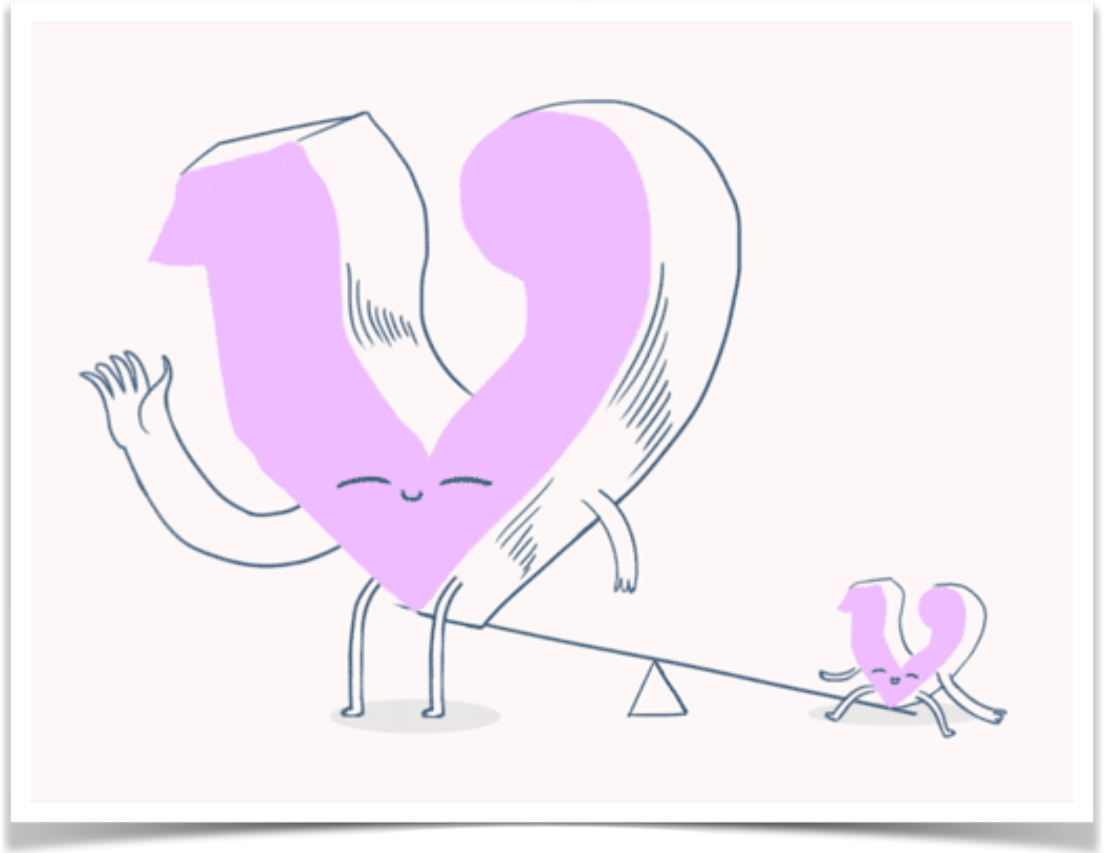
Flavour Puzzle





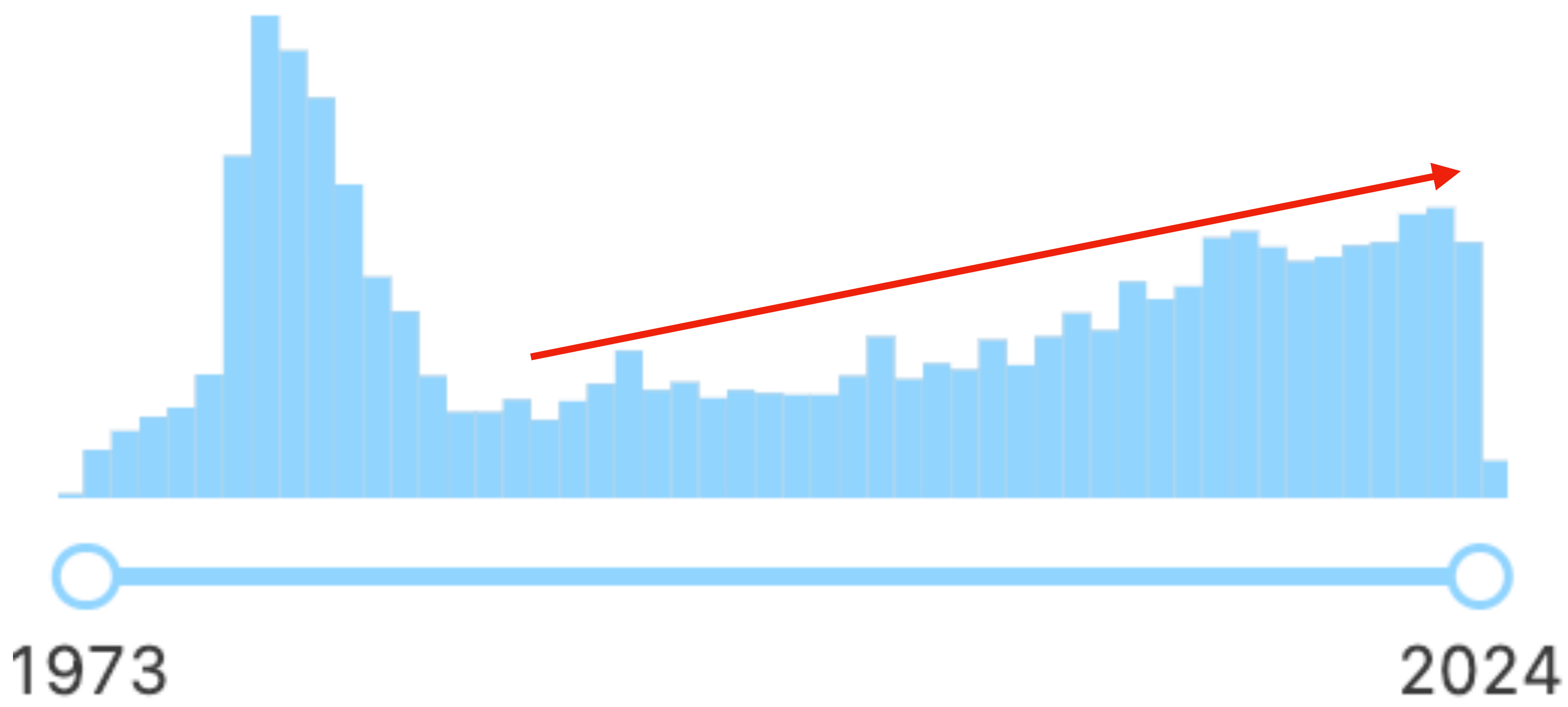
Matter-Antimatter Asymmetry

Neutrino Masses



Motivation for Grand Unification

Georgi & Glashow 1974

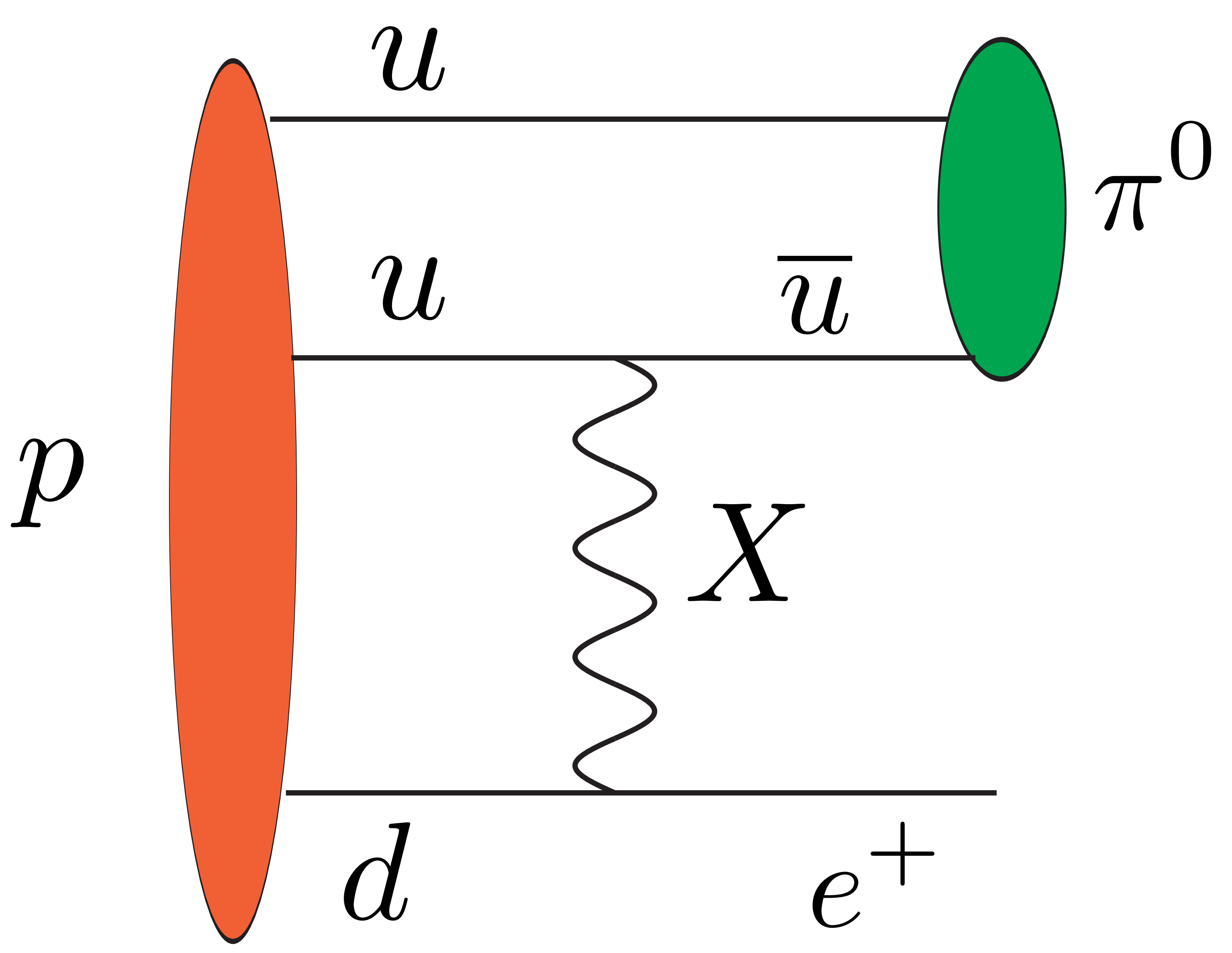


Reason 1: Proton Decay from GUTs

- GUTs unify leptons and quarks into common multiplets $\implies$ B & L not conserved

non-SUSY GUT

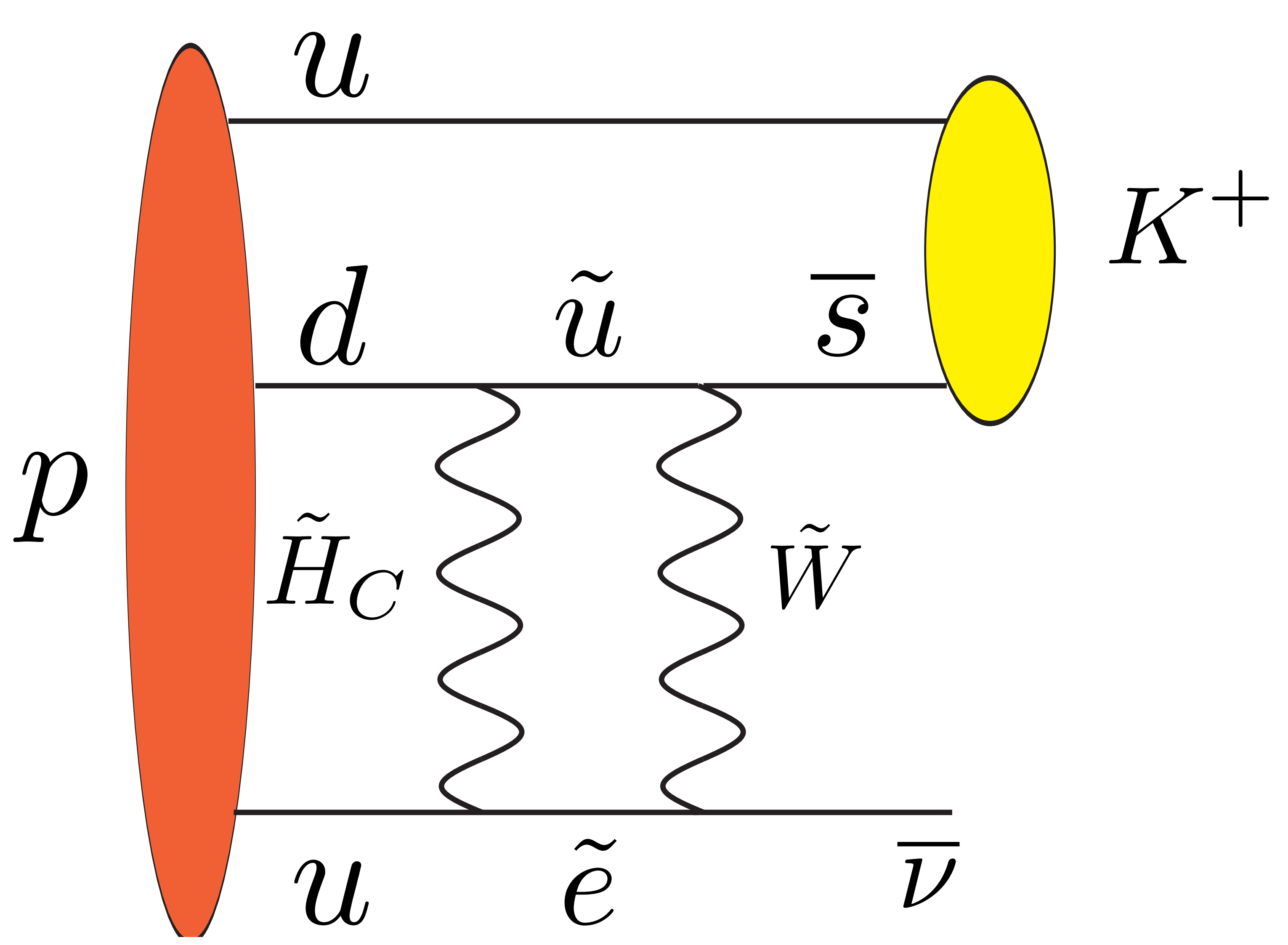
$$p \to \pi^0 \, e^+$$



$$\tau \propto \frac{1}{\alpha_X^2} \left(\frac{M_X}{10^{16}\mathrm{GeV}} \right)^4$$

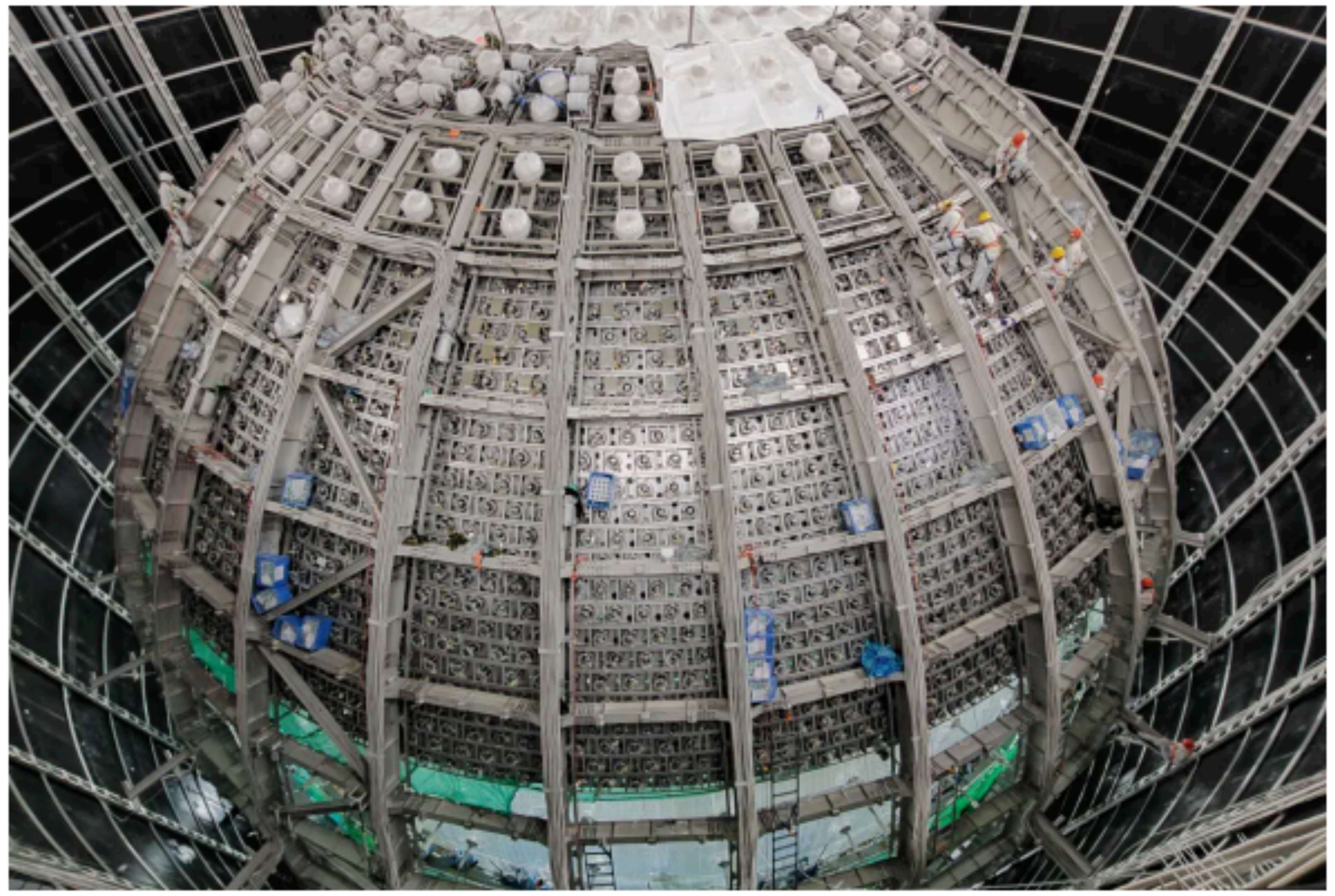
SUSY GUT

$$p \to K^+ \, \overline{\nu}$$



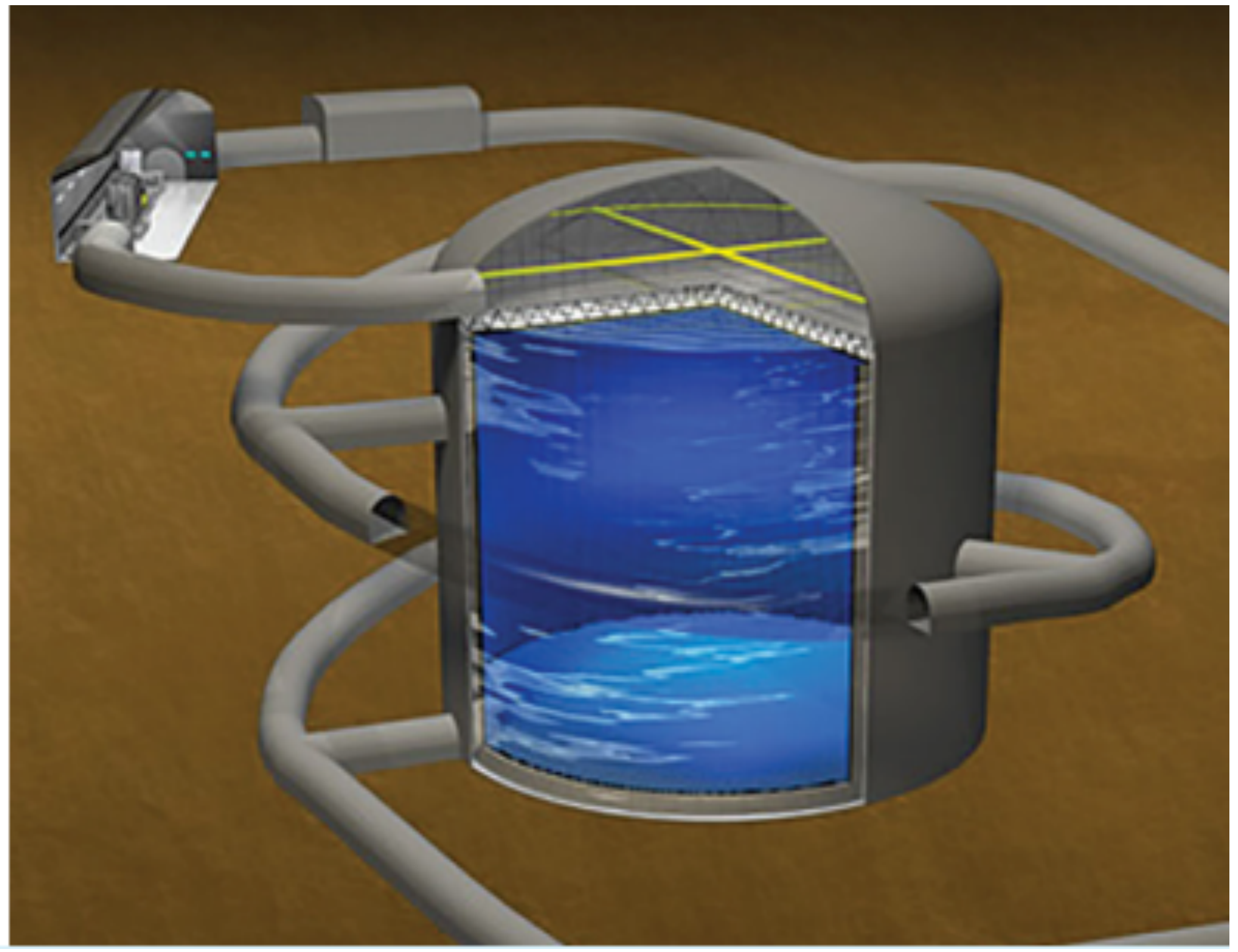
$$\tau \propto \frac{1}{\tilde{\alpha}^2} \left(\frac{M_{\tilde{W}} M_{\tilde{H}_C}}{10^{32}\,\mathrm{GeV}^2} \right)^2$$

Reason 1: Proton Decay from GUTs



JUNO, data taking end this year

20 kiloton $\sim 7 \times 10^{33}$ protons

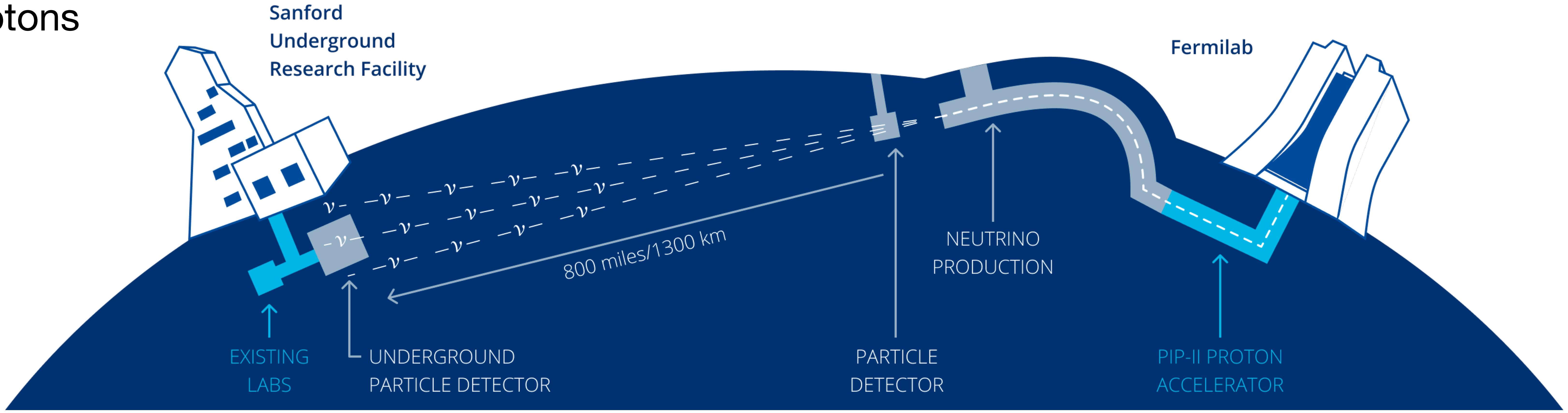


Hyper-Kamiokande
2027 expected data taking

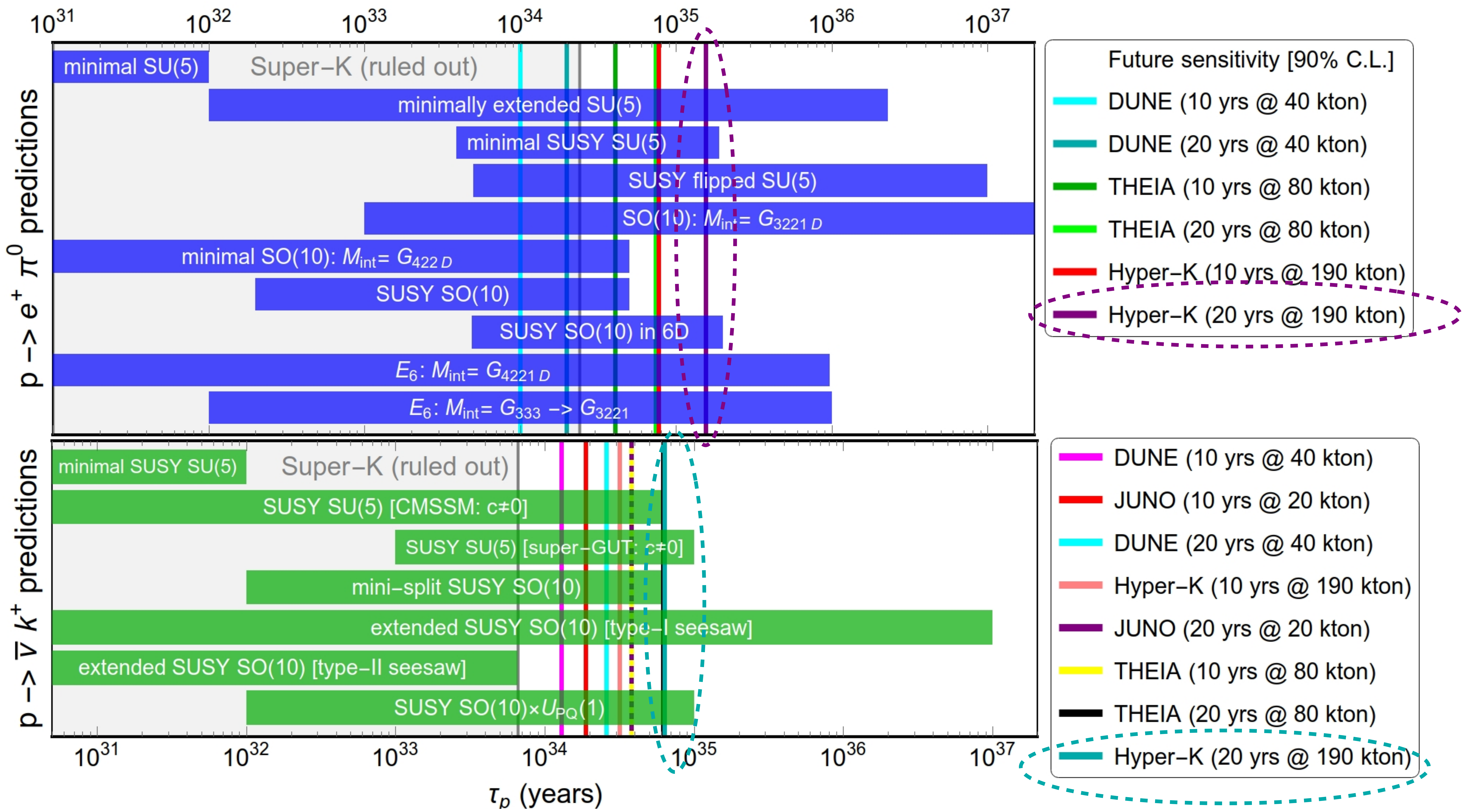
188 kiloton $\sim 7 \times 10^{34}$ protons

DUNE
2030(ish) expected data taking

40 kiloton $\sim 10^{34}$ protons

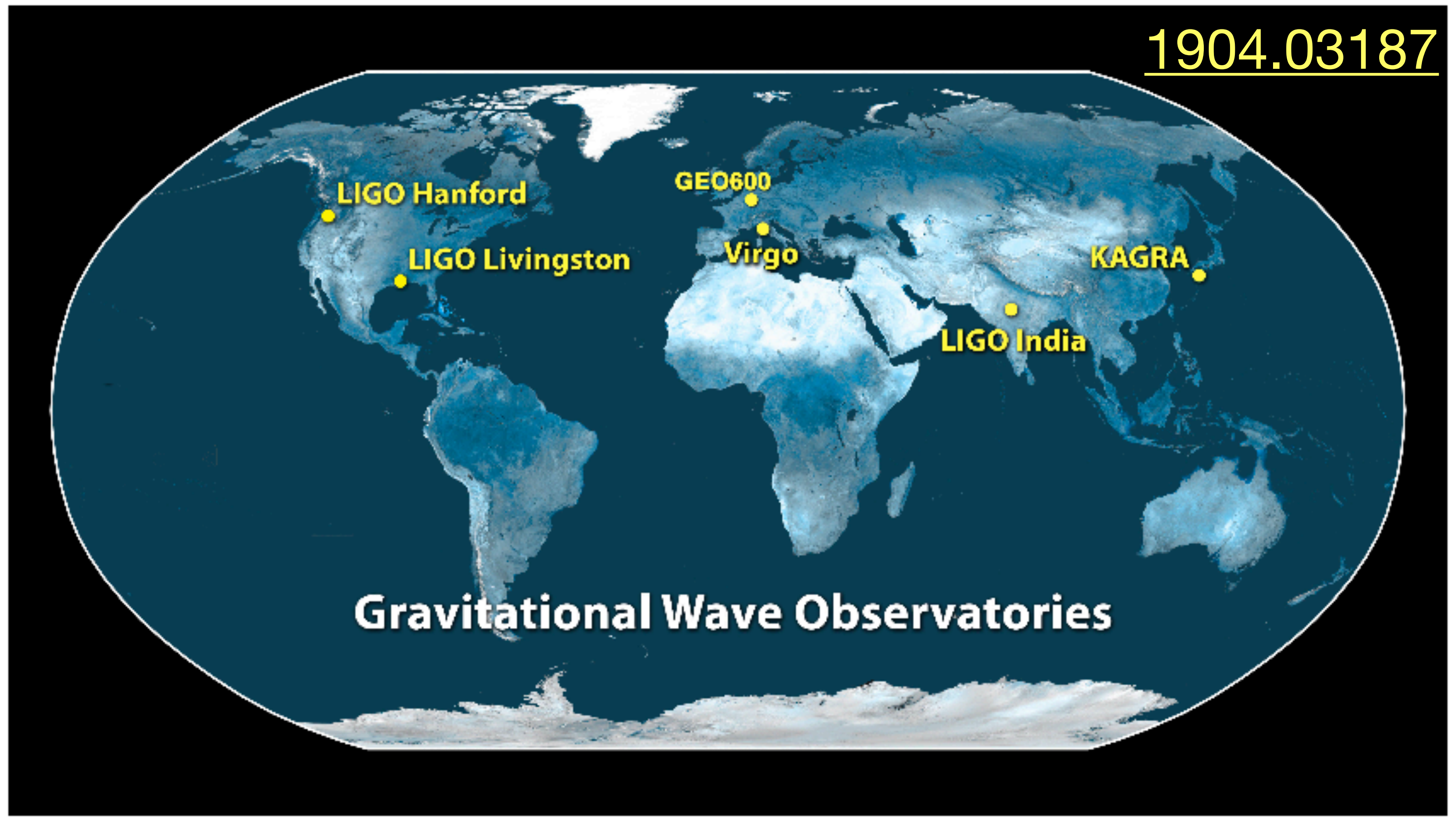


Searches for Baryon Number Violation in Neutrino Experiments: A White Paper



Reason 2: Gravitational waves from GUTs

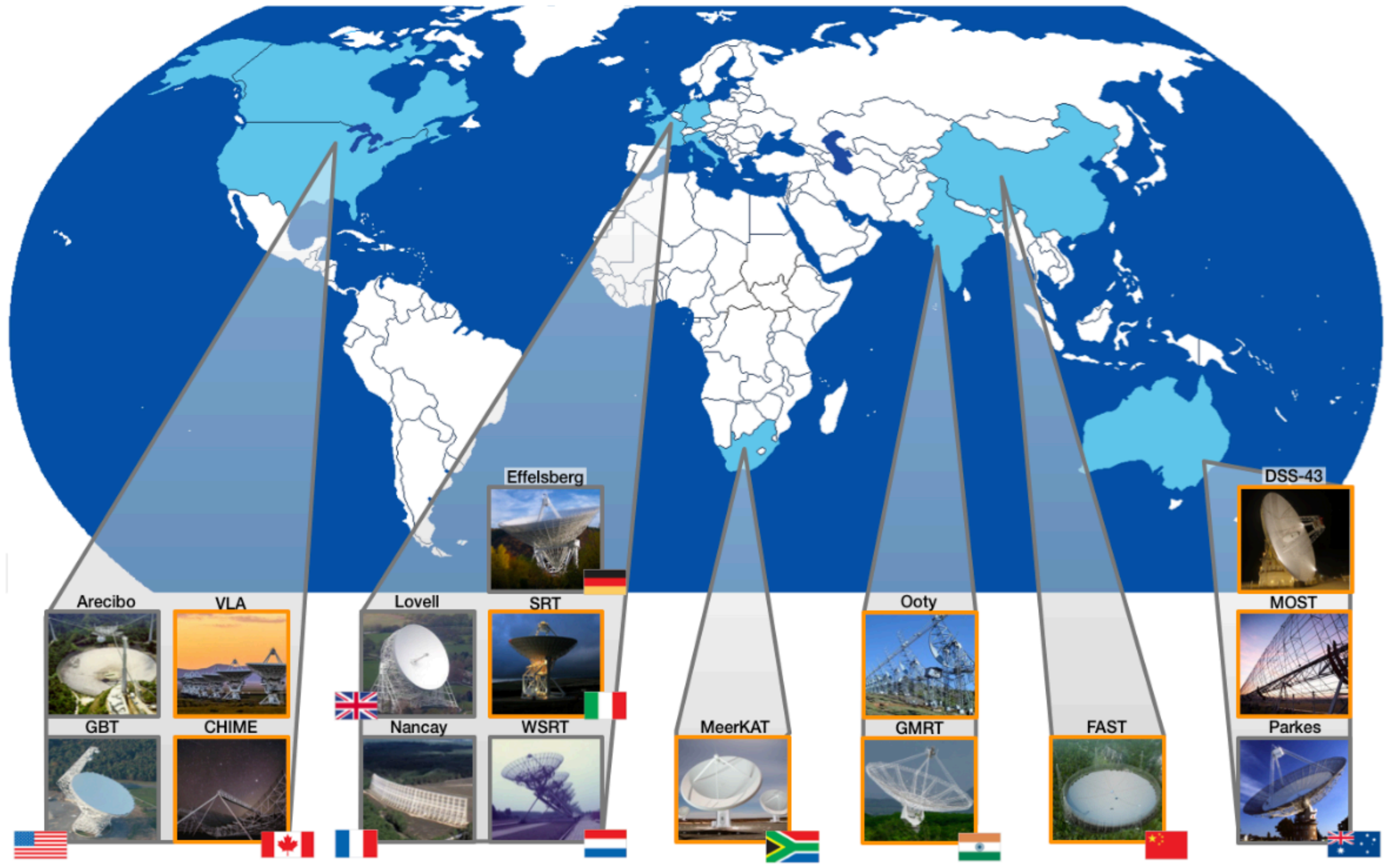
Ground Based Interferometers



$\mathcal{O}(1 - 10^3)$ Hz

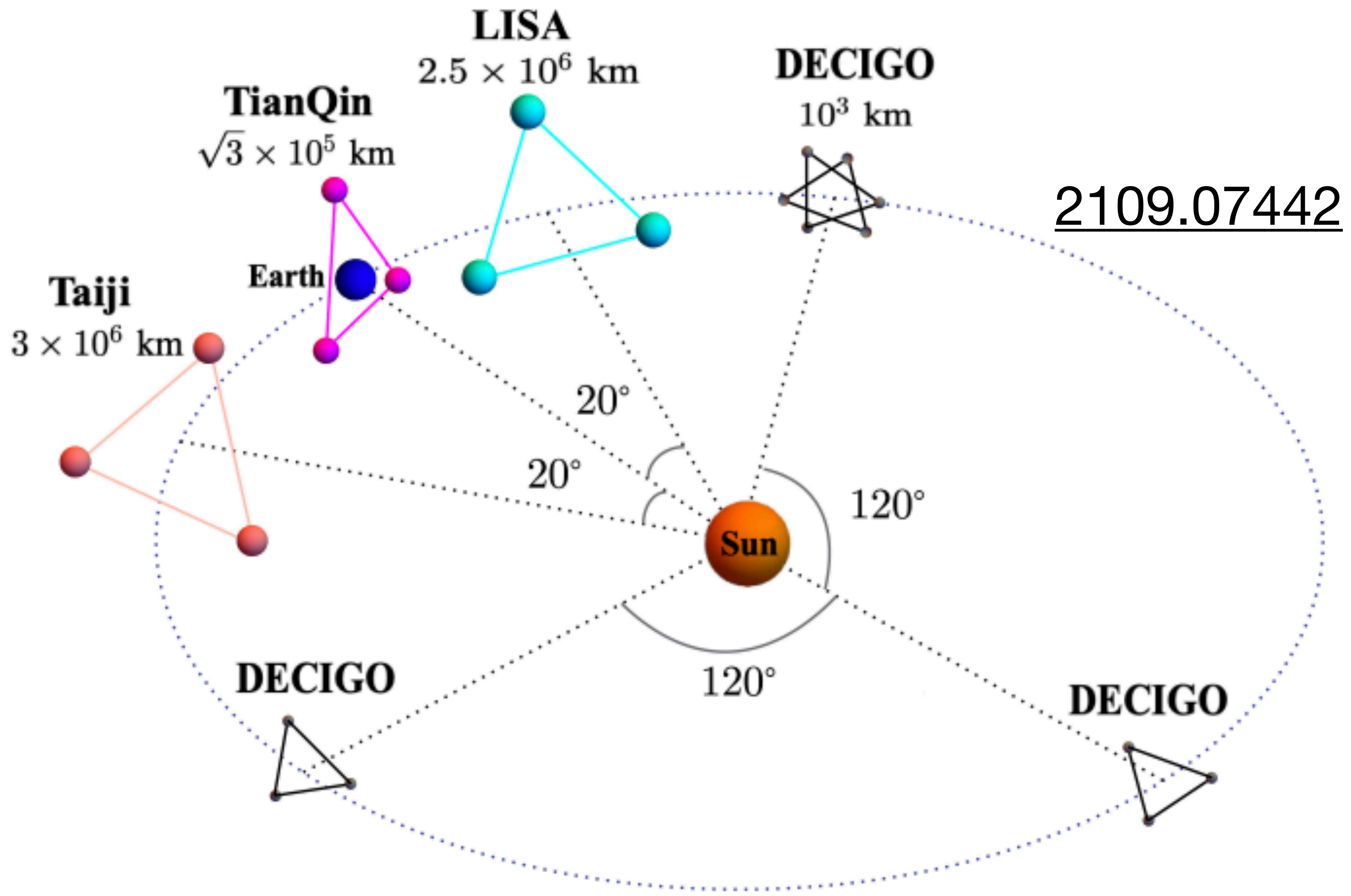
June 2023
NANOGrav 15 year dataset, European Pulsar Timing Array Parkes Pulsar Timing Array, Chinese Pulsar Timing Array has evidence of gravitational wave in nanoHertz regime

Pulsar Timing Arrays



$\mathcal{O}(10^{-9} - 10^{-6})$ Hz

Space Based Interferometers

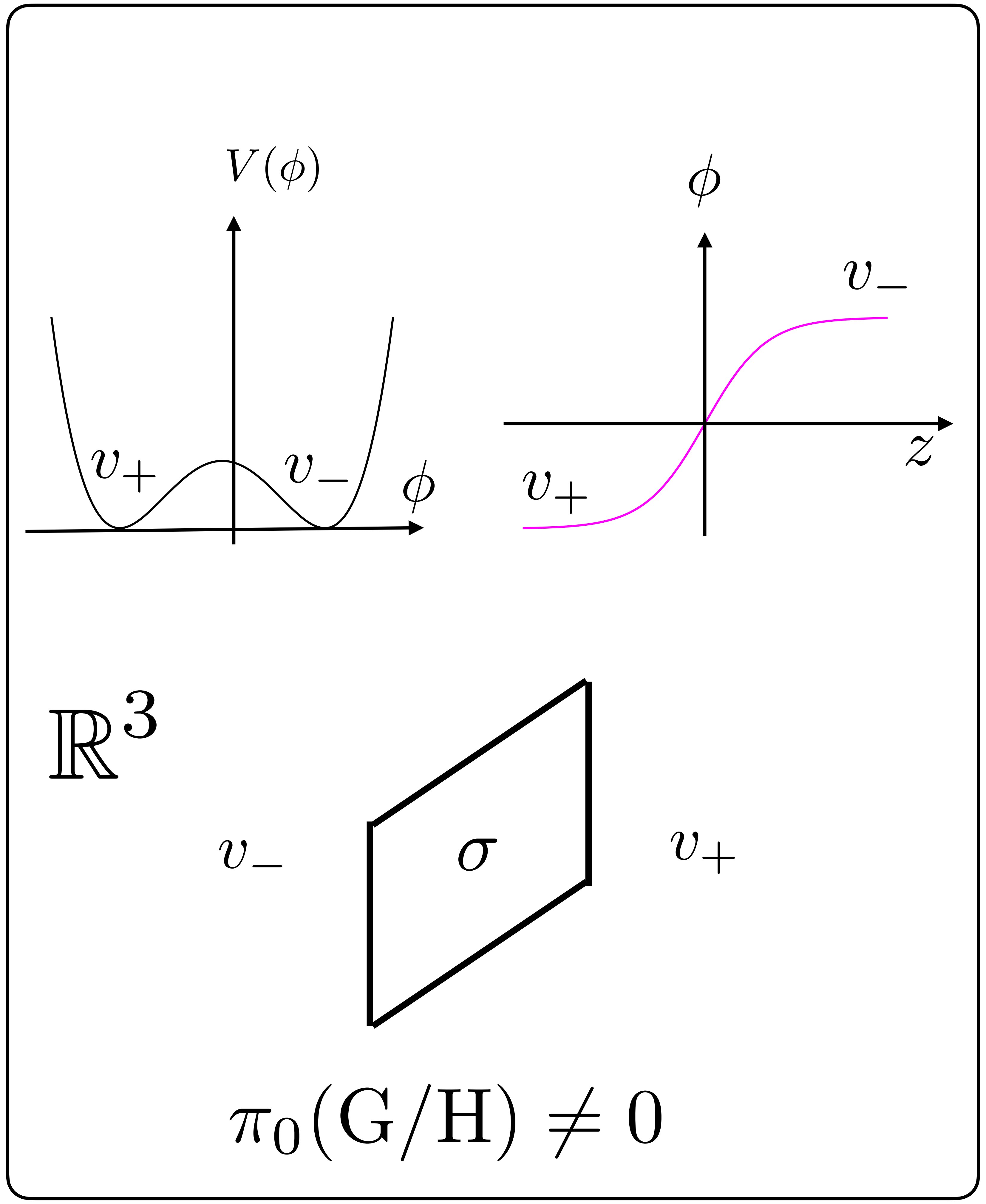


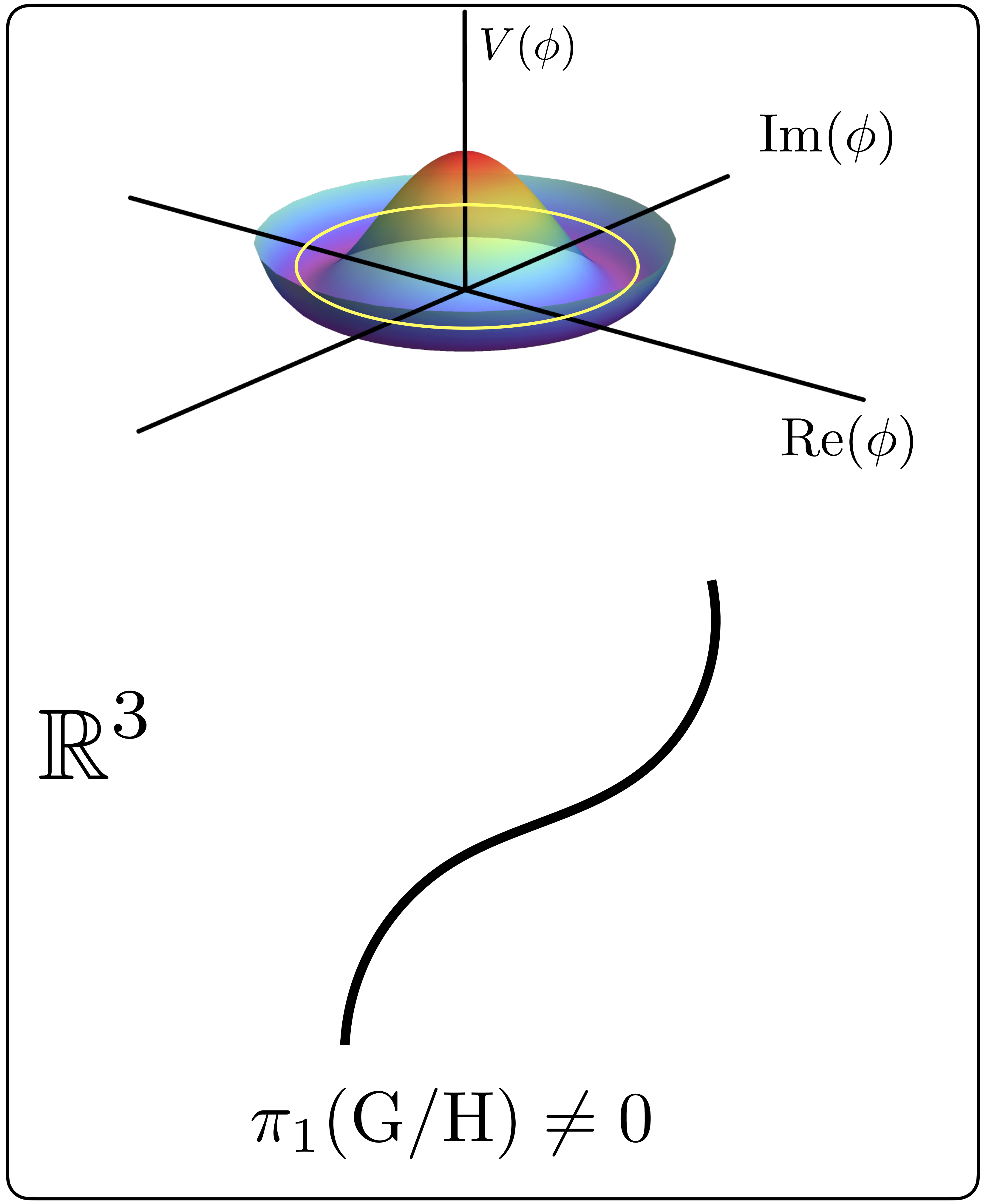
2109.07442

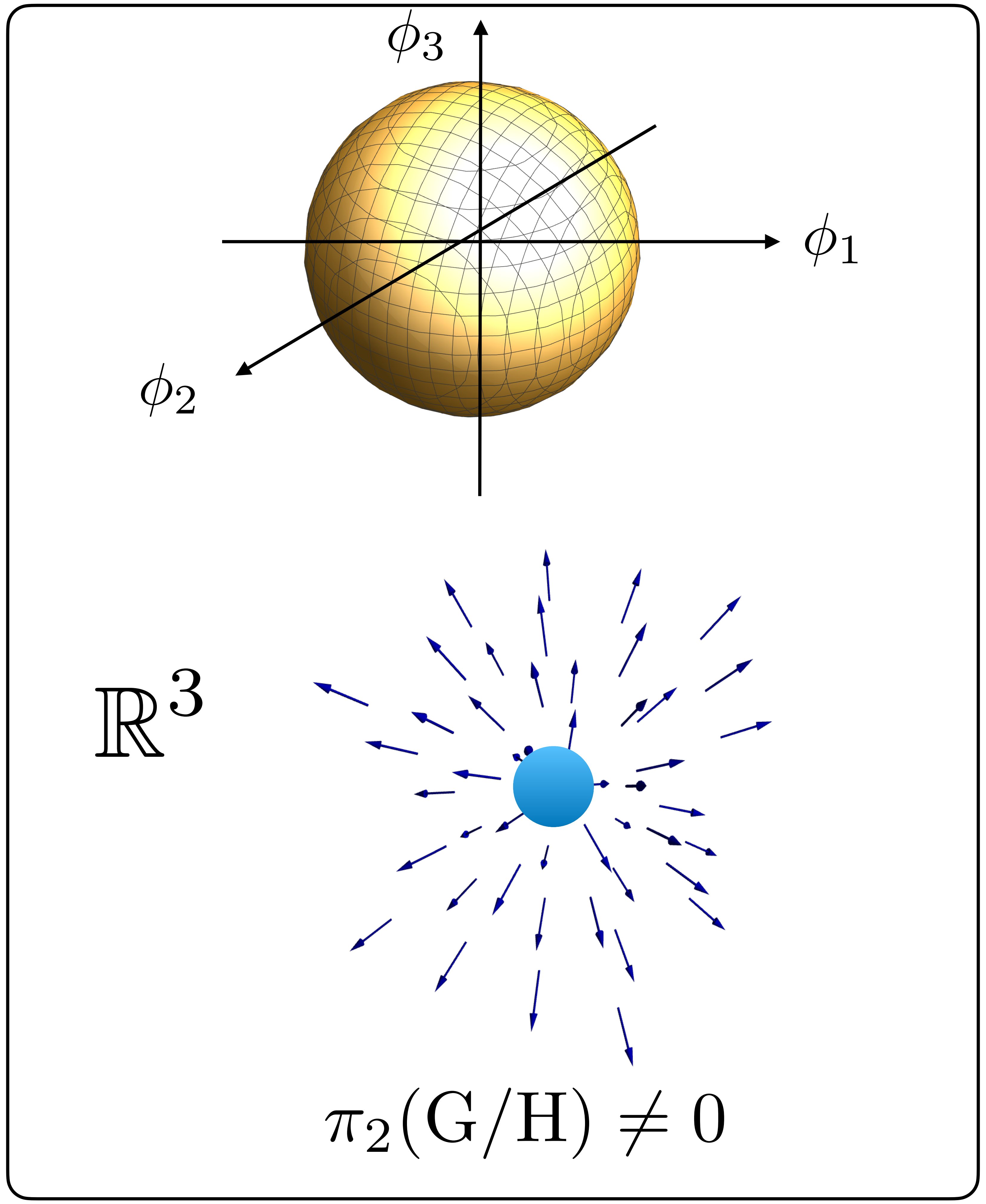
$\mathcal{O}(10^{-4} - 1)$ Hz

GUTs prediction: topological defects

During SSB from $G_{GUT} \rightarrow \cdots \rightarrow G_{SM}$ topological defects may form.

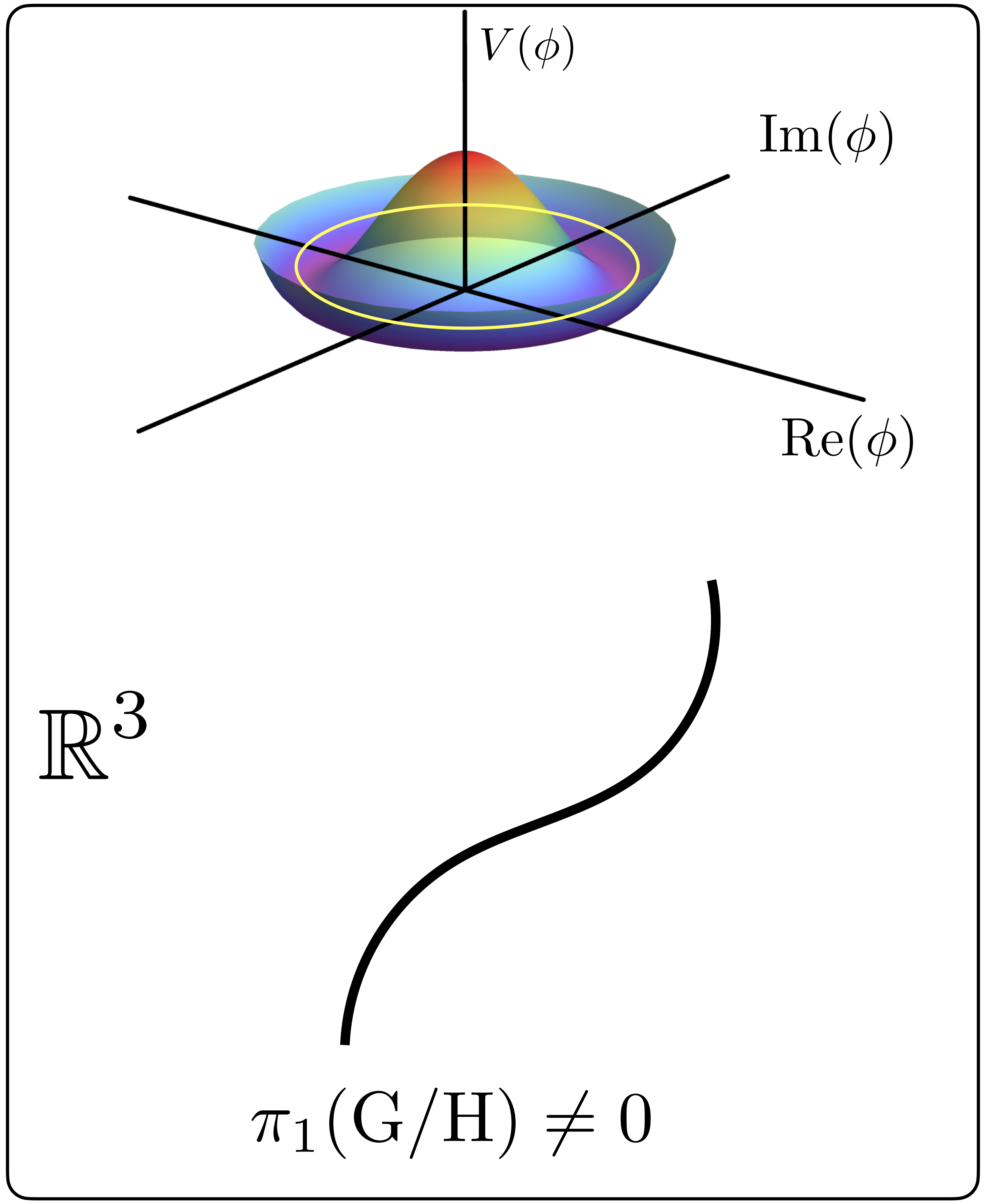






GUTs prediction: topological defects

During SSB from $G_{GUT} \to \cdots \to G_{SM}$ topological defects may form.



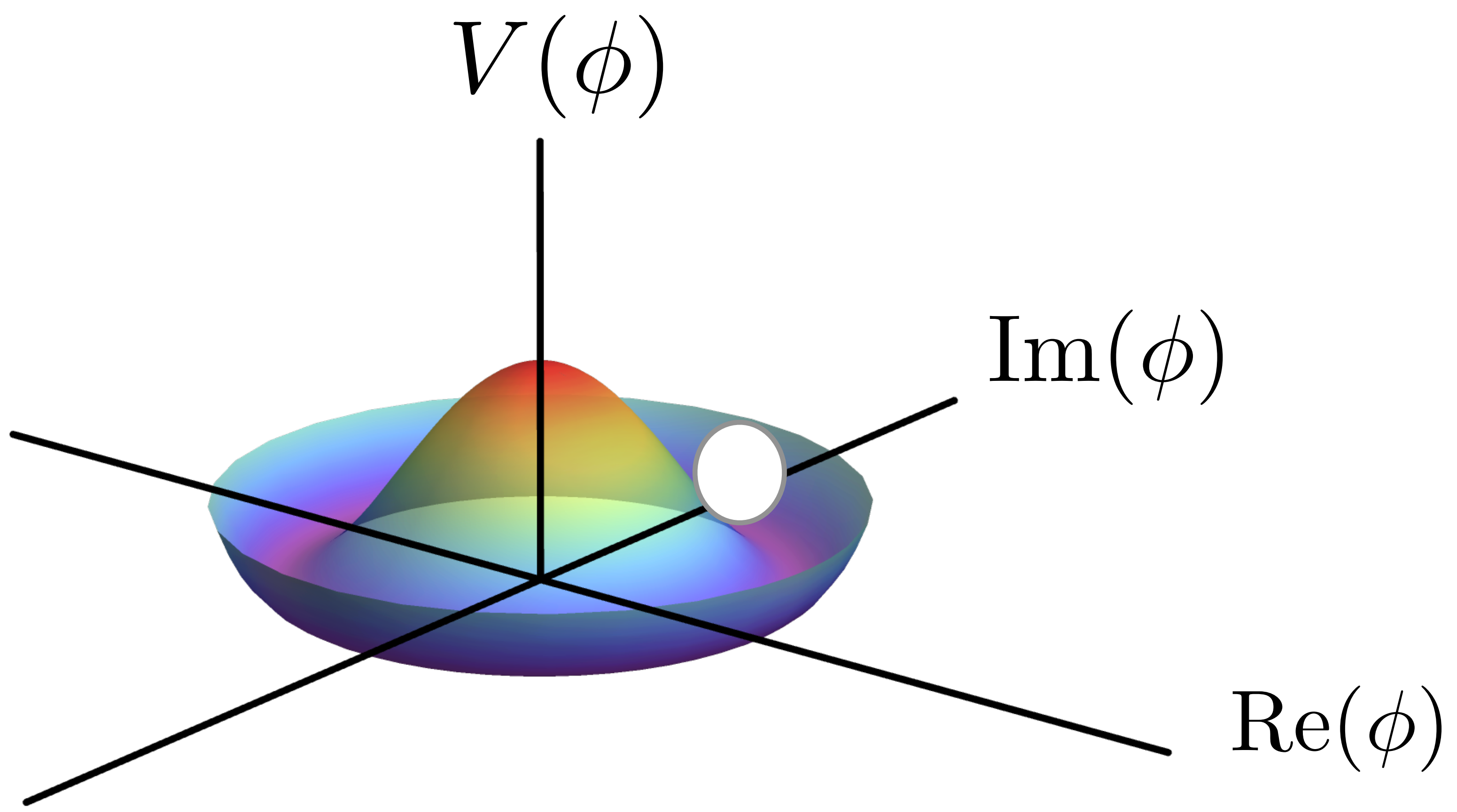
GUTs prediction: topological defects

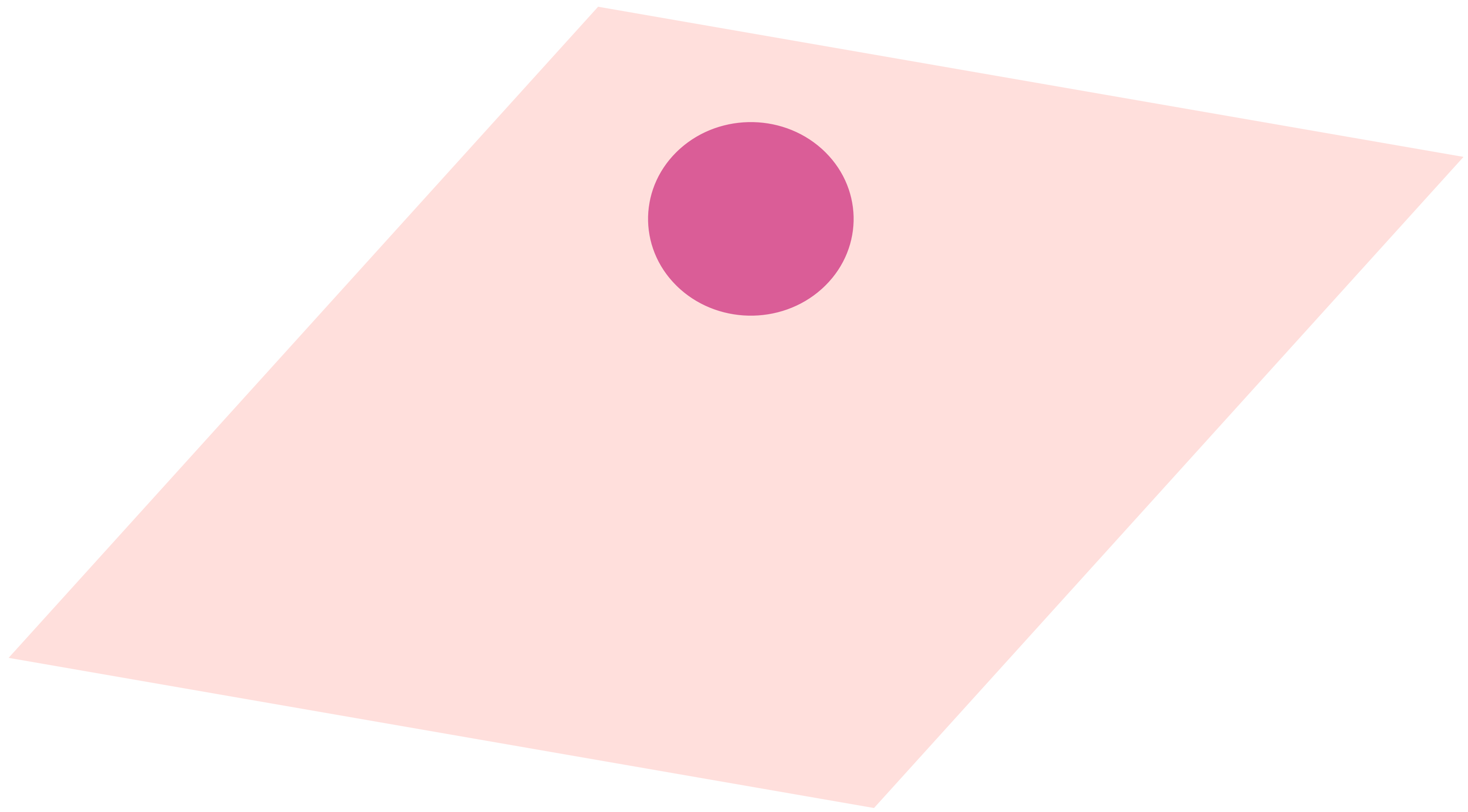
Abelian Higgs Model

Kibble Mechanism

$$S_{U(1)} = \int d^4x \left[\partial_\mu \phi \partial^\mu \phi^* - V\left(|\phi|^2\right)\right]$$

$$V(\phi) = \frac{\lambda}{4}\left(|\phi|^2 - \eta^2\right)^2$$





Fields at thermally independent regions of the Universe choose different θ

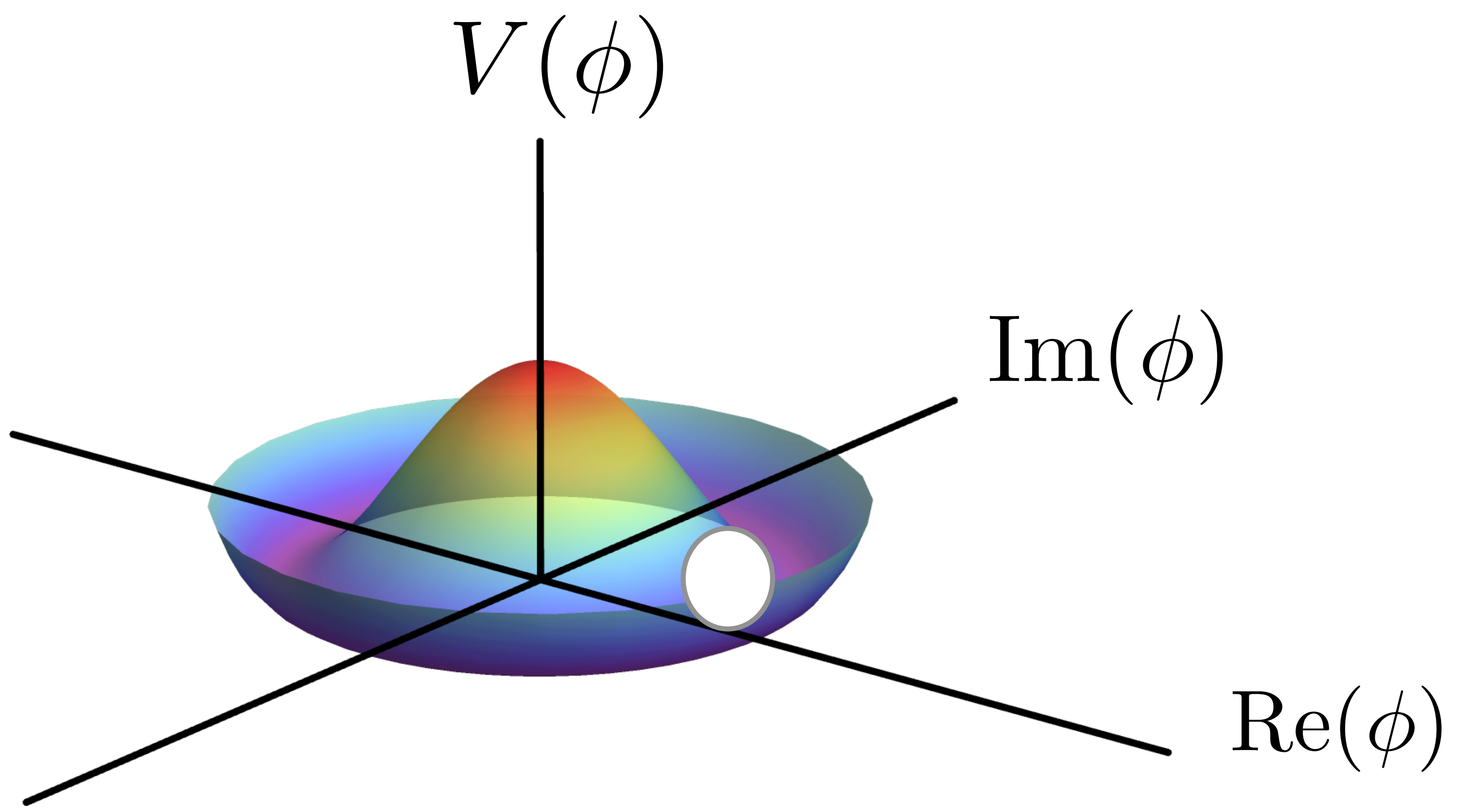
GUTs prediction: topological defects

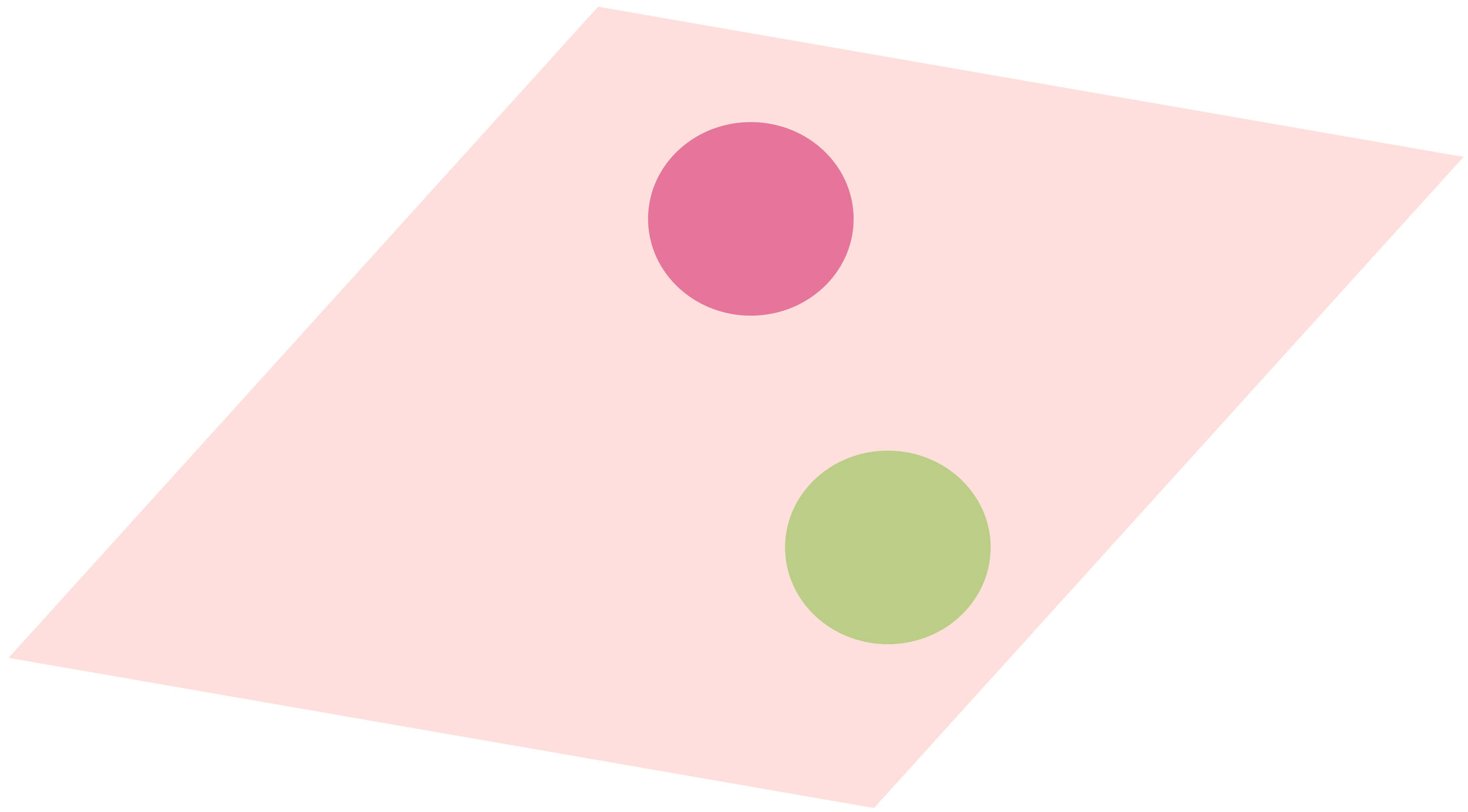
Abelian Higgs Model

Kibble Mechanism

$$S_{U(1)} = \int d^4x \left[\partial_\mu \phi \partial^\mu \phi^* - V\left(|\phi|^2\right)\right]$$

$$V(\phi) = \frac{\lambda}{4}\left(|\phi|^2 - \eta^2\right)^2$$





Fields at thermally independent regions of the Universe choose different θ

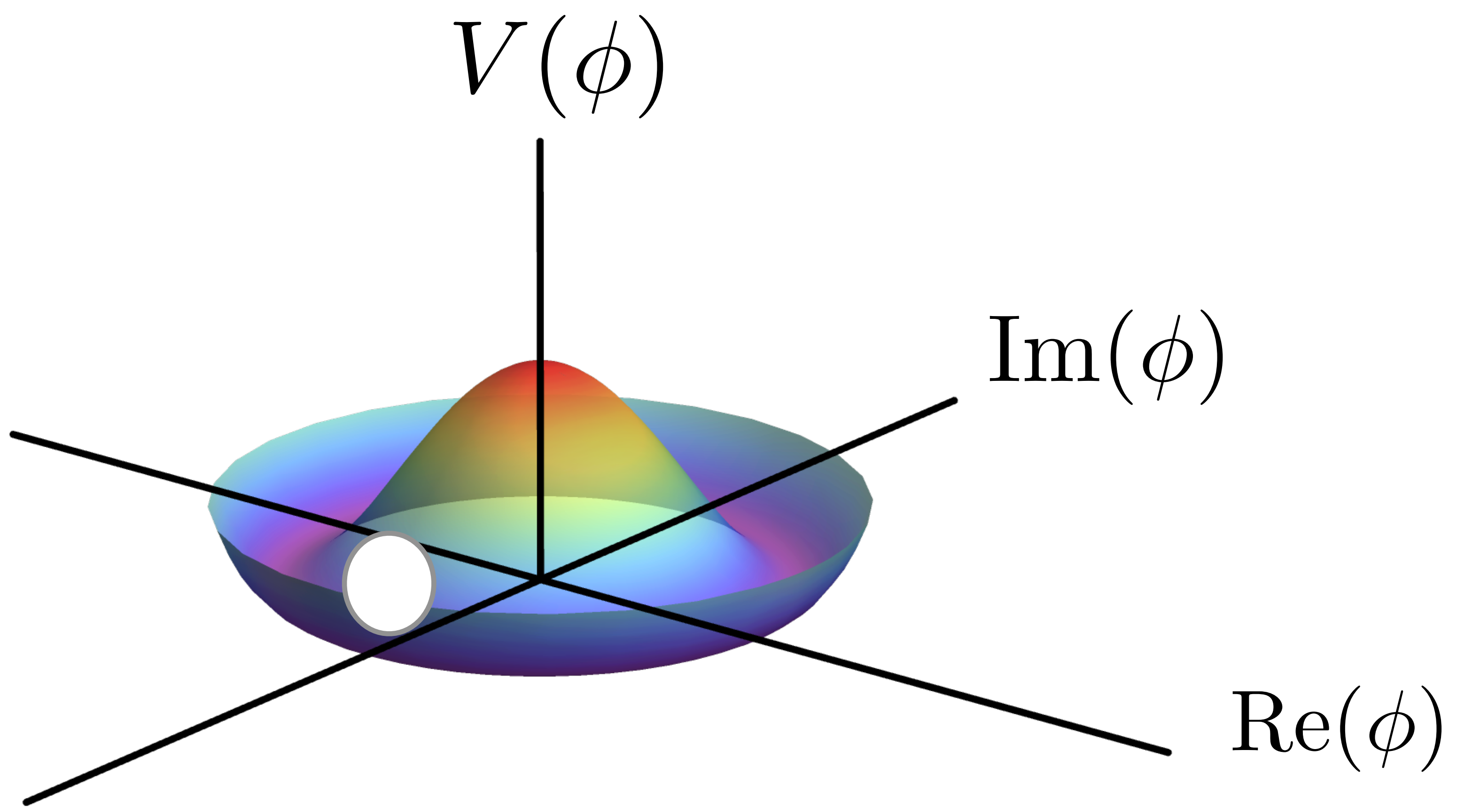
GUTs prediction: topological defects

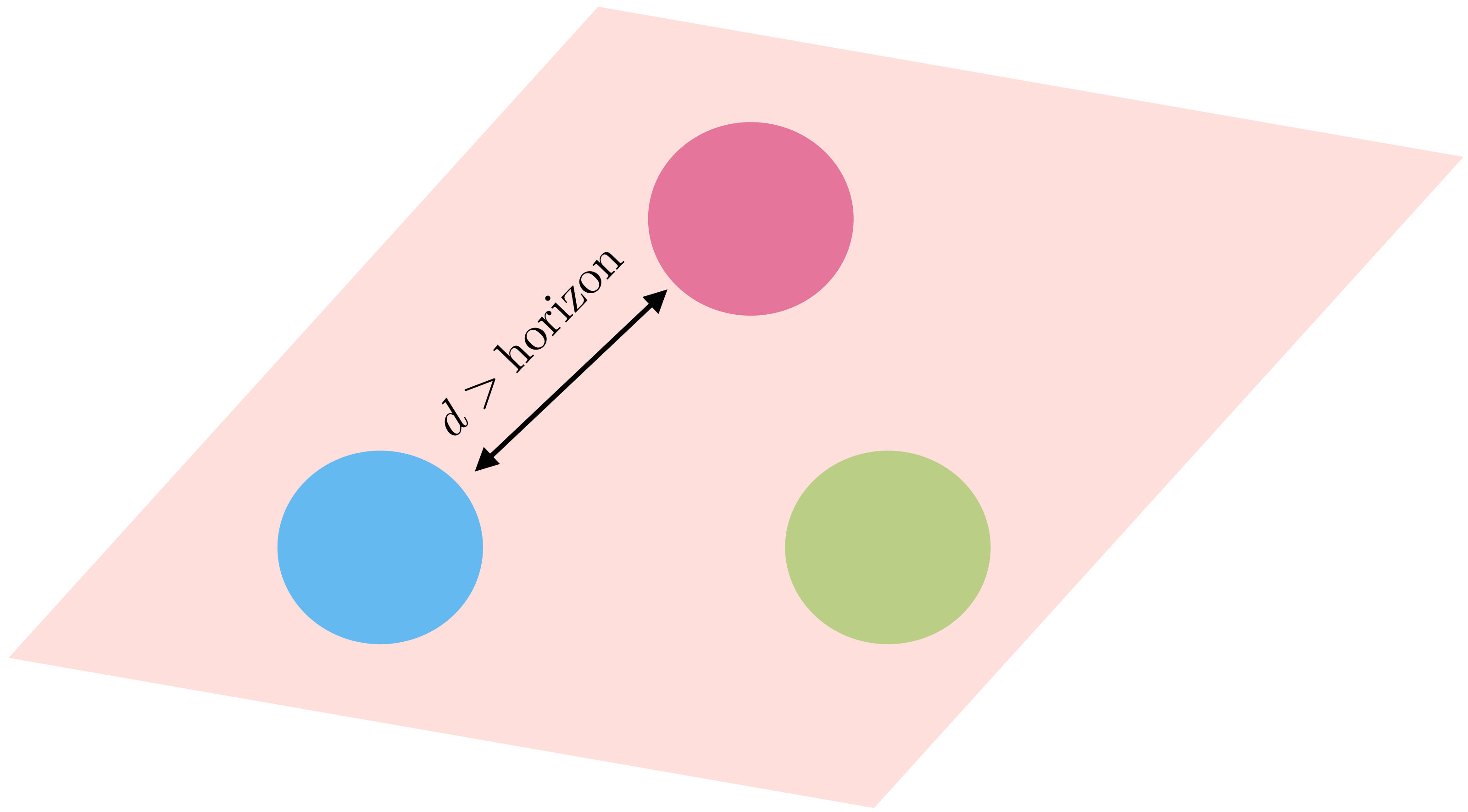
Abelian Higgs Model

Kibble Mechanism

$$S_{U(1)} = \int d^4x \left[\partial_\mu \phi \partial^\mu \phi^* - V\left(|\phi|^2\right)\right]$$

$$V(\phi) = \frac{\lambda}{4}\left(|\phi|^2 - \eta^2\right)^2$$





Fields at thermally independent regions of the Universe choose different θ

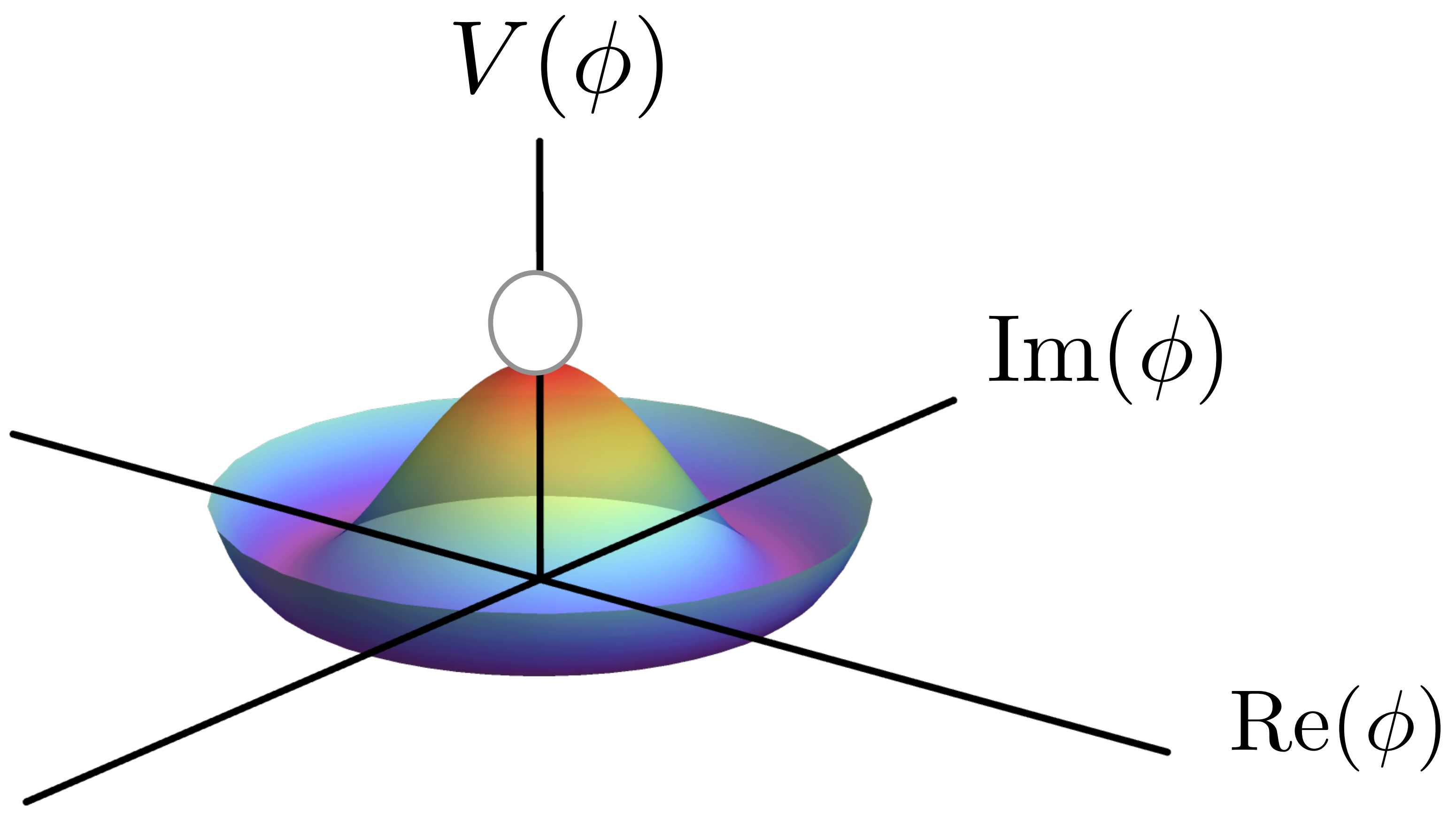
GUTs prediction: topological defects

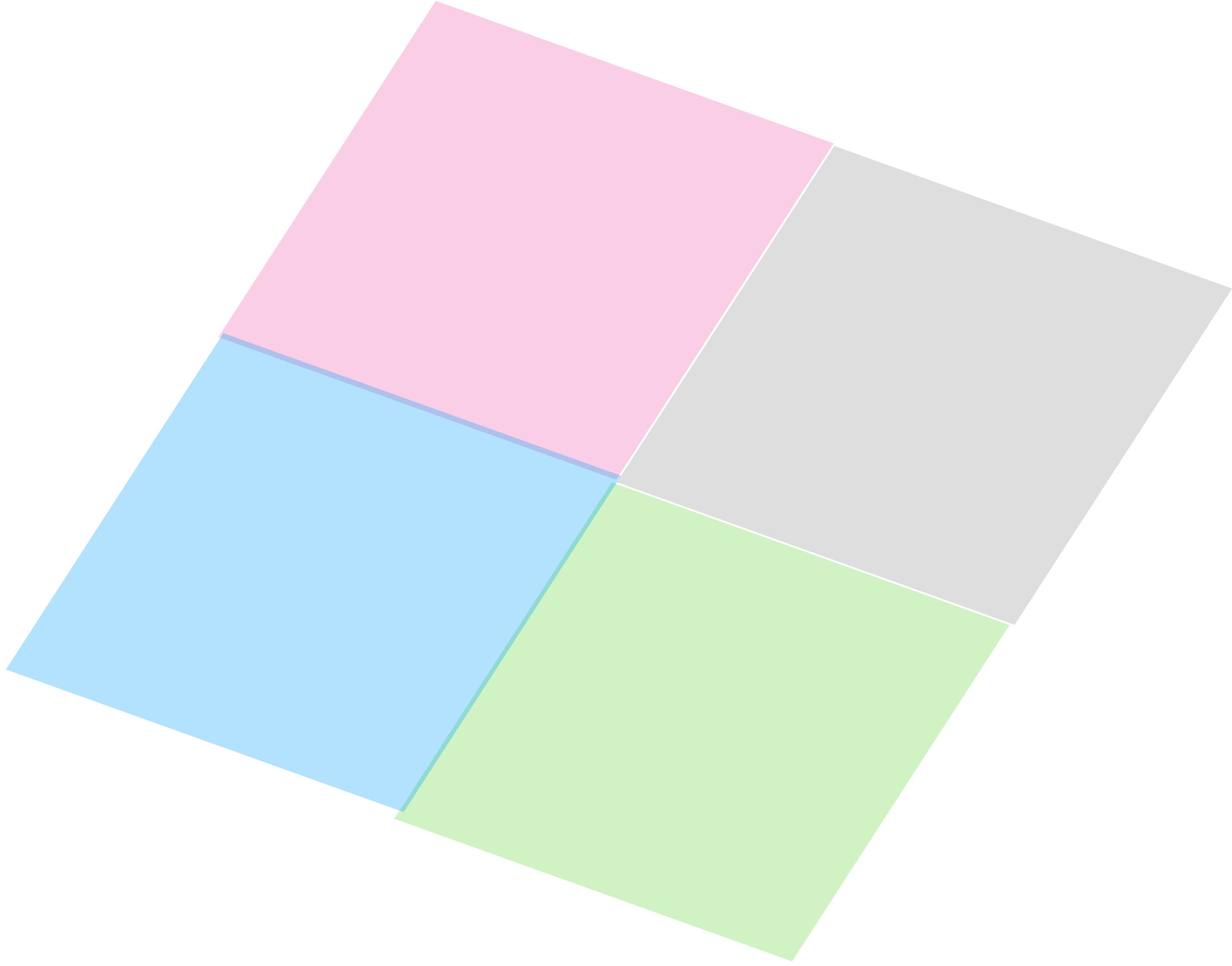
Abelian Higgs Model

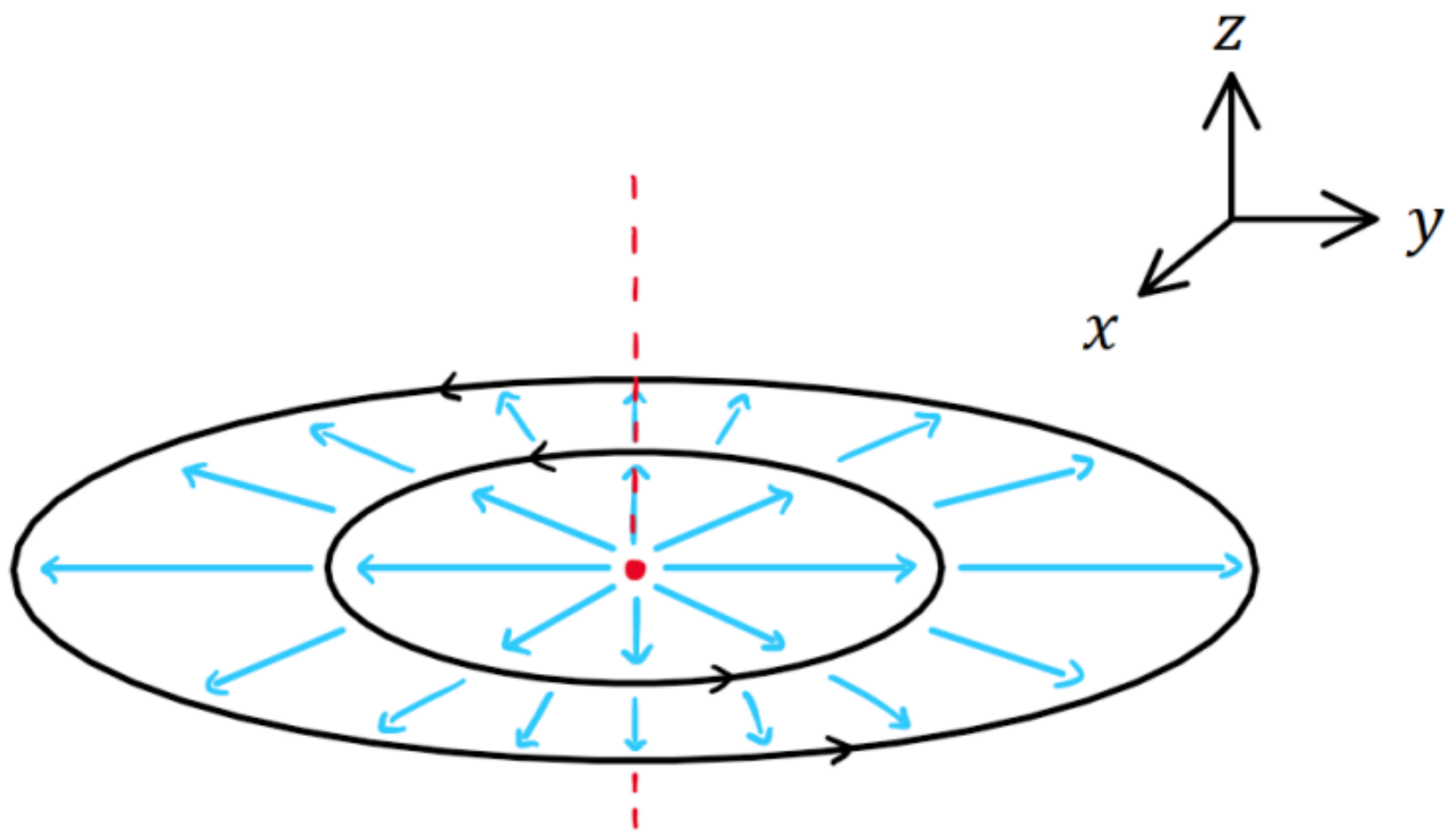
Kibble Mechanism

$$S_{U(1)} = \int d^4x \left[\partial_\mu \phi \partial^\mu \phi^* - V\left(|\phi|^2\right)\right]$$

$$V(\phi) = \frac{\lambda}{4}\left(|\phi|^2 - \eta^2\right)^2$$

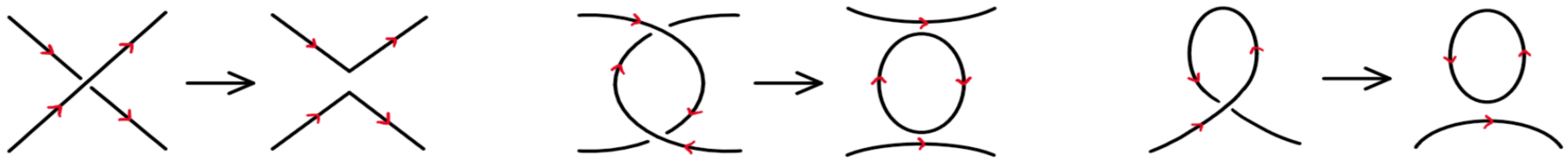






GUTs prediction: topological defects

- Assume Nambu-Goto strings, only coupling to massless mode is gravity
- String properties controlled by symmetry breaking scale, η
- $\eta = 10^{16}$ GeV $\implies$ $\delta = 10^{-30}$ cm and $\mu = 10^{22}$ gm/cm (string parameter)
- String intercommute, can swap partners and create loops



- Gravitational effect of GUT scale string $$G\mu = \left(\frac{\eta}{M_{Pl}}\right)^2 \sim 10^{-6}$$
- Emission of gravitational radiation by loops: $$\frac{dE^{(k)}_{GW}}{dt} = -\Gamma^{(k)} G\mu^2$$

- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) *1912.08832*

- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) *1912.08832*

$$\Omega_{\rm GW}(f) = \frac{G\mu^2}{\rho_{\rm crit}} \sum_{k=1}^{\infty} C_k(f) P_k$$

Pillado, Plum, Shlaer

Cui, Lewicki, Morrissey, Wells

- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) *1912.08832*

$$\Omega_{\mathrm{GW}}(f) = \frac{G\mu^2}{\rho_{\mathrm{crit}}} \sum_{k=1}^{\infty} C_k(f) \boxed{P_k}$$

- GW power emitted by single loop oscillating at $f = 2k/l$

Pillado, Plum, Shlaer

Cui, Lewicki, Morrissey, Wells

- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) *1912.08832*

$$\Omega_{\rm GW}(f) = \frac{G\mu^2}{\rho_{\rm crit}} \sum_{k=1}^{\infty} \boxed{C_k(f)}\,\boxed{P_k}$$

Pillado, Plum, Shlaer

Cui, Lewicki, Morrissey, Wells

- GW power emitted by single loop oscillating at $f = 2k/l$
- Number of loops that emit GW at frequency f today

- Inflation occurs **before** string formation → string network gives "scaling" solution

- Inflation occurs **after** string formation → string network diluted and **no GW signal**

- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) 1912.08832

$$\Omega_{\rm GW}(f) = \frac{G\mu^2}{\rho_{\rm crit}} \sum_{k=1}^{\infty} \boxed{C_k(f)} \boxed{P_k}$$

Pillado, Plum, Shlaer

Cui, Lewicki, Morrissey, Wells

- GW power emitted by single loop oscillating at $f = 2k/l$

- Number of loops that emit GW at frequency f today

- GW signal is superposition of GW emission from all oscillation modes, need to sum All modes for reliable result

- Inflation occurs **before** string formation → string network gives "scaling" solution

- Inflation occurs **after** string formation → string network diluted and **no GW signal**

- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) *1912.08832*

$$\Omega_{\rm GW}(f) = \frac{G\mu^2}{\rho_{\rm crit}} \sum_{k=1}^{\infty} \boxed{C_k(f)}\,\boxed{P_k}$$

Pillado, Plum, Shlaer

Cui, Lewicki, Morrissey, Wells

- GW power emitted by single loop oscillating at $f = 2k/l$

- Number of loops that emit GW at frequency f today

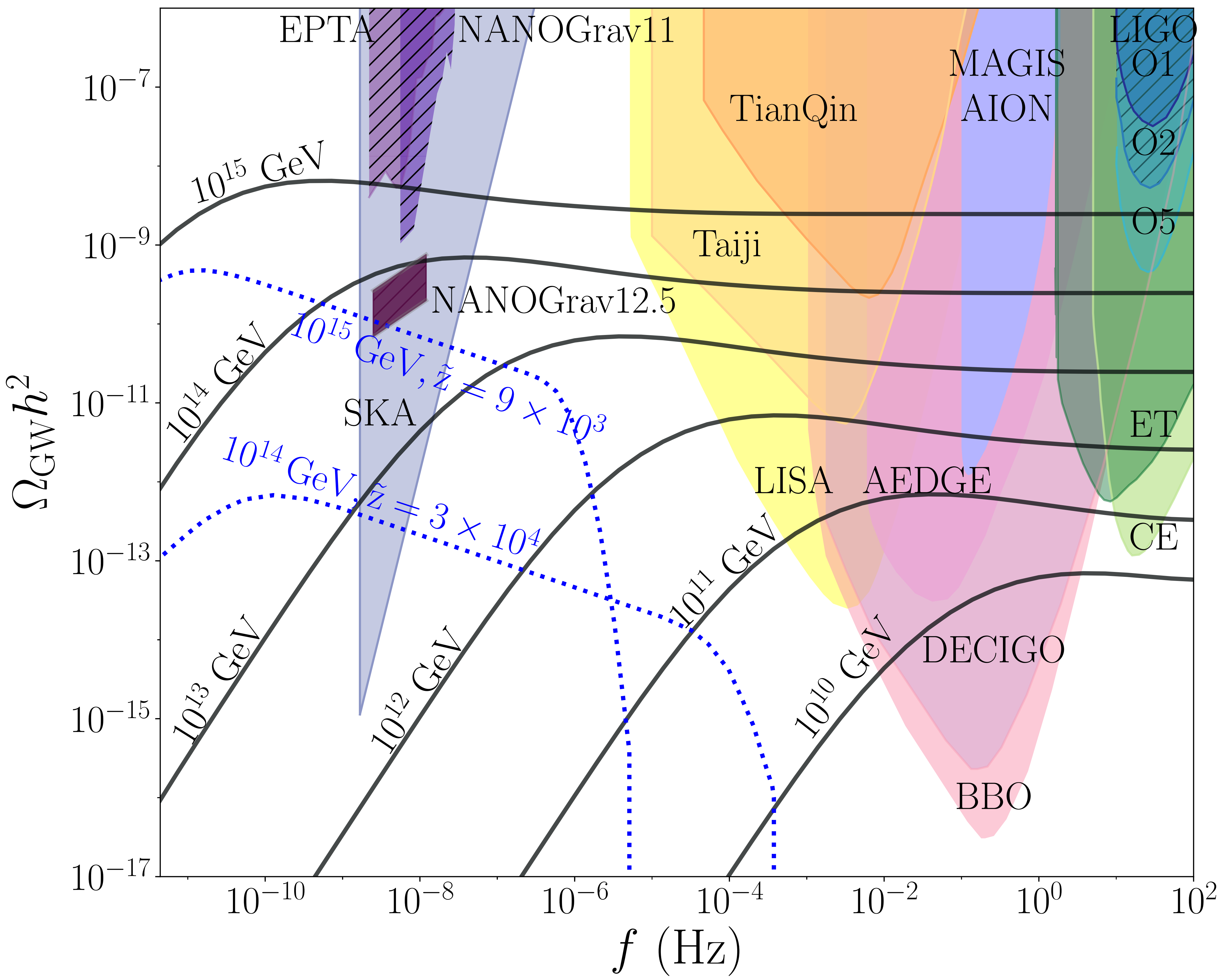
- GW signal is superposition of GW emission from all oscillation modes, need to sum All modes for reliable result

- All non-trivial physics contained in loop density function within $C_k(f)$

- Inflation occurs **before** string formation → string network gives “scaling” solution

- Inflation occurs **after** string formation → string network diluted and **no GW signal**

- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) *1912.08832*



SO(10) phenomenological predictions

- $SO(10)$ provides unification without SUSY and can explain neutrino masses

SO(10) phenomenological predictions

- $SO(10)$ provides unification without SUSY and can explain neutrino masses
- 51 “breaking chains” of $SO(10)$ to SM! *Jeannerot, 0308134*

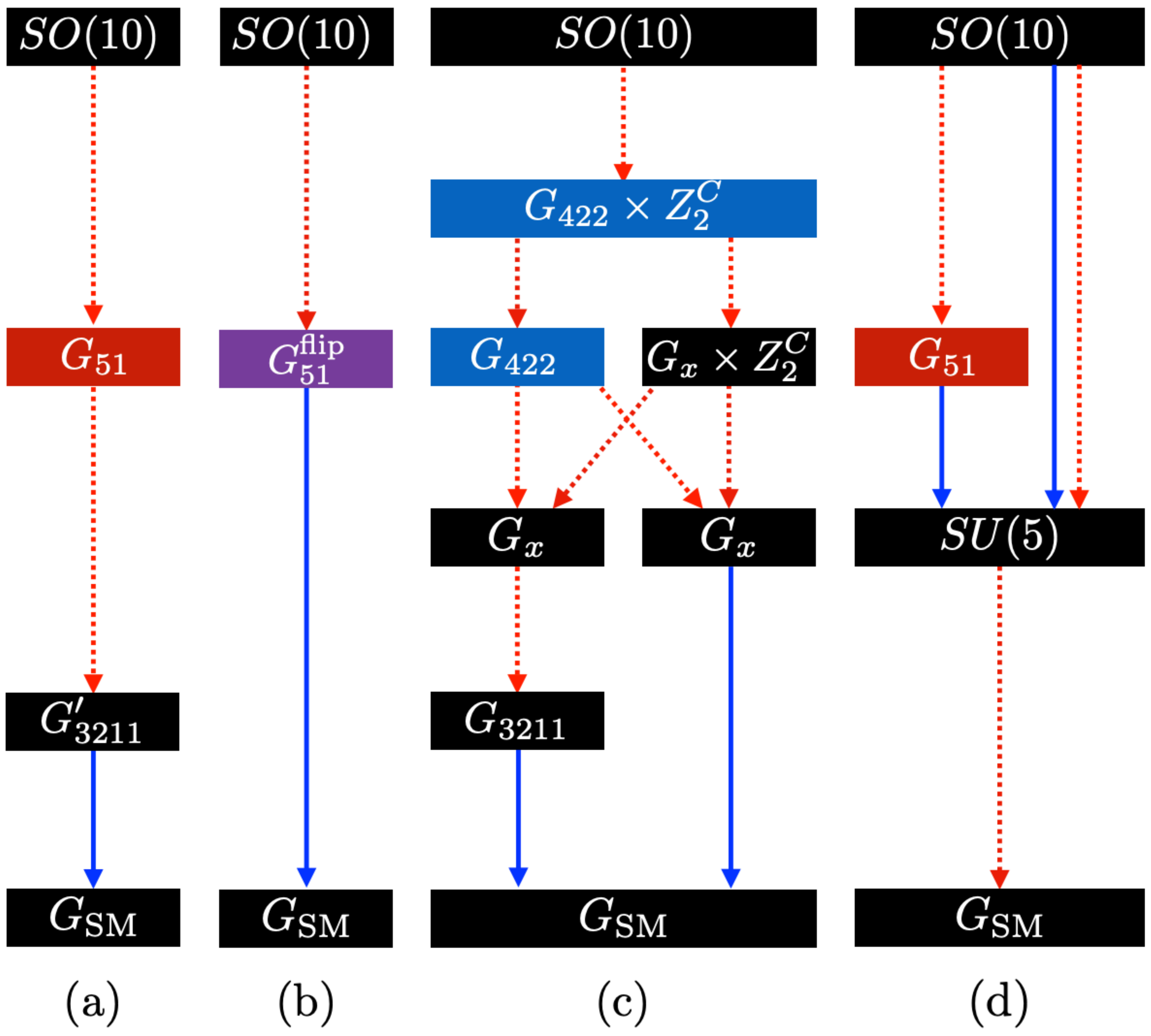
SO(10) phenomenological predictions

- $SO(10)$ provides unification without SUSY and can explain neutrino masses
- 51 "breaking chains" of $SO(10)$ to SM! *Jeannerot, 0308134*

$$\underbrace{SO(10)}_{G_X} \xrightarrow{M_X} \underbrace{G_{422}}_{G_2} \xrightarrow{M_2} \underbrace{G_{3221}}_{G_1} \xrightarrow{M_1} G_{SM}$$

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$
$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

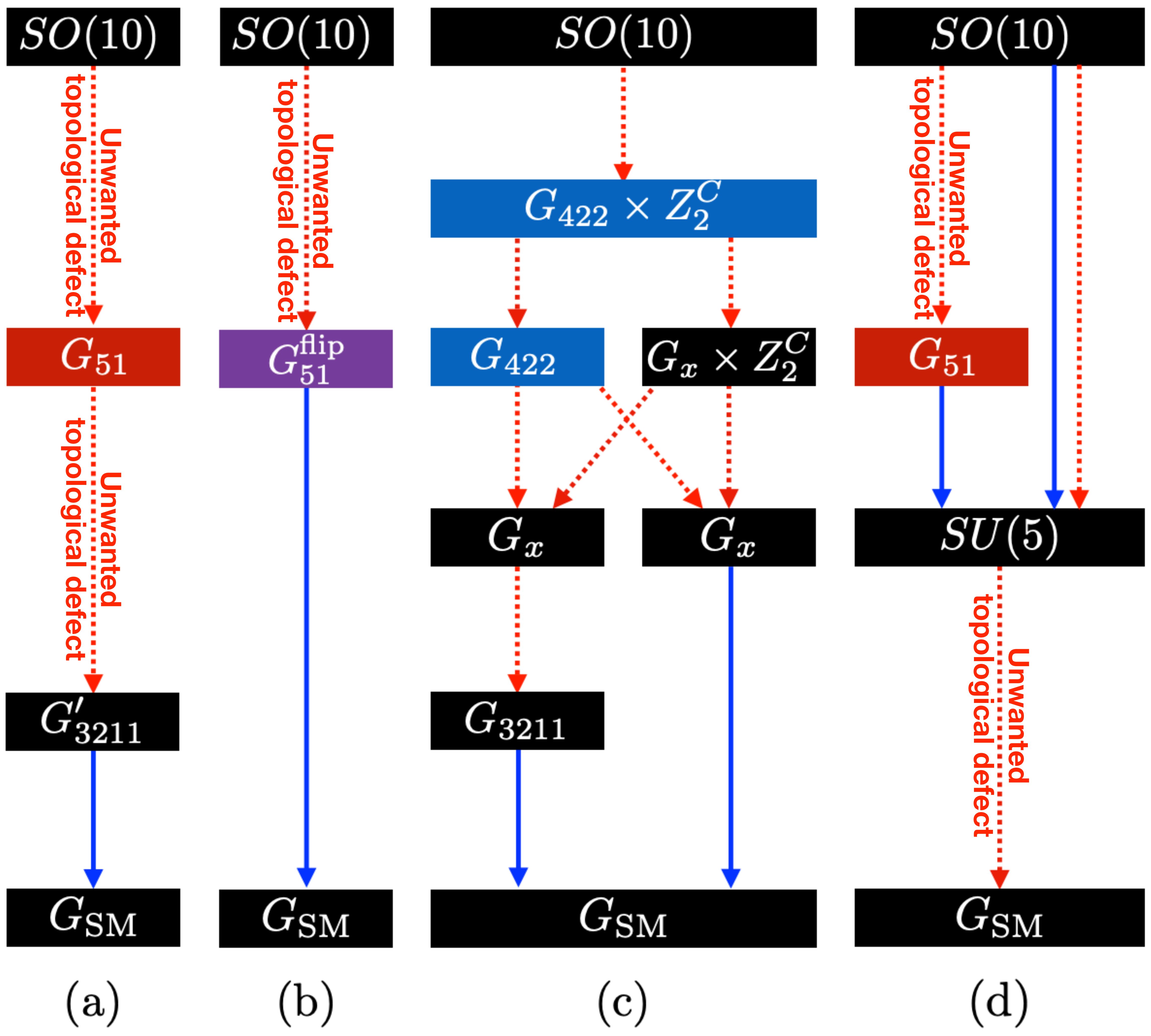
SO(10) phenomenological predictions



$$X = \sqrt{\frac{3}{4}}B - L$$
$$G_{51} = SU(5) \times U(1)_X$$
$$G_{51}^{\text{flip}} = SU(5)_{\text{flip}} \times U(1)_{\text{flip}}$$
$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
$$G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$$
$$G'_{3211} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$
$$G_{421} = SU(4)_C \times SU(2)_L \times U(1)_Y$$
$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R\,.$$

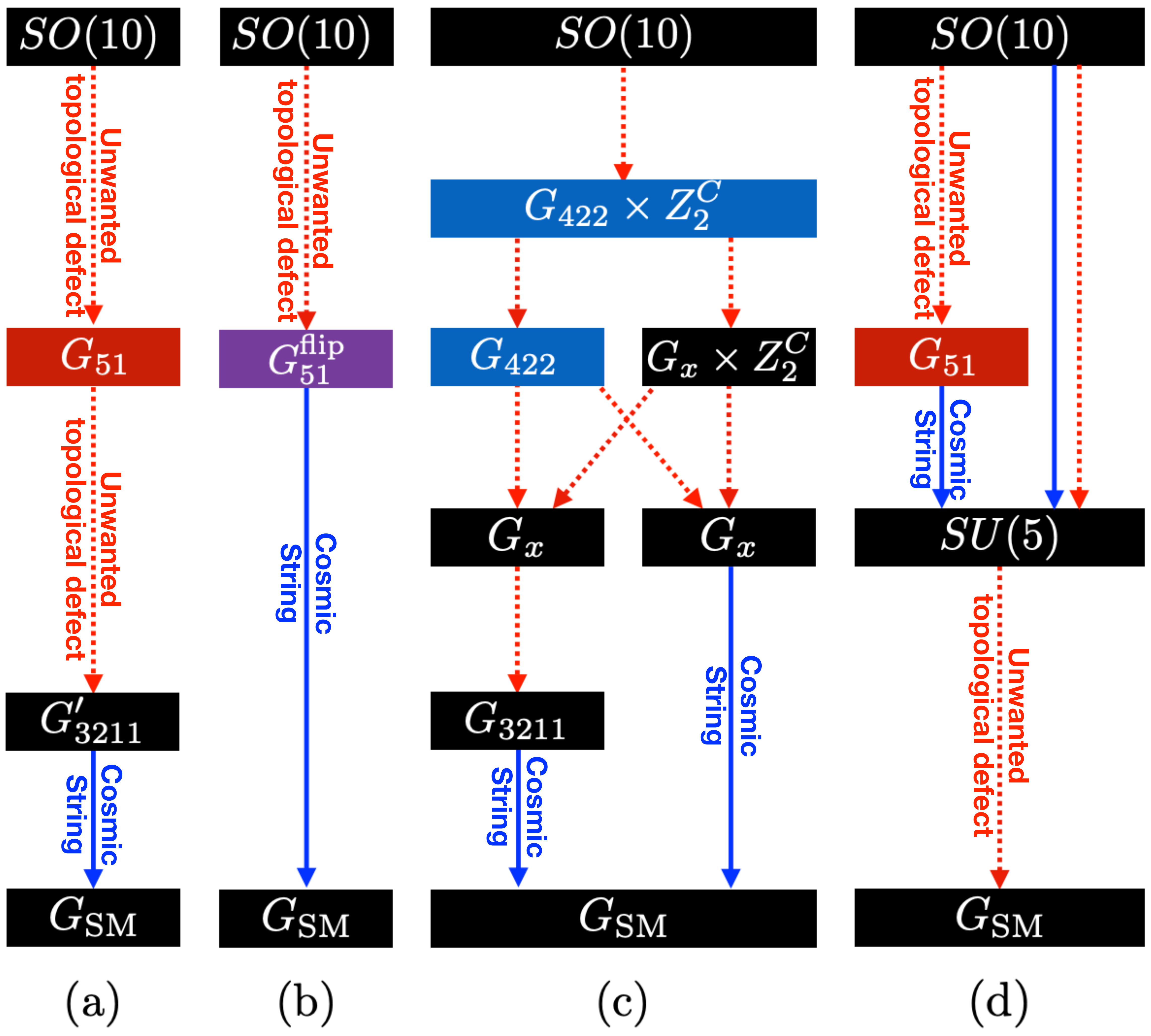
2005.13549 King, Pascoli, JT, Zhou

SO(10) phenomenological predictions



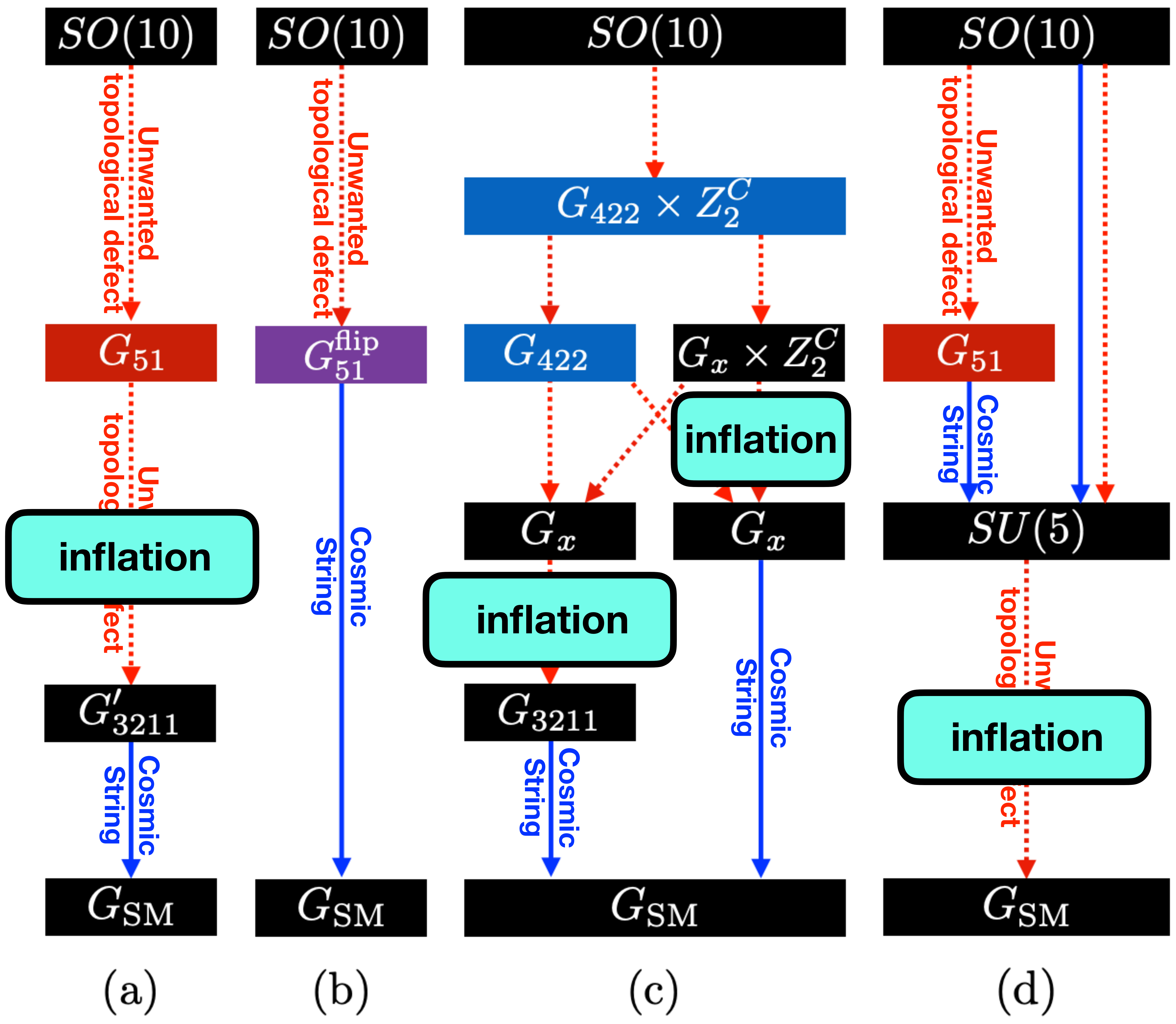
2005.13549 **King, Pascoli, JT, Zhou**

SO(10) phenomenological predictions



2005.13549 King, Pascoli, JT, Zhou

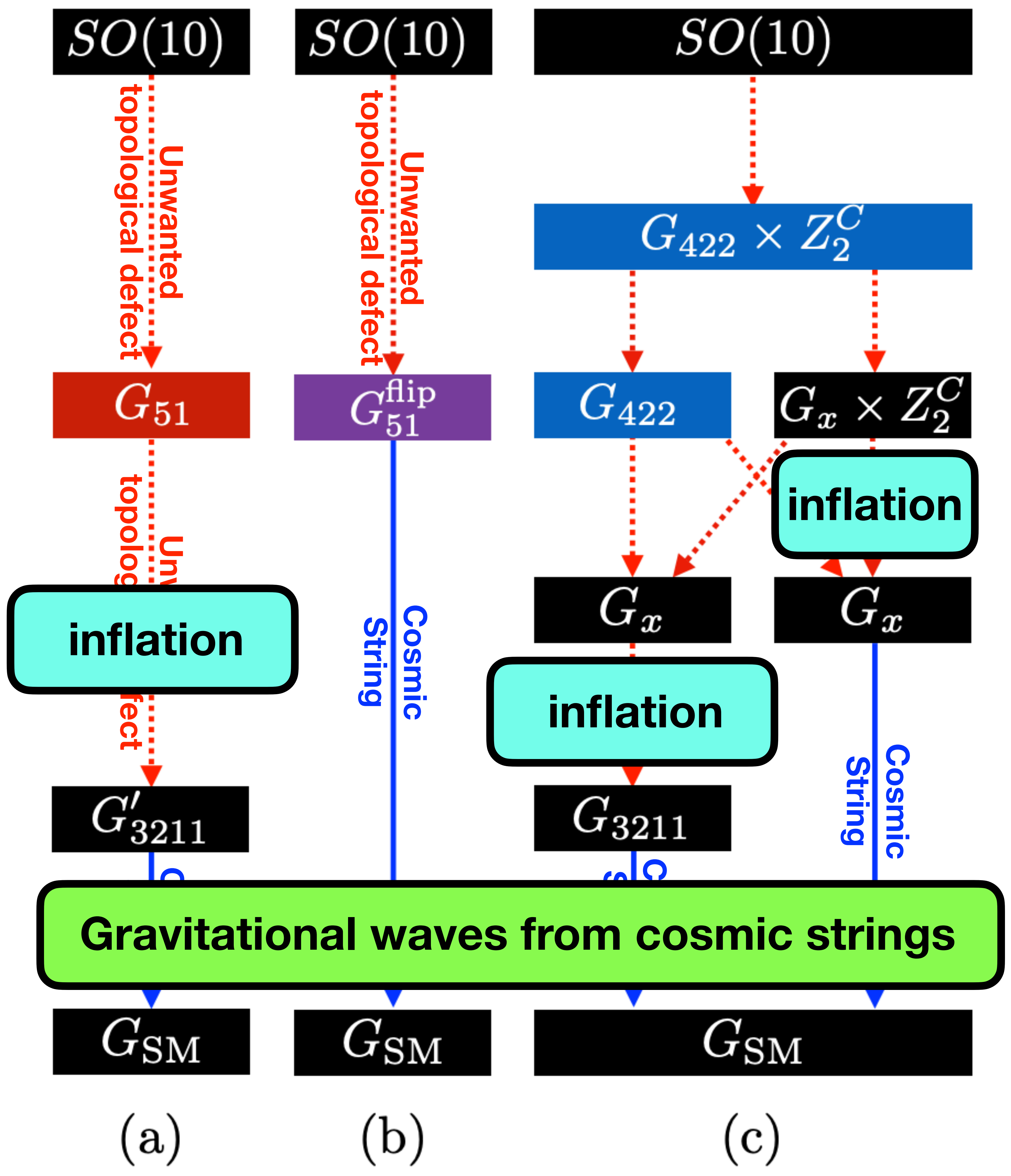
SO(10) phenomenological predictions



CMB data: $\Lambda_{\text{inf}} \lesssim 10^{16}\,\text{GeV}$

2005.13549 King, Pascoli, JT, Zhou

SO(10) phenomenological predictions

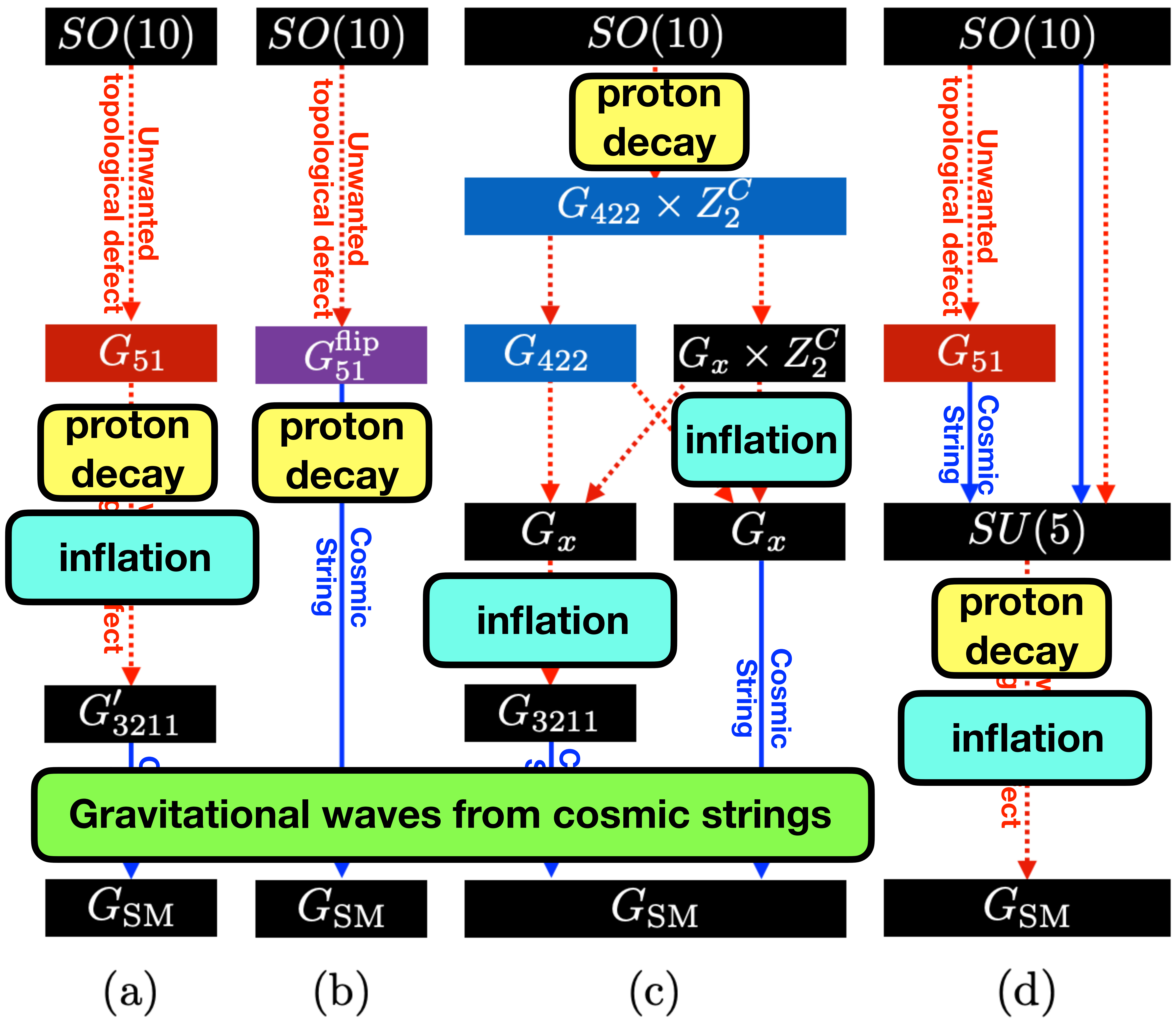


CMB data: $\Lambda_{\rm inf} \lesssim 10^{16}\,{\rm GeV}$

Non-observation GW: $\Lambda_{\rm cs} \lesssim 5 \times 10^{14}\,{\rm GeV}$

2005.13549 King, Pascoli, JT, Zhou

SO(10) phenomenological predictions



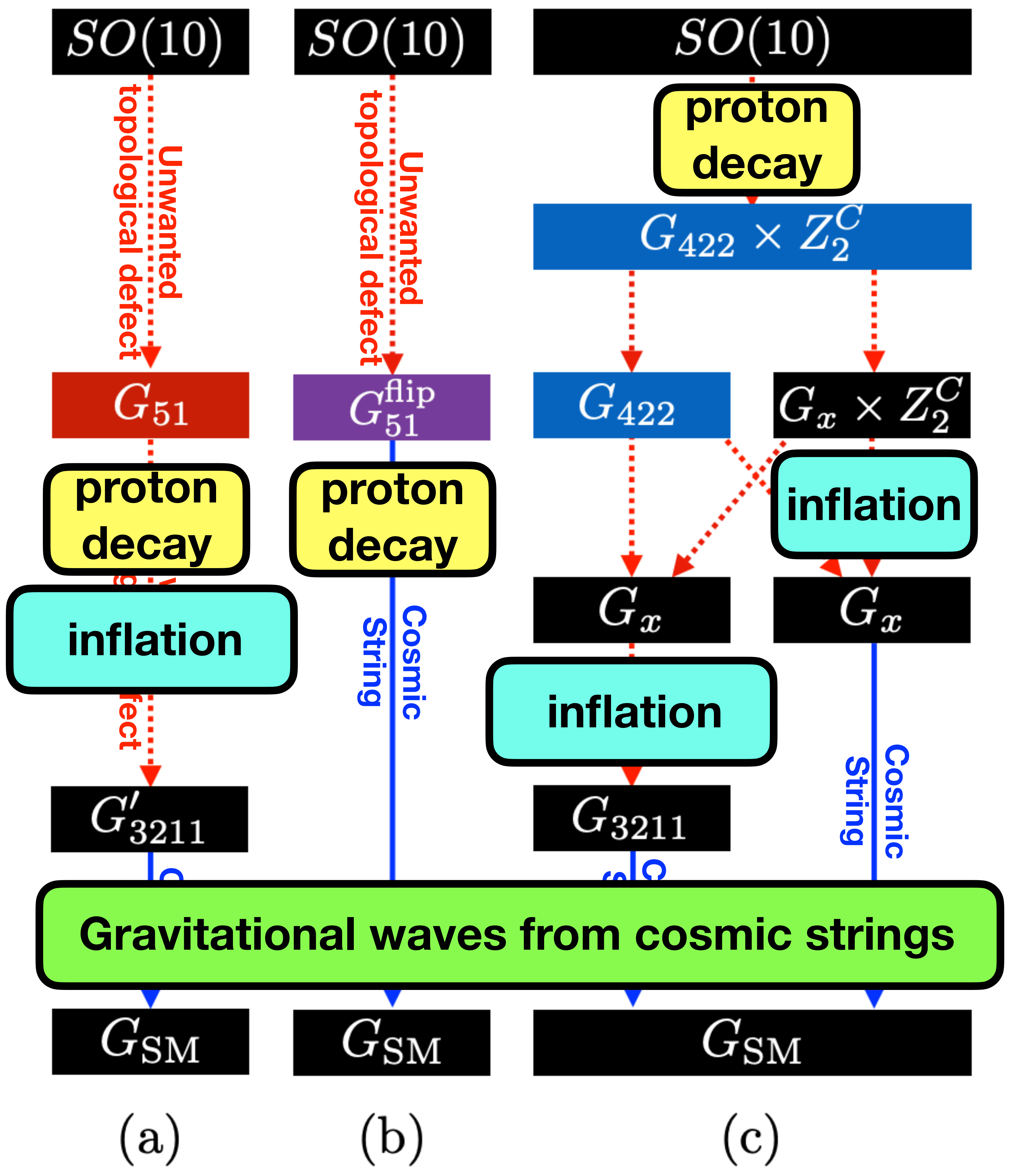
CMB data: $\Lambda_{\rm inf} \lesssim 10^{16}\,{\rm GeV}$

Non-observation GW: $\Lambda_{\rm cs} \lesssim 5 \times 10^{14}\,{\rm GeV}$

Non-observation proton decay: $\Lambda_{\rm pd} \gtrsim 10^{15}\,{\rm GeV}$

2005.13549 King, Pascoli, JT, Zhou

SO(10) phenomenological predictions



CMB data: $\Lambda_{\text{inf}} \lesssim 10^{16}\,\text{GeV}$

Non-observation GW: $\Lambda_{\text{cs}} \lesssim 5 \times 10^{14}\,\text{GeV}$

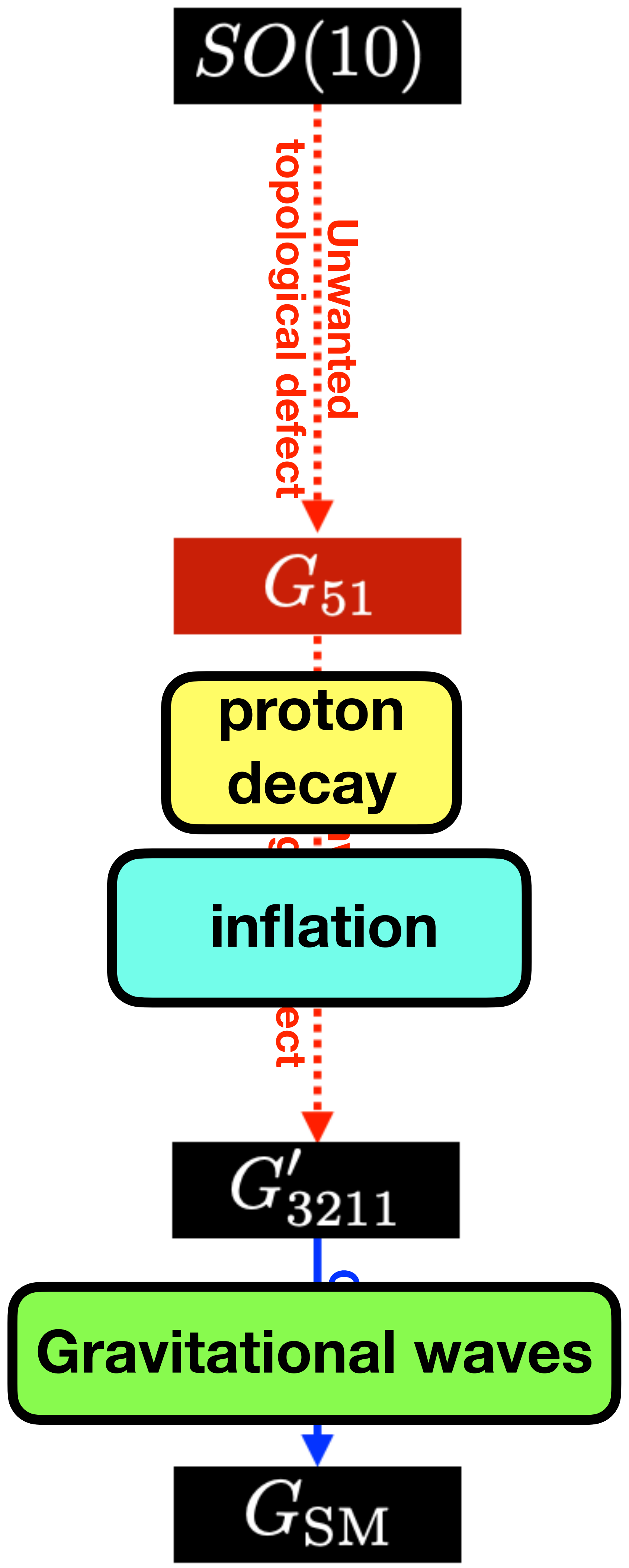
Non-observation proton decay: $\Lambda_{\text{pd}} \gtrsim 10^{15}\,\text{GeV}$

Certain scale ordering excluded e.g

$$\Lambda_{\text{inf}} \gg \Lambda_{\text{cs}} \gg \Lambda_{\text{pd}}$$

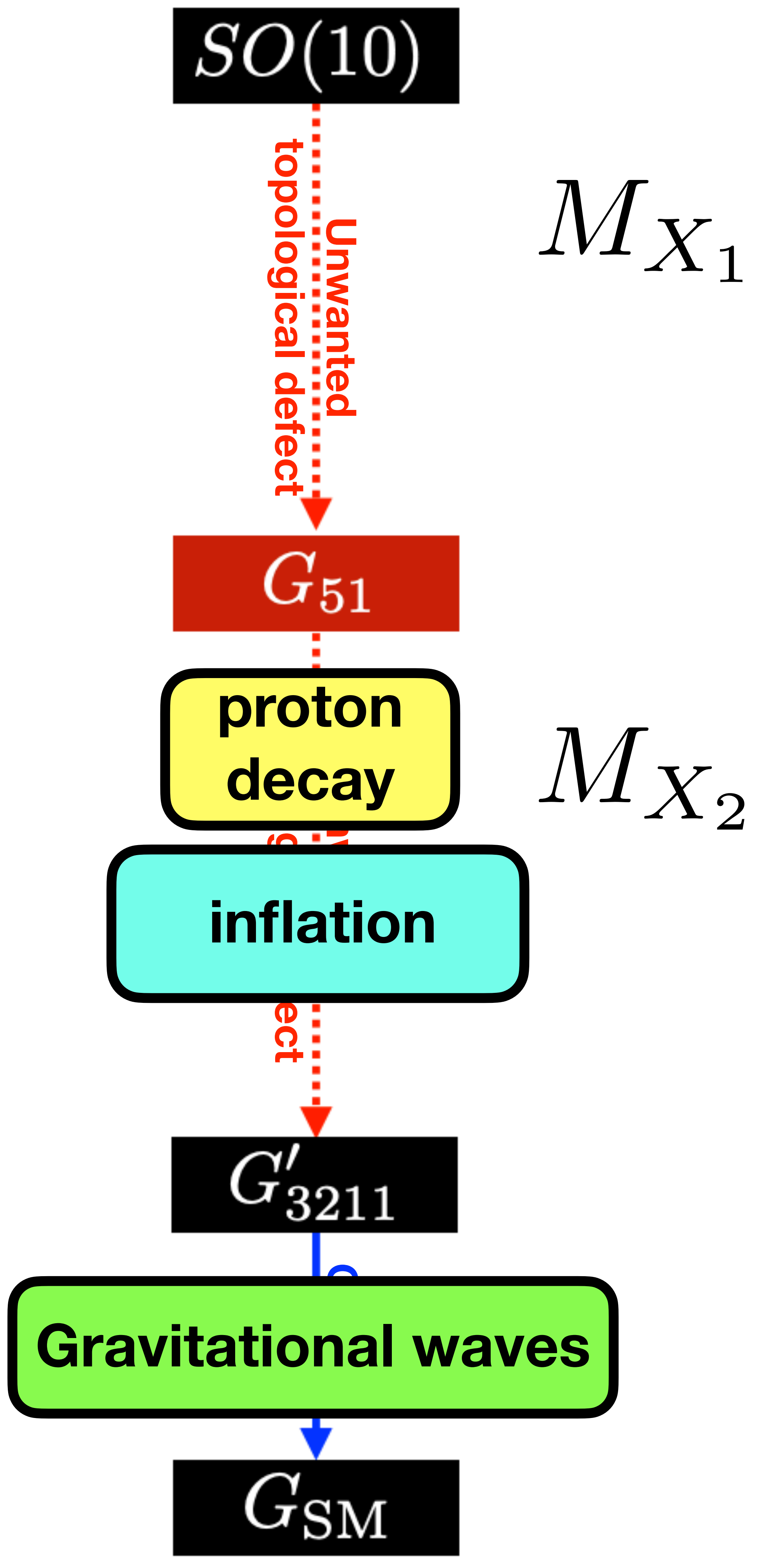
2005.13549 King, Pascoli, JT, Zhou

SO(10) phenomenological predictions



(a)

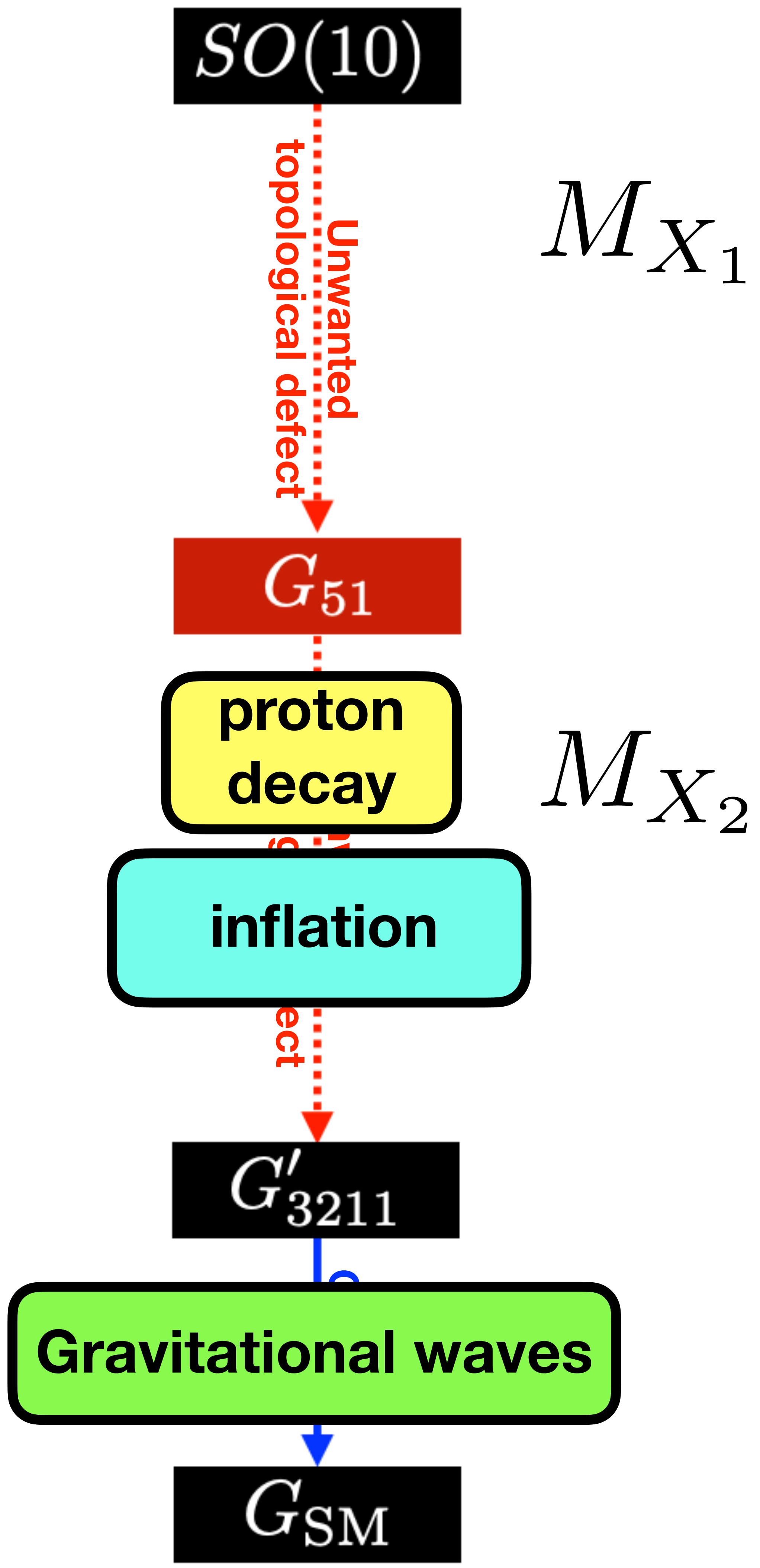
SO(10) phenomenological predictions



(a)

- Proton decay operators induced at M_{X_1} and M_{X_2}

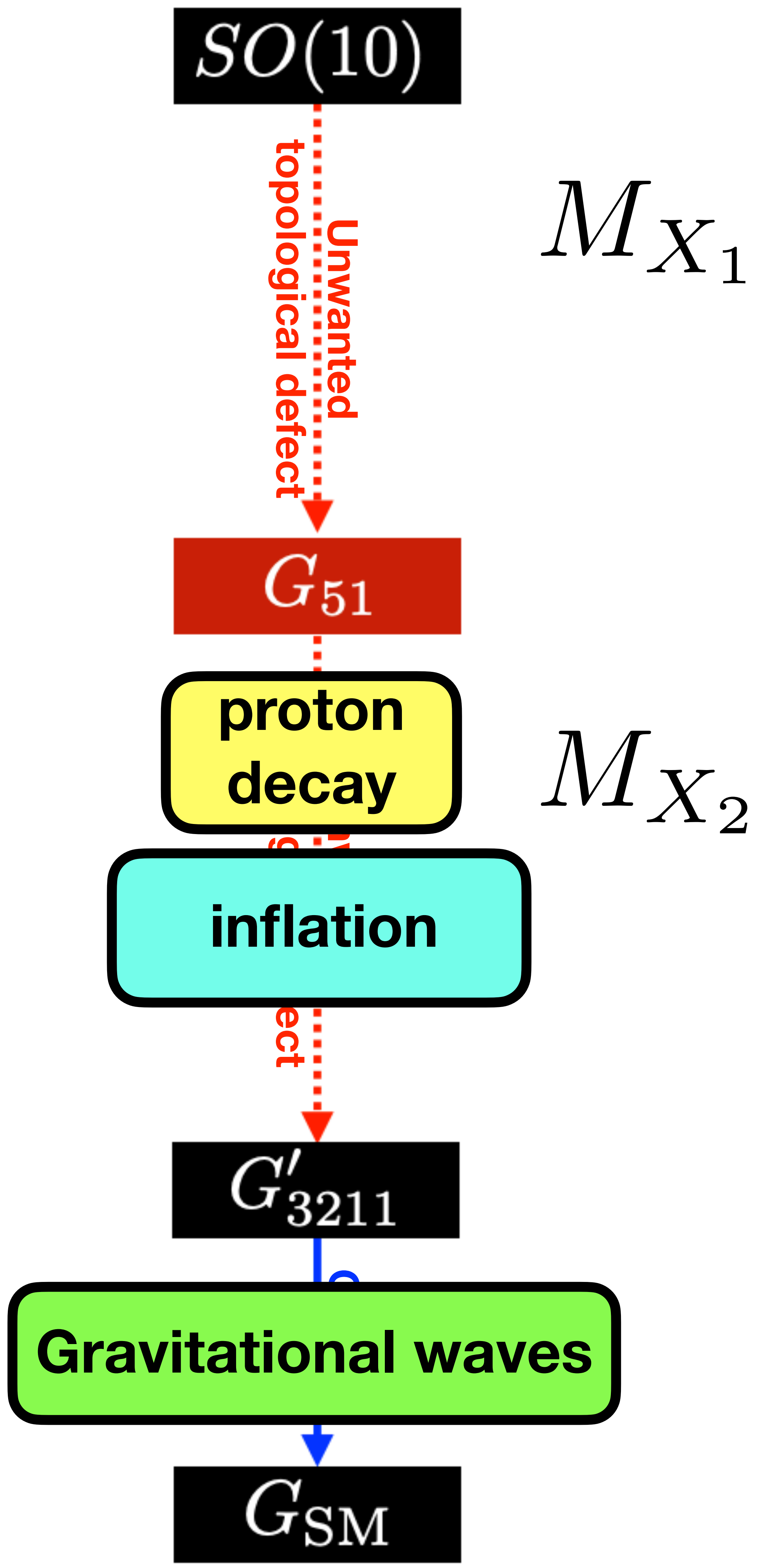
SO(10) phenomenological predictions



(a)

- Proton decay operators induced at M_{X_1} and M_{X_2}
- $M_{X_2} < M_{X_1} \implies$ main proton decay channel: $p \rightarrow e^+\pi^0$ at scale $\Lambda_{\text{pd}} = M_{X_2}$

SO(10) phenomenological predictions



(a)

- Proton decay operators induced at M_{X_1} and M_{X_2}

- $M_{X_2} < M_{X_1} \implies$ main proton decay channel: $p \to e^+\pi^0$ at scale $\Lambda_{\rm pd} = M_{X_2}$

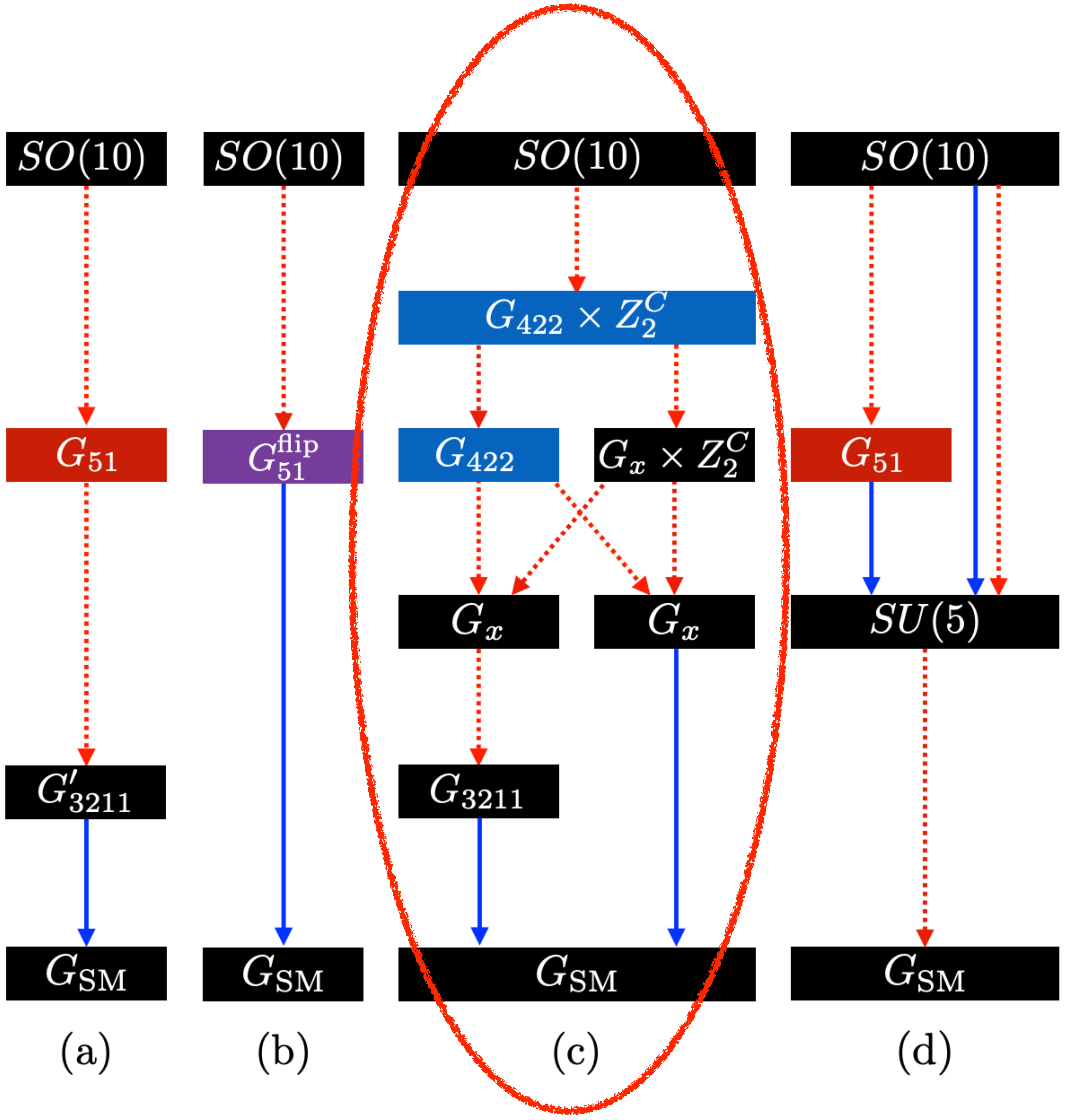
1. $\Lambda_{\rm pd} > \Lambda_{\rm inf} > \Lambda_{\rm cs}$: PD + undiluted GW observed (ideal case)
2. $\Lambda_{\rm pd} > \Lambda_{\rm inf} \sim \Lambda_{\rm cs}$: PD + diluted GW observed
3. $\Lambda_{\rm pd} > \Lambda_{\rm cs} > \Lambda_{\rm inf}$: PD + no associated GW

Proton decay and GWs as complementary windows

- Type (a): $\Lambda_{\text{pd}} > \Lambda_{\text{cs}}$
- Type (b): $\Lambda_{\text{pd}} \sim \Lambda_{\text{cs}}$
- Type (c): $\Lambda_{\text{pd}} > \Lambda_{\text{cs}}$
- Type (d): no GWs

<table>
<tr><td colspan="2" rowspan="2">Observables</td><td>Proton decays</td></tr>
<tr><td>$p \to \pi^0 e^+$ observed $\Rightarrow$ non-SUSY contribution indicated</td></tr>
<tr><td rowspan="2">GWs</td><td>Observed</td><td>• types (a) and (c) favoured
• types (b) and (d) excluded</td></tr>
<tr><td>Marginal</td><td>• types (a) and (c) favoured
• type (d) excluded
• type (b) allowed if $p \to K^+\bar{\nu}$ not observed and $\Lambda_{\text{pd}} \sim \Lambda_{\text{cs}}$</td></tr>
</table>

- Distinguish (a) & (c) requires model dependent study



- (d) cannot be tested with GWs since Unwanted defects formed in last SSB step
- Gauge unification not possible in (a) & (b) without SUSY
- Study (c) in more detail in **2106.15634** 31 breaking chains

Proton decay and GWs as complementary windows

	$SO(10) \xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
I1:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
I2:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
I3:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
I4:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m}}$		✗
I5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m,w}}$		✗
I6:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

Need to compute RGEs for all 31 chains

	$SO(10) \xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
II1:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
II3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II4:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II5:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II7:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m}}$		✗
II8:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II9:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II11:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II12:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

	$SO(10) \xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
III1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III4:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III7:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III8:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III9:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

	$SO(10) \xrightarrow[\text{Higgs}]{\text{defect}}$	G_4	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
IV1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

Proton decay and GWs as complementary windows

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
I1:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
I2:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
I3:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
I4:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m}}$		✗
I5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m,w}}$		✗
I6:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

Need to compute RGEs for all 31 chains

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
II1:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
II3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II4:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II5:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II7:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m}}$		✗
II8:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II9:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II11:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II12:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
III1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III4:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III7:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III8:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III9:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_4	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
IV1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

Proton decay and GWs as complementary windows

$$\mathrm{SO(10)} \xrightarrow{M_X} \mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R} \xrightarrow{M_2} \mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \xrightarrow{M_1} \mathrm{G_{SM}}$$

Monopole formation
Proton decay operators induced

Monopole formation

String formation

Proton decay and GWs as complementary windows

$$\mathrm{SO(10)} \xrightarrow{M_X} \mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R} \xrightarrow{M_2} \mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \xrightarrow{M_1} \mathrm{G_{SM}}$$

Monopole formation
Proton decay operators induced

Monopole formation

String formation

Assumptions

Proton decay and GWs as complementary windows

$$\mathrm{SO(10)} \xrightarrow{M_X} \mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R} \xrightarrow{M_2} \mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \xrightarrow{M_1} \mathrm{G_{SM}}$$

Monopole formation
Proton decay operators induced

Monopole formation

String formation

Assumptions

- Inflation after monopole formation & before cosmic string formation $\Longrightarrow$ observable GW

Proton decay and GWs as complementary windows

$$\mathrm{SO(10)} \xrightarrow{M_X} \mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R} \xrightarrow{M_2} \mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \xrightarrow{M_1} \mathrm{G_{SM}}$$

Monopole formation
Proton decay operators induced

Monopole formation

String formation

Assumptions

- Inflation after monopole formation & before cosmic string formation $\Longrightarrow$ observable GW
- **Minimal particle content**: SM, RH neutrinos and Higgs multiplet required for SSB

Proton decay and GWs as complementary windows

$$\mathrm{SO(10)} \xrightarrow{M_X} \mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R} \xrightarrow{M_2} \mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \xrightarrow{M_1} \mathrm{G_{SM}}$$

Monopole formation
Proton decay operators induced

Monopole formation

String formation

Assumptions

- Inflation after monopole formation & before cosmic string formation $\Longrightarrow$ observable GW

- **Minimal particle content**: SM, RH neutrinos and Higgs multiplet required for SSB

- **Assume gauge unification** at scale M_X. For each chain perform 2-loop RGE analysis to determine couplings & M_X, M_2, M_1

Proton decay and GWs as complementary windows

$$\mathrm{SO(10)} \xrightarrow{M_X} \mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R} \xrightarrow{M_2} \mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \xrightarrow{M_1} \mathrm{G_{SM}}$$

Monopole formation
Proton decay operators induced

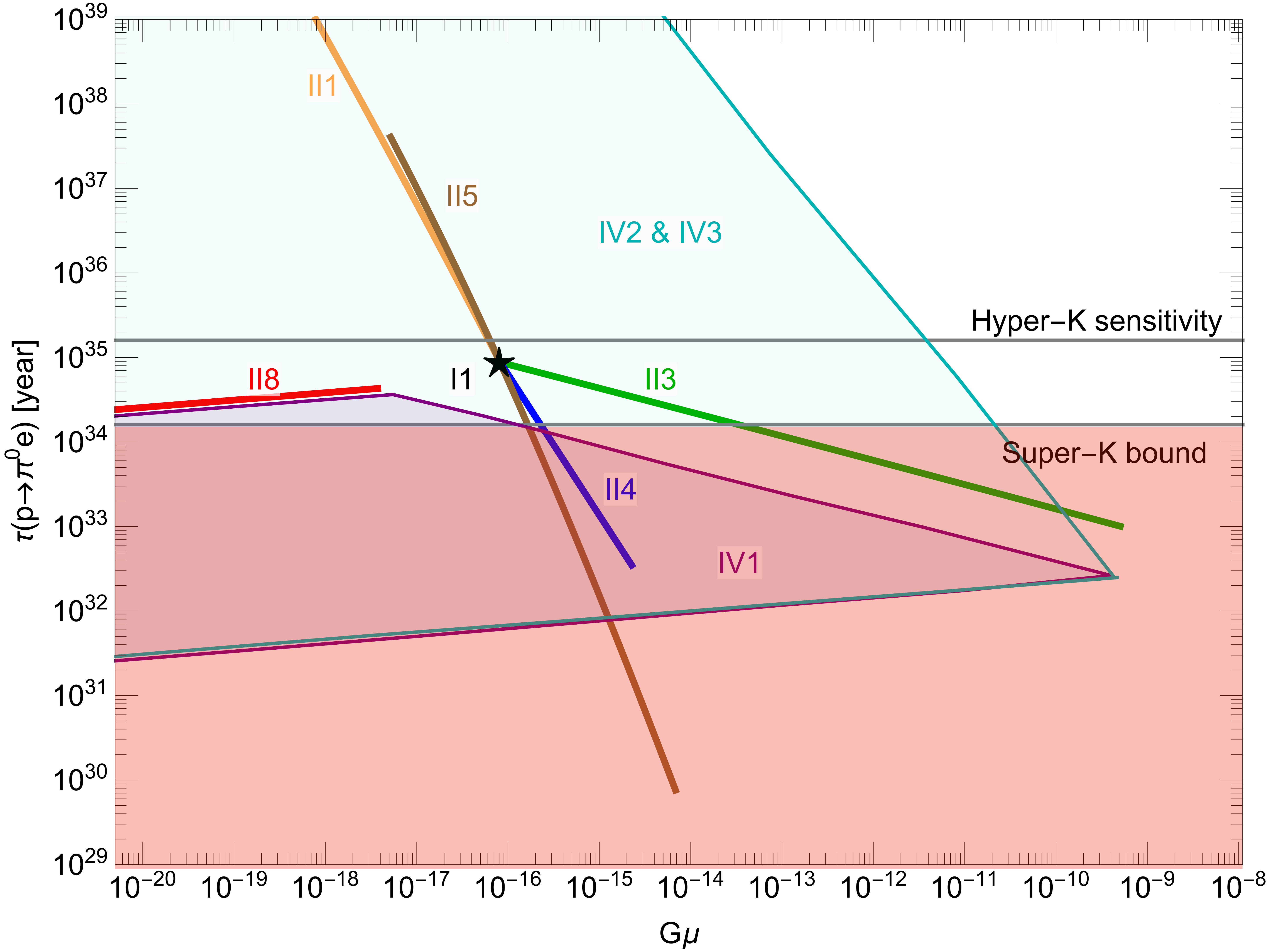
Monopole formation

String formation

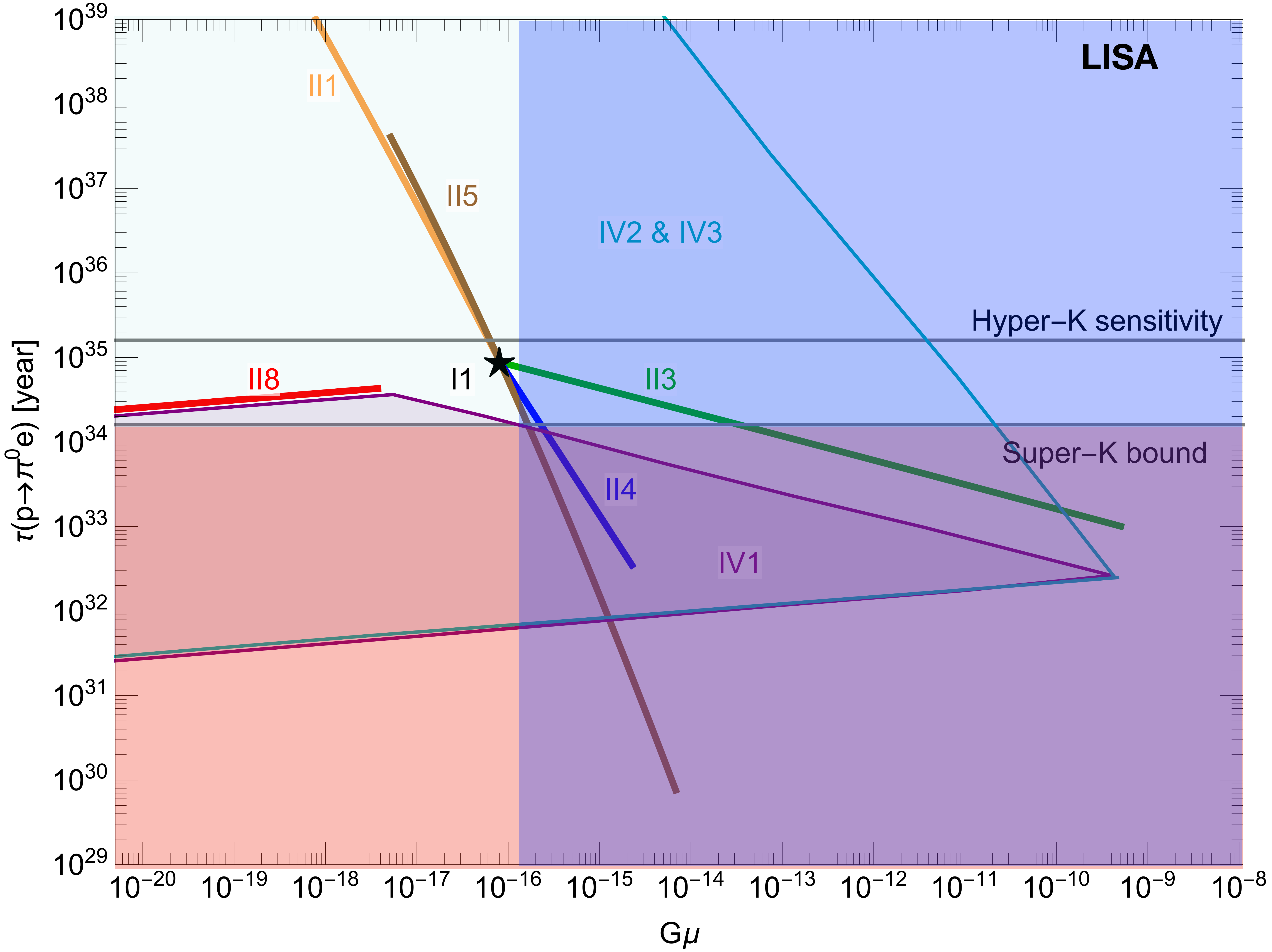
Assumptions

- Inflation after monopole formation & before cosmic string formation $\Longrightarrow$ observable GW
- **Minimal particle content**: SM, RH neutrinos and Higgs multiplet required for SSB
- **Assume gauge unification** at scale M_X. For each chain perform 2-loop RGE analysis to determine couplings & $M_X\,, M_2\,, M_1$
- Using these assumptions & RGE solutions we calculate proton lifetime & GW signal (see back up slides for details)

Correlation of GW and PD signals



Correlation of GW and PD signals



Correlation of GW and PD signals

- non-SUSY $\mathrm{SO}(10)$ Pati Salam type provide unification: **31 breaking chains**
- Two-loop RGE, **17 not excluded** by Super-K bound PD.

Chain	$G\mu$ after Hyper-K (no proton decay)
I1	excluded
II1:	$G\mu \lesssim 1.5 \times 10^{-17}$
II3:	excluded
II4:	excluded
II5:	$G\mu \simeq 5.1 \times 10^{-18} – 6.3 \times 10^{-17}$
II8:	excluded
III1:	$G\mu \simeq 1.3 \times 10^{-18} – 1.6 \times 10^{-15}$
III2:	$G\mu \lesssim 5.0 \times 10^{-12}$
III3:	$G\mu \lesssim 6.2 \times 10^{-14}$
III4:	excluded
III6:	excluded
III7:	excluded
III8:	excluded
III10:	$G\mu \lesssim 1.1 \times 10^{-21}$
IV1:	excluded
IV2:	$G\mu \lesssim 9.4 \times 10^{-13}$
IV3:	$G\mu \lesssim 9.4 \times 10^{-13}$

Testable by LIGO, DECIGO, AEDGE, C, ET, MAGIS..

- If HyperK **does not** observe PD $\Longrightarrow$ 9 chains excluded
- **8 survivors!** If we observe GW signal **larger than upper bounds** $\Longrightarrow$ exclude those breaking chains
- If we observe PD $\Longrightarrow M_1$ determined so is GW signal. Correlations between observables matter and need to be compared on case by case basis.

Correlation of GW and PD signals

- non-SUSY $\mathrm{SO}(10)$ Pati Salam type provide unification: **31 breaking chains**
- Two-loop RGE, **17 not excluded** by Super-K lower bound PD.

Chain	$G\mu$ after Hyper-K (no proton decay)
I1	excluded
II1:	$G\mu \lesssim 1.5 \times 10^{-17}$
II3:	excluded
II4:	excluded
II5:	$G\mu \simeq 5.1 \times 10^{-18} – 6.3 \times 10^{-17}$
II8:	excluded
III1:	$G\mu \simeq 1.3 \times 10^{-18} – 1.6 \times 10^{-15}$
III2:	$G\mu \lesssim 5.0 \times 10^{-12}$
III3:	$G\mu \lesssim 6.2 \times 10^{-14}$
III4:	excluded
III6:	excluded
III7:	excluded
III8:	excluded
III10:	$G\mu \lesssim 1.1 \times 10^{-21}$
IV1:	excluded
IV2:	$G\mu \lesssim 9.4 \times 10^{-13}$
IV3:	$G\mu \lesssim 9.4 \times 10^{-13}$

Study specific breaking chain 2209.00021

Why? **Can be tested by Hyper-K** & has an associated GW signal

SO(10) Model confronting data

- Treatment has been **model-independent** so far

SO(10) Model confronting data

- Treatment has been **model-independent** so far
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

SO(10) Model confronting data

- Treatment has been **model-independent** so far
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

$$
\begin{array}{c}
SO(10) \\
\downarrow \\
SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C \\
\downarrow \\
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C \\
\downarrow \\
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
\downarrow \\
SU(3)_c \times SU(2)_L \times U(1)_Y \,.
\end{array}
$$

SO(10) Model confronting data

- Treatment has been **model-independent** so far
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

$$
\begin{array}{c}
SO(10) \\
\downarrow \quad \longleftarrow \boxed{M_X\,, g_X} \\
SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C \\
\downarrow \\
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C \\
\downarrow \\
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
\downarrow \\
SU(3)_c \times SU(2)_L \times U(1)_Y \,.
\end{array}
$$

SO(10) Model confronting data

- Treatment has been **model-independent** so far
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

$$SO(10)$$
$$\downarrow$$
$$SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times U(1)_Y \,.$$

M_X, g_X → **Proton Lifetime**

SO(10) Model confronting data

- Treatment has been **model-independent** so far
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

$$
\begin{array}{c}
SO(10) \\
\downarrow \\
SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C \\
\downarrow \\
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C \\
\downarrow \\
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
\downarrow \\
SU(3)_c \times SU(2)_L \times U(1)_Y \,.
\end{array}
$$

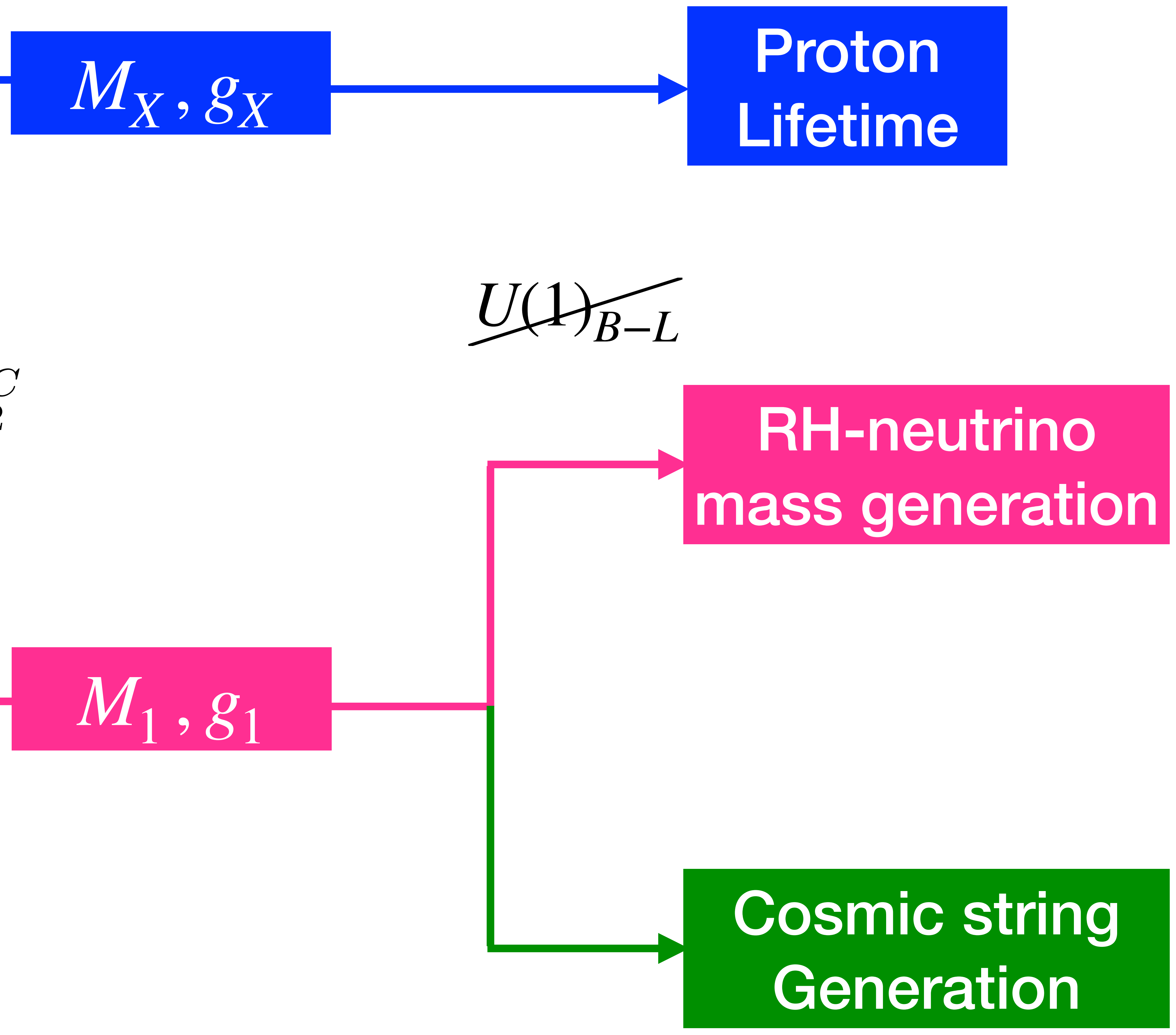
M_X, g_X → Proton Lifetime

M_1, g_1

SO(10) Model confronting data

- Treatment has been **model-independent** so far
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

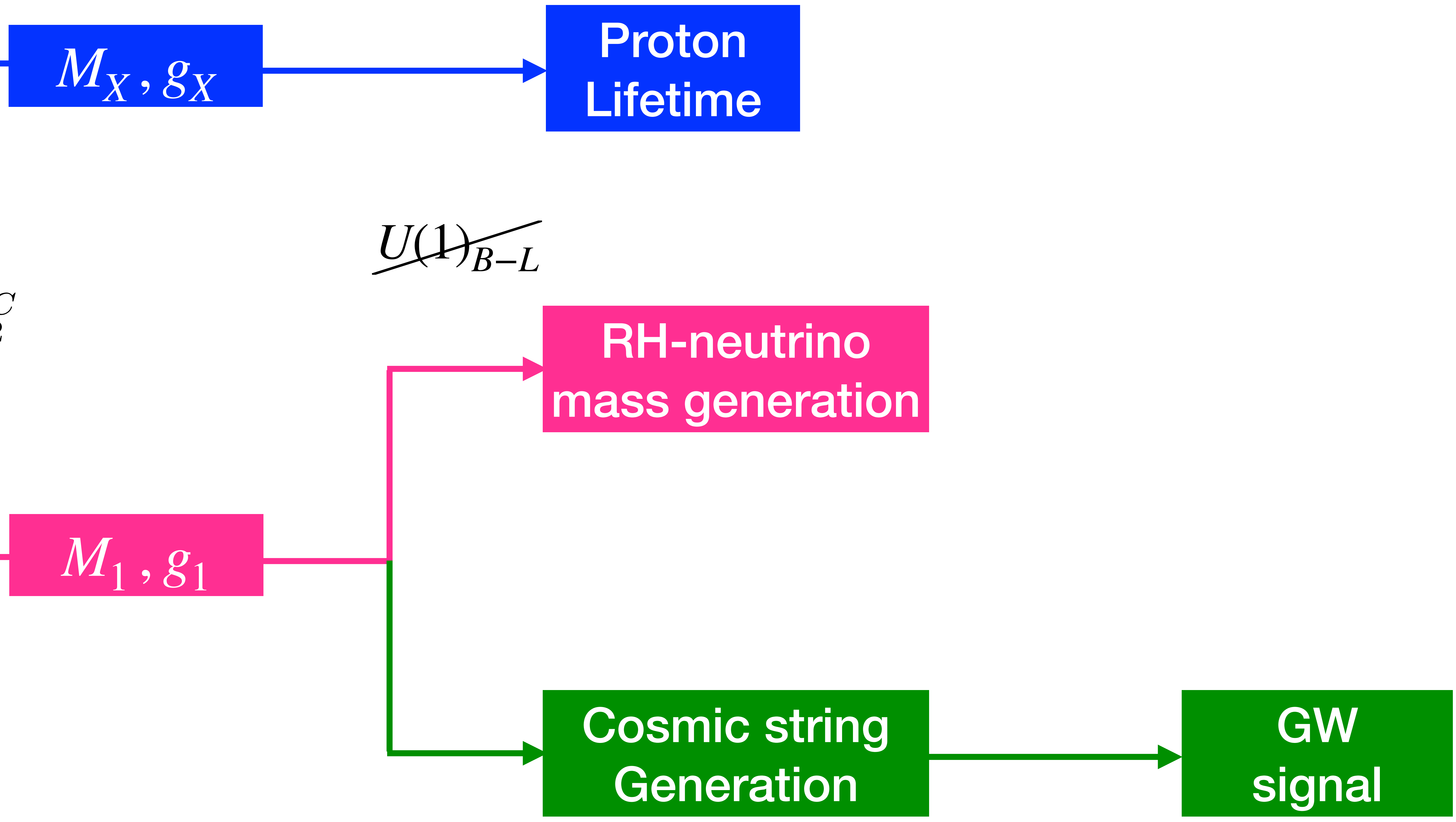
$$
\begin{aligned}
&SO(10)\\
&\downarrow\\
&SU(4)\times SU(2)_L\times SU(2)_R\times Z_2^C\\
&\downarrow\\
&SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_X\times Z_2^C\\
&\downarrow\\
&SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\\
&\downarrow\\
&SU(3)_c\times SU(2)_L\times U(1)_Y\,.
\end{aligned}
$$



SO(10) Model confronting data

- Treatment has been **model-independent** so far
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

$$
\begin{aligned}
&SO(10)\\
&\downarrow\\
&SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C\\
&\downarrow\\
&SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C\\
&\downarrow\\
&SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\\
&\downarrow\\
&SU(3)_c \times SU(2)_L \times U(1)_Y\ .
\end{aligned}
$$

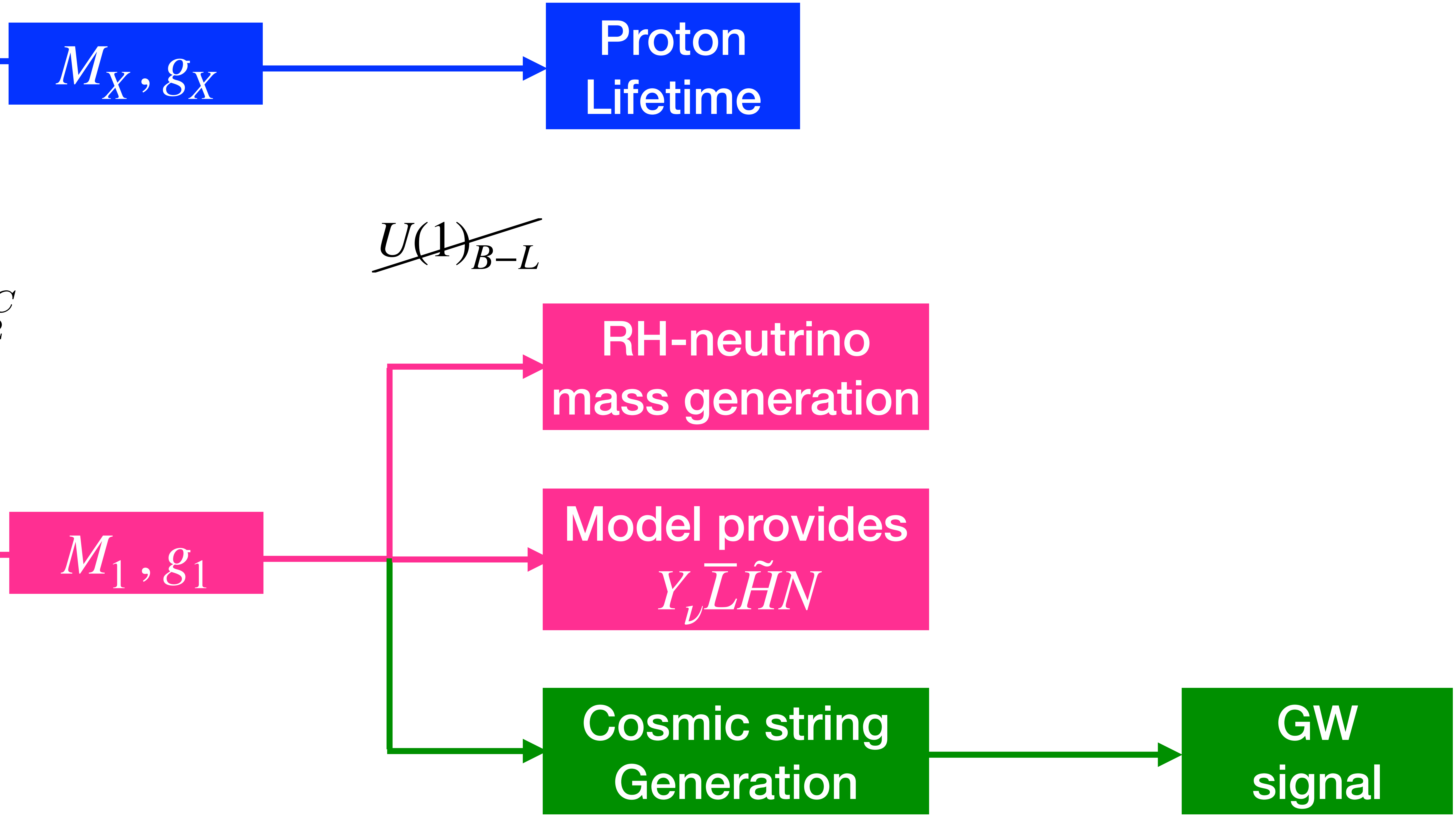


SO(10) Model confronting data

- Treatment has been **model-independent** so far

- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

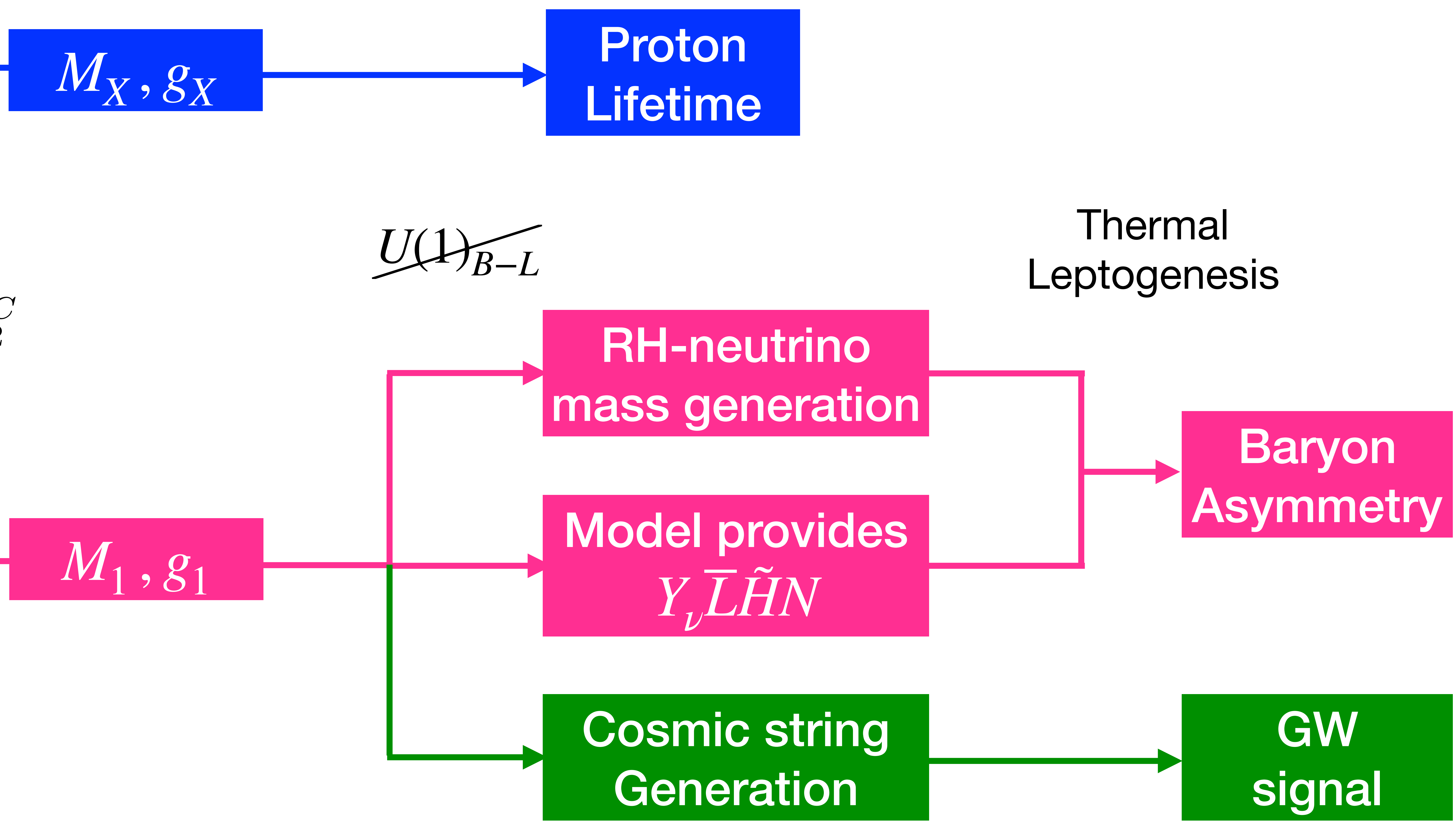
$$SO(10)$$
$$\downarrow$$
$$SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times U(1)_Y \, .$$



SO(10) Model confronting data

- Treatment has been **model-independent** so far
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

$$SO(10)$$
$$\downarrow$$
$$SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times U(1)_Y \, .$$



SO(10) Model confronting data

- Treatment has been **model-independent** so far
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

See Domcke et al (1912.03695) for connections between $\cancel{U(1)_{B-L}}$ breaking in GUTS & GWs & Blasi et al (2004.02889) for connections between $\cancel{U(1)_{B-L}}$, GWs & leptogenesis

SO(10) Model confronting data

- Model of Altarelli & Blankenburg
- Above GUT scale, Yukawa sector

$$Y^*_{\mathbf{10}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y^*_{\overline{\mathbf{126}}}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y^*_{\mathbf{120}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\mathrm{h.c.}\,,$$

SO(10) Model confronting data

- Model of Altarelli & Blankenburg
- Above GUT scale, Yukawa sector

$$Y^*_{\mathbf{10}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y^*_{\overline{\mathbf{126}}}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y^*_{\mathbf{120}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\mathrm{h.c.}\,,$$

- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R+Y_d\bar{Q}h_{\mathrm{SM}}d_R+Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R+Y_e\bar{L}h_{\mathrm{SM}}e_R+\ \mathrm{h.c.}$$

SO(10) Model confronting data

- Model of Altarelli & Blankenburg
- Above GUT scale, Yukawa sector

$$Y^*_{\mathbf{10}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y^*_{\overline{\mathbf{126}}}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y^*_{\mathbf{120}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\mathrm{h.c.}\,,$$

- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R+Y_d\bar{Q}h_{\mathrm{SM}}d_R+Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R+Y_e\bar{L}h_{\mathrm{SM}}e_R+\ \mathrm{h.c.}$$

$$Y_u=Y_{\mathbf{10}}V^*_{11}+\frac{1}{\sqrt{3}}Y_{\overline{\mathbf{126}}}V^*_{12}+Y_{\mathbf{120}}\left(V^*_{13}+\frac{1}{\sqrt{3}}V^*_{14}\right)$$

SM up Yukawa

SO(10) Model confronting data

- Model of Altarelli & Blankenburg
- Above GUT scale, Yukawa sector

$$Y_{\mathbf{10}}^{*}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10} + Y_{\overline{\mathbf{126}}}^{*}\, \mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}} + Y_{\mathbf{120}}^{*}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120} + \mathrm{h.c.}\,,$$

- After breaking to SM:

$$Y_u \bar{Q}\tilde{h}_{\mathrm{SM}} u_R + Y_d \bar{Q} h_{\mathrm{SM}} d_R + Y_\nu \bar{L}\tilde{h}_{\mathrm{SM}} \nu_R + Y_e \bar{L} h_{\mathrm{SM}} e_R + \ \mathrm{h.c.}$$

GUT Yukawa Parameter

$$Y_u = Y_{\mathbf{10}} V_{11}^{*} + \frac{1}{\sqrt{3}} Y_{\overline{\mathbf{126}}} V_{12}^{*} + Y_{\mathbf{120}} \left(V_{13}^{*} + \frac{1}{\sqrt{3}} V_{14}^{*} \right)$$

SM up Yukawa

SO(10) Model confronting data

- Model of Altarelli & Blankenburg
- Above GUT scale, Yukawa sector

$$Y_{\mathbf{10}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y_{\overline{\mathbf{126}}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y_{\mathbf{120}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\mathrm{h.c.}\,,$$

- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R+Y_d\bar{Q}h_{\mathrm{SM}}d_R+Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R+Y_e\bar{L}h_{\mathrm{SM}}e_R+\ \mathrm{h.c.}$$

GUT Yukawa Parameter

$$Y_u=Y_{\mathbf{10}}V_{11}^{*}+\frac{1}{\sqrt{3}}Y_{\overline{\mathbf{126}}}V_{12}^{*}+Y_{\mathbf{120}}\left(V_{13}^{*}+\frac{1}{\sqrt{3}}V_{14}^{*}\right)$$

SM up Yukawa

Mixing between Higgs doublets

SO(10) Model confronting data

- Model of Altarelli & Blankenburg
- Above GUT scale, Yukawa sector

$$Y^*_{\mathbf{10}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10} + Y^*_{\overline{\mathbf{126}}}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}} + Y^*_{\mathbf{120}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120} + \mathrm{h.c.}\,,$$

- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R + Y_d\bar{Q}h_{\mathrm{SM}}d_R + Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R + Y_e\bar{L}h_{\mathrm{SM}}e_R + \;\mathrm{h.c.}$$

GUT Yukawa Parameter

$$Y_u = Y_{\mathbf{10}}V^*_{11} + \frac{1}{\sqrt{3}}Y_{\overline{\mathbf{126}}}V^*_{12} + Y_{\mathbf{120}}\left(V^*_{13} + \frac{1}{\sqrt{3}}V^*_{14}\right)$$

SM up Yukawa

Mixing between Higgs doublets

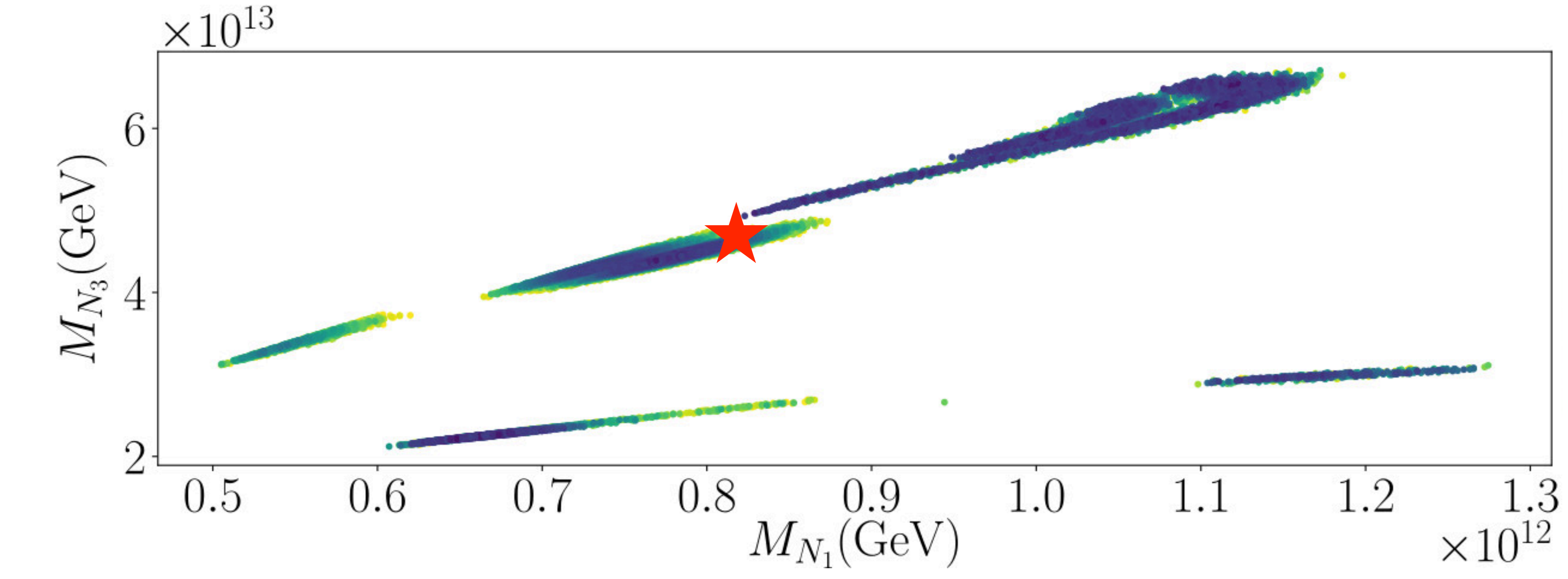
$$\hat{h}_i = \sum_j V_{ij}h_j \qquad h_{\mathrm{SM}} \equiv \hat{h}_1$$

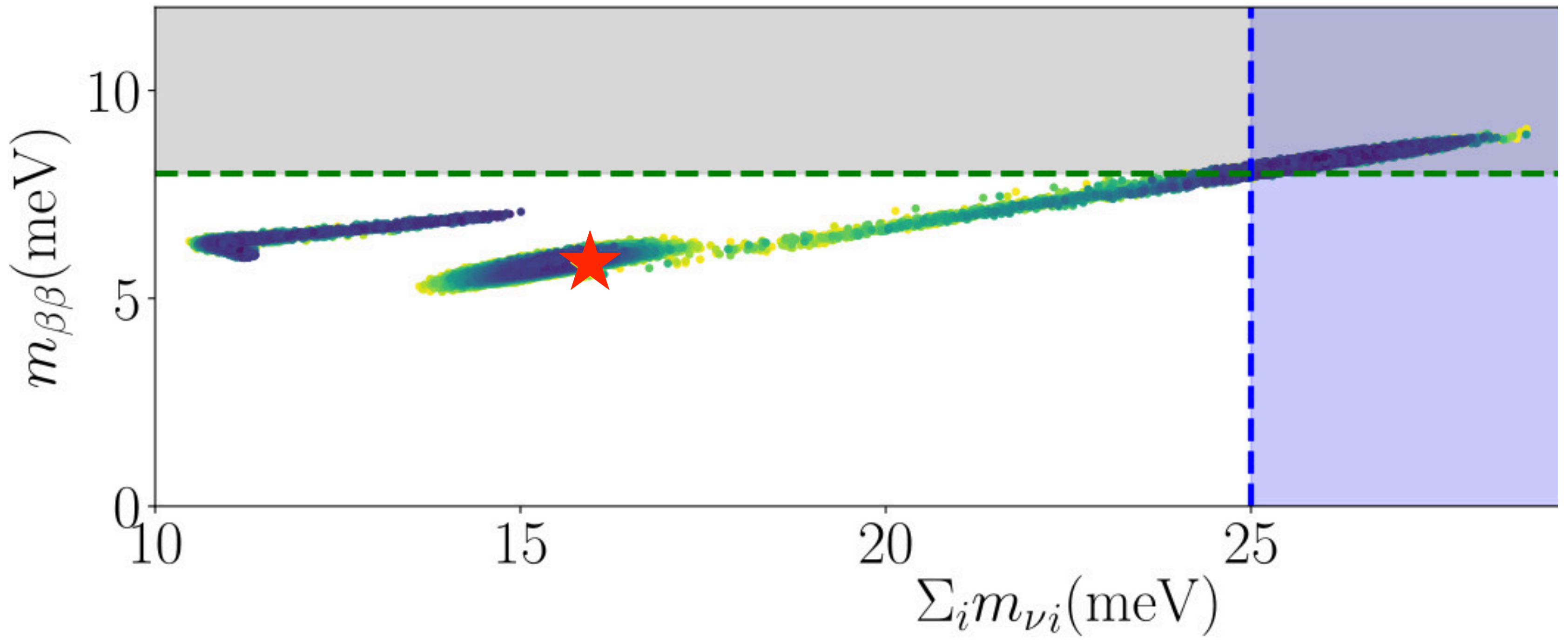
SO(10) Model confronting data

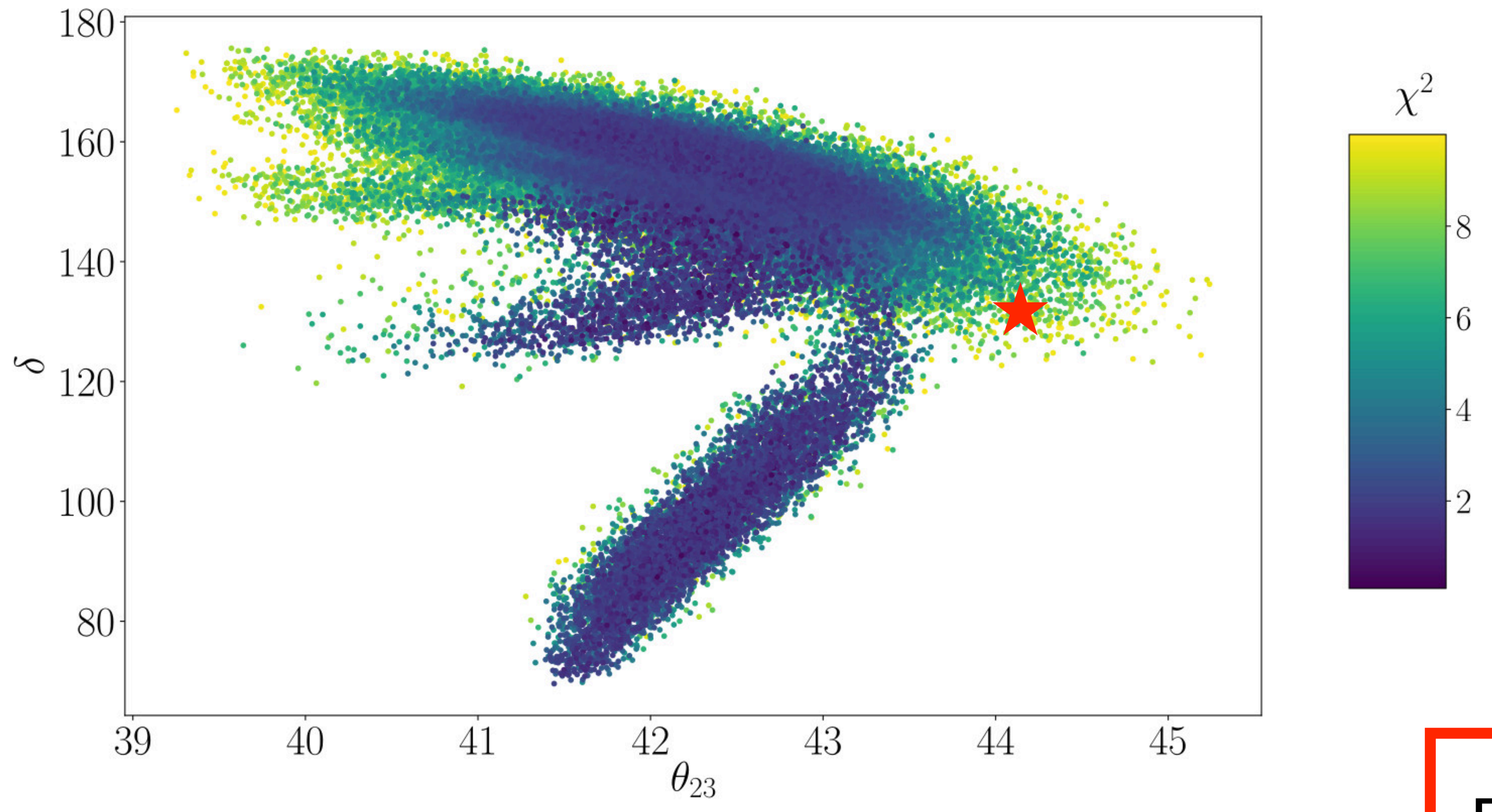
- **Input:** quark masses, mixing parameters, charged lepton Yukawa matrix
- **Theory Model parameters:** $\mathscr{P}_m \in \left\{ a_1, a_2, c_\nu, m_0, \eta_{q_{u,c,t,d,s,b}} \right\}$
- **Output:** predictions for $\mathscr{O}_m \in \left\{ \theta_{12}, \theta_{23}, \theta_{12}, \delta, \alpha_{21}, \alpha_{31}, \Delta m^2_{21}, \Delta m^2_{31} \right\}$

SO(10) Model confronting data

- **Input:** quark masses, mixing parameters, charged lepton Yukawa matrix
- **Theory Model parameters:** $\mathscr{P}_m \in \left\{ a_1, a_2, c_\nu, m_0, \eta_{q_{u,c,t,d,s,b}} \right\}$
- **Output:** predictions for $\mathscr{O}_m \in \left\{ \theta_{12}, \theta_{23}, \theta_{12}, \delta, \alpha_{21}, \alpha_{31}, \Delta m^2_{21}, \Delta m^2_{31} \right\}$



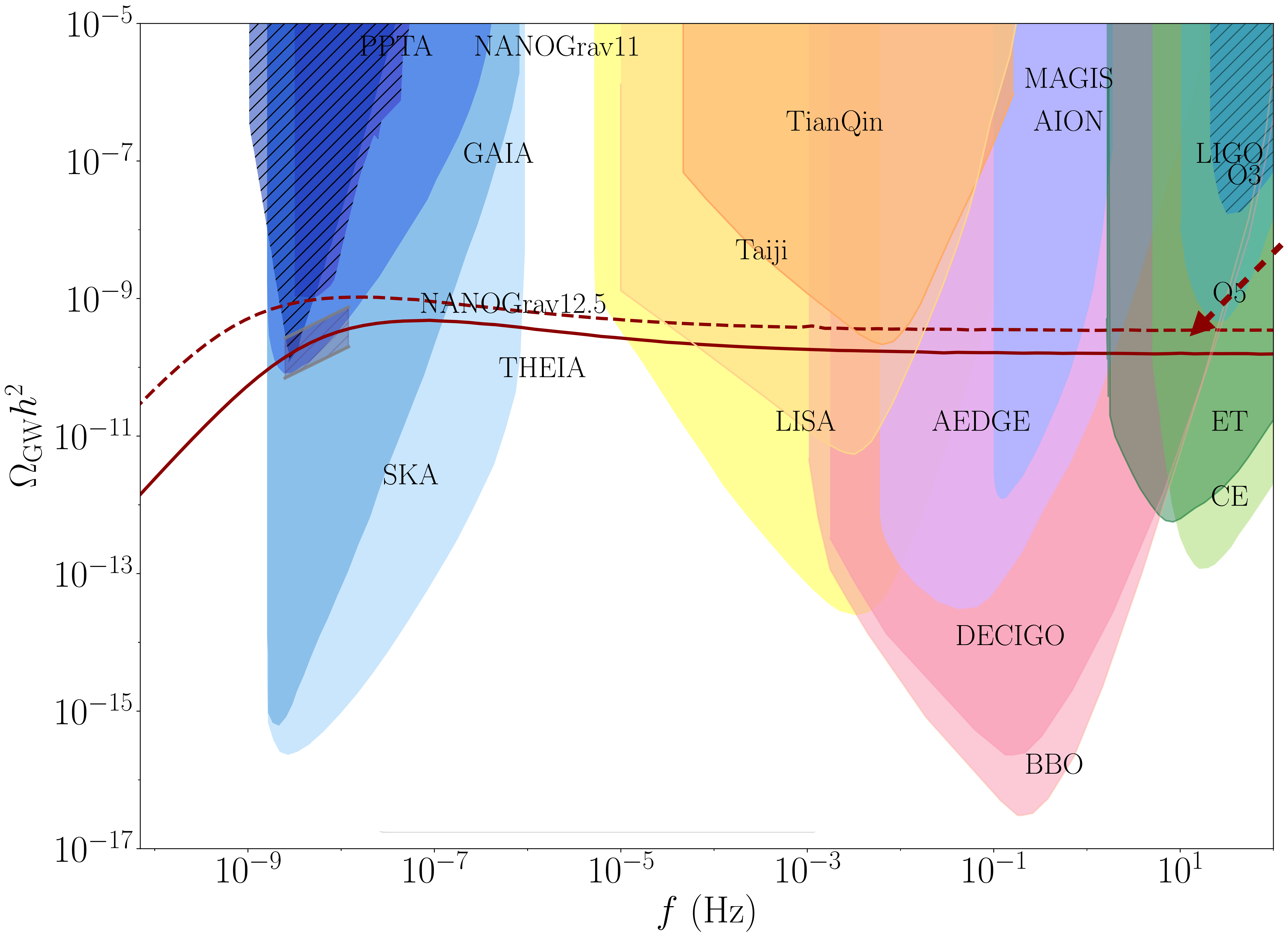




★ Point with $\eta_B \approx 6 \times 10^{-10}$

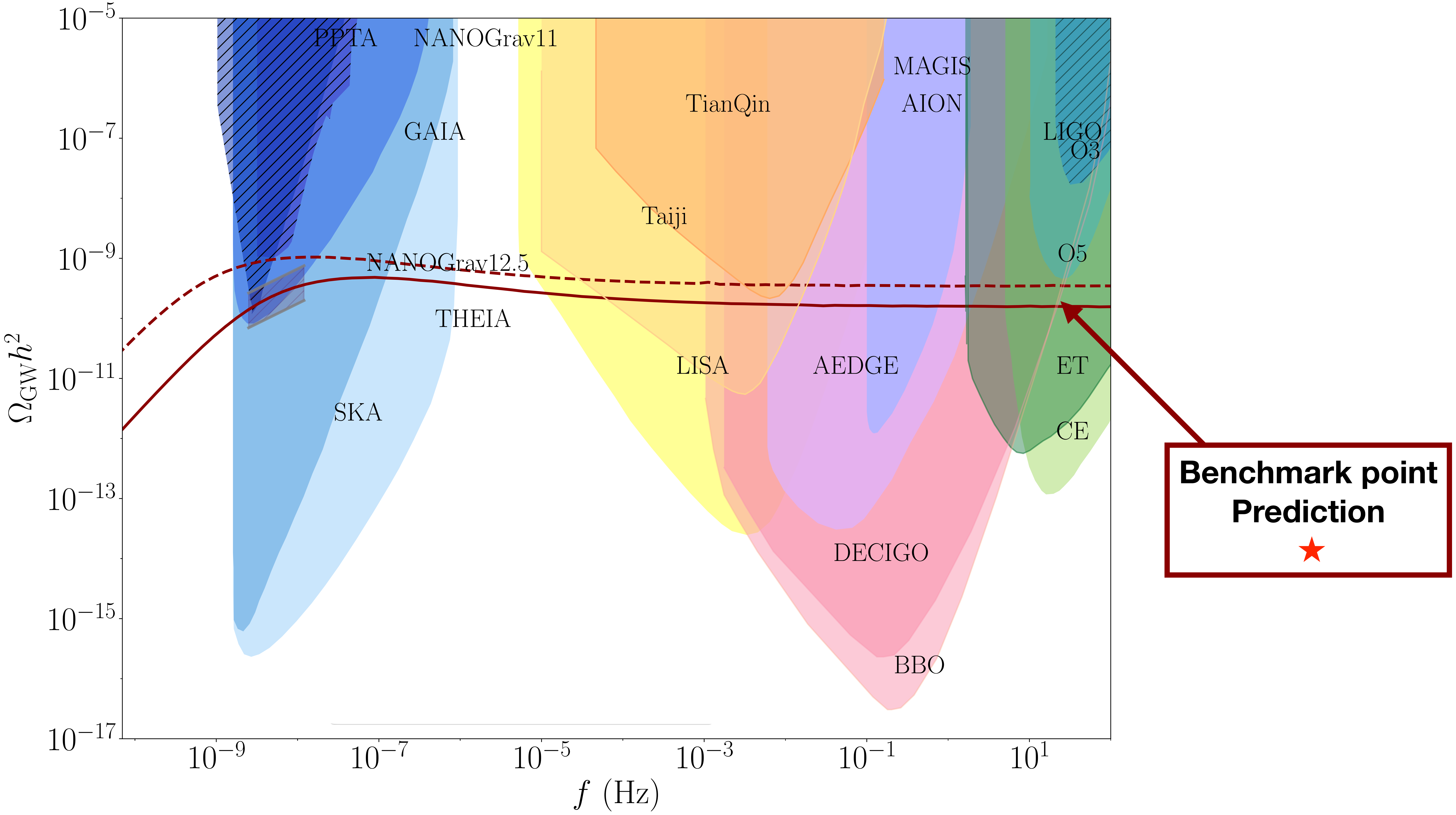
Parameter space fits neutrino data high stat significance

SO(10) Model confronting data



Upper bound
On $G\mu$ allowed
by proton decay limits

SO(10) Model confronting data



SUSY SO(10) Model confronting data

- Many GUTs are supersymmetric: gauge unification achieved in MSSM $\sim 10^{16}\,\mathrm{GeV}$

- In 2308.05799 we investigated a SUSY SO(10) GUT. See also 2307.04595

- Motivated by PTA observations

$$SO(10)\times\mathrm{SUSY}$$

Monopole Formation at M_X

$$\downarrow$$

$$SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times\mathrm{SUSY}$$

Cosmic String Formation at M_1

$$\downarrow$$

$$SU(3)_c\times SU(2)_L\times U(1)_Y\times\mathrm{SUSY}$$

Proton Decay from $\cancel{\mathrm{SUSY}}$

$\cancel{\mathrm{SUSY}}$ $\downarrow$

$$SU(3)_c\times SU(2)_L\times U(1)_Y$$

SUSY SO(10) Model confronting data

- Many GUTs are supersymmetric: gauge unification achieved in MSSM $\sim 10^{16}\,\mathrm{GeV}$
- In 2308.05799 we investigated a SUSY SO(10) GUT. See also 2307.04595
- Motivated by PTA observations

$$SO(10) \times \mathrm{SUSY}$$

Monopole Formation at M_X $\rightarrow$ $\downarrow$

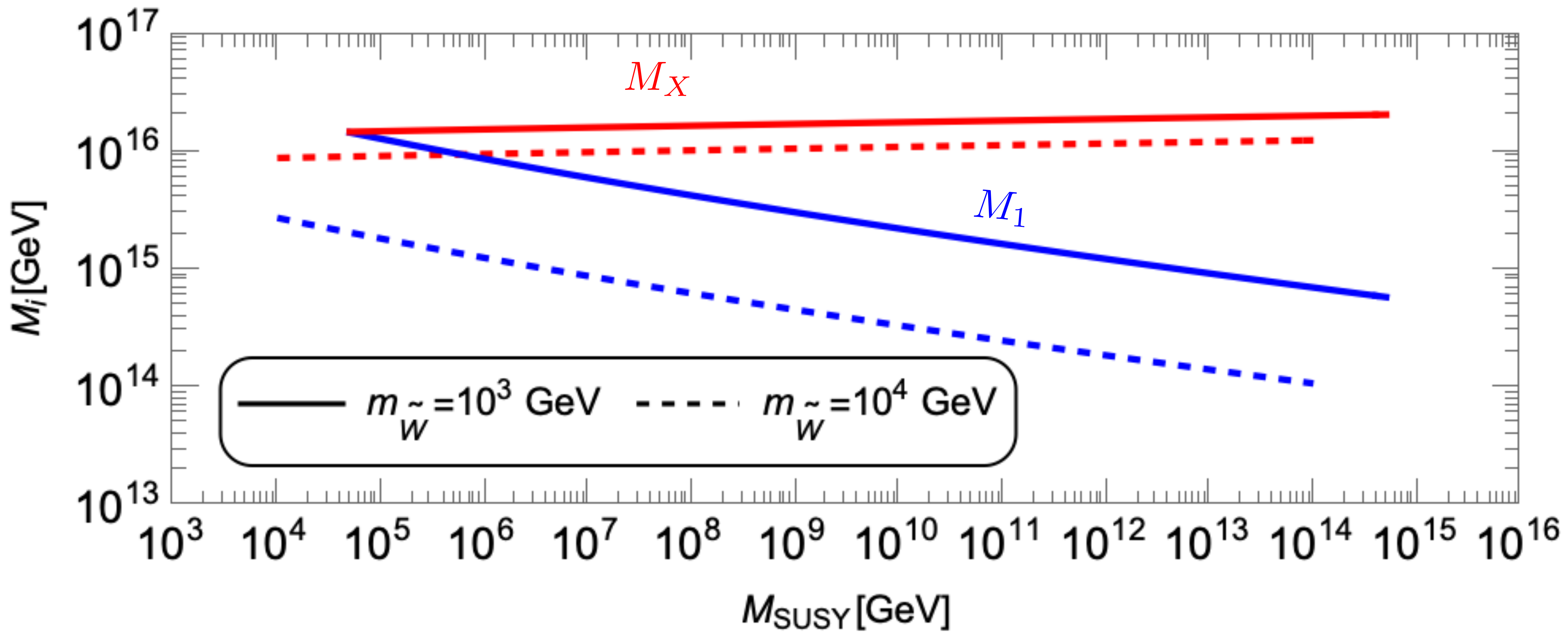
$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times \mathrm{SUSY}$$

Cosmic String Formation at M_1 $\rightarrow$ $\downarrow$

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times \mathrm{SUSY}$$

Proton Decay from SUSY $\rightarrow$ $\cancel{\mathrm{SUSY}}$ $\downarrow$

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$



Lower SUSY breaking $\implies$ higher the GUT and B-L breaking scale

SUSY SO(10) Model confronting data

$$SO(10) \times \text{SUSY}$$

Monopole Formation at M_X $\rightarrow$ $\downarrow$

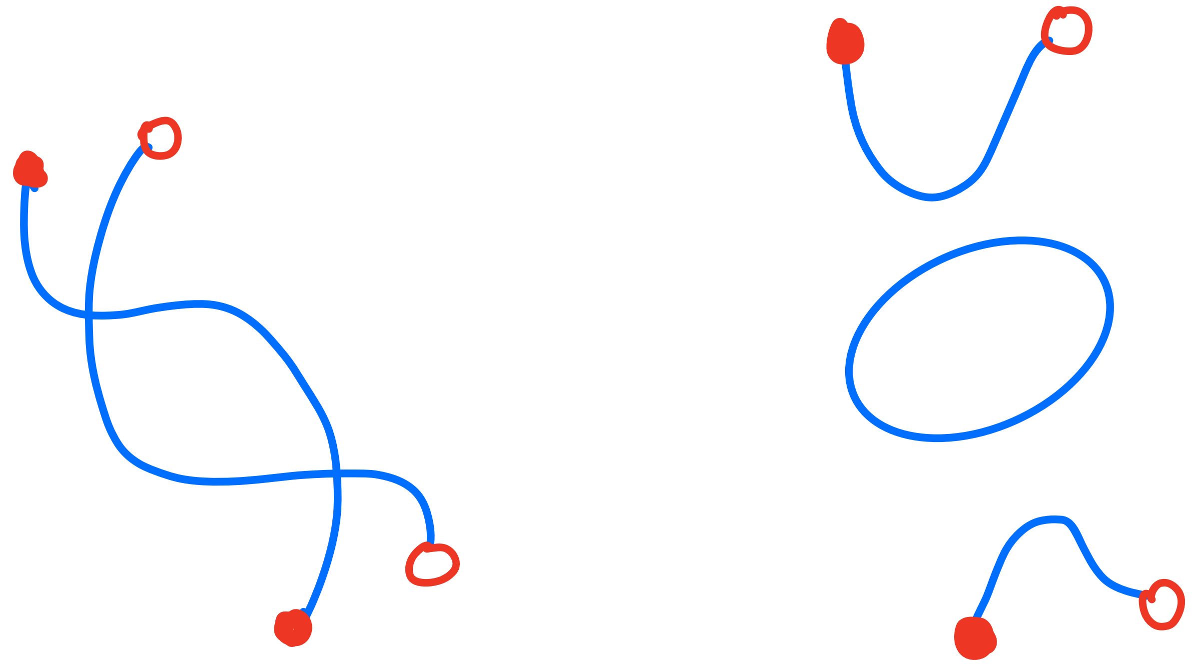
$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times \text{SUSY}$$

Cosmic String Formation at M_1 $\rightarrow$ $\downarrow$

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times \text{SUSY}$$

$\cancel{\text{SUSY}}$ $\downarrow$

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$



$m^2_{\text{mono}} \sim \mu \implies$ monopoles & antimonopoles can nucleate on string & annihilate $\implies$ **metastable string**

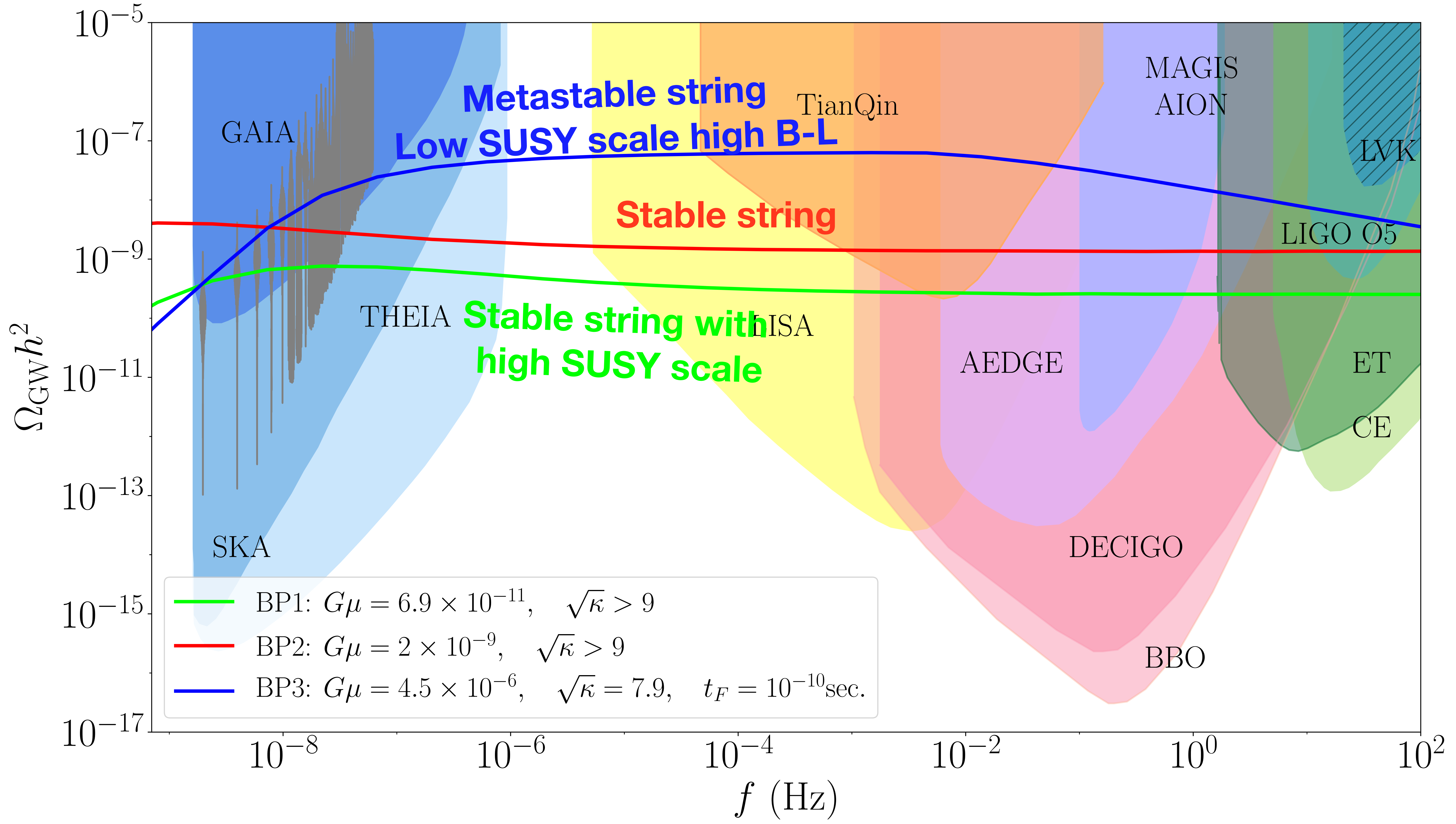
$$\Gamma_d = \frac{\mu}{2\pi} \exp(-\pi\kappa)$$

$$\kappa = \frac{m^2_{\text{mono}}}{\mu} \approx \frac{M_X^2}{\alpha_X \mu} = \frac{M_X^2}{\alpha_X M_1^2}$$

Vilenkin [1982], Leblond, Shlaer, Siemens [2009], Monin & Voloshin [2009], Buchmuller, Domcke, Schmitz [2021]

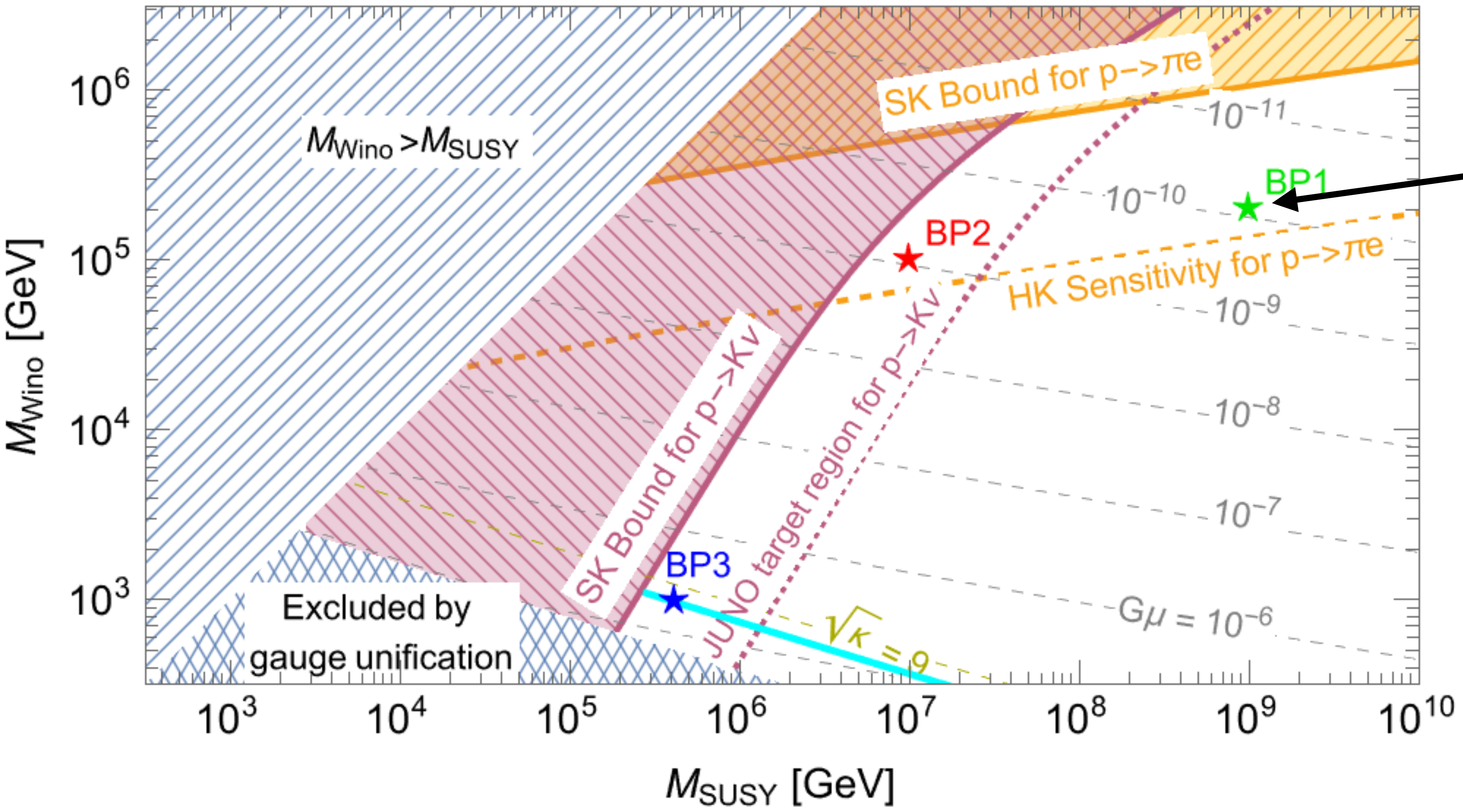
SUSY SO(10) Model confronting data

- Metastable strings are one possible explanation of the observed recent signal from Pulsar Timing Arrays. Signal requires $\sqrt{\kappa} \sim 7.9 \implies M_X \sim 1.5\, M_1$
- Achievable **with SUSY** but very hard in non-SUSY set up



SUSY SO(10) Model confronting data

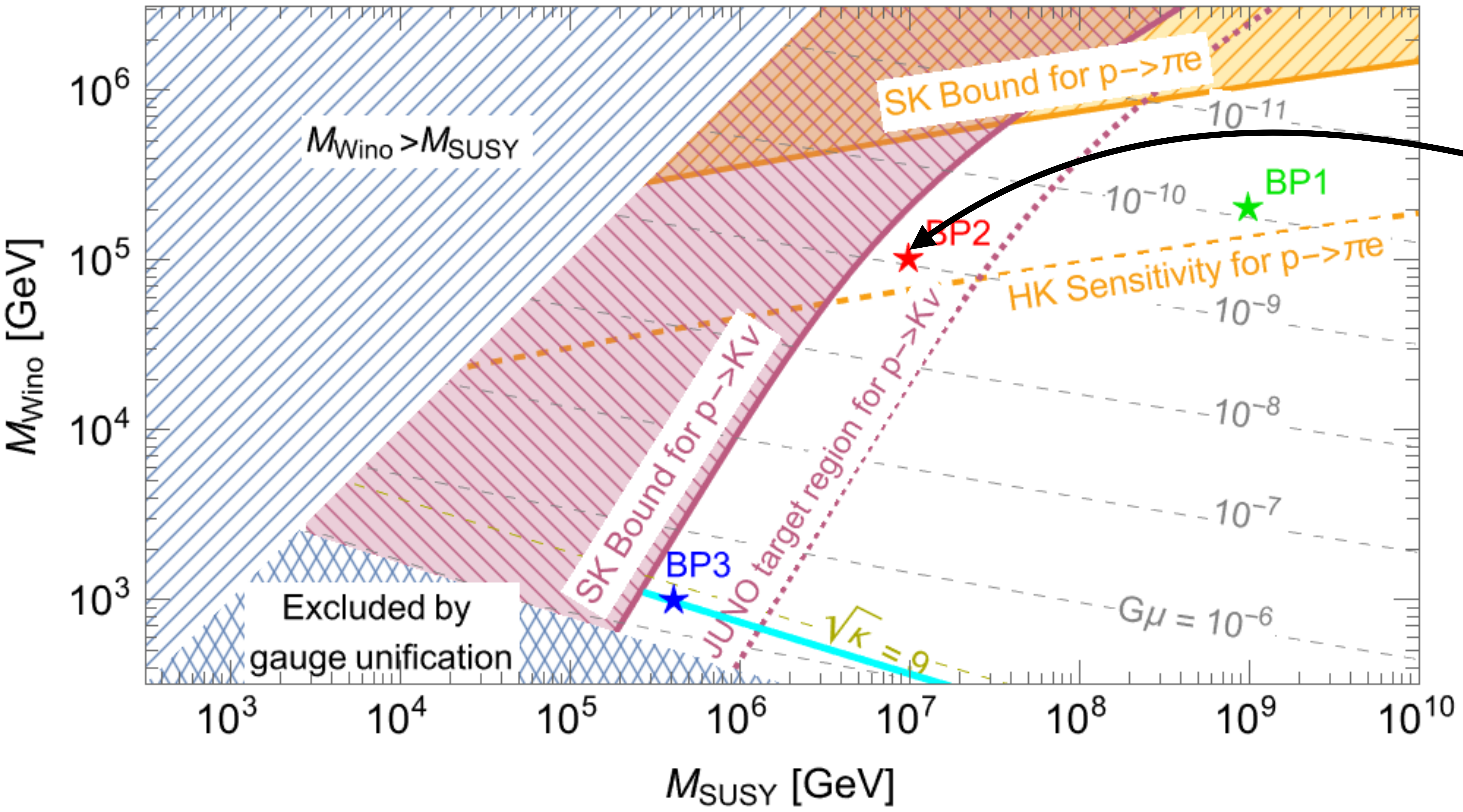
- Metastable strings are one possible explanation of the observed recent signal from Pulsar Timing Arrays. Signal requires $\sqrt{\kappa} \sim 7.9 \implies M_X \sim 1.5\, M_1$
- Achievable **with SUSY** but hard in non-SUSY set up



Testable by Hyper-K
$p \to \pi^0 e^+$
Cannot explain PTA signal

SUSY SO(10) Model confronting data

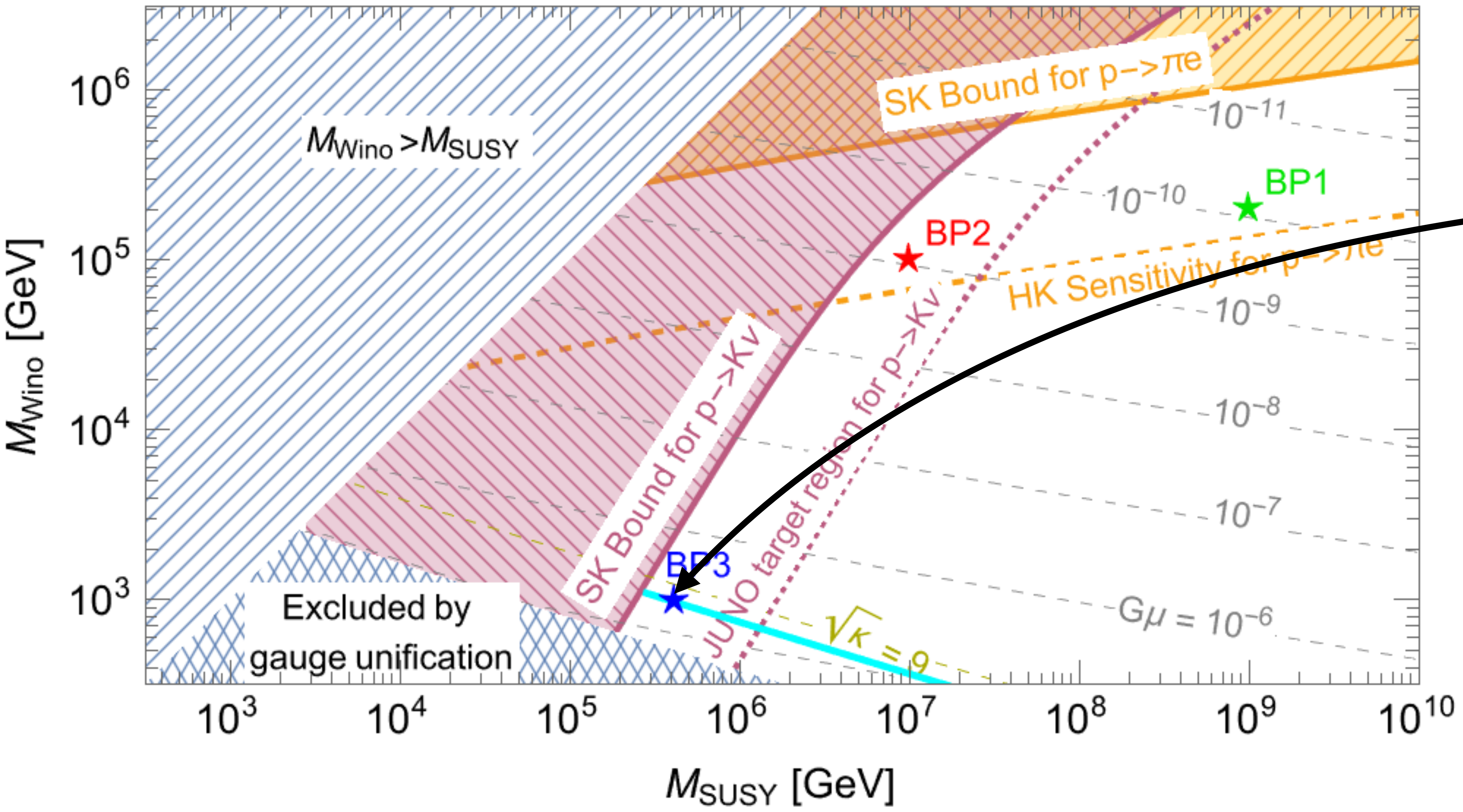
- Metastable strings are one possible explanation of the observed recent signal from Pulsar Timing Arrays. Signal requires $\sqrt{\kappa} \sim 7.9 \implies M_X \sim 1.5\, M_1$
- Achievable **with SUSY** but hard in non-SUSY set up



Testable by Hyper-K & JUNO
Should observe both $p \to \pi^0 e^+$ & $p \to K^+ \bar{\nu}$
cannot explain PTA signal

SUSY SO(10) Model confronting data

- Metastable strings are one possible explanation of the observed recent signal from Pulsar Timing Arrays. Signal requires $\sqrt{\kappa} \sim 7.9 \implies M_X \sim 1.5\, M_1$
- Achievable **with SUSY** but hard in non-SUSY set up



Testable by Hyper-K & JUNO
Should observe both $p \to \pi^0 e^+$ & $p \to K^+ \bar{\nu}$
Can explain PTA signal

Summary

- GUTs generically predict nucleon decay and the formation of topological defects. Interplay of these observables is a powerful way of constraining GUTs.
- Coming decade is an exciting time for GUTs as neutrino and GW experiments will constrain nucleon decay, the presence of GWs and neutrinoless double beta decay ($0\nu\beta\beta$).
- Studied non-SUSY & SUSY SO(10) breaking chains which can be tested by Hyper-K, GW detectors and $0\nu\beta\beta$.
- Parameter space consistent with fermionic masses and mixing & successful leptogenesis.
- SUSY SO(10) has some parameter space available to explain PTA signal and much of the PS can be tested by JUNO. PTA signal may be a constraint on non-SUSY GUTs and constraint many of them.

"we have entered an exciting era where new observations of GWs from the heavens and proton decay experiments from under the Earth can provide complementary windows to reveal the details of the unification of matter and forces at the highest energies."


Thank you for listening

Proton Decay

- From RGE, α_X and M_X determined

$$\Gamma\left(p \to \pi^0 e^+\right) = \frac{m_p}{32\pi}\left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right) A_L^2 \times \left[A_{SL}\Lambda_1^{-2}\left(1 + \left|V_{ud}\right|^2\right)\left|\left\langle \pi^0 \left|(ud)_R u_L\right| p\right\rangle\right|^2 \right.$$
$$\left. + A_{SR}\left(\Lambda_1^{-2} + \left|V_{ud}\right|^2 \Lambda_2^{-2}\right)\left|\left\langle \pi^0 \left|(ud)_L u_L\right| p\right\rangle\right|^2\right]$$

Proton Decay

- From RGE, α_X and M_X determined

$$\Gamma\left(p \to \pi^0 e^+\right) = \frac{m_p}{32\pi}\left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right) A_L^2 \times \left[A_{SL}\Lambda_1^{-2}\left(1 + \left|V_{ud}\right|^2\right)\left|\left\langle \pi^0 \left| (ud)_R u_L \right| p\right\rangle\right|^2 \right.$$
$$\left. + A_{SR}\left(\Lambda_1^{-2} + \left|V_{ud}\right|^2 \Lambda_2^{-2}\right)\left|\left\langle \pi^0 \left| (ud)_L u_L \right| p\right\rangle\right|^2\right]$$

Hadronic matrix element from lattice

1705.01338

Proton Decay

- From RGE, α_X and M_X determined

$$\Gamma\left(p \to \pi^0 e^+\right) = \frac{m_p}{32\pi}\left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right) A_L^2 \times \left[A_{SL}\Lambda_1^{-2}\left(1+|V_{ud}|^2\right)\left|\langle \pi^0 |(ud)_R u_L| p\rangle\right|^2 + A_{SR}\left(\Lambda_1^{-2} + |V_{ud}|^2 \Lambda_2^{-2}\right)\left|\langle \pi^0 |(ud)_L u_L| p\rangle\right|^2\right]$$

$$\Lambda_1 \simeq \Lambda_2 = \frac{g_X M_X}{2}$$

Hadronic matrix element from lattice

1705.01338

Proton Decay

- From RGE, α_X and M_X determined

$$\Gamma\left(p \to \pi^0 e^+\right) = \frac{m_p}{32\pi}\left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right) A_L^2 \times \Big[A_{SL}\Lambda_1^{-2}\left(1 + |V_{ud}|^2\right)\left|\langle \pi^0 \,|(ud)_R u_L|\, p\rangle\right|^2$$
$$+A_{SR}\left(\Lambda_1^{-2} + |V_{ud}|^2\,\Lambda_2^{-2}\right)\left|\langle \pi^0 \,|(ud)_L u_L|\, p\rangle\right|^2\Big]$$

Long & short range effects from renormalisation

$$\Lambda_1 \simeq \Lambda_2 = \frac{g_X M_X}{2}$$

Hadronic matrix element from lattice

1705.01338

Gravitational Waves

- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
- Correlate vev of Higgs breaking $U(1)$ with string tension, μ
- Assume ideal Nambu-Goto string $\Longrightarrow$ gravitational radiation primary emission

$$\mu \approx 2\pi v^2$$

Vilenkin & Shellard

Gravitational Waves

- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
- Correlate vev of Higgs breaking $U(1)$ with string tension, μ
- Assume ideal Nambu-Goto string $\Longrightarrow$ gravitational radiation primary emission

$$\mu \approx 2\pi v^2$$

Vilenkin & Shellard

$$M_1^2 \sim g_1^2 v^2 \implies G\mu = \frac{1}{M_{\rm Pl}^2}\frac{2\pi M_1^2}{g_1^2} = \frac{M_1^2}{2\alpha_1 M_{\rm Pl}^2}$$

Gravitational Waves

- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
- Correlate vev of Higgs breaking $U(1)$ with string tension, μ
- Assume ideal Nambu-Goto string $\Longrightarrow$ gravitational radiation primary emission

$$\mu \approx 2\pi v^2$$

Vilenkin & Shellard

$$M_1^2 \sim g_1^2 v^2 \quad \Longrightarrow \quad G\mu = \frac{1}{M_{\rm Pl}^2}\frac{2\pi M_1^2}{g_1^2} = \frac{M_1^2}{2\alpha_1 M_{\rm Pl}^2}$$

Determined from RGE

SO(10) Model confronting data

- Model of Altarelli & Blankenburg
- Above GUT scale, Yukawa sector

$$Y^*_{\mathbf{10}}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10} + Y^*_{\overline{\mathbf{126}}}\, \mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}} + Y^*_{\mathbf{120}}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120} + \mathrm{h.c.}\,,$$

$$Y_u = Y_{10}V^*_{11} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V^*_{12} + Y_{120}\left(V^*_{13} + \frac{1}{\sqrt{3}}V^*_{14}\right)$$

$$Y_d = Y_{10}V_{15} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} + \frac{1}{\sqrt{3}}V_{18}\right)$$

$$Y_\nu = Y_{10}V^*_{11} - \sqrt{3}Y_{\overline{126}}V^*_{12} + Y_{120}\left(V^*_{13} - \sqrt{3}V^*_{14}\right)$$

$$Y_e = Y_{10}V_{15} - \sqrt{3}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} - \sqrt{3}V_{18}\right).$$

SO(10) Model confronting data

- Model of Altarelli & Blankenburg

- Above GUT scale, Yukawa sector

$$Y^*_{\mathbf{10}}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10} + Y^*_{\overline{\mathbf{126}}}\, \mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}} + Y^*_{\mathbf{120}}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120} + \mathrm{h.c.}\,,$$

- Right handed neutrino masses predicted & generate light neutrino masses via seesaw

$$Y_{\overline{126}}\bar{\nu}_R \phi_S \nu_R^c \implies M_{\nu_R} = Y_{\overline{126}} v_S \implies M_\nu = \frac{Y_\nu Y_\nu^T v_{\mathrm{SM}}^2}{M_{\nu_R}}$$

- RHN masses and Y_ν predicted $\implies$ leptogenesis parameter space fixed and η_B predicted

Renormalisation Group Equations

Beta function coefficients 1 and 2-loop respectively

$$b_i = -\frac{11}{3} C_2(H_i) + \frac{2}{3} \sum_F T(F_i) + \frac{1}{3} \sum_S T(S_i)\,,$$

$$b_{ij} = -\frac{34}{3} [C_2(H_i)]^2 \delta_{ij} + \sum_F T(F_i)[2C_2(F_j) + \frac{10}{3} C_2(H_i)\delta_{ij}] + \sum_S T(S_i)[4C_2(S_j) + \frac{2}{3} C_2(H_i)\delta_{ij}]\,,$$

Two-loop RGE equation

Bertolini, di Luzio, Malinsky

$$\alpha_i(\mu)^{-1} = \alpha_i(\mu_0)^{-1} - \frac{b_i}{2\pi} \log \frac{\mu}{\mu_0} + \sum_j \frac{b_{ij}}{4\pi b_i} \log \left(1 - b_j \alpha_j(\mu_0) \log \frac{\mu}{\mu_0}\right)\,,$$

Matching condition

$$H_i \to H_j\,, \quad \frac{1}{\alpha_{H_i}(M_I)} - \frac{C_2(H_i)}{12\pi} = \frac{1}{\alpha_{H_j}(M_I)} - \frac{C_2(H_j)}{12\pi}\,.$$

- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales $\Longrightarrow$ PD rate and GW signal

Breaking chains with **one intermediate scale** has fixed prediction from unification

I1: $SO(10) \xrightarrow[M_X]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$

Mass of GB from SSB

$\Lambda_{\rm pd}$

- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales $\Longrightarrow$ PD rate and GW signal

Breaking chains with **one intermediate scale** has fixed prediction from unification

I1: $SO(10) \xrightarrow[M_X]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$

Mass of GB from $U(1)$ SSB parametrises Λ_{cs}

- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales $\Longrightarrow$ PD rate and GW signal

Breaking chains with **one intermediate scale** has fixed prediction from unification

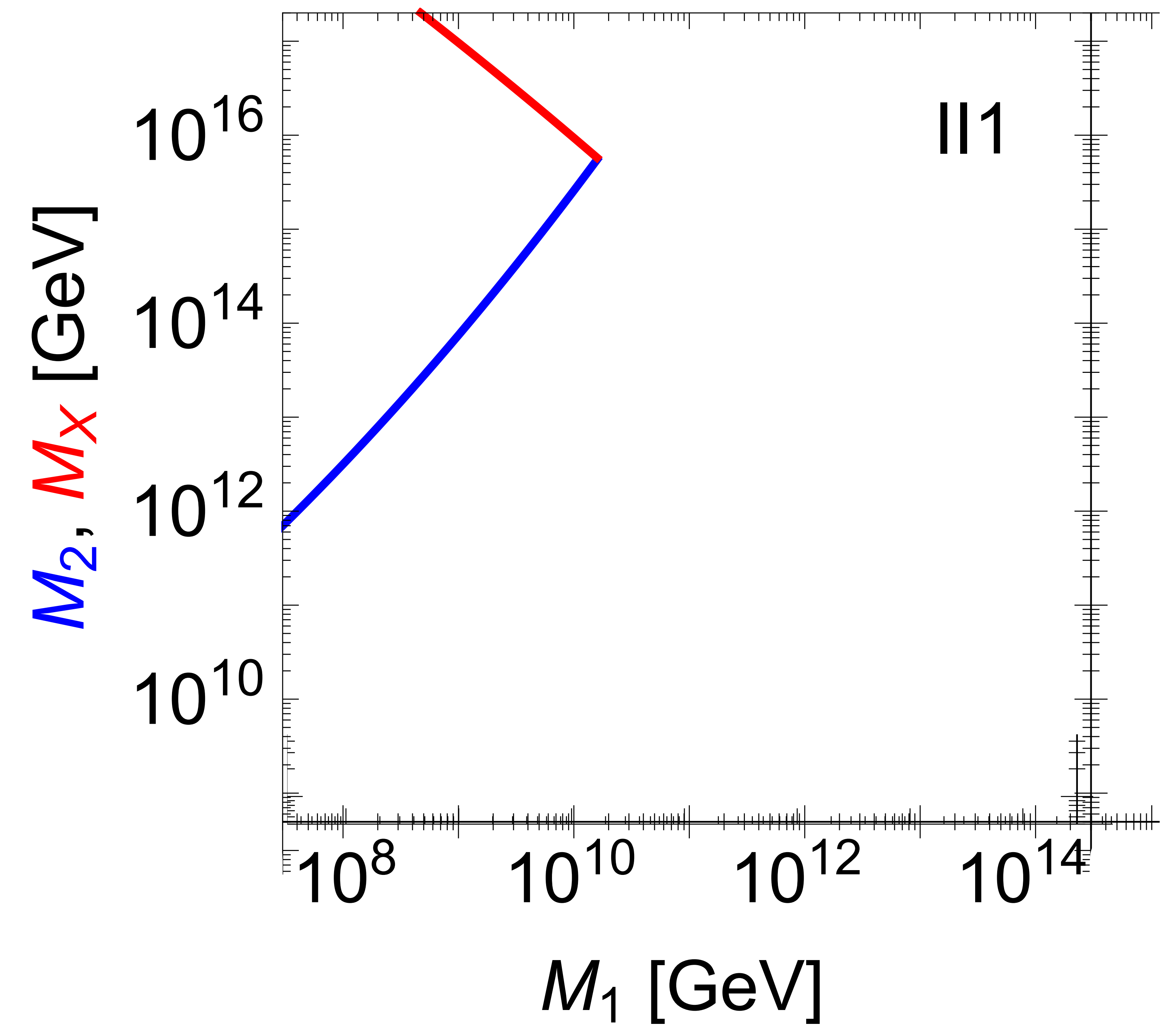
I1: $SO(10) \xrightarrow[M_X]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$

Chains	M_X [GeV]	M_1 [GeV]
I1	$\underline{5.660} \times 10^{15}$	1.617×10^{10}
I2	1.410×10^{15}	8.630×10^{10}
I3	2.902×10^{14}	1.634×10^{11}
I4	3.500×10^{16}	4.368×10^{9}
I5	2.722×10^{14}	1.143×10^{13}
I6	excluded	

- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales $\Longrightarrow$ PD rate and GW signal

Breaking chains with **two intermediate scales** can have a range of scales

II1 : $SO(10) \xrightarrow[M_X]{} G_{422} \xrightarrow[M_2]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$



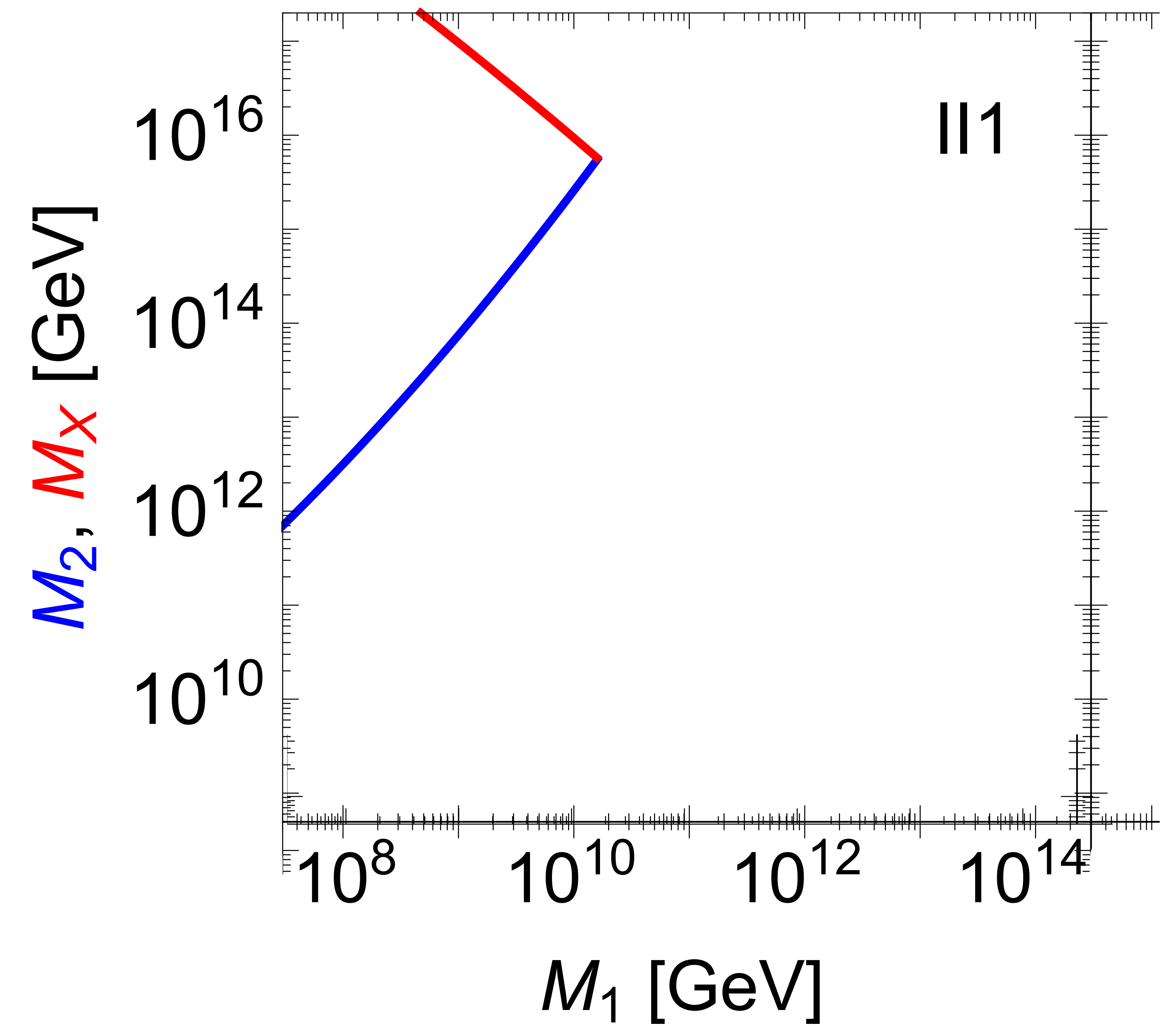
- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales $\Longrightarrow$ PD rate and GW signal

Breaking chains with **two intermediate scales** can have a range of scales

II1 : $SO(10) \xrightarrow[M_X]{} G_{422} \xrightarrow[M_2]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$

$M_2 = M_X$ recover I1

$$SO(10) \xrightarrow[M_X]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$$



- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
- Correlate vev of Higgs breaking $U(1)$ with string tension, μ
- Assume ideal Nambu-Goto string $\Longrightarrow$ gravitational radiation primary emission

Vilenkin & Shellard

$$\mu \approx 2\pi v^2$$

$$M_1^2 = M_{Z'}^2 \sim g^2 v^2 \implies G\mu \approx \frac{1}{M_{\mathrm{PL}}^2} \cdot \frac{2\pi M_1^2}{g^2} = \frac{M_1^2}{2\alpha M_{\mathrm{pl}}^2}$$

- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
- Correlate vev of Higgs breaking $U(1)$ with string tension, μ
- Assume ideal Nambu-Goto string $\implies$ gravitational radiation primary emission

Vilenkin & Shellard

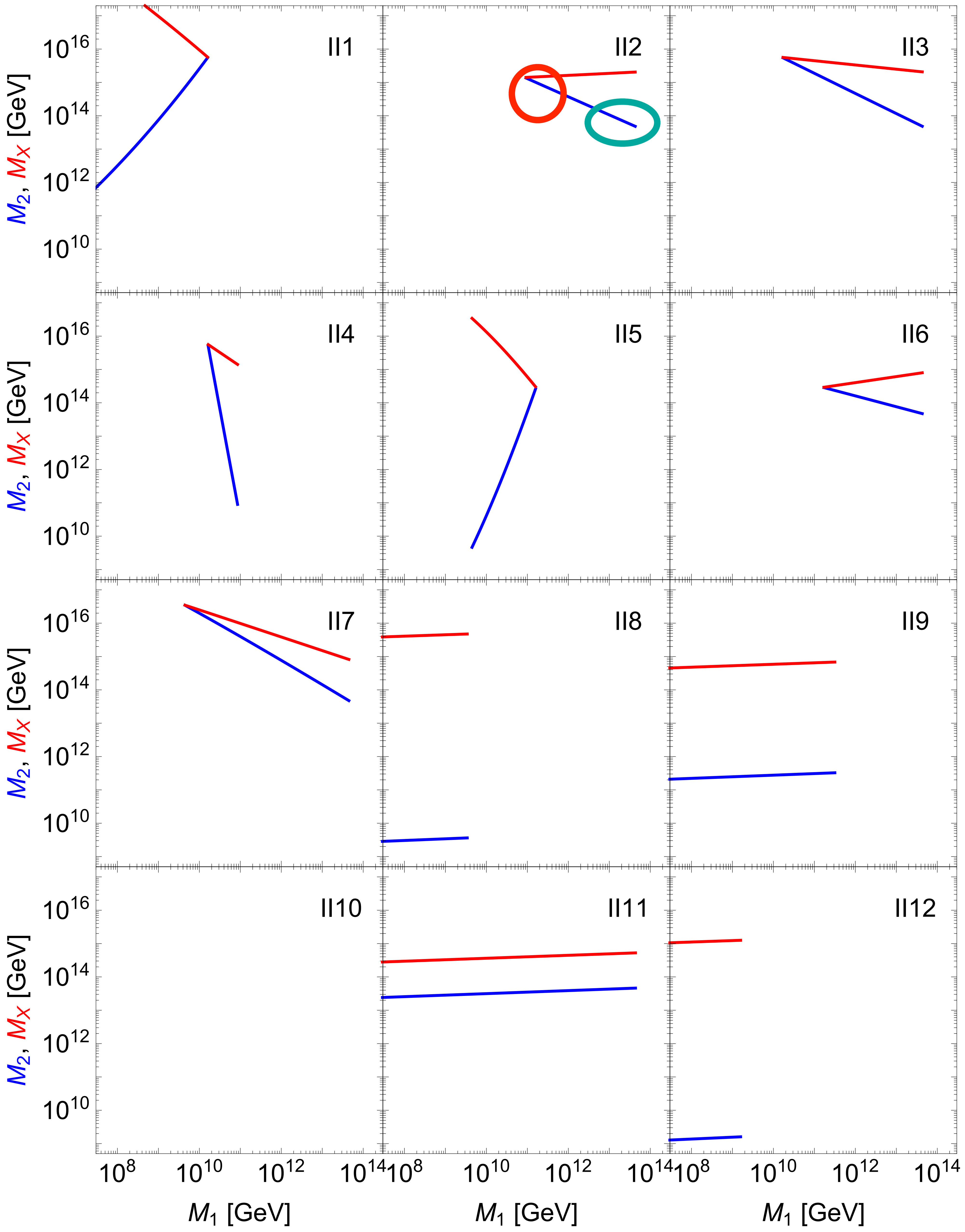
$$\mu \approx 2\pi v^2$$

$$M_1^2 = M_{Z'}^2 \sim g^2 v^2 \implies G\mu \approx \frac{1}{M_{\rm PL}^2} \cdot \frac{2\pi M_1^2}{g^2} = \frac{M_1^2}{2\alpha M_{\rm pl}^2}$$

Example

$$G_{3211} \longrightarrow G_{SM} \quad U(1)_R \times U(1)_X \to U(1)_Y$$

$$G\mu \simeq \frac{1}{2\left(\alpha_{1R}\left(M_1\right) + \alpha_{1X}\left(M_1\right)\right)} \frac{M_1^2}{M_{\rm pl}^2}$$



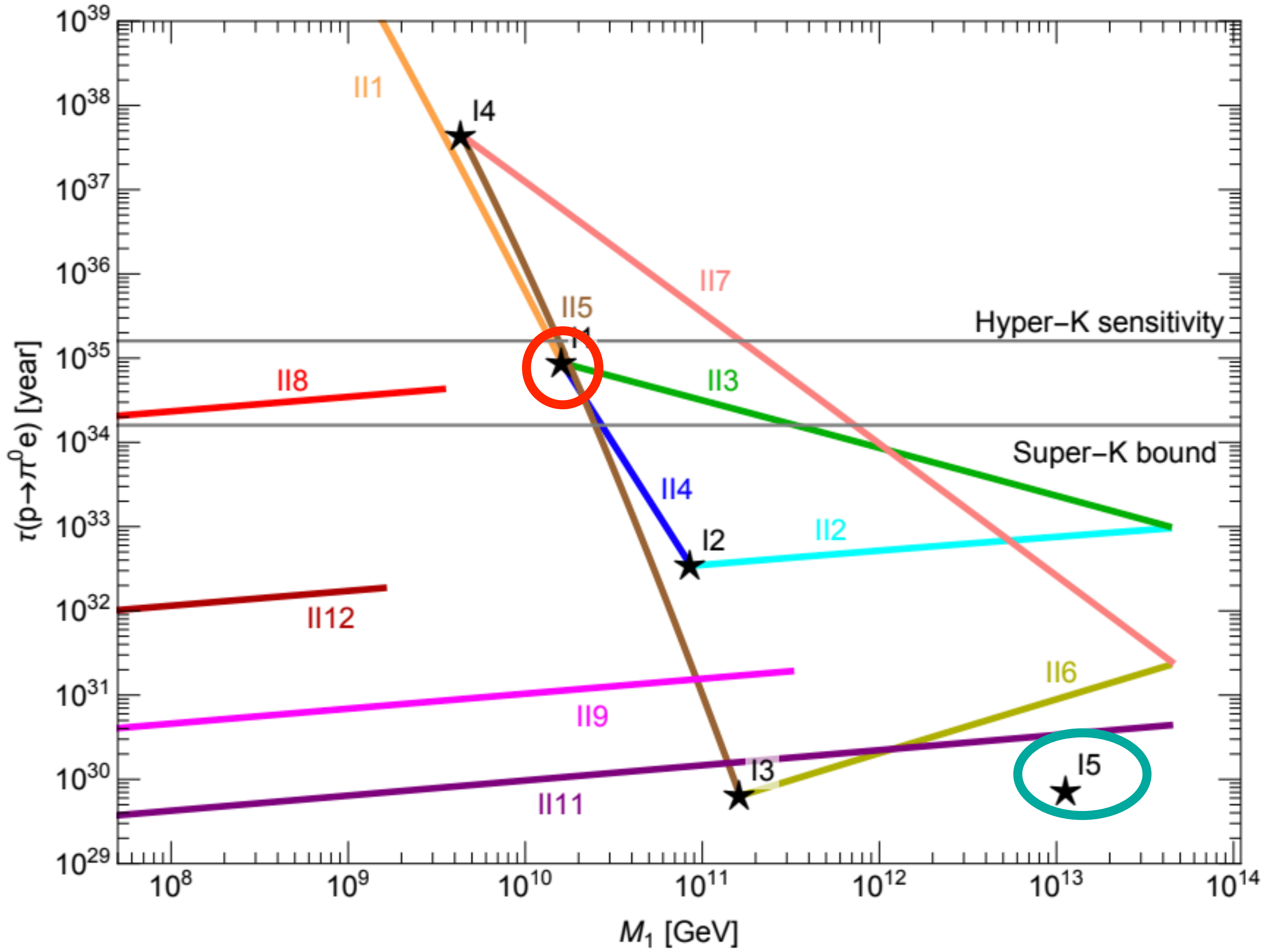
$$\mathrm{II2}: SO(10) \xrightarrow{M_X} G^C_{422} \xrightarrow{M_2} G^C_{3221} \xrightarrow{M_1} G_{\mathrm{SM}}$$

Intersection of M_2 and M_X reduces II2 to I2

$$\mathrm{I2}: SO(10) \xrightarrow{M_X} G^C_{3221} \xrightarrow{M_1} G_{\mathrm{SM}} \qquad M_X \equiv M_2$$

At right side blue curve II2 becomes I5

$$\mathrm{I5}: SO(10) \rightarrow G^C_{422} \rightarrow G_{\mathrm{SM}} \qquad M_2 \equiv M_1$$



Proton Lifetime

$$\epsilon^{ijk}\epsilon_{\alpha\beta}\Big(\frac{1}{\Lambda_1^2}(\overline{u_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{d_R^{ic}}\gamma_\mu L_\beta)+\frac{1}{\Lambda_1^2}(\overline{u_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{e_R^{c}}\gamma_\mu Q_\beta^i)$$
$$+\frac{1}{\Lambda_2^2}(\overline{d_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{u_R^{ic}}\gamma_\mu L_\beta)+\frac{1}{\Lambda_2^2}(\overline{d_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{\nu_R^{c}}\gamma_\mu Q_\beta^i)+\text{h.c.}\Big)$$

$$\Lambda_1=\Lambda_2\simeq(g_X M_X)/2$$

$$\Gamma(p\to\pi^0+e^+)=\frac{m_p}{32\pi}\left(1-\frac{m_{\pi^0}^2}{m_p^2}\right)^2 A_L^2\times\Big[A_{SL}\Lambda_1^{-2}(1+|V_{ud}|^2)\,|\langle\pi^0\,|(ud)_R u_L|p\rangle|^2$$
$$+A_{SR}(\Lambda_1^{-2}+|V_{ud}|^2\Lambda_2^{-2})\,\left|\langle\pi^0|(ud)_L u_L|p\rangle\right|^2\Big]$$

$$A_{SL(R)}=\prod_A^{M_Z\leqslant M_A\leqslant M_X}\prod_i\left[\frac{\alpha_i(M_{A+1})}{\alpha_i(M_A)}\right]^{\frac{\gamma_{iL(R)}}{b_i}}$$

Anomalous dimension

One-loop Beta coefficient

Gravitational Wave Calculation

$$l(t) = l_i - \Gamma G\mu \,(t - t_i) \qquad\qquad l_i = \alpha t_i \text{ with } \alpha \simeq 0.1$$

Frequencies of GW released from the loops are given by $2k/l_i$ where $k = 1, 2, \cdots$

Loops are found to emit energy in the form of gravitational radiation at a constant rate

$$\frac{dE}{dt} = -\Gamma G\mu^2 \qquad\qquad \Gamma \sim 50$$

Assuming the fraction of the energy transfer in the form of large loops is $F_\alpha \sim 0.1$

$$\Omega_{\mathrm{GW}}(f) = \sum_k \Omega_{\mathrm{GW}}^{(k)}(f) = \frac{1}{\rho_c}\frac{2k}{f}\frac{\mathcal{F}_\alpha \Gamma^{(k)} G\mu^2}{\alpha(\alpha + \Gamma G\mu)}$$

$$= \int_{t_F}^{t_0} dt \frac{C_{\mathrm{eff}}\left(t_i^{(k)}\right)}{t_i^{(k)4}} \frac{a^2(t) a^3\left(t_i^{(k)}\right)}{a^5\left(t_0\right)} \theta\left(t_i^{(k)} - t_F\right)$$

$$C_{\mathrm{eff}} = 5.7, 0.5$$

1101.5173 **1808.08968** **0003298**

GW from SSB

$$\mu = 2\pi v^2 \epsilon \qquad \epsilon = \frac{m_\phi^2}{{m'_Z}^2}$$

Without model information $m_\phi \sim m'_Z$

U(1) GB mass: $M_{Z'}^2 = 4\pi\alpha v^2$ $\qquad M_1^2 = M_{Z'}^2 \qquad G\mu \simeq \frac{GM_{Z'}^2}{(2\alpha)} = \frac{GM_1^2}{(2\alpha)}$

1101.5173 **1808.08968** **0003298**

GUT Model

In the Yukawa sector, couplings above the GUT scale are given by

$$Y^*_{\mathbf{10}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y^*_{\overline{\mathbf{126}}}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y^*_{\mathbf{120}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\text{h.c.}\,,$$

After breaking to G_{SM}

$$Y_{\mathbf{10}}\Big[(\overline{Q}u_R+\overline{L}\nu_R)h^u_{\mathbf{10}}+(\overline{Q}d_R+\overline{L}e_R)h^d_{\mathbf{10}}\Big]+\frac{1}{\sqrt{3}}Y_{\overline{\mathbf{126}}}\left[(\overline{Q}u_R-3\overline{L}\nu_R)h^u_{\overline{\mathbf{126}}}+(\overline{Q}d_R-3\overline{L}e_R)h^d_{\overline{\mathbf{126}}}\right]$$
$$+Y_{\mathbf{120}}\Big[(\overline{Q}u_R+\overline{L}\nu_R)h^u_{\mathbf{120}}+(\overline{Q}d_R+\overline{L}e_R)h^d_{\mathbf{120}}+\frac{1}{\sqrt{3}}(\overline{Q}u_R-3\overline{L}\nu_R)h^{u'}_{\mathbf{120}}+(\overline{Q}d_R-3\overline{L}e_R)h^{d'}_{\mathbf{120}}\Big]+\text{h.c.}$$

Rotating the Higgs fields to their mass basis, we derive Yukawa couplings to the SM Higgs

$$Y_u\bar{Q}\tilde{h}_{\rm SM}u_R+Y_d\bar{Q}h_{\rm SM}d_R+Y_\nu\bar{L}\tilde{h}_{\rm SM}\nu_R+Y_e\bar{L}h_{\rm SM}e_R+\ \text{h.c.}$$

$$Y_u=Y_{10}V^*_{11}+\frac{1}{\sqrt{3}}Y_{\overline{126}}V^*_{12}+Y_{120}\left(V^*_{13}+\frac{1}{\sqrt{3}}V^*_{14}\right)$$
$$Y_d=Y_{10}V_{15}+\frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16}+Y_{120}\left(V_{17}+\frac{1}{\sqrt{3}}V_{18}\right)$$
$$Y_\nu=Y_{10}V^*_{11}-\sqrt{3}Y_{\overline{126}}V^*_{12}+Y_{120}\left(V^*_{13}-\sqrt{3}V^*_{14}\right)$$
$$Y_e=Y_{10}V_{15}-\sqrt{3}Y_{\overline{126}}V_{16}+Y_{120}\left(V_{17}-\sqrt{3}V_{18}\right).$$

GUT Model

$$Y_u = Y_{10}V_{11}^* + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{12}^* + Y_{120}\left(V_{13}^* + \frac{1}{\sqrt{3}}V_{14}^*\right)$$
$$Y_d = Y_{10}V_{15} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} + \frac{1}{\sqrt{3}}V_{18}\right)$$
$$Y_\nu = Y_{10}V_{11}^* - \sqrt{3}Y_{\overline{126}}V_{12}^* + Y_{120}\left(V_{13}^* - \sqrt{3}V_{14}^*\right)$$
$$Y_e = Y_{10}V_{15} - \sqrt{3}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} - \sqrt{3}V_{18}\right).$$

$$Y_u = h + r_2 f + i r_3 h', \quad Y_d = r_1\left(h + f + i h'\right), \quad Y_\nu = h - 3r_2 f + i c_\nu h'$$
$$Y_e = r_1\left(h - 3f + i c_e h'\right), \quad M_{\nu_R} = f\,\frac{\sqrt{3}r_1}{V_{16}}v_S$$

$$h = Y_{\mathbf{10}}V_{11}\,,\; f = Y_{\overline{\mathbf{126}}}\frac{V_{16}}{\sqrt{3}}\frac{V_{11}^*}{V_{15}}\,, c_e = \frac{V_{17} - \sqrt{3}V_{18}}{V_{17} + V_{18}/\sqrt{3}}\,, \qquad c_\nu = \frac{V_{13}^* - \sqrt{3}V_{14}^*}{V_{17} + V_{18}/\sqrt{3}}\frac{V_{15}}{V_{11}^*}\,,$$
$$r_1 = \frac{V_{15}}{V_{11}^*}\,, \quad r_2 = \frac{V_{12}^*}{V_{16}}\frac{V_{15}}{V_{11}^*}\,, \qquad r_3 = \frac{V_{13}^* + V_{14}^*/\sqrt{3}}{V_{17} + V_{18}/\sqrt{3}}\frac{V_{15}}{V_{11}^*}\,, h' = -i\,Y_{\mathbf{120}}\left(V_{17} + V_{18}/\sqrt{3}\right)\frac{V_{11}^*}{V_{15}}\,,$$

$$Y_u = h + r_2 f = \mathrm{diag}\{\eta_u y_u, \eta_c y_c, \eta_t y_t\}$$

$$Y_d = P_a V_{\mathrm{CKM}}\, \mathrm{diag}\{\eta_d y_d, \eta_s y_s, \eta_b y_b\}\, V_{\mathrm{CKM}}^\dagger P_a^* \qquad P_a = \mathrm{diag}\{e^{ia_1}, e^{ia_2}, 1\}$$

$$Y_\nu = -\frac{3r_2+1}{r_2-1} Y_u + \frac{4r_2}{r_1\left(r_2-1\right)} \operatorname{Re} Y_d + i\frac{c_\nu}{r_1} \operatorname{Im} Y_d$$

$$Y_e = -\frac{4r_1}{r_2-1} Y_u + \frac{r_2+3}{r_2-1} \operatorname{Re} Y_d + i c_e \operatorname{Im} Y_d$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_q} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_q} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_q} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_q} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_q} & c_{13}c_{23} \end{pmatrix},$$

$$M_\nu = m_0 \Bigg(\frac{8r_2\left(r_2+1\right)}{r_2-1} Y_u - \frac{16r_2^2}{r_1\left(r_2-1\right)} \operatorname{Re} Y_d + \frac{r_2-1}{r_1} \left(r_1 Y_u + i c_\nu \operatorname{Im} Y_d\right) \left(r_1 Y_u - \operatorname{Re} Y_d\right)^{-1} \left(r_1 Y_u - i c_\nu \operatorname{Im} Y_d\right) \Bigg)$$

RHN mass matrix obtained from inverting Seesaw Fomula: i.e. we have light neutrino Yukawa, light neutrino masses

GUT Model Particle Content

	Multiplet	Role in the model
Fermions	$\mathbf{16}$	Contains all SM fermions and RH neutrinos
Higgses	$\mathbf{10}$	Generates fermion masses
	$\mathbf{45}$	Triggers intermediate symmetry breaking
	$\mathbf{54}$	Triggers GUT symmetry breaking
	$\mathbf{120}$	Generates fermion masses
	$\overline{\mathbf{126}}$	Generates fermion masses & intermediate symmetry breaking
	$\mathbf{210}$	Triggers intermediate symmetry breaking

$SO(10)$	$\mathbf{54}$	$\mathbf{210}$	$\mathbf{45}$	$\overline{\mathbf{126}}$
G_3	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{15},\mathbf{1},\mathbf{1})_1$	$(\mathbf{15},\mathbf{1},\mathbf{1})_2$	$(\mathbf{10},\mathbf{1},\mathbf{3})+(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})$
G_2	–	$(\mathbf{1},\mathbf{1},\mathbf{1},0)_1$	$(\mathbf{1},\mathbf{1},\mathbf{1},0)_2$	$(\mathbf{1},\mathbf{1},\mathbf{3},-1)+(\mathbf{1},\mathbf{3},\mathbf{1},1)$
G_1	–	–	$(\mathbf{1},\mathbf{1},\mathbf{1},0)_2$	$(\mathbf{1},\mathbf{1},\mathbf{3},-1)$
$G_{\rm SM}$	–	–	–	$(\mathbf{1},\mathbf{1},0)_S$

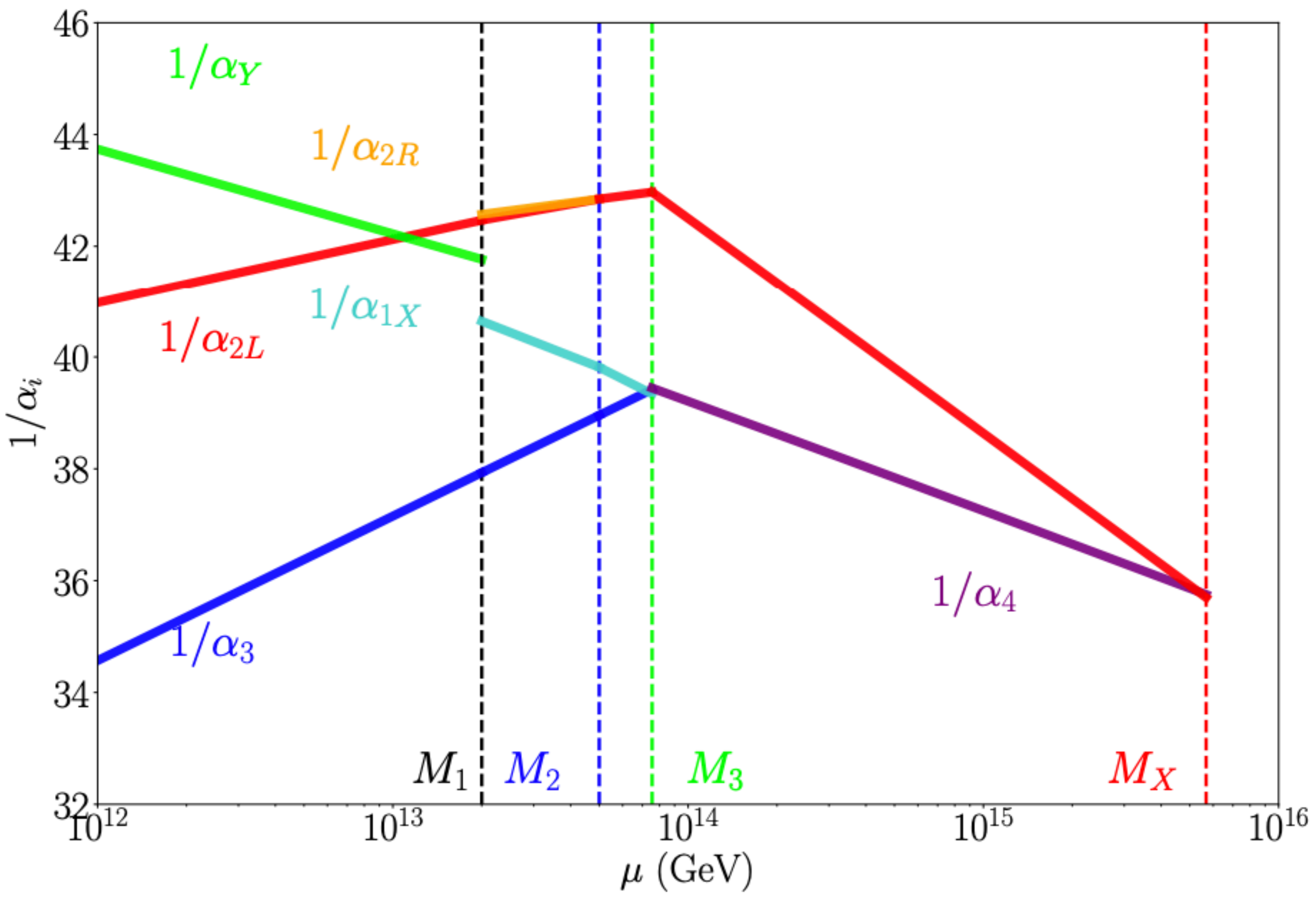
SO(10) Higgs reps for SSB

$SO(10)$	$\mathbf{16}$
G_3	$(\mathbf{4},\mathbf{2},\mathbf{1})_L+(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{R^c}$
G_2	$(\mathbf{3},\mathbf{2},\mathbf{1},1/6)_{Q_L}+(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},-1/6)_{Q_R^c}$ $+(\mathbf{1},\mathbf{2},\mathbf{1},-1/2)_{l_L}+(\mathbf{1},\mathbf{1},\mathbf{2},1/2)_{l_R^c}$
G_1	$(\mathbf{3},\mathbf{2},\mathbf{1},1/6)_{Q_L}+(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},-1/6)_{Q_R^c}$ $+(\mathbf{1},\mathbf{2},\mathbf{1},-1/2)_{l_L}+(\mathbf{1},\mathbf{1},\mathbf{2},1/2)_{l_R^c}$
$G_{\rm SM}$	$(\mathbf{3},\mathbf{2},1/6)_{Q_L}+(\overline{\mathbf{3}},\mathbf{1},-2/3)_{u_R^c}+(\overline{\mathbf{3}},\mathbf{1},1/3)_{d_R^c}$ $+(\mathbf{1},\mathbf{2},-1/2)_{l_L}+(\mathbf{1},\mathbf{1},0)_{\nu_R^c}+(\mathbf{1},\mathbf{1},1)_{e_R^c}$

Matter field decomposition

$SO(10)$	$\mathbf{10}$	$\overline{\mathbf{126}}$	$\mathbf{120}$
G_3	$(\mathbf{1},\mathbf{2},\mathbf{2})_1$	$(\mathbf{15},\mathbf{2},\mathbf{2})_1$ $+(\mathbf{10},\mathbf{1},\mathbf{3})+(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2},\mathbf{2})_2+(\mathbf{15},\mathbf{2},\mathbf{2})_2$
G_2	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_1$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_2$ $+(\mathbf{1},\mathbf{1},\mathbf{3},-1)+(\mathbf{1},\mathbf{3},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_{3,4}$
G_1	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_1$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_2$ $+(\mathbf{1},\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_{3,4}$
$G_{\rm SM}$	$(\mathbf{1},\mathbf{2},-1/2)_{h^u_{\mathbf{10}}}$ $+(\mathbf{1},\mathbf{2},+1/2)_{h^d_{\mathbf{10}}}$	$(\mathbf{1},\mathbf{2},-1/2)_{h^u_{\overline{\mathbf{126}}}}$ $+(\mathbf{1},\mathbf{2},+1/2)_{h^d_{\overline{\mathbf{126}}}}$ $+(\mathbf{1},\mathbf{1},0)_S$	$(\mathbf{1},\mathbf{2},-1/2)_{h^u_{\mathbf{120}},h^{u\prime}_{\mathbf{120}}}$ $+(\mathbf{1},\mathbf{2},+1/2)_{h^d_{\mathbf{120}},h^{d\prime}_{\mathbf{120}}}$

SO(10) Higgs reps for fermion mass generation



$$M_1 = 2 \times 10^{13}\ \text{GeV}\,, \quad M_2 = 5 \times 10^{13}\ \text{GeV}\,, \tag{1}$$

where the remaining scales and gauge coupling α_X, are then determined via the gauge unification,

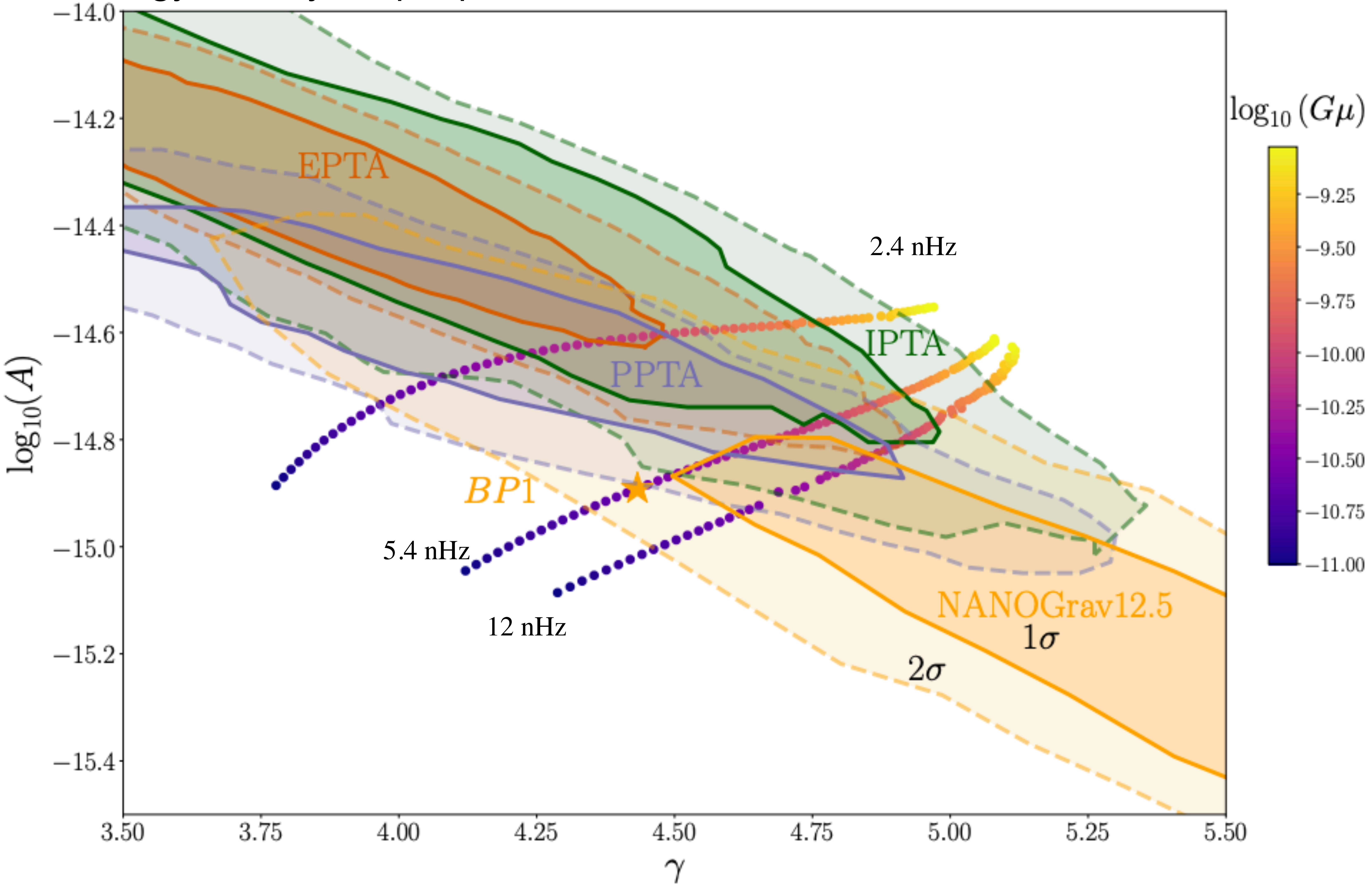
$$M_3 = 7.55 \times 10^{13}\ \text{GeV}\,, \quad M_X = 5.68 \times 10^{15}\ \text{GeV}\,, \quad \alpha_X = 0.0279\,. \tag{2}$$

$$\begin{array}{c} SO(10) \\ \mathbf{54} \big\downarrow \\ SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C \\ \mathbf{210} \big\downarrow \\ SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C \\ \mathbf{45} \big\downarrow \\ SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \\ \overline{\mathbf{126}} \big\downarrow \\ SU(3)_c \times SU(2)_L \times U(1)_Y\,. \end{array}$$

Overlap with PTA experiments

A ≡ amplitude parameter of correlation between pulsars.

γ ≡ related to GW energy density freq dependence



Gravitational Wave Calculation

Use approach of Cui et al (1711.03104, 1808.08968) which is based on simulation of Pillado, Plum & Shlaer

$$l(t) = l_i - \Gamma G\mu \left(t - t_i\right) \qquad l_i = \alpha t_i \text{ with } \alpha \simeq 0.1$$

Loop formation rate per unit volume per unit time

$$n(l,t) = \frac{C_{\text{eff}}\left(t_i\right)}{\alpha^2 t_i^4} \frac{a^3\left(t_i\right)}{a^3(t)}$$

Each loop radiates GW at constant rate:

$$\frac{dE}{dt} = -\Gamma G\mu^2, \quad \Gamma \approx 50$$

Gravitational Wave Calculation

Loops shrink as they radiate GWs

$$l = \alpha t_i - \Gamma G\mu\left(t - t_i\right)$$

Redshift GW emitted from loop of length l with where k is the oscillation mode $$f = \frac{a(\tilde{t})}{a\left(t_0\right)} \frac{2k}{l}$$

Energy density of GWs today

$$\Omega_{GW}(f) = \frac{f}{\rho_c} \frac{d\rho_{GW}}{df} = \sum_k \Omega_{GW}^{(k)}(f)$$

$$\Omega_{\mathrm{GW}}^{(k)}(f) = \frac{1}{\rho_c} \frac{2k}{f} \frac{\mathcal{F}_\alpha \Gamma^{(k)} G\mu^2}{\alpha(\alpha + \Gamma G\mu)} \int_{t_F}^{t_0} d\tilde{t} \frac{C_{\mathrm{eff}}\left(t_i^{(k)}\right)}{t_i^{(k)4}} \left[\frac{a(\tilde{t})}{a\left(t_0\right)}\right]^5 \left[\frac{a\left(t_i^{(k)}\right)}{a(\tilde{t})}\right]^3 \Theta\left(t_i^{(k)} - t_F\right)$$

Gravitational Wave Calculation

More generally

$$\Omega_{\rm GW} \equiv \frac{d\rho_{\rm GW}}{d\ln f}\frac{1}{\rho_{\rm crit}} \qquad\qquad \frac{d\rho_{\rm GW}(t)}{df} = \int dt' \frac{a\left(t'\right)^4}{a(t)^4}\int dl \frac{dn\left(l,t'\right)}{dl}\frac{dP\left(l,t'\right)}{df'}\frac{df'}{df}$$

There are different models for the number density of loops

VOS model velocity-dependent one-scale model: describe long string network as a function of characteristic length & average velocity of string. Tractable analytically once scaling is reached

Blanco-Pillado-Olum-Shlaer based on simulation of network, include backreaction of intersections On loops

Lorenz-Ringeval-Sakellariadou based on simulation of network with a focus on small-scale structure

First two approaches match better with each other than the latter. In the BOS approach large loops Tend to dominate GW spectrum but in the the LRS approach small loops tend to dominate