

BSM with cosmological neutrinos

Invisibles Workshop 2024

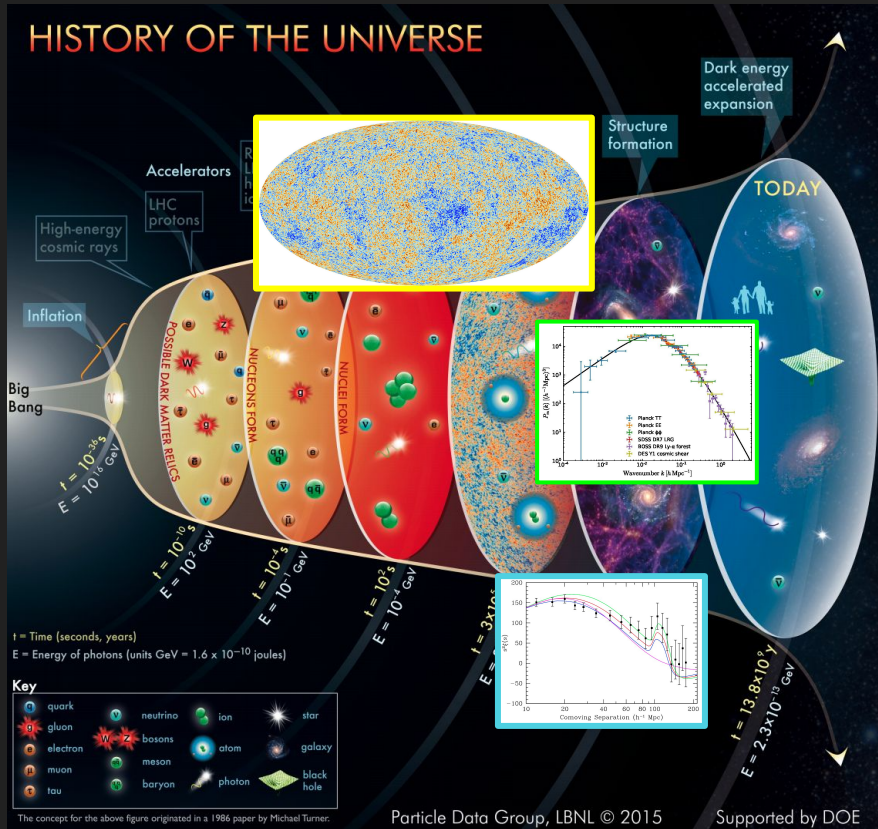
Bologna



Jordi Salvado



Let's use the whole universe



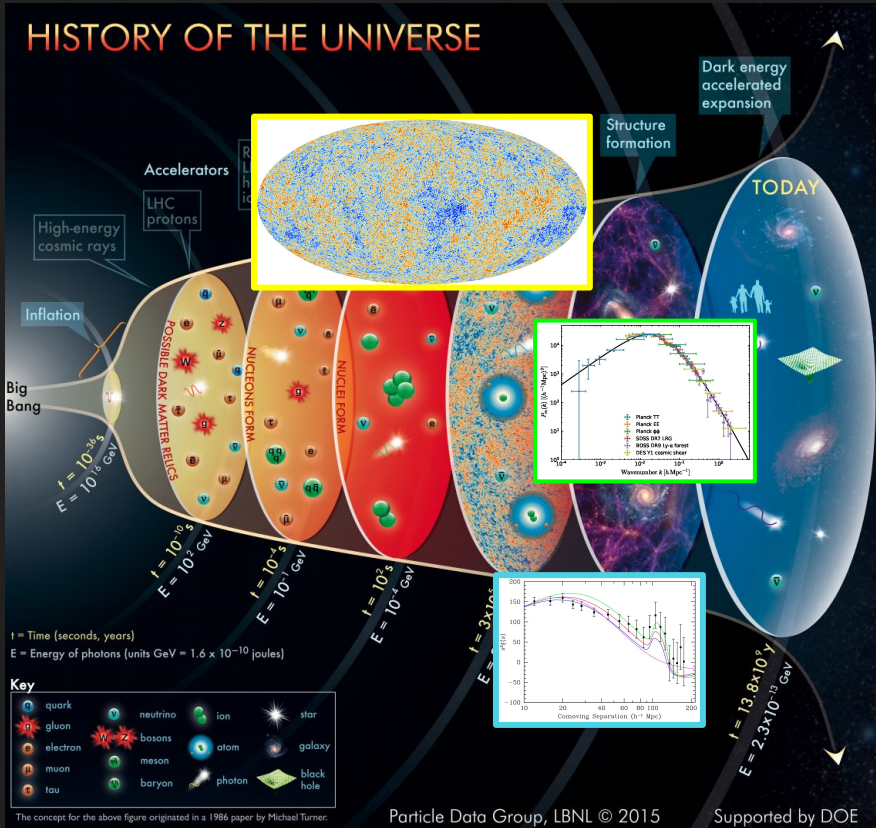
- What do we look at:

CMB data is one of the most powerful observations **Planck-2018**

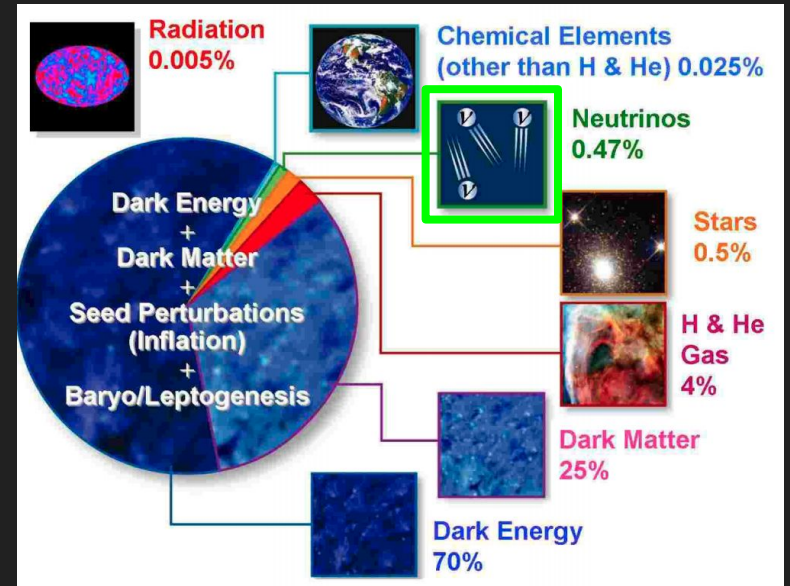
The **matter spectrum**, fourier transform of the two point correlation function.

Imprint of the CMB acoustic oscillations in the large scale structure of the universe **BAO**. Late expansion history test.

Let's use the whole universe



- (Today) The ingredients are:



Cosmology and the Unexpected

Edward W. Kolb (Chicago U., Astron. Astrophys. Ctr. and Chicago U., EFI and Chicago U., KICP)

BSM with Cosmological Neutrinos

- **The Mass**

Affects different scales.

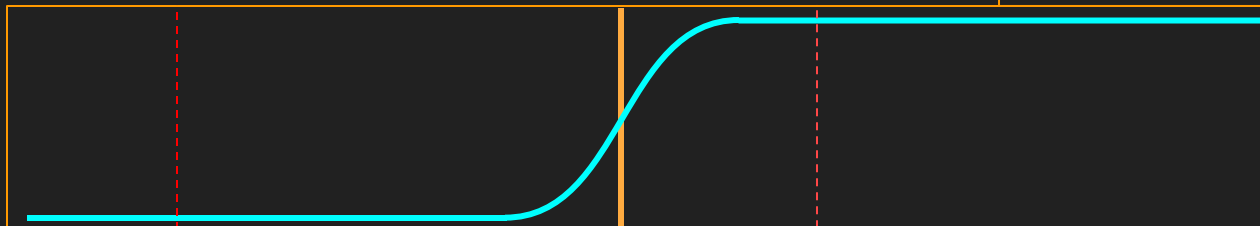
- Neutrino-Neutrino Interactions

Mass Scale: Background Effect.

Equation of State

$w=1/3$

$w=0$



m_ψ

T_ψ

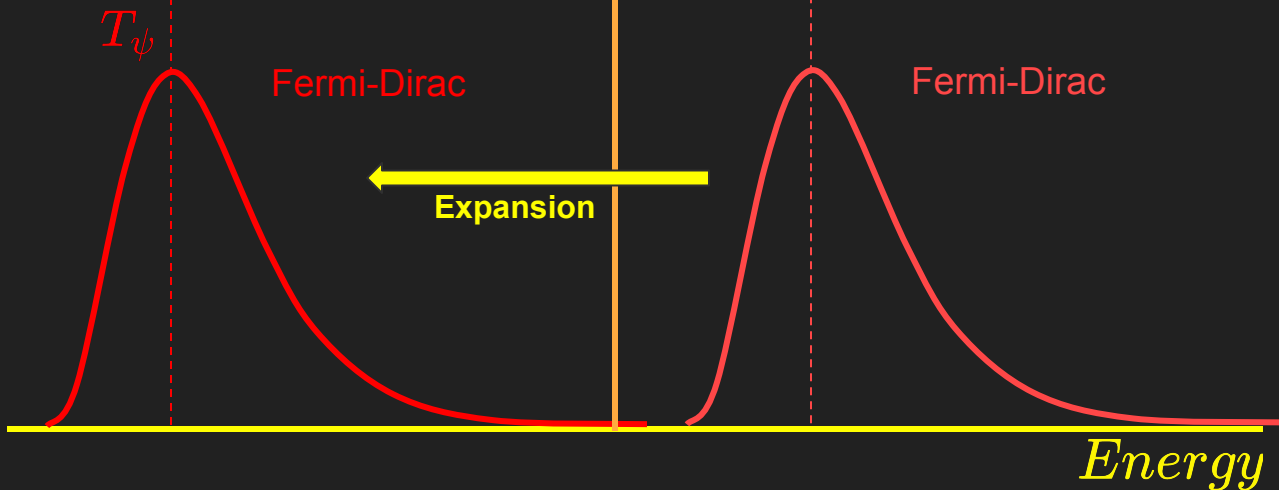
Fermi-Dirac

Fermi-Dirac

Expansion

Today, the best bound for the value of neutrino masses comes from a background effect.

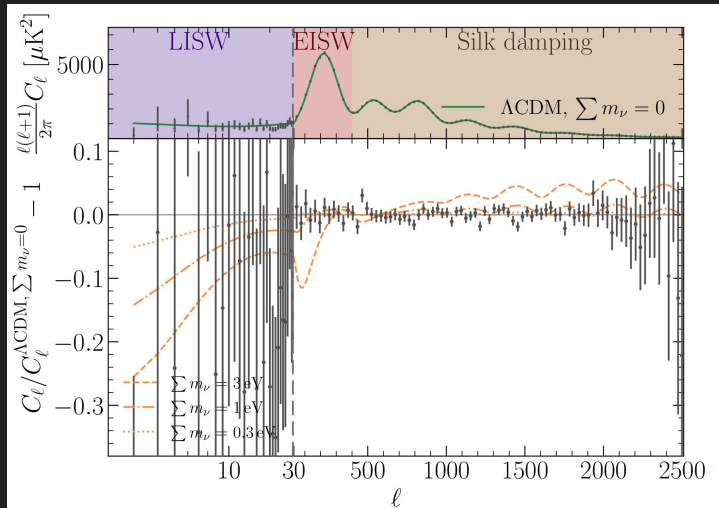
Energy



Mass Scale: Background Effect.

CMB is very sensitive to the expansion history.

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho$$

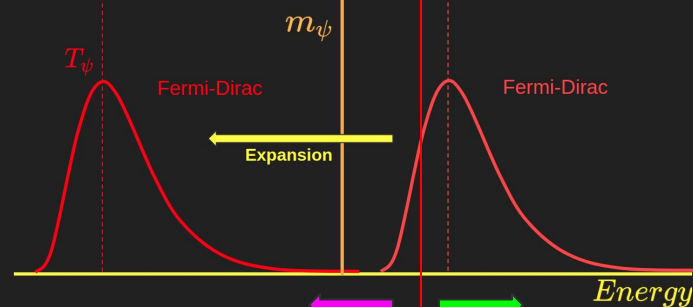
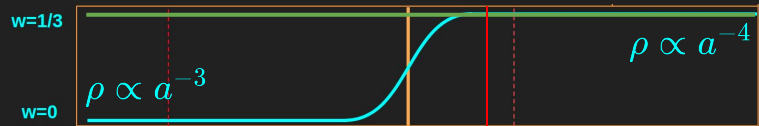


$$m_\psi < T_{rec}$$

$$H \quad H$$

$$\rho_\Lambda \downarrow + \rho_m \uparrow \quad \rho_r =$$

Effect low l (LISW)



Recombination

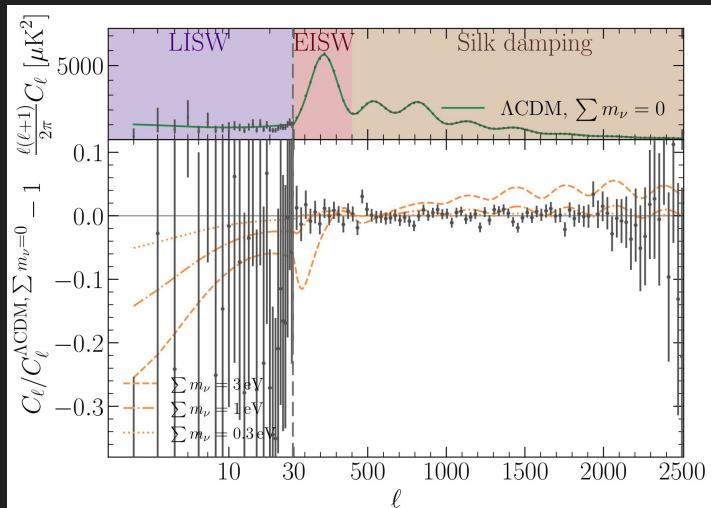
$$\rho_\Lambda + \rho_m + \rho_r$$

$$\theta_s = \int_{z_{rec}}^{\infty} c_s(z) \frac{dz}{H(z)} \times \left[\int_0^{z_{rec}} \frac{dz}{H(z)} \right]^{-1}$$

Mass Scale: Background Effect.

CMB is very sensitive to the expansion history.

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho$$



$$m_\psi < T_{rec}$$

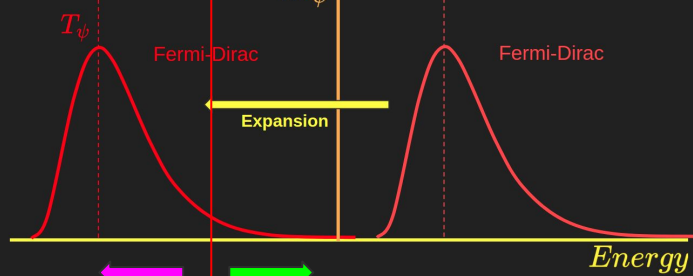
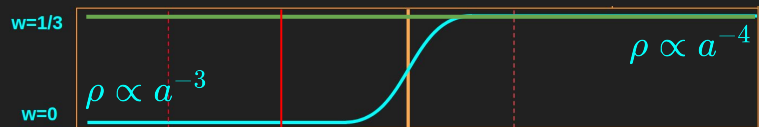
$$H \quad \rho_\Lambda \downarrow + \rho_m \uparrow$$

$$H \quad \rho_r =$$

Effect low l (LISW)

$$m_\psi > T_{rec}$$

$$H \quad \rho_\Lambda \downarrow + \rho_m \uparrow \quad H \quad \rho_r \downarrow + \rho_m \uparrow \quad \rho_\Lambda + \rho_m + \rho_r$$



Recombination

Very sensitive (EISW and Damping)

$$\theta_s = \left[\int_{z_{rec}}^{\infty} c_s(z) \frac{dz}{H(z)} \right] \times \left[\int_0^{z_{rec}} \frac{dz}{H(z)} \right]^{-1}$$

Large scale structure (BAO) will more sensitive to small neutrino masses.

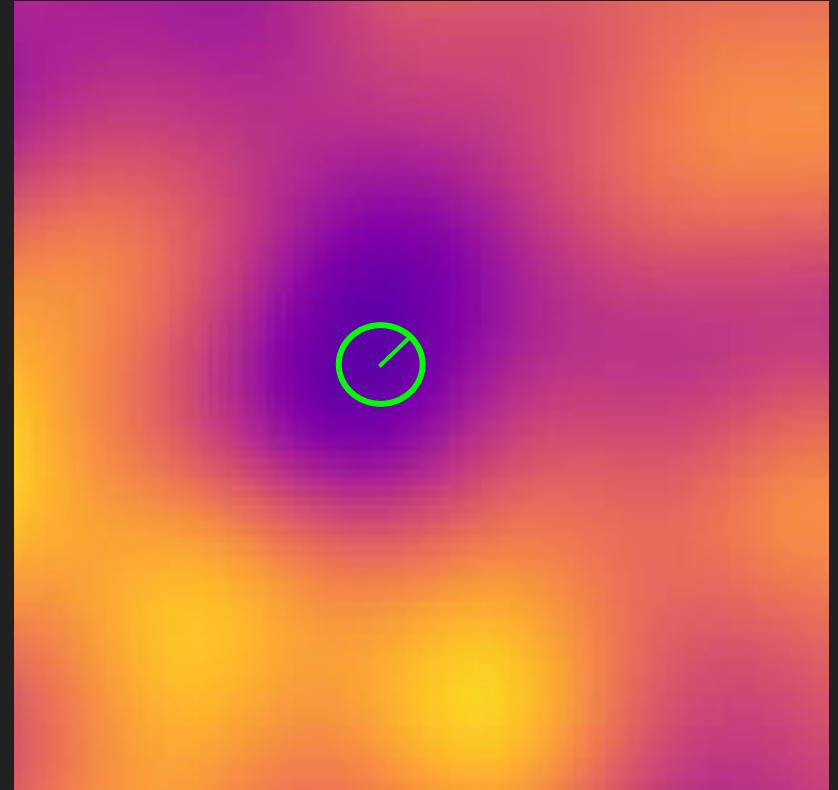
Mass Scale: Perturbations effect.

Small scale physics has an effect on the evolution of the cosmological perturbations.

$$d \approx \frac{T_\nu}{m_\nu} \frac{1}{H}$$

Larger perturbations don't get affected.

More on this “fluid” in M. Bauer Talk.



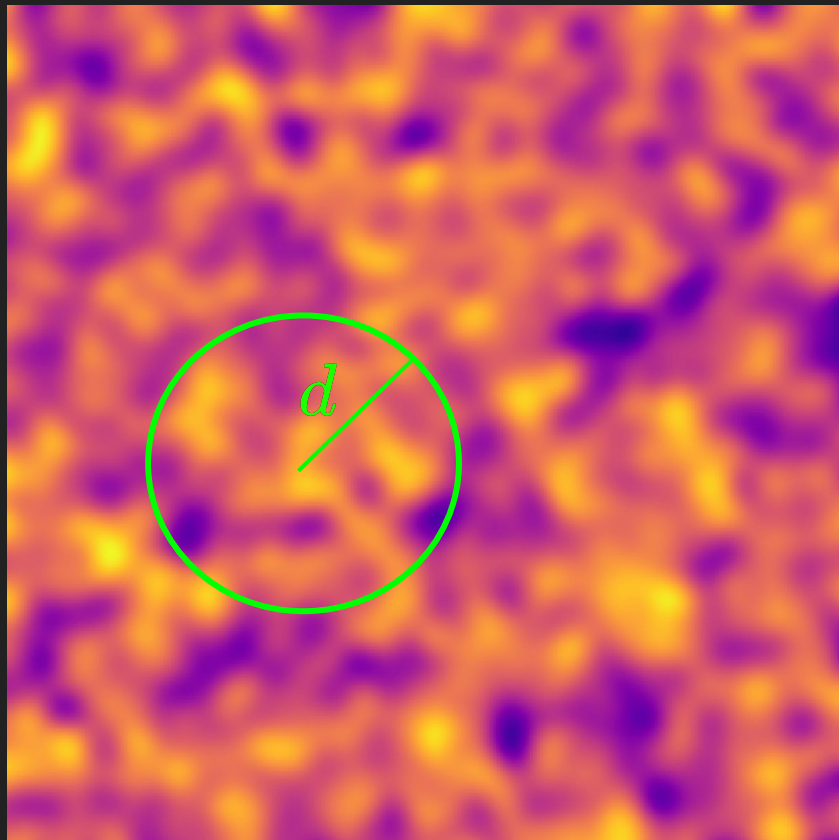
Mass Scale: Perturbations effect.

Small scale physics has an effect on the evolution of the cosmological perturbations.

$$d \approx \frac{T_\nu}{m_\nu} \frac{1}{H}$$

Perturbation at smaller scales get suppressed.

This is called **free streaming length**, don't confuse with **mean free path**, this second is related with scattering interactions (has essentially the same effect).



Precision CMB constraints on eV-scale bosons coupled to neutrinos

Stefan Sandner (Valencia U., IFIC), Miguel Escudero (CERN), Samuel J. Witte (Amsterdam U. and U. Amsterdam, GRAPPA and ICC, Barcelona U.)

Where in the equations are these effects?

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

$$\nabla_{\mu} T^{\mu\nu} = 0$$

By using this for the first order perturbations we generically get.

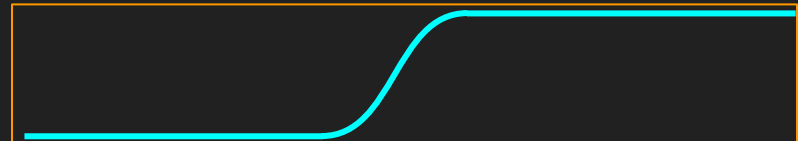
$$\dot{\delta} = -(1 + w)(\theta - 3\dot{\phi}) - 3\frac{\dot{a}}{a}(c_s^2 - w)\delta$$

$$\dot{\theta} = -\frac{\dot{a}}{a}(1 - 3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{c_s^2}{1+w}k^2\delta - k^2\sigma + k^2\psi$$

Variable that relates how energy change when we change the volume (EOS) **The only one that appears at $O(0)$.**

$w=1/3$

$w=0$



Where in the equations are these effects?

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Variables associated to small scale physics, involved in damping and propagation of the initial **perturbations**. **Not Constants**.

Where in the equations are these effects?

$$\dot{\Psi}_0 = -\frac{q}{\epsilon}\Psi_1 - \frac{d \ln f_0}{d \ln q}\dot{\phi}$$

$$\dot{\Psi}_1 = \frac{q}{3\epsilon}(\Psi_0 - 2\Psi_2) - \frac{\epsilon}{3q}\frac{d \ln f_0}{d \ln q}\dot{\psi}$$

$$\dot{\Psi}_\ell = \frac{q}{(2\ell+1)\epsilon}[\ell\Psi_{\ell-1} - (\ell+1)\Psi_{\ell+1}]$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T^{\mu\nu}$$

All can be computed from the Boltzmann eq.

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\dot{\delta} = -(1+w)(\theta - 3\dot{\phi}) - 3\frac{\dot{a}}{a}(c_s^2 - w)\delta$$

$$\dot{\theta} = -\frac{\dot{a}}{a}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{c_s^2}{1+w}k^2\delta - k^2\sigma + k^2\psi$$

Neutrinos are far from being a perfect fluid, they do have “viscosity” due to the efficient transfer of energy damping the density fluctuations. They do not interact!

Using a more sophisticated fluid approx.

Flows for the masses: A multi-fluid non-linear perturbation theory for massive neutrinos

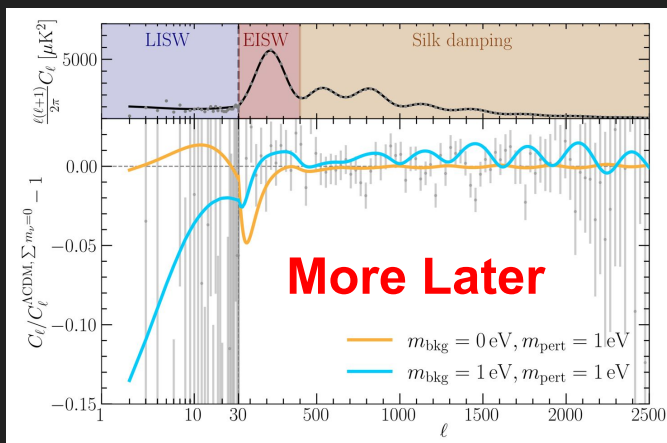
Joe Zhiyu Chen (New South Wales U.), Amol Upadhye (New South Wales U. and Liverpool John Moores U., ARI), Yvonne Y.Y. Wong (New South Wales U.)
Oct 28, 2022

An accurate fluid approximation for massive neutrinos in cosmology

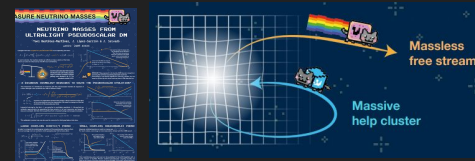
Gaio Nascimento

Consistency of the mass

- Cosmology can **test the mass** in **two physically different ways**.
- If no other **(scale dependent)** effect is present, both should be consistent with the same value of the mass.
- It will be great to measure both effects separately, the consistency would make the **measurement of the mass much robust** or the inconsistency **point towards new physics**.



We can use **two different mass parameters** consistently in the formalism and perform the full cosmological fit.



For some preliminary results on this + nu-dm interactions: **Toni's talk and poster**.

BSM with Cosmological Neutrinos

- The Mass

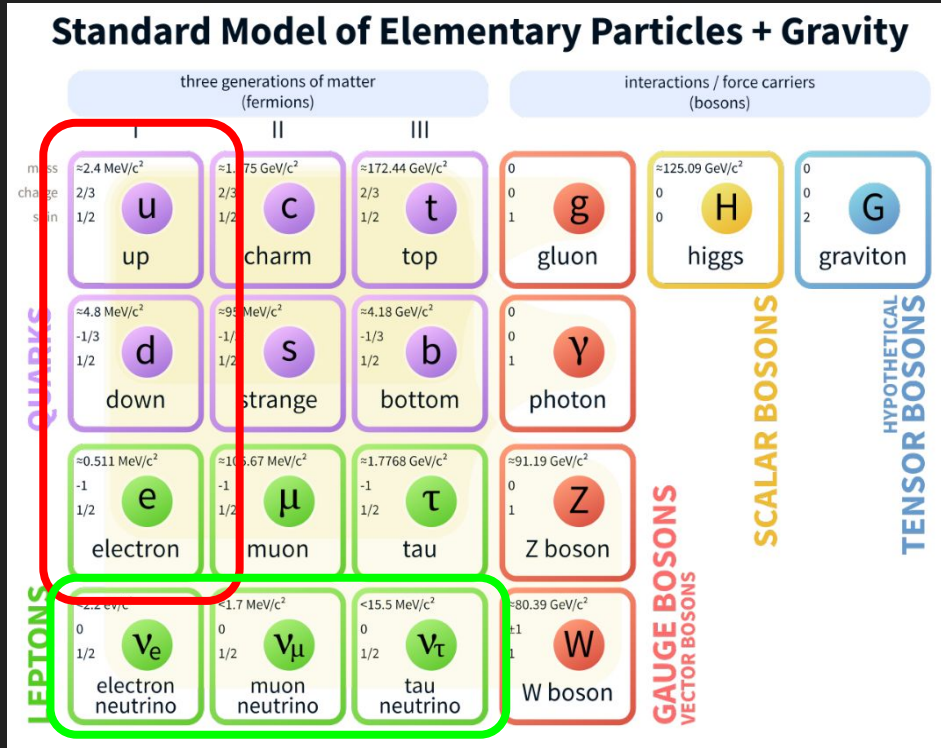
- Neutrino-Neutrino Interactions

One of many possible realization of a mismatch between background and perturbations (scale dependent physics).

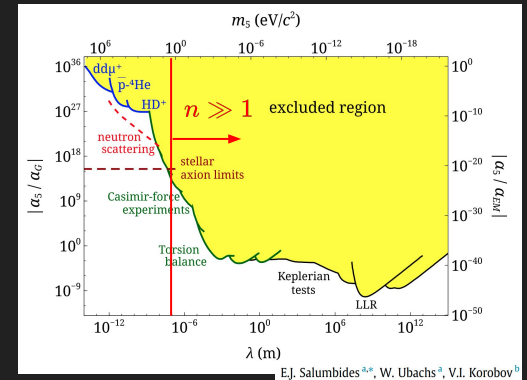
Interactions by light mediators



$$\frac{M_{Earth}}{m_p} \approx 10^{51}$$



Currently different experiments and observations also put bounds to fifth forces.



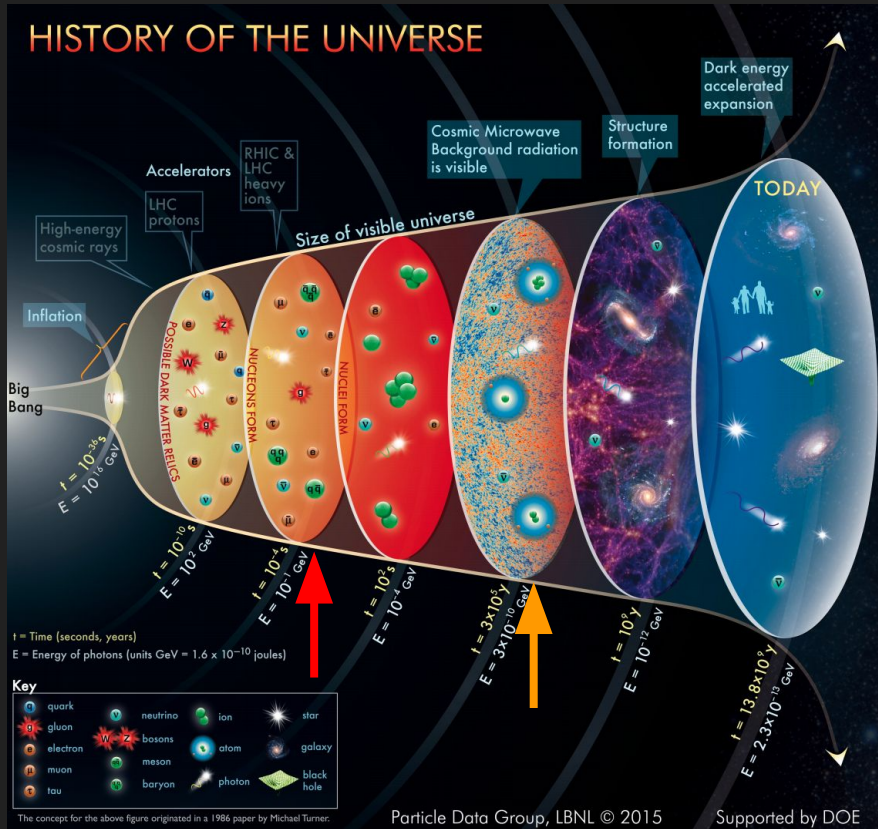
But all of them test only the first family.

Can we take advantage of a large number of particles but with the other families?

The neutrino background should be in mass eigenstates:

$$n_{\nu_e} \approx n_{\nu_\mu} \approx n_{\nu_\tau}$$

There are many neutrinos in the universe



The fact that the universe expands tells us:

- Densities were higher earlier in the expansion history, some examples:

At CMB : 10^{11} cm^{-3}

At BBN : 10^{35} cm^{-3}

Can we test neutrino properties (beyond the mass)?

Widely studied!

Massless

$$D_t$$

$$f(x, p, t) = C[f(x, p, t)]$$

Heavy

$$C[f(x, p, t)]$$

Gravity and Fifth forces

Can we test neutrino properties (beyond the mass)?

Widely studied!

Massless

Heavy

$$D_t f(x, p, t) = C[f(x, p, t)]$$

Gravity and Fifth forces

There is room for neutrinos, but high energy physics may be better to test this part.

$$L_{\text{Horizon}} \approx 10^{34} \text{ eV}^{-1} \longleftrightarrow L_{\text{int}} \approx 10^{-9} \text{ eV}^{-1}$$

A few orders of magnitude to be explored!

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Can we test properties beyond the mass?

Widely studied!

Massless

Heavy

$$D_t f(x, p, t) = C[f(x, p, t)]$$

We want to neglect cosmological fifth force scenarios

Let's explore the central region where both effects: cosmology large scales and scatterings can be neglected.

We want to neglect the scattering interactions.

$$L_{\text{Horizon}} \approx 10^{34} eV^{-1}$$

$$1/\text{Mpc} \approx 10^{-29} eV$$

$$g < 10^{-7} \ \& \ \frac{gm_\phi}{M_\phi} \approx O(10^3) \rightarrow L_\phi \approx 10^2 \text{ km}$$

$$L_{\text{int}} \approx 10^{-9} eV^{-1}$$

$$M_\phi \gg H$$

$$L_\phi \in \{10^2 \text{ km}, 10^{19} \text{ km}\}$$

We want to stay in this range to isolate and better illustrate the relevance of this effects.

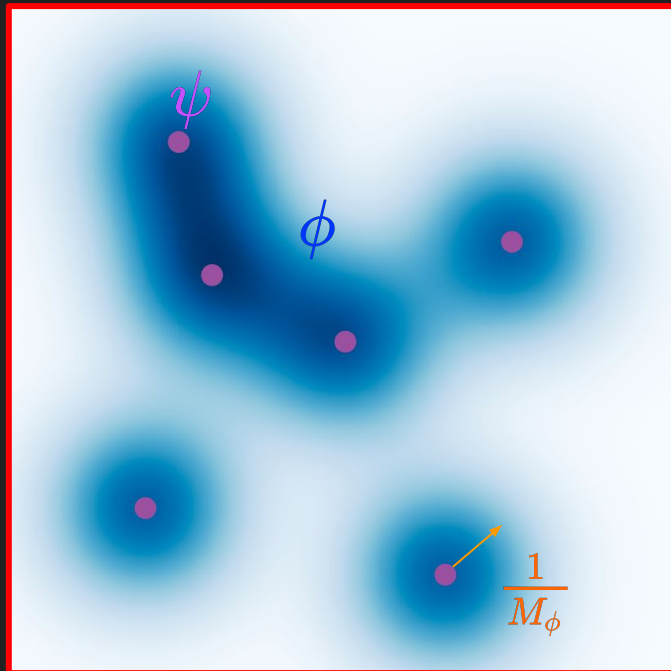
A simple setup

We will study the “simplest” case, a light scalar field:

Long Range Interactions in Cosmology: Implications for Neutrinos

Ivan Esteban (Ohio State U., CCAPP and Ohio State U. and Barcelona U.), Jordi Salvado (Barcelona U.)

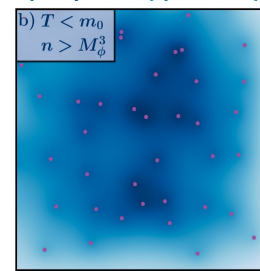
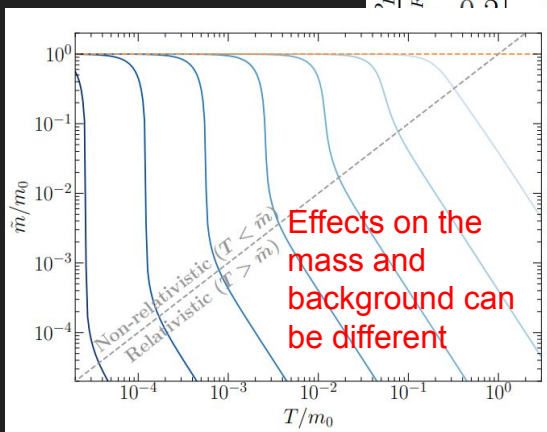
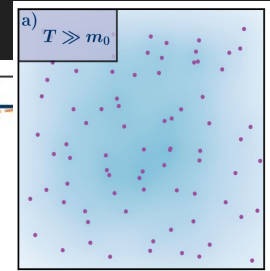
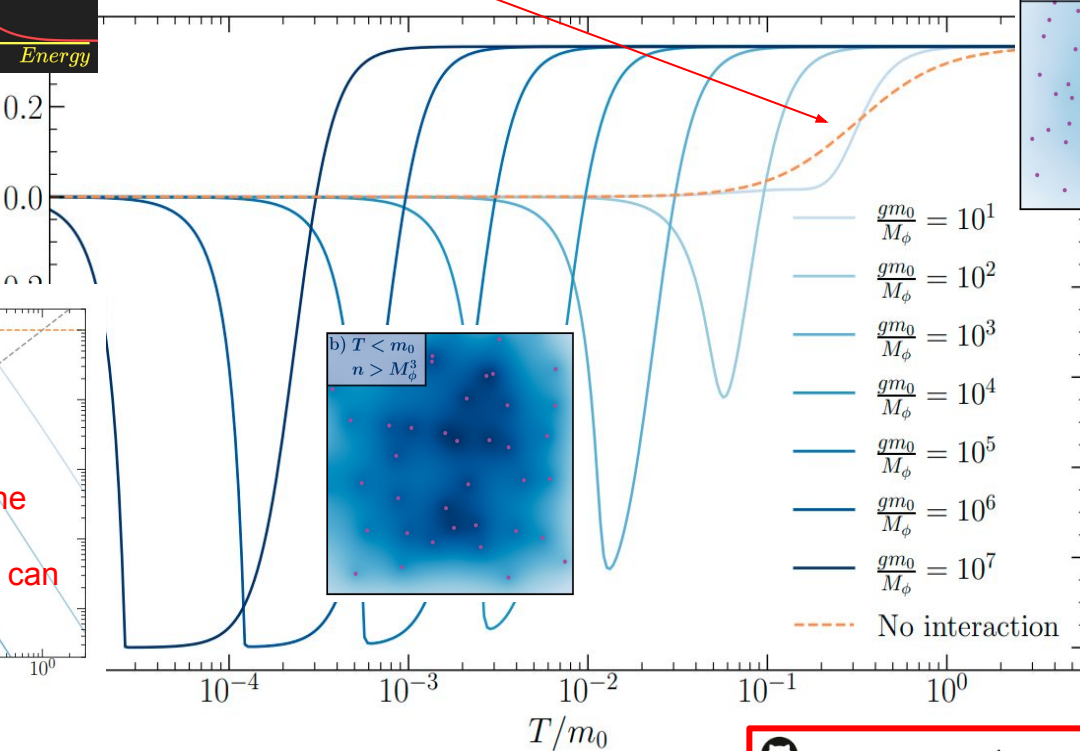
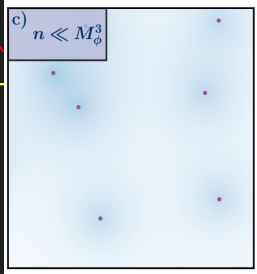
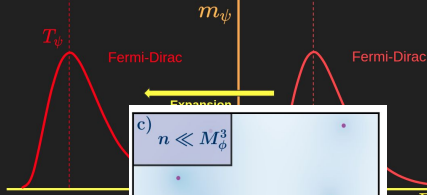
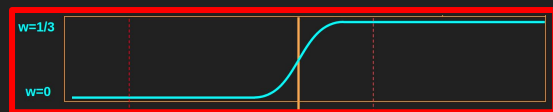
$$\mathcal{S} = \int \sqrt{-g} d^4x \left(-\frac{1}{2} D_\mu \phi D^\mu \phi - \frac{1}{2} M_\phi^2 \phi^2 + i \bar{\psi} \not{D} \psi - m_0 \bar{\psi} \psi - g \phi \bar{\psi} \psi \right)$$



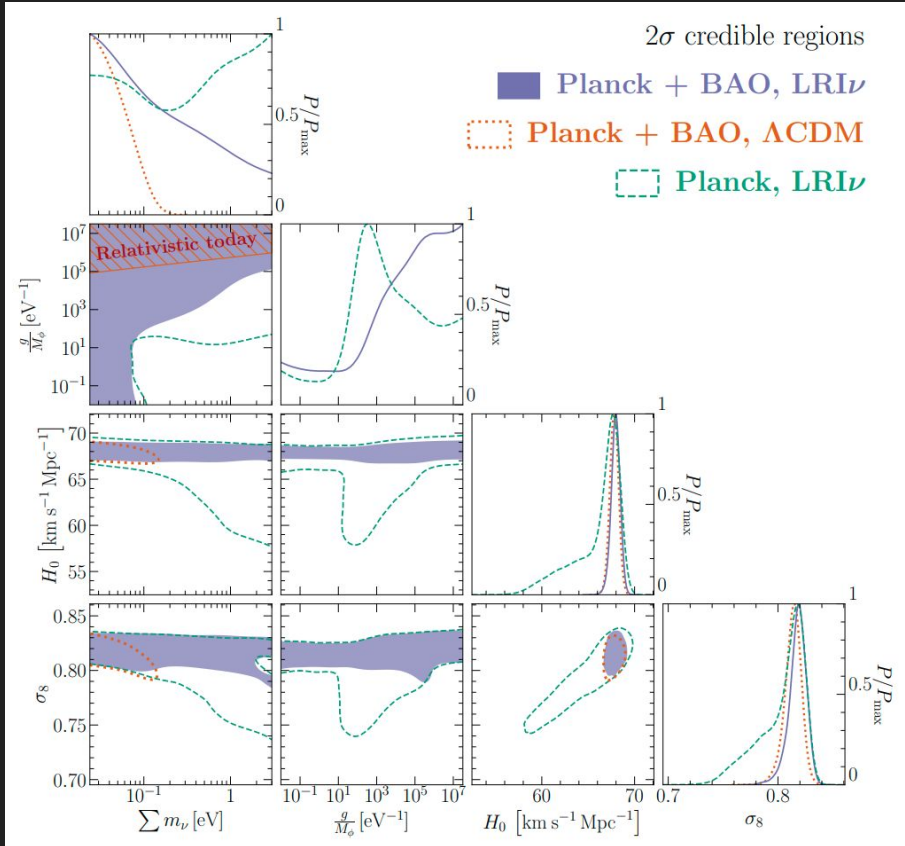
- The scalar field will extend in a range given by its mass.
- Non trivial effects are expected when the interaction range is of order of the interparticle distance.

$$M_\phi^2 \phi = -g \int d^3p \frac{\tilde{m}(\phi)}{\sqrt{p^2 + \tilde{m}(\phi)^2}} f(p)$$

How the Universe Evolves?



What is the current data telling us?



- BAO data plays an important role.
- But the new long range weakly coupled physics still relaxes drastically the cosmological mass bound.

Complementarity with other experiments:

- Neutrino oscillations can tell something about a part of the parameter space, **they can not have a mass such they are relativistic today.**
- A positive result by KATRIN is possible and will point to new physics.

Let's speculate about the Future (Euclid)

- We will reach a precision for the large-scale structure that may reveal the **mass scale** for neutrinos!



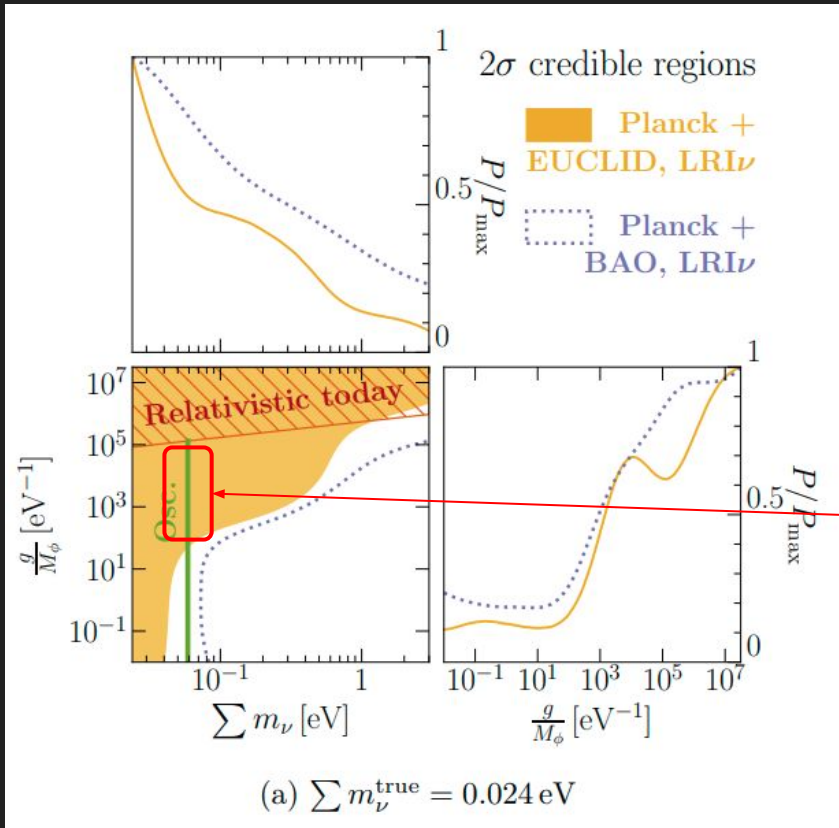
Forecast: Possible outcomes for EUCLID on neutrino masses

We create data consistent with:

No neutrino mass!
The bound is apparently excluding oscillation physics

We measure the mass scale! What do we learn?

Apparent contradiction between cosmology and experiments

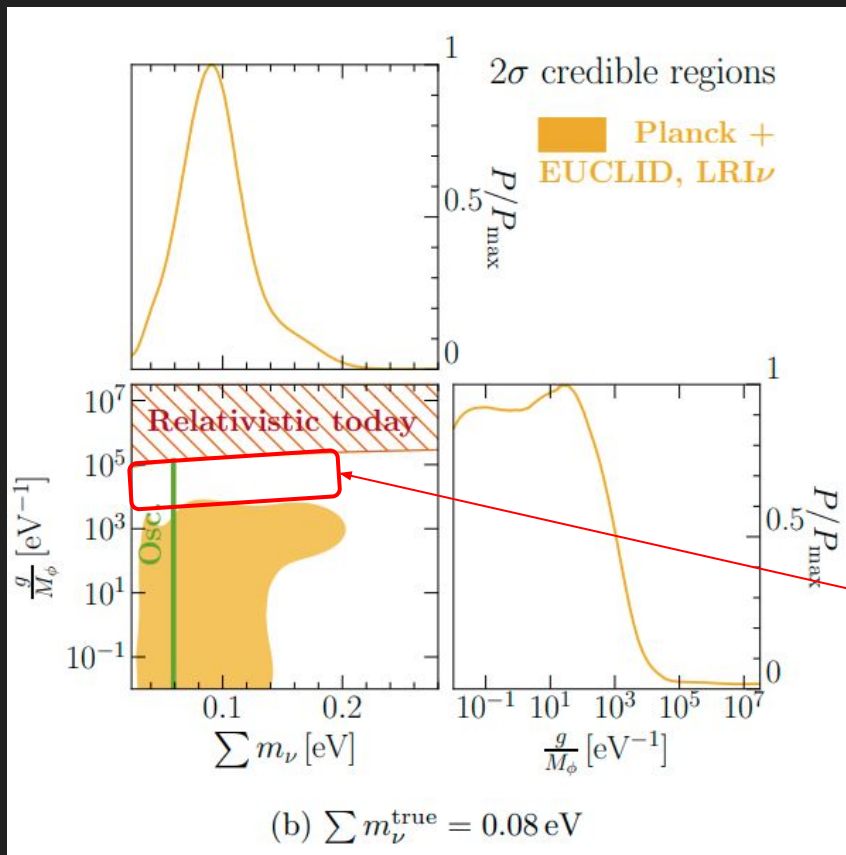


Complementarity with experiments:

- EUCLID may exclude the minimum mass allowed by oscillation physics.

Hint for new physics

The future is even better? (“expected” result by Euclid)



Complementarity with experiments:

- EUCLID may explore part of an still unexplored parameter space.

We may exclude part of the new-physics parameter space.

Lensing Anomaly

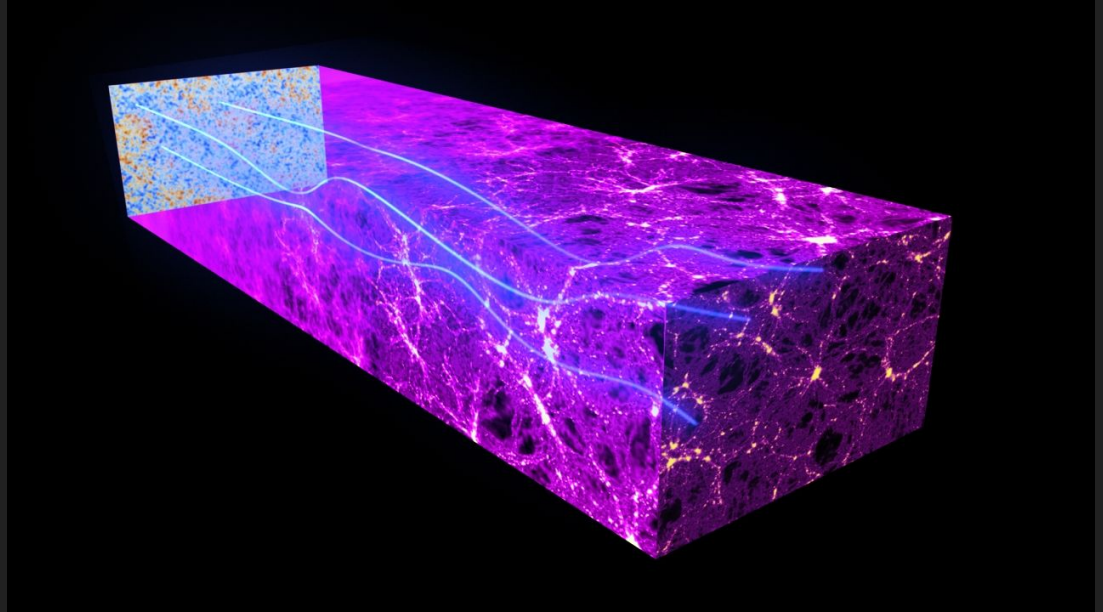
The light from the **CMB** can get a lensing effect due to traveling in the large scale structure of the universe.

This effect can be scaled with a parameter A_{lens}

If all is consistent: $A_{lens} = 1$

$$\tilde{T}(\hat{n}) = T(\hat{n}') = T(\hat{n} + \nabla\phi(\hat{n}))$$

$$\phi(\hat{n}) = -2 \int_0^{\chi_*} d\chi \frac{\chi_* - \chi}{\chi_* \chi} \Psi(\chi \hat{n}; \eta_0 - \chi)$$



Non-standard neutrino cosmology dilutes the lensing anomaly

Ivan Esteban,^{1,2,*} Olga Mena,^{3,†} and Jordi Salvado^{4,‡}

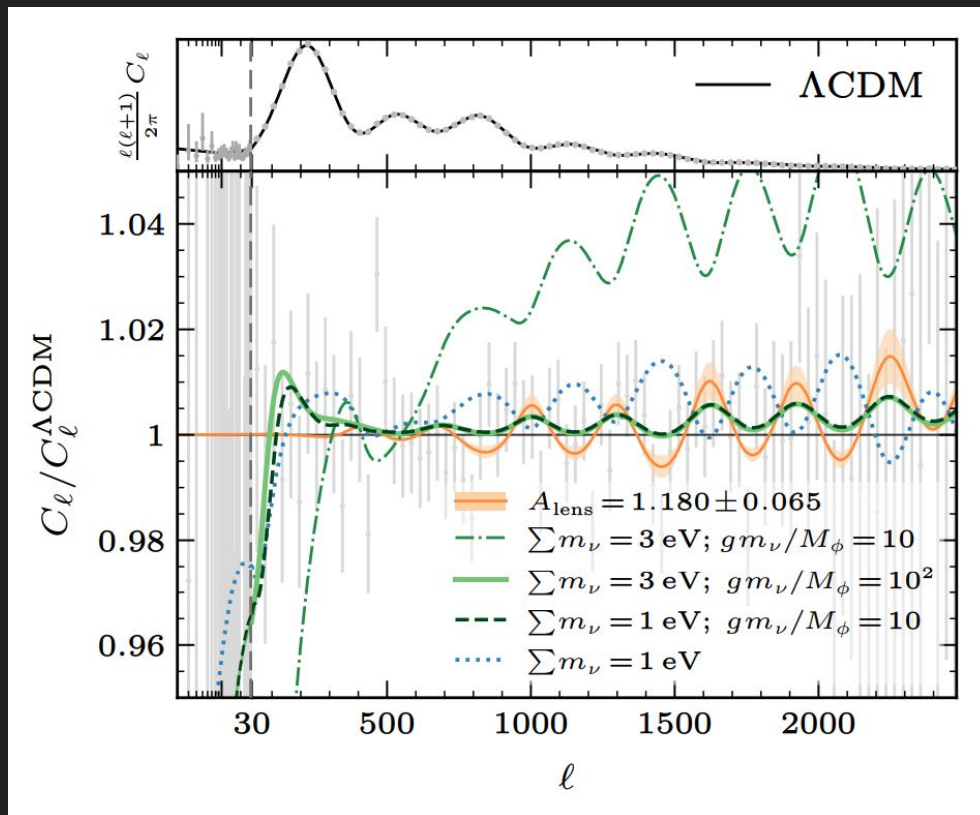
Lensing Anomaly and long range interactions

Neutrino masses go in the opposite direction.

The light scalar interactions produce the right effect.

Lensing anomaly artificially enhance the mass bound.

S.Petcov talk conservative bound let A_{eff} free.



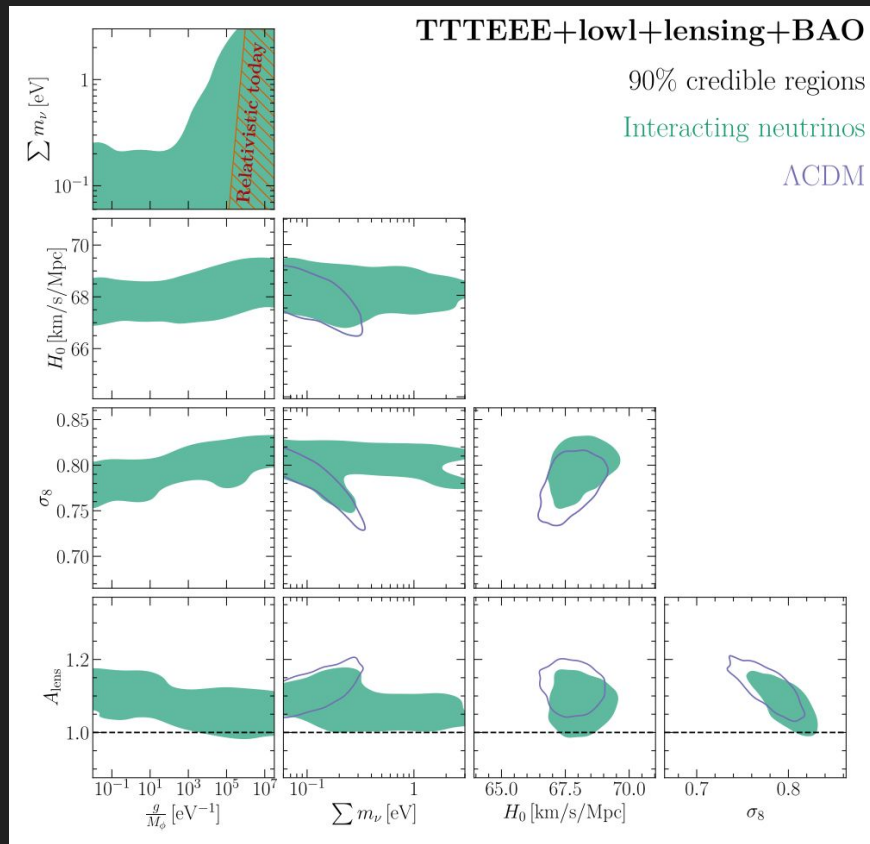
Lensing Anomaly and long range interactions

High values of g/M help to alleviate the tension.

Lensing anomaly depends on the likelihood, but still.

$$\Delta\chi_{\text{eff}}^2 = \chi_{\text{eff}}^2(A_{\text{lens}} = 1) - \chi_{\text{eff}}^2(A_{\text{lens}} \neq 1)$$

	TTTEEE+lowl	TTTEEE+lowl+lensing	TTTEEE+lowl+lensing+BAO
Plik (reference): Λ CDM	9.66 $p = 0.2\%$	3.43 $p = 6\%$	4.26 $p = 4\%$
Plik (reference): Self-interactions	4.87 $p = 3\%$	0.76 $p = 38\%$	2.71 $p = 10\%$
CamSpec (alternative): Λ CDM	4.82 $p = 3\%$	2.01 $p = 16\%$	1.96 $p = 16\%$
CamSpec (alternative): Self-interactions	2.06 $p = 15\%$	1.39 $p = 24\%$	1.79 $p = 18\%$



Conclusions

- The discovery of neutrino oscillations is telling us that in the next few years **LSS measurement** in cosmology are going to be highly **relevant in neutrino physics**.
- A **robust measurement** of a **mass scale** **must be consistent** along the physics of the cosmic evolution **at different scales**. (Soon)
DESI bound of 0.072 eV is a background driven bound.
- Cosmology, due to the **large occupation number** can test physics that can **not** be tested **with other experiments** (especially for neutrinos).
- **Working progress**: nu-dm interactions with **Toni Bertolez and Justo López-Sarrión**. (Toni's poster, ask him)

Thanks!

BKP

From Boltzmann to a Fluid

$$\dot{\delta} = -(1+w)(\theta - 3\dot{\phi}) - 3\frac{\dot{a}}{a}(c_s^2 - w)\delta$$

$$\dot{\theta} = -\frac{\dot{a}}{a}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{c_s^2}{1+w}k^2\delta - k^2\sigma + k^2\psi$$

$$\epsilon = \sqrt{q^2 + a^2 m^2}$$

$$c_s^2 = \frac{\delta P}{\delta \rho} = \frac{\frac{4\pi a^{-4}}{3} \int dq q^2 f_0(q) (q^2/\epsilon) \Psi_0(k, q, \tau)}{4\pi a^{-4} \int dq q^2 f_0(q) \epsilon \Psi_0(k, q, \tau)}$$

$$\sigma = \frac{8\pi a^{-4}}{3\bar{\rho}(1+w)} \int dq q^2 \frac{q^2}{\epsilon} f_0(q) \Psi_2(k, q, \tau)$$

$$\dot{\Psi}_0 = -\frac{q}{\epsilon} \Psi_1 - \frac{d \ln f_0}{d \ln q} \dot{\phi}$$

$$\dot{\Psi}_1 = \frac{q}{3\epsilon} (\Psi_0 - 2\Psi_2) - \frac{\epsilon}{3q} \frac{d \ln f_0}{d \ln q} \psi$$

Neutrinos are far from being a perfect fluid, they do have “viscosity” due to the efficient “effective” transfer of energy in the small degrees of freedom. They do not interact!

$$\dot{\Psi}_\ell = \frac{q}{(2\ell+1)\epsilon} [\ell \Psi_{\ell-1} - (\ell+1) \Psi_{\ell+1}]$$

From Boltzmann to a Fluid

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More on fluid approx:

Flows for the masses: A multi-fluid non-linear perturbation theory for massive neutrinos

Joe Zhiyu Chen (New South Wales U), Amol Upadhye (New South Wales U, and Liverpool John Moores U, AR), Yvonne Y.Y. Wong (New South Wales U), Oct 28, 2022

An accurate fluid approximation for massive neutrinos in cosmology

Caio Nascimento

A simple setup

We will study the “simplest” case, a light scalar field:

$$\mathcal{S} = \int \sqrt{-g} d^4x \left(-\frac{1}{2} D_\mu \phi D^\mu \phi - \frac{1}{2} M_\phi^2 \phi^2 + i \bar{\psi} \not{D} \psi - m_0 \bar{\psi} \psi - g \phi \bar{\psi} \psi \right)$$

$$\bar{\psi} \psi \longrightarrow f(p) = \frac{g}{(2\pi)^3} \frac{1}{e^{p/T} + 1}$$

Let's have a look at the equations of motion:

Dirac Equation:

$$i \not{D} \psi - (m_0 + g \phi) \psi = 0$$

Klein-Gordon Equation:

$$\underbrace{-D_\mu D^\mu \phi + M_\phi^2 \phi}_{\supset 3H\dot{\phi}} = -g \bar{\psi} \psi$$

- Fermions have an effective mass given by the value of the scalar field.

$$\tilde{m}(\phi) \equiv m_0 + g \phi$$

- The scalar field is suppressed by large momentum (relativistic fermions)

$$M_\phi^2 \phi = -g \int d^3p \frac{\tilde{m}(\phi)}{\sqrt{p^2 + \tilde{m}(\phi)^2}} f(p)$$

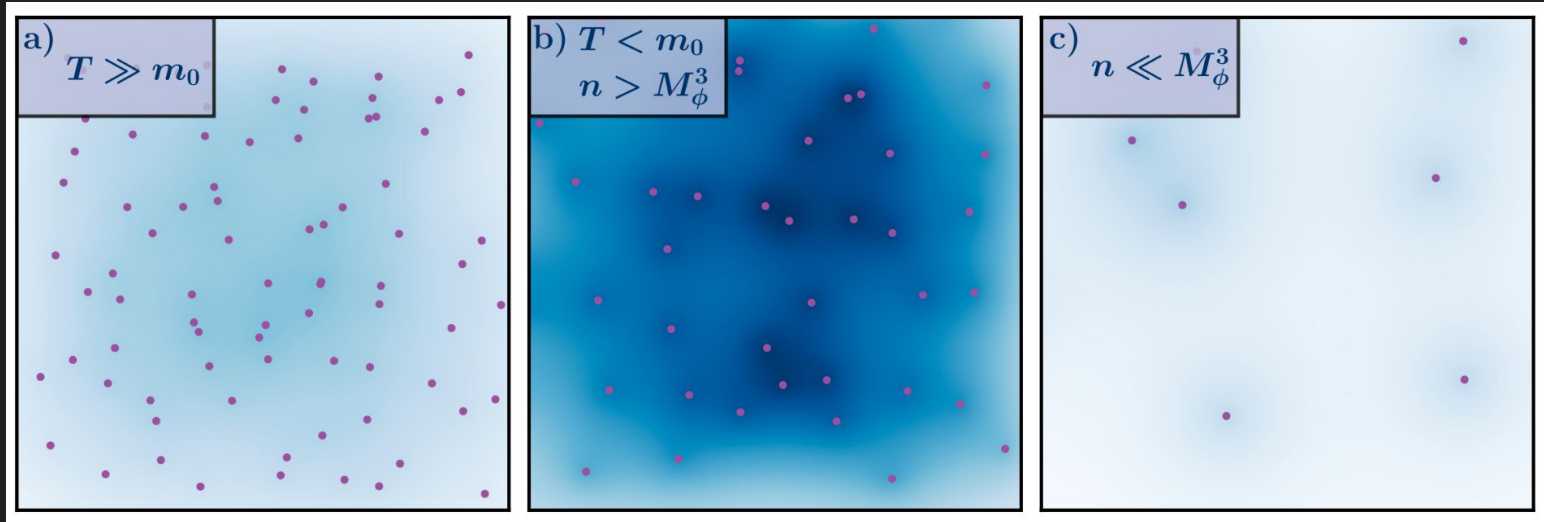
Phenomenological Regimes

We will study the “simplest” case, a light scalar field:

$$\tilde{m}(\phi) \equiv m_0 + g\phi$$

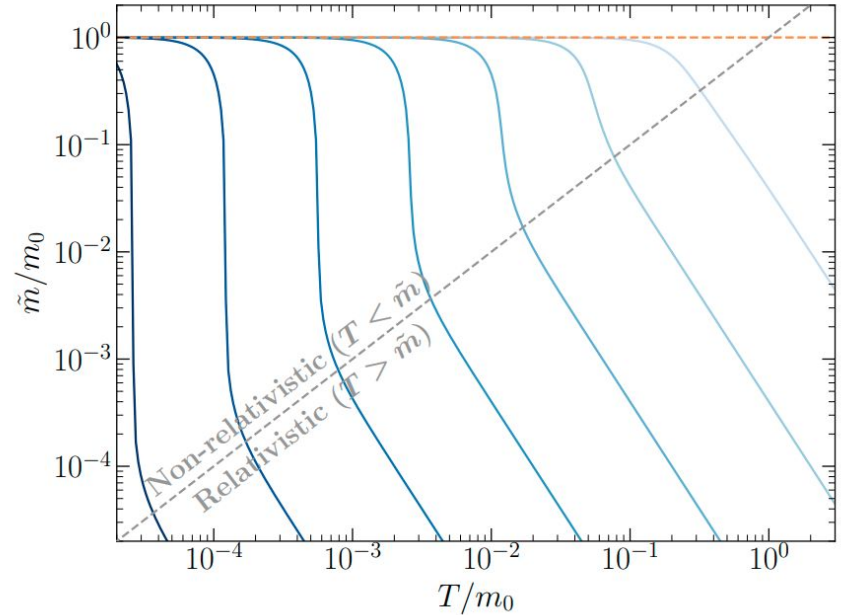
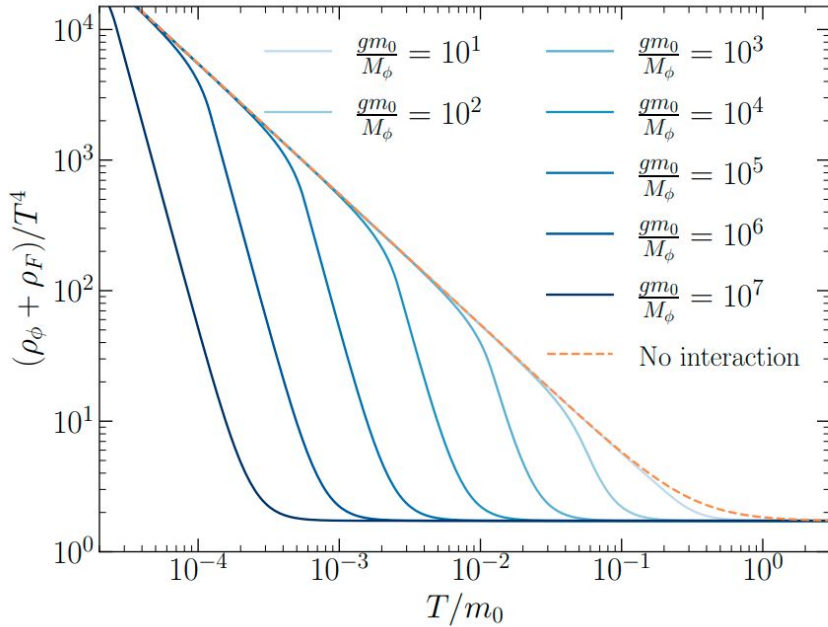
$$\mathcal{S} = \int \sqrt{-g} d^4x \left(-\frac{1}{2} D_\mu \phi D^\mu \phi - \frac{1}{2} M_\phi^2 \phi^2 + i\bar{\psi} \not{D} \psi - m_0 \bar{\psi} \psi - g\phi \bar{\psi} \psi \right)$$

$$M_\phi^2 \phi = -g \int d^3p \frac{\tilde{m}(\phi)}{\sqrt{p^2 + \tilde{m}(\phi)^2}} f(p)$$



How the Universe Evolves?

← Expansion



Energy density

$$\rho_\phi = \frac{1}{2} M_\phi^2 \phi^2 ; \rho_F = \int d^3 p \sqrt{p^2 + \tilde{m}^2} f(p)$$

Effective neutrino mass

Perturbations (instability)

$$f = f_0(q)[1 + \Psi(\vec{q}, \tau, \vec{x})]$$

$$\Psi'_0 = -\frac{qk}{\varepsilon} \Psi_1 - \phi' \frac{d \log f_0}{d \log q},$$

$$\Psi'_1 = \frac{qk}{3\varepsilon} (\Psi_0 - 2\Psi_2) - \left[\varepsilon \psi + g \delta \phi \frac{\tilde{m}}{\varepsilon} a^2 \right] \frac{k}{3q} \frac{d \log f_0}{d \log q},$$

$$\Psi'_\ell = \frac{qk}{(2\ell + 1)\varepsilon} [\ell \Psi_{\ell-1} - (\ell + 1) \Psi_{\ell+1}] \quad \forall \ell \geq 2.$$

The new interaction is much stronger than gravity.

It's unstable for most of the parameter space.

For our region of interest: Neutrinos will collapse much faster than any cosmological scale $O(100\text{yrs})$ and in structures much smaller than any cosmological scale $O(100\text{km-pc})$

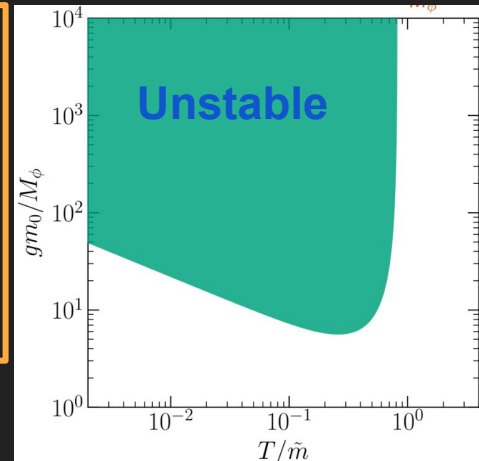
$$\phi = \phi_0(\tau) + \delta\phi(\vec{x}, \tau)$$

For $M_\phi \gg H$,

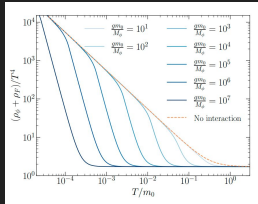
$$\delta\phi \simeq \frac{-g \frac{4\pi}{a^2} \int dq q^2 \frac{\tilde{m}}{\varepsilon} f_0(q) \Psi_0(\vec{q}, \tau, \vec{k})}{(k/a)^2 + M_\phi^2 + M_T^2}$$

$$M_T^2 \equiv g^2 \int d^3p \frac{p^2}{[p^2 + \tilde{m}^2]^{3/2}} f_0(p).$$

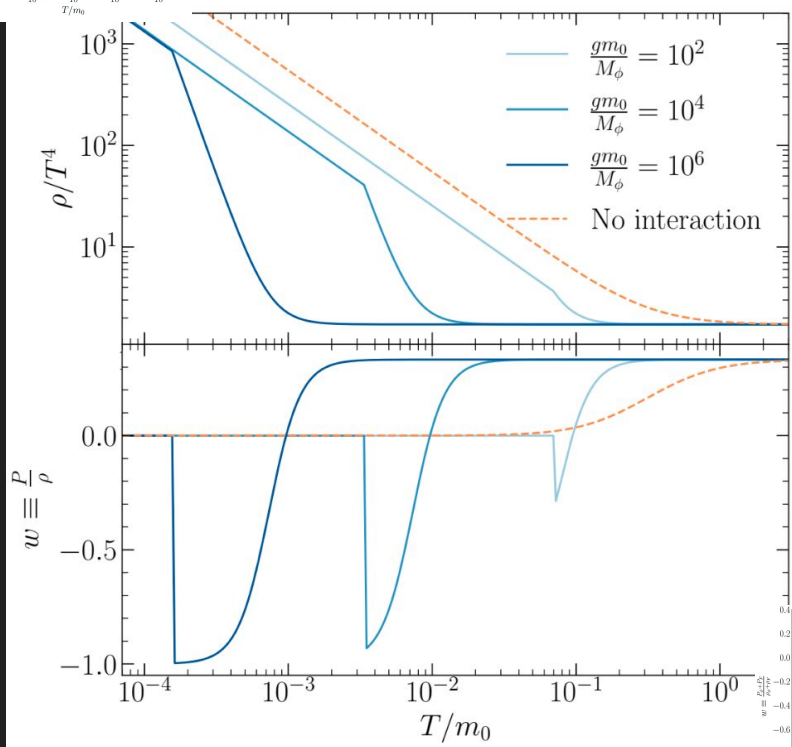
From a cosmological perspective: we can just switch to a non-relativistic “dust” made of neutrino nuggets when the instability happens.



How the Universe Evolves (instability)?




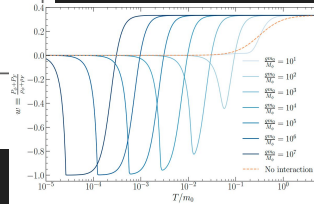
← Expansion



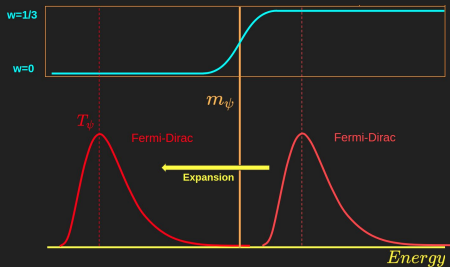
- Essentially the same as before where we perform the instantaneous transition when the system becomes unstable.

Physics implemented in:

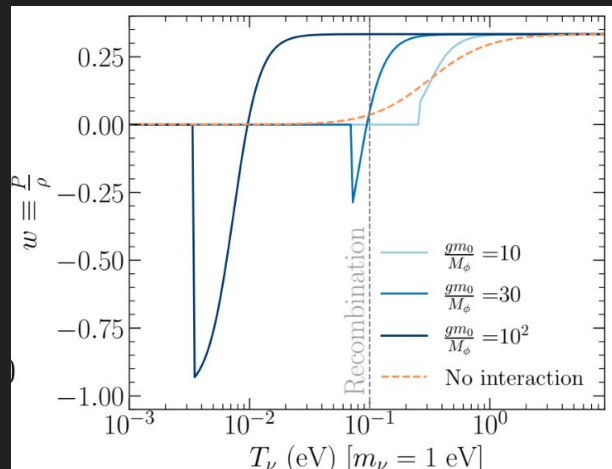
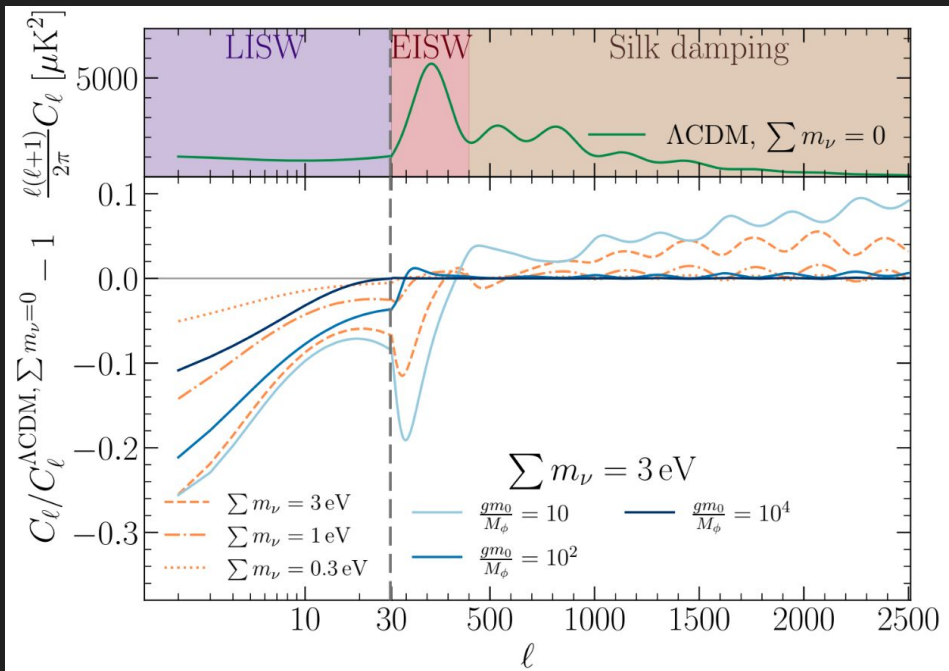
 github.com/jsalvado/class_public_lrs



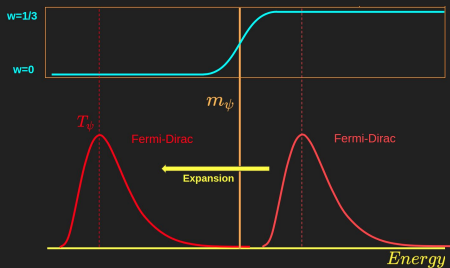
Impact in the CMB



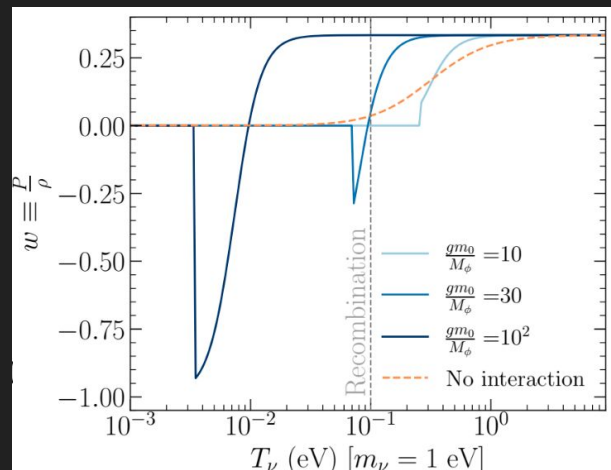
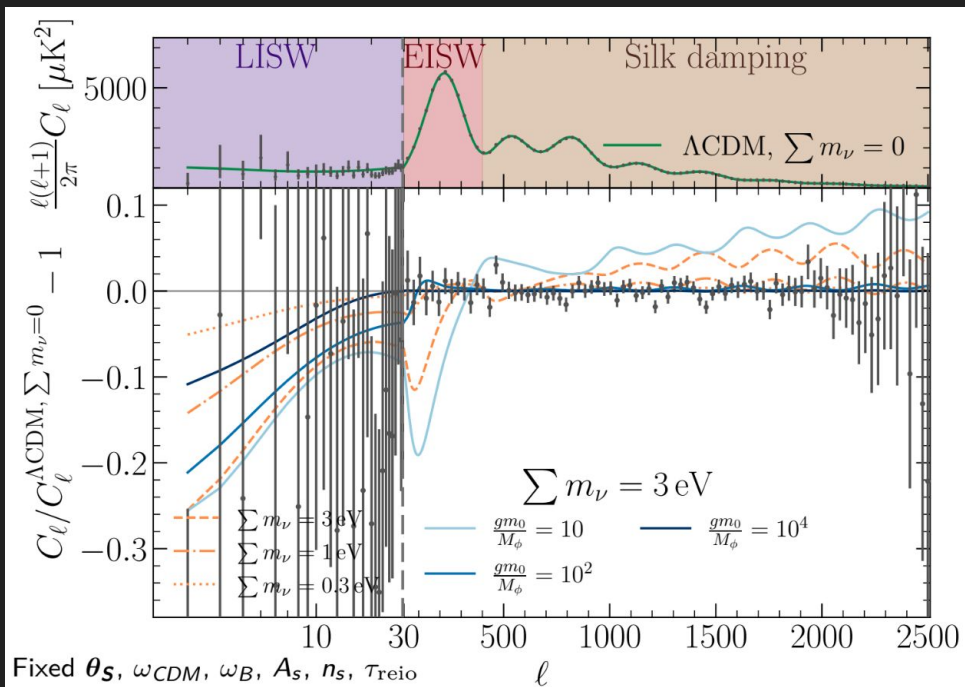
- The new physics dramatically change the equation of state, i.e. the transition to non-relativistic. This may strongly affect the CMB

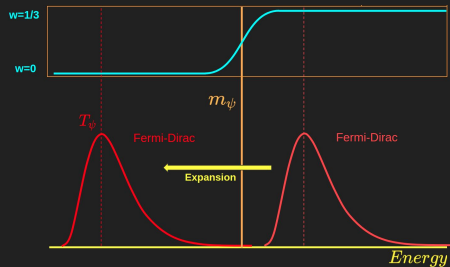


Impact in the CMB



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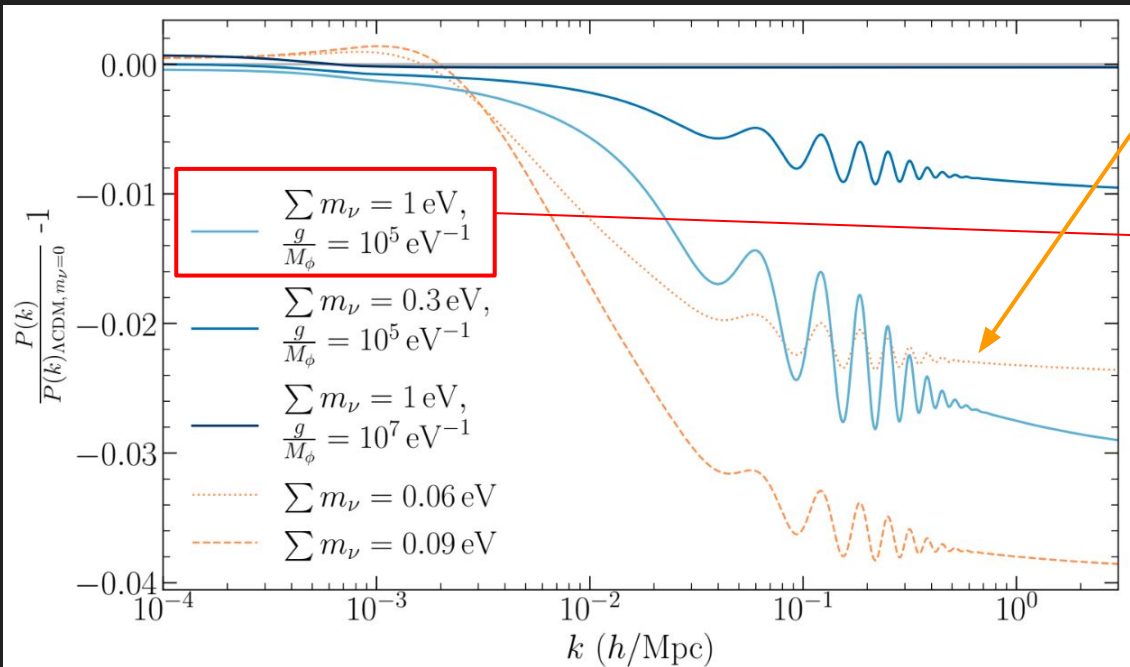




Full Power Spectrum by EUCLID



- Potential future sensitivity of 0.06eV



Fixed $\Omega_M, \omega_{\text{CDM}}, \omega_B, A_s, n_s, \tau_{\text{reio}}, z = 0$.

