INVISIBLES24

Dark matter in QCD-like theories with a theta vacuum
Cosmological and Astrophysical implications

Giacomo Landini

based on

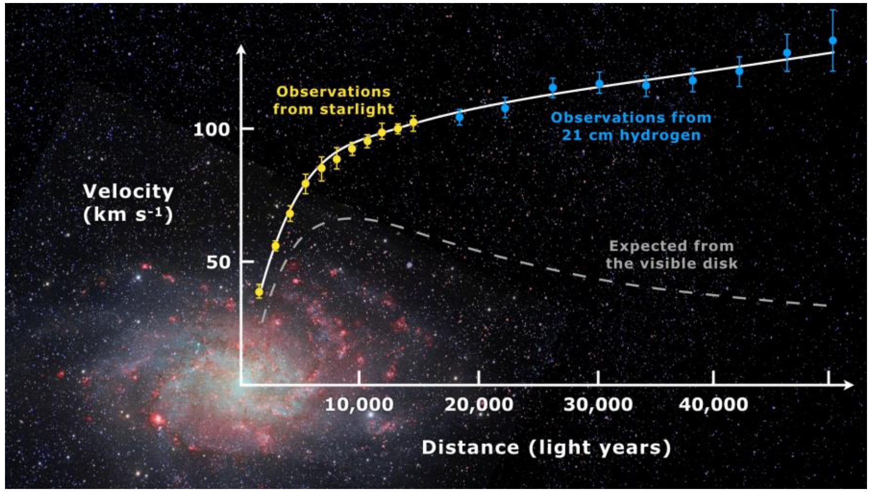
Camilo García-Cely, GL, Óscar Zapata, 2405.10367 and work in progress

Bologna, 04/07/2024

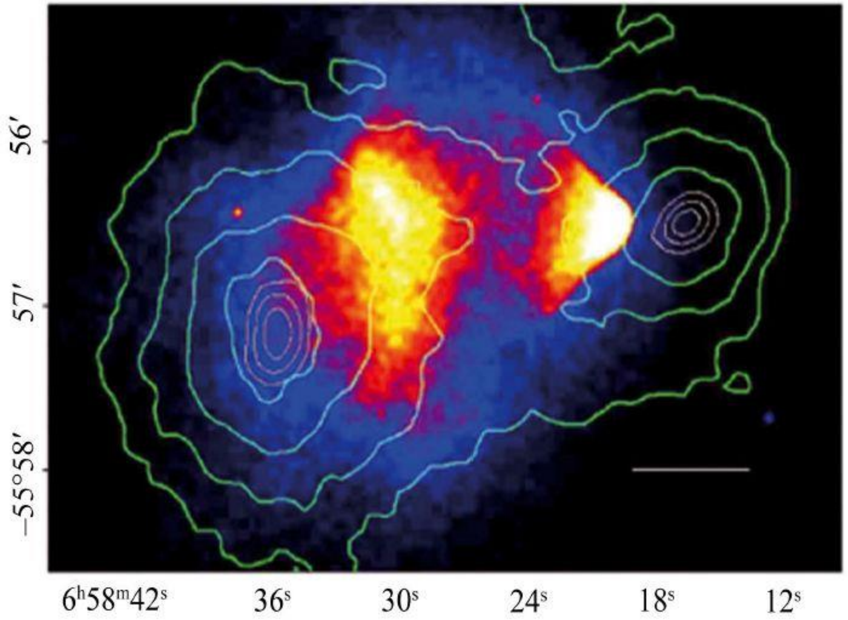
CSIC
Universitat de València | IFIC Institut de Física Corpuscular
Generalitat Valenciana

Dark Matter evidence

Dark Matter existence is supported by astrophysics and cosmology



Rotation curves of galaxies Rubin, Ford (1970)



Bullet Cluster and other galaxy clusters

Clowe *et al.* (2006)
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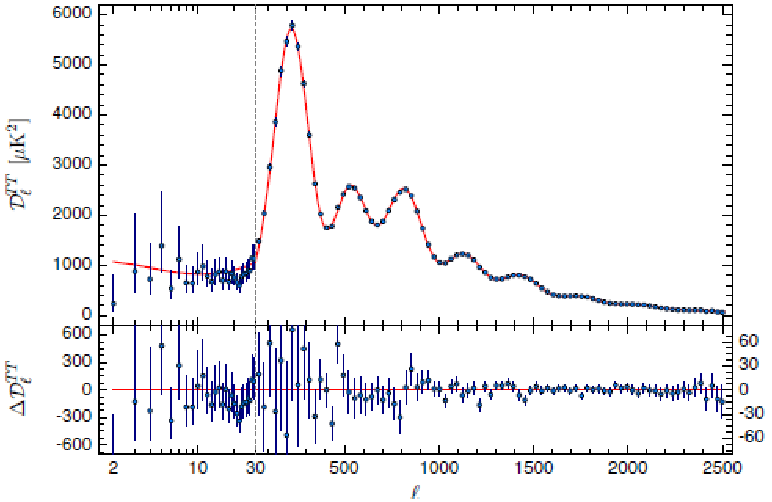
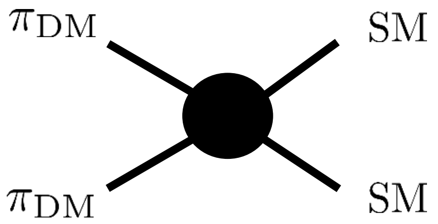


CMB

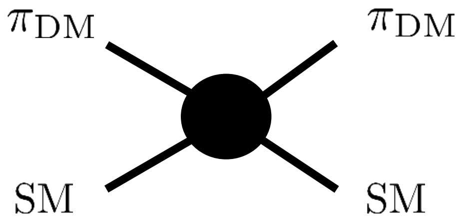
PLANCK (2020)

Dark Matter relic abundance: WIMP

Dark Matter is in thermal equilibrium with the SM bath in the early Universe



DM relic

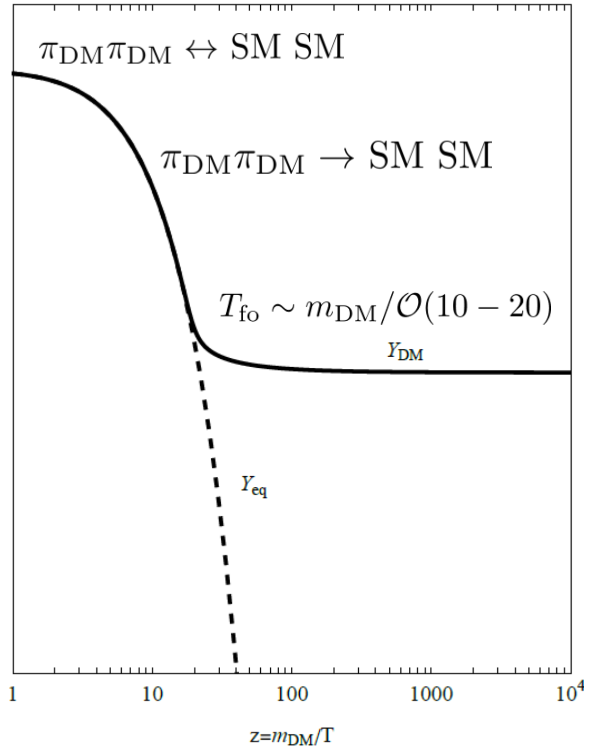


Thermal equilibrium

$$\frac{dY}{dz} = -\frac{s\langle\sigma_{22}v\rangle}{zH} Y_{\rm eq}^2 \left(\frac{Y^2}{Y_{\rm eq}^2} - 1\right) \qquad \begin{array}{l} z = m_{\rm DM}/T \\ Y = n/s \end{array}$$

DM relic abundance $Y_{\rm DM}^{-1} \simeq m_{\rm DM} M_{\rm Pl} \langle\sigma_{22}v\rangle$

DM mass $m_{\rm DM} \sim \sqrt{T_{\rm eq} M_{\rm Pl}} \sim {\rm TeV}$



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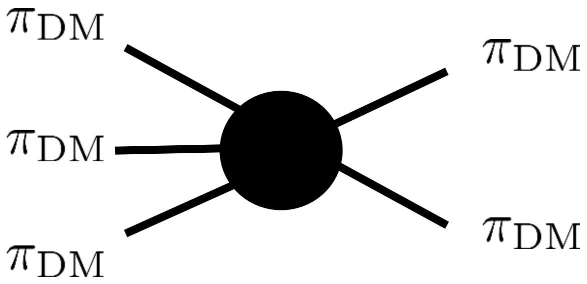

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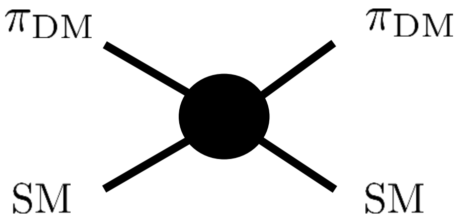
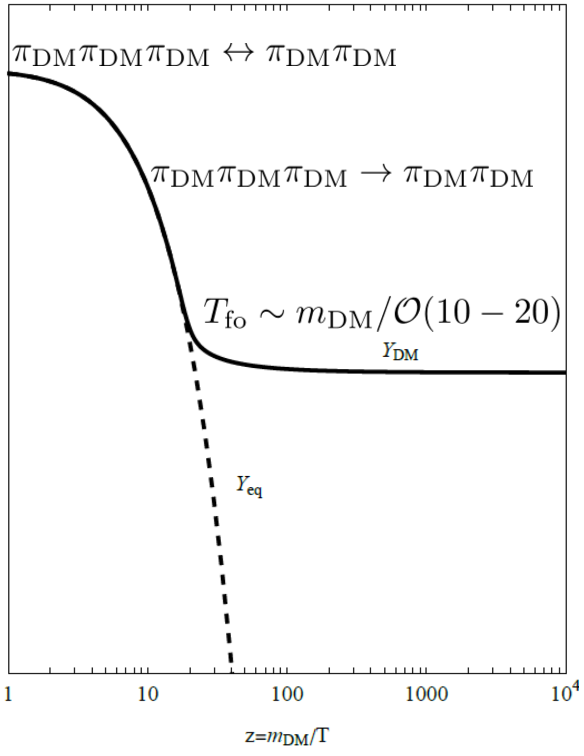


Thermal equilibrium

First proposed as *The SIMP Miracle* by Y. Hochberg,E.Kuflik, T.Volanksy,J.G.Wacker (2014)



$$\frac{dY}{dz} \simeq -\frac{s^2\langle\sigma_{32}v^2\rangle}{zH}Y_{\rm eq}^3\left(\frac{Y^3}{Y_{\rm eq}^3}-\frac{Y^2}{Y_{\rm eq}^2}\right) \qquad z=m_{\rm DM}/T,\quad Y=n/s$$

DM relic abundance $Y_{\rm DM}^{-1} \simeq m_{\rm DM}^2\sqrt{M_{\rm Pl}\langle\sigma_{32}v^2\rangle}$

DM mass $m_{\rm DM} \sim (T_{\rm eq}^2 M_{\rm Pl})^{1/3} \sim 100\ {\rm MeV}\ \text{-}\ {\rm GeV}$

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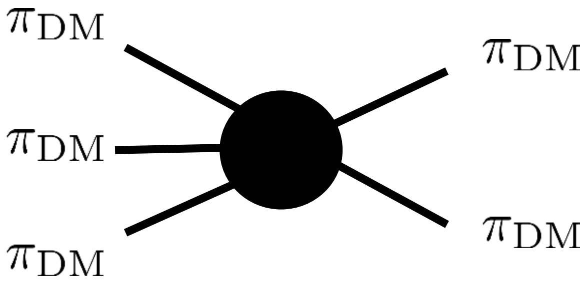

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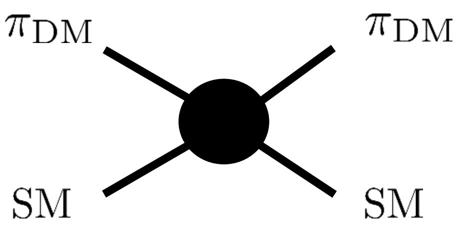

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See also
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Y. Hochberg,E.Kuflik,H.Murayama (2015)
A.Kamada,H.Kim,T.Sekiguchi (2017)
A.Katz,E.Salvioni,B.Shakya (2020)
Chu, Nikolic, Pradler (2024)
...

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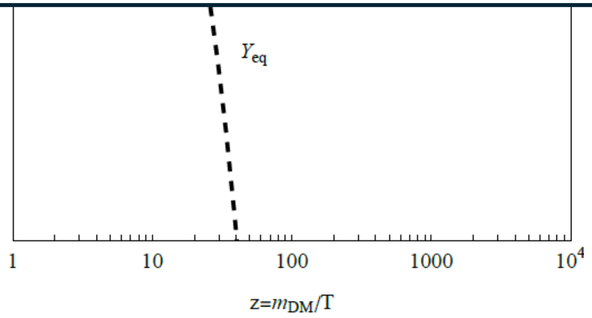

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Small scale issues

N-body simulations of collision-less DM on small scales (< 100 kpc)

Small scale galaxies – large DM density
DM velocity $v \sim 20 - 200\ \mathrm{Km/s}$

Universal halo profiles with large central density $\rho(r) \propto r^{-\beta}$ in the central regions $\beta \simeq 1$

In contrast to numerous observations (dwarf galaxies): $\beta \simeq 0$

<u>Observations</u>
Moore (1994)
Flores and Primack (1994)
Walker and Penarrubia (2011)

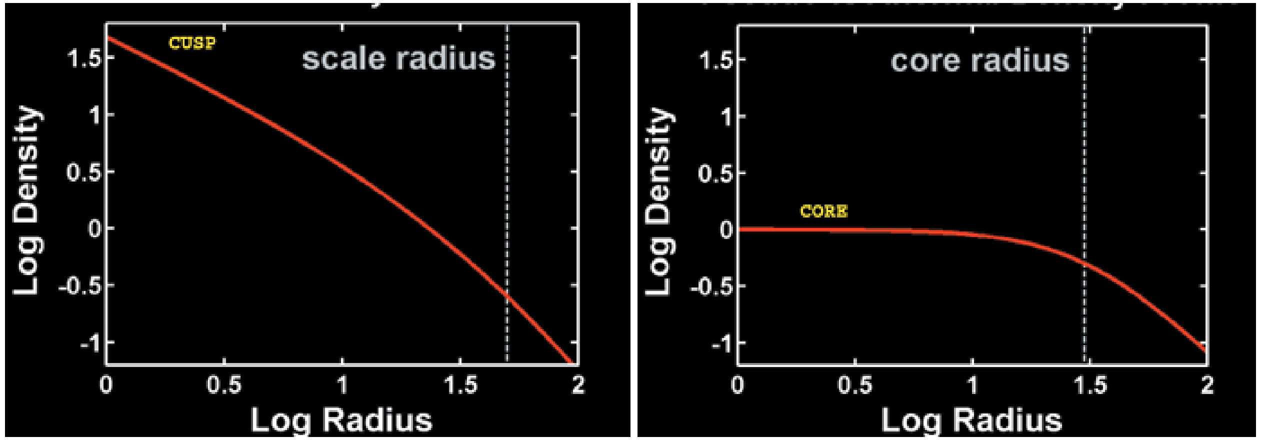

Figure by *Del Popolo, Le Delliou [1606.07790]*

+ diversity + prediction of very massive satellite halos in the Milky Way (not observed)

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Un

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In c

Observa

Moore (

Flores and Pri

Walker and Penar

Possible solutions

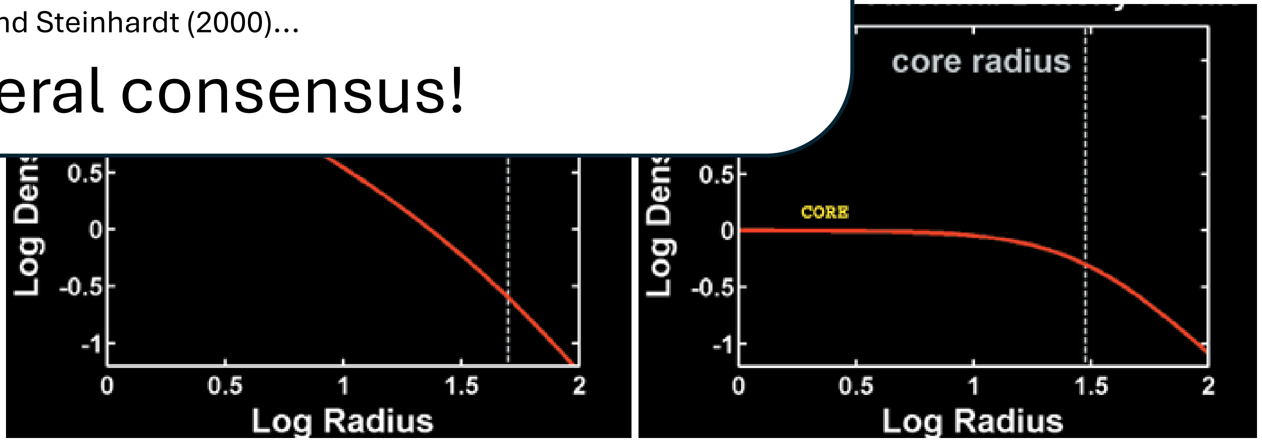
Systematic uncertaintes in deriving DM distributions from observations

Inclusion of dissipative baryonic processes in simulations

DM self-interactions

Spergel and Steinhardt (2000)...

Still no general consensus!



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Self-interacting Dark Matter (SIDM)

Elastic Dark Matter scatterings

$$\pi_{\rm DM}\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM}$$

Spergel and Steinhardt (2000)
Dave *et al*. (2001)
Vogelsberger *et al*. (2001)
...

Reduction of central density at small scales if

$\sigma(v)/m_{\rm DM} \sim 1 - 10\ {\rm cm}^2/{\rm g}$ for $v \sim 20 - 200\ {\rm Km/s}$

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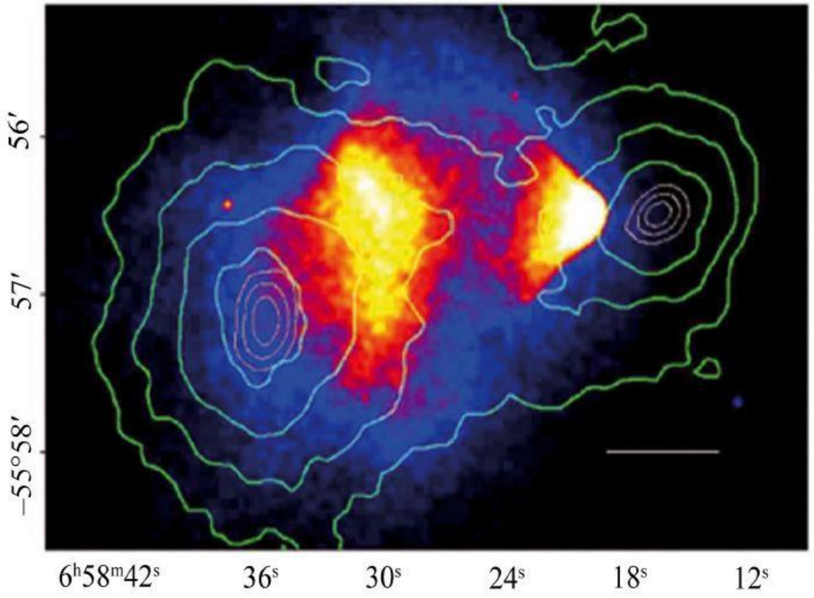

Bullet Cluster and other galaxy clusters

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for $v \sim 2000\ {\rm Km/s}$

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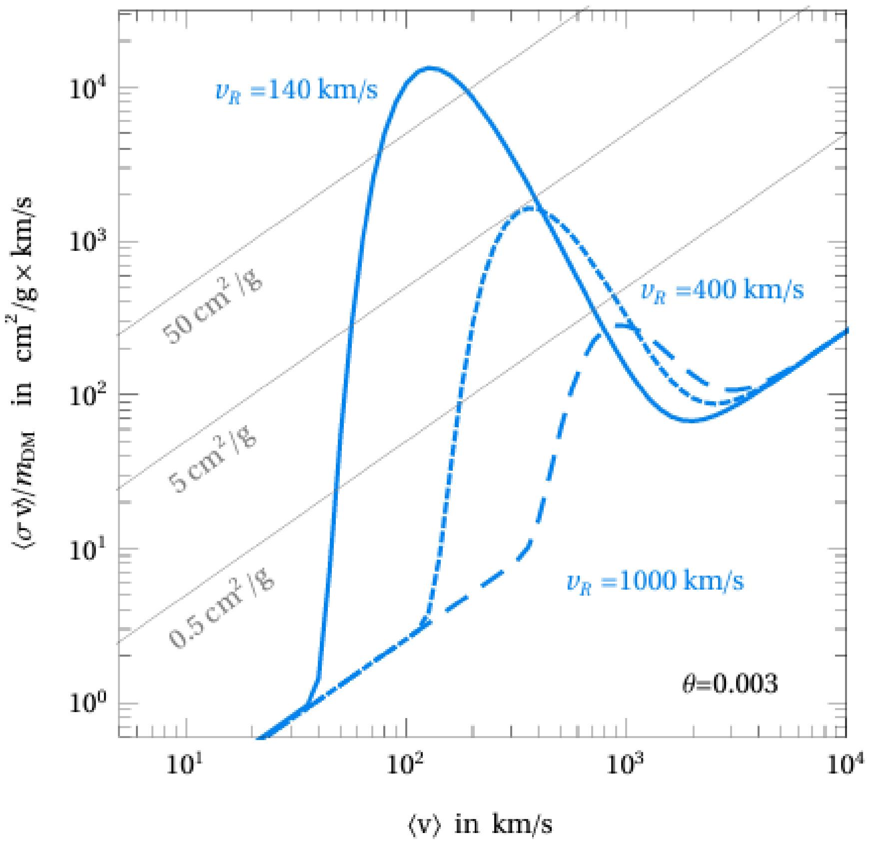
Need for a **velocity-dependent** self-interaction cross section



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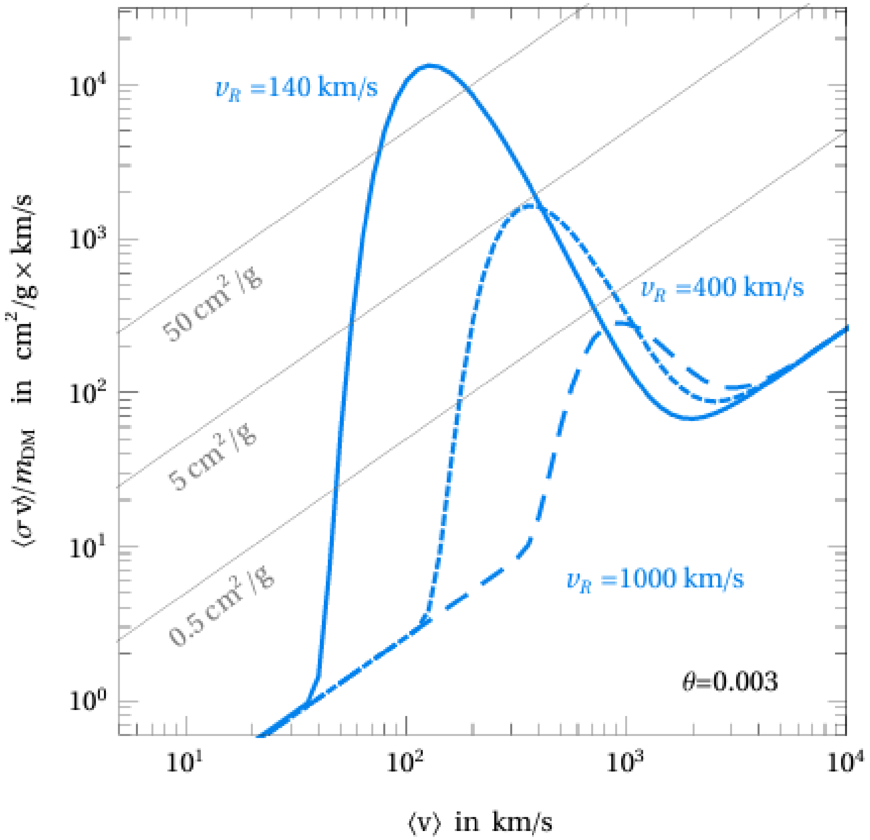

Possible realizations: light (MeV) mediator, resonant self-interactions (see also Chu, García-Cely, Murayama 2019),...

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QCD-like theories

We introduce a new dark gauge interaction (e.g a $SU(N_c)$ sector)

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{q}i\not{D}q - (\bar{q}_L M q_R + h.c.) + \frac{g^2\theta}{32\pi^2}F\widetilde{F}$$

usually ignored

N_f flavors of *light* quarks $M = \mathrm{diag}(m_1, \cdots, m_{N_f})$

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$N_f^2 - 1$ light pseudo-goldstone bosons π^a

$M \longrightarrow m_\pi \sim \sqrt{m_q \Lambda}$

Pion-like Dark Matter!

QCD-like theories ($\theta = 0$)

The low-energy dynamics of dark pions is described by ChPT

$$\mathcal{L}_{\text{eff}} = \frac{f_\pi^2}{4} Tr[\partial_\mu U^\dagger \partial^\mu U] + \frac{f_\pi^2}{2} B_0 Tr[M\ U + U^\dagger M^\dagger] + \mathcal{L}_{\text{WZW}}$$

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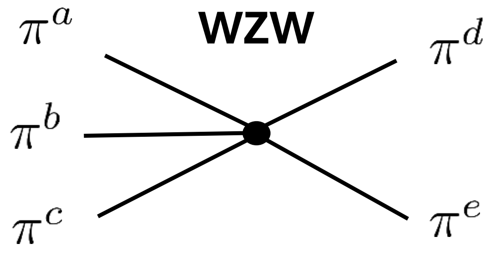
The SIMPlest Miracle (2014)

$$\mathcal{L}_{\text{WZW}} = -\frac{N_c}{240\pi^2 f_\pi^5} \epsilon^{\mu\nu\rho\sigma} Tr[\pi \partial_\mu \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi]$$

DM number changing processes



SIMP $\quad \langle \sigma_{32} v^2 \rangle \propto \left(\frac{m_\pi^5}{f_\pi^{10}}\right) v^2 \sim \left(\frac{m_\pi^5}{f_\pi^{10}}\right) \left(\frac{T}{m_\pi}\right)^2$

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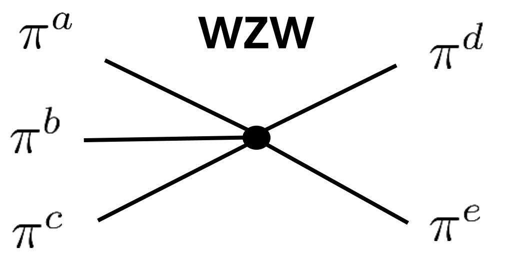

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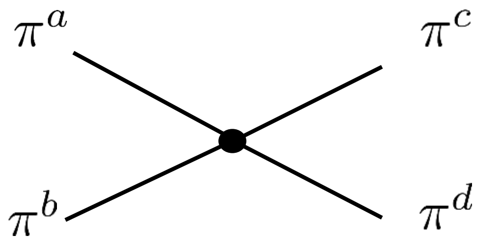
NO cubic vertex

DM self-interactions



$\sigma / m_\pi \propto \frac{m_\pi}{f_\pi^4}$ constant

QCD-like theories ($\theta = 0$)

This framework predicts MeV DM but...

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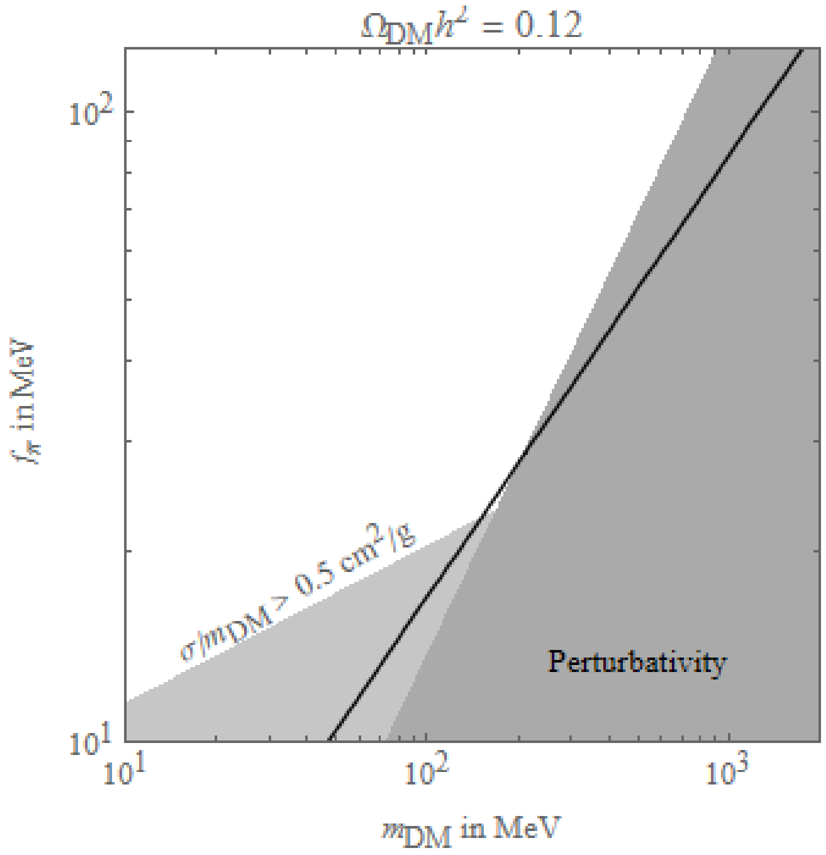
Tension among DM relic and Bullet Cluster bound

Tension among DM relic and perturbativity



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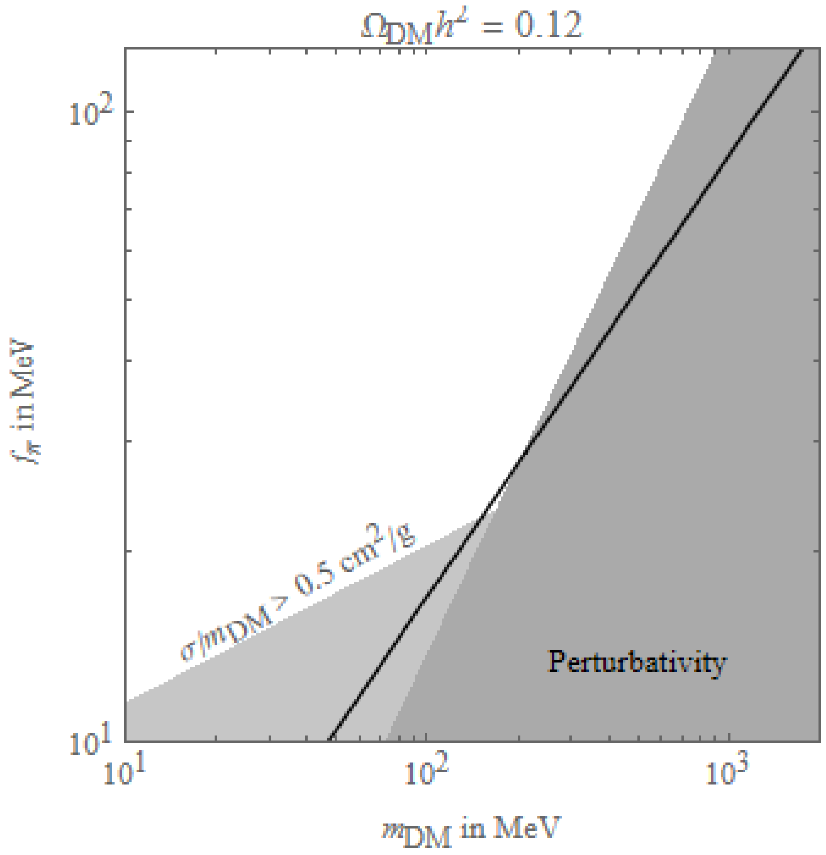
The self-interactions cross section is constant

$$\sigma/m_\pi \propto \frac{m_\pi}{f_\pi^4}$$

No SIDM realization



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New *odd* interactions induced by θ

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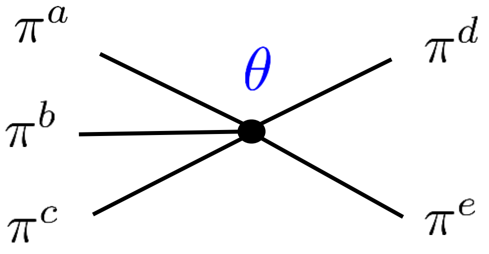
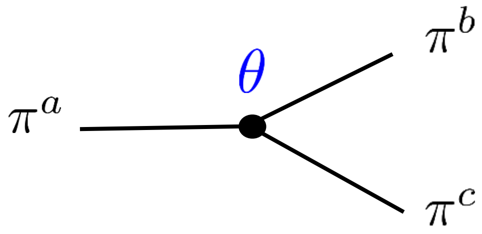
Camilo García-Cely, GL, Óscar Zapata [2405.10367]

$$\mathcal{L}_\theta = \frac{B_0\,\theta}{3 f_\pi Tr M^{-1}} \left(d_{abc}\pi_a\pi_b\pi_c - \frac{c_{abcde}}{10 f_\pi^2}\pi_a\pi_b\pi_c\pi_d\pi_e \right) \qquad d_{abc} = \mathrm{Tr}(\{\lambda_a, \lambda_b\}\lambda_c)/4$$

See also
A.Kamada, H.J.Kim.Kuflik, T.Sekiguchi (2017)





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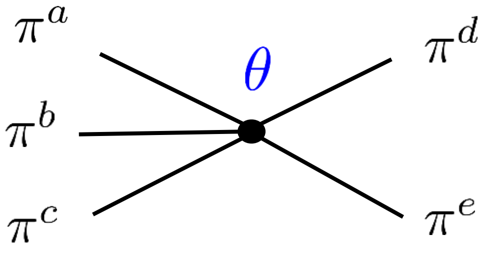

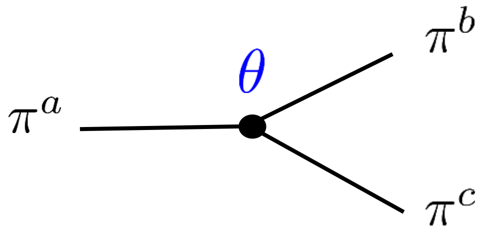

NEW TYPE OF VERTEX!!!

QCD-like theories ($\theta \neq 0$)

For non-degenerate quarks the spectrum can account for a resonance

$$(M_\pi^2)_{ab} \propto \mathrm{Tr}\left[M\{\lambda^a \lambda^b\}\right]$$

$$M = \begin{pmatrix} m_1 & & \\ & \ddots & \\ & & m_{N_f} \end{pmatrix} \longrightarrow \pi^a \begin{cases} \eta \\ \\ \pi_{\mathrm{DM}} \end{cases}$$

η: $m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi \quad v_R \lesssim 0.1$

m_K co-annihilating DM partner $n_K \ll n_\pi \sim n_{\mathrm{DM}}$

π_{DM}: m_π Dark Matter

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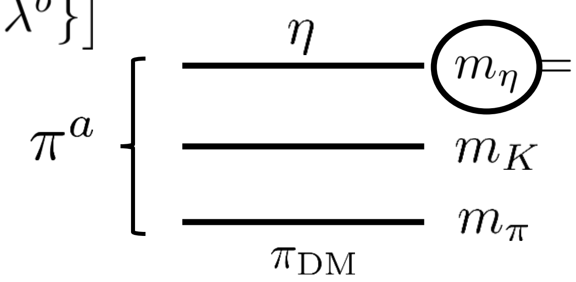
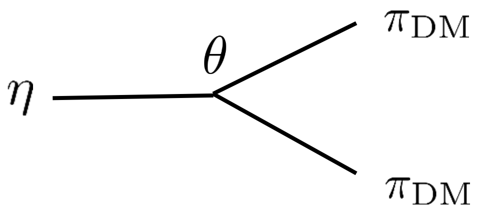


$$m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi \qquad v_R \lesssim 0.1$$

Unstable resonance

$$\Gamma(\eta \to \pi\pi) = \frac{\theta^2 B_0^2 \xi}{24\pi f_\pi^2 m_\eta (Tr M^{-1})^2} \sqrt{1 - \frac{4m_\pi^2}{m_\eta^2}}$$

Decay induced by θ



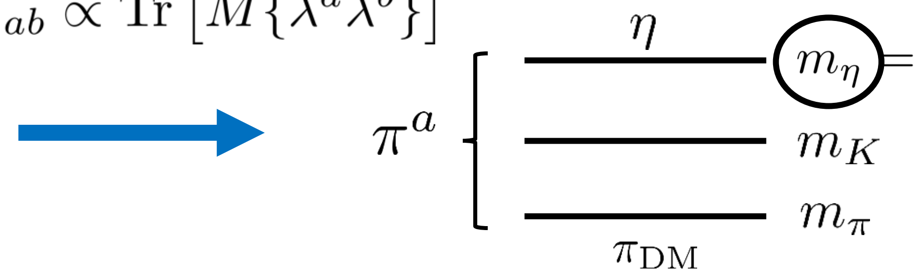
In the SM: Crewther, Di Vecchia, Veneziano, Witten (1979)

QCD-like theories ($\theta \neq 0$)

For non-degenerate quarks the spectrum can account for a resonance

$$(M_\pi^2)_{ab} \propto \mathrm{Tr}\left[M\{\lambda^a \lambda^b\}\right]$$

$$M = \begin{pmatrix} m_1 & & \\ & \ddots & \\ & & m_{N_f} \end{pmatrix}$$



$$m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi \qquad v_R \lesssim 0.1$$

Similar to $\frac{m(^8\mathrm{Be}) - 2m(\alpha)}{m(^8\mathrm{Be})} = 0.000012$ in the SM

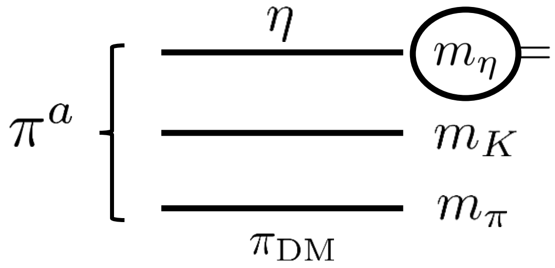
Explicit benchmark model in the following

The small splitting v_R <u>may</u> originate from $\mathcal{O}(\theta^2)$ corrections to the masses

$$v_R \sim 0.1\theta$$

QCD-like theories ($\theta \neq 0$)

For non-degenerate quarks the spectrum can account for a resonance



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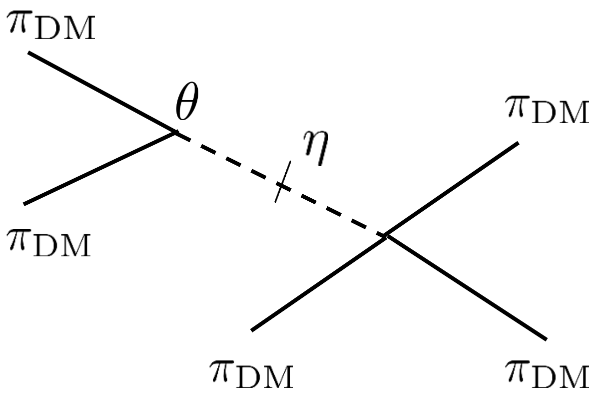
θ induces the following resonant interactions

Resonant 3-to-2



Camilo García-Cely, GL, Óscar Zapata [2405.10367]

Early Universe → DM relic

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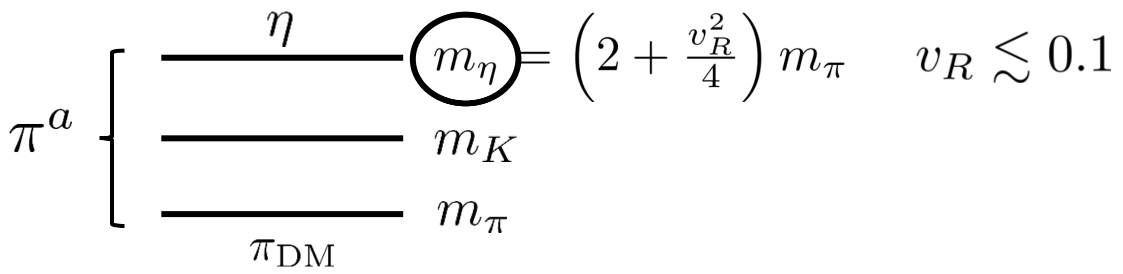

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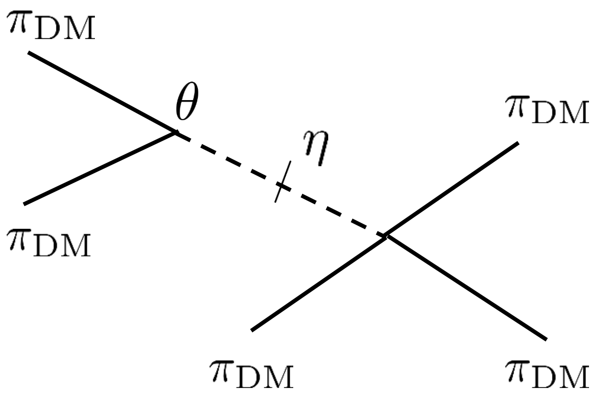
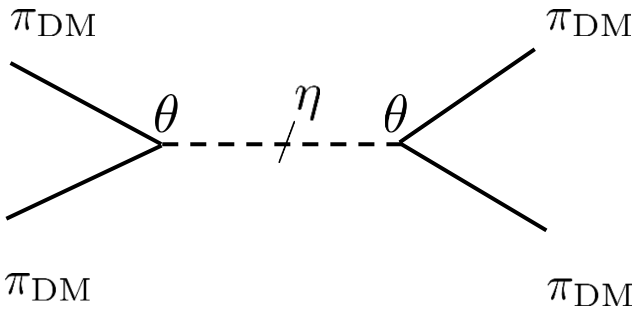


Resonant self-scattering



Camilo García-Cely, GL, Óscar Zapata [2405.10367]

Early Universe → DM relic

Today in halos → SIDM

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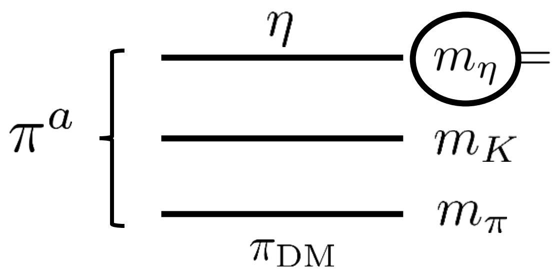

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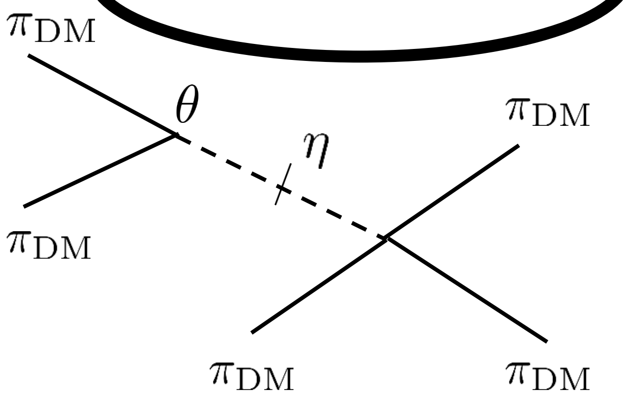

Early Universe → DM relic

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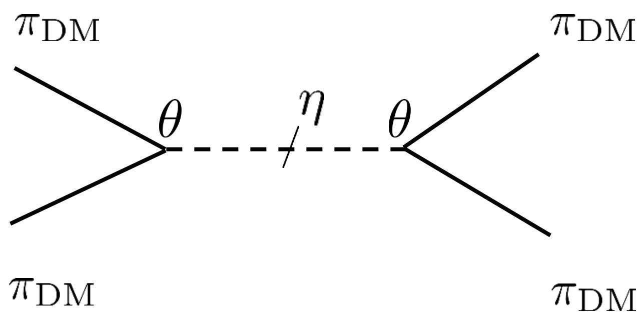


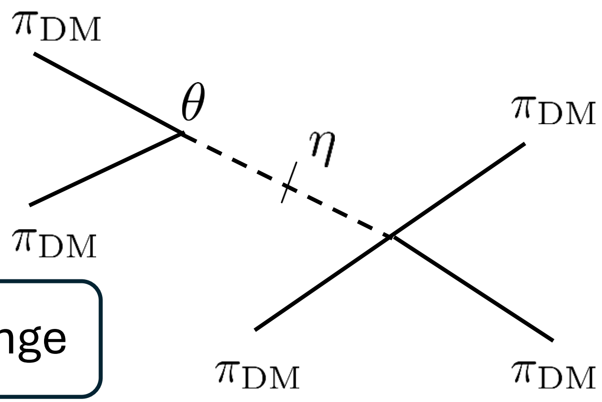
Today in halos → SIDM

Camilo García-Cely, GL, Óscar Zapata [2405.10367]

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

Camilo García-Cely, GL, Óscar Zapata [2405.10367]

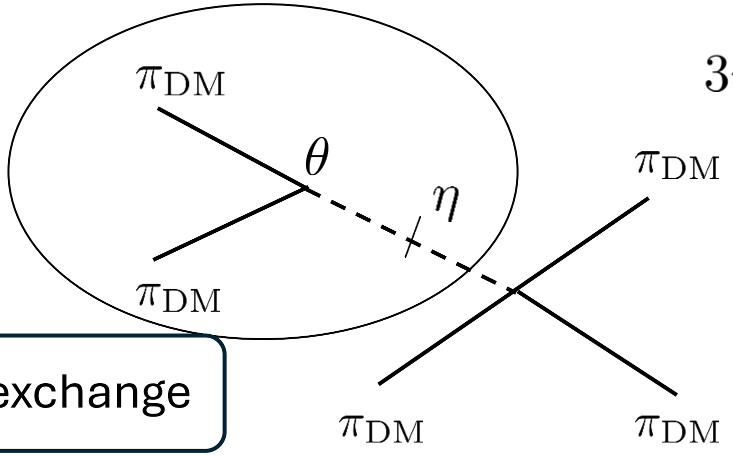


$\theta \neq 0$ → *On-shell* η resonance exchange

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

Camilo García-Cely,
GL, Óscar Zapata
[2405.10367]



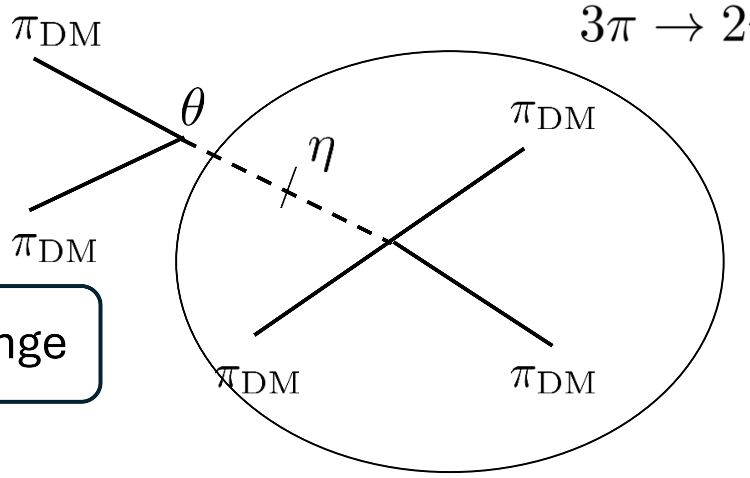
$3\pi \to 2\pi \overset{\rm res}{\simeq} (2\pi \to \eta)$ - - - ▸ $\eta\pi \to 2\pi$

$\theta \neq 0$ ➡ *On-shell* η resonance exchange

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

$3\pi \to 2\pi \overset{\text{res}}{\simeq} 2\pi \to \eta \dashrightarrow \eta\pi \to 2\pi$



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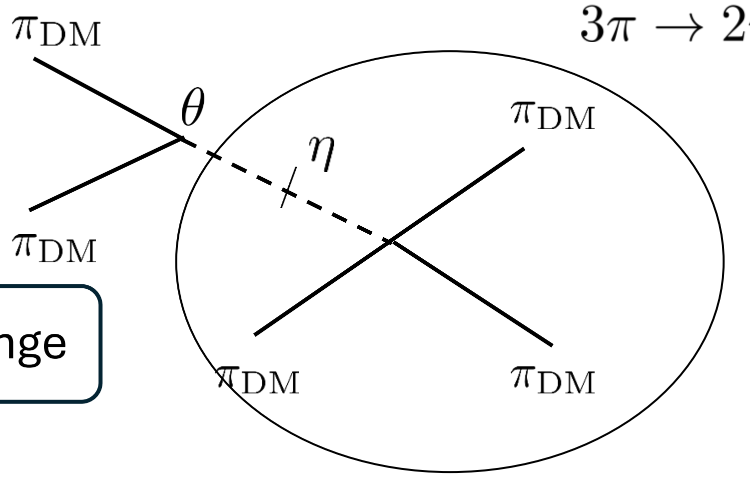

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η acts as a **catalyzer**

DM relic abundance with $\theta \neq 0$

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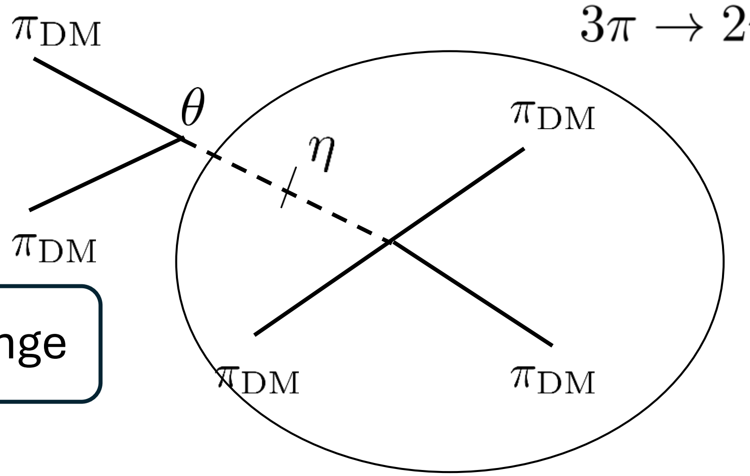


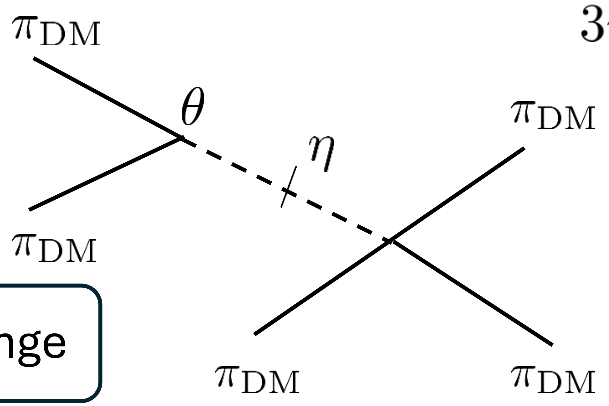
$\theta \neq 0$ ➡ *On-shell* η resonance exchange

η acts as a **catalyzer**

Similar to resonant triple- α reactions in stellar burning $3\alpha \to {}^{12}\text{C}^* \overset{\text{res}}{\simeq} \alpha\alpha \to {}^{8}\text{Be} \dashrightarrow {}^{8}\text{Be}\,\alpha \to {}^{12}\text{C}^*$

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes



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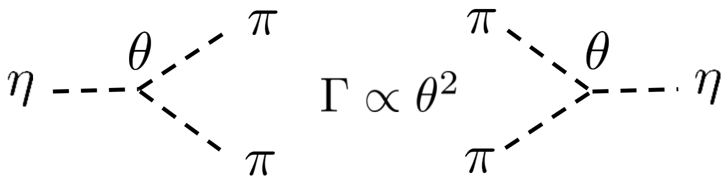
η acts as a **catalyzer**

$\theta \neq 0$ ➡ *On-shell* η resonance exchange

<u>Chemical equilibrium</u>

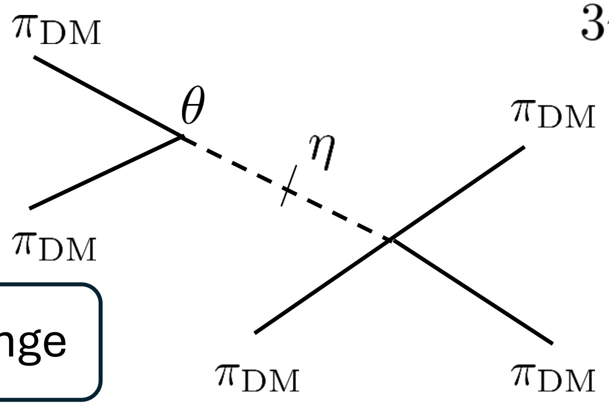


$$\langle \Gamma(\eta \leftrightarrow \pi\pi) \rangle > H \quad \Rightarrow \quad \frac{Y_\eta}{Y_{\eta,\rm eq}} = \frac{Y_\pi^2}{Y_{\pi,\rm eq}^2}$$

$$\theta > \theta_{\rm min} \sim 10^{-4}$$

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes



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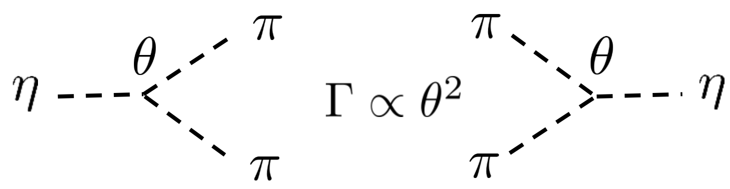


$\langle \Gamma(\eta \leftrightarrow \pi\pi) \rangle > H$ ➡ $\frac{Y_\eta}{Y_{\eta,\rm eq}} = \frac{Y_\pi^2}{Y_{\pi,\rm eq}^2}$

$\theta > \theta_{\rm min} \sim 10^{-4}$

<u>Boltzmann Equation</u>

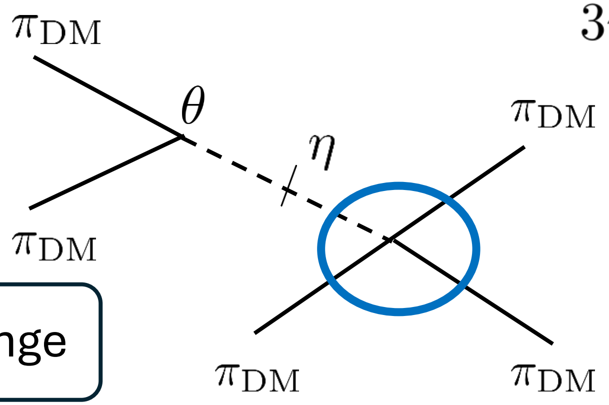
$$\frac{dY}{dz} = -\langle \sigma_{\eta\pi} v \rangle \frac{s Y_{\eta,\rm eq}}{z H} \left(\frac{Y_{\pi_{\rm DM}}^3}{Y_{\pi_{\rm DM},\rm eq}^2} - \frac{Y_{\pi_{\rm DM}}^2}{Y_{\pi_{\rm DM},\rm eq}} \right)$$

$Y = Y_{\pi_{\rm DM}} + 2Y_\eta \simeq Y_{\pi_{\rm DM}}$ $\quad \langle \sigma_{\eta\pi} v \rangle \propto m_\pi^2 / f_\pi^4$

$z = m_\pi / T$

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes



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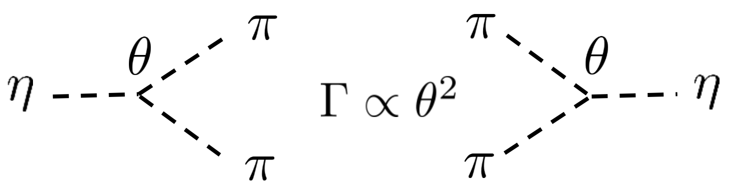
η acts as a **catalyzer**

Chemical equilibrium



$$\langle \Gamma(\eta \leftrightarrow \pi\pi) \rangle > H \quad \rightarrow \quad \frac{Y_\eta}{Y_{\eta,\rm eq}} = \frac{Y_\pi^2}{Y_{\pi,\rm eq}^2}$$

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Boltzmann Equation

2-to-2 annihilations **indipendent** on θ

$$\frac{dY}{dz} = -\langle \sigma_{\eta\pi} v \rangle \frac{s Y_{\eta,\rm eq}}{z H} \left(\frac{Y_{\pi_{\rm DM}}^3}{Y_{\pi_{\rm DM},\rm eq}^2} - \frac{Y_{\pi_{\rm DM}}^2}{Y_{\pi_{\rm DM},\rm eq}} \right)$$

$Y = Y_{\pi_{\rm DM}} + 2Y_\eta \simeq Y_{\pi_{\rm DM}}$ $\quad \langle \sigma_{\eta\pi} v \rangle \propto m_\pi^2 / f_\pi^4$

$z = m_\pi / T$

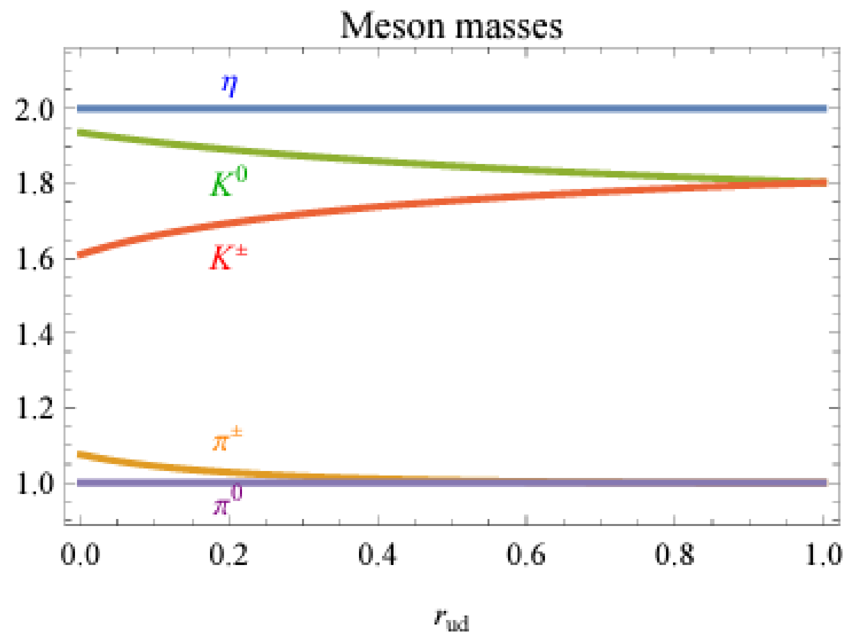
Explicit benchmark model

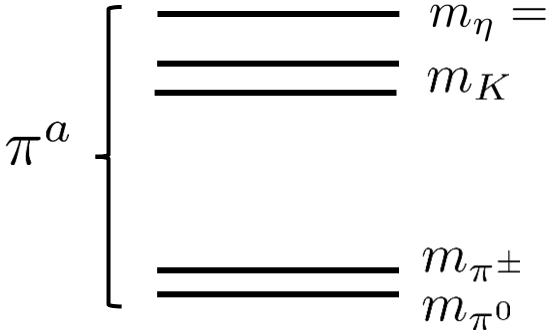
$$M = (m_u, m_d, m_s) \qquad r_{ud} = m_u/m_d$$

$$0 \leq r_{ud} \leq 1$$

m_u/m_s fixed as a function of (r_{ud}, v_R) so that $m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi$

with $v_R \lesssim 0.1$





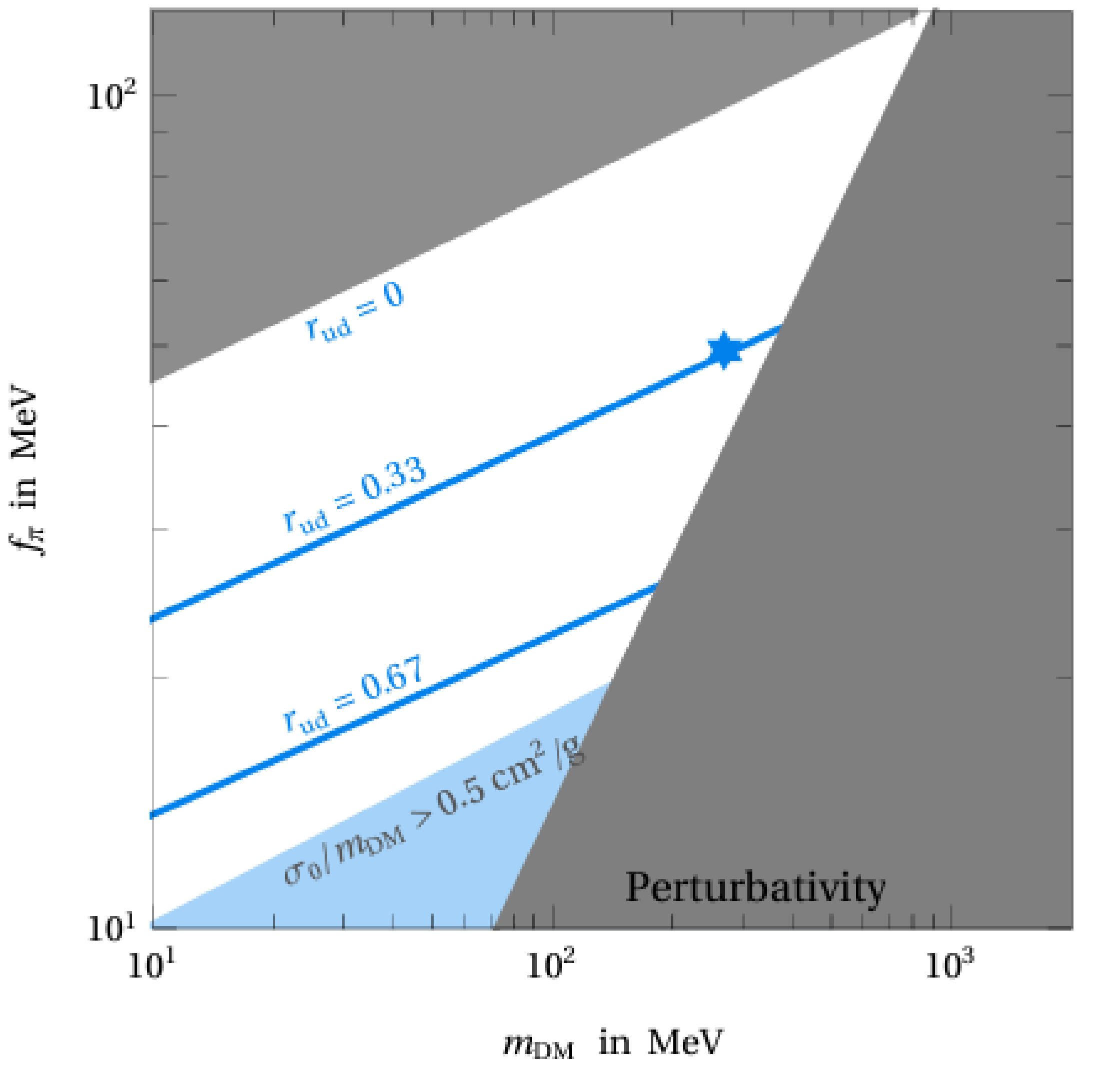
$$\frac{dY}{dz} = -\langle \sigma_{\eta\pi} v \rangle \frac{s Y_{\eta,\mathrm{eq}}}{zH} \left(\frac{Y^3_{\pi\mathrm{DM}}}{Y^2_{\pi\mathrm{DM,eq}}} - \frac{Y^2_{\pi\mathrm{DM}}}{Y_{\pi\mathrm{DM,eq}}} \right)$$

$$\langle \sigma_{\eta\pi} v \rangle = \frac{445\sqrt{5} m^2_{\pi^0} \delta(r_{ud})}{5184 \pi f_\pi^4}$$

$\delta(r_{ud}) = m_{\pi^\pm}/m_{\pi^0} - 1$ with $0 \lesssim \delta(r_{ud}) \lesssim 0.075$

DM relic abundance with $\theta \neq 0$

Camilo García-Cely, GL, Óscar Zapata [2405.10367]



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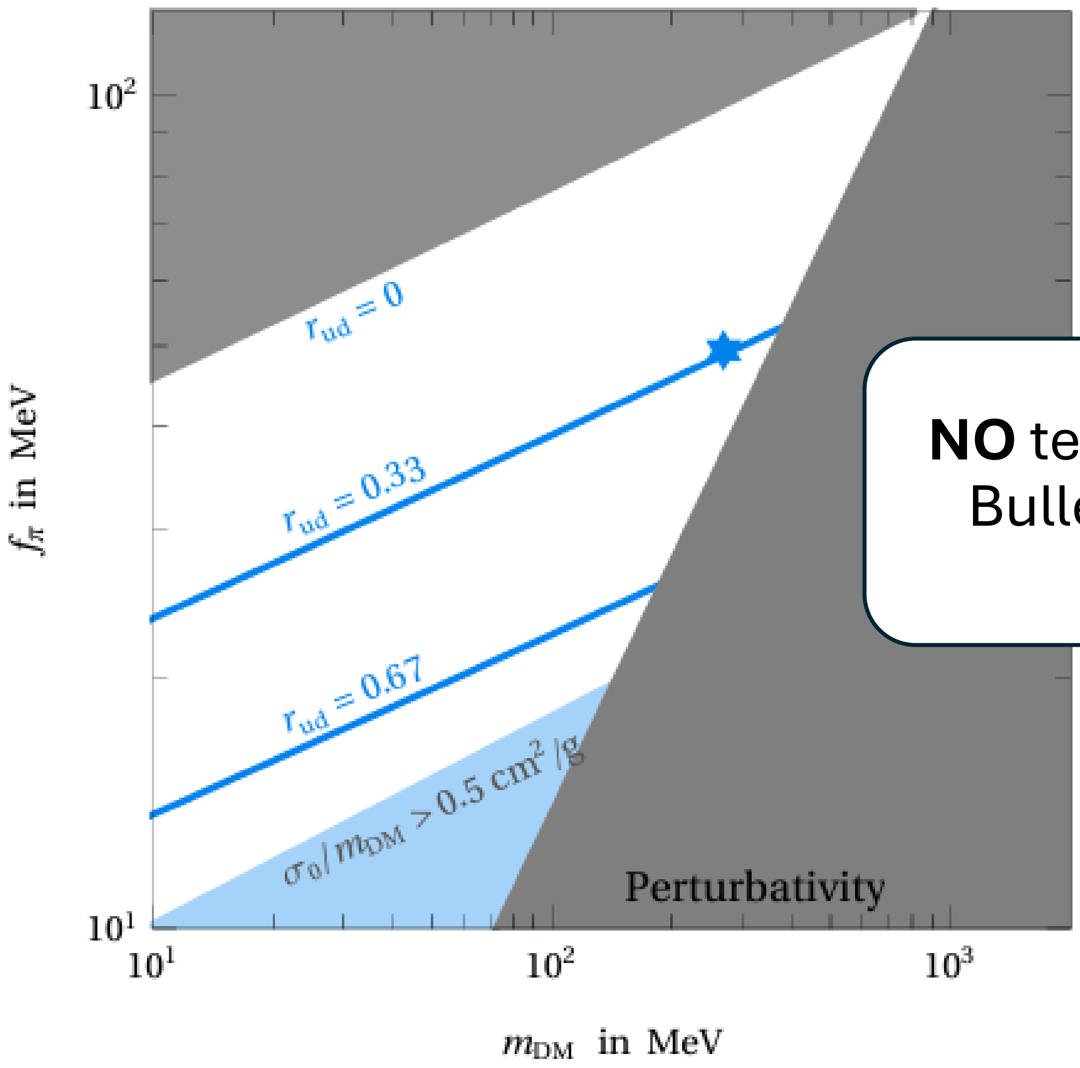

NO tension among DM relic, Bullet Cluster bound and perturbativity!

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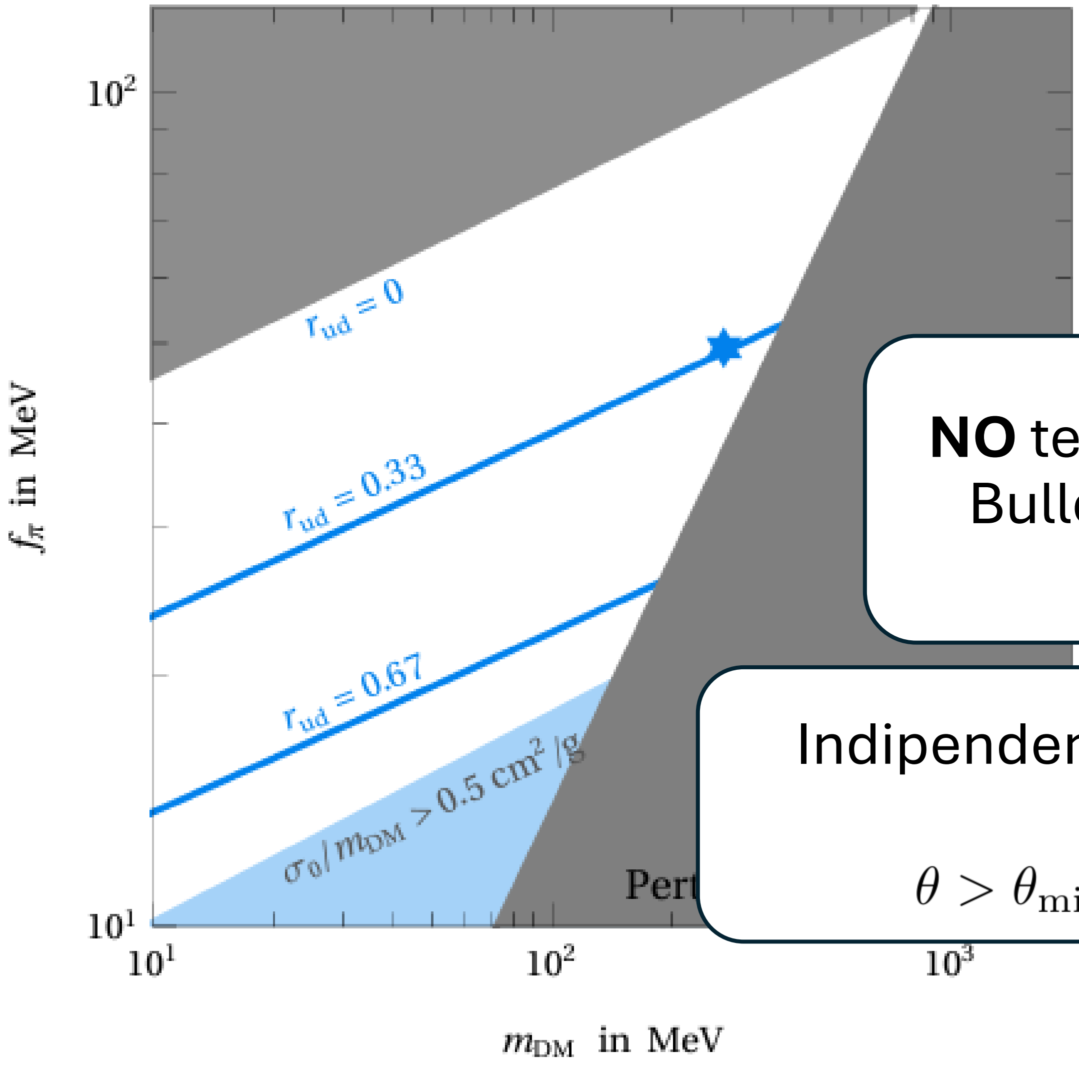

NO tension among DM relic, Bullet Cluster bound and perturbativity!

Indipendent on θ, v_R as long as

$$\theta > \theta_{\min} \sim 10^{-4} \ , \ v_R \lesssim 0.1$$

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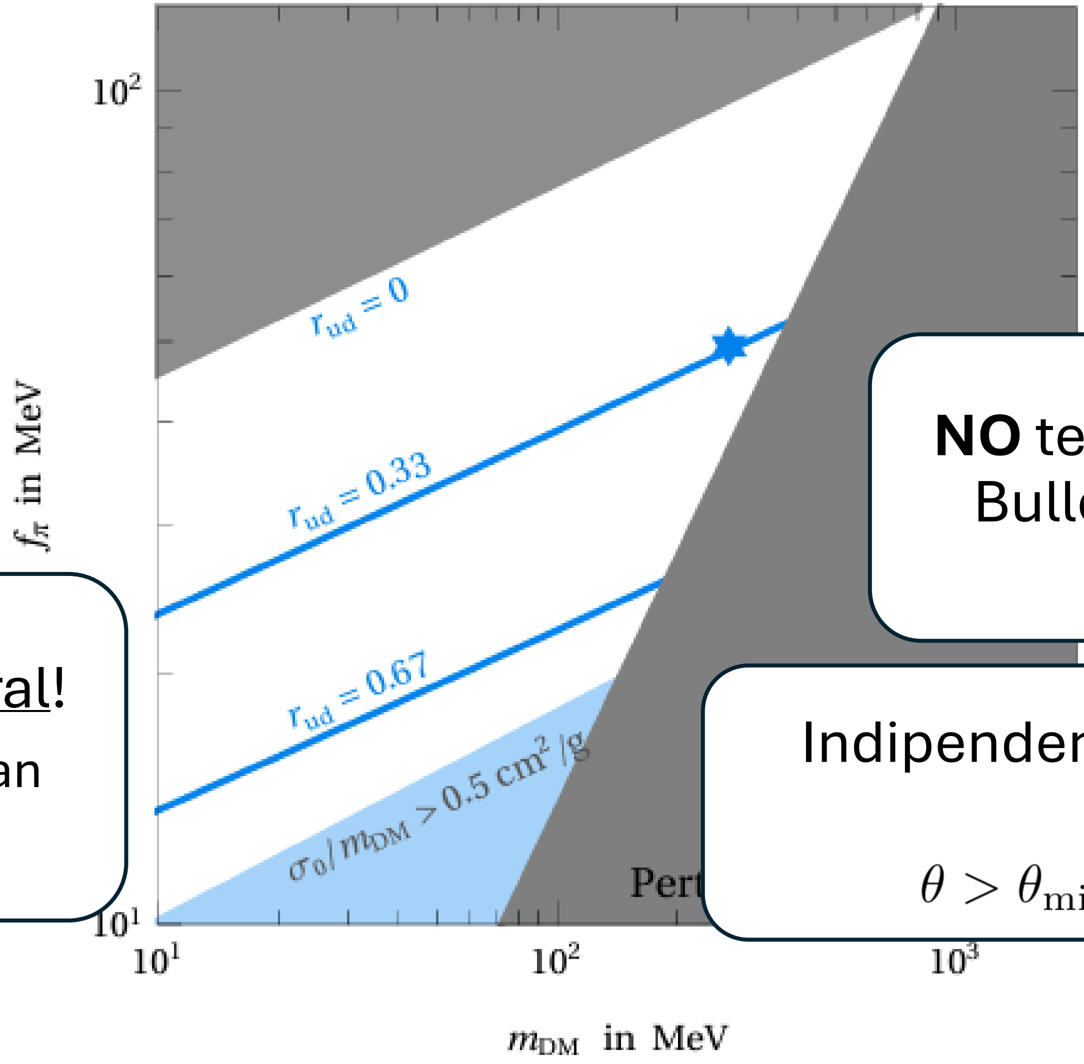


NO tension among DM relic, Bullet Cluster bound and perturbativity!

The mechanism is general!

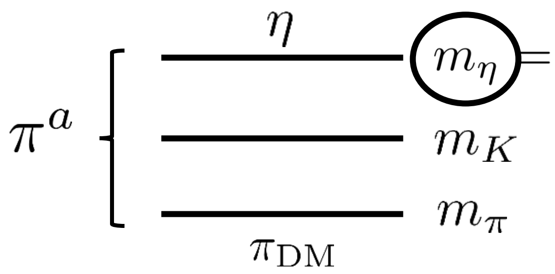
(Benchmark model is just an illustrative example)

Indipendent on θ, v_R as long as

$$\theta > \theta_{\min} \sim 10^{-4}\ ,\ v_R \lesssim 0.1$$

QCD-like theories ($\theta \neq 0$)

For non-degenerate quarks the spectrum can account for a resonance



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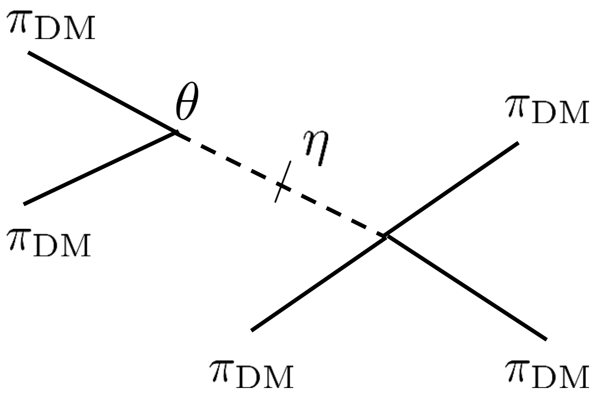
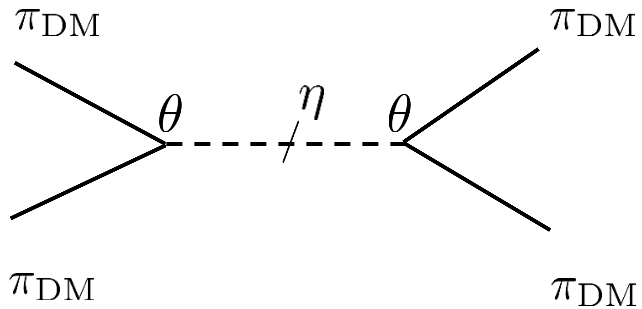
θ induces the following resonant interactions

Resonant 3-to-2



Early Universe → DM relic

Resonant self-scattering



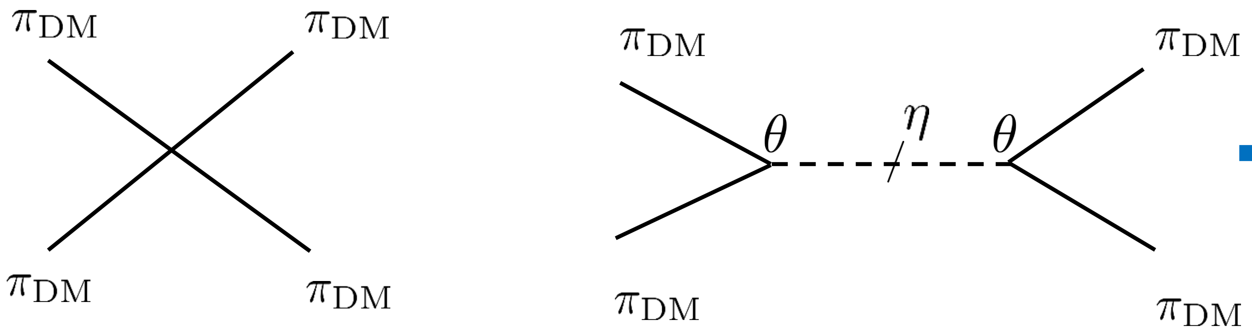
Today in halos → SIDM

Camilo García-Cely, GL, Óscar Zapata [2405.10367]

SIDM with $\theta \neq 0$

$\theta \neq 0$ induces velocity dependent resonant self-interaction cross section

Already present in Hochberg *et al* (2014)



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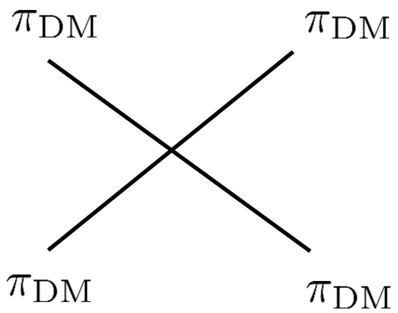

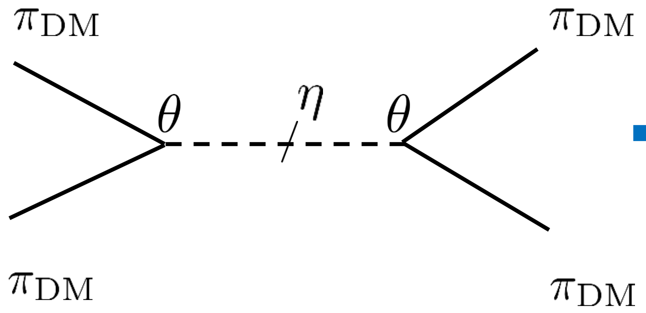

Camilo García-Cely, GL, Óscar Zapata [2405.10367]

$$\sigma(v) = \sigma_0 + \frac{128\pi}{m_\pi^2 v_R^2} \frac{\Gamma^2}{m_\pi^2 (v^2 - v_R^2)^2 + 4\Gamma^2 v^2 / v_R^2}$$

$v =$ DM velocity

constant term

resonant term

$v \simeq v_R$

in the benchmark model $\sigma_0 = \frac{m_\pi^2}{128\pi f_\pi^4}$

$$m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi \quad v_R \ll 1$$

$$\Gamma(\eta \to \pi\pi) = \frac{\theta^2 B_0^2 \xi}{24\pi f_\pi^2 m_\eta (TrM^{-1})^2} \sqrt{1 - \frac{4m_\pi^2}{m_\eta^2}}$$

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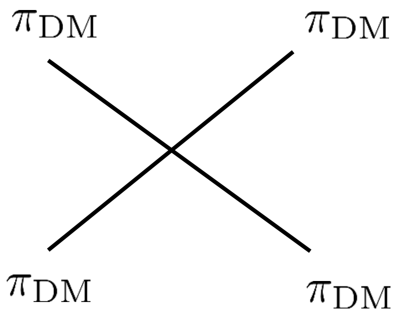
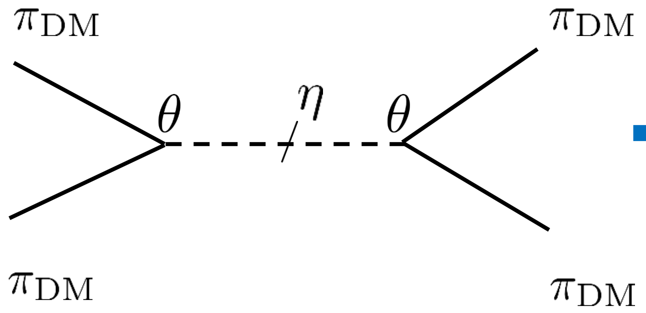




Camilo García-Cely, GL, Óscar Zapata [2405.10367]

$$\sigma(v) = \sigma_0 + \frac{128\pi}{m_\pi^2 v_R^2} \frac{\Gamma^2}{m_\pi^2 (v^2 - v_R^2)^2 + 4\Gamma^2 v^2 / v_R^2}$$

$v =$ DM velocity

constant term

resonant term

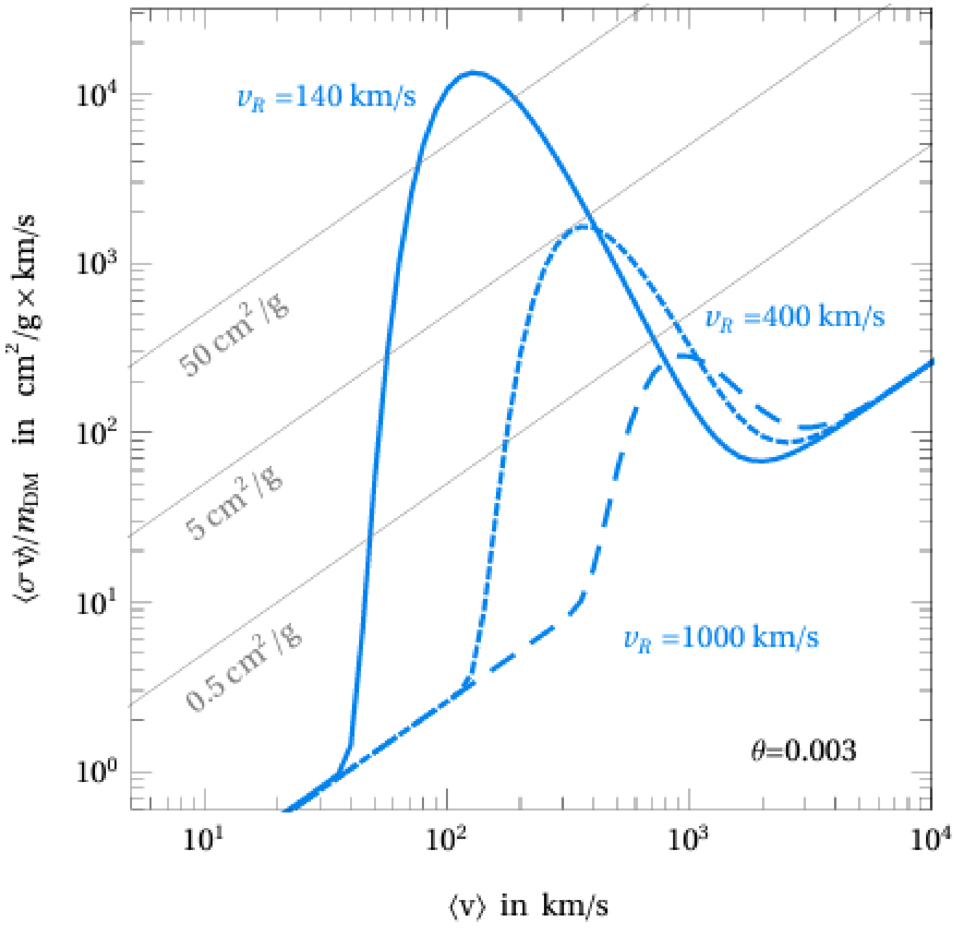
$v \simeq v_R$

in the benchmark model $\sigma_0 = \dfrac{m_\pi^2}{128\pi f_\pi^4}$

$v_R \sim 100 \text{ km/s} \sim 0.0003$ → resonance at small scales

SIDM with $\theta \neq 0$

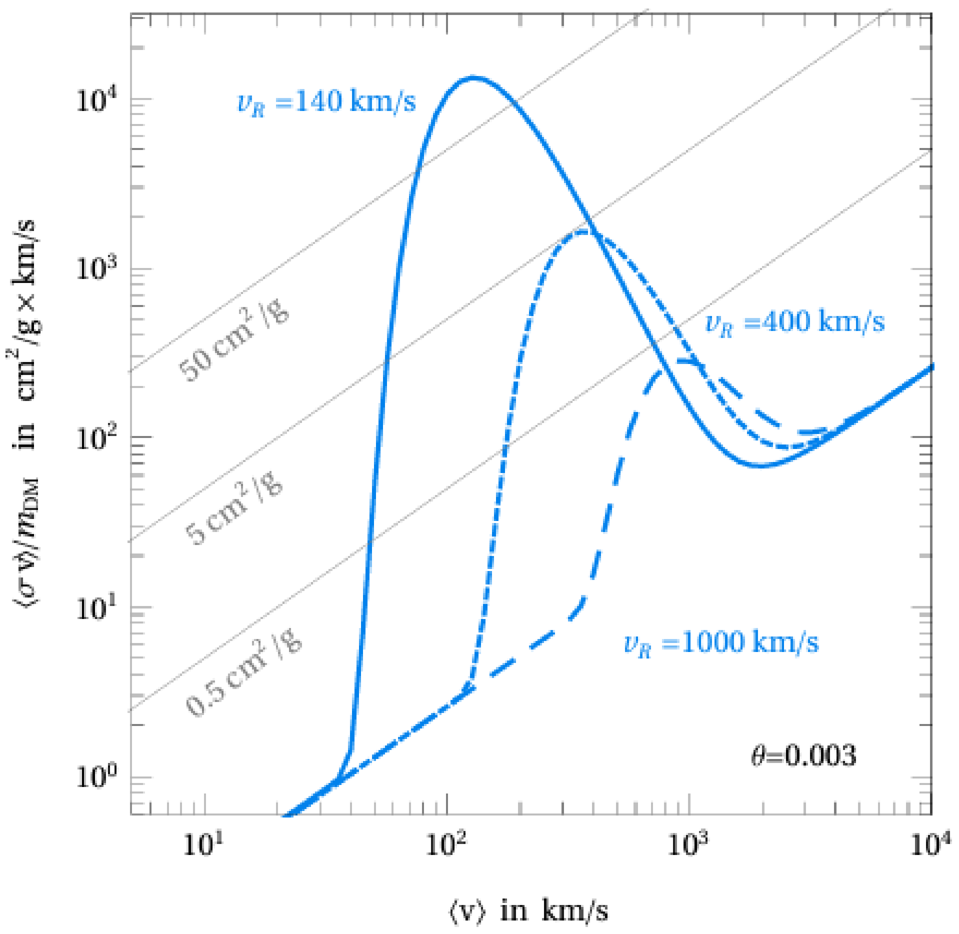
Camilo García-Cely, GL, Óscar Zapata [2405.10367]



Realization of SIDM for $v_R \sim 100$ km/s ~ 0.0003

SIDM with $\theta \neq 0$

Camilo García-Cely,
GL, Óscar Zapata
[2405.10367]



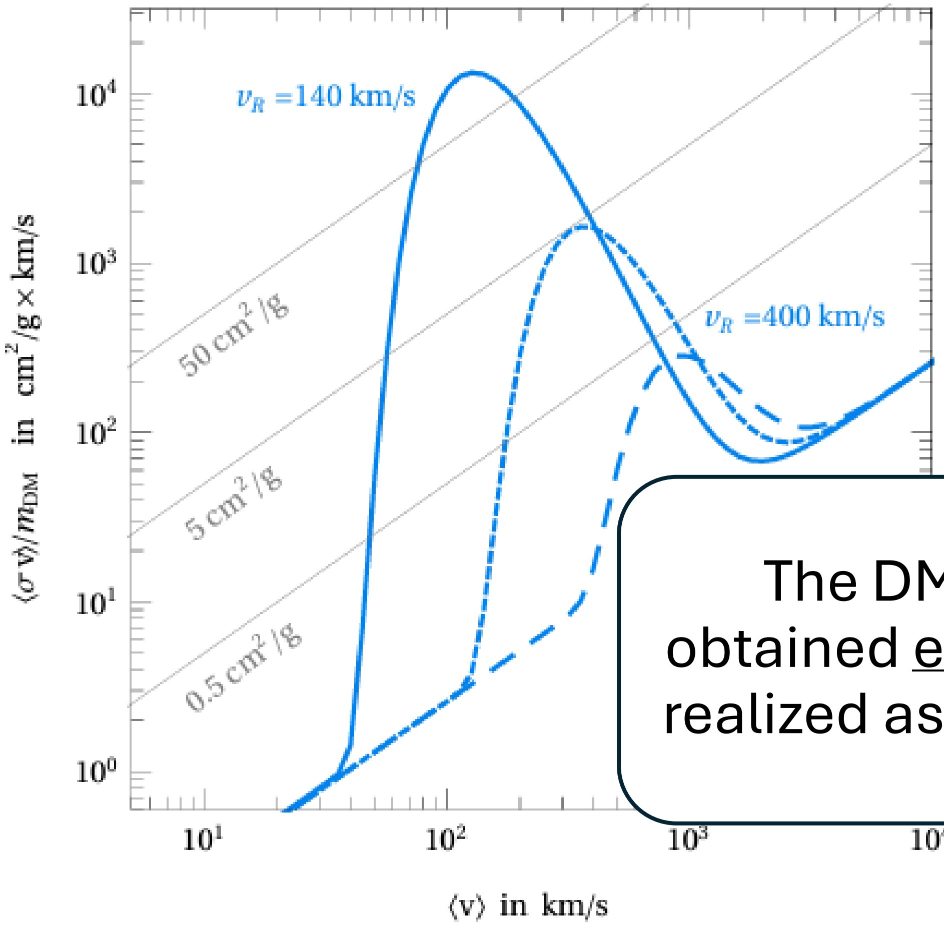
It works better for
$\theta \sim 10^{-\{4,2\}}$

Compatible with
$v_R \sim \{0.01, 1\}\theta$

Realization of SIDM for $v_R \sim 100 \text{ km/s} \sim 0.0003$

SIDM with $\theta \neq 0$

Camilo García-Cely, GL, Óscar Zapata [2405.10367]



It works better for $\theta \sim 10^{-\{4,2\}}$

Compatible with $v_R \sim \{0.01, 1\}\theta$

The DM relic abundance can be obtained <u>even if</u> the SIDM picture is not realized as long as $v_R \lesssim 0.1 \sim 30000\ \mathrm{Km/s}$ $\theta \gtrsim 10^{-4}$

Realization of SIDM for $v_R \sim 100\ \mathrm{km/s} \sim 0.0003$

Comments on the results

DM is a pion of a QCD-like dark sector

$\theta \gtrsim 10^{-4}$ ↘ ↙ Resonance in the spectrum

We reproduce the **relic abundance** with a resonant 3-to-2 process avoiding tensions with BC and perturbativity

only small amount of tuning $v_R \lesssim 0.1$

We can solve **small-scale issues** with resonant self-scatterings

larger tuning is required $v_R \sim 100\ \mathrm{km/s} \sim 0.0003$ ...which may originate from $v_R \sim \theta$

Outlook

Straightforward

- Generalize to other gauge groups

$$\mathcal{L}_\theta = \frac{B_0\theta}{3f_\pi TrM^{-1}}\left(d_{abc}\pi_a\pi_b\pi_c - \frac{c_{abcde}}{10f_\pi^2}\pi_a\pi_b\pi_c\pi_d\pi_e\right) \neq 0 \text{ if } \begin{cases} N_f \geq 3 & \text{for } SU(N_c) \\ N_f \geq 3 & \text{for } SO(N_c) \\ N_f \geq 6 & \text{for } Sp(N_c) \end{cases}$$

- Generalize to other benchmark models

Different choices of N_f and $M = (m_1, \cdots, m_{N_f})$

  - Small DM representations are preferred from BC bound

    Easily obtained breaking mass degeneracies

  - Anomalous axial U(1) with resonant η'

Outlook

Moderate

- More systematic analysis of the spectrum dependence on θ

Symmetry-based arguments for $m_\eta = 2m_\pi$

Origin of the small splitting from $v_R \sim \theta$

- More realistic model with SM portal (ALP? Dark Photon?...?)

Which portals can establish efficient thermal equilibrium?

What are the phenomenological consequences of the portal?

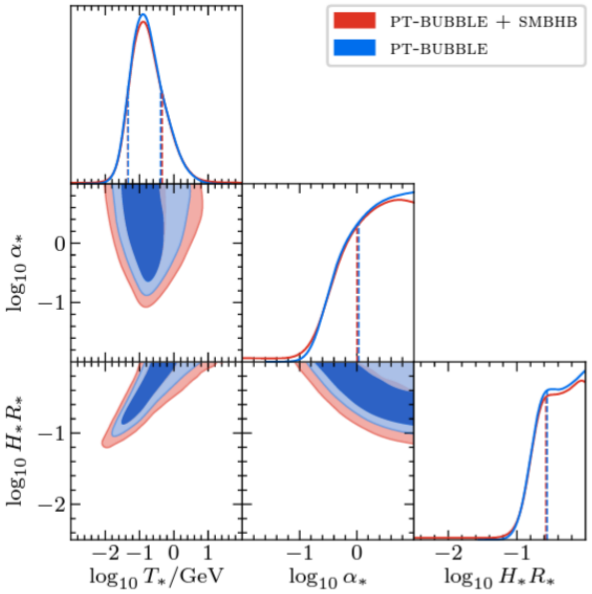
Outlook

Elaborate

- Gravitational Waves signal?

Chiral Phase Transition is first-order if $N_f \geq 3$

The PT critical temperature is $T_* \sim f_\pi \sim \mathcal{O}(10-100)\ \mathrm{MeV}$

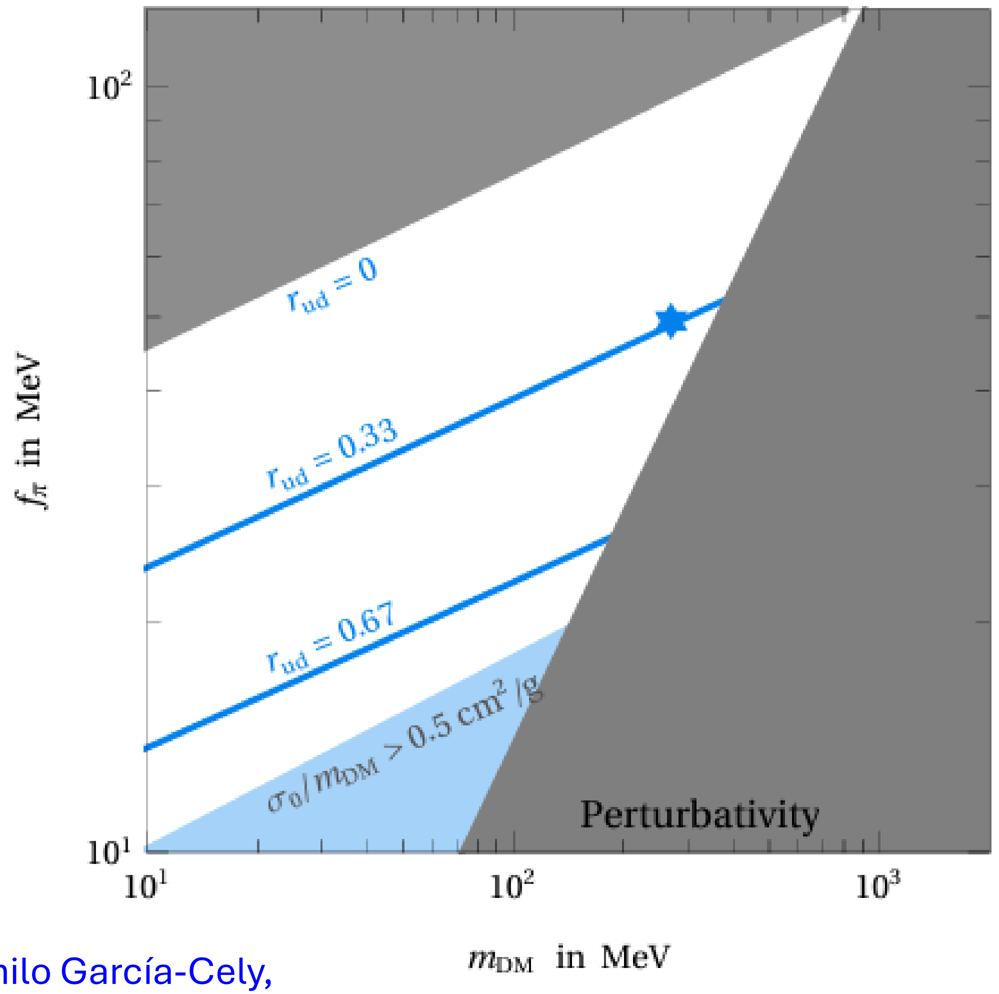


It could be relevant in view of **PTA signal!**

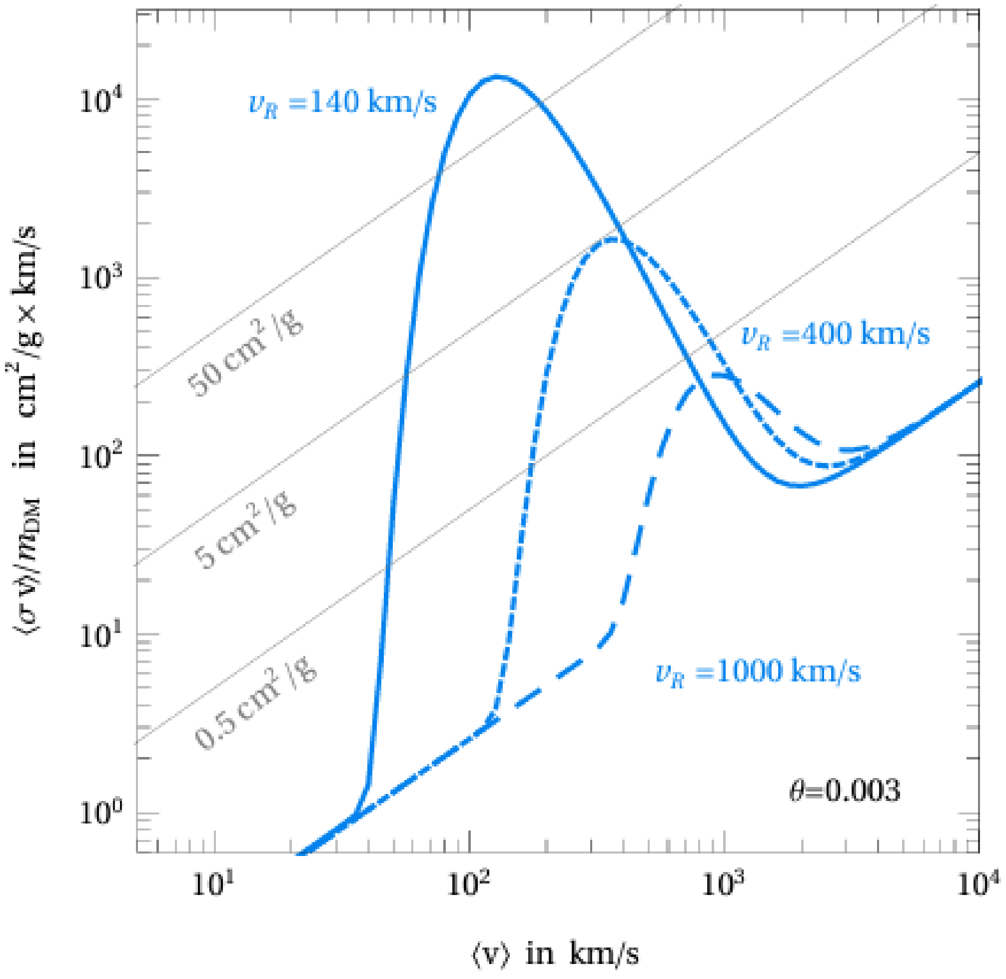
NANOGrav collaboration 2023

Figure by ***NANOGrav*** *collaboration [2306.16219]*

Relic abundance



Self-scatterings



Camilo García-Cely, GL, Óscar Zapata [2405.10367]

Thank you for the attention!

Backup slides

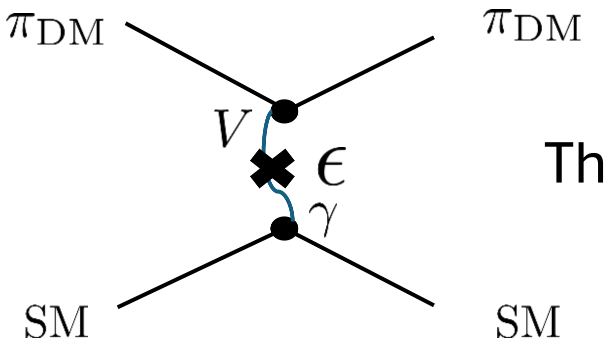
Portals with the SM

Dark photon portal

Hochberg *et al. (2015)*

Gauging a $U(1)_D$ subgroup of unbroken global symmetry $G \to H$ $\quad U(1)_D \supset H$

$$SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$$



Thermalization $\quad \epsilon F^{\mu\nu} F'_{\mu\nu}$

Kinetic mixing

Choose $Q_D(q)$
to eliminate axial anomaly
→ stable π^0

ALP portal

Kamada *et al. (2017)*

Hochberg *et al. (2018)*

$$\mathcal{L}_{aq} = -\frac{1}{2} m_a^2 a^2 - \left(\frac{1}{2} m_Q e^{ia/f_{a\pi}} J^{ij} q_i q_j + \text{h.c.} \right) \rightarrow -\frac{1}{2}\left(m_a^2 a^2 + \frac{2 m_\pi^2 f_\pi^2}{f_{a\pi}^2} \right) a^2 + \frac{m_\pi^2}{8 f_{a\pi}^2} a^2 \text{Tr}\pi^2$$

$$\mathcal{L}_{a\gamma} = \frac{1}{4 f_{a\gamma}} a F^{\mu\nu} \tilde{F}_{\mu\nu}$$

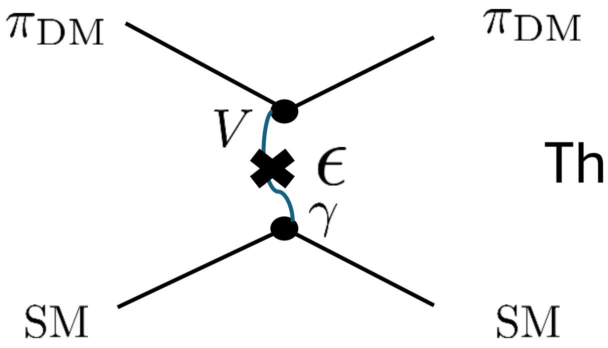
Thermalization
$$a\pi \to a\pi$$
$$a \leftrightarrow \gamma\gamma$$

Portals with the SM

Dark photon portal   Hochberg *et al.* (2015)

Gauging a $U(1)_D$ subgroup of unbroken global symmetry $G \to H$ $U(1)_D \supset H$

$SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$



Thermalization $\epsilon F^{\mu\nu} F'_{\mu\nu}$

Kinetic mixing

Choose $Q_D(q)$
to eliminate axial anomaly
→ stable π^0

Make sure that: thermalization is efficient

$$\pi\pi \to \pi V$$
$$\pi\pi \to V \to e^+e^-$$

are subdominant for DM relic

Allowed by bounds on DP and indirect detection (p-wave)

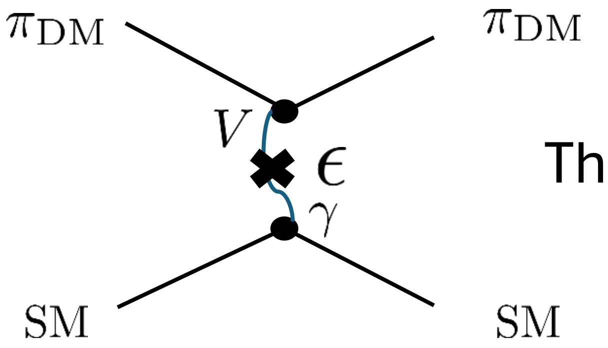
Portals with the SM

Dark photon portal

Hochberg *et al. (2015)*

Gauging a $U(1)_D$ subgroup of unbroken global symmetry $G \to H$ $\quad U(1)_D \supset H$

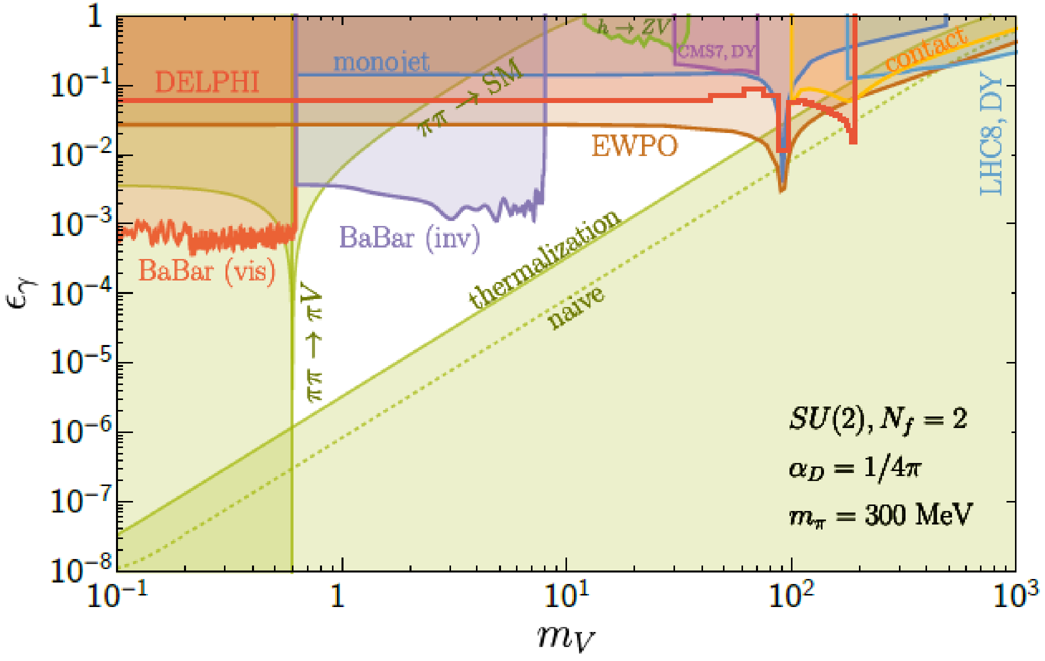
$$SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$$



Thermalization $\quad \epsilon F^{\mu\nu} F'_{\mu\nu}$

Choose $Q_D(q)$
to eliminate axial anomaly
→ stable π^0

Y. Hochberg, E. Kuflik, H. Murayama (2015)



Resonance and θ

Example: $M = (m, m, m, 5m)$

$$\rightarrow \quad \begin{cases} m_\pi^2 = 2B_0 m(1 - 25\theta^2/512) \\ m_\eta^2 = 8B_0 m(1 - 5\theta^2/1024) \end{cases}$$

$$\rightarrow \quad m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi \qquad v_R \sim 0.4\ \theta$$

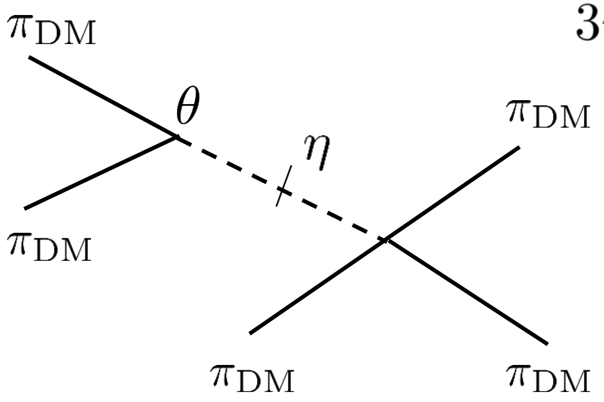
It works for DM relic if $10^{-4} \lesssim \theta \lesssim 0.5$

It works for small scale anomalies if $\theta \sim 10^{-3}$

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

$\theta \neq 0$ → *On-shell* η resonance



$$3\pi \to 2\pi \overset{\rm res}{\simeq} 2\pi \to \eta \dashrightarrow \eta\pi \to 2\pi$$

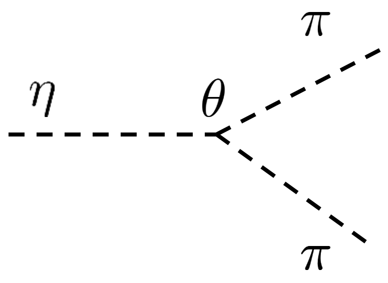
η acts as a **catalyzer**

$$\begin{cases} sHz\dfrac{dY_{\pi_{\rm DM}}}{dz} = +2\gamma_D(\eta \to \pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) + \gamma_2(\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}}\dfrac{Y_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) \\ sHz\dfrac{dY_\eta}{dz} = -\gamma_D(\eta \to \pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) - \gamma_2(\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}}\dfrac{Y_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) \end{cases}$$

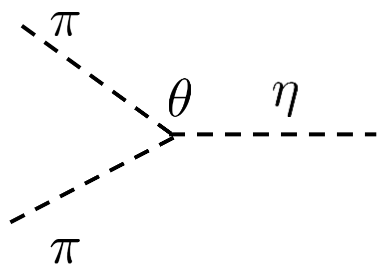
Neglecting the non-resonant piece of 3-to-2 processes

DM relic abundance with $\theta \neq 0$

Condition for chemical equilibrium



$$\Gamma \propto \theta^2$$



$$\langle \Gamma(\eta \leftrightarrow \pi\pi) \rangle > H \quad \longrightarrow \quad \theta > \theta_{\text{min}} \sim 10^{-4}$$

evalutated at freeze-out of $\eta\pi \to 2\pi$

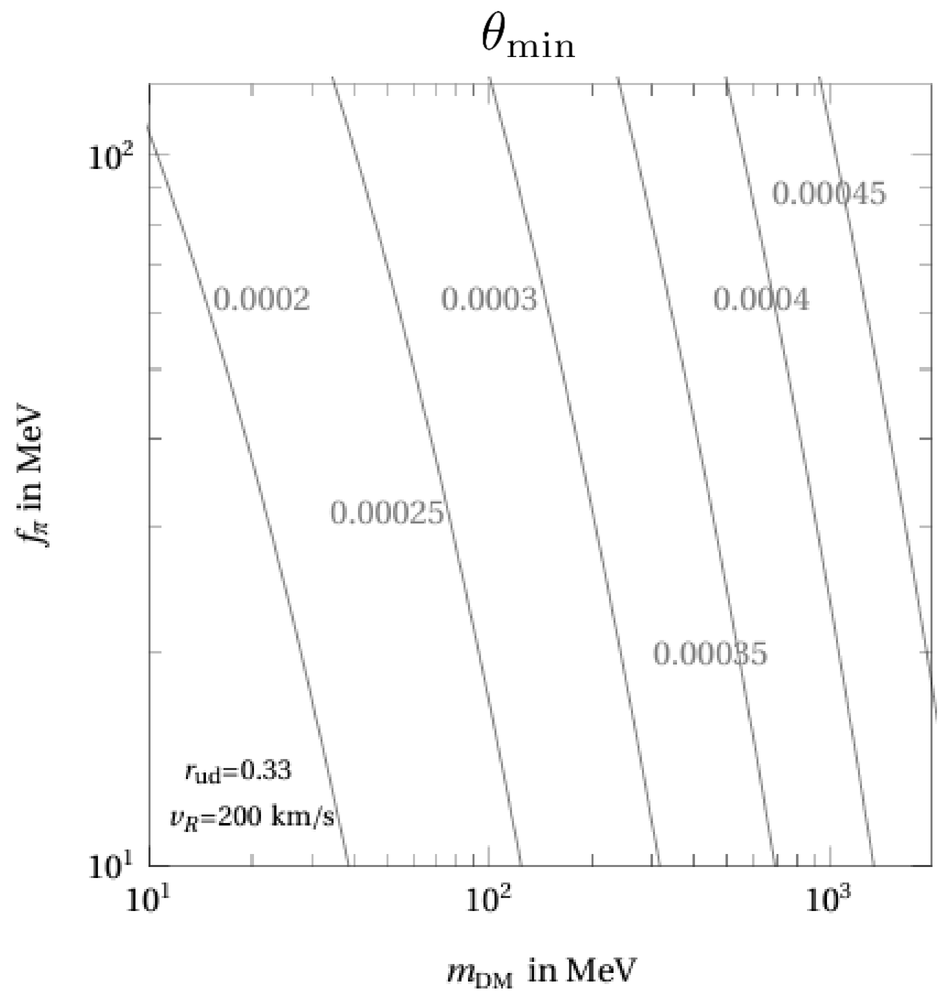
defined as $n_{\eta,\text{eq}}(z_{\text{fo}}) \langle \sigma_{\eta\pi} v \rangle \sim H(z_{\text{fo}})$

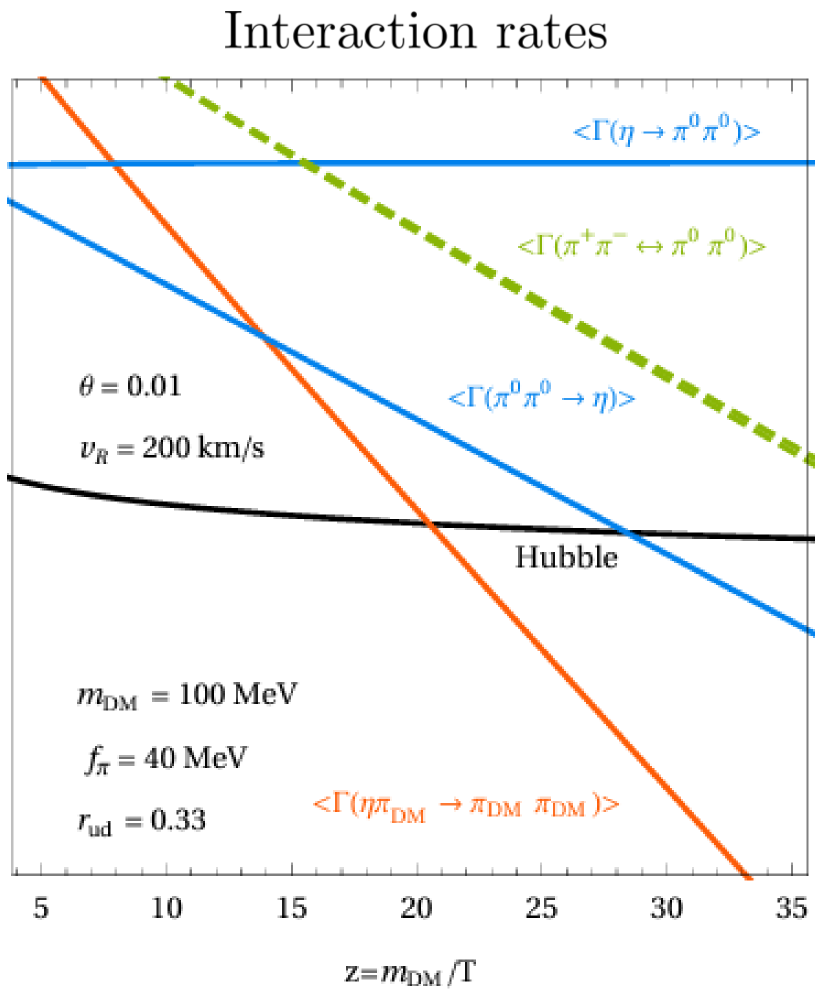
$$\frac{Y_\eta}{Y_{\eta,\text{eq}}} = \frac{Y_\pi^2}{Y_{\pi,\text{eq}}^2}$$

The Boltzmann equations simplify!

DM relic abundance with $\theta \neq 0$

Condition for chemical equilibrium



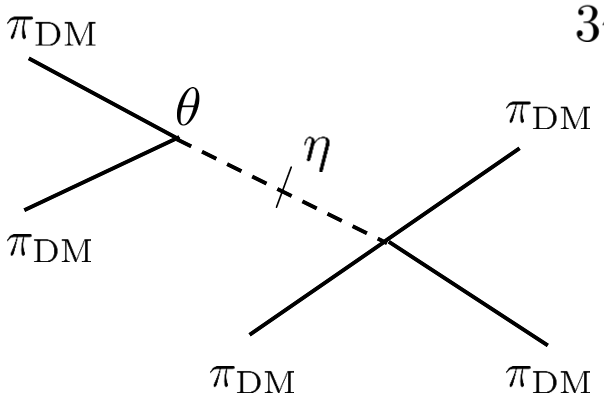


DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

$$3\pi \to 2\pi \overset{\text{res}}{\simeq} 2\pi \to \eta \dashrightarrow \eta\pi \to 2\pi$$

$\theta \neq 0$ ➡ *On-shell* η resonance



if $\theta > \theta_{\rm min} \sim 10^{-4}$

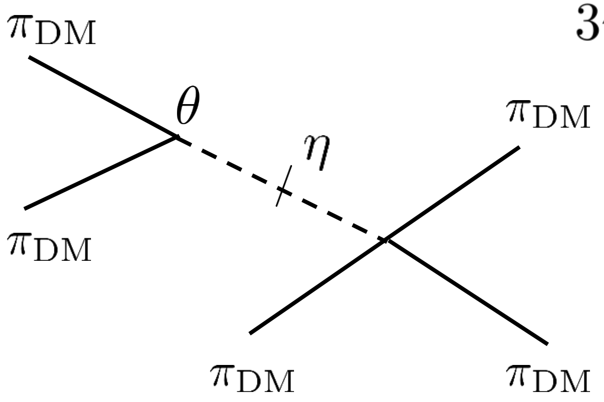
$$\begin{cases} sHz\dfrac{dY_{\pi_{\rm DM}}}{dz} = +2\gamma_D(\eta \to \pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) + \gamma_2(\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}}\dfrac{Y_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) \\ sHz\dfrac{dY_\eta}{dz} = -\gamma_D(\eta \to \pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) - \gamma_2(\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM})\left(\dfrac{Y_\eta}{Y_{\eta,\rm eq}}\dfrac{Y_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}} - \dfrac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}}\right) \end{cases}$$

In this regime the relic abundance is indipendent on θ, v_R

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

$\theta \neq 0$ → *On-shell* η resonance



$3\pi \to 2\pi \overset{\rm res}{\simeq} 2\pi \to \eta$ - - - → $\eta\pi \to 2\pi$

if $\theta > \theta_{\rm min} \sim 10^{-4}$

$$\frac{dY}{dz} = -\langle\sigma_{\eta\pi}v\rangle \frac{sY_{\eta,\rm eq}}{zH}\left(\frac{Y^3_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}} - \frac{Y^2_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}}\right)$$

$$z = m_\pi/T$$

$$\langle\sigma_{\eta\pi}v\rangle \propto m_\pi^2/f_\pi^4$$

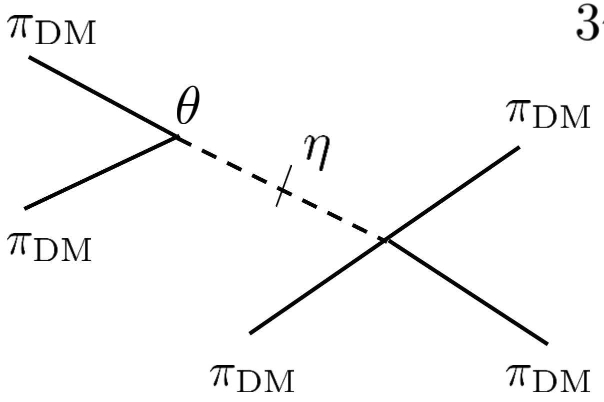
$$Y = Y_{\pi_{\rm DM}} + 2Y_\eta \simeq Y_{\pi_{\rm DM}}$$

In this regime the relic abundance is indipendent on θ, v_R

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

$\theta \neq 0$ ➡ *On-shell* η resonance



$$3\pi \to 2\pi \overset{\rm res}{\simeq} 2\pi \to \eta \dashrightarrow \eta\pi \to 2\pi$$

if $\theta > \theta_{\rm min} \sim 10^{-4}$

The relic is fixed by 2-to-2 annihilations

$$\frac{dY}{dz} = -\langle \sigma_{\eta\pi} v \rangle \frac{s Y_{\eta,\rm eq}}{zH} \left(\frac{Y^3_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}} - \frac{Y^2_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}} \right)$$

$z = m_\pi / T$

$\langle \sigma_{\eta\pi} v \rangle \propto m_\pi^2 / f_\pi^4$

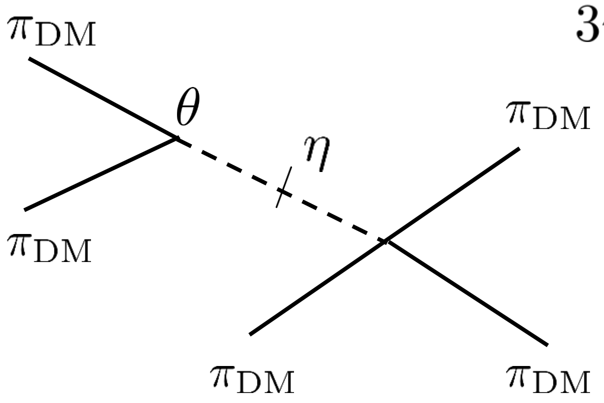
$$Y = Y_{\pi_{\rm DM}} + 2Y_\eta \simeq Y_{\pi_{\rm DM}}$$

In this regime the relic abundance is indipendent on θ, v_R

DM relic abundance with $\theta \neq 0$

Resonant 3-to-2 processes

$\theta \neq 0$ ➡ *On-shell* η resonance



$3\pi \to 2\pi \overset{\rm res}{\simeq} 2\pi \to \eta$ - - - -▸ $\eta\pi \to 2\pi$

if $\theta > \theta_{\rm min} \sim 10^{-4}$

$$\frac{dY}{dz} = -\langle\sigma_{\eta\pi}v\rangle \frac{sY_{\eta,\rm eq}}{zH}\left(\frac{Y^3_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}} - \frac{Y^2_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}}\right)$$

$z = m_\pi/T$

$\langle\sigma_{\eta\pi}v\rangle \propto m_\pi^2/f_\pi^4$

$$Y = Y_{\pi_{\rm DM}} + 2Y_\eta \simeq Y_{\pi_{\rm DM}}$$

We can integrate the Boltzmann Equation (both analytically and numerically)

$Y_{\pi_{\rm DM}} \simeq Y_{\pi_{\rm DM},\rm eq}(z_{\rm fo})$ defined as $n_{\eta,\rm eq}(z_{\rm fo})\langle\sigma_{\eta\pi}v\rangle \sim H(z_{\rm fo})$

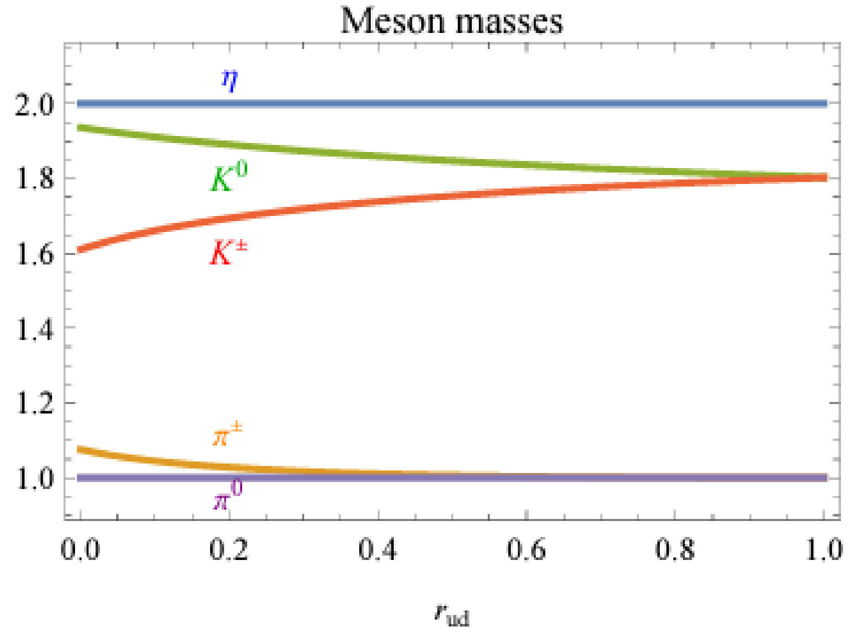
Explicit benchmark model

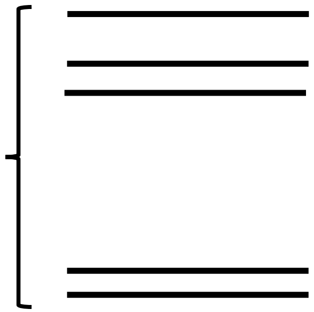
$M = (m_u, m_d, m_s)$ $\qquad r_{ud} = m_u/m_d$

$0 \leq r_{ud} \leq 1$

m_u/m_s fixed as a function of (r_{ud}, v_R) so that $m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi$

with $v_R \lesssim 0.1$





π^a

$m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_{\pi^0}$

m_K $\quad n_K \ll n_\pi \sim n_{\rm DM}$ always negligible

$m_{\pi^\pm}$

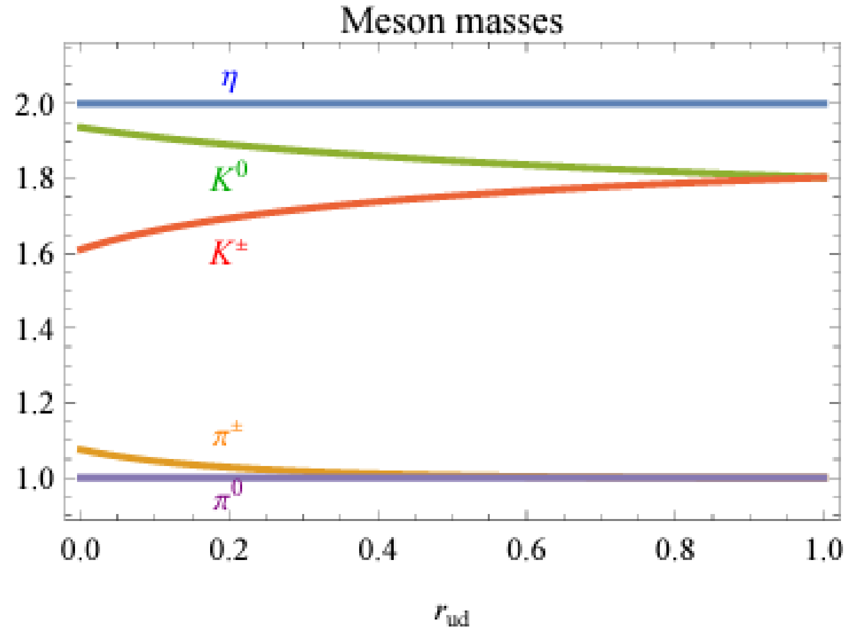
m_{π^0}

$\delta(r_{ud}) = m_{\pi^\pm}/m_{\pi^0} - 1$ with $0 \lesssim \delta(r_{ud}) \lesssim 0.075$

Explicit benchmark model

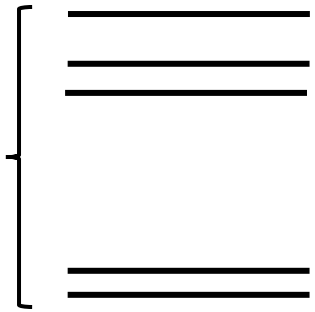
$M = (m_u, m_d, m_s)$ $\qquad r_{ud} = m_u/m_d$

$0 \leq r_{ud} \leq 1$



m_u/m_s fixed as a function of (r_{ud}, v_R) so that $m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi$

with $v_R \lesssim 0.1$



$m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_{\pi^0}$

m_K

π^a

$m_{\pi^\pm}$ Contribute to DM in the Early Universe but negligible today $\pi^+\pi^- \to \pi^0\pi^0$

m_{π^0}

$\delta(r_{ud}) = m_{\pi^\pm}/m_{\pi^0} - 1$ with $0 \lesssim \delta(r_{ud}) \lesssim 0.075$

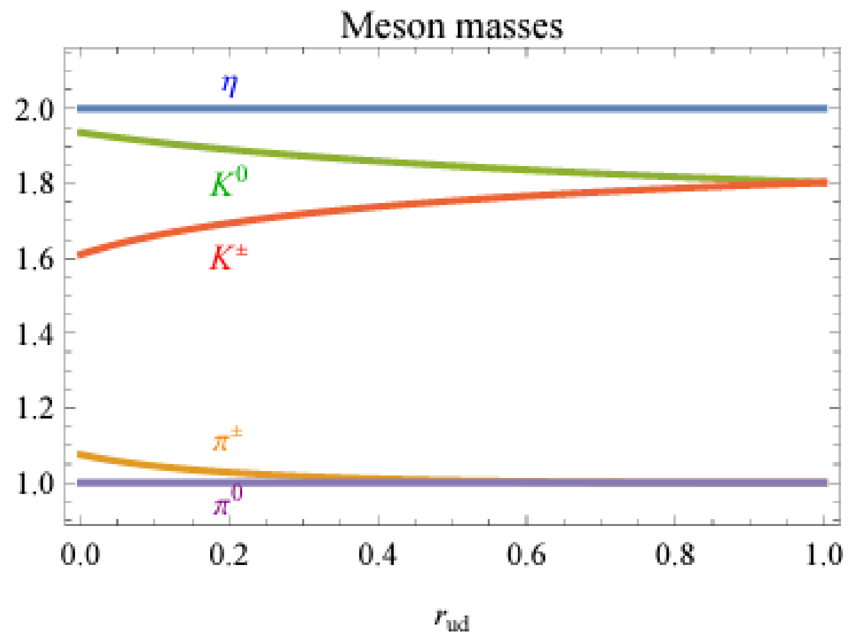
Explicit benchmark model

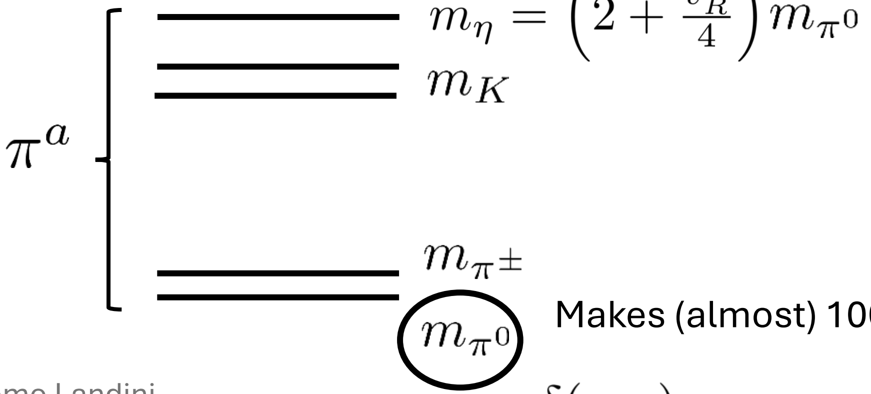
$$M = (m_u, m_d, m_s) \qquad r_{ud} = m_u/m_d$$

$$0 \leq r_{ud} \leq 1$$

m_u/m_s fixed as a function of (r_{ud}, v_R) so that $m_\eta = \left(2 + \frac{v_R^2}{4}\right) m_\pi$

with $v_R \lesssim 0.1$



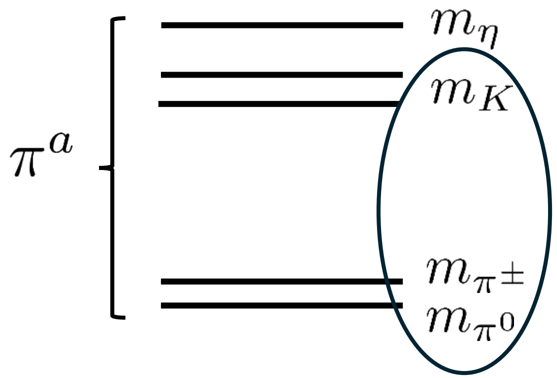


Makes (almost) 100% of DM today

$\delta(r_{ud}) = m_{\pi^\pm}/m_{\pi^0} - 1$ with $0 \lesssim \delta(r_{ud}) \lesssim 0.075$

Co-annihilations

$$\text{Stable states} = \{\pi^0, \pi^\pm, K\}$$



Stable co-annihilating partners

$$\begin{cases} KK \leftrightarrow \pi\pi \\ \pi^+\pi^- \leftrightarrow \pi^0\pi^0 \end{cases}$$

Keep the species in chemical equilibrium

$$Y_i/Y_{\pi^0} \sim \exp[-(m_i - m_{\pi^0})z/m_{\pi^0}] \qquad i = \pi^\pm, K$$

$m_K \sim 1.7 m_{\pi^0}$ → $z_{\text{fo}} \sim 20$ → $Y_K/Y_{\pi^0} \overset{z_{\text{fo}}}{\approx} 10^{-6}$ Negligible contribution to DM relic

$m_{\pi^\pm} \lesssim 1.075 m_{\pi^0}$ → $z_{\text{fo}} \sim 20$ → $Y_{\pi^\pm}/Y_{\pi^0} \overset{z_{\text{fo}}}{\approx} 0.5$ Sizeable contribution to DM relic

$\pi^+\pi^- \xrightarrow{z \gg z_{\text{fo}}} \pi^0\pi^0$ → Negligible amount of $\pi^\pm$ today → 100% of DM $\approx \pi^0$

Boltzmann Equation benchmark model

$$\text{Stable states} = \{\pi^0, \pi^\pm, \not{K}\}$$

Negligible

$\pi^+\pi^- \to \pi^0\pi^0$ → Chemical equilibrium → $Y_{\pi_{\rm DM}} = Y_{\pi^0} + 2Y_{\pi^\pm}$

$$\dot{n} \; + \; 3Hn = -\left(n_\eta n_{\pi_{\rm DM}} \langle \sigma_{\eta\pi} v\rangle - n^2_{\pi_{\rm DM}} \langle \sigma_{\pi\pi} v\rangle\right) \qquad n = n_{\pi_{\rm DM}} + 2n_\eta$$

$z = m_\pi / T$

$Y = n/s$

Detailed balance $\quad n_\eta^{\rm eq} \langle \sigma_{\eta\pi} v\rangle = n^{\rm eq}_{\pi_{\rm DM}} \langle \sigma_{\pi\pi} v\rangle$

$+$

Chemical equilibrium $\quad n_\eta / n^2_{\pi_{\rm DM}} = (n_\eta / n^2_{\pi_{\rm DM}})_{\rm eq}$

$\eta \leftrightarrow \pi^0\pi^0$

→

$$\frac{dY}{dz} = -\langle \sigma_{\eta\pi} v\rangle \frac{sY_{\eta,\rm eq}}{zH} \left(\frac{Y^3_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},\rm eq}} - \frac{Y^2_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},\rm eq}}\right)$$

Boltzmann Equation benchmark model

All $\eta\pi \to \pi\pi$ involving the different pion species must be taken into account

$$\langle\sigma(\eta\pi^0 \to \pi_{1,2}\pi_{1,2})v\rangle = \frac{529\sqrt{5}\,m_{\pi^0}^2}{5184\pi f_\pi^4}\delta, \quad \langle\sigma(\eta\pi_{1,2} \to \pi^0\pi_{1,2})v\rangle = \frac{49\sqrt{5}\,m_{\pi^0}^2}{2592\pi f_\pi^4}\delta, \quad \langle\sigma(\eta\pi^0 \to \pi^0\pi^0)v\rangle = \frac{\sqrt{5}\,m_{\pi^0}^2}{64\pi f_\pi^4}\delta$$

$$\pi^\pm = \frac{\pi_1 \pm i\pi_2}{\sqrt{2}}$$

$$\langle\sigma_{\eta\pi}v\rangle \simeq \frac{1}{3}\Big(\langle\sigma(\eta\pi^0 \to \pi^0\pi^0)v\rangle + 2\langle\sigma(\eta\pi^0 \to \pi_1\pi_1)v\rangle + 2\langle\sigma(\eta\pi_1 \to \pi^0\pi_1)v\rangle\Big)$$

$$\delta(r_{ud}) = m_{\pi^\pm}/m_{\pi^0} - 1$$

$$\langle\sigma_{\eta\pi}v\rangle = \frac{445\sqrt{5}m_{\pi^0}^2\,\delta(r_{ud})}{5184\pi f_\pi^4}$$

DM self-interactions in halos

$\pi^0\pi^0 \to \pi^0\pi^0$

$$\sigma(v) = \sigma_0 + \frac{128\pi}{m_\pi^2 v_R^2} \frac{\Gamma^2}{m_\pi^2 (v^2 - v_R^2)^2 + 4\Gamma^2 v^2 / v_R^2}$$

$$\sigma_0 = \frac{m_\pi^2}{128\pi f_\pi^4}$$

$\pi^+\pi^- \to \pi^0\pi^0$

Efficient conversions in the Early Universe deplete the $\pi^+\pi^-$ population

Negligible amount of $\pi^\pm$ today in halos

$\pi^0\pi^0 \to \pi^+\pi^-$

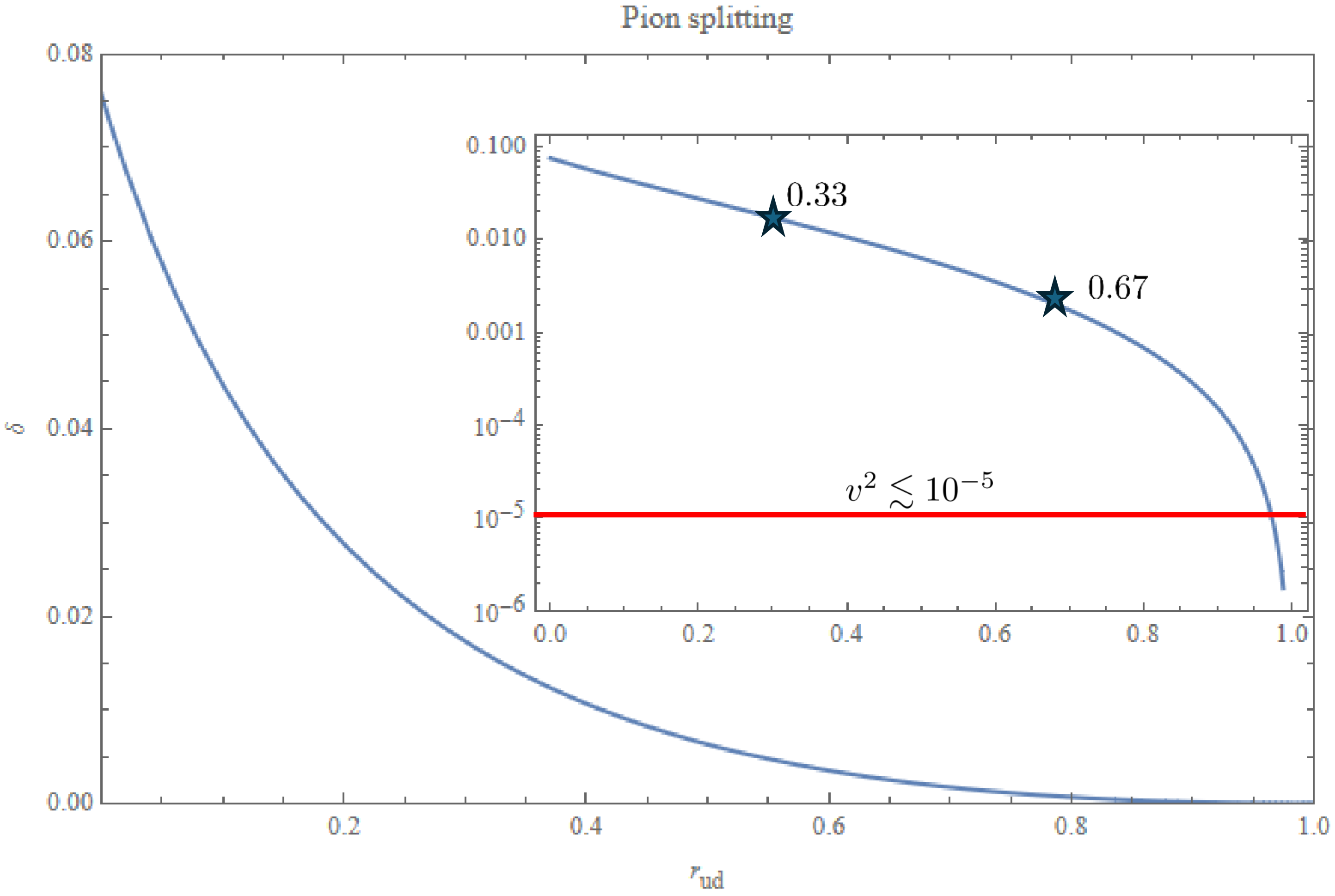
Up-scatterings are kinematically forbidden as $\delta \gg v^2$

$v \lesssim 0.0033 \longrightarrow v^2 \lesssim 10^{-5}$

DM velocity in clusters

$\delta \gtrsim 10^{-5}$ in (almost) all the parameter space (plot)

DM self-interactions in halos



Outlook

Elaborate

- Gravitational Waves signal?

Chiral Phase Transition is first-order if $N_f \geq 3$

The PT critical temperature is $T_* \sim f_\pi \sim \mathcal{O}(10 - 100)\ \text{MeV}$

It could be relevant in view of **PTA signal!**

NANOGrav collaboration 2023

Need to introduce extra d.o.f. to study PT dynamics (e.g. Linear sigma model)

A value $\theta \neq 0$ may deeply alter the PT properties!

SM QCD PT becomes first-order when $\theta \sim \pi$ *Bai, Chen, Korwar (2023)*

Outlook

Elaborate

- Gravitational Waves signal? PTA? $\theta \neq 0$?

- $\theta \neq 0$ is a new source of CP-violation

  observables? useful for baryogenesis?

DM relic abundance with $\theta \neq 0$

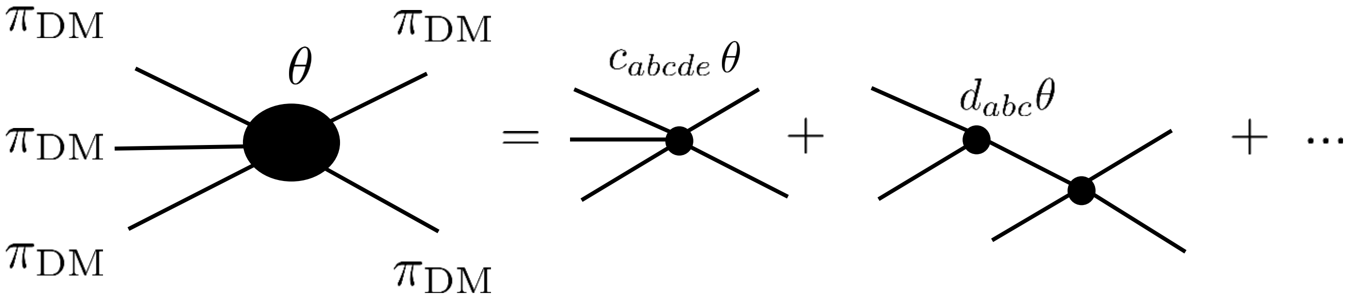
Degenerate quark spectrum gives degenerate pions

$$M = \begin{pmatrix} m & & \\ & \ddots & \\ & & m \end{pmatrix}$$

$$m_\pi^2 = 2B_0 m$$

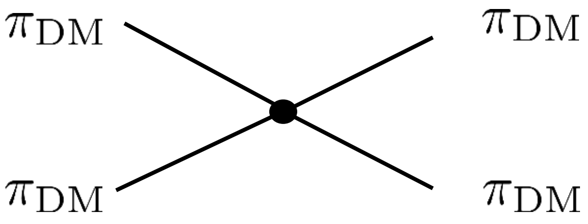
$$\mathcal{L}_\theta = \frac{B_0\theta}{3f_\pi TrM^{-1}}\left(d_{abc}\pi_a\pi_b\pi_c - \frac{c_{abcde}}{10f_\pi^2}\pi_a\pi_b\pi_c\pi_d\pi_e\right)$$

DM number changing processes



SIMP $\langle\sigma_{32}v^2\rangle \propto \theta^2 \frac{m_\pi}{f_\pi^6}$

DM self-interactions



$\sigma/m_\pi \propto \frac{m_\pi}{f_\pi^4}$ constant

indipendent on θ

DM relic abundance with $\theta \neq 0$

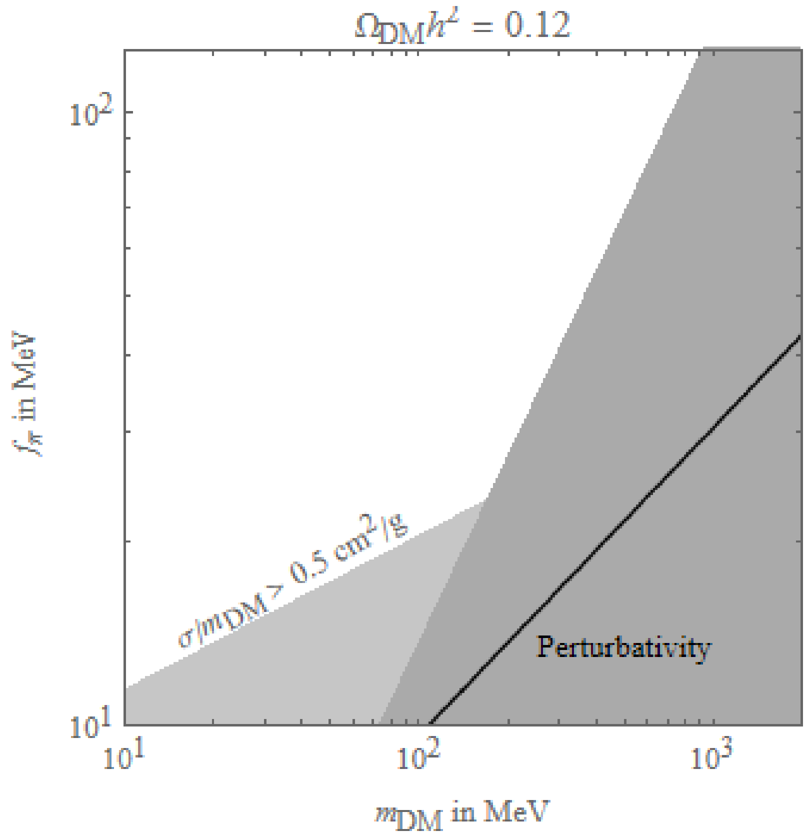
Tension among DM relic and Bullet Cluster bound

Tension among DM relic and perturbativity

The self-interactions cross section is constant

$$\sigma / m_\pi \propto \frac{m_\pi}{f_\pi^4}$$

No SIDM realization



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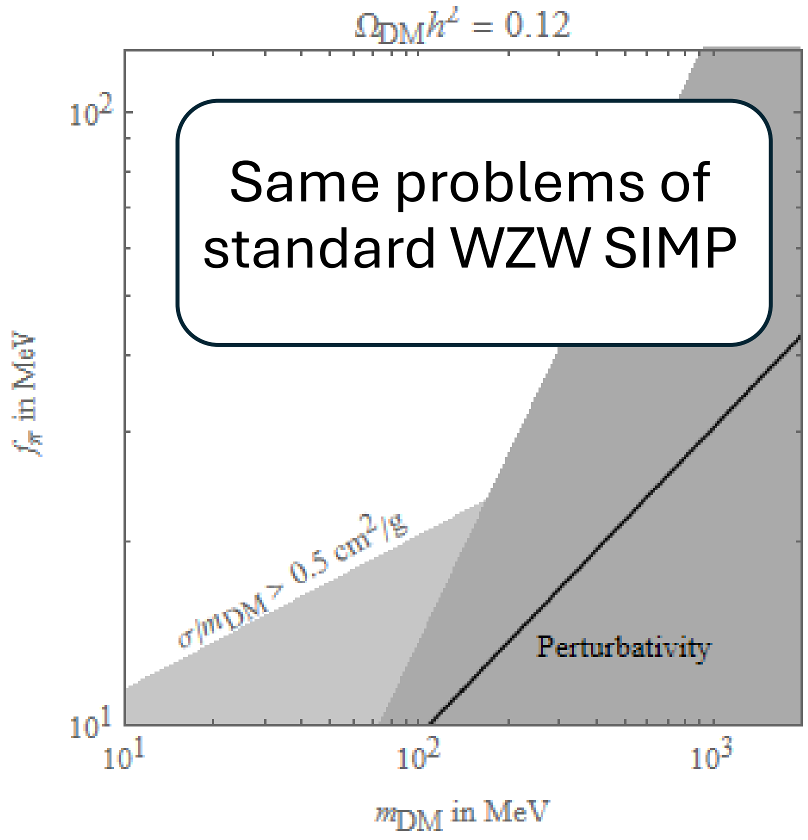
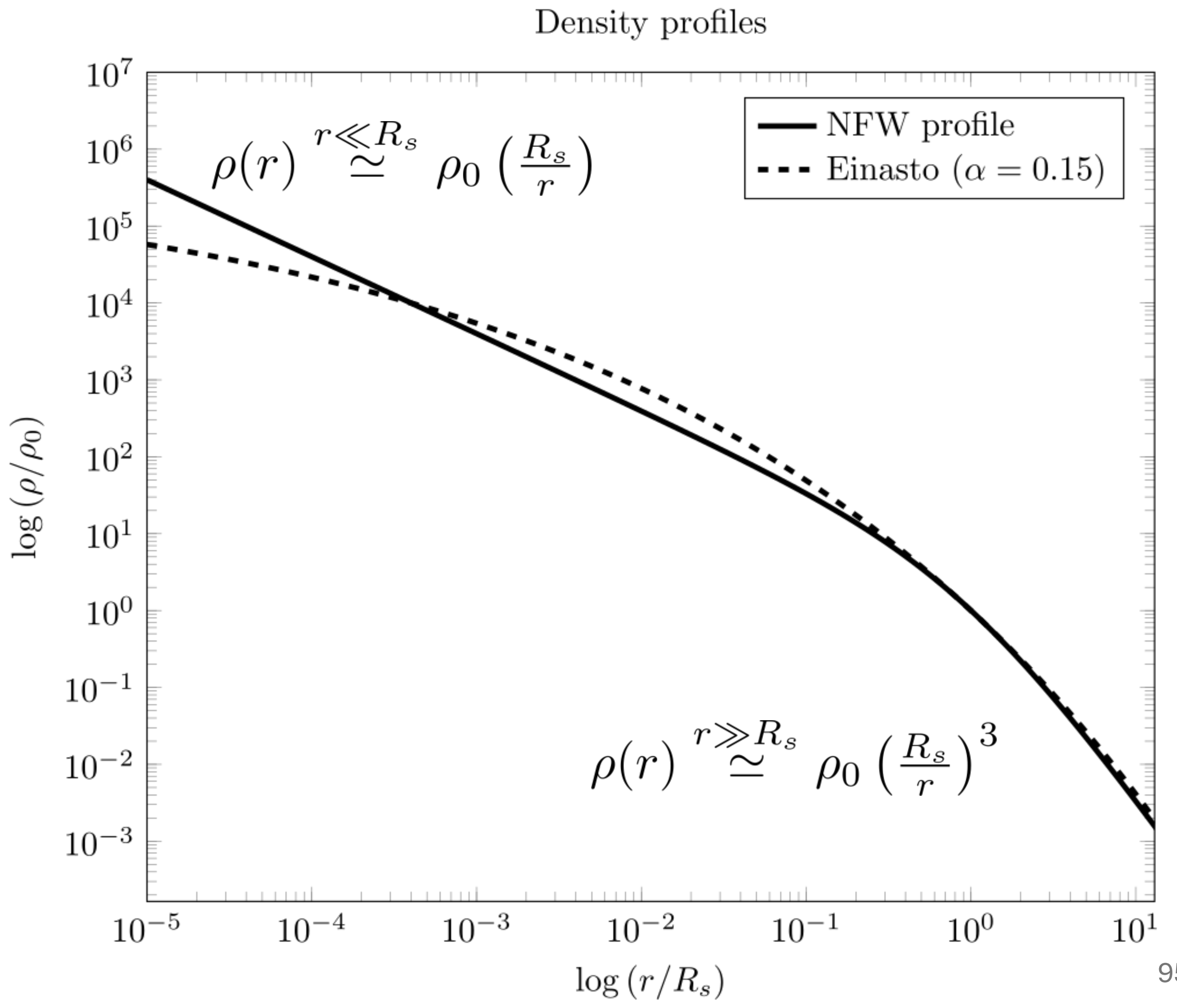


NFW profile

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$



Dark Baryons

Stable because of accidental global $U(1)$

$$m_{\mathcal{B}} \simeq \Lambda$$

$$\overline{\mathcal{B}}\mathcal{B} \to \pi\pi \quad \longrightarrow \quad Y_{\mathcal{B}} \sim \frac{\Lambda}{M_{\rm Pl}}$$

$$\downarrow$$

$$\frac{\Omega_{\mathcal{B}} h^2}{0.11} \simeq \left(\frac{\Lambda}{100\text{TeV}}\right)^2 \ll 1$$

$$\Lambda \sim 4\pi f_\pi/\sqrt{N_c} \sim \mathcal{O}(100 \text{ MeV} - 1 \text{ GeV})$$

Chiral rotation

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{q}i\not{D}q - (\bar{q}_L M q_R + h.c.) + \frac{g^2\theta}{32\pi^2}F\widetilde{F}$$

$$q_{L,R} \to e^{\mp i\theta Q/2}q_{L,R}$$

anomalous rotation →

$$\theta \to (1 - TrQ)\theta$$

$$M \to M_\theta = e^{i\theta Q/2}Me^{i\theta Q/2}$$

Choosing $TrQ = 1$ → Remove $F\widetilde{F}$

Choosing $Q = M^{-1}/TrM^{-1}$ → no linear terms in π in the chiral Lagrangian

Chiral rotation

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{q}i\not{D}q - (\bar{q}_L M q_R + h.c.) + \frac{g^2\theta}{32\pi^2}F\widetilde{F}$$

More generically one can start from

$$-(\bar{q}_L \underbrace{Me^{i\theta_M}}_{\mathcal{M}} q_R + h.c) + \frac{g^2\theta_F}{32\pi^2}F\widetilde{F}$$

anomalous rotation

$$q_{L,R} \to e^{\mp i\alpha} q_{L,R} \longrightarrow \begin{array}{l} \theta_F \to \theta_F - \alpha N_f \\ \theta_M \to \theta_M + \alpha \end{array}$$

$$\theta \equiv \theta_F + \arg\det\mathcal{M} = \theta_F + N_f\theta_M$$ Invariant

Physical quantity (if all quarks are massive)

$$\det\mathcal{M} \neq 0$$

Mass spectrum benchmark

$$SU(3)_L \otimes SU(3)_R \xrightarrow{8\pi_a} SU(3)_V$$

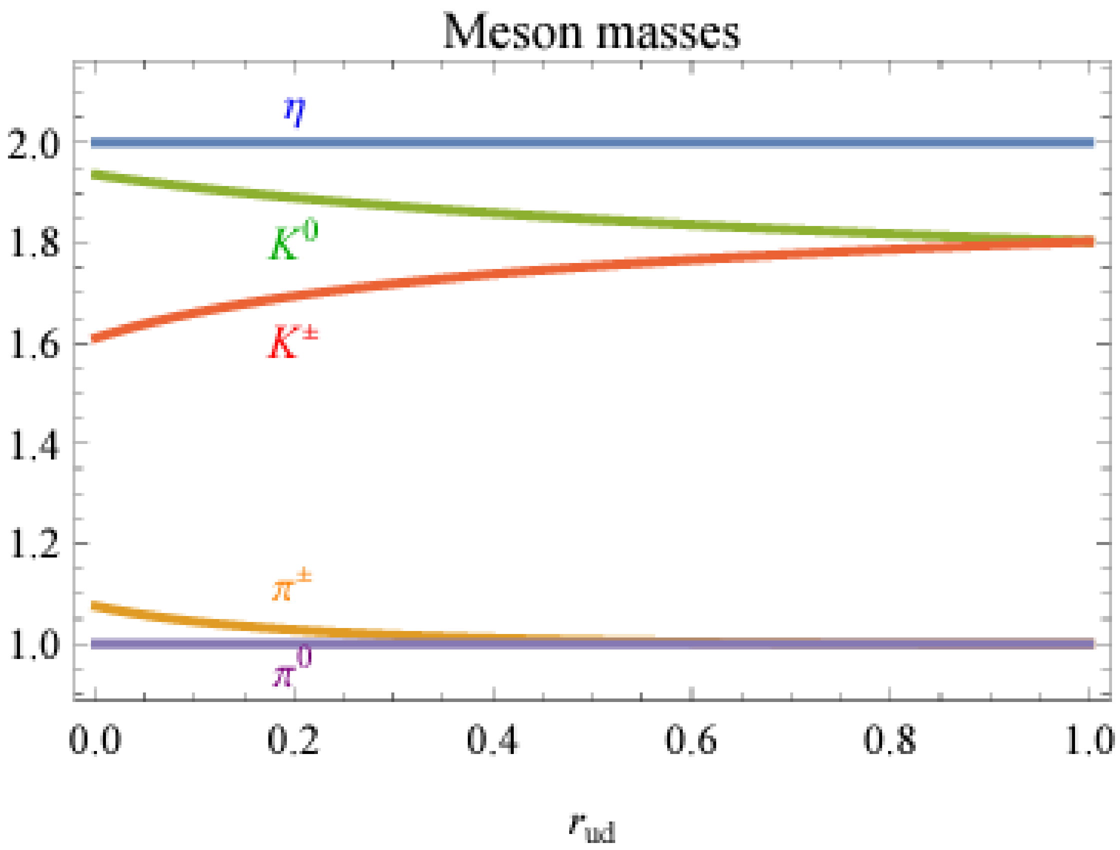
$$\pi^\pm = (\pi_1 \pm \pi_2)/\sqrt{2}\ ,\ K^\pm = (\pi_4 \pm i\pi_5)/\sqrt{2},\ K^0/\bar{K}^0 = (\pi_6 \pm i\pi_7)/\sqrt{2}$$

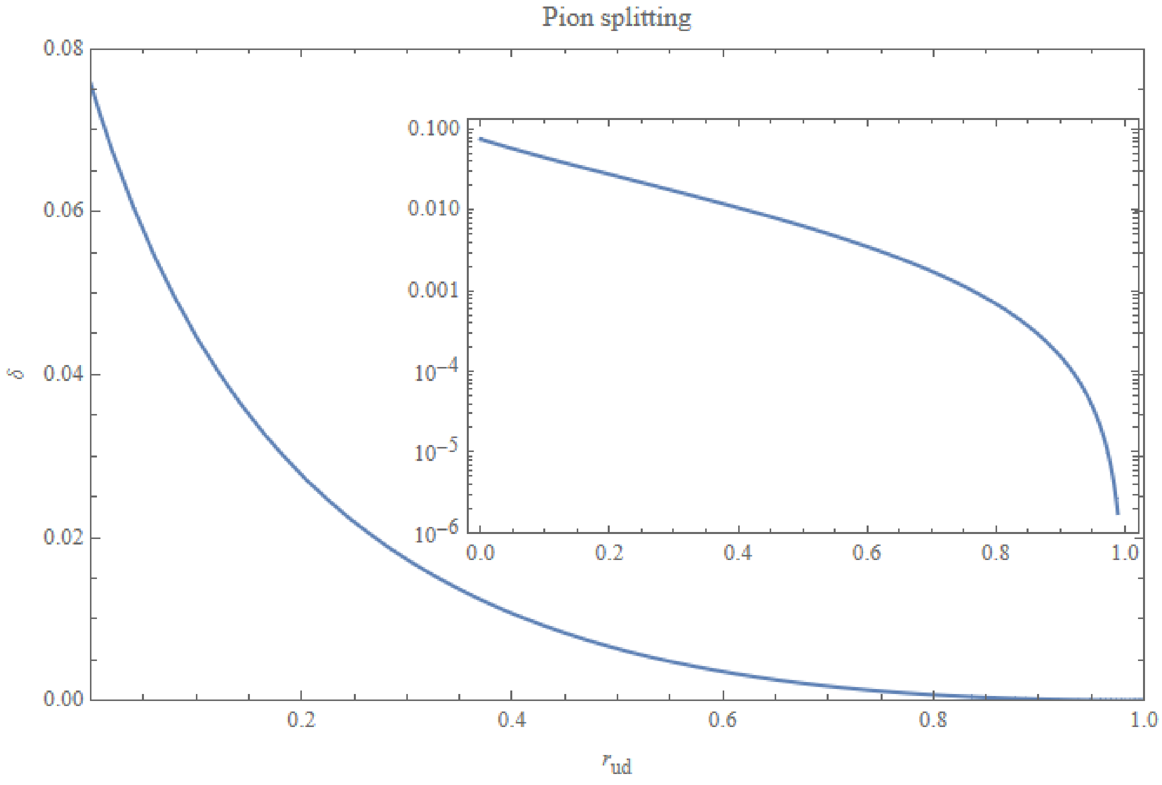
$$\begin{pmatrix} \pi^0 \\ \eta \end{pmatrix} = \begin{pmatrix} \cos\theta_{\eta\pi} & \sin\theta_{\eta\pi} \\ -\sin\theta_{\eta\pi} & \cos\theta_{\eta\pi} \end{pmatrix} \begin{pmatrix} \pi_3 \\ \pi_8 \end{pmatrix}, \qquad \text{with} \qquad \tan(2\theta_{\eta\pi}) = \frac{\sqrt{3}(m_u - m_d)}{(m_u + m_d - 2m_s)}. \tag{B1}$$

The masses squared of the mesons are $m^2_{\pi^\pm} = B_0(m_u + m_d)$, $m^2_{K^\pm} = B_0(m_u + m_s)$, $m^2_{K,\bar{K}^0} = B_0(m_d + m_s)$, while $m^2_{\pi^0}$ and m^2_η are the eigenvalues of

$$\mathcal{M}^2_{\pi^0,\eta} = \begin{pmatrix} B_0(m_u + m_d) & B_0(m_u - m_d)/\sqrt{3} \\ B_0/(m_u - m_d)/\sqrt{3} & B_0(m_u + m_d + 4m_s)/3 \end{pmatrix}. \tag{B2}$$

Mass spectrum benchmark





Cubic interactions

$$\mathcal{L}^{(\mathrm{BM1})}_{\eta\pi\pi} = \frac{B_0\theta}{\sqrt{3} f_\pi \mathrm{Tr} M^{-1}} \cos(3\theta_{\eta\pi}) \eta\pi^0\pi^0$$

$$\tan(2\theta_{\eta\pi}) = \frac{\sqrt{3}(m_u - m_d)}{(m_u + m_d - 2m_s)}.$$

$$\Gamma\left(\eta \to \mathrm{DM}\,\mathrm{DM}\right) = \frac{\theta^2 B_0^2 \xi}{24\pi f_\pi^2 m_\eta (\mathrm{Tr} M^{-1})^2} \sqrt{1 - \frac{4m_{\mathrm{DM}}^2}{m_\eta^2}}$$

$$\xi = \cos^2 3\theta_{\eta\pi}$$

Details of Symmetry Breaking

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{q}i\not{D}q - (\bar{q}_L M q_R + h.c.) + \frac{g^2\theta}{32\pi^2}F\widetilde{F}$$

$U(1)_V \qquad q_{L,R} \to e^{i\alpha} q_{L,R}$

$M \to 0$ →

$SU(N_f)_L \qquad q_L \to e^{i\alpha_L^a \lambda^a} q_L$

$SU(N_f)_R \qquad q_R \to e^{i\alpha_R^a \lambda^a} q_R$

$\langle \bar{q}_L q_R \rangle$ →

$SU(N_f)_V$

$\alpha_L^a = \alpha_R^a$

$U(1)_A \qquad q_{L,R} \to e^{\mp i\alpha} q_{L,R}$

Anomalous! → $N_f \alpha F\widetilde{F}$

Resonances in QCD

$$\frac{m(^8\mathrm{Be}) - 2m(\alpha)}{m(^8\mathrm{Be})} = 0.000012, \qquad \frac{m(^{12}\mathrm{C}^*) - m(^8\mathrm{Be}) - m(\alpha)}{m(^{12}\mathrm{C}^*)} = 0.000026.$$

$$\alpha\alpha \to {}^8\mathrm{Be} \quad \text{followed by} \quad {}^8\mathrm{Be}\,\alpha \to {}^{12}\mathrm{C}^*$$

Important process in stars

Similar to our resonant 3-to-2 processes

Other examples:

$$\frac{m(\phi) - 2m(K^0)}{m(\phi)} = 0.024, \qquad \frac{m(B_{s1}) - m(B^*) - m(K^0)}{m(B_{s1})} = 0.0011,$$

$$\frac{m(D^{0*}) - m(D^0) - m(\pi^0)}{m(D^{0*})} = 0.0035, \qquad \frac{m(\Upsilon(4S)) - 2m(B^0)}{m(\Upsilon(4S))} = 0.0019.$$

Instantons

$$\int d^4x F^a_{\mu\nu}\widetilde{F}^{\mu\nu a} = \int d^4x \partial_\mu K^\mu = \int_{S_3} d\sigma_\mu K^\mu$$

Total derivative

Instantons are (pure-gauge) field configurations which satisfies

$$\int d^4x F^a_{\mu\nu}\widetilde{F}^{\mu\nu a} = \frac{32\pi^2}{g^2}\nu$$

→ Integer (winding number)

Classical solution to (Euclidean) e.o.m.

Tunnelling among gauge configurations with different winding numbers

Instantons

$$\int d^4x F^a_{\mu\nu} \widetilde{F}^{\mu\nu a} = \int d^4x \partial_\mu K^\mu = \int_{S_3} d\sigma_\mu K^\mu$$

Total derivative

Theta vacuum

$$|\theta\rangle = \sum_{n=-\infty}^{+\infty} e^{in\theta} |n\rangle \,,$$

↳ Vacuum with winding number n

$$\langle \theta_+ | \theta_- \rangle_J = \sum_\nu \int \mathcal{D}A \, e^{-\int d^4x \, \frac{1}{4} G\tilde{G} + i\theta \frac{g_s^2}{32\pi^2} \int d^4x \, G\tilde{G} + J\text{-term}} \delta\left(\nu - \frac{g_s^2}{32\pi^2} \int d^4x \, G\tilde{G}\right)$$

Feedback

Q: PORTAL TO SM (ok from backup)

Q: why kaons are not relevant (ok backup)

Q: how changing the value of theta change the self-scattering plot (too large theta gives too much scatterings? **Comment on this. Maybe underline that a smaller value of theta (still > theta_min) is required and interestingly could explain the small vR of similar size!**

Obs: a bit confusing calling first all particles pions and then differentiate among pions and eta (maybe find a better notation)

Obs: Refs to observations of dwarfs (some more refs in general!) (how do they measure DM velocities?)(change a bit SIDM slides?)

Obs: Underline theta is crucial (no resonant even in presence of a resonance for theta = 0)